

Set A

(1)

~~***~~ In the Lecture note ***

(2)

(a)

$$0 < \lambda < 1$$

$$\lambda = |g'(x)|$$

$$g(x) = (4x-1)^3$$

$$g'(x) = 3(4x-1)^2 \times 4$$
$$= 12(4x-1)^2$$

$$0 < 12(4x-1)^2 \quad | \quad 12(4x-1)^2 < 1$$

$$0 < 4x-1$$

$$\frac{1}{4} < x$$

$$4x < \frac{1}{\sqrt{12}} + 1$$

$$x < \frac{6+\sqrt{3}}{24}$$

(b) $g(x) = (4x-1)^3$

$$g(3.5) = 2197 \text{ [upto 3 significant figures]}$$
$$= 219 \star$$

$$g(219) = 669$$

(3)

$$g(x) = \sqrt{2x-3}$$

$$\begin{aligned}\lambda &= |g'(x)| \\ &= \frac{1}{2} (2x-3)^{-1/2} \times 2 \\ &= (2x-3)^{-1/2}\end{aligned}$$

For $x = -1$

$\lambda = \text{undefined / Math error}$

$x = 3$

$\lambda = 0.577$ (linear convergence)

(4)

$$f(x) = x^3 + 2x - 4$$

$$f'(x) = 3x^2 + 2$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2$$

$$x_2 = 1.42857.$$

Set B
①

In the lecture notes.

②

a

Here, $\lambda = 0$

$$\boxed{f(x) = (4x-1)^3} \quad \lambda = |f'(x)| = 12(4x-1)^2$$

$$12(4x-1)^2 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

③ b

$$f(x) = (4x-1)^3$$

$$f(2.5) = 729$$

$$f(729) = 2.47$$

[upto 3 significant
figure]

(3)

$$g(x) = \sqrt{4x-3}$$
$$= (4x-3)^{1/2}$$

$$g'(x) = \frac{1}{2} (4x-3)^{-1/2} \times 4$$
$$= 2 (4x-3)^{-1/2}$$

For,

$$x = 1$$

$$\lambda = 2 \quad (\text{divergent})$$

$$x = 2$$

$$\lambda = 0.89 \quad (\text{linear convergence})$$

(4)

$$f(x) = x^3 + 2x - 4$$

$$f'(x) = 3x^2 + 2$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.4285$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.204$$