Week 10

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1 Question 1

(a) X is continuous so $P(X = 0.5) = 0$

(b) $P(0.25 \leq x \leq 0.5) = P(x < 0.5) - P(x < 0.25) = 0.25$

(c) $P(-1 \leq x \leq 0.5)$ is 0.5 as $P(x < -1) = 0$

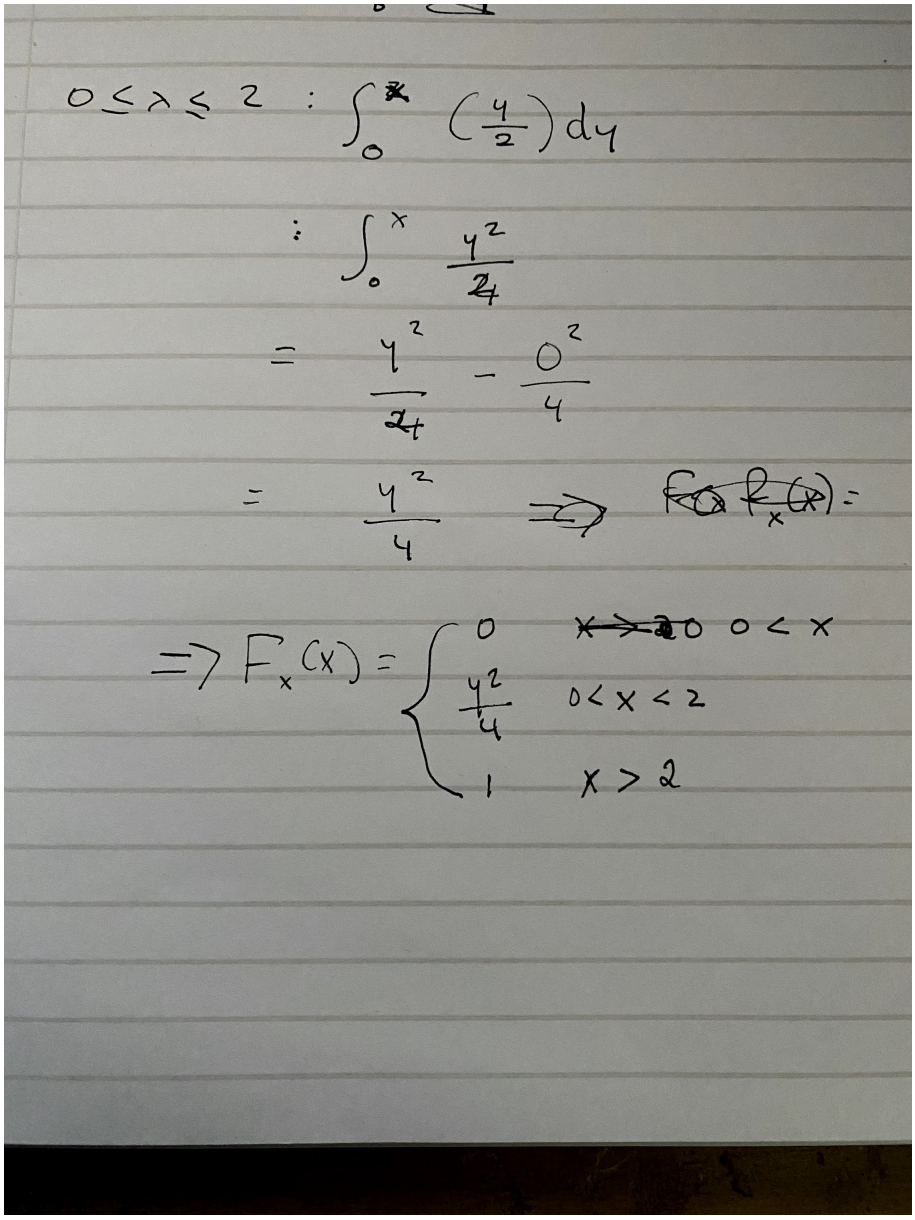
2 Question 2

(b)

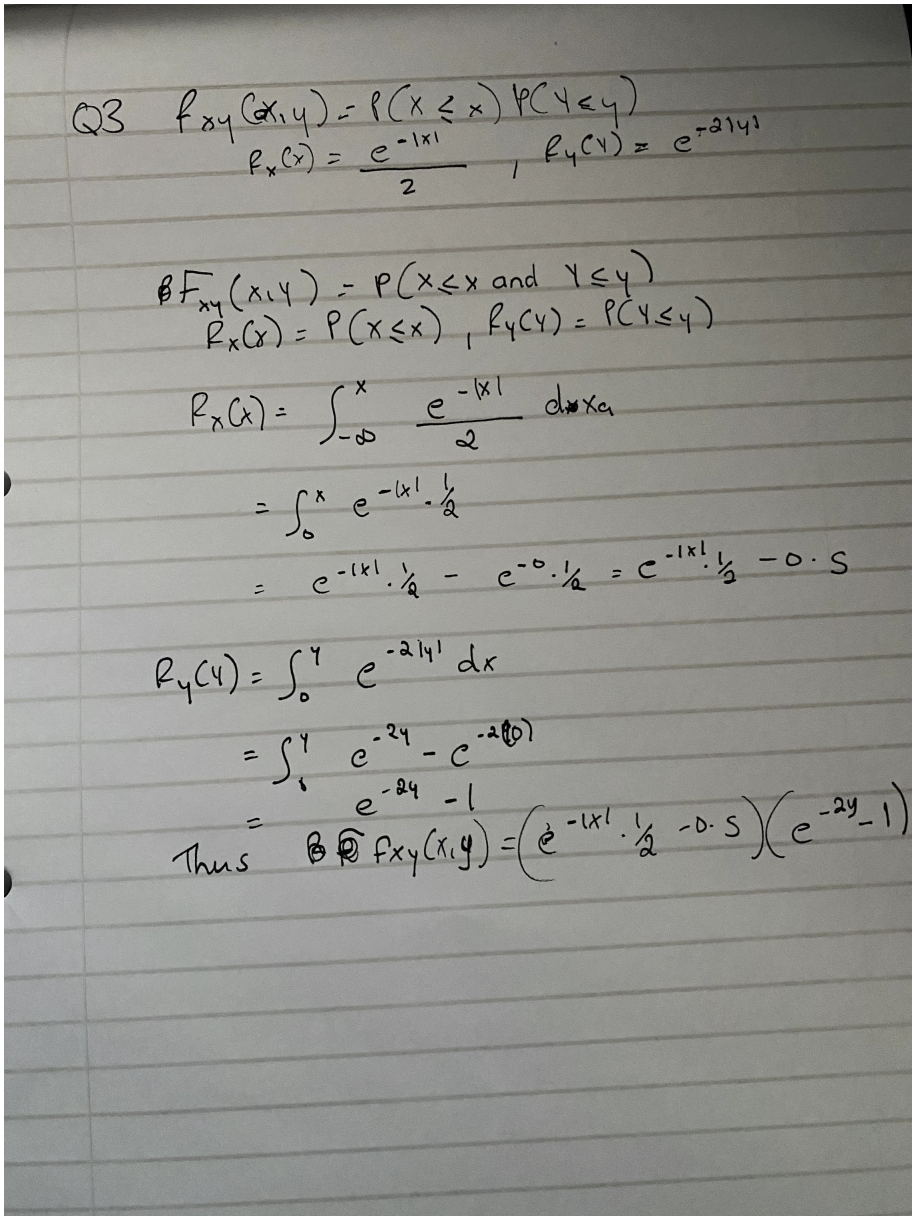
$$P(0.5 \leq X \leq 10) = P(X \leq 10) - P(X \leq 0.5)$$

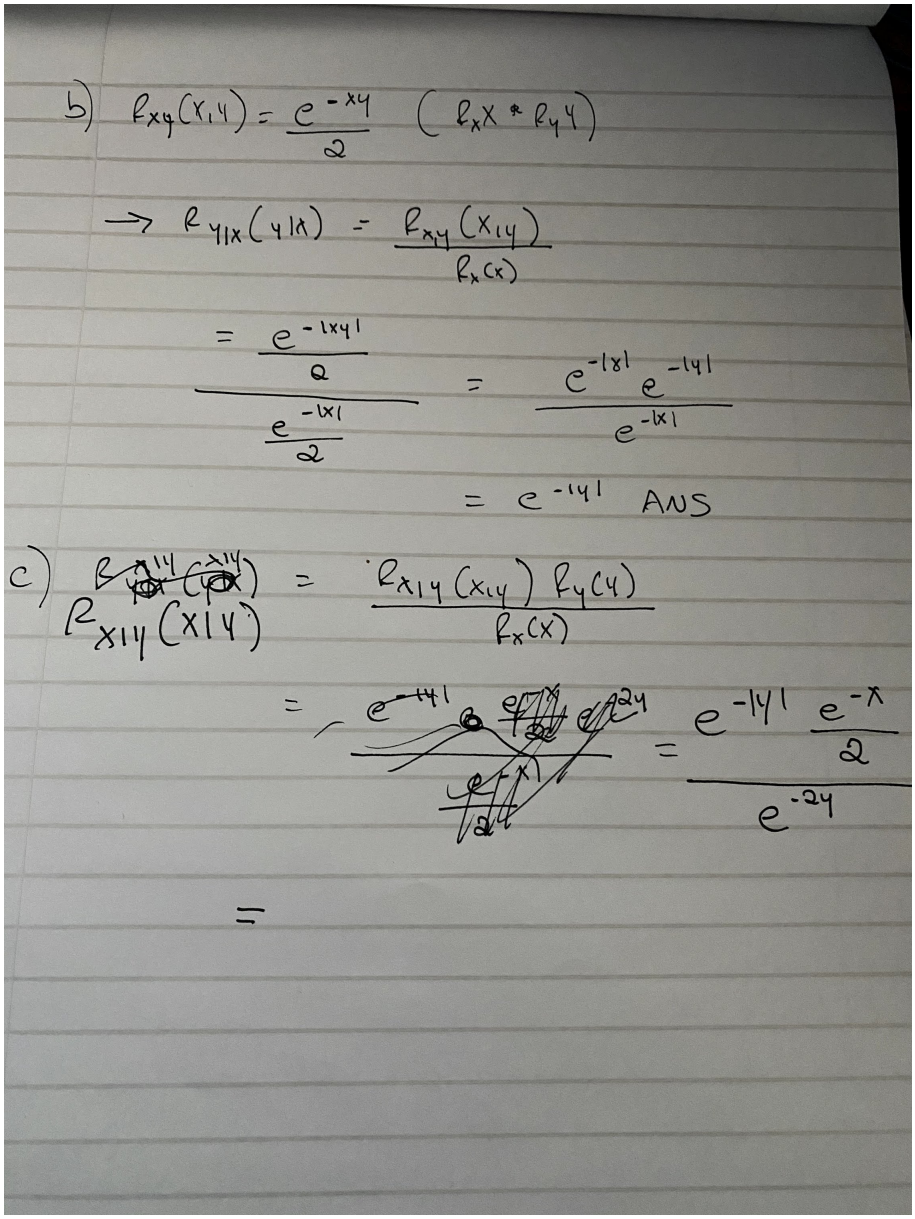
$$1 - \frac{0.5^2}{4} = 0.9375$$

(c)


$0 \le x \le 2 : \int_0^x \left(\frac{y}{2}\right) dy$
$: \int_0^x \frac{y^2}{4}$
$= \frac{y^2}{4} - \frac{0^2}{4}$
$= \frac{y^2}{4}$
$\Rightarrow F_x(x) = \begin{cases} 0 & 0 < x \\ \frac{y^2}{4} & 0 < x < 2 \\ 1 & x > 2 \end{cases}$

3 Question 3


Q3 $f_{XY}(x,y) = P(X \le x)\,P(Y \le y)$
$F_X(x) = \frac{e^{-|x|}}{2}$, $F_Y(y) = e^{-2|y|}$
$F_{XY}(x,y) = P(X \le x \text{ and } Y \le y)$
$F_X(x) = P(X \le x)$, $F_Y(y) = P(Y \le y)$
$F_X(x) = \int_{-\infty}^{x} \frac{e^{-|x|}}{2}\,dx$
$= \int_{0}^{x} e^{-|x|} \cdot \frac{1}{2}$
$= e^{-|x|} \cdot \frac{1}{2} - e^{-0} \cdot \frac{1}{2} = e^{-|x|} \cdot \frac{1}{2} - 0.5$
$F_Y(y) = \int_{0}^{y} e^{-2|y|}\,dx$
$= \int_{0}^{y} e^{-2y} - e^{-2(0)}$
$= e^{-2y} - 1$
Thus $f_{XY}(x,y) = \left(e^{-|x|} \cdot \frac{1}{2} - 0.5\right)\left(e^{-2y} - 1\right)$


b) $R_{XY}(X,Y) = \frac{e^{-XY}}{2}$ $(R_X X * R_Y Y)$
$\rightarrow R_{Y|X}(Y|X) = \frac{R_{XY}(X,Y)}{R_X(X)}$
$= \frac{\frac{e^{-|XY|}}{2}}{\frac{e^{-|X|}}{2}} = \frac{e^{-|X|}e^{-|Y|}}{e^{-|X|}}$
$= e^{-|Y|}$ ANS
c) $R_{X|Y}(X|Y) = \frac{R_{X|Y}(X,Y)\,R_Y(Y)}{R_X(X)}$
$= \frac{e^{-|Y|}\,e^{-X}}{2} = \frac{e^{-|Y|}\,\frac{e^{-X}}{2}}{e^{-2Y}}$
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