LACTU 2170 Project 23-24

February 12, 2024

The homework is a report of maximum 20 pages (appendix allowed for extra graphs-tables), written per group of 2 students maximum. This work counts for 4 points of the final mark. Notice that no resit project will be proposed and the mark will be reported for the summer session of exams. The report **and the code** must be uploaded on the Moodle website before Monday 15th of April, 12h00.

Pricing of GMAB and GMDB

We denote by $(S_t)_{t\geq 0}$, the market value of a fund of stocks & bonds, with $S_0 = 10000€$. We assume that S_t is driven by a geometric Brownian motion with $\sigma = 16\%$ (volatility). The risk free rate is equal to $r = 4.5\%$. We consider a GMAB of maturity T years. In case of survival, the insured receives the following payoff at expiry:

$$P(T, S_T) = \begin{cases} 10000€e^{0.07\,T} & \text{if } S_T \geq 10000€e^{0.07\,T} \\ S_T & \text{if } 10000€e^{0.02\,T} \leq S_T \leq 10000€e^{0.07\,T} \\ 10000€e^{0.02\,T} & \text{if } S_T \leq 10000€e^{0.02\,T} \end{cases} . \quad (1)$$

In case of death before maturity, the maximum between $S_0 e^{0.02\,t}$ and S_t is paid at the end of the year during which the insured dies (This is a GMDB).

The insured is 50 years old and the survival probality up to time t is provided by the formula:

$$ {}_tp_x \quad = \quad \exp\left(-0.002t - 0.00025\,t^2\right) . \quad (2)$$

1) (3 pts) Find the two analytical expressions of the GMAB at time $t \leq T$:

- using call options,
- using put options.

Clearly explain the reasoning and implement the formula in R or Python. Price GMAB at $t = 0$ with maturities from 1 to 20 years and plot them.

2) (3 pts) Implement a binomial tree to price a GMAB of maturity T=8 years. Use **at least** 10, 50, 100, 250 steps per year. Compare these prices with those obtained with the analytical formulas.

3) (3 pts) Find the two analytical expressions of the GMDB at time $t \leq T$:

- using call options,
- using put options.

Clearly explain the reasoning and implement the formula in R or Python. Price the GMDB at $t = 0$ with maturities from 1 to 20 years and plot them.

4) (3 pts) Implement a binomial tree to price a GMDB of maturity T=8 years. Use **at least** 10, 50, 100, 250 steps per year. Compare these prices with those computed with the analytical formulas.

Pricing of a maximum return insurance

We want to evaluate a maximum return insurance which delivers at expiry, **in case of survival**, the maximum return between the Eurostoxx 50 and AMEX composite index. The amount invested in this insurance is 15 000€. We use the same survival probabilities, Eq. (2) as in Exercise 1.

1) (3 pts) Download the file "Data_project.xlsx" from the moodle website and calculate

- Daily returns of each indice.
- Calculate averages, standard deviations and correlation of daily returns.
- Calculate the annual standard deviations, annual returns and covariance matrix.

2) (2 pts) The interest rate is equal $r = 3.75\%$ and the maturity is T. Using the parameters estimated from Q1, evaluate the maximum return insurance with a closed form expression (present and briefly comment it) for $T = 1, 2, ...10$ years ($t = 0$).

3) (1.5 pts) Price the contract by Monte-Carlo simulations for $T = 1, 2, ...10$ years. Compare your results with those obtained with the closed-form expression. Test different steps of time and check the convergence.

4) (1.5 pts) We assume that in case of death before expiry, the inheritor will receive the maximum between the initial premium (15 000€) and the funds tracking the Eurostoxx 50 and the AMEX. Evaluate this death insurance for $T = 1, 2, ...10$ years, using Monte-Carlo simulations. Test different steps of time and check the convergence.