1.   (i) Suppose a string of *n* bits is sent across a lossy link. In how many ways can 2 bit errors occur?   When *n=3*, list the possible set of bit error patterns.   [5 marks]

Solution

We're picking 2 items from a set of *n* items and we don't care about the order, so there are $\binom{n}{2}$ combinations.

When *n=3*, there are $\binom{3}{2}$ = 3 error patterns. The set of patterns is:

1 1 0

0 1 1

1 0 1

        (ii) In how many ways can the letters MISSISSIPPI be rearranged?   [10 marks]

Solution

Start with 11 blank slots for the letters.   Fill 4 slots with S, then 4 slots with I, 2 with P and one with M. So the answer is $\binom{11}{4}\binom{7}{4}\binom{3}{2} = \frac{11!}{4!4!2!1!}$

        (iii) In poker a straight flush consists of 5 cards in order, all of the same suit e.g. the 2,3,4,5,6 of clubs.   How many straight flushes are possible?   [10 marks]

Solution

Each suit has 10 straight flushes (the one starting with ace, the one starting with two, through to the one starting with 10). There are four suits, so there are 40 possible straight flushes.

2.   (i) Six people get into an elevator at the ground floor of a hotel which has 10 upper floors. Assuming each person gets off at a floor uniformly at random, what is the probability that no two people get off at the same floor?   [10 marks]

Solution

The first person has 10 floors to choose from, the second person has 9 floors to choose from and so on. So the number of combinations is 10.9.8.7.6.5. The total number of ways to for 6 people to choose from 10 floors is 10^6. So the probability is $10.9.8.7.6.5/10^6$

3.   (i) A random variable X has P(X=x)=x/15 for x=1, 2, 3, 4, 5, and P(X=x) is 0 for all other values. What is the mean and variance of X?   [5 marks]

Solution

$E[X]=1/15+..+5*5/15$. $E[X^2] = 1/15+..+5^2*5/15$. $Var(X)= E[X^2]- E[X]^2$

(ii) You perform the following experiment: you take a six-sided die, and roll it. If the number that comes up is six, you stop; otherwise you repeat.

a) What is the distribution of the number of times you roll the die?   [5 marks]

Solution

Let X be number of times rolled. $\text{Prob}(X=n) = (5/6)^{(n-1)}(1/6)$

b) What is the expected number of rolls?   [10 marks]

Solution

$E[X] = (1/6).1 + (5/6)(1/6).2 + (5/6)^2(1/6).3 + \ldots$