Chapter 2

2.2 Apply the intermediate value theorem to show that the function $f(x) = \cos x - x^2$ has a root in the interval $[0, \pi/2]$.

2.8 As a highway patrol officer, you are participating in a speed trap. A car passes your patrol car which you clock at 55 mph. One and a half minutes later, your partner in another patrol car situated two miles away from you, clocks the same car at 50 mph. Using the mean value theorem for derivatives (Eq. (2.4)), show that the car must have exceeded the speed limit of 55 mph at some point during the one and a half minutes it traveled between the two patrol cars.

2.22 Write the Taylor's series expansion of the function $f(x) = \sin(ax)$ about $x = 0$, where $a \neq 0$ is a known constant.

2.27 Write a user-defined MATLAB function that determines the cross product of two vectors $\vec{W} = \vec{V} \otimes \vec{U}$. For the function name and arguments, use `W = Cross(V,U)`. The input arguments `V` and `U` are the vectors to be multiplied. The output argument `W` is the result (three-element vector).
(*a*) Use `Cross` to determine the cross product of the vectors $v = i+2j+3k$ and $u = 3i+2j+k$.
(*b*) Use `Cross` to determine the cross product of the vectors $v = -2i+j-3k$ and $u = i+j+k$.

2.31 Write a user-defined MATLAB function that calculates the determinant of a square $(n \times n)$ matrix, where n can be 2, 3, or 4. For function name and arguments, use `D = Determinant(A)`. The input argument `A` is the matrix whose determinant is calculated. The function `Determinant` should first check if the matrix is square. If it is not, the output `D` should be the message "The matrix must be square."
Use `Determinant` to calculate the determinant of the following two matrices:

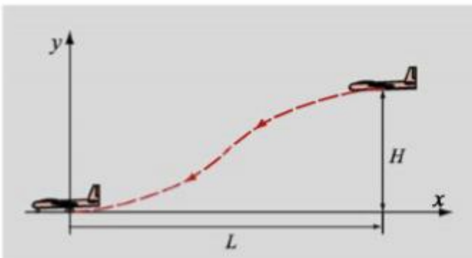
(*a*) $\begin{bmatrix} 1 & 5 & 4 \\ 2 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix}$. (*b*) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$.

2.32 One important application involving the total differential of a function of several variables is estimation of uncertainty.
(*a*) The electrical power P dissipated by a resistance R is related to the voltage V and resistance by $P = V^2/R$. Write the total differential dP in terms of the differentials dV and dR, using Eq. (2.63).
(*b*) dP is interpreted as the uncertainty in the power, dV as the uncertainty in the voltage, and dR as the uncertainty in the resistance. Using the answer of part (*a*), determine the maximum percent uncertainty in the power P for $V = 400$ V with an uncertainty of 2%, and $R = 1000\ \Omega$ with an uncertainty of 3%.

2.34 An aircraft begins its descent at a distance $x = L$ ($x = 0$ is the spot at which the plane touches down) and an altitude of H. Suppose a cubic polynomial of the following form is used to describe the landing:

$$y = ax^3 + bx^2 + cx + d$$

where y is the altitude and x is the horizontal distance to the aircraft. The aircraft begins its descent from a level position, and lands at a level position.

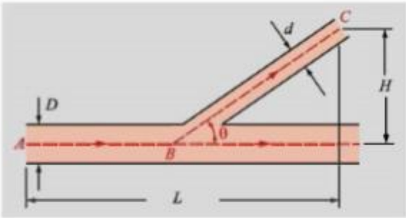


(*a*) Solve for the coefficients a, b, c, and d.
(*b*) If the aircraft maintains a constant forward speed ($\frac{dx}{dt} = u = constant$) and the magnitude of the vertical acceleration ($\frac{d^2y}{dt^2}$) is not to exceed a constant A, show that $\frac{6Hu^2}{L^2} \leq A$.
(*c*) If $A = 0.3\ \text{ft/s}^2$, $H = 15000$ ft, and $u = 200$ mph, how far from the airport should the pilot begin the descent?

2.36 An artery that branches from another more major artery has a resistance for blood flow that is given by:

$$R_{flow} = K\left(\frac{L - H\cot\theta}{D^4} + \frac{H\csc\theta}{d^4}\right)$$

where R_{flow} is the resistance to blood flow from the major to the branching artery along path ABC (see diagram), d is the diameter of the smaller, branching artery, D is the diameter of the major artery, θ is the angle that the branching vessel makes with the horizontal, or axis, of the major artery, and L and H are the distances shown in the figure.



(*a*) Find the angle θ that minimizes the flow resistance in terms of d and D.

(*b*) If $\theta = 45°$, and $D = 5$ mm, what is the value of d that minimizes the resistance to blood flow?

Chapter 3

3.2 Determine the root of $f(x) = x - 2e^{-x}$ by:

(*a*) Using the bisection method. Start with $a = 0$ and $b = 1$, and carry out the first three iterations.

(*b*) Using the secant method. Start with the two points, $x_1 = 0$ and $x_2 = 1$, and carry out the first three iterations.

(*c*) Using Newton's method. Start at $x_1 = 1$ and carry out the first three iterations.

3.8 Find the root of the equation $\sqrt{x} + x^2 = 7$ using Newton's method. Start at $x = 7$ and carry out the first five iterations.

3.14 Solve the following system of nonlinear equations:

$$x^2 + 2x + 2y^2 - 26 = 0$$
$$2x^3 - y^2 + 4y - 19 = 0$$

(*a*) Use Newton's method. Start at $x = 1$, $y = 1$, and carry out the first five iterations.

(*b*) Use the fixed-point iteration method. Start at $x = 1$, $y = 1$, and carry out the first five iterations.

3.26 Examine the differences between the True Relative Error, Eq. (3.8), and the Estimated Relative Error, Eq. (3.9), by numerically solving the equation $f(x) = 0.5e^{(2+x)} - 40 = 0$. The exact solution of the equation is $x = \ln(80) - 2$. Write a MATLAB program in a script file that solves the equation by using Newton's method. Start the iterations at $x = 4$, and execute 11 iterations. In each iteration, calculate the True Relative Error (TRE) and the Estimated Relative Error (ERE). Display the results in a four-column table (create a 2-dimensional array), with the number of iterations in the first column, the estimated numerical solution in the second, and TRE and ERE in the third and fourth columns, respectively.

3.27 When calculating the payment of a mortgage, the relationship between the loan amount, *Loan*, the monthly payment, *MPay*, the duration of the loan in months *Months*, and the annual interest rate, *Rate*, is given by the equation (annuity equation):

$$MPay = \frac{Loan \cdot Rate}{12\left(1 - \frac{1}{\left(1 + \frac{Rate}{12}\right)^{Months}}\right)}$$

Determine the rate of a 20 years, \$300,000 loan if the monthly payment is \$1684.57.

(*a*) Use the user-defined function `SteffensenRoot` from Problem 3.24.

(*b*) Use MATLAB's built-in function `fzero`.

Chapter 4

4.2 Given the system of equations $[a][x] = [b]$, where $a = \begin{bmatrix} 2 & -2 & 1 \\ 3 & 2 & -5 \\ -1 & 2 & 3 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 10 \\ -16 \\ 8 \end{bmatrix}$, determine the solution using the Gauss elimination method.

4.13 Find the inverse of the matrix $\begin{bmatrix} 10 & 12 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}$ using the Gauss–Jordan method.

4.19 Find the condition number of the matrix in Problem 4.13 using the 1-norm.

4.25 Write a user-defined MATLAB function that calculates the 1-norm of any matrix. For the function name and arguments use `N = OneNorm(A)`, where `A` is the matrix and `N` is the value of the norm. Use the function for calculating the 1-norm of:

(*a*) The matrix $A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1.5 \end{bmatrix}$. (*b*) The matrix $B = \begin{bmatrix} 4 & -1 & 0 & 1 & 0 \\ -1 & 4 & -1 & 0 & 1 \\ 0 & -1 & 4 & -1 & 0 \\ 1 & 0 & -1 & 4 & -1 \\ 0 & 1 & 0 & -1 & 4 \end{bmatrix}$.

4.34 A particular chemical substance is produced from three different ingredients *A*, *B*, and *C*, each of which have be dissolved in water first before they react to form the desired substance. Suppose that a solution containing ingredient *A* at a concentration of 2 g/cm^3 is combined with a solution containing ingredient *B* at a concentration of 3.6 g/cm^3 and with a solution containing ingredient *C* at a concentration of 6.3 g/cm^3 to form 25.4 g of the substance. If the concentrations of *A*, *B*, and *C* in these solutions are changed to 4 g/cm^3, 4.3 g/cm^3, and 5.4 g/cm^3, respectively (while the volumes remain the same), then 27.7 g of the substance is produced. Finally, if the concentrations are changed to 7.2, 5.5, and 2.3 g/cm^3, respectively, then 28.3 g of the chemical is produced. Find the volumes (in cubic centimeters) of the solutions containing *A*, *B*, and *C*.

Chapter 5

5.3 Find the eigenvalues of the following matrix by solving for the roots of the characteristic equation.

$$\begin{bmatrix} 10 & 0 & 0 \\ 1 & -3 & -7 \\ 0 & 2 & 6 \end{bmatrix}$$

5.7 Apply the power method to find the largest eigenvalue of the matrix from Problem 5.2 starting with the vector $[1\ 1\ 1]^T$.

5.9 Apply the inverse power method to find the smallest eigenvalue of the matrix from Problem 5.3 starting with the vector $[1\ 1\ 1]^T$. The inverse of the matrix in Problem 5.3 is:

$$\begin{bmatrix} 0.1 & 0 & 0 \\ 0.15 & -1.5 & -1.75 \\ -0.05 & 0.5 & 0.75 \end{bmatrix}$$

5.10 Write a user-defined MATLAB function that determines the largest eigenvalue of an $(n \times n)$ matrix by using the power method. For the function name and argument use `e = MaxEig(A)`, where `A` is the matrix and `e` is the value of the largest eigenvalue. Use the function `MaxEig` for calculating the largest eigenvalue of the matrix of Problem 5.8. Check the answer by using MATLAB's built-in function for finding the eigenvalues of a matrix.

5.12 Write a user-defined MATLAB function that determines all the eigenvalues of an $(n \times n)$ matrix by using the QR factorization and iteration method. For the function name and argument use `e=AllEig(A)`, where `A` is the matrix and `e` is a vector whose elements are the eigenvalues. Use the function `AllEig` for calculating the eigenvalues of the matrix of Problem 5.8. Check the answer by using MATLAB's built-in function for finding the eigenvalues of a matrix.

5.18 Suppose there are N web sites that are linked to each other. One (overly simplified) method to assess the importance of a particular web site i is as follows. If web site j links (or points) to web site i, a quantity $[a_{ij}]$ can be set to 1 whereas if j does not link to website k, than $[a_{jk}]$ is set to 0. Thus, if $[x_2]$ stands for the importance of web site 2, and web sites 1 and 4 point to web site 2, then $x_2 = x_1 + x_4$, and so on. Consider four web sites, larry.com, bill.com, steve.com, and mark.com linked as shown in the directed graph below.

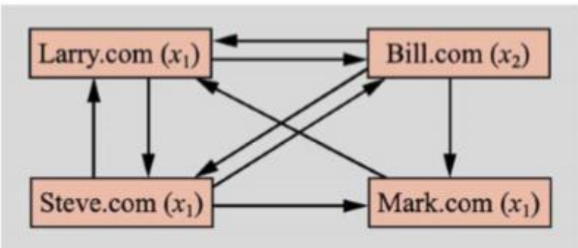
Let x_1 be the importance of larry, x_2 be the importance of bill, x_3 be the importance of steve, and x_4 be the importance of mark. The above directed graph[2] when converted to a set of equations using the scheme described before results in the following equations:

$x_1 = x_2 + x_3 + x_4$, $x_2 = x_1 + x_3$, $x_3 = x_1 + x_2$, and $x_4 = x_2 + x_3$, which can be written as:

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = [A][x]$$

(*a*) Find the eigenvalues and the corresponding eigenvectors of $[A]$, using MATLAB's built-in function `eig`.

(*b*) Find the eigenvector from part (*a*) whose entries are all real and of the same sign (it does not matter if they are all negative or all positive), and rank the web sites in descending order of importance based on the indices of the web sites corresponding to the largest to the smallest entries in that eigenvector.



Chapter 6

6.3 The following data give the approximate population of China for selected years from 1900 until 2010:

Year	1900	1950	1970	1980	1990	2000	2010
Population (millions)	400	557	825	981	1135	1266	1370

Assume that the population growth can be modeled with an exponential function $p = be^{mx}$, where x is the year and p is the population in millions. Write the equation in a linear form (Section 6.3), and use linear least-squares regression to determine the constants b and m for which the function best fits the data. Use the equation to estimate the population in the year 1985.

6.8 Water solubility in jet fuel, W_S, as a function of temperature, T, can be modeled by an exponential function of the form $W_S = be^{mT}$. The following are values of water solubility measured at different temperatures. Using linear regression, determine the constants m and b that best fit the data. Use the equation to estimate the water solubility at a temperature of 10° C. Make a plot that shows the function and the data points.

T (°C)	–40	–20	0	20	40
W_S (% wt.)	0.0012	0.002	0.0032	0.006	0.0118

6.13 The power generated by a windmill varies with the wind speed. In an experiment, the following five measurements were obtained:

Wind Speed (mph)	14	22	30	38	46
Electric Power (W)	320	490	540	500	480

Determine the fourth-order polynomial in the Lagrange form that passes through the points. Use the polynomial to calculate the power at a wind speed of 26 mph.

6.21 Write a MATLAB user-defined function that determines the best fit of a power function of the form $y = bx^m$ to a given set of data points. Name the function `[b m]= PowerFit(x,y)`, where the input arguments `x` and `y` are vectors with the coordinates of the data points, and the output arguments `b` and `m` are the values of the coefficients. The function `PowerFit` should use the approach that is described in Section 6.3 for determining the value of the coefficients. Use the function to solve Problem 6.3.

6.31 The percent of households that own at least one computer in selected years from 1981 to 2010, according to the U.S. census bureau, is listed in the following table:

Year	1981	1984	1989	1993	1997	2000	2001	2003	2004	2010
Household with computer [%]	0.5	8.2	15	22.9	36.6	51	56.3	61.8	65	76.7

The data can be modeled with a function in the form $H_C = C/(1 + Ae^{-Bx})$ (logistic equation), where H_C is percent of households that own at least one computer, C is a maximum value for H_C, A and B are constants, and x is the number of years after 1981. By using the method described in Section 6.3 and assuming that $C = 90$, determine the constants A and B such that the function best fit the data. Use the function to estimate the percent of ownership in 2008 and in 2013. In one figure, plot the function and the data points.

6.41 The following measurements were recorded in a study on the growth of trees.

Age (year)	5	10	15	20	25	30	35
Height (m)	5.2	7.8	9	10	10.6	10.9	11.2

The data is used for deriving an equation $H = H(Age)$ that can predict the height of the trees as a function of their age. Determine which of the nonlinear equations that are listed in Table 6-2 can best fit the data and determine its coefficients. Make a plot that shows the data points (asterisk marker) and the equation (solid line).

Chapter 8

8.3 The following data show estimates of the population of Liberia in selected years between 1960 and 2010:

Year	1960	1970	1980	1990	2000	2010
Population (millions)	1.1	1.4	1.9	2.1	2.8	4

Calculate the rate of growth of the population in millions per year for 2010.
(*a*) Use two-point backward difference formula.
(*b*) Use three-point backward difference formula.
(*c*) Using the slope in 2010 from part (*b*), apply the two-point central difference formula to extrapolate and predict the population in the year 2020.

8.8 A particular finite difference formula for the first derivative of a function is:

$$f'(x_i) = \frac{-f(x_{i+3})+9f(x_{i+1})-8f(x_i)}{6h}$$

where the points x_i, x_{i+1}, x_{i+2}, and x_{i+3} are all equally spaced with step size h. What is the order of the truncation or discretization error?

8.9 The following data show the number of female and male physicians in the U.S. for various years (American Medical Association):

Year	1980	1990	2000	2002	2003	2006	2008
# males	413,395	511,227	618,182	638,182	646,493	665,647	677,807
# females	54,284	104,194	195,537	215,005	225,042	256,257	276,417

(*a*) Calculate the rate of change in the number of male and female physicians in 2006 by using the three-point backward difference formula for the derivative, with unequally spaced points, Eq. (8.37).
(*b*) Use the result from part (*a*) and the three-point central difference formula for the derivative with unequally spaced points, Eq. (8.36), to calculate (predict) the number of male and female physicians in 2008.

8.19 Write a MATLAB user-defined function that determines the first and second derivatives of a function that is given by a set of discrete points with equal spacing. For the function name use `[yd, ydd] = FrstScndDeriv(x,y)`. The input arguments `x` and `y` are vectors with the coordinates of the points, and the output arguments `yd` and `ydd` are vectors with the values of the first and second derivatives, respectively, at each point. For calculating both derivatives, the function should use the finite difference formulas that have a truncation error of $O(h^2)$.

(*a*) Use the function `FrstScndDeriv` to calculate the derivatives of the function that is given by the data in Problem 8.18.

(*b*) Modify the function (rename it `FrstScndDerivPt`) such that it also creates three plots (one page in a column). The top plot should be of the function, the second plot of the first derivative, and the third of the second derivative. Apply the function `FrstScndDerivPt` to the data in Problem 8.18.

8.31 The altitude of the space shuttle during the first two minutes of its ascent is displayed in the following table (www.nasa.gov):

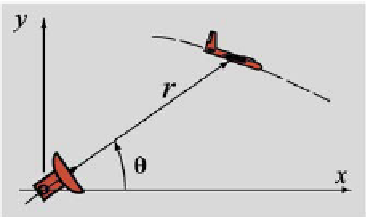
t (s)	0	10	20	30	40	50	60	70	80	90	100	110	120
h (m)	−8	241	1,244	2,872	5,377	8,130	11,617	15,380	19,872	25,608	31,412	38,309	44,726

Assuming the shuttle is moving straight up, determine its velocity and acceleration at each point. Display the results in three plots (*h* versus time, velocity versus time, and acceleration versus time).

(*a*) Solve by using the user-defined function `FrstScndDerivPt` that was written in Problem 8.19.

(*b*) Solve by using the MATLAB built-in function `diff`.

8.37 A radar station is tracking the motion of an aircraft. The recorded distance to the aircraft, *r*, and the angle θ during a period of 60 s is given in the following table. The magnitude of the instantaneous velocity and acceleration of the aircraft can be calculated by:



$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2} \qquad a = \sqrt{\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]^2 + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]^2}$$

Determine the magnitudes of the velocity and acceleration at the times given in the table. Plot the velocity and acceleration versus time (two separate plots on the same page). Solve the problem by writing a program in a script file. The program evaluates the various derivatives that are required for calculating the velocity and acceleration, and then makes the plots. For calculating the derivatives use

(*a*) the user-defined function `FrstScndDeriv` that was written in Problem 8.19;

(*b*) MATLAB's built-in function `diff`.

t (s)	0	4	8	12	16	20	24	28
r (km)	18.803	18.861	18.946	19.042	19.148	19.260	19.376	19.495

Chapter 9

9.1 The function $f(x)$ is given in the following tabulated form. Compute $\int_0^{1.8} f(x)dx$ with $h = 0.3$ and with $h = 0.4$.

(*a*) Use the composite rectangle method.

(*b*) Use the composite trapezoidal method.

(*c*) Use the composite Simpson's 3/8 method.

x	0	0.3	0.6	0.9	1.2	1.5	1.8
$f(x)$	0.5	0.6	0.8	1.3	2	3.2	4.8

9.5 The Head Severity Index (HSI) measures the risk of head injury in a car crash. It is calculated by:

$$HSI = \int_0^t [a(t)]^{2.5}\, dt$$

where $a(t)$ is the normalized acceleration (acceleration in m/s^2 divided by 9.81 m/s^2) and t is time in seconds during a crash. The acceleration of a dummy head measured during a crash test is given in the following table.

t (ms)	0	5	10	15	20	25	30	35	40	45	50	55	60
a (m/s^2)	0	3	8	20	33	42	40	48	60	12	8	4	3

Determine the HSI.
(*a*) Use the composite trapezoidal method.
(*b*) Use the composite Simpson's 1/3 method.
(*c*) Use the composite Simpson's 3/8 method.

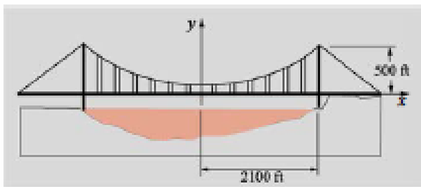
9.7 Evaluate the integral

$$I = \int_0^{2.4} \frac{2x}{1+x^2}\, dx$$

using the following methods:
(*a*) Simpson's 1/3 method. Divide the whole interval into six subintervals.
(*b*) Simpson's 3/8 method. Divide the whole interval into six subintervals.

The exact value of the integral is $I = \ln\frac{169}{25}$. Compare the results and discuss the reasons for the differences.

9.10 The central span of the Golden Gate bridge is 4200 ft long and the towers' height from the roadway is 500 ft. The shape of the main suspension cables can be approximately modeled by the equation:



$$f(x) = C\left(\frac{e^{x/C} + e^{-x/C}}{2} - 1\right) \quad \text{for} \quad -2100 \le x \le 2100 \text{ ft}$$

where $C = 4491$.

By using the equation $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$, determine the length of the main suspension cables with the following integration methods:
(*a*) Simpson's 1/3 method. Divide the whole interval into eight subintervals.
(*b*) Simpson's 3/8 method. Divide the whole interval into nine subintervals.
(*c*) Three-point Gauss quadrature.

9.23 Write a user-defined MATLAB function that uses the composite Simpson's 3/8 method for integration of a function $f(x)$ that is given in analytical form (equation). For the function name and arguments use `I=Simpsons38(Fun,a,b)`. `Fun` is a name for the function that is being integrated. It is a dummy name for the function that is imported into `Simpson38`. The actual function that is integrated should be written as an anonymous or a user-defined function that calculates, using element-by-element operations, the values of $f(x)$ for given values of x. It is entered as a function handle when `Simpson38` is used. `a` and `b` are the limits of integration, and `I` is the value of the integral. The integration function calculates the value of the integral in iterations. In the first iteration the interval $[a, b]$ is divided into three subintervals. In every iteration that follows, the number of subintervals is doubled. The iterations stop when the difference in the value of the integral between two successive iterations is smaller than 0.1%. Use `Simpsons38` to solve Problems 9.6 and 9.7.

9.26 The error function $erf(x)$ (also called the Gauss error function), which is used in various disciplines (e.g., statistics, material science), is defined as:

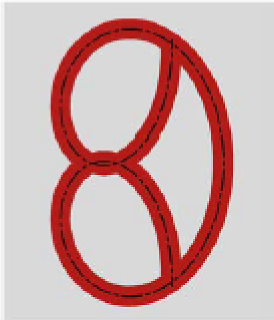
$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

Write a user-defined MATLAB function that calculates the error function. For function name and arguments use `ef=ErrorFun(x)`. Use the user-defined function `Simpson38` that was written in Problem 9.23 for the integration inside `ErrorFun`.

(*a*) Use `ErrorFun` to make a plot of the error function for $0 \le x \le 2$. The spacing between points on the plot should be 0.02.

(*b*) Use `ErrorFun` to make a two-column table with values of the error function. The first column displays values of x from 0 to 2 with spacing of 0.2, and the second column displays the corresponding values of the error function.

9.30 A pretzel is made by a robot that is programed to place the dough according to the curve given by the following parametric equations:



$$x = (2.5 - 0.3t^2)\cos(t) \qquad y = (3.3 - 0.4t^2)\sin(t)$$

where $-4 \le t \le 3$. The length of a parametric curve is given by the integral:

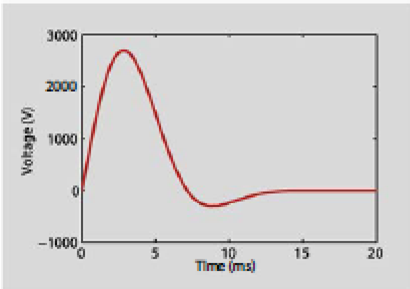
$$\int_a^b \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}\, dt$$

Determine the length of the pretzel. For the integration use:

(*a*) The user-defined function `SimpsonPoints` that was written in Problem 9.20.

(*b*) MATLAB's built-in function `trapz`.

9.35 The figure shows the output pulse from an MDS defibrillator. The voltage as a function time is given by:



$$v(t) = 3500\sin(140\pi t)e^{-63\pi t} \text{ V}$$

The energy, E, delivered by this pulse can be calculated by:

$$E = \int_0^t \frac{[v(t)]^2}{R}\, dt \text{ Joules.}$$

where R is the impedance of the patient. For $R = 50\Omega$, determine the time when the pulse has to be switched off if 250 J of energy is to be delivered.

(*a*) Use the composite Simpson's 3/8 method (user-defined function `Simpsons38` that was written in Problem 9.23).

(b) Use one of MATLAB's built-in functions.