3D Geometry Problem Set

Focus Learning

August 9, 2020

Abstract

These handout contains various problems using the techniques from 3D Geometry Lecture. The problems will generally increase in difficulty.

1 Problems

Exercise 1.1 (2000 AMC 12)**.** The point $P = (1, 2, 3)$ is reflected in the xy-plane, then its image Q is rotated by 180° about the x-axis to produce R, and finally, R is translated by 5 units in the positive-y direction to produce S. What are the coordinates of S?

Exercise 1.2 (2008 AMC 12)**.** A pyramid has a square base $ABCD$ and vertex E. The area of square $ABCD$ is 196, and the areas of $\triangle ABE$ and $\triangle CDE$ are 105 and 91, respectively. What is the volume of the pyramid?

Exercise 1.3 (2007 AMC 12)**.** A sphere is inscribed in a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surface area in square meters of the inner cube?

Exercise 1.4 (2011 AMC 12)**.** The circular base of a hemisphere of radius 2 rests on the base of a square pyramid of height 6. The hemisphere is tangent to the other four faces of the pyramid. What is the edge-length of the base of the pyramid?

Exercise 1.5 (2005 AMC 12)**.** A rectangular box P is inscribed in a sphere of radius r. The surface area of P is 384, and the sum of the lengths of its 12 edges is 112. What is r?

Exercise 1.6 (2013 AIME)**.** A rectangular box has width 12 inches, length 16 inches, and height $\frac{m}{n}$ inches, where m and n are relatively prime positive integers. Three faces of the box meet at a corner of the box. The center points of those three faces are the vertices of a triangle with an area of 30 square inches. Find $m + n$.

Exercise 1.7 (AOIME)**.** Two congruent right circular cones each with base radius 3 and height 8 have the axes of symmetry that intersect at right angles at a point in the interior of the cones a distance 3 from the base of each cone. A sphere with radius r lies withing both cones. The maximum possible value of r^2 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.