

Practice Problems: Chapter 2

1. Evaluate the interpolation polynomial of a function $f(x) = \sin(x)$ at the nodes $\{-\pi/2, 0, \pi/2\}$ by using the Vandermonde method. Note you need to show a detailed calculation to get full credit.
2. Consider the function $f(x) = xe^x$. Using Taylor Expansion a polynomial of degree 3 is calculated.
 - a. Using Taylor Expansion, write $f(x)$ as an infinite series.
 - b. Find the coefficients a_0, a_1, a_2 and a_3 for the polynomial $P_3(X)$.
 - c. Compute $f(0.1)$ and $P(0.1)$ up to seven significant figures.
 - d. Find the percentage of error using the answers got from Question (c)
3. Consider the function $f(x) = \ln(1+x)$. Using Taylor Expansion a polynomial of degree 3 is calculated.
 - a. Using Taylor Expansion, write $f(x)$ as an infinite series.
 - b. Find the coefficients a_0, a_1 and a_2 for the polynomial $P_2(X)$.
 - c. Compute $f(0.2)$ and $P(0.2)$ up to seven significant figures.
 - d. Find the percentage of error using the answers got from Question (c)
4. A function, $f(x) = x \sin(x)$, is to be interpolated at three nodes: $x_0 = -\pi/2, x_1 = 0$ and $x_2 = \pi/2$. Answer the following:
 - a. Draw a sketch of the function $f(x)$ and identify the nodes. Briefly explain what kind of interpolation polynomial is expected and discuss the properties of the polynomial.
 - b. Compute Lagrange bases.
 - c. Write the interpolation polynomial and compare the properties of the interpolation polynomial with the expectation in Part-(a).
5. Read the following and answer accordingly:
 - a. Find an interpolating polynomial of appropriate degree using Newton's divided-difference method for $f(x) = x \cos(x)$. Consider the nodes $[-\pi/2, 0, \pi/2]$.
 - b. Use the interpolated polynomial to find an approximate value at $\pi/4$, and compute the relative error at $\pi/4$.
 - c. Add a new node π to the above nodes, and find the interpolating polynomial.
6. Given, $f(x) = \sin(x)$ and Consider the nodes $[0, \pi/2, \pi]$. Use Newton's Divided-Difference method for the following questions-
 - a. Find the values of a_0, a_1 and a_2 .
 - b. Write down the interpolating polynomial.
 - c. Add a new node $3\pi/2$ to the existing nodes and find a new interpolating polynomial.
 - d. Estimate the upper bound of the interpolating error for the polynomial $P_3(x)$.

7. Consider the following table of data points/nodal points:

Time (sec) t	Velocity (ms^{-1}) $v(t)$
2	10
4	20
6	25

- Find an interpolating polynomial of velocity that goes through the above data points by using Vandermonde Matrix method. Also compute an approximate value of acceleration at Time, $t = 7$ sec.
 - Find an interpolating polynomial of velocity that goes through the above data points by using Lagrange method.
 - If a new data point is added in the above scenario, which method should you use in finding a new interpolating polynomial? Also what will be the degree of that new polynomial?
8. Use a Hermite polynomial that agrees with the data listed below to find the approximation of $f(1.5)$.

k	x_k	$f(x_k)$	$f'(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.5698959
2	1.9	0.2818186	-0.5811571

9. Find the value of y for $x = 2.1$ using 2nd order Lagrange polynomials with the appropriate data.

SI.	x	y
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4
5	3	25.8

10. Let $f(x) = \cos(x)$ and the nodes are $(-\pi/4, 0, \pi/4)$. Use Cauchy's theorem to find the upper bound of the error.

11. a. Compute the upper bound of error using Cauchy's Theorem for $f(x) = \sin^2(x/2)$ with nodes $\{-\pi/3, 0, \pi/3\}$ within the interval $[-1.2, 1.2]$.
 b. Why are Chebyshev nodes an optimal choice in Interpolation?

12. The following nodes come from the function $f(x) = \ln(5x+9)$:

x	f(x)
-0.5	1.87
0	2.20
0.5	2.44

- a. Using Newton's divided difference method, find the equation of a second degree polynomial which fits the above data points.
 b. Expand the function $f(x) = \ln(5x + 9)$ using Taylor Series, centered at 0. Include till the x^2 term of the Taylor series.
 c. Should the equation which you found in part (a) and part (b) match? Comment on why, or why not.

13. Consider the following nodes:

x	f(x)
0	5
3	9.5
6	5

- a. If an equation of a polynomial which fits through the above nodes is found using both the Vandermonde Matrix approach and the Lagrange approach, will both the equations match?
 b. Find the equation of a polynomial which fits through the above nodes using the Vandermonde matrix approach.
 c. Find the equation of a polynomial which fits through the above nodes using the Lagrange approach.

14. Given a Runge Function, $f(x) = \frac{8}{5+16x^2}$ between the interval $[-6,6]$ where $n=3$

- a. Calculate the equal angled points of θ 's.
 b. Calculate the values of the Chebychev's nodes.
 c. Find the Lagrange basis $l_2(x)$.

15. Consider the following dataset:

x	$f(x)$	$f'(x)$
0.1	-0.62050	3.58502
0.2	-0.28340	3.14033

Answer the following based on the above data:

- Compute the Hermite bases: $h_0(x)$, $h_1(x)$, $\hat{h}_0(x)$ and $\hat{h}_1(x)$.
- Write the Hermite polynomial and find the value at $x = 0.15$