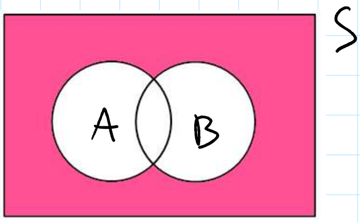
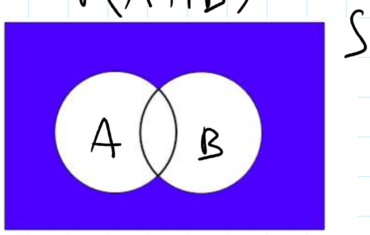
Oppg. 1

$P((A\cup B)')$

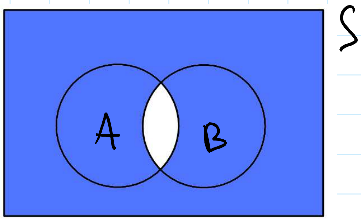


$=$

$P(A'\cap B)$

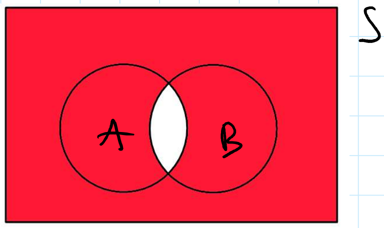


$P((A\cap B)')$



$=$

$P(A'\cup B')$



Oppg. 2

I) $P(7 \text{ røde}) = \frac{g}{m}$ pga. uniform modell

$g = \binom{7}{7}\binom{27}{0} = 1 \qquad m = \binom{34}{7} = 5379616$

$P(7 \text{ røde}) = \frac{g}{m} = \frac{1}{5379616} \approx \underline{\underline{1{,}86 \cdot 10^{-7}}}$

II) $P(4 \text{ av } 7 \text{ røde}) = \frac{g}{m}$

$g = \binom{7}{4}\binom{27}{3} = 102375$

$m = 5379616$

$P(4 \text{ av } 7 \text{ røde}) = \frac{g}{m} \approx \underline{\underline{0{,}0190}}$

III) $P(6/7 \text{ røde} + \text{rød ekstrakule})$

$= \frac{\binom{7}{6}\binom{27}{1}}{\binom{34}{7}} \cdot \frac{1}{27} = \underline{\underline{1{,}30 \cdot 10^{-6}}}$

$\binom{34}{7}$ 27 $\underline{\underline{1,30 \quad 10}}$

Oppg. 3

I) $P(1 \text{ type } A) = \dfrac{\binom{3}{1}\binom{297}{4}}{\binom{300}{5}} \approx \underline{\underline{0,0487}}$

II) $P(\text{minst } 1 \text{ type } A) = 1 - P(\text{Ingen type } A)$

$= 1 - \dfrac{\binom{297}{5}}{\binom{300}{5}} \approx \underline{\underline{0,0493}}$

III) $P(\text{minst } 1 \text{ gevinst}) = 1 - P(\text{Ingen gevinst})$

$= 1 - \dfrac{\binom{234}{5}}{\binom{300}{5}} \approx \underline{\underline{0,0567}}$

Oppg. 4

I) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) = \underline{\underline{0,1}}$

$P(B) + P(C) = P(B \cup C)$

$\Rightarrow$ B og C er disjunkte

$\Rightarrow P(A \cap B \cap C) = \underline{\underline{0}}$

$P(A|B) = \dfrac{P(A \cap B)}{\ } = \dfrac{0,1}{\ } = \dfrac{1}{\ }$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0,1}{0,3} = \underline{\underline{\frac{1}{3}}}$$

II) A og B er avhengige fordi

$$P(A) \neq P(A|B)$$

$$0,4 \neq \frac{1}{3}$$

III) A og b er ikke disjunkte fordi

$$P(A \cup B) \neq P(A) + P(B)$$

$$0,6 \neq 0,7$$

Oppg. 5

$$P(F|M) = 0,08$$

$$P(F|k) = 0,003$$

$$P(M) = \frac{1}{3}$$

$$P(k) = \frac{2}{3}$$

$$P(F) = P(F \cap k) + P(F \cap M)$$

$$= P(F|k) \cdot P(k) + P(F|M) \cdot P(M)$$

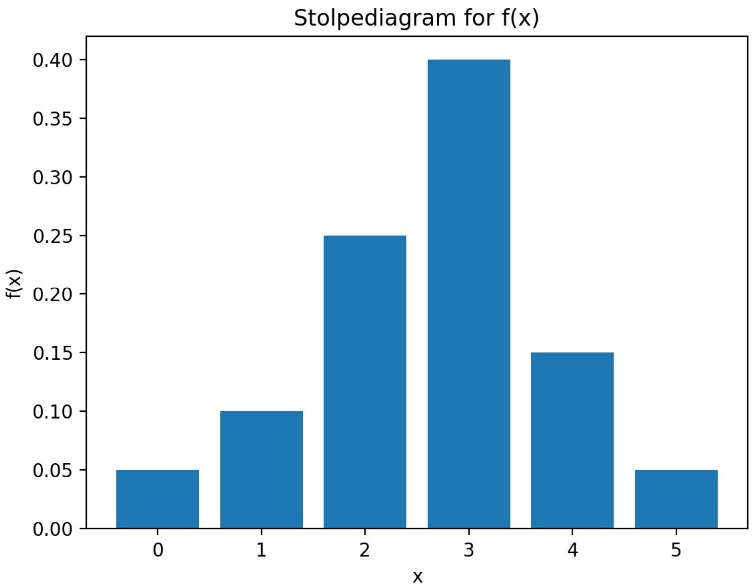
$$\approx 0,287$$

$$P(k|F) = \frac{P(F|k)P(k)}{P(F)}$$

$$= \frac{0,003 \cdot \frac{2}{3}}{0,287} = \underline{\underline{0,0698}}$$

Oppg. 6

I)



II) a) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0{,}05 + 0{,}10 + 0{,}25 = \underline{\underline{0{,}40}}$

$$P(X \leq 2 \mid X < 4) = \frac{P(X \leq 2 \cap X < 4)}{P(X<4)} = \frac{P(X \leq 2)}{P(X<4)}$$

$$= \frac{0{,}40}{P(X=0)+P(X=1)+P(X=2)+P(X=3)} = \frac{0{,}4}{0{,}8} = \underline{\underline{0{,}5}}$$

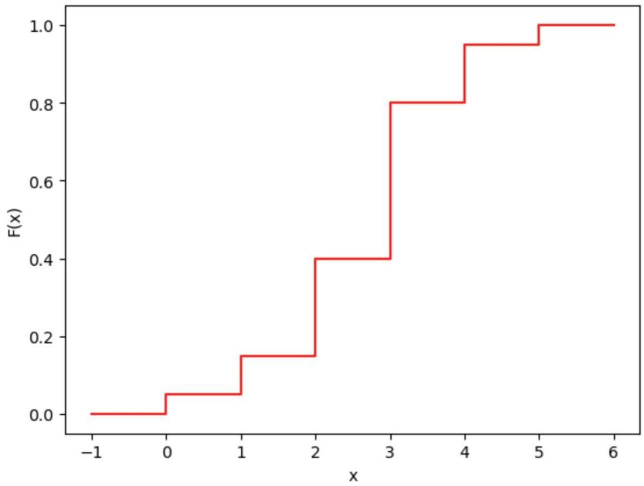
$$P(X \leq 2 \mid X \geq 1) = \frac{P(X \leq 2 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X=1 \cup X=2)}{0{,}10+0{,}25+0{,}40+0{,}15+0{,}05}$$

$$= \frac{0{,}1+0{,}25}{0{,}95} \approx \underline{\underline{0{,}368}}$$

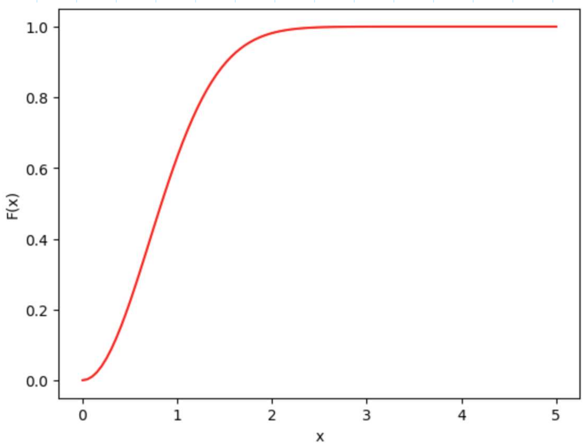
b)

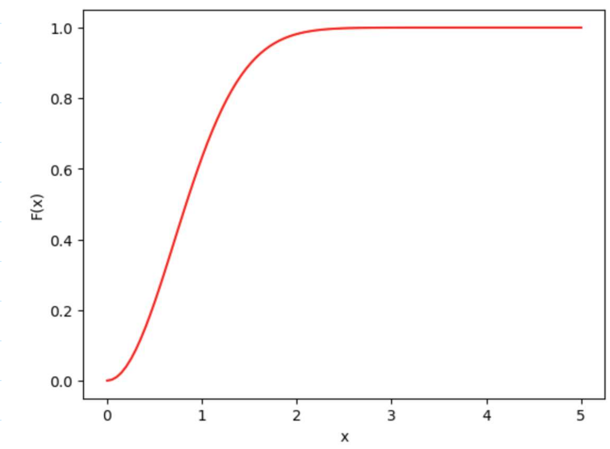
x	$F(x)$
$-\infty$	0
$\vdots$	$\vdots$
0	0,05
1	0,15
2	0,40
3	0,80
4	0,95
5	1
$\vdots$	$\vdots$
∞	1

c)



Oppg. 7 a)



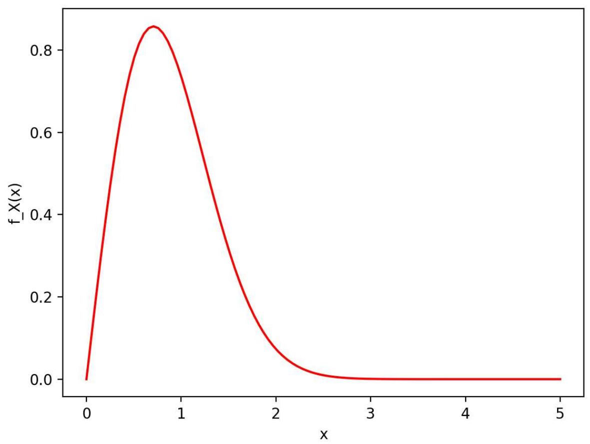


I) $f_X(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\left(1-e^{-\frac{x^2}{\alpha}}\right)$

$$= \underline{\underline{\frac{2x}{\alpha}e^{-\frac{x^2}{\alpha}}}}$$

II)

```python
import numpy as np
import matplotlib.pyplot as plt

def f_X(x, alpha):
    return (2*x/alpha)*np.exp((-x**2)/alpha)

x = np.linspace(0,5,100)

alpha = 1

plt.plot(x, f_X(x, alpha), color="red")
plt.xlabel("x")
plt.ylabel("f_X(x)")
plt.show()
```



b) I) Vi innfører S.V kalt X og Y.
De representerer levetiden til komponentene.

$F_X(x) = 1 - e^{-\frac{x^2}{\alpha}}$

$F_Y(y) = 1 - e^{-\frac{y^2}{\alpha}}$

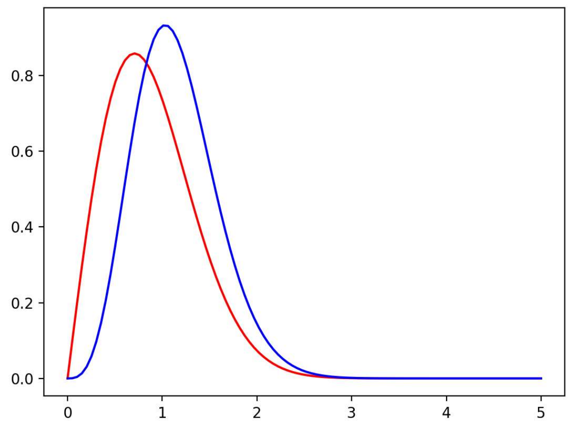
$F_Z(z) = F_X(z)F_Y(z) = \left(1-e^{-\frac{z^2}{\alpha}}\right)^2$

$1 - F_Z(z) = P(Z>z) = 1 - \left(1 - e^{-\frac{z^2}{\alpha}}\right)^2$

II) $f_Z(z) = \frac{d}{dz}\left(1-e^{-\frac{z^2}{\alpha}}\right)^2 = \underline{\underline{2\left(1-e^{-\frac{z^2}{\alpha}}\right)\left(\frac{2z}{\alpha}e^{-\frac{z^2}{\alpha}}\right)}}$

III)

```python
import numpy as np
import matplotlib.pyplot as plt

def f_X(x, alpha):
    return (2*x/alpha)*np.exp((-x**2)/alpha)

def f_Z(z, alpha):
    return 2*(1-np.exp((-z**2)/alpha))*((2*z/alpha)*np.exp((-z**2)/alpha))

x = np.linspace(0,5,100)

alpha = 1

plt.plot(x, f_X(x, alpha), color="red")
plt.plot(x, f_Z(x, alpha), color="blue")
plt.show()
```



Vi ser at f_Z er forskjøvet mot høyere. Dette betyr at det er høyere sjanse for at instrumentet varer lenger. Dette gir mening siden vi nå har to komponenter og vi kun trenger at den ene virker.

Oppg. 8 I) $f_X(x) = \dfrac{\binom{3}{x}\binom{297}{5-x}}{\binom{300}{5}}$

$$f_X(x) = \sum_y f_{XY}(x,y),$$

$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}.$$

II) $f_{XY}(x,y) = \dfrac{\binom{3}{x}\binom{3}{y}\binom{294}{5-x-y}}{\binom{300}{5}}$

III) $f_X(x) = \sum_y f_{XY}(x,y) = \sum_{y=0}^{5-x} \dfrac{\binom{3}{x}\binom{3}{y}\binom{294}{5-x-y}}{\binom{300}{5}}$

$$= \frac{\binom{3}{x}}{\binom{300}{5}} \sum_{y=0}^{5-x} \binom{3}{y}\binom{294}{5-x-y} = \frac{\binom{3}{x}\cdot\binom{297}{5-x}}{\binom{300}{5}}$$