CSE428: Image Processing

Lecture 5

Neighbourhood processing: Part 1

Contents

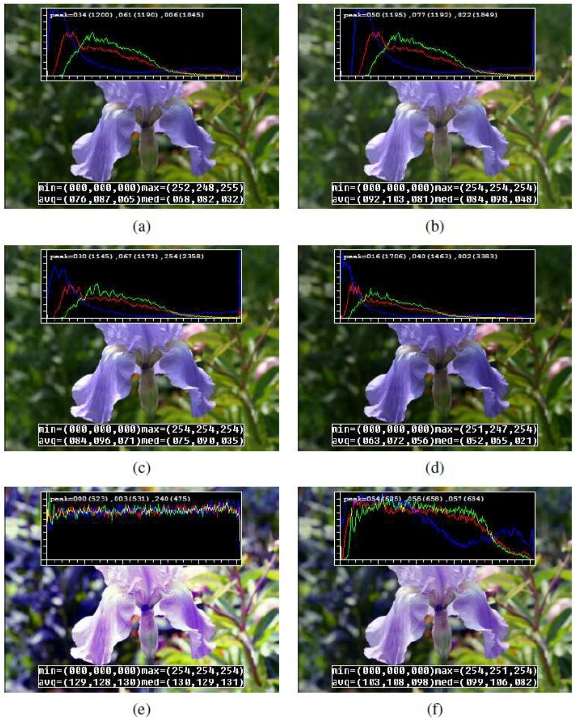
Spatial Operations

Can be subdivided into 3 broad categories:

1. Single-pixel operations or **point processing** (covered in the last lecture)
2. Neighborhood operations or **neighborhood processing**
3. Geometric spatial **transformations** (won’t go into too much detail)

Single-pixel operations

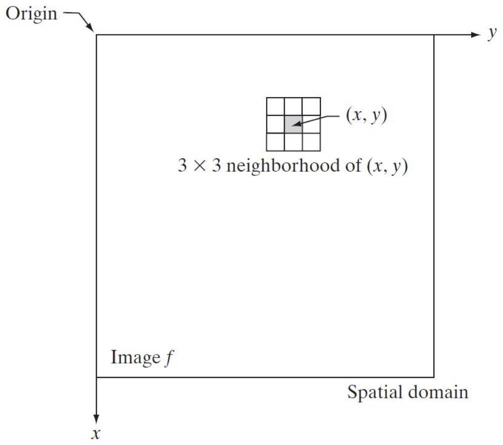
- Simplest possible operation on an digital image: alter the pixel intensities
- If *T* is a transfer function, *s* = *T(z)* or
  - z -> original image pixel intensity s -> mapped intensity
- Some local image processing operations:
  - (a) original image along with its three color histograms;
  - (b) brightness increased (additive offset, b = 16);
  - (c) contrast increased (multiplicative gain, a = 1:1);
  - (d) gamma (partially) linearized (γ = 1:2);
  - (e) full histogram equalization;
  - (f) partial histogram equalization



Computer Vision: Algorithms and Applications, 2nd Edition, Richard Szeliski

Neighborhood

- A pixel p at coordinates has four horizontal and vertical neighbors whose coordinates are given by
  - (x + 1, y), (x - 1, y), (x, y + 1), (x, y - 1)
  - This set of pixels, called the 4-neighbors of p, $N_4(p)$
- The four diagonal neighbors of p
  - (x + 1, y + 1), (x + 1, y - 1), (x - 1, y + 1), (x - 1, y - 1)
  - Diagonal neighbors of p, $N_D(p)$
- 4-neighbors together with the diagonal neighbors are called the 8-neighbors of p: $N_8(p)$



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Distance measures

- For pixels p, q, and z, with coordinates (x, y), (s, t), and (v, w)
- Euclidean distance between p and q
  - $D_e(p, q) = [(x - s)^2 + (y - t)^2]^{½}$
- D_4 distance (city-block distance) between p and q
  - $D_4(p, q) = |x - s| + |y - t|$
  - The pixels with $D_4 = 1$ are the 4-neighbors of (x, y)
- D_8 distance (chessboard distance) between p and q
  - $D_8(p, q) = \max(|x - s|, |y - t|)$
  - The pixels with $D_8 = 1$ are the 8-neighbors of (x, y)

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

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Linear vs Nonlinear Operations

Consider a general operator, *H* such that : $\boldsymbol{H}\,[\boldsymbol{f}(\mathbf{x, y})] = \boldsymbol{g}(\mathbf{x, y})$

H is said to be a linear operator if the following is true:

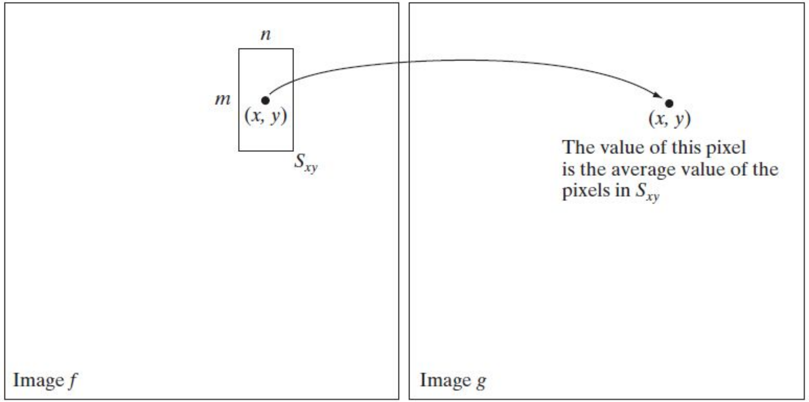
$$\boldsymbol{H}\,[a_1 f_1(x, y) + a_2 f_2(x, y)] = a_1 H\,[f_1(x, y)] + a_2 H\,[f_2(x, y)]$$

1. *Additivity* property: output of a linear operation due to the sum of two inputs is the same as performing the operation on the inputs individually and then summing the results
2. *Homogeneity* property: the output of a linear operation to a constant times an input is the same as the output of the operation due to the original input multiplied by that constant

Neighborhood operations

S_{xy} is the set of coordinates of a neighborhood centered on an arbitrary point (x, y) in an image f

- *Neighborhood processing* generates a corresponding pixel at the same coordinates in an output (processed) image, g, such that the value of that pixel is determined by a specified operation involving the pixels in the input image with coordinates in S_{xy}



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Neighborhood operations: Mechanism

Spatial filtering consists of

1. A **neighborhood** (defined by the filter kernel, or mask)
2. A **predefined operation** (which is performed on the neighborhood)
   - Linear operation
   - Nonlinear operation

Filtering creates a new pixel at the same position of the center of the neighborhood, and whose value is the result of the filtering operation

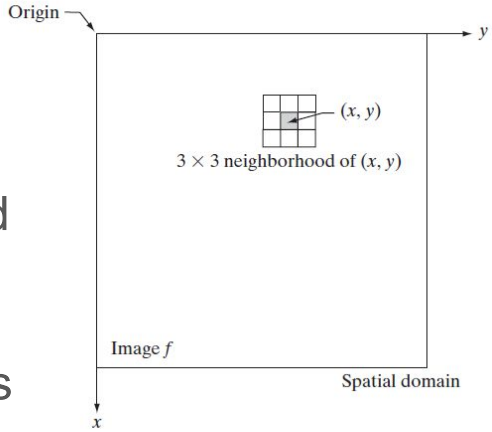
A processed (filtered) image is generated as the center of the filter visits each pixel in the input image

Spatial Filtering

Neighborhood processing is also called ***spatial filtering***

- $g(x, y) = T\,[f(x, y)]$
- Input image: $f(x, y)$
- Output image: $g(x, y)$
- *T* [.] is an operator in f, defined over a neighborhood
    - Linear operation -> linear spatial filtering
    - Nonlinear operation -> nonlinear spatial filtering
- *T* can be applied to a single image or multiple images

Example: a 3x3 neighborhood of (x, y)



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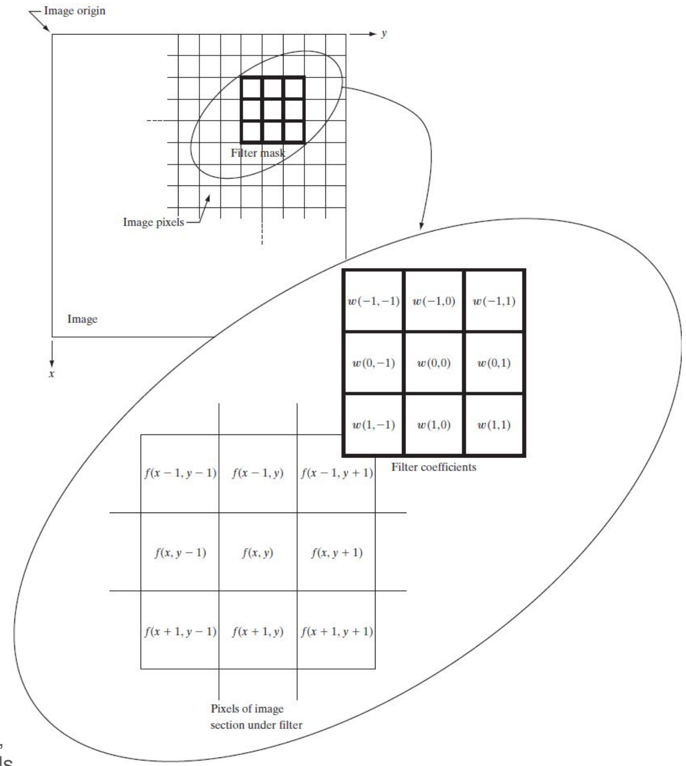
Linear Spatial Filtering

Linear spatial filtering in a m x n neighborhood

- Mask/filter/kernel size: m x n
  - Assume m = 2a + 1, n = 2b + 1 (both odd numbers)
- Image size: M x N

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Example: *g*(x, y) = *w*(-1, -1) *f*(x-1, y-1) + w(-1, 0) *f*(x -1, y) + … + *w*(0, 0) *f*(x, y) + ... + *w*(1, 1) *f*(x+1, y+1)



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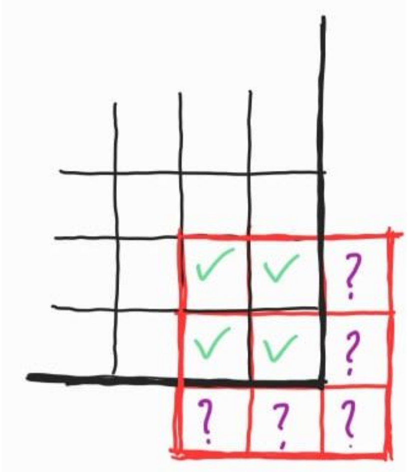
But.. what about the border pixels?



Solution 1: Don’t visit the border pixels!
Result: Image is ***shrinked***

Solution 2: *Padding*

Result: Image shape *stays the* ***same***

Image Padding

1. **Zero** padding: extend the borders of the original image with zeros
2. **Constant** padding: extend the borders of the original image with a constant pixel value
3. **Mirror** padding: extend the borders of the original image by reflecting the edge pixels
4. **Clamp/Edge/Nearest** padding: extend the borders of the original image by repeating edge pixels

More ways: https://scikit-image.org/docs/dev/api/skimage.util.html#skimage.util.pad

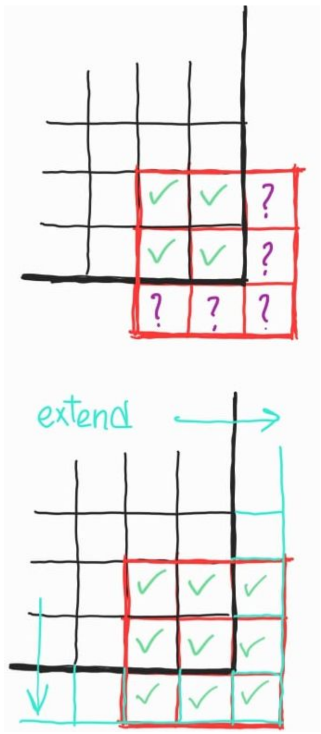
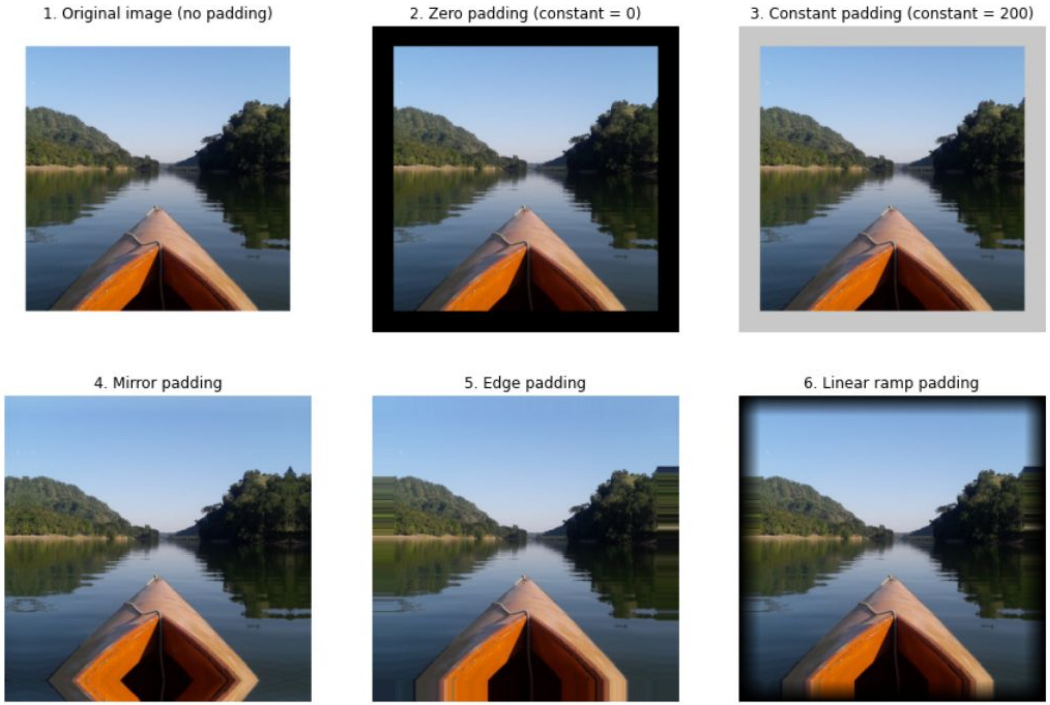


Image Padding

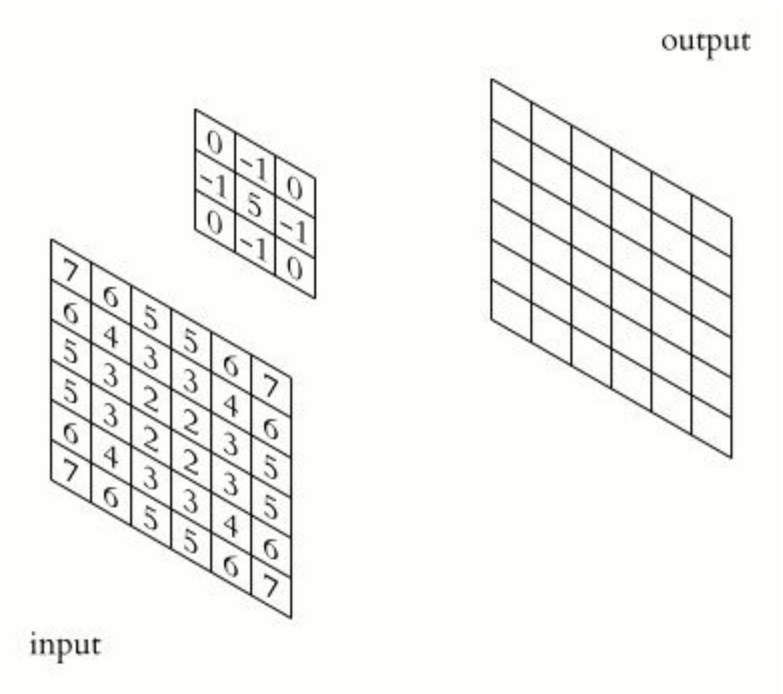
A 1448x1448x3 image padded with 200 pixels on each side:

1. Original image
2. Zero padding
3. Constant padding
4. Mirror padding
5. Edge padding
6. Linear(ramp) padding



Linear Spatial Filtering: Demo

- Kernel shape: 3x3
- Input image shape: 6x6
- Output image shape: 6x6
- Padding: Nearest
- Step (stride): 1
- Kernel, $w(x, y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$



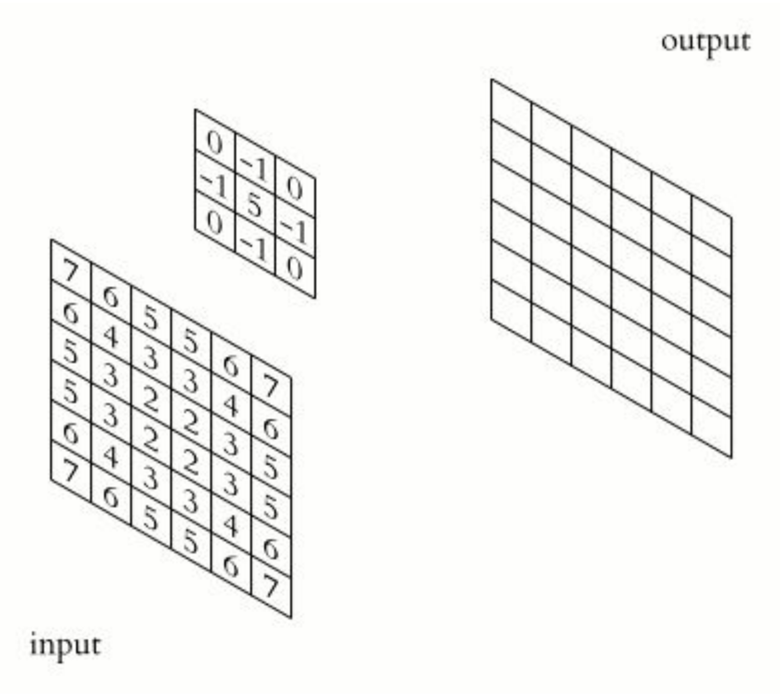
https://commons.wikimedia.org/wiki/File:2D_Convolution_Animation.gif

Linear Spatial Filtering: Output Shape

- Kernel shape: **m** x **n**
- Input image shape: **M** x **N**
- Number of pixels padded: **p**
- Stride: **s**
- Output image shape: **H** x **W**

$$H = floor\left(\frac{M + 2p - m}{s} + 1\right)$$

$$W = floor\left(\frac{N + 2p - n}{s} + 1\right)$$



https://commons.wikimedia.org/wiki/File:2D_Convolution_Animation.gif

Correlation

Correlation between two 2D signals *w*(x, y) & *f*(x, y) is defined as:

$$w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x+s, y+t)$$

- Outputs a ***measure of similarity*** of the two signals as a function of the displacement of one relative to the other
- Also called the cross-correlation
- Basically a sliding dot product (vector element wise multiplication)
- Used for searching a long signal [*f*(x, y)] for a shorter signal [*w*(x, y)] (a known feature)

Linear spatial filtering $\Leftrightarrow$ 2D signal correlation

Correlation to Convolution

- Linear spatial filtering is basically a 2D signal ***correlation*** operation
- If we rotate the original filter *w*(x, y) 180^0 and then proceed, it's called the ***convolution*** operation
- Mathematically,

$$Correlation: \; w(x,y) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

$$Convolution: \; w(x,y) \bigstar f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

For symmetric filters,
convolution and correlation both yield the same result!

Smoothing Spatial Filters

- Linear
  - Box/average filtering
  - Gaussian filtering
- Non-linear
  - Median filtering

Example: A 3 x 3 filter with coefficients w_1 through w_9

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

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Linear Smoothing Filters

1. Averaging filter
   a. Average of all the pixels encompassed by the filter kernel
   b. An m x n averaging *box kernel*: *w*(x, y) = 1 / m*n
2. Gaussian filter
   a. *Weighted* average of all the pixels encompassed by the gaussian filter kernel
   b. An m x n *gaussian kernel*: *w*(x, y) = exp(-(x^2+y^2)/2σ^2)

Example: a 3x3 Box filter and a 3x3 Gaussian filter

- (note that the filters are <u>normalized</u> so that the intensity of the output pixel remains unchanged)

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

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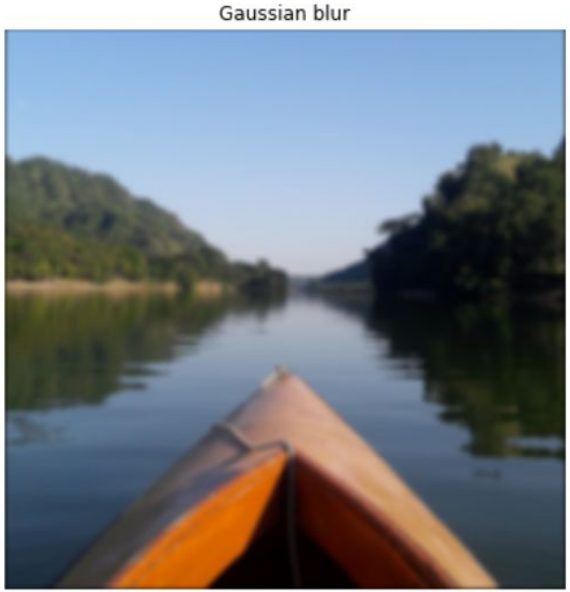
Linear Smoothing Filters

Applications

- Image **Blurring** (As the name suggests)
- Image **Sharpening** (wait, sharpening from blurring? what?)
- Image **Denoising**
- **Low pass filtering** (getting rid of high variations)

Gaussian Filtering:The Gaussian Blur

Result of filtering an image with with (25 x 25), $\sigma^2 = 8$ Gaussian kernel



“gaussian blur” memes

Gaussian Filtering:The Gaussian Blur

Increasing the kernel shape increases blurriness, borders become more prominent

(5 x 5)

(55 x 55)

(205 x 205)

Gaussian Filtering vs Average Filtering

For the same kernel size the gaussian blur *preserves more detail* of the image

Even though the image is blurred the edges are somewhat better preserved for the gaussian blurring (edge preserving blurring)

Order Statistic Filtering

Response is based on *ordering* (ranking) the pixels contained in the pixel neighborhood (S_{xy})

- **Median** filtering: $$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s, t)\}$$
- **Max** and **min** filtering: $$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}}\{g(s, t)\} \qquad \hat{f}(x, y) = \min_{(s,t)\in S_{xy}}\{g(s, t)\}$$
- **Midpoint** filtering: $$\hat{f}(x, y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s, t)\} + \min_{(s,t)\in S_{xy}}\{g(s, t)\}\right]$$
- **Alpha-trimmed** mean filtering: $$\hat{f}(x, y) = \frac{1}{mn - d}\sum_{(s,t)\in S_{xy}} g_r(s, t)$$

Median Filtering

Median Filter (Order-statistic filter)

- nonlinear spatial filter
- each pixel value is replaced by the median of the intensity values in the neighborhood
- excellent noise-reduction capabilities
- less blurring than linear filters
- particularly effective with impulse noise (salt and pepper noise)

Noise-corrupted Image

< Original image (left)

Noisy image > (right) (Gaussian noise)

Noise Reduction Using Linear Filtering

< Original grayscale image (left)

Noisy image **>** (right) (Salt and pepper noise, pretty common for old b&w cameras)

Noise Reduction Using Linear Filtering

Gaussian filter used to denoise both noisy images (25 x 25) σ^2=3

Output is blurred, s&p not properly removed

Noise Reduction Using Nonlinear Filtering

Median filter is used to denoise the salt and pepper noise

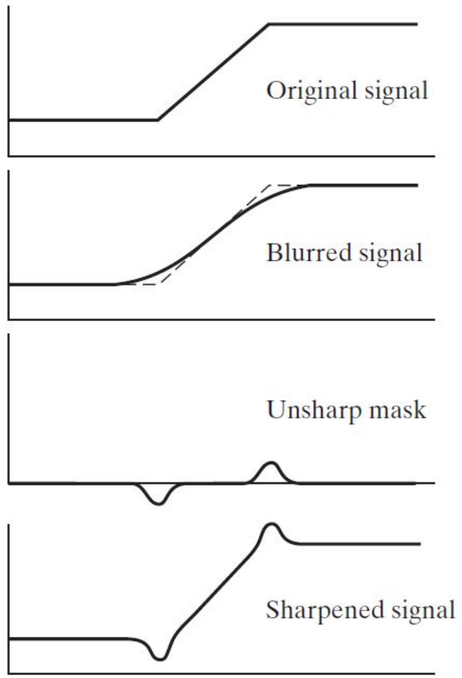
(can you tell the original from the filtered one?)

Unsharp Masking & High Boost Filtering

This unsharp masking process consists of the following steps:

1. Blur the original image
   a. $f'(x, y) = \text{Blur}[f(x, y)]$
2. Subtract the blurred image from the original (the resulting difference is called the mask)
   a. $g_{mask}(x, y) = f(x, y) - f'(x, y)$
3. Add the mask to the original
   a. $g(x, y) = f(x, y) + \boldsymbol{k} * g_{mask}(x, y)$

***k* = 1** is *unsharp masking*, ***k* > 1** is *high boost filtering*



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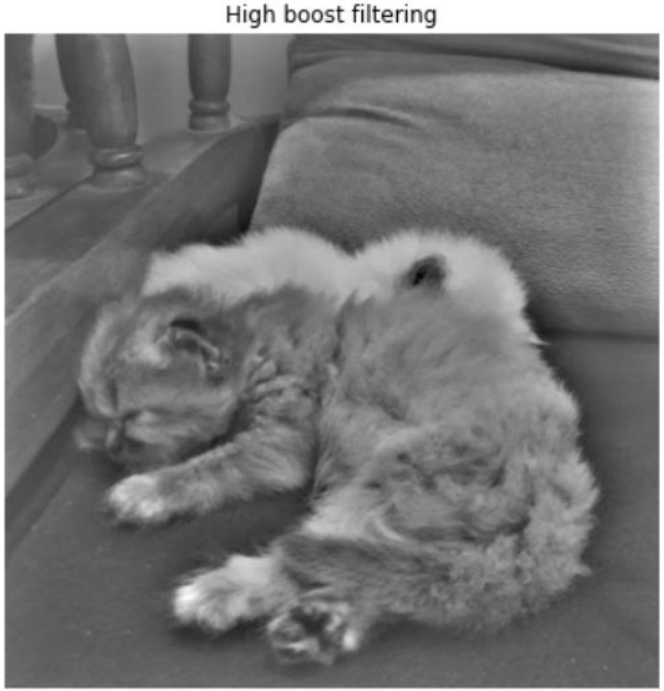
Unsharp Masking

High Boost Filtering



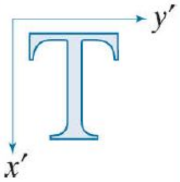
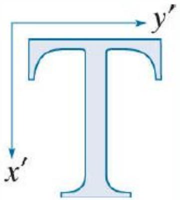
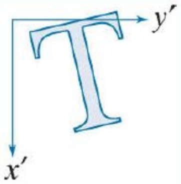
Geometric Transformations

- We use geometric transformations modify the spatial arrangement of pixels in an image
- These transformations are called rubber-sheet transformations because they may be viewed as analogous to “printing” an image on a rubber sheet, then stretching or shrinking the sheet according to a predefined set of rules

Geometric Transformations

- (x, y) are pixel coordinates in the original image
- (x’, y’) are the corresponding pixel coordinates of the transformed image
- Our interest is in so-called affine transformations, which include scaling, translation, rotation, and shearing
- The key characteristic of an affine transformation in 2-D is that it preserves points, straight lines, and planes
- $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Geometric Transformations

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
Scaling/Reflection (For reflection, set one scaling factor to -1 and the other to 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
Rotation (about the origin)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	

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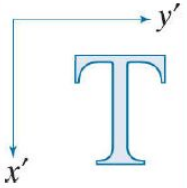
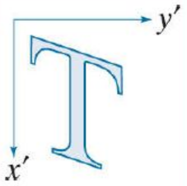
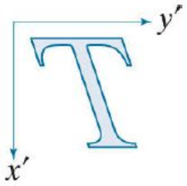
Geometric Transformations

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y' = y$	
Shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$	

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Tutorial

Basic python implementation of some of the algorithms taught in class can be found in the following google colab notebook: https://colab.research.google.com/drive/1JF3ww3l-bBzoC0iy9Ado7eQLcxC1hVFU?usp=sharing

Thank you!