Solution to Attribute-Based Assignment Problems

A Primer to the University Timetabling Problem

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Introduction

Claim

Given any set of objects $U = \{u_1, u_2, ..., u_n\}$ each possessing certain attributes, and a set of constraining conditions for assignment, we can:

1. Construct functions describing those constraints
2. Assign each object to a class of best fit

The Nature of Attributes

Definition

An Attribute is any characteristic of a group of objects that can serve as a basis for classification

Example

- Age
- State of Origin
- Courses offered in current Semester
- Body mass
- O'Level Results
- etc...

Partitions

Definition

A Partition, is a classification of objects based on an ideal value of one or more `attributes`

Example

- Teens, Young Adults,... (based on the Age attribute of objects)
- Hausa, Igbo, Yoruba,... (based on the State of Origin attribute)
- etc...

Constraints

Definition

A Constraint function, μ, matches the Ideal value of a partition, p_i, to the `attribute` value of an object, u_j, and assigns a membership value viz:

$$\mu(p_i, u_j) \mapsto [0, 1] \quad i,j \in \mathbf{Z}$$

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Example

NYSC: No student should be posted to their State of Origin.

ANALYSIS: Students should be posted as far as possible from their States of Origin

ATTRIBUTE: The distance of each state from a common point

Classes of Constraints

Definition

- **Binary Constraints** assign a membership value $v \in \{0, 1\}$ to each object
- **Continuous Constraints** assign a membership value $v \in [0, 1]$ to each object

Definition

- **Inclusion Constraints** admit objects based on proximity to the `ideal value`
- **Exclusion Constraints** admit objects based on distance from the `ideal value`

The NYSC Posting Problem

Problem Statement

Given a number of graduates, we seek to post them to Service States under the following constraints:

1. No students should be posted to their State of Origin
2. No students should be posted to their States of study
3. The maximum capacity of each state must not be exceeded
4. All students should be assigned

The States - Partitions

Serial	State	Capacity
1	Osun State	3
2	Kwara State	2
3	Enugu State	1
4	Akwa-Ibom State	5
5	Kano State	1

The Graduates - Objects

Serial	Name	State of Origin	University
u_1	GR01	Kwara(2)	OAU(1)
u_2	GR02	Kwara(2)	BUK(5)
u_3	GR03	Kwara(2)	NSUKKA(3)
u_4	GR04	Kwara(2)	UNILORIN(2)
u_5	GR05	Akwa-Ibom(4)	OAU(1)
u_6	GR06	Akwa-Ibom(4)	BUK(5)
u_7	GR07	Akwa-Ibom(4)	NSUKKA(3)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_{16}	GR16	Kano(5)	AKSU(4)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_{20}	GR20	Enugu(3)	AKSU(4)

Constraint A, $\mu_A \colon u \mapsto [0,1]$

Constraint A, μ_A: No student should be posted to their State of Origin

Illustration

$$\mu_{A_j}(u_i) = \frac{\left(j - s_A(u_i)\right)^2}{1 + \left(j - s_A(u_i)\right)^2}$$

Example

$$\begin{aligned}\mu_{A_1}(u_1) &= \frac{\left(1 - s_A(u_1)\right)^2}{1 + \left(1 - s_A(u_1)\right)^2} \\ &= \frac{(1-2)^2}{1+(1-2)^2} \\ &= 0.5 \\ &\vdots\end{aligned}$$

Constraint B, $\mu_B : u \mapsto [0, 1]$

Constraint B, μ_B: No student should be posted to their State of studies

Illustration

$$\mu_{B_j}(u_i) = \frac{(j - s_B(u_i))^2}{1 + (j - s_B(u_i))^2}$$

Example

$$\begin{aligned}\mu_{B_1}(u_1) &= \frac{(1 - s_B(u_1))^2}{1 + (1 - s_B(u_1))^2} \\ &= \frac{(1 - 1)^2}{1 + (1 - 1)^2} \\ &= 0 \\ &\vdots\end{aligned}$$

The Solution

Name	S. of Origin	University	Posting	Membership
GR06	Akwa-Ibom	BUK, Kano	Osun	85%
GR16	Kano	AKSU, Akwa-Ibom	Osun	84%
GR04	Kwara	UNILORIN, Kwara	Akwa-Ibom	64%
GR01	Kwara	OAU, Osun	Akwa-Ibom	45%
GR03	Kwara	NSUKKA, Enugu	Kano	88%
GR05	Akwa-Ibom	OAU, Osun	Kano	47%
GR07	Akwa-Ibom	NSUKKA, Enugu	Kano	45%
GR20	Enugu	AKSU, Akwa-Ibom	Kano	40%
⋮	⋮	⋮	⋮	⋮

The Presentation

Thank You!