BRILL | MENTIS

HISTORY OF PHILOSOPHY & LOGICAL ANALYSIS
(2023) 1–20

History of Philosophy
&
Logical Analysis
Philosophiegeschichte und
logische Analyse
brill.com/hpla

# Transcendental Knowability, Closure, Luminosity and Factivity: Reply to Stephenson

*Jan Heylen* | ORCID: 0000-0002-2809-3320
Centre for Logic and Philosophy of Science, Institute of Philosophy, KU Leuven, Leuven, Belgium
*jan.heylen@kuleuven.be*

*Felipe Morales Carbonell* | ORCID: 0000-0001-5492-0759
Departamento de Filosofía, Universidad de Chile, Maipú, Santiago, Chile
*ef.em.carbonell@gmail.com*

## Abstract

Stephenson (2022) has argued that Kant's thesis that all transcendental truths are transcendentally *a priori* knowable leads to omniscience of all transcendental truths. His arguments depend on luminosity principles and closure principles for transcendental knowability. We will argue that one pair of a luminosity and a closure principle should not be used, because the closure principle is too strong, while the other pair of a luminosity and a closure principle should not be used, because the luminosity principle is too strong. Stephenson's argument also depends on a factivity principle for transcendental knowability, which we will argue to be false.

## Keywords

transcendental knowability – luminosity – closure – factivity

There is a burgeoning literature on historical forms of antirealism (Kearns 2021; Heylen 2023b; Stephenson 2015, 2018, 2022; Kinkaid 2022). Each of these forms of antirealism comes with a knowability thesis, according to which all truths (of some kind) are knowable. There is an interest in finding out how to precisely formulate the various historical knowability theses. Much of the discussion is fuelled by a desire to know whether the various historical knowability theses are vulnerable to Church (2009)–Fitch (1963) type of *reductio*

Published with license by Brill Mentis | DOI:10.30965/26664275-BJA10073
 | ISSN: 2666-4283 (print) 2666-4275 (online)

arguments, e.g. do they entail a version of the omnscience thesis, according to which every truth (of some kind) is known? In this paper we will have a closer look at Kant's transcendental idealism and, in particular, the thesis that all transcendental truths are (a priori) knowable.

Transcendental truths are the principles of transcendental metaphysics. Kant claims that all those principles are *a priori* knowable. In other words, transcendental metaphysics is completable. Stephenson (2022) has argued that Kant's thesis that all transcendental truths are transcendentally *a priori* knowable leads to omniscience of all transcendental truths. In other words, the completability of transcendental metaphysics entails the completeness of transcendental metaphysics. Clearly, this is a Church–Fitch style objection to Kant's transcendental idealism. Stephenson's arguments depend on luminosity principles and closure principles for transcendental knowability. We will argue that one pair of a luminosity and a closure principle should not be used, because the closure principle is too strong, while the other pair of a luminosity and a closure principle should not be used, because the luminosity principle is too strong. Stephenson's argument also depends on a factivity principle for transcendental knowability, which we will argue to be false.

## 1 Transcendental Knowability

Stephenson (2022, 138) introduces a knowledge operator K and an *a priori* knowledge operator $K^a$. He clarifies that these are *explicit* knowledge operators (Stephenson 2022, 154). He also introduces a *feasible possibility* operator $\Diamonddot$, which differs from the standard possibility operator $\Diamond$.

First, there is a syntactical difference: $\Diamond$ does not need to be followed by K or $K^a$ (e.g. $\Diamond p$ is well-formed), whereas $\Diamonddot$ does need to be followed by K or $K^a$ (e.g. $\Diamonddot p$ is not well-formed). Stephenson (2022, 139) calls $\Diamonddot K$ the 'transcendental knowability' operator. Of course, $\Diamonddot K^a$ is then the transcendental *a priori* knowability operator. Alternatively, Stephenson talks about $\Diamonddot K$ expressing that

> given how things are with us now, in the current state of information, it would be feasible for someone to perform investigative procedures so as to come to know that $\phi$.
>
> STEPHENSON 2022, 139

The notion of feasibility used here is taken from Wright (2001, 60).

Second, there is a semantical difference: Williamson (1992, 67) suggests that to evaluate the truth of $\Diamonddot\phi$ at a possible world $w$ one should only consider possible worlds $w'$ where the ontic, non-epistemic truths are the same as in $w$ but the epistemic truths may differ,[1] whereas to evaluate the truth of $\Diamond\phi$ one should also consider those (accessible) possible worlds at which the ontic, non-epistemic truths may be different. Stephenson (2022, 139, n. 10) himself does not provide a semantics for the operator, but he explicitly refers to (Williamson 1992) and Tennant (2000, 2002).

Third, there is a logical difference: $\text{⟐}K\phi$ is factive (i.e. it entails $\phi$), whereas $\Diamond K\phi$ is not (i.e. it is consistent with $\neg\phi$). The factivity of transcendental knowability is the first principle stipulated by Stephenson (2022, 140):

(**FACT**) $\text{⟐}K\phi \rightarrow \phi$

Stephenson (2022, 141) provides the following textual evidence for the claim that Kant thinks that the notion of transcendental knowability is factive:[2]

> That there could be inhabitants on the moon, even though no human being has ever perceived them, must of course be conceded; but this means only that in the possible progress of experience we could encounter them; for anything is actual that stands in one context with a perception in accordance with laws of the empirical progression. Thus they are real when they stand in an empirical connection with my real consciousness, although they are not therefore real in themselves, i.e., outside this progress of experience.
>
> KANT, Kritik der reinen Vernunft (= KdrV) A493/B521[3]

Given that Kant writes 'it signifies no more than', which can be cashed out as an analytical equivalence, that $\phi$ is true is in this instance analytically equivalent to find out the truth of $\phi$ in the 'possible progress of experience' and, hence, the latter analytically entails the former. We will discuss the quote later.

1 Williamson (1992) does not use the $\text{⟐}$ symbol but rather the $\Diamond$ symbol. Moreover, in Williamson's language $\Diamond$ does not have to immediately followed by K.

2 Like Stephenson, we are using the Guyer & Wood translation (1998b), referencing the canonical A/B page numbers. Below we also provide the original German in footnotes for selected passages; for this we use Timmermann's critical edition (1998a).

3 Orig: 'Daß es Einwohner im Monde geben könne, ob sie gleich kein Mensch iemals wahrgenommen hat, muß allerdings eingeräumet werden, aber es bedeutet nur so viel: daß wir in dem möglichen Fortschritt der Erfahrung auf sie treffen könten; denn alles ist wirklich, was mit einer Wahrnehmung nach Gesetzen des empirischen Fortgangs in einem Context stehet. Sie sind also alsdenn wirklich, wenn sie mit meinem wirklichen Bewußtsein in einem empirischen Zusammenhange stehen, ob sie gleich darum nicht an sich, d.i. ausser diesem Fortschritt der Erfahrung wirklich sind.'

As a consequence of $\diamondcdot K$ being factive whereas $\Diamond K$ is not, $\Diamond K\phi$ does not entail $\diamondcdot K\phi$. But Stephenson (2022, 142) does think that transcendental (*a priori*) knowability entails metaphysical (*a priori*) knowability:[4]

(**TMK-1**) $\diamondcdot K\phi \rightarrow \Diamond K\phi$
(**TMK-2**) $\diamondcdot K^{a}\phi \rightarrow \Diamond K^{a}\phi$

Besides the factivity principle and the TMK principle, Stephenson also proposes additional principles. First, Stephenson (2022, 143) stipulates the following principle:

(**CLOS**) $(\diamondcdot K^{a}\phi \wedge \Box\,(K^{a}\phi \rightarrow K\psi)) \rightarrow \diamondcdot K\psi$

He notes that CLOS is similar to a theorem of modal logic:

$$(\Diamond\phi \wedge \Box\,(\phi \rightarrow \psi)) \rightarrow \Diamond\psi, \qquad (1)$$

although $\diamondcdot$ is not $\Diamond$, and therefore, $\diamondcdot$ is not the dual of $\Box$.[5] Despite the differences, Stephenson suggests that the intuitive plausibility of the theorem carries over to CLOS.[6]

Next, Stephenson (2022, 148) considers the following luminosity principle:

(**KK**) $\Box\,(K^{a}\phi \rightarrow K^{a}K^{a}\phi)$

But Stephenson notes that Kant would not accept KK, because he thinks that one can only have *a priori* knowledge of necessary truths and $K^{a}\phi$ is not a necessary truth, so the consequent would always be false. Moreover, for Kant

4 These labels are not found in Stephenson (2022).

5 There is an interesting question as to what the dual of $\diamondcdot K$ is. One may define $\boxdot$ as follows: $\neg\diamondcdot\neg$. However, $\diamondcdot$ is supposed to be followed immediately by K, not $\neg$. One may define the dual of $\diamondcdot K$ then as follows: $\neg\diamondcdot K\neg$. There is a further question as to whether this operator is then factive. This question is related to which knowability theses one accepts: if $\phi \rightarrow \diamondcdot K\phi$, for all $\phi$ belonging to a certain class, then also, $\neg\psi \rightarrow \diamondcdot K\neg\psi$ whence it follows that $\neg\diamondcdot K\neg\psi \rightarrow \psi$.

6 Stephenson (2022, 155) also considers the following stronger closure principle:
(CLOS*) $(\diamondcdot K\phi \wedge \Box(K\phi \rightarrow K\psi)) \rightarrow \diamondcdot K\psi$
Stephenson (2018, 3259) had used the same principle, except that he used $\Delta$ rather than $\diamondcdot$ (but the interpretation was the same) and that he had also allowed the $\Box$ operator to be read as a logical or conceptual necessity operator (in addition to its reading as a metaphysical necessity operator). However, Rosenkranz (2004) has argued that FACT and CLOS* are incompatible. Fortunately, CLOS does not entail CLOS*.

knowledge of *a priori* knowledge is empirical because it is knowledge of a mental state that is gained through inner *sense*, so again the consequent would always be false. One can avoid the latter objection by replacing $K^aK^a$ in KK by $K^eK^a$ (where $K^e$ is an empirical knowledge operator). But since empirical knowledge is obviously knowledge, one can replace $K^aK^a$ in KK by $KK^a$ instead, which yields the following weaker luminosity principle (Stephenson 2022, 149):

(**KK***) $\quad \Box (K^a\phi \rightarrow KK^a\phi)$

We will not review the case for why Kant might accept KK*, but we will just note that one common objection to luminosity principles does not gain traction (Sorensen 1988, 95): KK* does not entail

$$K^a\phi \rightarrow K \ldots KK^a\phi.$$

But another common objection does gain traction (Sorensen 1988, 95): the subject may lack the concept of knowledge. To forestall the latter objection, Williamson (2000, 95) suggests to replace $KK\phi$ by 'one is in a position to know that [$K\phi$]'. This modification also makes sense if one takes into account that acquiring knowledge of knowledge may require an act or process (e.g. using one's inner sense) and the latter may not start or it may halt before knowledge is reached. According to Stephenson (2022, 143, n. 14), the notion of transcendental knowability is intermediate in strength between the notions of being in a position to know and having the metaphysical possibility to know. Stephenson (2022, 158) uses the notion of transcendental knowability to weaken KK*:

(**KK*-**) $\quad \Box (K^a\phi \rightarrow \Diamond KK^a\phi)$

Notably, KK*- is weaker still than if Stephenson had used the notion of being in a position to know.[7]

It is important for Stephenson's purposes that KK* is a substitution instance of the second conjunct of the antecedent of CLOS. Clearly, KK* and CLOS can then be combined to derive $\Diamond KK^a\phi$ from $\Diamond K^a\phi$. Stephenson will use both principles for exactly that purpose. But if one has to abandon KK* in favour of KK*-, then one has to replace CLOS as well in order to justify the same inference. Stephenson (2022, 158) proposes the following new closure principle:

7 San (forthcoming) discusses versions of KK that are weaker still: if one knows that $\phi$, then it is possible that one knows that one knows that $\phi$, where the possibility may be interpreted as metaphysical possibility.

(**CLOS+**) $\Diamond K^a\phi \wedge \Box(K^a\phi \rightarrow \Diamond K\psi) \rightarrow \Diamond K\psi$

The new closure principle CLOS+ is a strengthening of CLOS, because $K\psi$ has been replaced by the weaker $\Diamond K\psi$. So, KK* and CLOS are a pair consisting of a stronger luminosity principle and a weaker closure principle, whereas KK*- and CLOS+ are a pair consisting of a weaker luminosity principle and a stronger closure principle.

## 2 The Completeness of Transcendental Metaphysics

By 'transcendental truths', Stephenson (2022, 136) means the truths of transcendental metaphysics, including the 'principles of nature in general' and 'the principles of corporeal nature'. As examples, Stephenson (2022, 136) mentions Kant's three 'analogies of experience' and his three 'laws of mechanics'. The three analogies of experience are the following:

> In all changes of appearances substance persists, and its quantum is neither increased not diminished in nature.
>
> KdrV B224

> All alterations occur in accordance with the law of the connection of cause and effect.
>
> KdrV B232

> All substances, insofar as they can be perceived in space as simultaneous, are in thoroughgoing interaction.
>
> KdrV B256

The three laws of mechanics are the following:

> Through all changes of corporeal nature, the total quantity of matter remains the same, neither increased nor diminished.
>
> KANT, *Metaphysische Anfangsgründe der Naturwissenschaft* (= MAN) 541; trans. FRIEDMAN

> Every change in matter has an external cause. (Every body persists in its state of rest or motion, in the same direction and with the same speed, if it is not compelled by an external cause to leave this state.)
>
> MAN 543; trans. FRIEDMAN

> In all communication of motion, action and reaction are always equal to one another.
>
> MAN 545; trans. FRIEDMAN

The list is not meant to be exhaustive, but the above examples are supposed to be paradigmatic.

Stephenson (2022, 144) introduces a transcendental truth operator T,[8] which he uses to formalize the thesis that all transcendental truths are transcendentally *a priori* knowable:

(**KPT**) $\mathrm{T}\phi \rightarrow \diamond \mathrm{K}^{a}\phi$

According to Stephenson (2022, 144), KPT says informally that:

> if $\phi$ is a transcendental truth, then given how things are with us now, in the current state of information, it would be feasible for someone to perform procedures such that they come to know a priori that $\phi$.

Stephenson (2022, 145) thinks that it is prudent to add two conditions on KPT. First, if humans did not exist, then it would not be feasible for a human to know anything. By contraposition, it would then follow from KPT that $\mathrm{T}\phi$ is false, for any sentence $\phi$. This might be an acceptable conclusion for Kant – see KdrV A383. But some might think that there could be transcendental truths even if humans were not to exist, so Stephenson prudently suggests to add as a condition that humans exist. Second, if matter did not exist, then humans could not feasibly acquire the empirical concept of matter. Again, this might simply entail that $\neg\mathrm{T}\phi$, for any sentence $\phi$ that use the concept of matter, and hence, KPT would still be true. But one might think that in that case $\mathrm{T}\phi$ is meaningless, for any sentence $\phi$ that contains a reference to matter. Then KPT would not be true. Stephenson does not make his reasoning explicit at this point, but here he also prudently suggests to add as a condition that matter exists. As a result, KPT is weakened to the following principle:

(KPT*) $\mathrm{T}\phi \rightarrow \diamond \mathrm{K}^{a}\phi$, whenever humans and matter exist

Stephenson (2022, 146) convincingly argues that Kant would accept KPT*. The two quotes that he provides are the following:

8 Stephenson erroneously talks about a transcendental truth *predicate*. A predicate should combine with a name of a sentence $\phi$, not with the sentence $\phi$ itself.

> In everything that is called metaphysics one can hope for the absolute completeness of the sciences, of such a kind one may expect in no other type of cognition. Therefore, just as in the metaphysics of nature in general, here also the completeness of the metaphysics of corporeal nature can confidently be expected. The reason is that in metaphysics the object is only considered in accordance with the general laws of thought.
>
> MAN 473; trans. FRIEDMAN

> For that this [a canon of pure reason] should be possible, indeed that such a system should not be too great in scope for us to hope to able entirely to complete it, can be assessed in advance from the fact that our object is not the nature of things, which is inexhaustible, but the understanding, which judges about the nature of things, and this in turn only in regard to its *a priori* cognition, the supply of which, since we do not need to search for it externally, cannot remain hidden from us, and in all likelihood is small enough to be completely recorded, its worth or worthlessness assessed, and subjected to a correct appraisal.
>
> KdrV A12–A13/B26–B27[9]

That 'the completeness of metaphysics [...] can confidently be expected' and that our supply of a priori cognition 'cannot remain hidden from us' do indeed support the claim that Kant thinks that transcendental metaphysics is completable.

The 'new knowability proof' of Stephenson (2022, 151–152) aims to show that the completeness of transcendental metaphysics follows from its completability. In other words, if all transcendental truths are feasibly known, then they are all known. The argument goes as follows:[10]

---

9 Orig: 'Denn daß dieses möglich sei, ja daß ein solches System von nicht gar großem Umfange sein könne, um zu hoffen, es ganz zu vollenden, läßt sich schon zum voraus daraus ermessen, daß hier nicht dier Natur der Dinge, welche unerschöpflich ist, sonder der Verstand, der über die Natur der Dinge urteilt, und auch dieser wiederum in Ansehung seiner Erkenntnis a priori den Gegenstand ausmacht, dessen Vorrat, weil wir ihn doch nicht auswärtig suchen dürfen, uns nicht verborgen bleiben kann, und allem Vermuten nach klein genug ist, um vollständig suchen dürfen, uns nicht verborgen beliben kann, und allem Vermuten nach klein genug ist, um vollständig aufgenommen, nach seinem Werte oder Unwerte beurteilt und unter richtige Schätzung gebracht zu werden.'

10 Stephenson (2022, 151, n. 24) notes that Brogaard and Salerno (2002) have given a similar argument. Indeed, Brogaard and Salerno (2002, 146) give a similar argument, with the following differences. First, the argument involves a knowability thesis for the 'basic truths' of Dummett, rather than the transcendental truths of Kant. This is a difference in content, but not so much in form. Second, they use a normal possibility operator $\Diamond$, $\Box$KK and

| | | |
|---|---|---|
| 1. | $T\phi \wedge \neg K\phi$ | assumption for *reductio* |
| 2. | $\Diamonddot K^a\phi$ | left conjunct of 1, KPT* |
| 3. | $\Box(K^a\phi \rightarrow KK^a\phi)$ | KK* |
| 4. | $\Diamonddot KK^a\phi$ | 2, 3, CLOS |
| 5. | $K^a\phi$ | 4, FACT |
| 6. | $K\phi$ | 5, knowing *a priori* is a way of knowing |
| 7. | $K\phi \wedge \neg K\phi$ | right conjunct of 1, 6 |
| 8. | $\neg(T\phi \wedge \neg K\phi)$ | |

In the above argument the combination of CLOS and KK* has been used, but the combination of KK*- and CLOS+ could also have been used: the sequence of steps is the same in both cases.

Stephenson (2022, 152) points out that the conclusion of the 'new knowability proof' holds for any knower at any time, including dogmatic, pre-critical metaphysicists and skeptics about metaphysics.[11]

## 3 Closure, Luminosity and Factivity Reconsidered

Stephenson's claim that the completability of transcendental metaphysics entails the completeness of metaphysics can be cast into doubt. First, we will have another look at the two pairs of luminosity and closure principles employed by Stephenson in his argument (section 3.1). Then we consider what the Kantian perspective on the modalities in the closure and luminosity principles might be (section 3.2). We will connect the Kantian perspective to a

FACT with $\Diamond$. Since the normal possibility operator is used, CLOS is not needed and it suffices to use (1) instead. Stephenson (2022, 151, n. 24) also notes that Williamson (1992) has also given a related argument. Williamson (1992, 67–68) has shown that, if the knowability thesis and FACT are both formulated using a normal possibility operator, then if one adds modal principle 4 ($\Diamond\Diamond\phi \rightarrow \Diamond\phi$) or KK, then one can derive that $\Diamond\phi \rightarrow \phi$. The latter is a modal collapse result, not an epistemic collapse result, i.e. an argument for the claim that knowability reduces to knowledge. Of course, if knowability is analyzed as possible knowledge, then the epistemic collapse result is just a special case of the modal collapse result.

11 Stephenson (2022, 159) observes that even Kant thinks that we are not omniscient about transcendental truths, if only because we 'may not know the answer to a question simply because it has never occurred to us, such as how synthetic a priori cognition is possible'. He refers to KdrV A764/B792, but KdrV A762/B790 seems to be the more appropriate reference. In that paragraph Kant writes the following: 'Now if someone cannot even make the possibility of these [principles of understanding] comprehensible to himself, then he may certainly begin to doubt whether they are really present in us *a priori*.'. Orig: 'Kann jemand nun die Möglichkeit derselben sich gar nicht begreiflich machen, so mag er zwar anfangs zweifeln, ob sie uns auch wirklich a priori beiwohnen'.

point made about one of the two closure principles. Finally, we will reconsider the factivity principle that is also employed in his argument (section 3.3).

### 3.1 *Closure and Luminosity*

Let us consider CLOS+ again. Suppose that $\Diamonddot K^a\phi$ and $\Box(K^a\phi \rightarrow \Diamonddot K\psi)$. By normal modal reasoning, it follows from the second conjunct that

$$\Diamond K^a\phi \rightarrow \Diamond\Diamonddot K\psi.$$

By the TMK-2 principle, it follows from the first conjunct that $\Diamond K^a\phi$. It then follows that $\Diamond\Diamonddot K\psi$. But the consequent of CLOS+ is stronger, namely $\Diamonddot K\psi$.

Let us go over it once more, but now we will use semantical reasoning. We will use Kripke models for the language. A formula $\Diamond\theta$ is true at a world $w$ if and only if there is a world $w'$ that is modally accessible from $w$ and $\theta$ is true at $w'$. A formula $\Diamonddot K\theta$ is true at a world $w$ if and only if there is a world $w'$ that is modal-epistemically accessible from $w$ and $K\theta$ is true at $w'$. For the latter type of accessibility relation, see the suggestion made by Williamson (1992, 67). Figure 1 illustrates the type of Kripke model at which the antecedent of CLOS+ is true at a world (i.e. $w_1$). The full arrow is the modal accessibility relation. The dashed arrows are the modal-epistemic accessibility relations. (We will not include epistemic accessibility relations.)

The following is both sufficient and necessary to derive CLOS+:

$$(\Diamonddot K^a\phi \wedge \Box(K^a\phi \rightarrow \Diamonddot K\psi)) \rightarrow (\Diamond\Diamonddot K\psi \rightarrow \Diamonddot K\psi). \qquad (2)$$

To be more precise, (2) and TMK-2 are jointly sufficient for CLOS+. As explained above, it follows in any normal modal logic from the antecedent of CLOS+ and TMK-2 that $\Diamond\Diamonddot K\psi$. By (2), it follows from the antecedent of CLOS+ that

$$\Diamond\Diamonddot K\psi \rightarrow \Diamonddot K\psi.$$

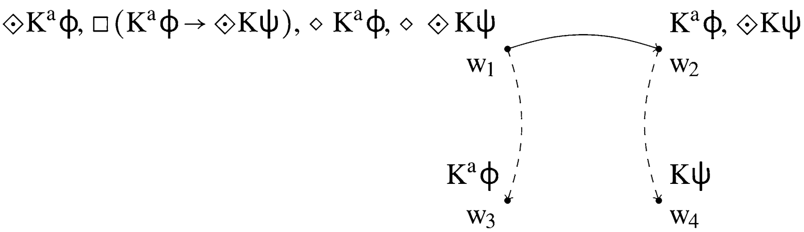


FIGURE 1 Kripke Model for the antecedent of CLOS+

By modus ponens, it follows that $\Diamond K\psi$. Moreover, (2) is also necessary for CLOS+: (2) is a tautological consequence of CLOS+. So, given TMK-2, (2) is both sufficient and necessary to derive CLOS+.

But (2) seems overly strong. The consequent of the above bridge principle states that, if it is (metaphysically) possible that it is feasibly known that $\psi$, then it is feasibly known that $\psi$. Now Stephenson is taking into account that the scope of feasibly possible knowledge is somewhat broader than what is feasibly possibly known by us at this particular moment:

> And we are also being asked to envisage 'finite extensions' of ourselves, which presumably allows some scope for development and refinement in our investigative capacities (albeit, in the current context, constrained by the essential natures of our cognitive capacities).
>
> STEPHENSON 2022, 139

But if we are talking about metaphysical possibility then much more radical developments and refinements in our investigative capacities come within reach. And then the assumption is that those more radical expansions and improvements in our investigative capacities are within the scope of feasibly possible knowledge now.

At some point in the past, humans did not have algorithms for solving mathematical problems yet, but a particular algorithm for solving some type of mathematical problem may have been in reach. So, at that point it was feasibly known that mathematical problems of that type problem could be solved. Once the algorithm was discovered, it became feasible to solve particular mathematical problems of that type. At the advent of a new era of algorithmic thinking it was metaphysically possible that it was feasibly known what the solution was to a particular mathematical problem of that type. But one may question that it was already feasibly known what the solution was before algorithms were even discovered. Similarly, the hypothetical development of large scale quantum computers means perhaps that the feasibility of some knowledge is metaphysically possible now although the knowledge itself is beyond what is feasibly known now.

While the above considerations do not sum up to a knock-down refutation of CLOS+, it does make clear that the defender of Kant's completability of metaphysics thesis has room for disagreement here. Of course, there is the version of the argument that makes use of the weaker closure principle CLOS. But the latter is used in combination with the stronger luminosity principle $KK\Box^*$. We have seen that there is also room for disagreement about the latter as well. We will now turn to how a Kantian might interpret the modalities in the closure and luminosity principles and then we will return to the point made about CLOS+.

### 3.2 *A Kantian Perspective on the Modalities in the Closure and Luminosity Principles*

The □ operator used in Stephenson's argument is intended to be the 'familiar metaphysical necessity[/possibility] operator' (Stephenson 2022, 143). However, it is not clear that this makes sense in the context of Kant's philosophy, or if it makes sense, how to make sense of it. For this reason we will briefly consider alternative readings of the □ operator used in Stephenson's argument that do make sense within the context of Kant's philosophy.

Kant recognizes and distinguishes several kinds of modality. Most importantly, Kant distinguishes between *logical modality*, which is elucidated in terms of logical consistency and is neutral to the existence of things, and *real modality*, which is ultimately grounded in the existence of things. This distinction is of central importance for Kant's critical project, and he often reminds us that it is inappropriate to derive the latter from the former: 'the unconditioned necessity of judgments […] is not an absolute necessity of things' (KdrV A593/B621).[12] Indeed, Kant warns us 'not to infer immediately from the possibility of the concept (logical possibility) to the possibility of the thing (real possibility)' (KdrV A596/B624n).[13] Even if what is logically possible is thinkable without contradiction, it may fail to be knowable.[14] Accordingly, we should we wary of principles from which real modalities can be derived from purely logical premises.

It is important to keep in mind that in the context of Stephenson's argument, the interpretation of the modality needs to be uniform throughout, otherwise the argument collapses. This means that if the □ and ◇ operators are logical in the relevant closure principle, they must also be logical in the relevant KK principle (and vice versa), and that if they are real in the closure principle, they must be real in the KK*/KK*- principles (and vice versa).

If we look at the KK*/KK*- principles, a potential problem with interpreting the □ operator as a real necessity operator emerges. The K operator is existential

12 Orig: 'Die unbedingte Notwendigkeit der Urteile aber ist nicht eine absolute Notwendigkeit der Sachen.'

13 Orig: 'Das ist eine Warnung, von der Möglichkeit der Begriffe (logische) nicht sofort auf die Möglichkeit der Dinge (reale) zu schließen'. See also KdrV BXXVI and B302.

14 For different accounts of Kant's distinction between these varieties of modality, see Stang (2016), Leech (2017), Abaci (2016, 2019) and Stephenson (forthcoming).

> Our operators implicitly quantify over subjects and times. *K*, for instance, says that 'it is known by someone at some time that'. The quantification over subjects should be understood throughout as restricted to adult human subjects (or at least to those with intellectual and sensible forms identical to our own).
>
> STEPHENSON 2022, 139, n. 9

So, the antecedents of KK*/KK*- state that it is *a priori* known by someone at some time that $\phi$. If no persons or times exist, then the antecedents of KK*/KK*- are false and, as a matter of tautological consequence, the material implications of KK*/KK*- are true. So, the material implications of KK*/KK*- do not depend on the existence of persons and times. Furthermore, the material implications of KK*/KK*- are also true in possible worlds in which persons or times do not exist. In other words, the $\Box$ operator in KK*/KK*- does not depend on the existence of persons and times. But then it is unclear in what sense the $\Box$ operator in KK*/KK*- is grounded in the existence of things, as is required for real necessity.

Alternatively, interpret the $\Box$ operator in the KK*/KK*- principles as a logical necessity operator. For the reason given above, the $\Box$ in CLOS/CLOS+ will then have to be interpreted as a logical necessity operator as well. As a matter of fact, Stephenson (2018, 3259) had allowed the $\Box$ operator in a similar closure principle (see n. 8) to be read as a logical or conceptual necessity operator. With $\Box_L$ as the logical necessity operator, the respective second conjuncts in the antecedents of CLOS/CLOS+ are the following:

$$\Box_L\,(K^a\phi \rightarrow K\psi)$$
$$\Box_L\,(K^a\phi \rightarrow \diamondsuit K\psi)$$

The consequent of both CLOS and CLOS+ has the form $\diamondsuit K\phi$. Given that knowability depends on the existence of cognizing subjects, knowability is a type of real possibility. Whenever some $\phi$ is knowable, there is a real possibility to know it. With $\diamondsuit$ as the real possibility operator, one can express the point formally as follows:

$$\diamondsuit K\phi \rightarrow \diamondsuit K\phi$$

So CLOS and CLOS+ allow us to derive real possibility. However, this may be problematic given Kant's strictures on deriving real necessity or possibility, $\nabla_R$, from logical necessity or possibility, $\nabla_L$: a real possibility, namely $\diamondsuit K\phi$, is then derivable from (among other things) a logical necessity, namely $\Box_L\,(K^a\phi \rightarrow K\psi)$ or $\Box_L\,(K^a\phi \rightarrow \diamondsuit K\psi)$. As a response, it may be pointed out that, while

Kant disallows inferring real necessity or possibility from merely logical necessity or possibility immediately, this leaves it open to allow inferring mediately, on the condition that other suitable conditions hold. So, for example, even though from $\nabla_L \phi$ it is not possible to infer $\nabla_R \psi$, there could be some $\rho$ such that $(\rho \wedge \nabla_L \phi) \rightarrow \nabla_R \psi$. This is the case with CLOS and CLOS+: the first conjunct in the antecedents of CLOS and CLOS+ also expresses a real possibility, namely $\diamondsuit K^a \phi$. In what conditions are such mixed preconditions sufficient for deriving real possibility? We raise this as an open question.

In any case, this is not sufficient to defuse all troubles with CLOS+. In this case, one needs

$$(\diamondsuit K^a \phi \wedge \Box_L (K^a \phi \rightarrow \diamondsuit K\psi)) \rightarrow (\diamondsuit_L \diamondsuit K\psi \rightarrow \diamondsuit K\psi).$$

But what is logically possible to be feasibly known goes well beyond 'some development and refinement in our investigative capacities', even more so than the extent to which what is metaphysically possible to be feasibly known goes beyond those developments and refinements. One can think without contradiction about changes to even the 'essential natures of our cognitive capacities'. To give one extreme example: what if we had infinite memory capacity?

### 3.3 *Factivity*

So far, we have considered the closure and luminosity principles. Now we will turn our attention to the factivity principle postulated by Stephenson (2022, 140):

(**FACT**) $\diamondsuit K\phi \rightarrow \phi$

Let $\phi = K\psi$. Then it follows that:

$$\diamondsuit KK\psi \rightarrow K\psi \tag{3}$$

Note that Stephenson (2022, 152) uses

$$\diamondsuit KK^a\phi \rightarrow K^a\phi \tag{4}$$

in his 'new knowability proof'.

Before we discuss the informal soundness of (3) and (4), let us consider it from a formal perspective. As mentioned before, Williamson (1992, 67) suggests that to evaluate the truth of $\diamondsuit\phi$ at a possible world $w$ only should only consider possible worlds $w'$ where the ontic, non-epistemic truths are the same as in but $w$ the epistemic truths may differ. His argument for doing so is

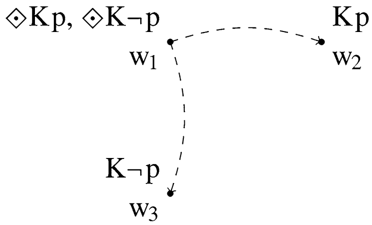


FIGURE 2
Feasible knowledge of an ontic formula and its negation

the following. Suppose that one is allowed to consider possible worlds where the ontic, non-epistemic truths may differ. For instance, consider a proposition $p$ that expresses that 'the number of tennis balls in [Williamson's] garden today, 4 July 1990, is even'. The proposition might be true and it might be false. There is an investigative procedure to find out its truth-value in either case. Then $p$ is feasibly known and $\neg p$ is feasibly known. More formally, it might be that at a world $w_2$ it is true that $p$ and at a world $w_3$ it is false that $p$ and both worlds are relevant for what is feasibly known at world $w_1$. Moreover, at world $w_2$ as a result of an investigative procedure one has come to know that $p$, and at $w_2$ also as a result of an investigative procedure one has come to know that $\neg p$. Then at world $w_1$ it is both true that $\Diamond Kp$ and $\Diamond K\neg p$. Figure 2 illustrates this. But if FACT holds for ontic, non-epistemic formulas, then it follows that at world $w_1$ it is both true that $p$ and $\neg p$. Contradiction. So, it makes sense to restrict the scope of $\Diamond K$ to worlds where the ontic, non-epistemic truths are the same.

Notably, Williamson's restriction leaves it open that the epistemic facts differ. Arguably, this is as it should be. Consider again Williamson's example, but now at all modal-epistemically relevant worlds it is true that the number of tennis balls in Williamson's garden on 4 July 1990 is even. However, at world $w_2$ the investigative procedure has been carried out and knowledge that $p$ has resulted, whereas at world $w_3$ no investigation into the truth-value of $p$ has been carried out and as a result there is no knowledge that $p$. Moreover, after some reflection on the reliability of the procedure it becomes known in $w_2$ that $p$ is known, and after a brief reflection it becomes known in $w_3$ that $p$ is not known. Figure 3 illustrates this. But if FACT holds for epistemic formulas, then it follows that at world $w_1$ it is both true that $Kp$ and $\neg Kp$. Contradiction. Of course, one might consider strengthening Williamson's restriction as follows: to evaluate the truth of $\Diamond K\phi$ at a possible world $w$ only should only consider possible worlds $w'$ where the truth-value of $\phi$ is the same at $w$ but the epistemic facts about $\phi$ may differ.

Whether the restrictions above really serve the purposes of antirealists (including transcendental idealists), is debatable. Heylen (2023a, 402) notes the following:

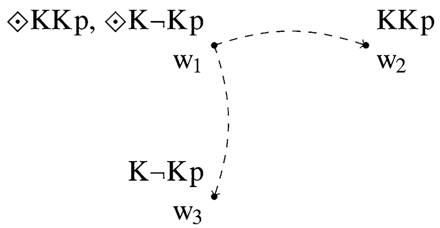


FIGURE 3
Feasible knowledge of an epistemic formula and its negation

> Given that acquiring knowledge often requires certain investigative acts (e.g., going into the garden and collecting all the tennis balls there), the non-epistemic facts are often going to be different (e.g., the investigator is going to be at a different time and place). If [⟐] ranges only over situations in which only epistemic facts differ, then those situations are outside the scope of [⟐]. However, this makes [any knowability thesis formulated with ⟐K] extremely implausible: it requires the existence of situations in which knowledge magically appears without any non-actual investigative acts that lead to that knowledge.

Likewise, one could note that acquiring second-order knowledge sometimes requires investigative acts and these can bring change in the first-order epistemic facts with them. If someone is asked whether they know that $\phi$, they may start by reflecting on $\phi$ and in the process come to know that $\phi$. Suppose that a mathematics teacher gives a test question to her students. She asks them to solve for a variable in an algebraic equation. One of the students writes down the correct answer to the question based on correct reasoning, but he has in the past frequently made errors in answering questions about mathematics and as a result he is unsure about his answer to such an extent that he does not know the answer. On submission of his answer the teacher asks him to first make sure that the answer is correct. The student returns to his seat and then remembers that one can check the correctness of the solution to in these cases by substituting the solution for the variable in the equation and then reason backwards to see if the new equation is true. It turns that it does and the student gains the needed confidence in his answer and he realizes that he knows the answer. If investigations into the presence of second-order knowledge sometimes change first-order epistemic facts and if FACT is to be maintained, then those investigations will have to be excluded as well, which seems overly restrictive.

The previous problem is about first-order knowledge that is generated when it is investigated whether one has first-order knowledge about something. The next problem is about an carrying out an investigative procedure that does not only lead to first-order knowledge but also second-order knowledge. As a general point, procedures can be combined. A recipe to make bolognese sauce can be combined with a recipe to (make and) cook spaghetti, resulting in a recipe to make spaghetti bolognese. Likewise, a procedure for solving for a variable in an algebraic equation can be combined with a procedure for checking the solution by plugging the solution back into the equation, which may be done even if one is sufficiently confident in the solution after the first part of the procedure. In many cases there is no reason to think that, if both the procedure for first-order knowledge is feasible and the the procedure for second-order knowledge is feasible, the combined procedure is not feasible. Sure, it may take longer and it may take more energy, but does not detract from its feasibility. The case where a procedure for solving an algebraic equation is combined with a procedure to check the answer is a feasible combined procedure. Likewise, one can combine procedures for solving arithmetical problems with procedures to check the answer (e.g. by using the inverse operations). In those cases the following holds:

**KK†** $\diamondsuit K^a\phi \rightarrow \diamondsuit KK^a\phi$

To be clear, it is not claimed that KK† is true in general or that it is a metaphysical truth.[15] But it is a principle that seems to be true for many arithmetical and algebraïc truths. Moreover, there are important classes of algebraic and arithmetical truths that are provable. For instance, the so-called '∃-rudimentary formulas' or the '$\Sigma^0_1$-formulas' of the language of arithmetic are provable within minimal arithmetic – see Boolos et al. (2007, 204, 207–210). For our purposes the details do not matter here. Provable truths in minimal arithmetic are paradigm examples of *a priori* knowable truths. So, one can also propose a knowability thesis:

$$\phi \rightarrow \diamondsuit K^a\phi \quad \text{for any } \Sigma^0_1\text{-formula } \phi \text{ of the language of arithmetic} \tag{5}$$

But now we have a problem. Suppose that $\phi$ is a true $\Sigma^0_1$ formula of the language of arithmetic. Hence, it follows by (5) that $\diamondsuit K^a\phi$. Clearly, KK† is true in this case: one can use a proof checker to verify the proof. It then follows that $\diamondsuit KK^a\phi$. Finally, one can use FACT to derive that $K^a\phi$. This means that anything that is *provable* in the standard axiomatic system of arithmetic is also (*a priori*) *known* to be true, not just *feasibly known* to be true. This is an epistemic

15 Given CLOS and KK*, one can derive KK†. But, as we have seen, KK* is deemed too strong. Given CLOS+ and KK*-, one can also derive KK†. But, as we have seen, CLOS+ is too strong.

collapse: knowability reduces to knowledge. There are infinitely many true $\Sigma^0_1$-formulas, because there are infinitely many true arithmetical equalities and inequalities. We do not presently have knowledge of all of them. Given the plausibility of the assumptions and the implausibility of the conclusion, we should put the blame at the feet of FACT.

Stephenson cannot retreat and restrict FACT to ontic, non-epistemic formulas, because doing so would block his 'new knowability proof'.[16] But is there is an exegetical reason for Stephenson to postulate the unrestricted factivity principle in the first place? Stephenson quoted Kant's *Kritik der reinen Vernunft* A493/B521, but that quote is about a hypothetical ontic, non-epistemic fact, namely that 'there may be inhabitants on the moon'. From the example one could generalize to a factivity principle for any ontic, non-epistemic statement. But to generalize it further and include epistemic statements as well is not motivated by Stephenson. Given the earlier comments on how the unrestricted factivity principle necessitates semantical restrictions, which negatively affect the scope of feasible knowledge, and how the unrestricted factivity principle, together with some plausible assumptions, entails that there is more *a priori* knowledge than there really is, one could have hoped for more.

## 4 Conclusion

Kant thinks that all transcendental truths are feasibly *a priori* known (KPT*). Stephenson has argued that, if one combines Kant's knowability thesis with principles of factivity, luminosity and closure, then one can derive that all transcendental truths are known by any agent at any time. But Kant does not think that all transcendental truths are known by any agent at any time. Stephenson's argument can be run with two different pairs of closure and luminosity principles, namely CLOS and KK* or CLOS+ and KK*-. We have argued that CLOS+ is too strong. It has already been argued in the literature that KK* is too strong. We have also argued that the factivity principle (FACT), in combination with

16 In contrast, Tennant (2009) retracted his earlier endorsement of an unrestricted factivity principle (Tennant 2000, 2002) and he investigated different restrictions on the factivity principle, including the restriction to ontic, non-epistemic formulas. The retraction was motivated by an argument developed by Brogaard and Salerno (2002), which made use of the KK principle, and which led to an epistemic collapse result. Williamson (1992, 65) allowed instances of the knowledge operator in his factivity principle, but he rejected the KK principle, because it would lead to a modal collapse result. See footnote 12 for on both arguments.

some plausible principles, entails that there should be *a priori* knowledge where there is not.

## Acknowledgements

Research for this article was funded by the Research Foundation – Flanders (Project Grant G088219N) and an ANID Beca FONDECYT de Postdoctorado 3220017. We would like to thank Kristine Grigoryan for her help and feedback.

## Bibliography

### *Primary Texts*

Kant, I. *Kritik Der Reinen Vernunft*. Timmermann, J. (ed.), 1998. Hamburg: Meiner. (= KdrV)

Kant, I. *The Critique of Pure Reason*. Guyer, P. & Wood, A. (eds., trs.), 1998. Cambridge: Cambridge University Press.

Kant, I. *Metaphysical Foundations of Natural Science*. Friedman, M. (tr.), 2004. Cambridge: Cambridge University Press. (= MAN)

Stephenson, A. 2022. Transcendental Knowability and a Priori Luminosity. *History of Philosophy & Logical Analysis* 25(1), 134–162.

### *Secondary Literature*

Abaci, U. 2016. The Coextensiveness Thesis and Kant's Modal Agnosticism in the 'Postulates'. *European Journal of Philosophy* 24(1), 129–158.

Abaci, U. 2019. *Kant's Revolutionary Theory of Modality*. Oxford: Oxford University Press.

Boolos, G., Burgess, J., Richard, P., & Jeffrey, C. 2007. *Computability and Logic*. Cambridge University Press.

Brogaard, B., & Salerno, J. 2002. Clues to the Paradoxes of Knowability: Reply to Dummett and Tennant. *Analysis* 62(2), 143–150.

Church, A. 2009. Referee Reports on Fitch's 'Definition of Value'. In: Salerno, J. 9ed.), *New Essays on the Knowability Paradox*. Oxford: Oxford University Press, 13–20.

Fitch, F. 1963. A Logical Analysis of Some Value Concepts. *Journal of Symbolic Logic* 28(2), 135–142.

Heylen, J. 2023a. Anti-Realism and Modal-Epistemic Collapse: Reply to Marton. *Erkenntnis* 88, 397–408.

Heylen, J. 2023b. Confusion in the Bishop's Church. *Philosophia*, forthcoming, 1–11.

Kearns, S. 2021. The Bishop's Church: Berkeley's Master Argument and the Paradox of Knowability. *Canadian Journal of Philosophy* 51(3), 175–190.

Kinkaid, J. 2022. Phenomenology, Anti-Realism, and the Knowability Paradox. *European Journal of Philosophy* 30(3), 1010–1027.

Leech, J. 2017. Kant's Material Condition of Real Possibility. In: Sinclair, M. (ed), *The Actual and the Possible: Modality and Metaphysics in Modern Philosophy*. Oxford: Oxford University Press, 94–116.

Rosenkranz, S. 2004. Fitch Back in Action Again? *Analysis* 64(1), 67–71.

San, W. K. forthcoming. KK, Knowledge, Knowability. *Mind*, forthcoming.

Sorensen, R A. 1988. *Blindspots*. Oxford, England: Oxford University Press.

Stang, N. F. 2016. *Kant's Modal Metaphysics*. Oxford: Oxford University Press.

Stephenson, A. forthcoming. Existence and Modality in Kant: Lessons from Barcan. *The Philosophical Review*, forthcoming.

Stephenson, A. 2015. Kant, the Paradox of Knowability, and the Meaning of 'Experience'. *Philosophers' Imprint* 15 (27), 1–19.

Stephenson, A. 2018. How to Solve the Knowability Paradox with Transcendental Epistemology. *Synthese* 198(Suppl 13), 3253–3278.

Stephenson, A. 2022. Transcendental Knowability and a Priori Luminosity. *History of Philosophy & Logical Analysis* 25(1), 134–162.

Tennant, N. 2000. Anti-Realist Aporias. *Mind* 109(436), 825–854.

Tennant, N. 2002. Victor Vanquished. *Analysis* 62(2), 135–142.

Tennant, N. 2009. Revamping the Restriction Strategy. In: Salerno, J. (ed), *New Essays on the Knowability Paradox*. Oxford: Oxford University Press, 223–238.

Williamson, T. 1992. On Intuitionistic Modal Epistemic Logic. *Journal of Philosophical Logic* 21(1), 63–89.

Williamson, T. 2000. *Knowledge and Its Limits*. Oxford University Press.

Wright, C. 2001. On Being in a Quandary. Relativism Vagueness Logical Revisionism. *Mind* 110(1), 45–98.