Obligatory assignment 1, MVE550, autumn 2024

Petter Mostad

November 12, 2024

NOTE: In the tasks below, you may have use for R functions like `rpois`, `dpois`, `ppois`, `qpois`, `rgamma`, `dgamma`, `pgamma`, `qgamma`, `rnorm`, `dnorm`, `pnorm`, `qnorm`, `plot`, `sample`, `integrate`, `hist`, `matrix`, `solve` etc. Use the help function in R to learn more.

1. Assume that the number of requests for data that a computer server receives in a day is Poisson(λ) distributed, where λ is the expected number of requests per day. Assume that during each of 6 days, the number of requests were 12, 21, 12, 13, 11, and 19, respectively.

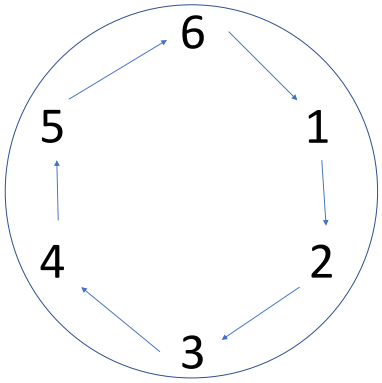
   (a) Using a prior that is proportional to $1/\lambda$, find the distribution for, and plot using R, the posterior distribution for λ given the data.

   (b) Find the distribution for, and plot using R, the predictive distribution for the number of requests for the 7th day, based on the given data.

   (c) Using discretization, re-do question (a) and re-compute the probability of getting exactly 18 requests on the seventh day. Make your choices so that you get satisfactory accuracy.

   (d) By using information from similar servers, one realizes that an informative prior distribution for λ (i.e., what one knows about λ before looking at the data given above) can be formulated as a normal distribution with expectation 15 and standard deviation 2, restricted (truncated) to positive values. Using discretization and this prior, re-do question (a) and re-compute the probability of getting exactly 18 requests on the seventh day.

   (e) Using the new prior, compute using numerical integration the probablity of getting exactly 18 requests on the seventh day.



2. A (not very popular) game is played on a circular board with numbers 1,2,3,4,5,6 along the edge. Players start at 6. For each round, a 4-sided dice is thrown and the player moves forward the corresponding number of steps. The game is won by a player when the player returns to 6 exactly. The only complication is that, whenever your move ends at 5, you move your piece immediately to 1.

   Find formulas for the answers to the questions below, and use R to compute the actual values:

   (a) Imagine a single player playing the game. What is the expected number of dice throws before the final winning throw?

   (b) What is the probability that the player will visit state 3 before the end of the game?

   (c) What is the probability that the player will visit state 3 two or more times before the end of the game?