PEIRCEAN KNOWABILITY AND FITCH-LIKE PARADOXES

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Plan

1. Introduce the concept of knowability
2. Give an overview of the knowability paradox
3. Reconstruct a Peircean notion of knowability
4. Peircean knowability falls prey to the paradox
5. Dabay on justifiability and how justifiability also falls to paradox
6. Pieterinen & Chiffi on conjecturability and how conjecturability also falls to paradox
7. Conclusions

This presentation gives an overview of some arguments published in:

- Grigoryan, K. & Morales Carbonell,F. (2025). Peircean knowability and Fitch-like paradoxes. *Res Philosophica*, 102(2), 163–190.
- Heylen, J. & Grigoryan, K. (2024). From knowability to conjecturability, and back again. *Contemporary Pragmatism*, 21(3), 287–298.

Knowability

The concept of knowability is key to many philosophical issues. Perhaps most famous of all, there is the observation that antirealist positions tend to entail versions of what is known as the knowability thesis:

Knowability thesis Every truth is knowable

For sure, if truth depended on the capacities of metaphysical subjects, we should expect the scope of truth to be determined by what subjects can know. This obviously holds for subjective idealism (where truths are subjectively constituted), but also for verificationists (every truth must, at a minimum, be understandable).

It may also be true in the case of pragmatist accounts of truth. Here, we examine the case of Peirce's philosophy.

The concept of knowability

A simple way to understand the concept of knowability is:

Possible knowledge *p* is knowable iff it is possible that *p* is known

This concept is not factive: something could be knowable in this sense without being true of the actual world.

We usually want a factive sense of knowability. For example, Edgington (1985) gives:

Counterfactual knowledge of actuality *p* is knowable iff it is possible to know that *p* is actually true.

There is a debate on what is the correct way to render the knowability concept. For our purposes we can omit the details.

For an overview of the literature on how to conceptualize knowability, see Heylen, J. & Morales Carbonell, F. (2023). Concepts of knowability. *Revista de Humanidades de Valparaíso*, 23, 287–308.

Fitch's paradox of knowability

It turns out that the possible knowledge concept of knowability leads to paradox. A famous argument by Fitch shows that from the knowability thesis along with some plausible assumptions, we can logically derive omniscience:

1. $\forall p(p \rightarrow \Diamond Kp)$
2. $\exists p(p \land \neg Kp)$
3. $Kp \rightarrow p$
4. $K(p \land q) \rightarrow (Kp \land Kq)$
5. $\varphi \vdash \Box\varphi$
6. $\Box\neg\varphi \vdash \neg\Diamond\varphi$
7. $p \land \neg Kp$ (inst. 2)
8. $p \land \neg Kp \rightarrow \Diamond K(p \land \neg Kp)$ (inst. 1)
9. $\Diamond K(p \land \neg Kp)$ (7, 8, MP)
10. $K(p \land \neg Kp)$ (supposition)
11. $Kp \land K\neg Kp$ (10, 4)
12. $Kp \land \neg Kp$ (11, 3 over right)
13. $\neg K(p \land \neg Kp)$ (10, 12, *reductio*)
14. $\Box\neg K(p \land \neg Kp)$ (13, 5)
15. $\neg\Diamond K(p \land \neg Kp)$ (14, 6)
16. $\neg(p \land \neg Kp)$ (7, 9, 15, *reductio*)
17. $\neg\exists p(p \land \neg Kp)$
18. $\forall p(p \rightarrow Kp)$

Peirce on knowledge

To do so, we need to make some observations about Peirce's concept of knowledge:

- Knowledge is a *belief-habit* that is settled through the application of an adequate method for settling beliefs.
- Whether a method is adequate depends on:
  a) whether it can provide reasons to cease our doubts,
  b) whether it is beholden to something that is not subject to our thinking—some 'external permanency' (CP 5.358–387, 'The fixation of belief').
- We must distinguish between knowledge for inquirers at the end of inquiry and for inquirers at the midst of inquiry.
  - Knowledge at the end of inquiry is a *regulative epistemic hope*, and it is infallible.
  - Knowledge at the midst of inquiry is fallible: 'our knowledge is never absolute but always swims, as it were, in a continuum of uncertainty and indeterminacy' (CP 1.171)
  - We can treat certain beliefs as infallible even in the midst of inquiry.
- Knowledge is factive: in the limit, we can only know what is true.
- Collective knowledge is superior to individual knowledge.

Peirce on knowability

Peirce seems to endorse some version of the knowability thesis:

> *Ignorance and error can only be conceived as correlative to a real knowledge and truth, which latter are of the nature of cognitions. Over against any cognition, there is an unknown but knowable reality; but over against all possible cognitions, there is only the self-contradictory. In short, cognizability (in its widest sense) and being are not merely metaphysically the same, but are synonymous terms. (CP 5.257)*

> *I may be asked what I have to say to all the minute facts of history, forgotten never to be recovered, to the lost books of the ancients, to the buried secrets. . . . Do these things not really exist because they are hopelessly beyond the reach of our knowledge? And then, after the universe is dead (according to the prediction of some scientists), and all life has ceased forever, will not the shock of atoms continue though there will be no mind to know it? To this I reply that, though in no possible state of knowledge can any number be great enough to express the relation between the amount of what rests unknown to the amount of the known, yet it is unphilosophical to suppose that, with regard to any given question (which has any clear meaning), investigation would not bring forth a solution of it, if it were carried far enough. (CP 5.409)*

Peircean knowability theses

IKI All truths are infallibly knowable (by the ideal community) at the end of inquiry.
For all *p*, if *p* then *p* is infallibly knowable (by the ideal community) at the end of inquiry.

FKI All truths are at least fallibly knowable (by the ideal community) at the end of inquiry.
For all *p*, if *p* then *p* is at least fallibly knowable (by the ideal community) at the end of inquiry.

IKA All truths are infallibly knowable (by the actual community) in the midst of inquiry.
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FKA All truths are at least fallibly knowable (by the actual community) in the midst of inquiry.
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For all *p*, if *p* then *p* is at least fallibly knowable (by the actual community) in the midst of inquiry.

For IKI, there is collapse but no paradox

It can be shown that IKI leads to the collapse of possible knowledge and knowlegede, and thus, to the omniscience thesis that all truths are known by the ideal community of inquirers at the end of inquiry.

(The argument is analogous to the traditional Fitch-like argument, so we omit it here).

However, this is not paradoxical: we should expect this result (Dabay 2016 agrees).

For FKA, there is collapse and paradox

However, FKA is also plausible in the context of Peirce's epistemology, and it does lead to paradox:

1. $\forall p(p \to \diamond K_F p)$
2. $\exists p(p \land \neg K_F p)$
3. $K_F p \to p$
4. $K_F(p \land q) \to (K_F p \land K_F q)$
5. $\varphi \vdash \Box\varphi$
6. $\Box\neg\varphi \vdash \neg\diamond\varphi$
7. $p \land \neg K_F p$ (inst. 2)
8. $p \land \neg K_F p \to \diamond K_F(p \land \neg K_F p)$ (inst. 1)
9. $\diamond K_F(p \land \neg K_F p)$ (7, 8, MP)
10. $K_F(p \land \neg K_F p)$ (supposition)
11. $K_F p \land K_F \neg K_F p$ (10, 4)
12. $K_F p \land \neg K_F p$ (11, 3 over right)
13. $\neg K_F(p \land \neg K_F p)$ (10, 12, *reductio*)
14. $\Box\neg K_F(p \land \neg K_F p)$ (13, 5)
15. $\neg\diamond K_F(p \land \neg K_F p)$ (14, 6)
16. $\neg(p \land \neg K_F p)$ (7, 9, 15, *reductio*)
17. $\neg\exists p(p \land \neg K_F p)$
18. $\forall p(p \to K_F p)$

Can this result be avoided?

Dabay on knowability

Dabay (2016) has argued that we can avoid the paradox. His argument is:

1. Knowledge is nothing more than ideally justified belief.
2. Actual individuals in the midst of inquiry can never attain knowledge concerning most all of their beliefs.
3. It is not only the case that they will never know, but they can never know.
4. Therefore, there is no reason to endorse a knowability thesis as it relates to inquiries in the midst of inquiry.

But: Peirce allows that we can have knowledge in the midst of inquiry. To say otherwise is unjustifiably skeptical!

Dabay recognizes the point:

> *For Peirce, knowledge is uncertain and has less-than-absolute epistemically normative standing, while I disagree with both characterizations. Going forward, I will continue to use "knowledge" to designate "ideally justified belief," and substitute the phrase "fallibly justified true belief" to designate what Peirce refers to as "knowledge." (2016, 248)*

Dabay on justifiability

While Dabay rejects the knowability thesis, he accepts

Justifiability thesis Every truth is justifiably believable.

Justified belief is not factive: we can be justified in thinking something on less than ideal grounds, so that our belief is only apparently true but false.

Because the typical formulation of the paradox requires the factivity of the relevant epistemic operator (typically knowledge), the analogous argument cannot work for justifiability.

Collapse for justifiability

Even though factivity is not valid for justifiability, a similar principle is plausible, namely, **Weak factivity**:

$$\forall p(J\neg Jp \to \neg Jp)$$

The argument then goes:

1. $\forall p(p \to \Diamond Jp)$
2. $\exists p(p \wedge \neg Jp)$
3. $J\neg Jp \to \neg Jp$
4. $J(p \wedge q) \to (Jp \wedge Jq)$
5. $\varphi \vdash \Box\varphi$
6. $\Box\neg\varphi \vdash \neg\Diamond\varphi$
7. $p \wedge \neg Jp$ (inst. 2)
8. $p \wedge \neg Jp \to \Diamond J(p \wedge \neg Jp)$ (inst. 1)
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17. $\neg\exists p(p \wedge \neg Jp)$
18. $\forall p(p \to Jp)$

Weak factivity

The key to our argument is weak factivity, and it can be reasonably asked if it is a principle that Peirce would plausibly endorse.

The principle can be derived from positive instrospection ($Jp \to JJp$) and consistency ($J\neg p \to \neg Jp$), both of which are not admissible on Peircean grounds.

We treat it as a primitive axiom, and rationalize it on the grounds of certain features of habitual states:

- Habits can be incompatible with each other.
- In some cases, having a justified habit to believe that it is not a justified habit to believe that p is incompatible with having the justified habit to believe that p.
- Not true in general: consider the case of a self-ignorant racist.
- However, if the subject has formed a habit to believe that they do not believe in something *reflectively*, their justified belief-habit is incompatible with having a belief that p.

The general picture

There is a class of habitual states for which the scheme $H\neg H'p \rightarrow \neg Hp$ holds, where H and H' are habitual states, and where H' is at least as strong as H in terms of epistemic support.

In general, for justified belief we can think of a family of habitual states with different degrees of groundedness (from the most well supported J_A to the least well supported J_N) that represent different degrees of habitual support. There is range of states within that range for which some version of weak factivity holds. We suppose that at least some variety of Peircean justified belief that is not all things considered falls within this class.

$$J_x \left\{ \begin{array}{l} \left.\begin{array}{l} J_A \\ \vdots \\ J_P \\ \end{array}\right\} = \text{Weak Factivity} \\ \vdots \\ J_N \end{array}\right.$$

Chiffi and Pietarinen on conjecturability

A different proposal comes from Pietarinen (2018) and Chiffi and Pietarinen (2020).

They try to avoid the paradox claiming that the key epistemological notion for Peirce is not knowledge, but *conjecture*, and they argue that a Fitch-like paradox does not arise for this notion.

We will argue that similar paradoxes arise for this proposal as well.

We also think that it is incorrect to say that Peirce would not be committed to something like the knowability thesis, and more fundamentally, that the notion of knowledge is merely secondary to the scheme. Peirce endorses both security and uberty, and the way he describes the process of inquiry leaves them on par in terms of importance. Inference requires deduction, induction and abduction.

Chiffi and Pietarinen on the logic of conjecturability

We are given a characterization of the logic of conjecturability.

- The conjecture operator C is constructed as a modality within a **S4**-like logic.
  - $\Box$ is understood as a proof operator, and $\Diamond$ as a hypothesis operator, so that $\Diamond p$ means that it has not been proven that p is false.
  - C is a gloss of $\Diamond\Box\Diamond$, that is, conjecturing that p is the provability of the hypothesis that p.
  - The logic of C does *not* have factivity ($Cp \to p$ is invalid), positive instrospection ($Cp \to CCp$), negative instrospection ($\neg Cp \to C\neg Cp$), or consistency ($\neg(Cp \wedge C\neg p)$). Thus, several ways to derive the paradox are blocked.

Issues with the logic of conjecturability

- While positive introspection is not valid for C, it is valid for the underlying $\Box$. This is not good in a Peircean framework.
- Hypothesis as the dual of the proof operator does not seem to be a genuine epistemic operator; it represents the absence of proof, not the presence of an attitude.
- It is provable in S4 that $\Diamond\Box p \to \Diamond\Box\Diamond p$, that is $\Diamond\Box p \to Cp$. But then, provability implies conjectures. This also sounds paradoxical.
  - Heylen and Grigoryan (2024) show that from the conjecturability thesis ($p \to \Diamond Cp$) we get the conjecture thesis ($p \to Cp$):
    1. $\Diamond Cp$ (assumption)
    2. $\Diamond\Diamond\Box\Diamond p$ (1 by the definition of C)
    3. $\Diamond\Box\Diamond p$ (2 by the dual of axiom 4)
    4. Cp (3 by the definition of C)

Weak factivity for conjectures

It may be possible to have a notion of conjecture for which some weak factivity principle is available:

Weak Factivity* $C\neg Cp \to \neg Cp$

That is, if it is conjectured that p is not conjectured, p is not conjectured.

To make this work we need to think of conjecture as some kind of habitual state for which certain compatibility restrictions holds (that is, such that certain conjectures prevent other conjectures to be made).

A lot depends on the strength of habitual support that conjecture has. Empirical conjectures are habitually weaker than mathematical conjectures—perhaps WF* is appropiate for the latter but not for the former.

The upshot

However, conjecture is at least more stable than hypothesis:

> *Being a meaningful scientific conjecture requires something more than just a status of a may-be, namely being connected to a rational expectation, in the sense of something akin to a scientific and intellectual "hope" . . . , that can be laid upon p. A rationally expected possibility would entitle scientific communities to assert p in the somewhat stronger sense of a may-be that could turn out to be the case in the future. (Chiffi and Pietarinen 2020, 212)*

In this sense of conjecture, perhaps if we expect that some *p* is not conjectured, we do not conjecture that *p* either.

If this is right, then we can run an analogous argument to that for justifiability, and derive that all truths are as a matter of fact conjectured.

If there is at least *some* version of conjecturability for which weak factivity is appropiate, there should be at leas some version of the conjecturability thesis that is susceptible to Fitch-like paradoxes. So this approach might be a no-go as well.

Conclusions

We are sorry to say we did not come with good news for Peircean epistemology.

Our results do not mean that nobody can come up with a way to capture a Peircean notion of knowability and other relevant notions without falling into paradoxes.

For example, Carrara et Chiff (2014) and Carrara et al (2017) introduce a different multimodal logic for illocutionary acts that can be used to deal with the paradox. Chiffi and Pietarinen (2018) use this to give a formal account of abductive inference along Peircean lines. We leave discussion of these for elsewhere.

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