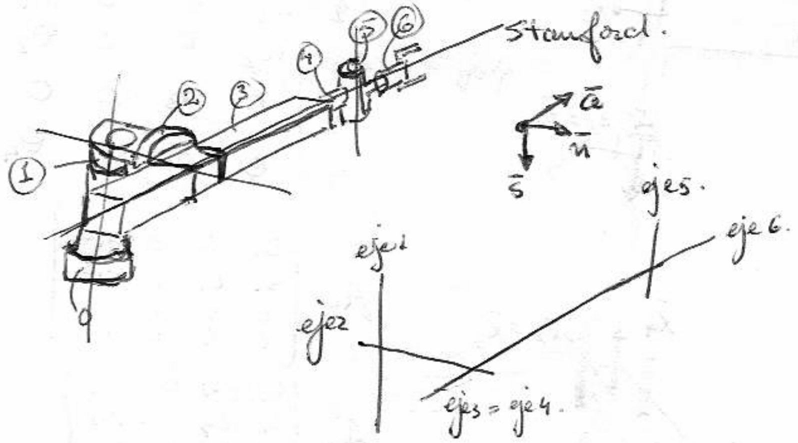
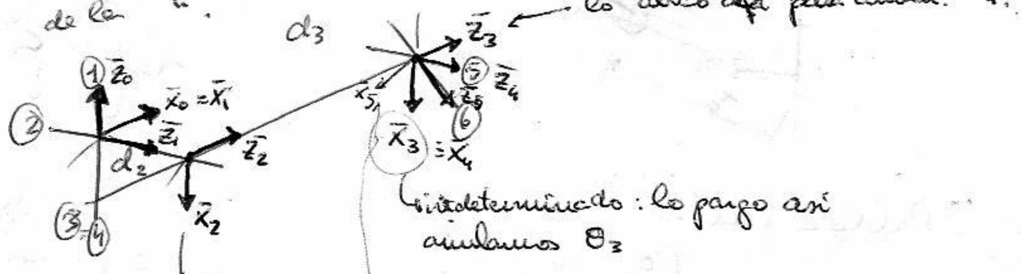
①



Podemos dibujar esto mismo con otra orientación de la herramienta para entender mejor cómo quedan las ternas de la " .



Indeterminado: lo pongo así anulamos θ_3

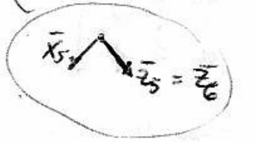
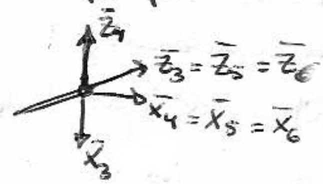


Tabla DH. para la pose inicial.

	a	d	α	θ
1	0	0	90	($\theta_1 = 0$)
2	0	d_2	−90	($\theta_2 = -90$)
3	0	(d_3)	0	0
4	0	0	90	(90) −90
5	0	0	−90	(0)
6	0	0	0	(0)

para la pose inicial.



$$A_{i-1}^{i} = \begin{bmatrix} c\theta_i & -s\theta_i\, c\alpha_i & s\theta_i\, s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i\, c\alpha_i & -c\theta_i\, s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^5 = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \end{bmatrix} \qquad A_5^6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \end{bmatrix}$$

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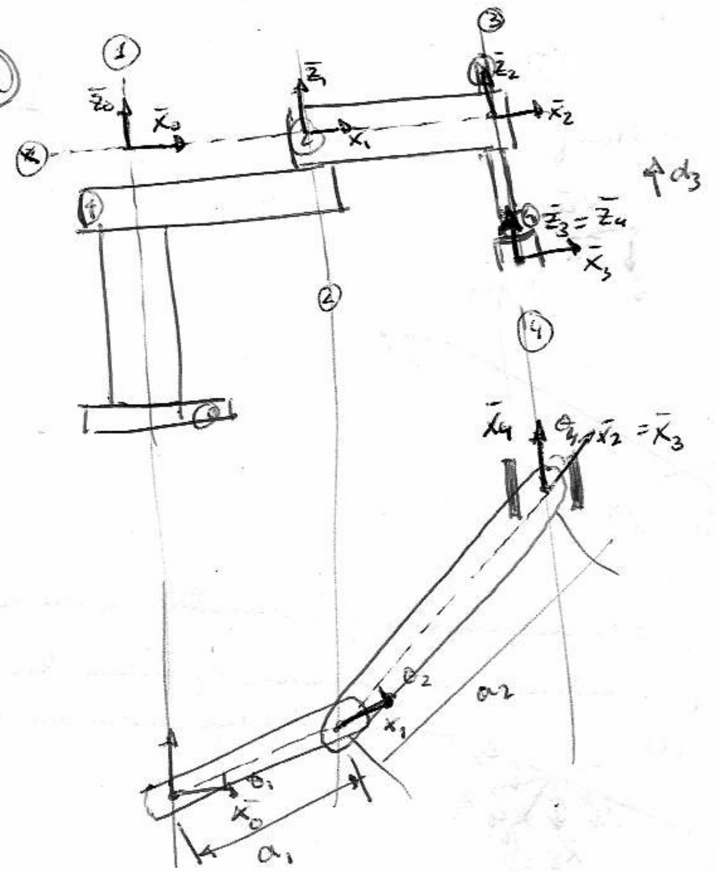


	a	d	α	θ
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	0	d_3	0	0
4	0	0	0	θ_4

$$A_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \left[\begin{array}{c|c} I & \begin{matrix} 0 \\ 0 \\ d_3 \end{matrix} \\ \hline \bar{0}^T & 1 \end{array}\right]$$

$$A_3^4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3 Problema inverso

$$A_2^4 = A_2^3 \cdot A_3^4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^4 = \left[\begin{array}{ccc|c} c_{124} & -s_{124} & 0 & a_2c_{12} + a_1c_1 \\ s_{124} & c_{124} & 0 & a_2s_{12} + a_1s_1 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$A_1^4 = A_1^2 \cdot A_2^4 = \begin{bmatrix} c_{24} & -s_{24} & 0 & a_2c_2 \\ s_{24} & c_{24} & 0 & a_2s_2 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Se complica rápidamente.

primero obtengo el P. inv. de la posición de la muñeca.

$$A_0^1 \cdot A_1^2 \cdot A_2^3 \cdot \underbrace{A_3^4\Big|_{4^a col}}_{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$A_0^1 \, A_1^2 \underbrace{\begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}}_{\begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

paso a la inversa

$$(A_0^1)^{-1} = \left[\begin{array}{ccc|c} c_1 & s_1 & 0 & -a_1 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} a_2c_2 \\ a_2s_2 \\ d_3 \\ 1 \end{bmatrix} = \begin{bmatrix} p_xc_1 + p_ys_1 - a_1 \\ -p_xs_1 + p_yc_1 \\ p_z \\ 1 \end{bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ \\ \end{matrix}$$

$$\boxed{p_z = d_3}$$

$$\begin{cases} p_x c_1 + p_y s_1 = a_2 c_2 + a_1 & (1) \\ -p_x s_1 + p_y c_1 = a_2 s_2 & (2) \end{cases}$$

Elevo al cuadrado cada una y sumo.

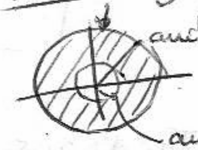
$$(p_x c_1 + p_y s_1)^2 = (a_2 c_2 + a_1)^2$$
$$(-p_x s_1 + p_y c_1)^2 = (a_2 s_2)^2$$

$$p_x^2 c_1^2 + p_y^2 s_1^2 + \cancel{2 p_x p_y c_1 s_1} + p_x^2 s_1^2 + p_y^2 c_1^2 - \cancel{2 p_x p_y c_1 s_1} = a_2^2 c_2^2 + a_1^2 + 2a_2 c_2 a_1 + a_2^2 s_2^2$$

$$p_x^2 + p_y^2 = a_2^2 + a_1^2 + 2 a_1 a_2 c_2.$$

$$c_2 = \frac{(p_x^2 + p_y^2) - (a_1^2 + a_2^2)}{2 a_1 a_2}$$

debo chequear que $|c_2| < 1$ para saber si la posición es alcanzable.



anillo de $(a_1+a_2)^2$

anillo de $(a_1-a_2)^2$.

$$\boxed{\theta_2 = \mathrm{ATAN2}\left[\left(\pm\right)\sqrt{1-c_2^2}\ ;\ c_2\right]}$$

$\mathrm{sgn}(s_2) = \mathrm{sgn}(\theta_2) = g_1$ — codo positivo / codo negativo.

Para despejar θ_1 multiplico por p_x y p_y sumo y despejo s_1

$$p_y(p_x c_1 + p_y s_1) = (a_2 c_2 + a_1) p_y$$
$$- \quad p_x(-p_x s_1 + p_y c_1) = (a_2 s_2)\, p_x$$

$$\cancel{p_y p_x c_1} + p_y^2 s_1 + p_x^2 s_1 - \cancel{p_x p_y c_1} = (a_2 c_2 + a_1) p_y - a_2 s_2 p_x$$

$$s_1 = \frac{(a_2 c_2 + a_1) p_y - a_2 s_2 p_x}{p_x^2 + p_y^2}$$

debemos cotejar que $p_x^2 + p_y^2 > 0$.

$$s_1 = \frac{a_2 (c_2 p_y - s_2 p_x) + a_1 p_y}{p_x^2 + p_y^2}$$

Ahora despejo c_1

$$p_x (p_x c_1 + p_y s_1) = (a_2 c_2 + a_1) p_x$$
$$+ \quad p_y (-p_x s_1 + p_y c_1) = a_2 s_2 p_y$$

$$p_x^2 c_1 + p_y^2 c_1 + \cancel{p_x p_y s_1} - \cancel{p_x p_y s_1} = (a_2 c_2 + a_1) p_x + a_2 s_2 p_y$$

$$\boxed{\theta_1 = \mathrm{ATAN2}(s_1, \theta_1)}$$

Ahora queda por resolver la orientación de la herramienta.

$$R_0^4 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{I} \begin{bmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^4 = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^4 = \begin{bmatrix} c_{124} & -s_{124} & 0 \\ s_{124} & c_{124} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{pmatrix} \bar{n} & \bar{s} & \bar{a} \end{pmatrix}$$

$$\left.\begin{aligned} c_{124} &= n_x \\ s_{124} &= n_y \end{aligned}\right\} \quad \theta_1 + \theta_2 + \theta_4 = \mathrm{ATAN2}(n_y ; n_x)$$

$$\boxed{\theta_4 = \mathrm{ATAN2}(n_y ; n_x) - \theta_1 - \theta_2.}$$

(4) Jacobiano de velocidades.

$$J_{\theta_4}^2 = \begin{bmatrix} \bar{p}_{04} \wedge \bar{x}_2^0|_z & \bar{p}_{14} \wedge \bar{x}_2^1|_z & \bar{x}_2^2|_z & \bar{p}_{34} \wedge \bar{x}_2^3|_z \\ \bar{p}_{04} \wedge \bar{y}_2^0|_z & \bar{p}_{14} \wedge \bar{y}_2^1|_z & \bar{y}_2^2|_z & \bar{p}_{34} \wedge \bar{y}_2^3|_z \\ \bar{p}_{04} \wedge \bar{z}_2^0|_z & \bar{p}_{14} \wedge \bar{z}_2^1|_z & \bar{z}_2^2|_z & \bar{p}_{34} \wedge \bar{z}_2^3|_z \\ \bar{x}_2^0|_z & \bar{x}_2^1|_z & 0 & \bar{x}_2^3|_z \\ \bar{y}_2^0|_z & \bar{y}_2^1|_z & 0 & \bar{y}_2^3|_z \\ \bar{z}_2^0|_z & \bar{z}_2^1|_z & 0 & \bar{z}_2^3|_z \end{bmatrix}$$

para esto voy a necesitar $A_0^4, A_1^4, A_3^4, A_2^3, A_1^2, A_0^3$

me falta $A_0^2 = A_0^1 A_1^2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_2 c_{12} + a_1 c_1 \\ s_{12} & c_{12} & 0 & a_2 s_{12} + a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\bar{p}_{04} \wedge \bar{x}_2^0|_z = \left.\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}\right|_z = a \cdot -y + b \cdot x = (a_2 c_{124} + a_1 c_1) s_{12} - (a_2 s_{124} + a_1 s_1) c_{12}$$

$$= a_2 (c_{124} s_{12} - s_{124} c_{12}) + a_1 (c_1 s_{12} - s_1 c_{12})$$

$$\begin{pmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \end{pmatrix} \quad \begin{bmatrix} c_{12} \\ s \end{bmatrix}$$

$$\wedge\ \bar{Y}_2^0\Big|_z = (a_2 c_{12} + a_1 c_1)\, c_{12} + (a_2 s_{12} + a_1 s_1) . s_{12}$$

$$= \underbrace{a_2 c_{12}^2 + a_2 s_{12}^2}_{a_2} + \underbrace{a_1 c_1 c_{12} + a_1 s_1 s_{12}}_{+ a_1 c_{12-1}}$$

$$= a_1 c_2 + a_2$$

$$\bar{P}_{04} \wedge \bar{Z}_2^0\Big|_z = 0$$

$$\bar{X}_2^0\Big|_z = 0 \qquad \bar{Y}_2^0\Big|_z = 0 \qquad \bar{Z}_2^0\Big|_z = 1$$

Segunda columna: el aporte del ~~eje~~ 2.

$$\bar{P}_{14} \times \bar{X}_2^1\Big|_z = \begin{pmatrix} a_2 c_2 \\ a_2 s_2 \\ d_3 \end{pmatrix} \times \begin{pmatrix} c_2 \\ s_2 \\ 0 \end{pmatrix}\Bigg|_z = a_2 s_2 c_2 - a_2 c_2 s_2 = 0 .$$

$$\bar{P}_{14} \times \bar{Y}_2^1\Big|_z = \begin{pmatrix} a_2 c_2 \\ a_2 s_2 \\ d_3 \end{pmatrix} \times \begin{pmatrix} -s_2 \\ c_2 \\ 0 \end{pmatrix}\Bigg|_z = a_2 c_2^2 + a_2 s_2^2 = a_2 .$$

$$\bar{P}_{14} \times \bar{Z}_2^1\Big|_z = \begin{pmatrix} a_2 c_2 \\ c_2 s_2 \\ d_3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 .$$

$$\bar{X}_2^1\Big|_z = 0 \qquad \bar{Y}_2^1\Big|_z = 0 \qquad \bar{Z}_2^1\Big|_z = 1 .$$

Tercera col.

$$\bar{X}_2^2\Big|_z = 0 \quad . \ \bar{Y}_2^2\Big|_z = 0 \quad \bar{Z}_2^2\Big| = 1$$

Cuarta col.

$$\bar{P}_{34} \wedge \bar{X}_2^3\Big|_z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \wedge \{\ = 0 .$$

$$\bar{P}_{34} \wedge \bar{Y}_2^3\Big|_z \ - - - - = 0$$

$$\bar{P}_{34} \wedge \bar{Z}_2^3\Big|_z \ \_\_\_\_\_\_ = 0 .$$

$$\text{con } \bar{Z}_3 = \bar{Z}_2 \Rightarrow \bar{Z}_3^2\Big|_z = \bar{Z}_2^3\Big|_z = 1$$

$$J_{04}^2 = \begin{bmatrix} a_1 s_2 & 0 & 0 & 0 \\ a_1 c_2 + a_2 & a_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Son dos grados de libertad que no tengo.

puntos singulares:

$\det(J_{o_4}^{2}) = 0.$

$(a_1 s_2) a_2 = 0 \Rightarrow s_2 = 0.$ brazo extendido

$\theta_2 = 0$

$$J_{o_4}^{(2)}(\theta_2 = 0) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_1 + a_2 & a_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Exploro el núcleo de $J_{o_4}^{(2)}$

$$J_{o_4}^{(2)} \cdot \begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot d_3 \\ \dot\theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \dot\theta_1 (a_1 + a_2) + \dot\theta_2 \cdot a_2 \\ \dot d_3 \\ \dot\theta_1 + \dot\theta_2 + \dot\theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} \dot\theta_1 (a_1 + a_2) + \dot\theta_2 a_2 = 0. \\ \dot d_3 = 0 \\ \dot\theta_4 = -\dot\theta_1 - \dot\theta_2 \end{array}$$

$$\dot\Theta_N = \begin{bmatrix} +a_2 \\ (a_1 + a_2) \\ 0 \\ +a_1 \end{bmatrix}$$

Ahora veo el núcleo transp.

$$\begin{pmatrix} 0 & a_1 + a_2 & 0 & 1 \\ 0 & a_2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{array}{l} (a_1 + a_2) \cdot V_y = 0 \Rightarrow V_y = 0. \\ a_2 \cdot V_y = 0 \Rightarrow V_y = 0. \\ V_z = 0. \\ \omega_z = 0 \end{array}$$

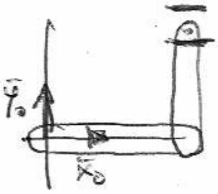
Entonces el núcleo de $J_{o_4}^{(2)\,T}$

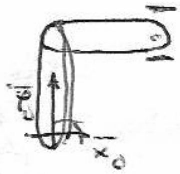
es $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ no puedo moverme en $\bar{X}^{(2)}$

La sing. es tipo 2 porque si muevo en el núcleo salgo de ella (pues cambio θ_2)

las 2 soluciones:

$$\bar{\Theta}^1 = \begin{bmatrix} 0 \\ \pi/2 \\ 0 \\ -\pi/2 \end{bmatrix} \qquad \bar{\Theta}^2 = \begin{bmatrix} \pi/2 \\ -\pi/2 \\ 0 \\ 0 \end{bmatrix}$$





Para ir a $\left[\begin{array}{c|c} I & \begin{matrix} 2a \\ 0 \\ 0 \end{matrix} \\ \hline \bar{0}^T & 1 \end{array}\right]$ obtengo $\bar{\Theta}^{fin} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$c_2 = \frac{4\cancel{a^2} - 2\cancel{a^2}}{2\cancel{a^2}} = 1$$

$$\theta_2 = \text{ATAN2}(0,1) = 0$$

$$s_1 = 0$$

$$c_1 = \frac{a(2a) + a \cdot 2a}{4a^2} = 1 \qquad \Big\} \rightarrow \theta_1 = 0.$$

$$\theta_4 = 0.$$

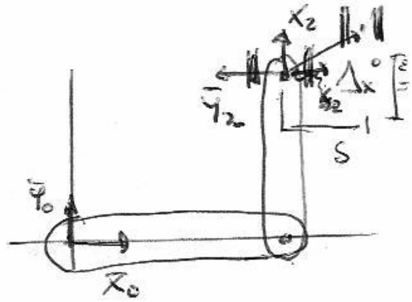
$$d_3 = 0.$$

$$\Delta\bar{\Theta}^1 = \bar{\Theta}^{fin} - \bar{\Theta}^1 = \begin{bmatrix} 0 \\ -\pi/2 \\ 0 \\ -\pi/2 \end{bmatrix} \qquad \max_t \left[\frac{|\Delta\bar{\Theta}^1|}{\Delta\theta_{max}}\right] = \max_t \begin{bmatrix} 0 \\ +\pi/2/2 \\ 0 \\ +\pi/2/3 \end{bmatrix} = \frac{\pi}{4} \text{ seg.}$$

$$\Delta\bar{\Theta}^2 = \bar{\Theta}^{fin} - \bar{\Theta}^2 = \begin{bmatrix} \pi/2 \\ \pi/2 \\ 0 \\ 0 \end{bmatrix} \qquad \max_t \left[\frac{|\Delta\bar{\Theta}^2|}{\dot{\theta}_{max}}\right] = \max_t \begin{bmatrix} \pi/2 \\ \pi/4 \\ 0 \\ 0 \end{bmatrix} = \frac{\pi}{2} \text{ seg.} \leftarrow$$

Esta es la Sn. que tarda más en llegar.

5



$$\Delta \bar{x}^0 = \begin{bmatrix}\delta\\ \varepsilon\\ 0\end{bmatrix} \qquad \Delta \bar{x}^2 = \begin{bmatrix}\varepsilon\\ -\delta\\ 0\end{bmatrix}$$

$$\left[\frac{\Delta \bar{x}^{(2)}}{\Delta \bar{\alpha}}\right] = J^{(2)}_{o_4} \cdot \Delta\bar{\theta}$$

$$\begin{bmatrix}\varepsilon\\ -\delta\\ 0\\ 0\end{bmatrix} = J^{(2)}_{o_4}\ \Delta\bar{\theta} = \begin{bmatrix} a_1 & 0 & 0 & 0\\ a_2 & a_2 & 0 & 0\\ 0 & 0 & 1 & 0\\ 1 & 1 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta\theta_1\\ \Delta\theta_2\\ \Delta d_3\\ \Delta\theta_4\end{bmatrix}$$

$\boxed{\Delta d_3 = 0.}$

$\Delta\theta_4 + \Delta\theta_1 + \Delta\theta_2 = 0$

$\Delta\theta_1\, a_1 = \varepsilon \Rightarrow \boxed{\Delta\theta_1 = \frac{\varepsilon}{a_1}}$

$\Delta\theta_1 . a_2 + \Delta\theta_2 . a_2 = -\delta \Rightarrow \boxed{\Delta\theta_2 = \frac{-\delta}{a_2} - \frac{\varepsilon}{a_1}}$

$$\boxed{\Delta\theta_4 = \cancel{\frac{-\varepsilon}{a_1}} + \frac{\delta}{a_2} + \cancel{\frac{\varepsilon}{a_1}} = \frac{\delta}{a_2}.}$$

6

$$POSE = \left[\begin{array}{c|c} I & \begin{matrix} a\\ a\\ 0\end{matrix}\\ \hline 0\ 0\ 0 & 1\end{array}\right]$$

P. inverso:

$c_2 = 2a^2 - 2a^2 = 0.$

$\theta_2 = ATAN2(\pm\sqrt{1}, 0)$

$\theta_2^1 = \pi/2$

$\theta_2^2 = -\pi/2.$

Si $\theta_2 = \pi/2$

$c_1 = \frac{a(a) + a\,a}{a^2 + a^2} = 1$

$s_1 = a^2 - a^2 = 0.$

$\rightarrow \boxed{\theta_1 = ATAN2(0, 1) = 0.}$

$d_3 = 0.$

$\theta_4 = \underbrace{ATAN2(0,1)}_{0} - 0 - \pi/2 = -\pi/2.$

Si $\theta_2 = -\pi/2.$

$c_1 = -a^2 + a^2 = 0.$

$s_1 = \frac{a^2 + a^2}{2a^2} = 1$

$\rightarrow \theta_1 = \pi/2.$

$\theta_4 = 0. - \frac{\pi}{2} + \frac{\pi}{2} = 0.$