Distribution-Free Checkpoint Placement Algorithms Based on Min-Max Principle

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Abstract—In this paper, we consider two kinds of sequential checkpoint placement problems with infinite/finite time horizon. For these problems, we apply approximation methods based on the variational principle and develop computation algorithms to derive the optimal checkpoint sequence approximately. Next, we focus on the situation where the knowledge on system failure is incomplete, i.e., the system failure time distribution is unknown. We develop the so-called min-max checkpoint placement methods to determine the optimal checkpoint sequence under an uncertain circumstance in terms of the system failure time distribution. In numerical examples, we investigate quantitatively the proposed distribution-free checkpoint placement methods, and refer to their potential applicability in practice.

Index Terms—Checkpoint/restart, fault-tolerance, high availability, modeling and prediction, performance evaluation, maintenance, incomplete failure information.

1 Introduction

It is well-known that the system failure in large scaled computer systems can lead to a huge economic or critical social loss. Checkpointing and rollback recovery is a commonly used technique for improving the reliability/availability of fault-tolerant computing systems, and is regarded as a low-cost dependability technique from the standpoint of environment diversity. Especially when the file system to write and/or read data is designed in terms of preventive maintenance, checkpoint generations periodically back up the significant data on a primary medium to a safe secondary medium and play a significant role in limiting the amount of data processing for recovery actions after system failures occur. If checkpoints are frequently taken, a larger overhead will be incurred. Conversely, if only a few checkpoints are taken, a larger overhead after system failures will be required in rollback recovery actions. Hence, it is important to determine the optimal checkpoint sequence taking account of the trade-off between two kinds of overhead factors above. In many cases, the system failure phenomenon is described with a probability distribution called the system failure time distribution, and the optimal checkpoint sequence is determined based on any stochastic model [1].

Young [2] obtains the optimal checkpoint interval approximately for a computation restart after system failures. Baccelli [3], Chandy [4], Chandy et al. [5], Dohi et al. [6], Gelenbe and Derochette [7], Gelenbe [8], Gelenbe and Hernandez [9], Goes and Sumita [10], Grassi et al. [11], Kulkarni et al. [12], Nicola and Van Spanje [13], and Sumita et al. [14] propose performance evaluation models for database recovery, and calculate the optimal checkpoint intervals which maximize the system availability or minimize the mean overhead during the normal operation. L'Ecuyer and Malenfant [15] formulate a dynamic checkpoint placement problem by a Markov decision process. Ziv and Bruck [16] reconsider a checkpoint placement problem under a random environment, by taking account of the change of operation circumstance. Vaidya [17] examines the impact of checkpoint latency on overhead ratio for a simple checkpoint model. Recently, Okamura et al. [18] reformulate the Vaidya model [17] with a semi-Markov decision process.

In almost all checkpoint placement models for transaction-based systems [3], [8], [9], [10], [14], it is proven that the constant checkpoint intervals maximizing the system availability are always better than the randomized checkpoint ones which are given by independent and identically distributed random variables. Because the constant checkpoint intervals cannot always be validated in many cases, some authors discuss sequential checkpoint placement problems. Since the way to place the optimal checkpoint sequence depends on the kind of objective functions (system availability, mean overhead, etc.) and the failure time distribution, in fact, the sequential checkpoint can provide a general framework on the checkpoint placement. Toueg and Babaoglu [19] develop a dynamic programming algorithm which minimizes expected execution time of tasks placing checkpoints between two consecutive tasks under very general assumptions. Recently, Ling et al. [20] propose an approximate method, called *the variational calculus approach*, to calculate the cost-optimal checkpoint sequence. This method was originally developed by Fukumoto et al. [21], [22] for seeking the nearly optimal checkpoint sequence in a database recovery. In the sequential checkpoint placement problem, it is assumed

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that the system failure time obeys an arbitrary probability distribution, i.e., it does not always follow the exponential distribution, and that the checkpoint time intervals are not constant. Actually, it is reported that some system failures in operating systems and middleware are caused by the software aging [23] and that the system failure time can have the increasing failure rate property.

From the viewpoints of practical file management, however, it is quite hard to estimate the system failure time distribution, because the system failure is a rare event and the corresponding time data are not available, especially, in the initial operational phase. Even if we experience a few system failures during the operation, is it really possible to observe a sufficient number of data to estimate the probability distribution function with higher accuracy and select the best candidate with satisfactory significance level from several theoretical distributions through the goodness-of-fit test? In general, the checkpoint decision is made under the incomplete failure information, where the incomplete information implies that the type of system failure time distribution is not specified but only its moment information is available. Nevertheless, in the past literature the arbitrary but completely known probability distribution has been assumed to obtain robust and general results on checkpoint placement. We often encounter such an operational environment for real-life system applications like database systems, with incomplete failure information. As an extreme but ad hoc example, the reader can image that almost all computer users seldom specify the system failure time distribution in their computing circumstance with the aim of checkpointing.

In this paper, we consider two kinds of sequential checkpoint placement problems with infinite/finite time horizons. For these problems, we apply the approximation methods based on the variational principle and develop the computation algorithms to derive the optimal checkpoint sequence approximately. Fukumoto et al. [21], [22] and Ling et al. [20] use the variational calculus approach to only the sequential checkpoint placement problem with infinite-time horizon, provided that the system failure time distribution is known. We generalize their results mathematically and propose a checkpoint placement algorithm for a finite time horizon problem. Next, we focus on the situation where the knowledge on system failure is incomplete, i.e., the system failure time distribution is unknown. Dohi et al. [24] develop an optimal checkpoint model with media failures and propose statistical estimation algorithms of the optimal checkpoint interval, based on the total time on test concept, in the situation where the media failure time distribution is unknown, but the corresponding complete data are available. Okamura et al. [18] propose an online adaptive checkpoint algorithm based on the reinforcement learning called the Q-learning (see, e.g., [25]), and revisit the Vaidya's model [17]. Since this algorithm is a statistical nonparametric algorithm, one does not need to specify the system failure time distribution in advance. The main advantages of the adaptive checkpoint algorithm based on the Q-learning are that the implement of the algorithm is quite easy on computers and that the asymptotic convergence to the real optimal policy can be guaranteed. However, since the convergence speed for the Q-learning is rather slow, i.e., a number of data are needed in estimation, it is difficult to apply this algorithm to real-time applications for practical use.

In order to overcome the difficulty on the online checkpoint generation under the incomplete failure information, we develop an alternative distribution-free approach based on the min-max principle to determine the optimal checkpoint sequence under the uncertain circumstance in terms of the system failure time distribution. Barzilovich et al. [26] and Derman [27] provide the theoretical framework of min-max surveillance schedules for hardware products. In the min-max principle, one does place the nearly optimal checkpoint with incomplete knowledge on system failure time distribution. More specifically, the min-max policy leads to the cost-optimal checkpoint sequence minimizing the expected operating cost under the most pessimistic situation, i.e., the system failure tends to occur most frequently. For two kinds of sequential checkpoint placement problems with infinite/finite time horizons, we develop the min-max sequential checkpoint placement algorithms. In numerical examples, we investigate quantitatively the proposed distribution-free checkpoint placement methods, and refer to their potential applicability in practice.

2 Sequential Checkpoint Placement

2.1 Infinite-Time Horizon Problem

Consider a simple file system with sequential checkpointing over an infinite time horizon. The system operation starts at time $t = 0$, and the checkpoint (CP) is sequentially placed at time $\{t_1, t_2, \cdots, t_n, \cdots\}$. At each CP, t_j $(j = 1, 2, \cdots)$, all the file data on the main memory is saved to a safe secondary medium like CD-Rom, where the cost (time overhead) c_0 (> 0) is needed per each CP placement. System failure occurs according to an absolutely continuous and nondecreasing probability distribution function $F(t)$ having density function $f(t)$ and finite mean $1/\mu$ (> 0). Upon a system failure, a rollback recovery takes place immediately where the file data saved at the last CP creation is recovered. Next, a checkpoint restart is performed and the file data is recovered to the state just before the system failure point. The time length for the checkpoint restart is given by the function $L(\cdot)$, which depends on the system failure time and is assumed to be differentiable and increasing. Without any loss of generality, it is assumed that no failure occurs during the recovery period with probability one.

Then, the problem is to derive the optimal CP sequence $\mathbf{t}_\infty = \{t_1, t_2, t_3 \cdots\}$ minimizing the expected operating cost function:

$$\min_{\mathbf{t}_\infty} : C(\mathbf{t}_\infty) = \sum_{n=0}^{\infty} \int_{t_n}^{t_{n+1}} \Big[c_0(n+1) + L(t - t_n)\Big] dF(t), \quad (1)$$

where $t_0 = 0$. In the above formulation, it is seen that an additional checkpointing is carried out just after completing the recovery operation and that the total CP cost becomes $c_0(n + 1)$. From the analogy to the inspection problems for hardware systems (see, e.g., [28]), it can be easily found that the optimal CP sequence $\mathbf{t}_\infty$ is a nonincreasing sequence, i.e.,

$t_1 \geq t_2 - t_1 \geq t_3 - t_2 \geq \cdots$ if the system failure time distribution $F(t)$ is PF_2 (Pólya frequency function of order 2) [28]:

$$\begin{vmatrix} f(u_1 - v_1) & f(u_1 - v_2) \\ f(u_2 - v_1) & f(u_2 - v_2) \end{vmatrix} \geq 0 \qquad (2)$$

for arbitrary $u_1 < u_2$ and $v_1 < v_2$. If $F(t)$ is PF_2, then it has to be IFR (increasing failure rate), i.e., the failure rate $r(t) = f(t)/\overline{F}(t)$ is increasing in t, where in general $\overline{\psi}(\cdot) = 1 - \psi(\cdot)$. With no loss of generality, it is assumed that the system failure time distribution belongs to the class of PF_2, because the expected operating cost diverges for an increasing CP sequence.

Following [4], [6], [7], [8], [9], [10], [14], [20], [21], [22], [24], [29], suppose that the recovery function $L(t)$ is the affine form of the system failure time t; $L(t) = a_0 t + b_0$, where $a_0\ (> 0)$ and $b_0\ (> 0)$ are constants. The first term $a_0 t$ denotes the time necessary to reexecute the lost file data in time interval $[0, t)$ since the last CP, and the second term is a fixed time associated with the CP restart. It is well known that the optimal CP interval is constant, i.e., $t_1 = t_2 - t_1 = \cdots = t_{n+1} - t_n = \cdots$, if $F(t)$ is the exponential distribution with mean $1/\mu$. Under the assumptions that $a_0 = 1$ and $b_0 = 0$, Young [2] considers a checkpoint restart model with constant CP interval under the exponential assumption, and derives the following nonlinear equation which satisfies the optimal CP interval, t_∞^{exp}:

$$e^{c_0\mu} - t_\infty^{exp}\mu - e^{-t_\infty^{exp}\mu} = 0. \qquad (3)$$

Based on the second order approximation

$$\exp(-t\mu) \approx 1 - \mu t + \mu^2 t^2/2,$$

he obtains an approximate form of the optimal CP interval:

$$t_\infty^{exp} \approx \sqrt{2(e^{c_0\mu} - 1)}/\mu \approx \sqrt{2c_0/\mu}, \qquad (4)$$

which is due to $\exp(c_0\mu) \approx 1 + c_0\mu$.

For the general system failure time distribution, the first order condition of optimality for the minimization problem in (1) is given by

$$t_n - t_{n-1} = \frac{F(t_{n+1}) - F(t_n)}{f(t_n)} + \frac{c_0}{a_0}, \quad n = 1, 2, 3, \cdots. \qquad (5)$$

From the condition of optimality, an algorithm to derive the optimal CP sequence $\mathbf{t}_\infty^* = \{t_1^*, t_2^*, \cdots\}$ which minimizes $C(\mathbf{t}_\infty)$ can be derived, where the bisection method is used for adjustment of t_1. We call this algorithm *Algorithm 0* in this paper. As seen intuitively, this algorithm strongly depends on the initial value t_1, and is unstable to determine the optimal CP sequence with higher accuracy. More effectively, the quasi-Newton method can be applied to calculate numerically the optimal CP sequence $\mathbf{t}_\infty^* = \{t_1^*, t_2^*, \cdots\}$. Nevertheless, since the problem on the adjustment of initial parameter t_1 remains, it is not so easy to determine automatically and quickly the optimal CP sequence in real-time systems.

Algorithm 0:

Step 1: Set z_l and z_u as lower and upper bounds for the initial value t_1, e.g., $z_l := 0$ and $z_u := F^{-1}(0.99)$.

Step 2: Let $t_1 := (z_l + z_u)/2$.

Step 3: Compute the CP sequence $\{t_2, t_3, \cdots, t_{M+1}\}$ using

$$t_{n+1} := F^{-1}\left(F(t_n) + (t_n - t_{n-1})f(t_n) - \frac{c_0}{a_0}\right), \quad n = 1, \cdots, M,$$

where M is a sufficient large integer value.

Step 4: For $j = 1, 2, \cdots, M$,

Step 4.1: if $t_{j+1} - t_j > t_j - t_{j-1}$ then $z_u := t_1$ and **Go** to **Step 2**.

Step 4.2: if $t_{j+1} - t_j < 0$ then $z_l := t_1$ and **Go** to **Step 2**.

Step 5: **Stop** the procedure.

2.2 Finite-Time Horizon Problem

Next, consider the finite-time horizon problem which is a natural extension of the infinite-time horizon problem. Suppose that the time horizon of operation for the file system is finite, say $T\ (> 0)$. For a finite sequence $\mathbf{t}_N = \{t_1, t_2, \cdots, t_N\}$, the expected operating cost is formulated as

$$TC(\mathbf{t}_N) = \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} \Big[c_0(n+1) + L(t - t_n)\Big] dF(t), \qquad (6)$$

where $N = \min\{n : t_{n+1} > T\}$. To simplify the notation, we define $t_{N+1} = T$ in this paper. Since the finite-time horizon problem involves constraints on the number of CPs, it is impossible to apply directly *Algorithm 0* mentioned before. Formally, differentiating (6) with respect to $t_n\ (n = 1, 2, \cdots, N)$ and setting it equal to 0 yields

$$t_n - t_{n-1} = \frac{F(t_{n+1}) - F(t_n)}{f(t_n)} + \frac{c_0}{a_0}, \quad n = 1, 2, \cdots, N, \qquad (7)$$

where $t_n - t_{n-1} > 0\ (n = 1, 2, 3, \cdots, N+1)$. We develop an exact computation algorithm of the optimal CP sequence for an arbitrary number of CPs, N, over a finite-time horizon as follows:

Algorithm 1:

Step 1: Set $z_l := 0$ and $z_u := T$ as lower and upper bounds for the initial value t_1.

Step 2: Let $t_1 := (z_l + z_u)/2$.

Step 3: Compute the CP sequence $\{t_2, t_3, \cdots, t_{N+1}\}$ using

$$t_{n+1} := F^{-1}\left(F(t_n) + (t_n - t_{n-1})f(t_n) - \frac{c_0}{a_0}\right), \quad n = 1, \cdots, N.$$

Step 4: For $j = 1, \cdots, N$,

Step 4.1: if $t_{j+1} - t_j > t_j - t_{j-1}$ then $z_u := t_1$ and **Go** to **Step 2**.

Step 4.2: if $t_{j+1} - t_j < 0$ then $z_l := t_1$ and **Go** to **Step 2**.

Step 5: If $t_{N+1} - T < -\epsilon$ then $z_l := t_1$ and **Go** to **Step 2**, where ϵ is a sufficient small value.

Step 6: If $t_{N+1} - T > \epsilon$ then $z_u := t_1$ and **Go** to **Step 2**.

Step 7: **Stop** the procedure.

For all possible combinations of N, we can choose the optimal number of CPs, N^*, minimizing expected operating cost and obtain the resulting CP sequence

$\mathbf{t}_N^* = \{t_1^*, t_2^*, \cdots, t_N^*\}$. It should be noted that the optimal CP sequence $\mathbf{t}_N^*$ may not be constant even if the system failure time is the exponential. This is because the first order condition of optimality in (7) is violated at the boundary T. In other words, our CP model with the optimal CP sequence $\mathbf{t}_N^{exp} = \{t_1^{exp}, t_2^{exp}, \cdots, t_N^{exp}\}$ under the exponential assumption is an extension of the classical Young model [2], since $t_N^{exp} \to t_\infty^{exp}$ as $T \to \infty$. This is an alternative motivation to consider the finite-time horizon problem. In the following section, we develop approximate methods based on the variational calculus for both finite and infinite-time horizon problems, and investigate those properties.

3 Approximate Algorithms

3.1 Infinite-Time Horizon Problem

Following Fukumoto et al. [21], [22] and Lin et al. [20], we derive the approximate form of the expected operating cost in (1). Let $D(t)$ be an absolutely continuous function of time t. We approximate the number of CPs placed per unit time by the continuous function $D(t)$, where $1/D(t)$ means the mean time interval between successive CPs. We call $D(t)$ *the checkpoint density* in this paper. The expected cost associated with the CP placement and the mean recovery cost from the system failure are approximately given by

$$\begin{aligned} c_0 \int_0^\infty \int_0^t D(y) dy dF(t) &= c_0 \int_0^\infty \int_0^t D(y) f(t) dy dt \\ &= c_0 \int_0^\infty \int_y^\infty D(y) f(t) dt dy \\ &= c_0 \int_0^\infty D(t) \overline{F}(t) dt \end{aligned} \tag{8}$$

and

$$D(t) \int_0^{D(t)^{-1}} L(x) dx \approx L(\{2D(t)\}^{-1}), \tag{9}$$

respectively, where $D(t)^{-1} = 1/D(t)$. Then, the expected operating cost over an infinite time horizon can be approximated as

$$\begin{aligned} C(\mathbf{t}_\infty) &\approx C(D(t), F(t)) \\ &= \int_0^\infty \Big[\int_0^t c_0 D(x) dx + L(\{2D(t)\}^{-1}) \Big] dF(t) \\ &= c_0 \int_0^\infty D(t) \overline{F}(t) dt + \int_0^\infty L(\{2D(t)\}^{-1}) dF(t). \end{aligned} \tag{10}$$

Hence, the problem is reduced to a variational problem (10) to derive the optimal $D(t)$ minimizing $C(D(t),\ F(t))$ for a given system failure time distribution $F(t)$.

Proposition 1. *For the variational problem*

$$\min_{D(t)} C(D(t), F(t)),$$

the corresponding Euler equation of this problem is given by

$$c_0 \overline{F}(t) - \frac{1}{2D(t)^2} L'(\{2D(t)\}^{-1}) f(t) = 0, \tag{11}$$

where $L'(t) = dL(t)/dt$. Then, the optimal CP density minimizing the expected operating cost over an infinite time horizon is given by

$$D^*(t) = \sqrt{\frac{L'(\{2D(t)\}^{-1}) f(t)}{2c_0 \overline{F}(t)}} \tag{12}$$

with

$$\begin{aligned} C(D^*(t), F(t)) = \sqrt{c_0} \int_0^\infty \sqrt{\frac{L'(\{2D(t)\}^{-1}) f(t) \overline{F}(t)}{2}} dt \\ + \int_0^\infty L\Big(\sqrt{\frac{c_0 \overline{F}(t)}{2L'(\{2D(t)\}^{-1})) f(t)}} \Big) dF(t). \end{aligned} \tag{13}$$

For the proof, see [20], [21], [22]. From Proposition 1, the optimal CP sequence can be calculated by $\mathbf{t}_\infty^* = \{t_1^*, t_2^*, \cdots\}$ so as to satisfy

$$n = \int_0^{t_n} D^*(t) dt, \quad n = 1, 2, \cdots. \tag{14}$$

As a special case, when $L(t) = a_0 t + b_0$, the optimal CP density $D^*(t)$ is given by $D^*(t) = \sqrt{a_0 r(t)/2c_0}$. If $F(t)$ is the exponential distribution with mean $1/\mu\ (>0)$, then we obtain

$$1 = \int_0^{t_1} \sqrt{\frac{a_0 \mu}{2c_0}} dt = \int_{t_1}^{t_2} \sqrt{\frac{a_0 \mu}{2c_0}} dt = \cdots \tag{15}$$

and the constant CP policy, $t_j = j\sqrt{2c_0/a_0\mu}\ (j = 1, 2, \cdots)$, is optimal. This can be reduced to the Young model [2] when $a_0 = 1$.

3.2 Finite-Time Horizon Problem

Next, let us consider an approximate method for the finite-time horizon problem. Define

$$X(t) = \int_0^t D(x) dx, \quad t \geq 0. \tag{16}$$

For the finite-time horizon problem in (6), the expected operating cost can be approximated as

$$\begin{aligned} TC(\mathbf{t}_N) &\approx TC(X(t), F(t)) \\ &= \int_0^T \Big[c_0 X(t) + L(\{2X'(t)\}^{-1}) \Big] dF(t), \end{aligned} \tag{17}$$

where $X(0) = 0$ and $X(T) = N + 1$.

Theorem 1. *The optimal CP density minimizing the expected operating cost over a finite time horizon is given by*

$$D^*(t) = \sqrt{\frac{L'(\{2D(t)\}^{-1}) f(t)}{2c_0(\beta - F(t))}}, \tag{18}$$

where β is a constant and is determined so as to satisfy $X(T) = N + 1$.

Proof. The Euler equation for the variational problem $\min_{X(t)} TC(X(t), F(t))$ is given by

$$c_0 f(t) + \frac{d}{dt}\left[\frac{L'(\{2D(t)\}^{-1})f(t)}{2D(t)^2}\right] = 0. \tag{19}$$

From (19), it is immediate to obtain

$$\frac{L'(\{2D(t)\}^{-1})f(t)}{D(t)^2} = 2c_0[\beta - F(t)], \quad 0 \le t \le T, \tag{20}$$

where β is an arbitrary integral constant. Since $F(t)$ is nondecreasing in t, it is evident that $f(t) \ge 0$ iff $\beta \ge F(T)$ for an increasing $L(\cdot)$. Since the optimal $X^*(t)$ minimizing $TC(X(t), F(t))$ satisfies $X^*(T) = N + 1$, solving (20) yields (18) for $\beta > F(T)$. The proof is completed. □

In Theorem 1, for an arbitrary N, we seek β so as to satisfy $N + 1 = \int_0^T D^*(x)dx$ (see (18)). For all possible combinations of N, we calculate all βs satisfying $\beta > F(T)$, the optimal number of CPs, N^*, and the corresponding optimal CP density, $D^*(t)$. Since β monotonically increases as N decreases, the search space on N can be rather limited. If $L(t) = a_0 t + b_0$, then the optimal CP density is given by $D^*(t) = \sqrt{a_0 f(t)/2c_0(\beta - F(t))}$. It can be easily seen that the above CP density approaches to the infinite case as $T \to \infty$ because of $\beta \to 1$. In other words, even when the system failure time is the exponential, the optimal CP density is the function of t and the optimal CP time with relatively small T is not constant. This fact has not been known in the past literature [2], [4], [5], [19], [20], [21], [22]. In fact, it can be checked numerically that the optimal solution of the finite-time horizon problem in (7), $\mathbf{t}_N^*$, is not a constant sequence for the exponential failure case.

In this section, we considered two CP placement problems with infinite/finite time horizons, when the system failure time distribution is completely known. If the system failure time distribution $F(t)$ can be specified in advance, then the optimal CP sequence and the nearly optimal CP sequence based on the approximate methods can be obtained numerically. However, if the information on the failure time distribution is incomplete, the computation algorithm as well as the variational approach to seek the approximate CP sequence cannot be applied for the practical use. In the following section, we develop the min-max CP placement methods in the case where the system failure time distribution is unknown.

4 Min-Max Checkpoint Policies

4.1 Infinite-Time Horizon Problem

Next, we consider an estimation problem of the optimal CP sequence $\mathbf{t}_\infty^*$ which minimizes the expected operating cost. Suppose that the system failure time $F(t)$ is unknown in the operational phase of the file system. Under such an incomplete information on the system failure time distribution, the most pessimistic approach is to derive the optimal CP sequence under the circumstance where the system failures may occur most frequently. In other words, it will be appropriate to derive the CP sequence satisfying

$$\max_{F(t)} \min_{D(t)} C(D(t), F(t)). \tag{21}$$

In this paper, we call the above CP sequence the min-max CP sequence.

Lemma 1. *The Euler equation for the maximization problem* $\max_{F(t)} C(D^*(t), F(t))$ *is given by*

$$\begin{aligned}
&\frac{1}{4}\sqrt{c_0\mathcal{L}(t)r(t)} - \frac{f'(t)}{4}\sqrt{\frac{c_0\mathcal{L}(t)}{r(t)f(t)^2}} \\
&- \frac{\mathcal{L}'(t)}{8D(t)^2}\sqrt{\frac{c_0}{\mathcal{L}(t)r(t)}} + L'\left(\sqrt{\frac{c_0}{\mathcal{L}(t)r(t)}}\right) \\
&\times\left[-\frac{\mathcal{L}'(t)D'(t)}{8D(t)^2}\sqrt{\frac{c_0}{L'(\{2D(t)\}^{-3})r(t)}} + \frac{1}{4}\sqrt{\frac{c_0 r(t)}{\mathcal{L}'(t)}}\right. \\
&\left.- \frac{1}{4}\sqrt{\frac{c_0}{\mathcal{L}'(t)r(t)f(t)^2}}\cdot f'(t)\right] \\
&+ L''\left(\sqrt{\frac{c_0}{\mathcal{L}'(t)r(t)}}\right) \\
&\times\left[-\frac{\mathcal{L}'(t)\{D(t)\}^{-2}D'(t)}{8}\sqrt{\frac{c_0}{L'(\{2D(t)\}^{-3})r(t)}}\right. \\
&\left.- \frac{1}{2}\sqrt{\frac{c_0 r(t)}{\mathcal{L}'(t)}} - \frac{f'(t)}{2}\sqrt{\frac{c_0}{\mathcal{L}(t)r(t)f(t)^2}}\right] = 0,
\end{aligned} \tag{22}$$

where

$$\mathcal{L}(t) = L'(\{2D(t)\}^{-1}).$$

Proof. The Euler equation with respect to $F(t)$ in (13) is given by

$$\begin{aligned}
&-\frac{1}{2}\sqrt{\frac{c_0 L'(\{2D(t)\}^{-1})f(t)}{\overline{F}(t)}} \\
&-\frac{1}{2}L'\left(\sqrt{\frac{c_0\overline{F}(t)}{L'(\{2D(t)\}^{-1})f(t)}}\right) \\
&\times\frac{1}{2}\sqrt{\frac{c_0 L'(\{2D(t)\}^{-1})f(t)}{\overline{F}(t)}} \\
&-\frac{d}{dx}\left[\frac{1}{2}\sqrt{\frac{c_0 L'(\{2D(t)\}^{-1})\overline{F}(t)}{f(t)}}\right. \\
&+L\left(\sqrt{\frac{c_0\overline{F}(t)}{L'(\{2D(t)\}^{-1})f(t)}}\right) \\
&-\frac{1}{2}L'\left(\sqrt{\frac{c_0\overline{F}(t)}{L'(\{2D(t)\}^{-1})f(t)}}\right) \\
&\left.\times\sqrt{\frac{c_0\overline{F}(t)}{L'(\{2D(t)\}^{-1})f(t)}}\right] = 0.
\end{aligned} \tag{24}$$

This will lead to (22) directly. □

Lemma 2. *Suppose that the recovery function is given by the affine form* $L(t) = a_0 t + b_0$. *Then, the expected operating cost over an infinite time horizon is simplified as*

$$C(D^*(t), F(t)) = \sqrt{2c_0 a_0}\int_0^\infty \sqrt{f(t)\overline{F}(t)}dt. \tag{25}$$

Proof. Substituting $L(t) = a_0t + b_0$ to (13), we have

$$C(D^*(t), F(t)) = \sqrt{c_0}\int_0^\infty \sqrt{\frac{a_0 f(t)\overline{F}(t)}{2}}dt + \int_0^\infty a_0\sqrt{\frac{c_0\overline{F}(t)}{2a_0 f(t)}}dF(t). \quad (26)$$

Then, (25) can be derived. □

Theorem 2. *Suppose that the recovery function is given by the affine form $L(t) = a_0t + b_0$. Then, the min-max CP policy is given by $\mathbf{t}_\infty^{**} = \{t_1^{**}, t_2^{**}, \cdots, \}$, where*

$$n = \int_0^{t_n^{**}} D^{**}(t)dt, \; n = 1, 2, \cdots, \quad (27)$$

$$D^{**}(t) = \frac{1}{2}\sqrt{\frac{a_0\lambda}{c_0(1-\lambda t)}}, \; 0 \le t \le \frac{1}{\lambda}, \quad (28)$$

and $\lambda\ (> 0)$ is a positive constant. The corresponding expected operating cost over an infinite time horizon is given in the following:

$$\min_{D(t)}\max_{F(t)} C(D(t), F(t)) = \sqrt{c_0 a/\lambda}. \quad (29)$$

Proof. From the assumption, the Euler equation for the variational problem $\max_{F(t)} C(D^*(t),\ F(t))$ is given by

$$\frac{d}{dt}\left[\sqrt{\frac{\overline{F}(t)}{f(t)}}\right] + \sqrt{\frac{f(t)}{\overline{F}(t)}} = 0 \quad (30)$$

and its solution has to satisfy the following differential equation:

$$\overline{F}(t)F''(t) = \{F'(t)\}^2. \quad (31)$$

Solving the differential equation above yields (29) and the proof is completed. □

Theorem 3. *The min-max checkpoint policy is symmetric, i.e., the order of* $\min - \max$ *operations in (21) is exchangeable:*

$$\max_{F(t)}\min_{D(t)} C(D(t), F(t)) = \min_{D(t)}\max_{F(t)} C(D(t), F(t)). \quad (32)$$

Then, the resulting policy is a saddle point in the min-max game given by (32), if it exists.

Proof. The Euler equation for the variational problem $\max_{F(t)} C(D(t), F(t))$ is given by

$$c_0 D(t) + \frac{d}{dt}\left[L(\{2D(t)\}^{-1})\right] = 0, \quad (33)$$

and is calculated as

$$c_0 D(t) - L'(\{2D(t)\}^{-1})\frac{1}{2D(t)^2}\frac{d}{dx}D(t) = 0. \quad (34)$$

A few algebraic manipulations yield

$$\int \frac{dD(t)}{D(t)^3} = \frac{2c_0}{L'(\{2D(t)\}^{-1})}\int dt + \lambda_1, \quad (35)$$

where λ_1 is an integral constant. By solving (35), it follows that

$$\frac{1}{D(t)^2} = \frac{-4c_0 t - 2L'(\{2D(t)\}^{-1})\lambda_1}{L'(\{2D(t)\}^{-1})}. \quad (36)$$

Hence, the min-max CP density $\hat{D}^{**}(t)$ can be derived in the following form:

$$\hat{D}^{**}(t) = \frac{1}{2}\sqrt{\frac{L'(\{2D(t)\}^{-1})\lambda}{c_i(1-\lambda t)}}, \quad 0 \le t \le \frac{1}{\lambda}, \quad (37)$$

where $\lambda = 2c_0/L'(\{2D(t)\}^{-1})\lambda_1$. Then, the expected operating cost is given by

$$\min_{D(t)}\max_{F(t)} C(D(t), F(t)) = \sqrt{c_0 L'(\{2D(t)\}^{-1})/\lambda}. \quad (38)$$

When $L(t) = a_0t + b_0$ is assumed, (38) is reduced to (29). □

Remark 1. The solution of the Euler equation for the variational problem $\max_{F(t)} C(D^*(t), F(t))$ is given by $F^*(t) = 1 - \sqrt{1-\lambda t}$. Hence, when an estimate of the mean time to failure (MTTF), $1/\hat{\mu}$, is given, the constant λ is represented by $\lambda = 2\hat{\mu}/3$. Even if the arithmetic mean of system failure time data is not available in the earlier phase of the system operation, the parameter λ can be subjectively estimated from the MTTF, i.e., $\int_0^{1/\lambda}\overline{F^*}(t)dt$.

4.2 Finite-Time Horizon Problem

Next, consider the min-max CP sequence corresponding to $\mathbf{t}_N^*$. For the finite-time horizon problem, unfortunately, we cannot apply the similar technique to the infinite case, because the second variational problem on $F(t)$, $\max_{F(t)} TC(D^*(t), F(t))$, cannot be solved in the closed form. More precisely, the Euler equation for $\max_{F(t)} TC(D^*(t), F(t))$ is, after a few algebraic manipulations, given by

$$\frac{(1+\beta-2F(t))f'(t)}{f^2(t)} + \frac{\overline{F}(t)}{\beta - F(t)} - 3 = 0. \quad (39)$$

Since this is a nonlinear differential equation, the analytical treatment may be impossible except for the case of $\beta = 1$. Here, we derive the min-max CP sequence from the different point of view.

In (6), letting $g_n(x, y) = c_0(n+1) + L(y-x)$, the expected operating cost over the finite time horizon $(0, T]$ is approximately represented by

$$TC(\mathbf{t}_N, F) = \sum_{n=0}^{N}\int_{t_n}^{t_{n+1}} g_n(t_n, t)dF(t). \quad (40)$$

Lemma 3. *Let $TC(\mathbf{t}_N) = \max_F TC(\mathbf{t}_N, F)$ be the maximum expected operating cost over a finite-time horizon with respect to $F(t)$. Then,*

$$TC(\mathbf{t}_N) = \max_{n=0,1,\cdots,N} g_n(t_n, t_{n+1}). \quad (41)$$

Proof. Let k be the maximizer of $g_n(t_n, t_{n+1}),\ n = 1, 2, \cdots, N$, i.e., $g_k(t_k, t_{k+1}) = \max_n g_n(t_n, t_{n+1})$. From the intuitive argument, the system failure can occur during the period between $[t_k, t_{k+1}]$ with the highest probability so as to maximize $TC(\mathbf{t}_N)$. Hence, it follows that

$$\max_F TC(\mathbf{t}_N, F) = \max_{n=0,1,\cdots,N} g_n(t_n, t_{n+1}). \quad (42)$$

□

From this result, the problem to derive the min-max CP sequence is reduced to $TC(\mathbf{t}_{N^*}) = \min_N TC(\mathbf{t}_N)$ in (41).

Lemma 4. *There exists a unique min-max solution t_N* satisfying*

$$g_0(0, t_1) = g_1(t_1, t_2) = \cdots = g_N^*(t_{N^*}, T). \quad (43)$$

Proof. Let N^* be the optimal CP number minimizing $TC(\mathbf{t}_N)$, which satisfies $\sum_{n=0}^{N} L^{-1}(c_0 n) < T$, where $L^{-1}(x)$ is the inverse function of $L(x)$. Then, it is evident that the expected operating cost is $c_0(N^*+1) + L(T)$ and is independent of the CP sequence if N^* is given. Since both the expected operating cost and the number of CPs are fixed, the minimization of the maximized $g_n(t_n, t_{n+1})$ $(n = 0, 1, \cdots, N)$ leads to (43). □

Finally, we obtain the following min-max CP policy over a finite time horizon without specifying the failure time distribution.

Theorem 4. *Suppose that $L(t) = a_0 t + b_0$. Then, the min-max CP sequence for the finite time horizon problem is given by*

$$t_n^{**} = n\Big[\frac{T}{N^*+1} + \frac{c_0}{2a_0}(N^* - n + 1)\Big], \quad (44)$$

where N^* is the maximum number of CPs minimizing the resulting expected operating cost and satisfies

$$N^*(N^*+1) < \frac{2a_0 T}{c_0}. \quad (45)$$

Proof. From Lemma 4 and the definition of $g_n(x, y)$, we obtain

$$a_0(t_{n+1} - t_n) = a_0(t_1) - nc_0, \; n = 0, 1, \cdots, N. \quad (46)$$

It is straightforward to see that

$$\sum_{i=0}^{n} a_0(t_{i+1} - t_i) = a_0 t_n = n a_0 t_1 - \frac{c_0 n(n-1)}{2} \quad (47)$$

for $n = 0, 1, \cdots, N$. By substituting $t_1 = a_0 T/(a_0(N+1)) + c_0 N/(2a_0)$ for $n = N$ into (42), we have (44). Also, noting that $L(t_n) > nc_0$ and $\sum_{n=0}^{N+1} t_n = T$ for N, the optimal N^* has to be the maximum integer satisfying $\sum_{n=0}^{N^*} L^{-1}(c_0 n) < T$. The proof is completed. □

Theorem 5. *The min-max checkpoint policy is symmetric, i.e.,*

$$\max_{F(t)} \min_N TC(\mathbf{t}_N, F(t)) = \min_N \max_{F(t)} TC(\mathbf{t}_N, F(t)). \quad (48)$$

Proof. Let $TC(\mathbf{t}_N) = \min_F TC(\mathbf{t}_N, F)$ be the minimum expected operating cost over a finite-time horizon with respect to $F(t)$. Then, we have

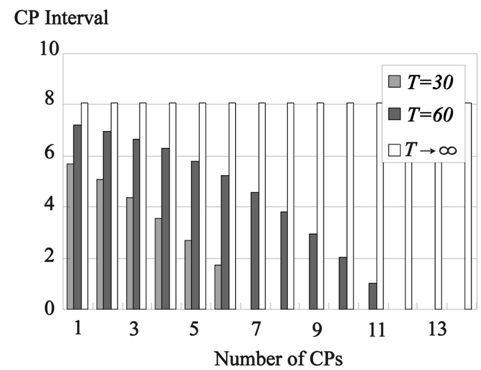

Fig. 1. Optimal CP sequence for the exponential failure case: $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, $1/\mu = 30$.

$$TC(\mathbf{t}_N) = \min_{n=0,1,\cdots,N} g_n(t_n, t_{n+1}). \quad (49)$$

From the above equation, the problem to derive the max-min CP sequence is reduced to $TC(\mathbf{t}_{N^*}) = \max_N TC(\mathbf{t}_N)$. Then, there exists a unique max-min solution $\mathbf{t}_{N^*}$ satisfying (43). The proof is completed. □

In this section, we developed two min-max CP policies under the incomplete failure information. It should be noted that both CP sequences are distribution free, i.e., they take account of the most pessimistic failure circumstance in terms of the min-max game. Our next interest is the investigation of how well the min-max CP policies will function when compared with the real optimal CP sequence. In the following section, we calculate the optimal CP sequence numerically and investigate the approximate performance of the proposed methods.

5 Numerical Examples

First, consider the case where the system failure time is the exponential. Fig. 1 shows the comparison of the optimal CP sequences with a different time horizon, where the each CP time is the real optimal solution for the problem in (1) or (6) and is calculated by carefully adjusting the initial values. From this result, it is observed that the optimal CP sequence with the finite-time horizon is monotonically decreasing and approaches to the infinite case as T becomes larger. From these observations, the well-known Young CP policy may be regarded as an upper bound of the optimal CP interval with an arbitrary time horizon, and provides rather optimistic CP interval. The expected loss of operating costs (the difference between the expected operating costs with constant and nonconstant CP sequences) for $T = 30$ and $T = 60$ are $3.67809 - 3.34918 = 0.32951$ and $5.96441 - 5.8119 = 0.15251$, respectively, in this example.

Next, we compare the approximate CP sequences with the real optimal ones. For the infinite-time horizon problem, we call the approximate methods based on the variational calculus and the min-max method *Algorithm A* and *Algorithm B*, respectively. On the other hand, we call the approximate methods based on the variational calculus and the min-max method for the finite-time horizon problem *Algorithm A′* and *Algorithm B′*, respectively. In Figs. 2, 3, and 4, we calculate the relative errors between the real optimal CP sequence and the approximate one for the infinite and

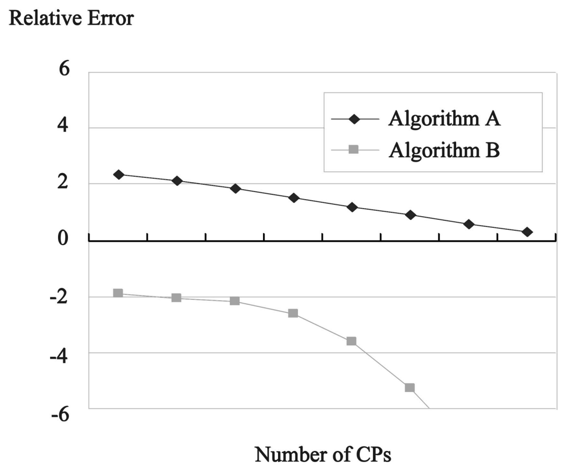

Fig. 2. Comparison of optimal CP sequence with infinite-time horizon: $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, and $\eta = 30$.

finite-time horizon problems, where the system failure time distribution obeys the Weibull distribution:

$$F(t) = 1 - e^{-(t/\eta)^m} \tag{50}$$

with shape parameter m (≥ 1) and scale parameter η (> 0). The relative error in Figs. 2, 3, and 4 is calculated as the difference between the approximate CP time and the real one, that is,

$$\text{Relative Error } (\%) = \frac{|\text{ approximate} - \text{CP interval }|}{|\text{ real CP interval }|} \times 100. \tag{51}$$

From these figures, the relative error of approximate CP sequences based on the variational calculus decreases for both cases as the number of CPs increases. This property may be attractive because the Weibull distribution mentioned above has IFR property, i.e., the system failure tends to occur frequently as the operation time goes on.

On the other hand, it should be noted in Fig. 2 that the min-max method for the infinite-time horizon case tends to underestimate the CP sequence as the number of CPs increases. This is because the degree of uncertainty increases as the operation time elapses. The most interesting result in Fig. 3 is that the min-max CP sequence with finite-time horizon can give almost the same performance as the variational method in spite of the incomplete failure information. Since the time horizon is assumed to be the same value as the MTTF in this example, two algorithms provide the closed CP intervals at the end of operation period. In Fig. 4 with longer operation time period ($T = 60$), *Algorithm A′* shows the similar tendency to Fig. 3, but *Algorithm B′* overestimates the CP interval in earlier phase and makes it smaller as the operating time goes on. The main reason of this result is that the min-max CP sequence does not always have monotone property with respect to the relative error from the real optimal solution. That is, since the relative error behaves with both the decreasing and increasing trends if the operation period is longer, it is not easy to know the timing when the min-max CP method should be applied to estimate the nearly optimal CP sequence with high accuracy. This will be the limitation of distribution-free CP placement approach.

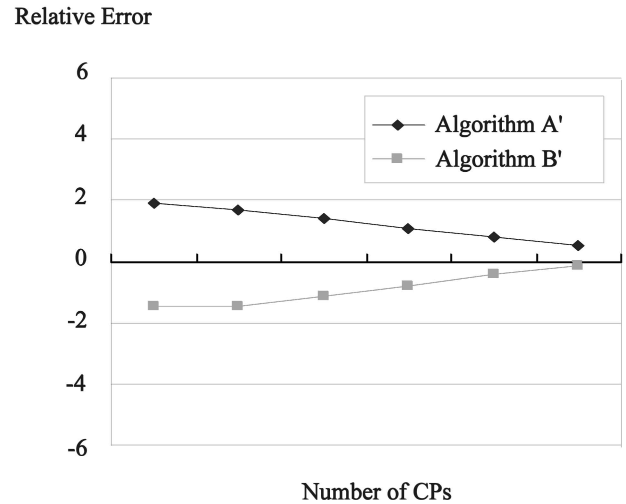

Fig. 3. Comparison of optimal CP sequence with finite-time horizon ($T = 30$): $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, and $\eta = 30$.

Finally, we compare the CP algorithms in terms of the expected operating cost. Instead of calculating the expected operating costs $C(\mathbf{t}_\infty)$ and $TC(\mathbf{t}_N)$, we use the normalized costs (expected costs per unit operating time); $\mu C(\mathbf{t}_\infty)$ and $TC(\mathbf{t}_N)/\int_0^T \overline{F}(t)dt$. Of course, the optimal CP policies minimizing them are equivalent to those for $C(\mathbf{t}_\infty)$ and $TC(\mathbf{t}_N)$. One of the reasons to apply such normalized costs is that the comparison should be made by the time average costs. Tables 1, 2, and 3 present the dependence of shape parameter on the minimum expected operating costs in respective cases, where the approximate expected operating cost is calculated by substituting the approximate CP sequence to (1) or (6), and the relative error is defined by

$$\text{Relative Error } (\%) = \frac{|\text{ approximate operation cost} - \text{minimum operation cost }|}{|\text{ real operation cost }|} \times 100. \tag{52}$$

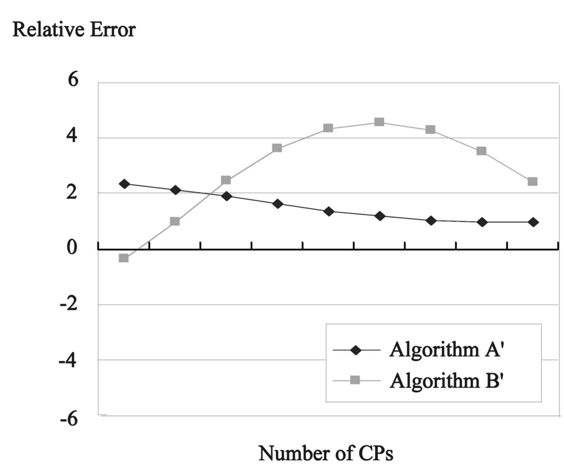

Fig. 4. Comparison of optimal CP sequence with finite-time horizon ($T = 60$): $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, and $\eta = 30$.

TABLE 1
Dependence of Shape Parameter on the Minimum Expected Operating Cost with Infinite-Time Horizon ($T \to \infty$): $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, and $\eta = 30$

	Algorithm A		Algorithm B	
m	operating cost	relative error (%)	operating cost	relative error (%)
1.0	0.288258	6.84	0.365124	35.33
1.1	0.294170	6.45	0.359157	29.96
1.2	0.297928	6.12	0.352236	25.46
1.3	0.300035	5.81	0.345375	21.80
1.4	0.300961	5.54	0.338776	18.80
1.5	0.301036	5.29	0.332535	16.31
1.6	0.300497	5.07	0.326706	14.23
1.7	0.299515	4.88	0.321296	12.50
1.8	0.298215	4.71	0.316301	11.06
1.9	0.296692	4.42	0.311860	9.76
2.0	0.294028	4.10	0.307630	8.91
3.0	0.278177	4.70	0.280170	5.45
4.0	0.273171	9.46	0.268028	7.40
5.0	0.271894	15.11	0.262343	11.06
6.0	0.273462	21.42	0.259465	15.21
7.0	0.277091	28.25	0.257704	19.28
8.0	0.282216	35.50	0.256425	23.11
9.0	0.288442	43.07	0.255413	26.69
10.0	0.295491	50.92	0.254540	30.01

TABLE 2
Dependence of Shape Parameter on the Minimum Expected Operating Cost with Finite-Time Horizon Problem ($T = 30$): $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, and $\eta = 30$

	Algorithm A'		Algorithm B'	
m	operating cost	relative error (%)	operating cost	relative error (%)
1.0	0.422637	0.03	0.425627	0.74
1.1	0.434393	0.01	0.436449	0.49
1.2	0.444455	0.04	0.445553	0.29
1.3	0.453124	0.19	0.453293	0.15
1.4	0.460009	0.09	0.459954	0.08
1.5	0.465728	0.07	0.465754	0.07
1.6	0.473095	0.60	0.470868	0.12
1.7	0.478129	0.80	0.475431	0.23
1.8	0.482609	1.04	0.479547	0.40
1.9	0.486625	1.30	0.483301	0.61
2.0	0.487153	0.95	0.486754	0.86
3.0	0.515098	5.61	0.512005	4.98
4.0	0.535396	11.57	0.528882	10.21
5.0	0.548962	17.20	0.541378	15.58
6.0	0.580031	26.86	0.551069	20.53
7.0	0.593253	32.92	0.558859	25.22
8.0	0.609079	39.68	0.565295	29.64
9.0	0.627507	47.10	0.570729	33.79
10.0	0.648490	55.13	0.575396	37.64

As the shape parameter in the Weibull distribution increases, the failure rate $r(t) = (m/\eta)(t/\eta)^{m-1}$ monotonically increases in $m\ (>1)$ and the $\mathrm{MTTF} = \eta\Gamma(1 + 1/m)$ decreases, where $\Gamma(\cdot)$ is the standard gamma function. In the infinite-time horizon case (Table 1), it can be seen that the relative error in both cases decreases first and turns to the increasing trend from $m = 3.0 \sim 4.0$. The similar tendency can be observed in the finite-time horizon cases (Tables 2 and 3), that is, the relative error trend changes from decreasing to increasing in the range of $m = 1.2 \sim 1.6$. Usually, if the amount of information on system failure is less, it will be expected that the accuracy of estimation becomes lower. However, it should be noted in the finite-time horizon cases that *Algorithm B′* is better than *Algorithm A′*. This point should be attractive for the uncertain file management with incomplete failure information, though *Algorithm B′* is not linked to *Algorithm A′* directly. The main reason that *Algorithm B* (*Algorithm B′*) based on the min-max analysis outperforms *Algorithm A* (*Algorithm A′*) for larger shape parameter m is that the former can be characterized as the worst-case analysis method and never provide the lowest cost performance. On the other hand, since Algorithm A has a monotone property on the shape parameter, it can provide the larger error on the expected operating cost even if the system failure time distribution function is completely known. In Table 2 with relatively smaller planning horizon T, it can be shown that the maximum relative error for the min-max policy decreases up to 20 percent for $T = 30$ and $m = 1.0 \sim 5.0$.

In Tables 4, 5, and 6, we investigate the dependence of scale parameter on the minimum expected operating costs in respective cases. As the scale parameter in the Weibull distribution increases, the failure rate $r(t)$ monotonically decreases and the MTTF also increases. In the infinite-time horizon case (Table 4), it can be seen that the relative error in Algorithm A increases first and begins decreasing from the point $\eta = 50 \sim 55$, but the relative error in Algorithm B continues decreasing in almost all cases. In the finite-time horizon cases (Tables 5 and 6), although the relative error in Algorithm A′ does not change extremely, the relative error in Algorithm B′ decreases first and turns to the increasing trend for $\eta = 30 \sim 50$. From these observations in Tables 4, 5, and 6, we can conclude that the dependence of scale parameter on the minimum expected operating costs in both algorithms is not sensitive except for the case where the scale parameter takes a very small value like $\eta = 5 \sim 10$. In such a situation, these algorithms are applicable to the real CP placement under incomplete failure information.

TABLE 3
Dependence of Shape Parameter on the Minimum Expected Operating Cost with Finite-Time Horizon Problem ($T = 60$): $a_0 = 1$, $b_0 = 0$, $c_0 = 1$, and $\eta = 30$

	Algorithm A'		Algorithm B'	
m	operating cost	relative error (%)	operating cost	relative error (%)
1.0	0.326332	0.06	0.337132	3.37
1.1	0.341169	0.03	0.351601	3.09
1.2	0.354314	0.05	0.364255	2.85
1.3	0.365885	0.10	0.375268	2.67
1.4	0.375989	0.19	0.384792	2.53
1.5	0.384723	0.31	0.392962	2.46
1.6	0.392181	0.46	0.399904	2.43
1.7	0.398459	0.63	0.405734	2.47
1.8	0.403654	0.83	0.410563	2.56
1.9	0.407868	1.05	0.414502	2.70
2.0	0.411203	1.30	0.417658	2.89
3.0	0.417890	4.71	0.426290	6.81
4.0	0.416845	9.45	0.426473	11.98
5.0	0.420283	15.10	0.427150	16.98
6.0	0.386998	10.02	0.427818	21.63
7.0	0.436370	28.25	0.428072	25.82
8.0	0.447437	35.49	0.427428	29.43
9.0	0.416512	29.41	0.425710	32.27
10.0	0.473265	50.80	0.423119	34.83

TABLE 4
Dependence of Scale Parameter on the Minimum Expected Operating Cost with Infinite-Time Horizon ($T \to \infty$)

	Algorithm A		Algorithm B	
η	operating cost	relative error (%)	operating cost	relative error (%)
5	0.804722	3.72	0.858182	10.61
10	0.541145	3.99	0.570455	9.62
15	0.431361	4.16	0.452427	9.24
20	0.368032	4.28	0.384860	9.05
25	0.325741	4.37	0.340654	9.15
30	0.294028	4.10	0.307630	8.91
35	0.271428	4.51	0.282595	8.81
40	0.252602	4.56	0.262523	8.67
45	0.237136	4.61	0.246525	8.75
50	0.224142	4.65	0.232894	8.73
55	0.205697	1.08	0.221247	8.72
60	0.196272	1.05	0.211157	8.71
65	0.188000	1.02	0.202300	8.70
70	0.180668	1.00	0.194453	8.70
75	0.174109	0.97	0.187432	8.70
80	0.168199	0.95	0.181109	8.70
85	0.162838	0.93	0.175371	8.70
90	0.157948	0.91	0.170137	8.70

TABLE 6
Dependence of Scale Parameter on the Minimum Expected Operating Cost with Finite-Time Horizon Problem ($T = 60$)

	Algorithm A'		Algorithm B'	
η	operating cost	relative error (%)	operating cost	relative error (%)
5	0.195370	1.26	0.298081	54.50
10	0.262554	1.45	0.366444	41.59
15	0.313365	1.41	0.381157	23.35
20	0.355761	1.34	0.395543	12.68
25	0.390296	1.32	0.409512	6.31
30	0.411203	1.30	0.417658	2.89
35	0.414384	1.27	0.414745	1.36
40	0.402304	1.27	0.400584	0.84
45	0.380386	1.33	0.378380	0.79
50	0.353140	1.31	0.351864	0.95
55	0.324396	1.31	0.323944	1.17
60	0.296228	1.32	0.296492	1.41
65	0.269742	0.90	0.270559	1.20
70	0.245685	1.33	0.246646	1.73
75	0.223611	1.45	0.224914	2.04
80	0.203807	1.44	0.205334	2.20
85	0.186113	1.44	0.187777	2.35
90	0.170332	1.44	0.172071	2.47

6 Conclusions

In this paper, we have considered two sequential checkpoint placement problems with finite and infinite time horizons, and generalized the approximate method based on the variational calculus. Further, we have developed the min-max checkpoint placement algorithms and compared them with the real optimal policies in numerical examples. Even if one cannot obtain the failure time information in actual file management, it has been shown that the distribution-free algorithms can provide the reasonable checkpoint decision for a relatively small shape parameter in the Weibull case. Also, we have shown numerically that the checkpoint interval is not constant in the exponential failure time case if the planning horizon is finite. Though in industry, the constant checkpoint placement which is hand-tuned by some system expert is very often employed, this will not be always optimal when the planning horizon is finite even under the exponential assumption.

TABLE 5
Dependence of Scale Parameter on the Minimum Expected Operating Cost with Finite-Time Horizon Problem ($T = 30$)

	Algorithm A'		Algorithm B'	
η	operating cost	relative error (%)	operating cost	relative error (%)
5	0.439176	1.26	0.588366	35.66
10	0.590003	1.45	0.652460	12.19
15	0.679409	1.41	0.687791	2.66
20	0.664623	1.47	0.658621	0.55
25	0.583972	1.55	0.577976	0.51
30	0.487153	0.95	0.486754	0.86
35	0.406453	1.64	0.404787	1.22
40	0.338416	1.97	0.336912	1.52
45	0.282888	1.96	0.282292	1.75
50	0.238705	1.96	0.238627	1.92
55	0.203405	1.96	0.203618	2.06
60	0.174972	1.95	0.175344	2.17
65	0.151850	1.95	0.152304	2.26
70	0.132862	1.96	0.133352	2.33
75	0.117118	1.96	0.117616	2.39
80	0.103942	1.96	0.104433	2.44
85	0.092821	1.96	0.093296	2.48
90	0.083358	1.96	0.083812	2.51

In the future, the min-max checkpoint placement algorithms should be applied to the other problems with different dependability measures. For instance, when a transaction oriented system with checkpointing is considered in the renewal reward process framework, any approximate method has to be used to represent the expected operating cost [3], [6], [8], [10], [13], [14]. Then, the min-max approach will be useful to derive the most pessimistic checkpointing policy. In this paper, we have made an assumption that no failures occur during the recovery operation. However, this assumption might be strong in some actual cases, so that the failure caused by checkpointing, like a human error, may occur independently from the failure time distribution. This problem can be considered by introducing the concept of imperfect checkpointing, under just a little bit different model assumption. Also, the min-max checkpoint algorithms developed in this paper should be implemented on the real-life systems and be evaluated experimentally from the standpoint of environment diversity. Since the min-max checkpoint placement algorithms does not include a statistical estimation procedure such as parameter estimation, goodness-of-fit test, etc., they can be applied to automatically place the checkpoint on computer.

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