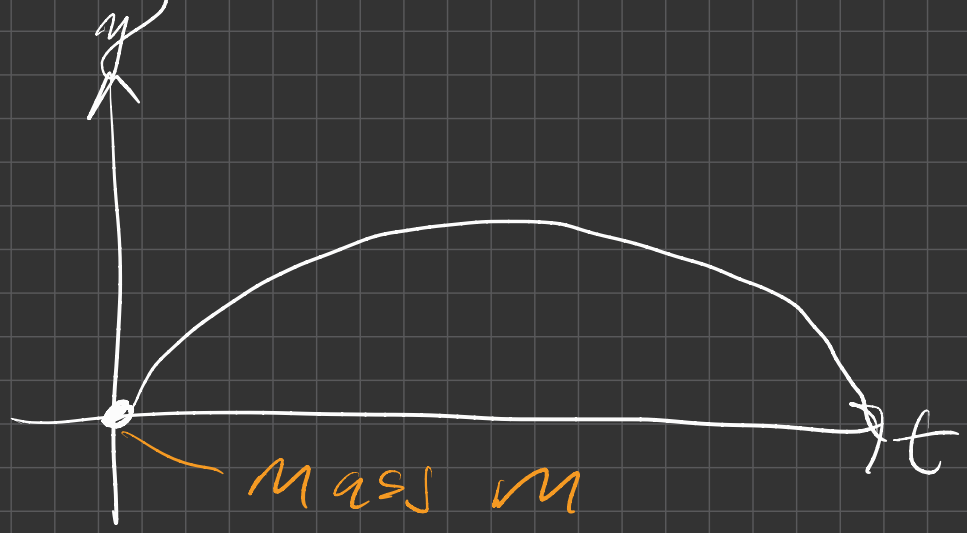
Example

$$E = T + V$$

$$T \equiv \frac{1}{2} m \dot{y}^2$$

$$V = \frac{dy}{dt}$$

$$\dot{} \equiv \frac{d}{dt}$$



$$V = mgy$$

$$\mathcal{L}(y; \dot{y}) = \frac{1}{2} m \dot{y}^2 - mgy$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{y}} = \frac{\partial \mathcal{L}}{\partial y}$$

$$\frac{\partial \mathcal{L}}{\partial y} = -mg$$

$$\frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y}, \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) = m\ddot{y}$$

$m\ddot{y} = -mg$

$y(t) = ?$

$\ddot{y} = -g$

$$\frac{d^2y}{dt^2} = \boxed{-g = \frac{dv}{dt}}$$

$$\int -g\,dt = \int dv$$

$$-g\int 1\,dt = \int 1\,dv$$

$$-g(t + C_1) = v + D_1$$

$$v = -gt + k_1 = \frac{dy}{dt}$$

$$\frac{dy}{dt} = -gt + k_1$$

$$dy = (-gt + k_1)\,dt$$

$$\int dy = \int (-gt + k_1)\,dt$$

$$y + C_2 = \int (-gt)\,dt + \int k_1\,dt$$

$$= -\frac{1}{2}gt^2 + D_2 + k_1 t + E_2$$

$$\boxed{y(t) = -\frac{1}{2}gt^2 + k_1 t + k_2}$$ E.O.M.