

Fundamentals of Linear Algebra and Optimization

Jean Gallier

Project 1

Adapt N=5 Case

(C1)

$$\begin{aligned}b_0^1 &= d_0 \\b_1^1 &= d_1 \\b_2^1 &= \frac{1}{2}d_1 + \frac{1}{2}d_2 \\b_3^1 &= \frac{1}{4}b_1^1 + \frac{1}{2}b_2^1 = \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3\end{aligned}$$

(C2)

$$\begin{aligned}b_0^2 &= \frac{1}{2}b_2^1 + \frac{1}{2}b_1^2 \\&= \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3 \\b_1^2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\b_2^2 &= \frac{1}{3}d_2 + \frac{2}{3}d_3 \\b_3^2 &= \frac{1}{2}b_2^2 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4\end{aligned}$$

(C3)

$$\begin{aligned}b_0^3 &= \frac{1}{2}b_2^2 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4 \\b_1^3 &= \frac{1}{2}d_3 + \frac{1}{2}d_4 \\b_2^3 &= d_4 \\b_3^3 &= d_5\end{aligned}$$

Adapt N=6 Case

(C1)

$$\begin{aligned}
 b_0^1 &= d_0 \\
 b_1^1 &= d_1 \\
 b_2^1 &= \frac{1}{2}d_1 + \frac{1}{2}d_2 \\
 b_3^1 &= \frac{1}{4}b_1^2 + \frac{1}{2}b_1^2 = \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3
 \end{aligned}$$

(C2)

$$\begin{aligned}
 b_0^2 &= \frac{1}{2}b_2^1 + \frac{1}{2}b_1^2 \\
 &= \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3 \\
 b_1^2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\
 b_2^2 &= \frac{1}{3}d_2 + \frac{2}{3}d_3 \\
 b_3^2 &= \frac{1}{2}b_2^2 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4
 \end{aligned}$$

(C3)

$$\begin{aligned}
 b_0^3 &= \frac{1}{2}b_2^2 + \frac{1}{2}b_1^3 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4 \\
 b_1^3 &= \frac{2}{3}d_3 + \frac{1}{3}d_4 \\
 b_2^3 &= \frac{1}{3}d_3 + \frac{2}{3}d_4 \\
 b_3^3 &= \frac{1}{2}b_2^3 + \frac{1}{2}b_1^4 = \frac{1}{6}d^3 + \frac{7}{12}d_4 + \frac{1}{4}d_5
 \end{aligned}$$

(C4)

$$\begin{aligned}b_0^4 &= \frac{1}{2}b_2^3 + \frac{1}{2}b_4^1 = \frac{1}{6}d^3 + \frac{7}{12}d_4 + \frac{1}{4}d_5 \\b_1^4 &= \frac{1}{2}d_4 + \frac{1}{2}d_5 \\b_2^4 &= d_5 \\b_3^4 &= d_6\end{aligned}$$

Adapt N=4 Case Not yet updated

(C1)

$$\begin{aligned}b_0^1 &= d_0 \\b_1^1 &= d_1 \\b_2^1 &= \frac{1}{2}d_1 + \frac{1}{2}d_2 \\b_3^1 &= \frac{1}{4}b^1_2 + \frac{1}{2}b^2_1 = \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3\end{aligned}$$

(C2)

$$\begin{aligned}b_0^2 &= \frac{1}{2}b_2^1 + \frac{1}{2}b_1^2 \\&= \frac{1}{4}d_1 + \frac{7}{12}d_2 + \frac{1}{6}d_3 \\b_1^2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\b_2^2 &= \frac{1}{3}d_2 + \frac{2}{3}d_3 \\b_3^2 &= \frac{1}{2}b^2_2 + \frac{1}{2}b^3_1 = \frac{1}{6}d_2 + \frac{4}{6}d_3 + \frac{1}{6}d_4\end{aligned}$$