Chapter 1

Introduction

In everyday human and industrial activities, we are faced with the challenges of optimal resource assignment. In the face of limited resources, low tolerance for scrap and rework, need for greater degree of accuracy and scalability, optimization is paramount.

In introducing this study, we shall proceed under the following sub-headings:

- Preamble
- Motivation
- Objective
- Definition of terms

1.1 Preamble

Over the years, several Mathematical Optimization methods in Operations Research (OR) especially, have been developed to help with these class of problems with a good amount of success recorded. Nonetheless, there still remains a great room for improvement on these. Classical approaches to solving assignment problems include the well revered Linear Programming techniques.

"Linear Programming is a mathematical technique for determining the optimal allocation of resources and obtaining a particular objective(i.e., cost minimization or inversely profit maximization) when there are alternative uses of the resources: Land, Capital, Materials, Machines, etc"

Solving an assignment problem with Linear Programming techniques, often requires a clear statement of an Objective function, resources constraints, usually expressed as a set of multivariate linear equations to be solved. A key definition of Linear Programming goes thus:

For example, the general formulation of a Linear Programming Problem is as follows:

$$Let\ Z\ be\ a\ linear\ function\ defined\ by$$

$$(i)\ Z = c_i x_i + c_2 x_2 + \ldots + c_n x_n$$

$$where\ c_j's\ are\ constants.$$

$$Let\ (a_{ij})\ be\ mn\ constants\ \text{and}\ let\ (b_i)\ be\ a\ set\ of\ m\ constants\ such\ that$$

$$(ii)\ a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n (\leq, =, \geq)\ b_1$$

$$a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n (\leq, =, \geq)\ b_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + a_{mn} x_n (\leq, =, \geq)\ b_n$$

$$(iii)\ x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$

The problem of determining the values of $x_{1,} x_{2,\ldots}, x_n$, *which makes* Z *a minimum* (or *maxiumum*) and *which satisfies* (ii) and (iii) *is called the* ***General Linear Programming Problem***

Linear Programming techniques, as well as other Mathematical Optimization techniques have been reputed a great success, nonetheless there are several yet unaddressed perspectives of optimum assignmemt problems.

1.2 Motivation

This study takes a whole new look at the Mathematical Optimization: what if our optimization goal doesn’t involve minimizing (or maximizing) any single variable such as cost or profit, or even price? What if we’re on the other hand interested in

Allocation of objects to non-overlapping groups,

based on properties possessed by the objects,

and one or more constraining conditions

Mathematical Optimization techniques, have not been formulated to solve problems as this, and thus the motivation of this study.

<u>Sample Problem</u>: Deploying NYSC Corp Members To Serving States

In deploying intending NYSC members to a state, the following constraints should be satisfied:

- No student should be deployed to his state of origin
- No student should be deployed to his state of study
- All enlisted students must be deployed to a state
- No state should be allocated more student than it has resources to accommodate

The foregoing problem statement needs a rather new approach as the OR methods aren’t designed to solve problems like this, thus the motivation for this study.

1.3 Objective

The aim of this study is to

1. Develop a mathematical model to solve assignment problems involving distribution of candidate objects into non-overlapping groups putting into consideration several constraining conditions
2. Create a simulation putting this model to work, on a sample practical problem
3. Sketch a path to evolving this class of solution to more complex problems with similar formulation

1.4 Definitions

Below are definition of terms encountered severally in the text. The user is assumed familiar with the conventional set notations and operations.

1. *Search Space*

   The group of all items to be assigned to various group is called the search space of the problem

2. *Partitions of a set X*

   Let X be a set and U, V be disjoint subsets of X. The sets U and V are said to be partitions of X if $U \cup V = X$ and $U \cap V = \emptyset$. If such sets U and V exist, they are said to be the separation of the set X

3. *Feasible Solution*

   Any successful assignment of the objects to groups that satisfy all constraints is referred to as a *feasible solution*

4. *Constraints*

   These are conditions that restrict the domain of our feasible solution and thus state the conditions under which any given solution is deemed acceptable

   Example of constraints are as follows:

   - Each group has a maximum capacity that cannot be exceeded

   - All objects in the search space must be assigned

- No object must be assigned more than once
- Whenever possible, don’t leave a group empty, etc

Furthermore, all mathematical notations contained in the text retain their usual meanings, unless otherwise stated.