Assignment 03

1.a) To obtail a degree 3 polynomial for the same function with the given nodes we need to use hermite interpolation.

b) Give $f(x) = x \ln(x)$

x	$f(x)$	$f'(x)$
1	0	1
3	3.296	2.099

$$P_3(x) = h_0(x) f(x_0) + \hat{h}_0(x) f'(x_0) + h_1(x) f(x_1) + \hat{h}_1(x) f'(x_1)$$

$$= 0 + \hat{h}_0(x) + 3.296\, h_1(x) + 2.099\, \hat{h}_1(x)$$

$$\hat{h}_0(x) = (x - x_0)\{l_0(x)\}^2$$

$$= (x-1)\left\{-\tfrac{1}{2}(x-3)\right\}^2$$

$$= \frac{(x-1)(x-3)^2}{4}$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$$= \frac{x-3}{1-3}$$

$$= -\frac{1}{2}(x-3)$$

$$h_1(x) = \left\{1 - 2(x-3) \times \frac{1}{2}\right\} \left(\frac{x-1}{2}\right)^2$$

$$= (1 - x + 3) \frac{(x-1)^2}{4}$$

$$= \frac{(x-1)^2 (4-x)}{4}$$

$$l_1(x) = \frac{x-1}{3-1} = \frac{x-1}{2}$$

$$l_1'(x) = \frac{1}{2}$$

$$l_1'(x_1) = \tfrac{1}{2}$$

$$\hat{h}_1(x) = (x-3)\left(\frac{x-1}{2}\right)^2$$

$$= \frac{(x-3)(x-1)^2}{4}$$

c) $P_3(x) = \frac{(x-1)(x-3)^2}{4} + 3.296\,\frac{(x-1)^2(4-x)}{4} + 2.099 \times \frac{(x-3)(x}{4}$

2)

$$\theta_j = \frac{(2j+1)\pi}{2(n+1)}$$

$\theta_0 = \pi/10$ $\quad x_0 = 5\cos \pi/10$

$\theta_1 = 3\pi/10$ $\quad x_1 = 5\cos 3\pi/10$

$\theta_2 = 5\pi/10$ $\quad x_2 = 5\cos 5\pi/10$

$\theta_3 = 7\pi/10$ $\quad x_3 = 5\cos 7\pi/10$

$\theta_4 = 9\pi/10$ $\quad x_4 = 5\cos 9\pi/10$

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3a) forward difference:

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{f(1+0.1) - f(1)}{0.1}$$

$$= \frac{1.1 \ln(1.1) - 1 \ln(1)}{0.1}$$

$$= 1.04841 1978$$

b) For backward difference,

$$f(x) = x \ln(x)$$

$$f'(x) = \ln x + 1$$

$$f''(x) = 1/x$$

$$\text{Interval} = [x-h, x] = [1-0.1, 1] = [0.9, 1]$$

$$f''(0.9) = 1.1111$$

$$f''(1) = 1$$

Upper bound of the truncation error $: \left| \frac{f''(\xi)}{2} h \right|$

$$= \frac{1.1111}{2!} \times 0.1$$

$$= 0.055555$$

For Central difference.

$f'''(x) = -1/x^2$ and $x \in [0.9, 1.1]$

$f'''(0.9) = -1.2346$ and $f'''(1.1) = -0.82645$.

$\therefore$ upper bound $= \frac{f'''(\xi)}{3!} h^2 = |-1.2346 \times (0.1)^2| / 6 = 2.06 \times 10^{-3}$