1 Written Problems

$\boxed{\text{Problem 1}}$

$$x(m,n) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 2 & 1 \end{bmatrix} \cdot h(m,n) = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \overset{\text{answer}}{\begin{bmatrix} -2 & 1 \\ -3 & 7 \\ 2 & 12 \\ 2 & 5 \end{bmatrix}}$$

padded x : $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

work

① $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (0\cdot 2)+(0\cdot 1)+(0\cdot 0)+(0\cdot 1)+(1\cdot 0)+(2\cdot -1) = -2$

② $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (0\cdot 2)+(0\cdot 1)+(0\cdot 0)+(1\cdot 1)+(2\cdot 0)+(0\cdot -1) = 1$

③ $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (0\cdot 2)+(1\cdot 1)+(2\cdot 0)+(0\cdot 1)+(3\cdot 0)+(4\cdot -1) = -3$

④ $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (1\cdot 2)+(2\cdot 1)+(0\cdot 0)+(3\cdot 1)+(4\cdot 0)+(0\cdot -1) = 7$

⑤ $\begin{bmatrix} 0 & 3 & 4 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (0\cdot 2)+(3\cdot 1)+(4\cdot 0)+(0\cdot 1)+(2\cdot 0)+(1\cdot -1) = 2$

⑥ $\begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (3\cdot 2)+(4\cdot 1)+(0\cdot 0)+(2\cdot 1)+(1\cdot 0)+(0\cdot -1) = 12$

⑦ $\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (0\cdot 2)+(2\cdot 1)+(1\cdot 0)+(0\cdot 1)+(0\cdot 0)+(0\cdot -1) = 2$

⑧ $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = (2\cdot 2)+(1\cdot 1)+(0\cdot 0)+(0\cdot 1)+(0\cdot 0)+(0\cdot -1) = 5$

$\boxed{\text{Problem 2}}$

1D convolution between 2 signals $f, g \in \mathbb{R}^N$ is given by:

$$(f * g)[n] = \sum_{k=0}^{N-1} f[n-k]\,g[k]$$

a) Commutative property: $f*g = g*f$

let $t = n-k$

then $(g*f)[n] = \sum_{t=0}^{N-1} f[t]\,g[n-t]$

since k & t are initialized at 0 & go to $N-1$

$$\rightarrow (f*g)[n] = \sum_{k=0}^{N-1} f[n-k]\,g[k] = \sum_{t=0}^{N-1} g[n-t]\,f[t] = (g*f)[n]$$

hence, $(f*g)[n] = (g*f)[n]$ $\blacksquare$

Associative Property: $((f*g)*h)[n] = (f*(g*h))[n]$

$$((f*g)*h)[n] = \sum_{k=0}^{N-1} (f*g)[n-k]\,h[k] = \sum_{k=0}^{N-1}\left(\sum_{t=0}^{N-1} f[n-k-t]\,g[t]\right)h[k]$$

$$= \sum_{t=0}^{N-1}\left(\sum_{k=0}^{N-1} f[n-k-t]\,h[k]\right)g[t] \quad , \text{let } l = n-t$$

$$= \sum_{t=0}^{N-1} f[l-k]\,(g*h)[t] = (f*(g*h))[n]$$ $\blacksquare$