Quad-linear Voting (QLV)

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1 Background

When on the topic of voting games, like those of an organisational body, authority is usually denominated by shares or tokens. While one-share-one-vote (plutocratic) systems enable decisive decision-making they concentrate power among wealthy stakeholders, creating vulnerability to vote-buying and marginalising smaller participants. While efficient in aligning influence with financial stake, they often erode representation and increase systemic exposure to short-sighted governance.

Quadratic Voting, introduced by Weyl and Posner (2018), addresses plutocratic concentration by making voting costs increase quadratically while influence grows linearly, allowing expression of preference intensity, though its effectiveness depends critically on robust identity assurance to prevent sybil attacks. Conviction Voting assigns weight based on support duration, favouring sustained participation over opportunistic intervention, but suffers from voter inertia and fragmentation. Ostrom (1990) demonstrated that the design of governance institutions fundamentally determines the success of collective action, showing how carefully crafted rules can enable effective autonomous governance without requiring continuous external intervention. The design challenge remains: how to equitably balance influence, commitment, and protection from manipulation without compromising the governing system's capacity to adapt and act.

2 Theory

Quad-linear voting (QLV) is a progressive plutocratic voting model that combines several principles to create stages in the governance lifecycle, attempting to **(a) empower smaller actors** and **(b) normalise larger actors' influence over time**. This is achieved through a power ratio which derives the linear and quadratic weighting of voting power dependent on time, such as how long shares are locked or held by a stakeholder.

2.1 Effective Time Weight

When quantifying time the optimal solution relies in progressively scaling the multiplier proportional to base average voting power, this is done to promote behaviour of either absolute (singleton) or progressive power accumulation (sequential), see section 3.5. Where t_d is the delta time factor following unix time and a_i equates to an increment amount of shares held or deposited.

$$t_{\text{effective}} = \frac{\sum a_i \cdot t_d}{\sum a_i} \tag{1}$$

This introduces the time multiplier being diluted with the introduction of additional shares, the proportion of which is equal to the active share amount. For example, Bob has a balance of 1000 shares with an effective time weight of 100 days. Bob decides to deposit an additional 500 shares, which dilutes his effective time weight to 66.7 days, a reduction of 33.3%.

$$t'_{\text{effective}} = \frac{\sum a_i \cdot t_d - a_w \cdot t_{\text{effective}}}{\sum a_i - a_w} \tag{2}$$

However, in the case of withdrawing or removing shares, the effective time weight is preserved in order to not disenfranchise committed stakeholders given that the share balance is a non-zero value, where a_w represents the withdrawal amount.

2.2 Power Levels

Using predefined tranches (P), defined as $0 \leq P \leq 1$ and dependent on the effective time weight, translates natural bands or levels across the voting power curve, creating distinct tiers or stages to the governance lifecycle. The default preference defines five discrete power levels with base tranche values: $P_1 = 0.2$ (new), $P_2 = 0.4$ (early), $P_3 = 0.6$ (median), $P_4 = 0.8$ (mature), and $P_5 = 1.0$ (veteran).

$$P_{\text{culm}}(t_{\text{effective}}) = \begin{cases} 0.2 & \text{if } t_{\text{effective}} < 30 \\ 0.6 & \text{if } 30 \leq t_{\text{effective}} < 90 \\ 1.2 & \text{if } 90 \leq t_{\text{effective}} < 180 \\ 2.0 & \text{if } 180 \leq t_{\text{effective}} < 365 \\ 3.0 & \text{if } t_{\text{effective}} \geq 365 \end{cases} \tag{3}$$

Which are cumulatively aggregated as P_{culm} for efficient lookup in O(1) time complexity, result in voting power becoming normalised across these phases of governance, requiring cross-consensus for effective decision-making and ultimately creating a more nuanced governance process.

2.3 Time Weight

Given the effective time weight $t_{\text{effective}}$, the time weight is computed using precomputed cumulative tranche coefficients for optimal computational efficiency.

$$W = P_{\text{culm}}(t_{\text{effective}}) \cdot t_{\text{effective}} \tag{4}$$

Where $P_{\text{culm}}(t_{\text{effective}})$ is determined by tranche lookup, maintaining identical power progression as summation while achieving O(1) complexity.

2.4 Power Ratio

The power ratio is the proportion of quadratic to linear in the weighting of voting power, in QLV being dependent on time (t) but in theory can be applied to any arbitrary metric.

$$\alpha = \max(r_{\text{min}}, \min(r_{\text{max}}, r_{\text{max}} + (t_{\text{effective}} \cdot k))) \tag{5}$$

Where $\alpha = 1$ equates to fully linear weighting and $\alpha = 0$ to entirely quadratic weighting, with variable k being defined as time limiting factor.

$$k = \frac{r_{\text{max}}}{\log_2(t_{\text{max}})} \tag{6}$$

Where t_{max} is defined as the time value for maturity, here defined as 730 days or 24 months, using lograthmic time to provide diminishing returns over-time.

2.5 Voting Power

Where T is token amount, W is time weight, and α is the power ratio.

$$V_P = W(\alpha \cdot T + \sqrt{T}(1 - \alpha)) \tag{7}$$

3 Power Analysis

To replicate a more realistic distribution, shareholder actors from each tier are added gradually over 80 day period to portray the temporal dynamics of voting power when bootstrapping a governing body.

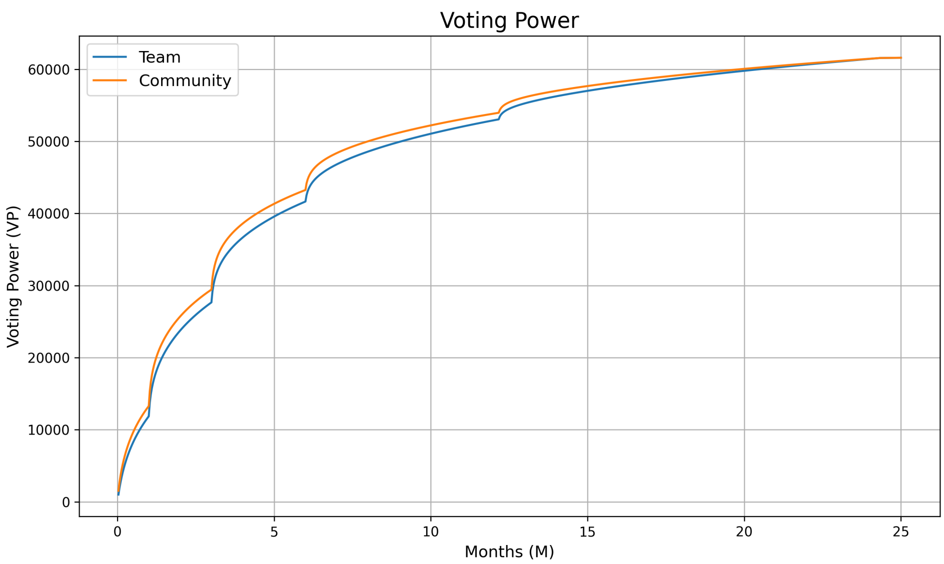

Figure 1: Voting power over time

Share distribution features tiered holder structure with factor-based scaling (4x, 2x, 1x, 0.4x, 0.1x) ensuring governance representation across wealth demographics for realistic assumptions with team cooperation and market liquidity.

Category	Tier	Shares per	Count	Total	Percent
	Major	2,000	5	10,000	10.0%
	Large	1,000	10	10,000	10.0%
Stakeholders	**Median**	500	20	10,000	10.0%
	Minor	200	40	8,000	8.0%
	Small	50	40	2,000	2.0%
	Subtotal	–	115	40,000	40.0%
Operations	–	40,000	1	40,000	40.0%
Liquidity	–	20,000	1	20,000	20.0%
Total Supply	–	–	117	100,000	100.0%

Day	Major	Large	Median	Minor	Small	Holders
0	1	2	3	6	2	14
20	2	4	7	14	8	35
40	3	7	12	25	18	65
60	4	9	17	35	30	95
80	5	10	20	40	40	115

Table 1: Share distribution and schedule

4 Strategies

4.1 Singleton

A stakeholder contributes a singular denomination stake, which results in the most absolute state of voting power accumulation and weight. For example, Bob locks 10,000 shares. A time-capital commitment, with no subsequent withdrawals until maturity.

4.2 Sequential

A stakeholder strategically obtains or locks shares gradually to dilute their time weight at an optimal threshold to allow voting power to compound effectively. For example, Alice locks 3,333 shares and subsequent deposits at; 1,667 shares at 90 days, 2,500 shares at 180 days and 2,500 at 270 days. This results a much more progressive power accumulation but is an operational strategy.

4.3 Splitting

A stakeholder aims to take advantage of the quadratic weighting by sybil attacking. For example, Claude has 10,000 shares and splits the balance to different stakeholder accounts. Amplifying their voting power temporally due to the quadratic weighting.

4.4 Benchmark

Each strategy assumes a total allocation of 10,000 shares, where P equals 0.2 at t_p to 30 days, then 0.4 for 90 days, 0.6 for 180 days, 0.8 for 365 days and 1 for 720 days.

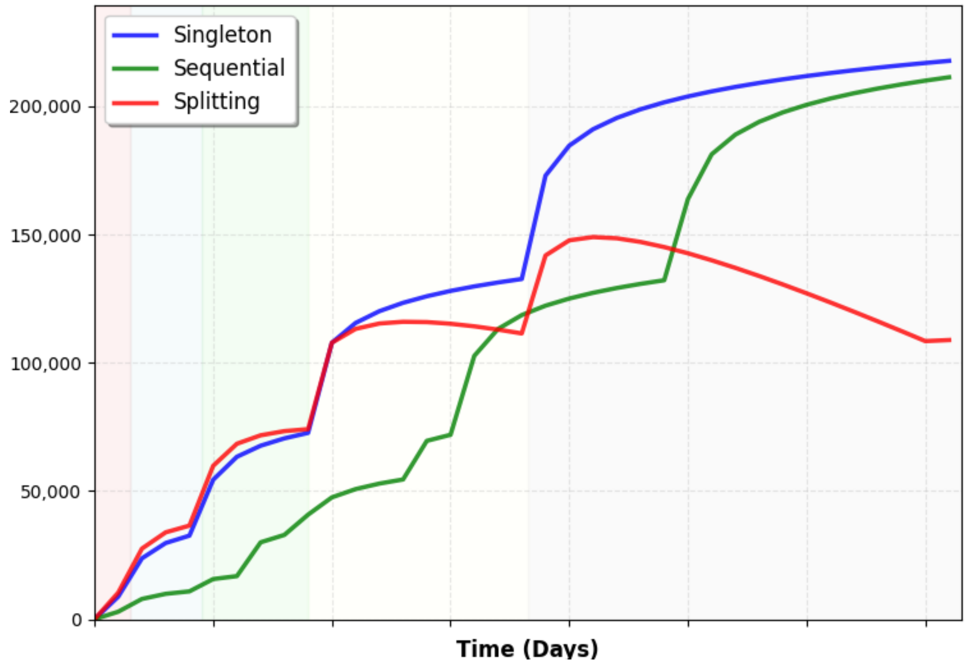

Figure 2: Strategies voting power series

Strategy	3 Months	6 Months	12 Months	24 Months	Impact
Singleton	3,938	6,563	7,884	10,000	+100%
Sequential	3,360	5,888	7,792	9,400	+88%
Splitting	4,860	6,480	7,200	5,000	-50%

Table 2: Strategy voting power

Strategy	3 Months	6 Months	12 Months	24 Months	Average
Singleton	1,313 V_p/s	1,094 V_p/s	657 V_p/s	417 V_p/s	870 V_p/s
Sequential	1,120 V_p/s	981 V_p/s	649 V_p/s	392 V_p/s	786 V_p/s
Splitting	1,620 V_p/s	1,080 V_p/s	600 V_p/s	208 V_p/s	877 V_p/s

Table 3: Strategy power rates

While token-splitting is advantageous in the short-term, it ultimately fails to increase voting power over time but actually expenses a stakeholder's governance influence by approximately 50% due to the time weighting loss. Following the Nash equilibrium theorem, we can identify the Singleton strategy as the dominant strategy, the Sequential strategy as the equilibrium strategy and the Splitting strategy as the suboptimal strategy.

5 Results

5.1 Wealth Dispersion

QLV significantly empowers smaller stakeholders through the quadratic dampening in the early phases of the governance cycle. By using the Gini coefficient, we can investigate how much of an impact this has.

Assuming the previous distribution defined in the voting power plot, we can see that it effectively

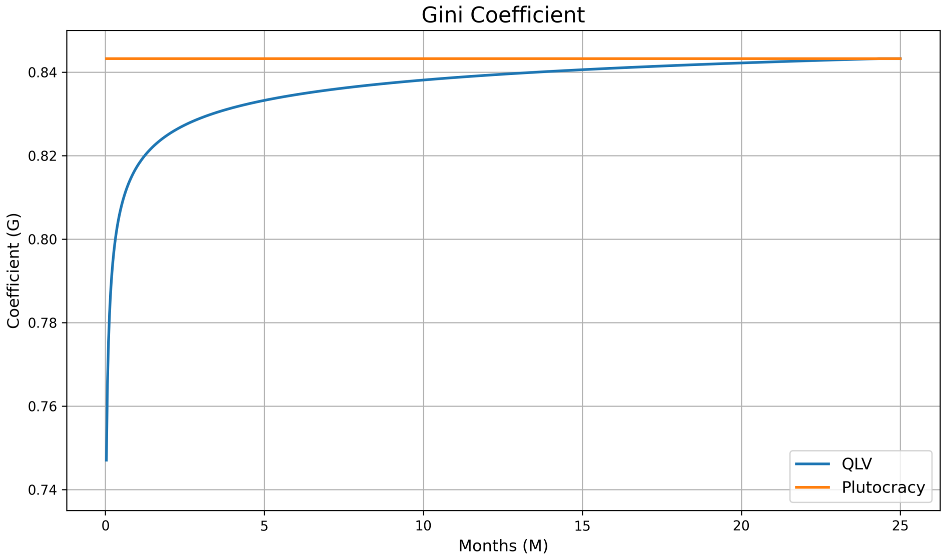

Figure 3: Gini analysis of QLV vs Plutocracy

empowers minor demographics in the early and median phases of the voting power curve until converging towards purely linear weighting.s

5.2 Byproducts

Coalition building: By introducing opacity in power progression, the model naturally incentivises cooperation, where participants within similar time-weighted tranches find strategic alignment. This creates an emergent governance landscape where shared experience and comparable power levels encourage collaborative strategies.

Graduated proposals: Given the nature of the nuance of voting demographics in QLV, the only logical decision to replicate that on the application layer is through tiered proposals by severity. Each with equivalent quorum and duration parameters to match, this mediates any short term advantages in voting power for any critical decisions through continuous voting periods plus caters to agile and effective decision making for lower severity ballots.

Delegation markets: While vote buying in stakeholder governance has long been scrutinized, QLV's time-stratified structure transforms these dynamics by creating significant friction for governance attacks. Unlike pure plutocracy where 51% control requires acquiring tokens equivalent to that voting threshold, QLV delegation attacks face compounding constraints: limited market depth as operational allocations are typically non-delegable, fragmented negotiations across stakeholders with heterogeneous time weights, and temporal requirements where high-influence votes must be developed over extended periods rather than purchased instantly. This creates indirect economic incentives for voter participation while substantially increasing coordination complexity and sustained costs for manipulation attempts, shifting governance attacks from capital allocation problems to multi-period coordination challenges with significantly higher execution risk. QLV's power levels actually are additionally benefactory for pricing in delegation markets due to how voting demographics are categorised.

6 Conclusion

Quad-linear voting is a governance model to align the interests of an organisation with its participants and makes power dynamics explicit, rather than opaque, creating more strategic and nuanced governance.

It provides a more robust and efficient voting model for organisations versus pure plutocracy and is the closest mathematical representation of human decision-making, commitment, and loyalty within organisational systems.

7 References

[1] Lalley, Posner, E. Glen Weyl. "Democracy Squared" Journal of Political Economy 126

[2] Ostrom, E. (1990). "Governing the Commons" The Evolution of Institutions for Collective Action. Cambridge University Press.