

# PUZZLES, GRAPHS, KNOW-HOW AND UNDERSTANDING

*towards a theory of content*

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# THE PLAN

- 1) Report on what has happened so far:
  - 1) Give an overview of earlier work on understanding.
  - 2) Present the results of the ANID/FONDECYT project 3220017: 'Saber cómo: Preguntas, Maneras e Intentos' (2022-2025).
- 2) Sketch what is next:
  - 1) Propose a hypothesis: we can treat graphical contents as a unifying element for a unified account of cognition.
  - 2) Sketch how graphical contents are a good substitute for propositional content (because they are more general, and it can incorporate inquisitive content).
  - 3) Deal with the issue of graphical inference.

# UNDERSTANDING

*Why should things be easy to understand? - THOMAS PYNCHON, in a *Playboy* article*

# UNDERSTANDING IN EPISTEMOLOGY

- Two traditions:
  - Understanding and explanation (Lipton: *the good of explanation is understanding*)
  - Understanding as a distinct epistemic/normative status
- I come to this from the angle of modal epistemology: perhaps understanding can be a source of modal knowledge.

# A MINIMALIST ACCOUNT OF UNDERSTANDING

- *A minimal characterization of understanding*: a positive epistemic state that:
  - a) is non factive ( $Ux \nvdash x$ ) (Elgin, 2017),
  - b) relates a subject to an object holistically (Zagzebski, 2001), and
  - c) gives them *cognitive control* concerning that object (Bengson, 2017; Hills, 2015).
- Knowing that something happened is not the same as understanding it. Knowing a phenomenon (in the sense of being acquainted), is not the same as understanding it. Knowing a person is not the same thing as understanding them.
- Understanding is in some cases more epistemically valuable than knowledge.
- Understanding comes in degrees.

# UNDERSTANDING ATTRIBUTIONS

- A theory of understanding must give an account of the truth/assertability conditions for statements of the form:

**COMPARATIVE JUDGMENTS**  $s_1$  understands  $x$  better than  $s_2$

**OUTRIGHT ATTRIBUTION**  $s_1$  understands  $x$

- Provided an account of comparative judgments (and then degrees), we give an account of outright attribution by giving an account of how crossing a threshold allows outright understanding.
  - We can say that  $s_1$  understands better than  $s_2$  without it being true that  $s_1$  understands or  $s_2$  understands.
  - I defend a higher-order contextualist account of understanding attribution: the context determines the necessary components of the relevant concept of understanding, and the threshold. This yields a unified scheme for understanding attribution across domains.

## UNDERSTANDING-LIKE STATES

- Given my preferred way to think of understanding attributions, it becomes natural to think that there are states short of understanding that are very much similar to it. Call these *understanding-like states*.
- Once accounted, this gives a more realistic picture of the process of acquiring and improving our understanding: we start from ignorance/incomprehension, and advance towards higher degrees of understanding.

# THE FORMAT OF UNDERSTANDING

- People have been drawn to the idea that understanding consists in some kind of grasp of *structure*.
- I take this seriously, and propose that understanding states have the properties that they have because the kind of format their content has allows for mappings to certain kinds of structures.
- I propose this as a property of *understanding-like* states, not just understanding states.
- We can then give a more precise characterization of these states:

**UNDERSTANDING-LIKE STATE** a state that can be characterized by a tuple  $\langle G, M, D \rangle$ , where  $G$  is a set of graphs,  $M$  is a mapping between these graphs, and  $D$  is a dispositional profile of capacities that involve those graphs.

- More on these graphs later, but in the usual case we will have a coupling of a set of internal graphs that represent structure, and external graphs that are proxies for the structure of the objects of understanding.
- This strategy yields a richer account of the measures of understanding.
- It also allows for a natural way to account for the role of compression in understanding (more on this later as well).

# COLLECTIVE UNDERSTANDING

- Most theories of understanding explain understanding as an individual status.
- I propose that we should also consider the possibility of collective understanding (cf Delarivi  re (2020)).
- As it turns out, the characterization of understanding-like states I give allows for the possession of understanding-like states to be distributed in groups.

# **KNOW-HOW**

*And we know how  
To use our know how  
And if you knew how  
You'd do it too - CAMBERWELL Now, "Know how"*

# KNOW HOW AND ABILITY

- The central discussion about know-how has traditionally concerned its nature, and the key debate has been between intellectualists (who argue that know-how is of a kind with theoretical knowledge), and anti-intellectualists (who argue that know-how is distinct from theoretical knowledge).
- I side with the anti-intellectualists: I think ability is key to the possession of know-how.
  - While usually the anti-intellectualist characterization of know-how is stated in terms like 's knows how to  $x$  iff s has the ability to  $x$ ', I think the relevant abilities are not as straightforward.
- Still, I think know-how often is linked to (mental) content.

# KNOW HOW IS UNDERSTANDING-LIKE

- The proposal is to adopt the strategy I used to account for understanding.
- Know-how is understanding-like:

**KNOW HOW AS UNDERSTANDING-LIKE** knowing how consists in being in an understanding-like state  $\langle P, A, M, D \rangle$ , where  $P$  is a set of internal graphs of procedures,  $A$  is a set of environmentally afforded procedures,  $M$  is mapping between  $P$  and  $A$ , and  $D$  is a set of dispositions to act in regards with  $P$ ,  $D$ , and  $M$ .

- The eagle-eyed should notice that this is actually a characterization of know-how-like understanding-like states, not a full account of know-how.
- This gives an account of gradability of know-how, the basis for an account of know-how attributions, an account of the content of know-how-like states, etc.

# KNOW HOW ATTRIBUTIONS

- Again, more or less higher-order contextualism:

**CONTEXTUALIST SUBJECT- AND FACT-SENSITIVE ANTI-INTELLECTUALISM** In context  $x$ ,  $s$  knows how to  $t$  if and only if  $P_c(s)$  (a contextually-defined set of people in some contextually-defined relation to  $s$ ) are able $_f$  to  $t$ , where able $_f$  is a type of ability relative to a contextually- defined set of facts  $f$ .

- This can handle cases where attributions seemingly follow the possession of propositional knowledge rather than abilities. The model predicts that for certain tasks, we do this because that the subject themselves have the relevant abilities does not matter.

# KNOW HOW AND QUESTIONS

- An observation that will matter later: know how attributions are usually understood as embedding questions:
  - 1) Felipe knows how to fix the blunderembeds the question ‘how to fix the blunder?’ (is an example of a wh-question, cf. Stanley (2011)).
- Habgood-Coote (2019) proposes an intellectualist account where know how consists in the possession of an answer to a question, and the ability to answer it (in a broad sense).
- This is an important insight, and my proposal tries to recover it in an anti-intellectualist frame.
- For reasons that should become clear later, this link between understanding-like states and questions is significant.

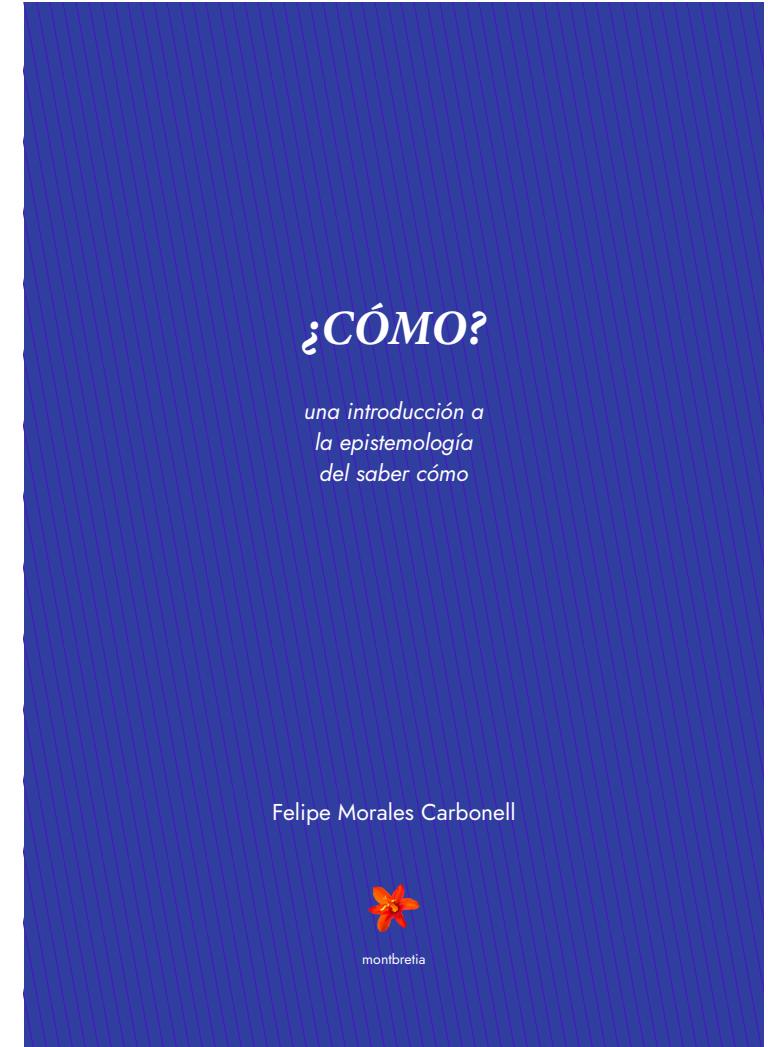
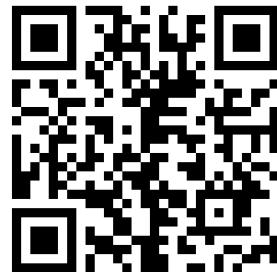
# COLLECTIVE KNOW HOW

- Again, there is an increasing interest in the social dimension of know-how (Birch, 2019; Habgood-Coote, 2022; Palermos & Tollefsen, 2018).
- The approach I take readily accommodates this: a group can have the relevant kinds of content, and possess the abilities that are relevant to the correctness of know-how attributions.

As an output of ANID/FONDECYT 3220017, I wrote a textbook on the epistemology of know-how:

Morales Carbonell, F. (2025). *¿Cómo? Una Introducción a la Epistemología Del Saber Cómo.*

It's available at:



# GRAPHS

*The carving and the paring of the land  
The quarter square, the graph divides  
Beneath the rule, a country hides - WIRE, "Map Ref. 41°N 93°W"*

# GRAPHICAL STRUCTURE

- The key component of both my account of understanding and know-how is the kind of content it assumes those states have: the proposal is that it is graphical, in the sense that it at the very least can be modeled using graph-theoretical structures.
- A *graph* (in the simplest sense) is a tuple  $\langle V, E \rangle$ , where  $V$  is a set of vertices, and  $E$  is a set of edges, which are pairs of elements of  $V$ .
- We can add more structure:
  - We can add different kinds of edges and vertices (distinguish between directed and undirected edges, allow more than one kind of each, etc.)
  - We can add labels,
  - We can allow more than 1 to 1 edges ( $n$  to 1, 1 to  $n$ ,  $n$  to  $n$ , etc. – here we have *hypergraphs*)
  - We can add topological information,
  - etc.
- The kinds of graphs that are interesting to me are heterogeneous.

# SYSTEMS

- The kinds of graphs we need depends on the kind of thing we want to represent. We can (supposedly) always use a graph, but not always the same kind of graph.
- Take the case of the system of cooking rice (Haslanger, 2015):

“Cooking rice is an instance of a more general practice of cooking, and regular engagement in the practice is constitutive of a social role: cook. Being a cook relates one in specific ways to other persons (not only the customer or family, but also the farmer, grocer, garbage collector, sources of recipes, including traditions, cookbooks, etc.), and also relates one in specific ways to things (foodstuffs, sources of heat, water, utensils). Cooking is only possible within a social structure that provides the ingredients, skills, tools; the norms for taste, texture and ingredients; the distribution of labor of cooks and consumers, etc.”

What is it to understand something like this?

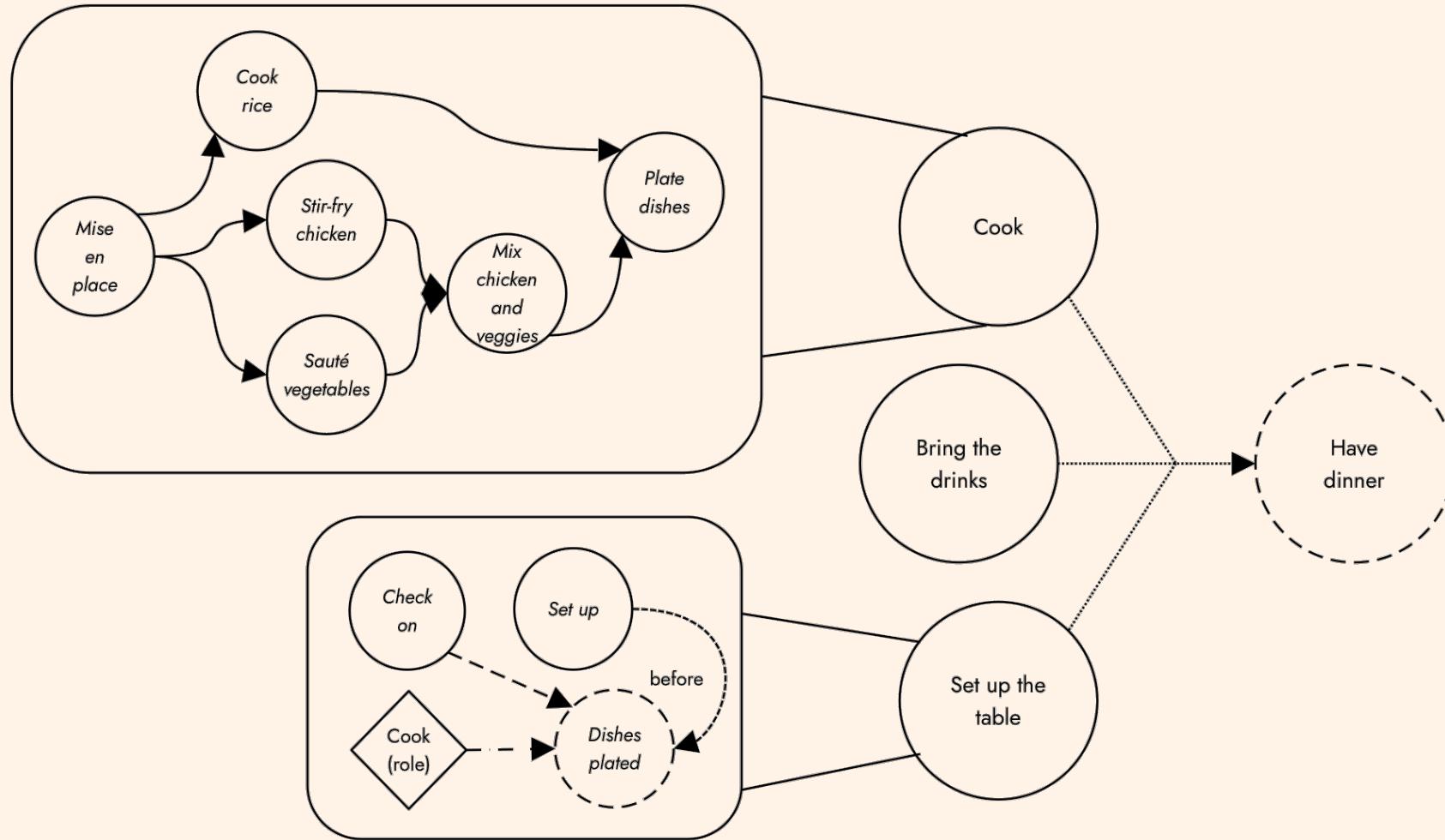
*How do we represent something like this?*

# SYSTEMS, THE PROPOSAL

- We need to make use of all the resources we sketched above:
  - we take concepts and objects of different kinds as vertices—we color them to capture types of objects (practices, resources, actions, objects are all different)
  - we allow n-t-n edges (social roles relate to  $n$  social roles).
  - we allow directed and undirected edges (resources are used by individuals in roles, but not vice versa)
  - we allow different kinds of edges (the relation between the practice of cooking rice and the practice of cooking is different to the relation between resources and the roles that use them, etc.)
  - we allow edges to connect vertices and edges (for example, to represent that the relation between a resource and a role is made possible by a certain social structure)
  - we allow grouping: a subgraph can be surrounded and put in a relation as a whole with other parts of the graph.

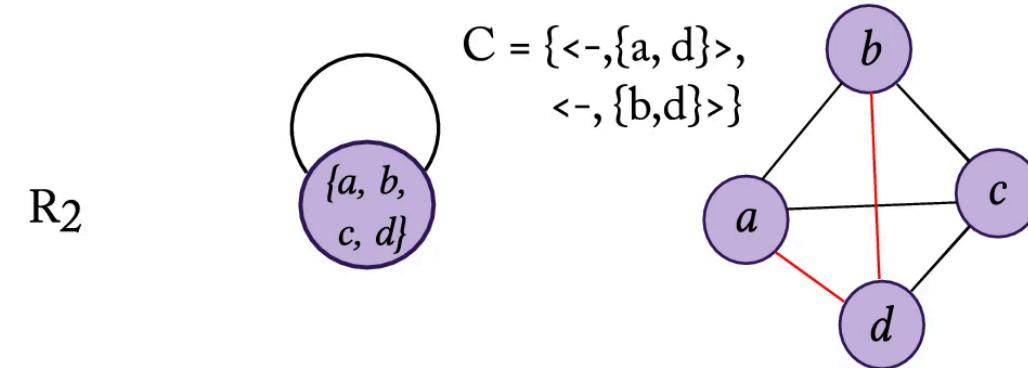
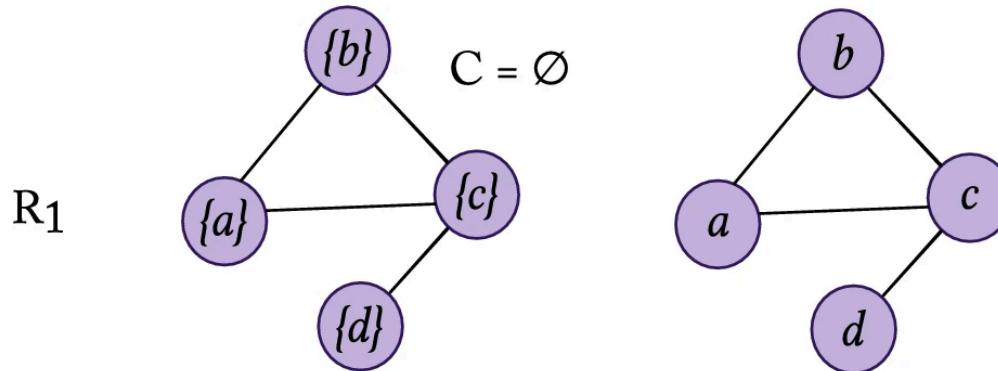
# PROCEDURES

- For know-how, the relevant graphical structures represent *procedures*.
- Procedures can be represented under different schemes: as temporally ordered sequences of actions, as sequences of states that are linked by actions, as a mix of both...
- Procedures need not be sequential (this is a limitation of certain models such as Stanley (2011) and Bengson & Moffett (2011). Some procedures branch out and join.



# COMPRESSION

- Something that can be incorporated into the model that is perhaps not obvious is that it allows very interesting kinds of *compression*.
- Why worry about compression? Some authors, such as (Wilkenfeld, 2019) have argued that compression matters for understanding: roughly, we can measure the degree of understanding by measuring how much it compresses relevant information.
- For the graphical format, we can appeal very naturally to *minimum description length* (MDL) compression schemes.



# INFERENCE

*Oh I know there's comfort in sadness  
But I try to distinguish these passing emotions  
My unimportant existence  
From the great machine of the world's indifference - MOMUS, "The Sadness of Things"*

# INFERENCE, CLASSICAL

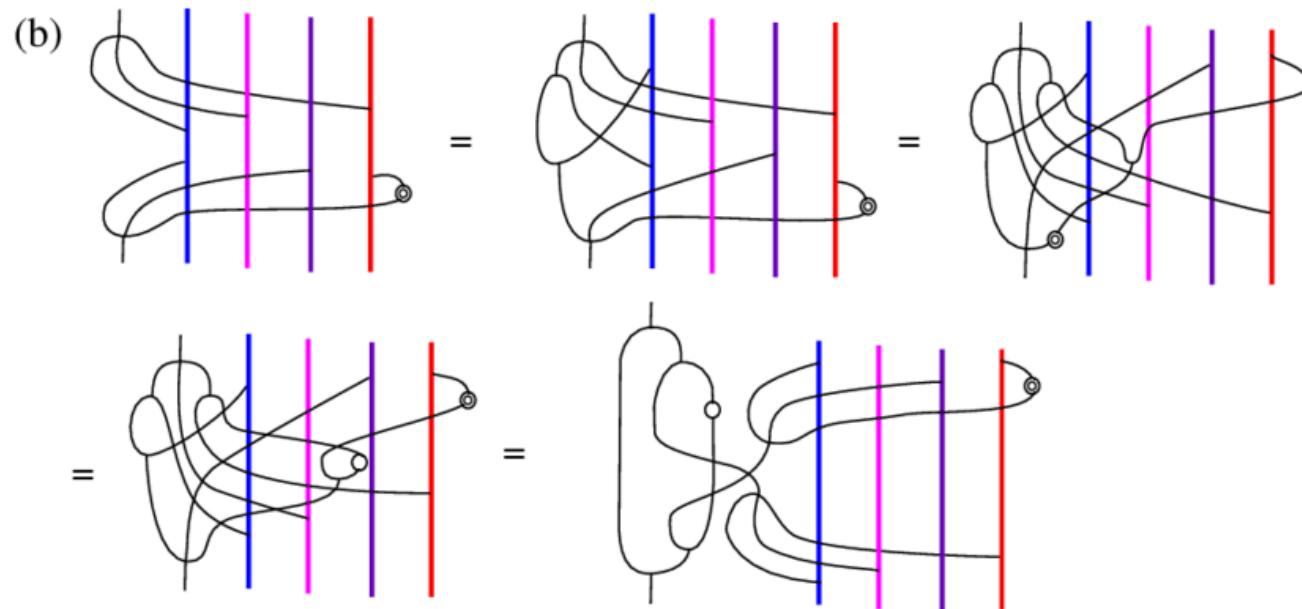
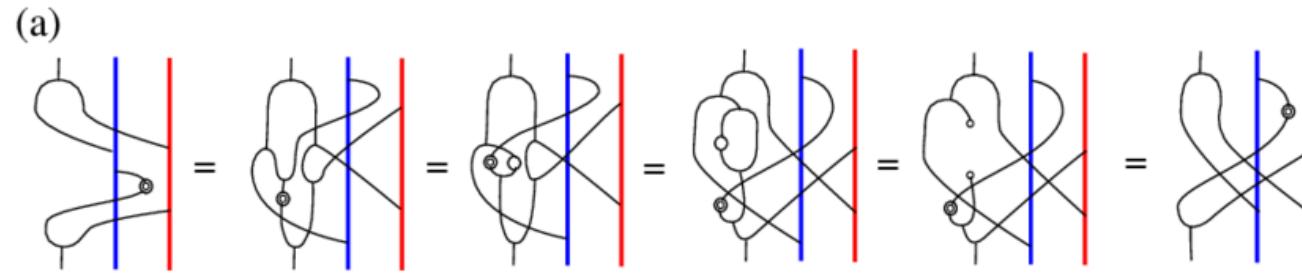
- Some of the abilities that understanding and know-how involve concern the manipulation of graphical representations (cf. Wilkenfeld (2013)), in the form of warranted transformations of representations into representations.
- Classical inference deals with the warranted transformations of (sets of) propositions into (sets of, although this is not so classical) propositions.

$$p \rightarrow q, p \vdash q$$

- Typically, traditional inferences are:
  - linear (non-branching) or hierarchical (tree-like)
  - homogeneous (from sets of propositions to sets of propositions through sequences of sets of propositions)
- Whenever we abandon purely indicative contexts (as in erotetic logic, where we can go from propositions to questions and vice versa), the relevant sets can still be correlated to propositional structures.

# INFERENCE, NON-PROPOSITIONAL?

- If the representations that we are manipulating when we are dealing with understanding-like states are not propositional or at least not entirely propositional, we need a different conception of inference.
- There are some antecedents: for example, there is Peirce's diagrammatic logic. But I think this is mostly just an alternative representation of a classical logic.
- In other cases, pictorial devices are used to capture propositional information in a surveyable manner.



From (Meusburger & Voß, 2021)

# HETEROGENEOUS INFERENCE WITH MAPS

- Some people have introduced the notion of *heterogeneous inference* to handle cases like this (Barwise & Etchemendy, 1996).
  - A well explored area of research on this has been the study of the logical properties of maps (Aguilera, 2021; Camp, 2018; Casati, 2024; Rescorla, 2009).
  - One of the debates in this area is how maps can encode predicative information. This is a concern because people have



from *Dancing and Singing: A Narrative Atlas of Boylan Heights*, by Denis Wood.

# SURROGATIVE REASONING

- One potential application of this approach to inference is to capture the kinds of inferences that we make use of when we use models, namely, surrogate reasoning.
- The most widely discussed form of surrogate reasoning is model-based inference: we use an object (either a concrete object, a mathematical structure, a computer simulation, etc.) to reason about a target object.
- Understanding-like states are like models, because they map inner structures to targets (cf. Frigg & Nguyen (2020) and their DEKI account of models).
  - We can treat models as potentially external understanding-like states of epistemic subjects.

# PUZZLES

*If everything fits into place*

*Seamlessly pieced together in the most terrible way*

*I couldn't keep it together*

*I couldn't make it last*

*I couldn't prove I was good*

*I wonder if anybody could - BLACK DRESSES, "(Can't) Keep it Together"*

# INQUIRY

- We locate all epistemic activity, and as a consequence all reasoning in the broad terms sketched above, in the context of *inquiry*.
- Inquiry aims at different goals, including knowledge and understanding. We arrive at understanding-like states through the process of inquiry.
- Inquiry, however, is not just a matter of possession of information.
- I propose that we see inquiry as structured by inquisitive states, attitudes and acts, that is, states, attitudes and acts that bear on questions.
- Accordingly, we need to explain the link between understanding-like states and the inquisitive dimension of inquiry.

# FRICITION

- A good starting point is to start from the basic experience of *epistemic friction* (Sher, 2016): we sometimes (often, perhaps) are presented with difficulties in making sense of our location in the world, which appears as something that is not up to us. A problem presents itself, and we feel *friction* that we cannot immediately pass through.

Cf. also Tichy: we don't know which world is the actual world, and science's only goal is to determine this.

- My proposal: it is an structural matter of our relation with the world. But then, the world might *afford* points of contact to start inquisitive episodes. The world has enough structure to allow for the experience of friction.
- The question here is: what is that structure, and how do we capture it?

# PUZZLES

- Let's consider what *puzzlement* as an understanding-like state could be:
  - It would involve being coupled to some structure in the target,
  - It would involve having an inner representation that did not satisfy certain epistemic goals.
- In a practical case, where we need to solve a puzzle, we can see the puzzle as an incomplete way, and we can represent it with a certain class of graphs (graphs with absent components).
- Knowing how to solve a puzzle is to have a solution to a puzzle, and learning how to solve a puzzle is a process by which, from having a representation of an incomplete way to solve it, we come to grasp more exhaustive ways to solve it. Each step of the process can be captured as an understanding-like state.

How do we go from being puzzled to being able to articulate a question?

# CONTENT

*I'm so full of ideas  
And here is a good one  
Lucia has a dream  
A dream that comes in colour of  
**International colouring**  
For your entry she's waiting - STEREOLAB, "International Colouring Contest"*

# GRAPHICAL CONTENT

- Given all this, it seems plausible that we can think about cognition in terms of the possession of graphical content and a set of abilities to deal with such content.
- So far, I have provided only a structural and functional account of graphical content: I postulate that there is such a thing, and describe what we could do with it.
- But I don't really have a *substantive* account of graphical content. Do I commit to representationalism about graphical content? Can I do without this commitment? I need to evaluate the options and take some kind of stance.
- For example, my proposal seems like an account of S(tructural)-representations. But these are contested.
- Likewise, more clarity needs to be had on the possibility of non-propositional intentionality (Grzankowski, n.d.; Grzankowski & Montague, 2018).
- I also need an account of the contents of graphical content. For example, there is a significant literature on the use of Bayesian networks to model cognitive states (Griffiths, 2024). I don't particularly like these models as a general account of cognition, they are too uniform. But allowing heterogeneity, as I wish, comes with costs.

# INQUISITIVE CONTENT

- A different question is how to model the inquisitive features of understanding-like states.
- There are some antecedents, most notably perhaps the project of inquisitive semantics (Ciardelli et al., 2019).
- My proposal is to introduce graphs with ‘absent’ components, where a graph can leave out pieces of information without definite values (so inquisitive contents are similar to what people have called ‘gappy’ propositions, cf. Mousavian (2022)).

## COLLECTIVE CONTENT

- Finally, another desideratum for a theory of graphical content is that it should allow for the distributed possession of content by collective entities, such as groups.
- Take the case of a musical band: they need to grasp the structure of the pieces they play together in a distributed way. So it seems plausible that whatever graphical content is, it needs to support such structures.
- It is an open question how to account for this, although I expect graphical content to be more suitable for this purpose than other forms of content.

# SUMMARY

- I am trying to examine the hypothesis that a theory of graphical content can provide a unified account of understanding, know-how, (some forms of) inference and (certain aspects of) inquiry.
- This project unifies research in epistemology, philosophy of mind and logic.

2011

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<https://fmoralesc.github.io>