Advanced applied econometrics

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Next week

1. next and final lecture: 11.7; 9-12
2. prepare Blundell et al. (2016)

Plan for today

1. Keane & Wolpin: Set-up
   - Structure of Model
   - Identification
   - Model fit & fix
2. K&W: Solution and Estimation
   - compare to Rust
   - Simulate & Interpolating value function
3. Practical session: Homework (Maxi online)

Motivation of Keane & Wolpin

What is the research question?

Main aims of paper

Explain **schooling & career choices** (“human K investment”)

- school
- white / blue collar work
- military
- home production

Policy Evaluations: Impact of tuition subsidy

Motivation of Keane & Wolpin

Assumptions ? Alternatives? Atheoretic answers?

What **choices** does paper model?

Rewards of choice

- How are the rewards of choices modelled?

Rewards of choice

(1) Wage

- occupation-specific skill heterogeneity $e_m(a)$
- schooling $g(a)$
- experience in occupation $x_m(a)$ quadratic
- unobserved endowments at age 16 $e_m(16)$
- **cross-experience** terms (what?)- see Section (III)

(2) non-monetary “rewards”

- **effort** cost of schooling
- **home** “production” (what?)

Unobserved heterogeneity

How modelled in Keane & Wolpin?

Unobserved heterogeneity (types)

Individual “fixed” effects very **interactive**

- e_{mk} are K types of initial endowments in diff occupations
- initial diffs create diff **comp advantage**
- link to initial schooling (initial conditions) ?

Schooling choices & technology

Schooling in model?

Schooling choices & technology

Costs of schooling

- monetary tuition fees (college & graduate)
- current period effort cost
- individual cost of schooling via $e_4(16)$

What are benefits of schooling?

State-space

What is state-space ?

State-space

At any point in time t, observe

- **past** choices
- initial **conditions**
- current **shocks**

... **how do individuals choose** what to do?

Choices

Choices made subject to

- draw shocks
- compare expected value of actions - Value functions.

(more on **solution method** in a minute)

Structure of Model

Assumptions, **Alternatives**, Atheoretic answers ?
What alternative assumptions could you imagine?

- Preferences
- Technology/ Market
- General structure - are processes missing?

Alternatives

Preferences

- non-monetary prefs over activities (edu ?) may change over time
- people care not only about themselves
  - people conform to **norms**
- people may care about risk
- people may be **myopic or backwardlooking**
- people may dislike specific occupation

Alternatives II

Technology

- information maybe imperfect
- hours choice may be possible

Market

- labour demand ?
- transitions not always **voluntary** (labor market frictions)
- single agent model (cf. BLP or search model)

Structure of Model

Assumptions, Alternatives, **Atheoretic answers** ?
What atheoretic strategies for this research question?

Less structural approaches

1. **OLS**
2. **Exogenous schooling & occupations**
   Quasi-experimental IV- and panel literature.
3. **Static discrete choice**: model multinomial choice

Less structural approaches II

Estimates of rates of return to education (RORE) are...

Less structural approaches II

OLS-estimated returns to educ'n/experience/occupations:
...biased if **unobserved factors** influence...

1. schooling choice & earnings
2. **selection** into work & experience.
3. **selection** into occupations

Here: **model selection** into schooling, experience, occupational choice.

Alternative?

Identification

How is the model identified ?

Identification

Transitions between activities over age

- schooling becomes less frequent
- working increases.

Occupational differences in transition trends

- blue-collar work occurs often at low ages
- blue-collars hardly increase after age of 22

Earnings evolution over age & transitions

- voluntary transitions / persistence must be incentivised
- what about **non-monetary costs/rewards** ?

Identification

- Data: Cohort study - NLSY 1979 - 1988
- What can explain **wage growth** in occupation?
- What role for **measurement error** in wages ?
- What can explain **high persistence** in activities ?
- What can identify **discount factor** ?

Identification

What can explain **wage growth** in occupation?

- experience & age
- no job-to-job transitions (no wage variance in occup'n)
- no bargaining, no promotions

What role for **measurement error** in wages ?

- variance of measurem't error **smooths predictions** (deal with outlier) - see FN 25.

Persistence?, **Discount factor?**

Percentage in state by age

How to get a **good fit?** see Figures 1-5 in paper.

Extended model: More persistence

Persistence in choices **stronger than model** predicts.

How to fix the fit ?

Extended model: More persistence

1. Occupation **switching costs.**
   Drop in earnings on leaving sector for a year
2. Occupation-specific **non-monetary costs** - interpretation ?
3. age-specific home term (young people couch premium ?) or involuntary unemployment ?
4. “psychic” **graduation effect**
   non-monetary schooling return

Results of augmented model

1. Importance of **initial endowments (types)** for w-var
2. Measurement error “accounts for” 40% of total w-var
3. Experience terms are important and heterogenous.
4. **No diploma effect** on wages (warm glow ? employment ?)
5. Monetary **job search costs** are significant.
6. How to interpret Table 12?

Solution methods

Compare **Rust** to **Keane & Wolpin** solution methods.

Keane & Wolpin

Dynamic Bellman equation:

$$V_t(S_{i,t},\theta) = max_{d(S_{i,t})} E\left[\sum_{t=\tau}^{T} \beta^{t-\tau} u_\tau(d_t, X_{i,t},\theta) + \varepsilon\right]$$
$$= max_{d(S_{i,t})} \left[u_t(d_t, X_{i,t},\theta) + \beta \; E\left[V_{t+1}(S_{i,t},\theta)|X_{i,t}, d_t\right]\right.$$

Conditional independence:

- Dynamic term $\beta \; E\left[V_{t+1}(S_{i,t},\theta)|X_{i,t}, d_t\right]$ indep of shock
- choice-reward combinations dep on **current state-space**

How to calculate $\beta \; E\left[V_{t+1}(S_{i,t},\theta)|X_{i,t}, d_t\right]$?

Solving for values

Infinite horizon (Rust)

- seek **fixed point**

Finite horizon (Keane & Wolpin)

- **backward induction**

Backward induction

(1) Optimal policy δ^* for **final period** problem simple

$$V_t(S_{i,t},\theta) = max_{d(S_{i,t})}\left[u_t(d_t, X_{i,t},\theta)\right]$$
$$\delta^*_T(S_{i,T},\theta) = argmax_{dT}\left[u_T(d_T, X_{i,T},\theta) + \varepsilon_{i,d_T}\right]$$

(2) in period $T-1$

$$E\left[V_t(S_{i,t},\theta)|X_{i,T-1}, d_{i,T-1}\right] =$$
$$\int\int\left[u_T(\delta^*_T(S_{i,T},\theta), X_{i,t},\theta) + \varepsilon_{i,\delta^*_T}\right]dF_\varepsilon\; dF_X(X_T|X_{i,T-1}, d_{T-1})$$

What to do about those integrals?

Backward induction II

What to do about **those integrals**?

1. Integrate idiosyncratic shock: assume ε **is EV(1)**

$$\int \left[u_T(.), X_{i,t}, \theta) + \varepsilon_{i,\delta_T^*}\right] dF_\varepsilon = log \left(\sum_{d_T} e^{u_T(d_T, X_{i,T,\theta})} \right) + \gamma$$

2. Integrate **future state space** (over $F(X_T | X_{i,T-1}, d_{T-1})$)?

Backward induction II

Theory: **Evaluate** choice prob's at **all points.**

- if $X_{i,t}$ has 5 binary variables
- if $X_{i,t}$ has 10 variables with 3 values

By how much does state space grow?

Backward induction III

Theory: **Evaluate** choice prob's at **all points.**

- if $X_{i,t}$ has 5 binary variables
  $=2^5=$**32** values for each t.
- if $X_{i,t}$ has 10 variables with 3 values
  $=3^{10}=$**59,049** values for each t.

Numerical challenge increases (fast!) in state space

Interpolating

Backward induction in practice:

1. **Solve at subset** of grid points
2. **Impute value** (interpolate) at other points

With optimal policy at $T-1$, go to $T-2$...

Keane & Wolpin interpolate **both integrals** - why?

1. Draw R vectors of shocks (one for each alternative) $\varepsilon_{1,T}, ..., \varepsilon_{M,T}$ for T.
2. Evaluate the maximum for each draw of R.
3. Average of R draws: expected maximum (EMAX)

$$V_T(x) = \frac{1}{R}\sum_{r=1}^{R} max_d \; u_T(d, x, \epsilon_T^r(d))$$

4. then calculate $V_{T-1}(x)$ in $T-1$

$$= \frac{1}{R}\sum_{r=1}^{R} max_d \left[u_{T-1}(d, x, \epsilon_{T-1}^r(d)) + \beta \sum_x {}_{i,T} V_T(x_{i,T}) F(x_i, T | x_{i,T-1}, d) \right]$$

This simulates EMAX for R specific points ε.

Simulate & Interpolate II

Select R **grid-points** (specific values of X) and **impute EMAX for others**, e.g.

$$V_t(x_1 = 1, x_2 = 10, x_3 = ln(1000)) = 125$$
$$V_t(x_1 = 2, x_2 = 100, x_3 = ln(2000)) = 500$$

Establish link between **EMAX and state space**

$$V_t(x_{k,t}) = \alpha_1\ x + \alpha_2 x^2 + ... + \xi_{k,t}$$

Simulate & Interpolate (last slide)

Then, estimate predicted EMAX for actual individuals using $\hat{\alpha}$.

$$\hat{V}_t(x_{i,t}) = \hat{\alpha}_1 \; x_{i,t} + \hat{\alpha}_2 x^2 + ... \tag{1}$$

10 minutes break