6.8370 Final Project: Extended Deconvolution and Poisson Editing

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December 2025

1 Project Choice and Motivation

For my final project, I chose to build on a previous assignment: *Deconvolution and Poisson Image Editing*. This problem set is rich in numerical optimization and computational photography concepts, and it provides a clean framework for working with inverse problems, convolution operators, and Poisson blending, which is what initially drew me to working on it.

However, I selected this pset for three more specific reasons. Firstly, I think it offers a self-contained yet substantial pipeline, integrating both deconvolution and gradient-domain compositing. Secondly, it requires implementing iterative solvers from scratch, which makes it a natural starting point for exploring extensions. Finally, the pset aligns well with classical literature on image restoration, allowing me to extend it with well-known techniques that were not originally included.

After implementing all required components of the pset, I also added two significant extensions: (1) Richardson-Lucy deconvolution, a multiplicative method commonly used for Poisson noise, and (2) three denoising algorithms (Gaussian smoothing, Perona-Malik anisotropic diffusion, and Non-Local Means). These additions allow a more complete experimental comparison between different restoration families.

2 Background and Related Work

In the pset itself, there are a few main concepts worthy of discussion. The gradient-domain framework introduced by Perez et al. blends regions of one image into another by reconstructing a function whose Laplacian matches that of the source. Although our pset version is simplified, the principle remains the same, solving a Poisson equation under boundary constraints.

There are also a few other concepts I used in this pset. Richardson-Lucy (RL) deconvolution is an iterative method originally developed for astronomical imaging under a Poisson noise model. Each update multiplies the current estimate by a backprojected ratio of observed to predicted intensities. RL preserves positivity, tends to sharpen edges, and often produces crisper results than quadratic inverse filters, though at the cost of amplifying noise.

Perona-Malik introduced anisotropic diffusion, an edge-aware PDE that smooths homogeneous regions while preserving boundaries. The diffusivity decreases with increasing gradient magnitude, allowing the method to denoise without destroying salient edges. This inspired many modern edge-preserving denoisers.

Non-Local Means (NLM) is a patch-based denoiser that averages pixels with similar local neighborhoods, even if they are spatially distant. This enables the method to preserve repeated textures and reduce noise more effectively than local smoothers. Although computationally expensive, NLM remains a gold standard for high-quality denoising.

These works generally, I think, complement the pset content, which focuses instead on quadratic least-squares methods such as $M^T M$ solvers, Laplacian regularization, and Poisson equations.

3 Algorithm Summary

The project involves two main problems: deblurring and Poisson blending.

3.1 Deconvolution

Given a blurred image $y = Mx$ where M is convolution by a known kernel, the least-squares solution satisfies

$$M^T M x = M^T y.$$

Gradient Descent. The update rule is

$$r_i = M^T(y - Mx_i), \qquad x_{i+1} = x_i + \alpha_i r_i.$$

The step size α_i is computed analytically using $(r_i \cdot r_i)/(r_i \cdot M^T M r_i)$.

Conjugate Gradient. CG accelerates convergence by constructing conjugate search directions

$$d_{i+1} = r_{i+1} + \beta_i d_i.$$

Regularized CG. A Laplacian penalty produces the operator

$$A_{\text{reg}} = M^T M + \lambda L.$$

This improves conditioning and reduces noise artifacts.

3.2 Richardson-Lucy Deconvolution (Extension)

RL solves a different statistical problem, maximizing Poisson likelihood:

$$x^{(k+1)} = x^{(k)} \cdot M^T \left(\frac{y}{Mx^{(k)} + \epsilon} \right),$$

with elementwise multiplication. RL enforces positivity and can produce very sharp edges. Because it amplifies noise, Gaussian smoothing may be applied between iterations.

3.3 Poisson Image Editing

Poisson blending reconstructs a region whose Laplacian matches the source gradient field:

$$\Delta x = \Delta f \quad \text{inside mask.}$$

Our solver uses gradient descent and conjugate gradient applied to the discrete Laplacian under masked updates.

4 Implementation and Challenges

I implemented all convolution-based operators manually, using reflected boundaries. The main components were 2D convolution and its transpose (kernel flipping), the operators M, M^T, $M^T M$, and the Laplacian, iterative solvers: GD, CG, and regularized CG, Poisson editing via masked Laplacian solves, Gaussian blurring for denoising, and Richardson-Lucy with clamping and stabilized division.

My main difficulties or challenges when completing this pset were ensuring CG stability (division by small $d^T Ad$ values), balancing regularization strength in deblurring, preventing RL from blowing up on noisy inputs, and implementing mask-restricted Poisson updates correctly.

5 Results and Test Cases

I evaluated the algorithms on both assignment-provided imagery and synthetic inputs.

Figure 1: Original, blurred, gradient descent result, and conjugate gradient result.

5.1 Test Case 1: Real Image ("pru.png")

Observations I made here were that CG converges faster and yields cleaner edges, and RL (not shown) sharpens too aggressively on this image unless denoised.

5.2 Test Case 2: Synthetic Checkerboard Deconvolution

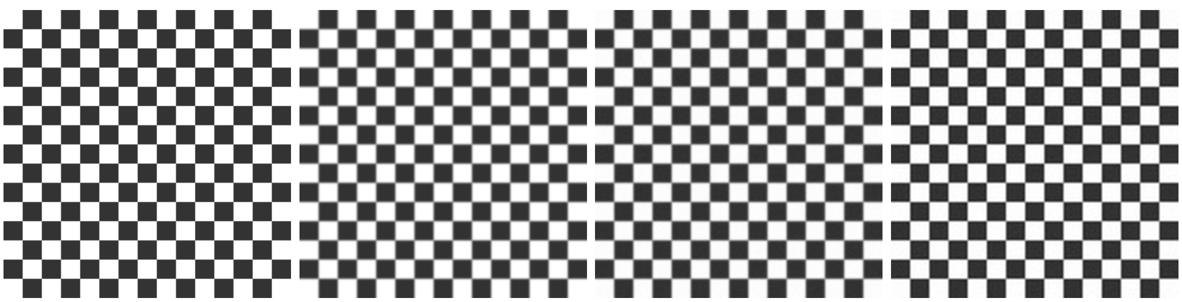

Figure 2: Synthetic checkerboard: original, blurred, noisy blur, and regularized CG recovery.

This case highlights the importance of regularization: unregularized CG oversharpens noise, while the Laplacian penalty stabilizes the reconstruction.

5.3 Test Case 3: Poisson Editing (Mask-Based Blending) - See figure 3

The Poisson solver smoothly integrates the source content while matching the target gradient field.

5.4 Test Case 4: Real Poisson Editing (bear → waterpool)

The original assignment includes a gradient-domain compositing task, where a foreground object is blended into a background using the Poisson equation. I reproduced the canonical test setup using the provided `bear.png`, `waterpool.png`, and `mask.png`.

The naive composite simply pastes the bear into the scene using linear alpha-blending and therefore produces strong boundary artifacts. I then applied both Poisson solvers and found that, qualitatively, the CG-based Poisson solver produces smoother transitions and fewer ringing artifacts. Boundary gradients match the background more naturally, and the inserted region appears more photometrically consistent with the waterpool environment.

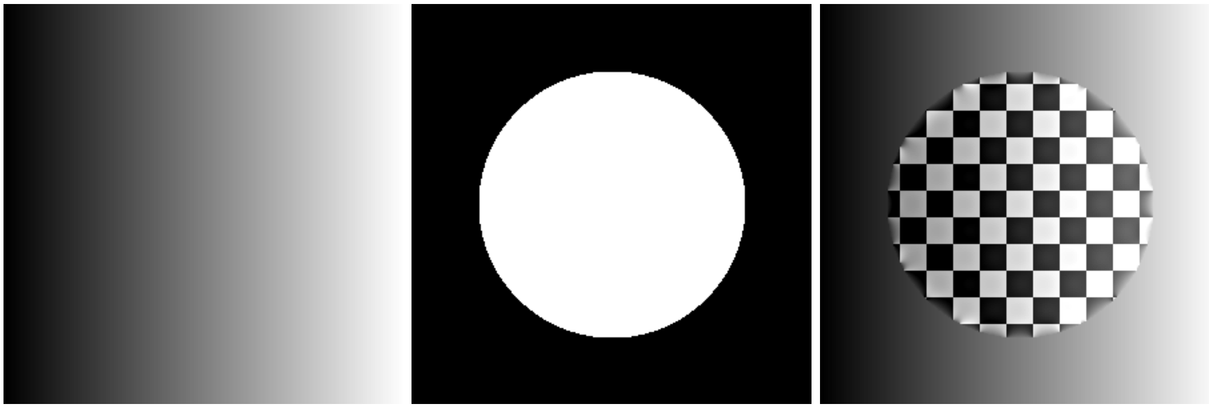

Figure 3: Gradient background, circular mask, and Poisson-blended output.

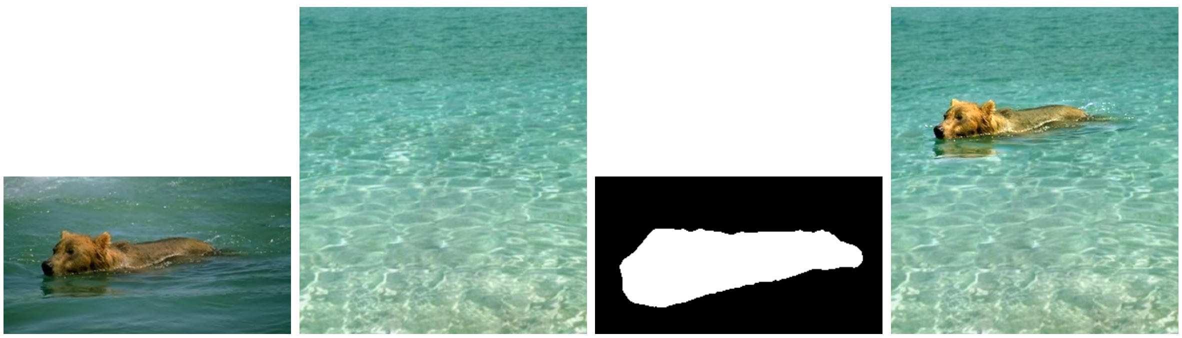

Figure 4: Foreground (*bear*), background (*waterpool*), mask, and the CG-based Poisson solver.

6 Conclusion

This project extended the original deconvolution and Poisson editing assignment with an additional classical deconvolution algorithm (Richardson-Lucy) and new synthetic evaluation cases. Implementing solvers from scratch clarified how convolution operators, adjoints, Laplacian regularization, and masked Poisson constraints interact in practical image restoration pipelines.

7 Ethical Issues

(1) Specific changes (such as skin smoothing, slimming, etc.) can perpetuate unrealistic beauty standards and remove markers of individuality. Mainly, it also can reinforce biases about race, gender, and age. Automated attractiveness enhancement can be used without consent to alter identities or misrepresent subjects in news or even in legal contexts. It risks psychological harm by normalizing a narrow standard of beauty and can have disparate impacts across demographic groups (e.g., lightening/darkening skin tones by automatic white-bias in processing).

(2) I would have a few ideas for rescoping to avoid ethical problems. These include focusing on tools that restore image fidelity (remove motion blur, correct exposure) rather than modifying identity features. We could also provide explicit controls and informed consent since the subject should be aware of which edits are automated and allowed to accept/reject. We can additionally offer presets that preserve diversity and avoid value judgements (e.g., "realistic correction", not "beautify"). Finally, we could evaluate how algorithms perform across skin tones, ages, and genders and mitigate biased behaviors (for example, avoid global color transforms that consistently bias one skin tone) and record edit metadata to trace transformations. These rescopings align the project with restoration and accessibility aims rather than normative aesthetic judgments.

8 References

- MIT 6.837 problem set (Deconvolution and Poisson Editing) — assignment spec used as the base.
- Richardson, W. H. (1972). Bayesian-based iterative method of image restoration. JOSA.
- Lucy, L. B. (1974). An iterative technique for the rectification of observed distributions. *Astronomical Journal*.
- Perona, P., & Malik, J. (1990). Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*.
- Buades, A., Coll, B., & Morel, J.-M. (2005). A non-local algorithm for image denoising. *CVPR*.