Problem 1

(Exercise 5.1.6)

Let $\{X(t)\}_{t \geq 0}$ be a Poisson process with rate λ.

Given $s, t > 0$, calculate

$$X(t) \mid X(t+s) = n$$

$$\frac{X(t) = k,\ X(t+s) = n}{X(t+s) = n}$$

$$\sum_{i=0}^{n} \frac{(\lambda s)^i e^{-\lambda s}}{i!} \cdot \frac{(\lambda t)^{n-i} e^{-\lambda t}}{(n-i)!}$$

Problem 2

Ex. 5.1.7

If the building has survived at time t, then it must have suffered $1, \dots, K$ shocks and survived all. Since the survival of shocks are independent, we get:

$$P(T = t) = \sum_{i=1}^{K} \frac{(\lambda t)^K a^K e^{-\lambda t}}{i!}$$

Problem 3

5.1.10

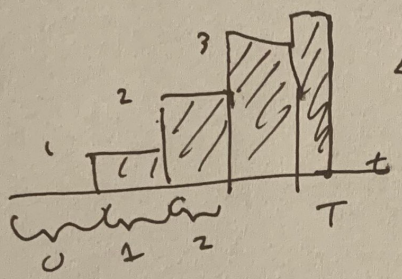
(a) We incur cost K at the end of the cycle.

(b) $T = kt$, t = unit of time.

$\mathbb{E}[\#\text{ of customers}] = K\lambda$.

The cost will equal

$f(x) = 0 + 1 + \cdots + \text{Floor}\{x\} \cdot (\text{decimals of } x)$



← integrate over steps (discontinuous) above formula gives the area

(c) $K + f(\mathbb{E}[\#\text{ of customers}])$

$= K + f(K\lambda)$

(d) $T = \left(\frac{K}{\lambda}\right)$ rounded

Problem 4

(Exercise 5.1.11)

rescaling poisson

Set $T = q * t_i$ t_i = unit of time.

Then $P(T = t) = 1 - \sum_{j=k}^{\infty} \frac{(\lambda q)^k e^{-\lambda q}}{k!}$

Problem 5