TRINITY COLLGE DUBLIN

School of Computer Science and Statistics

Extra Questions ST3009: Statistical Methods for Computer Science

Question 1. Suppose two random variables X and Y have PDFs $f_X(x) = e^{-x}$, $f_Y(y) = 0.5e^{-0.5y}$ and conditional PDF $f_{Y|X}(y|x) = e^{-|x-y|}$. Using Bayes Rule for PDFs write an expression for $f_{X|Y}(x|y)$.

Question 2. (a) Give Bayes rule for PDFs

(b) Explain the difference between the maximum likelihood and the MAP estimate of a random variable

(c) Suppose after observing data the likelihood of parameter θ is $L(\theta) = e^{-(\theta-1)^2}$. What is the maximum likelihood estimate of θ ?

Question 3. Suppose and urn contains balls and that fraction θ of the balls are white and the rest are red. I draw n balls, with replacement, from the urn and let X be the number of white balls observed.

(a) Give an expression for the likelihood $P(X = x|\theta)$

(b) Suppose $n = 100$ and I observe 25 white balls. What is the maximum likelihood estimate for θ (use matlab to plot the value of $P(X = x|\theta)$ for a range of values of θ).

(c) Suppose now that before drawing the balls my prior probability was $P(\theta) = \frac{1}{20\pi}e^{-100(\theta-0.5)^2}$ and for simplicity assume that $P(X = 25) = 1$ (since it just scales the posterior). Give an expression for the posterior $P(\theta|X = x)$ (use Bayes rule).

(d) What is the MAP estimate for θ (use matlab to plot the value of $P(\theta|X = x)$ for a range of values of θ). Discuss why it differs from the maximum likelihood estimate.

Question 4. We observe data $(x^{(i)}, y^{(i)})$, $i = 1, 2, \ldots, n$ from n people, where $x^{(i)}$ is the persons height and $y^{(i)}$ is the persons weight.

1. Explain how to construct a linear regression model for this data.

2. Suppose we suspect that the weight of a person is not linearly related to their height but rather is related to the square root of their height. Explain how to modify the linear regression model to accommodate this.

Question 5. Explain the principle of the gradient descent algorithm. Accompany your explanation with a diagram and pseudo-code.