

## Solution to exercise

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fsB([Node|_], Node, _, Target) :-  
    goal(Node, Target).  
  
fsB([Node|Rest], Found, Seed, Target) :-  
    findall(Next, arc(Node, Next, Seed), Children),  
    append(Rest, Children, NewF),  
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F=39 ? ;
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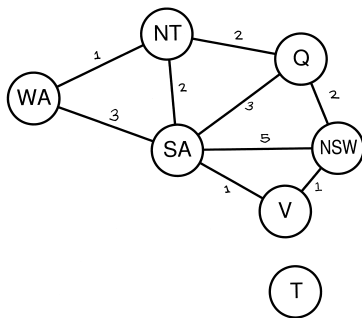
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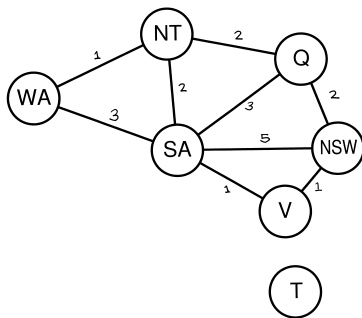
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## Min-cost



## Min-cost $\neq$ breadth-first

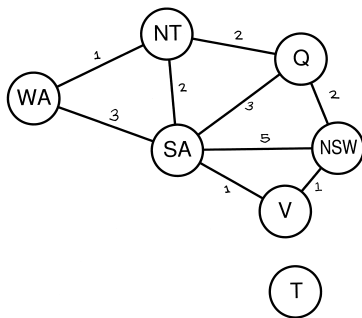


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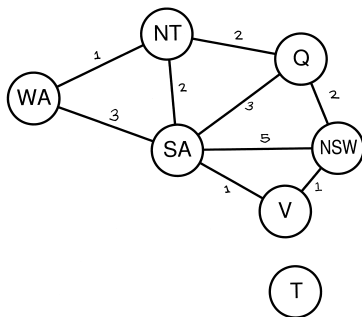
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`cost(Start $\cdots$ Head)  $\leq$  cost(Start $\cdots$  $n$ ) for each  $n$  in Tail ?`

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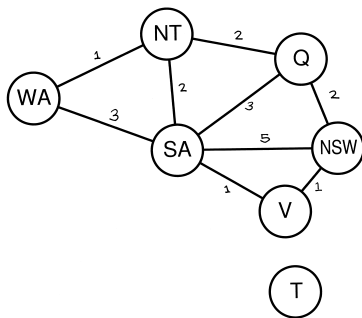
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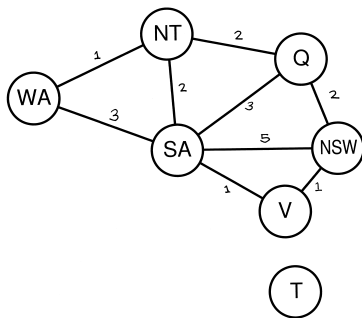
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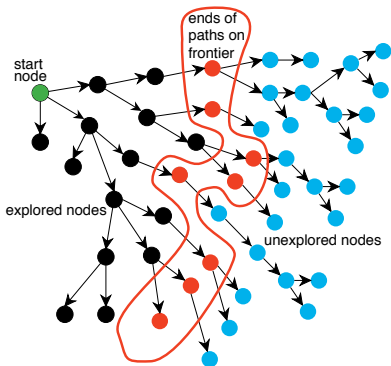
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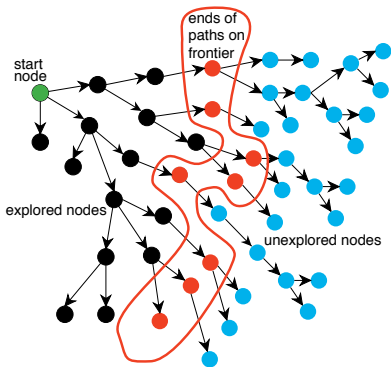
- ▶ node  $\rightsquigarrow$  path or pair  $(n, \text{cost}(\text{Start} \cdots n))$
- ▶ what about proximity to goal?

$h(n)$  = estimate of min cost path  $n \cdots \text{goal}$



$$\text{solution} = \underbrace{\text{start} \cdots n}_{\text{explored}} \cdots \text{goal}$$

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Ensure Frontier = [Head|Tail] where Head has minimal  $f$

- ▶  $h(n) = 0$  for every  $n \rightsquigarrow$  min-cost
- ▶  $\text{cost}(\text{start} \cdots n) = 0$  for every  $n \rightsquigarrow$  best-first  
(disregarding the past)

## Admissibility

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(ii)  $c = c_n$  for some  $n$  s.t. the head of  $F_n$  is a solution.