Transmission Expansion Planning Using Cycle Flows

Fabian Neumann and Tom Brown

fabian.neumann@kit.edu
Institute for Automation and Applied Informatics
Karlsruhe Institute of Technology (KIT), Germany

June 12, 2020

HELMHOLTZ
RESEARCH FOR GRAND CHALLENGES

Karlsruhe Institute of Technology

Long-Term Power System Planning

Cost-effective pathways to reduce greenhouse gas emissions → optimisation models

Simultaneous investment planning of generation, storage and **transmission** → trade-offs

$$\textbf{Minimise}\left[\begin{array}{c}\textbf{Yearly}\\ \textbf{system costs}\end{array}\right] = \textbf{Minimise}\left[\sum_n \left(\begin{array}{c}\textbf{Annualised}\\ \textbf{capital costs}\end{array}\right) + \sum_{n,t}\left(\begin{array}{c}\textbf{Operational}\\ \textbf{costs}\end{array}\right)\right]$$

subject to **linear optimal power flow (LOPF)** constraints,
the spatio-temporal **variability & potentials** of renewable energy,
and **emission reduction** targets.

Fast equivalent reformulation based on **cycle decomposition**

Capacity Limits and Nodal Power Balance

Decision Variable Parameter

Capacity Limit:

$$|f_\ell^0| \leq F_\ell^0 \qquad \forall \ell \in \mathcal{L}^0$$

Kirchhoff's Current Law (KCL) enforces nodal power balance:

$$p_i = \sum_\ell K_{i\ell} f_\ell^0 \qquad \forall i \in \mathcal{N}$$

where p_i is the active power injected or consumed at node i, f_ℓ^0 is the active power flow on branch ℓ, and $K \in \mathbb{R}^{|\mathcal{N}| \times |\mathcal{L}^0|}$ is the **incidence matrix of the network graph** which has non-zero values $+1$ if branch ℓ starts on node i and -1 if branch ℓ ends on node i.

Remarks

- KCL provides $|\mathcal{N}|$ linear equations for the $|\mathcal{L}^0|$ unknown flows (one is linearly dependent).
- not sufficient to uniquely determine the flows unless the network is a **tree**
- $|\mathcal{L}^0| - |\mathcal{N}| + 1$ additional independent equations needed

$\rightarrow$ **Kirchhoff's Voltage Law (KVL)**

$\rightarrow$ Linearisation of nonconvex AC power flow in voltage-polar coordinates

Angle-based LOPF

KVL is commonly formulated with **auxiliary** voltage angle variables:

$$f_\ell^0 = \frac{\theta_i - \theta_j}{x_\ell^0} = \frac{1}{x_\ell^0} \sum_i K_{i\ell}\theta_i \qquad \forall \ell \in \mathcal{L}^0$$

where x_ℓ^0 is the reactance and θ_i is the voltage angle at node i.

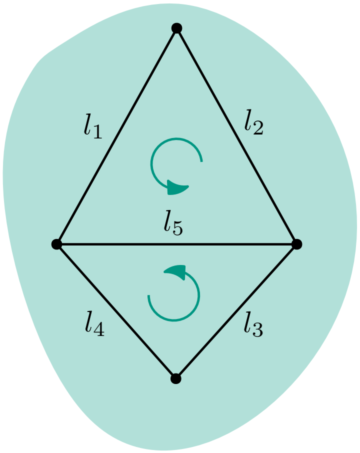
Select a slack bus to avoid **rotational degeneracy**:

$$\theta_0 = 0$$

$|\mathcal{N}|$ constraints from KCL

$|\mathcal{L}^0| + 1$ constraints from KVL

Cycle-based LOPF



Kirchhoff's Voltage Law (KVL)

The sum of voltage angle differences across branches around all cycles must sum to zero.

Consider the **example**:

- 3 simple cycles
- but only 2 independent simple cycles
- Third cycle covered by linear combination of the other two cycles

Cycle-based LOPF

There are $|\mathcal{L}^0| - |\mathcal{N}| + 1$ independent simple cycles in a connected graph.

We can express this **cycle basis** in a **cycle incidence matrix**:

$$C^0_{\ell c} = \begin{cases} 1 & \text{if edge } \ell \text{ is element of cycle } c, \\ -1 & \text{if reversed edge } \ell \text{ is element of cycle } c, \\ 0 & \text{otherwise.} \end{cases}$$

Then our condition for KVL becomes:

$$\sum_{\ell} C^0_{\ell c}(\theta_i - \theta_j) = 0 \qquad \forall c = 1, \ldots, |\mathcal{L}^0| - |\mathcal{N}| + 1.$$

Cycle-based LOPF

$$\sum_\ell C^0_{\ell c}(\theta_i - \theta_j) = 0$$

Let's substitute what we know from the angle-based formulation.

$$f^0_\ell = \frac{\theta_i - \theta_j}{x^0_\ell} \quad \Leftrightarrow \quad \theta_i - \theta_j = x^0_\ell f^0_\ell$$

$$\sum_\ell C^0_{\ell c} x^0_\ell f^0_\ell = 0 \qquad \forall c = 1, \ldots, |\mathcal{L}^0| - |\mathcal{N}| + 1.$$

$|\mathcal{N}|$ constraints from KCL

$|\mathcal{L}^0| - |\mathcal{N}| + 1$ constraints from KVL

Comparison

	variables	constraints
angle-based	$\lvert\mathcal{L}^0\rvert + \lvert\mathcal{N}\rvert$	$\lvert\mathcal{L}^0\rvert + \lvert\mathcal{N}\rvert + 1$
cycle-based	$\lvert\mathcal{L}^0\rvert$	$\lvert\mathcal{L}^0\rvert + 1$

Benchmarks: Jonas Hörsch, Henrik Ronellenfitsch, Dirk Witthaut, Tom Brown, *Linear optimal power flow using cycle flows*, Electric Power Systems Research, Volume 158, 2018, Pages 126-135, 10.1016/j.epsr.2017.12.034, arXiv:1704.01881

Transmission Expansion Planning (TEP) with LOPF

We want to use fast cycle-based LOPF formulation in TEP problems.

What's different now? LP $\rightarrow$ MI(N)LP

So far we neglected that

- transmission line expansion is often formulated as a **discrete problem** and
- line impedance depends **nonlinearly** on the line capacity.

Let $\mathcal{L}^1$ be a set of candidate lines.

Introduce binary investment variables $i_\ell \in \mathbb{B} \; \forall \, \ell \in \mathcal{L}^1$.

Formulate limit on the flow depending on the investment decision.

$$|f_\ell^1| \leq i_\ell F_\ell^1 \qquad \forall \ell \in \mathcal{L}^1.$$

Karlsruhe Institute of Technology

Angle-based TEP

KVL with **Big-M disjunctive relaxation** $f_\ell^1 = (\theta_i - \theta_j)x_\ell^{-1}$ when built

$$\left| f_\ell^1 - \frac{\theta_i - \theta_j}{x_\ell^1} \right| \leq M_\ell^{\text{KVL}}(1 - i_\ell) \qquad \forall \ell \in \mathcal{L}^1$$

Synchronous zones may be connected $\rightarrow$ one of the slack bus constraints must be lifted

$$|\theta_i| \leq \sum_{\ell \in \mathcal{L}_i^1} i_\ell M_\ell^{\text{slack}}$$

$\rightarrow$ complicated, see paper

1. Find big-M parameters M_ℓ^{KVL}.
2. Find big-M parameters M_ℓ^{slack}.

Big-M parameters can cause numerical issues $\rightarrow$ choose as small as possible

Karlsruhe Institute of Technology

Big-M Parameters M_ℓ^{KVL}

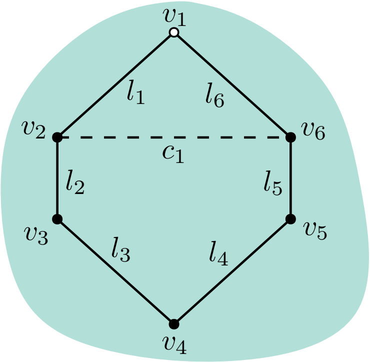
Within **one synchronous zone**:

$$M_\ell^{\text{KVL}} \geq \frac{|\mathcal{P}_{i,j}^{\text{min}}|}{x_\ell^1}$$

where $|\mathcal{P}_{i,j}^{\text{min}}|$ is the shortest path between the buses i and j along edges k of the existing network graph $\mathcal{G} = (\mathcal{N}, \mathcal{L}^0)$ with weights $F_k^0 x_k^0$.

Across **two synchronous zones**:

$$M_\ell^{KVL} \geq \frac{\sum_{k\in\mathcal{L}^0\cup\mathcal{L}^1} F_k x_k}{x_\ell^1}$$

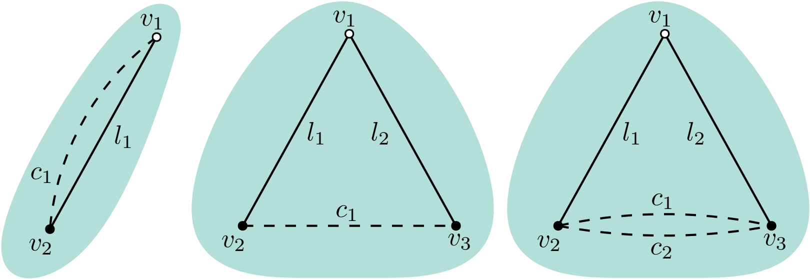


→ for proof see paper

Source: Silvio Binato, Mario Pereira, Sergio Granville, *A new Benders decomposition approach to solve power transmission network design problems*, IEEE Transactions on Power Systems, Volume 16, Issue 2, 2001, Pages 235-240, 10.1109/59.918292

Cycle-based TEP

Candidate lines can incur **new cycles** for which KVL must hold only if all its lines are built.



Initial cycle basis can be used! Candidate cycles only extend it.

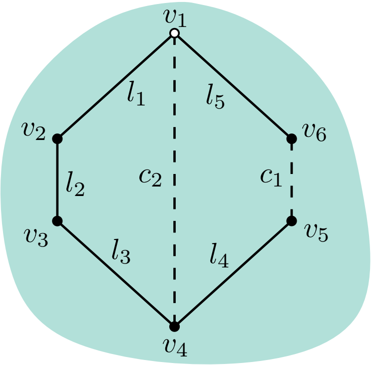
Cycle-based TEP

Given the **candidate cycles** as a matrix $C^1_{\ell c}$ we can formulate KVL as before, but have to make sure it is only binding if all candidate lines of that cycle are built; i.e.

$$\left| \sum_{\ell \in \mathcal{L}^0 \cup \mathcal{L}^1} C^1_{\ell c} x_\ell f_\ell \right| \leq M_c^{\text{KVL}} \cdot \sum_{\ell \in \mathcal{L}^1} C^1_{\ell c}(1 - i_\ell) \qquad \forall\, c \in \mathcal{C}^1$$

1. Find candidate cycle matrix $C^1_{\ell c}$.
2. Find big-M parameters M_c^{KVL}.

Candidate Cycle Matrix $C^1_{\ell c}$

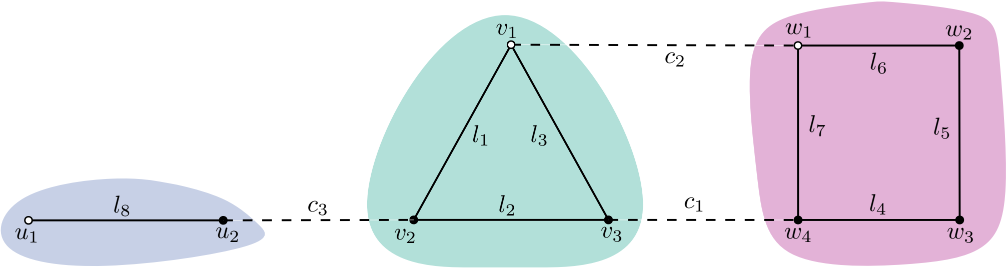


Candidate line within **same synchronous zone**:

- Find shortest path through existing network.
  - Sparsest constraints.
  - No investment interdependencies.
- Shortest path plus candidate line form candidate cycle.

Candidate Cycle Matrix $C^1_{\ell c}$

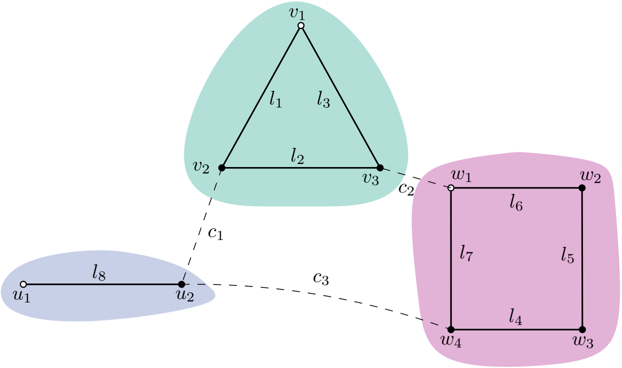
Candidate line(s) across **two synchronous zones**:

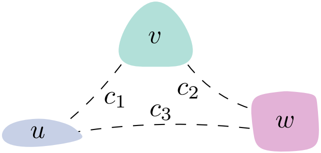


- c_3 introduces no new cycle.
- c_1 and c_2 form a candidate cycle together with the connecting shortest paths in each zone.

Candidate Cycle Matrix $C^1_{\ell c}$

Candidate lines across **multiple synchronous zones**:

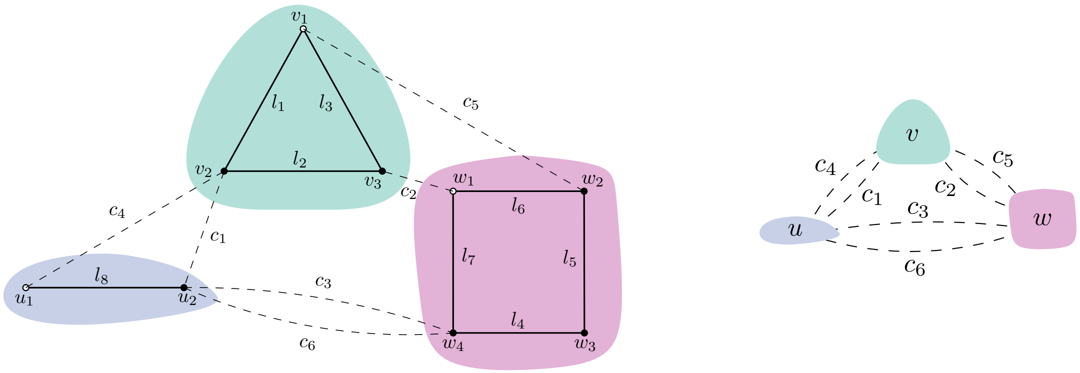




There could be more **complex synchronisation options**, extending beyond two candidate lines.

Candidate Cycle Matrix $C^1_{\ell c}$

Across **multiple synchronous zones**:



Need to consider **all simple cycles** of subnetwork graph (plus shortest paths within zones).

Big-M Parameters M_c^{KVL}

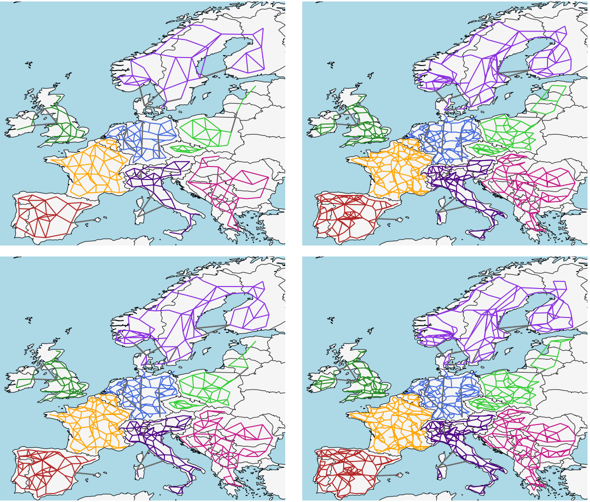
Find an **upper bound** that holds regardless of investment decisions:

$$M_c^{\text{KVL}} \geq \sum_{\ell \in \mathcal{L}^0 \cup \mathcal{L}^1} C_{\ell c}^1 x_\ell F_\ell \quad \geq \quad \sum_{\ell \in \mathcal{L}^0 \cup \mathcal{L}^1} C_{\ell c}^1 x_\ell f_\ell$$

$\rightarrow$ for proof see paper

Agnostic whether candidate lines are within or across synchronous zones!

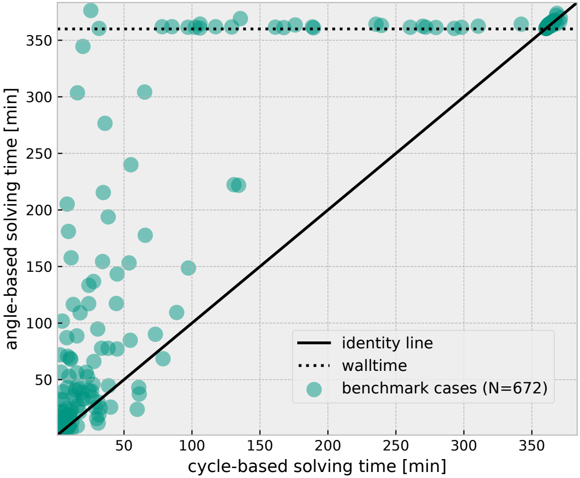
Large Set of Realistic Test Cases



- open-source implementation in PyPSA
- open-source European power transmission system model PyPSA-Eur
  - co-optimisation of generation and transmission with ambitious CO_2 targets
  - variety of clustered regional extracts
  - time series reduction to $\leq$ 75 snapshots
  - up to 2 candidate lines per corridor
  - no synchronisation options
  - tolerated optimality gap $\leq$ 1%
  - commercial solver Gurobi 9.0

Karlsruhe Institute of Technology

Solving Times



→ for more sensitivity analyses see paper

Conclusion

Introduced an alternative but equivalent **cycle-based TEP formulation with LOPF**, which

- omits unnecessary auxiliary variables for bus voltage angles,
- builds KVL constraints based on a cycle decomposition of the network graph,
- can easily consider the connection of multiple disconnected networks,
- solves on average four times faster than common angle-based formulation!

Resources

Find the slides:

https://neumann.fyi/files/cycletep.pdf

Send an email:

fabian.neumann@kit.edu

Find the energy system model:

Code: https://github.com/pypsa/pypsa-eur

Documentation: https://pypsa-eur.readthedocs.io

Karlsruhe Institute of Technology