STU33009 Mid-Term Assignment

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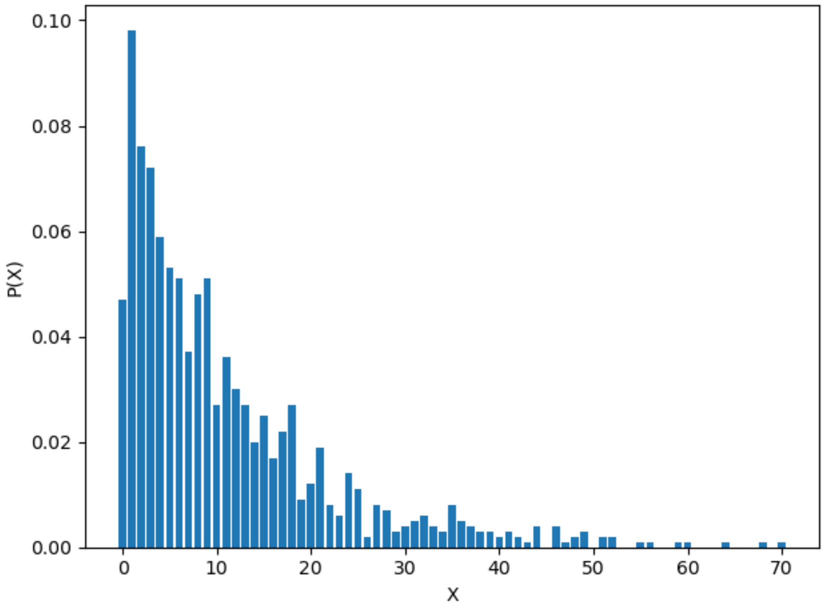
1 Question 1

(a)



(b)
To obtain this result I converted the times to dummy variables and set the sum of 1's over the total amount equal to the probability.

$$P(X_0 = 1) = 0.38100$$

(c)
One of the useful features of python is it calculates the standard deviation and mean for us alowing us just to plug in values for doing confidence intervals To calculate it normally we'd use $(1 - P)(0 - P)^2 + (P)(1 - P)^2$ where $P = P(X_0 = 1)$ We get

$$\sigma^2 = 0.235839$$

To get the 95 percent interval for each method we get: where mean is = 0.381 and N = 1000
Chebyshevs: Provides an actual bound for all values of N but is loose in general.

$$.381 \pm \frac{\sqrt{.235839}}{\sqrt{.05(1000)}}$$

which equals

$$.381 \pm 0.06867$$

CLT: Full distribution of X hat but accuracy depends on size of N

$$-2(.485633) \leq X - 0.381 \leq +2(.485633)$$

$$-.972 \leq X - 0.381 \leq +.972$$

2 Question 2

Same method as Q1(b)

$$P(X_1 = 1) = 0.574$$

$$P(X_2 = 1) = 0.523$$

$$P(X_3 = 1) = 0.561$$

3 Question 3

Using the calculated $P(X_i = 1)$ and the given $P(U_n = i)$ we can calculate $P(Z_n > 10)$ using marginalisation by doing the following:

$P(Z_n > 10) = P(X_0 = 1)P(U_n = 0) + P(X_1 = 1)P(U_n = 1) + ... + P(X_3 = 1)P(U_n = 3)$

giving us $P(Z_n > 10) = 0.5066$

4 Question 4

$$P(U_n = 0|Z_n > 10) = \frac{P(Z_n > 10|U_n = 0)P(Z_n > 10)}{P(U_n = 0)}$$

$P(Z_n > 10|U_n = 0)$ is the probability of user 0 sending a bad signal which is the same as $P(X_0 = 1)$ Answer:

$$P(U_n = 0|Z_n > 10) = \frac{.381 * 0.5066}{0.27616531089096} = 0.6989$$

5 Question 5

For this question, I generated a list of 1000 requests from a user i picked at random given the probabilities in the data given. Given the user a 0 or 1 is added to the list of requests based of $P(X_i = 1)$. I then calculated $P(Z_n > 10)$ by counting the number of 1's in the list and putting it over 1000(the number of requests).I then ran this 1000 times also to get the average value of $Z_n > 10$ This gave me:

$$P(Z_n > 10) = 0.5071$$

Compared to Zn we calculated earlier, our stochastic simulation gives us a value very close to what we had estimated initially. The more times we run the simulation, we can see our accuracy improving over time due to the law of large numbers. Our answer is just an approximation of what Zn should be given N trials though leading to the different values.

6 Code

```
def graphMaker():
    dataFile = open("statsData", "r")
    user0Vals = []
    user1vals = []
    user2vals = []
    user3vals = []
    firstLine = True;
    for i in dataFile:
        if (not firstLine):
            vals = i.split(" ")
            user0Vals.append(int(vals[0]))
            user1vals.append(int(vals[1]))
            user2vals.append(int(vals[2]))
            user3vals.append(int(vals[3]))
        firstLine = False
    user_0 = 0.27616531089096
    user_1 = 0.21892050946049
    user_2 = 0.19773040565225
    user_3 = 0.3071837739963

    df = pd.DataFrame.from_records([user0Vals, user1vals, user2vals, user3vals])
    df = df.transpose()
    data = pd.DataFrame(df[0].value_counts())
    data.columns = ["Counts"]
    data["Prob"] = data["Counts"] / 1000

    df['Indicator1'] = 0
    df['Indicator2'] = 0
    df['Indicator3'] = 0
    df['Indicator4'] = 0

    df.loc[df[0] > 10, 'Indicator1'] = 1
    df.loc[df[1] > 10, 'Indicator2'] = 1
    df.loc[df[2] > 10, 'Indicator3'] = 1
    df.loc[df[3] > 10, 'Indicator4'] = 1

   # pmf = plt.bar(data.index.values, data["Prob"])
   # plt.xlabel("X")
   # plt.ylabel("P(X)")
  #  plt.show()
    u0_stats = df['Indicator1'].agg(['mean', 'std'])
```

```
    u1_stats = df['Indicator2'].agg(['mean', 'std'])
    u2_stats = df['Indicator3'].agg(['mean', 'std'])
    u3_stats = df['Indicator4'].agg(['mean', 'std'])

    p_Zn = (u0_stats['mean'] * user_0) + (u1_stats['mean'] * user_1) + (u2_stats['mean']
           + (u3_stats['mean'] * user_3)  # current guess how to solve it
    print((p_Zn))
    print(u0_stats,'\n',u1_stats,'\n',u2_stats,'\n',u3_stats)
    return([u0_stats['mean'],u1_stats['mean'],u2_stats['mean'],u3_stats['mean']])


def stochastic_sim(means):
    order = np.random.choice(['user0','user1','user2','user3'],10000,
                     p=[0.27616531089096,0.21892050946049,
                        0.19773040565225,0.3071837739963])
    Zlist = []
    znList = []
    for s in range(1000):
        for i in order:
            if i == 'user0':
                Zlist.append(np.random.choice([0, 1], 1, p=[1 - means[0], means[0]])[0])
            elif i == 'user1':
                Zlist.append(np.random.choice([0, 1], 1, p=[1 - means[1], means[1]])[0])
            elif i == 'user2':
                Zlist.append(np.random.choice([0, 1], 1, p=[1 - means[2], means[2]])[0])
            elif i == 'user3':
                Zlist.append(np.random.choice([0, 1], 1, p=[1 - means[3], means[3]])[0])
        zFrame = pd.DataFrame(Zlist)
        znList.append(zFrame[0].agg(['mean']))
        Zlist = []
        print(s)
    print(sum(znList)/1000)
means = graphMaker()
stochastic_sim(means)
```