MAE6220 FINAL EXAM

Name:___________________________________ Time: 2hrs

Question 1:

a) Consider the following finite-difference formula for the second derivative $\delta^2\phi/\delta x^2$ on a uniform grid with spacing h:

$$\frac{\delta^2\phi}{\delta x^2} = \frac{\frac{35}{12}\phi_i - \frac{26}{3}\phi_{i+1} + \frac{19}{2}\phi_{i+2} - \frac{14}{3}\phi_{i+3} + \frac{11}{12}\phi_{i+4}}{h^2}$$

Using Taylor series expansions demonstrate the formal order of accuracy of the above formula.

b) Consider the uniform grid shown in the figure. Construct a one-sided finite-difference formula, which is 2nd order accurate, to compute the first derivative, $\partial\varphi/\partial x$, at point i on the wall (Figure 1).

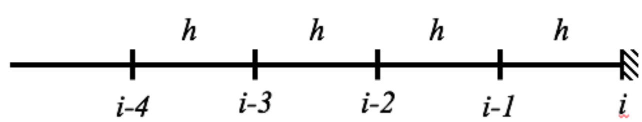

Figure 1.

Question 2: Consider the following equation where c is a constant.

$$\frac{\partial\phi}{\partial t} - c\frac{\partial\phi}{\partial x} = 0 \quad (Eq. 1)$$

a) If the equation is discretized using the implicit scheme below, determine the consistency of the discretization scheme.

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} - c\frac{\phi_{i+1}^{n+1} - \phi_{i-1}^{n+1}}{2\Delta x} = 0$$

b) Examine the stability of Eq.1 for the grid shown in Figure 2 using a forward differencing scheme for discretization in space for the boundary condition $\phi(t, L) = a$.

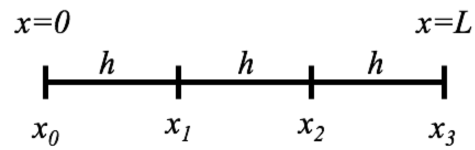

Figure 2.

Question 3: Consider the two-dimensional energy transport equation for an incompressible inviscid fluid.

$$\rho C_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

where T is the absolute temperature, C_p is specific heat for constant pressure, k is thermal conductivity of the fluid and ρ is density. ρ, C_p, k, μ are all constants.

a) Derive the Reynolds average of the equation and describe all terms. State all <u>relevant</u> assumptions.
b) Which term(s) need to be modeled? What are the possible strategies to model the term(s).