Transmission Expansion Planning with Linear Optimal Power Flow Using Cycle Flows

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HELMHOLTZ

RESEARCH FOR GRAND CHALLENGES

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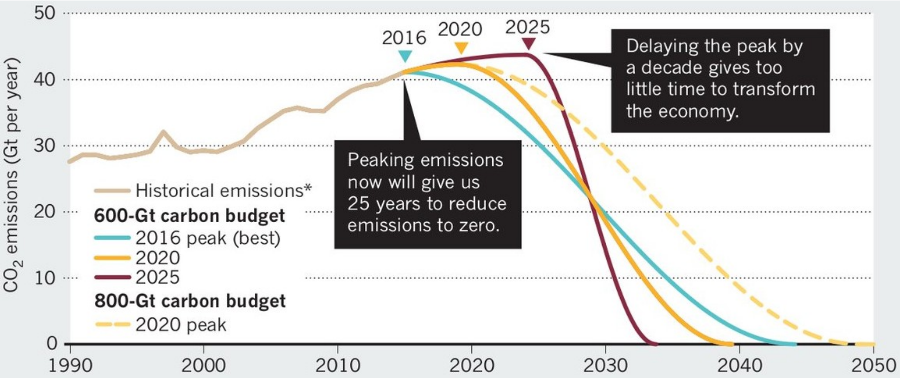
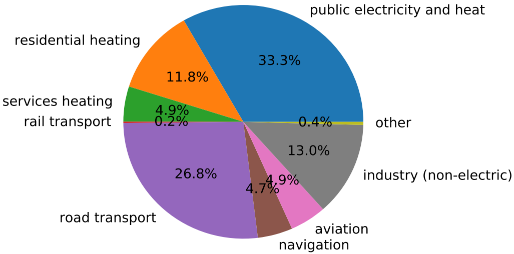
The Global Climate Change Challenge



Source: 'Three years to safeguard our climate,' Nature, 2017

It's not just about electricity demand...

EU28 CO_2 emissions in 2018 (total 3.2 Gt CO_2, 8% of global):



...but electrification of other sectors is critical for decarbonisation

...because **wind and solar** dominate the potentials for low-carbon energy.

Source: Tom Brown, data from EEA

Research Questions for Energy System Design

1. What **infrastructure** (onshore/offshore wind, solar, hydro, battery or hydrogen storage, power and gas networks) does a highly renewable energy system require, **where** should it go, and **when** should it be built?

2. Given a desired CO_2 emissions reduction (e.g. 95% compared to 1990), what is the **cost-optimal** combination of technologies (generation, storage, transmission)?

3. How do we deal with the **variablility** of wind and solar energy?

Assess the multitude of trade-offs in the energy system!

Source: adapted from Tom Brown

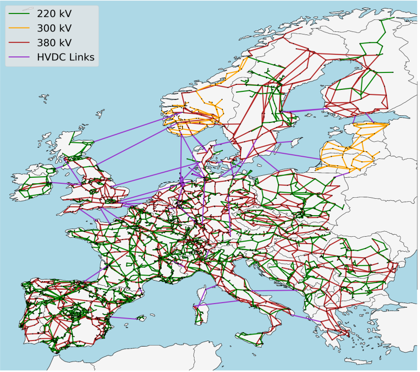
Approaches to Dealing with Variable Renewables

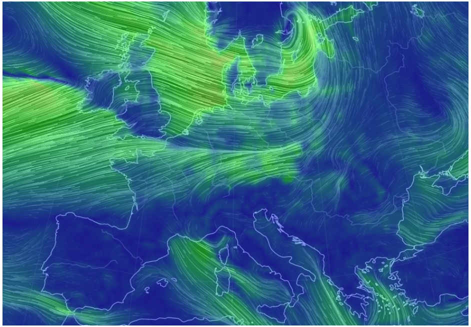
1. Smoothing renewable feed-in **in space** with transmission networks.
2. Smoothing the variability **in time** with storage.
3. Coupling with other energy **sectors** like heating, transport and industrial demand, and demand-side management.

Source: adapted from Tom Brown

Challenge 1: Spatial Resolution and Scope

Need high **spatial resolution** to represent renewables variations and transmission bottlenecks, as well as **continental** scope to capture large-scale weather patterns.





Source: adapted from Tom Brown

Challenge 2: Temporal Resolution and Scope

Need high **temporal resolution** but also **multi-year time series** to represent load and renewables' variability, correlations, patterns, extreme events, and the use of storage.

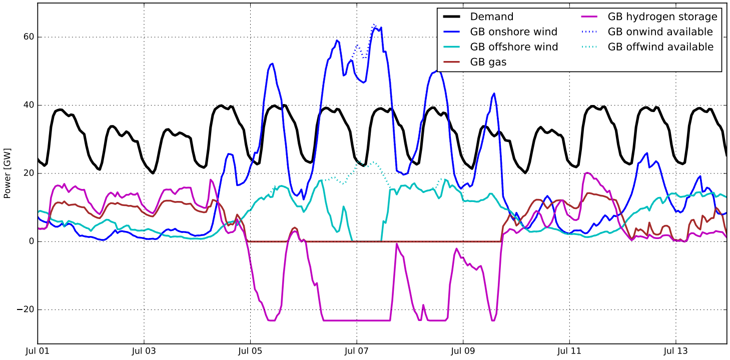
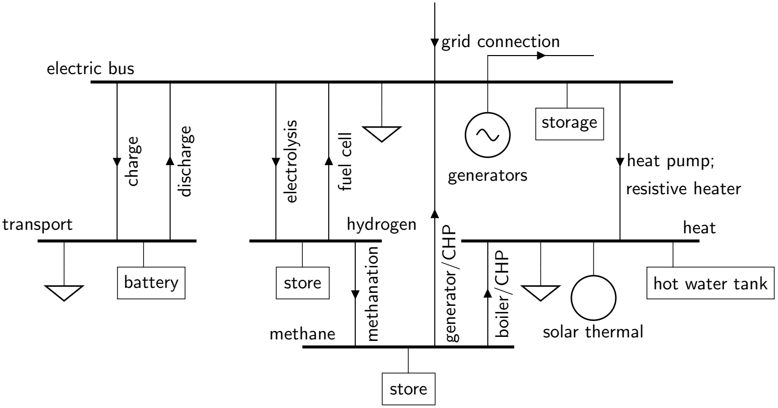


Figure: An exemplary energy system time profile for Great Britain in July 2013

Source: adapted from Tom Brown

Challenge 3: Model Complexity of Sector Coupling

What can we **simplify** while retaining accuracy?



Source: Tom Brown

Overarching Goal

Find the **sweet spot** where

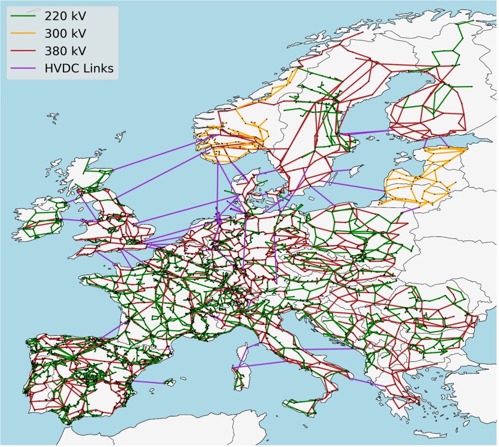
- Computation time and resource requirements are finite (i.e. a week, 250 GB RAM)
- Temporal resolution is “good enough”
- Spatial resolution is “good enough”
- Model detail is “good enough”

and **quantify the error** we make by only being “good enough”

(e.g. are important metrics $\pm 10\%$ or $\pm 50\%$ correct?)

Source: Tom Brown

PyPSA-Eur: Open Energy System Modelling



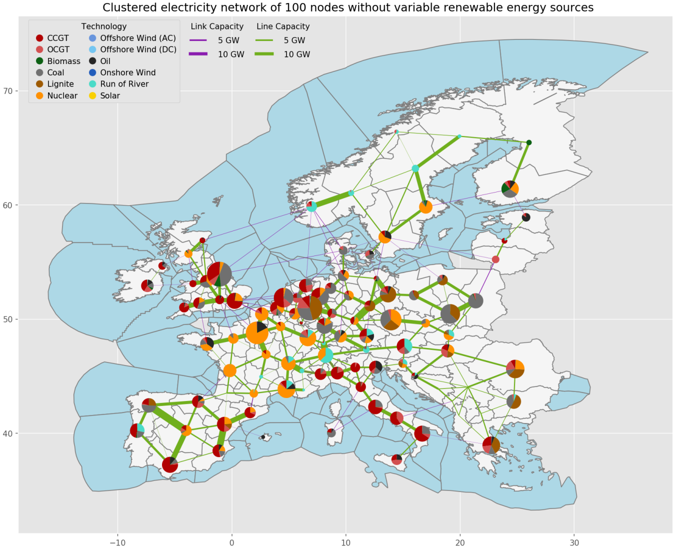
- Grid data based on network operator maps
- Power plant database combines multiple open databases using matching algorithms
- Renewable energy time series from reanalysis (historical) weather data (ERA-5)
- Geographic potentials from land use databases
- Time series aggregation (usually 8760h).
- Network clustering using *k*-means algorithm

Code and Documentation

- `https://pypsa-eur.readthedocs.io`
- `https://github.com/PyPSA/pypsa-eur`

Source: Tom Brown

PyPSA-Eur: Open Energy System Modelling



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PyPSA: Long-Term Investment Planning Problem (LP)

Find the long-term cost-optimal energy system, including
investments (generation, transmission and storage) and **short-term costs** (mainly fuel costs):

$$\textbf{Minimise} \left(\begin{array}{c}\textbf{Yearly}\\ \textbf{system costs}\end{array} \right) = \sum_{n} \left(\begin{array}{c}\textbf{Annualised}\\ \textbf{capital costs}\end{array} \right) + \sum_{n,t} \left(\begin{array}{c}\textbf{Marginal}\\ \textbf{costs}\end{array} \right)$$

subject to

- meeting **energy demand** at each node n (e.g. region) and time t (e.g. hour of year)
- **transmission constraints** between nodes and (linearised) power flow
- wind, solar, hydro (variable renewables) **availability time series** $\forall\, n, t$
- installed capacity $\leq$ **geographical potentials** for renewables
- **CO_2 constraint** (e.g. 95% reduction w.r.t. 1990 emission levels)
- **Flexibility** from gas plants, battery and H_2 storage, controllable HVDC links

Source: Tom Brown

Capacity Limits and Nodal Power Balance

Decision Variable **Parameter**

Capacity Limit:

$$|f_\ell^0| \leq F_\ell^0 \qquad \forall \ell \in \mathcal{L}^0$$

Nodal Power Balance (KCL):

$$p_i = \sum_\ell K_{i\ell} f_\ell^0 \qquad \forall i \in \mathcal{N}$$

where p_i is the active power injected or consumed at node i, f_ℓ^0 is the active power flow on branch ℓ, and $K \in \mathbb{R}^{|\mathcal{N}| \times |\mathcal{L}^0|}$ is the **incidence matrix of the network graph** which has non-zero values $+1$ if branch ℓ starts on node i and -1 if branch ℓ ends on node i.

Remarks

Power Conservation

KCL directly implies power conservation $\sum_i p_i = 0$ because $\sum_i K_{i\ell} = 0$ for all lines ℓ.

Equation System

- KCL provides $|\mathcal{N}|$ linear equations for the $|\mathcal{L}^0|$ unknown flows f_ℓ^0, of which one is linearly dependent.
- This is not sufficient to uniquely determine the flows unless the network is a **tree**!
- Hence, $|\mathcal{L}^0| - |\mathcal{N}| + 1$ additional independent equations are needed.

Recap: Linearisation of the AC power flow equations

Nonconvex AC power flow in voltage-polar coordinates:

$$f_\ell = p_\ell = g_\ell|v_i|^2 - |v_i||v_j|(g_\ell \cos(\theta_i - \theta_j) - b_\ell \sin(\theta_i - \theta_j))$$
$$q_\ell = b_\ell|v_i|^2 - |v_i||v_j|(g_\ell \sin(\theta_i - \theta_j) - b_\ell \cos(\theta_i - \theta_j))$$

Linearised "DC" power flow assumptions:

1. All voltage magnitudes $|v_i|$ are close to one per unit.
2. Conductances $|g_\ell|$ are negligible relative to susceptances b_ℓ.
3. Voltage angle differences are small enough such that $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$
4. Reactive power flows q are negligible relative to real power flows p.

Angle-based Linear Optimal Power Flow Formulation – angles

With these assumptions, the (active) power flow is commonly formulated with phase angles

$$f_\ell^0 = \frac{(\theta_i - \theta_j)}{x_\ell^0} = \frac{1}{x_\ell^0} \sum_i K_{i\ell}\theta_i \qquad \forall \ell \in \mathcal{L}^0$$

where x_ℓ^0 is the reactance and $(\theta_i - \theta_j)$ is the voltage angle difference between nodes i and j.

Due to **rotational degeneracy** we need a slack bus (reference bus):

$$\theta_0 = 0$$

Together, how many power flow constraints are there?

Answer: $|\mathcal{N}| + |\mathcal{L}^0| + 1$

Cycle-based Linear Optimal Power Flow – `kirchhoff`

KVL states that the sum of voltage angle differences across branches around all cycles in the network must sum to zero.

It follows from graph theory that there are $|\mathcal{L}^0| - |\mathcal{N}| + 1$ independent simple cycles (cycle basis) in a connected graph, which can be expressed in a **cycle incidence matrix**:

$$C^0_{\ell c} = \begin{cases} 1 & \text{if edge } \ell \text{ is element of cycle } c, \\ -1 & \text{if reversed edge } \ell \text{ is element of cycle } c, \\ 0 & \text{otherwise.} \end{cases}$$

Then our condition for KVL becomes:

$$\sum_\ell C^0_{\ell c}(\theta_i - \theta_j) = 0 \qquad \forall c = 1, \ldots, |\mathcal{L}^0| - |\mathcal{N}| + 1.$$

Cycle-based Linear Optimal Power Flow – `kirchhoff`

In addition to...

$$\sum_{\ell} C^0_{\ell c}(\theta_i - \theta_j) = 0 \qquad \forall c = 1, \ldots, |\mathcal{L}^0| - |\mathcal{N}| + 1.$$

... we also know *(from the angle-based formulation)* that ...

$$f^0_\ell = \frac{(\theta_i - \theta_j)}{x^0_\ell} \quad \Leftrightarrow \quad (\theta_i - \theta_j) = x^0_\ell f^0_\ell$$

... and by substitution we get ...

$$\sum_{\ell} C^0_{\ell c} x^0_\ell f^0_\ell = 0 \qquad \forall c = 1, \ldots, |\mathcal{L}^0| - |\mathcal{N}| + 1.$$

Cycle-based Linear Optimal Power Flow – `kirchhoff`

Together, how many power flow variables are there?

Angle-based Formulation: $|\mathcal{N}| + |\mathcal{L}^0| \quad \rightarrow \quad f_\ell, \theta_i$

Cycle-based Formulation: $|\mathcal{L}^0| \quad \rightarrow \quad f_\ell$

Together, how many power flow constraints are there?

Angle-based Formulation: $|\mathcal{N}| + |\mathcal{L}^0| + 1$

Cycle-based Formulation: $|\mathcal{L}^0| + 1$

If needed, the voltage angles can be calculated post-facto by solving:

$$p_i = \sum_\ell K_{i\ell} \frac{1}{x_\ell} \sum_j K_{j\ell} \theta_j = \sum_j KBK^\top \theta_j \qquad \forall i \in \mathcal{N}$$

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Benchmark Results

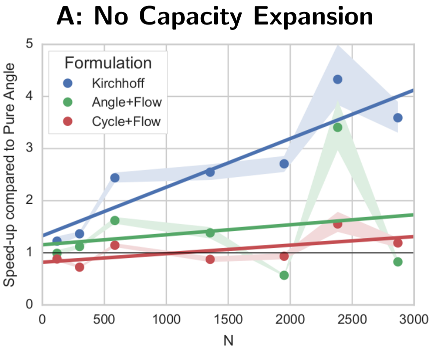

Figure 5: Speed-up of LOPF compared to Pure Angle per buses, shown are the mean values with 99% confidence interval and the result of a linear regression of all values for the three fastest formulations in mode ‘r’.

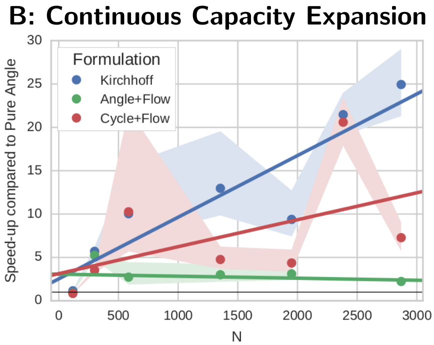

Figure 7: Speed-up of LOPF with capacity optimization compared to Pure Angle per buses. Shown are the mean values with 99% confidence interval and the result of a linear regression of all values.

Source: Jonas Hörsch, Henrik Ronellenfitsch, Dirk Witthaut, Tom Brown, *Linear optimal power flow using cycle flows*, Electric Power Systems Research, Volume 158, 2018, Pages 126-135, 10.1016/j.epsr.2017.12.034, arXiv:1704.01881

Transmission Network Expansion Planning with LOPF

What's different now? LP $\rightarrow$ MI(N)LP

So far we have neglected that transmission line expansion is usually a **discrete problem** and that the **line impedance depends on the line capacity**.

In transmission expansion planning we consider the discrete reinforcement of transmission lines based on a set of candidate projects $\mathcal{L}^1$.

We introduce a binary investment variable $i_\ell \in \mathbb{B}$ for each candidate line $\ell \in \mathcal{L}^1$ and then formulate a limit on the power flow depending on the investment decision.

$$|f_\ell^1| \leq i_\ell F_\ell^1 \qquad \forall \ell \in \mathcal{L}^1.$$

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Angle-based Transmission Expansion Planning – (angles)

Using a **Big-M disjunctive relaxation** we get for KVL:

$$f_\ell^1 - \frac{(\theta_i - \theta_j)}{x_\ell^1} \geq -M_\ell^{\text{KVL}}(1 - i_\ell)$$
$$\leq +M_\ell^{\text{KVL}}(1 - i_\ell) \qquad \forall \ell \in \mathcal{L}^1$$

Asynchronous power grids may get synchronised by building new lines and on of the reference angle constraints must be lifted in such case:

$$|\theta_i| \leq \sum_{\ell \in \mathcal{L}_i^1} i_\ell M_\ell^{\text{slack}}$$

The Big-Ms can easily incur numerical challenges $\rightarrow$ choose as small as possible

Big-M Parameters (angle-based)

The value of the disjunctive constant M_ℓ^{KVL} for a candidate line ℓ that connects two buses i and j of the same synchronous zone can be chosen following

$$M_\ell^{\text{KVL}} \geq \frac{|\mathcal{P}_{i,j}^{\text{min}}|}{x_\ell^1}$$

where $|\mathcal{P}_{i,j}^{\text{min}}|$ is the shortest path between the buses i and j along edges k of the existing network graph $\mathcal{G} = (\mathcal{N}, \mathcal{L}^0)$ with weights $F_k^0 x_k^0$.

Asynchronous Power Networks

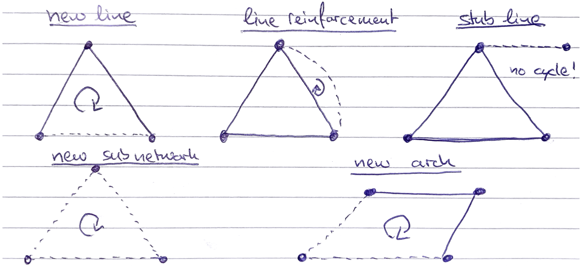
No path to infer an existing limit $\rightarrow$ value independent of slack bus

$$M_\ell^{KVL} \geq \frac{\sum_{k\in\mathcal{L}^0\cup\mathcal{L}^1} F_k x_k}{x_\ell^1}$$

Source: Silvio Binato, Mario Pereira, Sergio Granville, *A new Benders decomposition approach to solve power transmission network design problems*, IEEE Transactions on Power Systems, Volume 16, Issue 2, 2001, Pages 235-240, 10.1109/59.918292

Cycle-based Transmission Expansion Planning – (`kirchhoff`)

Investing in candidate lines can incur new cycles for which KVL must hold, if and only if all candidate lines that would form a new cycle are built.



Handling synchronisation becomes easier!

No slack buses & fewer Big-M parameters/constraints & KCL may sometimes be sufficient.

Cycle-based Transmission Expansion Planning – (`kirchhoff`)

Given the *candidate cycles* as a matrix $C^1_{\ell c}$ we can formulate the corresponding KVL analogously as before and make sure it holds if and only if all candidate lines of that cycle are built; i.e.

$$\sum_{\ell \in \mathcal{L}^0 \cup \mathcal{L}^1} C^1_{\ell c} x_\ell f_\ell \geq -M^{\text{KVL}}_c \left(\sum_{\ell \in \mathcal{L}^1} C^1_{\ell c} (1 - i_\ell) \right)$$
$$\leq +M^{\text{KVL}}_c \left(\sum_{\ell \in \mathcal{L}^1} C^1_{\ell c} (1 - i_\ell) \right) \qquad \forall\, c \in \mathcal{C}^1$$

How to Determine the Candidate Cycle Matrix?

For each candidate line find a shortest path (most sparse) through the network graph that includes both existing and candidate transmission infrastructure but not itself.

Try to minimise the number of other candidate lines in the path (to avoid formulating constraints for combinations of investments where unnecessary) and remove duplicates.

The edges of the shortest path and the respective candidate line form a **candidate cycle**.

Synchronising more than 2 asynchronous zones is more challenging!

Need to consider all combinations of synchronisation by candidate lines.

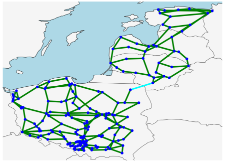
Big-M Parameters (cycle-based)

Constraint must be inactive even if an investment decision for only one candidate line is missing to complete the cycle.

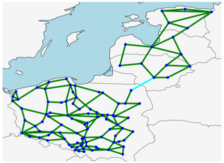
$$M_c^{\text{KVL}} \geq \sum_{\ell \in \mathcal{L}^0 \cup \mathcal{L}^1} C^1_{\ell c} x_\ell F_\ell$$

Benchmark: Test Cases

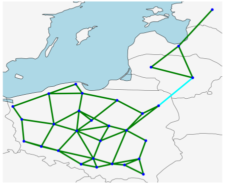
Baltics + Poland (180 nodes)



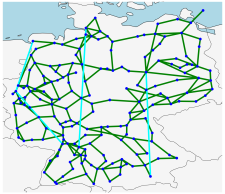
Baltics + Poland (100 nodes)



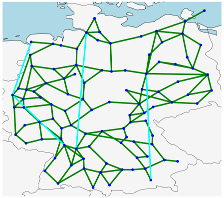
Baltics + Poland (30 nodes)



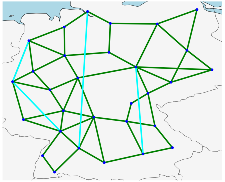
Germany (180 nodes)



Germany (100 nodes)

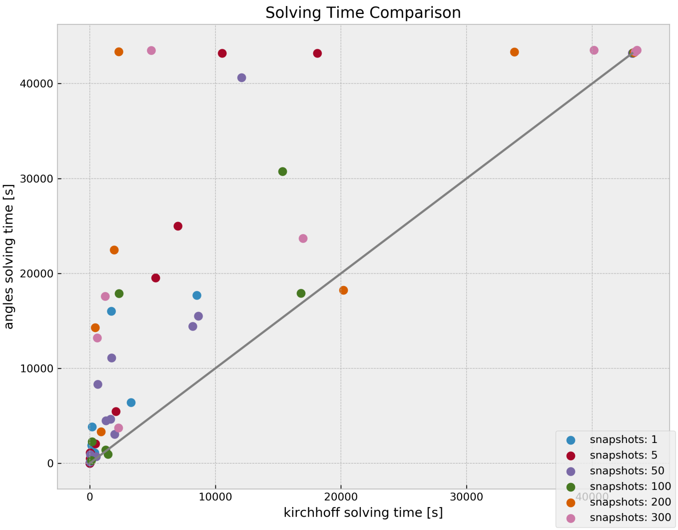


Germany (30 nodes)



- 2 regions
- 30, 100, 150, or 180 nodes
- 1, 5, 50, 100, or 200 snapshots
- 0.1% or 0.5% MIP optimality gap tolerance
- 1 or 2 candidate lines per corridor

Benchmark: Computation Times



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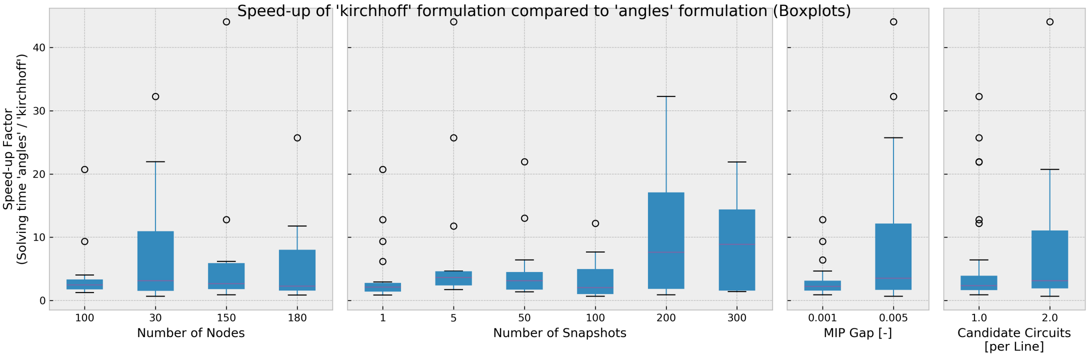
Benchmark: Computation Times



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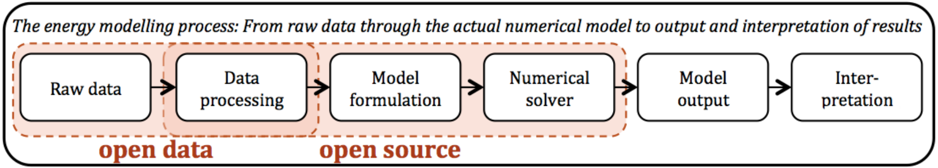
The Idea of Open Energy Modelling

The whole chain from raw data to modelling results should be open:



Why is openness important?

- Transparency
- Quality
- Credibility
- Avoiding double-work
- Encouraging cooperation
- Visibility
- Education
- Public funding
- Public engagement
- Unknown benefits and synergies

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Open Energy Modelling Initiative (openmod)

The **Open Energy Modelling Initiative** is a grass roots community of open energy modellers from universities, research institutions and the interested public that promotes open code, open data and open publishing in energy modelling.

openmod open energy modelling initiative

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Open Software: PyPSA and PyPSA-Eur

Our free software **PyPSA** is online at `pypsa.org` and on GitHub.

- It can do **total energy system investment optimisation** with linear optimal power flow including generation expansion, transmission expansion, storage expansion, sector coupling, unit commitment, and security-constraints)
- It has **models** for power plants, storage, meshed AC grids, meshed DC grids, hydro plants, variable renewables and sector coupling at European scale.

PyPSA-Eur is documented at `pypsa-eur.readthedocs.io` and available on GitHub.

- It is the corresponding **open model dataset** of the European power system at the transmission network level.

Resources

Find the slides:

https://neumann.fyi/files/cycletep.pdf

Send an email:

fabian.neumann@kit.edu

Find the energy system model:

Code: https://github.com/pypsa/pypsa-eur

Documentation: https://pypsa-eur.readthedocs.io