1. Assume that $f(a,b) = f(c,d)$.
Then $2^a(2b+1) = 2^c(2d+1)$.
Since $2b+1$ & $2d+1$ are ~~odd~~ odd,
we have that $a=c$. This follows
from ~~2~~ ~~being~~ the highest power of 2
dividing both sides being the same.
Thus $a=b$. From this it follows
that $b=d$.

2. Note that the sum of n integers is:
$S_n = \frac{n}{2}(n+1)$. Substitute $m+n$.

$$S_{m+n} = \frac{m+n}{2}(m+n+1)$$

$$= \frac{1}{2}((m+n)^2 + (m+n))$$

$$J(m,n) = \frac{1}{2}[(m+n)^2 + (m+n)] + m$$

$$= \frac{1}{2}((m+n)^2 + (m+n) + 2m)$$

$$= \frac{1}{2}((m+n)^2 + 3m + n)$$

3. Consider $f(x) = \frac{1}{x}$.

Injectivity: Suppose $f(x_1) = f(x_2)$.
$\frac{1}{x_1} = \frac{1}{x_2} \rightarrow x_1 = x_2$

Thus we have our desired
map

5. We define $f(x) = x$. This function will be injective, but not surjective.

6. This is equivalent to proving that the cardinality of sets defines an equivalence relation.

Reflexivity: For any set A, we take the identity function.

~~Reflexivity:~~

Symmetry: Assume $A \approx B$. There must exist a bijective function $f: A \to B$. This implies that ~~B~~ $f^{-1}: B \to A$ is injective. Thus $B \approx A$.

Transitivity: This follows from the composition of two bijections being bijective.

7. Assume $\operatorname{ran} f \neq A$. Then there must exist some $x_1, x_2 \in \operatorname{dom} f$ s.t. $f(x_1) = f(x_2)$, $x_1 \neq x_2$. For the other way, assume that f is injective. Then $f(x_1) = f(x_2) \to x_1 = x_2$. From this it follows that each x_1 must map to a unique $a_1 \in \operatorname{ran}(A)$. Thus $\operatorname{ran} f = A$.