Advanced Applied Econometrics
Static discrete choice with market-level data

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Organization

- Next class on June 25, 15:30-18:30
  - Single agent dynamic discrete choice
  - Read Rust (1987): "Optimal replacement of GMC bus engines: an empirical model of Harold Zurcher"
  - Graded problem set will be distributed this week. Due by Sunday, July 6, 23:59.

Plan for today

- Recap BLP: The Random Coefficient Logit Model of Demand
- Go through BLP code
- Discuss addition of supply-side moments

Recap BLP

- Individual choice model using market-level data with
  - horizontal (ϵ, μ) and vertical (δ, ξ) product differentiation
  - (price) endogeneity
    - Isolate mean utility to estimate β coefficients by linear IV estimation
  - unobserved consumer heterogeneity
  - flexible substitution patterns
    - In homogenous logit, only market shares matter
    - In heterogenous logit, closeness in characteristics space $\rightarrow$ product differentation matters
  - static pricing game on supply side can improve identification
  - quasilinear preferences allow computing changes in consumer welfare

Recap BLP: Identification

- exogenous variation in observed characteristics
- choice set variation (if data on multiple markets available)
- variation in preferences $\rightarrow$ variation in choice probabilities across consumer groups
- formal positive nonparametric identification results by Berry and Haile (Ecta 2014, ARE 2016), Fox and Gandhi (Rand 2016), Fox, Kim, Ryan, and Bajari (JoE 2012)
  $\rightarrow$ functional forms and distributional assumption not necessary for identification, standard IV conditions are sufficient
- Intuition for BLP instruments: assuming $E[\xi_j Z_j] = 0$ shuts off opportunity for model to explain data by systematic correlation between ξ_j and observable local market structure.
- Moment conditions with such IVs, in principle, allow identification of $\{\sigma, \delta, \alpha, \beta\}$

BLP widely used but problems

- Econometric
  - Identifying unobserved heterogeneity with aggregate data can be hard in practice, Petrin (JPE 2002), BLP (JPE 2004) add microdata
  - Weak instrumental variables due to lack of cost shifters - Armstrong (Ecta 2016), Reynaert and Verboven (JoE 2014), Berry and Haile (2014), Gandhi and Houde (2019), Gandhi and Houde (2020), handbook chapter Gandhi and Nevo (2021)
  - "Logit" assumption / Welfare analysis with many products, entry/exit, Ackerberg and Rysman (Rand 2005), Berry and Pakes (IER 2007)
  - Measurement error in market shares (small T, large J), moment inequalities, Gandhi, Lu, and Shi (QE 2023)

BLP widely used but problems

- Numerical
  - Error tolerance in inner loop δ, premature 'convergence', speed of convergence
    Dubé, Fox, and Su (Ecta 2012), Knittel and Metaxoglu (ReStat 2012), Reynaerts, Varadhan, and Nash (2012)
  - Quality of solvers in estimation, importance of starting values,
    Dubé, Fox, and Su (Ecta 2012), Knittel and Metaxoglu (ReStats 2014)
  - Numerical integration techniques and simulation error,
    Judd and Skrainka (2011), Chiou and Walker (JoE 2007)
- Implications for elasticity estimates and, in consequence, for measure of market power, merger evaluation, welfare gains from new products/technologies, ...?
- Solutions to most of these problems implemented in Conlon and Gortmaker (2020):
  https://github.com/jeffgortmaker/pyblp

BLP Code

Nested fixed point algorithm (BLP 1995)

Functions to be called (directly and indirectly) from main script

- Contraction mapping: $\delta^{h+1} = \delta^h + ln(S) - ln(s(\delta^h, \sigma))$
- Market shares: $s_{jt}(\delta_t, \sigma) \approx \sum_{i=1}^{n} \phi_i \frac{\exp(\delta_{jt}+\mu_{jt}(\nu_i))}{1+\sum_{l=1}^{J} \exp(\delta_{lt}+\mu_{lt}(\nu_i))}$
- GMM objective function,
  minimization problem based on moment conditions $E[g_{jt}(z_{jt})\xi_{jt}] = 0$:

$$\min_{\theta} \xi(\theta)' g(z)' A g(z) \xi(\theta)$$

- For simplicity, no analytical gradients. In practice, use if possible!

Nested fixed point algorithm (BLP 1995)

For data generation, functions needed for

- Equilibrium prices, using FOC: $0 = c_t - p_t - \Delta_t^{-1} s_t$
- Market share derivatives Δ_t:

$$\sum_{i=1}^{n} \phi_i \left[-\alpha s_{ijt}(1 - s_{ijt})\right] \;\forall\, j = k, \qquad \sum_{i=1}^{n} \phi_i \left[\alpha s_{ijt} s_{ikt}\right] \;\forall\, j \neq k$$

Notes on numerical integration

Stochastic / Monte Carlo

- "Monte Carlo is the art of approximating an expectation by the sample mean of a function of simulated random variables" - Statistical Genetics lecture notes, UC Berkeley.
- Pseudo-random - standard random number generator
- Quasi-random - more uniform coverage, e.g. Halton, modified Latin hypercube sampling

Notes on numerical integration

Non-stochastic / Quadrature

- Gaussian Hermite product rule
    - Difficult for high-dimensional distributions and complicated integrands
- Sparse grid integration (Heiss and Winschel, JoE 2008)
    - Subset of nodes from product rule
- Simulation bias in MSL and MSS (ln in simulated $\ln P_n(\theta)$ is a nonlinear transformation), not in MSM

Supply-side equilibrium Model

- How are prices determined in oligopolistic markets with differentiated products?
- $\rightarrow$ counterfactual policy analysis

- Increased efficiency of demand estimates by adding structure on the supply side
- Assume prices are result of firms' optimization problem
- Use information from first-order conditions for estimation
- Single-product vs. multi-product firms
- Specify cost function and equilibrium notion

Supply-side equilibrium Model

Berry (1994) single-product firms

- Profits of firm j are given by $\Pi_j(\mathrm{p}) = p_j q_j(\mathrm{p}) - C_j(q_j(\mathrm{p}))$
- Conduct assumption: compete à la Nash in prices
- The first-order condition is:
$$q_j + (p_j - mc_j)\frac{\partial q_j}{\partial p_j} = 0$$
- which, by rewriting, implies the *markup* term
$$b_j = p_j - mc_j = \frac{-q_j}{\partial q_j \backslash \partial p_j}$$

Supply-side equilibrium Model

- For simplicity, assume that marginal costs are constant and take the form:

$$ln(mc_j) = w_j\gamma + \omega_j \tag{1}$$

- The equation to be estimated is therefore

$$ln(p_j - b(p_j, x_j, \theta_1, \theta_2)) = w_j\gamma + \omega_j$$

  where w_j are observed and ω_j unobserved product characteristics
- The markup $b(p, x, \theta_1, \theta_2) = \Delta(p, x, \theta_1, \theta_2)^{-1}s(p, x, \theta_1, \theta_2)$ depends on demand parameter and, through the equilibrium price, on the unobserved cost component ω

GMM estimation with supply-side

- Estimate demand and supply simultaneously. Define additional instruments $z_j = (x_j, w_j)$.
- Additional moment conditions

$$E[\xi_j|z] = E[\omega_j|z] = 0$$

where $\omega = mc - [x, w]\gamma$

Multiproduct Firms

BLP (1995) look at the more general case of a multiproduct firm f producing a subset $\Im_f$ of the J products in the market

- Profits of firm f are given by:

$$\Pi_f = \sum_{j \in \Im_f} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2)$$

- The usual assumption is that firms compete à la Nash in prices

Multiproduct Firms

- The first-order condition is:

$$s_j(p, x, \theta_1, \theta_2) + \sum_{r \in \Im_f} (p_r - mc_r) \frac{\partial s_r(p, x, \theta_1, \theta_2)}{\partial p_j} = 0$$

- which can be rewritten in vector notation as:

$$p_j = mc_j + \Delta(p, x, \theta_1, \theta_2)^{-1} s(p, x, \theta_1, \theta_2)$$

where the (j, r) element of the J by J matrix $\Delta(p, x, \theta_1, \theta_2)$ is

$$\Delta_{jr} \begin{cases} 1 \times \frac{-\partial s_r}{\partial p_j} = \frac{-\partial s_r}{\partial p_j} & \text{if r and j are produced by the same firm} \\ 0 \times \frac{-\partial s_r}{\partial p_j} = 0 & \text{otherwise} \end{cases}$$

Multiproduct Firms: note on conduct

- Rewrite profit maximization problem:

$$\Pi_f = \sum_{j \in \Im_f} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2) + \kappa_{fg} \sum_{j \in \Im_g} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2)$$

- The first-order condition is then:

$$s_j(p, x, \theta_1, \theta_2) + \sum_{r \in (\Im_f, \Im_g)} \kappa_{fg} (p_r - mc_r) \frac{\partial s_r(p, x, \theta_1, \theta_2)}{\partial p_j} = 0$$

- Instead of 0's and 1's, now $\kappa_{fg} \in [0, 1]$ represents how much firm f cares about profits of g
- In reality, $\kappa_{fg} \in (0, 1)$, but evidence that $\kappa_{fg} > 0$ not necessarily "anti-competitive" – just deviation from static Bertrand pricing
- Berry and Haile (2014): κ_{fg} can be identified using IV that shifts demand but not supply ("rotate demand")