Brac University
Department of Computer Science and Engineering
Spring 2025

Name:
Student ID:
Section:

10 Marks
25 Minutes

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x	f(x)	f'(x)
0.4	-0.785	8.719
0.8	-0.415	8.315

Answer the following based on the above data:

a. ~~[5.5 Marks] Using~~ Hermite Polynomial Interpolation ~~find f(0.9)~~

b. [3.5 Marks] Using Newton's Divided Difference method find f(0.9)

a) $h_3(x) = h_0(x)f(x_0) + h_1(x)f(x_1) + \hat{h}_0(x)f'(x_0) + \hat{h}_1(x)f'(x_1)$

$l_0(x) = \frac{x-x_1}{x_0-x_1} = \frac{x-0.8}{0.4-0.8} = -\frac{x}{0.4} + 2 = 2 - \frac{x}{0.4}$

$l_0'(x) = -\frac{1}{0.4} \quad l_1'(x_0) = -\frac{1}{0.4}$

$h_0(x) = \left\{1 - 2(x-0.4)\left(-\frac{1}{0.4}\right)\right\} \times \left(2 - \frac{x}{0.4}\right)^2$

$= \left(1 + \frac{2x}{0.4} - 2\right)\left(2 - \frac{x}{0.4}\right)^2 = \left(\frac{2x}{0.4} - 1\right)\left(2 - \frac{x}{0.4}\right)^2$

$\hat{h}_0(x) = (x-0.4)\left(2 - \frac{x}{0.4}\right)^2$

$h_1(x) = \frac{x-x_0}{x_1-x_0} = \frac{x-0.4}{0.8-0.4} = \frac{x}{0.4} - 1$

$l_1'(x) = \frac{1}{0.4} \quad l_1'(x_1) = \frac{1}{0.4}$

$h_1(x) = \left\{1 - 2(x-0.8)\frac{1}{0.4}\right\}\left(\frac{x}{0.4} - 1\right)^2$

$= \left(1 - \frac{2x}{0.4} + 4\right)\left(\frac{x}{0.4} - 1\right)^2 = \left(5 - \frac{2x}{0.4}\right)\left(\frac{x}{0.4} - 1\right)^2$

$\hat{h}_1(x) = (x-0.8)\left(\frac{x}{0.4}-1\right)^2$

$\therefore P_3(x) = \left(\frac{2x}{0.4}-1\right)\left(2-\frac{x}{0.4}\right)^2(-0.785) + (x-0.4)\left(2-\frac{x}{0.4}\right)^2 8.719$

$+\left(5-\frac{2x}{0.4}\right)\left(\frac{x}{0.4}-1\right)^2(0.415) + (x-0.8)\left(\frac{x}{0.4}-1\right)^2 8.319$

$\therefore P_3(0.9) = \left(\frac{2\times0.9}{0.4}-1\right)\left(2-\frac{0.9}{0.4}\right)^2(-0.785) + (0.9-0.4)\left(2-\frac{0.9}{0.4}\right)^2 8.719$

$+\left(5-\frac{2\times0.9}{0.4}\right)\left(\frac{0.9}{0.4}-1\right)^2(-0.415) + (0.9-0.8)\left(\frac{0.9}{0.4}-1\right)^2 8.319$

$= -0.17171875 + 0.27246875 - 0.259375$

$- 0.32421875 + 1.29984375$

$= 1.076375$

b) $x_0 = 0.4 \quad f[x_0] = -0.785$

$x_1 = 0.8 \quad f[x_1] = -0.415$

$f[x_0, x_1] = \frac{-0.415+0.785}{0.8-0.4} = 0.925$

$P_1(x) = -0.785 + 0.925(x-0.4)$

$P_1(0.9) = -0.785 + 0.925(0.9-0.4) = -0.3225$

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x	f(x)	f'(x)
0.3	-0.785	8.719
0.6	-0.415	8.315

Answer the following based on the above data:
a. [6.5 Marks] Using Hermite Polynomial Interpolation find f(0.9)
b. [3.5 Marks] Using Newton's Divided Difference method find f(0.9)

(a) $h_3(x) = h_0(x) f(x_0) + h_1(x) f(x_1) + \hat{h}_0(x) f'(x_0) + \hat{h}_1(x) f'(x_1)$

$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - 0.6}{0.3 - 0.6} = -\frac{x}{0.3} + \frac{0.6}{0.3} = 2 - \frac{x}{0.3}$

$l_0'(x) = -\frac{1}{0.3}$ $\quad l_0'(x_0) = l_0'(0.3) = -\frac{1}{0.3}$

$h_0(x) = \{1 - 2(x - x_0) l_0'(x_0)\}\{l_0(x)\}^2$

$= \{1 - 2(x - 0.3)(-\frac{1}{0.3})\} \left(2 - \frac{x}{0.3}\right)^2$

$= \left(1 + \frac{2x}{0.3} - 2\right)\left(2 - \frac{x}{0.3}\right)^2 = \left(\frac{2x}{0.3} - 1\right)\left(2 - \frac{x}{0.3}\right)^2$

$\hat{h}_0(x) = (x - x_0)\{l_0(x)\}^2 = (x - 0.3)\left(2 - \frac{x}{0.3}\right)^2$

~~$h_1(x) = 1 - 0$~~

$$l_1(x) = \frac{x - x_1}{x_1 - x_1} = \frac{x - 0.3}{0.6 - 0.3} = \frac{x}{0.3} - 1$$

$$l_1'(x) = \frac{1}{0.3} \qquad l_1'(x_1) = l_1'(0.6) = \frac{1}{0.3}$$

$$h_1(x) = \{1 - 2(x - x_1) l_1'(x_1)\}\{l_1(x)\}^2$$

$$= \left\{1 - 2(x - 0.6)\frac{1}{0.3}\right\}\left(\frac{x}{0.3} - 1\right)^2$$

$$= \left(1 - \frac{2x}{0.3} + 4\right)\left(\frac{x}{0.3} - 1\right)^2 = \left(5 - \frac{2x}{0.3}\right)\left(\frac{x}{0.3} - 1\right)^2$$

$$\hat{h}_1(x) = (x - x_1)\{l_1(x)\}^2 = (x - 0.6)\left(\frac{x}{0.3} - 1\right)^2$$

$$\therefore P_3(x) = (x - 0.3)\left(2 - \frac{x}{0.3}\right)^2 (8.719) + (x - 0.6)\left(\frac{x}{0.3} - 1\right)^2 (-0.415) + \left(\frac{2x}{0.3} - 1\right)\left(2 - \frac{x}{0.3}\right)^2 (-0.785) + \left(5 - \frac{2x}{0.3}\right)\left(\frac{x}{0.3} - 1\right)^2 (-0.415)$$

$$P_3(0.9) = (0.9 - 0.3)\left(2 - \frac{0.9}{0.3}\right)^2 (8.719) + (0.9 - 0.6)\left(\frac{x}{0.3} - 1\right)^2 8.315 + \left(\frac{2 \times 0.9}{0.3} - 1\right)\left(2 - \frac{0.9}{0.3}\right)^2 (-0.785) + \left(5 - \frac{2 \times 0.9}{0.3}\right)\left(\frac{0.9}{0.3} - 1\right)^2 (-0.415)$$

$$= 0.6 \times (2 - 3)^2 \times 8.719 + 0.3 \times (3 - 1)^2 \times 8.315 + 5 \times (2 - 3)^2 (-0.785) + -1 \times (3 - 1)^2 \times -0.415$$

$$= 5.2314 + 9.978 - 3.925 + 1.66 = 12.9444$$

b) $x_0 = 0.3 \quad f[x_0] = 0.785$

$x_1 = 0.6 \quad f[x_1] = -0.415$

$$f[x_0, x_1] = \frac{-0.415 + 0.785}{0.6 - 0.3} = 1.23333$$

$$P_1(x) = -0.785 + 1.23333(x - 0.3)$$

$$P_1(0.9) = -0.785 + 1.23333(0.9 - 0.3) = -0.045$$