Structural Econometrics in Labor and IO.
Problem Set: Dynamic Discrete Choice Estimation.

Description

Superintendent Harry Zurcher manages the bus fleet at the Madison Metropolitan Bus Company. Notably, at every point in time t, he decides whether to replace an old bus engine with a new one or to keep operating the old one.

We are interested in estimating the cost of engine maintenance and replacement, using observed data on: (1) mileage x_t at period t, (2) observed replacement decision i_t at period t, and (3) mileage x_{t+1} next period.

The data generating process is as follows:

- x_t can take on 11 values, $x_t \in \{0, 1, 2, \dots, 10\}$ (i.e. mileage discretized into 11 categories).
- At any time period,

$$x_{t+1} = \begin{cases} \min\{x_t + 1,\ 10\}, & \text{with probability } \lambda, \\ x_t, & \text{with probability } 1 - \lambda \end{cases}$$

  Note that λ does not depend on the replacement decision i_t.

- The agent's replacement decision is made at the beginning of each period and is effective immediately (i.e. if $i_t = 1$, the agent uses machine with mileage 0 and the next period state x_{t+1} is 1 with probability λ and 0 with probability $1 - \lambda$).
- The per-period maintenance cost for a bus with mileage x is $C(x, \theta) = \theta_1 x + \theta_2 x^2$.
- The cost of replacement is θ_3.
- The per-period utility function is given by

$$u(x_t, i_t, \epsilon_{1t}, \epsilon_{2t}; \theta) = \begin{cases} -\theta_1 x_t - \theta_2 x_t^2 + \epsilon_{0t}, & \text{if } i_t = 0, \\ -\theta_3 + \epsilon_{1t}, & \text{if } i_t = 1 \end{cases}$$

  where $(\epsilon_{0t}, \epsilon_{1t}) \overset{iid}{\sim}$ Type 1 Extreme Value.

- Assume that the decision-maker's discount factor is known, $\beta = 0.95$.

Problems

Together with your answers, please submit your well-commented computer code for the questions below before next session. If possible, please send all files compressed into one single zip-file.

1. Simulate a fake data set with 1000 observations of x_t, x_{t+1}, and i_t. For this, set $\theta_1 = 0.5$, $\theta_2 = 0.04$, $\theta_3 = 2$, and $\lambda = 0.82$. Hint: All busses are assumed to be operated and maintained independently.

2. Now, forget the chosen parameter values and use the simulated data to:

   - report an estimate for λ (this does not require any nonlinear optimization).
   - report the parameters of the model estimated using MLE with a nested fixed point algorithm:
     (a) Guess initial parameter values;
     (b) Solve dynamic programming problem;
     (c) Calculate the probability of replacement at each state;
     (d) Use model predictions (the probability calculated in the previous step) and data to compute the (log-)likelihood function;
     (e) Search over parameter values (e.g. using scipy's *minimize*) by repeating steps $(b) - (d)$.
   - Report the probability of replacement for every state: $P(1|x_t, \theta)$.