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<pre>splitAt recursively splitAt :: Int -> [a] -> ([a], [a])</pre>	·	

- Let (xs1, xs2) = splitAt n xs below
- Then xs1 is the first n elements of xs
- Then xs2 is xs with the first n elements removed
- If $n \ge length xs then (xs1, xs2) = (xs, [])$
- If $n \le 0$ then (xs1, xs2) = ([], xs)

```
splitAt n xs | n \le 0 = ([], xs)
splitAt _ []
                      = ([], [])
splitAt n (n:xs)
 = let (xs1, xs2) = splitAt (n-1) xs
   in (x:xs1, xs2)
```

- How long does splitAt n xs take to run?
- It takes time proportional to n or length xs, whichever is shorter, which is twice as fast as te version using take and drop explicitly

take and drop

- Can we implement take and drop in terms of $\mathtt{splitAt}$
- The prelude provides the following

```
fst :: (a, b) -> a
snd :: (a, b) -> b
```

• Solution

```
take n xs = fst (splitAt n xs)
drop n xs = snd (splitAt n xs)
```

 How does the runtime of these definitions compare to the firect recursive ones?

Higher Order Functions

What is the difference between these two functions?

```
add x y = x + y
add2 (x, y) = x + y
```

We can see it in the types; add is curried, taking one argument at a time

```
add :: Integer -> Integer -> Integer
add2 :: (Integer, Integer) -> Integer
```

Any type $a \rightarrow a \rightarrow a$ can also be written $a \rightarrow b$. The function type arrow associates to the right.

In Haskell, functions are *first class citizens*. In other words, they occupy the same status in the langauge as values: you can pass them as arguments, make them part of data structions, compute them as the result of functions...

```
add3 :: (Integer -> (Integer -> Integer))
add3 :: add
> add3 1 2
```

Notice that there are no parameters in the definition of add3.

A function with multiple arguments can be viewed as a function of one argument, which computes a new function

```
add 3 4
==> (add 3) 4
==> ((+) 3) 4
```

The first place you might encounter this is the notion of partial application

```
increment :: Integer -> Integer
increment = add 1

If the type of add is Integer -> Integer -> Integer, and the type of add 1
2 is Integer, the the type of add 1 is?

It is Integer -> Integer
```

Examples of Partial Application

```
second :: [a] -> a
second = head . tail
> second [1, 2, 3]
```

An infix operator can be partially applied by taking a section:

```
increment = (1, +) -- or (+, 1)
addnewline = (++"\n")
double :: Integer -> Integer
double = (+2)
> [double x | x <- [1..10] ]
[2, 4, 6, 8, 10, 12, 14, 16, 18, 20]</pre>
```

Functions can be taken as parameters as well

```
twice :: (a -> a) -> a -> a
twice f x = f (f x)
addtwo = twice increment
```

This could also be written

```
twice f = f \cdot f
```

Composition

In fact, we can define composition using this technique:

```
compose :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c

compose f g x = f (g x)

twice f = f `compose` f
```

Super-bonus hack. Haskell permits the definition of infix functions:

$$(f ! g) x = f (g x)$$

twice $f = f!f$

Function composition is in fact part of the Haskell Prelude