Non-Linear State Estimation Exercises

The provided code consists of a skeleton for comparing the Extended Kalman filter (EKF) with the Unscented Kalman Filter (UKF). The exercise is to complete the code and perform a comparison. The exercise has the following configuration:

- Target state is $\mathbf{x}_k = (x, y, \dot{x}, \dot{y})'$
- Measurements are of range and angle: $\mathbf{z}_k = (r, \theta)'$ where r is range and θ is angle
- The radar producing the measurements is located at $x_r = -600$ and $y_r = 800$
- The measurement function is:

$$\mathbf{h}_k = \begin{bmatrix} ((x - x_r)^2 + (y - y_r)^2)^{0.5} \\ \text{atan2}((y - y_r)/(x - x_r)) \end{bmatrix}$$

- The measurements are corrupted by zero mean Gaussian noise, described by the covariance matrix:

$$\mathbf{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

  where $\sigma_r^2 = 5m$ and $\sigma_\theta^2 = 0.0698$ radians (equivalent to 4°)
- Nearly constant velocity model with a process noise intensity $\tilde{q} = 0.01$
- The root mean squared error is calculated over 1000 Monte Carlo runs
- As the state transitions are linear, the standard Kalman filter prediction step can be used

Task 1 – Complete the state transition matrix and the transition noise covariance matrix (like last exercise)

Task 2 – Complete the measurement error covariance (like last exercise)

Task 3 – Complete the function kalmanPrediction (like last exercise)

Extended Kalman Filter

Task 4 – Complete the function MeasurementMatrix. This function should take a state and the radar location as input and return the approximate measurement matrix used in the extended Kalman filter as a result of the local linearization.

Task 5 – Use the completed MeasurementMatrix function from Task 4, complete the function EKFUpdate.

Unscented Kalman Filter

Task 6 – Complete the function UKFUpdate with $\kappa = 0$.
(Remember: The square root in the UKF is the matrix square root, which is sqrtm in Matlab)

Analysis

Task 7 – Run the complete code and interpret the results. Vary the angular measurement error standard deviation (e.g. 2°, 4°, 6°), what can be observed about the difference between the EKF and UKF?