

# Heuristics for Transmission Expansion Planning in Low-Carbon Energy System Models

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# Long-Term Energy System Optimisation

Cost-effective pathways to reduce greenhouse gas emissions → optimisation models

Simultaneous investment planning of generation, storage and **transmission** → trade-offs

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$$\text{Minimise} \left( \begin{array}{c} \text{Yearly} \\ \text{system costs} \end{array} \right) = \sum_n \left( \begin{array}{c} \text{Annualised} \\ \text{capital costs} \end{array} \right) + \sum_{n,t} \left( \begin{array}{c} \text{Operational} \\ \text{costs} \end{array} \right)$$

subject to **linear optimal power flow** constraints,  
the **variability & potentials** of renewable energy,  
and **emission reduction** targets.

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# Linearised Power Flow without capacity expansion

Decision Variable

Parameter

Susceptance

Labels of Heuristics

power flow between nodes  
i and j at time t

$$\overbrace{|f_{ij,t}|}$$

$$\leq$$

line capacity

$$\overbrace{F_{ij,t}}$$

line susceptance

$$b_{ij} = \frac{1}{x_{ij}}$$

voltage angle difference

$$f_{ij,t} = \overbrace{b_{ij}} \cdot \overbrace{(\theta_{i,t} - \theta_{j,t})}$$

# Transmission Expansion Planning Problem (MINLP)

If line capacity can be **extended** discretely, the problem becomes **nonconvex**...

$$|f_{ij,t}| \leq \overbrace{\left(1 + \frac{\Gamma_{ij}}{\tilde{\gamma}_{ij}}\right)}^{\substack{\text{relative increase of capacity} \\ \text{by investment } (\Gamma_{ij} \in \mathbb{N}) \\ \text{w.r.t. initial \# circuits } (\tilde{\gamma}_{ij})}} \cdot \underbrace{\tilde{F}_{ij}}_{\text{original line capacity}}$$

... and **nonlinear** ...

$$f_{ij,t} = \overbrace{\left(1 + \frac{\Gamma_{ij}}{\tilde{\gamma}_{ij}}\right)}^{b_{ij}} \tilde{b}_{ij} \cdot (\theta_{i,t} - \theta_{j,t})$$

... which we can transform to an **MILP** using a **Big-M disjunctive relaxation**.

# Heuristic 1: Relaxation of Line Investment Variables

Reminder: **Labels of Heuristics**

**Discrete** investment decisions

heur-int

$$\Gamma_{ij} \in \mathbb{N}_{\geq 0}$$

are relaxed to allow each line to be **expanded continuously**, i.e.

heur

$$\Gamma_{ij} \in \mathbb{R}$$

## Heuristic 2: Iterative Update of (Discretized) Line Impedances

Pursue an **iterative** approach (convergence tolerance or iteration limit)

iter(-seqdisc)

$$f_{ij,t} = \overbrace{\left(1 + \frac{\Gamma_{ij}}{\tilde{\gamma}_{ij}}\right)}^{b_{ij}} \tilde{b}_{ij} \cdot (\theta_{i,t} - \theta_{j,t}) \quad \longrightarrow \quad f_{ij,t} = b_{ij}^{(k)} \cdot (\theta_{i,t} - \theta_{j,t})$$

First iteration  $\rightarrow$  **initial susceptances**

$$b_{ij}^{(1)} = \tilde{b}_{ij}$$

Subsequent iterations  $\rightarrow$  current **optimal line investment**  $\Gamma_{ij}^{*(k)}$  (or rounded)

$$b_{ij}^{(k+1)} = \left(1 + \frac{\Gamma_{ij}^{*(k)}}{\tilde{\gamma}_{ij}}\right) \tilde{b}_{ij} \quad \forall k > 1$$

## Heuristic 3: Post-facto Discretization of Line Investment Variables

Following the iterations,  
fractional decisions must be **fitted to valid investment choices**:

postdisc

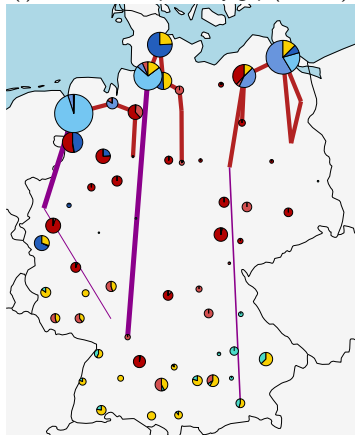
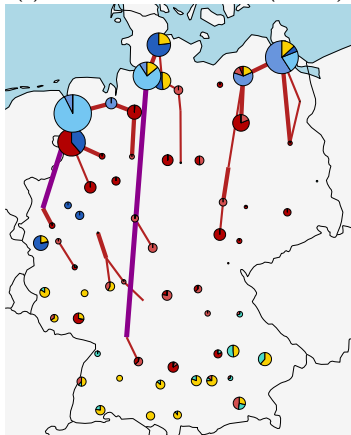
- 1 Round relaxed line capacities nearest discrete choice (threshold  $z = 0.3$ )
- 2 Fix line capacities & rerun generation expansion only.

Optionally:

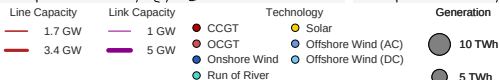
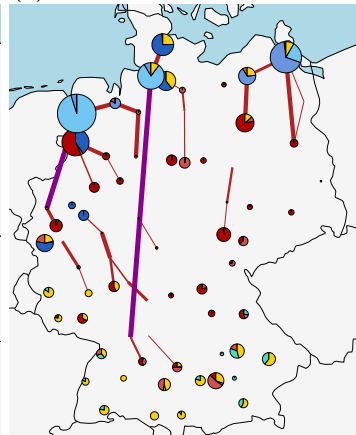
postdisc\_mult

- 3 Repeat for multiple discretization thresholds ( $z$ ) & choose configuration with lowest costs.

# Results: Generation and Transmission Expansion

(i) MILP at 1% optimality gap (*feasible*)(ii) Discrete iterative heuristic (*feasible*)

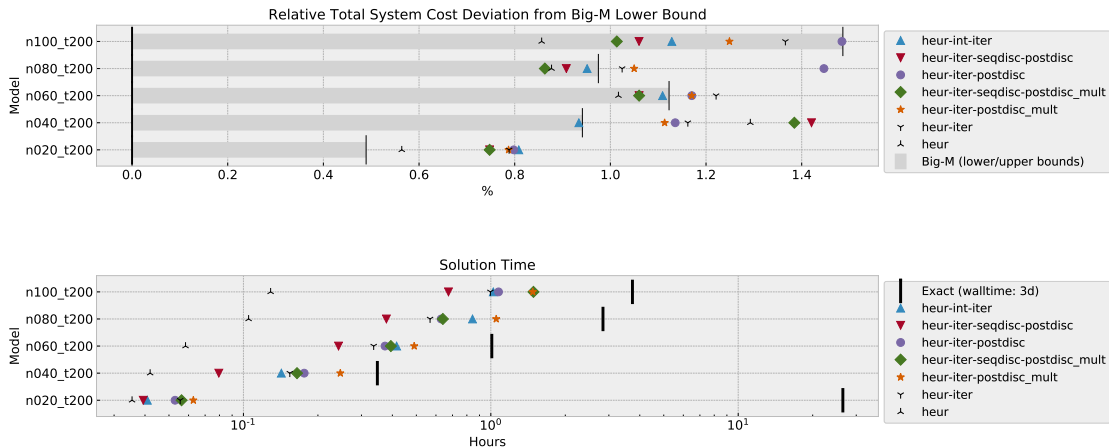
(iii) Continuous relaxation with iterations



Source: Energy system model: Hörsch et al. [PyPSA-Eur: An Open Optimisation Model of the European Transmission System](#)



# Results: Accuracy & Speed



# Conclusion

When **co-optimizing** generation, transmission and storage infrastructure **discrete line expansion** is computationally prohibitive (to date).

The heuristics **mirror optimal line investment** of the MINLP close enough in view of **attainable optimality gaps** but with **immense time savings!**

# Resources



16<sup>TH</sup> INTERNATIONAL CONFERENCE ON  
THE EUROPEAN ENERGY MARKET  
18-20 September 2019, Ljubljana, Slovenia



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## Find the slides:

<https://neumann.fyi/files/eem19-tepheuristics.pdf>

## Send an email:

[fabian.neumann@kit.edu](mailto:fabian.neumann@kit.edu)

## Find the energy system model:

Code: <https://github.com/pypsa/pypsa-eur>

Documentation: <https://pypsa-eur.readthedocs.io>