Chapter 3

Q 3.2

$$f(x) = x - 2e^{-x}$$

(a) Bisection

(1) $a = 0 \quad b = 1$

$$\text{mid} = \frac{0+1}{2} = 0.5$$

$f(a)f(\text{mid})$

$= (-2)(-0.713) > 0$

$\therefore$ Between mid + b

(2) $\text{new mid} = \frac{0.5+1}{2} = 0.75$

$f(0.5)f(0.75)$

$= (-0.713)(-0.19) > 0$

$\therefore$ Between mid + b

(3) $\text{mid} = \frac{0.75+1}{2} = 0.875$

$f(0.75)f(0.875)$

$= (-0.19)(0.04) < 0$

$\therefore 0.75 < x < 0.875$

(b) Secant Method

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_2) - 0}{x_2 - x_3}$$

$$x_2 - x_3 = \frac{(f(x_2) - 0)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = x_2 - \frac{(f(x_2) - 0)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$x_1 = 0 \qquad f(x_1) = -2$

$x_2 = 1 \qquad f(x_2) = 0.264$

$$x_3 = 1 - \frac{(0.264)(1)}{2.264}$$

$$= 0.883$$

$$x_4 = x_3 - \frac{(f(x_3) - 0)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

$$= 0.883 - \frac{0.056\,(0.883 - 1)}{0.056 - 0.264}$$

$$x_4 = 0.8515$$

$$x_5 = x_4 - \frac{f(x_4)\,(x_4 - x_3)}{f(x_4) - f(x_3)}$$

$$= 0.8515 - \frac{(-.002)(0.8515 - 0.883)}{(-.002) - 0.056}$$

$$x_5 = 0.8526$$

(c) Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = x - 2e^{-x}$$

$$f'(x) = 1 + 2e^{-x}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 0.8478$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.8478 - \frac{f(0.8478)}{f'(0.8478)}$$

$$= 0.8526$$

$$x_4 = 0.8526$$

Need to do

- Regular Falsi
- Fixed point iteration

Q3.8

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = \sqrt{x} + x^2 - 7$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 2x$$

$$x_2 = 7 - \frac{f(7)}{f'(7)}$$

$$= 3.85$$

$$x_3 = 2.62$$

$$x_4 = 2.35$$

$$x_5 = 2.33$$

$$x_6 = 2.3389$$

Q 3.14 Newtons Method for non-linear eq's.

$$x^2 + 2x + 2y^2 - 26 = 0 \qquad f_1(x,y)$$
$$2x^3 - y^2 + 4y - 19 = 0 \qquad f_2(x,y)$$

Start @ $(1,1)$

$$\frac{\partial f_1}{\partial x} = 2x + 2 \qquad \frac{\partial f_1}{\partial y} = 4y$$

$$\frac{\partial f_2}{\partial x} = 6x^2 \qquad \frac{\partial f_2}{\partial y} = -2y + 4$$

$$J\left(f_1(x_1,y_1),\, f_2(x_1,y_1)\right)$$

$$= \det \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \frac{\partial f_1}{\partial x}\left(\frac{\partial f_2}{\partial y}\right) - \frac{\partial f_1}{\partial y}\left(\frac{\partial f_2}{\partial x}\right)$$

$$= (2x+2)(-2y+4) - \left(4y\left(6x^2\right)\right)$$

$$= -4xy + 8x - 4y + 8 - 24x^2y$$

$$\underbrace{f_1(x_2,y_2)}_{0} = f_1(x_1,y_1) + \underbrace{(x_2 - x_1)}_{\Delta x}\frac{\partial f_1}{\partial x} + \underbrace{(y_2 - y_1)}_{\Delta y}\frac{\partial f_1}{\partial y}$$

$$\underbrace{f_2(x_2,y_2)}_{0} = f_2(x_1,y_1) + \underbrace{(x_2 - x_1)}_{\Delta x}\frac{\partial f_2}{\partial x} + \underbrace{(y_2 - y_1)}_{\Delta y}\frac{\partial f_2}{\partial y}$$

$$\underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}}_{M} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -f_1 \\ -f_2 \end{bmatrix}$$

$(1,1)$

Cramer's rule:

$$M_1 = \begin{bmatrix} -f_1(1,1) & \frac{\partial f_1}{\partial y} \\ -f_2(1,1) & \frac{\partial f_2}{\partial y} \end{bmatrix} \qquad M_2 = \begin{bmatrix} \frac{\partial f_1}{\partial x} & -f_1(1,1) \\ \frac{\partial f_2}{\partial y} & -f_2(1,1) \end{bmatrix}$$

$$\Delta x = \frac{\det[M_1]}{\det[M]} = \frac{-f_1(1,1)\left(\frac{\partial f_2}{\partial y}\right) - \left(\frac{\partial f_1}{\partial y}\right)\left(-f_2(1,1)\right)}{J(f_1(1,1),\ f_2(1,1))}$$

$$(J = \det[M])$$

$$\Delta y = \frac{\det[M_2]}{\det[M]} = \frac{-f_1(1,1)\left(\frac{\partial f_2}{\partial y}\right) - \left(\frac{\partial f_1}{\partial y}\right)\left(-f_2(1,1)\right)}{J(f_1(1,1),\ f_2(1,1))}$$

Then $x_2 = x_1 + \Delta x$

$y_2 = y_1 + \Delta y$

Repeat for new $\Delta x, \Delta y$

Q4.2 Gauss elimination

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array}\begin{bmatrix} 2 & -2 & 1 \\ 3 & 2 & -5 \\ -1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -16 \\ 8 \end{bmatrix}$$

$$M_{21} = \frac{3}{2} = \frac{a_{21}}{a_{11}}$$

$$R1 \times M_{21} = [\,2(3/2) \;\; -2(3/2) \;\; 1(3/2)\,] \quad [\,10(3/2)\,]$$

$$R2 = R2 - (R1(M_{21})) = 0 \quad 5 \quad -6.5 \qquad -31$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 5 & -6.5 \\ -1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -31 \\ 8 \end{bmatrix}$$

$$R3 = R3 - (R1(M_{31}))$$

$$M_{31} = \frac{-1}{2} = \frac{a_{31}}{a_{11}}$$

$$R1' = 2(-1/2) \quad -2(-1/2) \quad 1(-1/2) \qquad 10(-1/2)$$

$$-1 \qquad 1 \qquad -1/2 \qquad -5$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 5 & -6.5 \\ 0 & 1 & 3.5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -31 \\ 13 \end{bmatrix}$$

Now $R2$ = pivot eq.

$$M_{32} = \frac{1}{5} = \frac{a_{32}}{a_{22}}$$

$$R2' = 0 \quad 1 \quad -1.3 \qquad -6.2$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 5 & -6.5 \\ 0 & 0 & 4.8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -31 \\ 19.2 \end{bmatrix}$$

$$x_3 = \frac{19.2}{4.8} = 4$$

$$5x_2 - 6.5x_3 = -31$$

$$x_2 = \frac{-31 + 26}{5} = -1$$

$$2x_1 - 2x_2 + x_3 = 10$$

$$2x_1 + 2 + 4 = 10$$

$$x_1 = 2$$

Q4.13

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array}\begin{bmatrix} 10 & 12 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Normalise R1

$$\begin{bmatrix} 1 & 1.2 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$M_{21} = \frac{0}{1} = 0$

a_{21} already $= 0$, skip.

$M_{31} = \frac{2}{1} = 2$

$R1' = 2 \quad 2.4 \quad 0 \quad\quad 0.2$

$R3 = R3 - R1'$

$$\begin{bmatrix} 1 & 1.2 & 0 \\ 0 & 2 & 8 \\ 0 & 1.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

$R2$ = pivot eq.

Normalise R2

$$\begin{bmatrix} 1 & 1.2 & 0 \\ 0 & 1 & 4 \\ 0 & 1.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

$M_{12} = \frac{1.2}{1} = 1.2$

$R2' = 0 \quad 1.2 \quad 4.8 \quad\quad 0$

$R1 = R1 - R2'$

$$\begin{bmatrix} 1 & 0 & -4.8 \\ 0 & 1 & 4 \\ 0 & 1.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

$M_{32} = \frac{1.6}{1} = 1.6$

$R2' = 0 \quad 1.6 \quad 6.4 \quad\quad 0$

$R3 = R3 - R2'$

$$\begin{bmatrix} 1 & 0 & -4.8 \\ 0 & 1 & 4 \\ 0 & 0 & 1.6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

Normalise R3

$$\begin{bmatrix} 1 & 0 & -4.8 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ c \\ -0.125 \end{bmatrix}$$

$M_{13} = \frac{-4.8}{1} = -4.8$

$R3' = 0 \quad 0 \quad -4.8 \quad\quad 0.6$

$R1 = R1 - R3'$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.5 \\ c \\ -0.125 \end{bmatrix}$$

$M_{23} = \frac{4}{1} = 4$

$R3' = 0 \quad 0 \quad 4 \quad\quad -0.5$

$R2 = R2 - R3'$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \\ -0.125 \end{bmatrix}$$

$$[a^{-1}] = \begin{bmatrix} -0.5 & DK & DK \\ 0.5 & DK & DK \\ -0.125 & DK & DK \end{bmatrix}$$

Repeat 2 more times to find columns 2 & 3.

Q5.3 Finding eigenvalues

$$a = \begin{bmatrix} 10 & 0 & 0 \\ 1 & -3 & -7 \\ 0 & 2 & 6 \end{bmatrix}$$

$\det[a - \lambda I] = 0$ Characteristic eq.

$$\det \begin{bmatrix} 10-\lambda & 0 & 0 \\ 1 & -3-\lambda & -7 \\ 0 & 2 & 6-\lambda \end{bmatrix} = 0$$

$$10-\lambda \left(\det \begin{vmatrix} -3-\lambda & -7 \\ 2 & 6-\lambda \end{vmatrix} \right) - 0(-) + 0(-)$$

$$10-\lambda \left((-3-\lambda)(6-\lambda) - (-7)(2) \right)$$

$$10-\lambda \left(-18 - 3\lambda + \lambda^2 + 14 \right) = 0$$

$10 - \lambda = 0$ $\qquad$ $\lambda^2 - 3\lambda - 4 = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\lambda = 10$

$\lambda = 4 \qquad \lambda = -1$

Q 5.7

$$a = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$av = \begin{bmatrix} 4 \\ 10 \\ 1 \end{bmatrix} \quad \text{Max} = 10$$

$$\text{Normalise} = 10 \begin{bmatrix} 0.4 \\ 1 \\ 0.1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.4 \\ 1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 1.6 \\ 3.1 \\ 0.1 \end{bmatrix}$$

$$\text{Max} = 3.1$$

$$\text{Normalise} = 3.1 \begin{bmatrix} 0.516 \\ 1 \\ 0.032 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.516 \\ 1 \\ 0.032 \end{bmatrix} = \begin{bmatrix} 1.58 \\ 3.224 \\ 0.032 \end{bmatrix}$$

$$\text{Max} = 3.224$$

$$\Rightarrow \quad 3.224 \begin{bmatrix} 0.49 \\ 1 \\ 0.009 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.49 \\ 1 \\ 0.009 \end{bmatrix} = \begin{bmatrix} 1.508 \\ 3.005 \\ 0.009 \end{bmatrix}$$

$$\text{Max} = 3.005$$

$$\Rightarrow \begin{bmatrix} 0.502 \\ 1 \\ 0 \end{bmatrix}$$

Keep going till max's approx. equal.