3D Scaling Toolbox
User Guide

Version 1 of July 2011
This toolbox is a freeware (without technical support)

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Description: This toolbox contains all the Matlab functions for 3D scaling from an initial position to a final position using three different methods:

- Affine
- Kriging
- Radial Basis Function (RBF)

Comments, suggestions or questions:
The toolbox has been widely used and tested by several researchers but some bugs or problems can still be present, please report to: raphael.dumas@univ-lyon1.fr

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Scaling_Example.m

Purpose:
3D scaling example

Synopsis:
N/A (i.e., main program)

Description:
Plotting of a scaled model (dummy leg) using the three scaling methods, comparison of geometrical parameters of the transformation (rotation, translation, homothety, stretch)

See also:
Affine_3D_Approximation.m
Kriging_3D_Interpolation.m
RBF_3D_Approximation.m

Affine_3D_Approximation.m

Purpose:
Non-rigid registration and scaling of an initial mesh based on a set of control points

Synopsis:
[Meshj,Rotation,Translation,Homothety,Stretch, SSE] = …
Affine_3D_Approximation(Meshi,Xi,Xj)

Inputs: Meshi (i.e., matrix *l**3 of the *l* initial 3D mesh nodes), Xi (i.e., matrix *n**3 of the *n* initial 3D control points), Xj (i.e., matrix *n**3 of the *n* final 3D control points)

Outputs: Meshj (i.e., matrix *l**3 of the *l* final 3D mesh nodes), Rotation (i.e., matrix 3*3 of the rigid part of the linear polynomial), Translation (i.e., vector 3*1 of the rigid part of the linear polynomial), Homothety (i.e., matrix 3*3 of the deformative part of the linear polynomial), Stretch (i.e., matrix 3*3 of the deformative part of the linear polynomial), SSE (i.e., sum of squared errors)

Description:
Determination of the linear transformation (least squares) from an initial position *i* to a final position *j*, extraction of geometrical information and application to the initial mesh nodes

Theoretical background:
The linear transformation from position *i* to position *j* is:

$$\underbrace{\begin{bmatrix} x_1^j & y_1^j & z_1^j \\ \cdots & \cdots & \cdots \\ x_p^j & y_p^j & z_p^j \\ \cdots & \cdots & \cdots \\ x_n^j & y_n^j & z_n^j \end{bmatrix}}_{\mathbf{X}^j_{(n\times 3)}} = \underbrace{\begin{bmatrix} 1 & x_1^i & y_1^i & z_1^i \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_p^i & y_p^i & z_p^i \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_n^i & y_n^i & z_n^i \end{bmatrix}}_{\bar{\mathbf{X}}^i_{(n\times 4)}} \underbrace{\begin{bmatrix} a_0^x & a_0^y & a_0^z \\ a_1^x & a_1^y & a_1^z \\ a_2^x & a_2^y & a_2^z \\ a_3^x & a_3^y & a_3^z \end{bmatrix}}_{\mathbf{A}_{(4\times 3)}}$$

The coefficients *a* of the linear polynomial represent geometrical information (translation $\vec{t}_0^{ij}$, rotation $\mathbf{R}^{ij}$, homothety $\mathbf{H}^{ij}$ and stretch $\mathbf{S}^{ij}$) from position *i* to position *j*:

$$\mathbf{A}^T = \begin{bmatrix} \vec{t}_0^{ij} & \mathbf{R}^{ij} * \mathbf{H}^{ij} * \mathbf{S}^{ij} \end{bmatrix}$$

References:
R Dumas, L Cheze. Soft tissue artifact compensation by linear 3D interpolation and approximation methods. Journal of Biomechanics 2009;42(13):2214–2217
HJ Sommer 3rd, NR Miller, GJ Pijanowski. Three-dimensional osteometric scaling and normative modelling of skeletal segments. Journal of Biomechanics 1982;15(3):171-80

Kriging_3D_Interpolation.m

Purpose:
Non-rigid registration and scaling of an initial mesh based on a set of control points

Synopsis:
[Meshj,Rotation,Translation,Homothety,Stretch,Model,SSE] = …
Kriging_3D_Interpolation(Meshi,Xi,Xj)

Inputs: Meshi (i.e., matrix *l**3 of the *l* initial 3D mesh nodes), Xi (i.e., matrix *n**3 of the *n* initial 3D control points), Xj (i.e., matrix *n**3 of the *n* final 3D control points)

Outputs: Meshj (i.e., matrix *l**3 of the *l* final 3D mesh nodes), Rotation (i.e., matrix 3*3 of the rigid part of the drift), Translation (i.e., vector 3*1 of the rigid part of the drift), Homothety (i.e., matrix 3*3 of the deformative part of the drift), Stretch (i.e., matrix 3*3 of the deformative part of the drift), Model (i.e., selected generalized covariance), SSE (i.e., sum of squared errors)

Description:
Determination of the Kriging transformation (generalized covariance K with the lower Kriging variance) from an initial position i to a final position j, extraction of geometrical information and application to the initial mesh nodes

Theoretical background:
For each coordinate of a point p of the n control points, from position i to position j, the Kriging transformation is:

$$\begin{cases} x_p^j = a_0^x + a_1^x x_p^i + a_2^x y_p^i + a_3^x z_p^i + \sum_{q=1}^{n} b_q^x K\left(\left\|\vec{x}_p^i - \vec{x}_q^i\right\|\right) \\ y_p^j = a_0^y + a_1^y x_p^i + a_2^y y_p^i + a_3^y z_p^i + \sum_{q=1}^{n} b_q^y K\left(\left\|\vec{x}_p^i - \vec{x}_q^i\right\|\right) \\ z_p^j = a_0^z + a_1^z x_p^i + a_2^z y_p^i + a_3^z z_p^i + \sum_{q=1}^{n} b_q^z K\left(\left\|\vec{x}_p^i - \vec{x}_q^i\right\|\right) \end{cases}$$

The coefficients a of the drift represent geometrical information (translation $\vec{t}_0^{ij}$, rotation $\mathbf{R}^{ij}$, homothety $\mathbf{H}^{ij}$ and stretch $\mathbf{S}^{ij}$) from position i to position j:

$$\begin{bmatrix} a_0^x \\ a_0^y \\ a_0^z \end{bmatrix} = \vec{t}_0^{ij} \text{ and } \begin{bmatrix} a_1^x & a_2^x & a_3^x \\ a_1^y & a_2^y & a_3^y \\ a_1^z & a_2^z & a_3^z \end{bmatrix} = \mathbf{R}^{ij} * \mathbf{H}^{ij} * \mathbf{S}^{ij}$$

References:
R Dumas, L Cheze. Soft tissue artifact compensation by linear 3D interpolation and approximation methods. Journal of Biomechanics 2009;42(13):2214–2217

F Trochu. A contouring program based on dual kriging interpolation. Engineering Computation 1993;9:160-177
JD Martin, TW Simpson. Use of kriging models to approximate deterministic computer models. American Institute of Aeronautics and Astronautics Journal 2005;43(4):853–863

See also:
Kriging.m

Kriging.m

Purpose:
Determination of the drift and fluctuation coefficients and of the Kriging variance

Synopsis:
[BA,Var] = Kriging(Xi,Xj,Model)

Inputs: Xi (i.e., matrix n*3 of the n initial 3D control points), Xj (i.e., matrix n*3 of the n final 3D control points), Model (i.e., generalized covariance)

Outputs: BA (i.e., matrix of the drift and fluctuation coefficients), Var (i.e., Kriging variances among the x, y and z axes in line)

Description:
Inversion of the Kriging system and computation of the Kriging variance

Theoretical background:
The Kriging system is:
$\begin{bmatrix} \mathbf{K} & \bar{\mathbf{X}}^i \\ \left(\bar{\mathbf{X}}^i\right)^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{B} \\ \mathbf{A} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^j \\ \mathbf{0} \end{bmatrix}$ where the elements (p, q) of the matrix $\mathbf{K}$ are $K\left(\left\|\vec{x}_p^i - \vec{x}_q^i\right\|\right)$

and $\mathbf{B} = \begin{bmatrix} b_1^x & b_1^y & b_1^z \\ \cdots & \cdots & \cdots \\ b_q^x & b_q^y & b_q^z \\ \cdots & \cdots & \cdots \\ b_n^x & b_n^y & b_n^z \end{bmatrix}$

The Kriging variance (matrix) is:
$$\boldsymbol{\sigma}^2 = \frac{1}{n}\left(\mathbf{X}^j - \bar{\mathbf{X}}^i\mathbf{A}\right)^T \mathbf{K}^{-1}\left(\mathbf{X}^j - \bar{\mathbf{X}}^i\mathbf{A}\right)$$

References:
R Dumas, L Cheze. Soft tissue artifact compensation by linear 3D interpolation and approximation methods. Journal of Biomechanics 2009;42(13):2214–2217
F Trochu. A contouring program based on dual kriging interpolation. Engineering Computation 1993;9:160-177
JD Martin, TW Simpson. Use of kriging models to approximate deterministic computer models. American Institute of Aeronautics and Astronautics Journal 2005;43(4):853–863

RBF_3D_Approximation.m

Purpose:
Non-rigid registration and scaling of an initial mesh based on a set of control points

Synopsis:
[Meshj,Rotation,Translation,Homothety,Stretch,Model,Centers,SSE] = ...
RBF_3D_Approximation(Meshi,Xi,Xj,Lambda_reference,Percentage)

Inputs: Meshi (i.e., matrix l*3 of the l initial 3D mesh nodes), Xi (i.e., matrix n*3 of the n initial 3D control points), Xj (i.e., matrix n*3 of the n final 3D control points), Lambda_reference (i.e., initial regularization parameter), Percentage (i.e., minimal improvement of cost function)

Outputs: Meshj (i.e., matrix l*3 of the l final 3D mesh nodes), Rotation (i.e., matrix 3*3 of the rigid part of the linear polynomial), Translation (i.e., vector 3*1 of the rigid part of the linear polynomial), Homothety (i.e., matrix 3*3 of the deformative part of the linear polynomial), Stretch (i.e., matrix 3*3 of the deformative part of the linear polynomial), Model (i.e., m selected models), Centers (i.e., m selected centers), SSE (i.e., sum of squared errors)

Description:
Determination of the RBF transformation (forward regularized selection of models h_f and centers $\vec{c}_f$) from an initial position i to a final position j, extraction of geometrical information and application to the initial mesh nodes

Theoretical background:
For each coordinate of a point p of the n control points, from position i to position j, the RBF transformation is:

$$\begin{cases} x_p^j = \sum_{f=1}^{m} w_f^x h_f\left(\left\|\vec{x}_p^i - \vec{c}_f\right\|\right) + a_0^x + a_1^x x_p^i + a_2^x y_p^i + a_3^x z_p^i \\ y_p^j = \sum_{f=1}^{m} w_f^y h_f\left(\left\|\vec{x}_p^i - \vec{c}_f\right\|\right) + a_0^y + a_1^y x_p^i + a_2^y y_p^i + a_3^y z_p^i \\ z_p^j = \sum_{f=1}^{m} w_f^z h_f\left(\left\|\vec{x}_p^i - \vec{c}_f\right\|\right) + a_0^z + a_1^z x_p^i + a_2^z y_p^i + a_3^z z_p^i \end{cases}$$

The coefficients a of the linear polynomial represent geometrical information (translation $\vec{t}_0^{ij}$, rotation $\mathbf{R}^{ij}$, homothety $\mathbf{H}^{ij}$ and stretch $\mathbf{S}^{ij}$) from position i to position j:

$$\begin{bmatrix} a_0^x \\ a_0^y \\ a_0^z \end{bmatrix} = \vec{t}_0^{ij} \text{ and } \begin{bmatrix} a_1^x & a_2^x & a_3^x \\ a_1^y & a_2^y & a_3^y \\ a_1^z & a_2^z & a_3^z \end{bmatrix} = \mathbf{R}^{ij} * \mathbf{H}^{ij} * \mathbf{S}^{ij}$$

References:

R Dumas, L Cheze. Soft tissue artifact compensation by linear 3D interpolation and approximation methods. Journal of Biomechanics 2009;42(13):2214–2217

MJL Orr. Introduction to Radial Basis Function Networks. 1996 http://www.anc.ed.ac.uk/rbf/intro/intro.html

See also:

RBF.m

RBF.m

Purpose:
Determination of the basis function coefficients, cost function and refined regularization parameter

Synopsis:
[W,SSE,F,Lambda] = RBF(Xi,Xja,Model,Center,Lambda)

Inputs: Xi (i.e., matrix *n**3 of the *n* initial 3D control points), Xja (i.e., matrix *n**3 of the *n* final 3D control points registrated by Affine), Model (i.e., *m* basis functions), Center (i.e., *m* centers), Lambda (i.e., regularization parameter)

Outputs: W (i.e., basis function coefficients), SSE (i.e., sum of squared errors), F (i.e., cost function among the *x*, *y* and *z* axes in column), Lambda (i.e., refined regularization parameter)

Description:
Pseudo-inversion of the RBF system and computation of the cost function and of the refined regularization parameter

Theoretical background:
The RBF system is:
$\begin{bmatrix} \mathbf{H} & \bar{\mathbf{X}}^i \\ \left(\bar{\mathbf{C}}\right)^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{A} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^j \\ \mathbf{0} \end{bmatrix}$ where the elements (p, f) of the matrix $\mathbf{H}$ are $h_f\left(\left\|\vec{x}_p^i - \vec{c}_f\right\|\right)$,

$$\mathbf{W} = \begin{bmatrix} w_1^x & w_1^y & w_1^z \\ \dots & \dots & \dots \\ w_f^x & w_f^y & w_f^z \\ \dots & \dots & \dots \\ w_m^x & w_m^y & w_m^z \end{bmatrix} \text{ and } \bar{\mathbf{C}} = \begin{bmatrix} 1 & x_1^i & y_1^i & z_1^i \\ \dots & \dots & \dots & \dots \\ 1 & x_f^i & y_f^i & z_f^i \\ \dots & \dots & \dots & \dots \\ 1 & x_m^i & y_m^i & z_m^i \end{bmatrix}$$

The cost function (matrix) is:
$\mathbf{F} = \left(\mathbf{X}^j - \bar{\mathbf{X}}^i\mathbf{A}\right)^T \mathbf{P}\left(\mathbf{X}^j - \bar{\mathbf{X}}^i\mathbf{A}\right)$ where $\mathbf{P} = \left(\mathbf{I} - \mathbf{HVH}^T\right)$ with $\mathbf{V} = \left(\mathbf{H}^T\mathbf{H} + \lambda\mathbf{I}\right)^{-1}$

The refined regularization parameter (matrix) is:
$$\boldsymbol{\lambda} = \frac{\left(\mathbf{X}^j - \bar{\mathbf{X}}^i\mathbf{A}\right)^T \left(\mathbf{P}\right)^2 \left(\mathbf{X}^j - \bar{\mathbf{X}}^i\mathbf{A}\right) \operatorname{trace}\left(\mathbf{V} - \lambda\left(\mathbf{V}\right)^2\right)}{\mathbf{W}\ \mathbf{V}\ \mathbf{W}\ \operatorname{trace}\left(\mathbf{P}\right)}$$

References:
R Dumas, L Cheze. Soft tissue artifact compensation by linear 3D interpolation and approximation methods. Journal of Biomechanics 2009;42(13):2214–2217
MJL Orr. Introduction to Radial Basis Function Networks. 1996
http://www.anc.ed.ac.uk/rbf/intro/intro.html