UNIVERSITY OF DUBLIN
TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

Integrated Computer Science Programme **Trinity Term 2014**

Year 3 Annual Examinations

Computational Mathematics

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Tuesday May 20th **Luce Upper** **09:30–11:30h**

Instructions to Candidates:

Answer **both** questions in **Part A** and **two** questions out of three in **Part B.** All questions carry equal marks.

Suggestion: Take 20 minutes to read all five questions. This leaves 100 minutes for answering questions worth 100 marks in total. So if a part of a question is worth five marks, spend five minutes answering it.

Materials permitted for this examination:
Log tables—available from the invigilators.
Graph paper—available from the invigilators.
Non-programmable calculator—indicate make and model.

Part A

Question 1. Consider three non-normalised finite precision floating point number systems:

$$F^{16} \text{ where } \beta = 2, s = 10, m = -\beta^5, M = +\beta^5$$
$$F^{32} \text{ where } \beta = 2, s = 23, m = -\beta^8, M = +\beta^8$$
$$F^{64} \text{ where } \beta = 2, s = 52, m = -\beta^{11}, M = +\beta^{11}.$$

(i) What are the minimum non-zero absolute values that can be represented in each of these systems? Express in decimal notation.

[6 marks]

(ii) How many non-unique numbers can be represented in each of these systems? Express in decimal notation.

[6 marks]

(iii) Express decimal number 0.1_{10} in binary. What problem arises? When expressed in the F^{16} and F^{32} systems, the errors are approximately 2.44×10^{-5} and 1.49×10^{-9} respectively. For both of these systems, find a number which when multiplied by 0.1_{10} results in an error approximately equal to the value of the least significant bit.

[10 marks]

(iv) Suggest how multiplication by 0.1_{10} could be avoided.

[3 marks]

Question 2. Wave motion can be approximated as a second order differential equation $\partial^2 U(t,x)/\partial t^2 = C^2\, \partial^2 U(t,x)/\partial x^2$ where $U(t,x)$ is the height of the wave at position x at time t and C is a constant representing propagation speed.

For the purposes of simulation over the temporal interval $[T_1, T_2]$ and the spatial interval $[X_1, X_2]$, assume the following initial and boundary conditions are given: $U(T_1, x)$; $\partial U(T_1, x)/\partial t$; $U(t, X_1)$; $U(t, X_2)$.

(i) Show how a second order central difference approximation may be used to find an expression for $U(t + \Delta t, x)$ which can then be used in a program to simulate wave motion at a time Δt after the current time—where the current time is *not* the start time of the simulation. [10 marks]

(ii) Describe an alternative solution with which you are familiar from working on a computer program implementation. Discuss any expected or observed differences in behaviour with the above solution. [15 marks]

Part B

Question 3. (i) Explain in detail how an iteration function and its approximate linearisation with a Taylor Series are used in the analysis of convergence of any iterative method which starts with an initial guess of the solution.

[20 marks]

(ii) Why is iterative computation so often required to approximate the solutions of mathematical problems arising in science and engineering?

[5 marks]

Question 4. (i) Use the composite trapeziodal rule to numerically integrate $\int_0^1 e^x dx$ with intervals $h_0 = 1, h_1 = \frac{h_0}{2}, h_2 = \frac{h_1}{2}$. Note the true solution is $e - 1$.

[12 marks]

(ii) Combine the above estimates using Richardson's deferred approach to the limit with h^2 extrapolation.

[13 marks]

Question 5. (i) Describe in detail a computer program to find the least-squared error fit of a 5th degree polynomial to a number of observed data points much greater than 5. Hint: Solve an overdetermined system of equations which is linear in the 6 unknown coefficients of the polynomial. Use a programming language of your choice and assume a library of appropriate numerical methods is available.

[20 marks]

(ii) Name a well-known matrix method that can be used in the solution of an overdetermined system of equations linear in the unknowns.

[5 marks]