1. A soccer team plays a series of three games. Sports analysts predict that independently of each other the team:

- Wins the 1st game with probability 0.6
- Wins the 2nd game with probability 0.5
- Wins the 3rd game with probability 0.1

Compute the probability that the team wins at least one game.

Solution

Outcomes are independent so the probability that win no games is (1-0.6)(1-0.5)(1-0.1)=0.18. Therefore probability that win at least one game is 1-0.18 = 0.82

2. There are four communication links between two servers to provide resilience to failures. Each link fails with probability 0.4 independently of the others. What is the probability that a message is successfully sent ?

Solution

Failures are independent, therefore probability that all four links fail is 0.4^4=0.0256. Therefore probability that at least one link is ok is 1-0.0256=0.9744

3. The following table reports the fraction of students getting a first class honours conditioned on whether their leaving cert points are above 500 or not.

	Points < 500	Points > 500
<1st class honours	0.8	0.2
1st class honours	0.2	0.8

In the general population the fraction of people getting Leaving Cert points of 500 or greater is 5%. Is getting a 1st class honours degree independent of getting more than 500 points in the leaving cert ?

Solution

Let E be the event that get a 1st class degree, and F the event that get >500 points in the leaving. We have $P(F)=0.05$, $P(E)=P(E|F)P(F)+P(E|F^c)P(F^c) = 0.8 \times 0.05 + 0.2 \times 0.95 = 0.23$. $P(E)P(F)=0.23 \times 0.05=0.0115$. We also have that $P(E \cap F) = P(E|F)P(F)=0.8 \times 0.05=0.04$ (by chain rule). Since $P(E \cap F)$ is not equal to $P(E)P(F)$ the events are not independent.

4. A bag contains 3 red balls and 4 white balls. One ball is drawn from the bag and put to one side. A second ball is now drawn from the bag. What is the probability that the first ball is red and the second ball is white ? Are the events of drawing are red ball and then a white ball independent ? Suppose now that we put the first ball back into the bag after drawing it. What is the probability the first ball is red and the second ball is white now ? Now are the events of drawing are red ball and then a white ball independent ?

<u>Solution</u>

There are 7 balls in the bag initially, 3 of which are red. All balls are equally likely to be drawn from the bag so the probability of a red ball is 3/7. After drawing the red ball there are now 6 balls in the bag, 2 red and 4 white. The probability of now drawing a white ball is therefore 4/6 and P(red then white) = 3/7 x 4/6 = 0.28.

P(red first) = 3/7

P(white first) = 4/7

P(white second) = P(white second | red first)P(red first) + P(white second|white first) P(white first) = 4/6x3/7 + 3/6x4/7 = 0.57

P(red first)P(white second) = 3/7 x 0.57 = 0.24

But P(red then white)=0.285 which is not equal to P(red first)P(white second)=0.24, so the events are not independent.

When the first ball is put back into the bag, the probability of the second ball being white is now 4/7 rather than 4/6 so P(red then white) = 3/7 x 4/7.

P(white second) = P(white second | red first)P(red first) + P(white second|white first) P(white first) = 4/7 x 3/7 + 4/7 x 4/7 = 4/7

P(red first)P(white second) = 3/7 x 4/7 = P(red then white) so the events are independent.

5. Write a Matlab simulation for drawing two balls from a bag. Run the experiment multiple times and record what fraction of time a red then a white ball is drawn. This output of the simulation is itself random. How does changing the number of times we run the experiment change the fluctuations in the simulation output ?

<u>Solution</u>

```
P=[]; N=10000;

for i=1:N,

        X=[1 1 1 0 0 0 0];

        r=randi(length(X)); ball1=X(r); X(r)=[];
```

```
        r=randi(length(X)); ball2=X(r);
        P=[P;ball1>0 && ball2==0];
end
sum(P)/N
```

As we increase N the fluctuations in the simulation output tend to decrease.