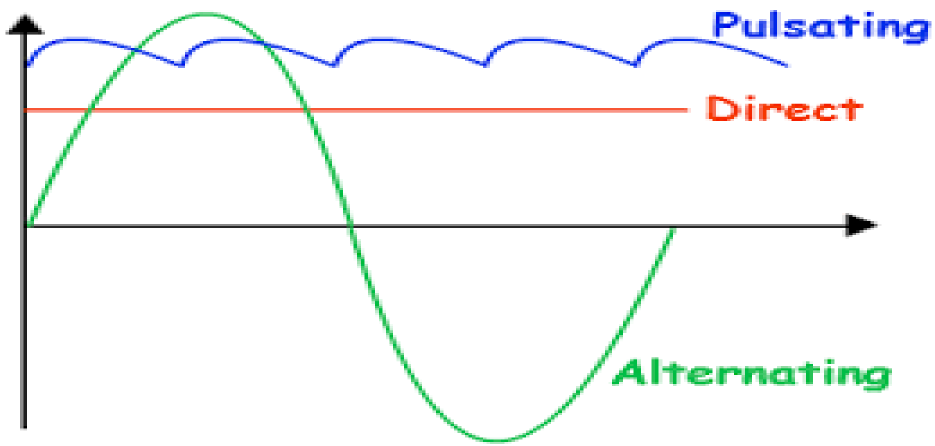
ALTERNATING CURRENT

AC Current:

In electricity, alternating current (AC) occurs when charge carriers in a conductor or semiconductor periodically reverse their direction of movement. Household utility current in most countries is AC with a frequency of 60 hertz (60 complete cycles per second), although in some countries it is 50 Hz.

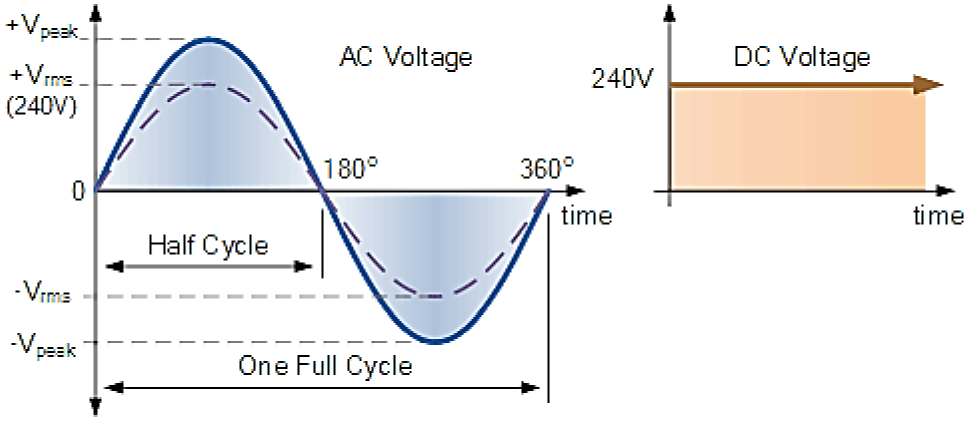


An AC waveform can be sinusoidal, square, or sawtooth-shaped.

R.M.S Voltage:

The term “RMS” stands for “Root-Mean-Squared”. Most books define this as the “amount of AC power that produces the same heating effect as an equivalent DC power”, or something similar along these lines, but an RMS value is more than just that. The RMS value is the square root of the mean (average) value of the squared function of the instantaneous values. The symbols used for defining an RMS value are V_{RMS} or I_{RMS}.

The term RMS, ONLY refers to time-varying sinusoidal voltages, currents or complex waveforms were the magnitude of the waveform changes over time and is not used in DC circuit analysis or calculations were the magnitude is always constant. When used to compare the equivalent RMS voltage value of an alternating sinusoidal waveform that supplies the same electrical power to a given load as an equivalent DC circuit, the RMS value is called the "effective value" and is generally presented as: V_{eff} or I_{eff}.



In other words, the effective value is an equivalent DC value which tells you how many volts or amps of DC that a time-varying sinusoidal waveform is equal to in terms of its ability to produce the same power. For example, the domestic mains supply in the United Kingdom is 240Vac. This value is assumed to indicate an effective value of "240 Volts RMS". This means then that the sinusoidal RMS voltage from the wall sockets of a UK home is capable of producing the same average positive power as 240 volts of steady DC voltage as shown below.

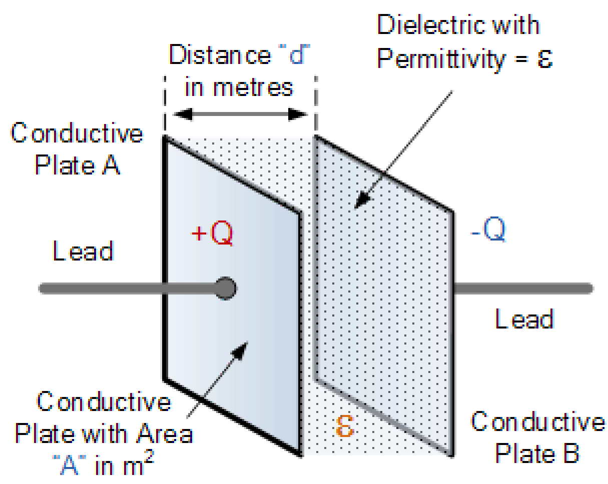
A periodic sinusoidal voltage is constant and can be defined as $V_{(t)} = Vm.\cos(\omega t)$ with a period of T. Then we can calculate the **root-mean-square** (rms) value of a sinusoidal voltage ($V_{(t)}$) as:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t)\, dt}$$

Capacitor:

The **Capacitor**, sometimes referred to as a **Condenser**, is a simple passive device that is used to "store electricity". The capacitor is a component which has the ability or "capacity" to store energy in the form of an electrical charge producing a potential difference (*Static Voltage*) across its plates, much like a small rechargeable battery.

Capacitor Construction:



$$\text{Capacitance, } C = \frac{\varepsilon_o \varepsilon_r A}{d} \text{ Farads}$$

In its basic form, a Capacitor consists of two or more parallel conductive (metal) plates which are not connected or touching each other, but are electrically separated either by air or by some form of a good insulating material such as waxed paper, mica, ceramic, plastic or some form of a liquid gel as used in electrolytic capacitors. The insulating layer between a capacitors plates is commonly called the **Dielectric**.

Also, because capacitors store the energy of the electrons in the form of an electrical charge on the plates the larger the plates and/or smaller their separation the greater will be the charge that the capacitor holds for any given voltage across its plates. In other words, larger plates, smaller distance, more capacitance.

By applying a voltage to a capacitor and measuring the charge on the plates, the ratio of the charge Q to the voltage V will give the capacitance value of the capacitor and is therefore given as: $C = Q/V$ this equation can also be re-arranged to give the more familiar formula for the quantity of charge on the plates as: $Q = C \times V$

Capacitance :

Capacitance is the electrical property of a capacitor and is the measure of a capacitors ability to store an electrical charge onto its two plates with the unit of capacitance being the **Farad** (abbreviated to F) named after the British physicist Michael Faraday.

Not only that, but capacitance is also the property of a capacitor which resists the change of voltage across it.

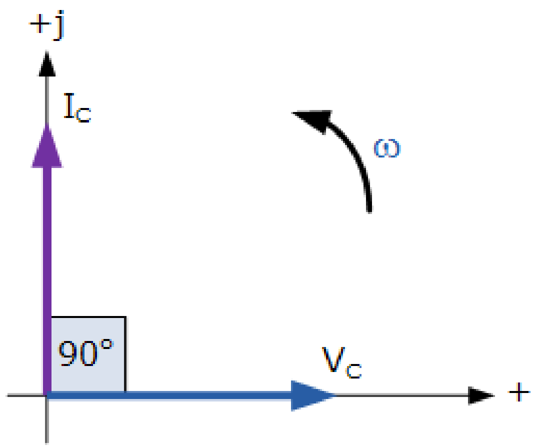
Capacitance is defined as being that a capacitor has the capacitance of **One Farad** when a charge of **One Coulomb** is stored on the plates by a voltage of **One volt**. Capacitance, C is always positive and has no negative units. However, the Farad is a very large unit of measurement to use on its own so sub-multiples of the Farad are generally used such as micro-farads, nano-farads and pico-farads, for example.

Basic Properties of a Capacitor:

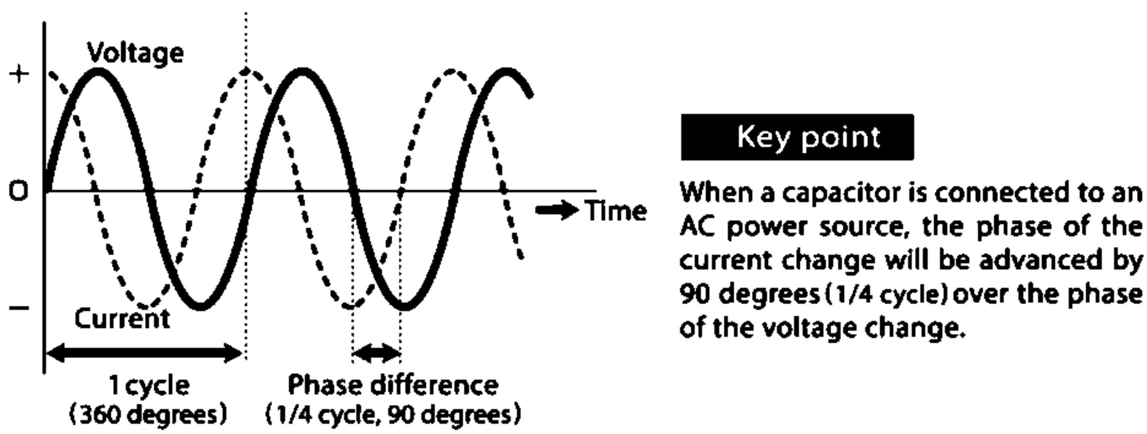
- Due to this insulating layer, DC current cannot flow through the capacitor as it blocks it allowing instead a voltage to be present across the plates in the form of an electrical charge.
- When used in a direct current or DC circuit, a capacitor charges up to its supply voltage but blocks the flow of current through it because the dielectric of a capacitor is non-conductive and basically an insulator. However, when a capacitor is connected to an alternating current or AC circuit, the flow of the current appears to pass straight through the capacitor with little or no resistance.
- There are two types of electrical charge, positive charge in the form of Protons and negative charge in the form of Electrons. When a DC voltage is placed across a capacitor, the positive (+ve) charge quickly accumulates on one plate while a corresponding negative (-ve) charge accumulates on the other plate. For every particle of +ve charge that arrives at one plate a charge of the same sign will depart from the -ve plate. Then the plates remain charge neutral and a potential difference due to this charge is established between the two

plates. Once the capacitor reaches its steady state condition an electrical current is unable to flow through the capacitor itself and around the circuit due to the insulating properties of the dielectric used to separate the plates.

➢



Voltage waveform and current waveform of sine wave alternating current



Where q=CV and i=dq/dt, Now if $V=V_m \sin\omega t$ then $q=C V_m \sin\omega t$ and $i=wCV_m \cos\omega t= I_m \sin(\pi/2 +\omega t)$

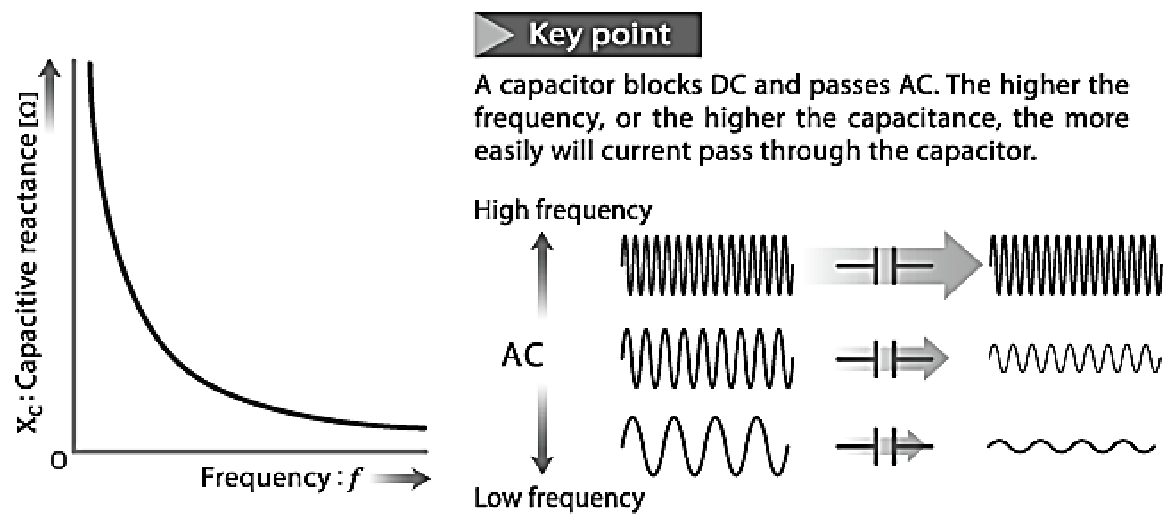
- ➢ One of the basic properties of a capacitor is that it blocks DC and passes AC. However, the ability to pass current is not the same for every kind of current. It depends on the frequency of the alternating current, as well as on the capacitance of the capacitor. The degree to which current can pass easily is indicated by a quantity called capacitive reactance (X_C)This is the resistance of the capacitor to alternating current, and it is expressed in ohms [Ω]. The equation for the capacitive reactance (X_C) of a capacitor is shown below.

< Capacitive reactance of a capacitor (X_c) >

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}\,[\Omega]$$

f : Frequency [Hz]
C : Capacitance [F]
$\omega = 2\pi f$

< Relationship between capacitive reactance and frequency characteristic of a capacitor >



- Instantaneous power of a capacitor

P=vi= V_m Sinωt * I_m Sin(π/2 +ωt)= V_m Sinωt* I_m Cosωt= (V_m I_m Sin 2ωt)/2

Now for whole cycle = V_m $I_m/2\int_0^{2\pi} Sin2wt$ = 0

So, No power dissipation in an Ideal Capacitor.

5.18. Charging of a Capacitor

In Fig. 5.29. (*a*) is shown an arrangement by which a capacitor C may be charged through a high resistance R from a battery of V volts. The voltage across C can be measured by a suitable voltmeter. When switch S is connected to terminal (*a*), C is charged but when it is connected to b, C is short circuited through R and is thus discharged. As shown in Fig. 5.29. (*b*), switch S is shifted to a for charging the capacitor for the battery. The voltage across C does not rise to V instantaneously but builds up slowly *i.e.* exponentially and not linearly. Charging current i_c is maximum at the start *i.e.* when C is uncharged, then it decreases exponentially and finally ceases when p.d. across capacitor plates becomes equal and opposite to the battery voltage V. At any instant during charging, let

v_c = p.d. across C; i_c = charging current

q = charge on capacitor plates

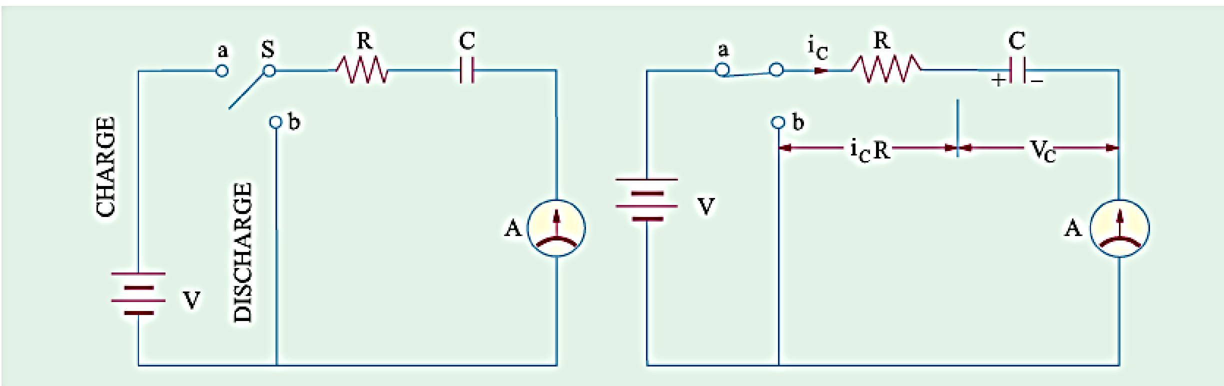


Fig. 5.29

The applied voltage V is always equal to the sum of :

(*i*) resistive drop ($i_c R$) and (*ii*) voltage across capacitor (v_c)

$\therefore$ $$V = i_c R + v_c \qquad \text{...(i)}$$

Now $$i_c = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C\frac{dv_c}{dt} \quad \therefore \; V = v_c + CR\frac{dv_c}{dt} \qquad \text{...(ii)}$$

or $$-\frac{dv_c}{V - v_c} = -\frac{dt}{CR}$$

Integrating both sides, we get $\int \frac{-d\,V_c}{V - v_c} = -\frac{1}{CR}\int dt$; $\therefore$ $\log_e (V - v_c) = -\frac{t}{CR} + K$ ...(*iii*)

where K is the constant of integration whose value can be found from initial known conditions. We

Substituting these values in *(iii)*, we get $\log_c V = K$

Hence, Eq. *(iii)* becomes $\log_e (V - v_c) = \dfrac{-t}{CR} + \log_e V$

or $$\log_c \frac{V - v_c}{V} = \frac{-t}{CR} = -\frac{1}{\lambda} \quad \text{where } \lambda = CR = \text{time constant}$$

$$\therefore \qquad \frac{V - v_c}{V} = e^{-t/\lambda} \quad \text{or} \quad v_c = V\,(1 - e^{-t/\lambda}) \qquad \text{...(iv)}$$

This gives variation with time of voltage across the capacitor plates and is shown in Fig. 5.27.(*a*)

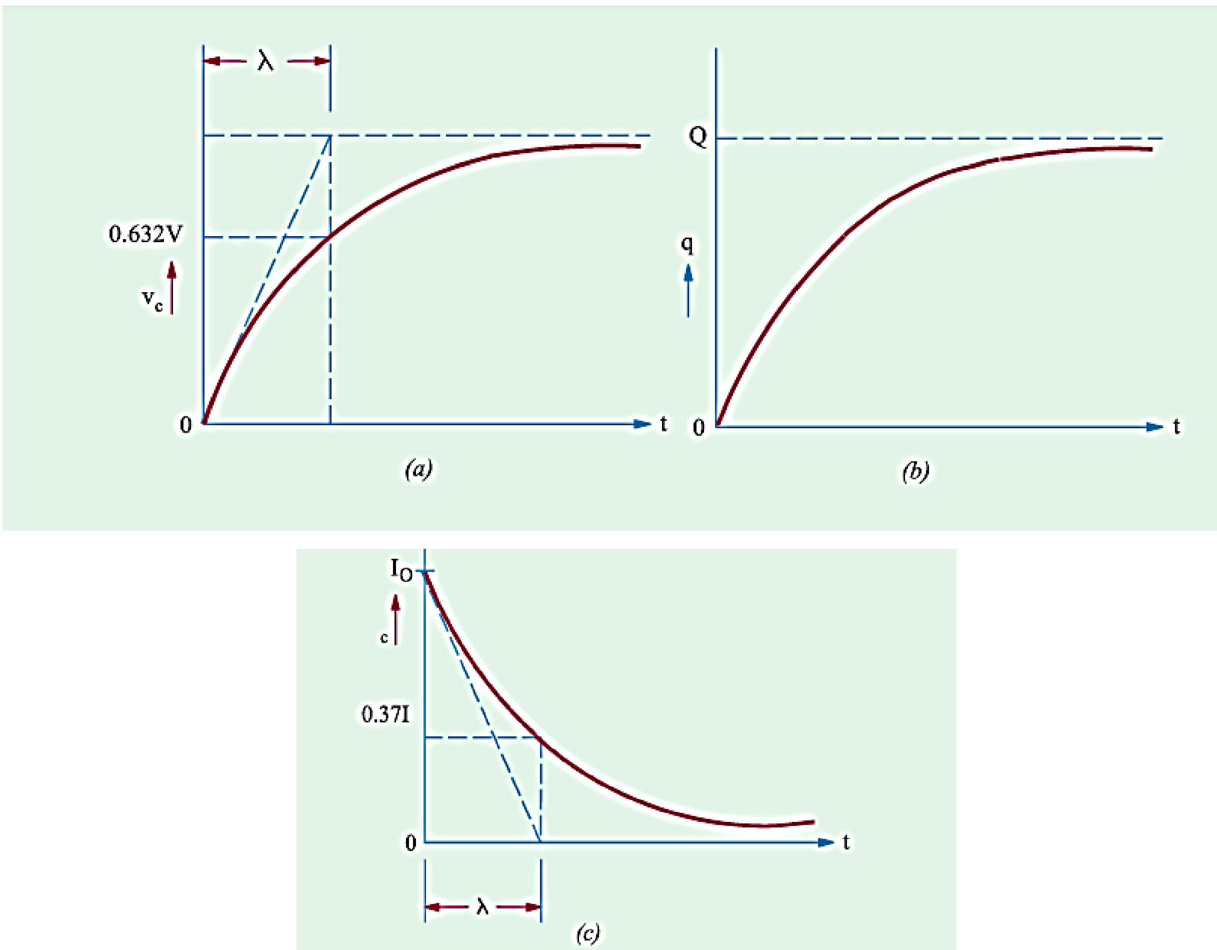


Fig. 5.30

Now $\qquad v_c = q/C$ and $V = Q/C$

Equation (*iv*) becomes $\dfrac{q}{c} = \dfrac{Q}{c}(1 - e^{-t/\lambda}) \;\therefore\; q = Q\,(1 - e^{-t/\lambda})$ ...(*v*)

We find that increase of charge, like growth of potential, follows an exponential law in which the steady value is reached after infinite time (Fig. 5.30 *b*). Now, $i_c = dq/dt$.

Differentiating both sides of Eq. (*v*), we get

$$\frac{dq}{dt} = i_c = Q\frac{d}{dt}(1 - e^{-t/\lambda}) = Q\left(+\frac{1}{\lambda}e^{-t/\lambda}\right)$$

$$= \frac{Q}{\lambda}e^{-t/\lambda} = \frac{CV}{CR}e^{-t/\lambda} \qquad (\because\ Q = CV \text{ and } \lambda = CR)$$

$$\therefore \qquad \boxed{i_c = \frac{V}{R}.e^{-t/\lambda} \text{ or } i_c = I_o\,e^{-t/\lambda}} \qquad \text{...(vi)}$$

where I_0 = maximum current = V/R

Exponentially rising curves for v_c and q are shown in Fig. 5.30 (*a*) and (*b*) respectively. Fig. 5.30 (*c*) shows the curve for exponentially decreasing charging current. It should be particularly noted that i_c decreases in magnitude only but its direction of flow remains the same *i.e.* positive.

As charging continues, charging current decreases according to equation (*vi*) as shown in Fig. 5.30 (*c*). It becomes zero when $t = \infty$(though it is almost zero in about 5 time constants). Under steady-state conditions, the circuit appears only as a capacitor which means it acts as an open-circuit. Similarly, it can be proved that v_R decreases from its initial maximum value of V to zero exponentially as given by the relation $v_R = V\,e^{-t/\lambda}$.

5.19. Time Constant

(*a*) Just at the start of charging, p.d. across capacitor is zero, hence from (*ii*) putting $v_c = 0$, we get

$$\boxed{V = CR\frac{dv_c}{dt}}$$

$\therefore$ initial rate of rise of voltage across the capacitor is* $= \left(\dfrac{dv_c}{dt}\right)_{t=0} = \dfrac{V}{CR} = \dfrac{V}{\lambda}$ volt/second

If this rate of rise were maintained, then time taken to reach voltage V would have been

Maximum voltage / initial rate of charging = V/ V/CR= CR which is known as time constant.

Hence, time constant of an *R-C* circuit is defined as ***the time during which voltage across capacitor would have reached its maximum value V had it maintained its initial rate of rise.***

(*b*) In equation (*iv*) if $t = \lambda$, then

$$v_c = V\,(1 - e^{-t/\lambda}) = V\,(1 - e^{-t/\lambda}) = V(1 - e^{-1}) = V\left(1 - \frac{1}{e}\right) = V\left(1 - \frac{1}{2.718}\right) = \mathbf{0.632\ V}$$

Hence, time constant may be defined as ***the time during which capacitor voltage actually rises to 0.632 of its final steady value.***

(*c*) From equaiton (*vi*), by putting $t = \lambda$, we get

$$i_c = I_0\, e^{-\lambda/\lambda} = I_0\, e^{-1} = I_0/2.718 \cong 0.37\, I_0$$

Hence, the constant of a circuit is also the ***time during which the charging current falls to 0.37 of its initial maximum value (or falls by 0.632 of its initial value).***

5.20. Discharging of a Capacitor

As shown in Fig. 5.31 (*a*), when *S* is shifted to *b*, *C* is discharged through *R*. It will be seen that the discharging current flows in a direction opposite to that the charging current as shown in Fig. 5.31(*b*). Hence, if the direction of the charging current is taken positive, then that of the discharging current will be taken as negative. To begin with, the discharge current is maximum but then decreases exponentially till it ceases when capacitor is fully discharged.

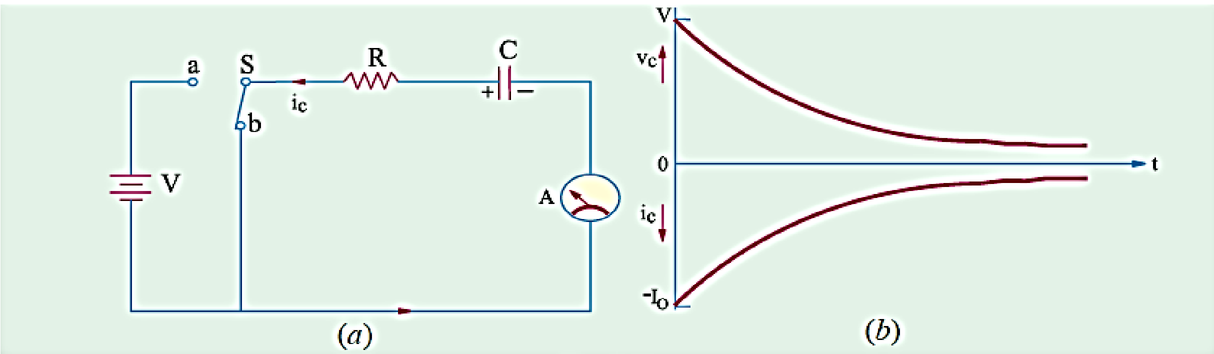


Fig. 5.31

Since battery is cut of the circuit, therefore, by putting $V = 0$ in equation (*ii*) of Art. 5.18, we get

$$0 = CR\frac{dv_c}{dt} \quad v_c \text{ or } v_c \quad CR\frac{dv_c}{dt} \qquad \left(i_c = C\frac{dv_c}{dt}\right)$$

$$\therefore \quad \frac{dv_c}{v_c} = \frac{dt}{CR} \text{ or } \frac{dv_c}{v_c} \quad \frac{1}{CR} \quad dt \quad \log_e v_e \quad \frac{t}{CR} \quad k$$

At the start of discharge, when $t = 0$, $v_c = V \therefore \log_e V = 0 + K$; or $\log_e V = K$

Putting this value above, we get

$$\log_e \quad v_c = -\frac{t}{\lambda} + \log_e V \text{ or } \log_e v_c/V = -t/\lambda$$

$$\text{or} \quad \frac{v_c}{V} = e^{-t/\lambda} \text{ or } v_c = Ve^{-t/\lambda}$$

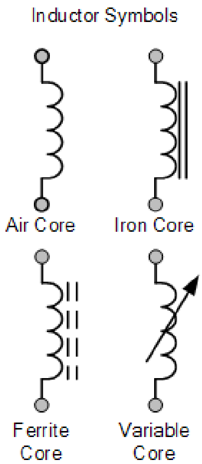
Similarly, $\quad q = Q\,e^{-t/\lambda}$ and $i_c = -I_0\,e^{-t/\lambda}$

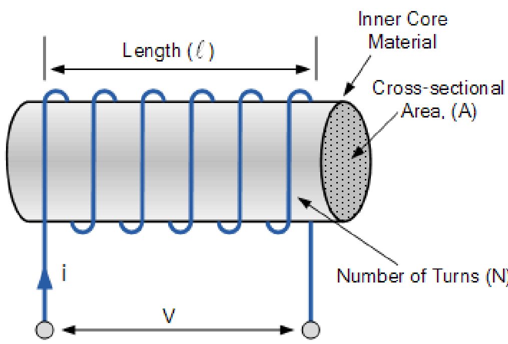
It can be proved that

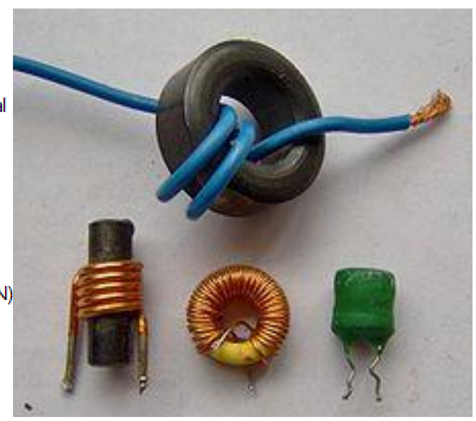
$$v_R = -V\,e^{-t/\lambda}$$

Inductor:

An inductor, also called a coil, choke or reactor, is a passive two-terminal electrical component which resists changes in electric current passing through it. It consists of a conductor such as a wire, usually wound into a coil. When a current flows through it, energy is stored temporarily in a magnetic field in the coil. When the current flowing through an inductor changes, the time-varying magnetic field induces a voltage in the conductor, according to Faraday's law of electromagnetic induction, which opposes the change in current that created it. As a result, inductors always oppose a change in current. Many inductors have a magnetic core made of iron or ferrite inside the coil, which serves to increase the magnetic field and thus the inductance.







Inductance:

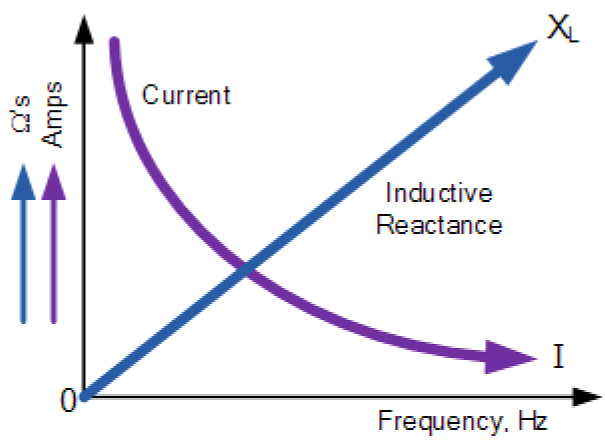
An inductor is characterized by its *inductance*, the ratio of the voltage to the rate of change of current, which has units of henries (H). Inductors have values that typically range from 1 µH (10^{-6}H) to 1 H.

Basic Properties of Inductor:

- When a current flows through it, energy is stored temporarily in a magnetic field in the coil. When the current flowing through an inductor changes, the time-varying magnetic field induces a voltage in the conductor, according to Faraday's law of electromagnetic induction, which opposes the change in current that created it. As a result, inductors always oppose a change in current.

- Along with capacitors and resistors, inductors are one of the three passive linear circuit elements that make up electric circuits. Inductors are widely used in alternating current (AC) electronic equipment, particularly in radio equipment. They are used to block AC while allowing DC to pass; inductors designed for this purpose are called chokes. As Inductive impedance $X_L = 2\pi fL$, where f is frequency and L is Inductance.

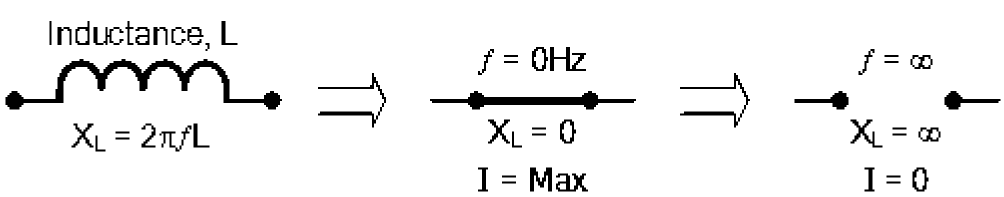
- Inductive Reactance against Frequency



The inductive reactance of an inductor increases as the frequency across it increases therefore inductive reactance is proportional to frequency ($X_L \alpha f$) as the back emf generated in the inductor is equal to its inductance multiplied by the rate of change of current in the inductor.

Also as the frequency increases the current flowing through the inductor also reduces in value.

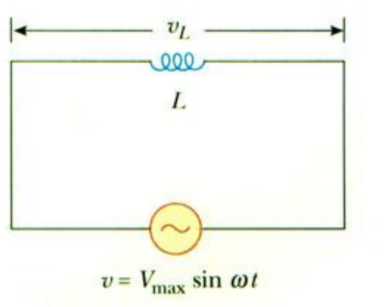
We can present the effect of very low and very high frequencies on a the reactance of a pure AC Inductance as follows:

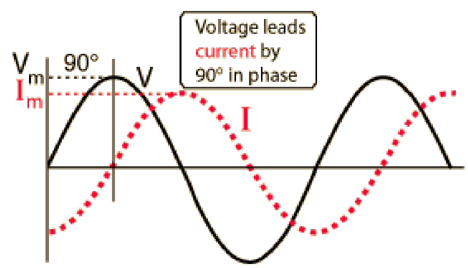


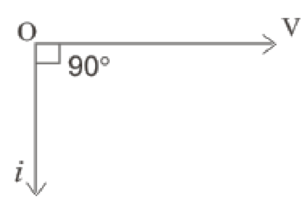
In an AC circuit containing pure inductance the following formula applies:

$$\text{Current, I} = \frac{\text{Voltage}}{\text{Opposition to current flow}} = \frac{V}{X_L}$$

- Suppose an ideal AC voltage source is connected across a pure inductor as shown:







The voltage source is

$$V = V_0 \sin(\omega t).$$

From Kirchhoff's loop rule, a pure inductor obeys

$$V_0 \sin(\omega t) = L\,di/dt,$$

So

$$di/dt = V_0 / L \sin(\omega t)$$

Whose solution is

$$I = -V_0/\omega L \cos(\omega t) + C$$

$$= I_o \sin(\omega t - \pi/2) + C$$

- Instantaneous power of a inductor

$P = vi = V_m \, Sin\omega t * I_m \, Sin(\pi/2 - \omega t) = -V_m \, Sin\omega t * I_m \, Cos\omega t = -(V_m I_m \, Sin\, 2\omega t)/2$

Now for whole cycle $= -V_m I_m/2 \int_0^{2\pi} Sin 2\omega t = 0$

So, No power dissipation in an Ideal inductor.

8.10. Rise of current in an Inductive Circuit

In Fig. 8.10 is shown a resistance of R in series with a coil of self-inductance L henry, the two being put across a battery of V volt. The R-L combination becomes connected to battery when switch S is connected to terminal 'a' and is short-circuited when S is connected to 'b'. The inductive coil is assumed to be resistanceless, its actual small resistance being included in R.

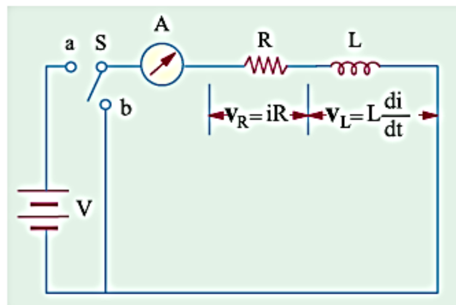


Fig. 8.10

When S is connected to 'a' the R-L combination is suddenly put across the voltage of V volt. Let us take the instant of closing S as the starting zero time. It is easily explained by recalling that the coil possesses electrical inertia *i.e.* self-inductance and hence, due to the production of the counter e.m.f. of self-inductance, delays the instantaneous full establishment of current through it.

We will now investigate the growth of current i through such an inductive circuit.

The applied voltage V must, at any instant, supply not only the ohmic drop iR over the resistance R but must also overcome the e.m.f. of self inductance *i.e.* $L di/dt$.

$$\therefore \qquad V = v_R + v_L = iR + \frac{di}{dt}$$

$$\text{or} \qquad (V - iR) = L\frac{di}{dt} \quad \therefore \quad \frac{di}{V - iR} = \frac{dt}{L}. \qquad \text{...}(i)$$

Multiplying both sides by $(-R)$, we get $(-R)\dfrac{di}{(V - iR)} = -\dfrac{R}{L}dt$

Integrating both sides, we get $\displaystyle\int \frac{(-R)\,di}{(V - iR)} = \int dt \;\; \therefore \;\; \log_e^{V - iR} = -\frac{R}{L}t + K$ ...(*ii*)

where e is the Napierian logarithmic base = 2.718 and K is constant of integration whose value can be found from the initial known conditions.

To begin with, when $t = 0$, $i = 0$, hence putting these values in *(ii)* above, we get

$$\log_e^{V} = K$$

Substituting this value of K in the above given equation, we have

$$\log_e^{V-iR} = \frac{R}{L}t + \log_e^{V} \quad \text{or} \quad \log_e^{V-iR} - \log_e^{V} = -\frac{R}{L}t$$

or
$$\log_e \frac{V-iR}{V} = -\frac{R}{L}t = -\frac{1}{\lambda} \quad \text{where } L/R = \lambda \text{ ‘time constant’}$$

$$\therefore \quad \frac{V-iR}{V} = e^{-t/\lambda} \text{ or } i = \frac{V}{R}(1-e^{-t/\lambda})$$

Now, V/R represents the maximum steady value of current I_m that would eventually be established through the R-L circuit.

$$\therefore \quad i = I_m\,(1 - e^{-t/\lambda}) \qquad \ldots(iii)$$

This is an exponential equation whose graph is shown in Fig. 8.11. It is seen from it that current rise is rapid at first and then decreases until at $t = \infty$ it becomes zero. Theoretically, current does not reach its maximum steady value I_m until infinite time. However, in practice, it reaches this value in a relatively short time of about 5λ.

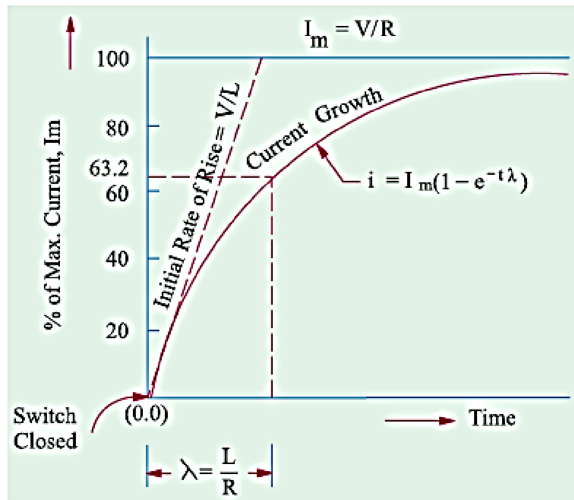


Fig. 8.11

The rate of rise of current di/dt at any stage can be found by differentiating Eq. *(ii)* above w.r.t. time. However, the *initial* rate of rise of

$$\therefore \quad \frac{i}{I_m} = e^{-t/\lambda}$$

or
$$i = I_m e^{-t/\lambda} \qquad \ldots(ii)$$

It is decaying exponential function and is plotted in Fig. 8.12. It can be shown again that theoretically, current should take infinite time to reach zero value although, in actual practice, it does so in a relatively short time of about 5λ.

Again, putting $t = \lambda$ in Eq. *(ii)* above, we get

$$i = \frac{I_m}{e} = \frac{I_m}{2.178} = 0.37\, I_m.$$

Hence, time constant (λ) of an R-L circuit may also be defined as the time during which current falls to 0.37 or 37% of its maximum steady value while decaying (Fig. 8.12).

current can be obtained by putting $t = 0$ and $i = 0$ in (*i*)* above.

$$\therefore \quad V = 0 \times R + L\frac{di}{dt} \quad \text{or} \quad \frac{di}{dt} = \frac{V}{L}$$

The constant $\lambda = L/R$ is known as the ***time-constant*** of the circuit. It can be variously defined as :

(*i*) It is the ***time*** during which current would have reached its maximum value of I_m (= V/R) had it maintained its initial rate of rise.

$$\text{Time taken} = \frac{I_m}{\text{initial rate of rise}} = \frac{V/R}{V/L} = \frac{L}{R}$$

But actually the current takes more time because its rate of rise decreases gradually. In actual practice, in a time equal to the time constant, it merely reaches 0.632 of its maximum values as shown below :

Putting $t = L/R = \lambda$ in Eq. (*iii*) above, we get

$$i = I_m\,(1 - e^{-\lambda/\lambda}) = I_m\left(1 - \frac{1}{e}\right) = I_m\left(1 - \frac{1}{2.718}\right) = 0.632\ I_m$$

(*ii*) Hence, the time-constant λ of an R-L circuit may also be defined as the time during which the current ***actually*** rises to 0.632 of its maximum steady value (Fig. 8.11).

This delayed rise of current in an inductive circuit is utilized in providing time lag in the operation of electric relays and trip coils etc.

8.11. Decay of Current in an Inductive Circuit

When the switch S (Fig. 8.10) is connected to point 'b', the R-L circuit is short-circuited. It is found that the current does not cease immediately, as it would do in a non-inductive circuit, but continues to flow and is reduced to zero only after an appreciable time has elapsed since the instant of short-circuit.

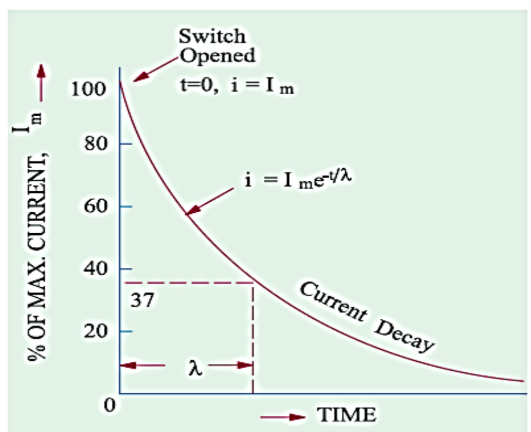


Fig. 8.12

The equation for decay of current with time is found by putting $V = 0$ in Eq. (*i*) of Art. 8.10

$$0 = iR + L\frac{di}{dt} \quad \text{or} \quad \frac{di}{i} = -\frac{R}{L}\,dt$$

Integrating both sides, we have

$$\int \frac{di}{i} = -\frac{R}{L}\int dt \quad \therefore \quad \log i = \frac{R}{L}t + K \qquad \text{...}(i)$$

Now, at the instant of switching off current, $i = I_m$ and if time is counted from this instant, then $t = 0$

$$\therefore \qquad \log_e I_m = 0 + K$$

Putting the value of K in Eq (*i*) above, we get,

$$\log_e i = -\frac{t}{\lambda} = \log_e I_m$$

$$\therefore \quad \log_e i/I_m = -\frac{t}{\lambda}$$

$$\therefore \qquad \frac{i}{I_m} = e^{-t/\lambda}$$

$$\text{or} \qquad i = I_m e^{-t/\lambda} \qquad \text{...}(ii)$$

It is decaying exponential function and is plotted in Fig. 8.12. It can be shown again that theoretically, current should take infinite time to reach zero value although, in actual practice, it does so in a relatively short time of about 5λ.

Again, putting $t = \lambda$ in Eq. (*ii*) above, we get

$$i = \frac{I_m}{e} = \frac{I_m}{2.178} = 0.37\ I_m.$$

Hence, time constant (λ) of an R-L circuit may also be defined as the time during which current falls to 0.37 or 37% of its maximum steady value while decaying (Fig. 8.12).