Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

Integrated Computer Science
BA (Mod) Computer Science and Business

Trinity Term 2017

Year 3 Annual examinations

Computational Mathematics

Wednesday 10th May 2017 RDS Main Hall 09.30-11.30

Dr. Eamonn O Nuallain

Instructions to Candidates:

(i) A total of FOUR questions should be attempted.

(ii) All questions carry equal marks.

Materials Permitted for this Examination:

(i) There is a Formula Sheet appended to this document

(ii) Use of non-programmable calculators and log tables are permitted.

Question 1.

The power output of a solar cell varies with the voltage it puts out. The voltage V_{mp} at which the output power is maximum is given by the equation:

$$e^{(qV_{mp}/k_BT)}\left(1+\frac{qV_{mp}}{k_BT}\right)=e^{(qV_{OC}/k_BT)}$$

where V_{OC} is the open circuit voltage, T is the temperature in Kelvin, q = 1.6022×10^{-19} C is the charge on an electron, and k_B = 1.3806×10^{-23} J/K is Boltzmann's constant.
For V_{OC} = 0.5 V and room temperature (T = 297 K), determine the voltage V_{mp} at which the power output of the cell is a maximum by writing a MATLAB program in a script file that uses the fixed-point iteration method to find the root.
For a starting point, use V_{mp}= 0.5 V. To terminate the iterations, use the Estimated Relative Error, $\varepsilon \leq 0.001$.

[25 Marks]

Question 2.

Write a user-defined MATLAB function that solves a system of n linear equations, $[a][x] = [b]$, with the Gauss–Jordan method. The program should include pivoting in which the pivot row is switched with the row that has a pivot element with the largest absolute numerical value. For the function name and arguments use x = GaussJordan(a,b), where a is the matrix of coefficients, b is the right-hand-side column of constants, and x is the solution.

[25 Marks]

Question 3.

The following data give the approximate population of China for selected years from 1900 until 2010:

Year	1900	1950	1970	1980	1990	2000	2010
Population (millions)	400	557	825	981	1135	1266	1370

Assume that the population growth can be modeled with an exponential function $p = be^{mx}$ where x is the year and p is the population in millions.

(a) Write the equation in linear form and use linear least-squares regression to determine the constants b and m for which the function best fits the data.

[20 Marks]

(b) Use the equation to estimate the population in the year 1985.

[5 Marks]

Question 4.

The following data is given for the stopping distance of a car on a wet road versus the speed at which it begins braking.

Speed (km/h)	12.5	25.0	37.5	50.0	62.5	75.0
Distance (m)	2.0	5.9	11.8	19.7	29.9	42.0

(a) Calculate the rate of change of the stopping distance at a speed of 62.5 km/h using

(i) The two-point backward difference formula
(ii) The three-point backward difference formula.

[10 Marks]

(b) Calculate an estimate for the stopping distance at 75 km/h by using the results from part (a) for the slope and the two-point central difference formula applied at the speed of 62.5 km/h. How does the estimate compare with the data?

[15 Marks]

Question 5.

The function f(x) is given in the following tabulated form:

x	0.0	0.3	0.6	0.9	1.2	1.5	1.8
f(x)	0.5	0.6	0.8	1.3	2.0	3.2	4.8

For elemental distances of h=0.3 and h=0.6 compute:

$$\int_{0}^{1.8} f(x)dx$$

using the Composite Simpson's 3/8 Method.

[25 Marks]

Formula Sheet

1.

For a line $y = a_1 x + a_0$, by Linear Least Squares Regression we get:

$$a_1 = \frac{nS_{xy} - S_x S_y}{nS_{xx} - (S_x)^2} \qquad a_0 = \frac{S_{xx} S_y - S_{xy} S_x}{nS_{xx} - (S_x)^2} \tag{6.14}$$

where:

$$S_x = \sum_{i=1}^{n} x_i, \quad S_y = \sum_{i=1}^{n} y_i, \quad S_{xy} = \sum_{i=1}^{n} x_i y_i, \quad S_{xx} = \sum_{i=1}^{n} x_i^2 \tag{6.13}$$

2.

The Two-Point Backward Difference Formula:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + \frac{f''(\xi)}{2!}h \tag{8.15}$$

The Three-Point Backward Difference Formula:

$$f'(x_i) = \frac{f(x_{i-2}) - 4f(x_{i-1}) + 3f(x_i)}{2h} + O(h^2) \tag{8.25}$$

The Two-Point Central Difference Formula:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2) \qquad (8.20)$$

where $h = x_i - x_{i-1}$ and $\xi \in \left]x_{i-1}, x_i\right[$

3.

The Composite Simpson's 3/8 Method the integral I(f) is approximately equal to:

$$(f) \approx \frac{3h}{8}\left[f(a) + 3\sum_{i=2,5,8}^{N-1} [f(x_i) + f(x_{i+1})] + 2\sum_{j=4,7,10}^{N-2} f(x_j) + f(b) \right] \qquad (9.22)$$

where:

$$h = x_i - x_{i-1}$$