Angle Transformations for Double Photoemission

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1 Convention for angles

The following convention is adopted for all global angles:

- Right Handed coordinate system
- positive Z-axis along the manipulator (vertical) axis
- positive X-axis along Analyser 1 away from the sample
- positive Y-axis along Analyser 2 away from the sample
- θ-ϕ coordinate system is constructed with
  - $\theta = 0$ corresponding to the positive Z-pole
  - $\phi = 0$ corresponding to the positive X-axis

The convention for the detector angles is as follows:

- θ-ϕ coordinate system with
  - $\theta = 0$ corresponding to the center of the Analyser
  - $\phi = 0$ corresponding to the direction towards the manipulator (vertical) axis with rotation in the anticlockwise direction as seen towards $\theta = \pi$

The convention for angles wrt to sample normal is

- alpha is the analogue of the polar angle (theta) away from the sample normal;
- beta is the analoguqe of the azimuthal angle (phi) measured from the direction towards the z-axis pole towards

2 Angle Transformation

2.1 Analyser Coordinates to Global Coordinates

$$\cos\theta_g = \sin\theta_i \cos\phi_i \tag{1}$$

$$\sin(\phi_g - \phi_d) = \frac{\sin\phi_i \sin\theta_i}{\sin\theta_g} \tag{2}$$

$$\tag{3}$$

2.2 Global Coordinates to Sample Coordinates

$$\cos\alpha = \cos\theta_g \cos\theta_s + \sin\theta_g \sin\theta_s \cos\left(\phi_g - \phi_s\right) \tag{4}$$
$$\tag{5}$$

Defining $\text{hav}x = \sin^2 \frac{x}{2}$

$$\text{hav}\alpha = \text{hav}\left(\theta_g - \theta_s\right) + \sin\theta_g \sin\theta_s \text{hav}\left(\phi_g - \phi_s\right) \tag{6}$$
$$\sin(\frac{\alpha}{2}) = \sqrt{\sin^2 \frac{(\theta_g - \theta_s)}{2} + \sin\theta_g \sin\theta_s \sin^2 \frac{(\phi_g - \phi_s)}{2}} \tag{7}$$

$$\text{hav}\theta_g = \text{hav}\left(\theta_s - \alpha\right) + \sin\theta_s \sin\alpha \text{hav}\beta \tag{8}$$
$$\text{hav}\beta = \frac{\sin^2 \frac{\theta_g}{2} - \sin^2 \frac{\theta_s - \alpha}{2}}{\sin\theta_s \sin\alpha} \tag{9}$$
$$\text{hav}\beta = \frac{1}{2} \frac{\cos\left(\theta_s - \alpha\right) - \cos\theta_g}{\sin\theta_s \sin\alpha} \tag{10}$$
$$2\sin^2 \frac{\beta}{2} = -2 \frac{\sin\left(\frac{\theta_s - \alpha - \theta_g}{2}\right) \sin\left(\frac{\theta_s - \alpha + \theta_g}{2}\right)}{\sin\theta_s \sin\alpha} \tag{11}$$
$$\sin^2 \frac{\beta}{2} = \frac{\sin\left(\frac{\theta_g - \theta_s + \alpha}{2}\right)}{\sin\alpha} \frac{\sin\left(\frac{\theta_s + \theta_g - \alpha}{2}\right)}{\sin\theta_s} \tag{12}$$

Define $\gamma = \frac{\theta_g - \theta_s - \alpha}{2}$

$$\sin^2 \frac{\beta}{2} = \frac{\sin\left(\alpha + \gamma\right)}{\sin\alpha} \cdot \frac{\sin\left(\theta_s + \gamma\right)}{\sin\theta_s} \tag{13}$$