Introduction to Probability

Joe Blitzsten

Fernando Náufel

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Sumário

Apresentação

- Página do livro: https://projects.iq.harvard.edu/stat110/home
- Strategic practice and homework: https://projects.iq.harvard.edu/stat110/strategic-practice-problems
- Handouts: https://projects.iq.harvard.edu/stat110/handouts — includes solutions to exercises marked with (s) in the book.
- Playlist: https://www.youtube.com/playlist?list=PL2SOU6wwxB0uwwH80KTQ6ht66KWxbzTIo

01: Probabilidade e contagem

Vídeo

https://youtu.be/KbB0FjPg0mw

Pascal e Fermat

- Ver artigo DEVLIN (2010).
- Ver originais em francês de toda a correspondência de Pascal.

Exercícios

Enunciados (pdf).

Practice

4. Norepeat words

A *norepeatword* is a sequence of at least one (and possibly all) of the usual 26 letters a, b, c, ...,z, with repetitions not allowed.

For example, "course" is a *norepeatword*, but "statistics" is not.

Order matters, e.g., "course" is not the same as "source".

A *norepeatword* is chosen randomly, with all *norepeatwords* equally likely. Show that the probability that it uses all 26 letters is very close to $1/e$.

- O denominador vai ser o total de todas as *norepeatwords* (NRW), que é a soma de
    - NRW de 1 letra: 26
    - NRW de 2 letras: $26 \cdot 25$
    - NRW de 3 letras: $26 \cdot 25 \cdot 24$
    - $\ldots$
    - NRW de 24 letras: $26 \cdot 25 \cdot 24 \cdot \dots \cdot 3$
    - NRW de 25 letras: $26 \cdot 25 \cdot 24 \cdot \dots \cdot 2$
    - NRW de 26 letras: $26 \cdot 25 \cdot 24 \cdot \dots \cdot 1$
- Ou seja,

$$\sum_{k=0}^{25} \frac{26!}{k!}$$

- Que é igual a

$$26! \left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{25!}\right)$$

- O total de NRW que usam as 26 letras é $26!$.
- A probabilidade procurada é

$$\begin{aligned} P &= \frac{26!}{26! \left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{25!}\right)} \\ &= \frac{1}{1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{25!}} \\ &= \frac{1}{e} \end{aligned}$$

- A última igualdade se justifica porque a série de Taylor para e^x é

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

- Numericamente:

```
1 / exp(1)
## [1] 0,3678794
```

```
1 / sum(1 / factorial(0:25))
## [1] 0,3678794
```

Homework

02: Histórias e axiomas

Vídeo

https://youtu.be/FJd_1H3rZGg

Referências

DEVLIN, K. The Pascal-Fermat Correspondence: How Mathematics Is Really Done. **The Mathematics Teacher**, v. 103, n. 8, p. 579–582, abr. 2010.