

Initial state and control path

$$Y_s, \quad X: [s, t] \rightarrow \mathbb{R}^d$$

Action of  $f$  on  
signature of  $X$

Taylor method

$$Y_t^{Taylor} := Y_s + \tilde{f}(Y_s) \text{Sig}_{s,t}^N(X)$$

$\approx$

Log-ODE method

$$Y_t^{Log} := z(1)$$

Action of  $f$  on  
log-signature of  $X$

$$\begin{aligned} z' &= \tilde{f}(z) \text{LogSig}_{s,t}^N(X) \\ z(0) &= Y_s \end{aligned}$$

Solve ODE on  $[0,1]$