Quasi-Newton Method / Secant form

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Two different function

* Computational cost is higher when we are calculating two different functions.

So, we will replace $f'(x_k)$ by computable function g_k.

$$f'(x) \approx g_k$$

This can be done in different approach.

1. Secant Method
2. steffensen's method [Not included in syllabus]

Replaces $f'(x)$ using Backward Difference formula.

Derivation of secant method formula :

We know,

Backward Difference, $$f'(x) = \frac{\overbrace{f(x)}^{x_k} - \overbrace{f(x-h)}^{x_{k-1}}}{h} \quad \ldots\ldots (1)$$

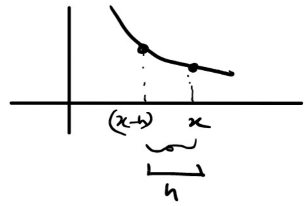
Let,

$$x_k = x$$

$$x_{k-1} = x-h$$

Putting these value in equation (1),

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$



$h = x - (x-h)$

$$f'(x) = \frac{f(x) - f(x-h)}{x - (x-h)}$$

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} \quad \text{—— (11)}$$

Newtons Raphson Formula,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{f(x_k)}{\frac{f(x_k) - f(x_{k-1})}{(x_k - x_{k-1})}}$$

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)\,(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}}$$

Formula of secant Method / Quasi - Newton Method

<u>Example:</u> Given, $f(x) = \frac{1}{x} - 0.5$, find the root of $f(x)$ using Quasi Newton Method / Secant method given initial point $x_1 = 0.5$, $x_0 = 0.25$.

$$x_{k+1} = x_k - \frac{f(x_k)\,(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{\left(\frac{1}{x_k} - 0.5\right)(x_k - x_{k-1})}{\left(\frac{1}{x_k} - 0.5\right) - \left(\frac{1}{x_{k-1}} - 0.5\right)}$$

iteration, k	x_k
0	$x_0 = 0.25$
1	$x_1 = 0.5$
2	$x_2 = 0.6875$
3	$x_3 = 1.01562$
4	$x_4 = 1.3540$
5	$x_5 = 1.68205$
⋮	⋮
12	$x_{12} = 2.00$
13	$x_{13} = 2.00$

$\left.\begin{matrix} x_{12} = 2.00 \\ x_{13} = 2.00 \end{matrix}\right] \rightarrow$ Same

$\therefore \; x^* = 2.00.$