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Horn-SAT: every clause has at most one positive literal — linear

Prolog and SAT

Prolog KB (definite clauses)

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$\texttt{x1} \lor \overline{\texttt{x2}} \lor \overline{\texttt{x4}}$

$\texttt{x2} \lor \overline{\texttt{x3}} \qquad \rightsquigarrow \qquad$ `[[1,-2,-4],[2,-3],[4]]`

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From proofs to unsatisfiability:

$$\underbrace{KB \text{ proves } \varphi}_{\text{Prolog}} \quad \text{iff} \quad \underbrace{KB, \overline{\varphi}}_{\text{Horn (linear SAT)}} \text{ is not satisfiable}$$