

**TRINITY COLLEGE DUBLIN
THE UNIVERSITY OF DUBLIN**

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

**Integrated Computer Science Programme
Year 3 Annual Examinations**

Trinity Term 2015

Statistical Analysis

Professor Myra O'Regan

Saturday 16th May, 2015

Luce Lower

14:00 – 16:00

Instructions to Candidates:

Answer all questions. All questions carry equal marks. Appendix A contains some useful formulae and normal tables.

Calculators may be used.

Question 1

- a) The number of customers visiting a site in an hour is known to have a Poisson distribution with an average of 3.4. What is the probability of having 5 visitors to the site in an hour? What is the probability of having at least 1 visitor to the site in an hour?

(17 marks)

- b) Seventy percent of light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator. Of the aircraft which were not discovered 90% do not have an emergency locator. Suppose that a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will be discovered?

(17 marks)

- c) The time to complete a job is normally distributed with a mean of 200ms and a standard deviation of 40ms. What is the probability of a job being completed in

- | | | |
|-----|------------------------|-------------------|
| i) | Less than 180ms | (8 marks) |
| ii) | Between 170 and 210 ms | (8 marks) |

Question 2

You have been asked to compare the lifetime of similar batteries from two companies A and B. You pick a random sample of 50 batteries from each company and measure their lifetime. The following summary statistics were obtained for the variable lifetime. Lifetimes were measured in hours.

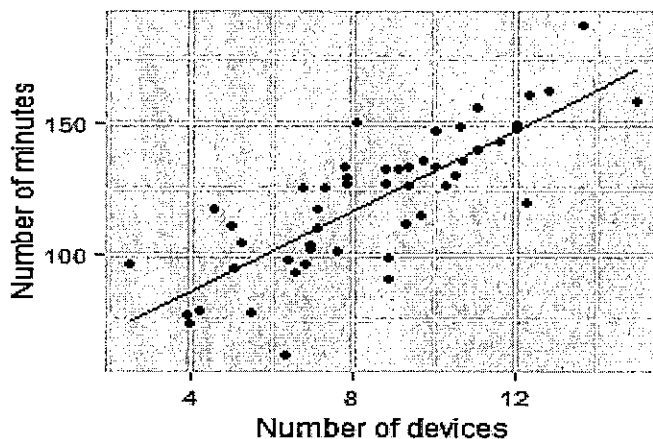
| Company | Mean | SD | max | min | 25th ile | 50th ile | 75th ile | n |
|---------|-------|------|-------|-------|----------|----------|----------|----|
| A | 213.7 | 20.0 | 255.1 | 177.2 | 195.3 | 216.7 | 229.1 | 50 |
| B | 200.6 | 17.5 | 241.2 | 156.1 | 192.1 | 198.7 | 213.1 | 50 |

You also obtained a bootstrapped confidence interval (6.09 to 20.20) using the percentile method for the difference in population means.

- Draw an appropriate graph for the above data. (5 marks)
- Explain how the bootstrapped interval is computed. (15 marks)
- Interpret the interval. (10 marks)
- How does this confidence interval relate to the classical hypothesis testing approach? (10 marks)
- You have been asked for advice on sample size for a study with a similar design, explain how you would approach the problem (10 marks)

Question 3

A company that provides a preventative maintenance and repair for computing devices has carried out a study of the times taken on service calls. Data were gathered for the last 50 calls on the number of devices serviced and the total number of minutes taken. A plot of the data together with the output from a simple linear regression model are shown below.



```
lm(formula = Minutes ~ Devices)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|--------|--------|--------|--------|
| -43.051 | -9.297 | 0.039 | 10.861 | 32.744 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|--------------|
| (Intercept) | 55.3492 | 7.2846 | 7.598 | 8.96e-10 *** |
| Devices | 7.6994 | 0.8147 | 9.451 | 1.56e-12 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.82 on 48 degrees of freedom

Multiple R-squared: 0.6504,

- Explain what a linear simple regression model is. (7 marks)
- How is the line in the above graph derived? (8 marks)
- Explain each of the figures in the Coefficients table. (20 marks)
- Explain how the above can be used to predict how long it would take to service 10 devices. (10 marks)
- In the context of simple linear regression, explain what a residual is? (5 marks)

Appendix for Section A

Poisson distribution $P(x) = e^{-\lambda} \frac{\lambda^x}{x!}$

Binomial distribution $P(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \text{hypothesised value}}{SE(\bar{x}_1 - \bar{x}_2)}$$

$$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\text{Chi-square} = \chi^2 = \sum_{\text{cells}} \frac{(\text{observed frequency} - \text{Expected frequency})^2}{\text{Expected Frequency}}$$

Proportion of Area to left of point for Standardised Normal Distribution

| | 0.0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 |
| 0.1 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 |
| 0.2 | 0.58 | 0.58 | 0.59 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.61 |
| 0.3 | 0.62 | 0.62 | 0.63 | 0.63 | 0.63 | 0.64 | 0.64 | 0.64 | 0.65 | 0.65 |
| 0.4 | 0.66 | 0.66 | 0.66 | 0.67 | 0.67 | 0.67 | 0.68 | 0.68 | 0.68 | 0.69 |
| 0.5 | 0.69 | 0.69 | 0.70 | 0.70 | 0.71 | 0.71 | 0.71 | 0.72 | 0.72 | 0.72 |
| 0.6 | 0.73 | 0.73 | 0.73 | 0.74 | 0.74 | 0.74 | 0.75 | 0.75 | 0.75 | 0.75 |
| 0.7 | 0.76 | 0.76 | 0.76 | 0.77 | 0.77 | 0.77 | 0.78 | 0.78 | 0.78 | 0.79 |
| 0.8 | 0.79 | 0.79 | 0.79 | 0.80 | 0.80 | 0.80 | 0.81 | 0.81 | 0.81 | 0.81 |
| 0.9 | 0.82 | 0.82 | 0.82 | 0.82 | 0.83 | 0.83 | 0.83 | 0.83 | 0.84 | 0.84 |
| 1.0 | 0.84 | 0.84 | 0.85 | 0.85 | 0.85 | 0.85 | 0.86 | 0.86 | 0.86 | 0.86 |
| 1.1 | 0.86 | 0.87 | 0.87 | 0.87 | 0.87 | 0.87 | 0.88 | 0.88 | 0.88 | 0.88 |
| 1.2 | 0.88 | 0.89 | 0.89 | 0.89 | 0.89 | 0.89 | 0.90 | 0.90 | 0.90 | 0.90 |
| 1.3 | 0.90 | 0.90 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.92 | 0.92 |
| 1.4 | 0.92 | 0.92 | 0.92 | 0.92 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 |
| 1.5 | 0.93 | 0.93 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 |
| 1.6 | 0.945 | 0.946 | 0.947 | 0.948 | 0.949 | 0.951 | 0.952 | 0.953 | 0.954 | 0.954 |
| 1.7 | 0.955 | 0.956 | 0.957 | 0.958 | 0.959 | 0.960 | 0.961 | 0.962 | 0.962 | 0.963 |
| 1.8 | 0.964 | 0.965 | 0.966 | 0.966 | 0.967 | 0.968 | 0.969 | 0.969 | 0.970 | 0.971 |
| 1.9 | 0.971 | 0.972 | 0.973 | 0.973 | 0.974 | 0.974 | 0.975 | 0.976 | 0.976 | 0.977 |
| 2.0 | 0.977 | 0.978 | 0.978 | 0.979 | 0.979 | 0.980 | 0.980 | 0.981 | 0.981 | 0.982 |
| 2.1 | 0.982 | 0.983 | 0.983 | 0.983 | 0.984 | 0.984 | 0.985 | 0.985 | 0.985 | 0.986 |
| 2.2 | 0.986 | 0.986 | 0.987 | 0.987 | 0.987 | 0.988 | 0.988 | 0.988 | 0.989 | 0.989 |
| 2.3 | 0.989 | 0.990 | 0.990 | 0.990 | 0.990 | 0.991 | 0.991 | 0.991 | 0.991 | 0.992 |
| 2.4 | 0.992 | 0.992 | 0.992 | 0.992 | 0.993 | 0.993 | 0.993 | 0.993 | 0.993 | 0.994 |
| 2.5 | 0.994 | 0.994 | 0.994 | 0.994 | 0.994 | 0.995 | 0.995 | 0.995 | 0.995 | 0.995 |
| 2.6 | 0.995 | 0.995 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 |
| 2.7 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 |
| 2.8 | 0.997 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 |
| 2.9 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.999 | 0.999 | 0.999 |
| 3.0 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |