1 Targets and bounds

Two types of targets are introduced: country level targets and a global restoration target.

Country targets

A country level target refers to the increase on the land exploitation that the country k needs to carry out to achieve its future proposes. As a proxy to quantify this exploitation, we define the total effective suitability S_k as the sum over all the land units within the country Π_k of the product of the suitability index s_i, the exploited area m_i, and the yield ratio γ_i of each land unit i:

$$S_k = \sum_{i \in \Pi_k} m_i s_i \gamma_i \tag{1}$$

The suitability index of a land unit is defined as the potential productivity of that unit normalised to the maximum possible potential productivity; it is then a measure of how good that land is producing a given commodity. The yield ratio represents how much a planning unit is exploited out of its full potential, and it is defined as the current productivity divided by the potential productivity. Magnitude s_i, and γ_i takes values between 0 and 1, as well as m_i if normalising it to the planning unit area (assuming that every planning unit has the same area). Note that it may exist different types of commodities j (e.g, agriculture and pasture) with different values of m_{ij}, s_{ij}, and γ_{ij}. If that is the case, we operate those magnitudes as vectors in such a way that $m_i s_i \gamma_i = \sum_j m_{ij} s_{ij} \gamma_{ij}$.

Following this approach, country targets T_{Sk} are defined as the difference between the effective suitability index of the present P and the future F. This difference can be positive or negative, implying the possibility of restoration or the need of deforestation, respectively. We assume that suitability indexes do not change with time $s_i^P = s_i^F = s_i$, whereas fractions m_i^P and m_i^F are extracted from data or predictive models [CITE]. Current yield ratio γ_i is also obtained from previous models [CITE], and it may change with time, reducing its gap on the future by a fraction f_δ, such as $\gamma_i^F = \gamma_i + (1-\gamma_i) f_\delta$. We will study scenarios where we propose to close present yield gaps by a fraction f_C and then the new yield ratio will be $\gamma_i^C = \gamma_i + (1-\gamma_i) f_C$. Note the difference between f_δ and f_C: whereas the former is given, the latter is part of the choices on our decision land use plan. Adding all these ingredients together:

$$T_{Sk} = \sum_{i \in \Pi_k} m_i^P s_i \gamma_i^C - \sum_{i \in \Pi_k} m_i^F s_i \gamma_i^F \tag{2}$$

and operating we obtain that

$$T_{Sk} = \Delta S_k + \Delta S_k^{1P} f_C - \Delta S_k^{1F} f_\delta \tag{3}$$

with

$$\Delta S_k = S_k^P - S_k^F \tag{4}$$

$$\Delta S_k^{1P} = S_k^{1P} - S_k^P \tag{5}$$

$$\Delta S_k^{1F} = S_k^{1F} - S_k^F \tag{6}$$

and

$$S_k^\tau = \sum_{i \in \Pi_k} m_i^\tau s_i \gamma_i \tag{7}$$

$$S_k^{1\tau} = \sum_{i \in \Pi_k} m_i^\tau s_i \tag{8}$$

where $\tau = \{P, F\}$.

Since the yield ratio is not known for those planning units that are not being exploited in the present, we will take the weighted average of the country when an unknown yield ratio is encountered:

$$\overline{\gamma}_k = \frac{\sum_{i \in \Pi_k} m_i^P \gamma_i}{\sum_{i \in \Pi_k} m_i^P} \tag{9}$$

From Eq. 3, it is straightforward to obtain the minimum fraction of gap to be closed for a country to avoid deforestation ($T_k = 0$):

$$f_{Ck}^{eq} = \max\left(0, -\frac{\Delta S_k}{\Delta S_k^{1P}} + f_\delta \frac{\Delta S_k^{1F}}{\Delta S_k^{1P}}\right) \tag{10}$$

Global restoration target

The global restoration target T_G is the amount of area in the world that is required to be restored at a given time. It is usually expressed as the fraction F_G of the area that is currently used for agriculture or pasture that needs to be reconverted in natural land:

$$T_G = F_G \sum_i m_i^P \tag{11}$$

Application of targets and bounds

The fraction of each planning unit to be restored or deforested is represented with a decision variable vector x_i, which is defined positive for restoration ($T_{Sk} > 0$) and negative for deforestation ($T_{Sk} < 0$), and has to satisfy

$$\text{For each country } k\text{: } \sum_{i \in \Pi_k} x_i \sigma_i^C \leq T_{Sk} \tag{12}$$

$$\text{Globally: } \sum_i x_i = T_G \tag{13}$$

Targets T_{Sk} and T_G come from Eqs. 3 and 11, respectively, and σ_i^C represents the product of suitability and yield ratio once gaps are closed by f_C:

$$\sigma_i^C = \sigma_i + \left(\sigma_i^{1P} - \sigma_i\right) f_C \tag{14}$$

Since the decision variable x_i is not distinguishing types of commodities (unlike m_{ij}), it is necessary to aggregate first the product of suitability and yield ratio $s_{ij}\gamma_{ij}$ of the different types of commodities j. We weight commodities using the area of the planning unit that they use in the present; if the planning unit is not exploited, we approximate these weights by their opportunity cost (i.e, benefit that each type of commodity would produce if the planning unit is used for its production). In this way:

$$\sigma_i = \frac{\sum_j \omega_{ij} s_{ij} \gamma_{ij}}{\sum_j \omega_{ij}} \tag{15}$$

$$\sigma_i^{1P} = \frac{\sum_j \omega_{ij} s_{ij}}{\sum_j \omega_{ij}} \tag{16}$$

where

$$\omega_{ij} = \begin{cases} m_{ij}^P & \text{if } \exists j : m_{ij}^P \neq 0 \\ o_{ij} & \text{if } \forall j : m_{ij}^P = 0 \end{cases} \tag{17}$$

Let us define l_i^{res} as the minimum amount of area of a planning unit that has to remain exploited, and l_i^{def} as the maximum area that can be natural within a planning unit, and therefore $0 \leq l_i^{res} \leq m_i \leq l_i^{def} \leq 1$. Then the fraction of planning unit to be converted is bounded by the following conditions in the case of restoration and deforestation, respectively:

$$\begin{cases} 0 \leq x_i \leq m_i - l_i^{res} & \text{if } T_{Sk} > 0 \text{ (restoration)} \\ 0 \leq -x_i \leq l_i^{def} - m_i & \text{if } T_{Sk} < 0 \text{ (deforestation)} \end{cases} \tag{18}$$