

# The Quadratic Discriminant

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## 1 Introduction

The discriminant is a crucial tool in solving problems related to quadratics, or polynomials of degree two.

Recall that the general form of a quadratic is  $ax^2 + bx + c = 0$  with  $a \neq 0$ . To solve for the discriminant, we first divide both sides by  $a$ .

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Adding  $(\frac{b}{2a})^2$ , or  $\frac{b^2}{4a^2}$ , to both sides of the equation allows us to complete the square.

$$\begin{aligned}x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} + \frac{c}{a} &= \frac{b^2}{4a^2} \\(x + \frac{b}{2a})^2 &= \frac{b^2}{4a^2} - \frac{c}{a}\end{aligned}$$

We arrive at the key equation:

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

**Definition 1.1.** The discriminant of a quadratic in the form  $ax^2 + bx + c = 0$  is the number  $b^2 - 4ac$ .

We know that a number  $c \in \mathbb{R}$  ( $c$  is a real number) has  $c^2 \geq 0$ . However, if  $c^2 < 0$ ,  $c \in \mathbb{C}$  and  $c \notin \mathbb{R}$  ( $c$  is a non-real complex number). The left-hand side of the key equation,  $(x + \frac{b}{2a})^2$ , follows the same rules. Note that since  $4a^2 > 0$ , the discriminant is the deciding factor for whether  $(x + \frac{b}{2a})^2 \in \mathbb{R}$  or not. We get the following theorem:

**Theorem 1.2.**

- If the discriminant is less than 0, then  $ax^2 + bx + c$  does not have real roots.
- If the discriminant is equal to 0, then  $ax^2 + bx + c$  has one real root.
- If the discriminant is greater than 0, then  $ax^2 + bx + c$  has two real roots.

## 2 Exercises