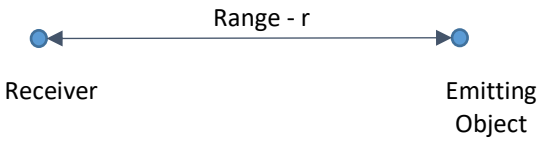
Exercise 1 – Estimators

Consider the following problem. An object emitting a signal is separated from a receiver that receives the signal.



The receiver can repetitively measure the signal power normalised by the receiver noise power (signal-to-noise ratio). However, the signal-to-noise ratio (SNR) fluctuates around a mean value $\mathrm{SNR}_{\mathrm{av}}$:

$$p(\mathrm{SNR};\mathrm{SNR}_{\mathrm{av}}) = \frac{1}{\mathrm{SNR}_{\mathrm{av}}}\exp\left(\frac{-\mathrm{SNR}}{\mathrm{SNR}_{\mathrm{av}}}\right)$$

The mean signal-to-noise ratio can be modelled as a function of the range r between the object and the receiver:

$$\mathrm{SNR}_{\mathrm{av}} = \left(\frac{r_0}{r}\right)^2$$

where r_0 is the range at which SNR = 1, taken as 10m.

Problem: We want to estimate the range to the object, based on a set of independent measurements of the signal-to-noise ratio. We have the prior information that the range is 5m with a standard deviation of 1m.

Tasks:

1. Complete the Matlab function for the measurement function
2. Complete the Matlab function for the likelihood function
3. Plot the likelihood as a function of range
4. What is the maximum likelihood estimate?
5. Plot the posterior as a function of range
6. What is the maximum a posteriori estimate?
7. How does it vary in comparison to the maximum likelihood estimate?
8. What is the least squares estimate?
9. What the is minimum mean squared error estimate?

Tips:

- Minimums can be found in Matlab by defining function handles and using fminbnd
- The object is not further than 10 away (for bounding fminbnd)