Q2.2

$$f(a) = \cos(x) - x^2$$
$$= \cos(0) - 0^2$$
$$= 1$$

$$f(b) = \cos\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{2}\right)^2$$

$$= -1.47$$

Pick any nuber between them

$$0.5 = \cos(x) - x^2$$

Questions

6.6 → 6.7 / 6.22 → 6.33 : Derivations on Σ

↳ Then drop -2 ?

Example:

Example 6-9: Curve fitting with linear combination of nonlinear functions.

The following data is obtained from wind-tunnel tests, for the variation of the ratio of the tangential velocity of a vortex to the free stream flow velocity $y = V_\theta / V_\infty$ versus the ratio of the distance from the vortex core to the chord of an aircraft wing, $x = R/C$:

x	0.6	0.8	0.85	0.95	1.0	1.1	1.2	1.3	1.45	1.6	1.8
y	0.08	0.06	0.07	0.07	0.07	0.06	0.06	0.06	0.05	0.05	0.04

Theory predicts that the relationship between x and y should be of the form $y = \frac{A}{x} + \frac{Be^{-2x^2}}{x}$. Find the values of A and B using the least-squares method to fit the above data.

SOLUTION

In the notation of Eq. (6.91) the approximating function is $F(x) = C_1 f_1(x) + C_2 f_2(x)$ with $F(x) = y$, $C_1 = A$, $C_2 = B$, $f_1(x) = \frac{1}{x}$, and $f_2(x) = \frac{e^{-2x^2}}{x}$. The equation has two terms, which means that $m = 2$, and since there are 11 data points, $n = 11$. Substituting this information in Eq. (6.97) gives the following system of two linear equations for A and B.

$$\sum_{i=1}^{11} A\frac{1}{x_i}\frac{1}{x_i} + \sum_{i=1}^{11} B\frac{e^{-2x_i^2}}{x_i}\frac{1}{x_i} = \sum_{i=1}^{11} y_i\frac{1}{x_i} \quad \text{for } k = 1$$

$$\sum_{i=1}^{11} A\frac{1}{x_i}\frac{e^{-2x_i^2}}{x_i} + \sum_{i=1}^{11} B\frac{e^{-2x_i^2}}{x_i}\frac{e^{-2x_i^2}}{x_i} = \sum_{i=1}^{11} y_i\frac{e^{-2x_i^2}}{x_i} \quad \text{for } k = 2$$

These two equations can be rewritten as:

$$A\sum_{i=1}^{11}\frac{1}{x_i^2} + B\sum_{i=1}^{11}\frac{e^{-2x_i^2}}{x_i^2} = \sum_{i=1}^{11} y_i\frac{1}{x_i}$$

$$A\sum_{i=1}^{11}\frac{e^{-2x_i^2}}{x_i^2} + B\sum_{i=1}^{11}\frac{e^{-4x_i^2}}{x_i^2} = \sum_{i=1}^{11} y_i\frac{e^{-2x_i^2}}{x_i}$$

The system can be written in a matrix form:

$$\begin{bmatrix} \sum_{i=1}^{11}\frac{1}{x_i^2} & \sum_{i=1}^{11}\frac{e^{-2x_i^2}}{x_i^2} \\ \sum_{i=1}^{11}\frac{e^{-2x_i^2}}{x_i^2} & \sum_{i=1}^{11}\frac{e^{-4x_i^2}}{x_i^2} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{11} y_i\frac{1}{x_i} \\ \sum_{i=1}^{11} y_i\frac{e^{-2x_i^2}}{x_i} \end{bmatrix}$$

The system is solved by using MATLAB. The following MATLAB program in a script file solves the system and then makes a plot of the data points and the curve-fitted function.

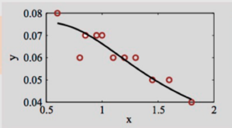
```
x = [0.6 0.8 0.85 0.95 1.0 1.1 1.2 1.3 1.45 1.6 1.8];
y = [0.08 0.06 0.07 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.04];
a(1,1) = sum(1./x.^2);
a(1,2) = sum(exp(-2*x.^2)./x.^2);
a(2,1) = a(1,2);
a(2,2) = sum(exp(-4*x.^2)./x.^2);
b(1,1) = sum(y./x);
b(2,1) = sum((y.*exp(-2*x.^2))./x);
AB = a\b
xfit = 0.6:0.02:1.8;
yfit = AB(1)./xfit + AB(2)*exp(-2*xfit.^2)./xfit;
plot(x,y,'o',xfit,yfit)
```

Why not b/a

When the program is executed, the solution for the coefficients is displayed in the Command Window (the two elements of the vector AB), and a plot with the data points and the curve-fitted function is created.

Command Window:

```
AB =
    0.0743      The coefficient A.
   -0.0597      The coefficient B.
```



Chapter 3

Q3.2

$$f(x) = x - 2e^{-x}$$

(a) Bisection

(1) $a = 0 \quad b = 1$

$$\text{mid} = \frac{0+1}{2} = 0.5$$

$f(a)f(\text{mid})$

$= (-2)(-0.713) > 0$

$\therefore$ Between mid + b

(2) $\text{new mid} = \frac{0.5+1}{2} = 0.75$

$f(0.5)f(0.75)$

$= (-0.713)(-0.19) > 0$

$\therefore$ Between mid + b

(3) $\text{mid} = \frac{0.75+1}{2} = 0.875$

$f(0.75)f(0.875)$

$= (-0.19)(0.04) < 0$

$\therefore \; 0.75 < x < 0.875$

(b) Secant Method

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_2) - 0}{x_2 - x_3}$$

$$x_2 - x_3 = \frac{(f(x_2) - 0)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = x_2 - \frac{(f(x_2) - 0)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$x_1 = 0 \qquad f(x_1) = -2$

$x_2 = 1 \qquad f(x_2) = 0.264$

$$x_3 = 1 - \frac{(0.264)(1)}{2.264}$$

$$= 0.883$$

$$x_4 = x_3 - \frac{(f(x_3) - 0)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

$$= 0.883 - \frac{0.056\,(0.883 - 1)}{0.056 - 0.264}$$

$$x_4 = 0.8515$$

$$x_5 = x_4 - \frac{f(x_4)\,(x_4 - x_3)}{f(x_4) - f(x_3)}$$

$$= 0.8515 - \frac{(-.002)(0.8515 - 0.883)}{(-.002) - 0.056}$$

$$x_5 = 0.8526$$

(c) Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = x - 2e^{-x}$$

$$f'(x) = 1 + 2e^{-x}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 0.8478$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.8478 - \frac{f(0.8478)}{f'(0.8478)}$$

$$= 0.8526$$

$$x_4 = 0.8526$$

Need to do

- Regular Falsi
- Fixed point iteration

Q3.8

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = \sqrt{x} + x^2 - 7$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 2x$$

$$x_2 = 7 - \frac{f(7)}{f'(7)}$$

$$= 3.85$$

$$x_3 = 2.62$$

$$x_4 = 2.35$$

$$x_5 = 2.33$$

$$x_6 = 2.3389$$

Q 3.14 Newtons Method for non-linear eq's.

$$x^2 + 2x + 2y^2 - 26 = 0 \qquad f_1(x,y)$$
$$2x^3 - y^2 + 4y - 19 = 0 \qquad f_2(x,y)$$

Start @ $(1,1)$

$$\frac{\partial f_1}{\partial x} = 2x + 2 \qquad \frac{\partial f_1}{\partial y} = 4y$$

$$\frac{\partial f_2}{\partial x} = 6x^2 \qquad \frac{\partial f_2}{\partial y} = -2y + 4$$

$$J(f_1(x_1,y_1), f_2(x_1,y_1))$$

$$= \det \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \frac{\partial f_1}{\partial x}\left(\frac{\partial f_2}{\partial y}\right) - \frac{\partial f_1}{\partial y}\left(\frac{\partial f_2}{\partial x}\right)$$

$$= (2x+2)(-2y+4) - (4y(6x^2))$$

$$= -4xy + 8x - 4y + 8 - 24x^2y$$

$$\underbrace{f_1(x_2,y_2)}_{0} = f_1(x_1,y_1) + \underbrace{(x_2 - x_1)}_{\Delta x}\frac{\partial f_1}{\partial x} + \underbrace{(y_2 - y_1)}_{\Delta y}\frac{\partial f_1}{\partial y}$$

$$\underbrace{f_2(x_2,y_2)}_{0} = f_2(x_1,y_1) + \underbrace{(x_2 - x_1)}_{\Delta x}\frac{\partial f_2}{\partial x} + \underbrace{(y_2 - y_1)}_{\Delta y}\frac{\partial f_2}{\partial y}$$

$$\underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}}_{M} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -f_1 \\ -f_2 \end{bmatrix} \qquad (1,1)$$

Cramer's rule:

$$M_1 = \begin{bmatrix} -f_1(1,1) & \frac{\partial f_1}{\partial y} \\ -f_2(1,1) & \frac{\partial f_2}{\partial y} \end{bmatrix} \qquad M_2 = \begin{bmatrix} \frac{\partial f_1}{\partial x} & -f_1(1,1) \\ \frac{\partial f_2}{\partial y} & -f_2(1,1) \end{bmatrix}$$

$$\Delta x = \frac{\det[M_1]}{\det[M]} = \frac{-f_1(1,1)\left(\frac{\partial f_2}{\partial y}\right) - \left(\frac{\partial f_1}{\partial y}\right)\left(-f_2(1,1)\right)}{J\left(f_1(1,1),\, f_2(1,1)\right)}$$

$$(J = \det[M])$$

$$\Delta y = \frac{\det[M_2]}{\det[M]} = \frac{-f_1(1,1)\left(\frac{\partial f_2}{\partial y}\right) - \left(\frac{\partial f_1}{\partial y}\right)\left(-f_2(1,1)\right)}{J\left(f_1(1,1),\, f_2(1,1)\right)}$$

Then $x_2 = x_1 + \Delta x$

$y_2 = y_1 + \Delta y$

Repeat for new $\Delta x, \Delta y$

Q4.2 Gauss elimination

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array} \begin{bmatrix} 2 & -2 & 1 \\ 3 & 2 & -5 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -16 \\ 8 \end{bmatrix}$$

$$M_{21} = \frac{3}{2} = \frac{a_{21}}{a_{11}}$$

$$R1 \times M_{21} = [\,2(3/2) \;\; -2(3/2) \;\; 1(3/2)\,] \quad [\,10(3/2)\,]$$

$$R2 = R2 - (R1(M_{21})) = \quad 0 \quad 5 \quad -6.5 \qquad -31$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 5 & -6.5 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -31 \\ 8 \end{bmatrix}$$

$$R3 = R3 - (R1(M_{31}))$$

$$M_{31} = \frac{-1}{2} = \frac{a_{31}}{a_{11}}$$

$$R1' = \quad 2(-1/2) \quad -2(-1/2) \quad 1(-1/2) \qquad 10(-1/2)$$

$$-1 \qquad 1 \qquad -1/2 \qquad -5$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 5 & -6.5 \\ 0 & 1 & 3.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -31 \\ 13 \end{bmatrix}$$

Now $R2$ = pivot eq.

$$M_{32} = \frac{1}{5} = \frac{a_{32}}{a_{22}}$$

$$R2' = \quad 0 \quad 1 \quad -1.3 \qquad -6.2$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 5 & -6.5 \\ 0 & 0 & 4.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -31 \\ 19.2 \end{bmatrix}$$

$$x_3 = \frac{19.2}{4.8} = 4$$

$$5x_2 - 6.5x_3 = -31$$

$$x_2 = \frac{-31 + 26}{5} = -1$$

$$2x_1 - 2x_2 + x_3 = 10$$

$$2x_1 + 2 + 4 = 10$$

$$x_1 = 2$$

Q4.13

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array}\begin{bmatrix} 10 & 12 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Normalise R1

$$\begin{bmatrix} 1 & 1.2 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$M_{21} = \frac{0}{1} = 0$

a_{21} already $= 0$, skip.

$M_{31} = \frac{2}{1} = 2$

$R1' = 2 \quad 2.4 \quad 0 \quad 0.2$

$R3 = R3 - R1'$

$$\begin{bmatrix} 1 & 1.2 & 0 \\ 0 & 2 & 8 \\ 0 & 1.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

$R2 =$ pivot eq.

Normalise R2

$$\begin{bmatrix} 1 & 1.2 & 0 \\ 0 & 1 & 4 \\ 0 & 1.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

$M_{12} = \frac{1.2}{1} = 1.2$

$R2' = 0 \quad 1.2 \quad 4.8 \quad 0$

$R1 = R1 - R2'$

$$\begin{bmatrix} 1 & 0 & -4.8 \\ 0 & 1 & 4 \\ 0 & 1.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

$M_{32} = \frac{1.6}{1} = 1.6$

$R2' = 0 \quad 1.6 \quad 6.4 \quad 0$

$R3 = R3 - R2'$

$$\begin{bmatrix} 1 & 0 & -4.8 \\ 0 & 1 & 4 \\ 0 & 0 & 1.6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}$$

Normalise R3

$$\begin{bmatrix} 1 & 0 & -4.8 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ c \\ -0.125 \end{bmatrix}$$

$M_{13} = \frac{-4.8}{1} = -4.8$

$R3' = 0 \quad 0 \quad -4.8 \quad 0.6$

$R1 = R1 - R3'$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.5 \\ c \\ -0.125 \end{bmatrix}$$

$M_{23} = \frac{4}{1} = 4$

$R3' = 0 \quad 0 \quad 4 \quad -0.5$

$R2 = R2 - R3'$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \\ -0.125 \end{bmatrix}$$

$$[a^{-1}] = \begin{bmatrix} -0.5 & DK & DK \\ 0.5 & DK & DK \\ -0.125 & DK & DK \end{bmatrix}$$

Repeat 2 more times to find columns 2 & 3.

Q5.3 Finding eigenvalues

$$a = \begin{bmatrix} 10 & 0 & 0 \\ 1 & -3 & -7 \\ 0 & 2 & 6 \end{bmatrix}$$

$$\det [a - \lambda I] = 0$$ Characteristic eq.

$$\det \begin{bmatrix} 10-\lambda & 0 & 0 \\ 1 & -3-\lambda & -7 \\ 0 & 2 & 6-\lambda \end{bmatrix} = 0$$

$$10-\lambda \left(\det \begin{vmatrix} -3-\lambda & -7 \\ 2 & 6-\lambda \end{vmatrix} \right) - 0(-) + 0(-)$$

$$10-\lambda \left((-3-\lambda)(6-\lambda) - (-7)(2) \right)$$

$$10-\lambda \left(-18 - 3\lambda + \lambda^2 + 14 \right) = 0$$

$$10 - \lambda = 0 \qquad \lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda = 10$$

$$\lambda = 4 \qquad \lambda = -1$$

Q 5.7 Power Method.

$$a = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \qquad v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$av = \begin{bmatrix} 4 \\ 10 \\ 1 \end{bmatrix} \qquad \text{Max} = 10$$

$$\text{Normalise} \quad = 10 \begin{bmatrix} 0.4 \\ 1 \\ 0.1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.4 \\ 1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 1.6 \\ 3.1 \\ 0.1 \end{bmatrix}$$

$$\text{Max} = 3.1$$

$$\text{Normalise} \quad = 3.1 \begin{bmatrix} 0.516 \\ 1 \\ 0.032 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.516 \\ 1 \\ 0.032 \end{bmatrix} = \begin{bmatrix} 1.58 \\ 3.224 \\ 0.032 \end{bmatrix}$$

$$\text{Max} = 3.224$$

$$\Rightarrow \quad 3.224 \begin{bmatrix} 0.49 \\ 1 \\ 0.009 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.49 \\ 1 \\ 0.009 \end{bmatrix} = \begin{bmatrix} 1.508 \\ 3.005 \\ 0.009 \end{bmatrix}$$

$$\text{Max} = 3.005$$

$$\Rightarrow \quad \begin{bmatrix} 0.502 \\ 1 \\ 0 \end{bmatrix}$$

Keep going till max's approx. equal.

Q6.14

$$f(x) = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2)\ldots$$

$$a_1 = y_1$$

$$a_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a_4 = \frac{\dfrac{\left(\dfrac{y_4-y_3}{x_4-x_3} - \dfrac{y_3-y_2}{x_3-x_2}\right)}{x_4-x_2} - \dfrac{\left(\dfrac{y_3-y_2}{x_3-x_2} - \dfrac{y_2-y_1}{x_2-x_1}\right)}{x_3-x_1}}{x_4 - x_1}$$

x	y
14	320
22	490
30	540
38	500
46	480

$a_1 = 320$

a_2:

$$\frac{490-320}{22-14} = 21.25$$

$$\frac{540-490}{30-22} = 6.25$$

$$\frac{500-540}{38-30} = -5$$

$$\frac{480-500}{46-38} = -2.5$$

$$\frac{6.25-21.25}{30-14}$$

$$\frac{-5-6.25}{38-22}$$

$\ldots$

$$f(x) = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2)$$

$$+ a_4(x-x_1)(x-x_2)(x-x_3) +$$

$$a_5(x-x_1)(x-x_2)(x-x_3)(x-x_4)$$

$$= 320 + 21.25(x-14) - 14.6(x-14)(x-22)$$

$$+ 13.04(x-14)(x-22)(x-30)$$

$$- 12.68(x-14)(x-22)(x-30)(x-38)$$

$$f(26) = 320 + 21.25(26-14) - 14.6(26-14)(26-22)$$

$$+ 13.04(26-14)(26-22)(26-30)$$

$$- 12.68(26-14)(26-22)(26-30)(26-38)$$

This is <u>wrong</u>

Q6.15

(a) $$\sum_{i=0}^{n} y_i \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

$$\frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)(x_1-x_6)} y_1$$

$$+ \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)(x_2-x_6)} y_2$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)(x-x_6)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)(x_3-x_6)} y_3$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)(x-x_6)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)(x_4-x_6)} y_4$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_6)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)(x_5-x_6)} y_5$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_6-x_1)(x_6-x_2)(x_6-x_3)(x_6-x_4)(x_6-x_5)} y_6$$

$$\frac{(x-2.2)(x-3.4)(x-4.8)(x-6)(x-7)}{(1-2.2)(1-3.4)(1-4.8)(1-6)(1-7)}\,2$$

$$+\frac{(x-1)(x-3.4)(x-4.8)(x-6)(x-7)}{(2.2-1)(2.2-3.4)(2.2-4.8)(2.2-6)(2.2-7)}\,2.8$$

$$+\frac{(x-1)(x-2.2)(x-4.8)(x-6)(x-7)}{(3.4-1)(3.4-2.2)(3.4-4.8)(3.4-6)(3.4-7)}\,3$$

$$+\frac{(x-1)(x-2.2)(x-3.4)(x-6)(x-7)}{(4.8-1)(4.8-2.2)(4.8-3.4)(4.8-6)(4.8-7)}\,3.2$$

$$+\frac{(x-1)(x-2.2)(x-3.4)(x-4.8)(x-7)}{(6-1)(6-2.2)(6-3.4)(6-4.8)(6-7)}\,4$$

$$+\frac{(x-1)(x-2.2)(x-3.4)(x-4.8)(x-6)}{(7-1)(7-2.2)(7-3.4)(7-4.8)(7-6)}\,5$$

Q 6.16

$$y = \frac{(x - x_{i+1})}{(x_i - x_{i+1})} y_i + \frac{(x - x_i)}{(x_{i+1} - x_i)} y_{i+1}$$

(a) $$f(24) = \frac{(24 - 30)}{(22 - 30)} 490 + \frac{(24 - 22)}{(30 - 22)} 540$$

Look at cubic, quadratic

Q8.9

$$\sum_{i=1}^{n} y_i \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x-x_i)}{(x_i-x_j)}$$

$$f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3$$

$$f(x) = \frac{x^2 - xx_3 - x_2x + x_2x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{x^2 - xx_3 - xx_1 + x_1x_3}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{x^2 - xx_2 - xx_1 + x_2x_3}{(x_3-x_1)(x_3-x_2)} y_3$$

$$f'(x) = \frac{2x - x_3 - x_2}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{2x - x_3 - x_1}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{2x - x_2 - x_1}{(x_3-x_1)(x_3-x_2)} y_3$$

For backwards diff. $= f'(x_3)$

$$f'(x_3) = \frac{2x_3 - x_3 - x_2}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{2x_3 - x_3 - x_1}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{2x_3 - x_2 - x_1}{(x_3-x_1)(x_3-x_2)} y_3$$

$$f'(x_3) = \frac{x_3 - x_2}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{x_3 - x_1}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{2x_3 - x_2 - x_1}{(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{2006 - 2003}{(2002-2003)(2002-2006)} \, 665.647 \quad \ldots$$

Sample paper

Q2

$$f(x) = a_1(x - x_2) + a_2(x - x_1)$$

Substituting known points into the above eq. gives us

$$y_1 = a_1(x_1 - x_2) + a_2(x_1 - x_1) \qquad a_1 = \frac{y_1}{(x_1 - x_2)}$$

$$y_2 = a_1(x_2 - x_2) + a_2(x_2 - x_1) \qquad a_2 = \frac{y_2}{(x_2 - x_1)}$$

Back substitution gives us:

$$f(x) = \frac{(x - x_2)}{(x_1 - x_2)} y_1 + \frac{(x - x_1)}{(x_2 - x_1)} y_2$$

Similarly for third order

$$f(x) = a_1(x - x_2)(x - x_3) + a_2(x - x_1)(x - x_3) + a_3(x - x_1)(x - x_2)$$

Filling in 3 known points then back substituting gives us:

$$f(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)} y_3$$

$\Rightarrow$ The $(n-1)^{th}$ polynomial is therefore given by

$$f(x) = \sum_{i=1}^{n} y_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Q9.1

(a) $0.3(0.5 + 0.6 + 0.8 \cdots\cdots) =$

(b) ...

(c) $I(f) \approx \frac{3h}{8}\left[f(a) + 3\sum_{i=2,5,8}^{N-1}[f(x_i) + f(x_{i+1})]\right.$

$\left. + 2\sum_{j=4,7,10}^{N-2} f(x_j) + f(b)\right]$

$$\frac{3(0.3)}{8}\left[0.5 + 3\left((0.6+0.8) + (2+3.2)\right) + \right.$$
$$\left. 2(1.3) + 4.8\right]$$

Q

5 points $\Rightarrow$ 4^{th} order polynomial

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} y_1$$

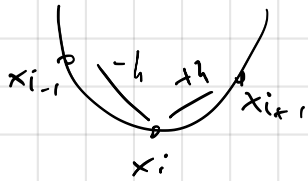
$$+ \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} y_2$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} y_3$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} y_4$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} y_5$$

2 point central derivation using Taylor Series



Get the 3 terms in relation to $f(x_i)$

$$f(x_i) = f(x_i)$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(\xi_1)}{3!}h^3$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(-h) + \frac{f''(x_i)(-h)^2}{2!} + \frac{f'''(\xi_2)}{3!}(-h)^3$$

These are the bits we want

(+ error)

$$f(x_{i+1}) - f(x_{i-1}) =$$

$$\cancel{f(x_i)} - \cancel{f(x_i)} + f'(x_i)h + f'(x_i)h + \overbrace{\cancel{\frac{f''(x_i)h^2}{2}} - \cancel{\frac{f''(x_i)h^2}{2}}}^{\text{This is what we wanted}}$$

$$+ \frac{f'''(\xi_1)}{3!}h^3 + \frac{f'''(\xi_2)}{3!}h^3$$

$$2f'(x_i)h = f(x_{i+1}) - f(x_{i-1})$$

$$- \frac{f'''(\xi_1)}{3!}h^3 - \frac{f'''(\xi_2)}{3!}h^3$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}) - \frac{f'''(\xi_1)}{3!}h^3 - \frac{f'''(\xi_2)}{3!}h^3}{2h}$$

$$= \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + \underbrace{\left\{ \frac{\left(-\frac{f'''(\xi_1)}{3!}h^2 - \frac{f'''(\xi_2)}{3!}h^2\right)}{2} \right\}}_{O(h^2)}$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

Jacobi Iterative / Gauss Siedel.