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CSC-366

Crypto: Heuristic Problem Solver - Demo

```
freddy@Freddy-Laptop:~/Documents/CSC-366/programming_assignments/heuristic_pr  
oblem_solver$ swipl
```

Welcome to SWI-Prolog (threaded, 64 bits, version 7.6.2)

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For built-in help, use `?- help(Topic).` or `?- apropos(Word).`

```
?- consult('hps.pro').
```

```
true.
```

```
?- demo(100).
```

Numbers = {5, 15, 9, 5, 2} Goal = 6

application of rule 5 produces ($6 + ((5 - 5) * (15 * 9))$)

Numbers = {1, 13, 8, 15, 4} Goal = 6

Numbers = {14, 7, 10, 8, 5} Goal = 14

Numbers = {4, 12, 2, 0, 1} Goal = 3

Numbers = {10, 15, 15, 13, 10} Goal = 15

application of rule 5 produces ($15 + ((10 - 10) * (15 * 13))$)

Numbers = {1, 4, 12, 13, 12} Goal = 14

application of rule 5 produces ($14 + ((12 - 12) * (1 * 4))$)

Numbers = {9, 15, 0, 0, 11} Goal = 2

application of rule 5 produces ($2 + ((0 - 0) * (9 * 15))$)

Numbers = {8, 0, 12, 3, 15} Goal = 4

Numbers = {13, 8, 13, 4, 2} Goal = 3

application of rule 5 produces ($3 + ((13 - 13) * (8 * 4))$)

Numbers = {8, 15, 3, 10, 14} Goal = 10

Numbers = {13, 4, 11, 9, 8} Goal = 6

Numbers = {7, 15, 3, 7, 10} Goal = 1

application of rule 4 produces ($(7 / 7) * (0 * (15 * 3))$)

Numbers = {15, 6, 3, 8, 13} Goal = 11

Numbers = {13, 12, 6, 11, 7} Goal = 0

Numbers = {13, 15, 8, 11, 3} Goal = 15
 Numbers = {10, 13, 8, 0, 12} Goal = 11
 application of rule 7 produces ($13 - ((8 - 10) + (0 * 12))$)
 Numbers = {3, 9, 15, 14, 2} Goal = 12
 Numbers = {9, 15, 1, 15, 2} Goal = 2
 application of rule 5 produces ($2 + ((15 - 15) * (9 * 1))$)
 Numbers = {2, 4, 4, 0, 14} Goal = 9
 application of rule 5 produces ($9 + ((4 - 4) * (2 * 0))$)
 Numbers = {3, 3, 5, 7, 8} Goal = 2
 application of rule 5 produces ($2 + ((3 - 3) * (5 * 7))$)
 Numbers = {15, 4, 9, 14, 15} Goal = 7
 application of rule 5 produces ($7 + ((15 - 15) * (4 * 9))$)
 Numbers = {6, 3, 7, 2, 7} Goal = 1
 application of rule 4 produces ($((7 / 7) * (0 * (6 * 3)))$)
 Numbers = {12, 9, 5, 3, 5} Goal = 7
 application of rule 5 produces ($7 + ((5 - 5) * (12 * 9))$)
 Numbers = {2, 12, 2, 12, 7} Goal = 8
 application of rule 5 produces ($8 + ((2 - 2) * (12 * 12))$)
 Numbers = {9, 13, 5, 2, 14} Goal = 8
 Numbers = {10, 10, 2, 8, 8} Goal = 5
 application of rule 5 produces ($5 + ((10 - 10) * (2 * 8))$)
 Numbers = {10, 12, 8, 13, 13} Goal = 15
 application of rule 5 produces ($15 + ((13 - 13) * (10 * 12))$)
 Numbers = {4, 9, 7, 11, 2} Goal = 14
 Numbers = {6, 8, 5, 11, 6} Goal = 13
 application of rule 5 produces ($13 + ((6 - 6) * (8 * 5))$)
 Numbers = {1, 1, 7, 2, 12} Goal = 15
 application of rule 5 produces ($15 + ((1 - 1) * (7 * 2))$)
 Numbers = {4, 6, 10, 1, 12} Goal = 2
 Numbers = {4, 4, 1, 4, 13} Goal = 11
 application of rule 5 produces ($11 + ((4 - 4) * (1 * 4))$)
 Numbers = {1, 4, 4, 12, 4} Goal = 7
 application of rule 5 produces ($7 + ((4 - 4) * (1 * 12))$)
 Numbers = {9, 10, 5, 15, 0} Goal = 6
 Numbers = {14, 4, 9, 8, 8} Goal = 15
 application of rule 5 produces ($15 + ((8 - 8) * (14 * 4))$)
 Numbers = {4, 0, 8, 12, 2} Goal = 6
 application of rule 7 produces ($8 - ((2 - 4) + (0 * 12))$)
 Numbers = {12, 5, 15, 3, 11} Goal = 1

Numbers = {1, 4, 13, 0, 3} Goal = 12
 Numbers = {10, 8, 1, 1, 15} Goal = 13
 application of rule 5 produces $(13 + ((1 - 1) * (10 * 8)))$
 Numbers = {12, 5, 14, 13, 8} Goal = 1
 Numbers = {1, 15, 4, 2, 5} Goal = 1
 Numbers = {1, 12, 3, 12, 8} Goal = 1
 application of rule 4 produces $((12 / 12) * (0 * (1 * 3)))$
 Numbers = {2, 12, 15, 13, 7} Goal = 9
 Numbers = {12, 9, 0, 0, 3} Goal = 1
 application of rule 4 produces $((0 / 0) * (0 * (12 * 9)))$
 Numbers = {4, 2, 11, 7, 11} Goal = 8
 application of rule 5 produces $(8 + ((11 - 11) * (4 * 2)))$
 Numbers = {7, 5, 3, 15, 0} Goal = 0
 application of rule 1 produces $(7 * (5 * (3 * (15 * 0))))$
 Numbers = {11, 5, 14, 14, 8} Goal = 15
 application of rule 5 produces $(15 + ((14 - 14) * (11 * 5)))$
 Numbers = {2, 15, 14, 7, 8} Goal = 1
 Numbers = {3, 1, 8, 2, 3} Goal = 0
 application of rule 3 produces $((3 - 3) * (1 * (8 * 2)))$
 Numbers = {8, 0, 11, 5, 2} Goal = 14
 Numbers = {5, 14, 2, 8, 12} Goal = 0
 Numbers = {15, 11, 8, 9, 3} Goal = 1
 Numbers = {10, 8, 7, 6, 8} Goal = 8
 application of rule 5 produces $(8 + ((8 - 8) * (10 * 7)))$
 Numbers = {0, 3, 5, 3, 12} Goal = 3
 application of rule 2 produces $(3 + (0 * (5 * (3 * 12))))$
 Numbers = {6, 11, 8, 13, 12} Goal = 3
 Numbers = {14, 7, 11, 9, 1} Goal = 13
 Numbers = {2, 2, 4, 2, 11} Goal = 9
 application of rule 5 produces $(9 + ((2 - 2) * (4 * 2)))$
 Numbers = {3, 4, 7, 10, 4} Goal = 11
 application of rule 5 produces $(11 + ((4 - 4) * (3 * 7)))$
 Numbers = {12, 10, 11, 6, 15} Goal = 4
 Numbers = {4, 0, 3, 14, 1} Goal = 11
 Numbers = {9, 3, 9, 8, 5} Goal = 5
 application of rule 5 produces $(5 + ((9 - 9) * (3 * 8)))$
 Numbers = {9, 2, 11, 11, 9} Goal = 4
 application of rule 5 produces $(4 + ((9 - 9) * (2 * 11)))$
 Numbers = {5, 11, 11, 8, 1} Goal = 4

application of rule 5 produces $(4 + ((11 - 11) * (5 * 8)))$
 Numbers = {7, 8, 9, 7, 13} Goal = 8
 application of rule 5 produces $(8 + ((7 - 7) * (9 * 13)))$
 Numbers = {15, 5, 6, 13, 6} Goal = 2
 application of rule 5 produces $(2 + ((6 - 6) * (15 * 5)))$
 Numbers = {6, 13, 10, 8, 14} Goal = 0
 Numbers = {11, 9, 0, 12, 12} Goal = 1
 application of rule 4 produces $((12 / 12) * (0 * (11 * 9)))$
 Numbers = {14, 3, 7, 9, 13} Goal = 13
 Numbers = {15, 0, 1, 1, 3} Goal = 3
 application of rule 2 produces $(3 + (15 * (0 * (1 * 1))))$
 Numbers = {5, 9, 3, 1, 2} Goal = 3
 Numbers = {4, 3, 6, 2, 1} Goal = 12
 Numbers = {5, 5, 11, 4, 11} Goal = 12
 application of rule 5 produces $(12 + ((5 - 5) * (11 * 4)))$
 Numbers = {1, 7, 13, 14, 2} Goal = 0
 Numbers = {2, 12, 6, 11, 0} Goal = 2
 application of rule 2 produces $(2 + (12 * (6 * (11 * 0))))$
 Numbers = {12, 0, 10, 0, 4} Goal = 4
 application of rule 2 produces $(4 + (12 * (0 * (10 * 0))))$
 Numbers = {11, 5, 9, 14, 10} Goal = 15
 Numbers = {11, 12, 9, 2, 12} Goal = 1
 application of rule 4 produces $((12 / 12) * (0 * (11 * 9)))$
 Numbers = {13, 15, 1, 15, 10} Goal = 2
 application of rule 5 produces $(2 + ((15 - 15) * (13 * 1)))$
 Numbers = {10, 2, 6, 15, 5} Goal = 2
 Numbers = {11, 13, 11, 14, 1} Goal = 13
 application of rule 5 produces $(13 + ((11 - 11) * (14 * 1)))$
 Numbers = {3, 11, 2, 13, 4} Goal = 3
 Numbers = {9, 11, 15, 9, 6} Goal = 7
 application of rule 5 produces $(7 + ((9 - 9) * (11 * 15)))$
 Numbers = {11, 13, 8, 8, 11} Goal = 13
 application of rule 5 produces $(13 + ((11 - 11) * (8 * 8)))$
 Numbers = {9, 0, 14, 11, 14} Goal = 11
 application of rule 2 produces $(11 + (9 * (0 * (14 * 14))))$
 Numbers = {8, 13, 7, 3, 10} Goal = 4
 Numbers = {4, 1, 8, 4, 3} Goal = 1
 application of rule 4 produces $((4 / 4) * (0 * (1 * 8)))$
 Numbers = {7, 15, 15, 15, 8} Goal = 2

application of rule 5 produces ($2 + ((15 - 15) * (7 * 15))$)
 Numbers = {4, 1, 6, 10, 8} Goal = 15
 Numbers = {5, 7, 5, 0, 12} Goal = 11
 application of rule 5 produces ($11 + ((5 - 5) * (7 * 0))$)
 Numbers = {2, 8, 10, 13, 8} Goal = 3
 application of rule 5 produces ($3 + ((8 - 8) * (2 * 10))$)
 Numbers = {15, 12, 2, 7, 9} Goal = 10
 Numbers = {1, 6, 11, 13, 7} Goal = 15
 Numbers = {1, 10, 12, 9, 8} Goal = 1
 Numbers = {11, 13, 1, 12, 1} Goal = 8
 application of rule 5 produces ($8 + ((1 - 1) * (11 * 13))$)
 Numbers = {8, 6, 7, 8, 0} Goal = 15
 application of rule 5 produces ($15 + ((8 - 8) * (6 * 7))$)
 Numbers = {6, 13, 3, 10, 15} Goal = 1
 Numbers = {0, 5, 7, 15, 0} Goal = 5
 application of rule 2 produces ($5 + (0 * (7 * (15 * 0)))$)
 Numbers = {10, 1, 8, 6, 8} Goal = 10
 application of rule 5 produces ($10 + ((8 - 8) * (1 * 6))$)
 Numbers = {8, 3, 12, 1, 14} Goal = 10
 Numbers = {9, 3, 13, 8, 1} Goal = 1
 true .

?- demo(100).

Numbers = {9, 11, 0, 3, 2} Goal = 12
 Numbers = {5, 9, 4, 13, 8} Goal = 3
 Numbers = {14, 4, 10, 13, 7} Goal = 11
 Numbers = {14, 4, 14, 11, 0} Goal = 9
 application of rule 5 produces ($9 + ((14 - 14) * (4 * 11))$)
 Numbers = {13, 6, 4, 0, 8} Goal = 5
 Numbers = {1, 14, 9, 4, 6} Goal = 9
 Numbers = {0, 10, 13, 12, 11} Goal = 0
 application of rule 1 produces ($0 * (10 * (13 * (12 * 11)))$)
 Numbers = {10, 11, 3, 3, 0} Goal = 6
 application of rule 5 produces ($6 + ((3 - 3) * (10 * 11))$)
 Numbers = {14, 1, 11, 6, 7} Goal = 2
 Numbers = {2, 6, 3, 2, 10} Goal = 1
 application of rule 4 produces ($(2 / 2) * (0 * (6 * 3))$)
 Numbers = {10, 14, 0, 8, 4} Goal = 13
 Numbers = {15, 3, 12, 11, 5} Goal = 6

Numbers = {8, 2, 5, 9, 8} Goal = 0
 application of rule 3 produces $((8 - 8) * (2 * (5 * 9)))$
 Numbers = {4, 11, 2, 0, 1} Goal = 1
 application of rule 2 produces $(1 + (4 * (11 * (2 * 0))))$
 Numbers = {11, 15, 4, 11, 0} Goal = 8
 application of rule 5 produces $(8 + ((11 - 11) * (15 * 4)))$
 Numbers = {12, 0, 1, 14, 14} Goal = 13
 application of rule 5 produces $(13 + ((14 - 14) * (12 * 0)))$
 Numbers = {0, 0, 2, 10, 10} Goal = 8
 application of rule 5 produces $(8 + ((0 - 0) * (2 * 10)))$
 Numbers = {4, 10, 14, 13, 4} Goal = 1
 application of rule 4 produces $((4 / 4) * (0 * (10 * 14)))$
 Numbers = {3, 5, 15, 13, 7} Goal = 10
 Numbers = {13, 15, 15, 1, 10} Goal = 14
 application of rule 5 produces $(14 + ((15 - 15) * (13 * 1)))$
 Numbers = {1, 6, 14, 3, 6} Goal = 2
 application of rule 5 produces $(2 + ((6 - 6) * (1 * 14)))$
 Numbers = {8, 11, 9, 3, 10} Goal = 7
 Numbers = {5, 5, 10, 0, 15} Goal = 9
 application of rule 5 produces $(9 + ((5 - 5) * (10 * 0)))$
 Numbers = {12, 1, 1, 0, 0} Goal = 1
 application of rule 2 produces $(1 + (12 * (1 * (0 * 0))))$
 Numbers = {1, 0, 10, 15, 4} Goal = 1
 application of rule 2 produces $(1 + (0 * (10 * (15 * 4))))$
 Numbers = {1, 7, 8, 11, 2} Goal = 2
 Numbers = {1, 6, 14, 4, 0} Goal = 5
 Numbers = {3, 2, 1, 2, 12} Goal = 6
 application of rule 5 produces $(6 + ((2 - 2) * (3 * 1)))$
 Numbers = {8, 3, 14, 12, 11} Goal = 2
 Numbers = {0, 11, 3, 0, 10} Goal = 12
 application of rule 5 produces $(12 + ((0 - 0) * (11 * 3)))$
 Numbers = {10, 4, 3, 15, 9} Goal = 2
 Numbers = {8, 6, 2, 6, 10} Goal = 1
 application of rule 4 produces $((6 / 6) * (0 * (8 * 2)))$
 Numbers = {8, 2, 6, 8, 5} Goal = 4
 application of rule 5 produces $(4 + ((8 - 8) * (2 * 6)))$
 Numbers = {0, 15, 12, 3, 5} Goal = 10
 application of rule 7 produces $(12 - ((5 - 3) + (15 * 0)))$
 Numbers = {1, 4, 13, 10, 8} Goal = 6

Numbers = {5, 14, 10, 6, 14} Goal = 7
 application of rule 5 produces (7 + ((14 - 14) * (5 * 10)))
 Numbers = {11, 4, 4, 4, 0} Goal = 15
 application of rule 5 produces (15 + ((4 - 4) * (11 * 4)))
 Numbers = {10, 12, 15, 12, 11} Goal = 4
 application of rule 5 produces (4 + ((12 - 12) * (10 * 15)))
 Numbers = {7, 7, 0, 4, 0} Goal = 12
 application of rule 5 produces (12 + ((7 - 7) * (0 * 4)))
 Numbers = {13, 10, 6, 3, 9} Goal = 3
 Numbers = {11, 6, 14, 3, 11} Goal = 10
 application of rule 5 produces (10 + ((11 - 11) * (6 * 14)))
 Numbers = {7, 4, 3, 6, 14} Goal = 9
 Numbers = {5, 14, 9, 14, 10} Goal = 11
 application of rule 5 produces (11 + ((14 - 14) * (5 * 9)))
 Numbers = {2, 1, 11, 7, 7} Goal = 14
 application of rule 5 produces (14 + ((7 - 7) * (2 * 1)))
 Numbers = {7, 3, 3, 8, 11} Goal = 11
 application of rule 5 produces (11 + ((3 - 3) * (7 * 8)))
 Numbers = {1, 1, 2, 10, 10} Goal = 13
 application of rule 5 produces (13 + ((1 - 1) * (2 * 10)))
 Numbers = {12, 2, 2, 4, 2} Goal = 12
 application of rule 5 produces (12 + ((2 - 2) * (4 * 2)))
 Numbers = {12, 7, 15, 12, 10} Goal = 8
 application of rule 5 produces (8 + ((12 - 12) * (7 * 15)))
 Numbers = {12, 3, 2, 4, 2} Goal = 15
 application of rule 5 produces (15 + ((2 - 2) * (12 * 3)))
 Numbers = {4, 0, 11, 6, 7} Goal = 11
 application of rule 2 produces (11 + (4 * (0 * (6 * 7))))
 Numbers = {15, 3, 0, 11, 0} Goal = 9
 application of rule 5 produces (9 + ((0 - 0) * (15 * 3)))
 Numbers = {3, 10, 3, 7, 4} Goal = 10
 application of rule 5 produces (10 + ((3 - 3) * (7 * 4)))
 Numbers = {8, 1, 15, 3, 0} Goal = 2
 Numbers = {6, 4, 6, 15, 7} Goal = 15
 application of rule 5 produces (15 + ((6 - 6) * (4 * 7)))
 Numbers = {9, 4, 1, 14, 2} Goal = 13
 Numbers = {9, 15, 11, 5, 1} Goal = 1
 Numbers = {9, 11, 15, 8, 9} Goal = 5
 application of rule 5 produces (5 + ((9 - 9) * (11 * 15)))

Numbers = {8, 7, 7, 12, 1} Goal = 4
 application of rule 5 produces (4 + ((7 - 7) * (8 * 12)))
 Numbers = {14, 0, 12, 1, 14} Goal = 13
 application of rule 5 produces (13 + ((14 - 14) * (0 * 12)))
 Numbers = {13, 5, 1, 1, 11} Goal = 1
 application of rule 4 produces ((1 / 1) * (0 * (13 * 5)))
 Numbers = {9, 8, 5, 2, 7} Goal = 7
 Numbers = {13, 3, 11, 0, 14} Goal = 10
 Numbers = {6, 13, 2, 4, 8} Goal = 15
 Numbers = {11, 7, 0, 1, 4} Goal = 11
 application of rule 2 produces (11 + (7 * (0 * (1 * 4))))
 Numbers = {8, 15, 12, 1, 3} Goal = 1
 Numbers = {7, 7, 13, 12, 12} Goal = 0
 application of rule 3 produces ((7 - 7) * (13 * (12 * 12)))
 Numbers = {12, 8, 8, 1, 14} Goal = 1
 application of rule 4 produces ((8 / 8) * (0 * (12 * 1)))
 Numbers = {11, 10, 15, 11, 9} Goal = 4
 application of rule 5 produces (4 + ((11 - 11) * (10 * 15)))
 Numbers = {6, 6, 6, 5, 5} Goal = 8
 application of rule 5 produces (8 + ((6 - 6) * (6 * 5)))
 Numbers = {14, 8, 0, 5, 4} Goal = 1
 Numbers = {9, 15, 3, 15, 13} Goal = 14
 application of rule 5 produces (14 + ((15 - 15) * (9 * 3)))
 Numbers = {0, 10, 14, 12, 0} Goal = 5
 application of rule 5 produces (5 + ((0 - 0) * (10 * 14)))
 Numbers = {7, 13, 12, 9, 5} Goal = 9
 Numbers = {14, 5, 1, 9, 2} Goal = 9
 Numbers = {2, 11, 4, 6, 15} Goal = 3
 Numbers = {2, 10, 9, 1, 15} Goal = 1
 Numbers = {8, 7, 9, 8, 3} Goal = 2
 application of rule 5 produces (2 + ((8 - 8) * (7 * 9)))
 Numbers = {11, 0, 0, 0, 0} Goal = 12
 application of rule 5 produces (12 + ((0 - 0) * (11 * 0)))
 Numbers = {6, 12, 0, 2, 14} Goal = 7
 Numbers = {15, 2, 0, 4, 2} Goal = 5
 application of rule 5 produces (5 + ((2 - 2) * (15 * 0)))
 Numbers = {4, 12, 9, 15, 4} Goal = 13
 application of rule 5 produces (13 + ((4 - 4) * (12 * 9)))
 Numbers = {3, 6, 12, 10, 15} Goal = 15

Numbers = {15, 2, 15, 14, 1} Goal = 6
 application of rule 5 produces (6 + ((15 - 15) * (2 * 14)))
 Numbers = {0, 14, 12, 4, 9} Goal = 14
 application of rule 2 produces (14 + (0 * (12 * (4 * 9))))
 Numbers = {0, 3, 15, 4, 4} Goal = 0
 application of rule 1 produces (0 * (3 * (15 * (4 * 4))))
 Numbers = {9, 9, 10, 6, 13} Goal = 2
 application of rule 5 produces (2 + ((9 - 9) * (10 * 6)))
 Numbers = {11, 7, 0, 3, 10} Goal = 6
 Numbers = {8, 2, 11, 13, 15} Goal = 4
 Numbers = {12, 11, 2, 15, 2} Goal = 11
 application of rule 5 produces (11 + ((2 - 2) * (12 * 15)))
 Numbers = {14, 0, 8, 6, 7} Goal = 6
 application of rule 2 produces (6 + (14 * (0 * (8 * 7))))
 Numbers = {11, 15, 2, 9, 0} Goal = 11
 application of rule 2 produces (11 + (15 * (2 * (9 * 0))))
 Numbers = {15, 5, 11, 11, 15} Goal = 10
 application of rule 5 produces (10 + ((15 - 15) * (5 * 11)))
 Numbers = {9, 2, 10, 11, 2} Goal = 3
 application of rule 5 produces (3 + ((2 - 2) * (9 * 10)))
 Numbers = {5, 6, 4, 9, 4} Goal = 2
 application of rule 5 produces (2 + ((4 - 4) * (5 * 6)))
 Numbers = {11, 13, 12, 7, 5} Goal = 13
 Numbers = {14, 8, 12, 9, 8} Goal = 7
 application of rule 5 produces (7 + ((8 - 8) * (14 * 12)))
 Numbers = {9, 12, 12, 15, 11} Goal = 11
 application of rule 5 produces (11 + ((12 - 12) * (9 * 15)))
 Numbers = {9, 4, 3, 13, 8} Goal = 13
 Numbers = {1, 8, 11, 1, 4} Goal = 14
 application of rule 5 produces (14 + ((1 - 1) * (8 * 11)))
 Numbers = {4, 4, 10, 4, 9} Goal = 9
 application of rule 5 produces (9 + ((4 - 4) * (10 * 4)))
 true .

?- demo(100).

Numbers = {15, 9, 1, 10, 4} Goal = 4
 Numbers = {4, 13, 6, 0, 9} Goal = 14
 Numbers = {13, 13, 4, 1, 10} Goal = 1
 application of rule 4 produces ((13 / 13) * (0 * (4 * 1)))

Numbers = {9, 9, 3, 15, 5} Goal = 4
 application of rule 5 produces (4 + ((9 - 9) * (3 * 15)))
 Numbers = {13, 3, 8, 10, 6} Goal = 1
 Numbers = {0, 5, 13, 5, 7} Goal = 14
 application of rule 5 produces (14 + ((5 - 5) * (0 * 13)))
 Numbers = {3, 15, 3, 4, 11} Goal = 4
 application of rule 5 produces (4 + ((3 - 3) * (15 * 11)))
 Numbers = {0, 12, 7, 13, 14} Goal = 4
 Numbers = {5, 15, 5, 8, 1} Goal = 9
 application of rule 5 produces (9 + ((5 - 5) * (15 * 8)))
 Numbers = {6, 1, 13, 2, 6} Goal = 15
 application of rule 5 produces (15 + ((6 - 6) * (1 * 13)))
 Numbers = {1, 2, 5, 8, 0} Goal = 4
 Numbers = {2, 9, 8, 15, 7} Goal = 13
 Numbers = {8, 8, 15, 8, 14} Goal = 2
 application of rule 5 produces (2 + ((8 - 8) * (15 * 8)))
 Numbers = {5, 8, 6, 7, 5} Goal = 0
 application of rule 3 produces ((5 - 5) * (8 * (6 * 7)))
 Numbers = {11, 4, 1, 12, 11} Goal = 2
 application of rule 5 produces (2 + ((11 - 11) * (4 * 1)))
 Numbers = {0, 13, 15, 14, 5} Goal = 5
 application of rule 2 produces (5 + (0 * (13 * (15 * 14))))
 Numbers = {10, 5, 6, 5, 1} Goal = 1
 application of rule 4 produces ((5 / 5) * (0 * (10 * 6)))
 Numbers = {12, 13, 11, 7, 0} Goal = 10
 application of rule 7 produces (12 - ((11 - 13) + (7 * 0)))
 Numbers = {7, 6, 15, 15, 7} Goal = 4
 application of rule 5 produces (4 + ((7 - 7) * (6 * 15)))
 Numbers = {10, 12, 13, 5, 9} Goal = 11
 Numbers = {10, 0, 13, 12, 5} Goal = 2
 Numbers = {15, 1, 14, 14, 7} Goal = 8
 application of rule 5 produces (8 + ((14 - 14) * (15 * 1)))
 Numbers = {9, 14, 15, 9, 13} Goal = 2
 application of rule 5 produces (2 + ((9 - 9) * (14 * 15)))
 Numbers = {7, 6, 14, 6, 9} Goal = 14
 application of rule 5 produces (14 + ((6 - 6) * (7 * 9)))
 Numbers = {13, 0, 8, 4, 6} Goal = 3
 Numbers = {15, 3, 12, 8, 6} Goal = 9
 Numbers = {6, 5, 14, 7, 5} Goal = 6

application of rule 5 produces $(6 + ((5 - 5) * (14 * 7)))$
 Numbers = {0, 5, 14, 11, 14} Goal = 8
 application of rule 5 produces $(8 + ((14 - 14) * (0 * 5)))$
 Numbers = {7, 9, 10, 13, 15} Goal = 7
 Numbers = {4, 2, 1, 3, 8} Goal = 10
 Numbers = {3, 12, 1, 1, 6} Goal = 11
 application of rule 5 produces $(11 + ((1 - 1) * (3 * 12)))$
 Numbers = {8, 14, 5, 12, 13} Goal = 14
 Numbers = {8, 5, 7, 1, 7} Goal = 15
 application of rule 5 produces $(15 + ((7 - 7) * (8 * 5)))$
 Numbers = {4, 12, 9, 12, 3} Goal = 13
 application of rule 5 produces $(13 + ((12 - 12) * (4 * 9)))$
 Numbers = {5, 11, 2, 9, 2} Goal = 9
 application of rule 5 produces $(9 + ((2 - 2) * (5 * 11)))$
 Numbers = {7, 11, 14, 9, 12} Goal = 5
 Numbers = {11, 11, 9, 6, 13} Goal = 5
 application of rule 5 produces $(5 + ((11 - 11) * (9 * 6)))$
 Numbers = {5, 7, 2, 11, 7} Goal = 14
 application of rule 5 produces $(14 + ((7 - 7) * (5 * 2)))$
 Numbers = {6, 4, 10, 13, 4} Goal = 0
 application of rule 3 produces $((4 - 4) * (6 * (10 * 13)))$
 Numbers = {9, 9, 11, 15, 12} Goal = 11
 application of rule 5 produces $(11 + ((9 - 9) * (15 * 12)))$
 Numbers = {3, 2, 0, 2, 14} Goal = 6
 application of rule 5 produces $(6 + ((2 - 2) * (3 * 0)))$
 Numbers = {8, 11, 3, 6, 13} Goal = 3
 Numbers = {2, 0, 13, 0, 4} Goal = 15
 application of rule 5 produces $(15 + ((0 - 0) * (2 * 13)))$
 Numbers = {5, 8, 6, 14, 2} Goal = 9
 Numbers = {10, 12, 7, 15, 5} Goal = 4
 Numbers = {10, 1, 4, 0, 4} Goal = 13
 application of rule 5 produces $(13 + ((4 - 4) * (10 * 1)))$
 Numbers = {8, 6, 8, 11, 9} Goal = 14
 application of rule 5 produces $(14 + ((8 - 8) * (6 * 11)))$
 Numbers = {12, 5, 0, 1, 13} Goal = 12
 application of rule 2 produces $(12 + (5 * (0 * (1 * 13))))$
 Numbers = {14, 5, 13, 2, 10} Goal = 3
 Numbers = {11, 0, 1, 8, 2} Goal = 10
 Numbers = {5, 5, 3, 2, 12} Goal = 14

application of rule 5 produces ($14 + ((5 - 5) * (3 * 2))$)
 Numbers = {12, 2, 10, 13, 10} Goal = 4
 application of rule 5 produces ($4 + ((10 - 10) * (12 * 2))$)
 Numbers = {9, 6, 9, 0, 0} Goal = 14
 application of rule 5 produces ($14 + ((9 - 9) * (6 * 0))$)
 Numbers = {6, 9, 0, 2, 2} Goal = 7
 application of rule 5 produces ($7 + ((2 - 2) * (6 * 9))$)
 Numbers = {7, 11, 0, 13, 9} Goal = 5
 application of rule 7 produces ($7 - ((13 - 11) + (0 * 9))$)
 Numbers = {13, 13, 6, 4, 7} Goal = 6
 application of rule 5 produces ($6 + ((13 - 13) * (4 * 7))$)
 Numbers = {11, 6, 4, 12, 3} Goal = 10
 Numbers = {15, 3, 0, 10, 13} Goal = 4
 Numbers = {4, 9, 13, 14, 8} Goal = 2
 Numbers = {3, 7, 1, 2, 6} Goal = 2
 Numbers = {7, 3, 3, 15, 9} Goal = 11
 application of rule 5 produces ($11 + ((3 - 3) * (7 * 15))$)
 Numbers = {2, 6, 2, 1, 3} Goal = 5
 application of rule 5 produces ($5 + ((2 - 2) * (6 * 1))$)
 Numbers = {11, 2, 6, 5, 12} Goal = 9
 Numbers = {1, 8, 8, 10, 11} Goal = 13
 application of rule 5 produces ($13 + ((8 - 8) * (1 * 10))$)
 Numbers = {12, 2, 15, 5, 9} Goal = 3
 Numbers = {2, 8, 5, 8, 0} Goal = 11
 application of rule 5 produces ($11 + ((8 - 8) * (2 * 5))$)
 Numbers = {5, 13, 5, 5, 10} Goal = 12
 application of rule 5 produces ($12 + ((5 - 5) * (13 * 5))$)
 Numbers = {14, 2, 9, 2, 5} Goal = 6
 application of rule 5 produces ($6 + ((2 - 2) * (14 * 9))$)
 Numbers = {15, 0, 11, 13, 12} Goal = 1
 Numbers = {10, 9, 3, 3, 2} Goal = 3
 application of rule 5 produces ($3 + ((3 - 3) * (10 * 9))$)
 Numbers = {7, 8, 5, 15, 13} Goal = 9
 Numbers = {2, 15, 15, 11, 13} Goal = 1
 application of rule 4 produces ($(15 / 15) * (0 * (2 * 11))$)
 Numbers = {15, 15, 5, 9, 6} Goal = 6
 application of rule 5 produces ($6 + ((15 - 15) * (5 * 9))$)
 Numbers = {4, 13, 10, 1, 3} Goal = 8
 Numbers = {3, 3, 15, 1, 2} Goal = 5

application of rule 5 produces $(5 + ((3 - 3) * (15 * 1)))$
 Numbers = {0, 4, 0, 2, 11} Goal = 5
 application of rule 5 produces $(5 + ((0 - 0) * (4 * 2)))$
 Numbers = {2, 13, 14, 5, 5} Goal = 14
 application of rule 5 produces $(14 + ((5 - 5) * (2 * 13)))$
 Numbers = {1, 11, 8, 8, 7} Goal = 10
 application of rule 5 produces $(10 + ((8 - 8) * (1 * 11)))$
 Numbers = {11, 8, 5, 5, 2} Goal = 13
 application of rule 5 produces $(13 + ((5 - 5) * (11 * 8)))$
 Numbers = {14, 15, 0, 7, 9} Goal = 6
 Numbers = {10, 14, 7, 4, 6} Goal = 6
 Numbers = {5, 14, 12, 13, 11} Goal = 5
 Numbers = {7, 7, 13, 5, 9} Goal = 3
 application of rule 5 produces $(3 + ((7 - 7) * (13 * 5)))$
 Numbers = {10, 0, 8, 3, 12} Goal = 5
 Numbers = {3, 12, 6, 1, 13} Goal = 3
 Numbers = {5, 3, 9, 11, 14} Goal = 13
 Numbers = {9, 2, 12, 1, 1} Goal = 2
 application of rule 5 produces $(2 + ((1 - 1) * (9 * 12)))$
 Numbers = {13, 3, 13, 3, 9} Goal = 12
 application of rule 5 produces $(12 + ((13 - 13) * (3 * 3)))$
 Numbers = {0, 14, 1, 1, 7} Goal = 10
 application of rule 5 produces $(10 + ((1 - 1) * (0 * 14)))$
 Numbers = {4, 3, 12, 11, 5} Goal = 9
 Numbers = {4, 12, 0, 5, 8} Goal = 3
 application of rule 7 produces $(5 - ((8 / 4) + (12 * 0)))$
 Numbers = {2, 8, 8, 15, 11} Goal = 15
 application of rule 5 produces $(15 + ((8 - 8) * (2 * 11)))$
 Numbers = {14, 14, 0, 13, 10} Goal = 5
 application of rule 5 produces $(5 + ((14 - 14) * (0 * 13)))$
 Numbers = {9, 6, 2, 3, 4} Goal = 3
 Numbers = {15, 14, 12, 9, 10} Goal = 13
 Numbers = {6, 15, 4, 10, 7} Goal = 7
 Numbers = {0, 15, 14, 0, 8} Goal = 3
 application of rule 5 produces $(3 + ((0 - 0) * (15 * 14)))$
 Numbers = {9, 3, 4, 14, 8} Goal = 0
 Numbers = {15, 13, 14, 3, 10} Goal = 7
 Numbers = {10, 7, 10, 11, 14} Goal = 13
 application of rule 5 produces $(13 + ((10 - 10) * (7 * 11)))$

true .

?- demo(100).

Numbers = {5, 14, 6, 5, 15} Goal = 2

application of rule 5 produces ($2 + ((5 - 5) * (14 * 6))$)

Numbers = {13, 15, 11, 2, 8} Goal = 10

Numbers = {14, 15, 4, 9, 8} Goal = 10

Numbers = {7, 3, 7, 13, 3} Goal = 14

application of rule 5 produces ($14 + ((7 - 7) * (3 * 13))$)

Numbers = {1, 15, 14, 9, 2} Goal = 9

Numbers = {8, 14, 15, 9, 12} Goal = 15

Numbers = {14, 3, 13, 9, 10} Goal = 12

Numbers = {8, 9, 0, 14, 11} Goal = 15

Numbers = {10, 6, 2, 14, 8} Goal = 4

Numbers = {14, 8, 7, 7, 7} Goal = 3

application of rule 5 produces ($3 + ((7 - 7) * (14 * 8))$)

Numbers = {8, 14, 1, 12, 13} Goal = 12

Numbers = {4, 10, 15, 12, 7} Goal = 1

Numbers = {15, 15, 15, 9, 6} Goal = 5

application of rule 5 produces ($5 + ((15 - 15) * (15 * 9))$)

Numbers = {15, 8, 6, 3, 11} Goal = 12

Numbers = {2, 15, 11, 12, 0} Goal = 4

Numbers = {9, 8, 3, 9, 8} Goal = 3

application of rule 5 produces ($3 + ((9 - 9) * (8 * 8))$)

Numbers = {11, 2, 5, 9, 6} Goal = 15

Numbers = {7, 15, 7, 7, 13} Goal = 13

application of rule 5 produces ($13 + ((7 - 7) * (15 * 7))$)

Numbers = {8, 9, 0, 8, 8} Goal = 14

application of rule 5 produces ($14 + ((8 - 8) * (9 * 0))$)

Numbers = {5, 8, 12, 8, 15} Goal = 5

application of rule 5 produces ($5 + ((8 - 8) * (12 * 15))$)

Numbers = {15, 3, 3, 12, 14} Goal = 8

application of rule 5 produces ($8 + ((3 - 3) * (15 * 12))$)

Numbers = {5, 3, 13, 13, 7} Goal = 13

application of rule 5 produces ($13 + ((13 - 13) * (5 * 3))$)

Numbers = {12, 14, 10, 1, 5} Goal = 0

Numbers = {12, 1, 15, 9, 8} Goal = 0

Numbers = {6, 10, 14, 10, 6} Goal = 0

application of rule 3 produces ($(6 - 6) * (10 * (14 * 10))$)

Numbers = {12, 2, 8, 14, 10} Goal = 0
 Numbers = {6, 5, 11, 7, 1} Goal = 0
 Numbers = {13, 8, 3, 3, 3} Goal = 8
 application of rule 5 produces $(8 + ((3 - 3) * (13 * 3)))$
 Numbers = {10, 4, 2, 13, 2} Goal = 0
 application of rule 3 produces $((2 - 2) * (10 * (4 * 13)))$
 Numbers = {0, 0, 4, 14, 10} Goal = 9
 application of rule 5 produces $(9 + ((0 - 0) * (4 * 14)))$
 Numbers = {5, 12, 7, 8, 5} Goal = 7
 application of rule 5 produces $(7 + ((5 - 5) * (12 * 8)))$
 Numbers = {7, 15, 7, 10, 5} Goal = 13
 application of rule 5 produces $(13 + ((7 - 7) * (15 * 10)))$
 Numbers = {1, 7, 4, 12, 4} Goal = 9
 application of rule 5 produces $(9 + ((4 - 4) * (1 * 7)))$
 Numbers = {6, 10, 10, 13, 7} Goal = 7
 application of rule 5 produces $(7 + ((10 - 10) * (6 * 13)))$
 Numbers = {8, 0, 14, 13, 9} Goal = 11
 Numbers = {10, 13, 13, 7, 9} Goal = 10
 application of rule 5 produces $(10 + ((13 - 13) * (7 * 9)))$
 Numbers = {1, 1, 2, 6, 8} Goal = 7
 application of rule 5 produces $(7 + ((1 - 1) * (2 * 6)))$
 Numbers = {5, 6, 11, 3, 3} Goal = 11
 application of rule 5 produces $(11 + ((3 - 3) * (5 * 6)))$
 Numbers = {4, 7, 6, 4, 13} Goal = 13
 application of rule 5 produces $(13 + ((4 - 4) * (7 * 6)))$
 Numbers = {3, 5, 10, 6, 7} Goal = 14
 Numbers = {15, 11, 14, 14, 8} Goal = 7
 application of rule 5 produces $(7 + ((14 - 14) * (15 * 11)))$
 Numbers = {9, 14, 11, 14, 8} Goal = 11
 application of rule 5 produces $(11 + ((14 - 14) * (9 * 8)))$
 Numbers = {10, 8, 0, 7, 0} Goal = 1
 application of rule 4 produces $((0 / 0) * (0 * (10 * 8)))$
 Numbers = {7, 7, 15, 15, 5} Goal = 3
 application of rule 5 produces $(3 + ((7 - 7) * (15 * 15)))$
 Numbers = {7, 5, 2, 8, 15} Goal = 14
 Numbers = {6, 10, 12, 1, 6} Goal = 15
 application of rule 5 produces $(15 + ((6 - 6) * (10 * 12)))$
 Numbers = {2, 2, 2, 7, 14} Goal = 6
 application of rule 5 produces $(6 + ((2 - 2) * (2 * 7)))$

Numbers = {9, 7, 7, 9, 6} Goal = 8
 application of rule 5 produces ($8 + ((9 - 9) * (7 * 7))$)
 Numbers = {3, 4, 9, 2, 12} Goal = 11
 Numbers = {9, 1, 12, 6, 3} Goal = 15
 Numbers = {15, 9, 2, 6, 9} Goal = 11
 application of rule 5 produces ($11 + ((9 - 9) * (15 * 2))$)
 Numbers = {11, 6, 4, 13, 1} Goal = 5
 Numbers = {11, 9, 14, 13, 6} Goal = 2
 Numbers = {14, 15, 13, 4, 2} Goal = 11
 Numbers = {9, 3, 14, 3, 5} Goal = 12
 application of rule 5 produces ($12 + ((3 - 3) * (9 * 14))$)
 Numbers = {2, 8, 0, 4, 13} Goal = 14
 Numbers = {5, 12, 5, 7, 3} Goal = 0
 application of rule 3 produces ($(5 - 5) * (12 * (7 * 3))$)
 Numbers = {8, 7, 4, 14, 14} Goal = 11
 application of rule 5 produces ($11 + ((14 - 14) * (8 * 7))$)
 Numbers = {4, 13, 10, 5, 9} Goal = 9
 Numbers = {11, 10, 14, 13, 10} Goal = 14
 application of rule 5 produces ($14 + ((10 - 10) * (11 * 13))$)
 Numbers = {6, 1, 10, 7, 7} Goal = 7
 application of rule 5 produces ($7 + ((7 - 7) * (6 * 1))$)
 Numbers = {1, 8, 5, 10, 6} Goal = 1
 Numbers = {2, 7, 10, 3, 14} Goal = 9
 Numbers = {15, 10, 14, 13, 3} Goal = 12
 Numbers = {13, 7, 8, 12, 9} Goal = 4
 Numbers = {5, 9, 5, 6, 14} Goal = 10
 application of rule 5 produces ($10 + ((5 - 5) * (9 * 6))$)
 Numbers = {3, 6, 11, 3, 11} Goal = 10
 application of rule 5 produces ($10 + ((3 - 3) * (6 * 11))$)
 Numbers = {10, 10, 7, 6, 13} Goal = 12
 application of rule 5 produces ($12 + ((10 - 10) * (7 * 6))$)
 Numbers = {12, 15, 8, 1, 1} Goal = 2
 application of rule 5 produces ($2 + ((1 - 1) * (12 * 15))$)
 Numbers = {5, 13, 6, 11, 5} Goal = 1
 application of rule 4 produces ($(5 / 5) * (0 * (13 * 6))$)
 Numbers = {5, 10, 13, 0, 12} Goal = 3
 application of rule 7 produces ($5 - ((12 - 10) + (13 * 0))$)
 Numbers = {13, 10, 1, 15, 3} Goal = 2
 Numbers = {13, 14, 6, 13, 9} Goal = 2

application of rule 5 produces $(2 + ((13 - 13) * (14 * 6)))$
 Numbers = {12, 5, 4, 0, 3} Goal = 7
 Numbers = {4, 0, 4, 10, 10} Goal = 6
 application of rule 5 produces $(6 + ((4 - 4) * (0 * 10)))$
 Numbers = {15, 0, 14, 1, 3} Goal = 1
 application of rule 2 produces $(1 + (15 * (0 * (14 * 3))))$
 Numbers = {4, 11, 8, 6, 5} Goal = 14
 Numbers = {0, 1, 4, 0, 15} Goal = 13
 application of rule 5 produces $(13 + ((0 - 0) * (1 * 4)))$
 Numbers = {0, 13, 3, 13, 15} Goal = 12
 application of rule 5 produces $(12 + ((13 - 13) * (0 * 3)))$
 Numbers = {12, 7, 4, 14, 4} Goal = 9
 application of rule 5 produces $(9 + ((4 - 4) * (12 * 7)))$
 Numbers = {5, 2, 14, 6, 0} Goal = 10
 Numbers = {5, 15, 9, 15, 9} Goal = 9
 application of rule 5 produces $(9 + ((15 - 15) * (5 * 9)))$
 Numbers = {10, 6, 8, 12, 13} Goal = 12
 Numbers = {14, 3, 0, 2, 11} Goal = 6
 Numbers = {11, 1, 0, 5, 10} Goal = 2
 Numbers = {13, 13, 11, 9, 13} Goal = 5
 application of rule 5 produces $(5 + ((13 - 13) * (11 * 9)))$
 Numbers = {12, 11, 5, 3, 2} Goal = 3
 Numbers = {15, 12, 11, 4, 15} Goal = 11
 application of rule 5 produces $(11 + ((15 - 15) * (12 * 4)))$
 Numbers = {8, 4, 12, 9, 11} Goal = 15
 Numbers = {4, 8, 3, 2, 3} Goal = 2
 application of rule 5 produces $(2 + ((3 - 3) * (4 * 8)))$
 Numbers = {10, 3, 6, 13, 15} Goal = 6
 Numbers = {14, 11, 11, 2, 2} Goal = 8
 application of rule 5 produces $(8 + ((11 - 11) * (14 * 2)))$
 Numbers = {4, 7, 3, 3, 0} Goal = 3
 application of rule 2 produces $(3 + (4 * (7 * (3 * 0))))$
 Numbers = {8, 6, 15, 13, 3} Goal = 10
 Numbers = {13, 10, 13, 6, 11} Goal = 2
 application of rule 5 produces $(2 + ((13 - 13) * (10 * 6)))$
 Numbers = {0, 8, 6, 2, 3} Goal = 8
 application of rule 2 produces $(8 + (0 * (6 * (2 * 3))))$
 Numbers = {4, 10, 9, 14, 7} Goal = 13
 Numbers = {14, 11, 11, 10, 6} Goal = 6

application of rule 5 produces $(6 + ((11 - 11) * (14 * 10)))$

Numbers = {11, 5, 13, 14, 6} Goal = 11

Numbers = {4, 11, 2, 9, 11} Goal = 5

application of rule 5 produces $(5 + ((11 - 11) * (4 * 2)))$

true .