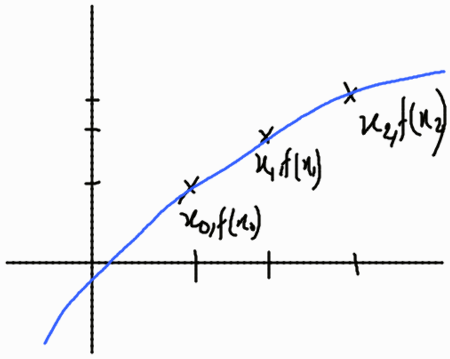
Polynomial Interpolation

Given $(n+1)$ datapoints



	house size	Rent	
x_0	20	200	$f(x_0)$
x_1	30	250	$f(x_1)$
x_2	40	275	$f(x_2)$

$n+1 = 3$

$$P_n(x) = P_2(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$$

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{bmatrix}$$

General form

Vandermond Matrix

Given,

$(n+1) = 3$

	(x) time	$f(x) = (y)$ velocity	
x_0	1	40	y_0
x_1	2	50	y_1
x_2	3	55	y_2

$$XA = Y$$

$$\begin{bmatrix} x_0^0 & x_0^1 & x_0^2 \\ x_1^0 & x_1^1 & x_1^2 \\ x_2^0 & x_2^1 & x_2^2 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

$$P_n(x) = P_2(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$$

$$a_0 \cdot 1^0 + a_1 1^1 + a_2 1^2 = 40 \quad \text{— (I)}$$
$$a_0 2^0 + a_1 2^1 + a_2 2^2 = 50 \quad \text{— (II)}$$
$$a_0 3^0 + a_1 3^1 + a_2 3^2 = 55 \quad \text{— (III)}$$

$$\begin{bmatrix} 1^0 & 1^1 & 1^2 \\ 2^0 & 2^1 & 2^2 \\ 3^0 & 3^1 & 3^2 \end{bmatrix} * \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 50 \\ 55 \end{bmatrix}$$

$$\underbrace{X}_{\text{Vandermond Matrix}} \qquad A \qquad Y$$

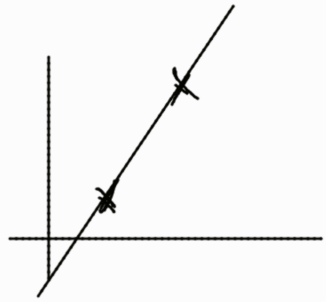
$$\Rightarrow \quad X \cdot A = Y$$

$$\Rightarrow \quad A = X^{-1} Y$$

Time	Velocity
15	362.8
20	517.3

node $= n+1$
$= 2$

degree $= 1$



$$P_1(x) = a_0 x^0 + a_1 x^1$$

$$\begin{cases} P_1(15) = a_0 + a_1 \cdot 15 = 362.8 \\ P_2(20) = a_0 + a_1 \cdot 20 = 517.3 \end{cases}$$

$$XA = Y$$

$$\Rightarrow \begin{vmatrix} x_0^0 & x_0^1 \\ x_1^0 & x_1^1 \end{vmatrix} \cdot \begin{vmatrix} a_0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 362.8 \\ 517.3 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 15 \\ 1 & 20 \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 362.8 \\ 517.3 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a_0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 1 & 15 \\ 1 & 20 \end{vmatrix}^{-1} \cdot \begin{vmatrix} 362.8 \\ 517.3 \end{vmatrix}$$

$$= \frac{1}{20-15} \begin{bmatrix} 20 & -15 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 362.8 \\ 517.3 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -100.85 \\ 30.91 \end{bmatrix}$$

$$\boxed{P_2(x) = -100.85 + 30.91x}$$

$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ A^{-1} : if $\det(A) \neq 0$

$\det(A) = ad - bc \neq 0$

$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

nodes = 3

degree = 2

time (t)		Velocity, v(t)	
x_0	3	11	y_0
x_1	5	21	y_1
x_2	7	26	y_2

Example: Given the time and velocity v(t) , find an interpolating Polynomial of velocity that goes through the datapoints using Vandermond Matrix. Also, find the approx. value of acceleration at Time t = 7 second.

$$P_2(x) = a_0x^0 + a_1x^1 + a_2x^2$$

$$XA = Y$$

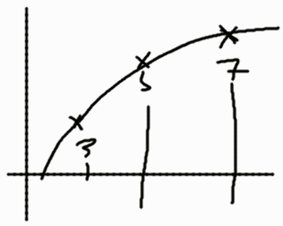
$$\begin{vmatrix} 3^0 & 3^1 & 3^2 \\ 5^0 & 5^1 & 5^2 \\ 7^0 & 7^1 & 7^2 \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} 11 \\ 21 \\ 26 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 9 \\ 1 & 5 & 25 \\ 1 & 7 & 49 \end{vmatrix}^{-1} \begin{vmatrix} 11 \\ 21 \\ 26 \end{vmatrix}$$

↓ using calculator

$$= \begin{vmatrix} 2.5 & -1.5 & 0 \\ -0.607 & 0.7142 & -0.107 \\ -0.0357 & -0.071 & -0.0357 \end{vmatrix} \begin{vmatrix} 11 \\ 21 \\ 26 \end{vmatrix}$$

$$\begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} -4 \\ 5.5392 \\ -0.17 \end{vmatrix}$$



$$P_2(x) = -4 + 5.5392x - 0.17x^2$$

$$\text{Accelaration} = \frac{d}{dx}\left(P_2(x)\right) = 5.5392 - 2*0.17x$$

$$= 5.5392 - 2*0.17*7$$

$$= 3.1592 \ ms^{-2}$$