Perception in Robotics

PS2: Localization

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Landmark localization

Task A:

1. Write the value for the covariance Q of the noise added to the observation function, knowing that the parameter *bearing_std* is its standard deviation.

`run.py`:

```
    parser.add_argument('-b',
                        '--bearing_std',
                        type=float,
                        action='store',
                        help='Diagonal of Standard deviations of the
Observation noise Q. (format: rad).',
                        default=0.35)
```

$Q = \begin{bmatrix} \sigma_{\phi}^2 \end{bmatrix} $, σ_{ϕ}^2 = `bearing_std` = 0.35.

2. Write the equation for the covariance R_t of the noise added to the transition function, as explained in class and their corresponding numeric values for the initial robot command $u=[\delta_{rot1},\delta_{trans},\delta_{rot2}]^\top=[0,10,0]^\top$. Find out the default values of α in *run.py* line 152.

For odometry motion model we have

$$\epsilon_t=\begin{bmatrix}\epsilon_{\delta_{rot1}}\\ \epsilon_{\delta_{trans}}\\ \epsilon_{\delta_{rot2}}\end{bmatrix}\sim N\left(0,\begin{bmatrix}\alpha_1\delta_{rot1}^2+\alpha_2\delta_{trans}^2 & ¿0 & ¿0\\ 0 & ¿\alpha_3\delta_{trans}^2+\alpha_4\left(\delta_{rot1}^2+\delta_{rot2}^2\right) & ¿0\\ 0 & ¿0 & ¿\alpha_1\delta_{rot2}^2+\alpha_2\delta_{trans}^2\end{bmatrix}\right)$$

As initial command $u=[\delta_{rot1},\delta_{trans},\delta_{rot2}]^\top=[0,10,0]^\top$ and default values of alphas $\alpha_{1\ldots4}=(0.05^2,0.001^2,0.05^2,0.01^2)$, then

$$R=\begin{bmatrix}100\alpha_2 & ¿0 & ¿0\\ 0 & ¿100\alpha_3 & ¿0\\ 0 & ¿0 & ¿100\alpha_2\end{bmatrix}=\begin{pmatrix}0.001^2*100 & 0 & 0\\ 0 & 0.05^2*100 & 0\\ 0 & 0 & 0.001^2*100\end{pmatrix}=\begin{pmatrix}0.0001 & 0 & 0\\ 0 & 0.25 & 0\\ 0 & 0 & 0.0001\end{pmatrix}$$

3. Derive the equations for the Jacobians G_t, V_t and H_t, and evaluate them at the initial mean state $\mu_1=[x,y,\theta]^\top=[180,50,0]^\top$ as it is considered in *run.py*.

$$G_t=\frac{\partial g(x_{t-1},u_t,\varepsilon_t)}{\partial x_{t-1}}\text{¿}_{\mu_{t-1},\varepsilon_t=0}=\begin{bmatrix}1 & 0 & -\delta_{trans}\sin(\theta+\delta_{rot1}) \\ 0 & 1 & \delta_{trans}\cos(\theta+\delta_{rot1}) \\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1\end{bmatrix}$$

$$V_t=\frac{\partial g(x_{t-1},u_t,\varepsilon_t)}{\partial u_t}\text{¿}_{\mu_{t-1},\varepsilon_t=0}=\begin{bmatrix}-\delta_{trans}\sin(\theta+\delta_{rot1}) & \cos(\theta+\delta_{rot1}) & 0 \\ \delta_{trans}\cos(\theta+\delta_{rot1}) & \sin(\theta+\delta_{rot1}) & 0 \\ 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 \\ 10 & 0 & 0 \\ 1 & 0 & 1\end{bmatrix}$$

$$H_t=\frac{\partial h(x_t)}{\partial \mu_t}=\begin{bmatrix}-\frac{(m_{j,x}-\mu_{t,x})}{\sqrt{(m_{j,x}-\mu_{t,x})^2+(m_{j,y}-\mu_{t,y})^2}} & -\frac{(m_{j,y}-\mu_{t,y})}{\sqrt{(m_{j,x}-\mu_{t,x})^2+(m_{j,y}-\mu_{t,y})^2}} & 0 \\ \frac{(m_{j,y}-\mu_{t,y})}{(m_{j,x}-\mu_{t,x})^2+(m_{j,y}-\mu_{t,y})^2} & -\frac{(m_{j,x}-\mu_{t,x})}{(m_{j,x}-\mu_{t,x})^2+(m_{j,y}-\mu_{t,y})^2} & -1\end{bmatrix}$$

In our case Output measurement (observation) vector is $y_t=[bearing, ID]^T$ but $\sigma_{ID}=0$ and then state vector is $x_t=[x,y,\theta]^T$

$$H_t=\frac{\partial h(x_t)}{\partial x_t}\text{¿}_{\acute{\mu}_t}=\begin{bmatrix}\frac{m_{i,y}-\acute{\mu}_{t,y}}{(m_{i,x}-\acute{\mu}_{t,x})^2+(m_{i,y}-\mu'_{t,y})^2} & \frac{-(m_{i,x}-\acute{\mu}_{t,x})}{(m_{i,x}-\acute{\mu}_{t,x})^2+(m_{i,y}-\mu'_{t,y})^2} & -1\end{bmatrix}$$

```
import numpy as np

mean_prior = np.array([180., 50., 0.])
_landmark_poses_x = np.array([21, 242, 463, 463, 242, 21])
_landmark_offset_y = np.array([0, 0, 0, 292, 292, 292])

b = (242 - 180)**2 + (50)**2

H = np.array([50/b, -(242 - 180)/b, -1])

print("H = ", H)

H =  [ 0.00788146 -0.00977301 -1.        ]
```

Task B

Implement EKF and PF-based robot localization using odometry and bearing-only observations to features in a landmark map. Remember to run the evaluation command to properly use the common created data file *evaluation-input.npy*.

Implemented in ekf.py and pf.py

Task C:

Create plots of pose error versus time i.e., a plot of $\hat{x} - x$ vs. t, $\hat{y} - y$ vs t, and $\hat{\theta} - \theta$ vs. t where $(\hat{x}, \hat{y}, \hat{\theta})$ is the filter estimated pose and (x, y, θ) is the ground-truth actual pose known only to the simulator. Plot the error in blue and in red plot the $\pm 3\sigma$ uncertainty bounds. Your state error should lie within these bounds approximately 99.73% of the time (assuming Gaussian statistics). For the PF, use the sample mean and variance.

```
import matplotlib.pyplot as plt
from tools.task import wrap_angle
from IPython.display import Video
#from tools.data import load_data

def plot_results(real_traj, pred_traj, cov, num_steps,
plt_title=None):
    error = real_traj - pred_traj

    for i, er in enumerate(error[:, -1]):
        if er < -np.pi or er > np.pi:
            error[i, -1] = wrap_angle(er)

    fig, axs = plt.subplots(1, 3, figsize=(25, 7))
    labels = [r'$\hat{x} - x$', r'$\hat{y} - y$', r'$\hat{z} - z$']
    titles = ['pose error x', 'pose error y', 'pose error theta']

    if plt_title:
        fig.suptitle(plt_title, fontsize=16)

    for i in range(error.shape[-1]):
        axs[i].plot(np.arange(num_steps), error[:, i],
label=labels[i])
        sigma = np.sqrt(cov[i])
        axs[i].fill_between(np.arange(num_steps), -3*sigma, 3*sigma,
color='red', alpha=0.5, label='$\pm$3sigma')
        axs[i].grid('on')
        axs[i].set_title(titles[i])
        axs[i].set_xlabel('time')
        axs[i].set_ylabel('error')
        axs[i].legend()

def load_data(filename_gt, filename_predict):
    input_data = np.load(filename_gt)
    output = np.load(filename_predict)

    covariance_matrices = output['covariance_trajectory']
    covs = np.array([np.diag(covariance_matrices[:, :, i]) for i in
range(covariance_matrices.shape[-1])]).T
    return input_data, output, covs
```

```
filename_gt = './ekf_out/input_data.npy'
filename_pf = './pf_out/output_data.npy'
filename_ekf = './ekf_out/output_data.npy'
```

EKF

```
input_data, output_ekf, covs_ekf = load_data(filename_gt,
filename_ekf)
plot_results(input_data['real_robot_path'],
output_ekf['mean_trajectory'], covs_ekf, input_data['num_steps'])
```

PF

```
input_data, output_ekf, covs_ekf = load_data(filename_gt, filename_pf)
plot_results(input_data['real_robot_path'],
output_ekf['mean_trajectory'], covs_ekf, input_data['num_steps'])
```

Comment: an estimation error lays within $\pm 3\sigma$ interval.

D. Once your filters are implemented, please investigate some properties of them.

1. How does EKF behaves when motion noise goes towards zero?

Let us assume following sets of motion noise constants in EKF

1: $ \alpha = 0.85 \cdot [0.05, 0.001, 0.05, 0.01] $

2: $ \alpha = 0.3 \cdot [0.05, 0.001, 0.05, 0.01] $

3: $ \alpha = 0 \cdot [0.05, 0.001, 0.05, 0.01] $

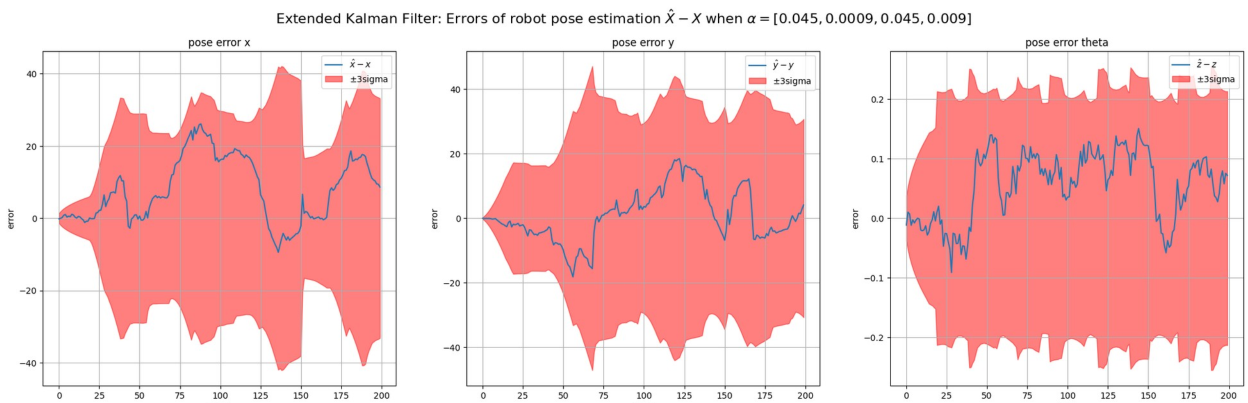
```
alpha0 = np.array([0.05, 0.001, 0.05, 0.01])
alpha1 = 0.9 * alpha0
alpha1
```

```
array([0.045 , 0.0009, 0.045 , 0.009 ])
```

```
alpha2 = 0.3 * alpha0
alpha2
```

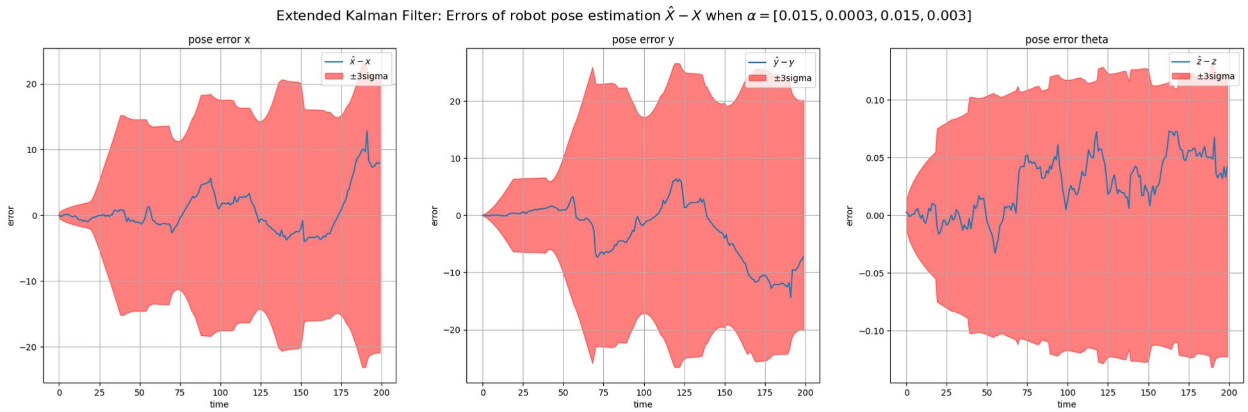
```
array([0.015 , 0.0003, 0.015 , 0.003 ])
```

```
# load the data with alpha = alpha1
alpha1_input, alpha1_output, covs1 =
load_data('Task_D/alpha1/input_data.npy',
'Task_D/alpha1/output_data.npy')

alpha1_title = r'Extended Kalman Filter: Errors of robot pose
estimation $\hat{X} - X$ when $\alpha=[0.045 , 0.0009, 0.045 , 0.009 ]
$'

plot_results(alpha1_input['real_robot_path'],
alpha1_output['mean_trajectory'], covs1, alpha1_input['num_steps'],
plt_title = alpha1_title)
```

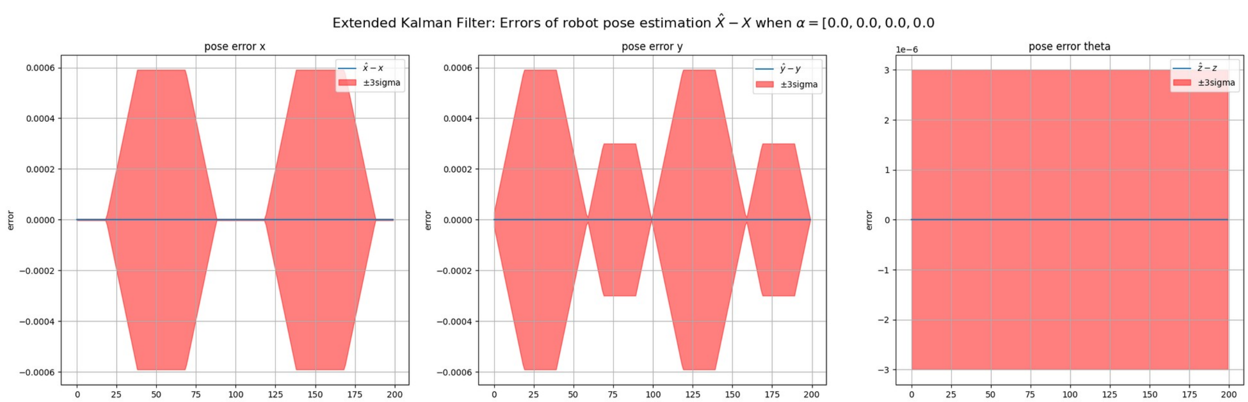



```
alpha2_input, alpha2_output, covs2 =
load_data('Task_D/alpha2/input_data.npy',
'Task_D/alpha2/output_data.npy')

alpha2_title = r'Extended Kalman Filter: Errors of robot pose
estimation $\hat{X} - X$ when $\alpha=[0.015 , 0.0003, 0.015 , 0.003 ]
$'

plot_results(alpha2_input['real_robot_path'],
```

```
alpha2_output['mean_trajectory'], covs2, alpha2_input['num_steps'],
plt_title = alpha2_title)
```




```
alpha3_input, alpha3_output, covs3 =
load_data('Task_D/alpha3/input_data.npy',
'Task_D/alpha3/output_data.npy')

alpha3_title = r'Extended Kalman Filter: Errors of robot pose
estimation $\hat{X} - X$ when $\alpha=[0.0, 0.0, 0.0, 0.0$'

plot_results(alpha3_input['real_robot_path'],
alpha3_output['mean_trajectory'], covs3, alpha3_input['num_steps'],
plt_title = alpha3_title)
```



Comment: the $\pm 3\sigma$ interval becomes less and hence the uncertainty in position estimation decreases.

2. How does EKF behaves when motion noise Q_t goes towards zero?

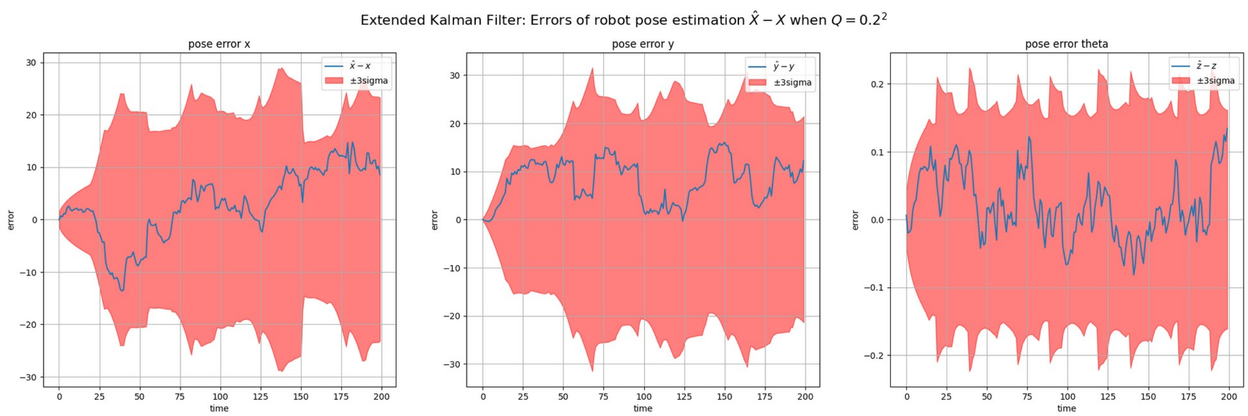
Let us assume following sets of measurement noise constants:

1: $ Q_1 = 0.2^2 $

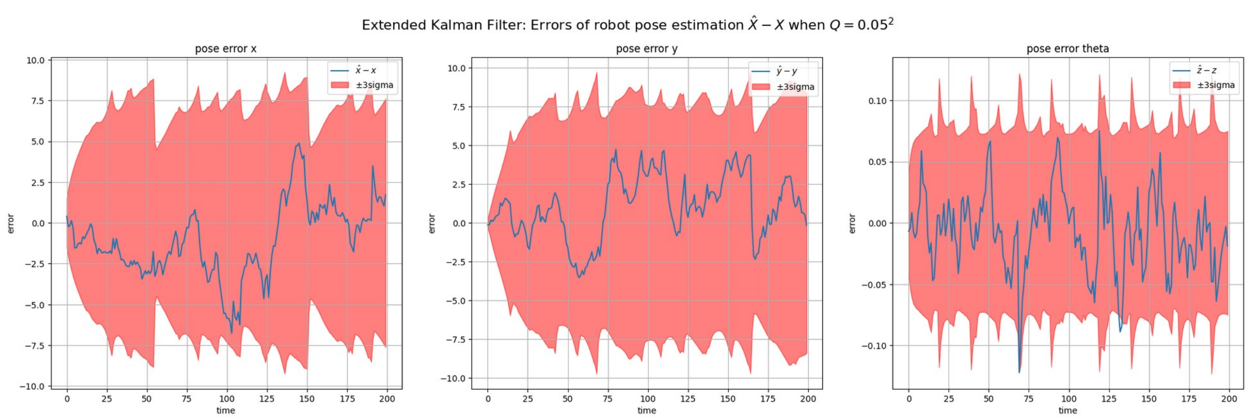
2: $ Q_2 = 0.05^2 $

3: $Q_3 = 0$

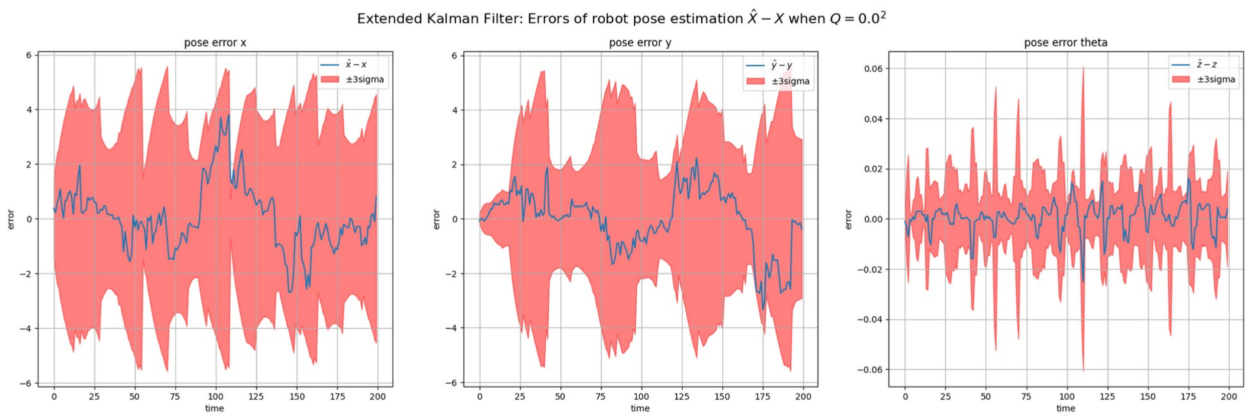
```
q1_input, q1_output, covs1 = load_data('Task_D/q1/input_data.npy',
'Task_D/q1/output_data.npy')

q1_title = r'Extended Kalman Filter: Errors of robot pose estimation
$\hat{X} - X$ when $Q=0.2^2$'

plot_results(q1_input['real_robot_path'],
q1_output['mean_trajectory'], covs1, q1_input['num_steps'], plt_title
= q1_title)
```




```
q2_input, q2_output, covs2 = load_data('Task_D/q2/input_data.npy',
'Task_D/q2/output_data.npy')

q2_title = r'Extended Kalman Filter: Errors of robot pose estimation
$\hat{X} - X$ when $Q=0.05^2$'

plot_results(q2_input['real_robot_path'],
q2_output['mean_trajectory'], covs2, q2_input['num_steps'], plt_title
= q2_title)
```



```
q3_input, q3_output, covs3 = load_data('Task_D/q3/input_data.npy',
'Task_D/q3/output_data.npy')

q3_title = r'Extended Kalman Filter: Errors of robot pose estimation
$\hat{X} - X$ when $Q=0.0^2$'

plot_results(q3_input['real_robot_path'],
q3_output['mean_trajectory'], covs3, q3_input['num_steps'], plt_title
= q3_title)
```



Comment: Here we can also see that uncertainty decreases, but since we still have motion noise than can not absolutely rely on measurements and eleminate estimation errors.

3. How does PF behaves when amount of particles is decreased ?

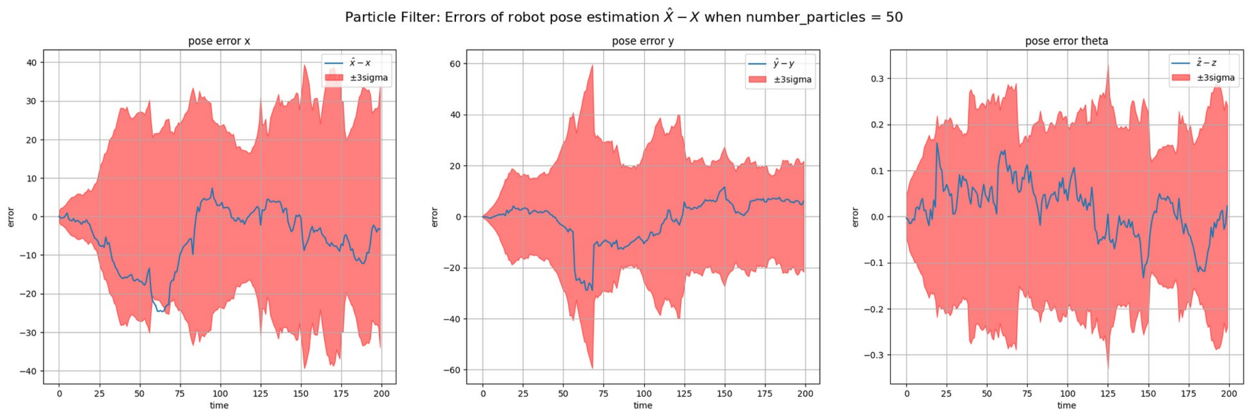
Let us assume following sets of number of particles in order to investigate properties of PF:

1: `num_particles`$ = 50$

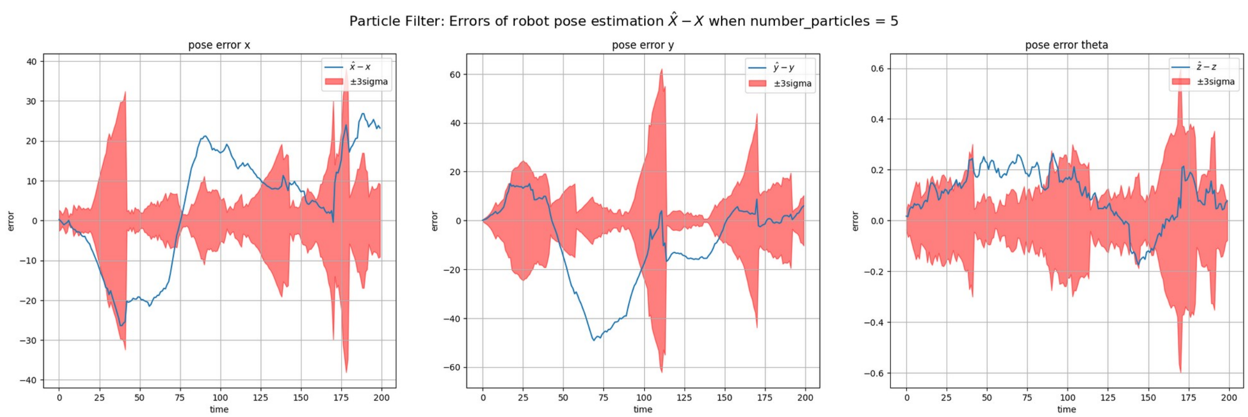
1: `num_particles`$ = 10$

3: `num_particles`$ = 5$

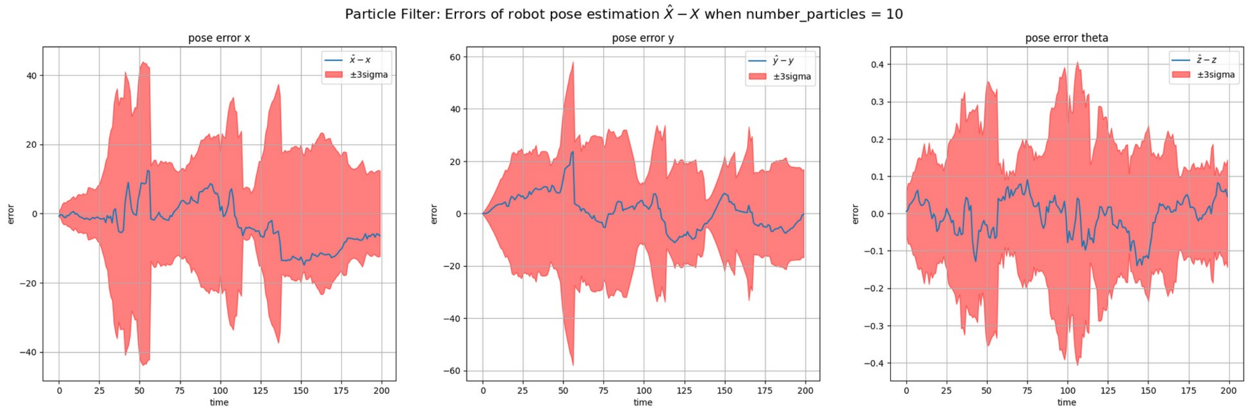
```
input, output, covs = load_data('Task_D/num_part50/input_data.npy',
'Task_D/num_part50/output_data.npy')

title = r'Particle Filter: Errors of robot pose estimation $\hat{X} -
X$ when number_particles = 50'

plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```




```
input, output, covs = load_data('Task_D/num_part5/input_data.npy',
'Task_D/num_part5/output_data.npy')

title = r'Particle Filter: Errors of robot pose estimation $\hat{X} -
X$ when number_particles = 5'

plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```




```
input, output, covs = load_data('Task_D/num_part10/input_data.npy',
'Task_D/num_part10/output_data.npy')

title = r'Particle Filter: Errors of robot pose estimation $\hat{X} -
X$ when number_particles = 10'

plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```



Comment: when number of particles is extremely decreased, we can not provide a good estimation, the distribution of particles tells provide us with wrong localaziton information.

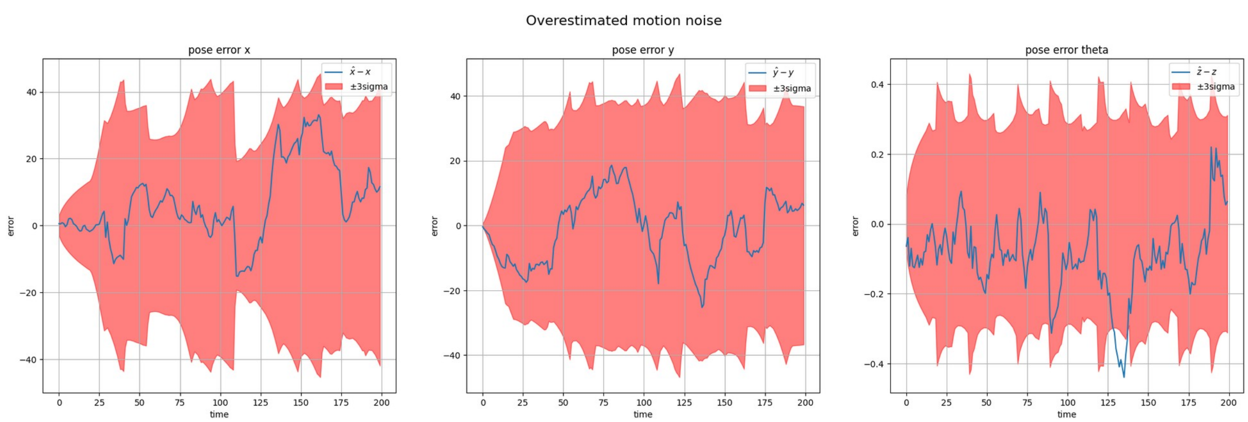
1. How does EKF behaves when we underestimate or overestimate motion noise and measurement noise?

Motion noise:

1: $ \alpha = 2 \cdot [0.05, 0.001, 0.05, 0.01] $ - Overestimated motion noise

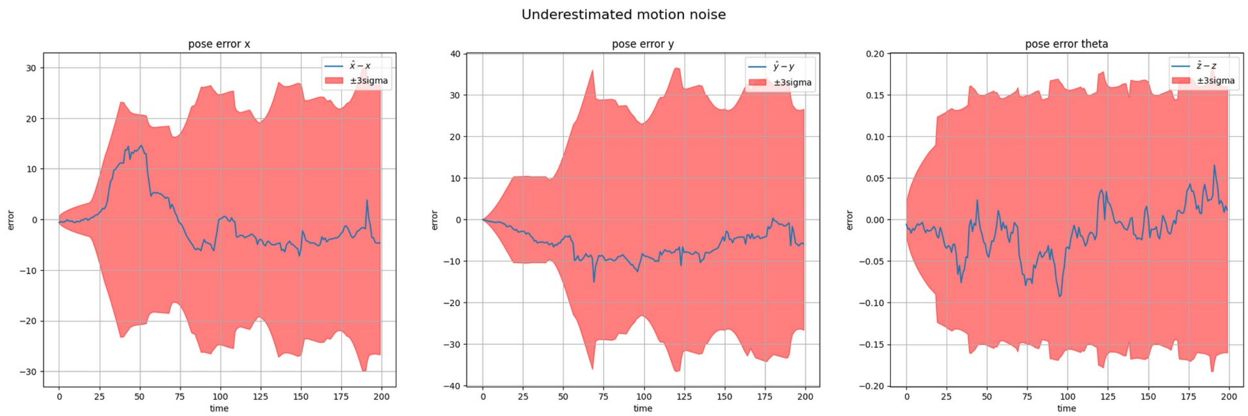
2: $ \alpha = 0.5 \cdot [0.05, 0.001, 0.05, 0.01] $ - Underestimated motion noise

```
input, output, covs = load_data('Task_D/part4/set1/input_data.npy',
'Task_D/part4/set1/output_data.npy')

title = r'Overestimated motion noise'

plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```




```
input, output, covs = load_data('Task_D/part4/set2/input_data.npy',
'Task_D/part4/set2/output_data.npy')

title = r'Underestimated motion noise'
```

```
plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```



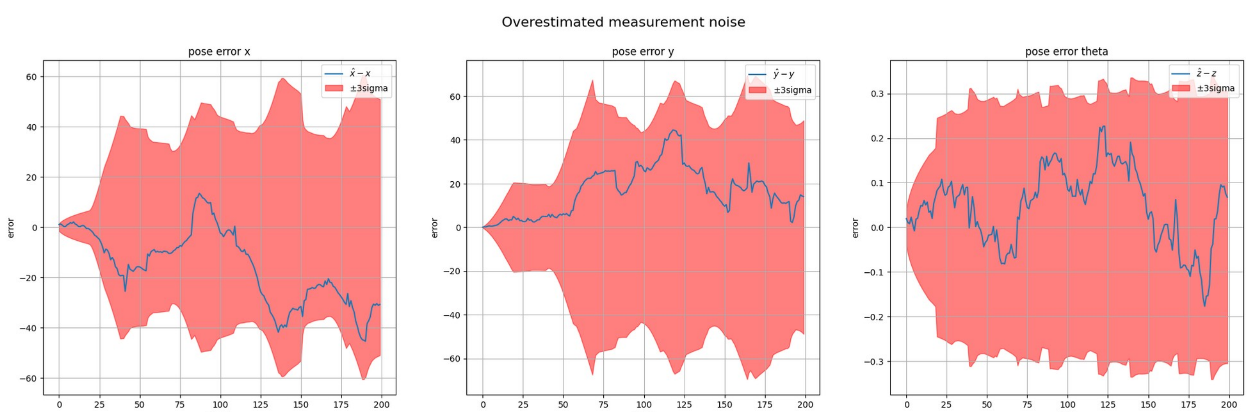
Comment: Underestimation or overestimation of the motion noise does not much affect the perfomance of EKF.

Observation noise:

1: $ Q_1 = 0.6^2 $ - Overestimated measurement noise

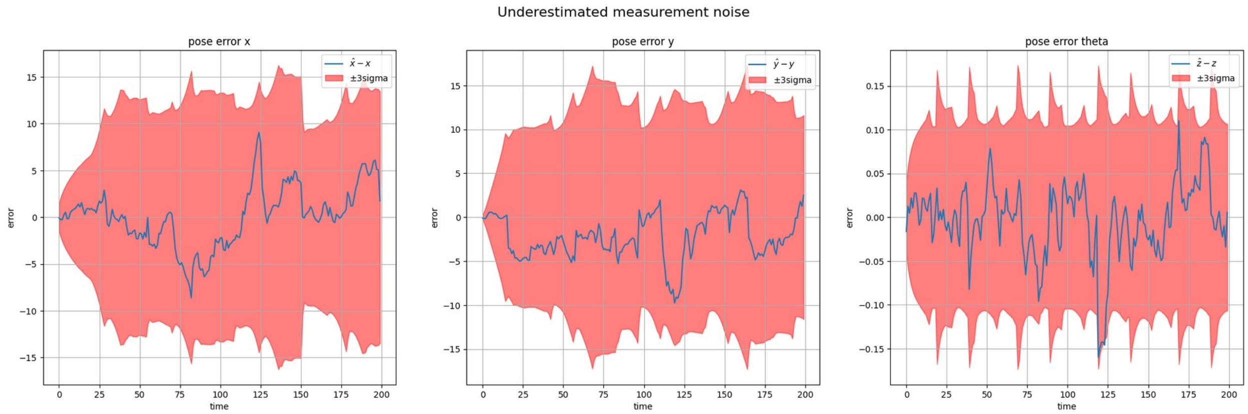
2: $ Q_2 = 0.1^2 $ - Underestimated measurement noise

```
input, output, covs = load_data('Task_D/part4/q1/input_data.npy',
'Task_D/part4/q1/output_data.npy')

title = r'Overestimated measurement noise'

plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```




```
input, output, covs = load_data('Task_D/part4/q2/input_data.npy',
'Task_D/part4/q2/output_data.npy')

title = r'Underestimated measurement noise'
```

```
plot_results(input['real_robot_path'], output['mean_trajectory'],
covs, input['num_steps'], plt_title = title)
```



Comment: we got the results of increased σ gap of possible estimation values