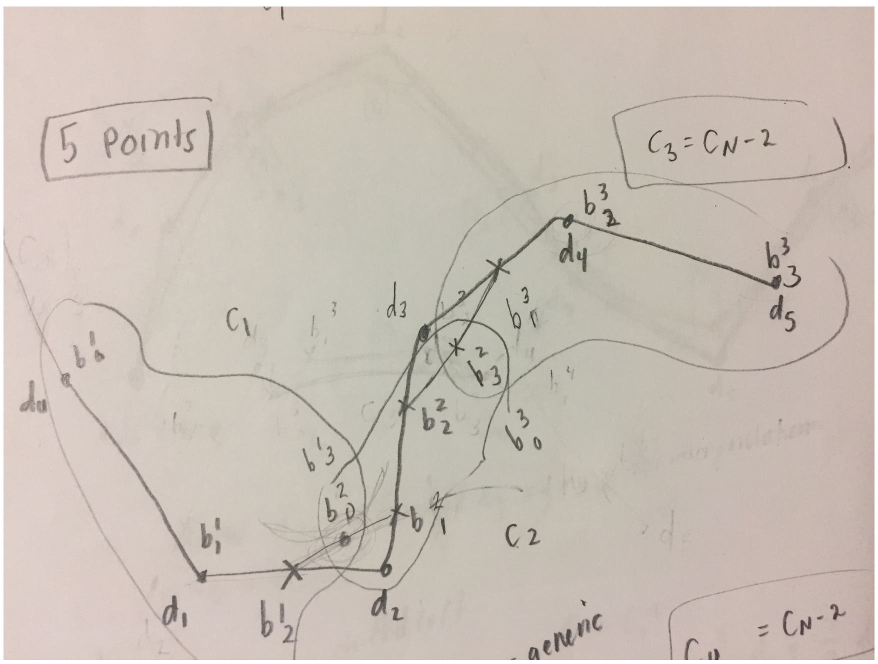
Fall 2016 CIS 515

Fundamentals of Linear Algebra and Optimization
Jean Gallier

Project 1

Problem 1 Part 1
Adapt N=5 Case



(C1)

$$
\begin{aligned}
b_0^1 &= d_0 \\
b_1^1 &= d_1 \\
b_2^1 &= \frac{1}{2}d_1 + \frac{1}{2}d_2 \\
b_3^1 &= \frac{1}{4}b^1 2 + \frac{1}{2}b_1^2 = \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3
\end{aligned}
$$

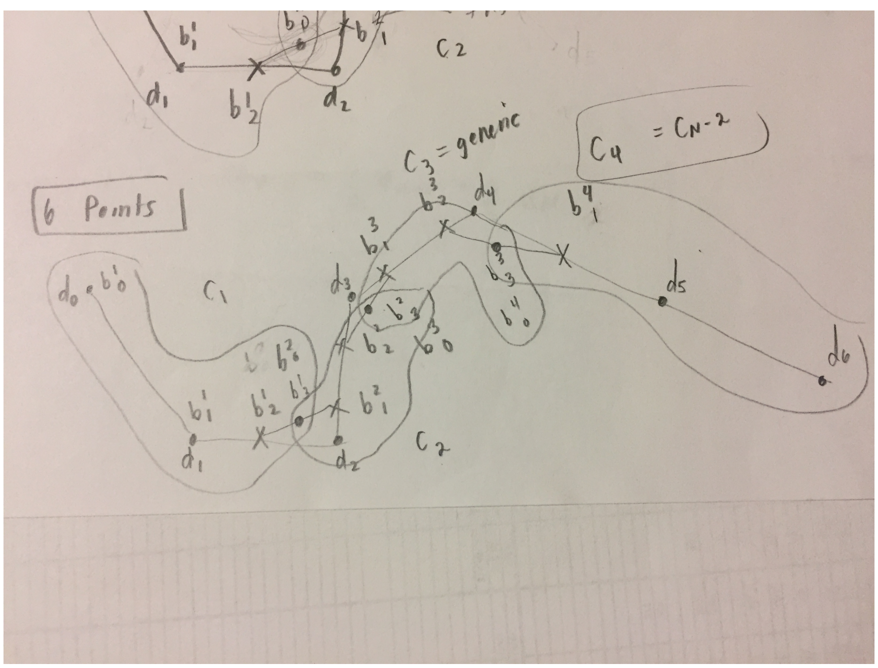
(C2)

$$
\begin{aligned}
b_0^2 &= \frac{1}{2}b_2^1 + \frac{1}{2}b_1^2 \\
&= \frac{1}{4}d_1 + + \frac{7}{12}d_2 + \frac{1}{6}d_3 \\
b_1^2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\
b_2^2 &= \frac{1}{3}d_2 + \frac{2}{3}d_3 \\
b_3^2 &= \frac{1}{2}b^2 2 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4
\end{aligned}
$$

(C3)

$$
\begin{aligned}
b_0^3 &= \frac{1}{2}b^2 2 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4 \\
b_1^3 &= \frac{1}{2}d_3 + \frac{1}{2}d_4 \\
b_2^3 &= d_4 \\
b_3^3 &= d_5
\end{aligned}
$$

Adapt N=6 Case



(C1)

$$
\begin{aligned}
b_0^1 &= d_0 \\
b_1^1 &= d_1 \\
b_2^1 &= \frac{1}{2}d_1 + \frac{1}{2}d_2 \\
b_3^1 &= \frac{1}{4}b^12 + \frac{1}{2}b_1^2 = \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3
\end{aligned}
$$

(C2)

$$
\begin{aligned}
b_0^2 &= \frac{1}{2}b_2^1 + \frac{1}{2}b_1^2 \\
&= \frac{1}{4}d_1 + +\frac{7}{12}d_2 + \frac{1}{6}d_3 \\
b_1^2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\
b_2^2 &= \frac{1}{3}d_2 + \frac{2}{3}d_3 \\
b_3^2 &= \frac{1}{2}b^22 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4
\end{aligned}
$$

(C3)

$$
\begin{aligned}
b_0^3 &= \frac{1}{2}b^22 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4 \\
b_1^3 &= \frac{2}{3}d_3 + \frac{1}{1}d_4 \\
b_2^3 &= \frac{1}{3}d_3 + \frac{2}{3}d_4 \\
b_3^3 &= \frac{1}{2}b_2^3 + \frac{1}{2}b_4^1 = \frac{1}{6}d^3 + \frac{7}{12}d_4 + \frac{1}{4}d_5
\end{aligned}
$$

(C4)

$$
\begin{aligned}
b_0^4 &= \frac{1}{2}b_2^3 + \frac{1}{2}b_4^1 = \frac{1}{6}d^3 + \frac{7}{12}d_4 + \frac{1}{4}d_5 \\
b_1^4 &= \frac{1}{2}d_4 + \frac{1}{2}d_5 \\
b_2^4 &= d_5 \\
b_3^4 &= d_6
\end{aligned}
$$

Adapt N=4 Case Not yet updated

(C1)

$$\begin{aligned}
b_0^1 &= d_0 \\
b_1^1 &= d_1 \\
b_2^1 &= \frac{1}{2}d_1 + \frac{1}{2}d_2 \\
b_3^1 &= \frac{1}{4}b^12 + \frac{1}{2}b_1^2 = \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3
\end{aligned}$$

(C2)

$$\begin{aligned}
b_0^2 &= \frac{1}{2}b_2^1 + \frac{1}{2}b_1^2 \\
&= \frac{1}{4}d_1 + +\frac{7}{12}d_2 + \frac{1}{6}d_3 \\
b_1^2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\
b_2^2 &= \frac{1}{3}d_2 + \frac{2}{3}d_3 \\
b_3^2 &= \frac{1}{2}b^22 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4
\end{aligned}$$

Problem 1 Part 2

The indices in the interpolation problem defined in the notes and slides are expressed in the form d_{-1}. Thus the indices in the project for Problem 1 are off by 1. The points d_1 to d_{N-1} are the control points on the Bezier cubic segments, and points d_{-1} and d_N are the first and last point de Boor control points respectively. Each of the $x_i = b_3^i$.

The first point, d_1, can be written in the form,

$$\begin{aligned}
x_1 &= b_3^1 \\
x_1 &= \frac{1}{4}d_0 + \frac{7}{12}d_1 + \frac{1}{6}d_2 \\
6(x_1 &= \frac{1}{4}d_0 + \frac{7}{12}d_1 + \frac{1}{6}d_2) \\
6x_1 &= \frac{3}{2}d_0 + \frac{7}{2}d_1 + d_2 \\
6x_1 - \frac{3}{2}d_0 &= \frac{7}{2}d_1 + d_2
\end{aligned}$$

The second point, d_2, can be written in the form,

$$\begin{aligned}
x_2 &= b_3^2 \\
x_2 &= \frac{1}{6}d_1 + \frac{4}{6}d_2 + \frac{1}{6}d_3 \\
6(x_2 &= \frac{1}{6}d_1 + \frac{4}{6}d_2 + \frac{1}{6}d_3 \\
6x_2 &= d_1 + 4d_2 + d_3
\end{aligned}$$

The last two points, d_{N-2} and d_{N-1} are the first two points in the Bezier cubic segments, not the last. Now x_{N-2} and x_{N-1} are equal to b_0^{N-2} and b_0^{N-1} respectively.

The second to last point, d_{N-2}, can be written in the form,

$$\begin{aligned}
x_{N-2} &= b_0^{N-2} \\
x_{N-2} &= \frac{1}{6}d_{N-3} + \frac{4}{6}d_{N-2} + \frac{1}{6}d_{N-1} \\
6(x_{N-2} &= \frac{1}{6}d_{N-3} + \frac{4}{6}d_{N-2} + \frac{1}{6}d_{N-1}) \\
6x_{N-2} &= d_{N-3} + 4d_{N-2} + d_{N-1}
\end{aligned}$$

The last point, d_{N-1}, can be written in the form,

$$\begin{aligned}
x_{N-1} &= b_0^{N-1} \\
x_{N-1} &= \frac{1}{6}d_{N-2} + \frac{7}{12}d_{N-1} + \frac{1}{4}d_N \\
6(x_{N-1} &= \frac{1}{6}d_{N-2} + \frac{7}{12}d_{N-1} + \frac{1}{4}d_N) \\
6x_{N-1} &= d_{N-2} + \frac{7}{2}d_{N-1} + \frac{3}{2}d_N \\
6x_{N-1} - \frac{3}{2}d_N &= d_{N-2} + \frac{7}{2}d_{N-1}
\end{aligned}$$