

2019-1 Computer Algorithms Homework #4 (Midterm Exam)

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(Deadline : May 6)

1. If $n \in \mathbb{N}$ then $(n+4)^4 + 4$ is not in PRIME. Prove or disprove.
2. Prove by mathematical induction that for any integer $n > 23$ there exist nonnegative integers x and y such that

$$n = 7x + 5y$$

3. Prove by mathematical induction that for a natural number $n > 1$

$$1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n+1)! - 1$$

4. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

Given $n \times n$, ($1 \leq n \leq 30$) matrix X and k , ($1 \leq k \leq 10^9$), calculate $R = X + X^2 + \dots + X^k$ and print R .

To avoid overflow problem, instead of printing each entry in r_{ij} of R , print the remainder of dividing r_{ij} by 32767.

The running time of the program should be less than 2 seconds.

Input

n=3

k=4

X={{1,2,3},{4,5,6},{7,8,9}}

Output

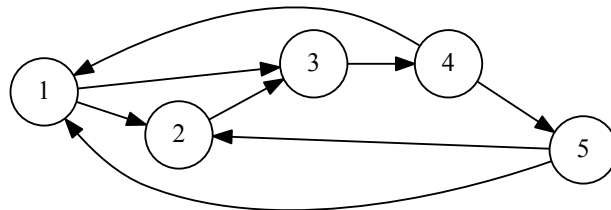
R={{8059, 9902, 11745},{18250, 22424, 26598},{28441, 2179, 8684}}

5. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

Given a directed graph $G = (V, E)$, where $1 \leq |V| \leq 100$, calculate the number of all distinct paths with length k , where $1 \leq k \leq 10^9$.

The running time of the program should be less than 2 seconds.

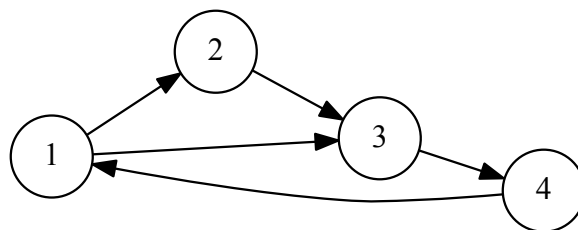
For example, for the following graph and $k = 3$:



Output

17

Another example for the following graph and $k = 2$:



Output

6

6. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

One Korean chaebol company tries to hire $1 \leq A \leq 10,000$ number of men and $1 \leq B \leq 10,000$ number of women. It costs \$10,000 to hire one person.

There is intimacy $1 \leq d \leq 9999$ between man and woman. So if there is intimacy d between a man and a woman and if one is hired, then the other can be hired with \$10,000− d . One person can have intimacies with multiple people, but we can use only one intimacy when hiring that particular person. The total number of intimacies D can be $1 \leq D \leq 50,000$.

Given A , B , D , and D number of (a, b, d) 's where a is a man, b is a woman, and d is their intimacy, calculate the minimum cost to hire all A men and B women.

Input

```
5 5 8
0 0 6590
0 1 3073
1 3 4573
1 3 2149
2 2 789
3 3 975
4 2 204
4 3 631
```

Output

83165

Input

```
5 5 10
0 4 4372
2 0 16
2 0 146
2 4 326
2 4 2133
2 4 9220
3 1 8364
3 2 6336
3 4 8833
4 1 2339
```

Output

60390

7. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

In a certain near future, Korea is united and we can drive to Europe from Korea. The straight line distance from Busan to Paris is about 9,307 km.

Mina is considering moving from Busan to a certain city. She is thinking of renting a truck for moving. The fuel price is very high, so she wants to minimize the fuel price for the movement.

For easy calculation, let's assume that the truck consumes 1 km/l, and the truck can contain upto 200 liters of fuel. When Mina rents the truck at Busan, the truck tank has a half of its capacity. When she returns the truck at the destination, she has to fill the fuel as much as the half of its capacity, otherwise she has to pay a lot of money to the rental company for the fuel handling charge. Mina wants to minimize the fuel cost, but doesn't want to run out of fuel during the trip.

Input will have the distance in kilometers ($\leq 10,000$) in the first line. The second line is the number of filling stations (≤ 100). From the third line, we have information about the filling stations, sorted by the distance from Busan. The information has two integers: the distance from Busan ($\leq 10,000$) and fuel price per liter in Korean Won ($\leq 2,000$).

Output will be the cost for the minimum fuel needed. If it is impossible to go to the city because of input condition, output "impossible".

The running time of the program should be less than 2 seconds.

Input

```
500
9
100 1200
150 1400
200 1300
250 1000
300 1100
350 1400
400 1300
450 1000
500 1399
```

Output

```
530000
```

8. (a) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

- (b) Generalize the answer for the optimal code when the frequencies are the first n Fibonacci numbers.

What is a Huffman code for the k^{th} character of n characters?

- (c) Using pseudo code, present Huffman algorithm for binary codes.

- (d) Using pseudo code, present Huffman algorithm for ternary codes.

9. The diameter of a tree $T = (V, E)$ is given by $\max_{u,v \in V} \delta(u, v)$, where $\delta(u, v)$ is the shortest-path distance from u and v .

Therefore the diameter of a tree is the largest of all shortest-path distances in the tree.

Provide an efficient algorithm to compute a diameter of a tree, and analyze the running time of your algorithm.