Web Structure Mining

Community detection and link prediction

— Session 3 —

Community Detection

Community

Community. It is formed by individuals such that those within a group interact with each other more frequently than with those outside the group, a.k.a. **group, cluster, cohesive subgroup, module** in different contexts

Community detection: discovering groups in a network where individuals' group memberships are not explicitly given

Why communities in social media?

- Human beings are social
- Easy-to-use social media allows people to extend their social life in unprecedented ways
- Difficult to meet friends in the physical world, but much easier to find friend online with similar interests
- Interactions between nodes can help determine communities

Community in Social Media

Two types of groups in social media

- **Explicit Groups:** formed by user subscriptions
- **Implicit Groups:** implicitly formed by social interactions

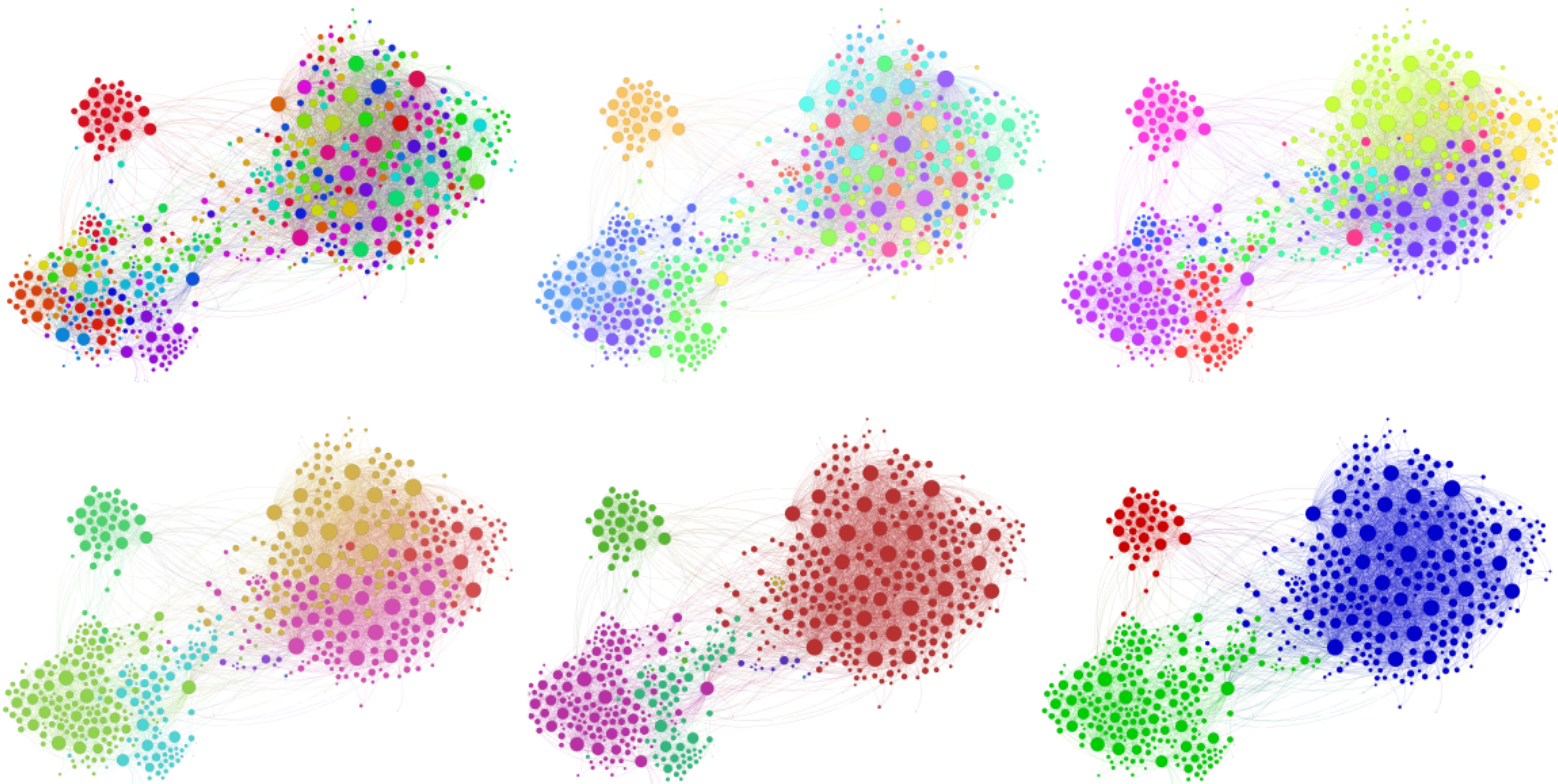
Some social media sites allow people to join groups, is it necessary to extract groups based on network topology?

- Not all sites provide community platform
- Not all people want to make effort to join groups
- Groups can change dynamically

Network interaction provides rich information about the relationship between users

- Can complement other kinds of information
- Help network visualization and navigation
- Provide basic information for other tasks

Subjectivity in Community Detection



Source: https://pegasusdata.com/2013/01/10/facebook-friends-network-mapping-a-gephi-tutorial/

Taxonomy of Community Criteria

Criteria vary depending on the tasks

Community detection methods can be divided into 4 categories (not exclusive):

1 - Node-Centric Community

Each node in a group satisfies certain properties

2 - Group-Centric Community

Consider the connections within a group as a whole. The group has to satisfy certain properties without zooming into node-level

3 - Network-Centric Community

Partition the whole network into several disjoint sets

4 - Hierarchy-Centric Community

Construct a hierarchical structure of communities

Node-Centric Community Detection

Nodes satisfy different properties

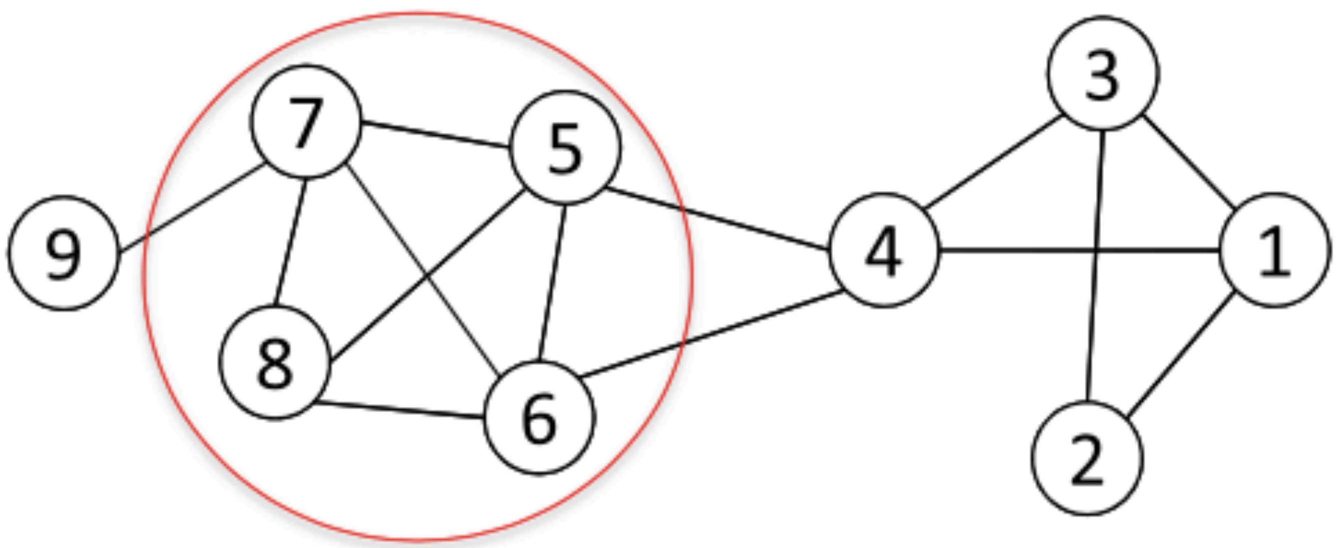
- Complete Mutuality
  - Cliques
- Reachability of members
  - k-clique, k-clan, k-club
- Nodal degrees
  - k-plex, k-core
- Relative frequency of Within-Outside Ties
  - LS sets, Lambda sets

Commonly used in traditional social network analysis

Here, we discuss some representative ones

Complete Mutuality: Cliques

A clique is a complete maximal subgraph



Nodes 5, 6, 7 and 8 form a clique

NP-hard to find the maximum clique in a network

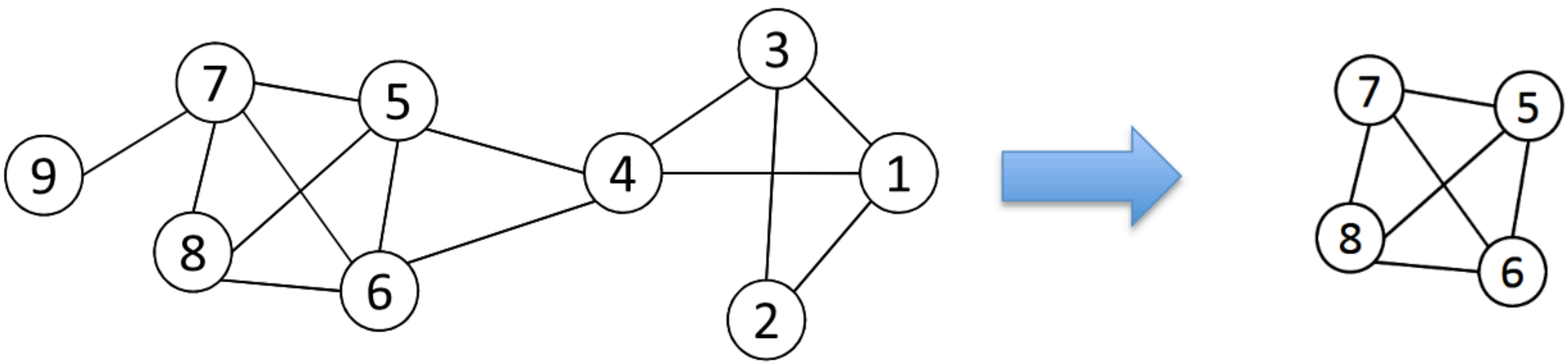
Straightforward implementation to find cliques is very expensive in time complexity

Finding the Maximum Clique

- In a clique of size k, each node has a degree >= k-1
- Nodes with degree < k-1 will not be included in the maximum clique
- Recursive pruning procedure :
    - Sample a sub-network from the given network, and find a clique in the sub-network, say, by a greedy approach
    - Suppose the clique above is size k, in order to find out a *larger* clique, all nodes with degree <= k-1 should be removed
- Repeat until the network is small enough
- In social media, many nodes are removed as social networks follow a power law distribution for node degrees

Maximum Clique

Example



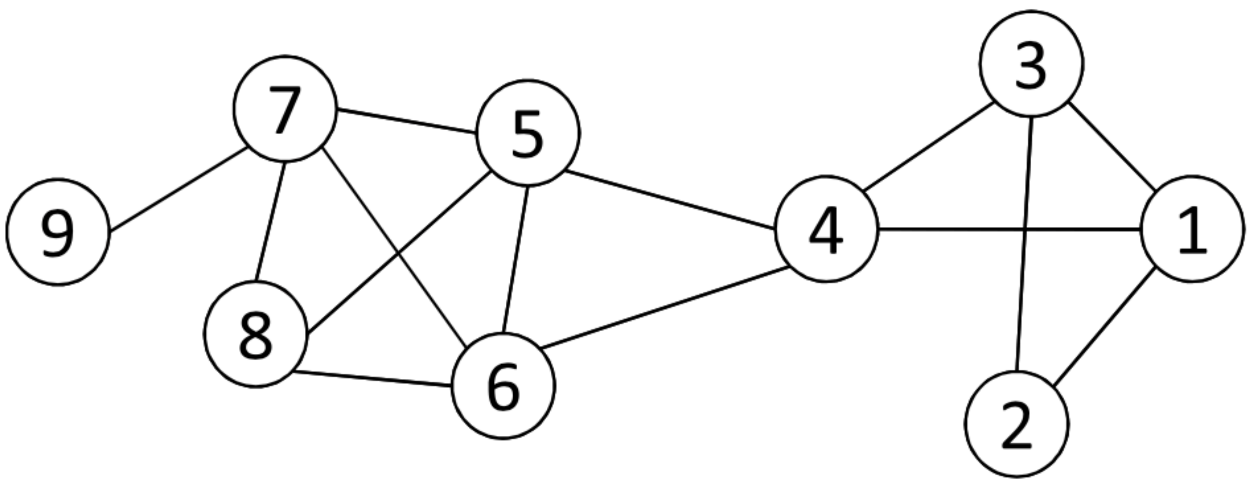
- Suppose we sample a sub-network with nodes 1 to 5 and find a 3-clique {1,2,3}
- In order to find a clique > 3, remove nodes with degree <= 2

Clique Percolation Method (CPM)

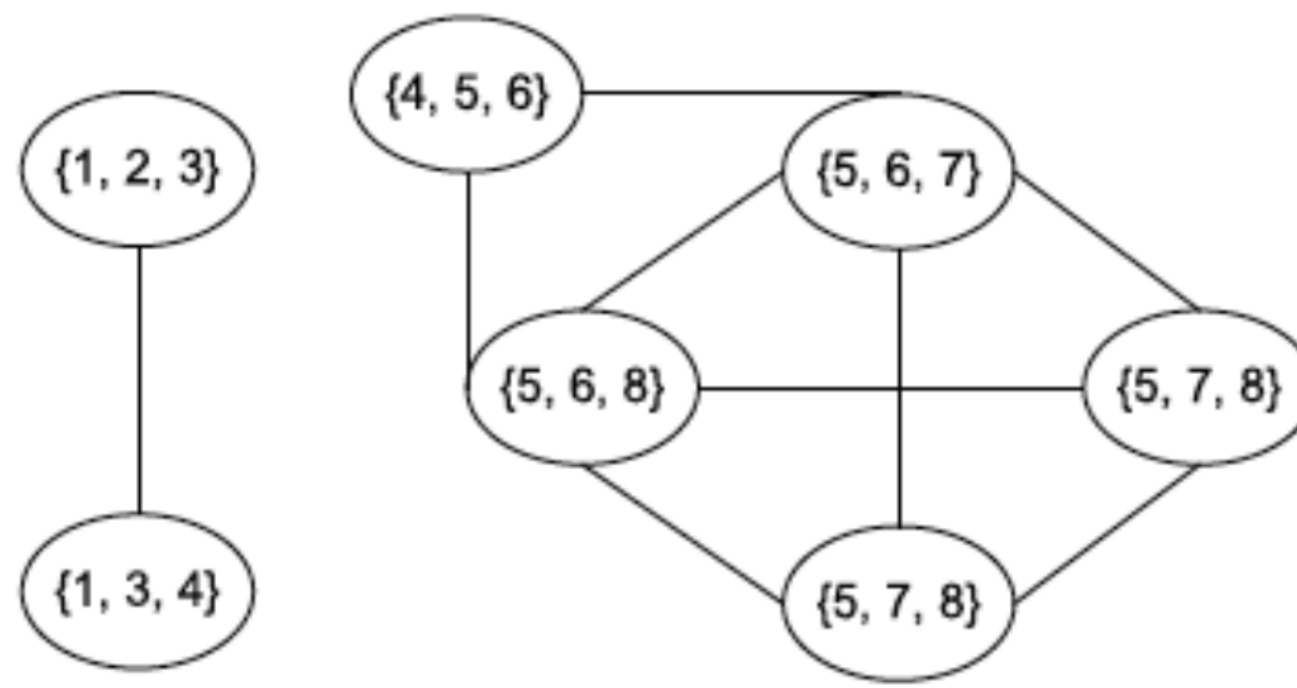
- Clique is a very strict definition, unstable
- Normally use cliques as a core or a seed to find larger communities

- CPM is such a method to find overlapping communities
  - **Input**
    - A parameter k, and a network
  - **Procedure**
    - Find out all cliques of size k in a given network
    - Construct a clique graph. Two cliques are adjacent if they share k-1 nodes
    - Each connected components in the clique graph form a community

CPM
Example



Cliques of size 3:
{1, 2, 3}, {1, 3, 4}, {4, 5, 6}, {5, 6, 7}, {5, 6, 8}, {5, 7, 8}, {6, 7, 8}



Communities:
{1, 2, 3, 4}
{4, 5, 6, 7, 8}

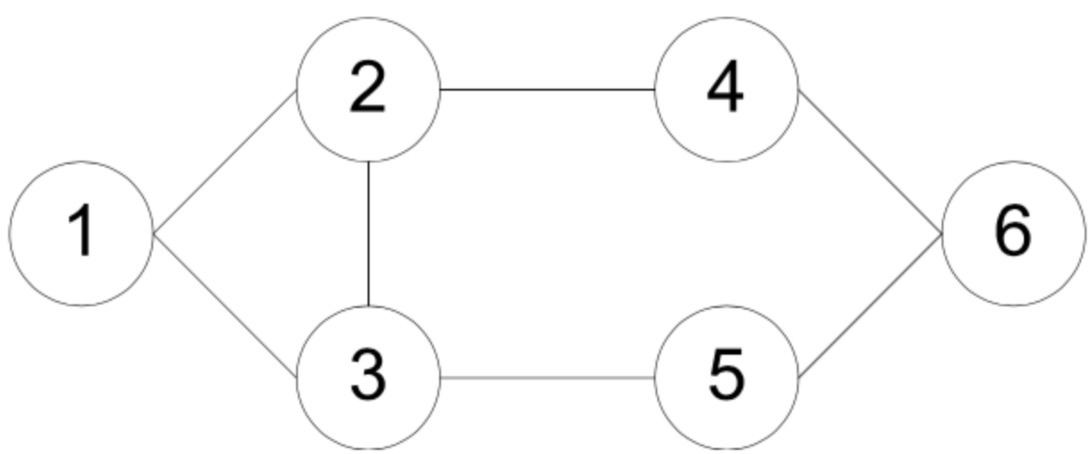
Reachability

k-clique and k-club

Any node in a group should be reachable in k hops

k-clique: a maximal subgraph in which the largest geodesic distance between any nodes <= k

k-club: a substructure of diameter <= k



Cliques: {1, 2, 3}
2-cliques: {1, 2, 3, 4, 5}, {2, 3, 4, 5, 6}
2-clubs: {1,2,3,4}, {1, 2, 3, 5}, {2, 3, 4, 5, 6}

A k-clique might have diameter larger than k in the subgraph

Commonly used in traditional SNA

Often involves combinatorial optimization

Group-Centric Community Detection

Density-Based Groups

The group-centric criterion requires the whole group to satisfy a certain condition

E.g., the group density >= a given threshold

A subgraph $G_s(V_s, E_s)$ is a $\gamma\text{-dense quasi-clique}$ if:

$$\frac{|E_s|}{|V_s|(|V_s|-1)/2} \geq \gamma$$

A similar strategy to that of cliques can be used

- Sample a subgraph, and find a maximal $\gamma\text{-dense quasi-clique}$ (say, of size k)
- Remove nodes with degree < $k\gamma$

Network-Centric Community Detection

Network-centric criterion needs to consider the connections within a network globally

Goal: partition nodes of a network into disjoint sets

Approaches:

- Clustering based on vertex similarity
- Latent space models
- Block model approximation
- Spectral clustering
- Modularity maximization

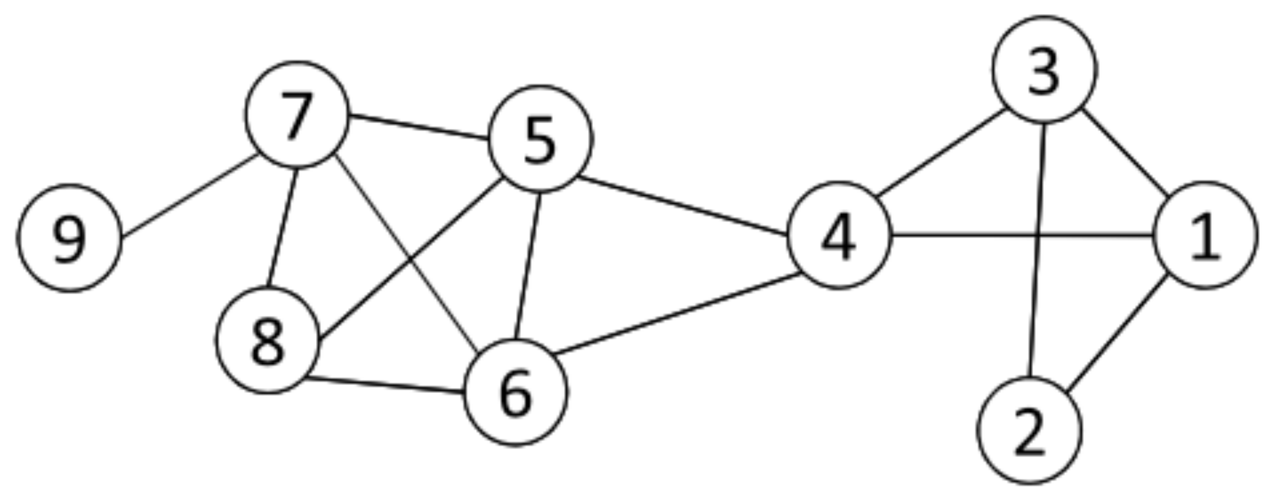
Clustering Based on Vertex Similarity

Apply k-means or similarity-based clustering to nodes

Vertex similarity is defined in terms of **the similarity of their neighborhood**

Structural equivalence: two nodes are structurally equivalent iff they are connecting to the same set of actors

Nodes 1 and 3 are structurally equivalent; So are nodes 5 and 7.

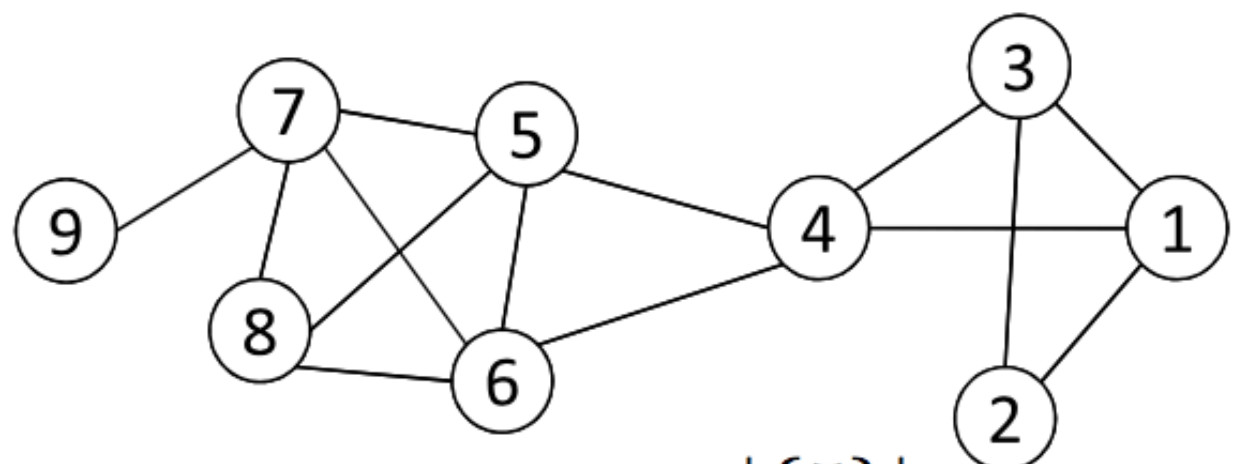


Structural equivalence is too restrict for practical use

Vertex Similarity

Jaccard Similarity $$Jaccard(v_i, v_j) = \frac{|N_i \cup N_j|}{|N_i \cap N_j|}$$

Cosine similarity $$cosine(v_i, v_j) = \frac{\sum_k A_{ik} A_{jk}}{\sqrt{\sum_k A_{ik}^2}\sqrt{\sum_k A_{jk}^2}}$$



$$Jaccard(4, 6) = \frac{|\{5\}|}{|\{1, 3, 4, 5, 6, 7, 8\}|} = \frac{1}{7}$$

$$cosine(4, 6) = \frac{1}{\sqrt{4}\sqrt{4}} = \frac{1}{4}$$

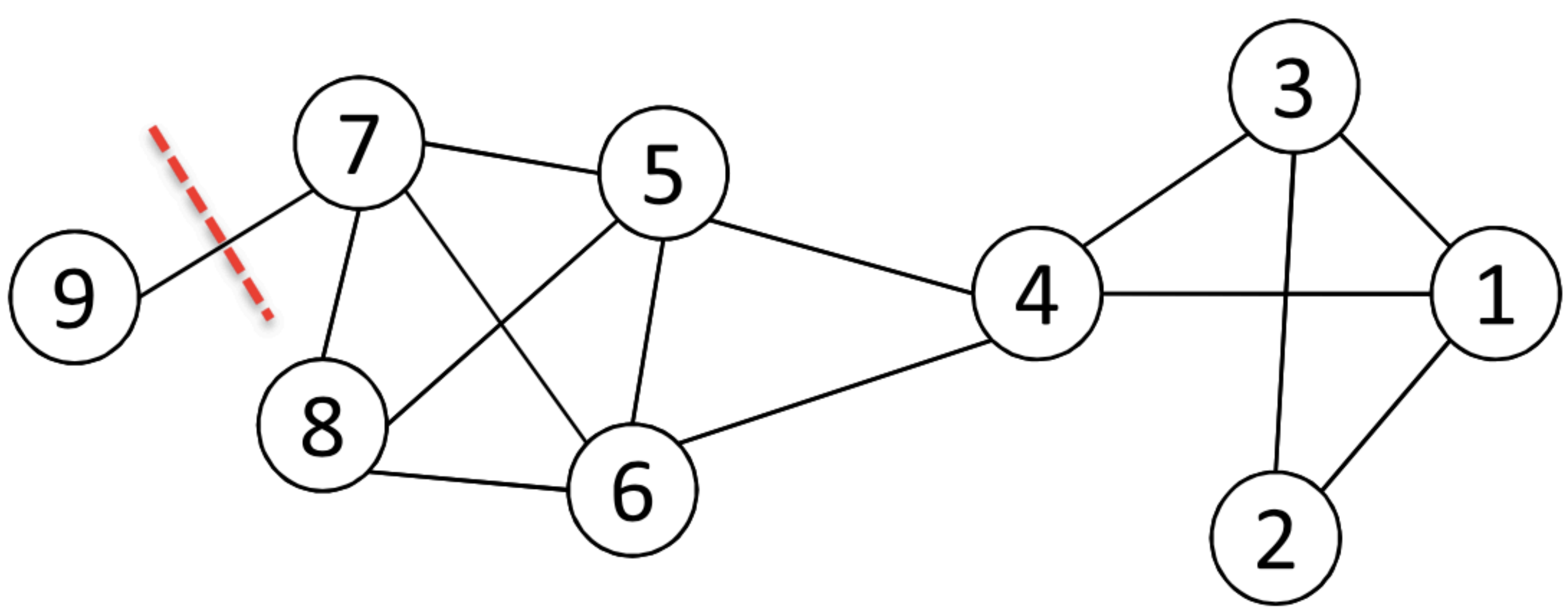
Cut

- Most interactions are within group whereas interactions between groups are few

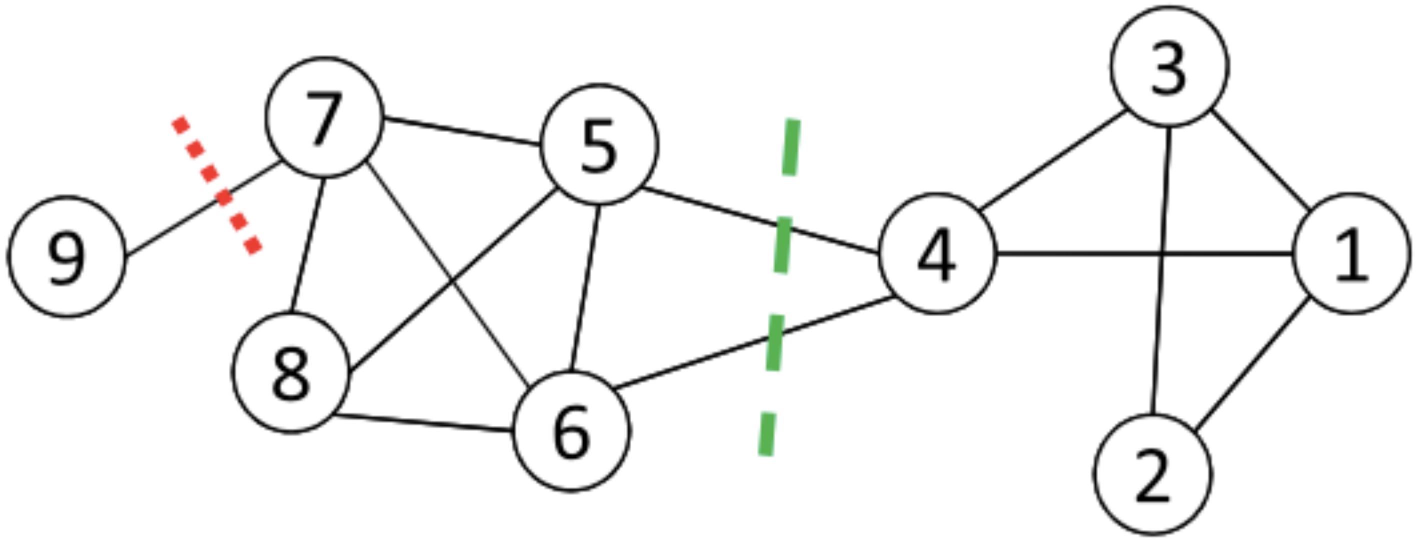
- **Community detection: minimum cut problem**

- **Cut:** A partition of ver-ces of a graph into two disjoint sets

- **Minimum cut problem:** find a graph partition such that the number of edges between the two sets is minimized



Ratio Cut and Normalized Cut



- Minimum cut often returns an unbalanced partition, with one set being a singleton
- **Change the objective function to take the size of the communities into account**

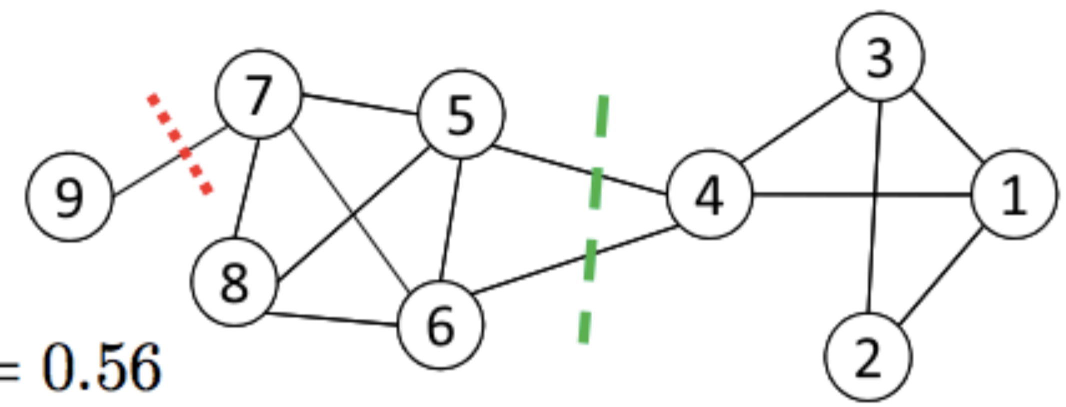
$$\text{Ratio Cut}(\pi) = \frac{1}{k}\sum_{i=1}^{k}\frac{cut(C_i,\bar{C}_i)}{|C_i|},$$

$$\text{Normalized Cut}(\pi) = \frac{1}{k}\sum_{i=1}^{k}\frac{cut(C_i,\bar{C}_i)}{vol(C_i)}$$

C_i,: a community
$|C_i|$: number of nodes in C_i
$vol(C_i)$: sum of degrees in C_i

Ratio Cut and Normalized Cut

Example



For partition in red: π_1

$$\text{Ratio Cut}(\pi_1) = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{8}\right) = 9/16 = 0.56$$

$$\text{Normalized Cut}(\pi_1) = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{27}\right) = 14/27 = 0.52$$

For partition in green: π_2

$$\text{Ratio Cut}(\pi_2) = \frac{1}{2}\left(\frac{2}{4} + \frac{2}{5}\right) = 9/20 = 0.45 < \text{Ratio Cut}(\pi_1)$$

$$\text{Normalized Cut}(\pi_2) = \frac{1}{2}\left(\frac{2}{12} + \frac{2}{16}\right) = 7/48 = 0.15 < \text{Normalized Cut}(\pi_1)$$

Both ratio cut and normalized cut prefer a balanced partition

Hierarchy-Centric Community Detection

Goal: build a hierarchical structure of communities based on network topology

Allow the analysis of a network at different resolutions

Representative approaches:

- Divisive Hierarchical Clustering
- Agglomerative Hierarchical clustering

Divisive Hierarchical Clustering

Divisive clustering

- Partition nodes into several sets
- Each set is further divided into smaller ones
- Network-centric partition can be applied for the partition

One particular example: recursively remove the "weakest" tie

- Find the edge with the least strength
- Remove the edge and update the corresponding strength of each edge

Recursively apply the above two steps until a network is discomposed into desired number of connected components.

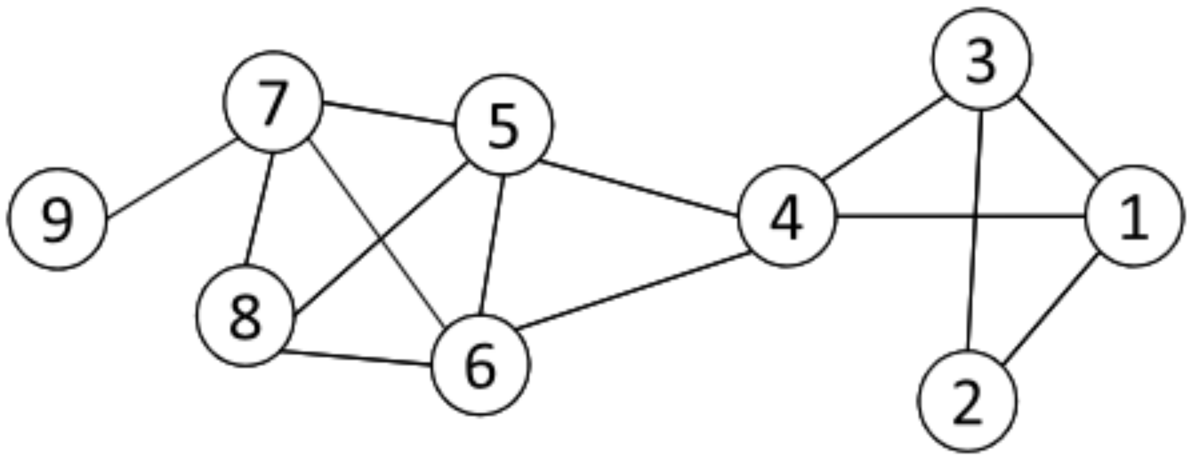
Each component forms a community

Edge Betweenness

The strength of a tie can be measured by **edge betweenness**

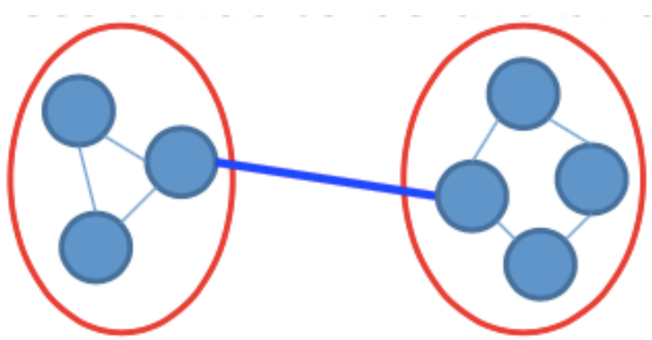
Edge betweenness: the number of shortest paths that pass along with the edge

$$\text{edge-betweenness}(e) = \Sigma_{s<t} \frac{\sigma_{st}(e)}{\sigma_{s,t}}$$

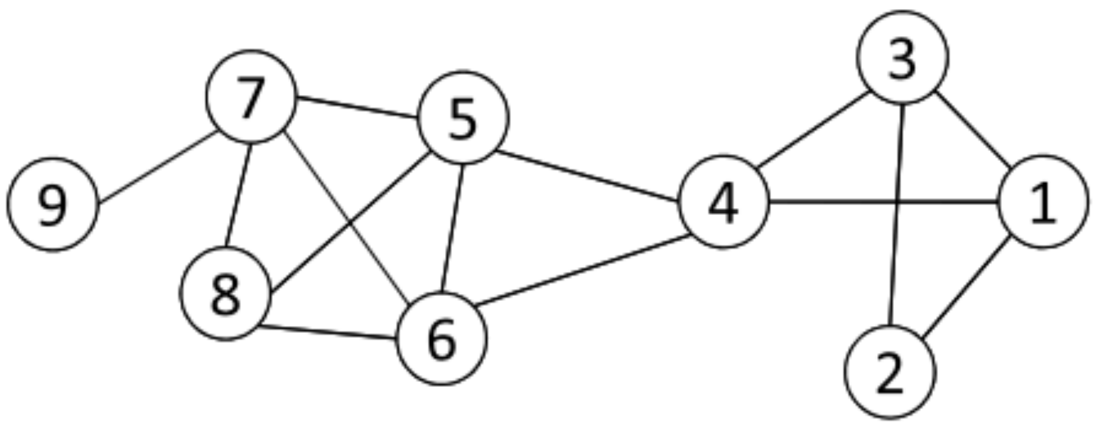


The edge betweenness of e(1, 2) is 4, as all the shortest paths from 2 to {4, 5, 6, 7, 8, 9} have to either pass e(1, 2) or e(2, 3), and e(1,2) is the shortest path between 1 and 2

The edge with the highest betweenness tends to be a bridge between 2 communities



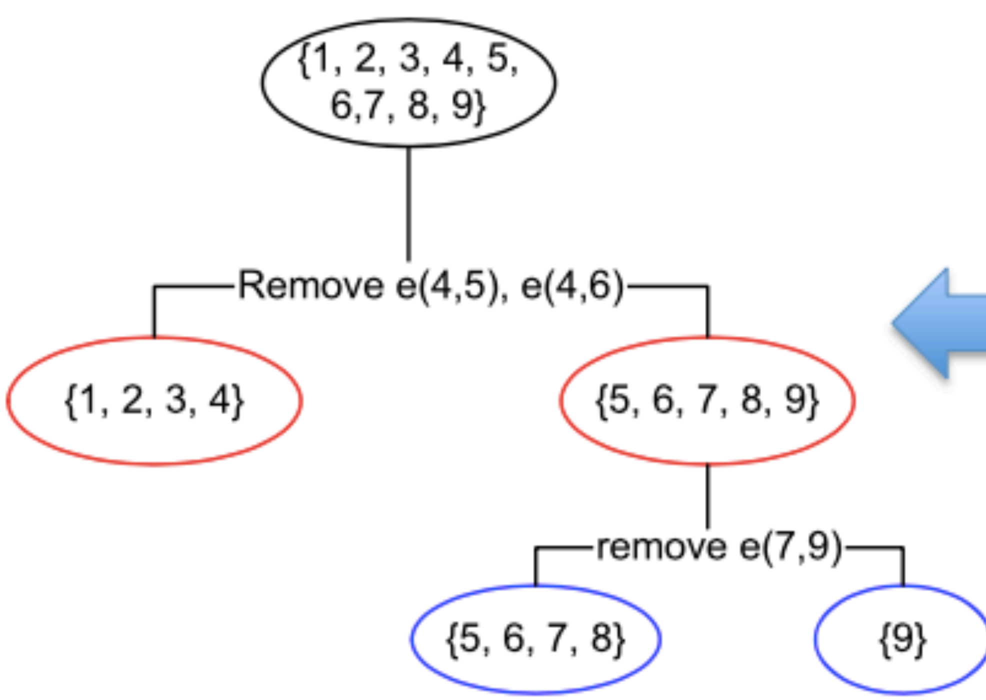
Divisive Clustering based on Edge Betweenness



Initial betweenness value

Table 3.3: Edge Betweenness

	1	2	3	4	5	6	7	8	9
1	0	4	1	9	0	0	0	0	0
2	4	0	4	0	0	0	0	0	0
3	1	4	0	9	0	0	0	0	0
4	9	0	9	0	10	10	0	0	0
5	0	0	0	10	0	1	6	3	0
6	0	0	0	10	1	0	6	3	0
7	0	0	0	0	6	6	0	2	8
8	0	0	0	0	3	3	2	0	0
9	0	0	0	0	0	0	8	0	0



After remove e(4,5), the betweenness of e(4, 6) becomes 20, which is the highest;

After remove e(4,6), the edge e(7,9) has the highest betweenness value 4, and should be removed.

Community Evaluation

Evaluating Community Detection

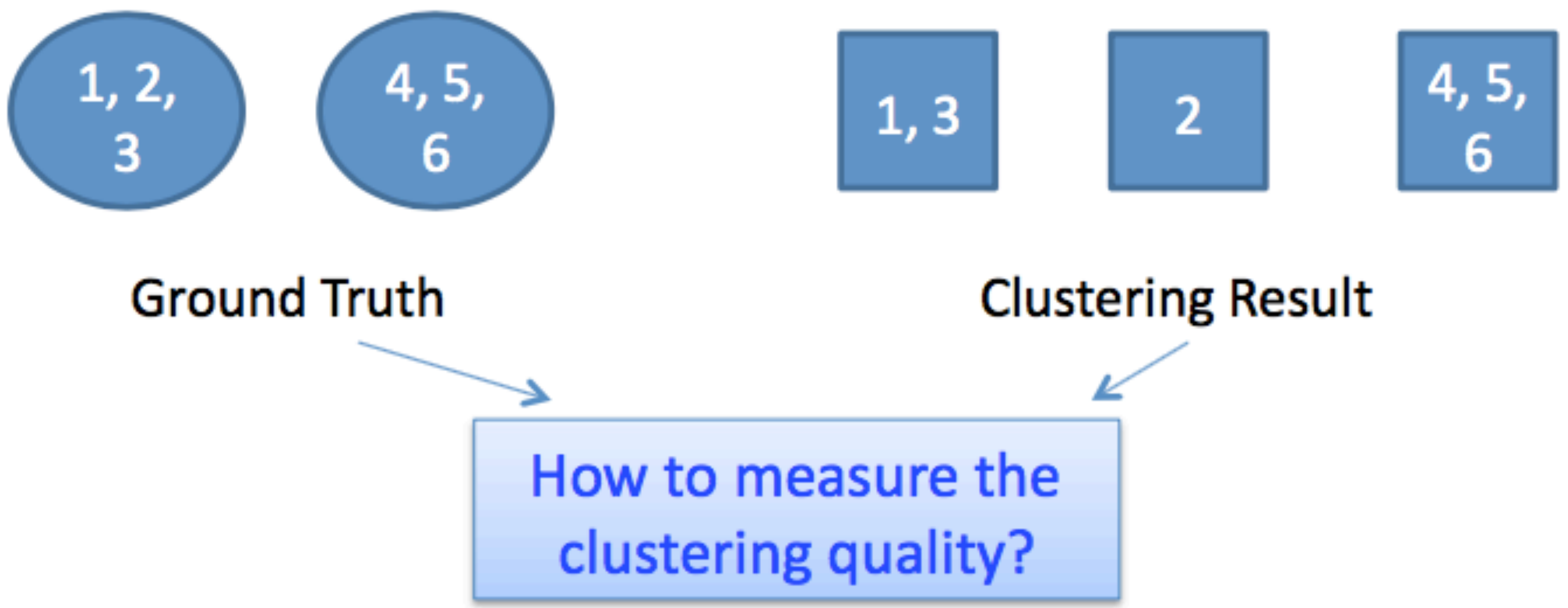
For groups with a clear and formal definition

- E.g., cliques, k-cliques, k-clubs, …
- Verify if the extracted communities satisfy the definition

For networks with ground truth information

- Normalized Mutual Information
- Accuracy of pairwise community memberships

Measuring a Clustering Result



- The number of communities after grouping can be different from the ground truth
- No clear community correspondance between clustering result and the ground truth
- Normalized Mutual Information can be used

Normalized Mutual Information

Entropy

- The information contained in a distribution

$$H(X) = \sum_{x \in X} p(x) \log p(x)$$

Mutual Information

- The shared information between two distributions

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log \left(\frac{p(x,y)}{p_1(x) p_2(y)} \right)$$

Normalized Mutual Information (between 0 and 1)

$$NMI(X;Y) = \frac{I(X;Y)}{\sqrt{H(X)H(Y)}}$$

Consider a partition as a distribution (probability of one node falling into one community), we can compute the matching between two clusterings

Normalized Mutual Information

$$H(X) = \sum_{x \in X} p(x) \log p(x)$$

$$H(\pi^a) = \sum_{h}^{k^{(a)}} \frac{n_h^a}{n} \log(\frac{n_h^a}{n})$$

$$H(\pi^b) = \sum_{\ell}^{k^{(b)}} \frac{n_\ell^b}{n} \log(\frac{n_\ell^b}{n})$$

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log \left(\frac{p(x,y)}{p_1(x) p_2(y)} \right)$$

$$I(\pi^a, \pi^b) = \sum_h \sum_\ell \frac{n_{h,\ell}}{n} \log \left(\frac{\frac{n_{h,\ell}}{n}}{\frac{n_h^a}{n} \frac{n_\ell^b}{n}} \right)$$

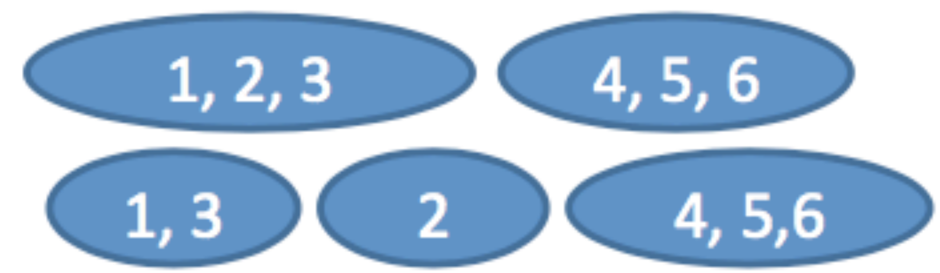
$$NMI(X;Y) = \frac{I(X;Y)}{\sqrt{H(X)H(Y)}}$$

$$NMI(\pi^a, \pi^b) = \frac{\sum_{h=1}^{k^{(a)}} \sum_{\ell=1}^{k^{(b)}} n_{h,\ell} \log \left(\frac{n \cdot n_{h,l}}{n_h^{(a)} \cdot n_\ell^{(b)}} \right)}{\sqrt{\left(\sum_{h=1}^{k^{(a)}} n_h^{(a)} \log \frac{n_h^a}{n} \right) \left(\sum_{\ell=1}^{k^{(b)}} n_\ell^{(b)} \log \frac{n_\ell^b}{n} \right)}}$$

Normalized Mutual Information

Example

- Partition a: [1, 1, 1, 2, 2, 2]
- Partition b: [1, 2, 1, 3, 3, 3]



$n = 6$

$k^{(a)} = 2$

$k^{(b)} = 3$

	n_h^a
h=1	3
h=2	3

	n_l^b
l=1	2
l=2	1
l=3	3

$n_{h,l}$	l=1	l=2	l=3
h=1	2	1	0
h=2	0	0	3

$$NMI(\pi^a, \pi^b) = \frac{\sum_{h=1}^{k^{(a)}} \sum_{\ell=1}^{k^{(b)}} n_{h,\ell} \log\left(\frac{n \cdot n_{h,l}}{n_h^{(a)} \cdot n_\ell^{(b)}}\right)}{\sqrt{\left(\sum_{h=1}^{k^{(a)}} n_h^{(a)} \log \frac{n_h^a}{n}\right)\left(\sum_{\ell=1}^{k^{(b)}} n_\ell^{(b)} \log \frac{n_\ell^b}{n}\right)}} = 0.8278$$

Evaluation with Semantics

- For networks with semantics
  - Networks come with semantic or attribute information of nodes or connections
  - Human subjects can check whether the extracted communities are coherent and homogeneous

- **Evaluation is qualitative**

- It is intuitive and helps in understanding a community

An *animal* community

A *health* community

Evaluation without Ground Truth

- For networks without ground truth or semantic information
- This is the most common situation
- A option is to resort cross-validation
  - Extract communities from a (training) network
  - Evaluate the quality of the community detection on a network constructed from a different date or based on a related type of interaction

- **Quantitative evaluation**
  - Modularity
  - Block model approximation error

Link Prediction

Outline

1. Link prediction problem

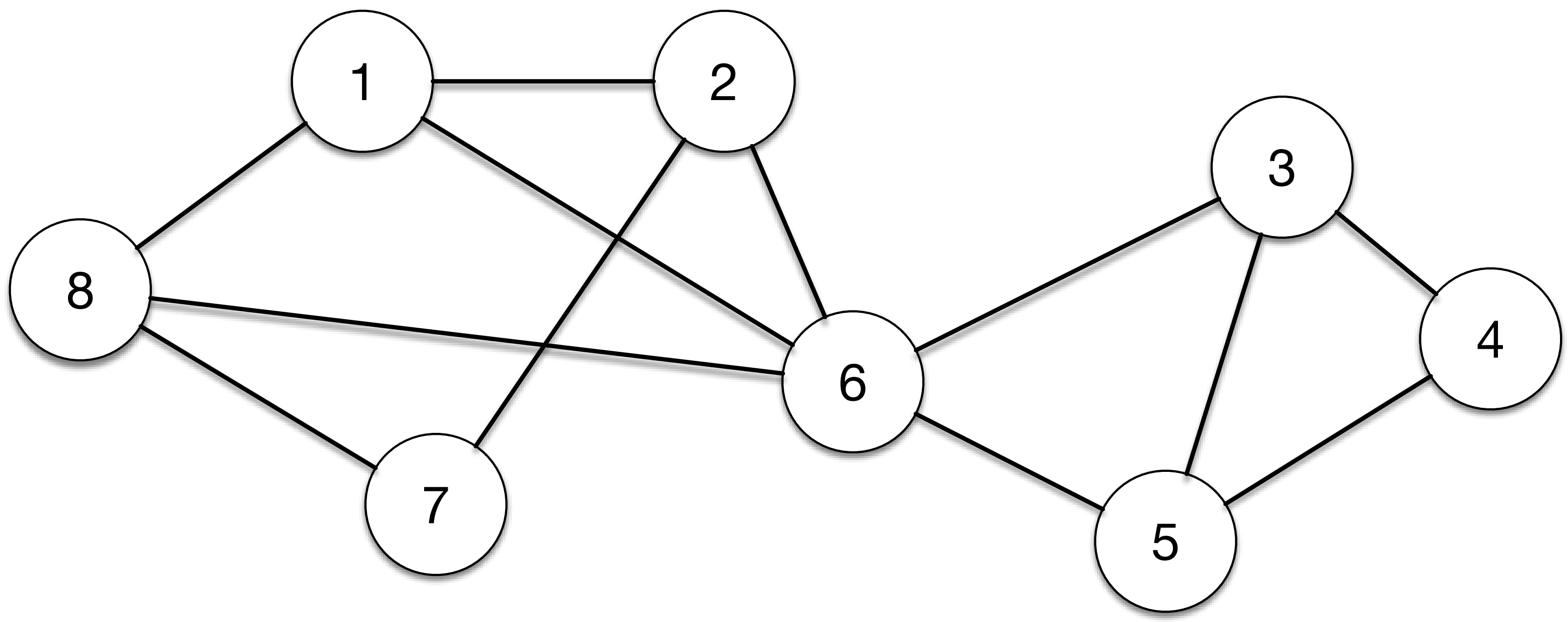
2. Proximity measures

3. Prediction by supervised learning

Link prediction

- **Link prediction.** Given a snapshot of a dynamic network at time t, predict edges added in the interval (t,t')
- **Link completion.** Given a network, infer links that are consistent with the structure, but missing
- **Link reliability.** Estimate the reliability of given links in the network
- **What to predict?**
  - Link existence
  - Link weight
  - Link type

Link prediction



- Number of missing edges = $|V| (|V| - 1)/2 - |E|$
- In sparse graphs, $|E| << |V|^2$
- Probability of correct random guess $O(1/|V|^2)$

Scoring algorithm

- Link prediction by proximity scoring
  1. For each pair of nodes compute proximity score c(v,v')
  2. Sort all pairs by the decreasing score
  3. Select top n pairs (or above some threshold) as new links

- Many metrics have been summarised in :

David Liben-Nowell and Jon Kleinberg. 2007. The link-prediction problem for social networks. *J. Am. Soc. Inf. Sci. Technol.* 58, 7 (May 2007), 1019-1031.

Scoring functions

- Based on the local neighbourhood of v_i and v_j
  - Number of common neighbours

$$|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|$$

  - Jaccard's coefficient

$$\frac{|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|}{|\mathcal{N}(v_i) \cup \mathcal{N}(v_j)|}$$

  - Adamic / Adar

$$\sum_{v \in \mathcal{N}(v_i) \cap \mathcal{N}(v_i)} \frac{1}{log|\mathcal{N}(v)|}$$

Scoring functions

- Based on paths and ensemble of paths between v_i and v_j
  - Shortest path

$$-min\{path_{ij} > 0\}$$

  - Katz score

$$\sum_{l=1}^{\infty} \beta^{(l)} |paths_{ij}^{(l)}|$$

  - Personalized (rooted) PageRank

$$PR = \alpha(D^{-1}A)^T PR + (1-\alpha)$$

Scoring functions

- Expected number of random walk steps:
  - Hitting time: $-H_{ij}$
  - Commute time: $-(H_{ij} + H_{ji})$
  - Normalized hitting / commute time: $-(H_{ij}\pi_j + H_{ji}\pi_i)$

  with π_i (resp. π_j) be the stationary probability of v_i (resp. v_j)

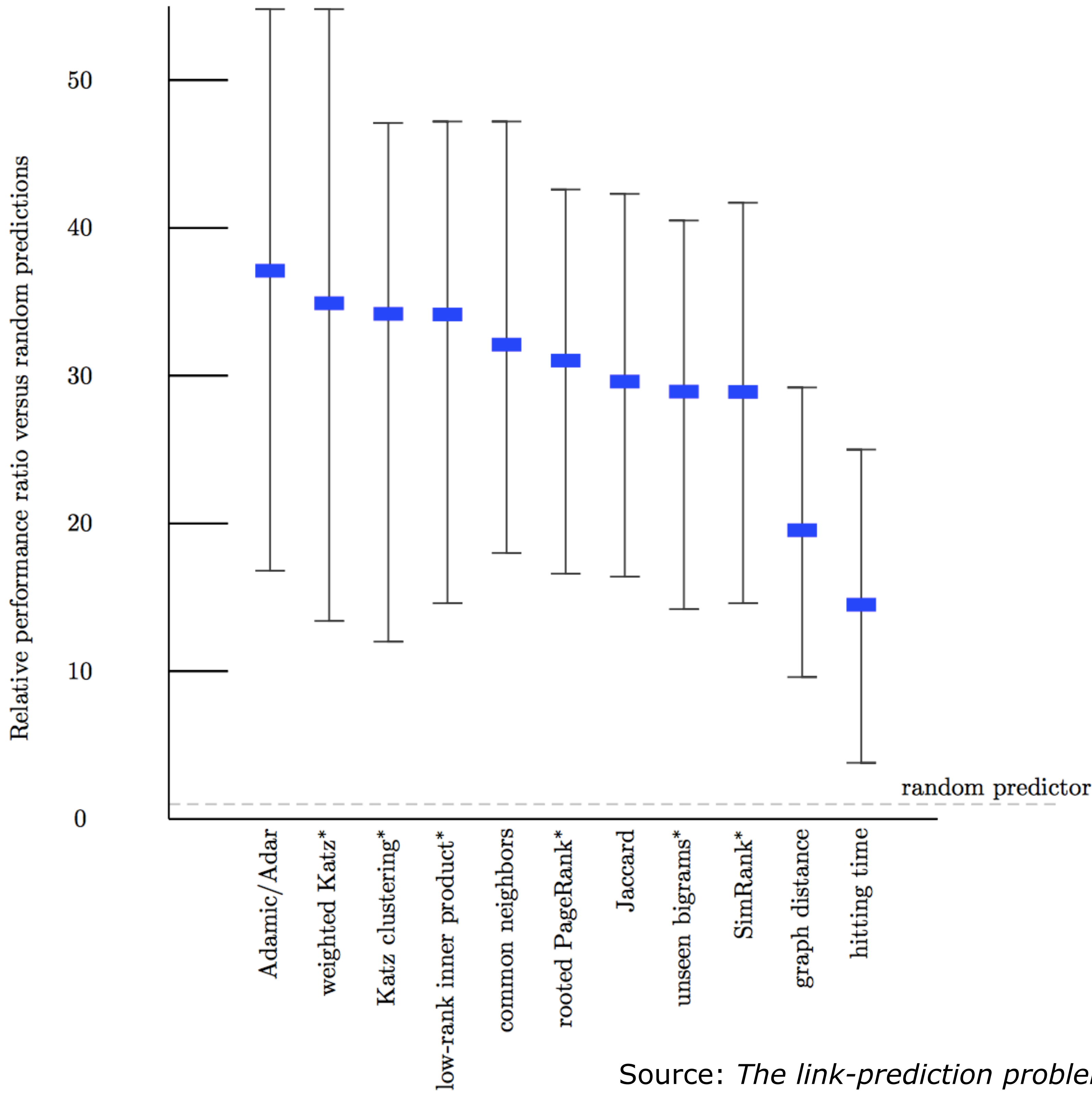
- SimRank:

$$SimRank(v_i, v_j) = \gamma . \frac{\sum_{a \in \mathcal{N}_i} \sum_{b \in \mathcal{N}_j} SimRank(a, b)}{|\mathcal{N}_i|.|\mathcal{N}_j|}$$

Scoring functions

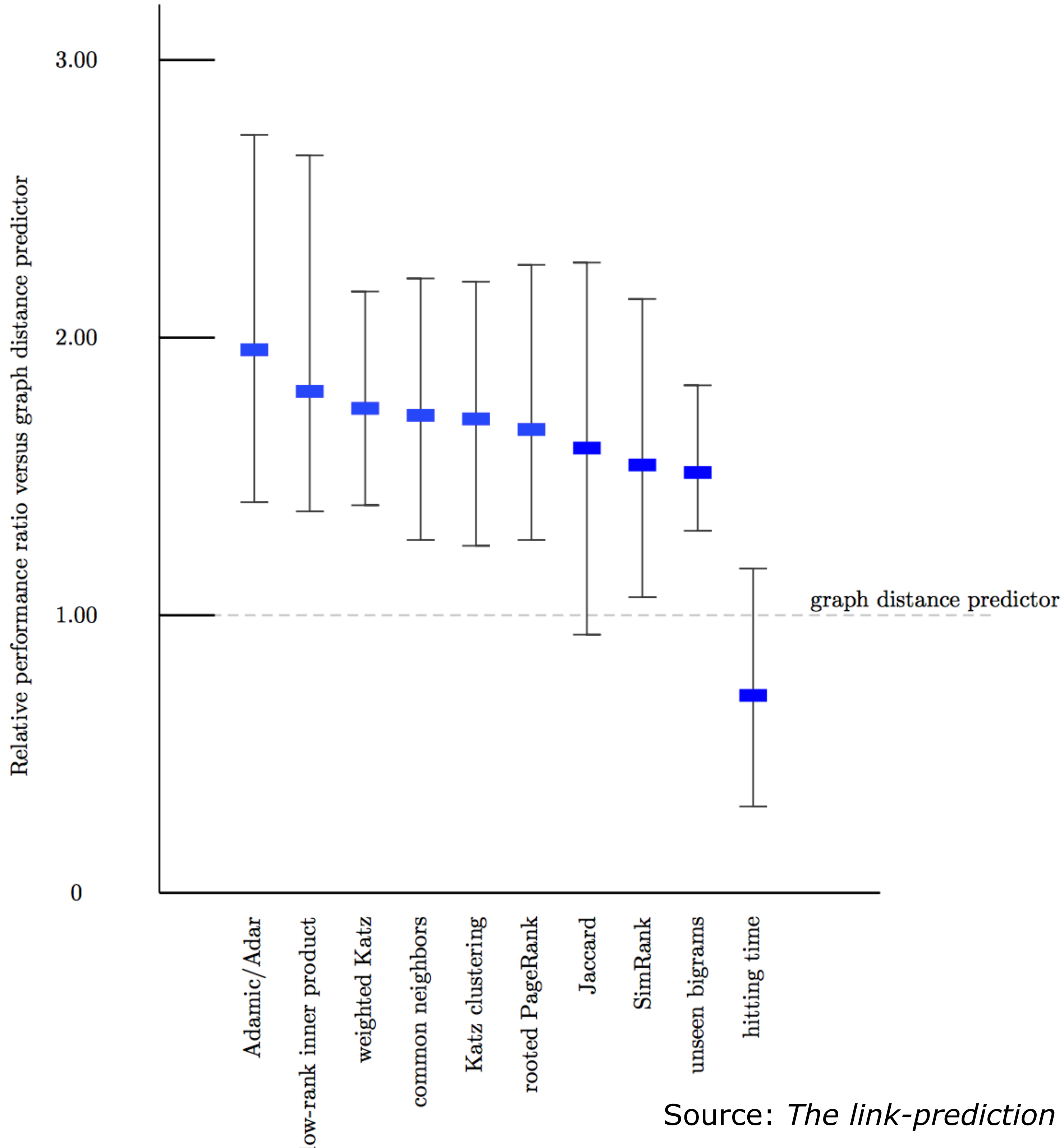
- Preferential attachment (2 alternative versions)
  - $k_i \ . \ k_j = |\mathcal{N}_i| \ . \ |\mathcal{N}_j|$
  - $k_i \ + \ k_j = |\mathcal{N}_i| \ + \ |\mathcal{N}_j|$
- Clustering coefficient
  - $CC(v_i) \ . \ CC(v_j)$
  - $CC(v_i) \ + \ CC(v_j)$

Some results



Source: *The link-prediction problem for social networks*

Some results



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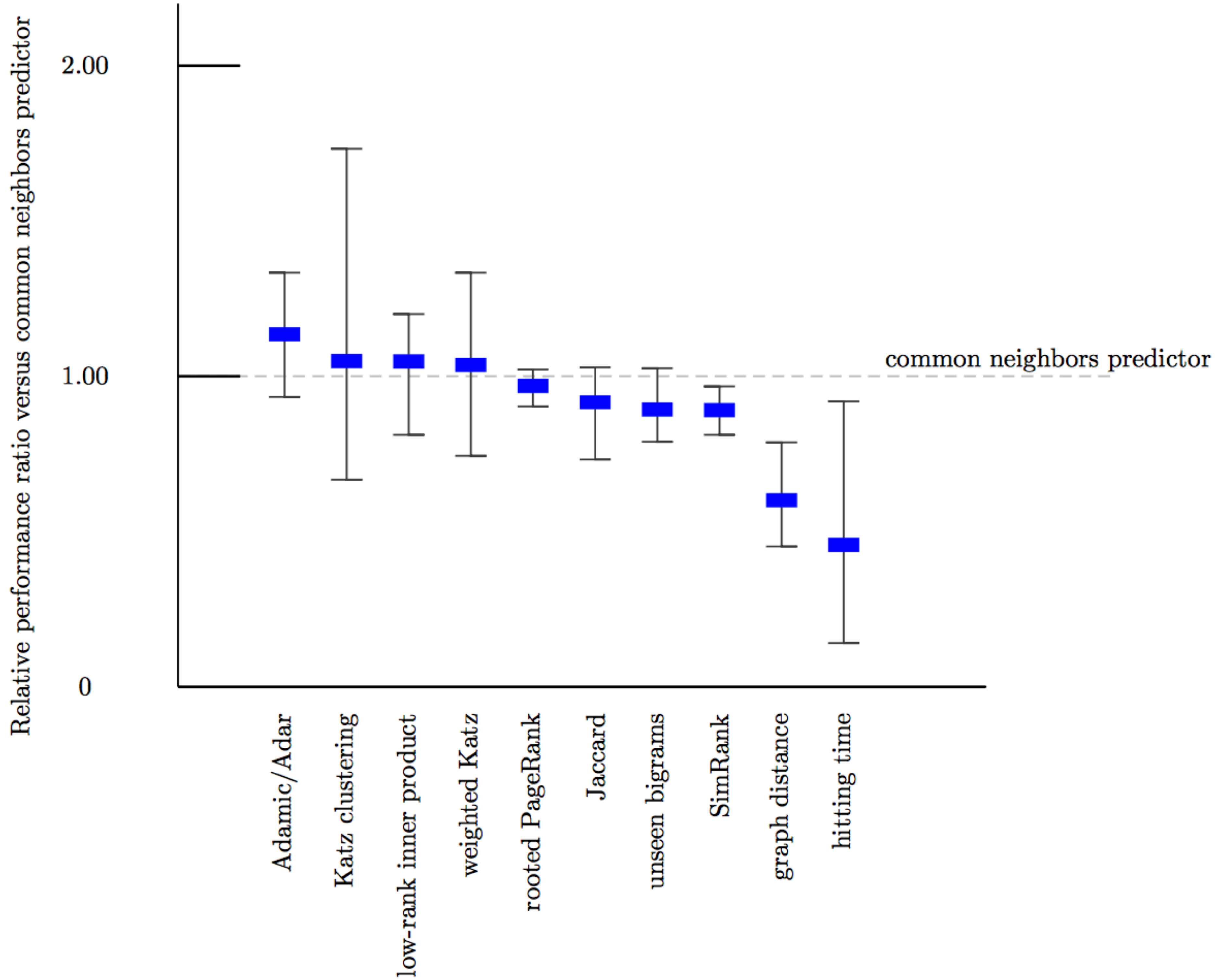
Some results



Source: *The link-prediction problem for social networks*

Take away message

- Node-based topological similarity measures (common neighbours, Jaccard, Adamic/Adar, preferential attachment) perform the best but does not scale well

- Path-based topological similarity measure (Katz, Hitting time, rooted PageRank) have to be preferred when dealing with relatively big networks (>10K vertices)

Binary classification

- A challenging classification problem:
  - A very large number of possible edges (quadratic in number of nodes)
  - Highly unbalanced class distribution
    - Positive examples : linear growth with number of nodes
    - Negative example : quadratic growth with number of nodes

A very challenging problem

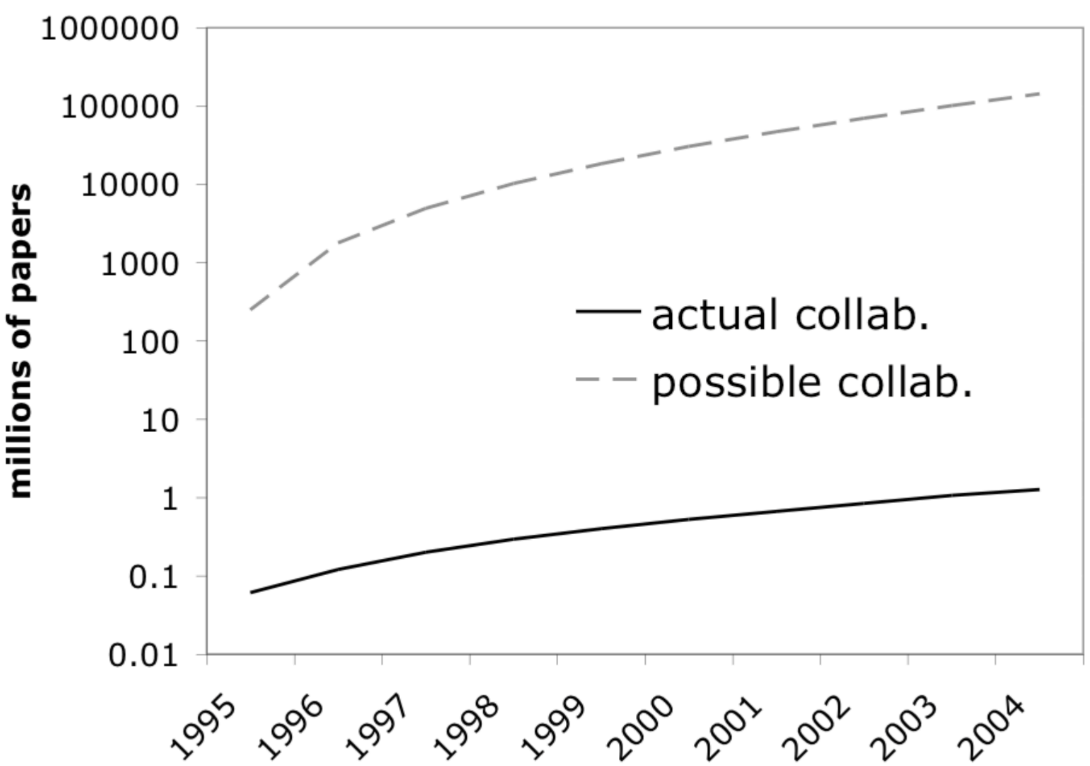


Figure 1. Logarithmic plot of actual and possible collaborations between DBLP authors, 1995-2004.

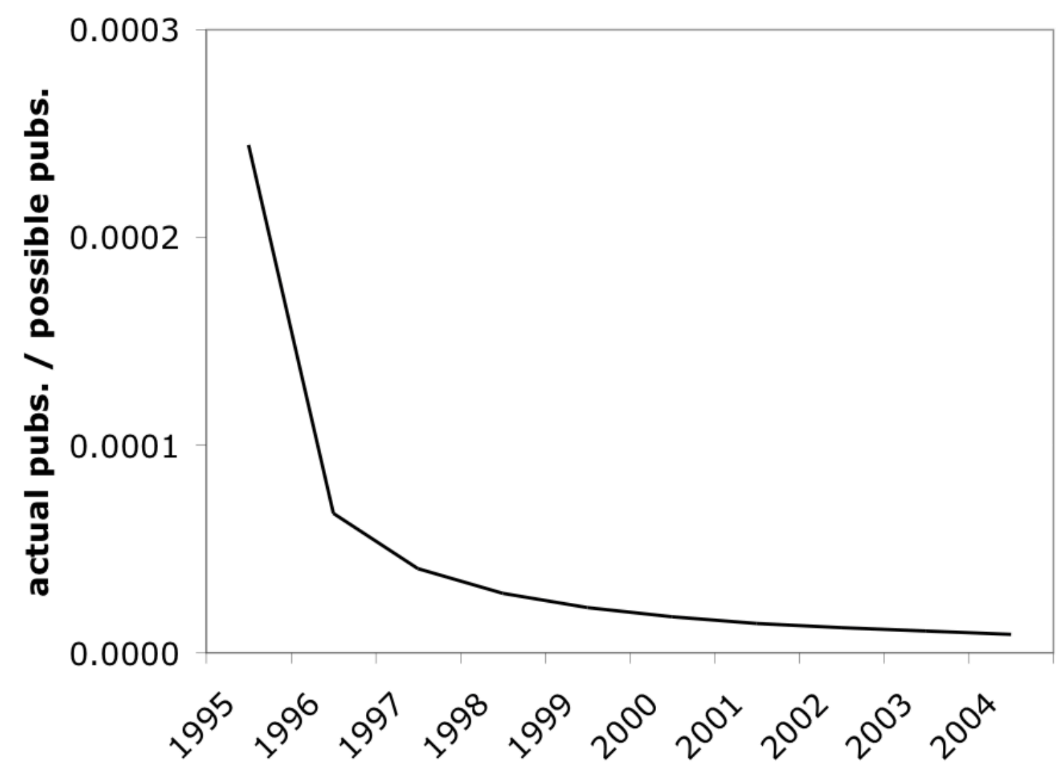


Figure 2. Publications of DBLP authors as a proportion of possible collaborations, 1995-2004.

Source: M. Rattigan, D. Jensen. The case for anomalous link discovery. ACM SIGKDD Explorations Newsletter. v 7, n 2, pp 41-47, 2005

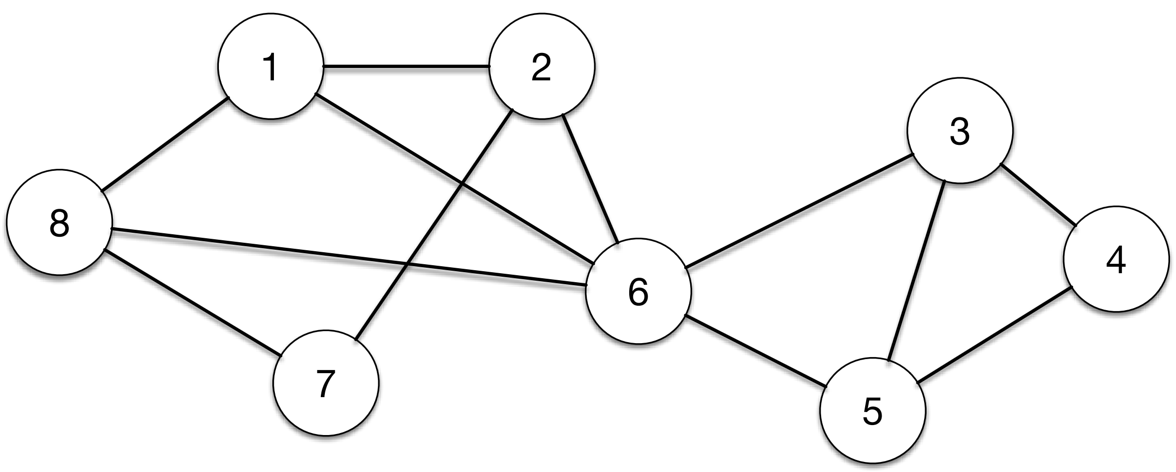
Link prediction by supervised learning

- Supervised learning process
  1. Feature generation
  2. Model training
  3. Testing

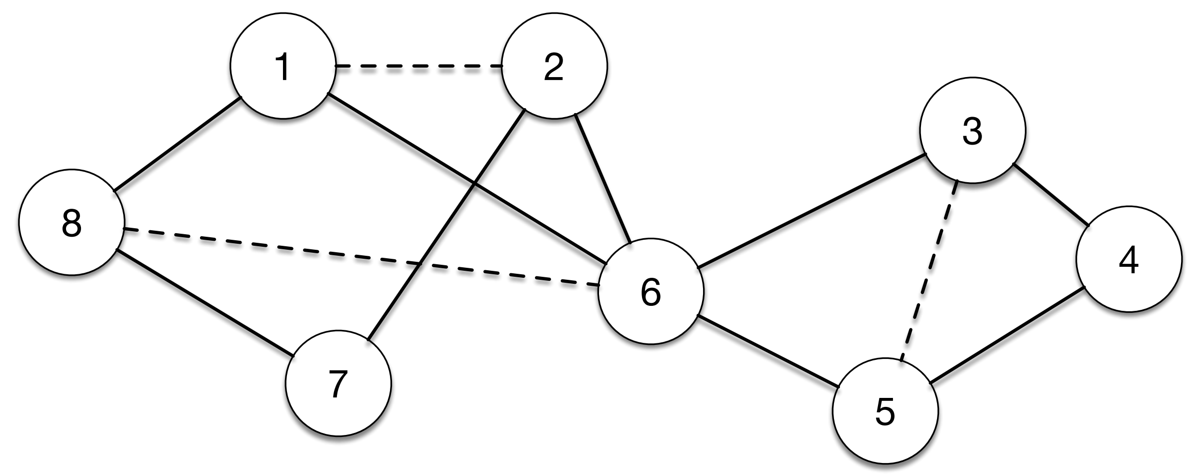
- Features
  - Topological proximity features
  - Aggregated features
  - Content based node proximity features

Evaluation

- Simple « hold out set » evaluation



Whole graph



Training graph

- More sophisticated evaluation method is preferable (cross-validation)

Evaluation metrics

- Precision, recall, F-measure

$$Precision = \frac{TP}{TP + FP}, \;\; Recall = \frac{TP}{TP + FN}$$

$$F = \frac{2\,.\,Precision\,.\,Recall}{Precision + Recall}$$

- True rate positive (TPR), False positive rate (FPR), ROC curve, AUC

$$TPR = \frac{TP}{TP + FN}, \;\; FPR = \frac{FP}{FP + TN}$$