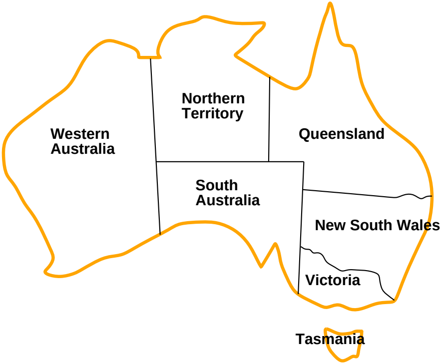
Graph modeling



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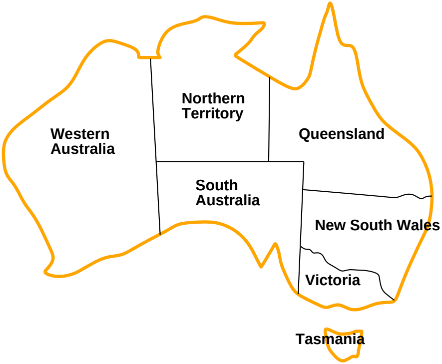
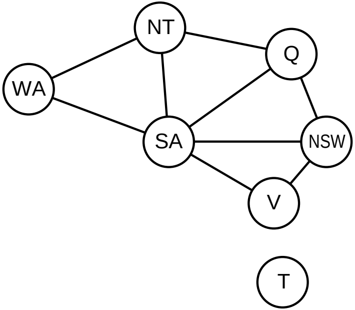
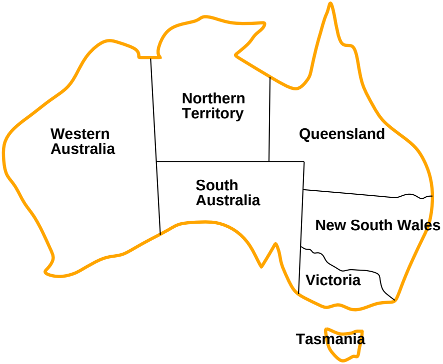
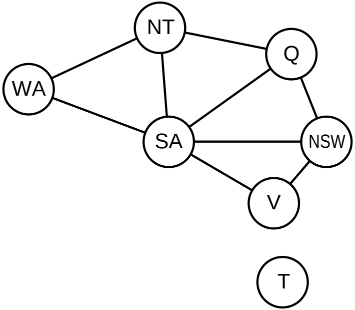




Russell & Norvig

Graph modeling





Russell & Norvig

```
arc(wa,nt).    arc(nt,q).    arc(q,nsw).
arc(wa,sa).    arc(nt,sa).   arc(sa,q).
arc(sa,nsw).   arc(sa,v).    arc(v,nsw).

arc2(X,Y) :- arc(X,Y) ; arc(Y,X).
```

Non-termination (due to poor choices)

```
i :- p,q.                    [i]

i :- r.

p :- i.

r.
```

```
| ?- i.
```

```
prove([],_).

prove([H|T],KB) :- member([H|B],KB), append(B,T,Next),
                   prove(Next,KB).

| ?- prove([i],[[i,p,q],[i,r],[p,i],[r]]).
```

Non-termination (due to poor choices)

```
i :- p,q.

i :- r.

p :- i.

r.
---------
| ?- i.
```

```
        [i]
      ↙     ↘
  [p,q]      [r]
```

```
prove([],_).

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________
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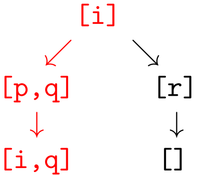
```

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r.
________

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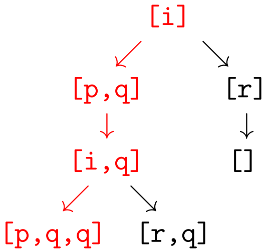
```

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```

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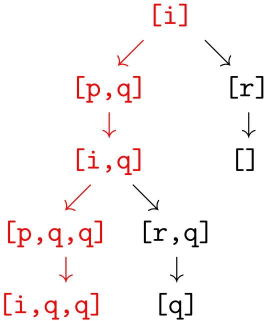
```
i :- p,q.

i :- r.

p :- i.

r.
_________
| ?- i.
```

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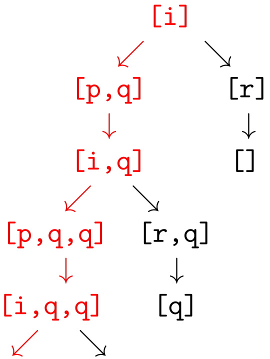
```
i :- p,q.

i :- r.

p :- i.

r.
________

| ?- i.
```

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| ?- prove([i],[[i,p,q],[i,r],[p,i],[r]]).
```

Determinization (eliminate choice)

A fsm [Trans, Final, Q0] such that

for all $\texttt{[Q,X,Qn]}$ *and* $\texttt{[Q,X,Qn}'\texttt{]}$ *in* $\texttt{Trans}, \quad \texttt{Qn} = \texttt{Qn}'$

is a *deterministic finite automaton* (DFA).

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Fact. Every fsm has a DFA accepting the same language.

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

for all `[Q,X,Qn]` *and* `[Q,X,Qn′]` *in* `Trans`, $\mathtt{Qn} = \mathtt{Qn}'$

is a *deterministic finite automaton* (DFA).

Fact. Every fsm has a DFA accepting the same language.

Proof: Subset (powerset) construction

```
arcD(NodeList,NextList) :-
        setof(Next, arcLN(NodeList,Next), NextList).

arcLN(NodeList,Next) :- member(Node,NodeList),
                        arc(Node,Next).
```

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

$$\textit{for all } \texttt{[Q,X,Qn]} \textit{ and } \texttt{[Q,X,Qn}'\texttt{]} \textit{ in } \texttt{Trans}, \quad \texttt{Qn} = \texttt{Qn}'$$

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arcLN(NodeList,Next) :- member(Node,NodeList),
                        arc(Node,Next).

goalD(NodeList) :- member(Node,NodeList), goal(Node).
```

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

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                        arc(Node,Next).

goalD(NodeList) :- member(Node,NodeList), goal(Node).

searchD(NL) :- goalD(NL);
               (arcD(NL,NL2), searchD(NL,NL2)).
```

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

for all `[Q,X,Qn]` *and* `[Q,X,Qn']` *in* `Trans`, $\texttt{Qn} = \texttt{Qn}'$

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Fact. Every fsm has a DFA accepting the same language.

Proof: Subset (powerset) construction

```
arcD(NodeList,NextList) :-
          setof(Next, arcLN(NodeList,Next), NextList).

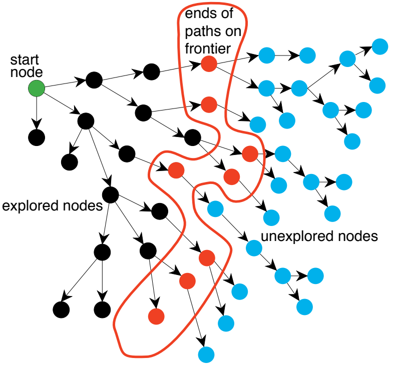
arcLN(NodeList,Next) :- member(Node,NodeList),
                        arc(Node,Next).

goalD(NodeList) :- member(Node,NodeList), goal(Node).

searchD(NL) :- goalD(NL);
               (arcD(NL,NL2), searchD(NL,NL2)).

search(Node) :- searchD([Node]).
```

Frontier search



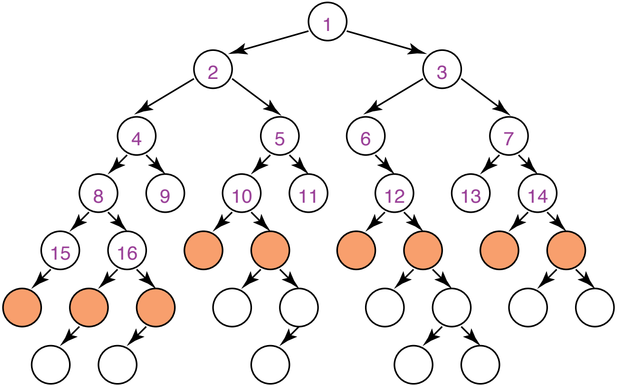
Poole & Mackworth

```
search(Node) :- frontierSearch([Node]).

frontierSearch([Node|_]) :- goal(Node).

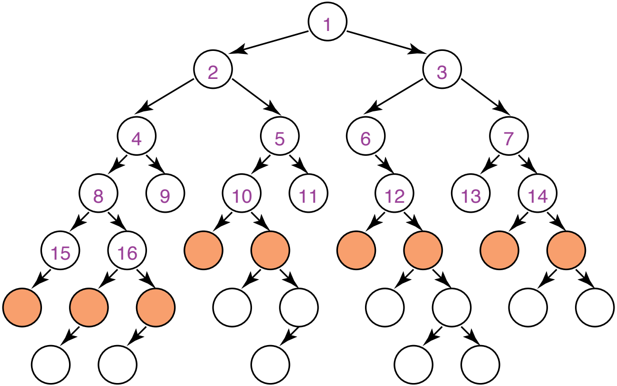
frontierSearch([Node|Rest]) :-
            findall(Next, arc(Node,Next), Children),
            add2frontier(Children,Rest,NewFrontier),
            frontierSearch(NewFrontier).
```

Breadth-first: queue (FIFO)



$$\texttt{[1]} \rightsquigarrow \texttt{[2,3]} \rightsquigarrow \texttt{[3,4,5]} \rightsquigarrow \texttt{[4,5,6,7]} \rightsquigarrow \cdots$$

Breadth-first: queue (FIFO)

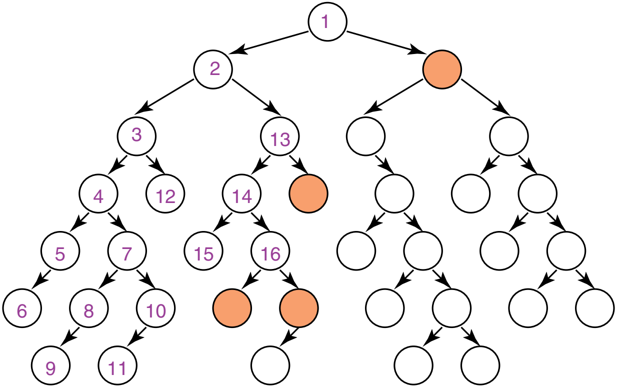


$[1] \leadsto [2,3] \leadsto [3,4,5] \leadsto [4,5,6,7] \leadsto \cdots$

```
add2frontier(Children,[],Children).

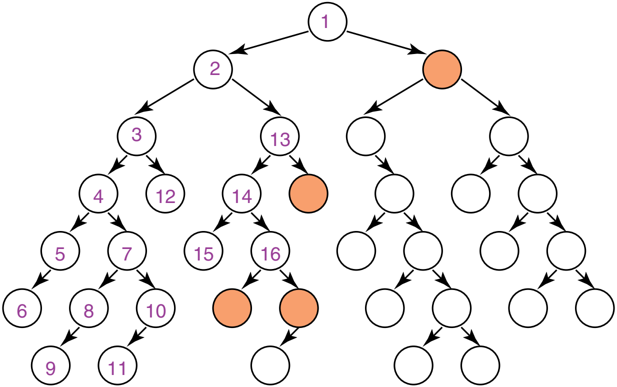
add2frontier(Children,[H|T],[H|More]) :-
                              add2frontier(Children,T,More).
```

Depth-first: stack (LIFO)



[1] $\rightsquigarrow$ [2,•] $\rightsquigarrow$ [3,13,•] $\rightsquigarrow$ [4,12,13,•] $\rightsquigarrow \cdots$

Depth-first: stack (LIFO)



[1] $\rightsquigarrow$ [2,•] $\rightsquigarrow$ [3,13,•] $\rightsquigarrow$ [4,12,13,•] $\rightsquigarrow$ $\cdots$

```
add2frontier([],Rest,Rest).

add2frontier([H|T],Rest,[H|TRest]) :-
                              add2frontier(T,Rest,TRest).
```

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

If-then-else and cut !

```
i :- p,!,q.                    [i]

i :- r.

p.

r.
```

```
| ?- i.
```

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$\begin{tikzcd}[column sep=small] & \texttt{[i]} \arrow[dl] \arrow[dr] & \\ \texttt{[p,!,q]} & & \texttt{[r]} \end{tikzcd}$$

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

```
           [i]
         ↙     ↘
   [p,!,q]      [r]
      ↓
   [!,q]
```

Cut ! is true but destroys backtracking.

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

[i]

↙

[p,!,q]

↓

[!,q]

↓

[q]

Cut ! is true but destroys backtracking.

If-then-else and cut !

```
i :- p,!,q.                        [i]
                                 ↙
i :- r.                     [p,!,q]
                               ↓
p.                           [!,q]
                               ↓
r.                            [q]
```

```
| ?- i.

no
```

Cut ! is true but destroys backtracking.

Review: Depth-first as frontier search

```
prove([],_).      % goal([]).
prove(Node,KB) :- arc(Node,Next,KB), prove(Next,KB).
```

Review: Depth-first as frontier search

```
prove([],_).      % goal([]).
prove(Node,KB) :- arc(Node,Next,KB), prove(Next,KB).
```

```
fs([[]|_],_).

fs([Node|More],KB) :- findall(X,arc(Node,X),L),
                      append(L,More,NewFrontier),
                      fs(NewFrontier,KB).
```

Review: Depth-first as frontier search

```
prove([],_).      % goal([]).
prove(Node,KB) :- arc(Node,Next,KB), prove(Next,KB).
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```
fs([[]|_],_).

fs([Node|More],KB) :- findall(X,arc(Node,X),L),
                      append(L,More,NewFrontier),
                      fs(NewFrontier,KB).
```

Cut?

Tracking the frontier

```
[[i]]
```

```
i :- p,!,q.                    [i]

i :- r.

p.

r.
```

```
| ?- i.
```

Tracking the frontier

`[[i]]` $\rightsquigarrow$ `[[p,!,q],[r]]`

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$\begin{tikzcd} & \texttt{[i]} \arrow[dl] \arrow[dr] & \\ \texttt{[p,!,q]} & & \texttt{[r]} \end{tikzcd}$$

Tracking the frontier

$\texttt{[[i]]} \rightsquigarrow \texttt{[[p,!,q],[r]]} \rightsquigarrow \texttt{[[!,q],[r]]}$

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$
\begin{tikzcd}
 & \texttt{[i]} \arrow[dl] \arrow[dr] & \\
\texttt{[p,!,q]} \arrow[d] & & \texttt{[r]} \\
\texttt{[!,q]} & &
\end{tikzcd}
$$

Tracking the frontier

$$\texttt{[[i]]} \rightsquigarrow \texttt{[[p,!,q],[r]]} \rightsquigarrow \texttt{[[!,q],[r]]} \rightsquigarrow \texttt{[[q]]}$$

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$\begin{tikzcd} & \texttt{[i]} \arrow[dl] \\ \texttt{[p,!,q]} \arrow[d] & \\ \texttt{[!,q]} \arrow[d] & \\ \texttt{[q]} & \end{tikzcd}$$

Tracking the frontier

$\texttt{[[i]]} \rightsquigarrow \texttt{[[p,!,q],[r]]} \rightsquigarrow \texttt{[[!,q],[r]]} \rightsquigarrow \texttt{[[q]]} \rightsquigarrow \texttt{[]}$

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$\begin{tikzcd}[column sep=small] & \texttt{[i]} \arrow[dl] \\ \texttt{[p,!,q]} \arrow[d] & \\ \texttt{[!,q]} \arrow[d] & \\ \texttt{[q]} & \end{tikzcd}$$

Tracking the frontier

`[[i]]` $\leadsto$ `[[p,!,q],[r]]` $\leadsto$ `[[!,q],[r]]` $\leadsto$ `[[q]]` $\leadsto$ `[]`

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.

no
```

`[i]`

$\swarrow$

`[p,!,q]`

$\downarrow$

`[!,q]`

$\downarrow$

`[q]`

Cut via frontier depth-first search

```
fs([[]|_],_).



fs([Node|More],KB) :-
                        findall(X,arc(Node,X),L),
                        append(L,More,NewFrontier),
                        fs(NewFrontier,KB).
```

Cut via frontier depth-first search

```
fs([[]|_],_).

fs([[cut|T]|_],KB)) :- fs([T],KB).

fs([Node|More],KB) :-
                    findall(X,arc(Node,X),L),
                    append(L,More,NewFrontier),
                    fs(NewFrontier,KB).
```

Cut via frontier depth-first search

```
fs([[]|_],_).

fs([[cut|T]|_],KB)) :- fs([T],KB).

fs([Node|More],KB) :-  Node = [H|_], H\== cut,
                       findall(X,arc(Node,X),L),
                       append(L,More,NewFrontier),
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Cut via frontier depth-first search

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                       fs(NewFrontier,KB).


if(p,q,r) :- (p,!,q); r.       % contra (p,q);r
```

Cut via frontier depth-first search

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fs([[]|_],_).

fs([[cut|T]|_],KB)) :- fs([T],KB).

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if(p,q,r) :- (p,!,q); r.      % contra (p,q);r

negation-as-failure(p) :- (p,!,fail); true.
```