Funiba Neba CV HW 3 Q1 #1-3

1 2D Transformations

i) $(a,b) \rightarrow$ homogeneous $\rightarrow \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$

$\rightarrow$ translation to origin $\rightarrow \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix} \rightarrow T_o$

$\rightarrow$ rotation of angle θ about origin $\rightarrow \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow R$

$\rightarrow$ translation back to $(a,b) \rightarrow \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \rightarrow T_p$

Full transformation $= T_o R T_p = \begin{bmatrix} \cos\theta & -\sin\theta & -a\cos\theta + a + b\sin\theta \\ \sin\theta & \cos\theta & -a\sin\theta - b\cos\theta + b \\ 0 & 0 & 1 \end{bmatrix}$

ii) $p_1 = (1,1) \sim \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$p_2 = (2,1) \rightarrow$ stays same since its about p_2, w/ $a=2, b=1$

$p_3 = (2,2) \rightarrow \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

$p_4 = (1,2) \longrightarrow \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$T_o R T_p p_1 = \begin{bmatrix} \frac{4-\sqrt{2}}{2} \\ \frac{2-\sqrt{2}}{2} \\ 1 \end{bmatrix}$, $T_o R T_p p_3 = \begin{bmatrix} \frac{4-\sqrt{2}}{2} \\ \frac{2+\sqrt{2}}{2} \\ 1 \end{bmatrix}$

$T_o R T_p p_4 = \begin{bmatrix} 2-\sqrt{2} \\ 1 \\ 1 \end{bmatrix}$

iii) $x' = ax + by + t_x + \alpha x^2 + \beta y^2$

$y' = cx + dy + t_y + \gamma x^2 + \theta y^2$

$\Rightarrow$ for (x_i, y_i) to (x_i', y_i') you have equations

$x_i' = ax_i + by_i + t_x + \alpha x_i^2 + \beta y_i^2$

$y_i' = cx_i + dy_i + t_y + \gamma x_i^2 + \theta y_i^2$, now set up linear system $Ax=b$

where A :

You have parameters $(a, b, c, d, t_x, t_y, \alpha, \beta, \gamma, \theta)$. If you group by x' & y' $\Rightarrow (a, b, t_x, c, d, t_y)$, then you have the quadratic contributions after that.

Construct A :

$$A = \begin{array}{cccccccc} a & b & t_x & c & d & t_y & x^2 & y^2 \end{array}$$
$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & x_i^2 & y_i^2 \\ 0 & 0 & 0 & x_i & y_i & 1 & x_i^2 & y_i^2 \\ & & & & \vdots & & & \end{bmatrix}$$

rows of form similar to row 1 correspond to x' equation & rows of form similar to row 2 correspond to y' equation. And obviously b in this system is

$$b = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ \vdots \\ x_n' \\ y_n' \end{bmatrix}$$

then x here is the collection of points. You'd need at least 10 equations to solve for each of the 10 parameters & since each point gives 2 equations, you'd need at minimum 5 points.