

Global Variables

```
In[1]:= prec = 100;

Nmax = 100;

x1 = 1.0;
x2 = 10.0;
x3 = 20.0;

xmax = 20.0;
xmin = -20.0;
xsize = 100;
dx = (xmax - xmin) / (xsize - 1);

Xvector = N[Range[xmin, xmax, dx], prec];
      ··· [intervalo de valores]
Nvector = Range[0, Nmax, 1];
      [intervalo de valores]
```

Wavefuntion_Wolfram_Mathematica_1

```
In[12]:= WavefunctionMathematica1[n_, x_, prec_] :=
Module[{nPrec, xPrec, norm, H, wavefunction},
  [módulo de código]
  SetPrecision[n, prec];
  [define precisão]
  SetPrecision[x, prec];
  [define precisão]
  norm = (2^(-0.5 * n)) * (Gamma[n + 1]^(-0.5)) * (Pi^(-0.25));
          [função gama de Euler]          [número pi]
  H = HermiteH[n, x];
      [polinômios de Hermite]
  wavefunction = SetPrecision[norm * Exp[-0.5 * x^2] * H, prec];
                  [define precisão]          [exponencial]
  wavefunction];
```

Wavefuntion_Wolfram_Mathematica_2

```

In[13]:= WavefunctionMathematica2[n_, x_, prec_] :=
Module[{wavefunction, xsize, i}, SetPrecision[x, prec];
  __módulo de código __define precisão
  wavefunction = Table[SetPrecision[0, prec], {n + 1}, {Length[x]}];
  __tabela __define precisão __comprimento
  wavefunction[[1]] = SetPrecision[Pi^(-1/4) Exp[-(x^2)/2], prec];
  __define precisão __número pi __exponencial
  wavefunction[[2]] = SetPrecision[(2 x wavefunction[[1]]) / Sqrt[2], prec];
  __define precisão __raiz quadrada
  For[i = 3, i ≤ n + 1, i++, wavefunction[[i]] = 2 x (wavefunction[[i - 1]] / Sqrt[2 (i - 1)]) -
    __para cada __raiz quadrada
    Sqrt[(i - 2) / (i - 1)] wavefunction[[i - 2]];];
  __raiz quadrada
  wavefunction[[n + 1]];

```

Wavefuncton_Wolfram_Mathematica_3

```

In[14]:= WavefunctionMathematica3[n_, x_, prec_] :=
Module[{wavefunction, i}, SetPrecision[x, prec];
  __módulo de código __define precisão
  wavefunction = Table[SetPrecision[0, prec], {n + 1}, {Length[x]}];
  __tabela __define precisão __comprimento
  wavefunction[[1]] = SetPrecision[Pi^(-1/4) Exp[-(x^2)/2], prec];
  __define precisão __número pi __exponencial
  For[i = 1, i ≤ n, i++,
    __para cada
    wavefunction[[i + 1]] =
      SetPrecision[2 x (wavefunction[[i]] / Sqrt[2 (i)]) -
        __define precisão __raiz quadrada
        Sqrt[(i - 1) / i] wavefunction[[i - 1]], prec];
    __raiz quadrada
  ]
  wavefunction];

```

Tests

★ Single-Mode and Onedimensional Function with $x = 1.0$

```

In[72]:= FastWaveSMOdN100x1 = SetPrecision[Normal[ExternalEvaluate["Python",
    [define precisão [normal [execução externa
    "from fast_wave.wavefunction import wavefunction_smod;import numpy
    as np;N_max = 100;x1 = 1.0;np.array([wavefunction_smod(n,x1)
    for n in range(N_max+1)])];"], prec];

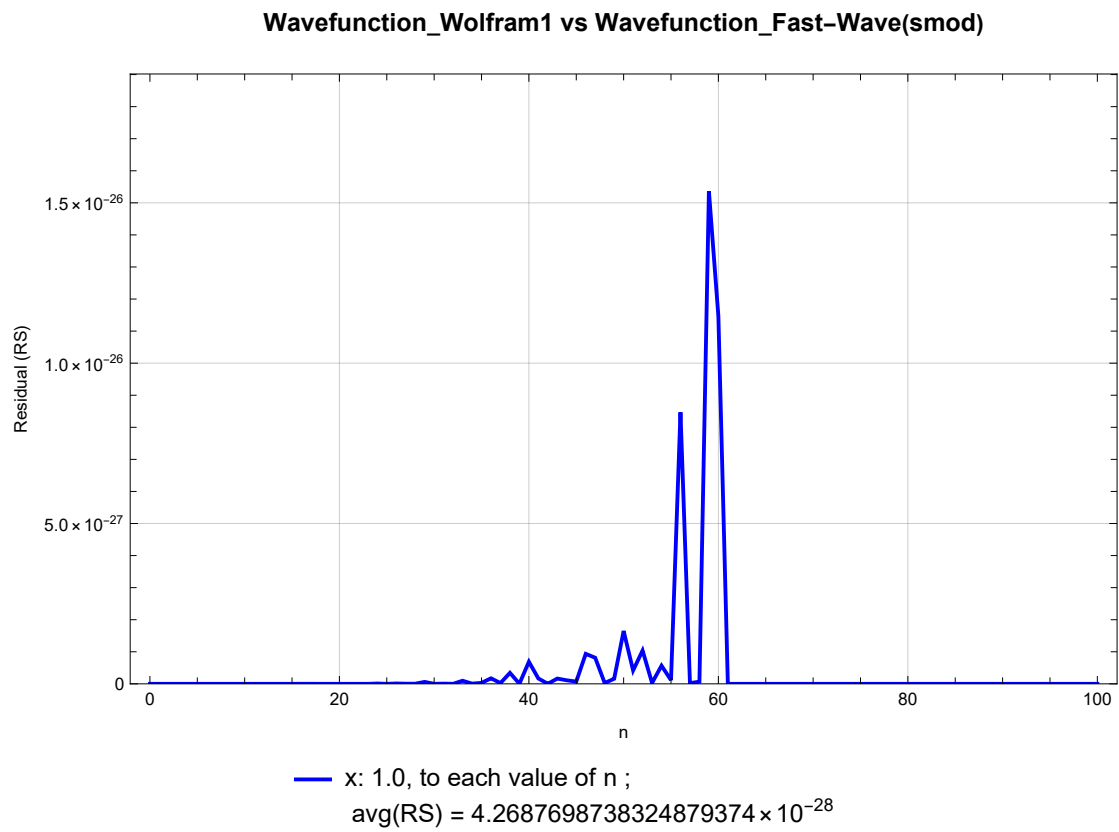
(*WolframSMOdN100x1 = WavefunctionMathematica3[Nmax,x1,prec];*)
WolframSMOdN100x1 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    [para cada
    AppendTo[WolframSMOdN100x1, WavefunctionMathematica1[i - 1, x1, prec]]];
    [adiciona a

Residual = (WolframSMOdN100x1 - FastWaveSMOdN100x1) ^ 2;

ListLinePlot[
    [gráfico de linha de uma lista de valores
    Transpose[{Nvector, Residual}],
    [transposição
    PlotStyle → {Thick, Blue},
    [estilo do gráfico [espessura [azul
    Frame → True,
    [quadro [verdadeiro
    FrameLabel → {"n", "Residual (RS)"},
    [legenda do quadro
    PlotLabel →
    [etiqueta de gráfico
    Style["Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod)\n", 13, Bold],
    [estilo [negrito
    GridLines → Automatic,
    [grade de linhas [automático
    PlotLegends → Placed[{"x: 1.0, to each value of n ; \n avg(RS) = " <>
    [legenda do gráfico [situado
    ToString[SetPrecision[Mean[Residual], 20],
    [converte... [define precisão [média
    TraditionalForm]], Below],
    [forma tradicional [abaixo
    ImageSize → Large,
    [tamanho da ... [grande
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-25.8)}}
    [intervalo do gráfico... [automático [automático
    ]
    Functionality Test Passed: True

```

Out[76]=



★ Single-Mode and Onedimensional Less Fast Function with $x = 1.0$

```

In[*]:= FastWaveSMOdN100x1 = SetPrecision[Normal[ExternalEvaluate["Python",
    [define precisão [normal [execução externa
    "from fast_wave.wavefunction import wavefunction_smod;import numpy as np;N_max
      = 100;x1 = 1.0;np.array([wavefunction_smod(n,x1,more_fast=False)
                                [falso
      for n in range(N_max+1)])];"], prec];

(*WolframSMOdN100x1 = WavefunctionMathematica3[Nmax,x1,prec];*)
WolframSMOdN100x1 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
  [para cada
    AppendTo[WolframSMOdN100x1, WavefunctionMathematica1[i - 1, x1, prec]]];]
  [adiciona a

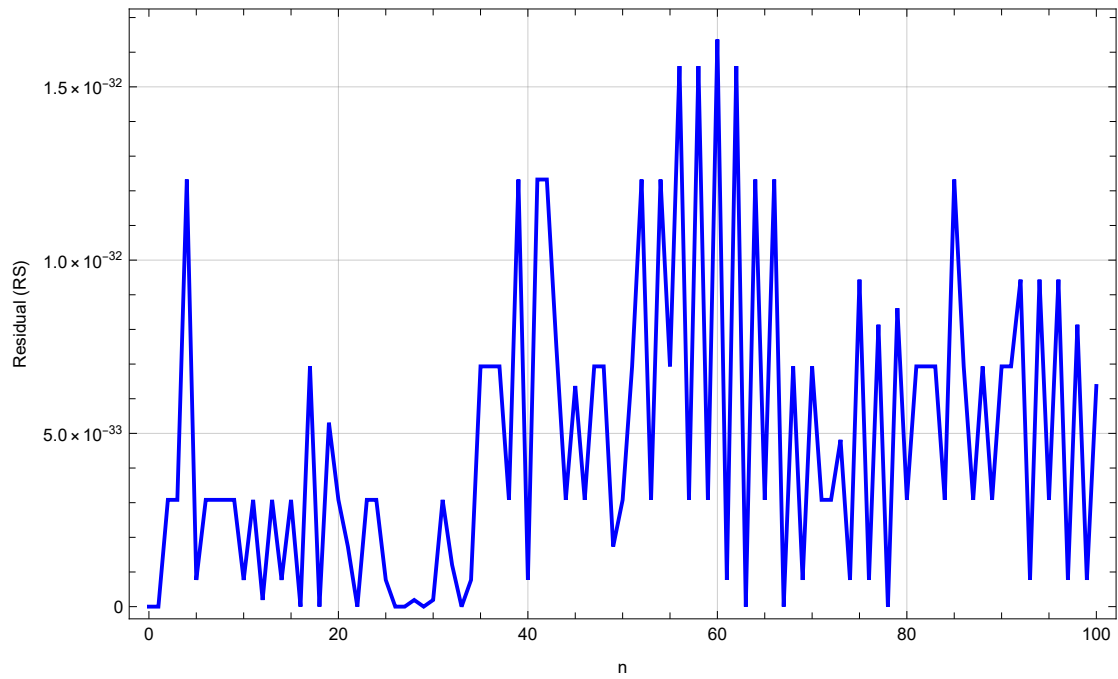
Residual = (WolframSMOdN100x1 - FastWaveSMOdN100x1) ^2;

ListLinePlot[
  [gráfico de linha de uma lista de valores
    Transpose[{Nvector, Residual}],
    [transposição
    PlotStyle → {Thick, Blue},
    [estilo do gráfico [espessc [azul
    Frame → True,
    [quadro [verdadeiro
    FrameLabel → {"n", "Residual (RS)"},
    [legenda do quadro
    PlotLabel → Style[
    [etiqueta de gr... [estilo
      "Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod; less_fast)\n", 13, Bold],
      [negrito

    GridLines → Automatic,
    [grade de linhas [automático
    PlotLegends → Placed[{"x: 1.0, to each value of n ; \n avg(RS) = " <>
    [legenda do gráfico [situado
      ToString[SetPrecision[Mean[Residual], 20],
      [converte... [define precisão [média
      TraditionalForm]], Below],
      [forma tradicional [abaixo
    ImageSize → Large
    [tamanho da ... [grande
  ]

Functionality Test Passed: True

```

Out[*n*]=**Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod; less_fast)**

— x: 1.0, to each value of n ;
 $\text{avg}(\text{RS}) = 4.8171283643533841588 \times 10^{-33}$

★ Single-Mode and Onedimensional Function with $x = 10.0$

In[102]:=

```

FastWaveSMOdN100x2 = SetPrecision[Normal[ExternalEvaluate["Python",
    "from fast_wave.wavefunction import wavefunction_smod;import numpy as
    np;N_max = 100;x2 = 10.0;np.array([wavefunction_smod(n,x2)
    for n in range(N_max+1)])];"], prec];

(*WolframSMOdN100x2 = WavefunctionMathematica3[Nmax,x2,prec];*)

WolframSMOdN100x2 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WolframSMOdN100x2, WavefunctionMathematica1[i - 1, x2, prec]];]

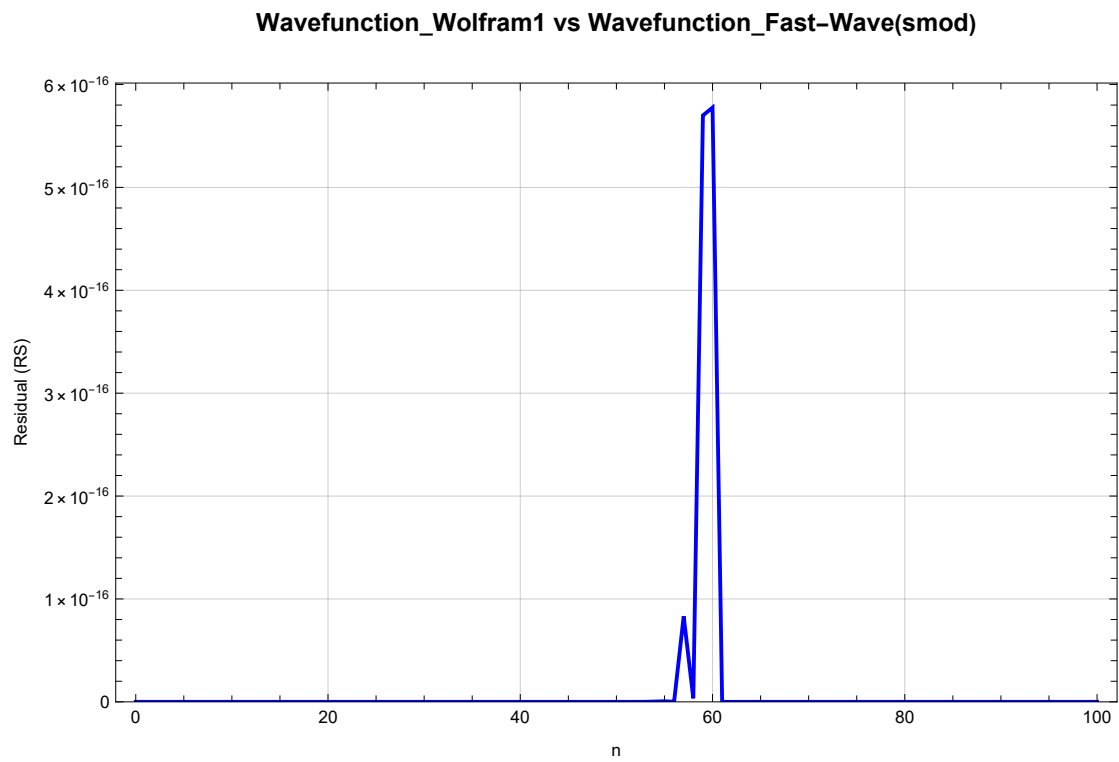
Residual = (WolframSMOdN100x2 - FastWaveSMOdN100x2) ^ 2;

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
        Style["Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{"x: 10.0, to each value of n ; \n avg(RS) = [" <>
        ToString[SetPrecision[Mean[Residual], 20],
        TraditionalForm]} <> "]", Below],
    ImageSize → Large,
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-15.3)}}]

```

Functionality Test Passed: True

Out[106]=



x: 10.0, to each value of n ;

avg(RS) = $[1.2227826509448020424 \times 10^{-17}]$

★ Single-Mode and Onedimensional Less Fast Function with x = 10.0

```

In[*]:= FastWaveSMOdN100x2 = SetPrecision[Normal[ExternalEvaluate["Python",
    "from fast_wave.wavefunction import wavefunction_smod;import numpy as np;N_max
    = 100;x2 = 10.0;np.array([wavefunction_smod(n,x2,more_fast=False)
    for n in range(N_max+1)])];"], prec];

(*WolframSMOdN100x2 = WavefunctionMathematica3[Nmax,x2,prec];*)

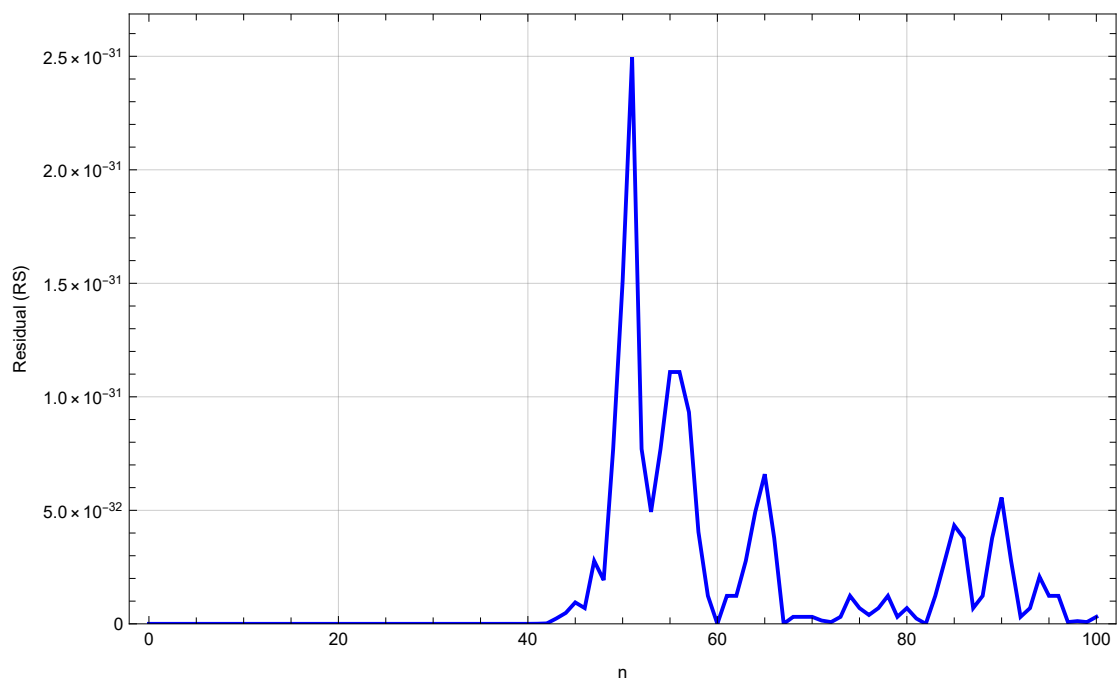
WolframSMOdN100x2 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WolframSMOdN100x2, WavefunctionMathematica1[i - 1, x2, prec]];]

Residual = (WolframSMOdN100x2 - FastWaveSMOdN100x2) ^ 2;

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel → Style[
        "Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod; less_fast)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{" x: 10.0, to each value of n ; \n avg(RS) = [" <>
        ToString[SetPrecision[Mean[Residual], 20],
        TraditionalForm]] <> "]" , Below],
    ImageSize → Large ,
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-30.65)}}]

Functionality Test Passed: True

```

Out[\ast]=**Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod; less_fast)**

x: 10.0, to each value of n ;

avg(RS) = [$1.7001583826933030519 \times 10^{-32}$]

★ Single-Mode and Onedimensional Function with $x = 20.0$

In[107]:=

```

FastWaveSMOdN100x3 = SetPrecision[Normal[ExternalEvaluate["Python",
    "from fast_wave.wavefunction import wavefunction_smod;import numpy as
    np;N_max = 100;x3 = 20.0;np.array([wavefunction_smod(n,x3)
    for n in range(N_max+1)])];"], prec];

(*WolframSMOdN100x3 = WavefunctionMathematica3[Nmax,x3,prec];*)
WolframSMOdN100x3 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WolframSMOdN100x3, WavefunctionMathematica1[i - 1, x3, prec]];

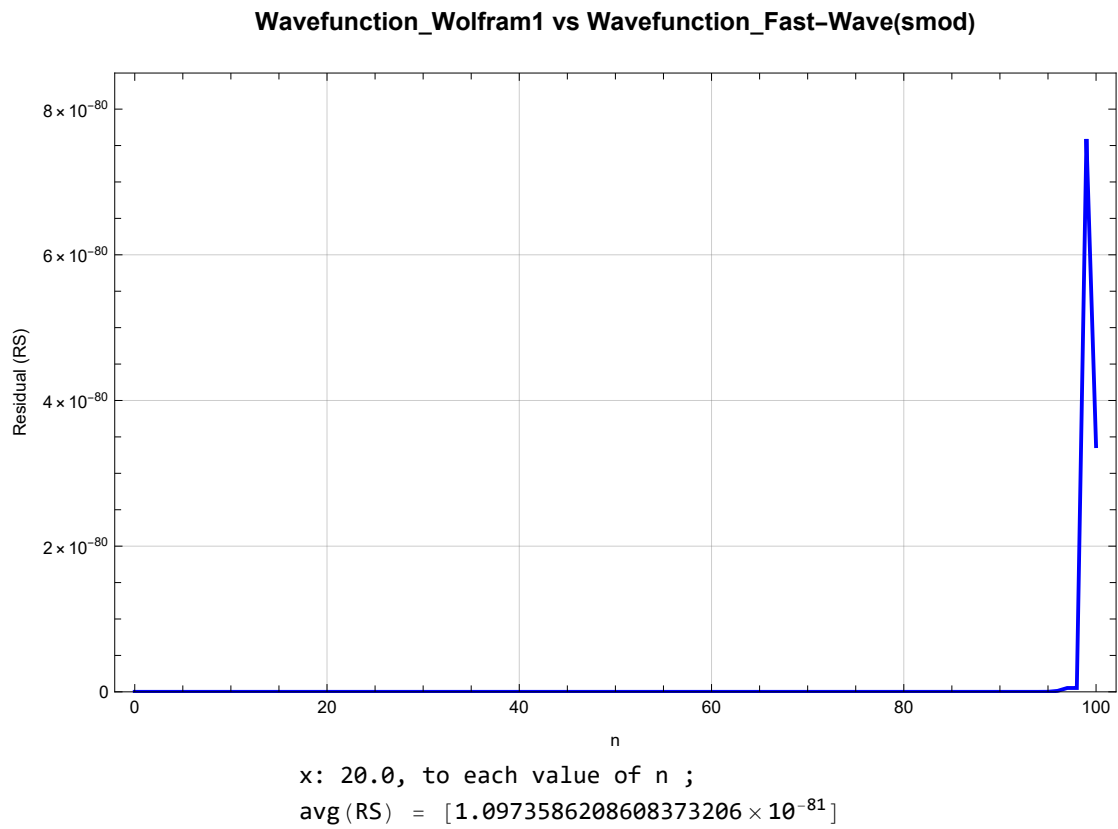
Residual = (WolframSMOdN100x3 - FastWaveSMOdN100x3)^2;

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
        Style["Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{"x: 20.0, to each value of n ; \n avg(RS) = [" <>
        ToString[SetPrecision[Mean[Residual], 20],
        TraditionalForm]] <> "]", Below],
    ImageSize → Large,
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-79.15)}}]

```

Functionality Test Passed: True

Out[111]=



★ Single-Mode and Onedimensional Less Fast Function with $x = 20.0$

```

In[*]:= FastWaveSMOdN100x3 = SetPrecision[Normal[ExternalEvaluate["Python",
    [define precisão [normal [execução externa
    "from fast_wave.wavefunction import wavefunction_smod;import numpy as
    np;N_max = 100;x3 = 20.0;np.array([wavefunction_smod(n,x3,
    more_fast= False) for n in range(N_max+1)])];"], prec];
    [falso

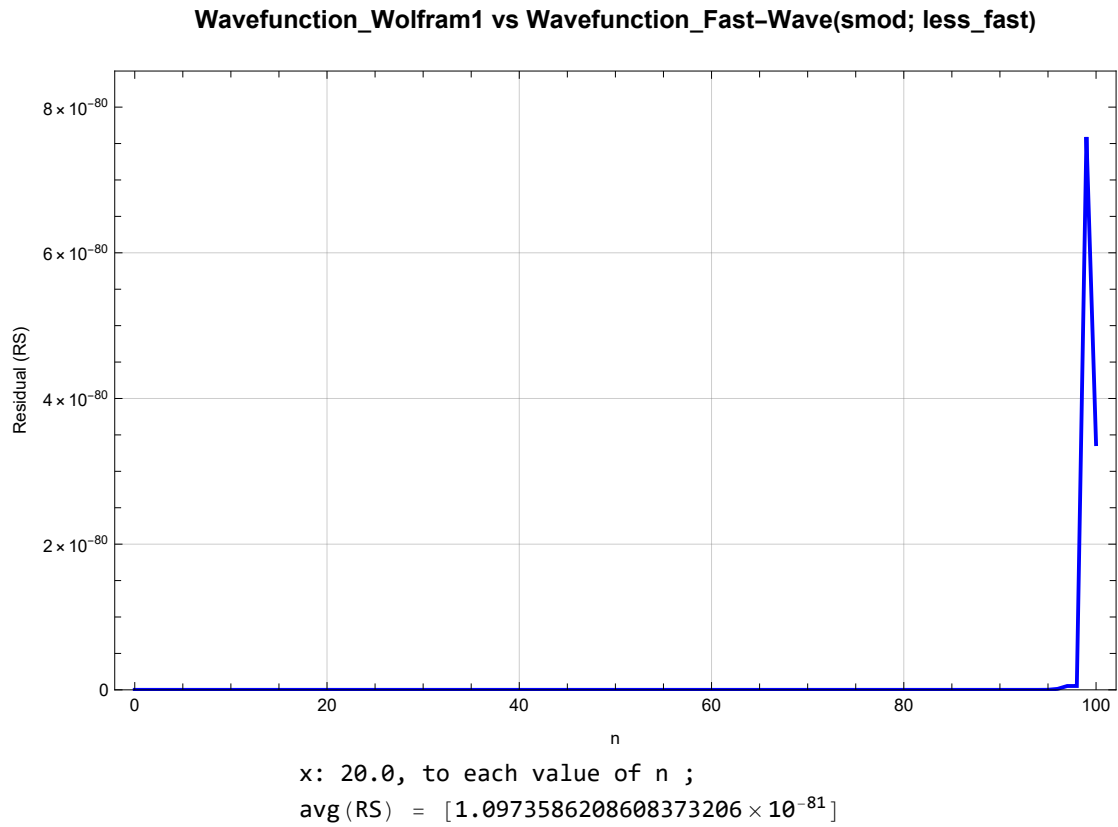
(*WolframSMOdN100x3 = WavefunctionMathematica3[Nmax,x3,prec];*)
WolframSMOdN100x3 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    [para cada
    AppendTo[WolframSMOdN100x3, WavefunctionMathematica1[i - 1, x3, prec]]];]
    [adiciona a

Residual = (WolframSMOdN100x3 - FastWaveSMOdN100x3) ^2;

ListLinePlot[
    [gráfico de linha de uma lista de valores
    Transpose[{Nvector, Residual}],
    [transposição
    PlotStyle → {Thick, Blue},
    [estilo do gráfico [espessura [azul
    Frame → True,
    [quadro [verdadeiro
    FrameLabel → {"n", "Residual (RS)"},
    [legenda do quadro
    PlotLabel → Style[
    [etiqueta de gr... [estilo
    "Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smod; less_fast)\n", 13, Bold],
    [negrito

    GridLines → Automatic,
    [grade de linhas [automático
    PlotLegends → Placed[{"x: 20.0, to each value of n ; \n avg(RS) = [" <>
    [legenda do gráfico [situado
    ToString[SetPrecision[Mean[Residual], 20],
    [converte... [define precisão [média
    TraditionalForm]] <> "]" , Below],
    [forma tradicional [abaixo
    ImageSize → Large ,
    [tamanho da ... [grande
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-79.15)}}
    [intervalo do gráf... [automático [automático
    ]
    Functionality Test Passed: True

```

Out[*n*]=

★ Single-Mode and Multidimensional Function with $X = [(-20) \rightarrow 20; 100]$

In[130]:=

```

FastWaveSMMdN100X = SetPrecision[
    Normal[ExternalEvaluate["Python", "from fast_wave.wavefunction
    import wavefunction_smmd;import numpy as np;N_max = 100;xmax =
    20.0;xmin=-20.0;xsize=100;X=np.linspace(xmin,xmax,xsize);np.array([
    wavefunction_smmd(n,X) for n in range(N_max+1)])];"],
    prec];

(*WolframSMMdN100X = WavefunctionMathematica3[Nmax,Xvector,prec];*)
WolframSMMdN100X = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WolframSMMdN100X, WavefunctionMathematica1[i - 1, Xvector, prec]];]

ResidualMatrix = (WolframSMMdN100X - FastWaveSMMdN100X) ^ 2;

Residual = Mean /@ ResidualMatrix;

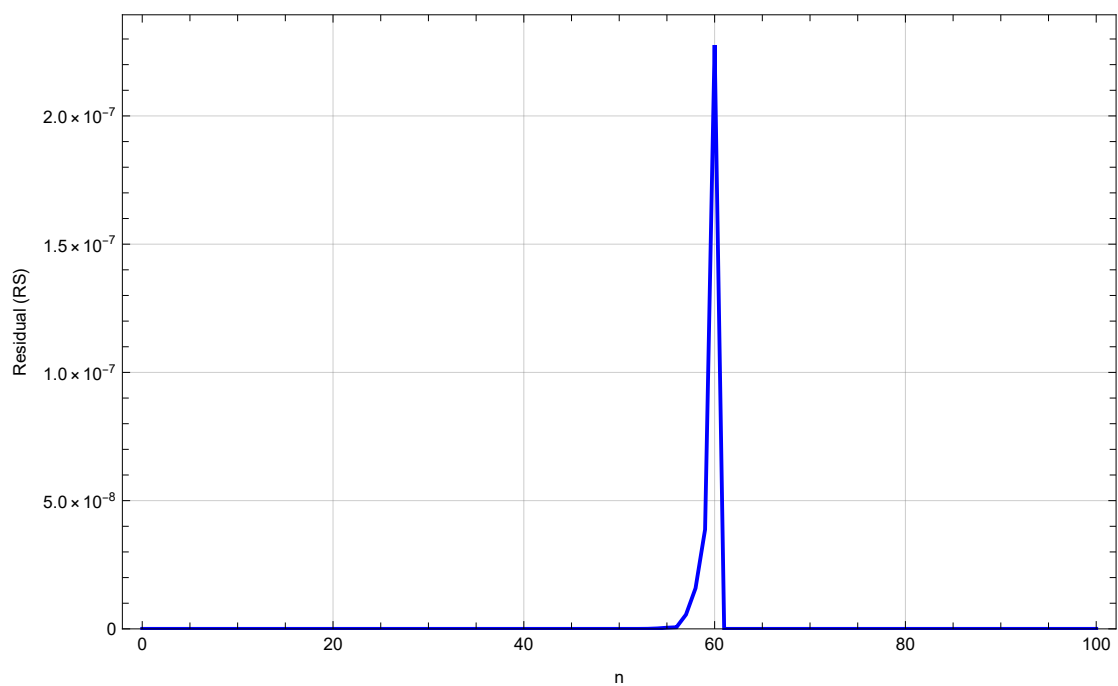
ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
        Style["Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smmd)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{"X: [(-20)→20;100], to each value of n ; \n avg(RS) = [" <>
        ToString[SetPrecision[Mean[Residual], 20],
        TraditionalForm]] <> "]", Below],
    ImageSize → Large,
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-6.7)}}]

```

Functionality Test Passed: True

Out[135]=

Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smmd)



X: $[(-20) \rightarrow 20; 100]$, to each value of n ;
 avg (RS) = $[2.8624974439968216501 \times 10^{-9}]$

★ Single-Mode and Multidimensional Less Fast Function with $X = [(-20) \rightarrow 20; 100]$

```

In[*]:= FastWaveSMMdN100X = SetPrecision[
    Normal[ExternalEvaluate["Python", "from fast_wave.wavefunction
    import wavefunction_smmd;import numpy as np;N_max = 100;xmax =
    20.0;xmin=-20.0;xsize=100;X=np.linspace(xmin,xmax,xsize);np.array([
    wavefunction_smmd(n,X,more_fast=False)
    for n in range(N_max+1)])];"], prec];

(*WolframSMMdN100X = WavefunctionMathematica3[Nmax,Xvector,prec];*)
WolframSMMdN100X = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WolframSMMdN100X, WavefunctionMathematica1[i - 1, Xvector, prec]];]

ResidualMatrix = (WolframSMMdN100X - FastWaveSMMdN100X) ^2;

Residual = Mean /@ ResidualMatrix;

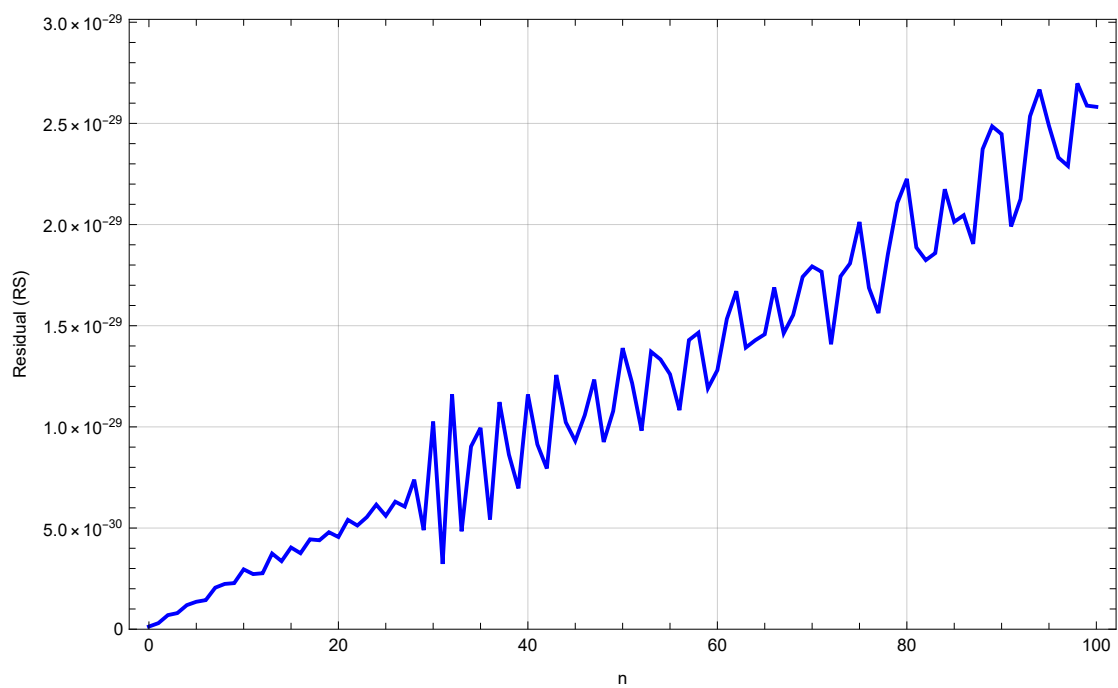
ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel → Style[
        "Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smmd; less_fast)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{" X: [(-20)→20;100], to each value of n ; \n avg(RS) = [" <>
        ToString[SetPrecision[Mean[Residual], 20],
        TraditionalForm]} <> "]", Below],
    ImageSize → Large,
    PlotRange → {{Automatic, Automatic}, {0, 1.2 * 10^(-28.6)}}]

```

Functionality Test Passed: True

Out[]=

Wavefunction_Wolfram1 vs Wavefunction_Fast-Wave(smmd; less_fast)



X: $[(-20) \rightarrow 20; 100]$, to each value of n ;
 avg(RS) = $[1.2113401822724537959 \times 10^{-29}]$

★ Multi-Mode and Onedimensional Function with $x = 1.0$

```

In[*]:= FastWaveMMOdN100x1 = SetPrecision[
    Normal[ExternalEvaluate["Python", "from fast_wave.wavefunction
    import wavefunction_mmod;import numpy as np;N_max = 100;x1
    =1.0;[wavefunction_mmod(n,x1) for n in range(N_max+1)];"], prec];

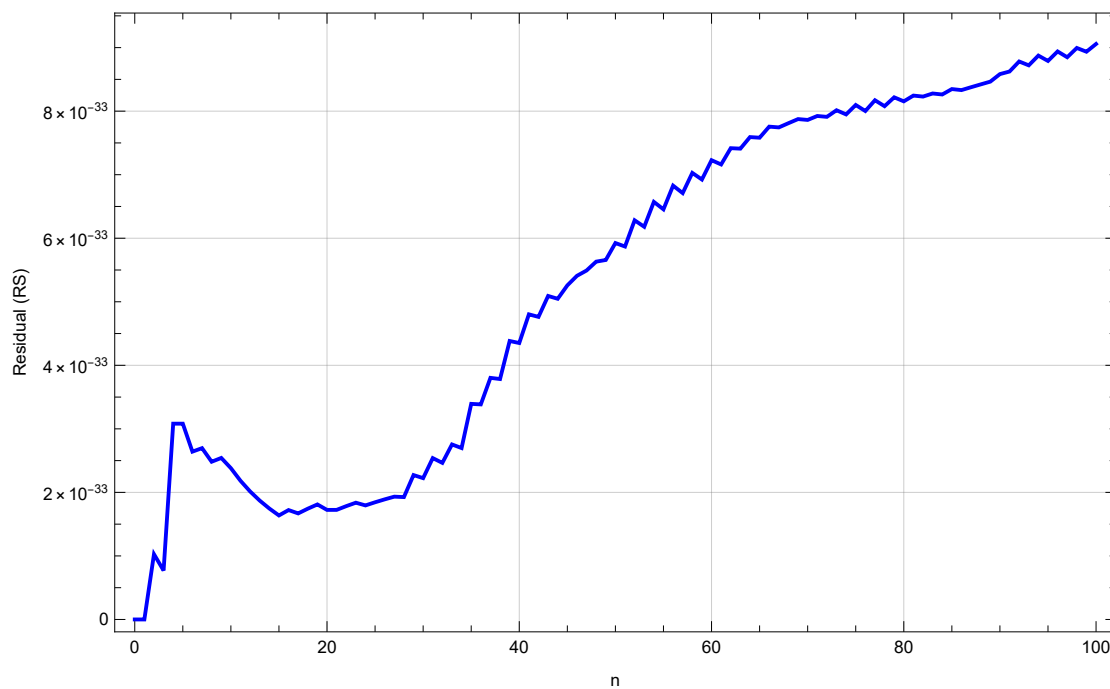
WavefunctionMMOdN100x1 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WavefunctionMMOdN100x1, WavefunctionMathematica3[i - 1, x1, prec]];
    ResidualList = (WavefunctionMMOdN100x1 - FastWaveMMOdN100x1) ^2;

Residual = Mean /@ ResidualList;

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
    Style["Wavefunction_Wolfram3 vs Wavefunction_Fast-Wave(mmod)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{" x: 1.0, to each value of n ; \n avg(RS) = [" <>
    ToString[SetPrecision[Mean[Residual], 20],
    TraditionalForm]} <> "]", Below],
    ImageSize → Large ]

Functionality Test Passed: True

```

Out[\ast]=**Wavefunction_Wolfram3 vs Wavefunction_Fast-Wave(mmod)**

x: 1.0, to each value of n ;

avg (RS) = $[5.3425573728452427011 \times 10^{-33}]$

★ Multi-Mode and Onedimensional Function with $x = 10.0$

```

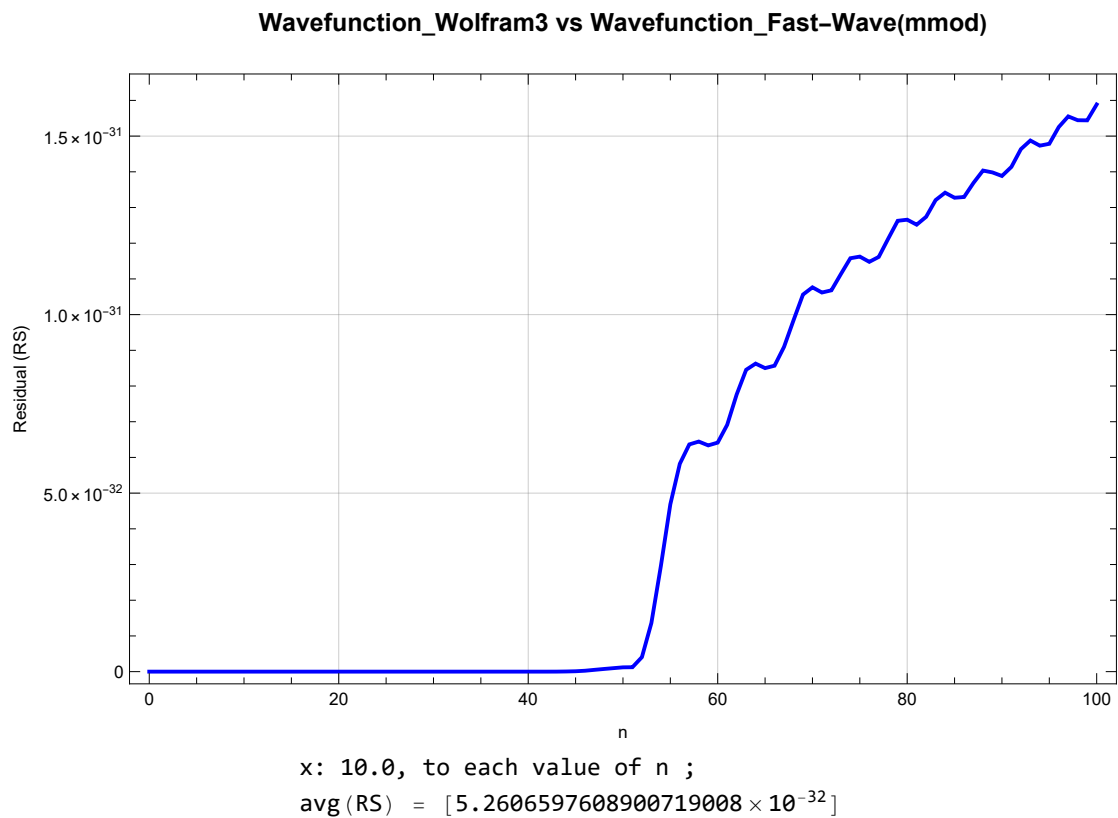
In[*]:= FastWaveMMOdN100x10 = SetPrecision[
    Normal[ExternalEvaluate["Python", "from fast_wave.wavefunction
    import wavefunction_mmod;import numpy as np;N_max = 100;x2
    =10.0;[wavefunction_mmod(n,x2) for n in range(N_max+1)];"], prec];

WavefunctionMMOdN100x10 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WavefunctionMMOdN100x10, WavefunctionMathematica3[i - 1, x2, prec]];]
ResidualList = (WavefunctionMMOdN100x10 - FastWaveMMOdN100x10)^2;

Residual = Mean /@ ResidualList;

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
    Style["Wavefunction_Wolfram3 vs Wavefunction_Fast-Wave(mmod)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{" x: 10.0, to each value of n ; \n avg(RS) = [" <>
    ToString[SetPrecision[Mean[Residual], 20],
    TraditionalForm]} <> "]", Below],
    ImageSize → Large ]
Functionality Test Passed: True

```

Out[*n*]=

★ Multi-Mode and Onedimensional Function with $x = 20.0$

```

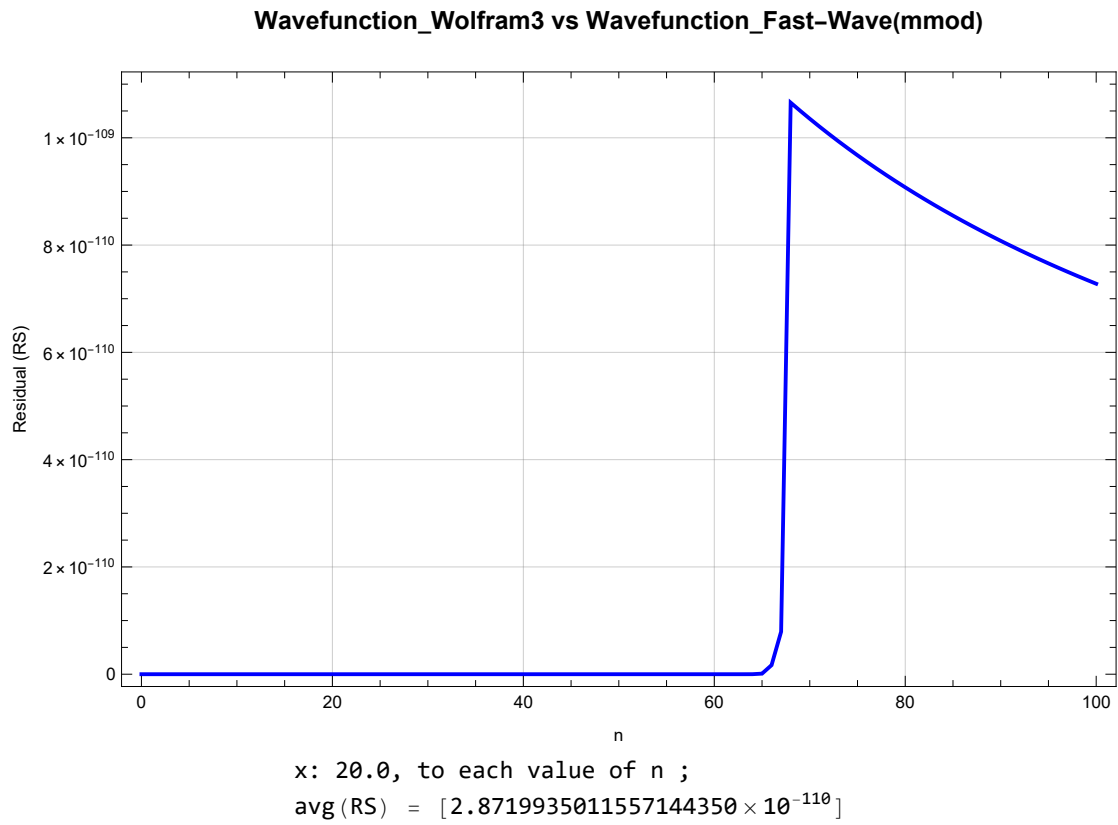
In[*]:= FastWaveMMOdN100x20 = SetPrecision[
    Normal[ExternalEvaluate["Python", "from fast_wave.wavefunction
    import wavefunction_mmod;import numpy as np;N_max = 100;x3
    =20.0;[wavefunction_mmod(n,x3) for n in range(N_max+1)];"], prec];

WavefunctionMMOdN100x20 = {};
For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WavefunctionMMOdN100x20, WavefunctionMathematica3[i - 1, x3, prec]];]
ResidualList = (WavefunctionMMOdN100x20 - FastWaveMMOdN100x20)^2;

Residual = Mean /@ ResidualList;

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
    Style["Wavefunction_Wolfram3 vs Wavefunction_Fast-Wave(mmod)\n", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{" x: 20.0, to each value of n ; \n avg(RS) = [" <>
    ToString[SetPrecision[Mean[Residual], 20],
    TraditionalForm]} <> "]", Below],
    ImageSize → Large ]
Functionality Test Passed: True

```

Out[\ast]=

★ Multi-Mode and Multidimensional Function with $X = [(-20) \rightarrow 20; 100]$

```

In[*]:= FastWaveMMdN100X = SetPrecision[
    Normal[ExternalEvaluate["Python", "from fast_wave.wavefunction
    import wavefunction_mmd;import numpy as np;N_max = 100;xmax =
    20.0;xmin=-20.0;xsize=100;X=np.linspace(xmin,xmax,xsize);[
    wavefunction_mmd(n,X) for n in range(N_max+1)]];"],
    prec];

WolframSMMdN100X = {};

For[i = 1, i ≤ (Nmax + 1), i++,
    AppendTo[WolframSMMdN100X, WavefunctionMathematica3[i - 1, Xvector, prec]];]

ResidualMatrixList = (WolframSMMdN100X - FastWaveMMdN100X)^2;

Residual = Mean /@ (Flatten /@ ResidualMatrixList);

ListLinePlot[
    Transpose[{Nvector, Residual}],
    PlotStyle → {Thick, Blue},
    Frame → True,
    FrameLabel → {"n", "Residual (RS)"},
    PlotLabel →
    Style["Wavefunction_Wolfram3 vs Wavefunction_Fast-Wave(mmd)", 13, Bold],
    GridLines → Automatic,
    PlotLegends → Placed[{" X: [(-20)→20;100], to each value of n ; \n avg(RS) = [" <>
    ToString[SetPrecision[Mean[Residual], 20],
    TraditionalForm]} <> "]", Below],
    ImageSize → Large ]

Functionality Test Passed: True

```

Out[*n*]=