Name:
Student ID:
Section:

10 Marks
25 Minutes

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1. Let $f(x) = x^2 \sin(x)$, where $x \in \{ \pi/3, \pi/2\}$.

   a. What will be the degree of the polynomial for above nodes? [1]

   b. Construct the Vandermonde Matrix, V for above nodes. [2]

   c. Using Lagrange Interpolation, find the interpolating polynomial. [3]

   d. Using the Vandermonde matrix found in answer (b) check if the interpolating polynomial is the same as the polynomial in answer (c) [4]

a) degree of the polynomial will be 1.

b)

$$V = \begin{bmatrix} 1 & \pi/3 \\ 1 & \pi/2 \end{bmatrix}$$

c) $P_1(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - \pi/2}{\pi/3 - \pi/2} = \frac{x - \pi/2}{\frac{2\pi - 3\pi}{6}} = \frac{-6(x - \pi/2)}{\pi}$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x - \pi/3}{\pi/2 - \pi/3} = \frac{x - \pi/3}{\frac{3\pi - 2\pi}{6}} = \frac{6(x - \pi/3)}{\pi}$$

$$P_1(x) = -\frac{6(x - \pi/2)}{\pi} \times (\pi/3)^2 \sin(\pi/3) + \frac{6(x - \pi/3)}{\pi} \times (\pi/2)^2 \sin(\pi/2)$$

$$= (-6x + 3\pi)\,\frac{\pi}{9}\sin \pi/3 + (6x - 2\pi)\,\frac{\pi}{4}\sin \pi/2$$

$$= -1.813799364\,x + 2.849109379$$

$$+ 4.71238898\,x - 4.934802201$$

$$= 2.898589616x - 2.085692822$$

d)
$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & \pi/3 \\ 1 & \pi/2 \end{bmatrix}^{-1} \begin{bmatrix} (\pi/3)^2 \sin(\pi/3) \\ (\pi/2)^2 \sin(\pi/2) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ -1.909859317 & 1.909859317 \end{bmatrix} \begin{bmatrix} 0.949703126 \\ 2.4674011 \end{bmatrix}$$

$$= \begin{bmatrix} -2.085692822 \\ 2.898589616 \end{bmatrix}$$

$$= 2.898589616\,x - 2.085692822$$

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   a. What will be the degree of the polynomial for above nodes? [1]

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a) degree of the polynomial will be 1

b) $V = \begin{bmatrix} 1 & \pi/2 \\ 1 & \pi/3 \end{bmatrix}$

c) $P_1(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$

$l_0(x) = \dfrac{x - x_1}{x_0 - x_1} = \dfrac{x - \pi/3}{\pi/2 - \pi/3} = \dfrac{x - \pi/3}{\frac{3\pi - 2\pi}{6}} = \dfrac{6(x - \pi/3)}{\pi}$

$l_1(x) = \dfrac{x - x_0}{x_1 - x_0} = \dfrac{x - \pi/2}{\pi/3 - \pi/2} = \dfrac{x - \pi/2}{\frac{2\pi - 3\pi}{6}} = \dfrac{-6(x - \pi/2)}{\pi}$

$P_1(x) = \dfrac{6(x - \pi/3)}{\pi} \left(\dfrac{\pi}{2}\right)^2 \sin(\pi/2) - \dfrac{6(x - \pi/2)}{\pi} \left(\pi/3\right)^2 \sin(\pi/3)$

$$= (6x - 2\pi)\frac{\pi}{4}\sin\frac{\pi}{2} + (-6x + 3\pi)\frac{\pi}{9}\sin\frac{\pi}{3}$$

$$= 2.898589616x - 2.085692822$$

d) $$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & \pi/2 \\ 1 & \pi/3 \end{bmatrix}^{-1} \begin{bmatrix} \left(\frac{\pi}{2}\right)^2 \sin\left(\frac{\pi}{2}\right) \\ \left(\frac{\pi}{3}\right)^2 \sin\left(\frac{\pi}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 \\ 1.909859317 & -1.909859317 \end{bmatrix} \begin{bmatrix} 2.4674011 \\ 0.949703126 \end{bmatrix}$$

$$= 2.898589616x - 2.085692822$$