ECG Classification with Deep Unfolding Variable Projection Network

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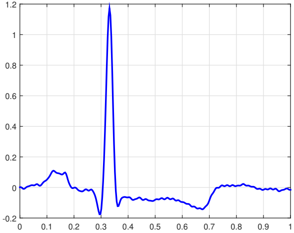
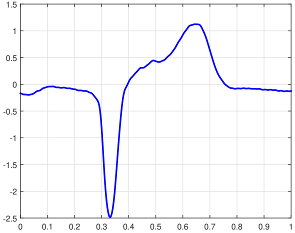
Outline

Motivations





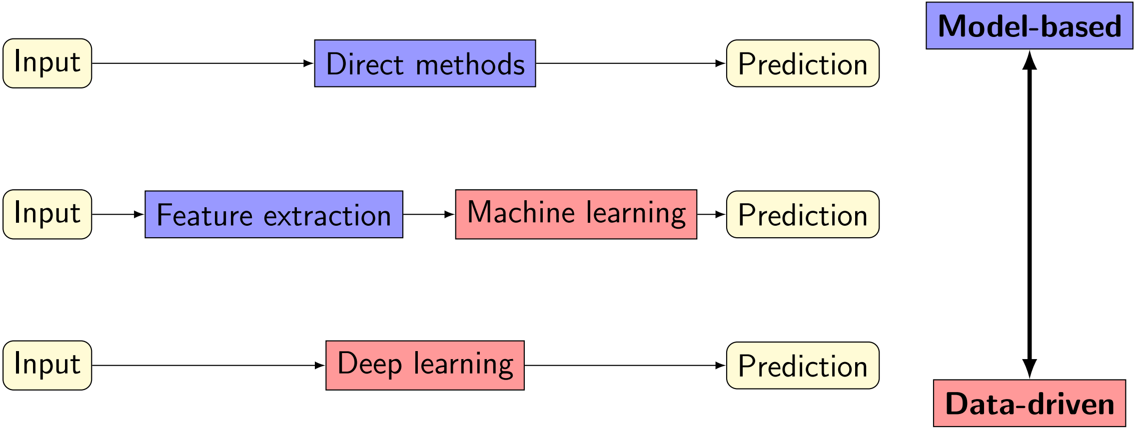
Tasks

- Biomedical signal processing via modeling and machine learning
- *ECG heartbeat classification for arrhythmia detection*

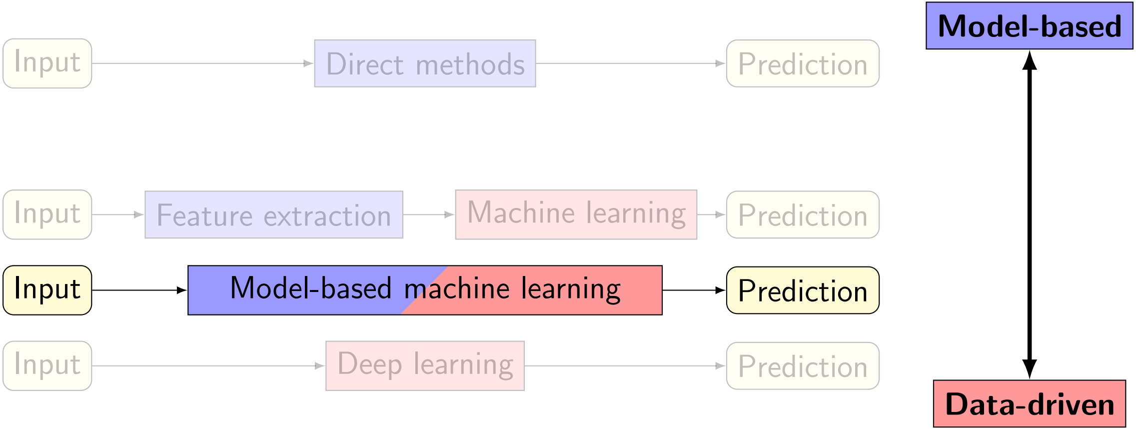
Expectations

- Accuracy, efficiency, explainability

Prediction techniques



Prediction techniques



Model-based machine learning

Advantages

- *Bridge between model-based direct methods and machine learning*
- Domain knowledge incorporation
- Model-based representation learning
- Compact, low-dimensional, optimized representation
- Interpretable parameters, explainable representation

Challenges

- *Why?* – modeling vs. learning
- *What?* – model selection, parametrization, mathematical description
- *How?* – specialized architecture development

Outline

Variable projections[1] (VP, VarPro)

Separable non-linear least squares

- Parametric function system: $\Phi_k(\theta) \in \mathbb{R}^m$, θ: non-linear system parameters
- Non-linear modeling problem:

$$x \approx \hat{x} = \sum_{k=1}^{n} c_k \Phi_k(\theta) = \Phi(\theta)c, \qquad r(c,\theta) := \|x - \Phi(\theta)c\|_2^2 \to \min_{c,\theta}$$

- VP functional, Hilbert space approximation:

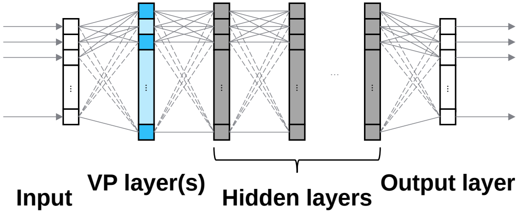
$$r_2(\theta) := \|x - \Phi(\theta)\Phi^+(\theta)x\|_2^2 \to \min_{\theta}, \qquad c = \Phi^+(\theta)x$$

  $\Phi^+(\theta)$: Moore–Penrose pseudoinverse of matrix $\Phi(\theta)$

- Gradient-based optimization possible (gradient descent, Gauss–Newton, Levenberg–Marquardt, ...)

[1] G. H. Golub, V. Pereyra: The Differentiation of Pseudo-Inverses and Nonlinear Least Squares Problems Whose Variables Separate, SIAM Journal on Numerical Analysis, 1973

VPNet: Variable Projection Networks[2]



- Model-based neural network with VP representation learning
- VP layers: VP projections for feature learning:

$$x \mapsto f^{(\mathrm{vp})}(x) = \Phi^{+}(\theta)x = c \qquad \text{(classification), or}$$

$$x \mapsto f^{(\mathrm{vp})}(x) = \Phi(\theta)\Phi^{+}(\theta)x = \hat{x} \qquad \text{(regression)}$$

- Different variants: autoencoder, spiking NN, SVM, ...

[2] P. Kovács, G. Bognár, C. Huber, M. Huemer: VPNet: Variable Projection Networks, International Journal of Neural Systems, 2022

Deep unfolding

Idea

- Projected gradient descent:

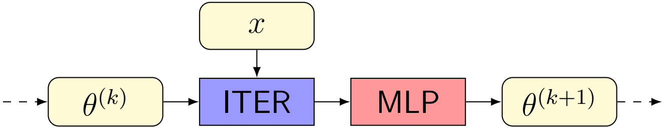
$$\theta^{(k+1)} := \Pi\left(\theta^{(k)} - \delta \cdot \nabla_\theta \|x - \hat{x}\|_2^2\right)$$

- Unfolding iterations to NN layers

$$\theta^{(k+1)} := \text{MLP}\left(\theta^{(k)} - \delta \cdot \nabla_\theta \|x - \hat{x}\|_2^2\right)$$

- Representation learning, combination with dense layers

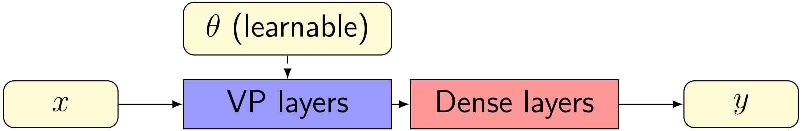
Deep unfolding layer structure[3]



[3] N. Samuel, T. Diskin, A. Wiesel: Learning to Detect, IEEE Trans. Sign. Proc., 2019

Proposed method

Original VPNet

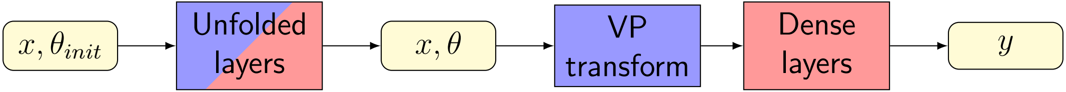


Deep unfolding variable projection network

- Motivation: expand VPNet to learn to learn (sic!) system parameters θ
- Unfolding the VP gradient iteration:

$$\theta^{(k+1)} := \mathrm{MLP}\left(\theta^{(k)} + 2\delta\left(x - \Phi(\theta)\Phi^{+}(\theta)x\right)^{T}\mathbf{D}\Phi(\theta)\Phi^{+}(\theta)x\right)$$

- Exact gradient (and gradient of gradient) computation for numerical stability



Outline

ECG application

Task

- ECG heartbeat classification on MIT-BIH Arrhythmia Database (PhysioNet)
- 5 AAMI classes, inter-patient paradigm (DS1 and DS2)[4]

Configuration

- Network input: preprocessed and segmented heartbeats
- Function system: combination of adaptive Hermite functions
  $\Phi_k(\tau, \lambda; x) := \sqrt{\lambda} \cdot \Psi_k\left(\lambda(x - \tau)\right)$ $\quad (x \in \mathbb{R}, \tau \in \mathbb{R}$: translation, $\lambda > 0$: dilation)
- Light-weight models: 2 system parameters, 1-3 unfolded layers, 4-16 linear coefficients, 67-685 NN weights
- RR interval information skip-connected to dense layers

[4] P. de Chazal, M. O'Dwyer, R. B. Reilly: Automatic classification of heartbeats using ECG morphology and heartbeat interval features, IEEE Trans Biomed Eng, 2004

Results

Method	Description		Accuracy
de Chazal et al.	Waveform + RR	LD	86.1%
Llamado et al.	Waveform + wavelet + RR	LD	93%
Ye et al.	Wavelet + ICA + RR	SVM	86%
Dózsa et al.	Hermite VP (LC + NLP + PRD) + RR	SVM	93.6%
Bognár et al.	Rational VP (LC + NLP) + RR	SVM	94.5%
	Hermite VPNet		91.9%
	Hermite VPNet + RR		93.2%
Proposed	Hermite VP Unfold		93.5%
Proposed	Hermite VP Unfold + RR		**94.7%**

Selected state-of-the-art: 5-class AAMI, inter-patient, complete database

Outline

Summary

- Model-based NN architecture with variable projections and deep unfolding
- Compact, low-dimensional representation learning
- Exact gradient computation, efficient implementation
- Application for ECG heartbeat classification for arrhythmia detection
- Explainable representation, parameters related to ECG morphology

Thank you for your attention!