TRINITY COLLEGE DUBLIN

School of Computer Science and Statistics

Week 4 Questions ST3009: Statistical Methods for Computer Science

For each problem, explain/justify how you obtained your answer in order to obtain full credit. In fact, most of the credit for each problem will be given for the derivation/model used as opposed to the final answer.

Question 1. A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you lose \$1.00. Calculate:

(a) The expected value of the amount you win

(b) The variance of the amount you win.

Question 2. Suppose you carry out a poll following an election. You do this by selecting n people uniformly at random and asking whether they voted or not, letting $X_i = 1$ if person i voted and $X_i = 0$ otherwise. Suppose the probability that a person voted is 0.6.

(a) Calculate $E[X_i]$ and $Var(X_i)$.

Let $Y = \sum_{i=1}^{n} X_i$.

(c) What is $E[Y]$? Is it the same as $E[X]$ or different, and why ?

(d) What is $E[\frac{1}{n}Y]$?

(e) What is the variance of $\frac{1}{n}Y$ (express in terms of $Var(X)$) ?

Hints: use linearity of the expectation and the fact that people are sampled independently.

Question 3. Suppose that 2 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X_i equal 1 if the i'th ball selected is white, and let it equal 0 otherwise.

(a) Give the joint probability mass function of X_1 and X_2

(b) Are X_1 and X_2 independent ? (Use the formal definition of independence to determine this)

(c) Calculate $E[X_2]$

(d) Calculate $E[X_2|X_1 = 1]$