A study of the DASH algorithm for software property checking

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DASH$_{int}$ overview

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int abs(int a)
{
  if(a < 0)
    a = -a;
  assert a ≥ 0;
  return a;
}
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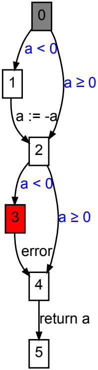

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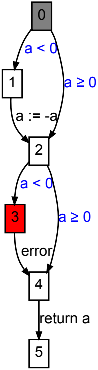

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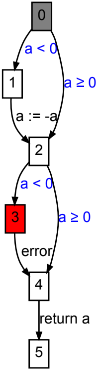

$\longrightarrow$ DASH$_{int}$

$\nearrow$ fail

$\searrow$ pass

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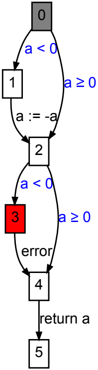
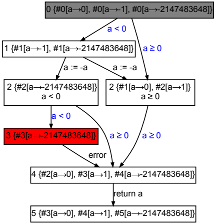


⟶ DASH$_{int}$

↗ fail



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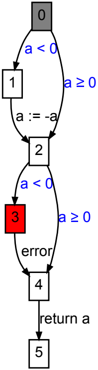

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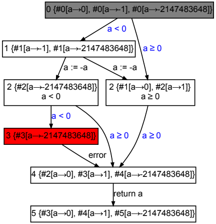

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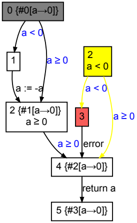

DashLoop - Test

DashLoop(P, $G = \langle \Sigma_{\simeq}, \rightarrow_{\simeq} \rangle$)

loop
  τ := FindAbstractErrorPath(G)
  if τ = NO-PATH **then**
    return (PASS, G)
  end if

  τ_c := ConvertToRegionTraceWithAbstractFrontier(τ, G)
  $\langle t, \rho \rangle$:= ExtendFrontier(τ_c, P)
  if $t \neq$ UNSAT **then**
    G := RunTest(t, P, G)
    if IsErrorRegionReached(G) **then**
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    end if
  else
    G := RefineGraph(ρ, τ_c, G)
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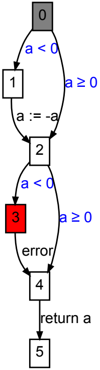

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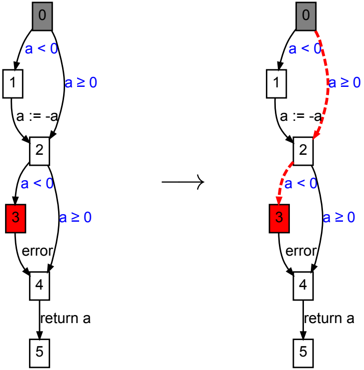

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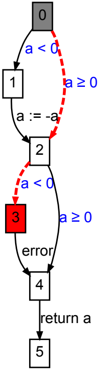

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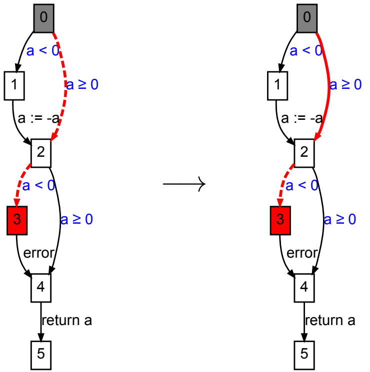

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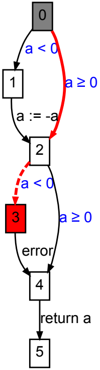

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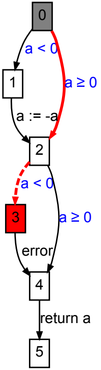

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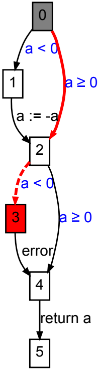

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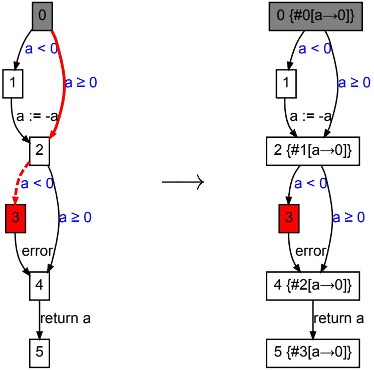

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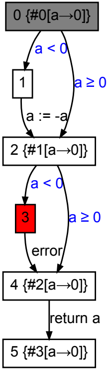

DashLoop - Refine

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    $\tau_c :=$ ConvertToRegionTraceWithAbstractFrontier(τ, G)
    $\langle t, \rho \rangle :=$ ExtendFrontier(τ_c, P)
    if $t \neq \text{UNSAT}$ **then**
        $G :=$ RunTest(t, P, G)
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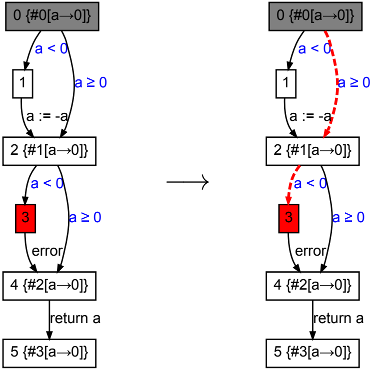

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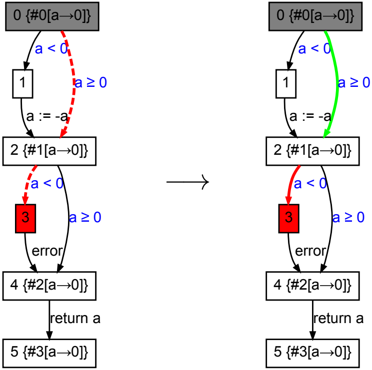

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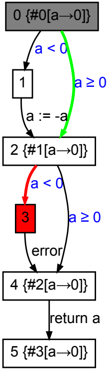

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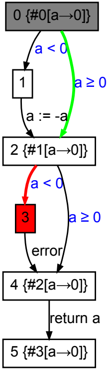

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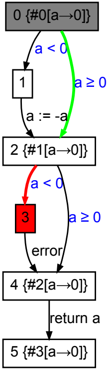

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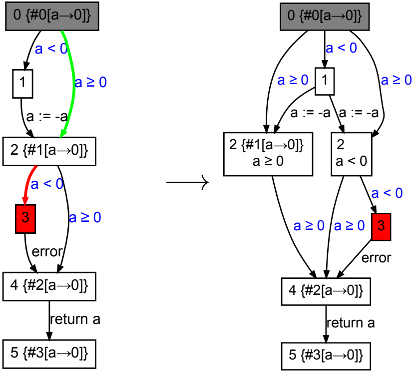

RefineGraph

RefineGraph$(\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle, G = \langle \Sigma_{\simeq}, \rightarrow_{\simeq} \rangle)$

let $\langle \rho_{k-1}, states \rangle = S_{k-1}$
if $k = 1$ **then**
  return $\langle \Sigma_{\simeq}, \rightarrow_{\simeq} \setminus (S_0, S_1) \rangle$
end if
$\Sigma^*_{\simeq} := \Sigma_{\simeq} \setminus \{S_{k-1}\}$
$\rightarrow^*_{\simeq} := \rightarrow_{\simeq} \setminus \{(S, S_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
$\rightarrow^*_{\simeq} := \rightarrow^*_{\simeq} \setminus \{(S_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
$\rho^*_{k-1} := \text{Simplify}(\rho_{k-1} \wedge \neg \rho)$
$S^*_{k-1} := \langle \rho^*_{k-1}, states \rangle$
$\Sigma^*_{\simeq} := \Sigma^*_{\simeq} \cup \{S^*_{k-1}\}$
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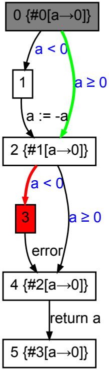
$\rho^{**}_{k-1} := \text{Simplify}(\rho_{k-1} \wedge \rho)$
$S^{**}_{k-1} := \langle \rho^{**}_{k-1}, \emptyset \rangle$
$\Sigma^*_{\simeq} := \Sigma^*_{\simeq} \cup \{S^{**}_{k-1}\}$
$\rightarrow^*_{\simeq} := \rightarrow^*_{\simeq} \cup \{(S^{**}_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
if IsSAT$(\rho^{**}_{k-1}) \neq$ UNSAT **then**
  $\rightarrow^*_{\simeq} := \rightarrow^*_{\simeq} \cup \{(S, S^{**}_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
end if
return $\langle \Sigma^*_{\simeq}, \rightarrow^*_{\simeq} \rangle$



$\longrightarrow$
$\rho \mapsto a < 0$

RefineGraph

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if IsSAT(ρ^{**}_{k-1}) $\neq$ UNSAT **then**
  $\rightarrow^*_\simeq := \rightarrow^*_\simeq \cup \{(S, S^{**}_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
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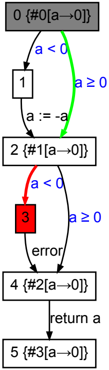

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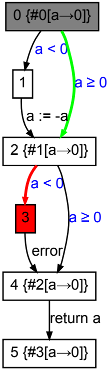
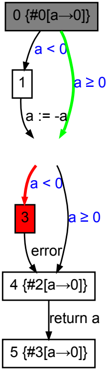
$\rightarrow^*_{\simeq} := \rightarrow^*_{\simeq} \cup \{(S^{**}_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
if IsSAT(ρ^{**}_{k-1}) $\neq$ UNSAT **then**
    $\rightarrow^*_{\simeq} := \rightarrow^*_{\simeq} \cup \{(S, S^{**}_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
end if
return $\langle \Sigma^*_{\simeq}, \rightarrow^*_{\simeq} \rangle$



$\longrightarrow$ $\rho \mapsto a < 0$



RefineGraph

RefineGraph$(\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k\rangle, G = \langle \Sigma_\simeq, \to_\simeq\rangle)$

let $\langle \rho_{k-1}, states\rangle = S_{k-1}$
if $k = 1$ **then**
    return $\langle \Sigma_\simeq, \to_\simeq \setminus (S_0, S_1)\rangle$
end if
$\Sigma^*_\simeq := \Sigma_\simeq \setminus \{S_{k-1}\}$
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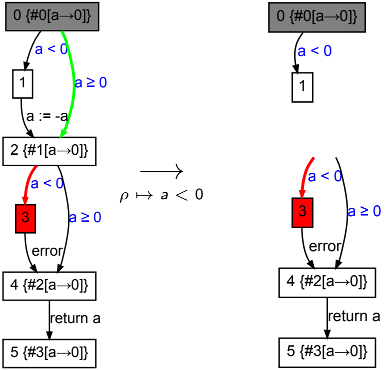

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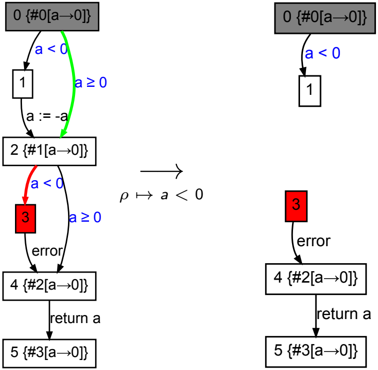

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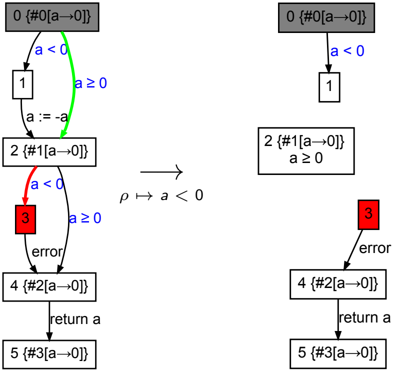

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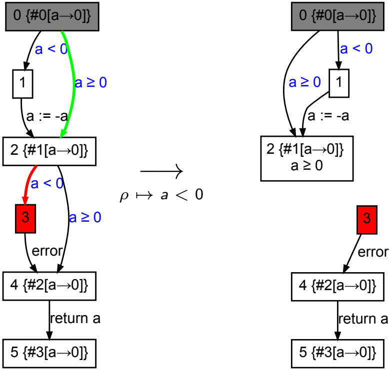

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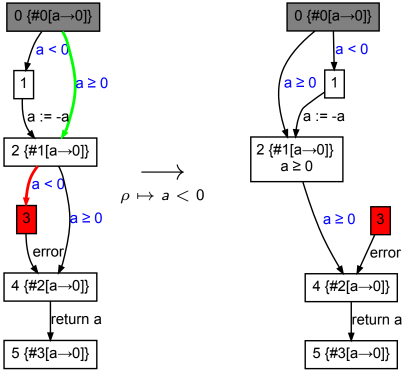

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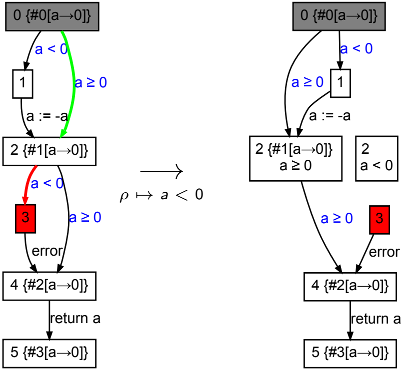

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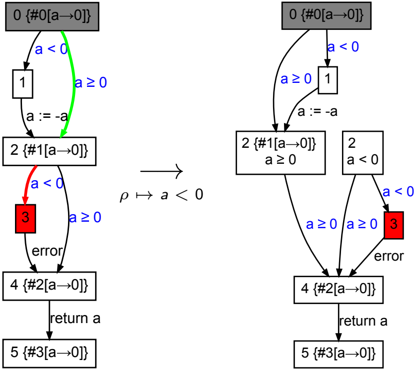

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if $\text{IsSAT}(\rho_{k-1}^{**}) \neq \text{UNSAT}$ **then**
  $\rightarrow_\simeq^* := \rightarrow_\simeq^* \cup \{(S, S_{k-1}^{**}) \mid S \in \text{Parents}(S_{k-1})\}$
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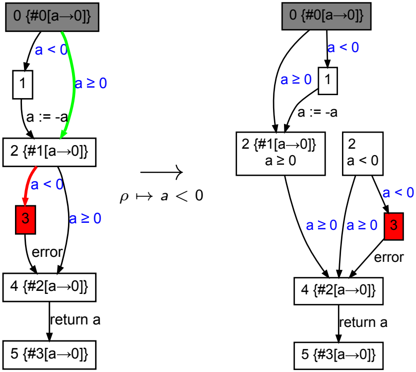

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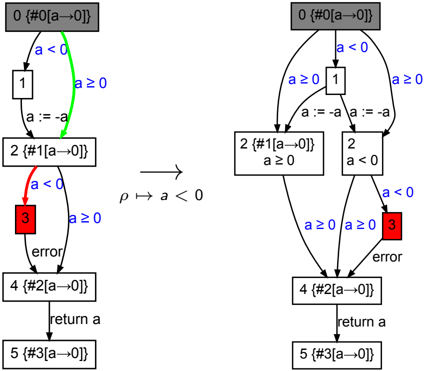

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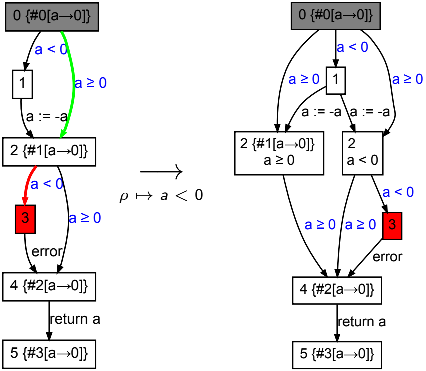
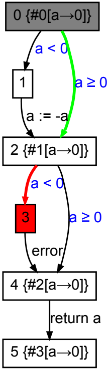


ExtendFrontier - Refine

ExtendFrontier(τ_c, P)

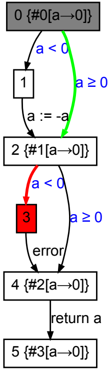
```
φ := ExecuteSymbolic(τc, P)
t := IsSAT(φ, P)
if t = UNSAT then
    ρ := RefinePred(τc)
else
    ρ := true
end if
return ⟨t, ρ⟩
```



ExtendFrontier - Refine

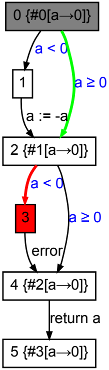
ExtendFrontier(τ_c, P)

ϕ := ExecuteSymbolic(τ_c, P)
t := IsSAT(ϕ, P)
if t = UNSAT **then**
    ρ := RefinePred(τ_c)
else
    ρ := *true*
end if
return $\langle t, \rho \rangle$



ExecuteSymbolic

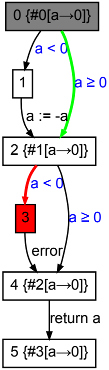
ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)
let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
    $op := \text{Op}(S_i, S_{i+1})$
    match op
        case($v := e$):
            $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
        case(assume c):
            $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
    let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
    $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match op
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(assume c):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



$\longrightarrow$

$$\mathcal{S} \rightarrow [a \mapsto a_0]$$
$$\phi \rightarrow true$$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
    $op := \text{Op}(S_i, S_{i+1})$
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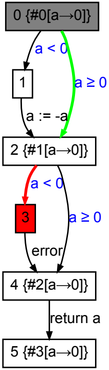
        case(assume c):
            $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
    let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
    $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



$\overrightarrow{op = a \geq 0}$

$\mathcal{S} \rightarrow [a \mapsto a_0]$

$\phi \rightarrow true$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
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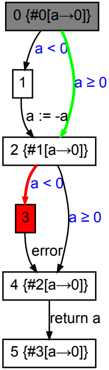
        case(assume c):
            $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
    let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
    $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



$\xrightarrow{}$
$op = a \geq 0$

$\mathcal{S} \rightarrow [a \mapsto a_0]$

$\phi \rightarrow a_0 \geq 0$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
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    case($v := e$):
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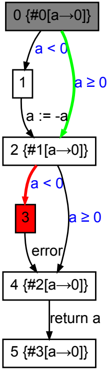
    case(assume c):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



$$\xrightarrow[\rho_{i+1} = true]{}$$

$$\mathcal{S} \rightarrow [a \mapsto a_0]$$
$$\phi \rightarrow a_0 \geq 0$$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$

$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$

$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$

for $i = 0$ **to** $k - 1$ **do**

  $op := \text{Op}(S_i, S_{i+1})$

  match op

    case($v := e$):

      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$

    case(assume c):

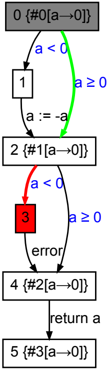
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$

  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$

  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$

end for

return ϕ



$\xrightarrow[\rho_{i+1} = true]{}$

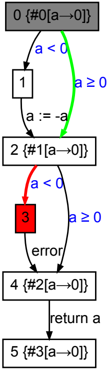
$\mathcal{S} \rightarrow [a \mapsto a_0]$

$\phi \rightarrow a_0 \geq 0$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match op
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(assume c):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



$\xrightarrow[op \,=\, a < 0]{}$

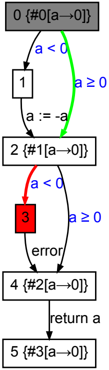
$\mathcal{S} \rightarrow [a \mapsto a_0]$

$\phi \rightarrow a_0 \geq 0$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match op
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(assume c):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



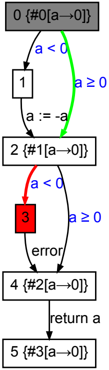
$$\xrightarrow[op \,=\, a < 0]{} \quad \mathcal{S} \rightarrow [a \mapsto a_0]$$

$$\phi \rightarrow a_0 \geq 0 \wedge a_0 < 0$$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match op
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(assume c):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



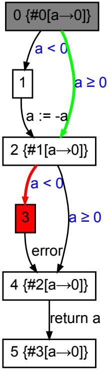
$\xrightarrow[\rho_{i+1} \,=\, true]{}$ $\mathcal{S} \rightarrow [a \mapsto a_0]$

$\phi \rightarrow a_0 \geq 0 \wedge a_0 < 0$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

```
let ⟨ρ0, _⟩ = S0
𝒮 := [v ↦ v0 | v ∈ params(P)]
φ := SymbolicEval(ρ0, 𝒮)
for i = 0 to k − 1 do
    op := Op(Si, Si+1)
    match op
        case(v := e):
            𝒮 := 𝒮[v ↦ SymbolicEval(e, 𝒮)]
        case(assume c):
            φ := φ ∧ SymbolicEval(c, 𝒮)
    let⟨ρi+1, _⟩ = Si+1
    φ := φ ∧ SymbolicEval(ρi+1, 𝒮)
end for
return φ
```



$\xrightarrow{\rho_{i+1} = true}$

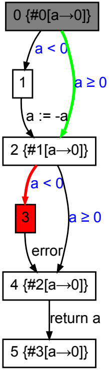
$\mathcal{S} \rightarrow [a \mapsto a_0]$

$\phi \rightarrow a_0 \geq 0 \wedge a_0 < 0$

ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
    $op := \text{Op}(S_i, S_{i+1})$
    match op
        case($v := e$):
            $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
        case(assume c):
            $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
    let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
    $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ



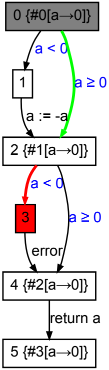
$\longrightarrow$

$$\mathcal{S} \to [a \mapsto a_0]$$
$$\phi \to a_0 \geq 0 \wedge a_0 < 0$$

ExtendFrontier - Refine

ExtendFrontier(τ_c, P)

```
φ := ExecuteSymbolic(τ_c, P)
t := IsSAT(φ, P)
if t = UNSAT then
    ρ := RefinePred(τ_c)
else
    ρ := true
end if
return ⟨t, ρ⟩
```

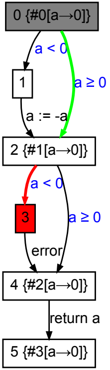


$\longrightarrow \quad \phi \to a_0 \geq 0 \wedge a_0 < 0$

ExtendFrontier - Refine

ExtendFrontier(τ_c, P)

ϕ := ExecuteSymbolic(τ_c, P)
t := IsSAT(ϕ, P)
if $t = \textsc{unsat}$ **then**
    ρ := RefinePred(τ_c)
else
    ρ := *true*
end if
return $\langle t, \rho \rangle$

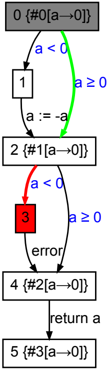


$$\phi \to a_0 \geq 0 \wedge a_0 < 0$$
$$t \to \text{UNSAT}$$

ExtendFrontier - Refine

ExtendFrontier(τ_c, P)

ϕ := ExecuteSymbolic(τ_c, P)
t := IsSAT(ϕ, P)
if $t = \text{UNSAT}$ **then**
    ρ := RefinePred(τ_c)
else
    ρ := *true*
end if
return $\langle t, \rho \rangle$



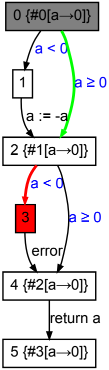
$\longrightarrow$

$$\phi \rightarrow a_0 \geq 0 \wedge a_0 < 0$$
$$t \rightarrow \text{UNSAT}$$

RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)
let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
op := Op(S_{k-1}, S_k)
if op **matches** assume c **then**
    if $k > 1 \wedge \forall s \in states_{k-1}$: Eval($\neg\rho_k, s$) $= true$ **then**
        return ρ_k
    end if
end if
return WP(op, ρ_k)

WP(op, ρ_k)
match op
    case(v := e):
        return $\rho_k[e/v]$
    case(assume c):
        return $c \wedge \rho_k$

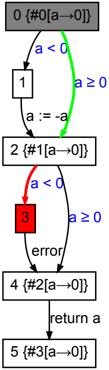


$\longrightarrow$

RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)
let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
    if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = true$ **then**
        return ρ_k
    end if
end if
return WP(op, ρ_k)

WP(op, ρ_k)
match op
    case(v := e):
        return $\rho_k[e/v]$
    case(assume c):
        return $c \wedge \rho_k$



$\longrightarrow$
$\rho_k \rightarrow true$

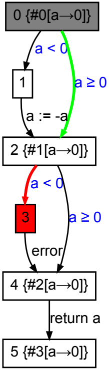
RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
    if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = true$ **then**
        return ρ_k
    end if
end if
return WP(op, ρ_k)

WP(op, ρ_k)

match op
    case(v := e):
        return $\rho_k[e/v]$
    case(assume c):
        return $c \wedge \rho_k$



$\longrightarrow$

$\rho_k \rightarrow true$
$op \rightarrow a < 0$

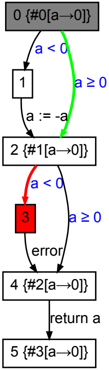
RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if *op* **matches** assume *c* **then**
    if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = true$ **then**
        return ρ_k
    end if
end if
return $\mathrm{WP}(op, \rho_k)$

WP(op, ρ_k)

match *op*
    case(v := e):
        return $\rho_k[e/v]$
    case(assume c):
        return $c \wedge \rho_k$



$\longrightarrow$

$\rho_k \rightarrow true$
$op \rightarrow a < 0$

RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
op := Op(S_{k-1}, S_k)
if op **matches** assume c **then**
  if $k > 1 \wedge \forall s \in states_{k-1}$: Eval($\neg \rho_k, s$) $= true$ **then**
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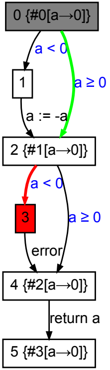
  end if
end if
return WP(op, ρ_k)

WP(op, ρ_k)

match op
  case(v := e):
    return $\rho_k[e/v]$
  case(assume c):
    return $c \wedge \rho_k$



$\longrightarrow$

$\rho_k \rightarrow true$
$op \rightarrow a < 0$

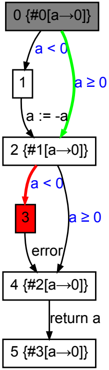
RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op :=$ Op(S_{k-1}, S_k)
if op **matches** assume c **then**
  if $k > 1 \wedge \forall s \in states_{k-1}$: Eval$(\neg\rho_k, s) = true$ **then**
    return ρ_k
  end if
end if
return WP(op, ρ_k)

WP(op, ρ_k)

match op
  case(v := e):
    return $\rho_k[e/v]$
  case(assume c):
    return $c \wedge \rho_k$



$\longrightarrow$

$\rho_k \rightarrow true$
$op \rightarrow a < 0$

RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \text{Op}(S_{k-1}, S_k)$
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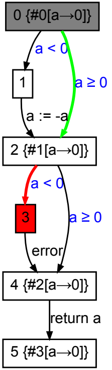
  end if
end if
return WP(op, ρ_k)

WP(op, ρ_k)

match op
  case(v := e):
    return $\rho_k[e/v]$
  case(assume c):
    return $c \wedge \rho_k$



$\longrightarrow$

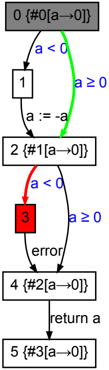
$\rho_k \to true$
$op \to a < 0$

$a < 0$

ExtendFrontier - Refine

ExtendFrontier(τ_c, P)

ϕ := ExecuteSymbolic(τ_c, P)
t := IsSAT(ϕ, P)
if $t = \text{UNSAT}$ **then**
    ρ := RefinePred(τ_c)
else
    ρ := *true*
end if
return $\langle t, \rho \rangle$

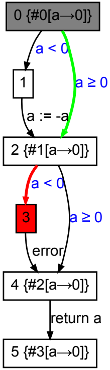


$\longrightarrow$

$$\phi \rightarrow a \geq 0 \wedge a < 0$$
$$t \rightarrow \text{UNSAT}$$
$$\rho \rightarrow a < 0$$

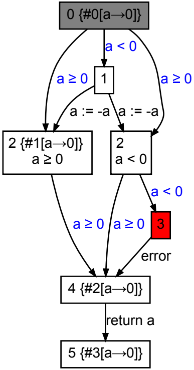
ExtendFrontier - Refine

ExtendFrontier(τ_c, P)
1: ϕ := ExecuteSymbolic(τ_c, P)
2: t := IsSAT(ϕ, P)
3: **if** t = UNSAT **then**
4: ρ := RefinePred(τ_c)
5: **else**
6: ρ := *true*
7: **end if**
8: **return** $\langle t, \rho \rangle$

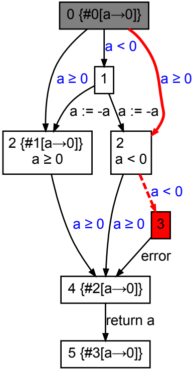


$\longrightarrow \quad \langle UNSAT, a < 0 \rangle$

Last steps of example



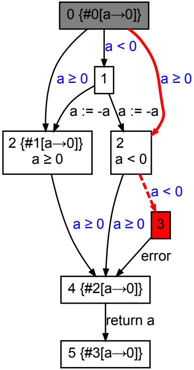
Last steps of example



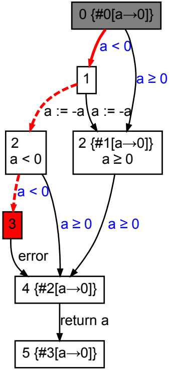
$$\longrightarrow$$

$$\phi \rightarrow a_0 \geq 0 \wedge a_0 < 0$$

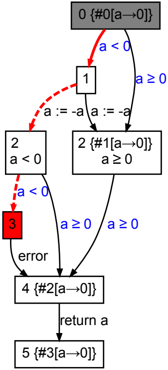
Last steps of example



$$\xrightarrow{\qquad}$$
$$\phi \to a_0 \geq 0 \wedge a_0 < 0$$

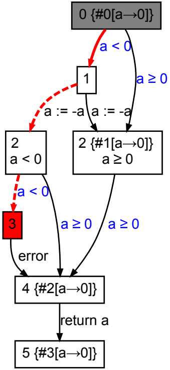


Last steps of example

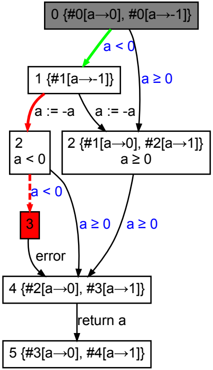


$$\xrightarrow{\quad}$$
$$\phi \rightarrow a_0 < 0$$

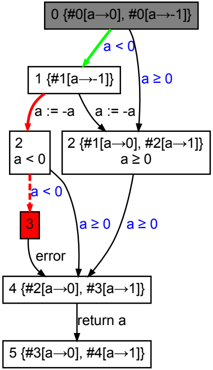
Last steps of example



$$\xrightarrow[\phi \rightarrow a_0 < 0]{}$$

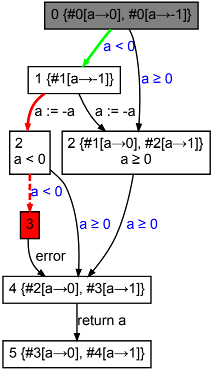


Last steps of example

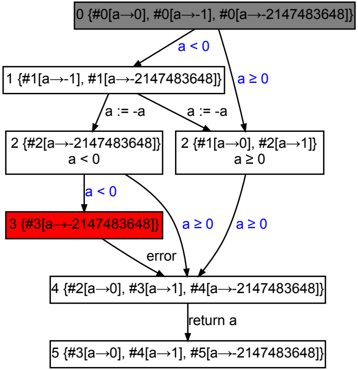


$$\xrightarrow[\phi \rightarrow a_0 < 0 \land -a_0 < 0]{}$$

Last steps of example



$$\xrightarrow{}$$
$$\phi \rightarrow a_0 < 0 \wedge -a_0 < 0$$



Modifications and Challenges

Modifications to DashLoop

loop
    τ := FindAbstractErrorPath(G)
    if τ = NO-PATH **then**
        return (PASS, G)
    end if

    τ_c := ConvertToRegionTraceWithAbstractFrontier(τ, G)
    $\langle t, \rho \rangle$:= ExtendFrontier(τ_c, P)
    if $t \neq$ UNSAT **then**
        G := RunTest(t, P, G)
        if IsErrorRegionReached(G) **then**
            return (FAIL, t)
        end if
    else
        G := RefineGraph(ρ, τ_c, G)
    end if
end loop

$\Sigma_{\simeq} := \bigcup_{l \in L} \{\{(pc, v) \in \Sigma \mid pc = l\}\}$
$\sigma^{I}{}_{\simeq} := \{S \in \Sigma_{\simeq} \mid \text{pc}(S) \textit{ is the the initial pc}\}$
$\rightarrow_{\simeq} := \{(S, S') \in \Sigma_{\simeq} \times \Sigma_{\simeq} \mid \text{Edge}(S, S') \in E\}$
$P_{\simeq} := \langle \Sigma_{\simeq}, \sigma^{I}_{\sigma}, \rightarrow_{\simeq} \rangle$
F:= Test(P)
loop
    if $\varphi \cap F \neq \emptyset$ **then**
        choose $s \in \varphi \cap F$
        t := TestForWitness(s)
        return ("fail", t)
    end if
    τ := GetAbstractTrace($P_{\simeq}$, φ)
    if $\tau = \epsilon$ **then**
        return ("pass", $\Sigma_{\simeq}$)
    else
        τ_0 := GetOrderedAbstractTrace(τ, F)
        $\langle t, \rho \rangle$:= ExtendFrontier(τ_0, F, P)
        if ρ = *true* **then**
            F := AddTestToForest(t, F)
        else
            Refinement of graph
        end if
    end if
end loop

Modifications to DashLoop

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- Remove creation of region graph

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- Remove creation of region graph
- Do no load test input

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- Remove creation of region graph
- Do no load test input
- No forest

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    if t ≠ UNSAT then
        G := RunTest(t, P, G)
        if IsErrorRegionReached(G) then
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        G := RefineGraph(ρ, τ_c, G)
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end loop
```

- Remove creation of region graph
- Do no load test input
- No forest
- Move test for error region reached

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Σ_≃ := ⋃_{l∈L} {{(pc, v) ∈ Σ | pc = l}}
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P_≃ := ⟨Σ_≃, σ^I_σ, →_≃⟩
F := Test(P)
loop
    if φ ∩ F ≠ ∅ then
        choose s ∈ φ ∩ F
        t := TestForWitness(s)
        return ("fail", t)
    end if
    τ := GetAbstractTrace(P_≃, φ)
    if τ = ϵ then
        return ("pass", Σ_≃)
    else
        τ_0 := GetOrderedAbstractTrace(τ, F)
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- Remove creation of region graph
- Do no load test input
- No forest
- Move test for error region reached
- The rest is nearly the same

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end loop
```

Modifications to DashLoop

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- Remove creation of region graph
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$\Sigma_{\simeq} := \bigcup_{l \in L} \{\{(pc, v) \in \Sigma \mid pc = l\}\}$
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  $\tau := \text{FindAbstractErrorPath}(G)$
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  $\tau_c := \text{ConvertToRegionTraceWithAbstractFrontier}(\tau, G)$
  $\langle t, \rho \rangle := \text{ExtendFrontier}(\tau_c, P)$
  if $t \neq \text{UNSAT}$ **then**
    $G := \text{RunTest}(t, P, G)$
    if $\text{IsErrorRegionReached}(G)$ **then**
      return (FAIL, t)
    end if
  else
    $G := \text{RefineGraph}(\rho, \tau_c, G)$
  end if
end loop

- Remove creation of region graph
- Do no load test input
- No forest
- Move test for error region reached
- The rest is nearly the same
- Refinement moved to later slide

$\Sigma_{\simeq} := \bigcup_{l \in L} \{\{(pc, v) \in \Sigma \mid pc = l\}\}$
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How to create a trace from a path

- What makes a trace ordered?
  - GetOrderedAbstractTrace and τ_o

How to create a trace from a path

- What makes a trace ordered?
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  - $\tau_o = S_0, \ldots, S_{k-1}, S_k, \ldots S_n$
- Should the trace τ_c follow the path τ?

```
void foo(int a)
{
  int i = 0;
  while(i < 1)
    i = i + 1;
  assert(a < 0);
}
```

How to create a trace from a path

- What makes a trace ordered?
  - GetOrderedAbstractTrace and τ_o
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  - $\tau_c = S_0, \ldots, S_{k-1}, S_k$
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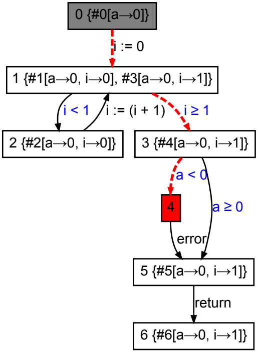

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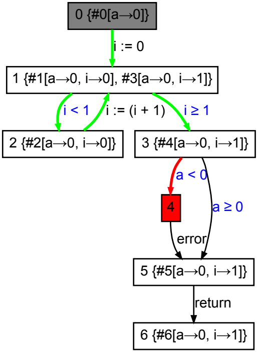
  - $\tau_o = S_0, \ldots, S_{k-1}, S_k, \ldots S_n$
- Should the trace τ_c follow the path τ?
  - No.



How to create a trace from a path

- What makes a trace ordered?
  - GetOrderedAbstractTrace and τ_o
- Why include the tail of the path in the trace?
  - $\tau_c = S_0, \ldots, S_{k-1}, S_k$
  - $\tau_o = S_0, \ldots, S_{k-1}, S_k, \ldots S_n$
- Should the trace τ_c follow the path τ?
  - No.
- Which state to pick when generating a trace?

```
void foo()
{
  int a = 1;
  int i = 0;
  while(i < 1000)
    i = i + 1;
  if(a == 0)
    error;
}
```

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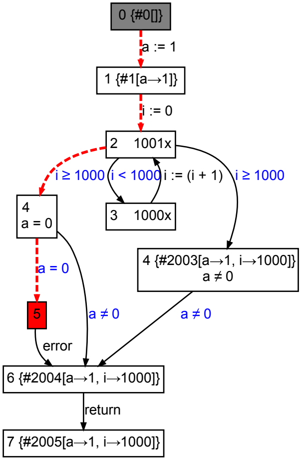

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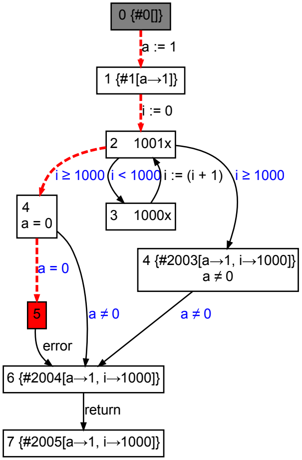
  - Pick random



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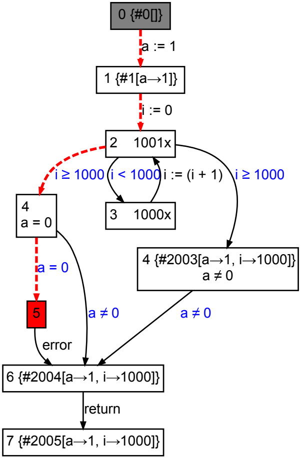
  - Pick the last state



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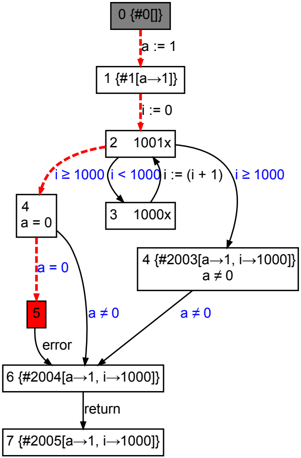

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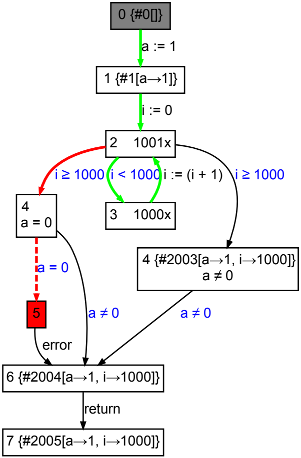

Modifications to refinement

RefineGraph$(\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle, G = \langle \Sigma_\simeq, \to_\simeq \rangle)$

let $\langle \rho_{k-1}, states \rangle = S_{k-1}$
if $k = 1$ **then**
  return $\langle \Sigma_\simeq, \to_\simeq \backslash (S_0, S_1) \rangle$
end if
$\Sigma^*_\simeq := \Sigma_\simeq \setminus \{S_{k-1}\}$
$\to^*_\simeq := \to_\simeq \backslash \{(S, S_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
$\to^*_\simeq := \to^*_\simeq \backslash \{(S_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
$\rho^*_{k-1} := \text{Simplify}(\rho_{k-1} \wedge \neg\rho)$
$S^*_{k-1} := \langle \rho^*_{k-1}, states \rangle$
$\Sigma^*_\simeq := \Sigma^*_\simeq \cup \{S^*_{k-1}\}$
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$\rho^{**}_{k-1} := \text{Simplify}(\rho_{k-1} \wedge \rho)$
$S^{**}_{k-1} := \langle \rho^{**}_{k-1}, \emptyset \rangle$
$\Sigma^*_\simeq := \Sigma^*_\simeq \cup \{S^{**}_{k-1}\}$
$\to^*_\simeq := \to^*_\simeq \cup \{(S^{**}_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
if IsSAT$(\rho^{**}_{k-1}) \neq$ UNSAT **then**
  $\to^*_\simeq := \to^*_\simeq \cup \{(S, S^{**}_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
end if
return $\langle \Sigma^*_\simeq, \to^*_\simeq \rangle$

let $S_0, S_1, \ldots, S_n = \tau_0$ and
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$\Sigma_\simeq := (\Sigma_\simeq \backslash \{S_{k-1}\}) \cup \{S_{k-1} \wedge \rho, S_{k-1} \wedge \neg\rho\}$
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$\to_\simeq := \to_\simeq \cup \{(S, S_{k-1} \wedge \rho) \mid S \in \text{Parents}(S_{k-1})\}$
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Modifications to refinement

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$\setminus \{(S_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
$\to_\simeq := \to_\simeq \cup \{(S, S_{k-1} \wedge \rho) \mid S \in \text{Parents}(S_{k-1})\}$
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- Disambiguate regions and region predicates

Modifications to refinement

RefineGraph($\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle, G = \langle \Sigma_\simeq, \to_\simeq \rangle$)

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if IsSAT(ρ^{**}_{k-1}) $\neq$ UNSAT **then**
  $\to^*_\simeq := \to^*_\simeq \cup \{(S, S^{**}_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
end if
return $\langle \Sigma^*_\simeq, \to^*_\simeq \rangle$

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$\to_\simeq := \to_\simeq \cup \{(S, S_{k-1} \wedge \rho) \mid S \in \text{Parents}(S_{k-1})\}$
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- Disambiguate regions and region predicates
- Spell out what happens

Modifications to refinement

RefineGraph(ρ, $\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$, $G = \langle \Sigma_\simeq, \rightarrow_\simeq \rangle$)

```
let⟨ρ_{k−1}, states⟩ = S_{k−1}
if k = 1 then
    return ⟨Σ_≃, →_≃ \(S_0, S_1)⟩
end if
Σ*_≃ := Σ_≃ \ {S_{k−1}}
→*_≃ := →_≃ \{(S, S_{k−1}) | S ∈ Parents(S_{k−1})}
→*_≃ := →*_≃ \{(S_{k−1}, S) | S ∈ Children(S_{k−1})}
ρ*_{k−1} := Simplify(ρ_{k−1} ∧ ¬ρ)
S*_{k−1} := ⟨ρ*_{k−1}, states⟩
Σ*_≃ := Σ*_≃ ∪ {S*_{k−1}}
→*_≃ := →*_≃ ∪{(S, S*_{k−1}) | S ∈ Parents(S_{k−1})}
→*_≃ := →*_≃ ∪{(S*_{k−1}, S) | S ∈ Children(S_{k−1})}
→*_≃ := →*_≃ \{(S*_{k−1}, S_k)}
ρ**_{k−1} := Simplify(ρ_{k−1} ∧ ρ)
S**_{k−1} := ⟨ρ**_{k−1}, ∅⟩
Σ*_≃ := Σ*_≃ ∪ {S**_{k−1}}
→*_≃ := →*_≃ ∪{(S**_{k−1}, S) | S ∈ Children(S_{k−1})}
if IsSAT(ρ**_{k−1}) ≠ UNSAT then
    →*_≃ := →*_≃ ∪{(S, S**_{k−1}) | S ∈ Parents(S_{k−1})}
end if
return ⟨Σ*_≃, →*_≃⟩
```

```
let S_0, S_1, ..., S_n = τ_0 and
(k − 1, k) = Frontier(τ_0) in
Σ_≃ := (Σ_≃\{S_{k−1}}) ∪ {S_{k−1} ∧ ρ, S_{k−1} ∧ ¬ρ}
→_≃ := (→_≃ \{(S, S_{k−1}) | S ∈Parents(S_{k−1})})
        \{(S_{k−1}, S) | S ∈ Children(S_{k−1})}
→_≃ := →_≃ ∪{(S, S_{k−1} ∧ ρ) | S ∈ Parents(S_{k−1})}
        ∪{(S, S_{k−1} ∧ ¬ρ) | S ∈ Parents(S_{k−1})}
        ∪{(S_{k−1} ∧ ρ, S) | S ∈ Children(S_{k−1})}
        ∪{(S_{k−1} ∧ ¬ρ, S) | S ∈ (Children(S_{k−1})\{S_k})}
```

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Modifications to refinement

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if IsSAT$(\rho^{**}_{k-1}) \neq$ UNSAT **then**
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let $\langle \rho_{k-1}, states \rangle = S_{k-1}$
if $k = 1$ **then**
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$\Sigma_{\simeq}^* := \Sigma_{\simeq} \setminus \{S_{k-1}\}$
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$\rho_{k-1}^{**} := \text{Simplify}(\rho_{k-1} \wedge \rho)$
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if IsSAT(ρ_{k-1}^{**}) $\neq$ UNSAT **then**
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$\to_{\simeq} := \to_{\simeq} \cup \{(S, S_{k-1} \wedge \rho) \mid S \in \text{Parents}(S_{k-1})\}$
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return $\langle \Sigma^*_{\simeq}, \rightarrow^*_{\simeq} \rangle$

1: let $S_0, S_1, \ldots, S_n = \tau_0$ and
2: $(k-1, k) = \text{Frontier}(\tau_0)$ in
3: $\Sigma_{\simeq} := (\Sigma_{\simeq} \setminus \{S_{k-1}\}) \cup \{S_{k-1} \wedge \rho, S_{k-1} \wedge \neg\rho\}$
4: $\rightarrow_{\simeq} := (\rightarrow_{\simeq} \setminus \{(S, S_{k-1}) \mid S \in \text{Parents}(S_{k-1})\})$
$\setminus \{(S_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
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- Disambiguate regions and region predicates
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Modifications to refinement

RefineGraph($\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle, G = \langle \Sigma_{\simeq}, \to_{\simeq} \rangle$)

let $\langle \rho_{k-1}, \mathit{states} \rangle = S_{k-1}$
if $k = 1$ **then**
  return $\langle \Sigma_{\simeq}, \to_{\simeq} \setminus (S_0, S_1) \rangle$
end if
$\Sigma_{\simeq}^{*} := \Sigma_{\simeq} \setminus \{S_{k-1}\}$
$\to_{\simeq}^{*} := \to_{\simeq} \setminus \{(S, S_{k-1}) \mid S \in \text{Parents}(S_{k-1})\}$
$\to_{\simeq}^{*} := \to_{\simeq}^{*} \setminus \{(S_{k-1}, S) \mid S \in \text{Children}(S_{k-1})\}$
$\rho_{k-1}^{*} := \text{Simplify}(\rho_{k-1} \wedge \neg\rho)$
$S_{k-1}^{*} := \langle \rho_{k-1}^{*}, \mathit{states} \rangle$
$\Sigma_{\simeq}^{*} := \Sigma_{\simeq}^{*} \cup \{S_{k-1}^{*}\}$
$\to_{\simeq}^{*} := \to_{\simeq}^{*} \cup \{(S, S_{k-1}^{*}) \mid S \in \text{Parents}(S_{k-1})\}$
$\to_{\simeq}^{*} := \to_{\simeq}^{*} \cup \{(S_{k-1}^{*}, S) \mid S \in \text{Children}(S_{k-1})\}$
$\to_{\simeq}^{*} := \to_{\simeq}^{*} \setminus \{(S_{k-1}^{*}, S_k)\}$
$\rho_{k-1}^{**} := \text{Simplify}(\rho_{k-1} \wedge \rho)$
$S_{k-1}^{**} := \langle \rho_{k-1}^{**}, \emptyset \rangle$
$\Sigma_{\simeq}^{*} := \Sigma_{\simeq}^{*} \cup \{S_{k-1}^{**}\}$
$\to_{\simeq}^{*} := \to_{\simeq}^{*} \cup \{(S_{k-1}^{**}, S) \mid S \in \text{Children}(S_{k-1})\}$
if $\text{IsSAT}(\rho_{k-1}^{**}) \neq \textsc{unsat}$ **then**
  $\to_{\simeq}^{*} := \to_{\simeq}^{*} \cup \{(S, S_{k-1}^{**}) \mid S \in \text{Parents}(S_{k-1})\}$
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return $\langle \Sigma_{\simeq}^{*}, \to_{\simeq}^{*} \rangle$

let $S_0, S_1, \ldots, S_n = \tau_0$ and
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Modifications to refinement

RefineGraph($\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle, G = \langle \Sigma_{\simeq}, \to_{\simeq} \rangle$)

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if IsSAT(ρ^{**}_{k-1}) $\neq$ UNSAT **then**
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Modifications to refinement

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$\rho^{**}_{k-1} := \text{Simplify}(\rho_{k-1} \wedge \rho)$
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if IsSAT$(\rho^{**}_{k-1}) \neq$ UNSAT **then**
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Modifications to refinement

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- Disambiguate regions and region predicates
- Spell out what happens
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Modifications to refinement

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- Disambiguate regions and region predicates
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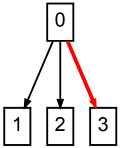
- Remove incoming edges (remove infeasible regions)
- Initial region refinement

Initial region refinement

- How should the initial region be refined?



Initial region refinement

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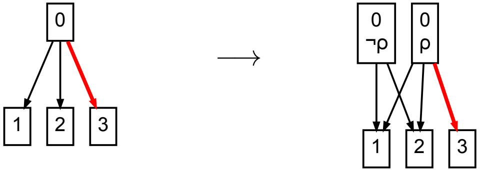
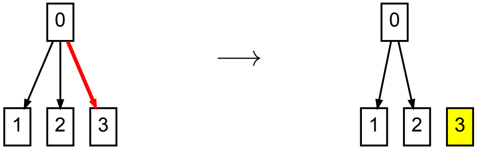


Initial region refinement

- How should the initial region be refined?
  - Like any other region (Multiple initial regions)
  - Remove infeasible edge



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RefineGraph($\rho, \tau_c = \langle S_0, \ldots, S_{k-1}, S_k\rangle, G = \langle \Sigma_{\simeq}, \to_{\simeq}\rangle$)

let $\langle \rho_{k-1}, \mathit{states}\rangle = S_{k-1}$
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Modifications to ExtendFrontier

ExtendFrontier(τ_c, P)

ϕ:= ExecuteSymbolic(τ_c, P)
t := IsSAT(ϕ, P)
if t = UNSAT **then**
  ρ := RefinePred(τ_c)
else
  ρ := *true*
end if
return $\langle t, \rho \rangle$

ExtendFrontier(τ, F, P)

$(k-1, k)$:= Frontier(τ)
$\langle \phi_1, S, \phi_2 \rangle$:= ExecuteSymbolic(τ, F, P)
t := IsSAT(ϕ_1, S, ϕ_2, P)
if $t = \epsilon$ **then**
  ρ := RefinePred(S,τ)
else
  ρ := *true*
end if
return $\langle t, \rho \rangle$

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(k − 1, k) := Frontier(τ)
⟨ϕ_1, S, ϕ_2⟩ := ExecuteSymbolic(τ, F, P)
t := IsSAT(ϕ_1, S, ϕ_2, P)
if t = ε then
    ρ := RefinePred(S, τ)
else
    ρ := true
end if
return ⟨t, ρ⟩
```

- Consistent naming of τ and τ_o/τ_c

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- Consistent naming of τ and τ_o/τ_c
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- Simpler path constraint $(\phi_1 \wedge S \wedge \phi_2)$

Modifications to ExtendFrontier

ExtendFrontier(τ_c, P)

ϕ:= ExecuteSymbolic(τ_c, P)
t := IsSAT(ϕ, P)
if $t = \textsc{unsat}$ **then**
  ρ := RefinePred(τ_c)
else
  ρ := *true*
end if
return $\langle t, \rho \rangle$

ExtendFrontier(τ, F, P)

$(k-1, k)$:= Frontier(τ)
$\langle \phi_1, S, \phi_2 \rangle$:= ExecuteSymbolic(τ, F, P)
t := IsSAT(ϕ_1, S, ϕ_2, P)
if $t = \epsilon$ **then**
  ρ := RefinePred(S,τ)
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return $\langle t, \rho \rangle$

- Consistent naming of τ and τ_o/τ_c
- Removal of Frontier since $k-1$ and k is unused
- Simpler path constraint $(\phi_1 \wedge S \wedge \phi_2)$
- Use $\textsc{unsat}$ to indicate unsatisfiability, instead of ϵ

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match *op*
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(*assumec*):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ

ExecuteSymbolic(τ_0, F, P)

$(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$
$S := [v \mapsto v_0 \mid *v \in \mathit{inputs}(P)]$
$\phi_1 := \text{SymbolicEval}(S_0, S)$
$\phi_2 := \mathit{true}$
$i := 0$
while $i \neq k - 1$ **do**
  $op := \lambda(\text{Edge}(S_i, S_{i+1}))$
  match *op*
    case($*$m=e):
      $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
    case(if e goto l):
      $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$
  $i := i + 1$
  $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$
end while
$op := \lambda(\text{Edge}(S_{k-1}, S_k))$
match *op*
  case($*$m=e):
    $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$
    $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
  case(if e goto l):
    $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$
    $S' = S$
$\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$
return $\langle \phi_1, S, \phi_2 \rangle$

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
    $op := \text{Op}(S_i, S_{i+1})$
    match op
        case($v := e$):
            $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
        case(*assumec*):
            $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
    let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
    $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ

- $\phi = \phi_1 \wedge \phi 2$

ExecuteSymbolic(τ_0, F, P)

$(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$
$S := [v \mapsto v_0 \mid *v \in \mathit{inputs}(P)]$
$\phi_1 := \text{SymbolicEval}(S_0, S)$
$\phi_2 := \mathit{true}$
$i := 0$
while $i \neq k - 1$ **do**
    $op := \lambda(\text{Edge}(S_i, S_{i+1}))$
    match op
        case($*$m=e):
            $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
        case(if e goto l):
            $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$
    $i := i + 1$
    $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$
end while
$op := \lambda(\text{Edge}(S_{k-1}, S_k))$
match op
    case($*$m=e):
        $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$
        $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
    case(if e goto l):
        $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$
        $S' = S$
$\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$
return $\langle \phi_1, S, \phi_2 \rangle$

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
    $op := \text{Op}(S_i, S_{i+1})$
    match op
        case($v := e$):
            $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
        case(*assumec*):
            $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
    let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
    $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ

- $\phi = \phi_1 \wedge \phi 2$
- Remove case for assigning to dereferenced pointer

ExecuteSymbolic(τ_0, F, P)

$(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$
$S := [v \mapsto v_0 \mid *v \in \textit{inputs}(P)]$
$\phi_1 := \text{SymbolicEval}(S_0, S)$
$\phi_2 := \textit{true}$
$i := 0$
while $i \neq k - 1$ **do**
    $op := \lambda(\text{Edge}(S_i, S_{i+1}))$
    match op
        case($*\text{m}=\text{e}$):
            $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
        case(if e goto l):
            $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$
    $i := i + 1$
    $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$
end while
$op := \lambda(\text{Edge}(S_{k-1}, S_k))$
match op
    case($*\text{m}=\text{e}$):
        $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$
        $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
    case(if e goto l):
        $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$
        $S' = S$
$\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$
return $\langle \phi_1, S, \phi_2 \rangle$

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match op
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(*assumec*):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ

- $\phi = \phi_1 \wedge \phi 2$
- Remove case for assigning to dereferenced pointer
  - No need to return the symbolic map

ExecuteSymbolic(τ_0, F, P)

$(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$
$S := [v \mapsto v_0 \mid *v \in \textit{inputs}(P)]$
$\phi_1 := \text{SymbolicEval}(S_0, S)$
$\phi_2 := \textit{true}$
$i := 0$
while $i \neq k - 1$ **do**
  $op := \lambda(\text{Edge}(S_i, S_{i+1}))$
  match op
    case($*$m=e):
      $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
    case(if e goto l):
      $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$
  $i := i + 1$
  $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$
end while
$op := \lambda(\text{Edge}(S_{k-1}, S_k))$
match op
  case($*$m=e):
    $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$
    $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
  case(if e goto l):
    $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$
    $S' = S$
$\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$
return $\langle \phi_1, S, \phi_2 \rangle$

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

let $\langle \rho_0, \_ \rangle = S_0$
$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
for $i = 0$ **to** $k - 1$ **do**
  $op := \text{Op}(S_i, S_{i+1})$
  match op
    case($v := e$):
      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    case(*assumec*):
      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
end for
return ϕ

- $\phi = \phi_1 \wedge \phi 2$
- Remove case for assigning to dereferenced pointer
  - No need to return the symbolic map
  - No special case for the last iteration

ExecuteSymbolic(τ_0, F, P)

$(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$
$S := [v \mapsto v_0 \mid *v \in \mathit{inputs}(P)]$
$\phi_1 := \text{SymbolicEval}(S_0, S)$
$\phi_2 := \mathit{true}$
$i := 0$
while $i \neq k - 1$ **do**
  $op := \lambda(\text{Edge}(S_i, S_{i+1}))$
  match op
    case($*$m=e):
      $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
    case(if e goto l):
      $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$
  $i := i + 1$
  $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$
end while
$op := \lambda(\text{Edge}(S_{k-1}, S_k))$
match op
  case($*$m=e):
    $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$
    $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
  case(if e goto l):
    $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$
    $S' = S$
$\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$
return $\langle \phi_1, S, \phi_2 \rangle$

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle, P$)

- **let** $\langle \rho_0, \_ \rangle = S_0$
- $\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$
- $\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$
- **for** $i = 0$ **to** $k - 1$ **do**
  - $op := \text{Op}(S_i, S_{i+1})$
  - **match** op
    - **case**($v := e$):
      - $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$
    - **case**($assume c$):
      - $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$
  - **let** $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$
  - $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$
- **end for**
- **return** ϕ

- $\phi = \phi_1 \wedge \phi 2$
- Remove case for assigning to dereferenced pointer
  - No need to return the symbolic map
  - No special case for the last iteration
- Add case for regular assignment

ExecuteSymbolic(τ_0, F, P)

- $(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$
- $S := [v \mapsto v_0 \mid *v \in inputs(P)]$
- $\phi_1 := \text{SymbolicEval}(S_0, S)$
- $\phi_2 := true$
- $i := 0$
- **while** $i \neq k - 1$ **do**
  - $op := \lambda(\text{Edge}(S_i, S_{i+1}))$
  - **match** op
    - **case**($*$m=e):
      - $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
    - **case**(if e goto l):
      - $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$
  - $i := i + 1$
  - $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$
- **end while**
- $op := \lambda(\text{Edge}(S_{k-1}, S_k))$
- **match** op
  - **case**($*$m=e):
    - $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$
    - $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$
  - **case**(if e goto l):
    - $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$
    - $S' = S$
- $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$
- **return** $\langle \phi_1, S, \phi_2 \rangle$

Modifications to ExecuteSymbolic

ExecuteSymbolic($\tau_c = \langle S_0, \ldots, S_k \rangle$, P)

let $\langle \rho_0, \_ \rangle = S_0$

$\mathcal{S} := [v \mapsto v_0 \mid v \in \text{params}(P)]$

$\phi := \text{SymbolicEval}(\rho_0, \mathcal{S})$

for $i = 0$ **to** $k - 1$ **do**

  $op := \text{Op}(S_i, S_{i+1})$

  match op

    case($v := e$):

      $\mathcal{S} := \mathcal{S}[v \mapsto \text{SymbolicEval}(e, \mathcal{S})]$

    case(*assumec*):

      $\phi := \phi \wedge \text{SymbolicEval}(c, \mathcal{S})$

  let $\langle \rho_{i+1}, \_ \rangle = S_{i+1}$

  $\phi := \phi \wedge \text{SymbolicEval}(\rho_{i+1}, \mathcal{S})$

end for

return ϕ

- $\phi = \phi_1 \wedge \phi 2$
- Remove case for assigning to dereferenced pointer
  - No need to return the symbolic map
  - No special case for the last iteration
- Add case for regular assignment
- Simpler handling of trace

ExecuteSymbolic(τ_0, F, P)

$(k-1, k) := \text{Frontier}(\tau_0 = \langle S_0, S_1, \ldots, S_n \rangle)$

$S := [v \mapsto v_0 \mid *v \in \mathit{inputs}(P)]$

$\phi_1 := \text{SymbolicEval}(S_0, S)$

$\phi_2 := \mathit{true}$

$i := 0$

while $i \neq k - 1$ **do**

  $op := \lambda(\text{Edge}(S_i, S_{i+1}))$

  match op

    case($*$m=e):

      $S := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$

    case(if e goto l):

      $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(e, S)$

  $i := i + 1$

  $\phi_1 := \phi_1 \wedge \text{SymbolicEval}(S_i, S)$

end while

$op := \lambda(\text{Edge}(S_{k-1}, S_k))$

match op

  case($*$m=e):

    $\phi_2 := \phi_2 \wedge *(\text{SymbolicEval}(m, S)) = \text{SymbolicEval}(e, S)$

    $S' := S + [\text{SymbolicEval}(m, S) \mapsto \text{SymbolicEval}(e, S)]$

  case(if e goto l):

    $\phi_2 := \phi_2 \wedge \text{SymbolicEval}(e, S)$

    $S' = S$

$\phi_2 := \phi_2 \wedge \text{SymbolicEval}(S_k, S')$

return $\langle \phi_1, S, \phi_2 \rangle$

Modifications to RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if *op* **matches** assume c **then**
    if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = \mathrm{true}$ **then**
        return ρ_k
    end if
end if
return $\mathrm{WP}(op, \rho_k)$

WP(op, p)

match *op*
    case(v := e):
        return $p[\mathrm{e}/\mathrm{V}]$
    case(assume c):
        return $\mathbf{C} \wedge p$

RefinePred(S,τ_o)

$(k-1, k) := \mathrm{Frontier}(\tau_o = \langle S_0, S_1, \ldots, S_m \rangle)$
$op := \lambda(\mathrm{Edge}(S_{k-1}, S_k))$
$\alpha := \mathrm{Aliases}(S, op, S_k)$
return $\mathrm{WP}_\alpha(op, S_k)$

$$WP_\alpha(op, \phi) = \neg(\alpha \wedge \neg(\alpha \wedge WP(op, \phi)))$$

Modifications to RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
    if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = \mathrm{true}$ **then**
        return ρ_k
    end if
end if
return WP(op, ρ_k)

WP(op, p)

match op
    case(v := e):
        return $p[\mathrm{e}/\mathrm{V}]$
    case(assume c):
        return $\mathrm{C} \wedge p$

RefinePred(S,τ_o)

$(k-1, k) := \mathrm{Frontier}(\tau_o = \langle S_0, S_1, \ldots, S_m \rangle)$
$op := \lambda(\mathrm{Edge}(S_{k-1}, S_k))$
$\alpha := \mathrm{Aliases}(S, op, S_k)$
return $\mathrm{WP}_\alpha(op, S_k)$

$$WP_\alpha(op, \phi) = \neg(\alpha \wedge \neg(\alpha \wedge WP(op, \phi)))$$

- Provide implementation for WP

Modifications to RefinePred

RefinePred($\tau_c = \langle S_0, \dots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op :=$ Op(S_{k-1}, S_k)
if op **matches** assume c **then**
  if $k > 1 \wedge \forall s \in states_{k-1}$: Eval($\neg\rho_k, s$) $=$ true **then**
    return ρ_k
  end if
end if
return WP(op, ρ_k)

WP(op, p)

match op
  case(v := e):
    return $p[\mathrm{e}/\mathrm{v}]$
  case(assume c):
    return $\mathbf{C} \wedge p$

RefinePred(S,τ_o)

$(k-1, k) :=$ Frontier($\tau_o = \langle S_0, S_1, \dots, S_m \rangle$)
$op := \lambda$(Edge(S_{k-1}, S_k))
$\alpha :=$Aliases(S, op, S_k)
return WP$_\alpha$(op, S_k)

$$WP_\alpha(op, \phi) = \neg(\alpha \wedge \neg(\alpha \wedge WP(op, \phi)))$$

- Provide implementation for WP
- Remove α

Modifications to RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op :=$ Op(S_{k-1}, S_k)
if op **matches** assume c **then**
    if $k > 1 \land \forall s \in states_{k-1} : \text{Eval}(\neg \rho_k, s) = \text{true}$ **then**
        return ρ_k
    end if
end if
return WP(op, ρ_k)

WP(op, p)

match op
    case(v := e):
        return $p[\mathrm{e}/\mathrm{v}]$
    case(assume c):
        return $\mathbf{C} \land p$

RefinePred(S, τ_o)

$(k-1, k) :=$ Frontier($\tau_o = \langle S_0, S_1, \ldots, S_m \rangle$)
$op := \lambda$(Edge(S_{k-1}, S_k))
$\alpha :=$Aliases(S, op, S_k)
return WP$_\alpha$(op, S_k)

$$WP_\alpha(op, \phi) = \neg(\alpha \land \neg(\alpha \land WP(op, \phi)))$$

- Provide implementation for WP
- Remove α
- Simpler trace

Modifications to RefinePred

RefinePred($\tau_c = \langle S_0, \ldots, S_{k-1}, S_k \rangle$)

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
  if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = \mathrm{true}$ **then**
    return ρ_k
  end if
end if
return WP(op, ρ_k)

WP(op, p)

match op
  case(v := e):
    return $p[\mathrm{e}/\mathrm{v}]$
  case(assume c):
    return $\mathbf{C} \wedge p$

RefinePred(S,τ_o)

$(k-1, k) := \mathrm{Frontier}(\tau_o = \langle S_0, S_1, \ldots, S_m \rangle)$
$op := \lambda(\mathrm{Edge}(S_{k-1}, S_k))$
$\alpha :=\mathrm{Aliases}(S, op, S_k)$
return $\mathrm{WP}_\alpha(op, S_k)$

$WP_\alpha(op, \phi) = \neg(\alpha \wedge \neg(\alpha \wedge WP(op, \phi)))$

- Provide implementation for WP
- Remove α
- Simpler trace
- Add the loop optimization

Loop optimization

1: **let** $\langle \_, states_{k-1} \rangle = S_{k-1}$
2: **let** $\langle \rho_k, \_ \rangle = S_k$
3: $op := \mathrm{Op}(S_{k-1}, S_k)$
4: **if** op **matches** assume c **then**
5: **if** $\forall s \in states_{k-1} : \mathrm{Eval}(\neg\rho_k, s) = \mathrm{true}$ **then**
6: **return** ρ_k
7: **end if**
8: **end if**
9: **return** $\mathrm{WP}(op, \rho_k)$

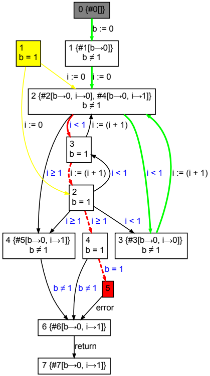
```
void test()
{
  int b = 0;
  int i = 0;
  while (i < 1)
    i++;
  if (b == 1)
    error;
}
```

- Loop optimization ignores assume on edge

Loop optimization

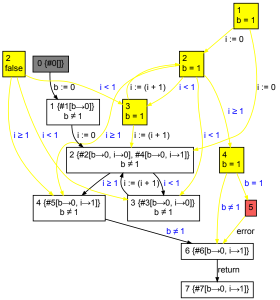
let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
  if $\forall s \in states_{k-1} : \mathrm{Eval}(\neg\rho_k, s) = \mathrm{true}$ **then**
    return ρ_k
  end if
end if
return WP(op, ρ_k)

- Loop optimization ignores assume on edge



Loop optimization

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
    if $\forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = \mathrm{true}$ **then**
        return ρ_k
    end if
end if
return $\mathrm{WP}(op, \rho_k)$



- Loop optimization ignores assume on edge

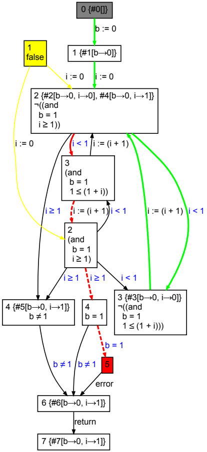
Loop optimization

```
let ⟨_, states_{k-1}⟩ = S_{k-1}
let ⟨ρ_k, _⟩ = S_k
op := Op(S_{k-1}, S_k)
if op matches assume c then
    if k > 1 ∧ ∀s ∈ states_{k-1} : Eval(¬ρ_k, s) = true then
        return ρ_k
    end if
end if
return WP(op, ρ_k)
```

- Loop optimization ignores assume on edge
- Disable loop optimization for initial region

Loop optimization

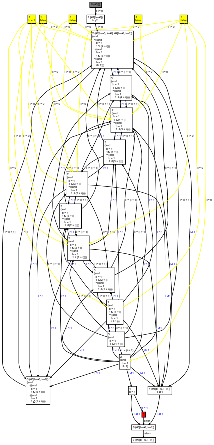
let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if op **matches** assume c **then**
  if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = \mathrm{true}$ **then**
    return ρ_k
  end if
end if
return $\mathrm{WP}(op, \rho_k)$



- Loop optimization ignores assume on edge
- Disable loop optimization for initial region
- Problem: Infinite refines, due to irrelevant predicates

Loop optimization

let $\langle \_, states_{k-1} \rangle = S_{k-1}$
let $\langle \rho_k, \_ \rangle = S_k$
$op := \mathrm{Op}(S_{k-1}, S_k)$
if *op* **matches** assume c **then**
    if $k > 1 \wedge \forall s \in states_{k-1} : \mathrm{Eval}(\neg \rho_k, s) = \text{true}$ **then**
        return ρ_k
    end if
end if
return $\mathrm{WP}(op, \rho_k)$



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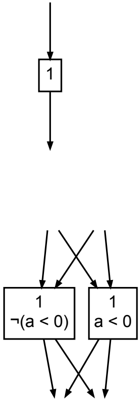
Optimizations overview

- Split regions initially
- SP heuristic (Loop optimization)
- CD heuristic
- Interprocedural
- Other

Split regions initially

Split the regions in the original region graph, based on conditionals that are needed to reach the error statement.



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  - Else add back assumes for the found error trace and then try again.

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3. Regular analysis

Other

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  - For Java this is native methods