# The Content of Understanding as Compressed Graphs

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# Compression

# Graphs

Compression + Graphs

Compression

+

Graphs

*Why?*
*How?*

Compression + Graphs

*Why?*
*How?*

*Why maybe not? But, actually, yes*

*1.*

# Compression and Understanding

"That's another thing we've learned from *your* Nation," said Mein Herr, "map-making. But we've carried it much further than *you*. What do you consider the *largest* map that would be really useful?"

"About six inches to the mile."

"Only *six inches*!" exclaimed Mein Herr. "We very soon got to six *yards* to the mile. Then we tried a *hundred* yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of *a mile to the mile*!"

"Have you used it much?" I enquired.

"It has never been spread out, yet," said Mein Herr: "the farmers objected: they said it would cover the whole country and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it dœs nearly as well. ..."

Lewis Carroll, *Sylvie and Bruno Concluded*, 1893, Ch. XI

# Compression

- *Representation to representation*: R' is a compressed representation of R iff R and R' encode the same relevant information, and the 'length' of R' is less than the length of R.
- *Object to representation*: R is a compressed representation of O iff R encodes information about O and the length of R is less than the length of a given or maximally lengthy representation of O.
- Lossless vs. lossy compression.

# Wilkenfeld’s (2018) theory

- **UC**: A person p 1 understands object o in context C more than another person p 2 in C to the extent that p 1 has a representation/process pair that can generate more information of a kind that is useful in C about o (including at least some higher order information about which information is relevant in C) from an accurate, more minimal description length [representation.]
- What Wilkenfeld does *not* offer is an account of the format of the relevant types of representation.

# 2.

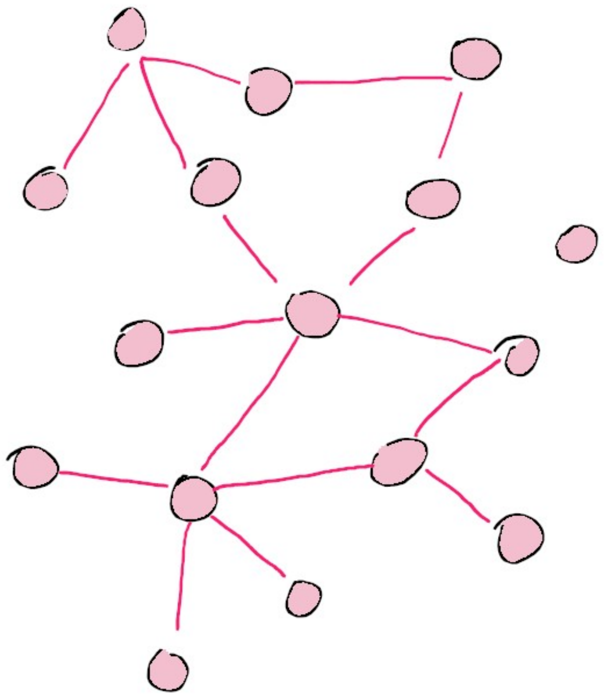
# **Graph**-based Models for Understanding

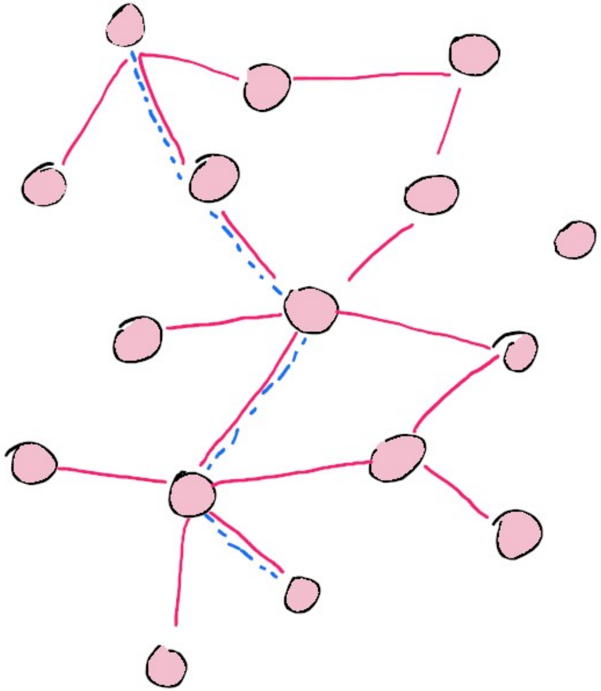
# A bit of history

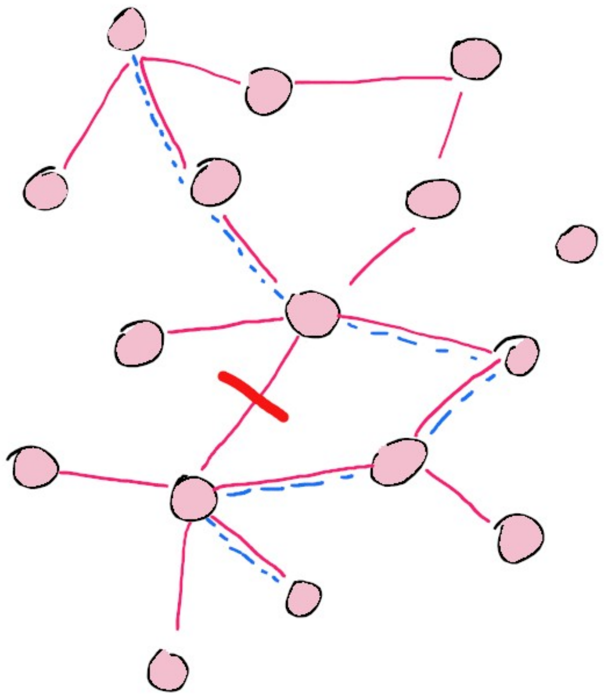
- Wittgenstein (1953) - "that kind of understanding which consists in 'seeing connections'." (122)
- The whole tradition of accounts of explanation as unification (Friedman (1974), Kitcher (1989), Schurz (1999), Bartelborth (2002), etc.)
- Stroud (1979) – the format of understanding cannot be a list of items of knowledge; connections are important. (Refers back to Carroll's regress about deduction.)
- Zagzebski (2001) - "[...] understanding is not directed toward a discrete object, but involves seeing the relation of parts to other parts and perhaps even the relation of part to a whole. It follows that the object of understanding is not a discrete proposition. One's mental representation of what one understands is likely to include such things as maps, graphs, diagrams, and three-dimensional models in addition to, or even in place of, the acceptance of a series of propositions." (241)
- Elgin (2009, elsewhere) - understanders need to grasp how various 'truths' relate to each other.
- Kelp (2021, elsewhere) – understanding is well connected, systematic, knowledge.
- Grimm (2021, elsewhere) – understanding as grasp of structure.

# Graph theory, the basics

- Graphs as sets
  - A *graph* G is a pair <V, E> of V: vertices, E: edges, where E is a function that maps vertices to vertices (so it defines a set of pairs)
  - The *degree* of a vertex is the number of adjacent vertices.
  - Graphs can be *directed* or not. In a directed graph, edges are ordered pairs.
  - There can be many kinds of edges and vertices.
  - Edges and vertices can have labels attached to them (and in general, metadata).
- Paths
  - An ordered list of adjacent edges between two points is a path.
- Connectedness
  - Graphs can be connected or disconnected. *How* connected or disconnected can be measured, for example, by the number of edges that need to be removed in order to disconnect the graph (more on this later).







# Towards Graph Maximalism (GM)

- Understanding as a contentful *state*
  - *Why a state?* So we can attribute it to subjects. Then we can define u-processes as processes that a subject goes through in relation to u-state. Note that states can have dispositional profiles.
  - *Why contentful?* Because it can be communicated, and it has other semantic properties (aboutness, truth-aptness, etc). (I'm assuming some form of representationalism.)
- GM's basic claim is that **the content of understanding states is formatted as a graph**

# But it needs to be a bit more complicated

- It must include Internal and External graphs:
  - *Internal graph*: the internal representation.
  - *External graph*: an external structure that the internal graph is measured against. (cf. Bueno & Colyvan (2011))
- Not just one graph of each:
  - Overall understanding states can be disconnected.
  - There might not be a unified structure that internal graphs are measured against (different topics are different structures)
  - This means some mapping needs to be stored to connect internal and external graphs.

# Graph Maximalism (GM)

- S understands at t only if
  - S is at t in a state U characterizable by a quadruple <$G_I$, $G_E$, *M*, D>, where $G_I$ is a non empty set of internal graphs, $G_E$ is a possibly empty set of external graphs, *M* is a mapping between elements in $G_I$ and $G_E$, and D is a dispositional profile associated to possible manipulations of $G_I$.
- Note that GM is a necessary condition for understanding, it is not a full account of when it is appropriate to assert an understanding attribution.
- D could include a set of dispositions X to act as if $G_I$ contained certain items E; in that case we can reduce U to a different U' where $G_{I'}$ does contain E and D' does not contain X. U and U' are *content-equivalent*.
  - It could be possible to project other 'virtual' graphs from U; for example, the graph that is the union of the graphs that one is in a position to understand.
  - Other virtual graphs could be constructed from the current internal graph as *views* of it. This is *latent* content.

# Why go graphical?

- It has intuitive appeal (although more on this later).
- Might help explain (at least partially) the role of “representational manipulability” (Wilkenfeld) and “cognitive control” (Hills).
- Offers an unified format of thought (although it is compatible with pluralism about this as well).
- The story about measuring understanding is the best (multiple measures, multiple dimensions, calculability [in principle]).
  - This leads to a richer *attribution* story.

# A case study: robustness and connectedness

- In certain cases, robustness will matter to understanding.
- We can think of robustness in terms of *graph connectedness*, how easy it will be to disconnect a subject's internal graph.
- We can calculate this:
  - Given the diagonal matrix D of the degrees of each node and A the adjacency matrix, the Laplacian matrix of a graph $L = D - A$. The second smallest eigenvalue $\lambda_2$ of the Laplacian matrix is 0 when the graph is disconnected, and if it is small, the graph is close to being disconnected. This gives us a measure of the degree to which the graph is disconnected, and so, a measure of well-connectedness.

*3.*

# Compressing Graphs
# (*Tight* and ***Real Tight***)

# First: we need to add dynamics

- The internal and external graphs change over time.
- The internal graph only changes as the result of internal processes.
- Updating the internal graph:
  - creating a new graph,
  - adding to the graph,
  - deleting from the graph,
  - modifying the graph-in-place,
  - copying the graph.

# Graph compression and summarization

- A graph G is a compressed representation of a graph G’ if G compresses G’.
- A summarization S of a graph G is a description of some relevant features of G (i.e., a lossy compression of G).

> “Graph summarization methods leverage compression in order to find a smaller representation of the input graph, while discovering structural patterns. In these cases, although compression is the means, finding the absolutely smallest representation of the graph is not the end goal. The patterns that are being unearthed during the process may lead to suboptimal compression. On the other hand, in graph compression works, the goal is to compress the input graph as much as possible in order to minimize storage space, irrespective of patterns.” (Liu et al 2018)

- Graph compression/summarization matters when dealing with big graphs, and it is plausible (but not necessary) that understanding-graphs are big.

# Compressible Graph Maximalism (CGM)

- We extend Graph Maximalism with two ideas:
  - The internal graph can be compressed
  - The internal graph can be a compression of the external graph*

* If we keep the understanding *as* compression model, then it *must* be.

# Compressing the external graph

- Given $U = \langle G_I, G_E, D \rangle$, it is possible that $G_I$ compresses $G_E$.
  - In fact, given the assumption that representation is compression, $G_I$ *must* compress $G_E$.
- One source of performance differential: For two subjects s and s' who compress an external graph E, by some measure u, $u(U_s) > u(U_{s'})$ only if the degree of compression of $U_s$ relative to E is greater than the degree of compression of $U_{s'}$ relative to E.

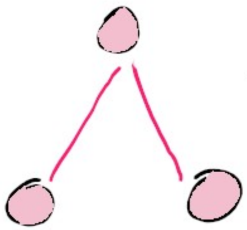
# Compressing the internal graph

- The same information can be compressed internally in different ways.*
- Different compression methods will have different computational/dispositional profiles.
- Another source of performance differential: For two subjects s and s' who compress an external graph E, by some measure u, $u(U_s) > u(U_{s'})$ only if the size of the internal graph of s is > the size of the internal graph of s'.
- ... and the compressed version is not worse according to some other criteria. Whether compression improves performance is a matter of trade-offs.

* I will assume that the same internal graph is stored only in one format, with other versions constructible as exercises of the subjects representational manipulation capacities. The same external graph can be mapped to different internal graphs, however.

# Graph representations

- The set representation of a graph is complete but large.
- An alternative is to use an *adjacency matrix*, which can be more compact.
- Another, which may be even more compact is an *adjacency list*.



*set*

G =
<{a,b,c}, {{a,b}, {b,c}}>
= {{a,b,c}, {{a,b,c}, {{a,b}, {b,c}}}}

*matrix*

G =

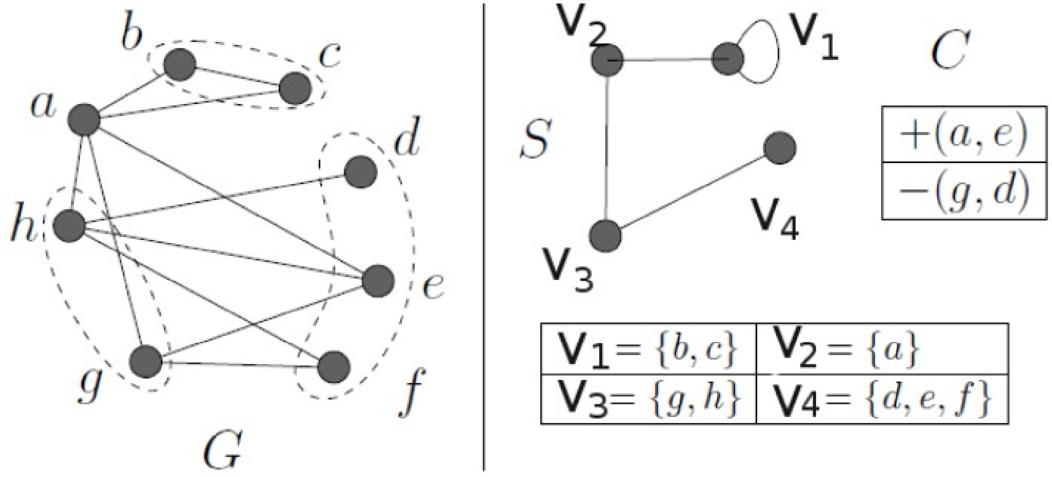
| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 0 |

= <<0,1,0>, <1,0,1>, <0,1,0>> = 3010101010

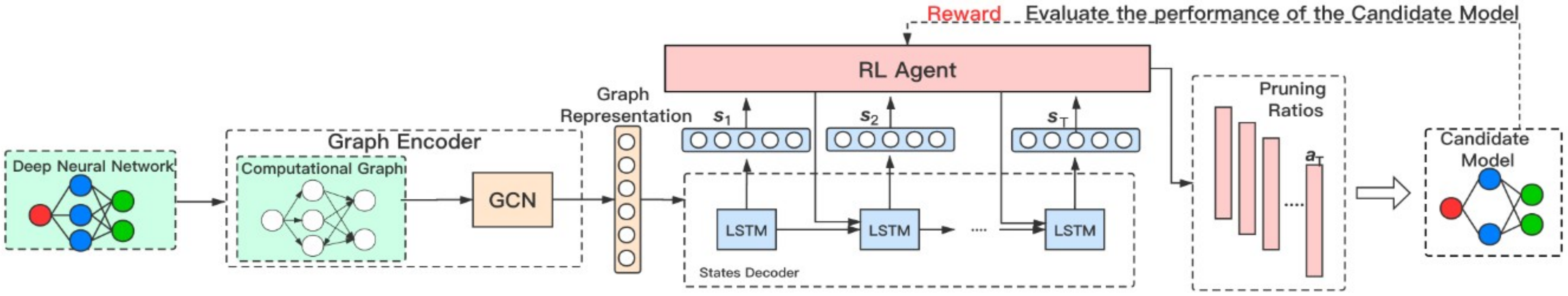
*list*

G = {<a, {b}>, <b , {c}>}
= {{{a}, {a, {b}}}, {{b}, {b, {c}}}}

# A way to compress graphs (Zhou 2010)



# Compressing graphs with neural networks




Yu et al (2021)

# *4.*

# Whether this is *Too Fancy* (or Engineering Understanding Systems)

# Excess mathematical structure

- Modeling understanding states in terms of graphical structure offers a rich way to measure them.
- But, as a consequence, we have a lot of mathematical structure available that does not seem to have any application to our understanding of understanding. This is, in a sense, *excess* mathematical structure.
- Now, it may be that our manifest image of understanding is simply impoverished (in fact, I think it is), or it makes use of shortcuts to describe the underlying structure, or it may be the case that only some of it is relevant to the kinds of task that we are worried about.

# Why have the graphs if you have dispositions?

- If part of the dispositional profile of states involves the ability to construct certain graphical representations, why couldn't we do away with the internal graphs entirely?
  - One answer: it is a matter of cognitive economy. Generating the graphs on demand could be expensive.

# What is *really* represented by the graphs?

- What *are* vertices and edges in the supposed graphs?
  - They can be *anything*. Graphical structure does not impose any constraints on content.
  - But our architecture might not feed it arbitrarily.

# Is *human* understanding *really* modeled by CGM?

- What is the *empirical* evidence to the idea that the format of understanding representations is graphical?
  - S-representations, maps as a format for thought, mental models (Cummins (1991), Ramsey (2007), Rescorla (2009), Blumson (2011), Gladziejewski (2015), Lee (2018), Camp (2018)), connectionist models of the mind, causal maps and Bayes nets (Gopnik & Glymour 2002), graph neural networks (Zhou et al 2020).
- What is the empirical evidence for the claim that the format of understanding representations is compressed?
  - Predictive coding theories of mind (Clark 2013, Hohwy 2014) seem to suggest that compression is common cognition, if not central to it; MDL literature on learning (Grünwald 2007, Robinet et al (2011), )

# CGM as a model for understanding systems

- I think we should view CGM as a model or framework for the *construction* of understanding systems.
  - We can bypass the question of whether CGM is a good model for human cognition.
  - It makes sense to allow the model to have excess structure, and to impose few constraints from the model; both things make it more flexible.

# Thanks!

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