3BA26 : Concurrent Systems Project
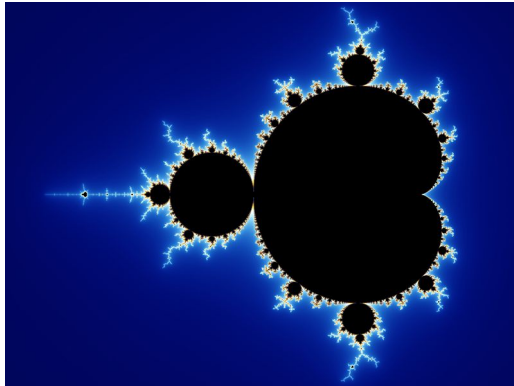
David Gregg

Department of Computer Science
Trinity College Dublin

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Introduction
The Mandelbrot Set
Rendering
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Render of the M Set
Complex Numbers Recap

The Mandelbrot Set

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Complex Numbers Recap

Complex numbers are numbers which have both a *real* and *imaginary* component. They are generally written as $x + iy$ where $i^2 = -1$.

As complex numbers contain two separable components we can represent a complex number as a point on a 2 dimensional graph where one axis represents the real component and the other the imaginary. Such a graph is called the *Complex Plane*.

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The M Set
Set Membership

The M Set

The Mandelbrot or *M* Set is the set of all complex numbers, c, which satisfy the criteria that they remain bounded under a complex quadratic polynomial:

$x_0 = 0$

$x_{n+1} = (x_n)^2 + c$

If *c* has the value 1, we get the sequence 0, 1, 2, 5, 26, 677....
Obviously this sequence is not bounded as it escapes to infinity and therefore 1 is not a member of the M set.

If we however take the example c = i we get the sequence 0, i, (-1 + i), -i, (-1 + i), -i which is cyclical and bounded

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Determining if a point is a member of M

The sequences generated by $x_{n+1} = (x_n)^2 + c$ can vary wildly depending on the input. For some points we escape towards infinity quickly. It can be easily shown in fact, that if the absolute value of X_n is greater than 2 then we are guaranteed to escape.
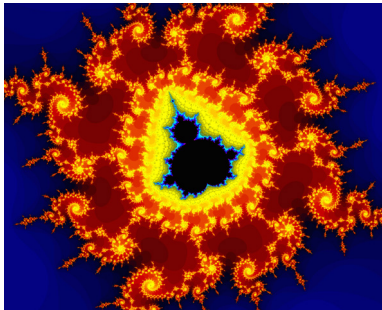
For other points we must go through many hundreds of iterations before we escape.

Some points will never escape no matter how many times we iterate. We therefore set a limit on the number of iterations. If we have not escaped by the time we reach the limit, we count that point as a member of the M set (which means that for the purposes of this lab, we are actually implementing an approximation of M).

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Rendering the M Set
Colouring the M Set

Rendering the M Set

The familiar Mandelbrot image is simply a representation of which points on the complex plane are members of the M set and which are not. If a point is member of the M set we colour the point on the complex plane black. If it's not we colour it another colour. We'll explain how the colourful images you may be familiar with are generated later.

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Colouring the M Set

From purely an aesthetic perspective, we may wish to colour the M set to produce an image more striking than a simple monotone one. If we colour a point based on how many iterations it took to escape, we can create some astounding images.

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EPAs
The Solution
The WRONG Solution
Points to Note

Embarrassingly Parallel Algorithms

An embarrassingly parallel algorithm is one for which we need to make virtually no effort to parallelise. It can be broken into many parallel tasks which have no dependence on each other very easily.

Rendering the Mandelbrot Set is a famous example as each point on the plane can be tested for set membership independently of any other point. Another famous example of one of these algorithms is the Ray Tracing Algorithm where rays can be traced independently and in parallel with other rays.

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The Proper Solution

The full solution will be generated by applying the following techniques:

- Vectorisation.
- Parallelisation using threads.
- Load balancing.
- General optimisation through profiling.

Please include a small report (no more than 3 pages) detailing your work. Please concentrate on your design decisions.

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The Wrong Solution

Please bear in mind that this class and project are aimed at teaching techniques for parallelisation. There are many other techniques that can be used to accelerate the rendering of the M Set including but not limited to:

- Advanced mathematics.
- Perceptual techniques.

If you choose to employ such techniques as an exercise, you *must* also hand up a version of the project using only the given algorithm to gain full marks.

You may NOT change the number of iterations or the basic design of the algorithm.

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Points to Note

- Taking a small performance hit on N processors with code that performs much better when you have more processors is OK.

- Don't forget WHY we are doing this project. Refer to the Proper Solution slide.

- Experimentation is good. Make a note of what happened when you tried a technique. Compare it to another.

- Don't make claims you can't backup without data or citations.