Advanced Applied Econometrics: static discrete choice with market-level data.
Problem Set: Preliminaries.

Description

The aim of this problem set is to refresh (or make you familiar with) some basic concepts – notably GMM estimation and root-finding for numerically solving systems of nonlinear equations - that will be essential in subsequent sessions and problem sets (and in large parts of the literature).

Main reference: Berry, Steven T. (1994), "Estimating Discrete Choice Models of Product Differentiation," *Rand Journal of Economics*, 25 (2), 242-262.

Problems

Together with your answers, please submit your computer code for the questions below.

1. **A simple demand estimation example.** We assume consumer i chooses one unit of product $j \in J$ or an outside good (e.g. no purchase) to obtain utility

$$u_{ijt} = x_{jt}\beta + \alpha p_{jt} + \xi_{jt} + \varepsilon_{ijt} = \delta_{jt} + \varepsilon_{ijt} \tag{1}$$

where (x_{jt}, p_{jt}) are observable characteristics and price, ξ_{jt} is an unobservable characteristic, and an idiosyncratic error term ε_{ijt} assumed i.i.d. extreme value type 1. The utility of the outside good is normalized such that $\delta_{0t} = 0$. The assumption of utility-maximizing consumers and the distribution of ε_{ijt} yields the logit choice probabilities:

$$s_{jt}(\delta_t) = \frac{\exp(\delta_{jt})}{1 + \sum_{l=1}^{J} \exp(\delta_{lt})}. \tag{2}$$

(a) Simulate product-level data based on the described model assuming the following

- $J = 10$ products are sold in $T = 25$ markets (size $L_t = 1$) by single-product firms.
- Two observable product characteristics $x_{jt} = (1, x^1_{jt})$, with $x^1_{jt} \sim U(1, 2)$.
- Preference parameters $\beta = (2, 2), \alpha = -2$.
- Marginal cost $c_{jt} = x_{jt}\gamma_1 + w_{jt}\gamma_2 + \omega_{jt}$.
- Three observable cost shifters $w_{jt} = (w^1_{jt}, w^2_{jt}, w^3_{jt})$, all i.i.d. $U(0, 1)$.
- Marginal cost parameters: $\gamma_1 = (0.7, 0.7)$ and $\gamma_2 = (1, 1, 1)$.
- Unobserved demand and cost characteristic $(\xi_{jt}, \omega_{jt}) \sim N(0, \sigma_c)$ with $\sigma_c = \begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$.

! To simulate the data, you must code a function computing a $(T \times J) \times 1$ vector containing market shares using equation (2). Assume perfect competition on price so that $p_{jt} = c_{jt}$.

(b) The simulated data at hand, forget parameter values and, following Berry (1994),

- estimate $\{\alpha, \beta\}$ using OLS and report the results.
- estimate $\{\alpha, \beta\}$ by GMM using as instrumental variables the observable characteristics x_{jt} and cost shifters w_{jt}, and report the results.

Do not use python's statsmodel package to estimate the parameters but formulate the moment conditions using the error term ϵ_{jt}, observables x_{jt}, and instruments $z_{jt} = \{x_{jt}, w_{jt}\}$. You do not need to compute standard errors.

2. **Numerical primer: root-finding.** Newton's method is an important tool in nonlinear optimization.[1] It is used to find roots of a function $f(x)$. Write a function finding the number $\sqrt{a}$ by iterating $x_{t+1} = x_t - \frac{f(x_t)}{f'(x_t)}$. Hint: $\sqrt{a}$ is the positive root of $f(x) = a - x^2$.

3. **Solving for a static industry equilibrium.** Now consider the setting in Problem 1 but with an imperfectly competitive industry, that is $p_{jt} \neq c_{jt}$. Take the market share equation, cost and demand parameters from above as given. Assume that single-product firms j maximize profits given by

$$\pi_{jt} = (p_{jt} - c_{jt}) s_{jt} L_t, \tag{3}$$

where c_{jt} are marginal cost and L_t market size, so that the system of FOC for a Nash equilibrium:

$$s_{jt} + (p_{jt} - c_{jt}) \frac{\partial s_{jt}}{\partial p_{jt}} = 0 \tag{4}$$

With multi-product firms, it is useful to write expression (4) in vector notation as $s_t + \Delta_t(p_t - c_t) = 0$, where $\Delta_t(j,k)$ denotes a diagonal matrix of own-price derivatives and off-diagonal elements according to market structure. With single-product firms, off-diagonal elements are equal to zero so that Δ_t can be collapsed to a vector. If marginal cost are known, we obtain the supply side by solving the system for c_t:

$$p_t + \Delta_t^{-1} s_t = c_t \tag{5}$$

For single-product firms, $\partial s_{jt} / \partial p_{jt}$ is given by $\alpha s_{jt}(1 - s_{jt})$.

(a) Generate a $(T \times J) \times 1$ vector Δ containing market share derivatives with respect to own-price price, and

(b) compute a $(T \times J) \times 1$ vector p containing (Bertrand-Nash) equilibrium prices, using equation (5) and the root-finding function imported in the python notebook, and report the mean, minimum, and maximum industry price.

[1] By the Contraction Mapping Principle (Banach Fixed Point Theorem) this method is a contraction under the conditions that 1) $f(x)$ has a continuous second derivative, 2) $f'(x) \neq 0 \;\forall\; x \in \mathbb{R}$, and 3) a $q \in (0,1)$ exists such that $|f(x)f''(x)| \leq q|f'(x)|^2 \;\forall\; x \in \mathbb{R}$.