UNIVERSIDAD NACIONAL DE INGENIERIA
FACULTAD DE CIENCIAS

Tema:
Simulación de MonteCarlo

Apellidos: Moreno Vera
Nombres: Felipe Adrian
Código: 20120354I

Curso: Modelamiento y Simulación
Codigo Curso: CC562

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1. Sean Sea $U_i \tilde{} U\langle 0,1\rangle$ para $i \geq 1$, y si se define a

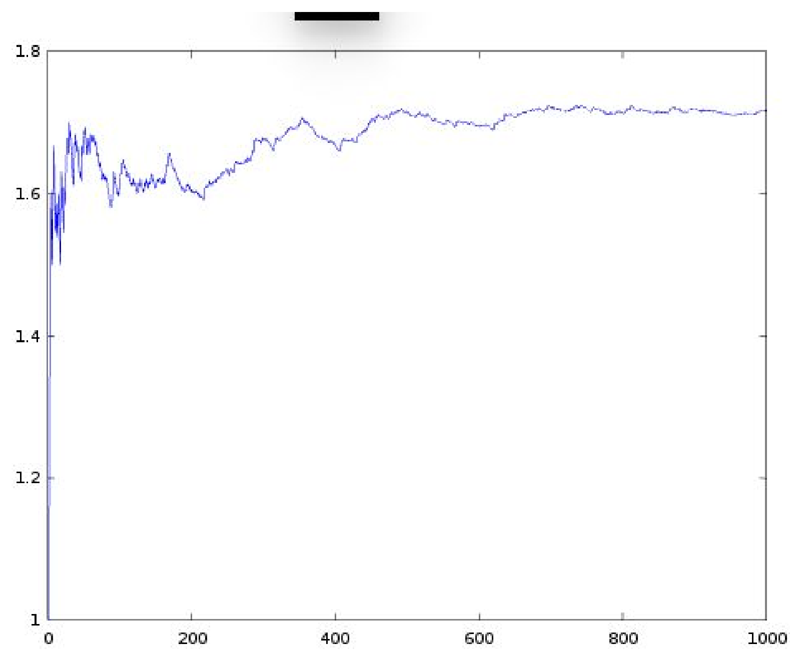
$$N = Min\{n : \sum_{i=1}^{n} U_i > 1\}$$

Estimar el $E[N]$ mediante simulación Monte Carlo.

Sol:

En MATLAB, Numericamente:

```
minNum=zeros(1000,1);n=1:length(minNum);n=n';sumatoria=zeros(length(minNum),1);

for m = 1:length(minNum)
    sum=0.0;i=0;x=rand();
    while (sum+x)<1
      i=i+1;sum=sum+x;x=rand();
    end
    sumatoria(m)=sum;
    minNum(m)=i;
end

E_N=cumsum(minNum)./n;
disp('----------------------');
disp('4.1 Determinar E[N]:');
fprintf('E[N]= %f \n',E_N(length(E_N)));
plot(n,E_N);
disp('----------------------');
ans =
----------------------
Determinar E[N]:
E[N]= 1.712000
----------------------
```



2. Aproximar la siguiente integral

$$\int_{-2}^{2} e^{x+x^2}\, dx$$

Comparar con su valor exacto.

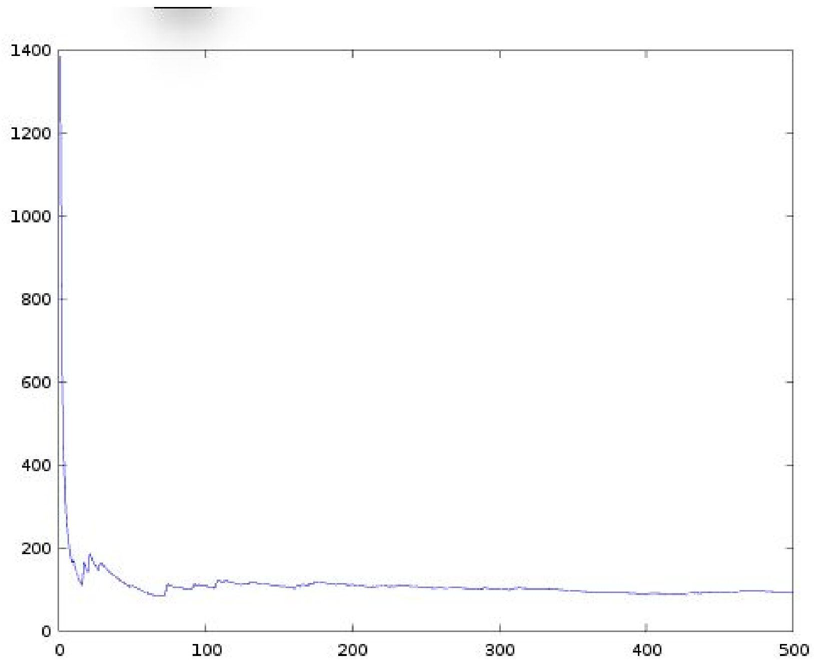
Sol:

En MATLAB, Numericamente:

Se hace el cambio por montecarlo:

$y = \frac{x+2}{4}$, entonces: $x = 4y - 2$, por lo que $e^{x+x^2} = e^{(4y-2)+(4y-2)^2}$ = $e^{16y^2-12y+2}$ y además se debe que dx = 4dy, entonces la integral queda: $4\int_0^1 e^{16y^2-12y+2} dy$

```
y=rand(10000,1);
n=1:length(y);
n=n';
int_aprox=4*(cumsum(exp(2)*(1./exp(12*y)).*exp(16.*y.^(2)))./n;
plot(n,int_aprox);
int_aprox(length(y))
ans=93.864
```



Valor Exacto, Analiticamente:

Usando el matlab para calcular el valor exacto (pues a mano sale con la función error de taylor)

```
f=@(x) exp(x+x*x);
 quad(f,-2,2)
ans =  93.163

h=@(y) 4*exp(16*y*y-12*y+2);
quad(h,0,1)
ans =  93.163
```

Error = 93.864 - 93.163 = 0.70100

3. Sea $U \sim U(0,1)$. Utilizar simulación para aproximar lo siguiente

Se tiene que la convarianza es:
$Cov(U,A) = E[UA] - E[U]E[A]$

3.1. $Cov(U,\sqrt{1-U^2})$

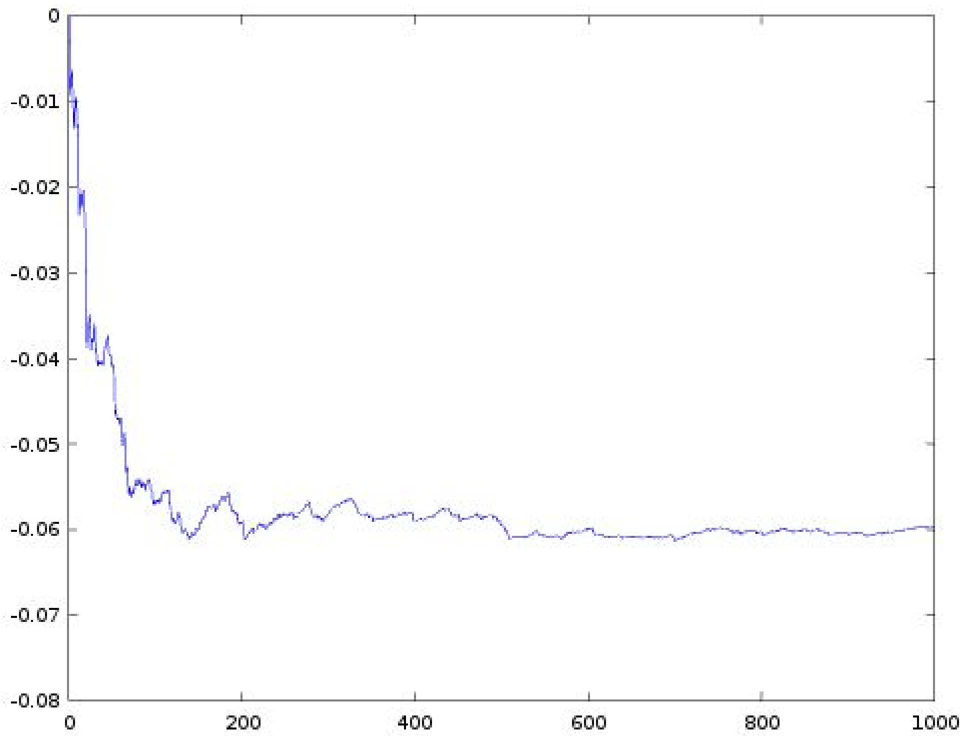
3.2. $Cov(U^2,\sqrt{1-U^2})$

Sol:

En MATLAB, Numericamente:

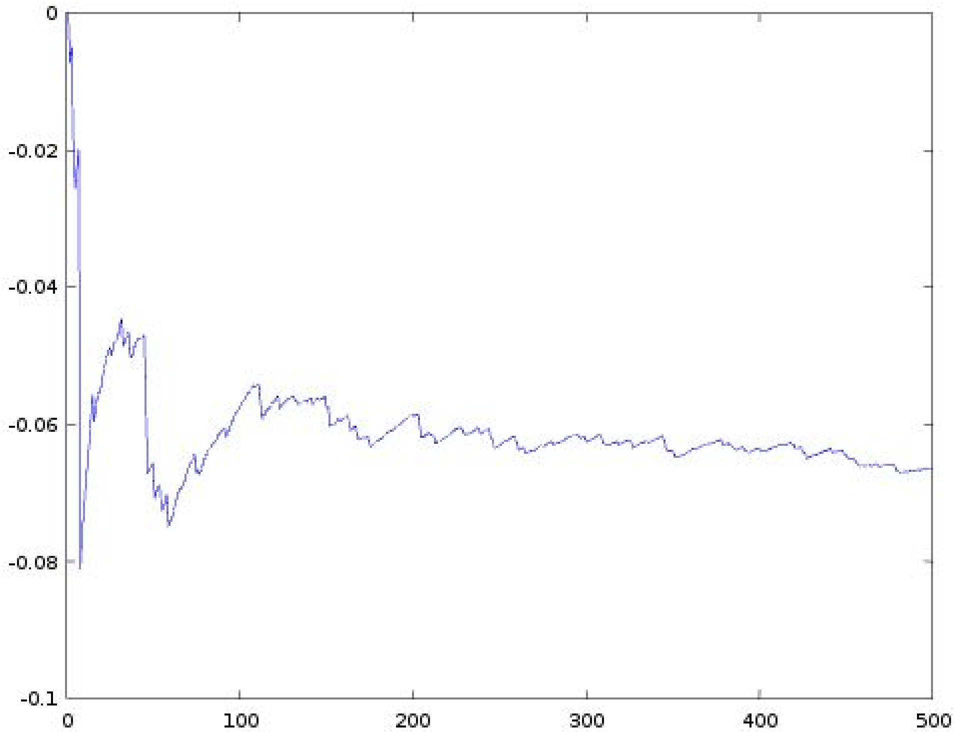
3.1

```
U=rand(1000,1);
n=1:length(U);
n=n';
aprox=((cumsum(U.*sqrt(1-U.*U)))./n)-((cumsum(U))./n).*((cumsum(sqrt(1-U.*U)))./n);
plot(n,aprox);
aprox(length(U))
ans = -0.059524
```



3.2

```
U=rand(1000,1);
n=1:length(U);n=n';
aprox=((cumsum(U.*U.*sqrt(1-U.*U)))./n)-((cumsum(U.*U))./n).*((cumsum(sqrt(1-U.*U))
)./n);
plot(n,aprox)
aprox(length(U))
ans = -0.066295
```



4. Sean Sea $U_i \tilde{} U\langle 0,1\rangle$ para $i \geq 1$, y si se define a

$$N = Max\{n : \prod_{i=o}^{n} U_i \geq e^{-3}\}$$

Donde $\prod_{i=1}^{0} U_i = 1$

4.1. Determinar $E[N]$

4.2. Determinar $P[N = i]$ para $i = 0, 1, 2, \ldots, 6$ por simulación.

Sol:

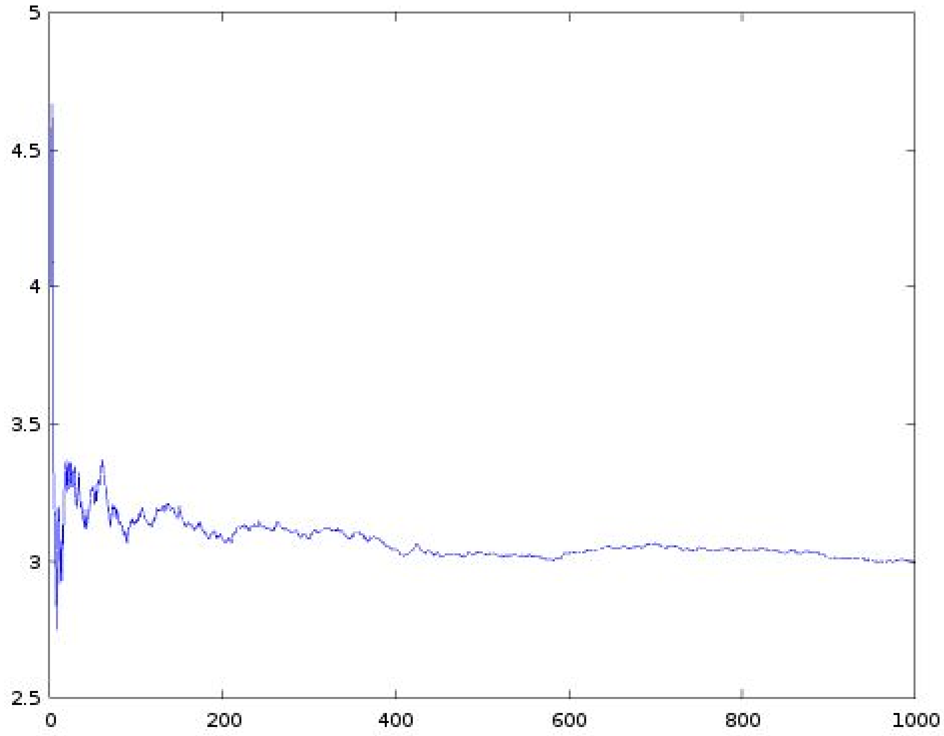
En MATLAB, Numericamente:

```
maxNum=zeros(1000,1);
n=1:length(maxNum);
n=n';
productoria=zeros(1000,1);

for m = 1:1000
    prod=1.0;
    i=0;
    x=rand();
    while (prod*x)>=exp(-3)
    i=i+1;
    prod=prod*x;
    x=rand();
    end
    productoria(m)=prod;
    maxNum(m)=i;
end

E_N=cumsum(maxNum)./n;
P_0=cumsum(maxNum==0)./n;
P_1=cumsum(maxNum==1)./n;
P_2=cumsum(maxNum==2)./n;
P_3=cumsum(maxNum==3)./n;
P_4=cumsum(maxNum==4)./n;
P_5=cumsum(maxNum==5)./n;
P_6=cumsum(maxNum==6)./n;
disp('----------------------');
disp('4.1 Determinar E[N]:');
fprintf('E[N]= %f \n',E_N(length(E_N)));
plot(n,E_N);
disp('----------------------');
disp('4.2 Determinar P[N] para i = 0,1, ..., 6:');
fprintf('P[0]= %f \n',P_0(length(P_0)));
plot(n,P_0);
fprintf('P[1]= %f \n',P_1(length(P_1)));
plot(n,P_1);
fprintf('P[2]= %f \n',P_2(length(P_2)));
plot(n,P_2);
fprintf('P[3]= %f \n',P_3(length(P_3)));
plot(n,P_3);
```

```
fprintf('P[4]= %f \n',P_4(length(P_4)));
plot(n,P_4);
fprintf('P[5]= %f \n',P_5(length(P_5)));
plot(n,P_5);
fprintf('P[6]= %f \n',P_6(length(P_6)));
plot(n,P_6);
disp('-----------------------');
ans =
-----------------------
```

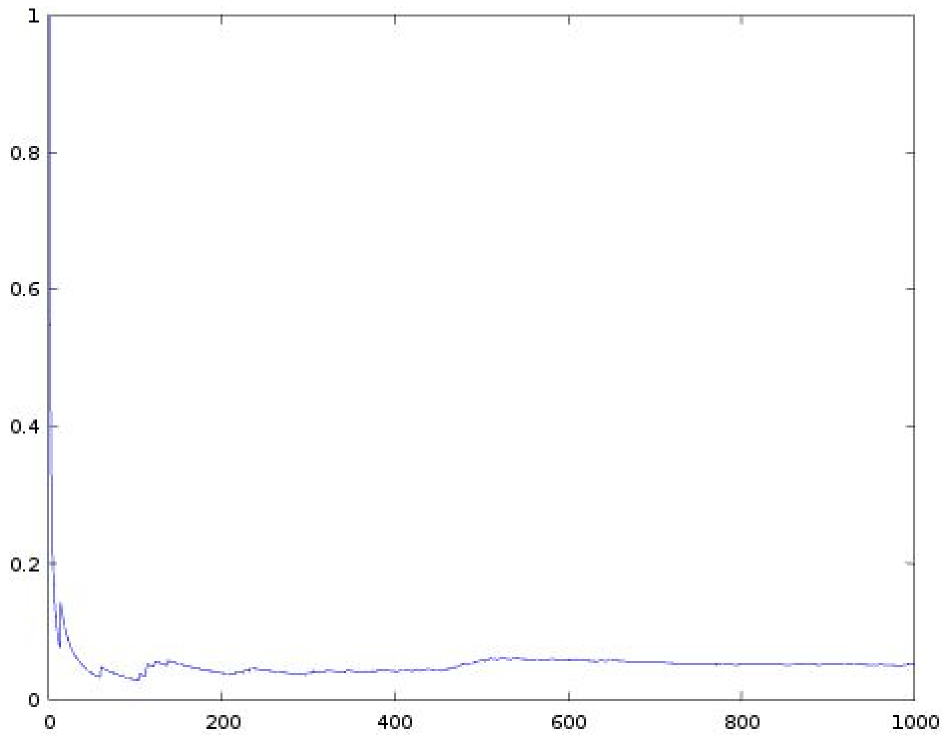
4.1 Determinar E[N]:

E[N]= 2.937000

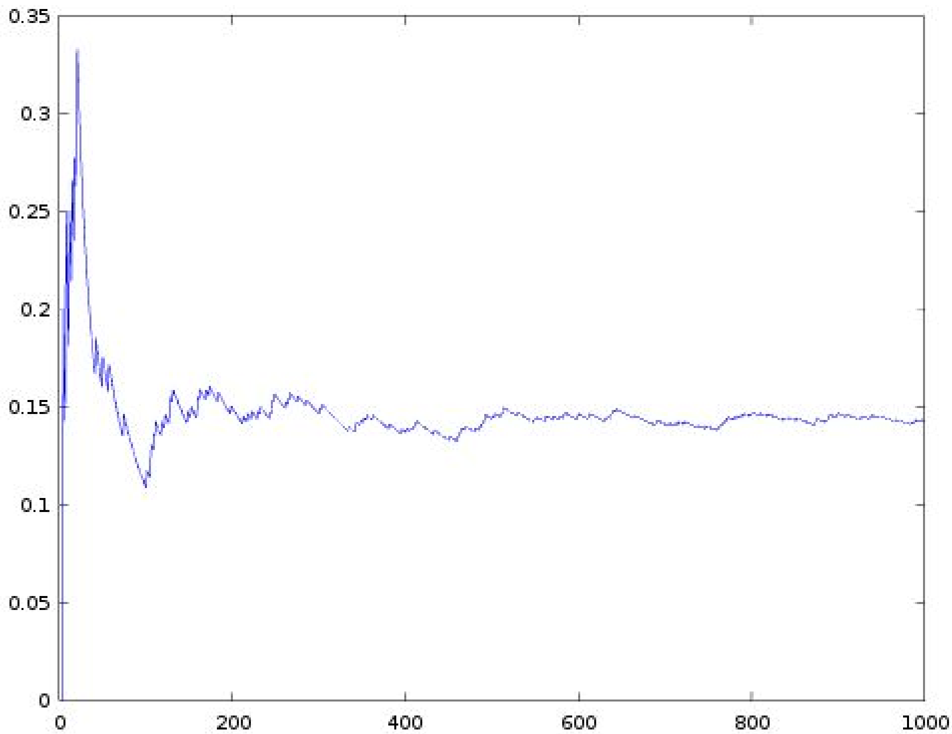


4.2 Determinar P[N] para i = 0,1, ..., 6:

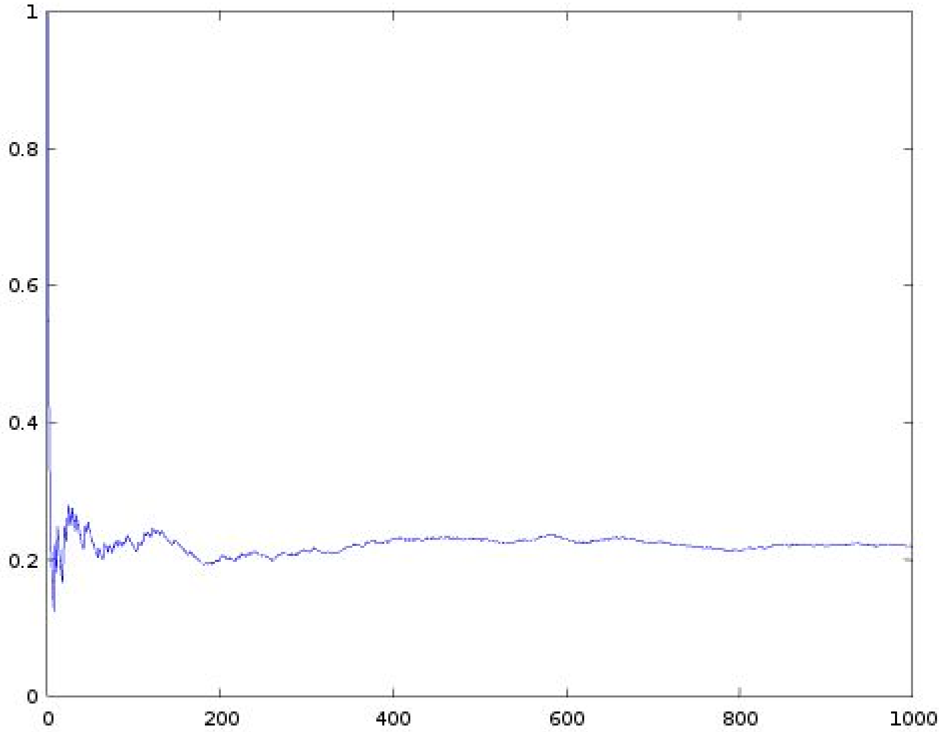
P[0]= 0.052000



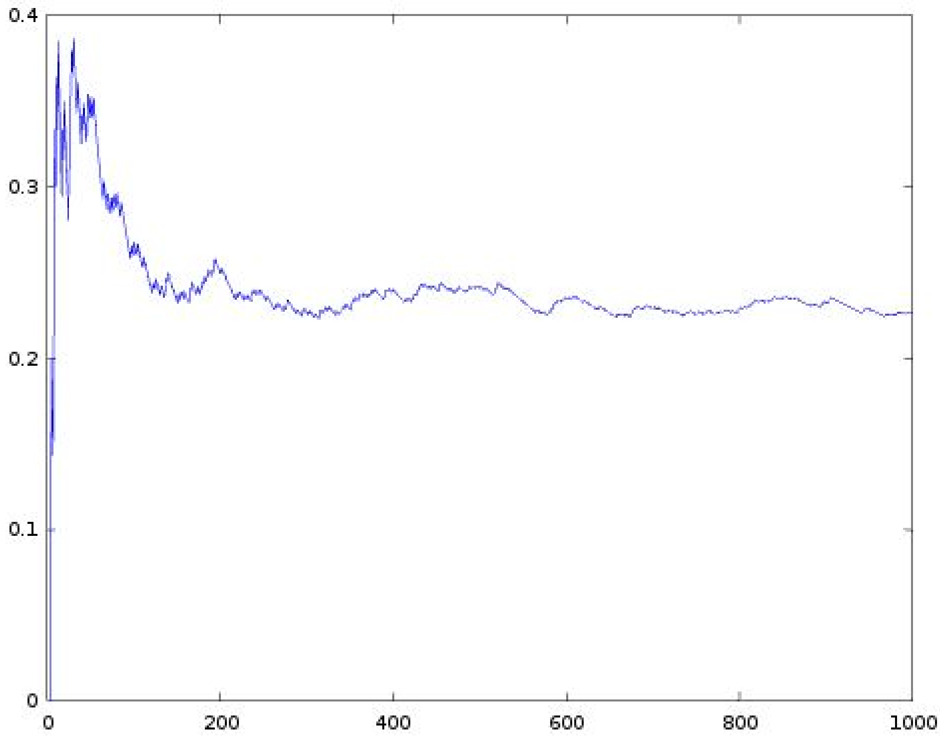
P[1]= 0.153000



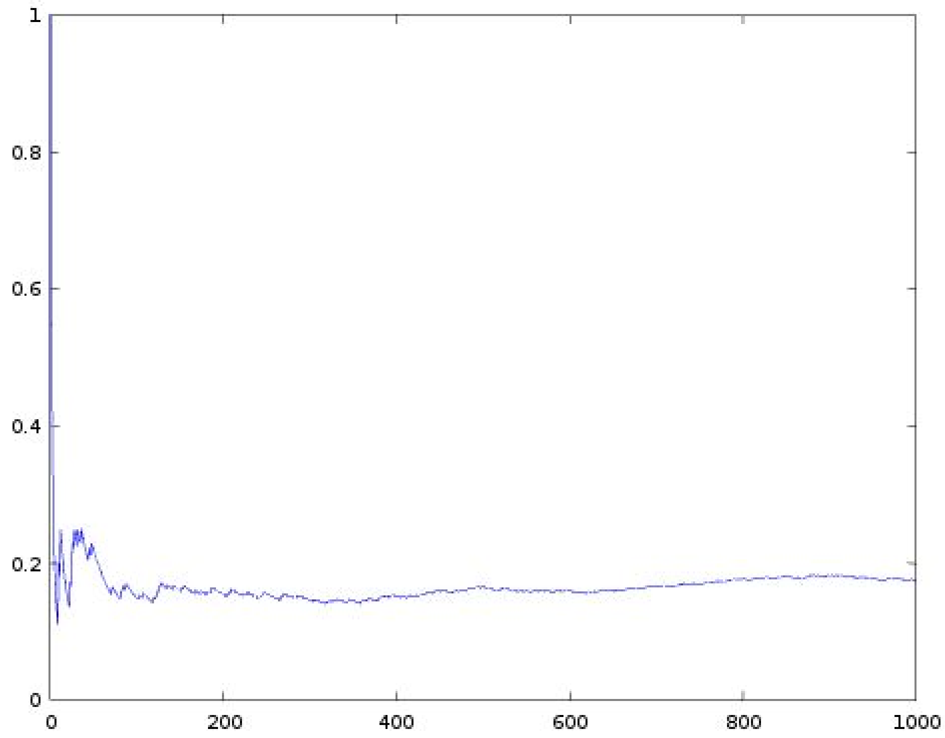
P[2]= 0.231000



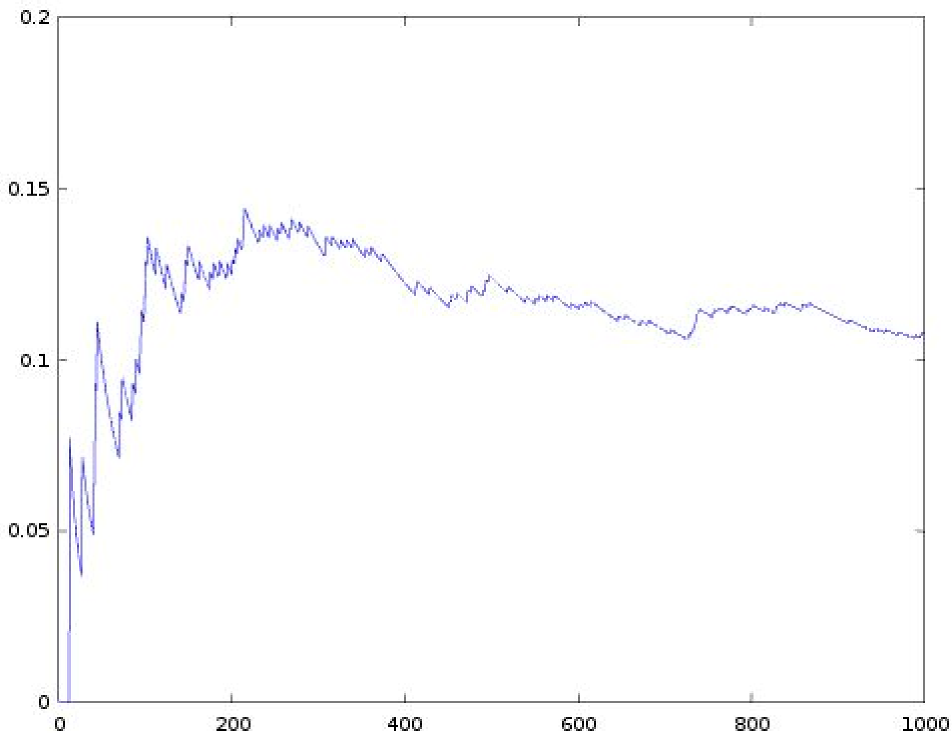
P[3]= 0.226000



P[4]= 0.163000



P[5]= 0.110000



P[6]= 0.036000