A SOLUTION TO ATTRIBUTE-BASED ASSIGNMENT PROBLEMS

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Certification

This is to certify that this project was carried out by FOLARIN, Wasiu Junior under my supervision during the course of his undergraduate studies in the Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria, in partial fulfilment of the requirements for the award of Bachelor of Science Degree in Mathematics.

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Dedication

This work is dedicated to the Creatives among us and all those who have a deep and rather insatiable love for the world of logic, analytical reasoning and critical thinking...

May all the good dreams come to fruition!

Acknowledgement

Great acknowledgement to the Most High - The ONLY Wise One. And to my family, friends and colleagues for their much love. And specially to my Supervisor, Dr. B. S. Ogundare, for giving me the freedom and support to develop the techniques presented in this work.

Abstract

In this work, we take a mathematical exploration of certain assignment problems which involve assigning a set of objects to non-overlapping subsets (partitions), in which assignment is based on certain properties possessed by the objects and the partitioning sets.

Several mathematical techniques are used to achieve these aims, principally including the Membership Functions of the Theory of Fuzzy Set, by constructing Constraint functions mapping each object-partition pair to a value in the unit interval based on the attributes of interest. Such that an object with a membership degree closer to 1 is a better fit than one with a lower value.

We apply the techniques developed in this text to solving a sample problem (NYSC Posting Problem). We observe that the algorithm excels in problems involving assigment of objects based on their attributes, achieving assignment accuracies of up to 100%.

We conclude this text by analyzing the assigment schedule plotted using the algorithm, and making several improvement recommendations to solving other problems in this category.

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Chapter 1

Introduction

1.1 Preamble

In everyday human, corporate and industrial activities, we are faced with the challenges of optimal resource assignment. In the face of limited resources, low tolerance for scrap and rework, need for greater degree of accuracy and scalability, optimization of resources is paramount. Over the years, several Mathematical Optimization techniques in Operations Research (OR) specifically, have been developed to help with these class of problems with a good amount of success recorded. Nonetheless, there still remains a great room for improvement on these. Classical approaches to solving assignment problems, among others include the well revered Linear Programming techniques.

1.2 Motivation

The University Timetable Problem: This study, though it doesn't directly proffer a solution to this problem, it attempts to solve a simplified variant of it.
In Universities, there are usually a good number of students taking courses from a large pool, and in varying mixes, based on Faculty and Institutional requirements. Typically, there are limited facilities such as lecture halls to assign to each course. The problem therefore arises in creating a lecture or examination timetable that caters for all the courses, eliminating, or at least minimizing stress and ensuring usability of the timetable. Studies show that, in order to prepare a well usable timetabling system, provision has to be made for such considerations as the following parameters:

1. Time Clash: A student group cannot have more than one exam at one time

2. Semester Clash: Student groups from the same major but in different semesters cannot have exams at the same time
3. Daily Maximum: A student group cannot have more than one exam per day
4. Core Papers: A student group cannot take more than one *core* exam a day
5. Difficulty Level: A student group cannot have two *difficult*[1] exams in two consecutive days
6. Capacity: The total number of students seating for exams in a particular time cannot exceed predefined limits
7. One-period Execution: Each course must occur at the same time for all student groups
8. Periodic Unavailability: Some exams may not be scheduled at particular time slots
9. Comprehensiveness of Coverage: All exams on the timetable have to be assigned
10. Pre-assignment Capabilities: A course may be assigned to a particular target time slot
11. Controllable Exams Conclusion: Student in all student groups should conclude their exams at approximately the same time

The foregoing attributes have been cited as some of the desirable properties of a credible timetabling system. Proffering credible solutions to the timetabling problem has been a major challenge in the wide, that annually, the International Timetabling Competition 2019 (ITC, 2019) holds to select a number of outstanding solutions to the timetabling problem.
Although several aspects of this problem have been attempted over the years, it is still a long way from being efficiently solved to tackle the highlighted problems

This study takes a whole new look at Mathematical Optimization: what if our optimization goal doesn't involve minimizing (or maximizing) any variable or group of variables such as cost, profit, or even price? What if we're on the other hand interested in the *Allocation of objects to non-intersecting groups, based on attributes possessed by the objects, and one or more constraining conditions*

[1]Difficulty level for each course is assessed by some Institution-wide accepted metric(s)

Studies show that Mathematical Optimization techniques have not been formulated to solve problems as this, and thus the motivation for this study.

1.3 A Reduction Problem

Consider the following problem statement: In deploying intending NYSC members to a serving state, the following constraints need be satisfied:

1. State of Origin: No student should be deployed to his/her state of origin
2. State of Study: No student should be deployed to his state of study
3. Comprehensiveness of Coverage: All enlisted students must be deployed
4. Capacity: No state should be allocated more students that it has resources to accommodate

The foregoing problem statement is an attempt at reducing the more complex University Timetabling problem to a simpler form where the parameters involved are better appreciated. This study focuses on resolving the simpler NYSC batching problem, and makes recommendations on applying the concepts herein in tackling the more complex University Timetabling Problem.

1.4 Objective

The specific objectives of this study are to:

(i) Develop a mathematical algorithm to solve assignment problems involving the distribution of candidate objects into non-overlapping groups, putting into considerations several constraining conditions

(ii) Exercise this model on a sample problem

(iii) Sketch a path to evolving this class of solution to the University Timetabling problem, owing to its more complex constraints formulation.

1.5 Scope of Work

This work explores the problem applying concepts of probabilities. It uses the concept of membership functions in the Theory of Fuzzy Set. While the theory of Fuzzy Set is a large space to explore, involving concepts such as Fuzzy Logic, Fuzzy Algebra, etc. This work only takes into consideration concepts of membership funtion from the Theory Fuzzy Set.

1.6 Definition of Terms and Concepts

1.6.1 Fuzzy Set Theory

Since its inception in 1965, the theory of fuzzy sets has advanced in a variety of ways and in many disciplines. Applications of this theory can be found, for example, in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, and robotics. Mathematical developments have advanced to a very high standard and are still forthcoming to day. In this review, the basic mathematical framework of fuzzy set theory will be described.

If X is a nonempty collection of objects denoted generically by $x_1, x_2, ..., x_n$ for $n = 1, 2, \ldots,$ then a fuzzy set A in X is a pair (A, μ_A) where $A \subset X$ and $\mu_A \colon \boldsymbol{R} \to [0, 1]$, called the membership function of A *(Lee, 2005).*

Examples of Fuzzy Sets

To better understand the concept of Fuzzy Sets, let us consider the following Examples

Example 1: Let us define a fuzzy set

$$A = \{\, x \mid x \in R, \text{ real numbers near } 0 \,\}$$

The boundary for set real number near 0 is pretty ambiguous. The possibility of real number x to be a member of prescribed set can be defined by the following membership function.

We define the membership mapping

$$\mu_A \colon \mathbf{R} \to [0, 1] \tag{1.1}$$

describing the membership of elements of **R** in **A**, which is defined as:

$$\mu_A(x) = \frac{1}{1+x^2} \tag{1.2}$$

Example 2: Let us define a similar fuzzy set

$$B = \{\, x \mid x \in R, \text{ real numbers very near 0 } \}$$

We define the membership function

$\mu_B\colon \mathbf{R} \to [0,1]$

Defining the membership of elements of $\mathbb{R}$ in $\mathbf{B}$ such that:

$$\mu_B(x) = \left(\frac{1}{1+x^2}\right)^2 \tag{1.3}$$

The Theory of Fuzzy Sets forms the basic framework upon which the solution proposed by this work is formulated

1.6.2 Membership Probability or Degree

Each candidate object in all cases studied in this text, belong to each partition with varying degrees of membership, described as a value in the unit interval [0,1]. An object with a membership degree of 0 is referred to as *unfit* for the class, while an object having a higher value has a better fit, and the membership value of 1 being a perfect fit into the class in question.

1.6.3 Partition

Let X be a set, and $A_1, A_2, ..., A_n$ be subsets of sets such that $A_1 \cap A_2 \cap \cdots \cap A_n = \emptyset$ and $A_1 \cup A_2 \cup \cdots \cup A_n = X$. The disjoint sets $A_1, A_2, \ldots, A_n$ are referred to as a *partition* of the set X

Chapter 2

Literature Review

Finding mathematical solutions to optimization problems has had a long history in coming. This becomes imperative when the problems become enormous both in terms of volume and dimension, where contraction isn't easily achieved based on popular heuristics.

In mathematics, computer science and operations research, mathematical optimization or mathematical programming is the selection of a best element (with regard to some criterion) from some set of available alternatives (Arjang & Saul, 2011).

In Operations Research, applied mathematics and theoretical computer science, combinatorial optimization is a topic that consists of finding an optimal object from a finite set of objects (Schrijver, 2017).

Operations research (OR) had its origins in the late 1930s when a group of British Royal Air Force officers and civilian scientists were asked to determine how recently developed radar technology could be used for controlled interception of enemy aircraft (Arjang & Saul, 2011). Several successful outcomes were attributed to this collaboration which also promulgated into developing strategies for the World War II, hence the popular remark that *OR helped the Allied forces win the war* (Arjang & Saul, 2011). At the end of WWII, the U.S. military services, recognizing the wartime contributions of OR, continued their support of OR groups with the problems now focused on logistics, combat modeling, and force planning. Similarly, senior scientists who had participated in wartime OR in the U.K and U.S. were convinced that OR could be used to solve management and operational problems of nonmilitary enterprises and government (Arjang & Saul, 2011).

2.1 Linear Programming (LP)

As highlighted in the preceding paragraphs, Linear Programming techniques often lend themselves to making optimum resource allocation in many cases. The following definition accurately captures these techniques.

Linear Programming is a mathematical technique for determining the optimal allocation of resources and obtaining a particular objective (i.e, cost minimization or inversely profit maximization) when there are alternative uses of resources: Land, Capital, Materials, Machines, etc (Sanchetti & Kapoor, 2010).

Solving an assignment problem with Linear Programming techniques, often requires a clear statement of an Objective function and resource constraints, usually expressed as a set of multivariate linear equations.

2.1.1 Classical LP Problems

Three problems have become classical illustrations in Linear Programming:

1. **The Diet Problem:** It is the problem of deciding how much of 'n' different foods to include in a diet, given the cost of each food, and the particular combination of nutrients each food contains. The objective is to minimize the cost of the diet such that it contains a certain minimum amount of each nutrient.

2. **Optimal Product Lines Problem:** How much of 'n' different products a firm should produce and sell. When each product requires a particular combination of labour, machine, time and warehouse space per unit of output and where there are fixed limits on the amounts of labour, machine-time and warehouse space available?

3. **Transportation Problem:** It is a problem of determining a shipping schedule for a commodity, say, steel or oil, from each of a number of plants (or oil-fields) at different locations to each of a number of markets (or refineries) at different locations in such a way as to minimize the total shipping cost subject to the constraints that (1) the demands at each market (refinery) will be satisfied, and (2) the supply at the plant (oil field) will not be exceeded (Sanchetti & Kapoor, 2010).

Other classifications include the Homemaker's Problem, Blending Problem, On-the-Job Training Problem, etc (Dantzig & Thapa, 1997, 2003).

The general formulation of a Linear Programming problem is given below

Let Z be a linear function defined by,

$$Z = c_1x_1 + c_2x_2 + ... + c_nx_n$$

where c_j's are constants.

And and let (a_{ij}) be $m \cdot n$ constants and let (b_i) be a set of m constants such that

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n(\leq, =, \geq)b_1 \\
a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n(\leq, =, \geq)b_2 \\
\vdots \\
a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n(\leq, =, \geq)b_m \\
x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0
\end{aligned}$$

The problem of determining the values of $x_1, x_2, ..., x_n$, which makes Z a minimum (or maximum) and which satisfies (ii) and (iii) is called the General Linear Programming Problem

Indeed, linear programming forms the hinge in the history of combinatorial optimization (Schrijver, 2001). Linear Programming techniques, as well as other Mathematical Optimization techniques have been reputed a great success, nonetheless, there are several yet unaddressed aspects of optimization problems, one of which, as introduced in the previous chapter, is the focus of this text.

Chapter 3

The Algorithm

3.1 Defining Resources and Objects

In this section we shall proceed to develop the algorithm needed to solve the reduction problem proposed in the previous section.

Let the set $\mathbb{U}$ be a collection of objects to be assigned to partitioning sets

$$\mathbb{U} = \{u_1, u_2, \ldots, u_n\} \tag{3.1}$$

where each $u_i, i = 1, 2, \ldots$,n are the objects possessing characteristics to be assigned to various partition classes, e.g say, the group of graduates to be posted to NYSC serving states

And let

$$p_1, p_2, \ldots, p_k \tag{3.2}$$

be the partitions to which objects u_i's are to be assigned. We observe that there are k such classes, eg, each p_i would be a state where graduates can be posted to for their Service Year

3.2 Adjusting for Capacities

We now let

$$J(p_i) = j_i \tag{3.3}$$

be the maximum capacity the partition p_i can contain. That is, the maximum number of u_i's each partition p_i can contain

Let V,

$$V = \sum_{i=1}^{k} J(p_i) = \sum_{i=1}^{k} j_i \tag{3.4}$$

be the total capacity of the system, that is, the total number of u_i's all the partitions can contain combined.

NB: It should be noted that for an optimum solution to be feasible, $\mid U \mid \leq V$ needs be satisfied

We now define the weight, w_i of each partition p_i

$$w_i = \frac{j_i}{V} = \frac{J(p_i)}{\sum_{i=1}^{k} J(p_i)} \; \textit{for i=1,2,3,\ldots,k} \tag{3.5}$$

The weight, w_i represents the proportion of each p_i's capacity in the whole

We now define the expected cardinality, $\mid p_i \mid$ of each partition p_i

$$\mid p_i \mid = w_i \cdot \mid U \mid = w_i \cdot n, \textit{for i=1,2,3,\ldots,k} \tag{3.6}$$

The cardinality, $\mid p_i \mid$ represents the number of objects proportionately assignable to each partition p_i

3.3 Working with the Constraints

We now proceed to define constraints functions, akin to membership function of a Fuzzy Set. But in this case, they membership functions are of two types: inclusion and exclusion types.

Inclusion Constraints require that the value of a characteristic measured on candidate objects, u_i match that prescribed by the constraints, while on the other hand, Exclusion constraints define characteristics that once a candidate object u_i possesses, it is disqualified (by assigning a value of 0) from being a probable member of the partition in question Constraints, like membership functions, are generally of the form

$$\mu \colon U \mapsto [0, 1]$$

Inclusion Constraints are defined thus:

$$\mu_j(u_i) = \frac{1}{1 + (j - s(u_i))^2} \tag{3.7}$$

And, Exclusion Constraints are defined thus:

$$\mu_j(u_i) = \frac{(j - s(u_i))^2}{1 + (j - s(u_i))^2} \tag{3.8}$$

for i*=1,2,...,*n and j*=1,2,...,*k

and

$$s\colon U \mapsto \{1, 2, \ldots, k\} \tag{3.9}$$

is a helper function defined by the individual Constraint function, μ_j to get the index of the partition class p_j with which each object u_i shares the characteristic being measured For instance, if s is the state of origin helper function, it returns the index of the partition class p_i to which the object u_i maps as a state of origin

NB: $p_1, p_2, \ldots, k$ are indexed based on proximity or similarity. That is, the difference between the indices of any two partitions is directly proportional to the distance between them.

Now, in situations where there are more than one Constraint functions defined, we obtain a single Constraint function by multiplying through viz:

$$\mu_i(u_j) = \prod_{t=1}^{s} \mu_{i_t}(u_j) \tag{3.10}$$

for i*=1,2,...,*k*;* j*=1,2,...,*n and s is the total number of Constraint functions defined

We now define the membership set for each partition. These sets list the probability of each object u_i being a member of the partition p_j

$$\mu_i = \{\mu_i(u_1), \mu_i(u_2), \ldots, \mu_i(u_n)\} \tag{3.11}$$

So, for example

$$\mu_1 = \{\mu_1(u_1), \mu_1(u_2), \ldots, \mu_1(u_n)\} \tag{3.12}$$

and,

$$\mu_2 = \{\mu_2(u_1), \mu_2(u_2), \ldots, \mu_2(u_n)\} \tag{3.13}$$

and,

$$\mu_3 = \{\mu_3(u_1), \mu_3(u_2), \ldots, \mu_3(u_n)\} \tag{3.14}$$

$$\vdots$$

$$\mu_k = \{\mu_k(u_1), \mu_k(u_2), \ldots, \mu_k(u_n)\} \tag{3.15}$$

3.4 Assigning the Objects

We now proceed to populate each partition with the objects with the highest membership probability, in turn

Let

$$\mu_{p_i} = \mu_i \backslash \bigcup_{j=1}^{u-1} \{\mu_j(u_x) \forall u_x \in p_j\} \tag{3.16}$$

That is, isolating the already assigned objects u_x from the membership set to avoid multiple membership

Now, sorting out the first $\mid p_k \mid$ objects with the highest membership points:

Let

$$Q_1 = u_x \mid \mu_i(u_x) = \max\{\mu_{p_i}\} \tag{3.17}$$

And

$$Q_2 = u_x \mid \mu_i(u_x) = \max\{\mu_{p_i} \backslash \mu_i(u_y) \mid u_y \in Q_1\} \tag{3.18}$$

And

$$Q_3 = u_x \mid \mu_i(u_x) = \max\left\{\mu_{p_i} \backslash \mu_i(u_y) \mid u_y \in Q_1 \bigcup Q_2\right\} \tag{3.19}$$

And

$$Q_4 = u_x \mid \mu_i(u_x) = \max\left\{\mu_{p_i} \backslash \mu_i(u_y) \mid u_y \in Q_1 \bigcup Q_2 \bigcup Q_3\right\} \tag{3.20}$$

$$\vdots$$

$$Q_{|p_i|} = u_x \mid \mu_i(u_x) = \max\left\{\mu_{p_i} \backslash \bigcup_{j=1}^{|p_i|-1} \{\mu_i(u_y) \mid u_y \in Q_j\}\right\} \tag{3.21}$$

Whence we finally have that

$$p_i = \bigcup_{j=1}^{|p_i|} \{Q_j\} \qquad \textit{for i=1,2,}\ldots\textit{,k} \tag{3.22}$$

Chapter 4

Solving A Sample Problem

In this section, we attempt to apply the algorithm developed in the previous chapter to solve a sample problem. The problem is an attempt to assign a group of 20 University graduates to one of 5 states for service.

4.1 Statement of Constraints

1. No students should be posted to their State of Origin
2. No students should be posted to their States of study
3. The maximum capacity of each state must not be exceeded
4. All students should be assigned

The 3rd and 4th constraints are innately adhered to by the algorithm, so we'll focus only on constructing Constraint Functions for the first two constraints.

4.2 The States

The States in this problem stand for the partitions to be filled. In order to simplify the problem, we focus only on 5 states and assume each of the 20 graduates schooled, and originated from one of these states:

Δs	**State**	**Capacity**
1	Osun State	3
2	Kwara State	2
3	Enugu State	1

Δs	**State**	**Capacity**
4	Akwa-Ibom State	5
5	Kano State	10

Table 4.1: Showing the States

4.3 The Graduates

Each graduate has a number of characteristics which include names, State of origin, and Alma Mater. Here is our list of 20 graduates which we shall consider in this sample problem:

Serial	**Name**	**State of Origin**	**College**
u_1	GR01	Kwara	OAU, Osun
u_2	GR02	Kwara	BUK, Kano
u_3	GR03	Kwara	NSUKKA, Enugu
u_4	GR04	Kwara	UNILORIN, Kwara
u_5	GR05	Akwa-Ibom	OAU, Osun
u_6	GR06	Akwa-Ibom	BUK, Kano
u_7	GR07	Akwa-Ibom	NSUKKA, Enugu
u_8	GR08	Akwa-Ibom	UNILORIN, Kwara
u_9	GR09	Osun	OAU, Osun
u_{10}	GR10	Osun	BUK, Kano
u_{11}	GR11	Osun	NSUKKA, Enugu
u_{12}	GR12	Osun	UNILORIN, Kwara
u_{13}	GR13	Kano	OUI, Osun
u_{14}	GR14	Kano	NSUKKA, Enugu
u_{15}	GR15	Kano	UNILORIN, Kwara
u_{16}	GR16	Kano	AKSU, Akwa-Ibom
u_{17}	GR17	Enugu	UNIOSUN, Osun
u_{18}	GR18	Enugu	BUK, Kano
u_{19}	GR19	Enugu	UNILORIN, Kwara
u_{20}	GR20	Enugu	AKSU, Akwa-Ibom

Serial	Name	State of Origin	College

Table 4.2: Showing the Graduates

4.4 Capacities

As indicated in the States data, each state has a capacity, j_i. So in this case,

$$J(p_1) = j_1 = 3 \tag{4.1}$$

$$J(p_2) = j_2 = 2 \tag{4.2}$$

$$J(p_3) = j_3 = 1 \tag{4.3}$$

$$J(p_4) = j_4 = 5 \tag{4.4}$$

$$J(p_5) = j_5 = 10 \tag{4.5}$$

Thus, the maximum capacity of the system is given as:

$$V = \sum_{i=1}^{5} J(p_i) = 3 + 2 + 1 + 5 + 10 = 21 \tag{4.6}$$

Since V is greater than the total number of graduates awaiting deployment (20), an optimum assignment is feasible.

The weight, w_i of each state is defined thus:

$$w_i = \frac{j_i}{V}, \qquad \textit{for i=1,2,...,5} \tag{4.7}$$

$$w_1 = \frac{j_1}{V} = \frac{3}{21} = 0.143 \tag{4.8}$$

$$w_2 = \frac{j_2}{V} = \frac{2}{21} = 0.095 \tag{4.9}$$

$$w_3 = \frac{j_3}{V} = \frac{1}{21} = 0.048 \tag{4.10}$$

$$w_4 = \frac{j_4}{V} = \frac{5}{21} = 0.238 \tag{4.11}$$

$$w_5 = \frac{j_5}{V} = \frac{10}{21} = 0.476 \tag{4.12}$$

We now define the expected cardinality, $\mid p_i \mid$ of each state. That is, the expected number of Corp members posted to each state

$$\mid p_i \mid = w_i \cdot \text{no. of graduates} = w_i \cdot n = w_i \cdot 20, \qquad \textit{for i=1,2,...,5} \tag{4.13}$$

$$\mid p_1 \mid = w_1 \times 20 = 0.143 \times 20 = 2.86 \approx 3 \tag{4.14}$$

$$\mid p_2 \mid = w_2 \times 20 = 0.095 \times 20 = 1.9 \approx 2 \tag{4.15}$$

$$\mid p_3 \mid = w_3 \times 20 = 0.048 \times 20 = 0.96 \approx 1 \tag{4.16}$$

$$\mid p_4 \mid = w_4 \times 20 = 0.238 \times 20 = 4.76 \approx 5 \tag{4.17}$$

$$\mid p_5 \mid = w_5 \times 20 = 0.476 \times 20 = 9.52 \approx 9 \tag{4.18}$$

4.5 Constraints

The sample problem has two statements of exclusion constraints. We will establish the Constraint functions defining the membership probability of each graduate into each state thus:

Constraint A, μ_A: No student should be posted to their State of Origin

$$\mu_{A_j}(u_i) = \frac{(j - s_A(u_i))^2}{1 + (j - s_A(u_i))^2} \tag{4.19}$$

where $\mu_{A_j}(u_i)$ measures the degree of object u_i's membership in State j under the consideration of the *state of origin* constraint and, $s_A(u_i)$ represents the index of graduate u_i's State of Origin

$$\begin{aligned} \mu_{A_1}(u_1) &= \frac{(1 - s_A(u_1))^2}{1 + (1 - s_A(u_1))^2}, \\ &= \frac{(1-2)^2}{1 + (1-2)^2}, \\ &= 0.5 \ . \end{aligned} \tag{4.20}$$

$$\begin{aligned}\mu_{A_1}(u_2) &= \frac{(1-s_A(u_2))^2}{1+(1-s_A(u_2))^2}, \\ &= \frac{(1-2)^2}{1+(1-2)^2}, \\ &= 0.5 \ \ .\end{aligned} \tag{4.21}$$

$$\begin{aligned}\mu_{A_1}(u_3) &= \frac{(1-s_A(u_3))^2}{1+(1-s_A(u_3))^2}, \\ &= \frac{(1-2)^2}{1+(1-2)^2}, \\ &= 0.5 \ \ .\end{aligned} \tag{4.22}$$

$$\vdots$$

$$\begin{aligned}\mu_{A_1}(u_{10}) &= \frac{(1-s_A(u_{10}))^2}{1+(1-s_A(u_{10}))^2}, \\ &= \frac{(1-1)^2}{1+(1-1)^2}, \\ &= 0 \ \ .\end{aligned} \tag{4.23}$$

$$\begin{aligned}\mu_{A_1}(u_{11}) &= \frac{(1-s_A(u_{11}))^2}{1+(1-s_A(u_{11}))^2}, \\ &= \frac{(1-1)^2}{1+(1-1)^2}, \\ &= 0 \ \ .\end{aligned} \tag{4.24}$$

$$\begin{aligned}\mu_{A_1}(u_{12}) &= \frac{(1-s_A(u_{12}))^2}{1+(1-s_A(u_{12}))^2}, \\ &= \frac{(1-1)^2}{1+(1-1)^2}, \\ &= 0 \ \ .\end{aligned} \tag{4.25}$$

$$\vdots$$

$$\mu_{A_1}(u_{18}) = \frac{(1 - s_A(u_{18}))^2}{1 + (1 - s_A(u_{18}))^2}, \tag{4.26}$$
$$= \frac{(1-3)^2}{1+(1-3)^2},$$
$$= 0.8 \ .$$

$$\mu_{A_1}(u_{19}) = \frac{(1 - s_A(u_{19}))^2}{1 + (1 - s_A(u_{19}))^2}, \tag{4.27}$$
$$= \frac{(1-3)^2}{1+(1-3)^2},$$
$$= 0.8 \ .$$

$$\mu_{A_1}(u_{20}) = \frac{(1 - s_A(u_{20}))^2}{1 + (1 - s_A(u_{20}))^2}, \tag{4.28}$$
$$= \frac{(1-3)^2}{1+(1-3)^2},$$
$$= 0.8 \ .$$

$$\mu_{A_2}(u_1) = \frac{(2 - s_A(u_1))^2}{1 + (2 - s_A(u_1))^2}, \tag{4.29}$$
$$= \frac{(2-2)^2}{1+(2-2)^2},$$
$$= 0 \ .$$

$$\mu_{A_2}(u_2) = \frac{(2 - s_A(u_2))^2}{1 + (2 - s_A(u_2))^2}, \tag{4.30}$$
$$= \frac{(2-2)^2}{1+(2-2)^2},$$
$$= 0 \ .$$

$$\mu_{A_2}(u_3) = \frac{(2 - s_A(u_3))^2}{1 + (2 - s_A(u_3))^2}, \tag{4.31}$$
$$= \frac{(2-2)^2}{1+(2-2)^2},$$
$$= 0 \ .$$

$$\vdots$$

$$\begin{aligned}
\mu_{A_2}(u_{10}) &= \frac{(2 - s_A(u_{10}))^2}{1 + (2 - s_A(u_{10}))^2}, \\
&= \frac{(2-1)^2}{1+(2-1)^2}, \\
&= 0.5 \ \ .
\end{aligned} \tag{4.32}$$

$$\begin{aligned}
\mu_{A_2}(u_{11}) &= \frac{(2 - s_A(u_{11}))^2}{1 + (2 - s_A(u_{11}))^2}, \\
&= \frac{(2-1)^2}{1+(2-1)^2}, \\
&= 0.5 \ \ .
\end{aligned} \tag{4.33}$$

$$\begin{aligned}
\mu_{A_2}(u_{12}) &= \frac{(2 - s_A(u_{12}))^2}{1 + (2 - s_A(u_{12}))^2}, \\
&= \frac{(2-1)^2}{1+(2-1)^2}, \\
&= 0.5 \ \ .
\end{aligned} \tag{4.34}$$

$$\vdots$$

$$\begin{aligned}
\mu_{A_2}(u_{18}) &= \frac{(2 - s_A(u_{18}))^2}{1 + (2 - s_A(u_{18}))^2}, \\
&= \frac{(2-3)^2}{1+(2-3)^2}, \\
&= 0.50 \ \ .
\end{aligned} \tag{4.35}$$

$$\begin{aligned}
\mu_{A_2}(u_{19}) &= \frac{(2 - s_A(u_{19}))^2}{1 + (2 - s_A(u_{19}))^2}, \\
&= \frac{(2-3)^2}{1+(2-3)^2}, \\
&= 0.50 \ \ .
\end{aligned} \tag{4.36}$$

$$\mu_{A_2}(u_{20}) = \frac{(2 - s_A(u_{20}))^2}{1 + (2 - s_A(u_{20}))^2}, \tag{4.37}$$
$$= \frac{(2-3)^2}{1 + (2-3)^2},$$
$$= 0.50 \ .$$

$$\mu_{A_3}(u_1) = \frac{(3 - s_A(u_1))^2}{1 + (3 - s_A(u_1))^2}, \tag{4.38}$$
$$= \frac{(3-2)^2}{1 + (3-2)^2},$$
$$= 0.5 \ .$$

$$\mu_{A_3}(u_2) = \frac{(3 - s_A(u_2))^2}{1 + (3 - s_A(u_2))^2}, \tag{4.39}$$
$$= \frac{(3-2)^2}{1 + (3-2)^2},$$
$$= 0.5 \ .$$

$$\mu_{A_3}(u_3) = \frac{(3 - s_A(u_3))^2}{1 + (3 - s_A(u_3))^2}, \tag{4.40}$$
$$= \frac{(3-2)^2}{1 + (3-2)^2},$$
$$= 0.5 \ .$$

$$\vdots$$

$$\mu_{A_3}(u_{10}) = \frac{(3 - s_A(u_{10}))^2}{1 + (3 - s_A(u_{10}))^2}, \tag{4.41}$$
$$= \frac{(3-1)^2}{1 + (3-1)^2},$$
$$= 0.8 \ .$$

$$\begin{aligned}
\mu_{A_3}(u_{11}) &= \frac{(3 - s_A(u_{11}))^2}{1 + (3 - s_A(u_{11}))^2}, \\
&= \frac{(3-1)^2}{1+(3-1)^2}, \\
&= 0.8 \ \ .
\end{aligned} \tag{4.42}$$

$$\begin{aligned}
\mu_{A_3}(u_{12}) &= \frac{(3 - s_A(u_{12}))^2}{1 + (3 - s_A(u_{12}))^2}, \\
&= \frac{(3-1)^2}{1+(3-1)^2}, \\
&= 0.8 \ \ .
\end{aligned} \tag{4.43}$$

$$\vdots$$

$$\begin{aligned}
\mu_{A_3}(u_{18}) &= \frac{(3 - s_A(u_{18}))^2}{1 + (3 - s_A(u_{18}))^2}, \\
&= \frac{(3-3)^2}{1+(3-3)^2}, \\
&= 0 \ \ .
\end{aligned} \tag{4.44}$$

$$\begin{aligned}
\mu_{A_3}(u_{19}) &= \frac{(3 - s_A(u_{19}))^2}{1 + (3 - s_A(u_{19}))^2}, \\
&= \frac{(3-3)^2}{1+(3-3)^2}, \\
&= 0 \ \ .
\end{aligned} \tag{4.45}$$

$$\begin{aligned}
\mu_{A_3}(u_{20}) &= \frac{(3 - s_A(u_{20}))^2}{1 + (3 - s_A(u_{20}))^2}, \\
&= \frac{(3-3)^2}{1+(3-3)^2}, \\
&= 0 \ \ .
\end{aligned} \tag{4.46}$$

$$\mu_{A_4}(u_1) = \frac{(4 - s_A(u_1))^2}{1 + (4 - s_A(u_1))^2}, \quad (4.47)$$
$$= \frac{(4-2)^2}{1 + (4-2)^2},$$
$$= 0.8 \quad .$$

$$\mu_{A_4}(u_2) = \frac{(4 - s_A(u_2))^2}{1 + (4 - s_A(u_2))^2}, \quad (4.48)$$
$$= \frac{(4-2)^2}{1 + (4-2)^2},$$
$$= 0.8 \quad .$$

$$\mu_{A_4}(u_3) = \frac{(4 - s_A(u_3))^2}{1 + (4 - s_A(u_3))^2}, \quad (4.49)$$
$$= \frac{(4-2)^2}{1 + (4-2)^2},$$
$$= 0.8 \quad .$$

$$\vdots$$

$$\mu_{A_4}(u_{10}) = \frac{(4 - s_A(u_{10}))^2}{1 + (4 - s_A(u_{10}))^2}, \quad (4.50)$$
$$= \frac{(4-1)^2}{1 + (4-1)^2},$$
$$= 0.9 \quad .$$

$$\mu_{A_4}(u_{11}) = \frac{(4 - s_A(u_{11}))^2}{1 + (4 - s_A(u_{11}))^2}, \quad (4.51)$$
$$= \frac{(4-1)^2}{1 + (4-1)^2},$$
$$= 0.9 \quad .$$

$$\mu_{A_4}(u_{12}) = \frac{(4 - s_A(u_{12}))^2}{1 + (4 - s_A(u_{12}))^2}, \quad (4.52)$$
$$= \frac{(4-1)^2}{1+(4-1)^2},$$
$$= 0.9 \ .$$

$$\vdots$$

$$\mu_{A_4}(u_{18}) = \frac{(4 - s_A(u_{18}))^2}{1 + (4 - s_A(u_{18}))^2}, \quad (4.53)$$
$$= \frac{(4-3)^2}{1+(4-3)^2},$$
$$= 0.50 \ .$$

$$\mu_{A_4}(u_{19}) = \frac{(4 - s_A(u_{19}))^2}{1 + (4 - s_A(u_{19}))^2}, \quad (4.54)$$
$$= \frac{(4-3)^2}{1+(4-3)^2},$$
$$= 0.50 \ .$$

$$\mu_{A_4}(u_{20}) = \frac{(4 - s_A(u_{20}))^2}{1 + (4 - s_A(u_{20}))^2}, \quad (4.55)$$
$$= \frac{(4-3)^2}{1+(4-3)^2},$$
$$= 0.50 \ .$$

$$\mu_{A_5}(u_1) = \frac{(5 - s_A(u_1))^2}{1 + (5 - s_A(u_1))^2}, \quad (4.56)$$
$$= \frac{(5-2)^2}{1+(5-2)^2},$$
$$= 0.9 \ .$$

$$
\begin{aligned}
\mu_{A_5}(u_2) &= \frac{(5 - s_A(u_2))^2}{1 + (5 - s_A(u_2))^2}, \\
&= \frac{(5-2)^2}{1+(5-2)^2}, \\
&= 0.9 \ \ .
\end{aligned} \tag{4.57}
$$

$$
\begin{aligned}
\mu_{A_5}(u_3) &= \frac{(5 - s_A(u_3))^2}{1 + (5 - s_A(u_3))^2}, \\
&= \frac{(5-2)^2}{1+(5-2)^2}, \\
&= 0.9 \ \ .
\end{aligned} \tag{4.58}
$$

$$\vdots$$

$$
\begin{aligned}
\mu_{A_5}(u_{10}) &= \frac{(5 - s_A(u_{10}))^2}{1 + (5 - s_A(u_{10}))^2}, \\
&= \frac{(5-1)^2}{1+(5-1)^2}, \\
&= 0.94 \ \ .
\end{aligned} \tag{4.59}
$$

$$
\begin{aligned}
\mu_{A_5}(u_{11}) &= \frac{(5 - s_A(u_{11}))^2}{1 + (5 - s_A(u_{11}))^2}, \\
&= \frac{(5-1)^2}{1+(5-1)^2}, \\
&= 0.94 \ \ .
\end{aligned} \tag{4.60}
$$

$$
\begin{aligned}
\mu_{A_5}(u_{12}) &= \frac{(5 - s_A(u_{12}))^2}{1 + (5 - s_A(u_{12}))^2}, \\
&= \frac{(5-1)^2}{1+(5-1)^2}, \\
&= 0.94 \ \ .
\end{aligned} \tag{4.61}
$$

$$\vdots$$

$$\mu_{A_5}(u_{18}) = \frac{(5 - s_A(u_{18}))^2}{1 + (5 - s_A(u_{18}))^2}, \tag{4.62}$$
$$= \frac{(5-3)^2}{1+(5-3)^2},$$
$$= 0.80 \ .$$

$$\mu_{A_5}(u_{19}) = \frac{(5 - s_A(u_{19}))^2}{1 + (5 - s_A(u_{19}))^2}, \tag{4.63}$$
$$= \frac{(5-3)^2}{1+(5-3)^2},$$
$$= 0.80 \ .$$

$$\mu_{A_5}(u_{20}) = \frac{(5 - s_A(u_{20}))^2}{1 + (5 - s_A(u_{20}))^2}, \tag{4.64}$$
$$= \frac{(5-3)^2}{1+(5-3)^2},$$
$$= 0.80 \ .$$

The foregoings, based on the first constraint, calculates the degree of membership of each object(graduate) u_i in each state

In a similar manner, we now proceed to calculate the degree of membership of each object(graduate) in each state, under the second constraint.

Constraint B, μ_B: No student should be posted to their State of Studies

$$\mu_{B_j}(u_i) = \frac{(j - s_B(u_i))^2}{1 + (j - s_B(u_i))^2} \tag{4.65}$$

where $\mu_{B_j}(u_i)$ measures the degree of object u_i's membership in State j under the consideration of the *state of origin* constraint and, $s_B(u_i)$ represents the index of graduate u_i's State of Study

$$\mu_{B_1}(u_1) = \frac{(1 - s_B(u_1))^2}{1 + (1 - s_B(u_1))^2}, \tag{4.66}$$
$$= \frac{(1-1)^2}{1+(1-1)^2},$$
$$= 0 \ .$$

$$
\begin{aligned}
\mu_{B_1}(u_2) &= \frac{(1-s_B(u_2))^2}{1+(1-s_B(u_2))^2}, \\
&= \frac{(1-2)^5}{1+(1-5)^2}, \\
&= 0.94 \ \ .
\end{aligned} \tag{4.67}
$$

$$
\begin{aligned}
\mu_{B_1}(u_3) &= \frac{(1-s_B(u_3))^2}{1+(1-s_B(u_3))^2}, \\
&= \frac{(1-3)^2}{1+(1-3)^2}, \\
&= 0.80 \ \ .
\end{aligned} \tag{4.68}
$$

$$\vdots$$

$$
\begin{aligned}
\mu_{B_1}(u_{10}) &= \frac{(1-s_B(u_{10}))^2}{1+(1-s_B(u_{10}))^2}, \\
&= \frac{(1-5)^2}{1+(1-5)^2}, \\
&= 0.94 \ \ .
\end{aligned} \tag{4.69}
$$

$$
\begin{aligned}
\mu_{B_1}(u_{11}) &= \frac{(1-s_B(u_{11}))^2}{1+(1-s_B(u_{11}))^2}, \\
&= \frac{(1-3)^2}{1+(1-3)^2}, \\
&= 0.80 \ \ .
\end{aligned} \tag{4.70}
$$

$$
\begin{aligned}
\mu_{B_1}(u_{12}) &= \frac{(1-s_B(u_{12}))^2}{1+(1-s_B(u_{12}))^2}, \\
&= \frac{(1-2)^2}{1+(1-2)^2}, \\
&= 0.50 \ \ .
\end{aligned} \tag{4.71}
$$

$$\vdots$$

$$\begin{aligned} \mu_{B_1}(u_{18}) &= \frac{(1 - s_B(u_{18}))^2}{1 + (1 - s_B(u_{18}))^2}, \\ &= \frac{(1-5)^2}{1+(1-5)^2}, \\ &= 0.94 \ . \end{aligned} \tag{4.72}$$

$$\begin{aligned} \mu_{B_1}(u_{19}) &= \frac{(1 - s_B(u_{19}))^2}{1 + (1 - s_B(u_{19}))^2}, \\ &= \frac{(1-2)^2}{1+(1-2)^2}, \\ &= 0.5 \ . \end{aligned} \tag{4.73}$$

$$\begin{aligned} \mu_{B_1}(u_{20}) &= \frac{(1 - s_B(u_{20}))^2}{1 + (1 - s_B(u_{20}))^2}, \\ &= \frac{(1-4)^2}{1+(1-4)^2}, \\ &= 0.90 \ . \end{aligned} \tag{4.74}$$

$$\begin{aligned} \mu_{B_2}(u_1) &= \frac{(2 - s_B(u_1))^2}{1 + (2 - s_B(u_1))^2}, \\ &= \frac{(2-1)^2}{1+(2-1)^2}, \\ &= 0.5 \ . \end{aligned} \tag{4.75}$$

$$\begin{aligned} \mu_{B_2}(u_2) &= \frac{(2 - s_B(u_2))^2}{1 + (2 - s_B(u_2))^2}, \\ &= \frac{(2-2)^5}{1+(2-5)^2}, \\ &= 0.9 \ . \end{aligned} \tag{4.76}$$

$$\begin{aligned} \mu_{B_2}(u_3) &= \frac{(2 - s_B(u_3))^2}{1 + (2 - s_B(u_3))^2}, \\ &= \frac{(2-3)^2}{1+(2-3)^2}, \\ &= 0.5 \ . \end{aligned} \tag{4.77}$$

$$\vdots$$

$$\begin{aligned}\mu_{B_2}(u_{10}) &= \frac{(2-s_B(u_{10}))^2}{1+(2-s_B(u_{10}))^2}, \\ &= \frac{(2-5)^2}{1+(2-5)^2}, \\ &= 0.9 \ \ .\end{aligned} \tag{4.78}$$

$$\begin{aligned}\mu_{B_2}(u_{11}) &= \frac{(2-s_B(u_{11}))^2}{1+(2-s_B(u_{11}))^2}, \\ &= \frac{(2-3)^2}{1+(2-3)^2}, \\ &= 0.5 \ \ .\end{aligned} \tag{4.79}$$

$$\begin{aligned}\mu_{B_2}(u_{12}) &= \frac{(2-s_B(u_{12}))^2}{1+(2-s_B(u_{12}))^2}, \\ &= \frac{(2-2)^2}{1+(2-2)^2}, \\ &= 0 \ \ .\end{aligned} \tag{4.80}$$

$$\vdots$$

$$\begin{aligned}\mu_{B_2}(u_{18}) &= \frac{(2-s_B(u_{18}))^2}{1+(2-s_B(u_{18}))^2}, \\ &= \frac{(2-5)^2}{1+(2-5)^2}, \\ &= 0.9 \ \ .\end{aligned} \tag{4.81}$$

$$\begin{aligned}\mu_{B_2}(u_{19}) &= \frac{(2-s_B(u_{19}))^2}{1+(2-s_B(u_{19}))^2}, \\ &= \frac{(2-2)^2}{1+(2-2)^2}, \\ &= 0 \ \ .\end{aligned} \tag{4.82}$$

$$\begin{aligned}\mu_{B_2}(u_{20}) &= \frac{(2-s_B(u_{20}))^2}{1+(2-s_B(u_{20}))^2}, \\ &= \frac{(2-4)^2}{1+(2-4)^2}, \\ &= 0.8 \ \ .\end{aligned} \tag{4.83}$$

$$\begin{aligned}\mu_{B_3}(u_1) &= \frac{(3-s_B(u_1))^2}{1+(3-s_B(u_1))^2}, \\ &= \frac{(3-1)^2}{1+(3-1)^2}, \\ &= 0.8 \ \ .\end{aligned} \tag{4.84}$$

$$\begin{aligned}\mu_{B_3}(u_2) &= \frac{(3-s_B(u_2))^2}{1+(3-s_B(u_2))^2}, \\ &= \frac{(3-2)^5}{1+(3-5)^2}, \\ &= 0.8 \ \ .\end{aligned} \tag{4.85}$$

$$\begin{aligned}\mu_{B_3}(u_3) &= \frac{(3-s_B(u_3))^2}{1+(3-s_B(u_3))^2}, \\ &= \frac{(3-3)^2}{1+(3-3)^2}, \\ &= 0 \ \ .\end{aligned} \tag{4.86}$$

$$\vdots$$

$$\begin{aligned}\mu_{B_3}(u_{10}) &= \frac{(3-s_B(u_{10}))^2}{1+(3-s_B(u_{10}))^2}, \\ &= \frac{(3-5)^2}{1+(3-5)^2}, \\ &= 0.8 \ \ .\end{aligned} \tag{4.87}$$

$$\mu_{B_3}(u_{11}) = \frac{(3 - s_B(u_{11}))^2}{1 + (3 - s_B(u_{11}))^2}, \tag{4.88}$$
$$= \frac{(3-3)^2}{1+(3-3)^2},$$
$$= 0 \quad .$$

$$\mu_{B_3}(u_{12}) = \frac{(3 - s_B(u_{12}))^2}{1 + (3 - s_B(u_{12}))^2}, \tag{4.89}$$
$$= \frac{(3-2)^2}{1+(3-2)^2},$$
$$= 0.5 \quad .$$

$$\vdots$$

$$\mu_{B_3}(u_{18}) = \frac{(3 - s_B(u_{18}))^2}{1 + (3 - s_B(u_{18}))^2}, \tag{4.90}$$
$$= \frac{(3-5)^2}{1+(3-5)^2},$$
$$= 0.80 \quad .$$

$$\mu_{B_3}(u_{19}) = \frac{(3 - s_B(u_{19}))^2}{1 + (3 - s_B(u_{19}))^2}, \tag{4.91}$$
$$= \frac{(3-2)^2}{1+(3-2)^2},$$
$$= 0.50 \quad .$$

$$\mu_{B_3}(u_{20}) = \frac{(3 - s_B(u_{20}))^2}{1 + (3 - s_B(u_{20}))^2}, \tag{4.92}$$
$$= \frac{(3-4)^2}{1+(3-4)^2},$$
$$= 0.50 \quad .$$

$$\mu_{B_4}(u_1) = \frac{(4 - s_B(u_1))^2}{1 + (4 - s_B(u_1))^2}, \tag{4.93}$$
$$= \frac{(4-1)^2}{1+(4-1)^2},$$
$$= 0.5 \ \ .$$

$$\mu_{B_4}(u_2) = \frac{(4 - s_B(u_2))^2}{1 + (4 - s_B(u_2))^2}, \tag{4.94}$$
$$= \frac{(4-2)^5}{1+(4-5)^2},$$
$$= 0.5 \ \ .$$

$$\mu_{B_4}(u_3) = \frac{(4 - s_B(u_3))^2}{1 + (4 - s_B(u_3))^2}, \tag{4.95}$$
$$= \frac{(4-3)^2}{1+(4-3)^2},$$
$$= 0.5 \ \ .$$

$$\vdots$$

$$\mu_{B_4}(u_{10}) = \frac{(4 - s_B(u_{10}))^2}{1 + (4 - s_B(u_{10}))^2}, \tag{4.96}$$
$$= \frac{(4-5)^2}{1+(4-5)^2},$$
$$= 0.5 \ \ .$$

$$\mu_{B_4}(u_{11}) = \frac{(4 - s_B(u_{11}))^2}{1 + (4 - s_B(u_{11}))^2}, \tag{4.97}$$
$$= \frac{(4-3)^2}{1+(4-3)^2},$$
$$= 0.5 \ \ .$$

$$\mu_{B_4}(u_{12}) = \frac{(4 - s_B(u_{12}))^2}{1 + (4 - s_B(u_{12}))^2}, \tag{4.98}$$
$$= \frac{(4-2)^2}{1+(4-2)^2},$$
$$= 0.80 \ .$$

$$\mu_{B_4}(u_{18}) = \frac{(4 - s_B(u_{18}))^2}{1 + (4 - s_B(u_{18}))^2}, \tag{4.99}$$
$$= \frac{(4-5)^2}{1+(4-5)^2},$$
$$= 0.50 \ .$$

$$\mu_{B_4}(u_{19}) = \frac{(4 - s_B(u_{19}))^2}{1 + (4 - s_B(u_{19}))^2}, \tag{4.100}$$
$$= \frac{(4-2)^2}{1+(4-2)^2},$$
$$= 0.80 \ .$$

$$\mu_{B_4}(u_{20}) = \frac{(4 - s_B(u_{20}))^2}{1 + (4 - s_B(u_{20}))^2}, \tag{4.101}$$
$$= \frac{(4-4)^2}{1+(4-4)^2},$$
$$= 0 \ .$$

$$\mu_{B_5}(u_1) = \frac{(5 - s_B(u_1))^2}{1 + (5 - s_B(u_1))^2}, \tag{4.102}$$
$$= \frac{(5-1)^2}{1+(5-1)^2},$$
$$= 0.94 \ .$$

$$\mu_{B_5}(u_2) = \frac{(5 - s_B(u_2))^2}{1 + (5 - s_B(u_2))^2}, \tag{4.103}$$
$$= \frac{(5-2)^5}{1+(5-5)^2},$$
$$= 0 \ .$$

$$\begin{aligned}\mu_{B_5}(u_3) &= \frac{(5 - s_B(u_3))^2}{1 + (5 - s_B(u_3))^2}, \\ &= \frac{(5-3)^2}{1+(5-3)^2}, \\ &= 0.80 \ .\end{aligned} \tag{4.104}$$

$$\vdots$$

$$\begin{aligned}\mu_{B_5}(u_{10}) &= \frac{(5 - s_B(u_{10}))^2}{1 + (5 - s_B(u_{10}))^2}, \\ &= \frac{(5-5)^2}{1+(5-5)^2}, \\ &= 0 \ .\end{aligned} \tag{4.105}$$

$$\begin{aligned}\mu_{B_5}(u_{11}) &= \frac{(5 - s_B(u_{11}))^2}{1 + (5 - s_B(u_{11}))^2}, \\ &= \frac{(5-3)^2}{1+(5-3)^2}, \\ &= 0.80 \ .\end{aligned} \tag{4.106}$$

$$\begin{aligned}\mu_{B_5}(u_{12}) &= \frac{(5 - s_B(u_{12}))^2}{1 + (5 - s_B(u_{12}))^2}, \\ &= \frac{(5-2)^2}{1+(5-2)^2}, \\ &= 0.90 \ .\end{aligned} \tag{4.107}$$

$$\vdots$$

$$\begin{aligned}\mu_{B_5}(u_{18}) &= \frac{(5 - s_B(u_{18}))^2}{1 + (5 - s_B(u_{18}))^2}, \\ &= \frac{(5-5)^2}{1+(5-5)^2}, \\ &= 0 \ .\end{aligned} \tag{4.108}$$

$$\begin{aligned}\mu_{B_5}(u_{19}) &= \frac{(5 - s_B(u_{19}))^2}{1 + (5 - s_B(u_{19}))^2}, \\ &= \frac{(5-2)^2}{1+(5-2)^2}, \\ &= 0.90 \ \ .\end{aligned} \tag{4.109}$$

$$\begin{aligned}\mu_{B_5}(u_{20}) &= \frac{(5 - s_B(u_{20}))^2}{1 + (5 - s_B(u_{20}))^2}, \\ &= \frac{(5-4)^2}{1+(5-4)^2}, \\ &= 0.50 \ \ .\end{aligned} \tag{4.110}$$

Now, since our sample problem set defines two Constraints, $\mu_{A_i}(u_j)$ and $\mu_{B_i}(u_j)$ *for i=1,2,...,5 and j=1,2,...,20*, we will now proceed to estimate a single membership probability $\mu_i(u_j)$ *for i=1,2,...,5 and j=1,2,...,20*, which is estimated by:

$$\mu_i(u_j) = \mu_{A_i}(u_j) \cdot \mu_{B_i}(u_j) \tag{4.111}$$

$$\begin{aligned}
\mu_1(u_1) &= \mu_{A_1}(u_1) \cdot \mu_{B_1}(u_1) && (4.112)\\
&= 0.50 \times 0,\\
&= 0\\
\mu_1(u_2) &= \mu_{A_1}(u_2) \cdot \mu_{B_1}(u_2) && (4.113)\\
&= 0.50 \times 0.94,\\
&= 0.47\\
\mu_1(u_3) &= \mu_{A_1}(u_3) \cdot \mu_{B_1}(u_3) && (4.114)\\
&= 0.50 \times 0.50,\\
&= 0.25\\
&\vdots\\
\mu_3(u_4) &= \mu_{A_3}(u_4) \cdot \mu_{B_3}(u_4) && (4.115)\\
&= 0.50 \times 0.80,\\
&= 0.40\\
&\vdots\\
\mu_5(u_{20}) &= \mu_{A_5}(u_{20}) \cdot \mu_{B_5}(u_{20}) && (4.116)\\
&= 0.80 \times 0.50,\\
&= 0.40\\
& && (4.117)
\end{aligned}$$

Which we can more concisely represent in the following sets:

$$\mu_i = \{\mu_i(u_1), \mu_i(u_2), \mu_i(u_3), \ldots, \mu_i(u_2 0)\} \qquad \textit{for i=1,2,\ldots,5} \tag{4.118}$$

So that

$$\begin{aligned}\mu_1 &= \{\mu_1(u_1), \mu_1(u_2), \mu_1(u_3), \ldots, \mu_1(u_20)\} \\ &= \{0, 0.47, 0.40, \ldots, 0.90\},\end{aligned} \tag{4.119}$$

$$\begin{aligned}\mu_2 &= \{\mu_2(u_1), \mu_2(u_2), \mu_2(u_3), \ldots, \mu_2(u_20)\} \\ &= \{0, 0, 0, \ldots, 0.40\},\end{aligned} \tag{4.120}$$

$$\begin{aligned}\mu_3 &= \{\mu_3(u_1), \mu_3(u_2), \mu_3(u_3), \ldots, \mu_3(u_20)\} \\ &= \{0.40, 0.40, 0, \ldots, 0\},\end{aligned} \tag{4.121}$$

$$\begin{aligned}\mu_4 &= \{\mu_4(u_1), \mu_4(u_2), \mu_4(u_3), \ldots, \mu_4(u_20)\} \\ &= \{0.40, 0.40, 0.40, \ldots, 0\},\end{aligned} \tag{4.122}$$

$$\begin{aligned}\mu_5 &= \{\mu_5(u_1), \mu_5(u_2), \mu_5(u_3), \ldots, \mu_5(u_20)\} \\ &= \{0.84, 0, 0.72, \ldots, 0.40\}\end{aligned} \tag{4.123}$$

i	μ_{A_1}	μ_{B_1}	$\mu_1 = \mu_{A_1} \cdot \mu_{B_1}$	μ_{A_2}	μ_{B_2}	$\mu_2 = \mu_{A_2} \cdot \mu_{B_2}$	μ_{A_3}	μ_{B_3}	$\mu_{A_3} \cdot \mu_{B_3}$	μ_{A_4}	μ_{B_4}	$\mu_{A_4} \cdot \mu_{B_4}$	μ_{A_5}	μ_{B_5}	$\mu_{A_5} \cdot \mu_{B_5}$
1.	0.5	0	0	0	0.5	0	0.5	0.8	0.4	0.8	0.5	0.4	0.9	0.94	0.84
2.	0.5	0.94	0.47	0	0.5	0	0.5	0.8	0.4	0.8	0.5	0.4	0.9	0	0
3.	0.5	0.80	0.40	0	0.5	0	0.5	0	0	0.8	0.5	0.4	0.9	0.8	0.72
4.	0.5	0.5	0.25	0	0	0	0.5	0.5	0.25	0.8	0.8	0.64	0.9	0.9	0.81
5.	0.9	0	0	0.8	0.5	0.4	0.5	0.8	0.4	0	0.9	0	0.5	0.94	0.47
6.	0.9	0.94	0.85	0.8	0.9	0.72	0.5	0.8	0.4	0	0.5	0	0.5	0	0
7.	0.9	0.8	0.72	0.8	0.5	0.4	0.5	0	0	0	0.5	0	0.5	0.8	0.4
8.	0.9	0.5	0.45	0.8	0	0	0.5	0.5	0.25	0	0.8	0	0.5	0.9	0.45
9.	0	0	0	0	0.5	0	0.8	0.8	0.64	0.9	0.9	0.81	0.94	0.94	0.88
10.	0	0	0	0.5	0.5	0.25	0.8	0.8	0.64	0.9	0.5	0.45	0.94	0	0
11.	0	0.8	0	0.5	0.5	0.25	0.8	0	0	0	0.5	0.45	0.94	0.8	0.75
12.	0	0.5	0	0.5	0.5	0.25	0.8	0.5	0.45	0.9	0.8	0.72	0.94	0.9	0.84
13.	0.94	0	0	0.9	0.5	0.45	0.8	0.8	0.64	0.5	0.9	0.45	0	0.94	0
14.	0.94	0	0	0	0.5	0	0.8	0	0	0.5	0.5	0.25	0	0.8	0
15.	0.94	0.8	0.75	0.7	0.5	0.45	0.8	0.5	0.4	0.5	0.8	0.4	0	0.90	0
16.	0.94	0.9	0.85	0.9	0.8	0.72	0.8	0.5	0.4	0.5	0	0	0	0.5	0
17.	0.8	0	0	0.5	0.5	0.25	0.9	0.8	0	0.5	0.9	0.45	0.8	0.94	0.75
18.	0.8	0.94	0.75	0.5	0.9	0.45	0	0.8	0	0.5	0.5	0.25	0.8	0	0
19.	0.8	0.5	0.4	0.5	0	0	0	0.5	0	0.5	0.8	0.4	0.8	0.9	0.72
20.	0.8	0.9	0.72	0.5	0.8	0.40	0	0.5	0	0.5	0	0	0.8	0.5	0.4

Table 4.3: Summary of the constraint values

4.6 Populating the States

Having achieved a well defined membership probability for each of the graduates into each of the states, in the previous section, we now proceed to assigning the most qualifying candidates to each state, while not exceeding each State's defined expected partition size, capacity that is. Recall that from equations 4.14 through 4.18, we defined the expected assignment size of each state to be:

$$| p_1 | = 3$$

$$| p_2 | = 2$$

$$| p_3 | = 1$$

$$| p_4 | = 5$$

$$| p_5 | = 9$$

Now, for each State,p_i, we select the first $| p_i |$ candidates, u_j with the highest membership $\mu_i(u_j)$ probability in order, for each State i=1,2,...,5:

$$p_1 = \{u_6(0.85), u_{16}(0.84), u_{14}(0.74)\} \tag{4.124}$$

$$p_2 = \{u_{10}(0.45), u_{13}(0.45)\} \tag{4.125}$$

$$p_3 = \{u_9(0.64)\} \tag{4.126}$$

$$p_4 = \{u_{12}(0.72), u_4(0.64), u_{11}(0.45), u_{17}(0.45), u_1(0.45)\} \tag{4.127}$$

$$p_5 = \{u_3(0.72), u_{19}(0.72), u_8(0.45), u_5(0.47), u_7(0.40), u_{20}(0.40), u_2(0), u_{18}(0), u_{15}(0)\} \tag{4.128}$$

After successfully applying the algorithm developed in the previous section to make the assignment on the sample problem, we have been able to reach an assignment explained thus:

4.6.1 The Assignment Table

Name	State of Origin	College	Posting	Membership Degree
GR06	Akwa-Ibom	BUK, Kano	Osun	85%
GR16	Kano	AKSU, Akwa-Ibom	Osun	84%
GR14	Kano	NSUKKA, Enugu	Osun	74%

Name	State of Origin	College	Posting	Membership Degree
GR10	Osun	BUK, Kano	Kwara	45%
GR13	Kano	OUI, Osun	Kwara	45%
GR09	Osun	OAU, Osun	Enugu	64%
GR12	Osun	UNILORIN, Kwara	Akwa-Ibom	72%
GR04	Kwara	UNILORIN, Kwara	Akwa-Ibom	64%
GR11	Osun	NSUKKA, Enugu	Akwa-Ibom	45%
GR17	Enugu	UNIOSUN, Osun	Akwa-Ibom	45%
GR01	Kwara	OAU, Osun	Akwa-Ibom	45%
GR03	Kwara	NSUKKA, Enugu	Kano	88%
GR19	Enugu	UNILORIN, Kwara	Kano	72%
GR08	Akwa-Ibom	UNILORIN, Kwara	Kano	72%
GR05	Akwa-Ibom	OAU, Osun	Kano	47%
GR07	Akwa-Ibom	NSUKKA, Enugu	Kano	45%
GR20	Enugu	AKSU, Akwa-Ibom	Kano	40%
GR02	Kwara	BUK, Kano	Kano	0%
GR18	Enugu	BUK, Kano	Kano	0%
GR15	Kano	UNILORIN, Kwara	Kano	0%

Table 4.4: Showing the NYSC Deployment Schedule

Chapter 5

Discussion and Conclusion

5.1 Discussion of Results

From the assignment achieved in the previous Chapter, the total optimal assignment is 17 out of 20, which is a score of 85%, with an average membership score of 51.6%. This algorithm can be assessed to have performed good on the task, nonethless, we make several points for possible improvements that could be explored to improve the result.

5.2 Improvement Suggestions

Although, the sample problem outlined in the previous chapter got a 85% assignment accuracy, by making a few adjustments, we observe that the accuracy could be raised as high as 100%

1. When choosing objects into each partition, keep a tab on the number of non-zero assignable members left for each class, if this number is equal to the expected partition size of any class, proceed to make assignments into the class in question. This precaution ensures that each class gets the opportunity to get the maximum number assignments with fitness value different from zero.

2. As would be observed, the assignment of probabilities to each partition proceeds partition-wise, that is, iterating from partition to partition and filling them up in turn with the highest-probability member. We also exercised the member-wise assignment, that is, iterating through each object, and assigning each based on its highest probability, taking caution not to exceed the partition width. Putting this method to practise improved the assignment from 85% to 90%, which is a significant improvement.

3. As cited in the previous point, several routes can be taken to assign objects to the appropriate partitions. Since the candidate-wise iteration introduces some improvements to the assignment, we would investigate further to identify a 'hybrid' algorithm which combines both partition-based and object-based algorithm to make assignment, and this promises to further raise the accuracy level of the assignment schedule.

5.3 Further Work

As must have been observed in the algorithm designed in this text, an attempt was made to solve a simplified version of the University Timetabling problem. Here, the Constraints are of a simpler construct with no inter-constraint interaction. Such that, by simply multiplying out the individual constraints, we were able to estimate the degree of membership of each object in each partition. In the case of the University Timetabling problem, several constraints have more complex interactions which may not combine simply by multiplying out. The successful assignmt schedule achieved in this text would be improved upon for this cause.

References

1. Arjang, A. A., & Saul, I. G. (2011). Profiles in operations research - pioneers and innovators. *International Series in Operations Research & Management Science*, *147*.
2. Dantzig, G. B., & Thapa, M. N. (1997). *Linear programming 1: Introduction.* Berlin, Heidelberg: Springer-Verlag.
3. Dantzig, G. B., & Thapa, M. N. (2003). *Linear programming 2: Theory and extensions.* Berlin, Heidelberg: Springer-Verlag.
4. ITC. (2019). *https://www.itc2019.org/home.* Retrieved from `https://www.itc2019.org/home`
5. Lee, K. H. (2005). *First course on fuzzy set theory and applications.* Berlin, Heidelberg: Springer-Verlag.
6. Sanchetti, D., & Kapoor, V. (2010). *Statistics (theory, methods and applications).* New Delhi, India: Sultan Chand and Sons.
7. Schrijver, A. (2001, 08). On the history of combinatorial optimization (till 1960). *Handbooks in Operations Research and Management Science*, *12*. doi: DOI10.1016/S0927-0507(05)12001-5
8. Schrijver, A. (2017). *A course in combinatorial optimization.* Berlin, Heidelberg: Springer-Verlag.