Web Structure Mining

Generalities, Complex Networks and Node-Centric Metrics

— Session 2 —

Outline

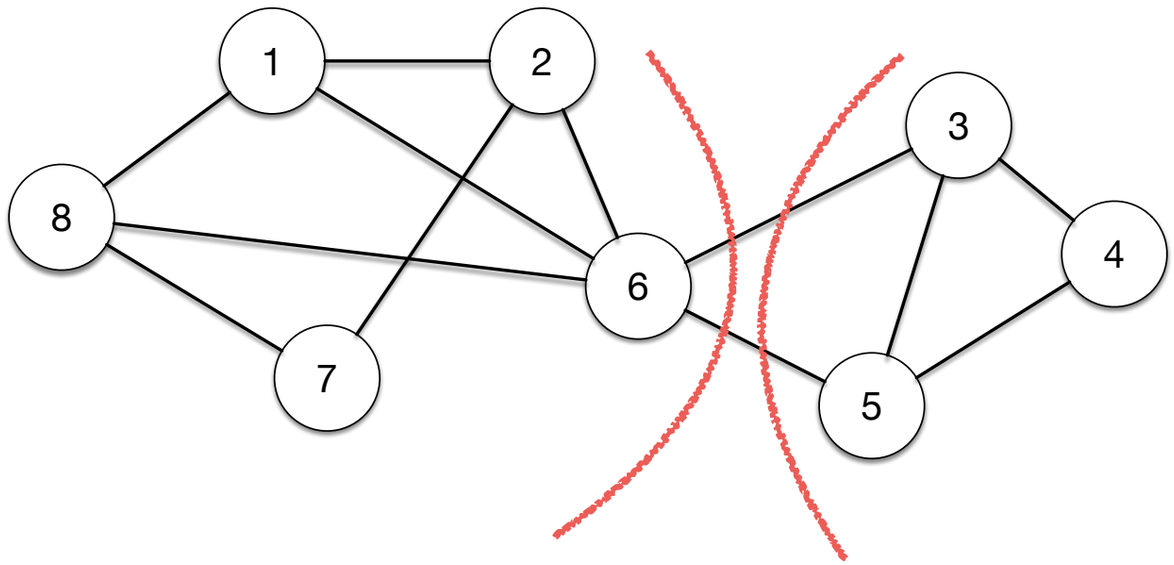
1. Introduction to network science
2. Complex network properties
    A. Long tail distribution
    B. Small world effect
    C. Community structure
3. Node-centric metrics

An interdisciplinary science

- Sociology (social networks)
- Mathematics (graphs)
- Computer science (graphs)
- Statistical physics (complex networks)
- Economics (networks)
- Bioinformatics (networks)

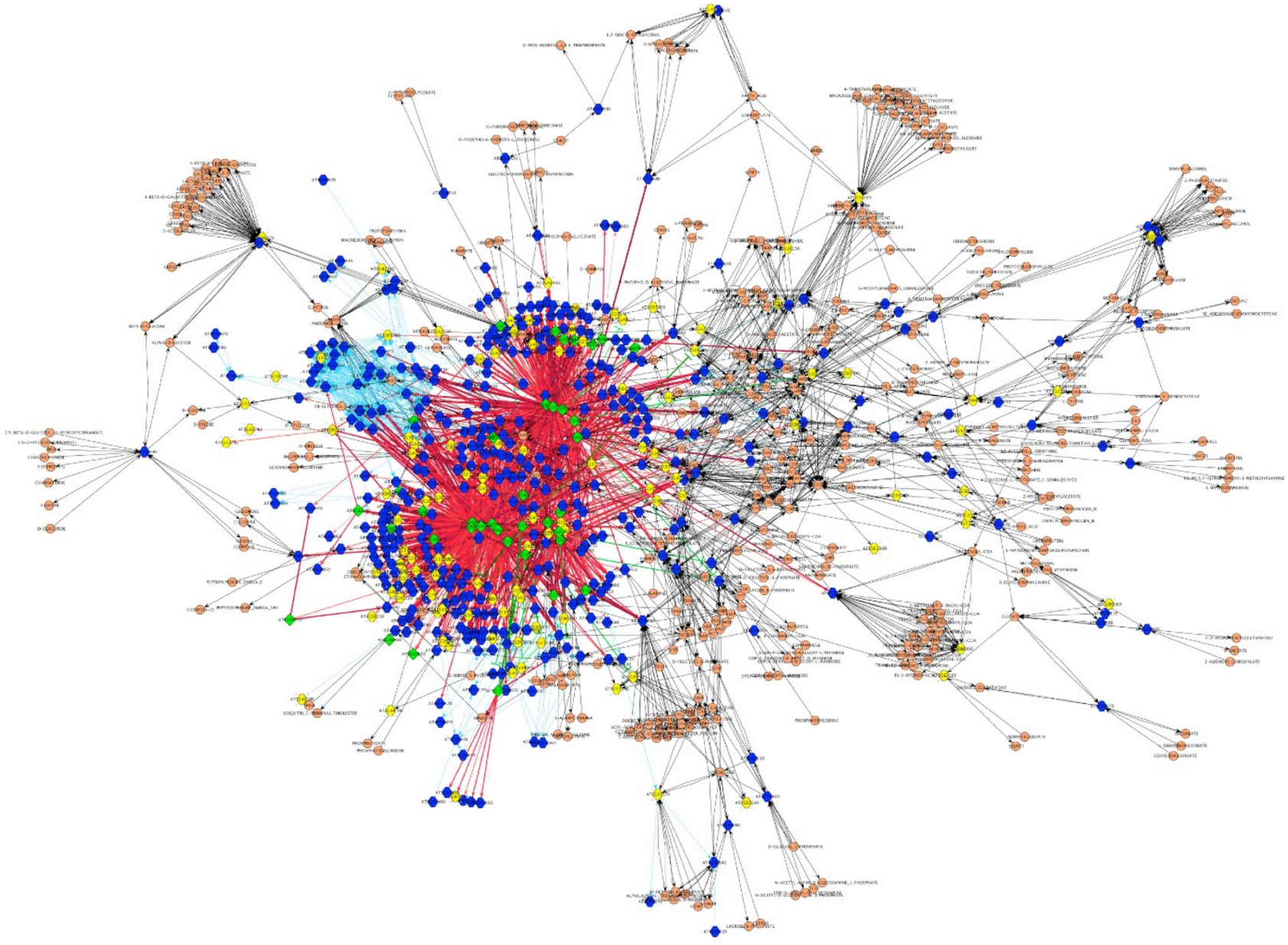
Let's speak the same language

- Network = graph
- Nodes = vertices, actors
- Links = ties, edges, relations
- Clusters = communities



Complex networks

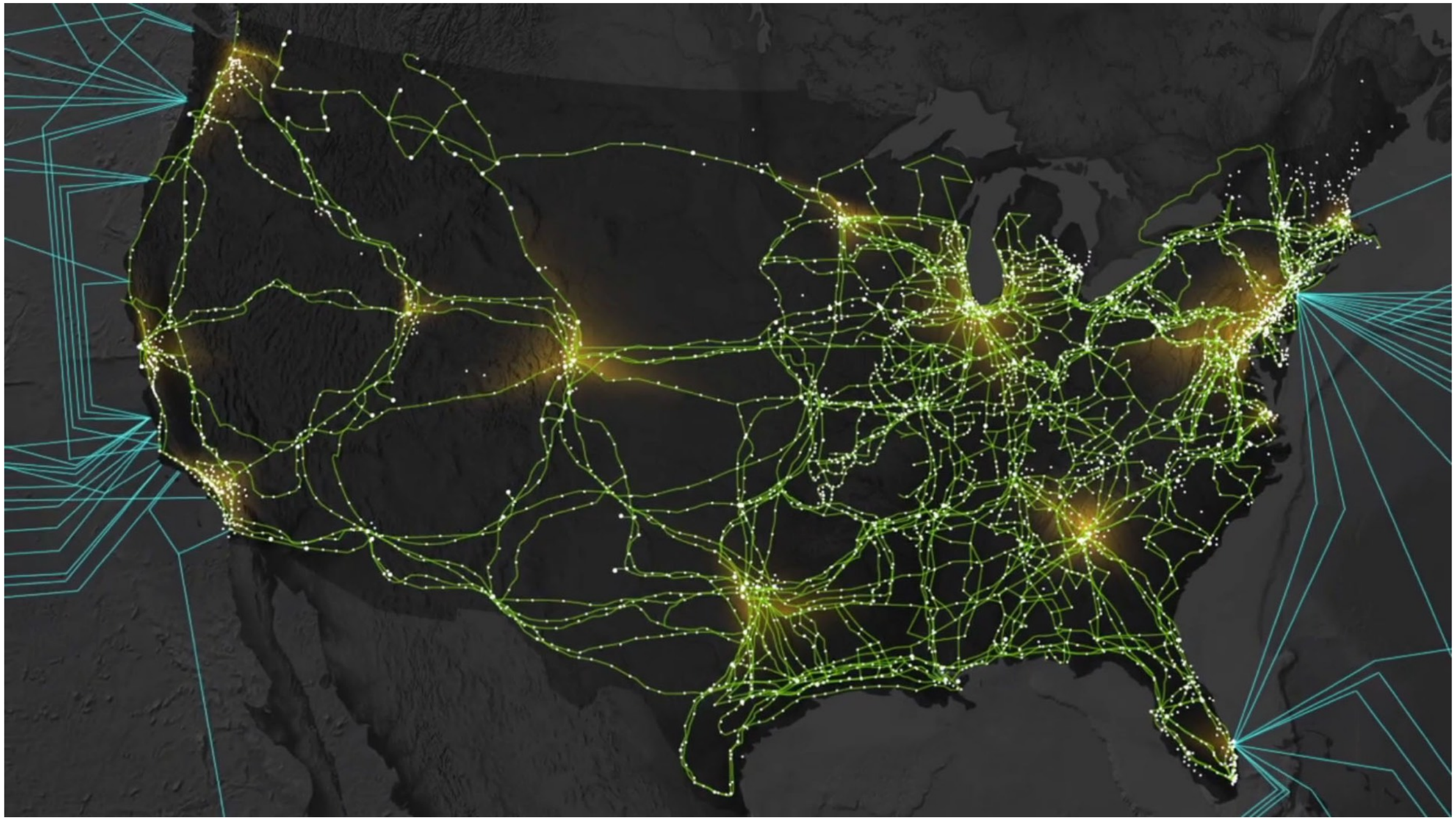
- Not regular
- Not random
- Scale-free networks
- Universal properties
- Very common
- Model complex systems



Source: http://images.biomedsearch.com

Complex networks

US internet infrastructure

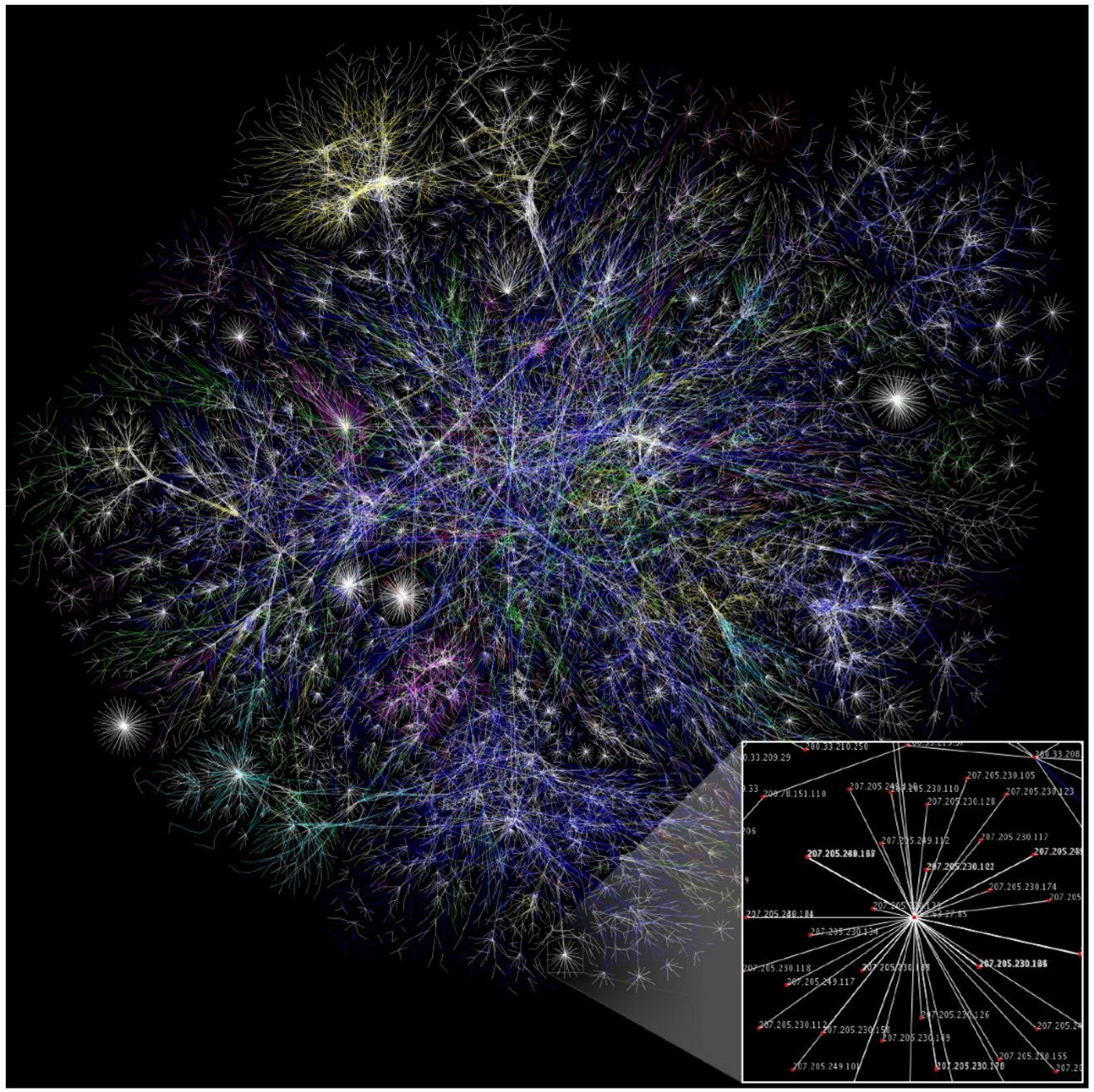


Source: http://www.aec.at/nextidea/tracing-the-digitalphysical/

Complex networks

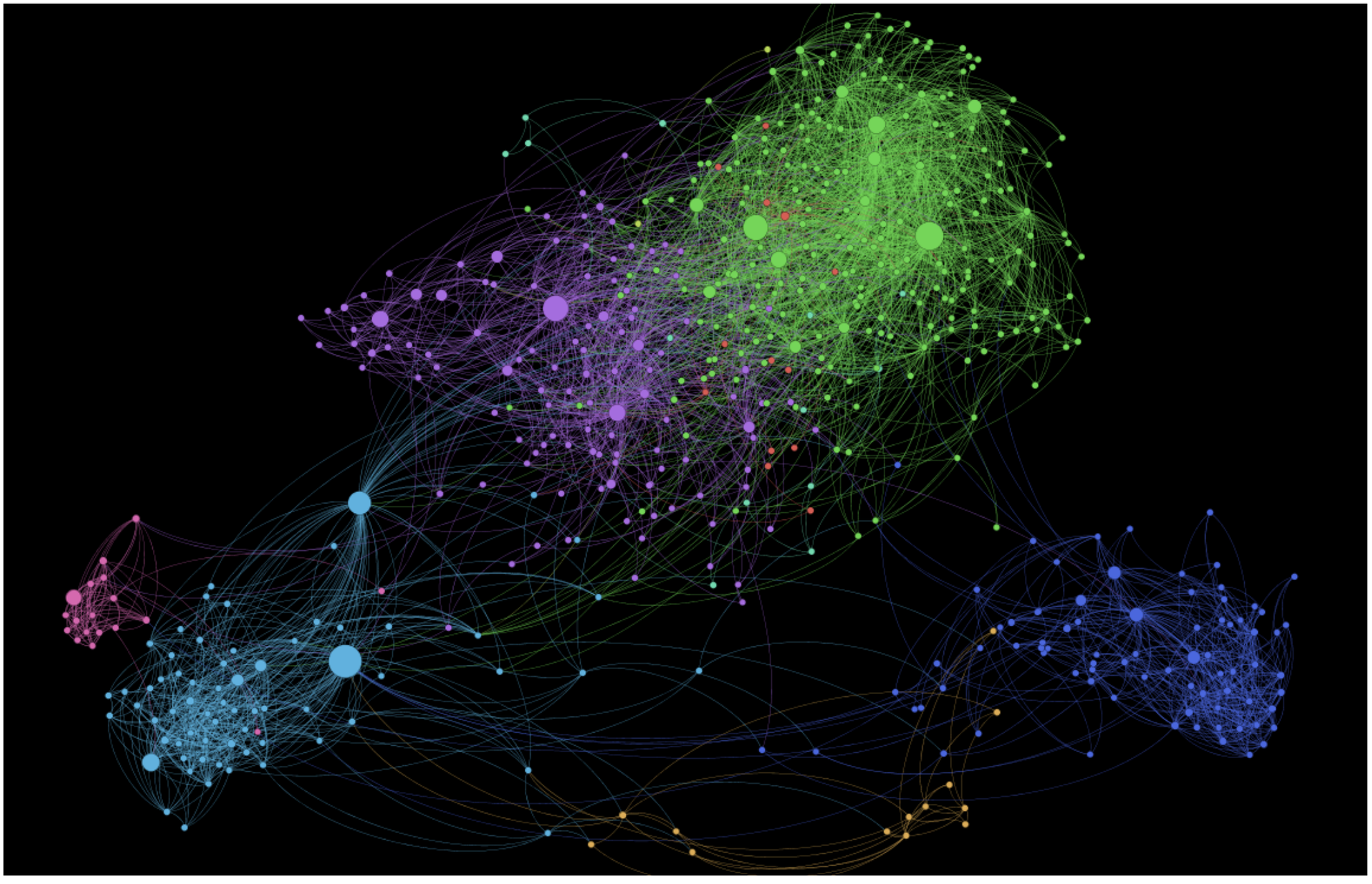
Class C networks

- Barrett Lyon, OPTE.org
- January 2015
- Node : IP address
- Link length : delay between two nodes



Complex networks

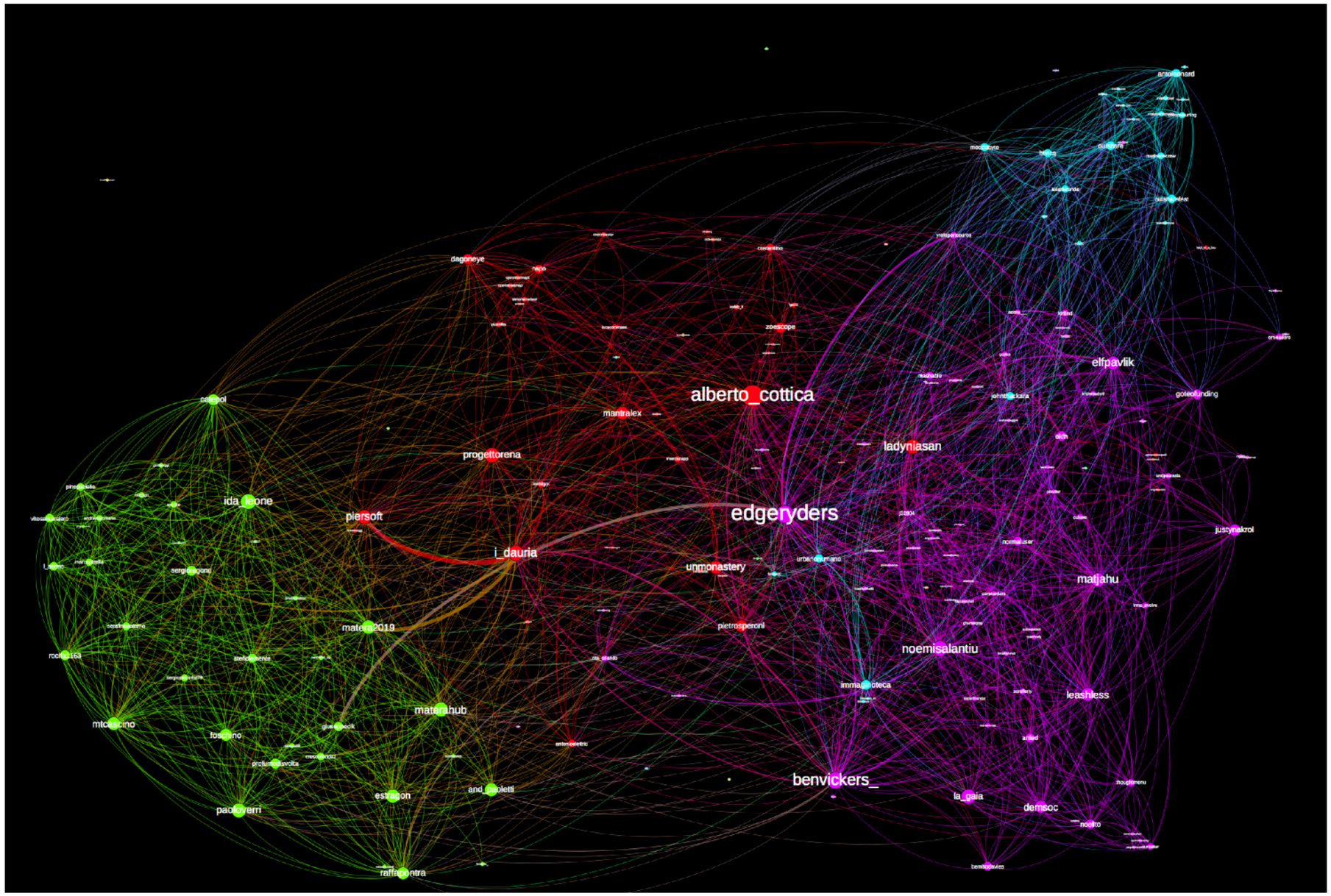
Study about Somalis from their Facebook networks



Source: https://kimoquaintance.com/2011/08/22/what-can-we-learn-about-somalis-from-their-facebook-networks/

Complex networks

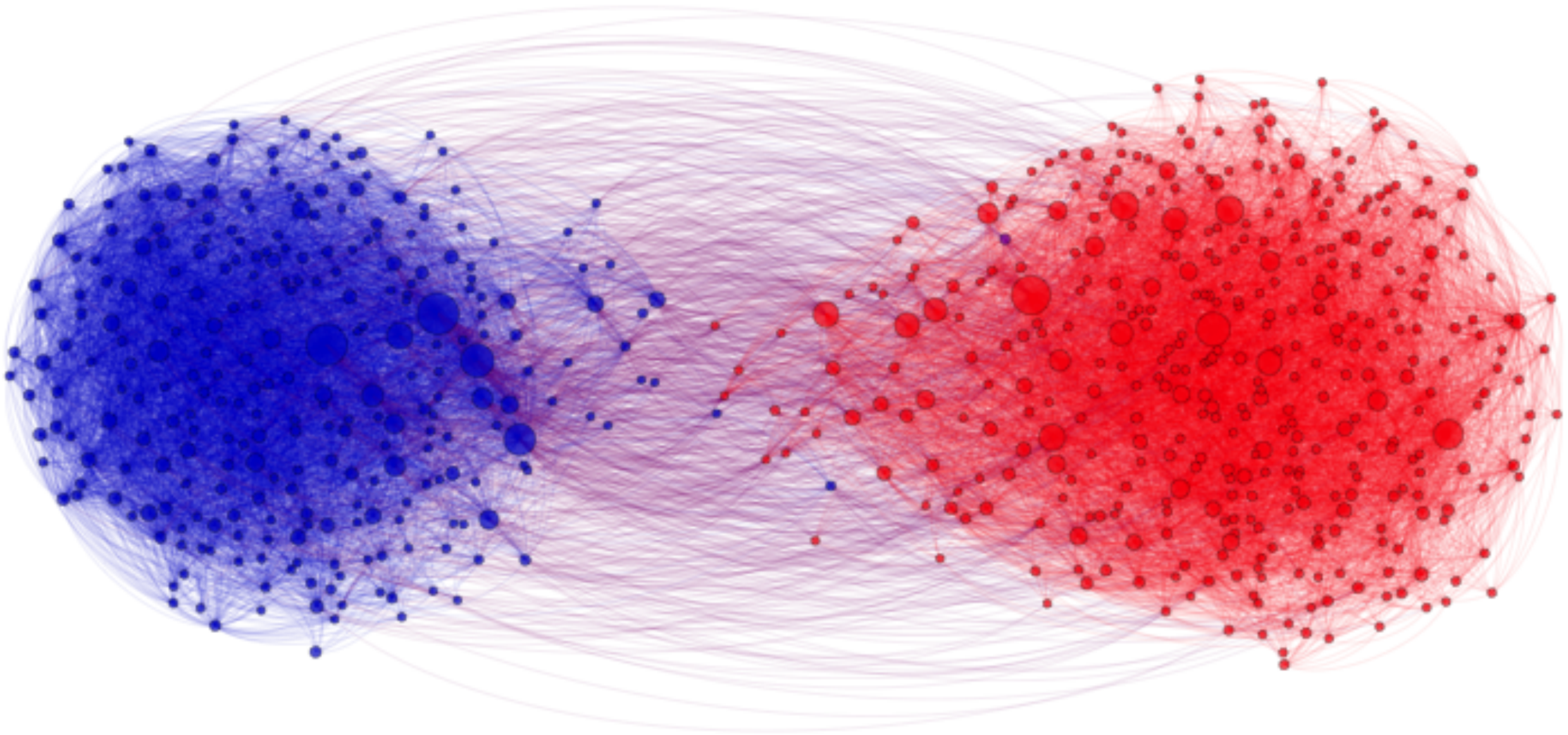
Twitter



Source: http://www.cottica.net/2013/10/19/learning-from-the-twitterstorm-an-architecture-for-effortless-collaboration/

Complex networks

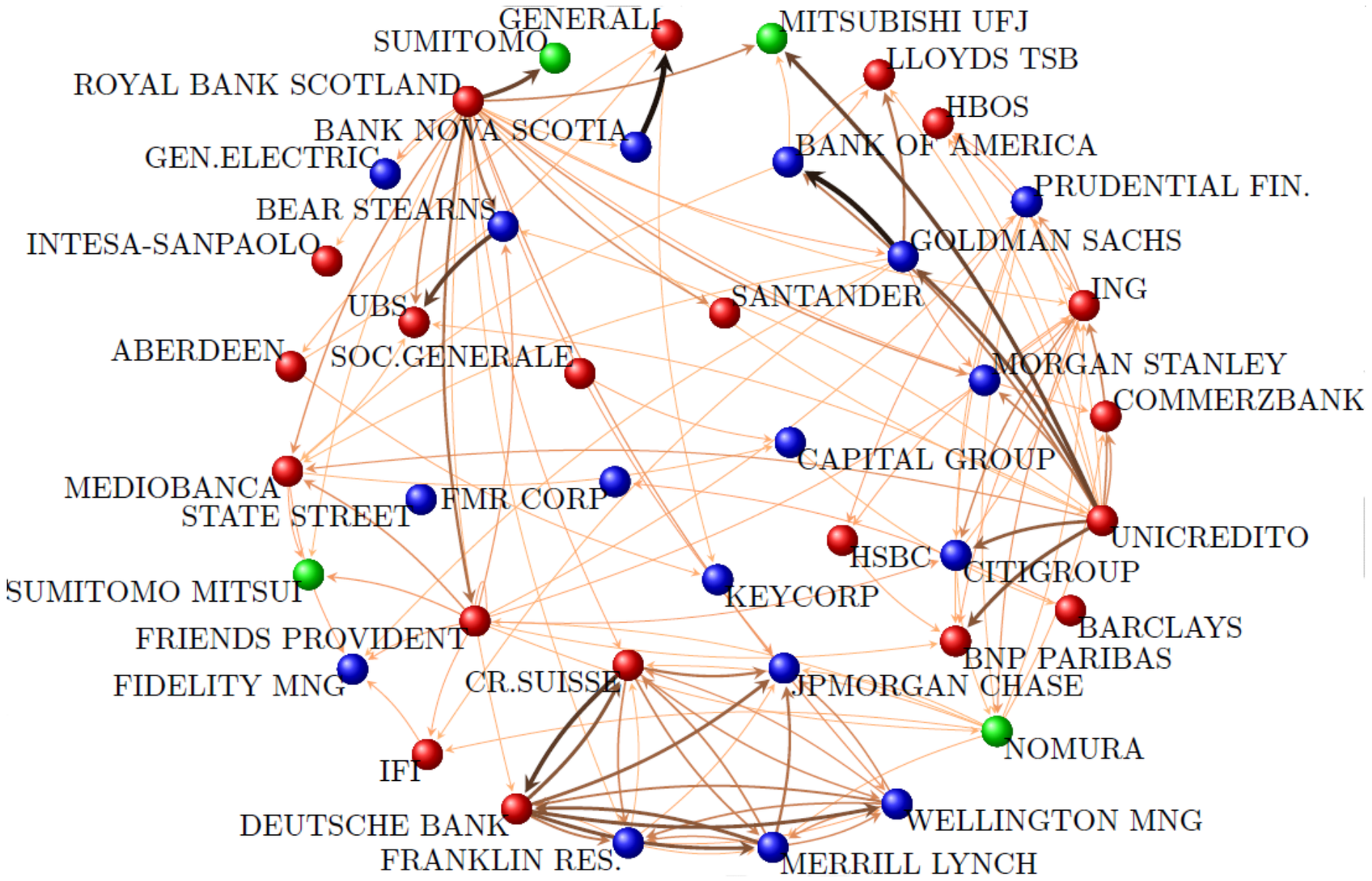
Political blogs



Source: http://allthingsgraphed.com/2014/10/09/visualizing-political-polarization/

Complex networks

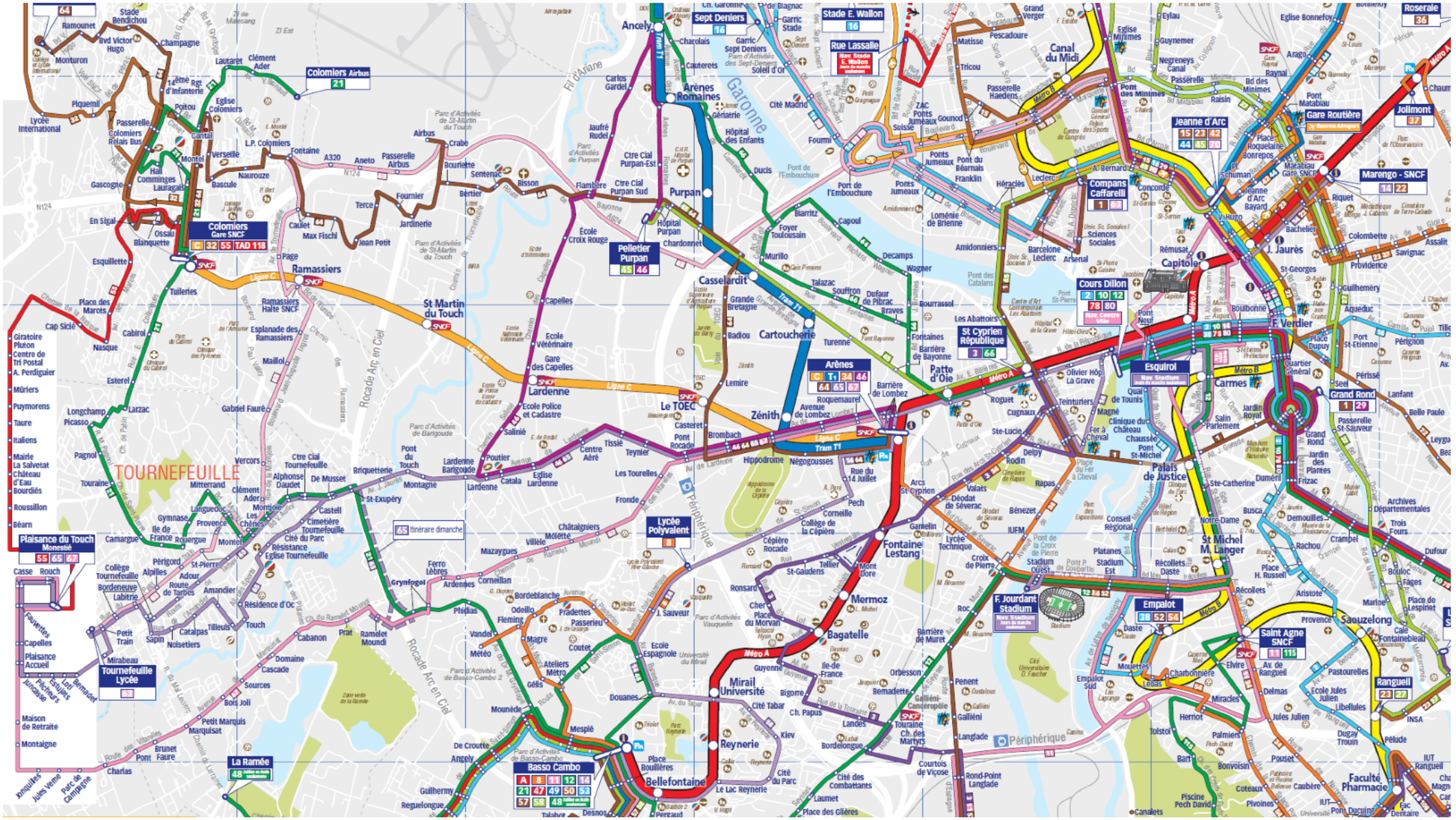
Finance



Source: http://j-node.blogspot.fr/2011/10/network-of-global-corporate-control.html

Complex networks

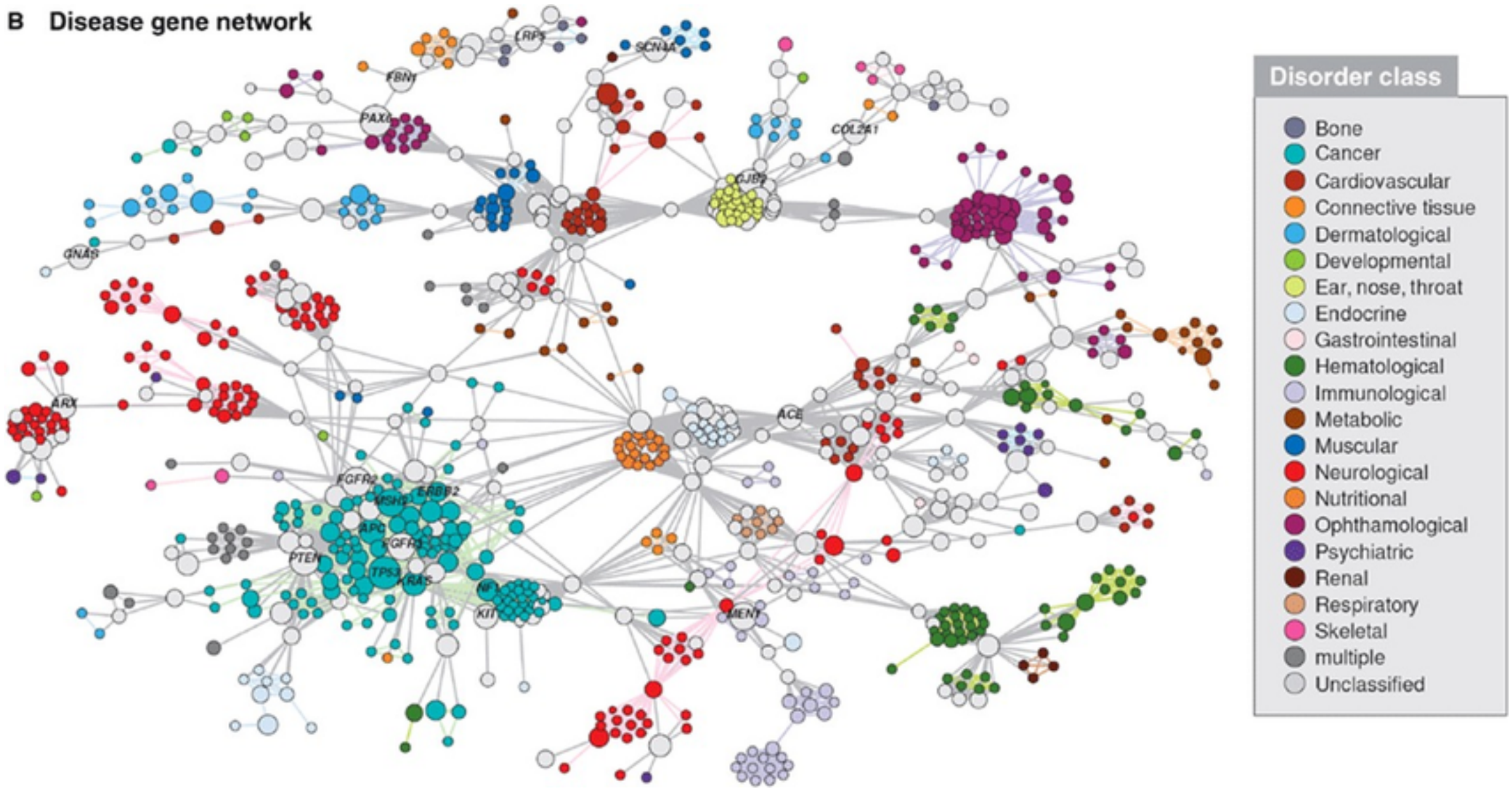
Transport



Source: Tisséo

Complex networks

Biology



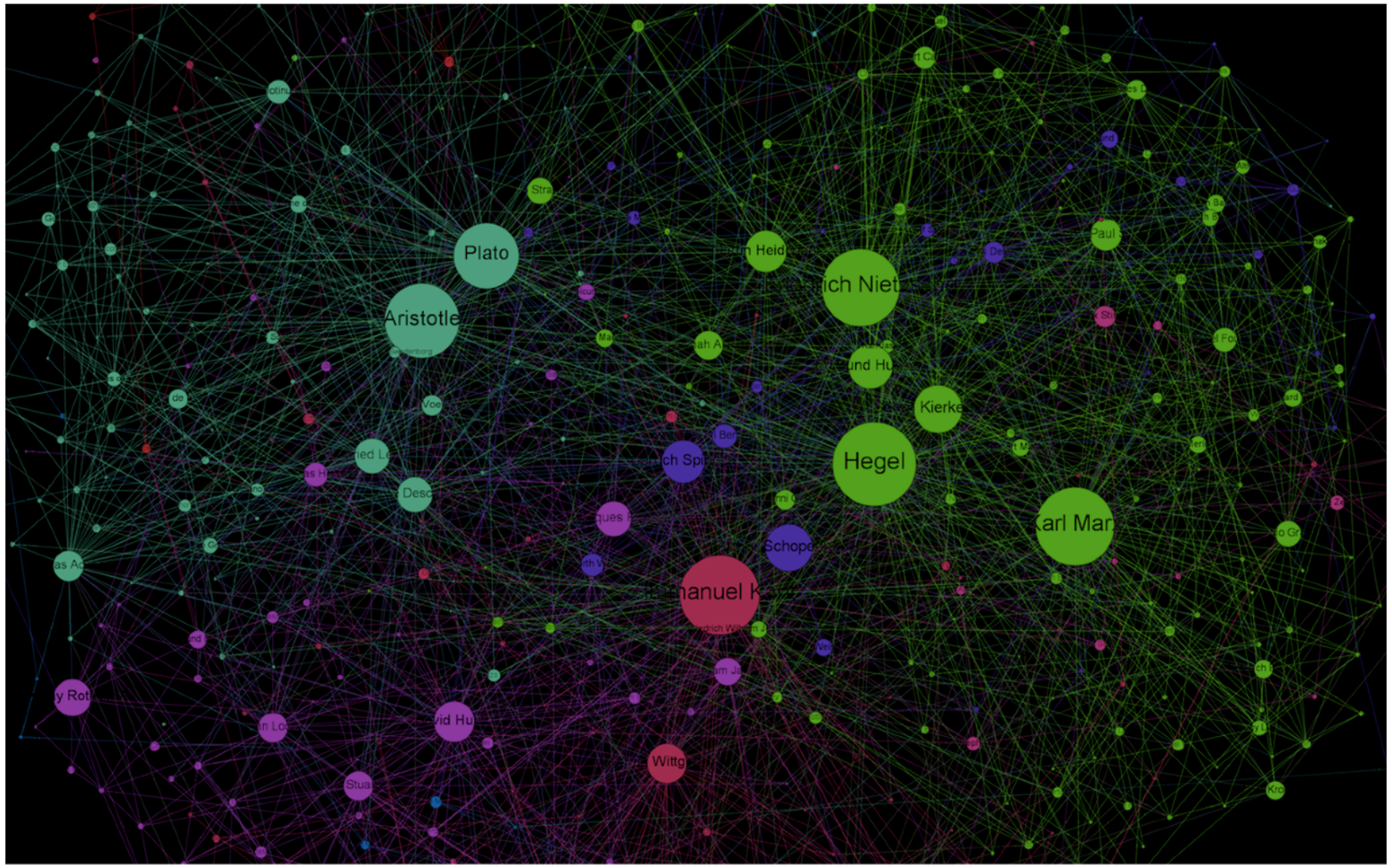
Source: Human disease classification in the postgenomic era: A complex systems approach to human pathobiology, 2007

Complex networks

Properties

- Complex network share common properties
  - Long-tail distribution (a.k.a. power law degree distribution)
  - Small world effect
  - Strong community structure

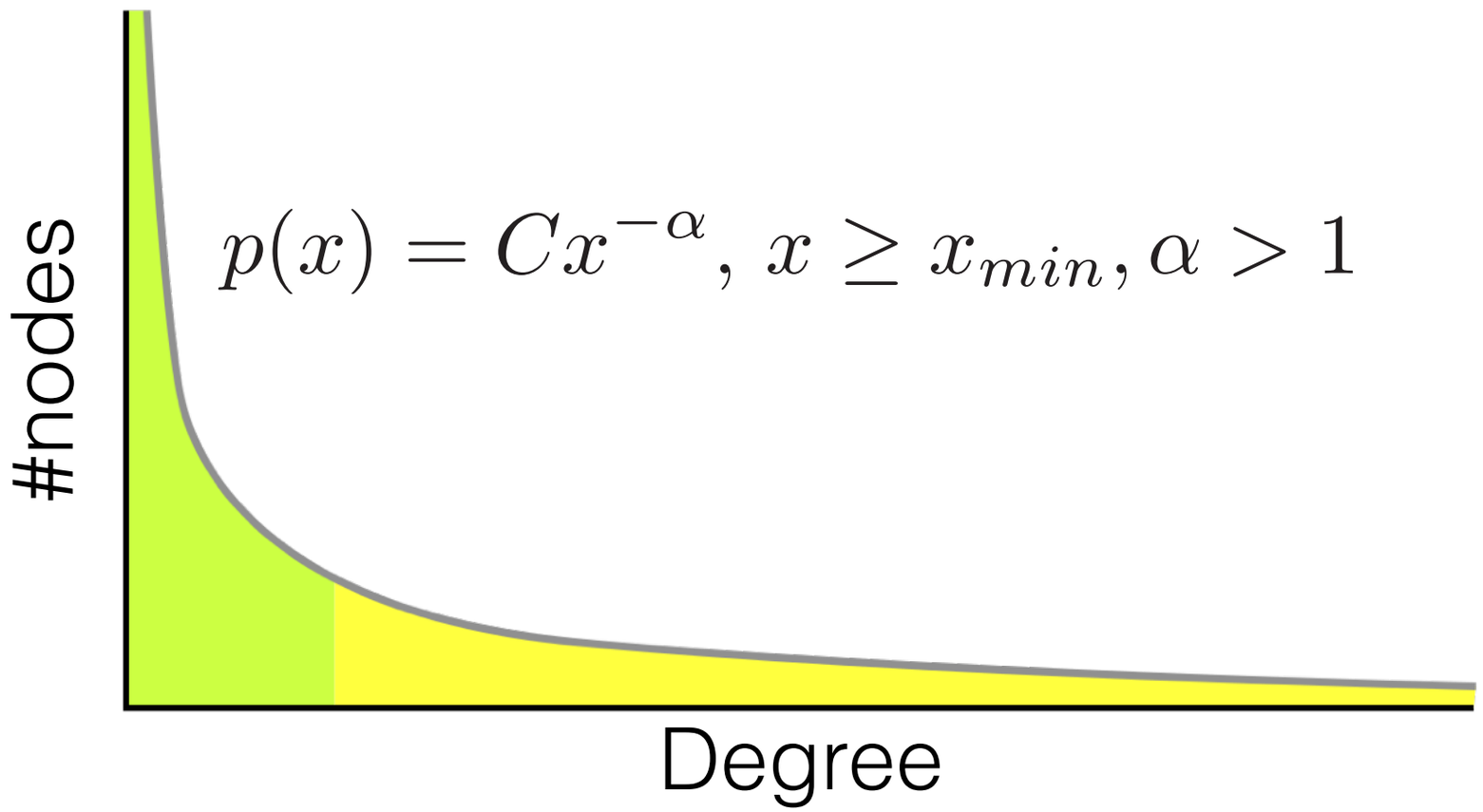
Long-tail distribution



- Few philosophers have many relations
- Many philosophers have few relations

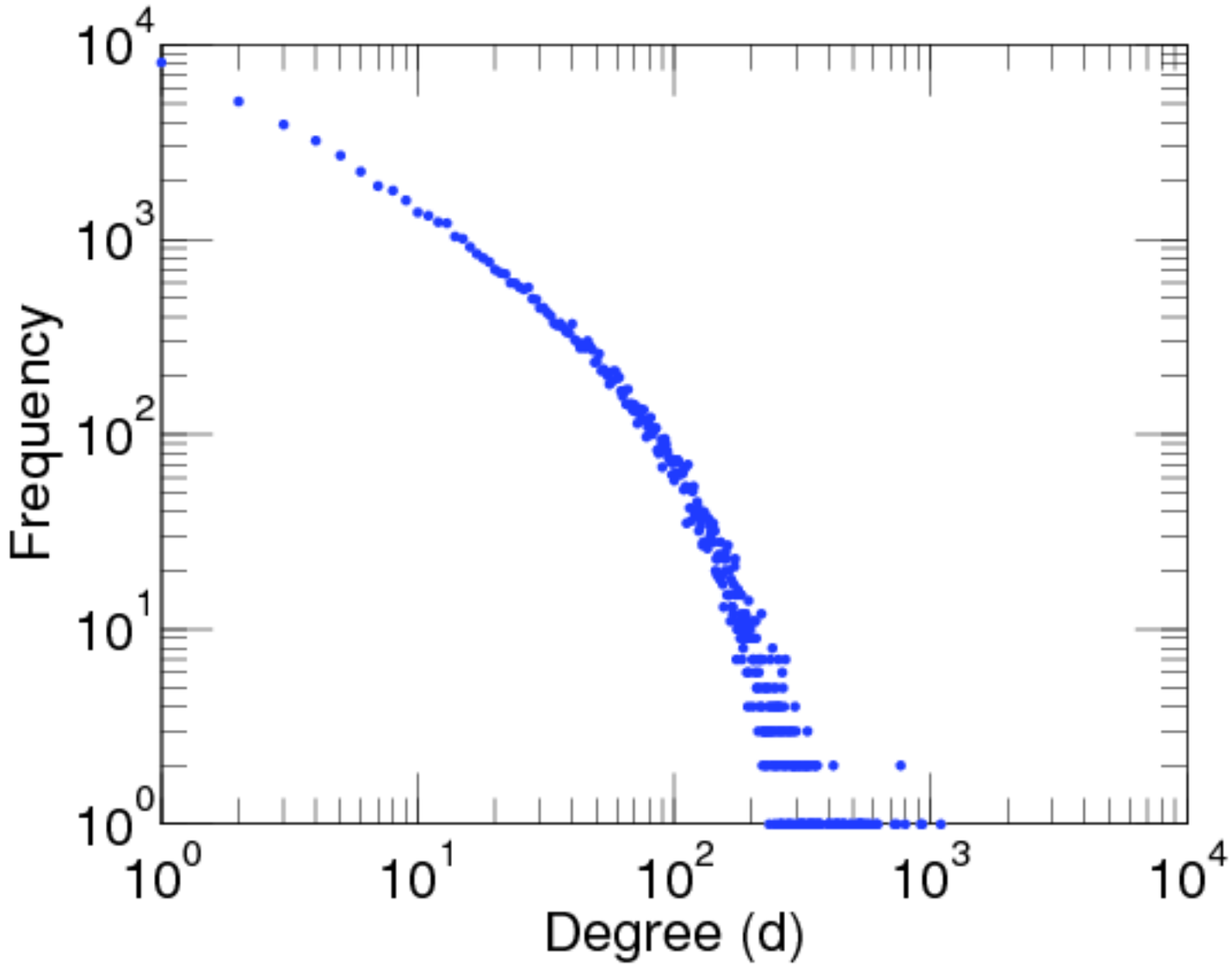
Long-tail distribution

- Degree distribution in large-scale networks often follows a power law (long-tail distribution or scale-free distribution)

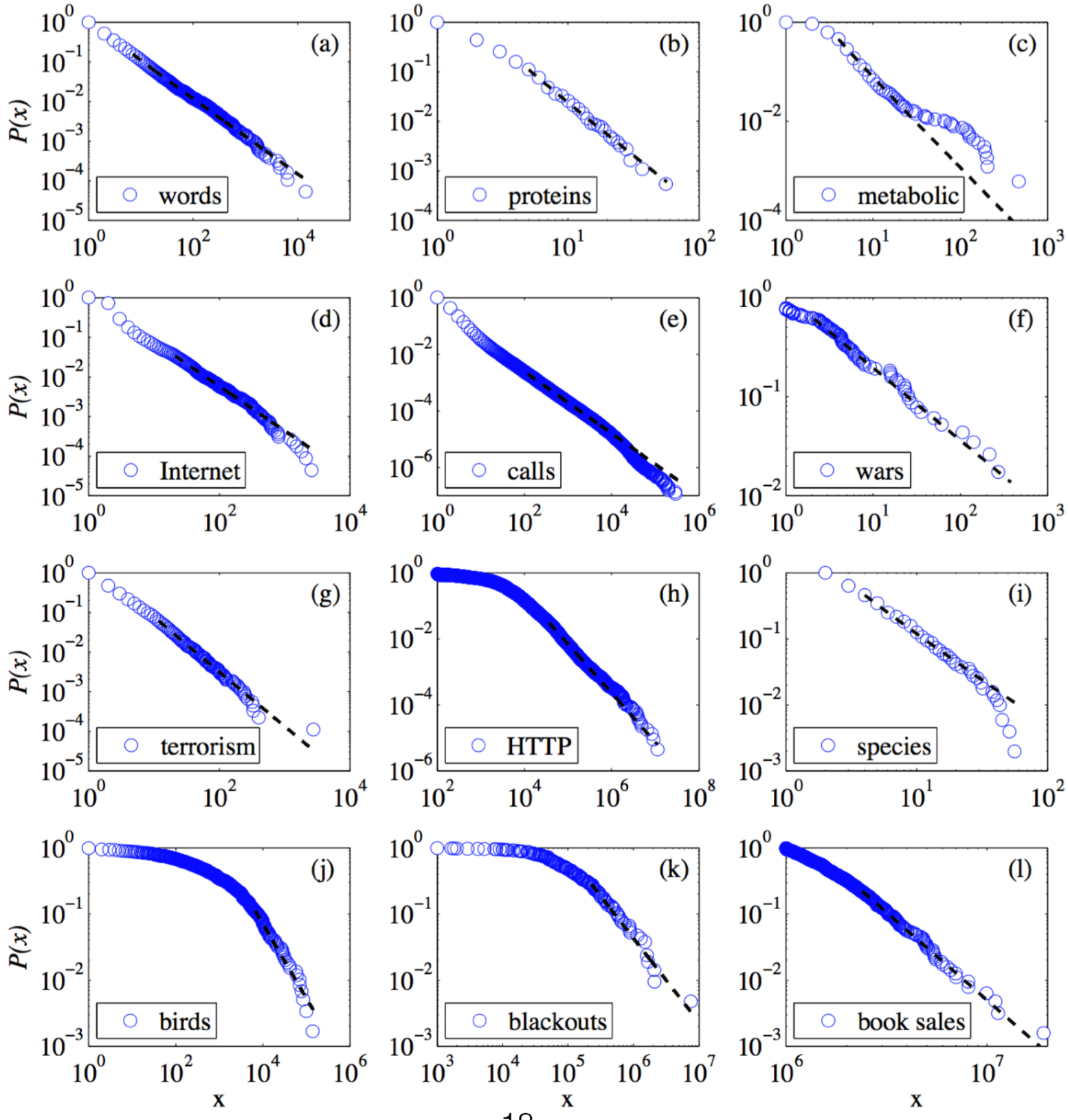


Long-tail distribution

- Long-tail distribution becomes a straight line if plot in log-log scale



Power-law is (almost) everywhere



Source: Power-law distributions in empirical data

Six degree of separation

- *« Any two people are on average separated no more that by six intermediate connections »*
- **A quite old topic**
  - *Chain-links* by Frigyes Karinthy in 1929 (short story)[1]
  - John Guare play (1991)
  - Fred Schepisi movie (1993)

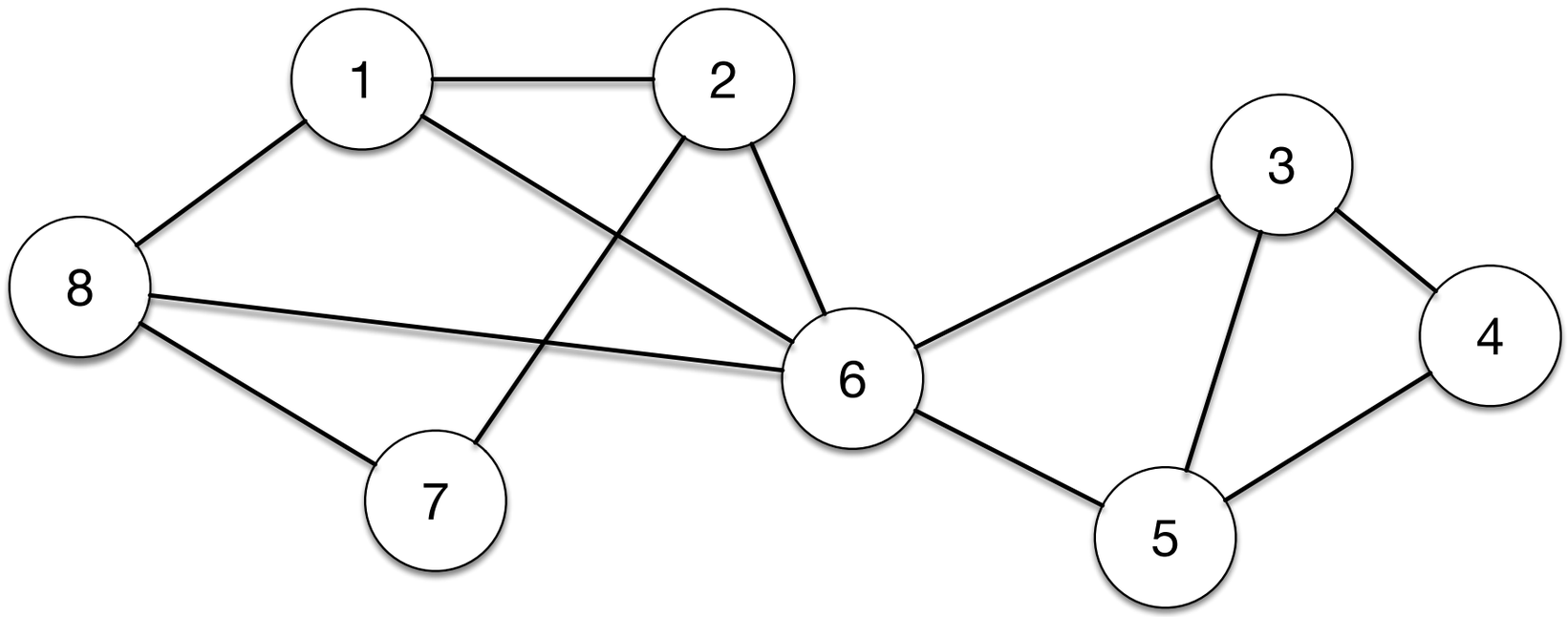
[1] An english translation is available at https://djjr-courses.wdfiles.com/local--files/soc180%3Akarinthy-chain-links/Karinthy-Chain-Links_1929.pdf

Small world effect

- Six Degrees of Separation
- A famous experiment conducted by Travers and Milgram in 1969
  - Subjects (296) were asked to send a chain letter to his/her acquaintance in order to reach a target person
  - The average path length is around 5.5
- Verified on a planetary-scale instant messenger network of 180M users (Leskovec and Horvitz 2008)
  - The average path length is 6.6

Diameter

- Common measures to calibrate the small world effect:
  - Diameter
  - Average shortest path length



Community structure

- Community:
  - People (nodes) who interact with each other more frequently than with those outside the community
  - Can be seen as a partition of dense subgraph

- Many different definitions of what a community is
  - Dozens of scientific papers to detect communities

One possible intuition behind community

- Friends of a friend are likely to be friends as well
- Clustering coefficient: a very popular and widely used network metric
- Clustering coefficient measures the above intuition
  - Can be seen as the density of connections among one's friends

Local clustering coefficient

Directed

$$C_i = \frac{|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

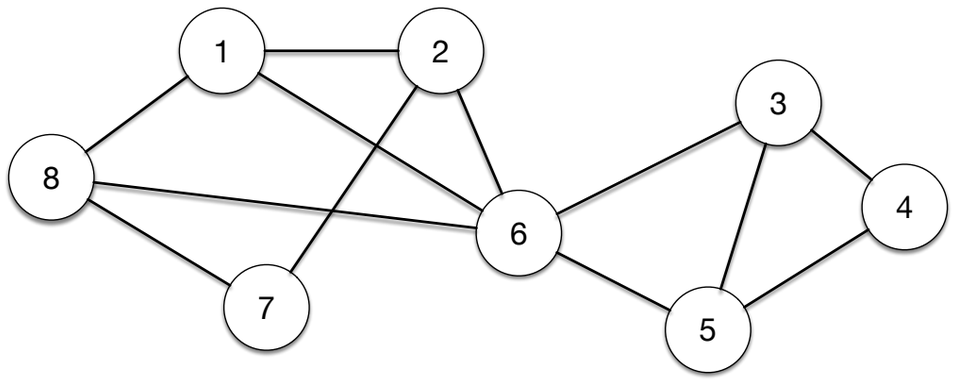
Undirected

$$C_i = \frac{2|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

with N_i the neighbourhood of i and $k_i = |N_i|$

Average Clustering Coefficient (a.k.a. transitivity)

$$\overline{C} = \frac{\sum_i C_i}{|V|}$$



C_1?

Average clustering coefficient

A graph has a strong community structure if its average clustering coefficient is much higher than the cluster coefficient of a random graph having the same number of vertices and a similar number of edges

A graph having an average clustering coefficient greater than 0.15 is usually considered as a graph having a strong community structure

Outline

3. Node-centric metrics

A. Centrality

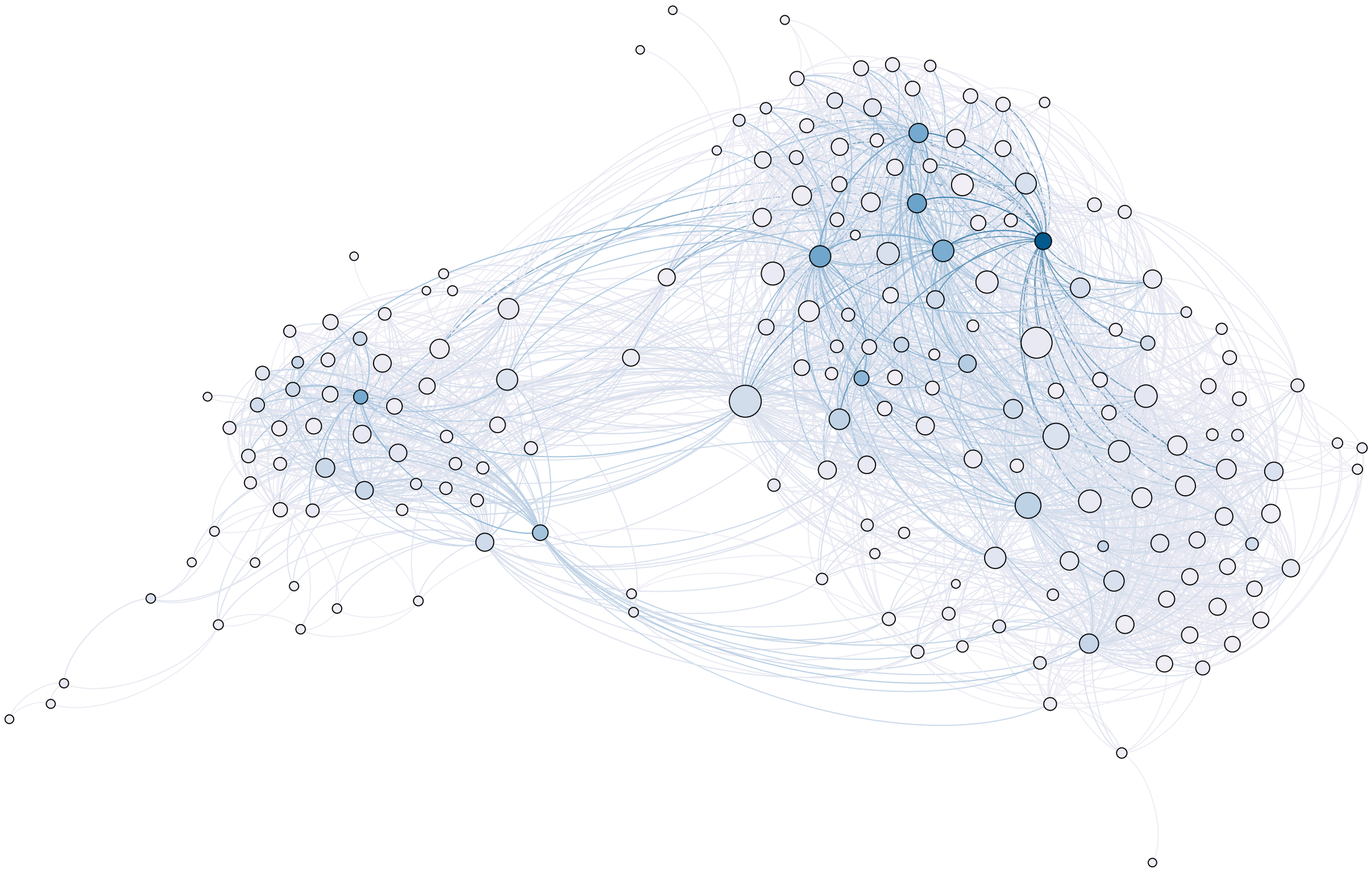
B. Prestige

C. Page Rank

D. Hubs and Authority

E. Metrics comparison

Importance of Nodes



Importance of Nodes

Centrality and Prestige

- Not all nodes are equally important
- An actor is prominent if the ties of the actor make the actor particularly visible to the other actors in the network
- *Visibility is not only measured by direct ties, but also by indirect ties through intermediaries*
- Knock and Burt distinguished two types of visibility: centrality and prestige
- **A** is the adjacency matrix of the network

Centrality and Prestige

- Sociologic point of view
- Both measures rely on the importance of actors in the social network
- Actor centrality
  - involvement with other actors, many ties, source or recipient
  - Mostly over undirected networks
- Actor prestige
  - Object of many ties, ties directed to an actor
  - Only over directed networks

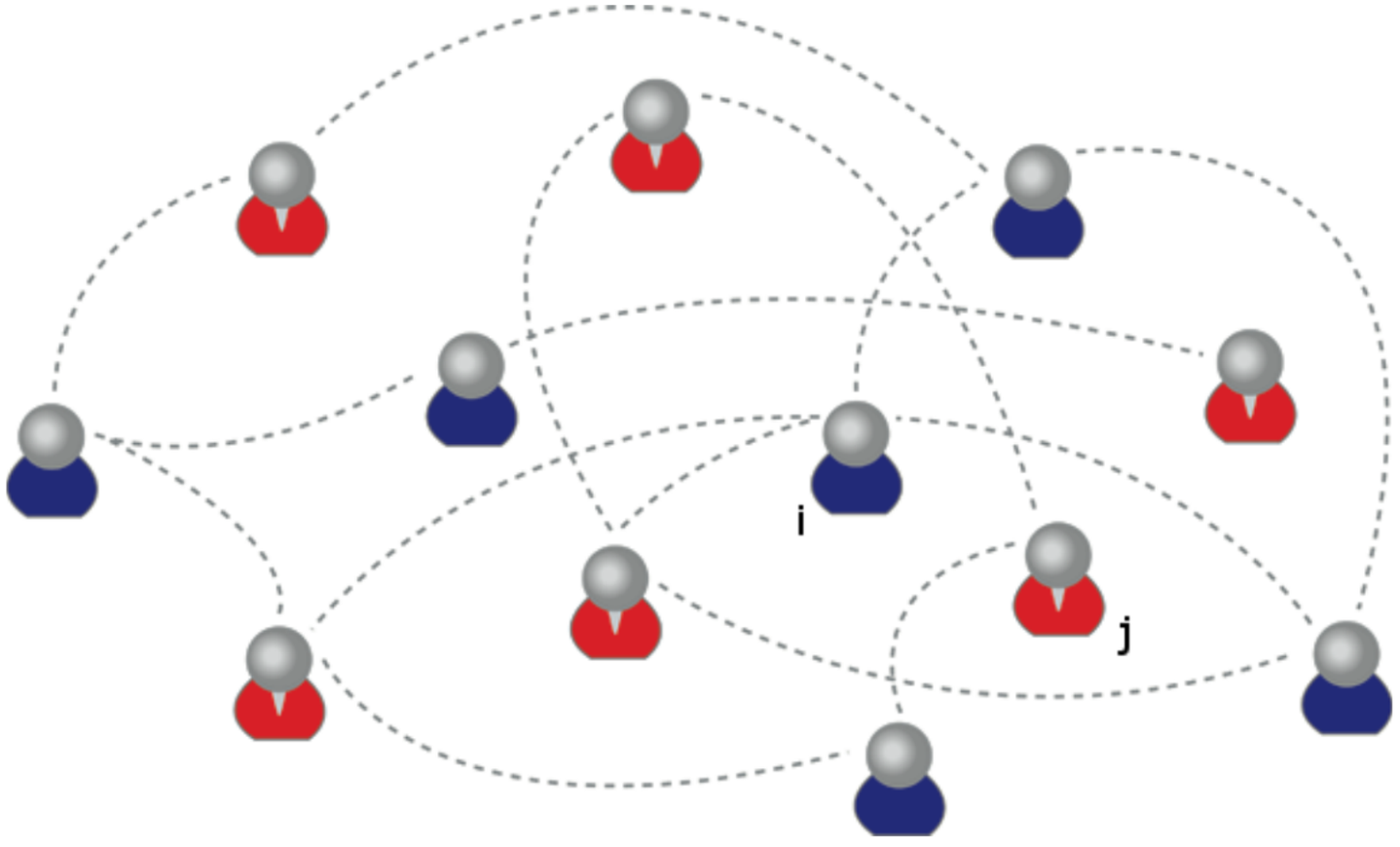
Centrality

- Not concerned with whether prominence is due to the receiving or the transmission of many ties
- What is important is that the actor is involved
- In undirected graphs, a central actor is involved in many ties.
- ***Commonly used prestige metrics***
  - Degree
  - Closeness
  - Betweenness
  - Eigenvector

Centrality

Intuition

The more connected a user, the more important



Centrality

Degree - Undirected

Degree centrality: number of nearest neighbours

$$C_D(i) = \sum_j A_{ij} = \sum_j A_{ji}$$

Normalized degree centrality

$$C_D^*(i) = \frac{1}{n-1} C_D(i)$$

Interpretation

- High value: direct contact with many other actors
- Low value: not active, peripheral actor

Centrality

Degree - Directed

Degree centrality: number of nearest neighbours

$$C_D(i) = \sum_j A_{ij}$$

Normalized degree centrality

$$C_D^*(i) = \frac{1}{n-1} C_D(i)$$

Interpretation

- High value: direct contact with many other actors
- Low value: not active, peripheral actor

Centrality

Closeness

Closeness centrality: how close an actor to all other actors in network

$$C_C(i) = \frac{1}{\sum_j d(i,j)}$$

Normalized closeness centrality

$$C_C^*(i) = (n-1)C_C(i)$$

Can be adapted to the directed scenario taking direction of edges into account

Interpretation

- High value: actor in the cantor can quickly interact with all others, short communication path to others, minimal number of steps to reach others

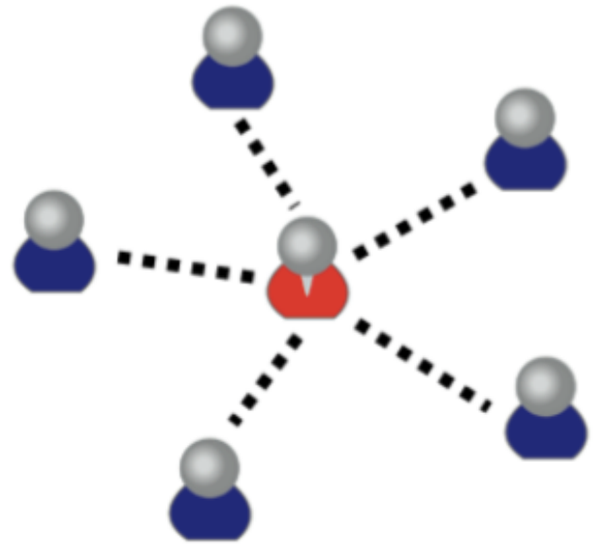
Centrality

Betweenness

Measures how much a node has control over all possible pairs of nodes. If it is part of the shortest paths of many pairs, it will be central

Undirected

$$C_B(i) = \sum_{i \neq j \neq k} \frac{p_{jk}(i)}{p_{jk}}$$



Can be normalized by (n-1) (n-2) / 2

Directed

Same as for undirected
Normalized by (n-1) (n-2)

Centrality

Eigenvector

Eigenvector centrality: the importance of a node depends on the importance of its neighbours (recursive definition)

$$C_E(i) = \frac{1}{\lambda} \sum_{j \neq i} A_{i,j} \, C_E(j)$$

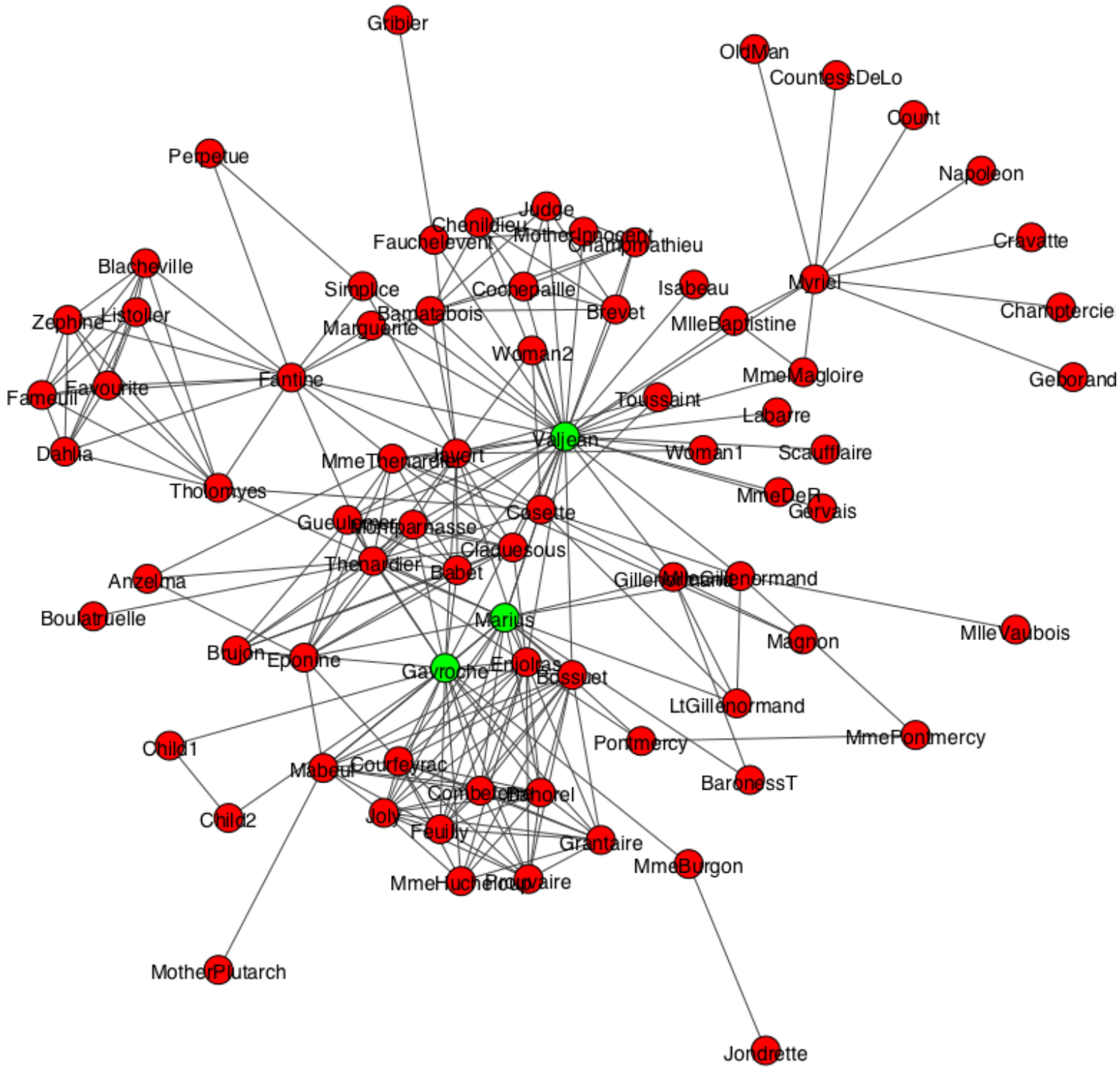
With the vector notation: $\mathbf{Ax} = \lambda \mathbf{x}$

Additional requirement : the eigenvector is non negative

Consequence (Perron-Frobenius theorem) : only the greatest eigenvalue works

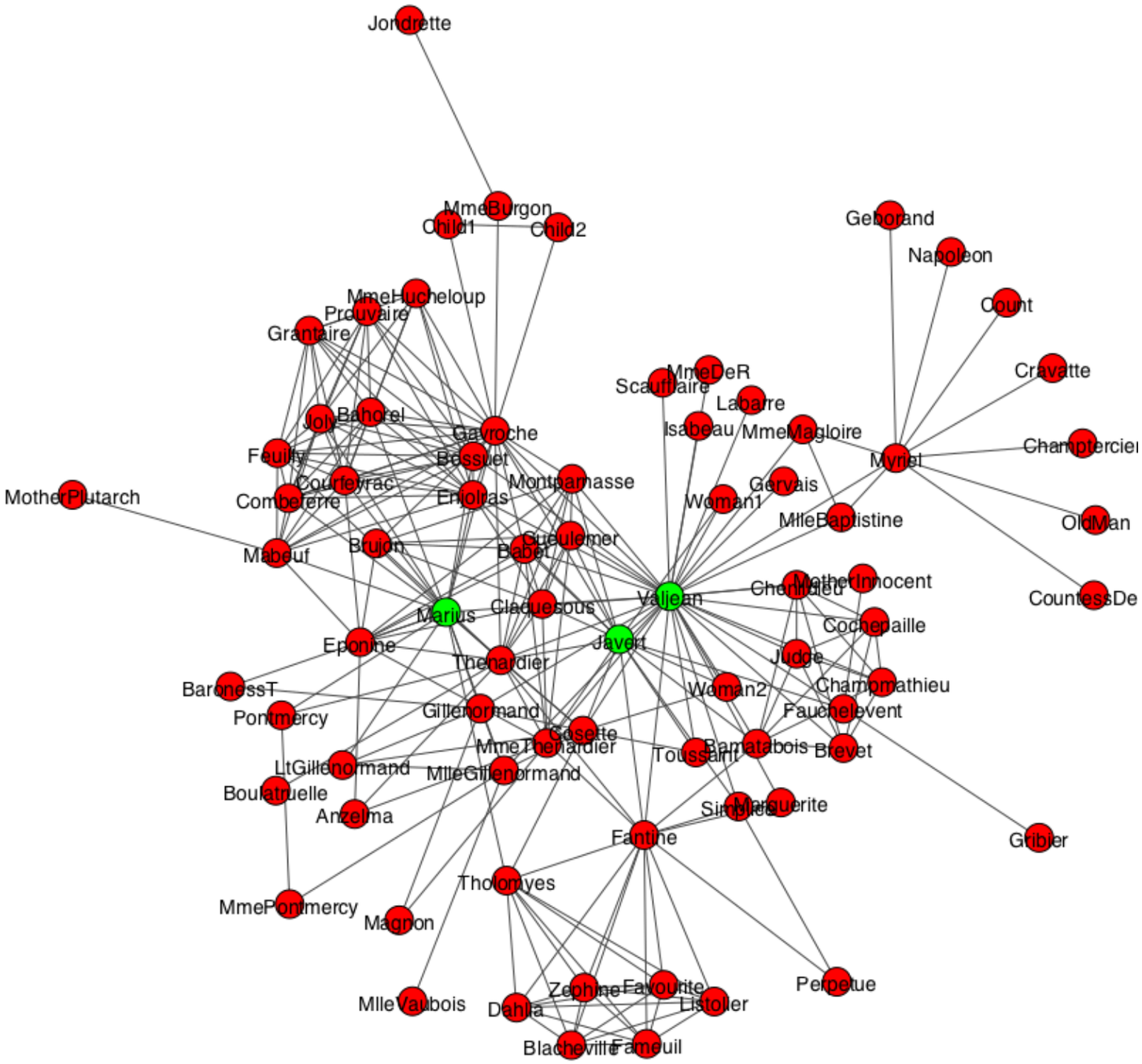
Centrality

Examples - Degree



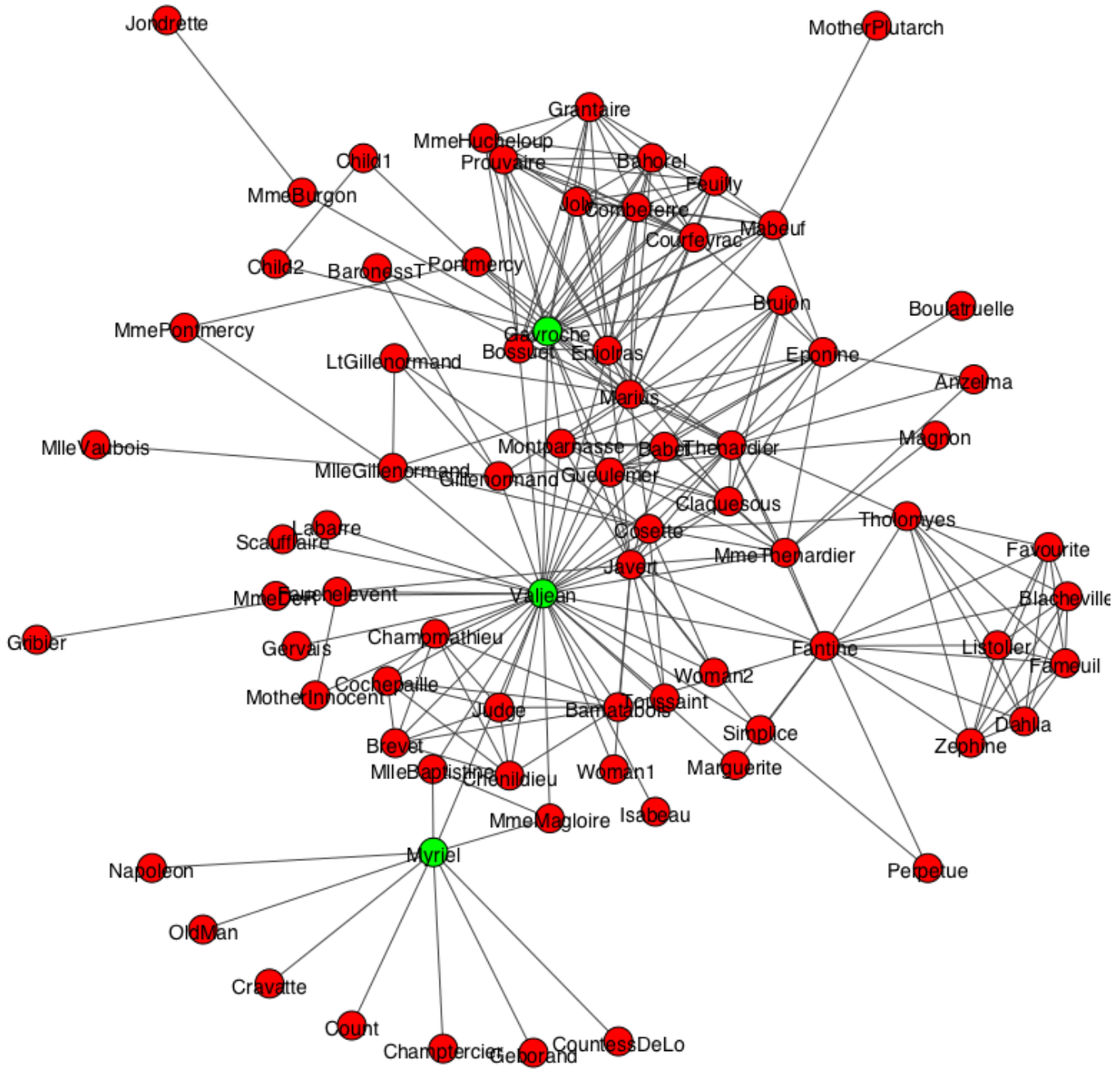
Centrality

Examples - Closeness



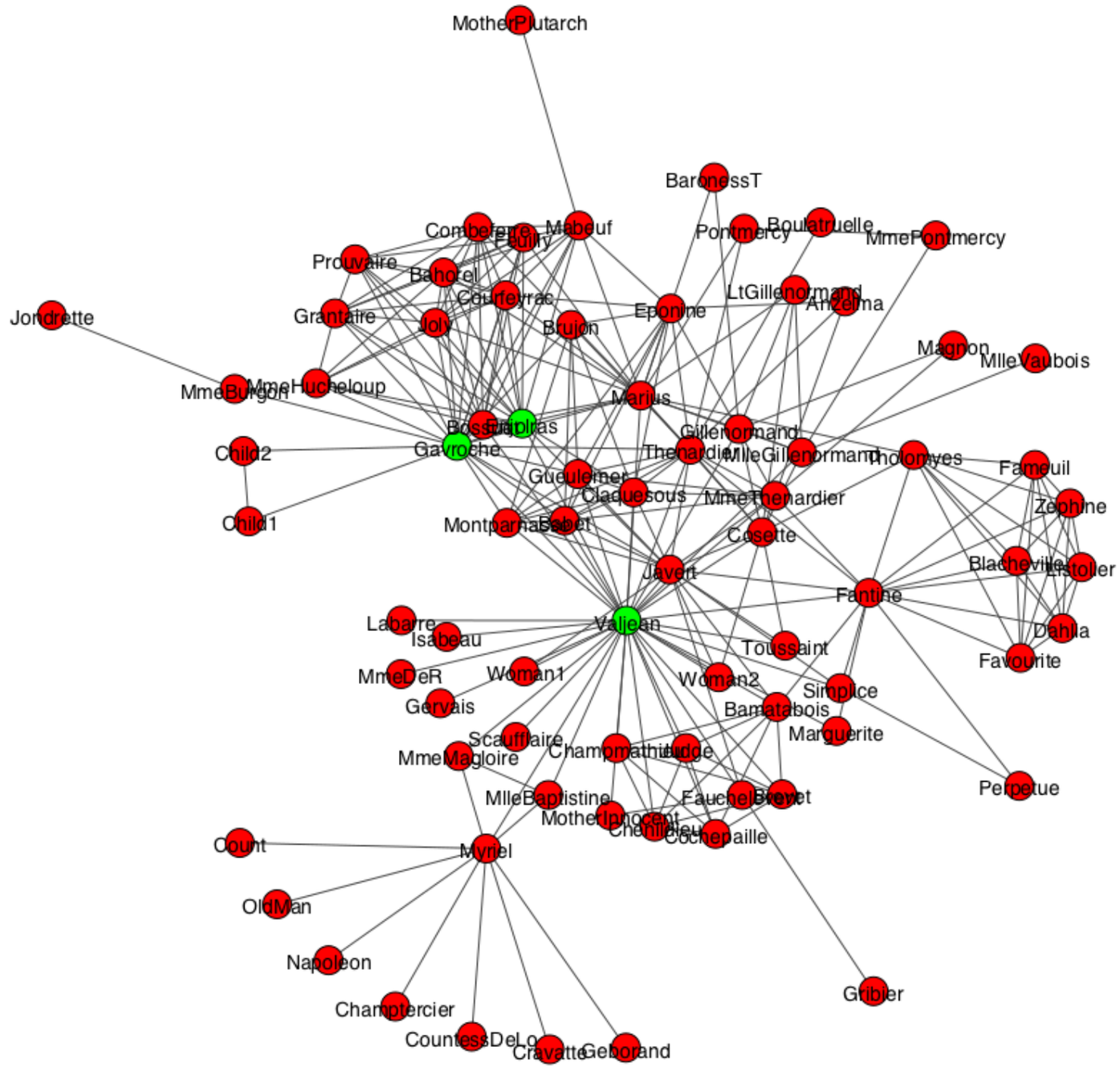
Centrality

Examples - Betweenness



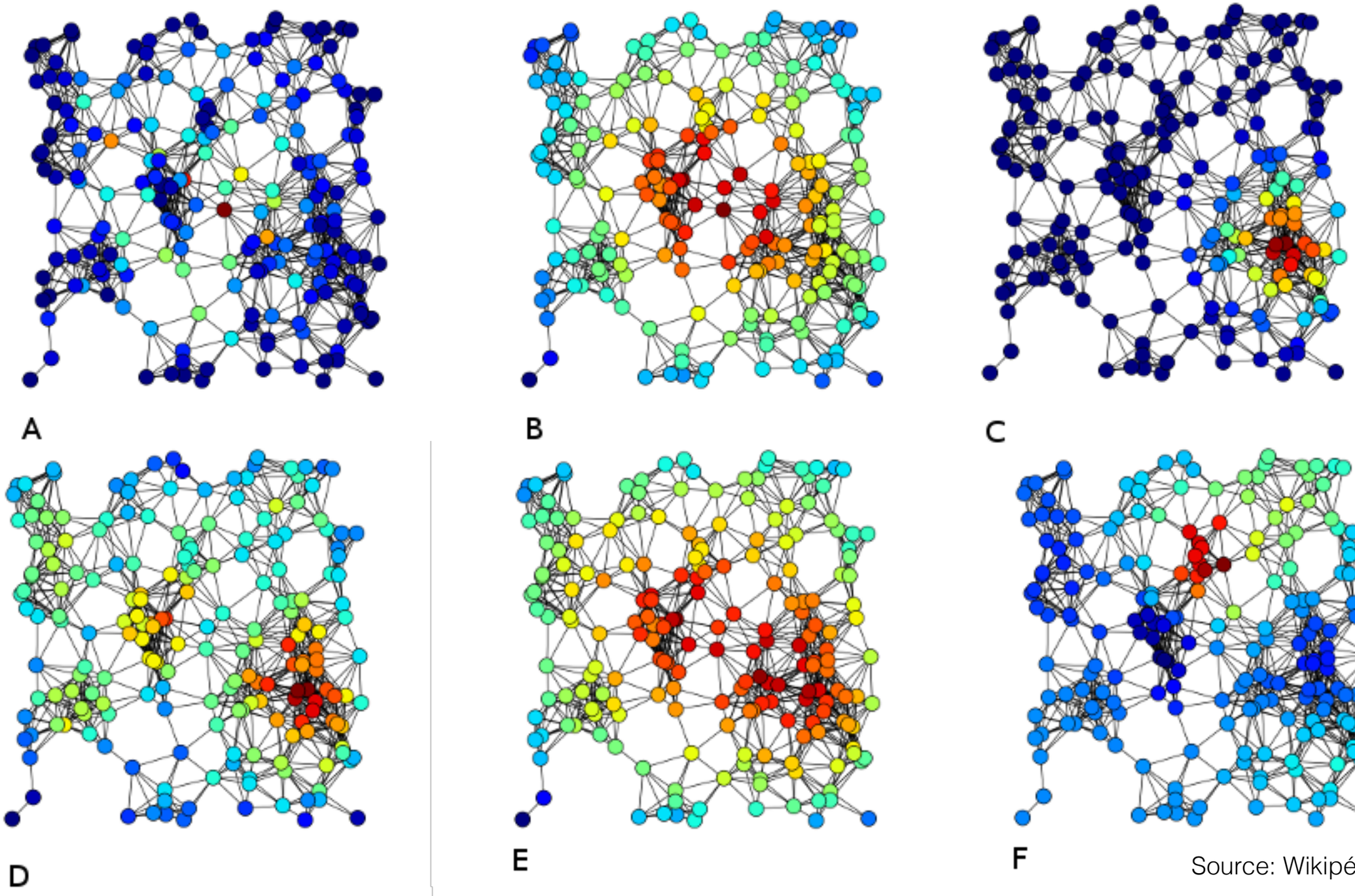
Centrality

Examples - Eigenvector



Centrality

Examples



Source: Wikipédia

A) Betweenness centrality, B) Closeness centrality, C) Eigenvector centrality, D) Degree centrality, E) Harmonic Centrality and F) Katz centrality

Prestige

Intuition

More subtle than centrality

- Distinguish incoming and outgoing links
- A prestigious node is often referenced
- Only incoming links are considered
- Only directed graphs

Commonly used prestige metrics

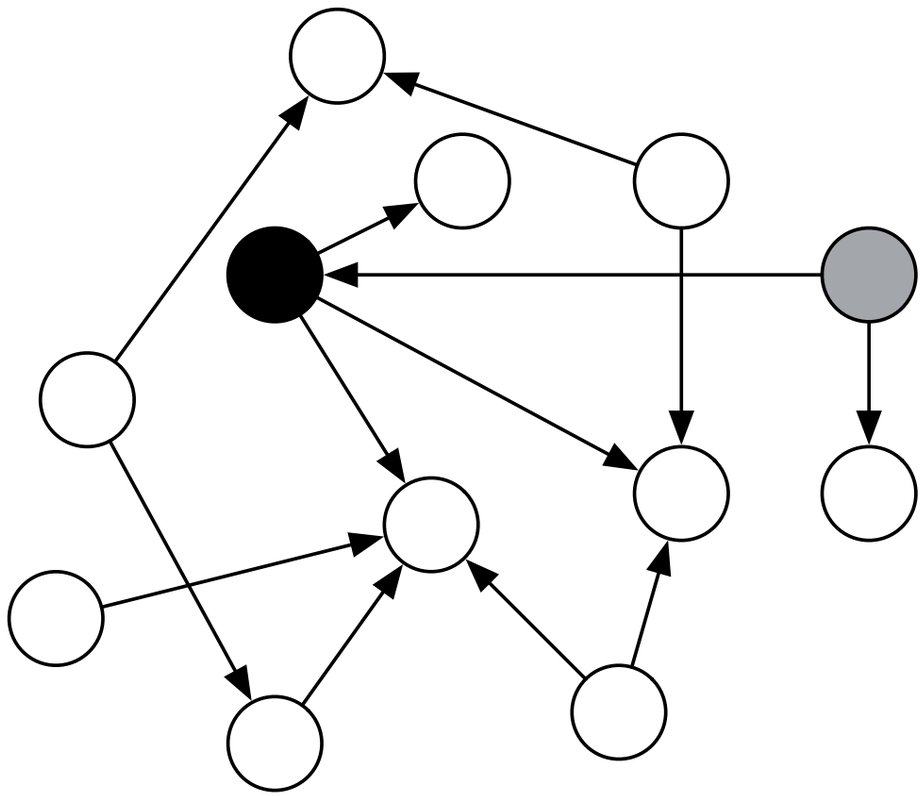
- Degree
- Closeness
- Rank

Prestige

Degree

Definition

$$P_D(i) = \frac{d_I(i)}{n-1}$$



Prestige of ● ?

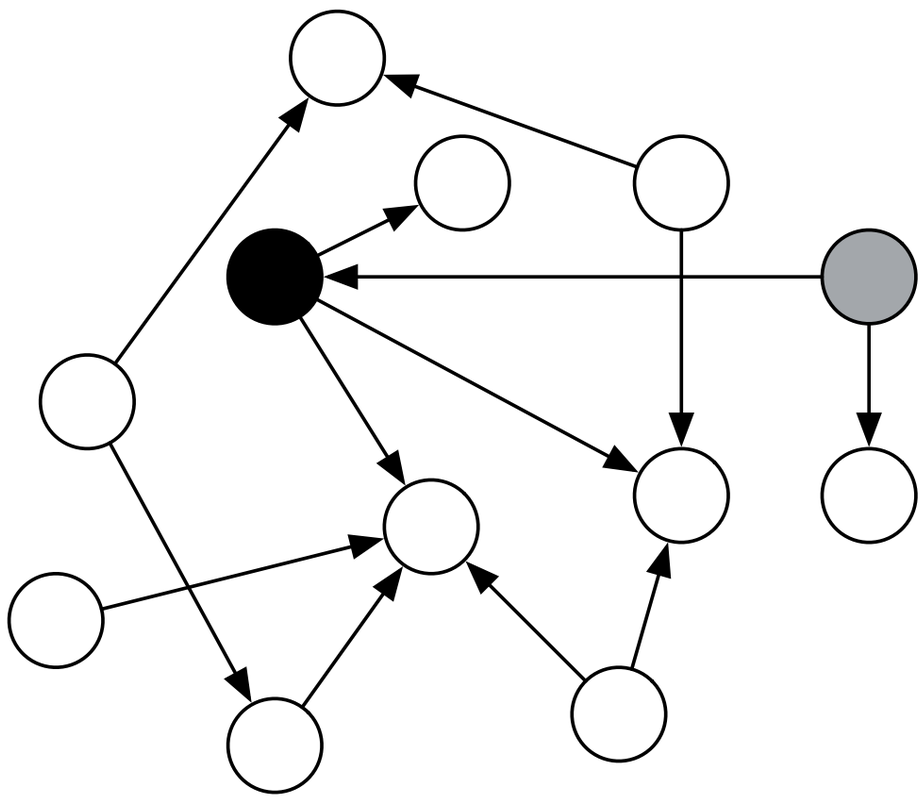
Prestige of ● ?

Prestige

Closeness

Intuition

Paths going to the node are considered



Definition

$$P_B(i) = \frac{\sum_{j \in I_i} d(j,i)}{|I_i|}$$

Prestige

Rank

Intuition

The prestige of a node depends on the importance of its predecessors.

Definition

$$P_R(i) = \sum_{j \neq i} A_{ji} P_R(j)$$

Matrix representation

$$\mathbf{P} = \mathbf{A}^T \mathbf{P}$$

Page Rank (1)

Intuition

”PageRank can be thought of as a model of user behavior. We assume there is a ”random surfer” who is given a web page at random and keeps clicking on links, never hitting ”back” but eventually gets bored and starts on another random page. The probability that the random surfer visits a page is its PageRank.”

Model

Create a stochastic matrix of the Web, denoted by **S**:

- Each page i corresponds to row i and column i of the matrix
- If page j has n successors (links) then the ijth cell of the matrix is equal to 1/n if page i is one of these n successors of page j, and 0 otherwise.

Page Rank (2)

- Initially each page has 1 unit of importance. At each round, each page shares importance it has among its successors, and receives new importance from its predecessors
- The importance of each page reaches a limit after some steps
- Random walk on a directed graph

$$\mathbf{p}^{t+1} = \mathbf{S}\mathbf{p}^{t}$$

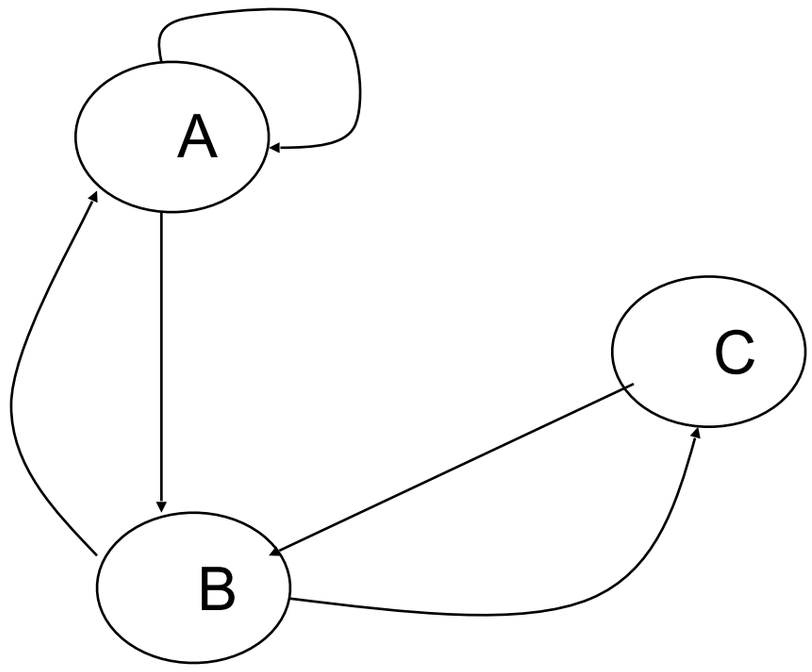
- Can also be seen as a Markov chain

Page Rank (3)

Example 1

Assume that the Web consists of only three pages - A, B, and C. The links among these pages are shown below.

Let [a, b, c] be the vector of importances for these three pages



	A	B	C
A	1/2	1/2	0
B	1/2	0	1
C	0	1/2	0

Page Rank (3)

Example 1

The equation describing the asymptotic values of these three variables is:

$$\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

We can solve the equations like this one by starting with the assumption a = b = c = 1, and applying the matrix to the current estimate of these values repeatedly. The first four iterations give the following estimates:

a =	1	1	5/4	9/8	5/4	…	**6/5**
b =	1	3/2	1	11/8	17/16	…	**6/5**
c =	1	1/2	3/4	1/2	11/16	...	**3/5**

Problems with Real Web Graphs

In the limit, the solution is a=b=6/5, c=3/5. That is, ***a*** and ***b*** each have the same importance, and twice of ***c***.

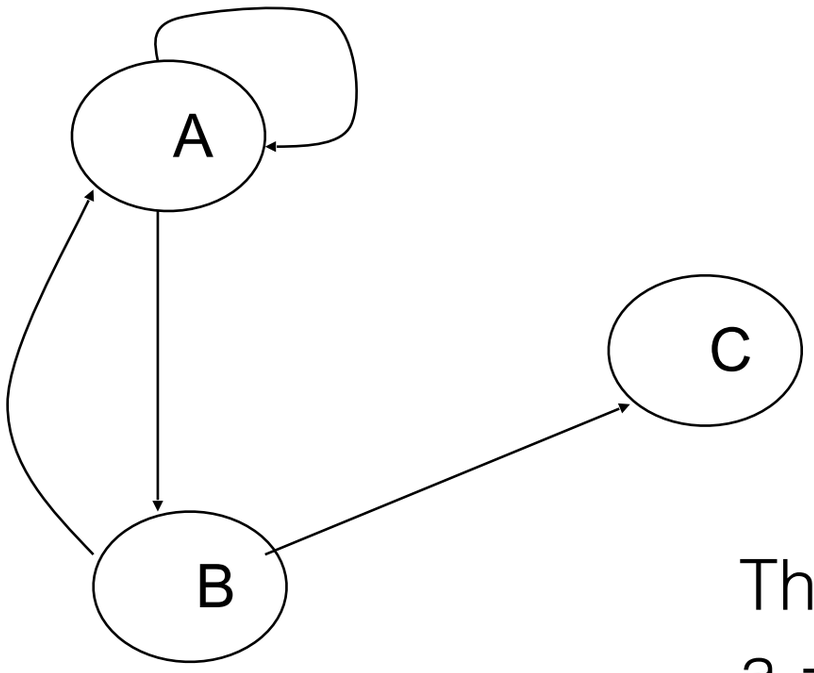
Problems with Real Web Graphs

- Dead ends: a page that has no successors has nowhere to send its importance.
- Spider traps: a group of one or more pages that have no links out.

Page Rank

Example 2

Assume now that the structure of the Web has changed. The new matrix describing transitions is:



$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} ½ & ½ & 0 \\ ½ & 0 & 0 \\ 0 & ½ & 0 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

The first four steps of the iterative solution are:

a = 1 1 3/4 5/8 1/2

b = 1 1/2 1/2 3/8 5/16

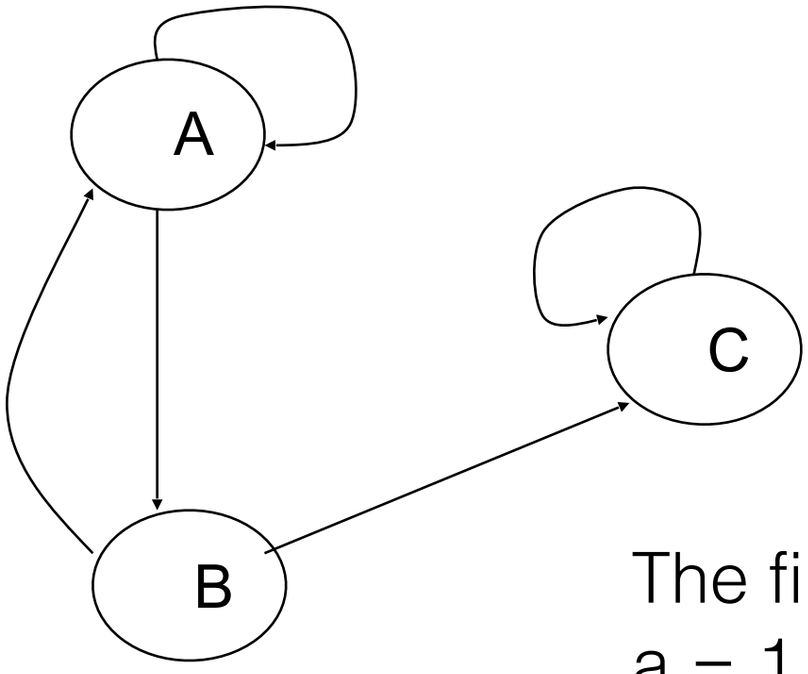
c = 1 1/2 1/4 1/4 3/16

Eventually, each of a, b, and c become 0.

Page Rank

Example 3

Assume now once more that the structure of the Web has changed. The new matrix describing transitions is:



$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} ½ & ½ & 0 \\ ½ & 0 & 0 \\ 0 & ½ & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

The first four steps of the iterative solution are:

a = 1 1 3/4 5/8 1/2

b = 1 1/2 1/2 3/8 5/16

c = 1 3/2 7/4 2 35/16

c converges to 3, and a=b=0.

Google Solution

Instead of applying the matrix directly, "tax" each page some fraction of its current importance, and distribute the taxed importance equally among all pages.

Example: if we use 20% tax, the equation of the previous example becomes:

a = 0.8 * (½*a + ½ *b +0*c)

b = 0.8 * (½*a + 0*b + 0*c)

c = 0.8 * (0*a + ½*b + 1*c)

The solution to this equation is a=7/11, b=5/11, and c=21/11

HITS Algorithm

Hyperlink-Induced Topic Search

Key Definitions

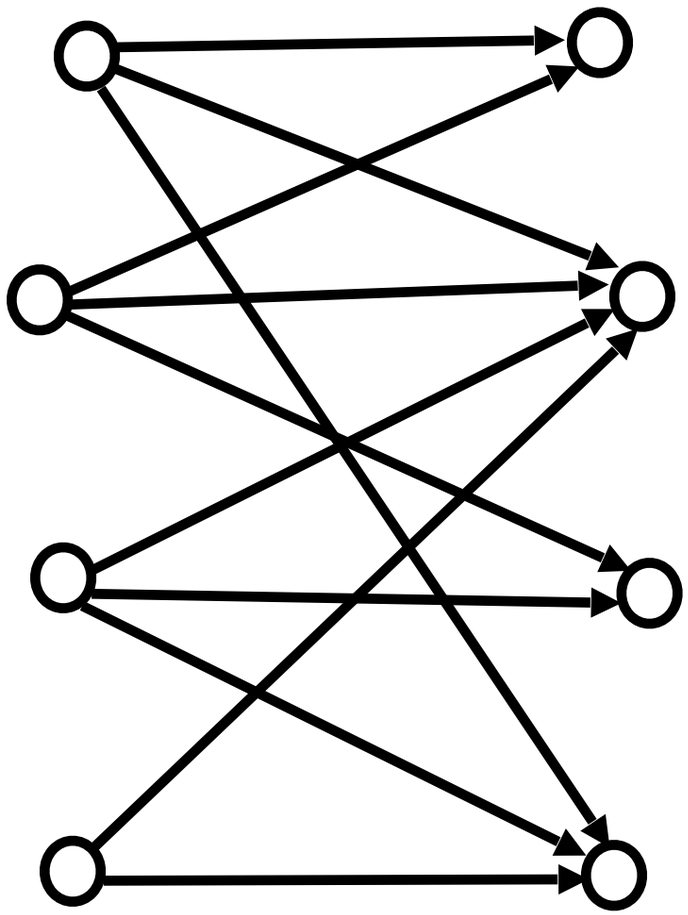
Authorities

- Relevant pages of the highest quality on a broad topic

Hubs

- Pages that link to a collection of authoritative pages on a broad topic

Hub-Authority Relations



Hyperlink-Induced Topic Search

(HITS)

The approach consists in two phases:

- It uses the query terms to collect a starting set of pages (200 pages) from an index-based search engine – root set of pages.

- The root set is expanded into a base set by including all the pages that the root set pages link to, and all the pages that link to a page in the root set, up to a designed size cutoff, such as 2000-5000.

- A weight-propagation phase is initiated. This is an iterative process that determines numerical estimates of hub and authority weights

Hub and Authorities

- Let **a** and **h** be vectors, whose i^{th} component corresponds to the degrees of authority and hubbiness of the ith page. Then:

- **h = A × a**. That is, the hubbiness of each page is the sum of the authorities of all the pages it links to.

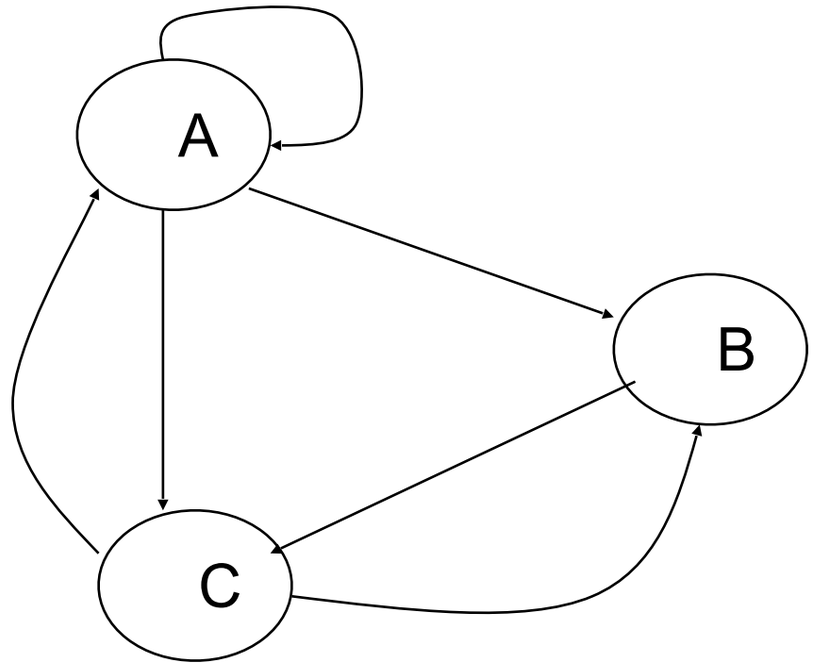
- **a = A^T × h**. That is, the authority of each page is the sum of the hubbiness of all the pages that link to it (**A^T** - transposed matrix).

- Then, **a = A^T × A × a h = A × A^T × h**

Hub and Authorities

Example

Consider the Web presented below.



$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Hub and Authorities

Example

If we assume that the vectors $h = [h_a, h_b, h_c]$ and $a = [a_a, a_b, a_c]$ are each initially [1,1,1], the first three iterations of the equations for a and h are the following:

a_a	=	1	5	24	114
a_b	=	1	5	24	114
a_c	=	1	4	18	84
h_a	=	1	6	28	132
h_b	=	1	2	8	36
h_c	=	1	4	20	96

Metrics comparison

- **Pearson correlation coefficient** shows linear dependence between variables

$$r = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_i (X_i - \bar{X})^2} \sqrt{\sum_i (Y_i - \bar{Y})^2}}$$

- **Spearman rank correlation** coefficient (Sperman's rho) shows strength of monotonic association
  - Convert raw scores to ranks (X_i^* is the rank of X_i)

$$\rho = 1 - \frac{6 \sum_i (X_i^* - Y_i^*)^2}{n(n^2 - 1)}$$

Ranking comparison

- The **Kendall tau rank** distance is a metric that count the number of pairwise disagreement between two ranking lists

$$\tau = \frac{n_c - n_d}{n(n-1)/2}$$

  - n_c: the number of concordant pairs, n_d: the number of discordant pairs

- **Example**

Rank1	A	B	C	D	E
Rank 2	A	C	B	E	D

$$\tau = \frac{8-2}{5(5-1)/2} = 0.6$$

Take away message

- Generally different centrality metrics will be positively correlated
- When they are not, there is likely something interesting about the network

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important/active players		Probably multiple paths in the network, ego is near many people, but so are many others
High Betweenness	Ego's few ties are crucial for network flow	Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others.	