Exam 2020 Computational Mathematics

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1 Q4

$$
\begin{aligned}
P_2(x) &= f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} \\
&= 3 - 17(2)^3 + -51(2)^2\frac{2.5-2}{1} + -102(2)\frac{(2.5-2)^2}{2!} \\
&= -133 - 102 - 25.5 = -260.5 \\
F(x) - P_2(x) &= \\
&= -262.625 + 260.5 = -2.125
\end{aligned}
\tag{1}
$$

2 Q5

$$f(x) = 16x^5 - 73x^2 - 133$$

$$x_0 = x$$

$$x_1 = 2.5$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_i - 1) - f(x_i)}$$

$$x_{i+1} = 2.5 - \frac{f(2.5)(3 - 2.5)}{f(3) - f(2.5)}$$

$$2.270973 = 2.5 - \frac{973.25(3 - 2.5)}{3098 - 973.25}$$

$$x_{i+1} = 2.270973 - \frac{f(2.270973)(2.5 - 2.270973)}{f(2.5) - f(2.270973)} \tag{2}$$

$$2.068259 = 2.270973 - \frac{456.966852(2.5 - 2.270973)}{456.966852 - 973.25}$$

$$1.958756 = 2.068259 - \frac{f(2.068259)(2.270973 - 2.068259)}{f(2.270973) - f(2.068259)}$$

$$1.911576 = 1.958756 - \frac{f(1.958756)(2.068259 - 1.958756)}{f(2.068259) - f(1.958756)}$$

$$1.90128 = 1.911576 - \frac{f(1.911576)(1.958756 - 1.911576)}{f(1.958756) - f(1.911576)}$$

$$1.900475 = 1.901285 - \frac{f(1.901285)(1.911576 - 1.901285)}{f(1.911576) - f(1.901285)}$$

2

3 Q6

$$\begin{pmatrix} 25 & 5 & 4 \\ 10 & 8 & 16 \\ 8 & 12 & 22 \end{pmatrix}$$

$$U = 25 - 0 = 25$$

$$L = (10 - 0)/25 = 0.4$$

$$L = (8 - 0)/25 = 0.32$$

$$U = 5 - 0 = 5$$

$$U = 8 - (0 + (5 * 0.4) = 2) = 6$$

$$U = \begin{pmatrix} 25 & 5 \\ 0 & 6 \\ 0 & 0 \end{pmatrix} L = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ .32 & & 1 \end{pmatrix} \quad (3)$$

$$L = (12 - (0 + (5x0.32) = 1.6))/6 = 1.73333$$

$$U = 4 - 0 = 4$$

$$U = 16 - (0 + (4x0.4) = 1.6) = 14.4$$

$$U = (0 + (4x0.32) = 1.28) - (1.28 + (14.4x1.73333) = 26.2400) = -4.2400$$

$$U = \begin{pmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.4 \\ 0 & 0 & -4.24 \end{pmatrix} L = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ .32 & 1.7333 & 1 \end{pmatrix}$$

4 Q7

$$x_1 = \frac{(2 - (7x_2 + 3x_3))}{12}$$

$$x_2 = \frac{(-5 - (1x_1 + 1x_3))}{5}$$

$$x_3 = \frac{(6 - (2x_1 + 7x_2))}{-11}$$

$$1st\ Iteration$$

$$x_1 = \frac{(2 - (7(3) + 3(5)))}{12} = -\frac{17}{6}$$

$$x_2 = \frac{(-5 - (1(-\frac{17}{6}) + 1(5)))}{5} = -\frac{43}{30}$$

$$x_3 = \frac{(6 - (2(-\frac{17}{6}) + 7(-\frac{43}{30})))}{-11} = -\frac{217}{110}$$

$$2nd\ Iteration \tag{4}$$

$$x_1 = \frac{(2 - (7(-\frac{43}{30}) + 3(-\frac{217}{110})))}{12} = \frac{1481}{990}$$

$$x_2 = \frac{(-5 - (1(\frac{1481}{990}) + 1(-\frac{217}{110})))}{5} = -\frac{2239}{2475}$$

$$x_3 = \frac{(6 - (2(\frac{1481}{990}) + 7(-\frac{2239}{2475})))}{-11} = -\frac{7766}{9075}$$

$$3rd\ Iteration$$

$$x_1 = \frac{(2 - (7(-\frac{2239}{2475}) + 3(-\frac{7766}{9075})))}{12} = 0.90666$$

$$x_2 = \frac{(-5 - (1(0.90666) + 1(-\frac{7766}{9075})))}{5} = -1.0115$$

$$x_3 = \frac{(6 - (2(0.90666) + 7(-1.0115)))}{-11} = -1.0243$$

5 Q8

$$\begin{pmatrix} 4 & 5 \\ 6 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \end{pmatrix} = 11 \begin{pmatrix} 9/11 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 5 \\ 6 & 5 \end{pmatrix} \begin{pmatrix} \frac{9}{11} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{91}{11} \\ \frac{109}{11} \end{pmatrix} = \frac{109}{11} \begin{pmatrix} \frac{91}{109} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 5 \\ 6 & 5 \end{pmatrix} \begin{pmatrix} \frac{91}{109} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{909}{109} \\ \frac{1091}{109} \end{pmatrix} = \frac{1091}{109} \begin{pmatrix} \frac{909}{1091} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 5 \\ 6 & 5 \end{pmatrix} \begin{pmatrix} \frac{9091}{1091} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{9091}{1091} \\ 10.00 \end{pmatrix} = 10.00 \begin{pmatrix} .8333 \\ 1 \end{pmatrix}$$

$$\tag{5}$$

6 Q9

$$f(a_0) = f(2) = 4 * log_2 2 = 4$$
$$f(a_1) = f(3) = 9 * log_2 3 = 14.2646 \tag{6}$$
$$f(a_2) = f(7) = 49 * log_2 7 = 137.5604$$

$$(A_0, y_0) : a_0 = y_0 = 4$$
$$(A_1, y_1) : \frac{y_2 - y_1}{x_2 - x_1} = \frac{14.2646 - 4}{3 - 2} = 10.26464$$
$$(A_2, y_2) : \frac{\frac{y_2 - y_1}{a_2 - a_1} - \frac{y_1 - y_0}{a_2 - a_0}}{a_2 - a_0} \tag{7}$$
$$= \frac{\frac{137.5604 - 14.2645}{7 - 3} - 10.26464}{7 - 2}$$
$$= 4.111185$$

7 Q10

$$\int_0^{2\pi} \frac{1}{2 + cosx} dx$$
$$x = \frac{1}{2}(t(b - a) + a + b) = \frac{1}{2}(t(2\pi - 0) + 0 + 2\pi)$$
$$\frac{2\pi t + 2\pi}{2} = \pi t + \pi$$
$$dx = \frac{1}{2}(b - a)dt = \frac{1}{2}(2\pi)dt = \pi dt \tag{8}$$
$$\int_{-1}^1 f(t)dt = \frac{\pi}{2 + cos(\pi t + \pi)}dt$$
$$= C_1 * f(t_1) + C_2 * f(t_2) + C_3 * f(t_3)$$
$$= .5555556 * f(-.77459667) + .8888889 * f(0) + .5555556 * f(.77459667)$$
$$= .6324614064 + 2.792526838 + .6324614064 = 4.05745$$