

# Matching Theory and Virtual Machines

Kristen Hines  
School of Electrical and  
Computer Engineering  
Virginia Tech  
Email: kphines@vt.edu

Ferdinando Romano  
School of Electrical and  
Computer Engineering  
Virginia Tech  
Email: fmromano@vt.edu

**Abstract**—The assignment of jobs to separate compute clusters can be approached using matching theory. The problem is modeled as a modified college admissions game where each institution has multiple quotas, each of which is of a specified type. An algorithm based on multiple iterations of the college admissions deferred acceptance algorithm is proposed. The algorithm is shown to terminate, result in a stable matching, and, under certain common conditions, approximate an optimal solution. The proposed algorithm is simulated and shown to result in either significant improvement or only minor regression compared to other approaches.

## I. INTRODUCTION

Cloud computing is a system that allows ubiquitous IT services, ranging from online social networking services to infrastructure outsourcing. This is because cloud computing is expected to be a cost-effective and flexible way to handle data and programs. These cloud computing services are packaged in the form of virtual machines through the use of virtualization technology. Virtualization technology allows for computing technologies to be virtualized by emulating processors, main memory, storage, and networking devices [resource needed]. The final product after this virtualization is the virtual machine.

One of the benefits of virtual machines, especially ubiquitous ones, is they can be configured to suit a specific application's needs, such as application isolation, security requirements, service level-agreements, and computational performance. [resource needed] These virtual machines and cloud computing servers are still housed on powerful physical machines at this point. [need to figure out the logic between here and our problem...]

This project is focused on turning a simple virtual machine job assignment problem into a college admissions game. The jobs will be the applicants, and the virtual machines will be institutions. The jobs will apply for spots on the virtual machines until either there are no more jobs in the queue, or there are no more virtual machines created and available for the users at this time. It is assumed that the virtual machines take a long enough time to create that a user will not want to just make one once all of the already made virtual machines are taken. In this paper, a computer-optimal algorithm will be proposed to provide a solution that is better for both the virtual machines and the jobs. More will be stated in section three, Method Description.

The paper will proceed as follows. The second section will cover the background knowledge for matching games and work that is related to virtual machines and cloud computing. The

third section will cover the problem formulation and proposed algorithm. The fourth section will cover the results. The fifth section will be the conclusion of the work here. The sixth section will give possible future routes this work can take.

## II. BACKGROUND

A review about the stable marriage problem is needed to help the reader understand the proposed solution. The stable marriage problem is a one-to-one matching model used to effectively match two groups of agents together, such as men and women for a marriage. These agents are two disjoint sets. Each agent has a complete, strict, and transitive preference over other individuals, which means the agent is not indifferent for any choices. In addition to this quality, each agent has a chance of being unmatched. For demonstration purposes, the two agent sets are men and women, whose sets are  $W = \{w_1, w_2, \dots, w_p\}$  and  $M = \{m_1, m_2, \dots, m_n\}$ , where  $p$  does not have to equal  $n$ .

Their preferences are arranged and represented as ranked ordered lists. An example of such a list is  $p_{m_n} = w_2, w_4, \dots, \emptyset$ , where  $w_2$  is the man's first choice for a partner,  $w_4$  is the second, and so on. In this case, the final choice represents when the man prefers to be single over his possible choices.

*Definition 1:* An outcome is a matching  $\mu : M \times W \rightarrow M \times W$  such that  $w = \mu(m)$  if and only if  $m = \mu(w) \in W \cup \emptyset$ ,  $\mu(m) \in M \cup \emptyset$  for all  $m, w$ .

This implies that agents from one set are matched to either the agents of the other set or to the null set. Agents' preferences over outcomes are determined only by their own preferences for certain partners.

*Definition 2:* A matching  $\mu$  is stable if and only if it is individual rational and not blocked by any pair of agents.

Individual rational and blocking pair are defined as followed.

*Definition 3:* A matching set is individual rational to all agents if and only if there does not exist an agent  $i$  who prefers being unmatched to being matched.

*Definition 4:* A matching  $\mu$  is blocked by a pair of agents  $(m, w)$  if they prefer each other to the partner they receive at  $\mu$ . That is,  $w \succ_m \mu(m)$  and  $m \succ_w \mu(w)$ . Such a pair is called a blocking set, where  $\succ$  represents an agent's preference of one individual over another.

This means that as long as a matching is not blocked and a matching set is individually rational, a matched set will be stable. Therefore:

*Theorem 1:* A stable matching pair exists for every marriage market.

This theorem was proposed and proved by Gale and Shapley by using their deferred acceptance algorithm, which was proven in [1].

The college applications game extends the concepts behind stable marriage. For a college applications game, the two sets of players are the institutions and the applicants. Institutions can be matched to multiple applicants at one time.

*Theorem 2:* Every applicant is at least as well off under the assignment given by the deferred acceptance procedure as he would be under any other stable arrangement.

The end result will be a stable and optimal pairing between the institutions and the applicants. This is also proven in [1]. One of the issues with the stable marriage problem and the college admissions problem is that they are applicant optimal. In other words, the set who is being applied to, such as the women in the earlier example, are not guaranteed their optimal choices. If the men and women switched, and the women were the ones proposing to the men, the men would not be guaranteed their optimal choices, either.

### III. METHOD DESCRIPTION

High level description if needed.

#### A. Problem Formulation

The problem this paper is exploring is how to optimize job assignment to separate computer clusters. Each of these jobs perform differently on different cores types. These core types can be graphical processors, computational processors, or something else. Each of these clusters have different core types. Each job can only be divided into a finite number of threads, and each job is assigned to one computer at a time.

This problem can be formed as a college admissions game. The differences are that the institutions have multiple quotas, each applicant can fill multiple slots of different types, applicants prefer different slot types over others, and an applicant cannot be divided among multiple institutions.

Key assumptions for this problem are: the virtual machines will be treated like flexible computers, jobs are submitted at the same time, chosen jobs are completed simultaneously, unchosen jobs will be submitted with the next round, no indifference, and no externalities.

#### B. Proposed Algorithm

Things to do: Write the math and assumptions, state why we are doing this, and show that this will be stable as well.

The proposed algorithm is based on a deferred acceptance college admissions algorithm with special modifications to optimize it for the situation where the institutions have multiple quotas of different types and the applicants can fill multiple slots.

*1) Algorithm:* Step 1: Calculate the preferences of the jobs (applicants) and the computers (institutions). A job's preference for a particular computer is determined simply: given the processors available on each computer, if a job would perform faster on one computer than another, then the first computer is preferred over the second. Given the relative speeds of each processor at performing a particular job, the speed of that job on a particular computer is calculated by first choosing the fastest available processors until the job's processor limit is reached or there are no more processors available on the computer. Then, the speeds of the chosen processors are summed together to give the computer's total speed at the job. A job prefers one computer over another if its total speed is higher than the other's.

A computer's preference for a particular job is based on the assumption that a computer wants to maximize utilization of its resources. In this case, a job that can utilize more processors than another is preferable. It is assumed that the number of processors that a job can use does not depend on the computer or the processor types and so each computer has the same preference ranking of jobs.

Step 2: Perform a 1-1 Matching. The jobs are matched to the computers according to the calculated preferences using a college admissions algorithm where the quota of a computer is set to 1 if it has at least 1 processor still available. Otherwise, its quota is set to 0.

Step 3: Determine the most important matching. A job that can use the greatest number of processors is the most highly preferred and so it is matched to its first choice of computer. Thus this pair is stable and can be assigned to the finalized matching of the algorithm. This job and the processors it uses are no longer available so they are removed from future iterations of the algorithm.

Step 4: Return to Step 1. The algorithm is repeated either until all processors are assigned a job or until all jobs are matched to a computer.

*2) Guaranteed of Termination:* The algorithm is guaranteed to terminate because there is a finite number of jobs and each iteration of the algorithm matches one job to a computer.

*3) Stability of Algorithm:* The proposed algorithm produces a stable matching because in each iteration, the college admissions game is used to find a set of stable matchings. Out of the jobs listed in the resultant set of stable matchings, the job that can use the most processors is preferred most by every computer. Thus, that job will be matched with its first choice and its matching to a computer is a stable matching. Thus, each pair produced by an iteration of the proposed algorithm is stable and therefore the final matching is stable.

*4) Optimality of Algorithm:* Whether the matching is optimal can be understood in multiple senses. In this section, three different approaches to optimality are discussed as they apply to the proposed algorithm.

Resource Utilization: A simple goal of the proposed algorithm would be to maximize processor utilization so that no computing resources go unused/wasted.

The proposed algorithm does not always maximize processor utilization. However, it does in every iteration where

the preferred computer of the job with the greatest possible processor utilization has at least as many processors available as either i) that job can use or ii) any other computer has. This situation is common because, often, a computer with more available processors will outperform one with fewer. The exceptions occurs where there is a computer that has special purpose processors that significantly outperform those available at other computers and this computer does not meet either of conditions i) or ii) listed above.

**Total Job Completion Time:** The total computation time, i.e., the sum of total computation times for each job, is another good measure of the optimality of the proposed algorithm.

Assuming individual jobs cannot take advantage of processors previously used by other jobs that have completed, the proposed algorithm minimizes total computation time whenever jobs that use more processors are jobs that would take longer to complete than any other job. By 'take longer to complete', we mean take longer than other jobs if the other jobs were to use a subset or superset of the processors used by the first job. When this condition is met, the job that takes the longest is given the greatest speed possible, the job that takes the 2nd longest is given the next greatest speed possible for it, and so on. Thus, total computation time is minimized.

In the proposed algorithm, this condition that jobs use more processors take longer is not guaranteed. However, it is strongly encouraged by the proportional fairness of the algorithm: Jobs that would take longer to complete are incentivized to be able to use more processors.

**Proportional Fairness:** In the proposed algorithm, jobs' individual computation times/total required processing are not factored into the preferences and so have no bearing on the matchings. Instead, it is the processor utilization ability of a job that effects its ranking. This leads to a proportional fairness in which jobs that are shorter are still given a fair amount of processing power so that they will not take very long. On the other hand, jobs that require more processing power, i.e. would take longer, are incentivized to be able to use more processors than jobs that do not take as long.

For a job that would take time to complete  $\tau_1 > \tau_2$ , where  $\tau_2$  is the completion time for a second job, Job 1 would reduce its completion time by an absolute amount  $\Delta\tau_1 = \tau_1 - \frac{\tau_1}{f}$  if it could increase its speed by a factor  $f$ . Similarly, for Job 2,  $\Delta\tau_2 = \tau_2 - \frac{\tau_2}{f}$ . Thus,  $\tau_1 = \Delta\tau_1(1 - \frac{1}{f})$  and  $\tau_2 = \Delta\tau_2(1 - \frac{1}{f})$ . Since  $\tau_1 > \tau_2$ , we have  $\frac{\Delta\tau_1}{1 - \frac{1}{f}} > \frac{\Delta\tau_2}{1 - \frac{1}{f}} \implies \Delta\tau_1 > \Delta\tau_2$ . Therefore, Job 1 has more to gain by increasing its speed by a given factor than Job 2 does and so Job 1 has a greater incentive to be able to use more processors.

#### IV. RESULTS AND DISCUSSION

Show results and discuss what they represent.

#### V. RELATED WORK

#### VI. CONCLUSION

The conclusion goes here.

	Comparison		
	I	II	III
Number of Simulations	500	100	100
Number of Computers	[1,20]	[1,50]	[1,20]
Number of Jobs	[1,50]	[1,100]	[1,100]
Number of Core Types	[1,5]	[1,10]	[1,5]
Max. Number of Cores per Type	[1,25]	[1,50]	[1,50]
Max. Number of Threads per Job	[1,50]	[1,200]	[1,50]
Speed Ratios	[1,100]	[1,200]	[1,200]

TABLE I. RANDOMIZED INPUTS TO COMPARISONS

	S. M.	D. A. 1	D. A. 2	Prop. Alg.
Avg. Time Score	780	722	942	989
Percent Jobs Assigned	24.92%	41.67%	37.1%	34.49%
Avg. Avail. Threads Used	71.41%	78.96%	65.57%	64.83%
Avg. Cores Used	63.63%	87.94%	98.57%	98.53%
Avg. Resources Score	54.49%	62.41%	50.02%	49.5%
Avg. Time Score Change from Simple Matching	0.00%	-7.496%	20.82%	26.78%

TABLE II. COMPARISON I

#### VII. FUTURE WORK

This paper studies an interesting extension of the college admissions algorithm where the institutions have multiple quotas of different types and the applicants can fill any of a particular institution's quotas. Other, similar modifications to the college admissions algorithm may be worthy of interest such as having each applicant only able fill certain quota types rather simply preferring some types over others. The proposed algorithm of this paper could also be extended to consider scenarios that are complicated by software licensing restrictions or the need to factor in job length and complexity. These considerations could lead to more realistic approximations so that the algorithm can more practically lend itself to real-world implementation.

#### VIII. INDIVIDUAL CONTRIBUTIONS

Kristen Hines formulated the problem, and tested the proposed algorithm against other approaches which she implemented along with the college admissions algorithm to produce the results of the project. She also researched the background literature related to the project and contributed to this written report.

Ferdinando Romano formulated the problem, and developed and implemented the proposed algorithm. He developed other implementations against which the proposed algorithm was tested and he also derived the results about the algorithm's properties relating to termination, stability, and optimality and contributed to this written report.

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	S. M.	D. A. 1	D. A. 2	Prop. Alg.
Avg. Time Score	8668	8185	10524	9161
Percent Jobs Assigned	57.93%	75.15%	74.45%	74.63%
Avg. Avail. Threads Used	88.65%	85.81%	85.00%	79.91%
Avg. Cores Used	72.25%	87.51%	99.21%	86.50%
Avg. Resources Score	67.09%	66.49%	61.90%	53.64%
Avg. Time Score Change from Simple Matching	0.00%	-5.57%	21.41%	5.69%

TABLE III. COMPARISON II

	S. M.	D. A. 1	D. A. 2	Prop. Alg.
Avg. Time Score	2742	2541	3724	3516
Percent Jobs Assigned	48.12%	68.78%	57.06%	59.73%
Avg. Avail. Threads Used	89.65%	95.17%	92.89%	84.35%
Avg. Cores Used	47.48%	86.52%	89.93%	88.44%
Avg. Resources Score	72.73%	80.30%	74.21%	72.60%
Avg. Time Score Change from Simple Matching	0.00%	-7.32%	35.79%	28.21%

TABLE IV. COMPARISON III

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