

Language And Translators - TP Correction

Francesco Nieri

June 2024 (last update: fdc3d86 on 2025-05-31)

Contents

1	TP1	2
1.1	Question 1	2
1.1.1	Question 1.1	2
1.1.2	Question 1.2	3
1.2	Question 2	4
1.2.1	Question 2.1	4
1.2.2	Question 2.2	4
1.3	Question 3	4
1.3.1	Question 3.1	4
1.3.2	Question 3.2	5
1.3.3	Question 3.3	5
1.3.4	Question 3.4	6
1.3.5	Question 3.5	7
1.3.6	Question 3.6	8
2	TP2	9
2.1	Question 1	9
2.1.1	Question 1.1	9
2.1.2	Question 1.2	9
2.1.3	Question 1.3	10
2.1.4	Question 1.4	11
2.2	Question 2	11
2.2.1	Question 2.1	11
2.2.2	Question 2.2	11
2.2.3	Question 2.3	12
2.3	Question 3	13
2.4	Question 4	13
2.4.1	Question 4.1	13
2.4.2	Question 4.2	14
2.4.3	Question 4.3	14

3	TP3	15
3.1	Question 1	15
3.2	Question 2	15
3.2.1	Question 2.1	15
3.2.2	Question 2.2	16
3.2.3	Question 2.3	16
3.3	Question 3	17
3.3.1	Question 3.1	17
3.3.2	Question 3.2	17
3.3.3	Question 3.3	18
4	TP4	21
4.1	Question 1	21
4.2	Question 2	21
4.2.1	Question 2.1	21
4.2.2	Question 2.2	23
4.2.3	Question 2.3	23
4.2.4	Question 2.4	23
5	TP5	25
5.1	Question 1	25
5.2	Question 2	25
5.3	Question 3	26
6	TP6	27
6.1	Question 1	27
6.2	Question 2	27
6.3	Question 3	31
7	TP7	32
7.1	Question 1	32
7.2	Question 2	33
7.3	Question 3	33
7.4	Question 4	34

1 TP1

1.1 Question 1

Consider this piece of code from a fictional C-like programming language:

```
1 int x2 = 25;
2 while(x2>0) {
3     // increment x2 by one
4     x2++;
5     int y = fread(file) % 10;
6     if(y<=4) {
7         x2 = x2 - 1;
8     }
9     printf("Value: %d", y);
10 }
```

1.1.1 Question 1.1

Give the symbol classes that a lexer would need to lex this code (in the course, we have seen some basic classes, like Identifier, or Number). Also make symbol classes for whitespaces (space, newline), and comments.

We will combine question 1 and 3 together, the item on the left is the symbol class, on the right we have the regular expression.

- Type: int
- Space: \w
- Identifier: $[a-zA-Z][a-zA-Z0-9]^*$
- Assignment op: =
- Number: $[0-9]^+$
- String: ".*"
- Special char: $[\{\}\{\}]$
- Keyword: (if|while)
- Comparison operator: $[>|\leq]$
- Increment operator: ++
- Comment : $//\text{.*}\backslash n$
- Math operator: $(-|\%)$
- EOL character: ;
- Param separator: ,

1.1.2 Question 1.2

Write the sequence of symbols (<Token,Attribute>) that a lexer would generate for this code.

```
11 <Type, int >
12 <Identifier, x2 >
13 <Assignment operator, = >
14 <Number, 25 >
15 <EOL character, \backslashn >
16 <Keyword, while>
17 <Special character, ( >
18 <Identifier, x2>
19 <Comparison operator, > >
20 <Number, 0 >
21 <Special character, ) >
22 <Special character, \{ >
23 <Comment, //...>
24 <Identifier, x2>
25 <Increment operator, ++>
26 <EOL character, ;>
27 <Type, int>
28 <Identifier, y>
29 <Assignment operator, =>
30 <Identifier, fread>
31 <Special character, (>
32 <Math operator, \%>
33 <EOL character, ;>
34 <Keyword, if >
35 <Special character, (>
36 <Identifier, y>
37 <Comparison operator, \le>
38 <Number, 4>
39 <Special character, )>
40 <Special character, \{>
41 <Identifier, x2>
42 <Assignment operator, =>
43 <Identifier, x2>
44 <Math operator, ->
45 <Number, 1>
46 <EOL character, ;>
47 <Special character, \}>
48 <Identifier, printf>
49 <Special character, (>
50 <String, "Value: \%d">
51 <Param separator, ,>
52 <Identifier, y>
53 <Special character, )>
54 <EOL character, ;>
```

1.2 Question 2

1.2.1 Question 2.1

The language of a RE can be described in English words. For example, one could say that the RE $(0|1)^*1$ stands for “all binary numbers ending with a 1”. Describe the RE

$$(0|1)^*0(0|1)^*0(0|1)^*$$

in English words.

Any binary sequence that contains at least two zeros

1.2.2 Question 2.2

Construct a RE for “all binary numbers starting with a 1 and containing exactly one pair of consecutive 0”.

$$1(01|1)^*00(1|10)^*$$

1.3 Question 3

1.3.1 Question 3.1

Draw the NFA for RE $a(bc)^*(d|ef)$. Indicate which state is the initial state and which states are the final states. See the NFA in question 3.5 on how to indicate the initial state and final states in an NFA diagram.

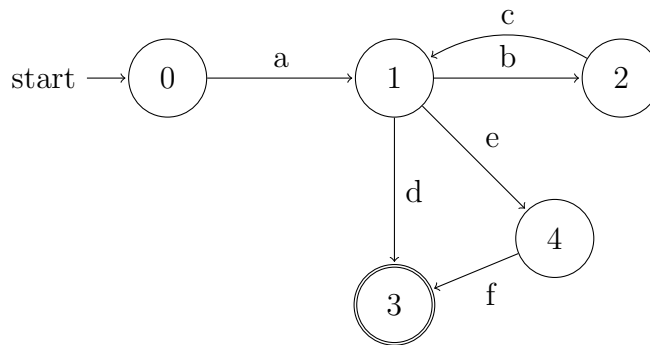


Figure 1: NFA for $a(bc)^*(d|ef)$

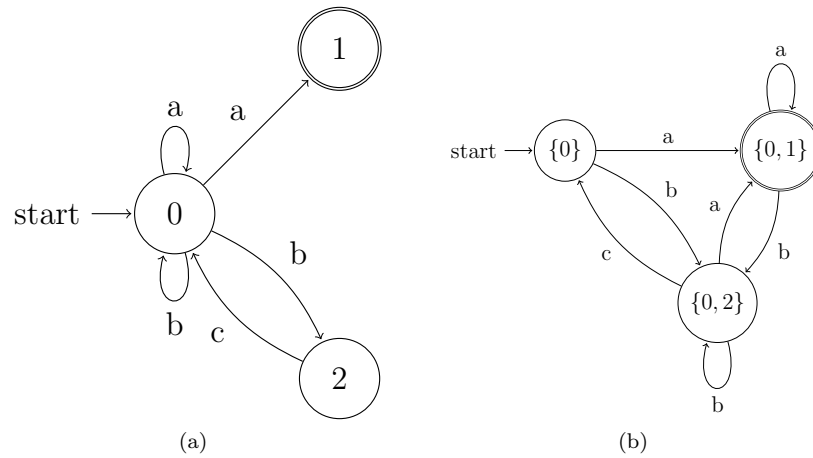


Figure 2: NFA and DFA for $(a|b|bc)^*a$

1.3.2 Question 3.2

Draw the NFA for $(a|b|bc)^*a$ and construct the DFA. The $+$ sign can be used to express that a pattern must appear at least one time. What would be the NFA for the RE $(a|b|bc)^+a$?

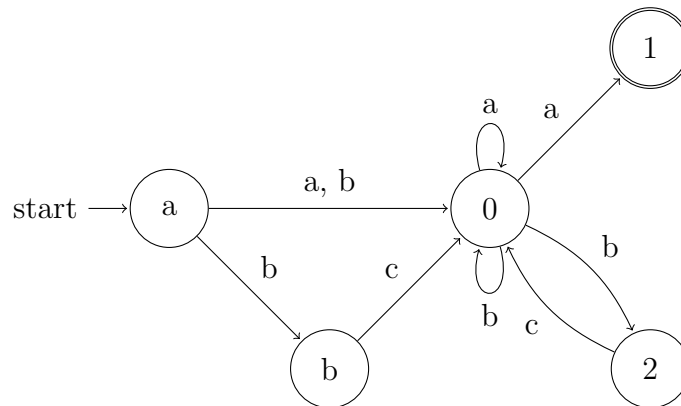


Figure 3: NFA for $(a|b|bc)^+a$

1.3.3 Question 3.3

ε -transitions are very convenient if you want to combine two NFAs. Draw first the NFAs for a^*b and c^*d and then draw an NFA for $(a^*b) \mid (c^*d)$ using ε -transitions.

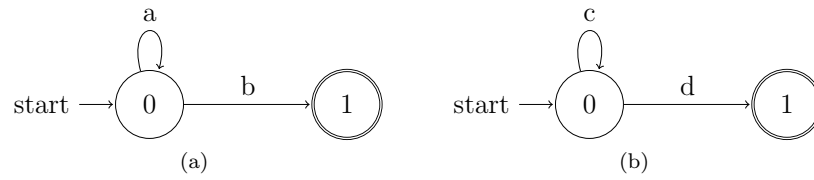


Figure 4: NFA and DFA for $(a|b|bc)^*a$

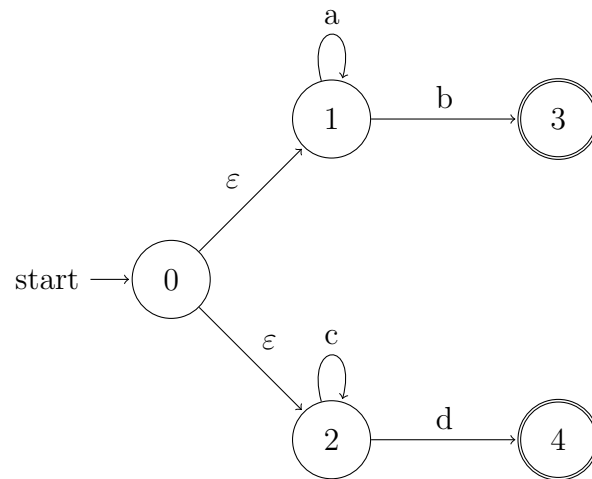


Figure 5: NFA for $(a^*b) \mid (c^*d)$

1.3.4 Question 3.4

Transform the NFA for $(a^*b) \mid (c^*d)$ from question 3.3 into an NFA without ϵ -transitions.

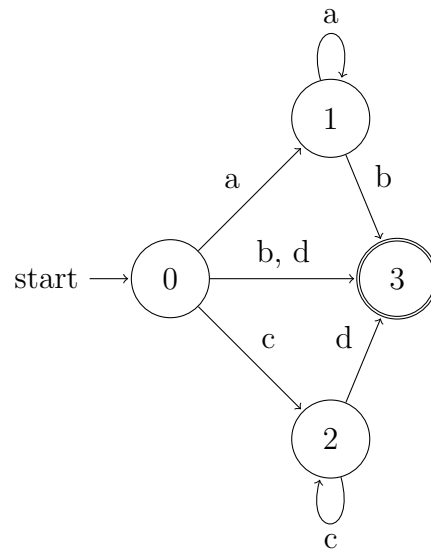


Figure 6: NFA for $(a^*b) \mid (c^*d)$ without ε -transitions

1.3.5 Question 3.5

When dealing with NFAs with ε -transitions, it is useful to think about the ε -closure $\varepsilon(s)$ of a state s , which is defined as the set of states that can be reached from that state s by doing one or several (!) ε -transition steps. For the below NFA over the alphabet $\Omega = 0,1$, give the ε -closures $\varepsilon(A)$, $\varepsilon(B)$, $\varepsilon(C)$ of the states A, B, C.

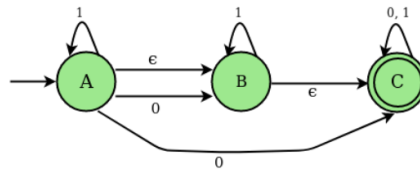


Figure 7

ε -closures:

$$\begin{aligned}\varepsilon\{A\} &= \{A, B, C\} \\ \varepsilon\{B\} &= \{B, C\} \\ \varepsilon\{C\} &= \{C\}\end{aligned}$$

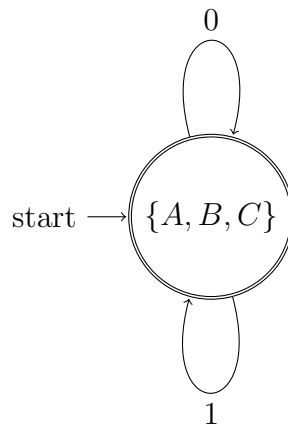


Figure 8: DFA for the above graph

1.3.6 Question 3.6

Here is a modification of the powerset construction method seen in the course that also works with NFAs containing ϵ -transitions:

- The initial state of the powerset automaton is $\epsilon(s_0)$.
- The set of transitions T' of the powerset automaton is defined as:
 $\forall Q \subseteq S, c \in \Omega : (Q, c, R) \in T' \text{ for } R = \epsilon(\{t \mid s \in Q, (s, c, t) \in T\})$ Compare this modified method with the method that we have seen in the course. What is the difference? Transform the NFAs from question 3.3 (the combined NFA) and from question 3.5 into DFAs using this modified powerset construction method.

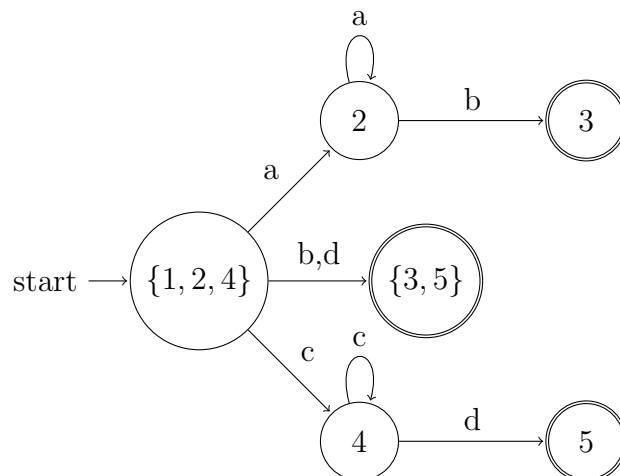


Figure 9: Combined DFA from NFAs

2 TP2

2.1 Question 1

Consider the CFG G with the rules:

$$S \rightarrow a \mid (S) \mid S \cdot S \mid S + S \mid -S$$

2.1.1 Question 1.1

Give the set of terminal symbols used in the rules of G

$$\Sigma = \{a, (,), -, \cdot, +\}$$

2.1.2 Question 1.2

Give a leftmost analysis of the input $a \cdot (-a + (a))$ and draw the syntax tree.

$$\begin{aligned}
&(a \cdot (-a + (a)), S, \varepsilon) \\
&(a \cdot (-a + (a)), S \cdot S, 3) \text{ Rule 3 to expand } S \\
&(a \cdot (-a + (a)), a \cdot S, 31) \text{ Rule 1 to expand } S \\
&(\cdot(-a + (a)), \cdot S, 31) \text{ Match } a \\
&((-a + (a)), S, 31) \text{ Match } \cdot \\
&((-a + (a)), (S), 321) \text{ Rule 2 to expand } S \\
&(-a + (a)), (S), 312) \text{ Match } (\\
&((-a + (a)), S + S), 3124) \text{ Rule 4 to expand } S \\
&(-a + (a)), (-S + S), 31245) \text{ Rule 5 to expand } S \\
&(a + (a)), (S + S), 31245) \text{ Match } - \\
&(a + (a)), (a + S), 312451) \text{ Rule 1 to expand } S \\
&(+ (a)), (+S), 312451) \text{ Match } a \\
&((a)), (S), 312451) \text{ Match } + \\
&((a)), (S)), 3124512) \text{ Rule 2 to expand } S \\
&((a)), (S)), 3124512) \text{ Match } (\\
&((a)), (a)), 31245121) \text{ Rule 1 to expand } S \\
&>((a)), ((a)), 31245121) \text{ Match } a \\
&>((a)), ((a)), 31245121) \text{ Match }) \\
&(\varepsilon, \varepsilon, 31245121) \text{ Match })
\end{aligned}$$

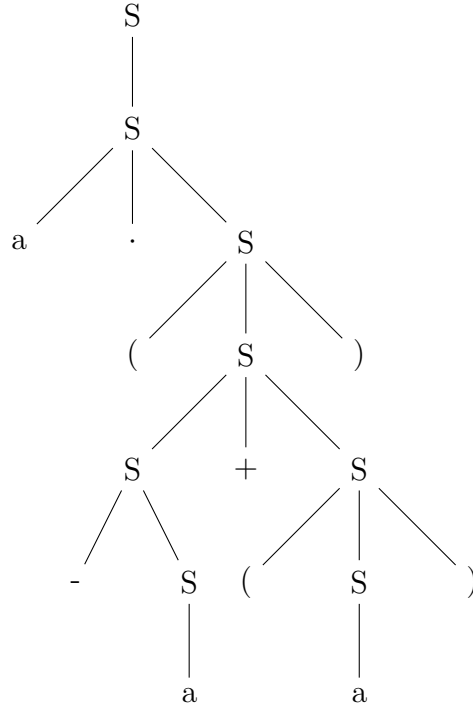


Figure 10: Syntax tree for $a \cdot (-a + (a))$

2.1.3 Question 1.3

Give a rightmost analysis of the input $a \cdot (-a + (a))$

$(a \cdot (-a + (a)), S, \varepsilon)$
 $(a \cdot (-a + (a)), S \cdot S, 3)$ Rule 3 to expand S
 $(a \cdot (-a + (a)), S \cdot (S), 32)$ Rule 2 to expand S
 $(a \cdot (-a + (a)), S \cdot (S, 32)$ Match)
 $(a \cdot (-a + (a)), S \cdot (S + S, 324)$ Rule 4 to expand S
 $(a \cdot (-a + (a)), S \cdot (S + S, 324)$ Rule 4 to expand S
 $(a \cdot (-a + (a)), S \cdot (S + (S), 3242)$ Rule 2 to expand S
 $(a \cdot (-a + (a), S \cdot (S + (S, 3242)$ Match)
 $(a \cdot (-a + (a), S \cdot (S + (a, 32421)$ Rule 1 to expand S
 $(a \cdot (-a + (, S \cdot (S + (, 32421)$ Match a
 $(a \cdot (-a +, S \cdot (S +, 32421)$ Match (
 $(a \cdot (-a, S \cdot (S, 32421)$ Match +
 $(a \cdot (-a, S \cdot (-S, 324215)$ Rule 5 to expand S
 $(a \cdot (-a, S \cdot (-a, 3242151)$ Rule 1 to expand S
 $(a \cdot (-, S \cdot (-, 3242151)$ Match a
 $(a \cdot (, S \cdot (, 3242151)$ Match -
 $(a \cdot, S \cdot, 3242151)$ Match (

$(a, S, 3242151)$ Match \cdot
 $(a, a, 32421511)$ Rule 1 to expand S
 $(\varepsilon, \varepsilon, 32421511)$ Match a

2.1.4 Question 1.4

Show that G is ambiguous.

With a leftmost derivation we can do the following:

$$S \xrightarrow{5} -S \xrightarrow{4} -S + S \xrightarrow{1} -a + S \xrightarrow{1} -a + a = 5411.$$

But also:

$$S \xrightarrow{4} S + S \xrightarrow{5} -S + S \xrightarrow{1} -a + S \xrightarrow{1} -a + a = 4511$$

2.2 Question 2

Extend the CFG for arithmetic expressions from the slides

$$\begin{aligned}
 E &\rightarrow E + T \mid T \\
 T &\rightarrow T \cdot F \mid F \\
 F &\rightarrow (E) \mid \textit{Number} \mid \textit{Identifier}
 \end{aligned}$$

by:

2.2.1 Question 2.1

adding the binary minus operator and the division operator

$$\begin{aligned}
 E &\rightarrow E + T \mid E - T \mid T \\
 T &\rightarrow T \cdot F \mid T / F \mid F \\
 F &\rightarrow (E) \mid \textit{Number} \mid \textit{Identifier}
 \end{aligned}$$

2.2.2 Question 2.2

Write the syntax tree for $3 + 4/5$. Then manually turn it into a more readable AST.

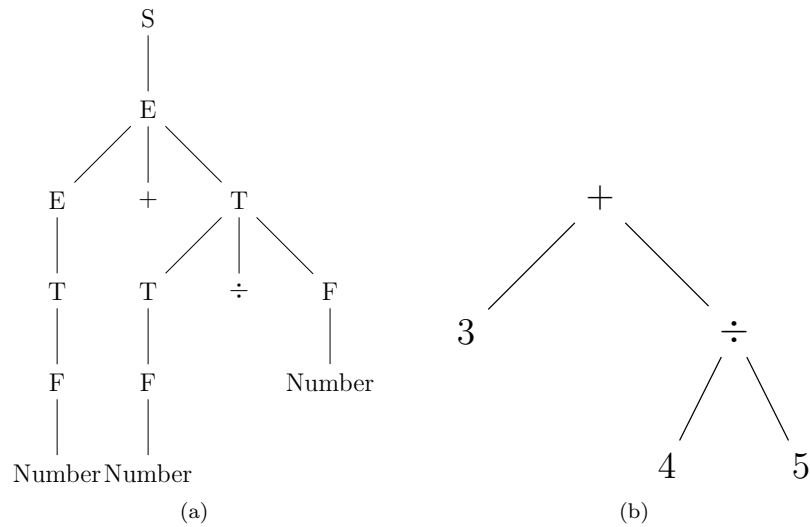


Figure 11: AST for $3 + 4/5$ and readable AST

2.2.3 Question2.3

You will notice that the AST in question 2 does not only reflect the syntactic structure of the input but also the precedence of the $/$ operator over the $+$ operator. Add a new operator $<$ to the grammar that has lower precedence than the $+$ operator and the $/$ operator and test the new grammar on the input $4 + 7 < 3 + 4/5$.

$$\begin{aligned}
 D &\rightarrow D < E \mid E \\
 E &\rightarrow E + T \mid E - T \mid T \\
 T &\rightarrow T \cdot F \mid T / F \mid F \\
 F &\rightarrow (E) \mid \text{Number} \mid \text{Identifier}
 \end{aligned}$$

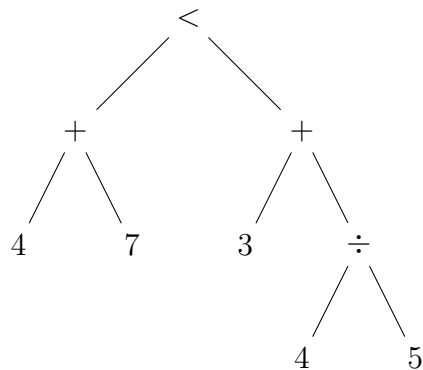


Figure 12

2.3 Question 3

Here is a CFG for regular expressions over the alphabet a, b. That means the CFG describes how regular expressions look like.

$$\begin{aligned} S &\rightarrow R \\ R &\rightarrow R + R \mid RR \mid R \cdot \mid T \\ T &\rightarrow (R) \mid a \mid b \end{aligned}$$

Note that we are using the plus-symbol “+” (e.g., “a+b”) to represent the choice in regular expression $(a|b)$ to avoid confusion with the $|$ used by the CFG itself. Run the NTA of this grammar for the input $a + (b \cdot)$

$(a + (b \cdot), S, \varepsilon)$
 $(a + (b \cdot), R, 1)$ Rule 1 to expand S
 $(a + (b \cdot), R + R, 12)$ Rule 2 to expand S
 $(a + (b \cdot), T + R, 125)$ Rule 5 to expand R
 $(a + (b \cdot), a + R, 1257)$ Rule 7 to expand T
 $(+(b \cdot), +R, 1257)$ Match a
 $((b \cdot), R, 1257)$ Match +
 $((b \cdot), T, 1257)$ Rule 5 to expand R
 $((b \cdot), (R), 125756)$ Rule 6 to expand T
 $(b \cdot), R, 125756)$ Match (
 $(b \cdot), R \cdot, 1257564)$ Rule 4 to expand R
 $(b \cdot), T \cdot, 1257645)$ Rule 5 to expand R
 $(b \cdot), b \cdot, 125767458)$ Rule 8 to expand R
 $(\cdot), \cdot, 125767458)$ Match b
 $(\cdot), \cdot, 125767458)$ Match ·
 $(\varepsilon, \varepsilon, 125767458)$ Match)

2.4 Question 4

Consider the CFG G with the two rules

$$S \rightarrow ab \mid ac$$

over the terminal symbols $\{a, b, c\}$.

2.4.1 Question 4.1

Explain why G is not LL(1). Is it LL(2) ?

A grammar is LL(1) if and only if, for all rules

$$\begin{aligned} A &\rightarrow \beta \mid \gamma \\ la(A \rightarrow \beta) \cap la(A \rightarrow \gamma) &= \emptyset \end{aligned}$$

with $(\beta \neq \gamma)$

It is not LL(1) as:

$$first_1(S \rightarrow ab) = first_1(S \rightarrow ac) = \{a\}$$

It is LL(2) as:

$$first_2(S \rightarrow ab) = \{ab\} \neq first_2(S \rightarrow ac) = \{ac\}$$

2.4.2 Question 4.2

Write a CFG that is LL(1) and that generates the same language as G. Hint: the new grammar has two non-terminal symbols.

$$\begin{aligned} S &\rightarrow aR \\ R &\rightarrow b \mid c \end{aligned}$$

2.4.3 Question 4.3

Give a CFG similar to G that is not LL(5)

$$S \rightarrow aaaaaab \mid aaaaaac$$

3 TP3

3.1 Question 1

Show that any regular language can be generated by an LL(1) language.

For a regular language, we can write a regular grammar. That grammar corresponds to an NFA that we can transform into a DFA. We will show that the DFA can be translated to a LL(1) grammar.

Since a LL(1) grammar generates (or recognizes) a LL(1) language, we have achieved our goal.

To translate a DFA into an LL(1) grammar, we do the following:

The states of the DFA become non-terminal symbols and the transitions become terminal symbols in the grammar.

Each transition in the DFA is translated to a rule in the grammar.

Example: the transition $A \xrightarrow{a} B$ of the DFA with two states A and B becomes the rule $A \rightarrow a B$.

In the course, we have seen the theorem: A grammar is LL(1) if and only if for all rules $A \rightarrow \beta | \gamma$ (with $\beta \neq \gamma$) $la(A \rightarrow \beta) \cap la(A \rightarrow \gamma) = \emptyset$. Having $la(A \rightarrow \beta) \cap la(A \rightarrow \gamma) \neq \emptyset$ is not possible in our case because that would mean that the DFA had a state with two transitions with the same symbol, i.e., a non-deterministic transition.

3.2 Question 2

Consider the following grammar. This a CFG for regular expressions over the alphabet {a, b}

$$\begin{aligned} S &\rightarrow R \\ R &\rightarrow R + T \mid RT \mid R \cdot \mid T \\ T &\rightarrow (R) \mid a \mid b \end{aligned}$$

3.2.1 Question 2.1

Transform the grammar into a grammar without left recursion and prove that the result is an LL(1) grammar.

$$\begin{aligned} S &\rightarrow R \\ R &\rightarrow TR' \\ R' &\rightarrow +TR' \mid TR' \mid \cdot R' \mid \varepsilon \\ T &\rightarrow (R) \mid a \mid b \end{aligned}$$

It is LL(1) as:

$$\begin{aligned} la(S \rightarrow R) &= \{ (, a, b \} \\ la(R \rightarrow TR') &= \{ (, a, b \} \end{aligned}$$

$$\cap = \emptyset \begin{cases} la(R' \rightarrow +TR') = \{+\$ \\ la(R' \rightarrow TR') = \{(, a, b\} \\ la(R' \rightarrow \cdot TR') = \{\cdot\} \\ la(R' \rightarrow \varepsilon) = \{\varepsilon,)\} \end{cases} \quad (1)$$

$$\cap = \emptyset \begin{cases} la(T \rightarrow (R)) = \{(\} \\ la(T \rightarrow a) = \{a\} \\ la(T \rightarrow b) = \{b\} \end{cases} \quad (2)$$

3.2.2 Question 2.2

Specify the Deterministic Top-Down Automaton for the transformed grammar. For the possible actions, either write the transitions or give the action table.

	S	R	T	R'	a	b	+	·	()	ε
a	(R, 1)	(TR', 2)	(a, 8)	(TR', 4)	pop						
b	(R, 1)	(TR', 2)	(b, 9)	(TR', 4)		pop					
+				(+TR', 3)			pop				
·				(·R', 5)				pop			
((R, 1)	(TR', 2)	((R), 7)	(TR', 4)					pop		
)				(ε, 6)						pop	
ε				(ε, 6)							accept

3.2.3 Question 2.3

Run the deterministic automaton on the input $a + (b \cdot)$

$(a + (b \cdot), S, \varepsilon)$
 $(a + (b \cdot), R, 1)$ Rule 1 to expand S
 $(a + (b \cdot), TR', 12)$ Rule 2 to expand R
 $(a + (b \cdot), aR', 128)$ Rule 8 to expand T
 $(+(b \cdot), R', 128)$ Match a
 $(+(b \cdot), +TR', 1283)$ Rule 3 to expand R'
 $((b \cdot), TR', 1283)$ Match +
 $((b \cdot), (R)R', 12837)$ Rule 7 to expand T
 $(b \cdot), R)R', 12837)$ Match (
 $(b \cdot), TR')R', 128372)$ Rule 2 to expand R
 $(b \cdot), bR')R', 1283729)$ Rule 9 to expand T
 $(\cdot), R')R', 1283729)$ Match b
 $(\cdot), \cdot R')R', 12837295)$ Rule 5 to expand R'
 $(\cdot), R')R', 12837295)$ Match ·
 $(\cdot), \varepsilon)R', 128372956)$ Rule 6 to expand R'
 $(\cdot), \varepsilon)R', 128372956)$ Match ε

($\varepsilon, R', 128372956$) Match)
 ($\varepsilon, \varepsilon, 1283729566$) Rule 6 to expand R'

3.3 Question 3

Here is a CFG for Boolean expressions:

Expression \rightarrow Factor | Expression or Factor | Expression and Factor
 Factor \rightarrow not Factor | (Expression) | true | false

3.3.1 Question 3.1

Show the syntax tree for the expression

true and false or (false and true)

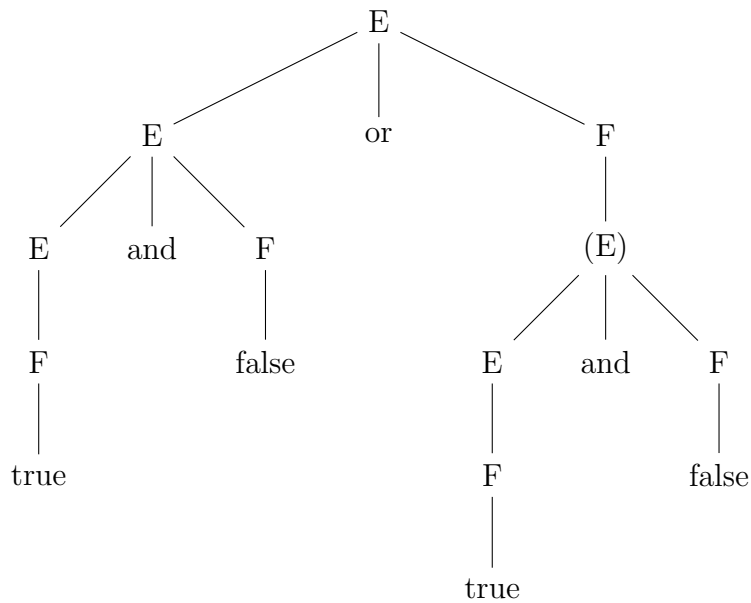


Figure 13: Syntax tree

3.3.2 Question 3.2

Transform the grammar into a grammar without left recursion.

$E \rightarrow FE'$
 $E' \rightarrow or FE' \mid and FE' \mid \varepsilon$
 $F \rightarrow not F \mid (E) \mid true \mid false$

3.3.3 Question 3.3

Write the outline of a recursive descent parser for the transformed grammar, similar to what we did in the video on LL(1) parsing. The main challenge here is to design good data structures for the abstract syntax tree.

```
56 enum Symbol {
57     Or,
58     And,
59     Not,
60     True,
61     False,
62     OpenParen,
63     ClosingParen,
64     END
65 }
66
67 class Expression {
68     Expression leftHand;
69     Symbol operation;
70     Expression rightHand;
71
72     public Expression(Expression left, Symbol op, Expression
73         right) {
74         leftHand = left;
75         operation = op;
76         rightHand = right;
77     }
78
79     public Symbol getReverseOperation() {
80         switch (operation) {
81             case Symbol.Or:
82                 return Symbol.And;
83             case Symbol.And:
84                 return Symbol.Or;
85             case Symbol.True:
86                 return Symbol.False;
87             case Symbol.False:
88                 return Symbol.True;
89             default:
90                 throw new ParseException(
91                     "Unsupported expression operation"
92                 );
93         }
94     }
95
96     public boolean isFactor() {
97         return operation == Symbol.True || operation ==
98             Symbol.False;
```

```

97     }
98
99     public void setLeftHand(Expression expr) {
100         leftHand = expr;
101     }
102 }
103
104 class Lexer {
105     List<Symbol> symbols;
106     Symbol lookahead;
107     Symbol nextSymbol(); // I can't be bothered to implement
108                          // that, so pretend it exists
109
110     public Lexer(List<Symbol> symbols) {
111         this.symbols = symbols;
112         this.lookahead = nextSymbol();
113     }
114
115     private Symbol match(Symbol symbol) {
116         if (lookahead != symbol) {
117             throw new ParseException("No match");
118         }
119         Symbol matchingSymbol = lookahead;
120         lookahead = lexer.nextSymbol();
121         return matchingSymbol;
122     }
123
124
125
126     private Factor parseFactor() {
127         switch (lookahead) {
128             case Symbol.True:
129                 match(Symbol.True);
130                 return new Expression(null, Symbol.True, null);
131             case Symbol.False:
132                 match(Symbol.False);
133                 return new Expression(null, Symbol.False, null)
134                     ;
135             case Symbol.Not:
136                 match(Symbol.Not);
137                 Expression factorToNegate = parseFactor();
138                 Symbol reverseOp = factorToNegate.
139                     getReverseOperation();
140                 return new Expression(null, reverseOp, null);
141             case Symbol.OpenParen:
142                 match(Symbol.OpenParen);
143                 Expression expr = parseExpression();
144                 match(Symbol.ClosingParen);
145                 return expr;

```

```

144         default:
145             throw new ParseException("No match");
146     }
147 }
148
149
150 private Expression parseExpression() {
151     Expression leftHand = parseFactor();
152     assert leftHand.isFactor();
153     Expression expr = parseExpressionPrime();
154     if (expr != null) {
155         expr.setLeftHand(leftHand);
156         return expr;
157     } else {
158         return leftHand;
159     }
160 }
161
162 private Symbol parseExpressionPrime() {
163     switch (lookahead) {
164         case Symbol.END:
165             match(Symbol.END);
166             return new Expression(null, Symbol.END, null);
167         case Symbol.And:
168             match(Symbol.And);
169             Expression rightHand = parseExpression();
170             return new Expression(null, Symbol.And,
171                                 rightHand);
172         case Symbol.Or:
173             match(Symbol.Or);
174             Expression rightHand = parseExpression();
175             return new Expression(null, Symbol.Or,
176                                 rightHand);
177         default: // epsilon rule
178             return null;
179     }
180 }
181
182 // Lexer entry point for outside users
183 public Expression parse() {
184     return parseExpression();
185 }

```

4 TP4

4.1 Question 1

Why is nobody interested in LL(0) parsers? How does an LL(0) grammar look like?

An LL(0) parser is a parser that doesn't do a lookahead to decide which rule to apply. It can only take the next input symbol and check whether it matches or not. Such a parser could only recognize one word.

4.2 Question 2

Consider the following grammar:

$$\begin{aligned} S' &\rightarrow S \\ S &\rightarrow A \mid Bd \\ A &\rightarrow aAb \mid Be \\ B &\rightarrow aBc \mid ac \end{aligned}$$

4.2.1 Question 2.1

Describe the deterministic LR(0) parsing automaton of this grammar and give the parsing table

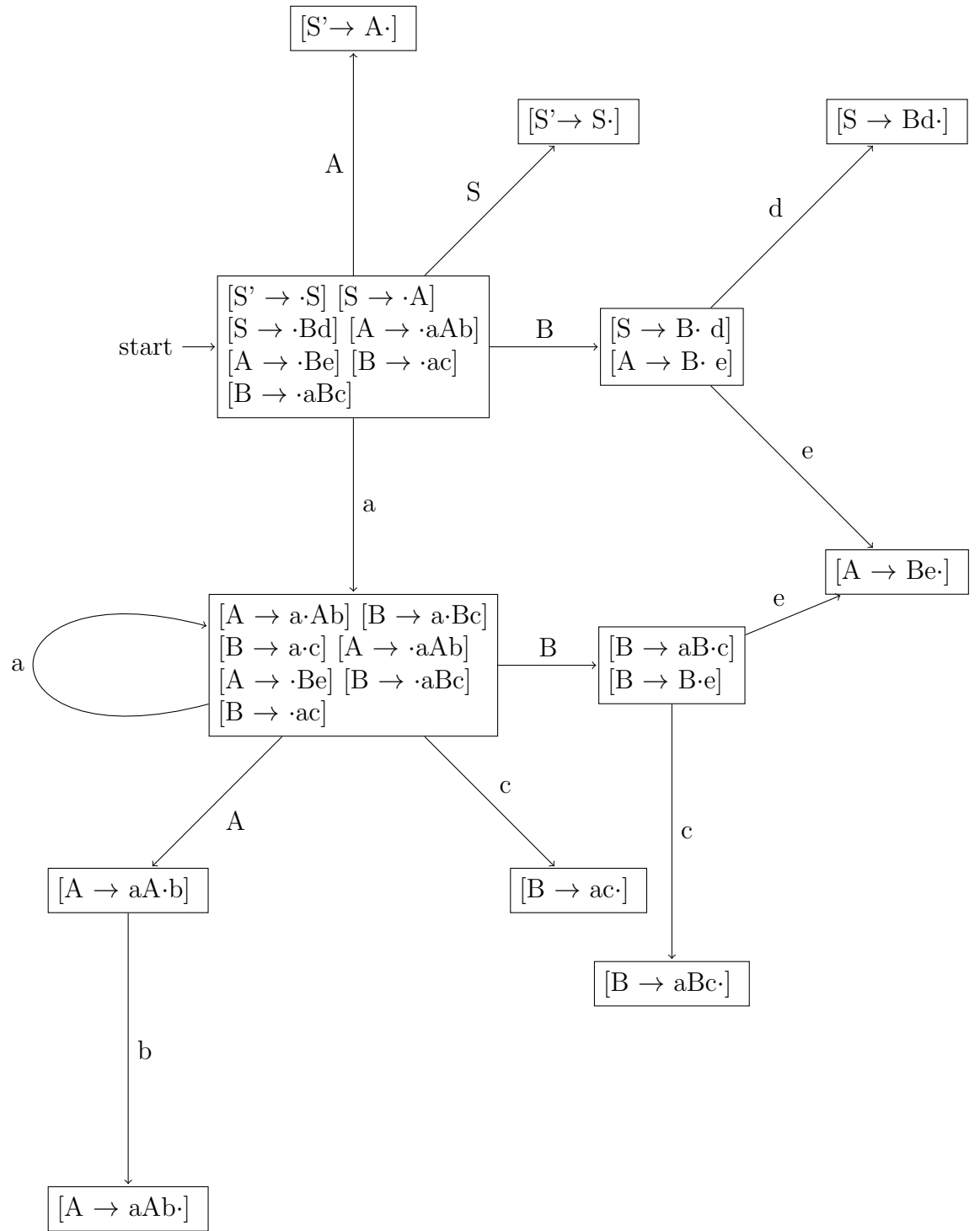


Figure 14: LR(0) goto automaton

	S	A	B	a	b	c	d	e	action
$I_0 = \varepsilon$	I_1	I_2	I_3	I_6					shift
$I_1 = S$									accept
$I_2 = A$									reduce 2
$I_3 = B$							I_4	I_5	shift
$I_4 = Bd$									reduce 3
$I_5 = Be$									reduce 5
$I_6 = a$		I_7	I_9	I_6		I_{11}			shift
$I_7 = aA$					I_8				shift
$I_8 = aAb$									reduce 4
$I_9 = aB$						I_{10}		I_5	shift
$I_{10} = aBc$									reduce 6
$I_{11} = ac$									reduce 7
$I_{12} = \emptyset$									error

Table 1: Parsing table for G

4.2.2 Question 2.2

Show that the grammar is not LL(1).

After reading a, it could be either rule 3 or rule 5/6, so the grammar isn't LL(1)

4.2.3 Question 2.3

Run the automaton on the input aaacebb

$(aaacebb, I_0, \varepsilon)$ Shift
 $(aacebb, I_0 I_6, \varepsilon)$ Shift
 $(acebb, I_0 I_6 I_6, \varepsilon)$ Shift
 $(cebb, I_0 I_6 I_6 I_6, \varepsilon)$ Shift
 $(ebb, I_0 I_6 I_6 I_6 I_{11}, \varepsilon)$ Shift
 $(ebb, I_0 I_6 I_6 I_9, 7)$ Reduce
 $(bb, I_0 I_6 I_6 I_9 I_5, 7)$ Shift
 $(bb, I_0 I_6 I_6 I_7, 75)$ Reduce
 $(b, I_0 I_6 I_6 I_7 I_8, 75)$ Shift
 $(b, I_0 I_6 I_7, 754)$ Reduce
 $(\varepsilon, I_0 I_6 I_7 I_8, 754)$ Shift
 $(\varepsilon, I_0 I_2, 7544)$ Reduce
 $(\varepsilon, I_0 I_1, 75442)$ Reduce
 $(\varepsilon, \varepsilon, 754421)$ Reduce

4.2.4 Question 2.4

Modify the grammar such that it has a shift/reduce or reduce/reduce conflict.

We can add some rules to create shift/reduce or reduce/reduce conflicts

Shift/reduce = When ε : $[A \rightarrow \cdot]$ in I.

Reduce/reduce = When a: $[A \rightarrow a\cdot]$, $[B \rightarrow C \cdot]$ in item set

$$\begin{array}{c} A \rightarrow aAb \mid Be \mid \varepsilon \\ \text{LR}(0)(aA) = \{[(A \rightarrow aA\cdot b, [A \rightarrow aA\cdot])]\} \end{array}$$

5 TP5

5.1 Question 1

Write type inference rules for the double type in Java

Double constants are of type double:

$$\frac{}{\vdash \text{any double constant: double}}$$

Adding two doubles:

$$\frac{\vdash e_1 : \text{double}, \vdash e_2 : \text{double}}{\vdash e_1 + e_2 : \text{double}}$$

Adding a double and an integer gives a double:

$$\frac{\vdash e_1 : \text{double}, \vdash e_2 : \text{int}}{\vdash e_1 + e_2 : \text{double}}$$

$$\frac{\vdash e_1 : \text{int}, \vdash e_2 : \text{double}}{\vdash e_1 + e_2 : \text{double}}$$

As you can see, allowing type conversions from int to double means that we have to repeat rules.

To reduce the number of rules, we can define that

$$\text{int} \leq \text{double}$$

In this example, we then only need two rules for the add operator:

$$\frac{\vdash e_1 : \text{double}, \vdash e_2 : T \leq \text{double}}{\vdash e_1 + e_2 : \text{double}}$$

$$\frac{\vdash e_1 : \text{int}, \vdash e_2 : \text{double}}{\vdash e_1 + e_2 : \text{double}}$$

5.2 Question 2

Write a type inference rule for array expressions. The rule should be applicable to expressions like `a[i+3]`

Let's assume we have already specified the type rules for variables and arithmetic expressions like $i + 3$. Then we can write for array expressions:

$$\frac{\begin{array}{l} \text{a is an identifier} \\ \text{a has the type of an array of int in the symbol table} \\ \vdash e : \text{int} \end{array}}{\vdash a[e] : T}$$

Write a type inference rule for the ternary conditional operator of Java and C, for example: $a < 3 ? b : 5$

For the ternary conditional operator, we have:

$$\frac{\begin{array}{l} \vdash b : \text{bool} \\ \vdash e_1 : T \\ \vdash e_2 : T \end{array}}{\vdash b ? e_1 : e_2 : T}$$

5.3 Question 3

How would you check whether a function uses return statements correctly? Think about situations where this can be sometimes difficult to check. For example, how would you do it for this function:

```
186 int min(int a, int b) {  
187     int m;  
188     if (a < b) {  
189         m = a;  
190         return m;  
191     }  
192     else  
193         m = b;  
194 }
```

If you represent the start of the function and the end of the function as nodes in the control flow graph, any path from the start node to the end node must contain a return statement.

6 TP6

6.1 Question 1

Give the intermediate representation in TAC for the following expression:

Give the intermediate representation in TAC for the following expression:

$$x = (-b + \sqrt{b^2 - 4 \cdot a \cdot c}) / (2 \cdot a)$$

```
195 x0 = b * b
196 x1 = 4 * a
197 x2 = x1 * c
198 x3 = x0 - x2
199 x4 = sqrt(x3)
200 x5 = -b
201 x6 = x5 + x4
202 x7 = 2 * a
203 x8 = x6 / x7
204 x = x8
```

6.2 Question 2

For each of the following C functions, give the control flow graph (CFG), the minimized SSA form, and the non-SSA form without parameterized labels (the form that can be used to generate assembly code)

```
205 int min(int a, int b) {
206     int m;
207     if(a<b)
208         m = a;
209     else
210         m = b;
211     return m;
212 }
213 int minAlternative(int a, int b) {
214     int m = b;
215     if(a<b)
216         m = a;
217     return m;
218 }
219 int squareSum(int n) {
220     int sum=0;
221     for(int i=1;i<=n;i++){
222         sum += i*i;
223     }
224     return sum;
225 }
```

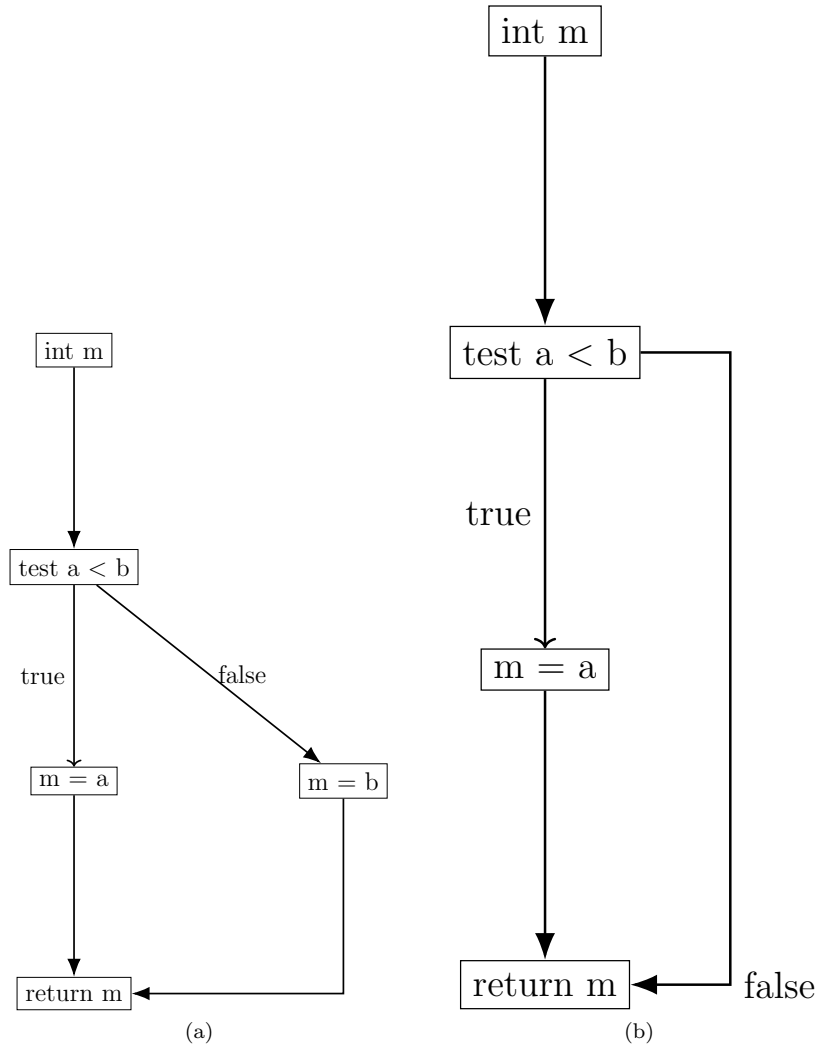


Figure 15: CFG for `min` and `minAlternative` $(a|b|bc)^*a$

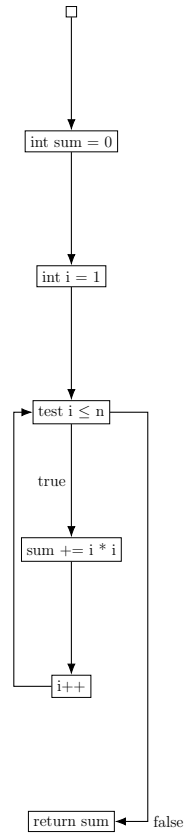


Figure 16: CFG for squareSum

Minimized SSA for min:

```

226 min(a0,b0):
227     if a0 < b0 then goto body else goto elsebody
228 body:
229     m0 = a0
230     goto done(m0)
231 elsebody:
232     m1 = b0
233     goto done(m1)
234 done(m2):
235     return m2

```

Minimized SSA for minAlternative:

```

236 minAlternative(a0,b0):
237     m0 = b0

```

```

238     if a0 < b0 then goto body else goto done(m0)
239 body:
240     m1 = a0
241     goto done(m1)
242 done(m3):
243     return m3

```

Non minimized SSA for squareSum:

```

244 squareSum(n0):
245     sum0 = 0
246     i0 = 1
247     goto loop(n0, sum0, i0)
248 loop(n1, sum1, i1):
249     if i1<=n1 then goto body(n1,sum1,i1) else goto end(n1,sum1,
        i1)
250 body(n2, sum2, i2):
251     t0 = i2*i2
252     sum3 = sum2 + t0
253     i3 = i2 + 1
254     goto loop(n2, sum3, i3)
255 end(n3, sum4, i4):
256     return sum4

```

Minimized SSA

```

257 squareSum(n0):
258     sum0 = 0
259     i0 = 1
260     goto loop(sum0, i0)
261 loop(sum1, i1):
262     if i1<=n0 then goto body else goto end
263 body:
264     t0 = i1*i1
265     sum3 = sum1 + t0
266     i3 = i1+1
267     goto loop(sum3, i3)
268 end:
269     return sum1

```

Non minimized non-SSA

```

270 squareSum(n):
271     sum = 0
272     i = 1
273     goto loop
274 loop:
275     if i<=n then goto body else goto end
276 body:
277     t = i*i
278     sum = sum + t

```

```

279     i = i + 1
280     goto loop
281 end:
282     return sum

```

Minimized non-SSA

```

283 squareSum(n0):
284     sum0 = 0
285     i0 = 1
286     sum1 = sum0
287     i1 = i0
288     goto loop
289 loop(sum1, i1):
290     if i1<=n0 then goto body else goto end
291 body:
292     t0 = i1*i1
293     sum3 = sum1 + t0
294     i3 = i1+1
295     sum1 = sum3
296     i1 = i3
297     goto loop
298 end:
299     return sum1

```

6.3 Question 3

Design a simple algorithm that verifies that a function has no missing return statement. The algorithm should also work for complex functions that contain many if-statements, loops, etc.

If you represent the start of the function and the end of the function as nodes in the control flow graph, any path from the start node to the end node must contain a return statement.

7 TP7

7.1 Question 1

Translate the following function to IR in SSA form and determine the liveness ranges of the variables. Draw the interference graph. Then, allocate registers for an ARM-like CPU and generate machine code, assuming that the parameter “c” of the function is passed in register r1 (without using the stack. That’s more efficient!) and that the return value of the function should be in register r0.

	t1	c1	a1	t0	a0	b0	c0
a0 = c0 · 2							live
b0 = a0 + 1					live		live
c1 = c0 + b0					live	live	live
t0 = b0 · 2		live			live	live	
a1 = t0 + a0		live		live	live		
t1 = c1 + a1		live	live				
return t1	live						

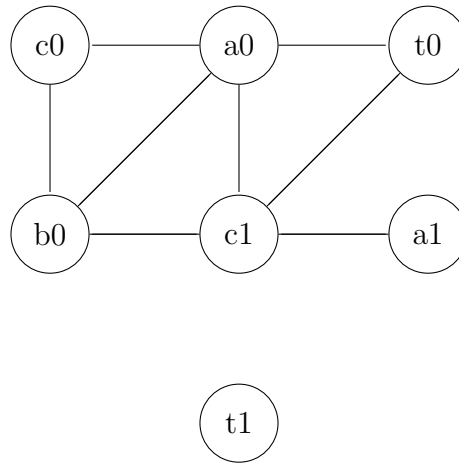


Figure 17: Interference graph

```

300 pop r0
301 c0 = r1 (required by the ABI)
302 a0 = r0
303 b0 = r2
304 c1 = r1
305 t0 = r2
306 a1 = r0
307 t1 = r0 (required by the ABI)

```

7.2 Question 2

In the course, we only saw code generation for functions with parameters and local variables. Now, let's look at an example with global variables:

```
308 int x;  
309 int y[10];  
310  
311 void add(int v) {  
312     y[x] = v;  
313     x++;  
314 }
```

Translate the above function “add” first to IR code and then to machine code. Assume that the global variables “x” and “y” start at address 0x10000 and 0x10004, respectively, in main memory.

```
315 // parameter is v0  
316 t0 = 0x10004  
317 t1 = *t0  
318 // read value of x  
319 t2 = t1 * 4  
320 // each array element is four bytes  
321 t3 = 0x10000  
322 t4 = t0 + t3  
323 // address of y[x]  
324 *t4 = v0  
325 // store v in y[x]  
326 //-----  
327 t5 = 0x10004  
328 t6 = *t5  
329 t7 = t6 + 1  
330 *t5 = t7  
331 // store x+1 in x
```

Of course, this code can be optimized. Instead of loading again the variable x in t6, we could just re-use t1.

7.3 Question 3

Function calls are expensive because they involve a lot of operations (pushing the arguments on the stack, making backups of registers, jumping to the function, etc.). Many compilers can perform an optimization called function inlining where the code of the called function is directly inserted at the call location, thus avoiding the call. Let's look at the following example:

```
332 int f(int v) {  
333     v = v + 2;  
}
```

```

334     return v*v;
335 }
336 int g(int a, int b) {
337     int c = f(a);
338     int d = 2*f(a+b);
339     return c+d;
340 }

```

Take the role of the compiler and inline the function `f` at the two places where it is called. Do this in the IR, not in the source code. Think about a strategy how to handle the variables. By the way, compilers only inline small functions. What could be the reason?

```

341 // parameters are a0 and b0
342 v0 = a0
343 // calling function f with argument a
344 v1 = v0 + 2
345 t0 = v1 * v1
346 c0 = t0
347 // c = f(a)
348 //-----
349 v2 = a0 + b0
350 // calling function f with argument a+b
351 v3 = v2 + 2
352 t1 = v3 * v3
353 d0 = 2 * t1
354 // d = 2*f(a+b)
355 //-----
356 t2 = c0 + d0
357 return t2

```

Why do compilers only inline small functions? Code size! Inlining increases the number of instructions in the code and therefore reduces the efficiency of the instruction cache.

7.4 Question 4

In most CPUs, integer multiplications are slower than additions or bit shifting. Think about ways to reduce the strength of multiplications, i.e., find ways to avoid the multiplication instruction for arithmetic expressions where one of the operands is a constant, for example

$$x \cdot 2$$

$$x \cdot 3$$

$$x \cdot 4$$

$$x \cdot 5$$

$$x \cdot 2 = x+x \text{ or } x \ll 1$$

$$x \cdot 3 = x+x+x \text{ or } x \ll 1 + x$$

$$\begin{aligned}
 x \cdot 4 &= x \ll 2 \\
 x \cdot 5 &= x \ll 2 + x \\
 x \cdot (2^n + 1) &= (x \ll n) + x
 \end{aligned}$$