Angle Chasing Problem Set

Focus Learning

July 25, 2020

Abstract

These handout contains various problems using the techniques from the Angle Chasing lecture. The problems will generally increase in difficulty.

1 Problems

Remark. Remember that angle chasing usually isn't much help on its own, use other techniques to finish the problems.

Exercise 1.1 (2007 AMC 12)**.** Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside $\triangle ABC$, $\angle ABC = 40°$, and $\angle ADC = 140°$. What is the degree measure of $\angle BAD$?

Exercise 1.2. In quadrilateral $ABCD$, the diagonals meet at E. If $\angle ABD = 20°, \angle AED = 70°$, and $\angle BDC = 50°$, prove or disprove that ABCD is cyclic.

Exercise 1.3 (1986 MATHCOUNTS)**.** ABC is an isosceles triangle such that $AC = BC$. CBD is an isosceles triangle such that $CB = DB$. BD meets AC at a right triangle. If $\angle A = 57°$, what is $\angle D$?

Exercise 1.4. Let $ABCD$ be a cyclic quadrilateral inscribed in circle ω. Let ℓ be a line tangent to ω at A. Prove that the angle formed by ℓ and AB is equal to $\angle ACB$.

Exercise 1.5 (1990 AHSME)**.** An acute isosceles triangle, ABC is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$, find the measure of $\angle A$.

Exercise 1.6 (Reflecting the Orthocenter Lemma)**.** Let O be the orthocenter (intersection of the three altitudes) of $\triangle ABC$. Prove that the reflections of O across AB, BC, and AC all lie on the circumcircle of $\triangle ABC$.

Exercise 1.7 (1999 BAMO)**.** Let $O = (0,0), A = (0,a)$, and $B = (0,b)$, where $0 < b < a$ are reals. Let Γ be a circle with diameter $\overline{AB}$ and let P be any other point on Γ. Line PA meets the x-axis again at Q. Prove that angle $\angle BQP = \angle BOP$.

Exercise 1.8 (2011 USAJMO)**.** Points A, B, C, D, E lie on a circle ω and point P lies outside the circle. The given points are such that (i) lines PB and PD are tangent to ω, (ii) P, A, C are collinear, and (iii) $DE \parallel AC$. Prove that BE bisects AC.

Exercise 1.9 (2014 Sharygin MO)**.** Let ABC be an isosceles triangle with base AB. Line ℓ touches its circumcircle at point B. Let CD be a perpendicular from C to ℓ, and AE, BF be the altitudes of ABC. Prove that D, E, and F are collinear.