New Pressure-Driven Approach for Modeling Water Distribution Networks

Herman A. Mahmoud[1]; Dragan Savić[2]; and Zoran Kapelan[3]

Abstract: A number of pressure-driven analysis (PDA)–based hydraulic solvers have been proposed in the literature to address issues of negative pressures estimated by demand-driven analysis (DDA) solvers. However, the PDA methods reported so far attempt to achieve this by either developing a new PDA methodology, which requires modifying the source code of hydraulic solvers, or using iterative-type approaches in which artificial elements (like suitably chosen reservoirs) are added to network nodes until convergence is achieved. None of this is ideal, because the former is difficult to implement and the latter results in computationally inefficient PDA solvers that are difficult, and sometimes even impossible, to use in larger networks, especially under extended period simulation conditions. The PDA modeling approach proposed here does not require either of the aforementioned, because it is based on a single iteration-type algorithm, which involves connecting a set of artificial elements to each network node with demand and deficient pressure. This set consists of a check valve, a flow control valve, and a flow emitter. The new PDA method developed was validated on a number of benchmark and real-life networks under different flow conditions, clearly demonstrating its advantages when compared to existing methods. The key advantages include the simplicity of its implementation and the ability to predict network pressures and flows in a consistently accurate, numerically stable, and computationally efficient manner under pressure-deficient and normal-flow conditions in both steady-state and extended period simulations. **DOI: 10.1061/(ASCE)WR.1943-5452.0000781.** *© 2017 American Society of Civil Engineers.*

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Introduction

Water distribution networks (WDNs) have been traditionally simulated using demand-driven analysis (DDA) methods (Wood and Charles 1970; Isaacs and Mills 1980; Todini and Pilati 1988). These methods assume that the demand required will be delivered irrespective of the system pressures available. The DDA hydraulic solvers are based on the well-known mass and energy balance equations that are used to compute pipe flows and nodal pressures in the network. These methods work well under normal flow conditions in which sufficient pressures are available in the pipe network. However, pressure can fall substantially under certain conditions, such as pipe bursts (or isolation) or during excessive water use (Wu and Walski 2006; Kapelan and Giustolisi 2006; Giustolisi et al. 2008). Under these flow conditions, DDA solvers cannot always deliver realistic predictions of pressures and flows, because it is not always possible to deliver all desired demands in such circumstances. As a result, DDA solvers may produce unrealistically low, sometimes negative nodal pressures that are, in some cases, physically impossible (Kapelan and Giustolisi 2006; Tanyimboh and Templeman 2010; Romano et al. 2014).

The pressure-driven analysis (PDA) solvers address the aforementioned by representing the water flow delivered as a function of available pressure (Bhave 1981; Germanopoulos 1985; Wagner et al. 1988; Chandapillai 1991; Fujiwara and Ganesharajah 1993; Gupta and Bhave 1996; Tanyimboh and Templeman 2010; Siew and Tanyimboh 2012). This is termed nodal head–flow relationship (NHFR). Having said this, some authors have proposed alternative approaches that do not make use of NHFR. Examples include (1) Collins et al. (1978), who proposed using optimization techniques instead; (2) Todini (2006), who developed a heuristic-type solution with three corrective steps to the demand-driven global gradient algorithm (GGA) (Todini and Pilati 1988), in order to correct nodal outflows whenever pressure is insufficient; and (3) Piller and Van Zyl (2007, 2009), who used mass balance–based constraints in the minimization of their content model.

Since the 1980s, many attempts have been made to simulate WDN under pressure-deficient conditions (Ozger and Mays 2003; Giustolisi et al. 2008; Wu et al. 2009; Tabesh et al. 2011; Siew and Tanyimboh 2012) and to formulate the NHFR. However, only a limited number of approaches succeeded in producing acceptable results under certain boundary conditions. Nevertheless, most of these approaches are not ideal, because they are either difficult to implement or computationally inefficient. This makes them difficult and sometimes impossible to use in larger networks, especially under extended period simulation (EPS) conditions (see the next section for details). Therefore, this paper aims to develop an improved, more robust, and generic PDA approach that is accurate, numerically stable, computationally efficient, and easy

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to implement, and so usable in real-life networks, especially under EPS conditions.

The rest of the paper is organized as follows: the relevant background information for solving and analyzing deficient pressure networks in demand-driven hydraulic solvers is provided. Then, an overview of commonly used functions for modeling the NHFR is presented. Following this, the new PDA methodological approach is described. The methodology is then applied to several literature networks, and the results are compared to results obtained from previous PDA methods. Finally, the main conclusions are drawn.

Background

Most of the currently available WDN hydraulic simulators, such as *EPANET* (Rossman 2000a), work as DDA-type solvers utilizing the GGA (Todini and Pilati 1988). However, these simulators can be used to analyze WDNs under pressure-deficient conditions as well, which can be done in two principal ways.

The first approach relies on using a specific pressure-demand relationship that requires changing the source code of the simulator. A number of examples of this approach exist. Ackley et al. (2001) and Yoo et al. (2012) presented optimization methods that maximized nodal outflow under abnormal operating conditions. Although they delivered results for simple WDNs, no generic solution approach was developed for practical applications. More recently, Goldstein's algorithm was used by Elhay et al. (2015) to upgrade different NHFRs in the GGA (Todini and Pilati 1988). This approach was successfully demonstrated on eight challenging (i.e., one has 20,000 pipes) case study networks. However, this involved only steady-state hydraulic analysis. Rossman (2000b) extended *EPANET* by implementing a flow emitter at a node to simulate demand delivered as a function of available pressure. However, a flow emitter produces negative outflow (i.e., inflow) when nodal pressure becomes negative and there is no upper limit for the discharge value. Cheung et al. (2005) modified the source code of *EPANET* by introducing emitters into the network model to simulate pressure-driven demands by using an object-oriented toolkit (OOTEN). Yet this approach failed to converge when attempting to model highly looped WDNs under low flow conditions and EPS analysis. A modification of the emitter methodology was proposed by Morley and Tricarico (2014) through *EPANETpdd* by allowing each emitter to have its own empirical exponent. However, *EPANETpdd* fails due to convergence-related issues when applied to medium and larger or more complex WDNs. Liu et al. (2011), Siew and Tanyimboh (2012), and Jun and Guoping (2013) introduced backtracking, line search, and relaxation of parameter techniques to correct the nodal heads only (i.e., not flows) in *EPANET*. Alternatively, Giustolisi et al. (2011) proposed an entirely new system tool, WDNetXL, for simulating WDN under both DDA and PDA conditions. However, all these approaches exhibit one or more of the following limitations for wider implementation: (1) they require underlying algorithm modifications to be made, (2) they are not in the public domain, (3) they are iterative in nature, (4) they have been demonstrated on limited case studies often involving small or simple networks, and (5) they are unable to handle EPS analysis.

A second principal way of simulating pressure-deficient conditions in a pipe network is by using methods that add artificial hydraulic elements to the network nodes (e.g., valves, emitters, reservoirs, and pipes) without the need to introduce a NHFR function. This is followed by a DDA run in an iterative manner until convergence is achieved. Ozger and Mays (2003), Ang and Jowitt (2006), Todini (2006), and Suribabu and Neelakantan (2011) connected (and removed, when necessary) artificial reservoirs to pressure-deficient nodes to calculate the actual flows delivered. However, this type of approach has the drawback of withholding demands until the minimum nodal pressure head is satisfied, which usually needs a large number of iterations to converge. Furthermore, it can be very difficult to apply this type of approach to large networks, because adding and removing artificial reservoirs at various stages of the iterative methodology is not an easy task, especially under EPS analysis (Wu 2007; Wu et al. 2009). This is because the network topology must be changed iteratively at each time step to identify the correct pressure-deficient and pressure-sufficient nodes. Hayuti et al. (2008) found that the number of iterations in the aforementioned approaches could be reduced by using the successive solution-seeking method, but even with this reduction, the overall computational time is still substantial. Also, Jinesh Babu and Mohan (2012), and Sivakumar and Prasad (2015), addressed the limitation of the Ang and Jowitt (2006) method by introducing artificial flow control and check valves to ensure that flows into artificially added reservoirs did not exceed the required demand and to restrict the negative pressure in the network, respectively. This led to a smaller number of iterations when compared to the Ang and Jowitt (2006) and Todini (2006) approaches. Nevertheless, the artificial reservoirs still could not simulate the important partial flow between the minimum and the desired pressure head levels. Gorev and Kodzhespirova (2013) tried to address this shortcoming by introducing an additional artificial pipe with a suitable resistance coefficient at all demand nodes. Although the results were obtained with a single hydraulic run, the approach only supported the specific, parabolic type of NHFR (Wagner et al. 1988). Sayyed et al. (2014, 2015) replaced the artificial reservoir and pipe with a suitably chosen flow emitter to reflect the properties of each node in the network. Both algorithms delivered good results under steady-state analysis. However, neither algorithm considered the effect of minimum pressure head level for a head value higher than zero. In summary, the implementation of existing PDA approaches by introducing artificial elements in all the network nodes results in the increased size and computational time of the pipe network problem. This, in turn, often leads to convergence failures and crashing hydraulic solvers, especially in large WDNs under EPS analysis.

Even though the single-iteration PDA approach (SIPDA) proposed in this paper makes use of the similar sequence of artificial elements (check valve–flow control valve–emitter) to model pressure-deficient conditions, unlike in other approaches (Sayyed et al. 2014, 2015), the artificial elements in SIPDA are added on a selective basis, i.e., only to pressure-deficient nodes with demands (i.e., demand nodes with pressure head less the required value: $H^{avl} < H^{req}$). These nodes are identified by running the DDA-type hydraulic solver (e.g., *EPANET*) once before PDA simulation. This is very important, especially in the case of larger, real-life networks, where it is well known that typically only a small part on the network (i.e., a small number of nodes) is likely to experience pressure-deficient conditions during a failure event (e.g., a pipe burst or equipment failure). Therefore, there is no need to add the proposed artificial elements to all demand nodes as other PDA methods currently suggest. As will be shown in the case studies that follow, this leads to several benefits, including (1) increased computational efficiency of hydraulic calculations; (2) more stable convergence of the PDA simulator, ending in more realistic results; and (3) less effort required to add artificial elements to the network, which, in this case, can be done manually. However, this is virtually impossible with other approaches. Furthermore, the approach works well under both steady-state and EPS conditions, even in the case of large and complex WDNs, as illustrated in the following sections.

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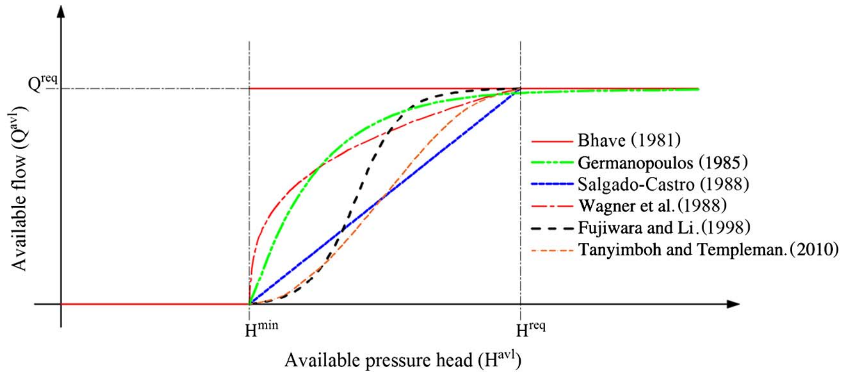

Fig. 1. Comparison of different head-flow relationships used in PDA

Nodal Head–Flow Relationship

Several attempts have been made to figure out the relationship between nodal head and flow. Probably the first function was suggested by Bhave (1981), who assumed that the actual (or available) nodal flow delivered (Q_j^{avl}) depends only on the minimum required head, $H_j^{\min}$, for flow to occur at node j, as shown in Fig. 1. This work was extended by Germanopoulos (1985) to avoid the discontinuity properties in the aforementioned function. He proposed two characteristic head values for defining the NHFR, namely, $H_j^{\min}$, the minimum head for flow to occur at node j, and H_j^{req}, the head required to provide full flow, i.e., the demand required (Q_j^{req}) at node j. Salgado-Castro (1988) assumed a linear relation for the NHFR.

Further, Wagner et al. (1988) proposed a continuous quadratic relationship for modeling partial flow between $H_j^{\min}$ and H_j^{req} as follows:

$$Q_j^{\mathrm{avl}} = Q_j^{\mathrm{req}} \left(\frac{H_j^{\mathrm{avl}} - H_j^{\min}}{H_j^{\mathrm{req}} - H_j^{\min}} \right)^{1/n_j} \quad \text{if } H_j^{\mathrm{req}} > H_j^{\mathrm{avl}} \geq H_j^{\min} \tag{1}$$

Ackley et al. (2001) claim that the Wagner et al. (1988) function is hydraulically significant but not differentiable when outflow begins (i.e., H^{avl} approaches $H^{\min}$) or when it reaches Q^{req}, which may sometimes lead to numerical oscillations and convergence problems in iterative methods that upgrade the NHFR in the GGA (Kovalenko et al. 2014). Thus, later, a differentiable function was suggested by Fujiwara and Li (1998) to simulate and assure a smooth transition for the partial flow between $H^{\min}$ and H^{req}. For the same reason Tanyimboh and Templeman (2010) proposed a logit function–based formulation to model partial flow.

All the formulations mentioned for modeling the NHFR have been derived primarily on the basis of mathematical considerations and have not been validated by experimental or laboratory data. Also, Todini (2006) stated "it is impossible practically to derive a realistic NHFR." However, the function proposed by Wagner et al. (1988) was highly recommended by Gupta and Bhave (1996), and Abdy Sayyed and Gupta (2013), among other existing functions at that time. Furthermore, Shirzad et al. (2012) investigated the performance of the existing NHFRs by comparing them to laboratory and field measurements collected from tap outlets under different pressure head scenarios. The NHFR functions presented by Wagner et al. (1988) matched the best experimental data. Kovalenko et al. (2014) and Vairagade et al. (2015) performed a further comparison between the Tanyimboh and Templeman (2010) and Wagner et al. (1988) functions for some networks and calibration parameters, and concluded that the Tanyimboh and Templeman (2010) function gives better convergence properties, especially when H^{avl} approaches $H^{\min}$.

It is also noteworthy that the PDA methodology presented here has the ability to model a more general NHFR and, hence, even though it is suggested here and by several other authors, using the aforementioned Wagner et al. (1988) function is not a must; this PDA approach can support other pressure-flow relationships, as shown in the next section.

Proposed PDA Methodology

The PDA method presented here is based on adding a specific sequence of artificial network elements to all pressure-deficient nodes with a nonzero demand (i.e., nodes where $Q_j^{\mathrm{req}} > 0$ and $H_j^{\mathrm{avl}} < H_j^{\mathrm{req}}$). These nodes are identified by running the DDA-based solver first. The sequence of elements added is composed of a check valve (CV), a flow control valve (FCV), an internal dummy node (DN), and an emitter (EM), as shown in Fig. 2. The role of a check valve is to prevent flow reversal (i.e., inflow at demand nodes when $H_j^{\mathrm{avl}} < H_j^{\min}$); the role of the flow control valve is to prevent delivery of a demand larger than required (i.e., limiting the flow in the artificial added elements to Q^{req}); the internal dummy node is used to link the check and flow control valves in series (an *EPANET* requirement); and the emitter is used to represent pressure-dependent demand delivery. Given this and the assumed NHFR shown in Eq. (1), the parameter settings of all these elements are allocated as specified in Table 1.

The small length, large diameter, and large Hazen-Williams coefficient values were chosen to ensure that all additional elements do not introduce (significant) head loss between demand node j and

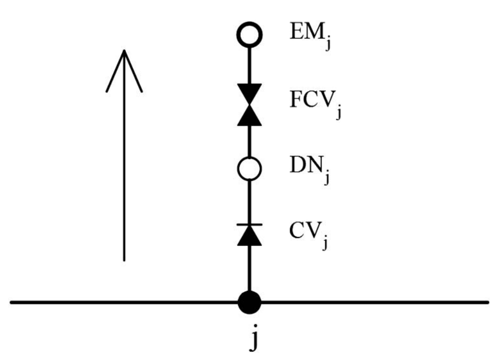

Fig. 2. Setup at each deficient demand node

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Table 1. Properties of Added Elements

Network element	Parameter	Setting
Demand node j	Required demand	Q_j^{req}
	Elevation	EL_j
Check valve, CV_j	Length	Small value (e.g., 0.01)
	Diameter	Large value (e.g., 1,000)
	Roughness	Large value (e.g., 140) for Hazen-Williams formula and small value (e.g., 0.001) for other formulas
Dummy node, DN_j	Elevation	EL_j
Flow control valve, FCV_j	Diameter	Large value (e.g., 1,000)
	Setting	Q_j^{req}
Emitter, EM_j	Elevation	$H_j^{\min} + EL_j$
	Emitter coefficient, C_d	Eq. (2)

the emitter. The delivered flow (Q_j^{avl}) to deficient nodes can now be estimated as follows:

$$Q_j^{\mathrm{avl}} = \begin{cases} 0 & \text{if } H_j^{\mathrm{avl}} < H_j^{\min} \\ C_d(H_j^{\mathrm{avl}} - H_j^{\min})^{\gamma} & \text{if } H_j^{\mathrm{req}} > H_j^{\mathrm{avl}} \geq H_j^{\min} \\ Q_j^{\mathrm{req}} & \text{if } H_j^{\mathrm{avl}} \geq H_j^{\mathrm{req}} \end{cases} \quad (2)$$

where C_d = emitter coefficient estimated as $C_d = Q_j^{\mathrm{req}}/(H_j^{\mathrm{req}} - H_j^{\min})^{\gamma}$; and γ = emitter exponent estimated as $\gamma = 1/n_j$. The values of both variables depend on the properties of each node that is defined in terms of empirical exponent coefficient n_j (value in the range between 0.5 and 2.5), and the characteristic heads (H_j^{req}, $H_j^{\min}$). The overall SIPDA algorithm is presented step by step in Fig. 3. The procedure shown in Fig. 3 is for the steady-state analysis in a pressure-deficient network. When performing the EPS analysis, the diurnal demand variation in nodes, the water level in storage tanks, and the valve/pump control settings need to be considered over a predefined simulation period. This involves changing the parameters of the connected FCVs and emitters according to the current values of the desired demands in deficient nodes. The number of deficient nodes is estimated by running the DDA solver prior to conducting the SIPDA analysis and it is equal to the number of demand nodes having pressure lower than H^{req} for the most critical time snapshot in the analyzed time period (usually the peak hour of the day). This will guarantee connecting the artificial elements to all potential pressure nodal demands during extended period simulation. Once these nodes are identified, they are able to handle changes from DDA to PDA and vice versa by using the sequence of artificial elements added to these nodes. For example, whenever pressure drops enough for conditions to change from DDA to PDA, the emitters start working on these nodes. Also, when pressure increases sufficiently (i.e., above H^{req}) and conditions change from PDA to DDA, the FCVs ensure that only the demands required are delivered at these nodes.

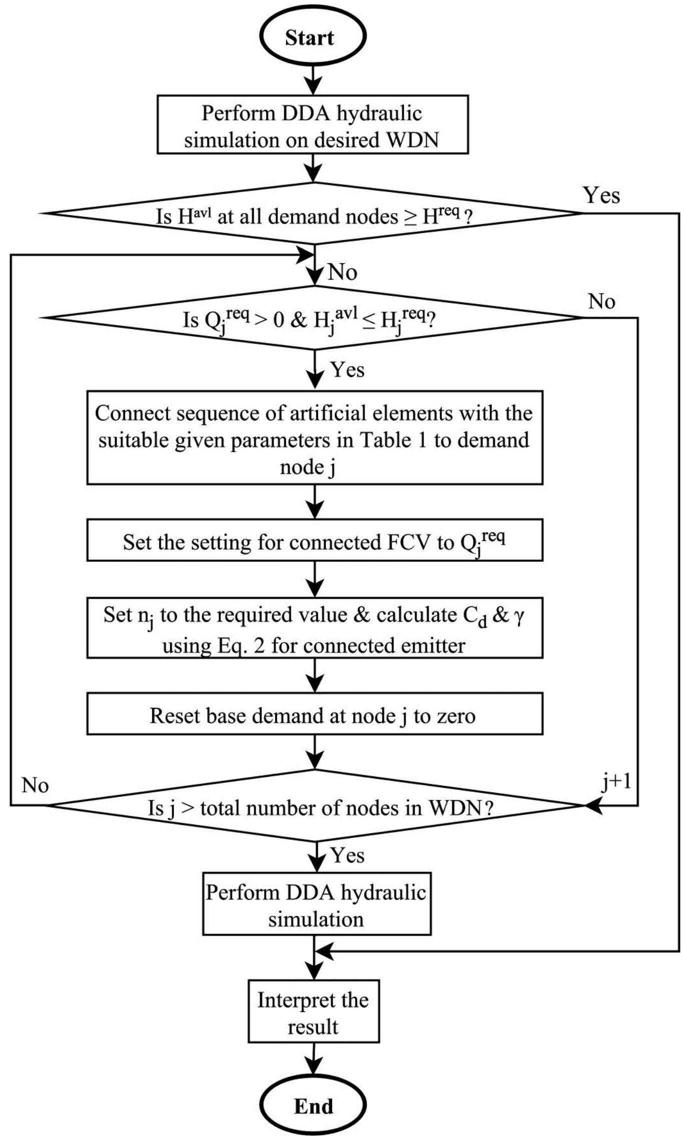

Fig. 3. Flowchart of the SIPDA approach

Case Studies

The SIPDA approach presented in this paper has been tested, verified, and illustrated under different flow conditions on a number of benchmarks and real-life case WDNs found in the literature. The case studies considered are divided into four groups based on the complexity of the WDNs analyzed. The results obtained are also compared to the results reported or obtained by using a number of reviewed PDA methodologies (or obtained by the DDA method in cases in which direct comparison was not possible) with the aim of assessing the overall performance and convergence of the SIPDA approach. The ability to simulate WDNs under EPS analysis is demonstrated in the last two case studies. All hydraulic simulations were conducted by using *EPANET2.0* and its toolkit functions for steady-state and EPS analyses, respectively. All simulations were conducted on a computer with an Intel processor, Core i5-4570 CPU at 3.2 GHz, and 64-bit Windows 7. The values of 0.001, 2, 10, and 0 have been assumed for accuracy, CHECKFREQ, MAXCHECH, and DUMPLIMIT parameters in *EPANET 2.0*. It should be noted that even though *EPANET* was used in all case studies here, the SIPDA method is generic in the sense that it can be replicated using any other existing hydraulic solvers.

Case Study 1: Steady-State Simulation in a Small, Simple, Looped Pipe Network

The first WDN analyzed is taken from the paper by Ang and Jowitt (2006) (Fig. 4) and later used by a number of researchers as a benchmark problem to validate their PDA methods. This network consists of a fixed head reservoir feeding six demand nodes through eight pipes in a looped configuration. A failure scenario is

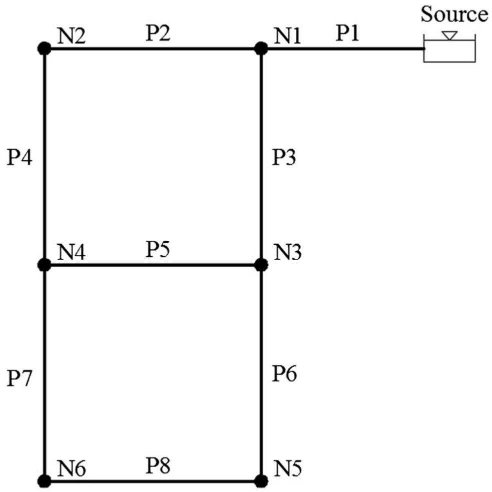

Fig. 4. Network layout for Case Study 1

introduced through a break in one of the pipes in the system. The empirical exponent coefficient (n_j) for all demand nodes is assumed equal to 1.85. Minimum nodal head is 50 m at Nodes N1 and N2, 45 m at Nodes N3 and N4, and 55 m at Nodes N5 and N6. The required head in the node head–flow relationship is assumed equal to 60 m for all demand nodes.

Table 2 shows the results obtained by the proposed PDA method in comparison with methods reported by Ackley et al. (2001), Hayuti et al. (2008), Jinesh Babu and Mohan (2012), Gorev and Kodzhespirova (2013), and Sayyed et al. (2015).

EPANET2.0 predicted available (i.e., actual) heads below the required threshold value (H^{req}) in all nodes, as shown in Table 2. Hence, the DDA results reported for the failure scenario are meaningless because the outflow at nodes cannot be met in full. All demand nodes delivered partial flow by using the SIPDA approach, the Ackley et al. (2001) method, the Hayuti et al. (2008) method, and the Gorev and Kodzhespirova (2013) method, and drew a total flow (i.e., total demand) of 206.0, 226.6, 206.1, and 208.2 l/s, respectively, from the source. In addition, the Jinesh Babu and Mohan (2012) method produced only two nodes with partially delivered demand with a total flow of 231.8 l/s. Note that the Jinesh Babu and Mohan (2012) method cannot model the partial flow because it is based on H^{min} only (i.e., it does not consider H^{req}).

The solutions by the SIPDA approach and the Hayuti et al. (2008) method are more realistic and produce pressure-dependent nodal flows similar to those derived by using the Wagner et al. (1988) model. However, the SIPDA approach is computationally more efficient than that of Hayuti et al. (2008), because it needs only a single internal iteration in *EPANET2.0* to converge, as compared to the 10 iterations used by Hayuti et al. (2008). Because the value of n_j for this network is 1.85, the assumption in Gorev and Kodzhespirova's (2013) paper caused errors in calculation of nodal flows and pressure heads, as shown in Table 2. Furthermore, the Ackley et al. (2001) method generated unrealistic results, in which the pressure heads at Nodes 5 and 6 were below the minimum H^{min} of 55 m. This could be because a formal optimization technique was used in this method to maximize nodal flows and the NHFR function was considered as an additional constraint. It is worth mentioning that the Sayyed et al. (2015) approach failed to change the DDA conditions to PDA conditions for all nodes, as shown in Table 2 (Columns 2 and 3), even though they both used the same artificial elements used in SIPDA. This is because the Sayyed et al. (2015) approach assumed that the elevations of the connected dummy nodes and emitters were equal to the elevation of demand nodes regardless of the minimum pressure head value (H^{min}). This assumption caused errors in pressures and outflow calculations for all nodes in which the connected emitters have H^{min} values greater than zero, as shown in Table 2. Ultimately, the SIPDA approach obtained the solution more quickly in terms of computational time (CT) than the Jinesh Babu and Mohan (2012) and Gorev and Kodzhespirova (2013) methods by 14 and 23%, respectively.

Case Study 2: Steady-State Simulations in Medium and Larger Pipe Networks

It was important to assess the performance of the SIPDA approach on large-scale and real-life water system case studies. To achieve this, a number of WDNs of different sizes and with different failure scenarios were considered. The results obtained were compared to those produced by using the DDA built in *EPANET2.0* and the PDA methods of Jinesh Babu and Mohan (2012), Gorev and Kodzhespirova (2013), and Sayyed et al. (2015) in terms of prediction accuracy, convergence, and computational time. For the purpose of comparison, the artificial sequence of elements proposed in this paper was connected to all nonzero demand nodes in Networks N3 and N4 (where no deficient pressure conditions were found), but only connected to pressure-deficient nodes in N1, N2, and N5. The details of these networks are summarized in Table 3.

The SIPDA approach and the Gorev and Kodzhespirova (2013) method produced the same demand satisfaction ratio [DSR: the ratio of water delivered to water required (Siew and Tanyimboh 2012)]

Table 2. Results of Case Study 1 Network

Node identified	DDA results; Sayyed et al. (2015)		PDA results: SIPDA approach		Ackley et al. (2001)		Hayuti et al. (2008)		Jinesh Babu and Mohan (2012)		Gorev and Kodzhespirova (2013)	
	Q^{avl} (l/s)	H^{avl} (m)	Q^{avl} (l/s)	H^{avl} (m)	Q^{avl} (l/s)	H^{avl} (m)	Q^{avl} (l/s)	H^{avl} (m)	Q^{avl} (l/s)	H^{avl} (m)	Q^{avl} (l/s)	H^{avl} (m)
N1	41.7	54.33	*35.0*	57.23	*34.1*	56.88	*35.0*	57.22	41.7	50.00	*35.4*	57.19
N2	41.7	51.75	*32.6*	56.32	*31.0*	55.77	*32.7*	56.32	41.7	50.00	*33.0*	56.27
N3	77.8	50.83	*66.0*	56.06	*55.9*	55.42	*66.0*	56.06	77.8	45.00	*66.6*	56.00
N4	41.7	50.40	*35.2*	55.98	*29.5*	55.26	*35.2*	55.98	41.7	45.00	*35.6*	55.92
N5	55.6	**43.40**	*14.8*	55.43	*29.7*	**53.14**	*14.8*	55.43	*15.1*	55.00	*15.0*	55.36
N6	88.9	**42.91**	*22.5*	55.39	*46.4*	**53.00**	*22.5*	55.39	*13.4*	55.00	*22.6*	55.32
Source	−347.4	59.00	−206.0	59.00	−226.6	59.00	−206.1	59.00	−231.4	59.00	−208.2	59.00
CT, s	—		4.2×10^{-5}		—		—		4.9×10^{-5}		5.5×10^{-5}	

Note: Italic numbers indicate partial flow; bold numbers indicate nodes with heads below minimum head limit, H^{min}; CT = computational time; CT not available for methods of Ackley et al. (2001) and Hayuti et al. (2008) due to complexity of obtaining these values by using both PDA approaches.

Table 3. Characteristics of Network Models Used in Performance Tests

Identifier	Network model	Nodes	Pipes	Reservoirs	Head-loss equation	n_j	$H^{\min}$ (m)	H^{req} (m)	Total required demand (l/s)
N1	BWSN 2	12,523	14,830	7	D-W	2.0	0	20	1,463.86
N2	Exeter	1,891	3,032	2	D-W	1.5	0	20	3,245.81
N3	Modena	268	317	4	H-W	2.0	7	20	406.93
N4	Pescara	68	99	3	H-W	2.0	7	25	257.4
N5	New York tunnel	19	21	1	H-W	2.0	0	77.72	71.25

Note: D-W = Darcy-Weisbach; H-W = Hazen-Williams.

for Networks N3, N4, and N5 with DSR values of 0.970, 1.0, and 1.0, respectively. However, the SIPDA approach obtained a smaller DSR value of 0.926 for N2 in comparison with the Gorev and Kodzhespirova (2013) method's value of 0.934. This is because the proposed SIPDA approach used the Wagner et al. (1988) pressure-dependent nodal flow function, in which $n_j = 1.5$, whereas the shape of the NHFR function in the Gorev and Kodzhespirova (2013) approach was fixed at $n_j = 2.0$. The Jinesh Babu and Mohan (2012) method produced DSR values of 0.988 and 1.0 for N2 and N5, the reason for this being that their method cannot simulate partial nodal flows. On the other hand, as expected, *EPANET2.0* produced unrealistic negative pressure heads in N2 and N5.

Furthermore, in order to check the accuracy of the reported results using the SIPDA approach, the *EPANET2.0* solver was applied to Networks N3 and N4, in which the DDA assumption (i.e., no pressure-deficient nodes) is satisfied. It was observed that the pressure heads and flows at all nodal demands were the same for both the SIPDA and the DDA approaches. As expected, *EPANET2.0* produced the solutions notably faster than the SIPDA approach for all networks, as shown in Table 4. Compared to the results of Gorev and Kodzhespirova (2013), the SIPDA approach showed better performance in terms of computational time for all networks, in which additional artificial elements were only added to the deficient nodes, and an extra calculation of friction loss in the CV pipes was not required. The computational performance of the SIPDA approach was also better than that of the Jinesh Babu and Mohan (2012) approach in large networks (N2 and N3). However, the Jinesh Babu and Mohan (2012) method was faster for smaller networks (N4 and N5). This is because of the large number of artificial reservoirs that need to be added to all nodal demands in the Jinesh Babu and Mohan (2012) method, which slows down computations increasingly in larger networks (to obtain relevant water balances between the reservoirs).

Finally, in order to illustrate the advantage of SIPDA over the other approaches that add artificial elements to all network nodes [such as Jinesh Babu and Mohan (2012), Gorev and Kodzhespirova (2013), and Sayyed et al. (2015)], the SIPDA and Sayyed et al. (2015) approaches were applied to Network N1. When the DDA solver (*EPANET*) was applied to this network it showed that only 185 nodes with demands experienced low-pressure conditions (i.e., $H^{\text{avl}} < H^{\text{req}} = 20$ m). Accordingly, in the SIPDA approach,

Table 4. Computational Performance Comparison

	Time (s)			
Network	*EPANET2.0*	SIPDA approach	Jinesh Babu and Mohan (2012)	Gorev and Kodzhespirova (2013)
N1	5.60×10^{-3}	5.60×10^{-2}	5.65×10^{-2}	5.75×10^{-2}
N2	4.60×10^{-4}	1.30×10^{-3}	5.90×10^{-3}	1.05×10^{-2}
N3	1.35×10^{-4}	3.85×10^{-4}	3.11×10^{-4}	3.98×10^{-4}
N4	3.20×10^{-5}	8.80×10^{-5}	6.60×10^{-5}	1.71×10^{-4}

artificial elements were connected only to these nodes, which represented about 1.5% of the total network of nodes, whereas in Sayyed et al. (2015) artificial elements are added to all 12,523 nodes. As a result, SIPDA was 68% faster in performing the PDA simulation than the Sayyed et al. (2015) approach. In addition, the Sayyed et al. (2015) approach failed to converge when the aforementioned network was exposed to more serious failure scenarios. For example, when a large burst was modeled on Pipe L887, *EPANET* showed that 1,396 demand nodes were subjected to pressure-deficient conditions (i.e., $H^{\text{avl}} < H^{\min}$). The SIPDA successfully changed the status of 232 of these nodes from no outflow to partial outflow under these conditions, whereas the Sayyed et al. (2015) approach failed to change the status of any of these deficient nodes. Therefore, the Sayyed et al. (2015) approach failed to converge and produced unrealistic predictions of nodal pressures and outflows.

Case Study 3: Extended Period Simulation in a Small Network

The validated SIPDA approach was applied to the single-source pumped WDN considering diurnal variation in demand to assess SIPDA performance under EPS conditions. The network consisted of a reservoir (Res1), a tank, eight demand nodes, and 12 pipes (Fig. 5); further detail for the network can be found in Rossman (2000a).

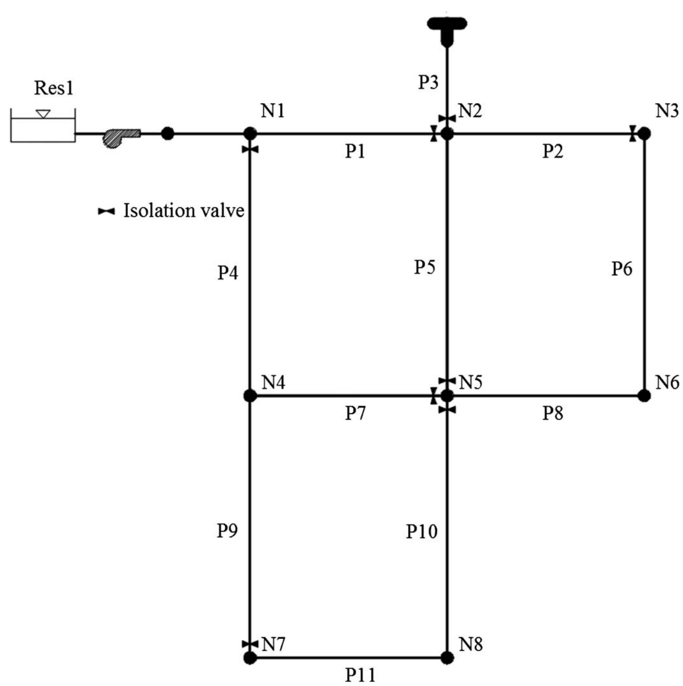

Fig. 5. Test Case Study 4 network (*EPANET2.0* Example Network 1)

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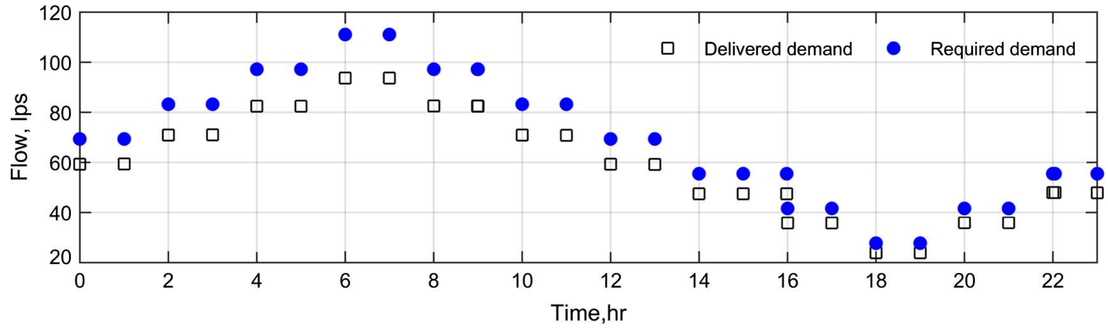

Fig. 6. Supply/demand balance in Case Study 4 network

For demonstration purposes, failure of P7 during the whole simulation time was considered to create a pressure-deficient condition. In order to isolate P7, the isolation valves located on P4, P7, and P9 were closed. As a result, P4 and P9 were also isolated from the other parts of the network and Demand Node N4 disconnected from the water sources (Jun and Loganathan 2007). In the SIPDA approach the settings of the added elements at the isolated demand nodes (e.g., N4) do not need to be reset to zero during hydraulic calculations for the isolated network nodes (which normally needs to be done). However, in order to apply the Ang and Jowitt (2006), Jinesh Babu and Mohan (2012), and Gorev and Kodzhespirova (2013) approach for analyzing isolation in a WDN, the additional artificial reservoirs should be removed, because they provide water to the isolated parts.

A 24-h EPS was carried out with a 1-h time step. Demand pattern values were changed every 2 h for all nodes. Furthermore, FCV and emitter settings were updated at the beginning of each pattern time step based on the corresponding required nodal demands (Q^{req}) for Deficient Nodes N7 and N8. The hydraulic simulation of the network by the SIPDA approach showed that shortfall in supply occurred over the whole simulation period, as shown in Fig. 6. This shortfall increased slightly with time and reached a peak of 17.7 l/s at 6:00 and 7:00, then dropped to around 3.0 l/s at 18:00 and 19:00 before increasing again to 7.5 l/s at 23:00, as shown in Fig. 6. The shortfall occurred mainly because of the isolation of N4 and partial flows at Nodes N7 and N8. Fig. 7 shows that Node N7 was unable to deliver the full demand from 0:00 until 17:00 because the pressure head in this period was below the required value (H^{req}). However, the pressure head from 17:00 to 24:00 was sufficient to supply the full demand, as shown in Fig. 7. Consequently, the network experiencing the aforementioned failure condition provided the required demand at all nodes during the low-demand period but not during the peak-demand period, when some pressure heads dropped below H^{req}. All this demonstrates that the PDA approach provides logical and numerically stable results under EPS conditions, which cannot be said for all the other PDA methods (see "Background" section).

Case Study 4: Extended Period Simulation in a Large Real-Life Network

In order to further validate the robustness of the SIPDA approach and measure its computational performance, a 24-h simulation was conducted on a large and more complex WDN, the C-Town network (Marchi et al. 2014). This was done under both normal and abnormal flow conditions simulated in respective scenarios. The results obtained by the SIPDA approach were compared with the DDA (*EPANET2.0*) hydraulic solver results.

The C-Town network consists of six district metred areas (DMAs), 443 pipes, 399 nodes, four pressure reducing valves (PRVs), and one throttle control valve (TCV). The water is supplied to the system by a large reservoir (RES1) with a constant head and via seven balancing water tanks. The water is pumped through Pumping Station S1 to Tanks T1 and T2, and the water supply to T2 is controlled by a TCV. Pumping Stations S2 and S3 draw water to Tanks T3 and T4, whereas Stations S4 and S5 pump supply from T1 to T5, T6, and T7, as shown in Fig. 8. The required pressure head (H^{req}), the minimum pressure head (H^{min}), and the emitter exponent for demand nodes are assumed to be 15 m, 0 m, and 2.0, respectively.

Normal Flow Scenario

Different values for the hydraulic time step were used, as shown in Fig. 9, to investigate computational time and numerical convergence for the SIPDA approach under different EPS conditions. The demand pattern time step was fixed at 1 h. Both the *EPANET2.0* and the SIPDA approaches yielded almost the same results in terms of nodal flows and pressures for all time steps. Fig. 9 also shows that the computational times for both solvers increased from 0.7 s to about 2.5 s with decreasing hydraulic time steps. As expected, the SIPDA approach is slower that the *EPANET2.0* solver in all cases,

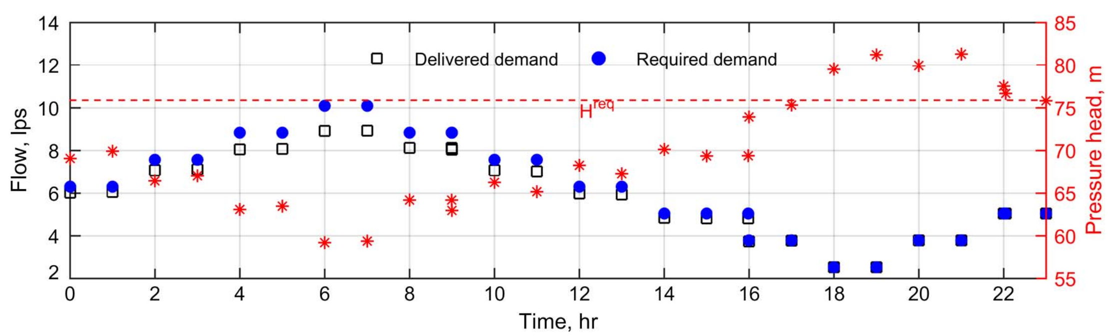

Fig. 7. Supply/demand balance and available pressure head at Node N7 in Case Study 4 network

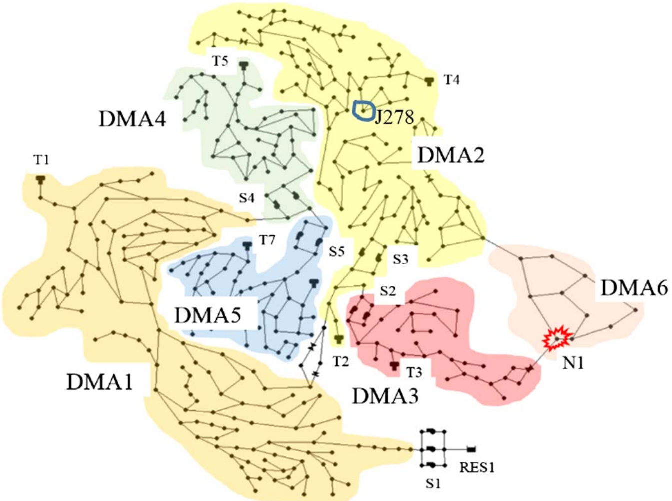

Fig. 8. Network layout for Case Study 4

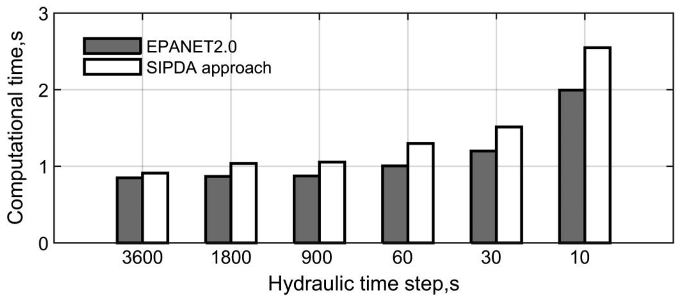

Fig. 9. Average computational time comparison for EPS

but not significantly, which is a good indication. For example, for a 60-s time step, SIPDA is 29% slower than *EPANET2.0*. This difference is acceptable and is due to the fact that in the SIPDA approach the flow settings for the FCV and the emitter coefficient for all demand nodes are updated at the beginning of each pattern time step (i.e., every hour in this case). In the *EPANET2.0* solver, the demand values are only updated at demand nodes (i.e., every 2 h in this case).

Abnormal Flow Scenario

The abnormal flow scenario was created by subjecting Node N1 in DMA6 (Fig. 8) to an additional constant demand of 15 l/s from 12:00 to 20:45, thus simulating a large burst (or abnormal demand) at that location. Accordingly, the base demand at Node N1 was changed from 0 to 15 l/s during this period, whereas the required demands for N1 were presented in the corresponding emitter connected to N1. The hydraulic time and pattern time steps were fixed at 15 min and 1 h, respectively. Simulations by *EPANET2.0* predicted pressure heads below H^{req} at all 10 nodes with demands in DMA6 during the whole overloading condition period. This number increased at 17:00 (the most severe time) to 26 nodes affected in DMA2. Because of these failure conditions, the proposed artificial elements were connected to all 36 deficient demand nodes for the whole simulation period considered. The SIPDA approach showed that only 24 nodes were affected by low-pressure conditions at 17:00, as illustrated in Fig. 10. This is because the water depth in Tank T4 dropped to zero, as shown in Fig. 11, and Pumping Stations S2 and S3 were unable to deliver the required demand to all nodes. The number of pressure-deficient nodes decreased slightly in both solver solutions during low-demand patterns. By 20:15 the number of pressure-deficient nodes increased again in *EPANET2.0* and the proposed PDA algorithm to 36 and 24, respectively. Therefore, the proposed SIPDA method estimated fewer

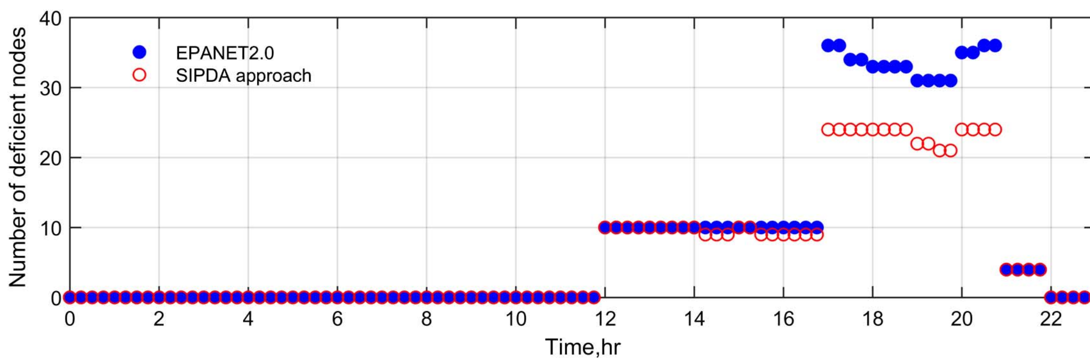

Fig. 10. Number of nodal demands deficient under abnormal flow scenario

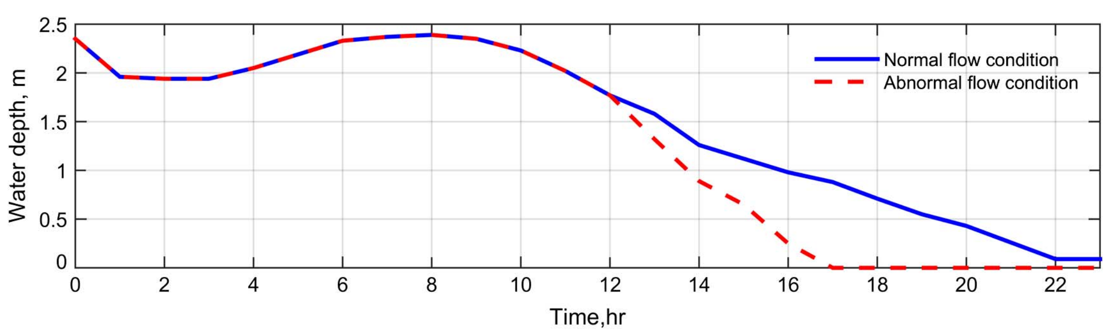

Fig. 11. Water depth at Tank T4

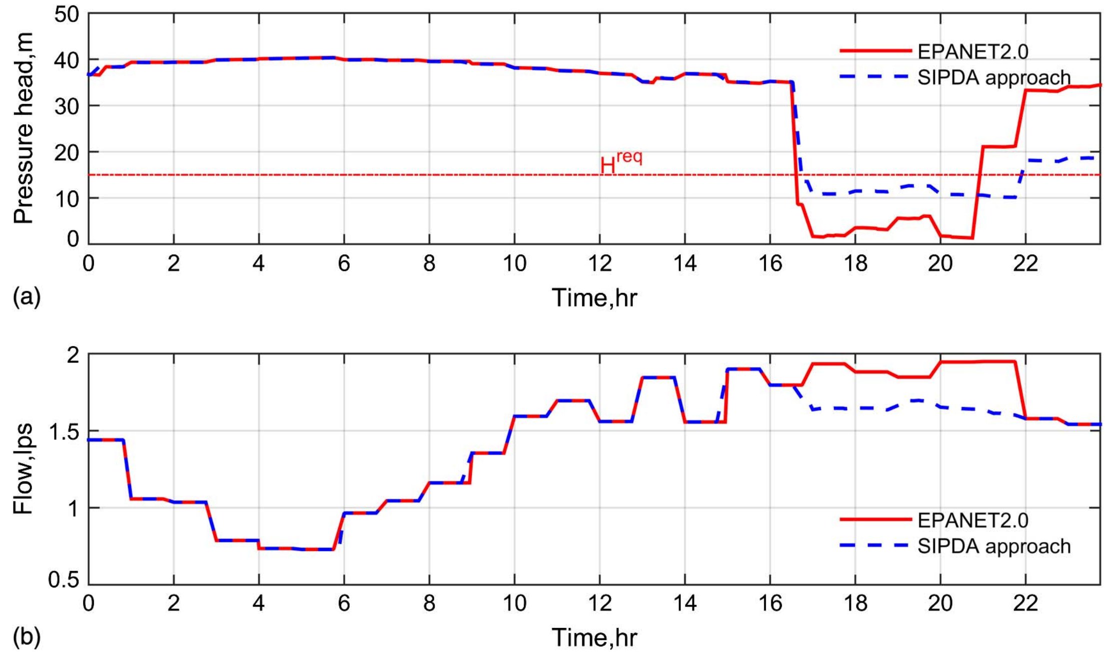

Fig. 12. Prediction of (a) pressure heads and (b) flows at Node J278

deficient nodes in comparison with *EPANET2.0*. This is because the SIPDA approach applied the NHFR function for each node individually, whereas *EPANET2.0* satisfied demands at all nodes regardless of the available nodal pressure heads, which produced more nodes with pressure heads below H^{req} or below H^{min}. Both solvers predicted four pressure-deficient nodes in DMA2 after the additional demand at N1 was removed (e.g., at 21:00). This is because the water depth in Tank T4 was zero, compared to around 0.5 m during normal loading conditions (Fig. 11). By 22:00 the system was back to normal pressure conditions. Note that the SIPDA approach converged to a numerically stable solution under deficient pressure conditions and it was faster by approximately 5% under normal flow conditions.

The prediction of delivered demands and pressure heads at Node J278 are shown in Fig. 12. As can be seen from this figure, both *EPANET2.0* and SIPDA delivered identical flows at this location when the available pressures were sufficient to deliver the required demand. On the other hand, as expected, only partial demand was delivered at this node in SIPDA when the pressure head went below the threshold required (H^{req}).

Summary and Conclusions

This paper proposes a new direct PDA modeling approach (SIPDA) that is based on using a specific set of elements added to each deficient pressure demand node in the network. This set is composed of a short pipe with a check valve, a flow control valve, and an emitter. This, in turn, enables modeling the minimum pressure head below which no flow occurs, the required pressure above which the full demand required is delivered, and the partial flow conditions for the pressure heads in between these two characteristic values. The SIPDA approach suggested here uses the Wagner et al. (1988) relationship for the latter, although other relationships could be modeled if deemed more suitable. The SIPDA approach was validated, demonstrated, and compared to several other PDA methods reported in the literature on a number of benchmark and real-life networks under both normal and pressure-deficient conditions. The results obtained lead to the following conclusions:

1. The proposed SIPDA approach is able to simulate effectively and efficiently the WDN under both normal and pressure-deficient conditions and in both steady-state and extended period simulations. The predicted flows and pressures in the network are accurate, numerically stable, and obtained in a computationally efficient manner.
2. When compared to other PDA methods reported in the literature, the approach proposed here possesses the following key advantages: (1) it predicts the same or more-accurate pressures and flows under pressure-deficient conditions; (2) it does this in a generally more computationally efficient manner, especially when applied to larger pipe networks; and (3) unlike some PDA methods, it can be used for extended period simulation, i.e., not just steady-state hydraulic analysis.
3. The proposed SIPDA approach implemented here works with the widely used *EPANET2.0* software, although there is no reason why it should not work with other popular hydraulic solvers, including commercial options. The reasons for this are the generic nature and relative simplicity of the approach proposed and the fact that it does not require any hydraulic solver source code modifications.

Based on the aforementioned, the SIPDA approach proposed lends itself naturally to tackling practical problems such as WDN reliability, optimal location of isolation valves, and multiobjective optimization.

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