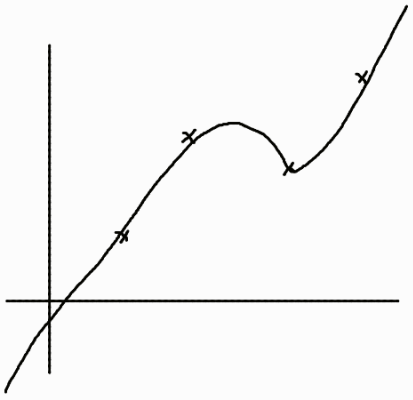
Lagrange Interpolation



$(n+1)$ nodes $\rightarrow$ 4

x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$

degree $= n = 3$

$$P_3(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3$$

$$\begin{bmatrix} & \end{bmatrix}^{-1}$$

$l_0 \rightarrow$	x_0	$f(x_0)$
$l_1 \rightarrow$	x_1	$f(x_1)$
	x_2	$f(x_2)$
	x_3	$f(x_3)$

$$P_3(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2) + l_3(x) f(x_3)$$

Lagrange Basis

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \frac{x - x_3}{x_0 - x_3}$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3}$$

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1} \cdot \frac{x - x_3}{x_2 - x_3}$$

$$l_3(x) = \frac{x - x_0}{x_3 - x_0} \cdot \frac{x - x_1}{x_3 - x_1} \cdot \frac{x - x_2}{x_3 - x_2}$$

Example:

node = 3

degree = 2

	Time (x)	Velocity $f(x)$	
x_0	15	227.04	$f(x_0)$
x_1	20	362.78	$f(x_1)$
x_2	22.5	517.35	$f(x_2)$

$x = 17$

$P_n(x) = ?$

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$$

$$l_0(x) = \frac{x-x_1}{x_0-x_1} \cdot \frac{x-x_2}{x_0-x_2} = \frac{x-20}{15-20} * \frac{x-22.5}{15-22.5} = \frac{2(x-20)(x-22.5)}{75}$$

$$l_1(x) = \frac{x-x_0}{x_1-x_0} \cdot \frac{x-x_2}{x_1-x_2} = \frac{x-15}{20-15} * \frac{x-22.5}{20-22.5} = \frac{-2(x-15)(x-22.5)}{25}$$

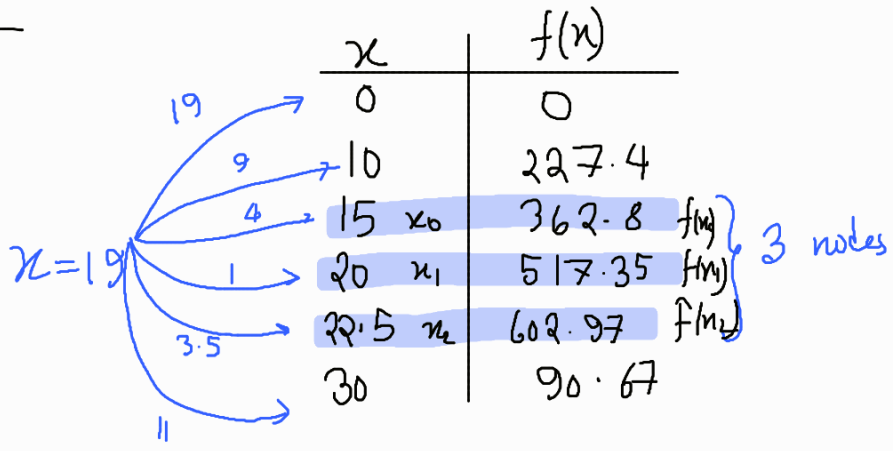
$$l_2(x) = \frac{x-x_0}{x_2-x_0} \cdot \frac{x-x_1}{x_2-x_1} = \frac{x-15}{22.5-15} * \frac{x-20}{22.5-20} = \frac{4(x-15)(x-20)}{75}$$

$$P_2(x) = \frac{2}{75}(x-20)(x-22.5)\cdot(227.04) - \frac{2}{25}(x-15)(x-22.5)\cdot(362.78) + \frac{4}{75}(x-15)(x-20) * (517.35)$$

$$P_2(17) = \frac{2}{75}(17-20)(17-22.5)\cdot(227.04) - \frac{2}{25}(17-15)(17-22.5)\cdot(362.78) + \frac{4}{75}(17-15)(17-20) * (517.35)$$

$$= 197.228$$

Ex:



$x = 19$, $P_n(x) = ?$

You are only allowed to use a polynomial of degree 2.

datapoint $= 2n+1 = 3$

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} = \cdots\cdots$$

do it yourself.

Advantages

* No need to inverse a matrix

Disadvantages

* If we want to add new nodes, we need to do the whole calculation from the beginning.