Prosumer Markets: A Unified Formulation

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***Abstract*—The increasing share of more proactive actors in the electricity market supports the proposal of novel schemes to accommodate a more decentralized paradigm to power system and electricity market operation. The resulting prosumer markets may be identified as belonging to various classes such as prosumer grid integration, peer-to-peer models and prosumer community groups. We show here that they all can be modelled within a unified peer-to-peer market model but with different communication structures. Consequently, we profit of this unified formulation to compare these structures in terms of efficiency and convergence speed, as well as by generally looking at the effect of sparsification of those communication structures on market outcomes and convergence speed. The open-source simulation platform developed for that purpose may be readily used by others to study and analyze prosumer markets.**

***Index Terms*—Electricity markets, Economic dispatch, Prosumer markets, Decentralized optimization.**

I. Introduction

Resource allocation in electricity markets is traditionally solved with a centralized clearing mechanism, where agents participate in a pool market. The efficiency of this organization is challenged as Distributed Energy Resources (DERs), as well as demand response mechanisms and distributed storage, introduce new small-sized agents that can both net generate or consume energy: the so-called prosumers. As of current practice, small-sized prosumers are managed at retail level, since existing mechanisms, such as real time markets for DERs and demand response [1], require thresholds on agents' size and often a strict dichotomy between consumers and producers. Extending these existing mechanisms to small-sized prosumers is not an option, since the amount of communications and data required can quickly become too large to be handled efficiently by a central agent.

The aforementioned reasons, as long as the need of privacy, justify the need for adapting electricity market designs to more decentralized organizations. Decentralized electricity markets were first introduced by Wu and Varaiya as coordinated multilateral transactions [2], now better known as peer-to-peer (P2P) trades when solely involving two parties. In this framework, each market participant directly negotiates with a set of trading partners with the objective of minimizing their energy procurement costs. In view of large scale applications, regulation and other economic arguments – such as licensing and certification, data and employment regulation – are fundamental but still open topics [3]. Depending on the overall objectives and potential regulation, alternative organizations may be considered. An attempt of categorizing some of the possible organization layouts of decentralized electricity markets is proposed in [4], where additionally to a P2P market, the authors identify two other market organizations. In the first one, prosumers are connected to microgrids which can either be isolated or interconnected; while in the second one, prosumers are organized in groups, namely energy communities, in which resources, not necessarily geographically located close to each other, are managed in small pools. Other recent literature proposes P2P energy-trading markets either to incentivize prosumers to form virtual power plants [5] or for microgrid management [6]. Each organization has been investigated independently and through different market mechanisms. On one hand, P2P energy trading is proposed in the form of matching contracts [7], consensus-based optimization [8], microgrid management [9] and control systems [10]. On the other hand, community-based mechanisms are designed as control strategies [11], coalition games [12] and distributed optimization [13].

In this paper, we propose a market formulation, based on distributed optimization techniques, that generalizes decentralized electricity markets. We formulate the market clearing problem such that different layouts can be simulated only by modifying the communication links among agents and not the underlying negotiation algorithm. We implement this unified formulation of decentralized electricity markets as an open-source simulation platform in the form of a web app, where different market layouts can be quickly designed, shared and simulated.

The paper proceeds in the following steps. First, the P2P formulation, along with its associated decentralized negotiation mechanism are expressed in Section II. Section III describes the structure and functioning of the open-source simulation platform. Simulations results are then analyzed in Section IV to investigate the impact of different organization layouts on market equilibrium and convergence speed. Finally, Section V gathers conclusions and perspectives regarding future works.

II. Prosumer Markets Model

We conceive decentralized electricity markets as consensus-based decentralized optimization of agents' energy procurement on a communication graph. As displayed in Fig. 1, the nodes of the graph represent market participants while edges are placed to connect two agents who can trade energy

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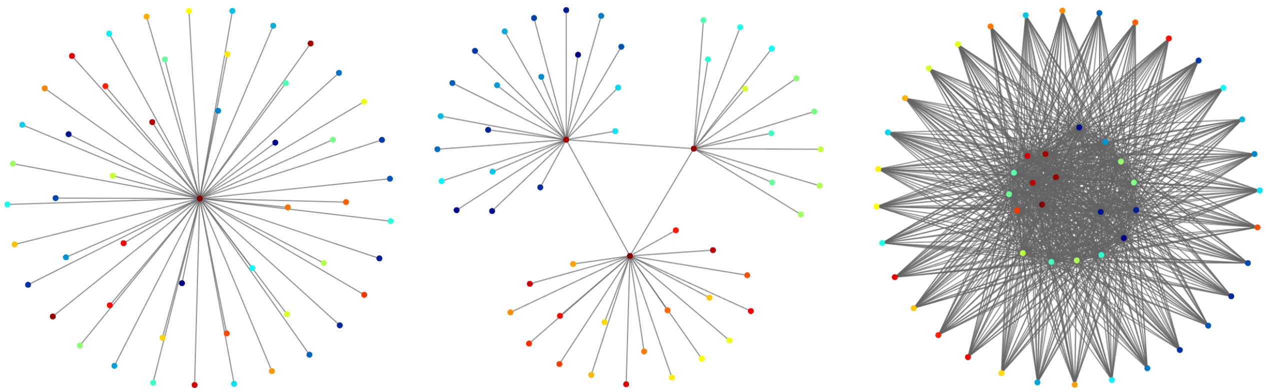

(a) Pool (b) Connected communities (c) Peer-to-peer

Fig. 1: Decentralized electricity market layouts

between each other. In this framework, one can interpret a pool market as a radial decentralized market (a). The market is cleared in a decentralized fashion, where agents do not disclose their assets' information but negotiate with a central agent, i.e. market maker, to minimize their energy costs. In the same way, energy communities (b) can be seen as smaller pools, where market makers, or community managers as defined in [13], operate as interfaces with the outside world. Communities can operate in isolated mode, mimicking stand-alone microgrid operations, or connected with other market agents. At the extreme, P2P layouts (c) can be seen as singleton communities connected to any, or a subset, of market participants.

In this section, we first describe the problem formulation of a decentralized electricity market as a generalization of our previous work in [13] and [14]. We then present the associated decentralized negotiation mechanism, based on the consensus version of the Alternating Direction Method of Multipliers (ADMM).

A. Problem formulation

Let Ω be the set of market participants, in a classical economic dispatch problem the goal of each agent is to minimize its energy costs (or maximize its payoffs). The costs of each participant $i \in \Omega$ are calculated as the sum over all its available assets $a \in \mathcal{A}_i$ of the respective cost functions f_i^a as in (1a). Each power set-point p_i^a is constrained to a feasible set $\mathscr{P}_i^a$ and the net generation of each agent is described as $p_i = \sum_{a \in \mathcal{A}_i} p_i^a$ (positive when generated and negative when consumed). Considering multi-bilateral trades calls for a split of this net energy (note we indifferently refer to power and energy as we assume an hourly dispatch) into a set of multiple bilateral trades t_{ij} as in [8]. Hence, power balance between net generation and total traded energy is enforced by (1c) for each agent i, with the associated dual variable μ_i representing the perceived energy price. Moreover, for a bilateral trade to be valid it needs to be reciprocal both in quantity and price. Reciprocity of trades in quantity, noted $\mathbf{T} = (t_{ij})_{i,j}$, is enforced by constraint (1b). On the other hand, the reciprocity of trade prices, noted $\mathbf{\Lambda} = (\lambda_{ij})_{i,j}$, i.e. dual variables of (1b), is verified at optimality and implicitly granted by the solving algorithm presented in Section II-B.

In this paper market participants are assumed rational [15] and non-strategic. In other words, agents are assumed to always take decisions beneficial for themselves but can neither anticipate actions nor reactions of other agents. The proposed formulation is for a single time step and deterministic market, but it can readily be extended to multiple time units, with temporally binding constraints, and uncertainty, with a scenario based approach. The overall procurement for all agents can be written as

$$
\begin{aligned}
&\min_{\mathbf{T},P} \sum_{i\in\Omega} \left[\sum_{a\in\mathcal{A}_i} f_i^a(p_i^a) + \sum_{j\in\omega_i} \gamma_{ij}|t_{ij}| \right] && && \text{(1a)}\\
&\text{s.t. } \mathbf{T} = -\mathbf{T}^\intercal && [\mathbf{\Lambda}] && \text{(1b)}\\
&\qquad \sum\nolimits_{a\in\mathcal{A}_i} p_i^a = \sum\nolimits_{j\in\omega_i} t_{ij} && [\mu_i]\ i\in\Omega && \text{(1c)}\\
&\qquad p_i^a \in \mathscr{P}_i^a && a\in\mathcal{A}_i,\ i\in\Omega && \text{(1d)}
\end{aligned}
$$

where $P = (p_i^a)_{i,a}$ lists the power set-points of all assets involved in the market. Operators $\cdot^\intercal$ and $|\cdot|$ respectively denotes the matrix transpose and the absolute value. Note that in the case of multi-bilateral trades one can add a specific cost on each trade. For example, agent i could decide to penalize each of its partners j with a coefficient γ_{ij} ,in accord to the concept of product differentiation as in [8]. In other words, these coefficients allows to express preferences (the smaller the more favorable the associated trade) and to model transaction costs. The impact of these coefficients on trade prices as well as other market properties, such as market efficient, incentive compatibility, cost recovery and revenue adequacy, are analyzed in Appendix A.

B. Decentralized solving algorithm

Several algorithms exist for solving consensus problems in a decentralized fashion. For the sake of this paper, we use ADMM consensus techniques to find market equilibrium. This algorithm is preferred over the Consensus + Innovation algorithm for its convergence speed and its resilience to asynchronous behaviours [16]. However, the architecture of the simulation platform is such that different solving algorithms can be easily integrated, in case users wanted to compare performances.

We decompose the optimization problem (1), following [17], into subproblems at agent level as follows. Let's define a global variable $\mathbf{C} = (\mathbf{T} - \mathbf{T}^\mathsf{T})/2$, as the average between the energy trade proposed from agent i to agent j and from agent j to agent i (note that these trades are equal at optimality). The variable $\mathbf{C}$ is skew-symmetric, $\mathbf{C} = -\mathbf{C}^\mathsf{T}$, and verifies

$$\left(\mathbf{C} - \mathbf{C}^\mathsf{T}\right)/2 = \mathbf{T}. \tag{2}$$

Therefore, problem (1) can equivalently be written with (2) instead of (1b) as pointed in [14]. Yet, by considering this complicating constraint, agents seek consensus between their local trade values, lines of $\mathbf{T}$, and their estimate of what the optimal trades will be, i.e. $\mathbf{C}$. Thus, as detailed in Appendix B, this simple change of variables leads to a fully decentralized negotiation algorithm composed of the two following iterative steps

$$\begin{aligned}
\begin{pmatrix} P_i \\ T_i \end{pmatrix}^{k+1} = \underset{P_i,T_i}{\operatorname{argmin}} \quad & \sum_{a\in\mathcal{A}_i} f_i^a(p_i^a) + \sum_{j\in\omega_i} \Bigg[\gamma_{ij}|t_{ij}| \\
& + \frac{\rho}{2}\left(\frac{t_{ij}^k - t_{ji}^k}{2} - t_{ij} + \lambda_{ij}^k/\rho\right)^2\Bigg] \\
\text{s.t.} \quad & \textstyle\sum_{a\in\mathcal{A}_i} p_i^a = \sum_{j\in\omega_i} t_{ij} \\
& p_i^a \in \mathscr{P}_i^a \qquad\qquad a \in \mathcal{A}_i
\end{aligned} \tag{3a}$$

$$\lambda_{ij}^{k+1} = \lambda_{ij}^k - \rho(t_{ij}^{k+1} + t_{ji}^{k+1})/2 \tag{3b}$$

where $\rho > 0$ is the penalty factor. Vectors $P_i = (p_i^a)_{a\in\mathcal{A}_i}$ and $T_i = (t_{ij})_{j\in\omega_i}$ are agent i's power set-points and trade proposals, respectively. One can note that (3b) leads to reciprocal prices $\lambda_{ij} = \lambda_{ji}$ when $\lambda_{ij}^0 = \lambda_{ji}^0$, $\forall ij$. The stopping criteria of (3) can be written as

$$\sum\nolimits_{i\in\Omega} r_i^{k+1} \leqslant {\epsilon^{\text{pri}}}^2 \quad \text{and} \quad \sum\nolimits_{i\in\Omega} s_i^{k+1} \leqslant {\epsilon^{\text{dual}}}^2 \tag{4}$$

where ϵ^{pri} and ϵ^{dual} are respectively primal and dual feasibility tolerances. Local primal and dual residuals are given by

$$r_i^{k+1} = \sum\nolimits_{j\in\omega_i} \left(t_{ij}^{k+1} + t_{ji}^{k+1}\right)^2 \tag{5a}$$

$$s_i^{k+1} = \sum\nolimits_{j\in\omega_i} \left(t_{ij}^{k+1} - t_{ij}^{k}\right)^2 \tag{5b}$$

The overall negotiation algorithm is decentralized such that each agent $i \in \Omega$ concurrently executes the following steps. First, agent i updates power set-points $(p_i^a)_a$ and trade proposals $(t_{ij})_j$ using local optimization (3a). Secondly, trade proposals are sent individually to each of its partners $j \in \omega_i$. Once all counter proposals $(t_{ji})_j$ are received the agent can update trading prices $(\lambda_{ij})_j$ and local residuals (r_i, s_i) with (3b) and (5), respectively. Finally, after broadcasting its local residuals and receiving all other local residuals $(r_l, s_l)_{l\in\Omega\setminus\{i\}}$, agent i checks global stopping criteria (4). This process is repeated as long as the global stopping criteria are not satisfied.

III. Simulation Platform

To facilitate design and simulation of decentralized electricity markets, we developed an open-source platform which is accessible at https://gitlab.com/fmoret/P2PApp.git. This platform takes the form of a web based application. Developed in Python, the platform is bound to evolve and be complemented with additional features proposed by new contributors. The platform is currently composed of two modules. The simulation module allows to design and simulate test cases, while the test case generator creates synthetic test cases.

A. Simulation module

This primary module is split in two tabs. The first tab is dedicated to the design of the communication graph among market participants. Accordingly with Section II each node of the market graph can either represent a community manager or an agent with multiple assets. Assets are currently limited to quadratic cost functions with lower and upper power boundaries. In addition, there are two types of links between graph nodes such that a community manager can differentiate between community members and outer trading partners. Designed market graphs can be saved, in a specific *.pyp2p* file format, such that they can be shared or reused. Moreover, the editor supports *.csv* files to facilitate compatibility with other programs. Once defined the market layout, users can run the simulation in the second tab. Naturally, essential parameters such as the maximum number of iterations, the penalty factor and primal/dual tolerances can be adjusted. Two additional global commission fees can be applied from this tab. The first type is limited to P2P trades, i.e. outside of communities, while the second uniformly impacts power exchanges within communities, i.e. between managers and their community members. The decentralized negotiation mechanism of Section II-B is currently the sole algorithm implemented. In spite of that, we encourage users to contribute to the development by integrating other solving methods.

B. Test case generator

The second module of the platform is devoted to the generation of one or multiple test cases. The cases can be composed by consumers, producers and four types of prosumers. Prosumer possess an asset which consumes and an uncontrollable production unit, such as PV panels. Assets' characteristics are randomly selected within a range defined by the user for each category of prosumers. Only considering quadratic cost functions, the user chooses ranges of lower and upper prices, and of lower and upper power boundaries for each consumption and production assets. Finally, the number of agents per category can either be the same for all cases or randomly chosen to add variability among cases. Cases are then saved in separate *.csv* files which can be loaded in the main module. The platform currently suffers of two main limitations. First, the simulation can not be launched for a set of cases, which would be suitable to test a high number of cases issued from the generator. Secondly, the application lacks of a post analysis module, which would be convenient to compare multiple simulation results.

IV. Simulation Results

After a brief description of the test cases, this section analyzes the influence of different organization layouts on market equilibrium. Test cases are first simulated without transaction costs while, secondly, increasing transaction costs are introduced to analyze their influence on both market outcomes and the convergence speed of negotiations. Finally, a Monte Carlo analysis outlines the influence of partnerships sparsity on market outcomes and convergence speed.

A. Test case description

As highlighted in the introduction, all prosumer market structures of [4] can be grouped in two families, namely fully P2P and community based structures. We propose here to compare them to the classical pool organization. Note that the pool organization is solved with the prosumer market model of Section II when all agents are grouped in a single community. To solely evaluate the impact of communication structure, the same data for market participants are used in all simulations. These agents were created by means of the test case generator described in Section III-B. Thus, a total of 50 agents, gathering 62 assets overall, were considered and then split into three communities, one lacking generation, another with extra non-dispatchable power, the last being balanced. These communities can be seen respectively as district areas such as: *(i)* a city center with flats and office buildings, *(ii)* an industrial district with factories and power plants, and *(iii)* a suburb composed of households, stores and power plants. Using such imbalanced communities requires energy exchange among each other. Note that agents and simulated organizations are saved as examples on the simulation platform.

B. In absence of transaction costs

We first consider a market setup without any transaction cost or preference criteria, i.e. with all γ_{ij} equal to zero in problem (1). According to the market properties derived in Appendix A, in this situation trading prices are uniform and equal to the price of the equivalent pool market. The communication graph is therefore expected to have no effect on the global social welfare. This fact is confirmed by the simulations as detailed in Table I. Pool, P2P and community organizations reach the same social welfare optimum for a price of 15.22 c\$/kW. The three models reach the same level of total consumption and production of 1.44 MW confirming that, overall, prosumers obtain the same set-points. It can also be observed that the total power exchanged, i.e. $\sum |\mathbf{T}|/2$, is higher for community based structure than the P2P one. This translates the presence of managers whose trades are also encompassed in the sum. Using a single community structure, the same remark can be done for the pool organization.

TABLE I: Simulation results without transaction costs (with $\epsilon^{\text{pri}} = \epsilon^{\text{dual}} = 10^{-4}$)

	Pool	P2P	Community
Social Welfare (\$)	125	125	125
No. of iterations	82	37	134
Penalty factor ρ	5.10^{-4}	0.01	5.10^{-4}
Avg. trading price (c\$/kW)	15.22	15.22	15.22
Cons./prod. power (kW)	1440	1440	1440
Total traded power (kW)	2620	1900	2620

C. In presence of transaction costs

Whenever transaction costs are not null, different market outcomes occur depending on the market layout. For the sake of this study we consider trade-based transaction costs, hence not effective within communities. Consequently, the pool based model is not affected by this transaction cost as it behaves as a single community. As expected, in Table IIa the social welfare of the P2P approach is negatively impacted by the use of a 1 c\$/kW trade-based transaction cost. The effect is largest on prices and, hence, power set-points. It can be noted that, since the transaction costs are uniform over P2P trades, in the P2P structure all participants are equally affected. Since inter-community exchanges are considered as P2P trades, the community-based simulation also shows a decrease of social welfare. As described in Appendix A, transaction costs introduce a difference of price between the inside and the outside of a community. Table IIb shows that communities with a positive balance of trade perceive a lower price within the community, while communities with a negative balance are penalized with a higher interior price as they need to import power. By increasing the value of trade-based transaction costs, one can observe that these differences follows the same trend. In fact, the average trading price grows linearly in both P2P and community layouts. The social welfare is less impacted in the community layout than in the P2P one, as already pictured in Table II. In addition, convergence speed of the negotiation mechanism appeared to linearly increase with transaction costs' intensity for both P2P and community approaches. However, in the community case the slope is rather flat compared to the P2P. A broader study should be conducted to evaluate in more comprehensively the influence of transaction costs on the negotiation mechanism.

TABLE II: Simulation results for 1 c\$/kW transaction costs on P2P trades (with $\epsilon^{\text{pri}} = \epsilon^{\text{dual}} = 10^{-4}$)

	Pool	P2P	Community
Social Welfare (\$)	125	102	105
No. of iterations	82	80	195
Penalty factor ρ	5.10^{-4}	0.01	5.10^{-4}
Avg. trading price (c\$/kW)	15.22	15.58	15.28
Cons./prod. power (kW)	1440	1152	1192
Tot. exchanged power (kW)	2620	1021	2122

(a) Overview

	Balance of trade	Interior price
Community 1	-908 kW	16.50 c\$/kW
Community 2	590 kW	14.50 c\$/kW
Community 3	319 kW	14.50 c\$/kW

(b) Focus on communities balance

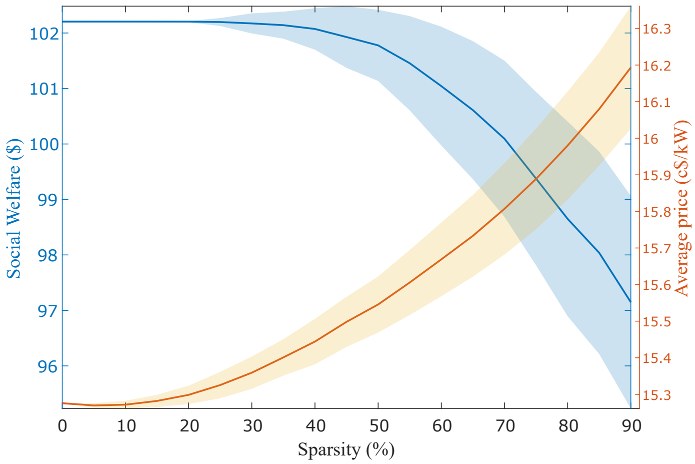

Fig. 2: Effects of sparsity on P2P market outcomes

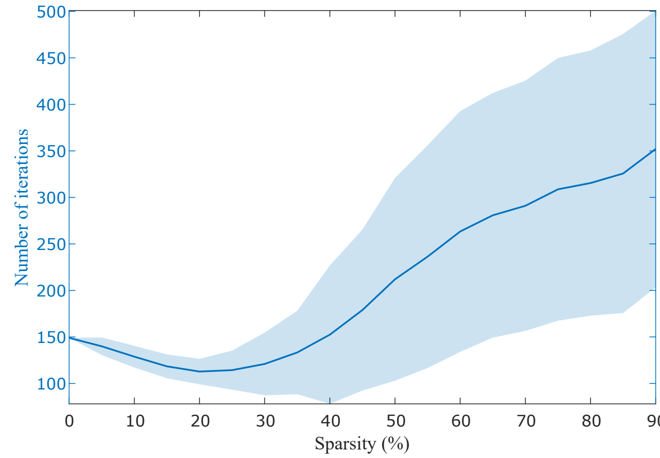

Fig. 3: Effects of sparsity on convergence speed

D. *Effects of sparsity*

To evaluate the influence of communication structures on market outcomes, we carry out a Monte Carlo analysis on the P2P design for different levels of sparsity of the communication graph. Starting from a fully connected P2P market, i.e. each agent is connected to all others, we progressively alter the communication graph by randomly deleting links. The Monte Carlo analysis, over 1000 cases per step of 5%, allows to describe how P2P market outcomes evolve as communication links get more sparse. Obtained in the presence of a unitary trade-based transaction cost, Fig. 2 outlines means (lines) and standard deviations (shadows) of social welfare and average trade prices. The more sparse communications are, the more likely market outcomes are to be affected and with a larger variety. This correlates with the increased possibility of agents to be unsatisfied, e.g. when a consumer is solely partnered to other consumers. As it is harder for agents to match their requirements, the convergence speed of negotiations can be significantly slowed down as shown in Fig. 3. However, comparing Fig. 2 and Fig. 3, it is possible to notice that there exist situations where the trade-off between increased convergence speed (less iterations and less communications) and loss of social welfare is optimal. Hence, the development of methods for retrieving communication layouts that optimize this trade-off becomes fundamental in order to enhance the feasibility in real world implementations of decentralized electricity markets.

V. Conclusion

With the deployment of distributed energy sources and home management systems, the role of prosumers in power systems will soon be fundamental. In the literature a variety of market structures adapted to particular situations are presented. We aimed for a general and comprehensive formulation of decentralized electricity markets. Based on consensus ADMM, the negotiation mechanism solves any market configuration as defined by its communication matrix. An open-source platform has been developed in order to facilitate the simulation of different organization layouts. With the help of this platform, the paper analyzed simulation results of two layouts: a fully peer-to-peer model and a community based approach. It is outlined that the communication structure influences market outcomes and the time required to reach it. In the presence of trade-based transaction costs, the community approach seemed better suited in terms of optimality while the P2P converged faster. However, convergence speed of the P2P structure appeared much more sensitive to transaction costs intensity than with communities. A Monte Carlo analysis revealed that sparsity of the communication matrix influences market outcomes and convergence speed in a non-trivial way. Therefore, the development of methods for exploiting sparsity to improve convergence speed while limiting the optimality gap is a fundamental future work, which would improve the feasibility of prosumer markets in real world implementations. Naturally, a better understanding of sensitivities to transaction costs and agents' flexibility would also allow to improve the robustness of the negotiation mechanism.

References

[1] Q. Wang, C. Zhang, Y. Ding, G. Xydis, J. Wang and J. Østergaard, "Review of real-time electricity markets for integrating Distributed Energy Resources and Demand Response", *Applied Energy*, vol. 138, pp. 695–706, 2015.

[2] F. F. Wu and P. Varaiya, "Coordinated multilateral trades for electric power networks: theory and implementation", *International Journal of Electrical Power & Energy Systems*, vol. 21, no. 21, pp. 75–102, 1999.

[3] L. Einav, C. Farronato and J. Levin, "Peer-to-Peer Markets", *Annual Review of Economics*, 2016.

[4] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era", *Nature energy*, vol. 1, pp. 16032, 2016.

[5] T. Morstyn, N. Farrell, S. J. Darby and M. D. McCulloch, "Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants", *Nature Energy*, vol. 3, no. 2, pp. 94–101, 2018.

[6] C. Zhang, and J. Wu, Y. Zhou, M. Cheng and C. Long, "Peer-to-Peer energy trading in a Microgrid", *Applied Energy*, vol. 220, pp. 1–12, 2018.

[7] T. Morstyn, A. Teytelboym and M. D. McCulloch, "Bilateral contract networks for peer-to-peer energy trading", *IEEE Transactions on Smart Grid*, vol. PP, no. 99, pp. 1–1, 2018.

[8] E. Sorin, L. Bobo and P. Pinson, "Consensus-based Approach to Peer-to-Peer Electricity Markets with Product Differentiation", *IEEE Transactions on Power Systems*, pp. 1–1, 2018.

[9] E. Munsing, J. Mather and S. Moura, "Blockchains for decentralized optimization of energy resources in microgrid networks", *IEEE Conference on Control Technology and Applications (CCTA)*, pp. 2164–2171, 2017.
[10] Y. Guo, M. Pan, Y. Fang and P. P. Khargonekar, "Decentralized coordination of energy utilization for residential households in the smart grid", *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1341–1350, 2013.
[11] F. Olivier, D. Marulli, D. Ernst and R. Fonteneau, "Foreseeing new control challenges in electricity prosumer communities", *Proc. of the 10th Bulk Power Systems Dynamics and Control Symposium–IREP2017*, 2017.
[12] W. Lee, L. Xiang, R. Schober, and V. WS Wong, "Direct electricity trading in smart grid: A coalitional game analysis", *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 7, pp. 1398–1411, 2014.
[13] F. Moret and P. Pinson, "Energy Collectives: a Community and Fairness based Approach to Future Electricity Markets", *IEEE Transactions on Power Systems*, vol. PP, no. 99, pp. 1, 2018.
[14] T. Baroche, P. Pinson, R. Le Goff Latimier and H. Ben Ahmed, "Exogenous Cost Allocation in Peer-to-Peer Electricity Markets", *IEEE Transactions on Power Systems* (Under Review), 2018.
[15] R. H. Day, "Rational choice and economic behavior", *Theory and Decision*, vol. 1, no. 3, pp. 229–251, 1971.
[16] F. Moret, T. Baroche, E. Sorin and P. Pinson, "Negotiation Algorithms for Peer-to-Peer Electricity Markets: Computational Properties", *Proc. of Power Systems Computation Conference (PSCC)*, pp. 1–7, 2018.
[17] S. Boyd, N. Parikh, E. Chu and J. Eckstein, "Distributed optimization and statistical learning via the Alternating Direction Method of Multipliers", *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2010.
[18] D. Monderer and L. S. Shapley, "Potential Games", *Games and Economic Behavior*, vol. 14, no. 1, pp. 124–143, 1996.
[19] G. D. Lã, Y. H. Chew and B.-H. Soong, "Potential Games", *Potential Game Theory*, pp. 23–69, 2016.

APPENDIX

A. Market Properties

If an equilibrium problem is taken such that each agent i solves

$$\min_{\mathbf{T},P} \sum_{a\in\mathcal{A}_i} f_i^a(p_i^a) + \sum_{j\in\omega_i} \left(\gamma_{ij}|t_{ij}| + \lambda_{ij}t_{ij}\right) \tag{6a}$$

$$\text{s.t.} \sum_{a\in\mathcal{A}_i} p_i^a = \sum_{j\in\omega_i} t_{ij} \qquad [\mu_i]\ i\in\Omega \tag{6b}$$

$$p_i^a \in \mathscr{P}_i^a \qquad a\in\mathcal{A}_i,\ i\in\Omega \tag{6c}$$

where λ_{ij} is the (i,j) component of the dual variable $\mathbf{\Lambda}$ from equilibrium

$$\mathbf{T} = -\mathbf{T}^{\mathsf{T}} \qquad [\mathbf{\Lambda}]. \tag{7}$$

Then its corresponding KKT conditions

$$\frac{\partial \tilde{f}_i^a(p_i^a)}{\partial p_i^a} - \mu_i = 0 \qquad \forall i\in\Omega, \forall a\in\mathcal{A}_i \tag{8a}$$

$$\gamma_{ij}\text{sign}(t_{ij}) - \lambda_{ij} + \mu_i = 0 \qquad \forall i\in\Omega, \forall j\in\omega_i \tag{8b}$$

$$\sum_{a\in\mathcal{A}_i} p_i^a = \sum_{j\in\omega_i} t_{ij} \qquad \forall i\in\Omega \tag{8c}$$

$$\mathbf{T} = -\mathbf{T}^{\mathsf{T}} \tag{8d}$$

are identical to those of (1), where $\tilde{f}_i^a$ is defined as in Appendix B.

In consequence, the prosumer market model proposed in II-A is equivalent to an equilibrium problem which is a potential game [18]. Thus, it inherits every traits of a potential game such as the existence of a Nash equilibrium [19], towards which negotiation algorithm (3) converges. This proves that the market-clearing mechanism in Section II is efficient in the presence of rational, non-strategic agents. However, this may not hold in the presence of strategic agents. For example, a market agent n may exercise market power by not truthfully offering quantities p_{nm} or preferences γ_{nm}. Hence, incentive compatibility and market efficiency are not ensured in the presence of strategic agents. Note that market failure may occur if at least one agent considers an asset with a non-convex cost function, in such case the ADMM based algorithm may converge to a local optimum which is not global.

Moreover, two other desirable properties can be proven for this market-clearing mechanism: *(i)* cost recovery, and *(ii)* revenue adequacy. Power reciprocity constraint (1b) imposes power balance of each trade. Yet, as discussed in II-B, negotiation algorithm (3) ensures symmetrical prices $\lambda_{ij} = \lambda_{ji}$ as long as $\lambda_{ij}^0 = \lambda_{ji}^0$. Hence, each bilateral trade is budget balanced, ensuring budget balance of the overall market.

In addition, KKT condition (8b) stipulates that

$$\mu_i = \lambda_{ij} - \text{sign}(t_{ij})\gamma_{ij} \qquad \forall i\in\Omega, \forall j\in\omega_i \tag{9a}$$

$$\Leftrightarrow\ \mu_i + \text{sign}(t_{ij})\gamma_{ij} = \lambda_{ij} \qquad \forall i\in\Omega, \forall j\in\omega_i \tag{9b}$$

where μ_i is both agent i's average generation price and perceived trading price. For purely rational agent and in absence of taxes parameters γ_{ij} are all equal to zero. Then, there would be an exact cost recovery of each agent's operational cost. However, in the presence of commission fees collected through coefficients γ_{ij}, (9b) shows that both operational and commission costs are recovered through trading prices. On the other hand, (9a) indicates that when agents have preferences, favored partners are subsidizes through price. For example, if one wants to penalize CO_2 intense industries, it penalizes its perceived price with γ_{ij},

B. ADMM Algorithm

Suppose penalty factor $\rho > 0$, functions $\tilde{f}_i = \sum_{a\in\mathcal{A}_i} \tilde{f}_i^a$ with $\tilde{f}_i^a$ the extended-value of f_i^a, in the sense of [17], defined on $\mathscr{P}_i^a$. The augmented Lagrangian of (1) with (2) reads

$$L_\rho\left((P_i,T_i,\mu_i)_i, \mathbf{C}, \mathbf{\Lambda}\right) = \sum_{i\in\Omega} L_\rho^i\left(P_i,T_i,\mu_i,\mathbf{C},\Lambda_i\right) \tag{10}$$

$$L_\rho^i\left(P_i,T_i,\mu_i,\mathbf{C},\Lambda_i\right) = \tilde{f}_i(P_i) + \mu_i\Big(\sum_{j\in\omega_i} t_{ij} - \sum_{a\in\mathcal{A}_i} p_i^a\Big)$$

$$+ \sum_{j\in\omega_i} \gamma_{ij}|t_{ij}| + \frac{\rho}{2}\left(\frac{c_{ij}-c_{ji}}{2} - t_{ij} + \frac{\lambda_{ij}}{\rho}\right)^2 - \frac{1}{\rho}\lambda_{ij}^2 \tag{11}$$

Hence, the ADMM of (1) with (2) reads

$$(P_i,T_i,\mu_i)^{k+1} = \underset{P_i,T_i,\mu_i}{\text{argmin}}\, L_\rho^i\left(P_i,T_i,\mu_i,\mathbf{C}^k,\Lambda_i^k\right) \tag{12a}$$

$$\mathbf{C}^{k+1} = \underset{\mathbf{C}}{\text{argmin}} \sum_{i\in\Omega} L_\rho^i\left((P_i,T_i,\mu_i)_i^{k+1}, \mathbf{C}, \Lambda_i^k\right) \tag{12b}$$

$$\mathbf{\Lambda}^{k+1} = \mathbf{\Lambda}^k - \left(\mathbf{T}^{k+1} + \mathbf{T}^{\mathsf{T},k+1}\right)/2 \tag{12c}$$

As proposed in [17], step (12b) can be written

$$\mathbf{C}^{k+1} = \left(\mathbf{T}^{k+1} - \mathbf{T}^{\mathsf{T},k+1}\right)/2 - \Big(\mathbf{\Lambda}^k - \mathbf{\Lambda}^{\mathsf{T},k}\Big)/2\rho \tag{13}$$

where $\mathbf{\Lambda}^k - \mathbf{\Lambda}^{\mathsf{T},k}$ equals zero after the first iteration. Hence, when replacing $\mathbf{C}^k$ with $\mathbf{T}^k$, the final simplified version of the consensus ADMM solving (1) with (2) reads as in (3).