TOPIC 7

Adjacency

Spatially-explicit optimisation

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Time table

10:15-10:25 **-A-** SOLUTION: Scheduling in Natural Resources Management

10:25-10:45 **-B-** Representation of space for optimisation purposes

10:45-11:30 **-C-** Spatially explicit scheduling of harvesting operations

C1 - Conceptualisation

C2 - Implementation

11:30-15:00 **-D-** Implement and solve problem
[computer lab]

- A -

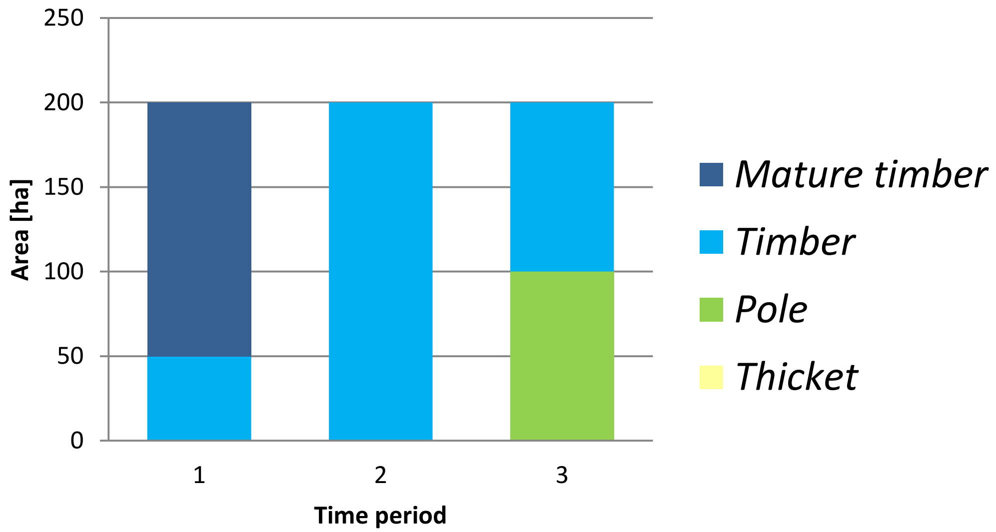
SOLUTION

Scheduling in natural resources management

Scheduling of timber harvest

												State variables (what age class, when, how much)																				
			time period 1			time period 2			time period 3			time period 1				time period 2				time period 3				time period 4							
			x_{12}	x_{13}	x_{14}	x_{22}	x_{23}	x_{24}	x_{32}	x_{33}	x_{34}	y_{11}	y_{12}	y_{13}	y_{14}	y_{21}	y_{22}	y_{23}	y_{24}	y_{31}	y_{32}	y_{33}	y_{34}	y_{41}	y_{42}	y_{43}	y_{44}				
Decision variables		x_{ti}: Area cut [ha] in age class i at period t y_{ti}: Area of age class i at period	0	50	150	0	200	0	100	100	0	100	200	50	150	200	100	200	0	200	200	100	0	200	200	100	0				
																															UNITS
Objective function		v_{ti}: projected revenues in age class i at period t [CHF/ha]	7500	#####	#####	7500	35000	45000	7500	35000	45000																		MAX	19750000	[CHF]
Constraints																												LHSsu	SIGN	RHS	
[A] Initial state		stage 1: thicket										1																100	==	100	[ha]
		stage 2: pole											1															200	==	200	[ha]
		stage 3: timber												1														50	==	50	[ha]
		stage 4: mature timber													1													150	==	150	[ha]
[B] Dynamic model	*Period 1*	stage 1	1	1	1											-1												0	==	0	[ha]
		stage 2										1					-1											0	==	0	[ha]
		stage 3	-1										1					-1										0	==	0	[ha]
		stage 4		-1	-1									1	1				-1									0	==	0	[ha]
	Period 2	stage 1				1	1	1												-1								0	==	0	[ha]
		stage 2														1					-1							0	==	0	[ha]
		stage 3				-1											1					-1						0	==	0	[ha]
		stage 4					-1	-1										1	1				-1					0	==	0	[ha]
	Period 3	stage 1							1	1	1													-1				0	==	0	[ha]
		stage 2																		1					-1			0	==	0	[ha]
		stage 3							-1												1					-1		0	==	0	[ha]
		stage 4								-1	-1											1	1				-1	0	==	0	[ha]
[C] Restrict harvest to	*Period 1*	stage 2	-1										1															200	=>	0	[ha]
		stage 3		-1										1														0	=>	0	[ha]
		stage 4			-1										1													0	=>	0	[ha]
	Period 2	stage 2				-1											1											100	=>	0	[ha]
		stage 3					-1											1										0	=>	0	[ha]
		stage 4						-1											1									0	=>	0	[ha]
	Period 3	stage 2							-1												1							100	=>	0	[ha]
		stage 3								-1												1						0	=>	0	[ha]
		stage 4									-1												1					0	=>	0	[ha]
[D] Steady-state condition		stage 1																		1				-1				0	==	0	[ha]
		stage 2																			1				-1			0	==	0	[ha]
		stage 3																				1				-1		0	==	0	[ha]
		stage 4																					1				-1	0	==	0	[ha]

Scheduling of timber harvest



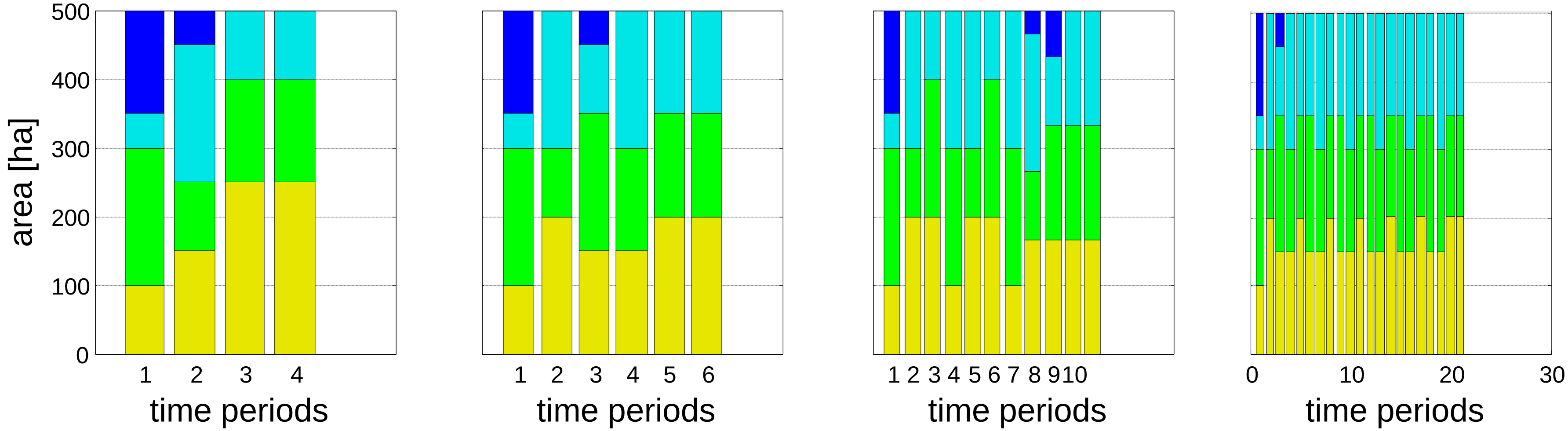
Estimated revenues

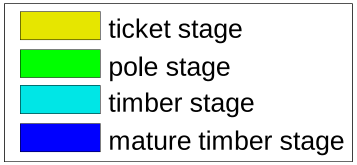
CHF 19’750’000

Sensitivity – Count of planning periods

Count of planning periods

3 5 10 20





Vanish of «mature timber» is not a matter of limited time periods!

➲ **Add constraint for state variable which ensures a minimum area of «mature timber» at any time period**

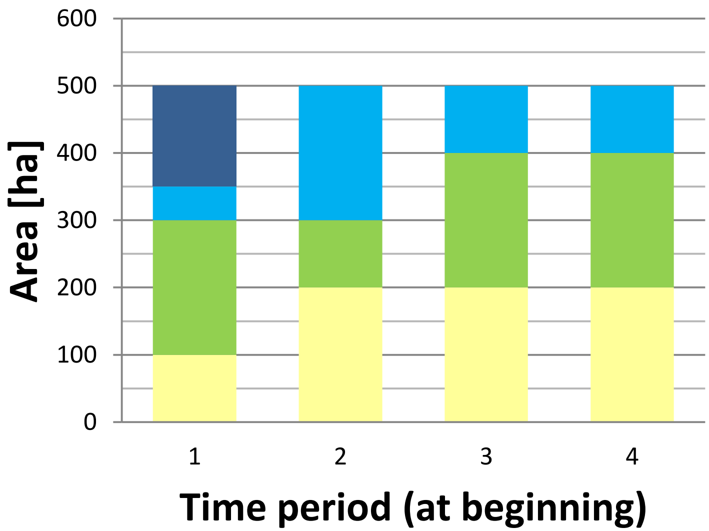
$$y_{t,mature\ timber} \geq area\ threshold$$

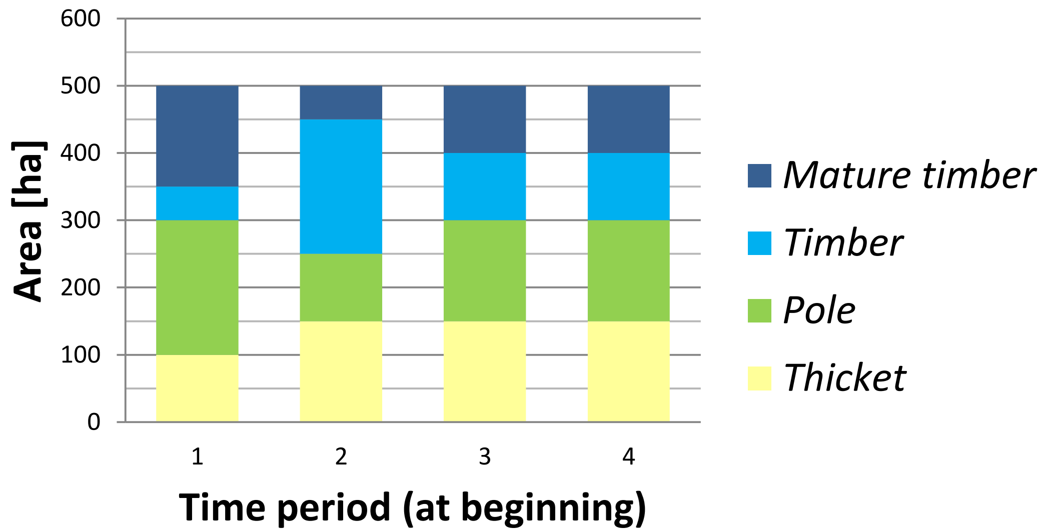
Sensitivity - Estimated revenues [CHF per ha]

pole	timber	mature timber
7500	35'000	45'000

pole	timber	mature timber
7500	35'000	62’000

Increase revenues for
MATURE TIMBER





- B -

Representation of space

Learning goals

CONCEPTUALIZE AND REPRESENT

Learn to formulate optimization models that include…

… decision units that refer to polygons

… control of spatial pattern of selected polygons

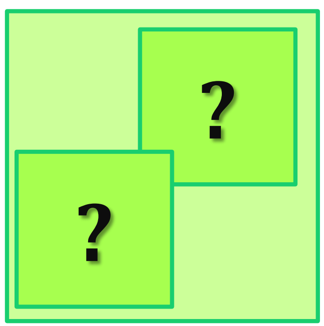
IMPLEMENT

- Automate the creation of large matrix notation optimisation models
- Learn to [1] import, [2] manipulate and [3] export shapefiles in MATLAB

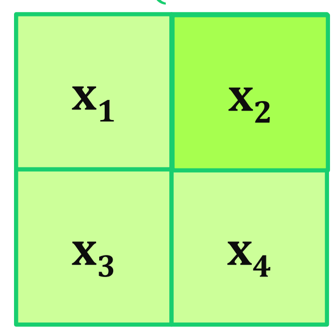
Representation of discrete spatial units

x_i : take action on polygon i

share of area where action will be applied to	apply action to polygon i
$x_i \in [0, 1]$ *real*	$x_i = \begin{cases} 1, yes \\ 0, no \end{cases}$ *integer*



Decompose into planning units



Example $x_i = 0.25$

$x_1 = 0$
$x_2 = 1$
$x_3 = 0$
$x_4 = 0$

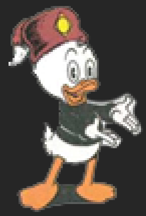
Many units result into many decision variables

Control shape of spatial patterns

Adjacency : list of neighboring polygons ***i*** and ***j***

Relationships between polygons facilitate optimisation of spatial patterns

Compactness

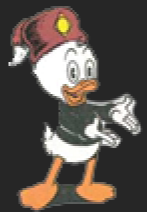


Selected polygons create shapes characterized by low perimeter/area ratio

Contiguity

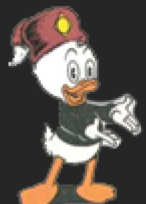
Any selected polygon can be reached from any other selected polygon within the shape created by the selected polygons

Dispersion



Selected polygons are distant from each other

Edge-focused



Selected polygons create edges relevant to the problem

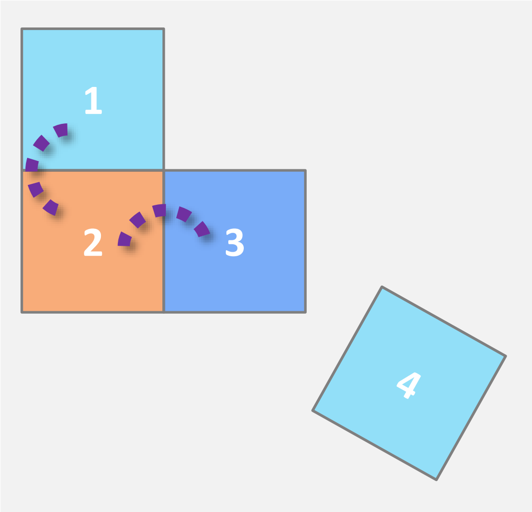
Summary

Compactness	Contiguity	Dispersion	Edge-focused
Spatial concentration of efforts	**Corridor creation**	**Distribute obnoxious* actions in space**	**Edge-related goals**
[1] management [2] nature conservation	e.g., nature conservation problems	[1] siting of power plants, waste depots, etc. [2] harvest operations	[1] nature conservation (forest edge) [2] manage interactions between neigh-boring polygons

Exercise 7

** unliebsam*

Representation of spatial data



Adjacency definition:
«Polygons share an edge»

geometry

ID	shape	volume
1	polygon	300
2	polygon	500
3	polygon	400
4	polygon	300

adjacency

ID_i	ID_j
1	2
2	3

Example: Attribute table in ArcGIS

GEOMETRY formats

ArcGIS [shapefile]

FID	Shape *	property
0	Polygon	300
1	Polygon	400
2	Polygon	500
3	Polygon	300

MATLAB

`shaperead`

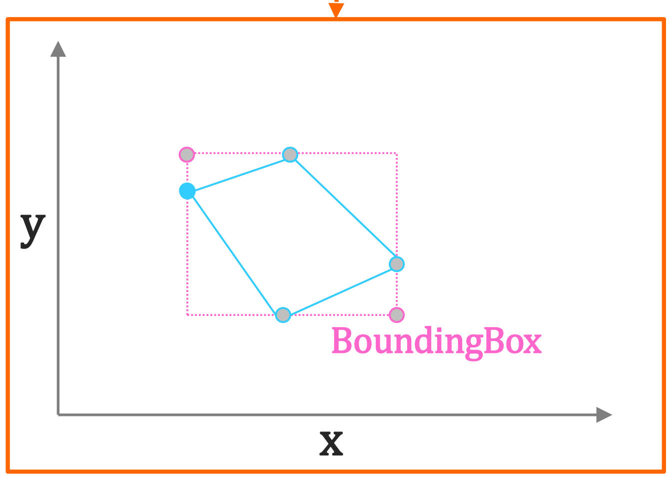
MATLAB [struct]

4x1 struct with 5 fields

Fields	Geometry	BoundingBox	X	Y	property
1	'Polygon'	[6.7823e+05,2.46...	[6.7823e+05...	[2.4657e+05...	300
2	'Polygon'	[6.7810e+05,2.46...	[6.7810e+05...	[2.4660e+05...	400
3	'Polygon'	[6.7800e+05,2.46...	[6.7800e+05...	[2.4660e+05...	500
4	'Polygon'	[6.7800e+05,2.46...	[6.7800e+05...	[2.4670e+05...	300

geometry

attributes



➲ *MRM M4*

ADJACENCY formats

ArcGIS

Polygon neighbors

ArcGIS [Excel spreadsheet]

	A	B	C	D	E
1	OBJECTID	src_OBJECTID	nbr_OBJECTID	LENGTH	NODE_COUNT
2	1	1	2	100	0
3	2	2	3	100	0

adjacency-ID

parcel ***i***

parcel ***j***

length of shared edge

neighborhood type
0: edge
1: vertex

MATLAB

xlsread

MATLAB [2D array = matrix]

2x5 double

	1	2	3	4	5
1	1	1	2	100	0
2	2	2	3	100	0

parcel ***i***

parcel ***j***

- C -

Spatially explicit scheduling of harvesting operations

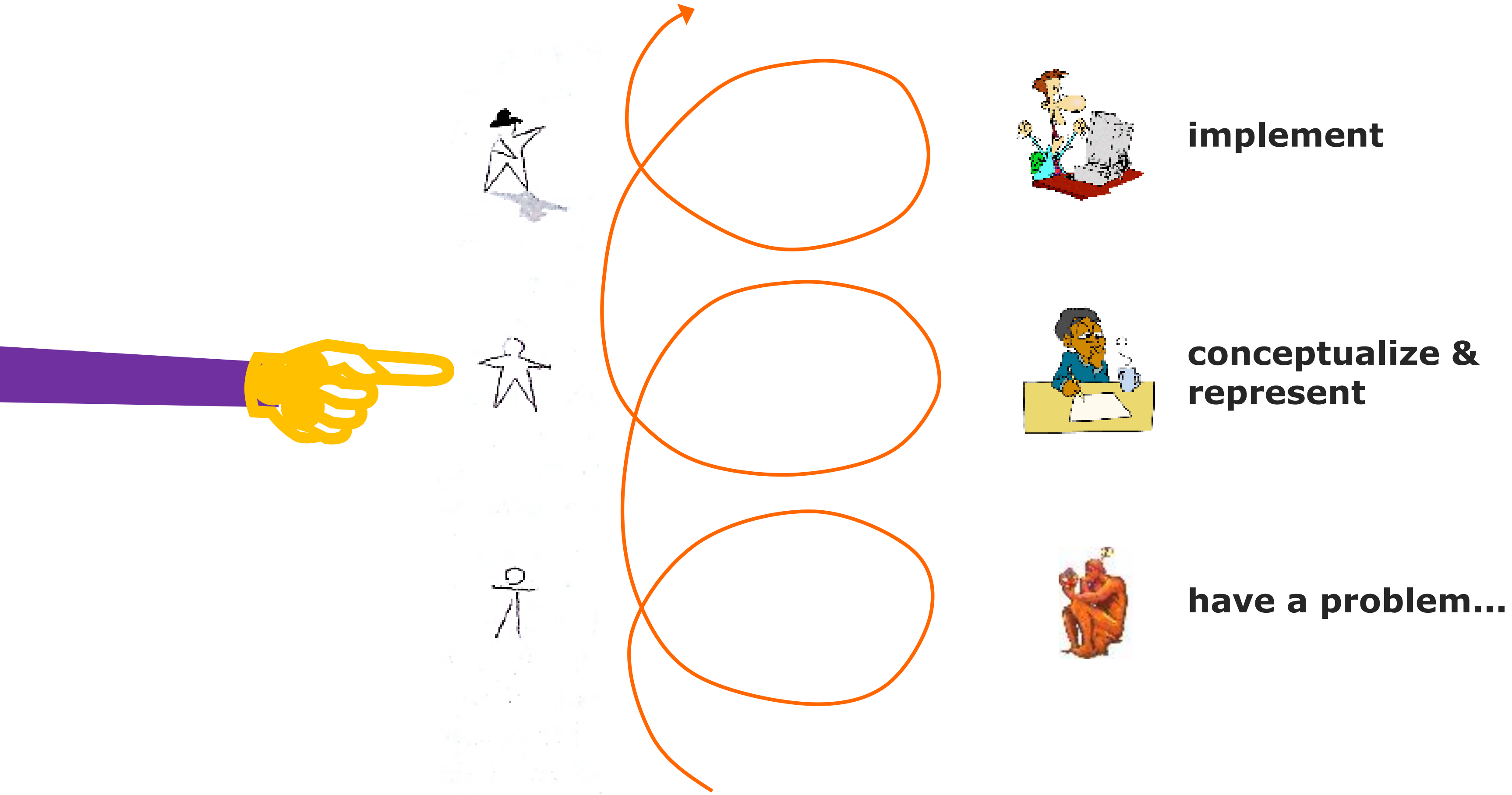
The Uetliberg forest is divided up into more than 600 stands which are used as decision units for managing actions. All stands in the timber stage (n=266) have been selected for harvest in the next three upcoming periods.

The stands should be harvested in order to maximize harvested timber volume under the restriction that the size of openings after harvest is limited. Therefore, the management authority has set the policy that harvesting of neighboring stands during the same period is not allowed. The authority would like to know how to schedule harvest of the stands in order to concurrently meet the objective and the harvesting constraint. Come up with a map that indicates which stands are subject to harvest at which period!

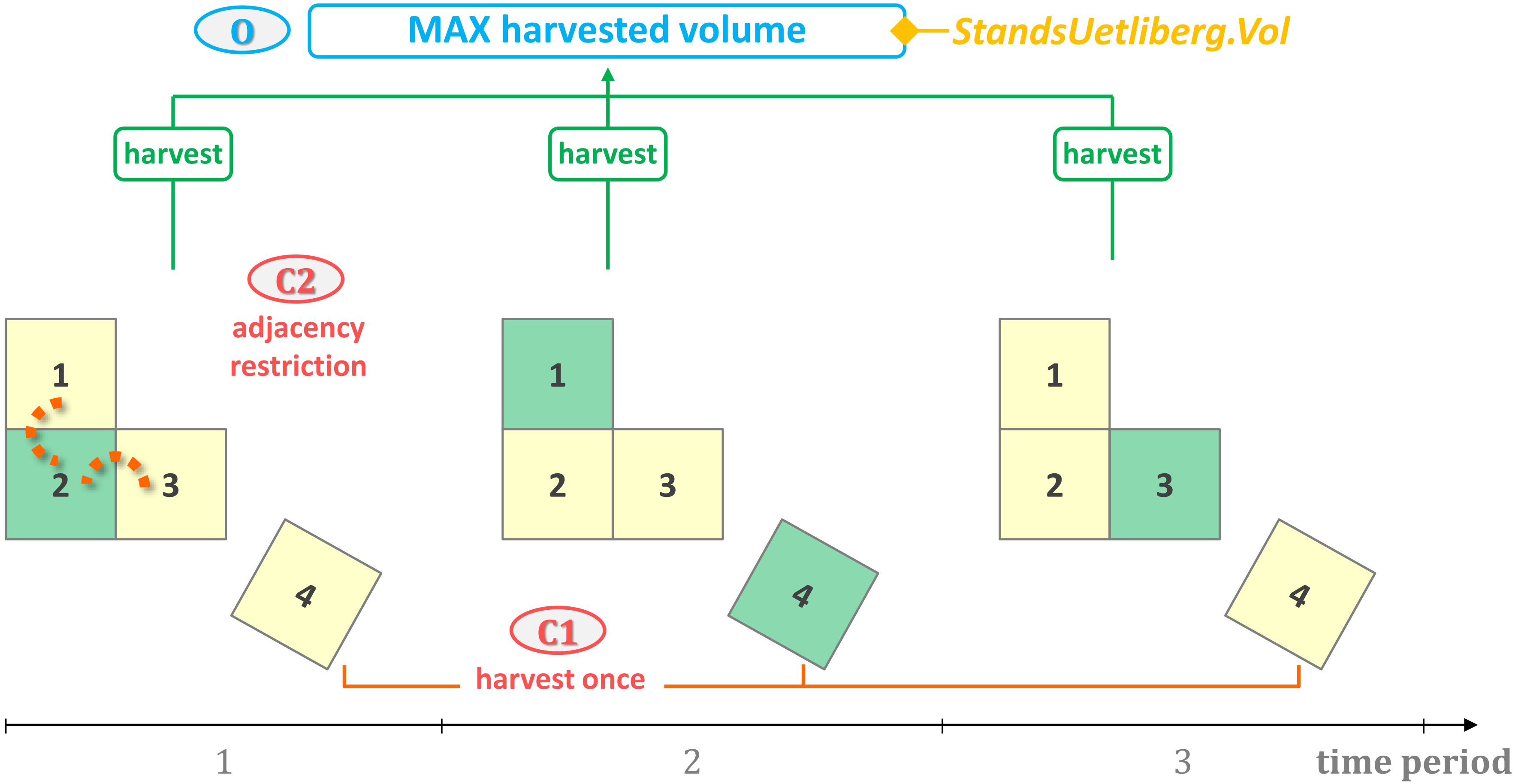
StandsUetliberg [polygon feature]

attributes (incomplete)

```
STAND_ID:  Stand identifier [-]
SOD:       Stage of development [classes]
Vol:       timber volume of the stand [m3]
```

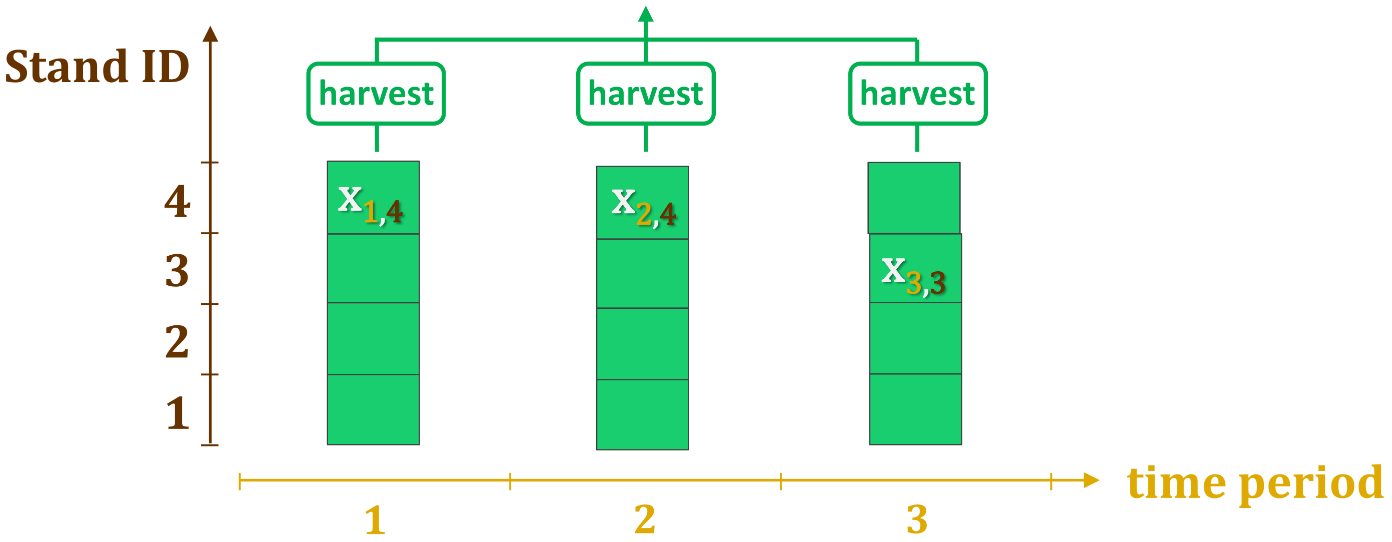

implement
conceptualize & represent
have a problem...

Conceptual model for the spatial scheduling problem



Use constraints for control of spatial patterns!

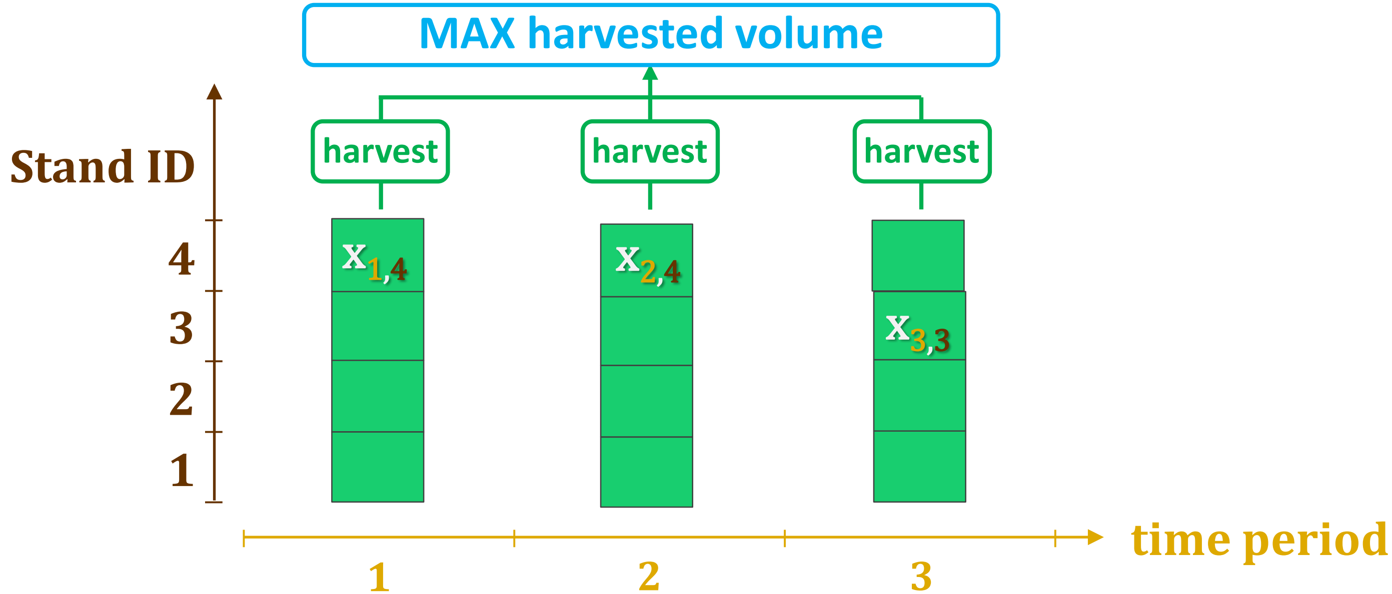
Decision variables



Harvest stand ***i*** at period ***t***

$$x_{t,i} = \begin{cases} 1, \text{yes} \\ 0, \text{no} \end{cases}$$

O - Objective function



f_i $[m^3]$
timber volume in stand ***i***

$$\text{MAX} \quad f_1x_{1,1} + f_2x_{1,2} + f_3x_{1,3} + f_4x_{1,4}$$ *period 1*

$$f_1x_{2,1} + f_2x_{2,2} + f_3x_{2,3} + f_4x_{2,4}$$ *period 2*

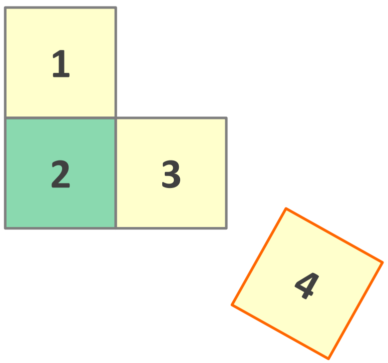
$$f_1x_{3,1} + f_2x_{3,2} + f_3x_{3,3} + f_4x_{3,4}$$ *period 3*

or…

$$\text{MAX} \sum_{t=1}^{m=3} \sum_{i=1}^{n=4} f_i x_{t,i} \qquad [\text{m}^3]$$

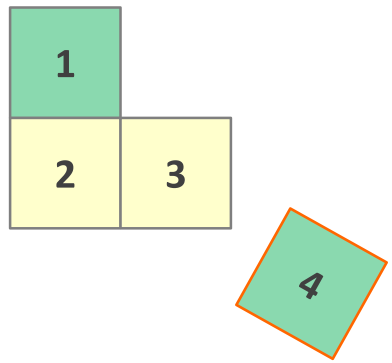
C1 - Harvest once

period 1



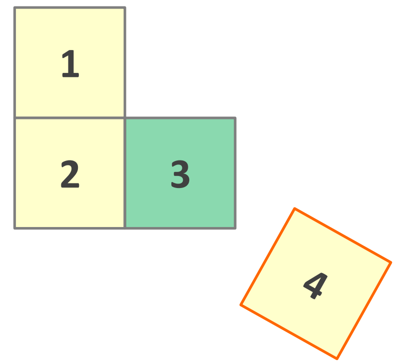
$= 0$

period 2



$= 1$

period 3



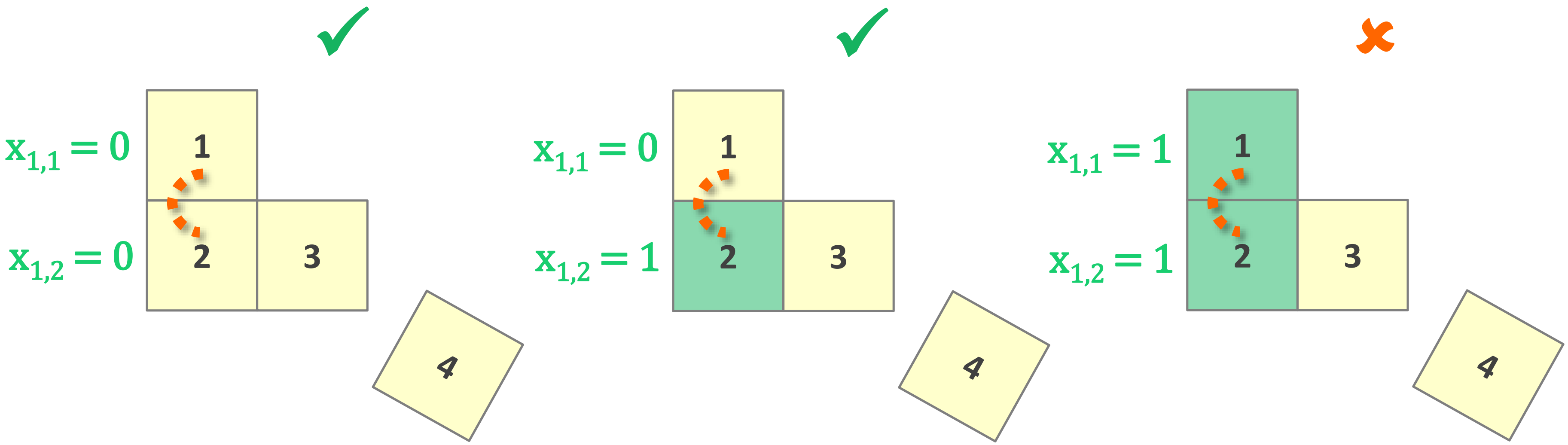
$= 0$

$$x_{1,4} + x_{2,4} + x_{3,4} \leq 1$$

short notation

$$\sum_{t=1}^{m=3} x_{t,i} \leq 1$$ for all stands i=1,…,n

C2 - Adjacency restriction



$$x_{1,1} + x_{1,2} \leq 1$$

---

generalised notation

$$x_{t,i} + x_{t,j} \leq 1$$ for all periods $t=1,\dots,m$ and for all neighbors $(i,j) \in \mathbb{A}$

$\mathbb{A}$: adjacency list, a set of pairs of stands **(i,j)** which share an edge

Optimisation model

Decision variables

$$x_{t,i} = \begin{cases} 1, \text{ stand } i \text{ harvested in period } t \\ 0, \text{ else} \end{cases}$$

McDill ME, Rebain SA, Braze J (2002). Harvest scheduling with area-based adjacency constraints. *Forest Science*, *48*(4), 631-642.

Parameters

f_i : timber volume in stand i [m^3]

O **Maximize harvested volume** MAX $\sum_{t=1}^{m}\sum_{i=1}^{n} f_i x_{t,i}$

C1 *Harvest once* **s.t.** $x_{1,i} \; \ldots \; + x_{t,i} \; \ldots \; + x_{m,i} \leq 1$ for all i=1,..,n

C2 *Adjacency restriction* $x_{t,i} + x_{t,j} \leq 1$ for all t=1,...,m and all (i,j) ∈ $\mathbb{A}$

$x_{t,i} \in \{0, 1\}$

$\mathbb{A}$: Adjacency list, set of pairs of stands **(i,j)** which share an edge

Link spatial information with optimisation model

SPATIAL INFORMATION

geometry

ID	shape	volume
1	polygon	300
2	polygon	500
3	polygon	400
4	polygon	300

Count of stands

adjacency

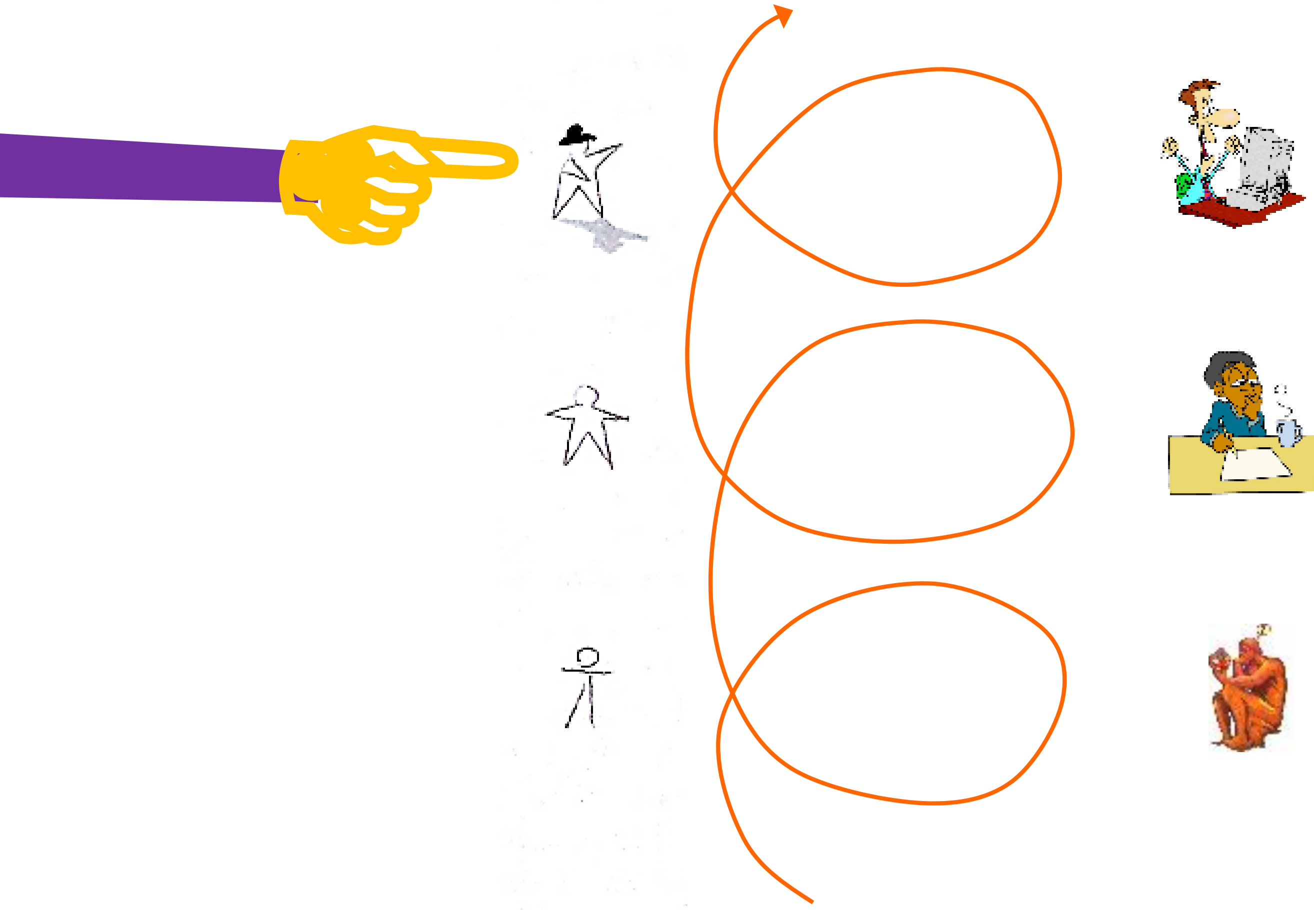
ID_i	ID_j
1	2
2	4

Count of adjacencies

$$\begin{array}{lccccl} & x_{t,1} & x_{t,2} & x_{t,3} & x_{t,4} & \\ \text{adjacency (1,2)} & 1 & 1 & 0 & 0 & \leq 1 \\ \text{adjacency (2,4)} & 0 & 1 & 0 & 1 & \leq 1 \end{array}$$

OPTIMISATION MODEL

Do for all adjacencies $\in \mathbb{A}$!



implement

conceptualize & represent

have a problem…

Problem sizes

Exercise 6
Multiperiod planning problem

Exercise 7
Spatially explicit scheduling problem

⇨ **need for automation of manual work!**

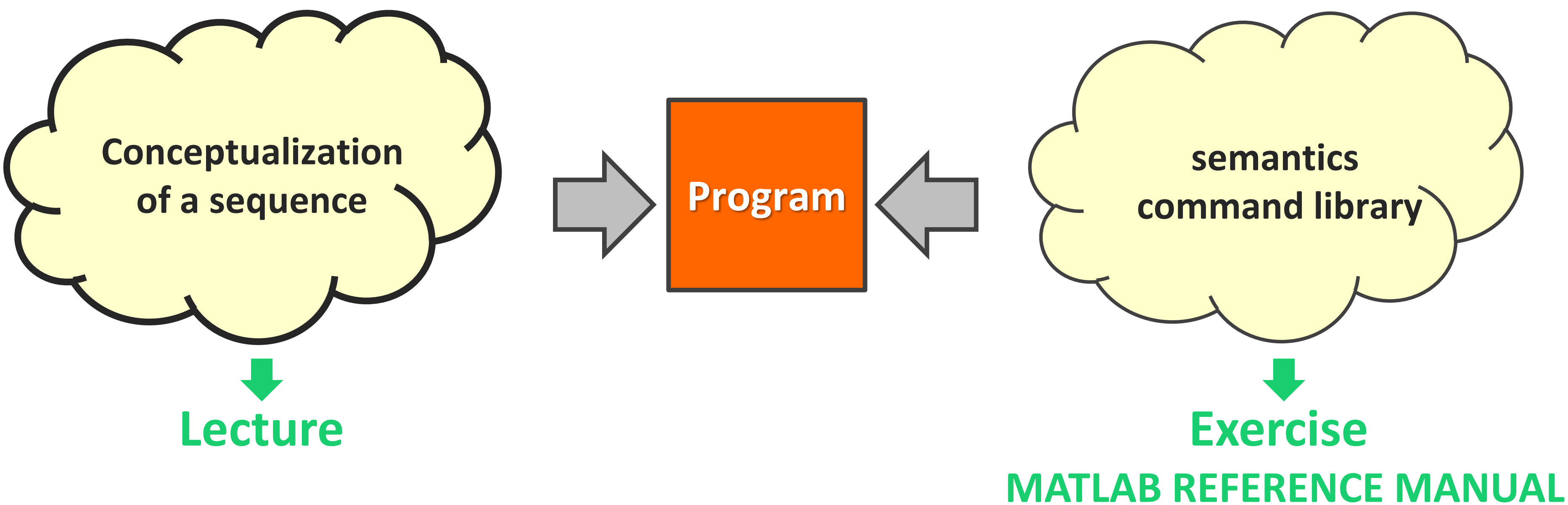
Programming - what is it about?

What is the purpose of programming?

Identify a sequence of instructions that will automate performing a specific task [wikipedia]

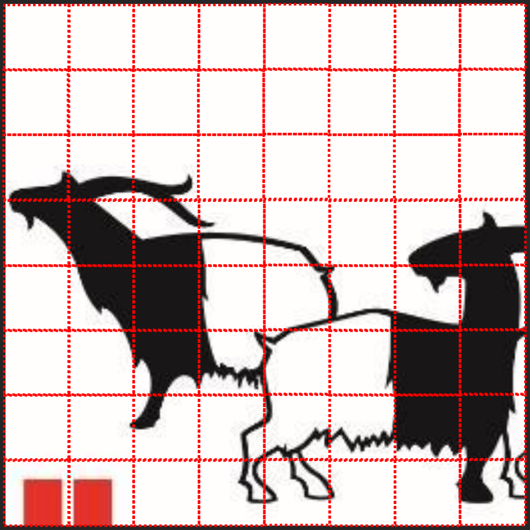
What is the act of programming?

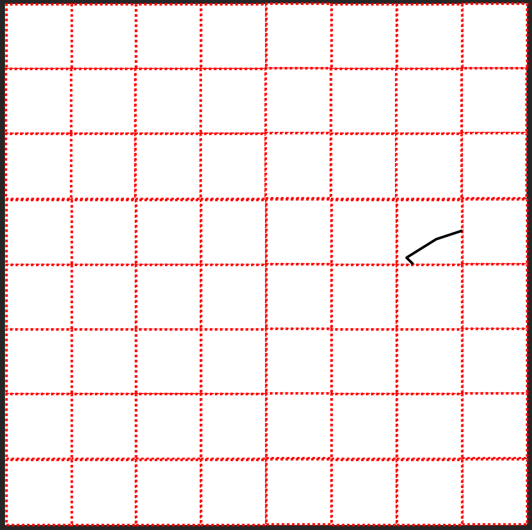
Transfer a calculation problem into an executable computer program



"The Pen is mightier than the sword" (i.e., computer)

How to copy picture by hand ?

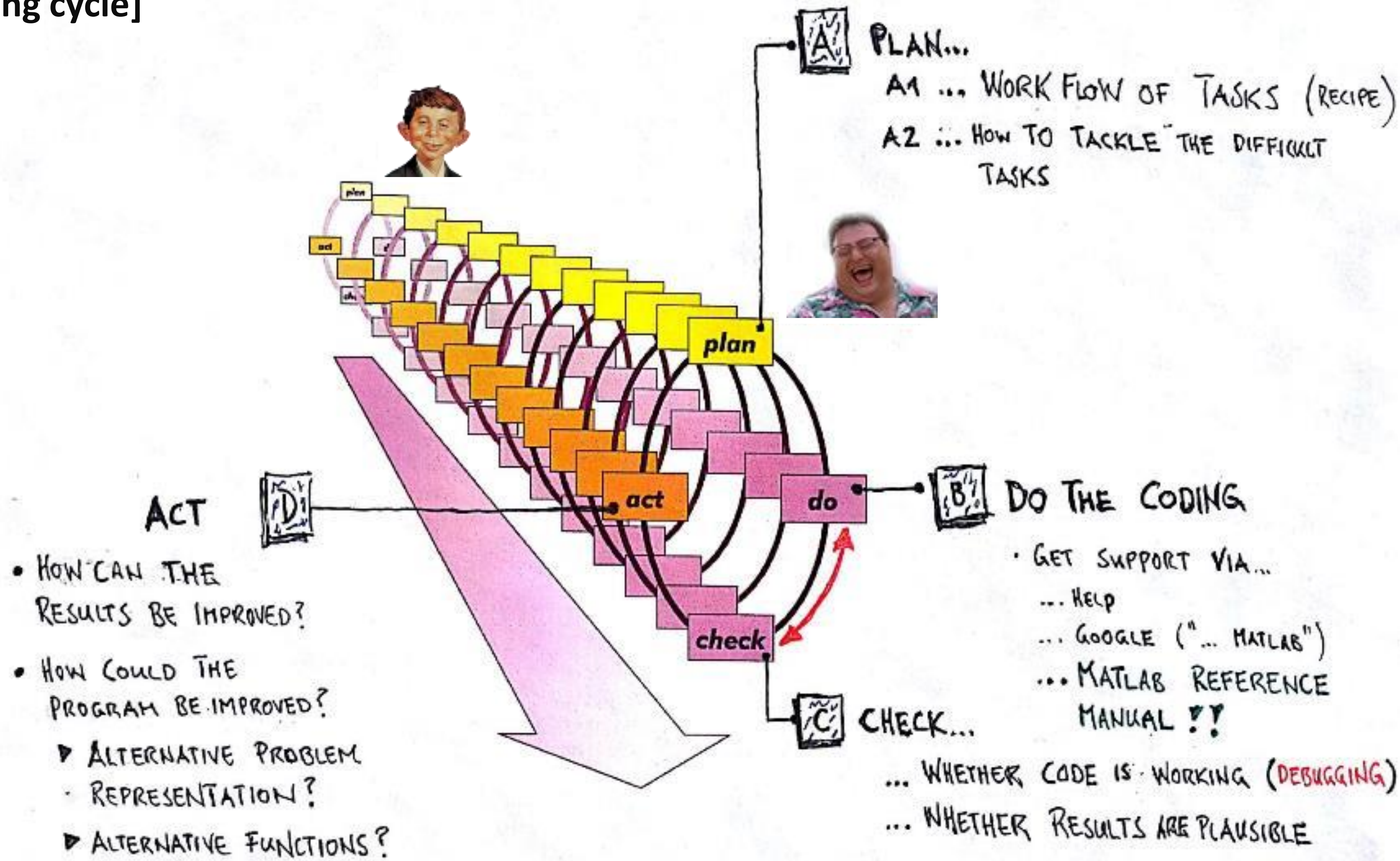




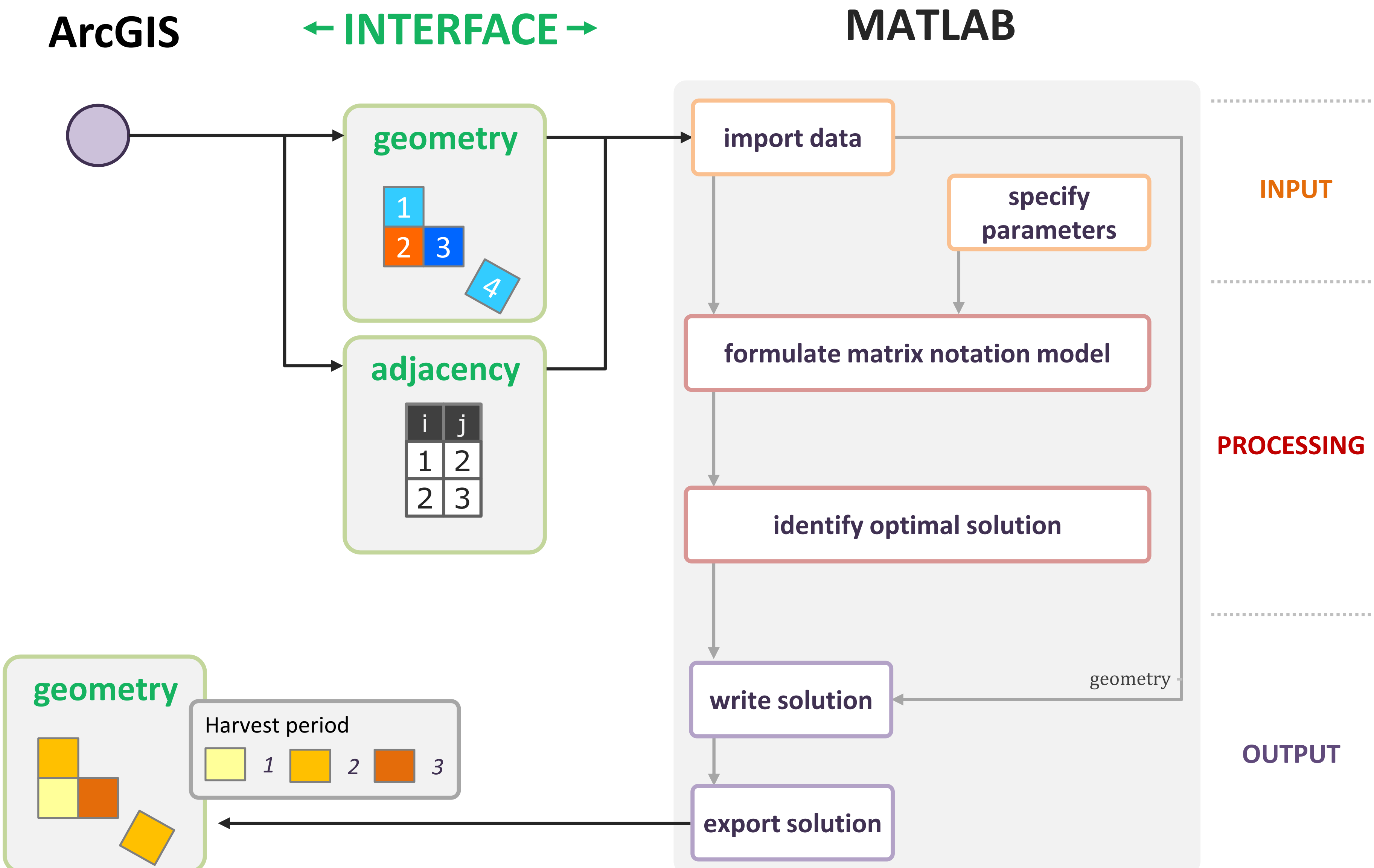
Divide and conquer!

Iteratively improve your programming skills

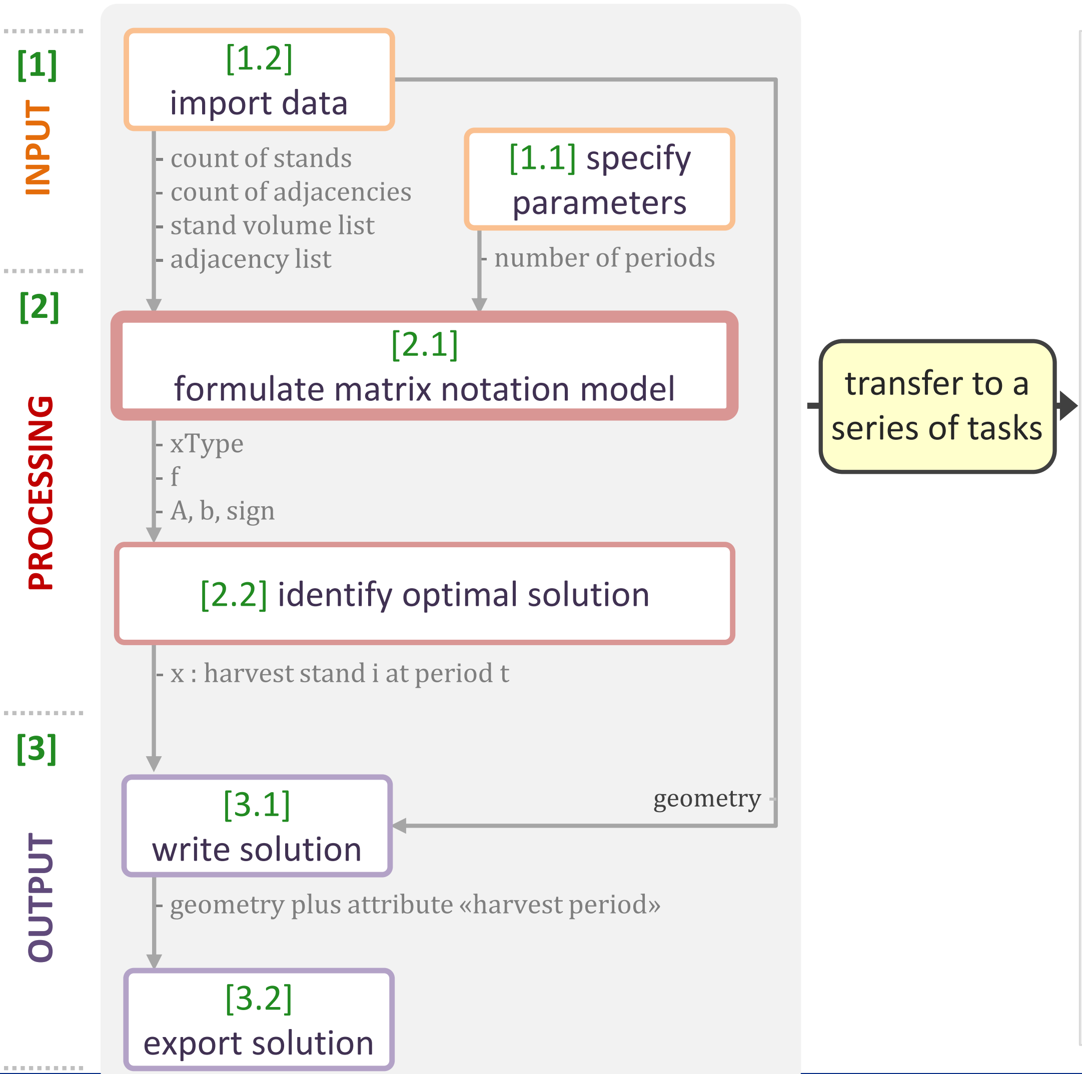
[Deming cycle]



Conceptualisation ➲ work flow



Implementation ➲ «chapter structure»



IMPLEMENTATION

```
%% [1] Input
%  [1.1] Specify parameters
%  [1.2] Import data

%% [2] Processing
%  [2.1] Formulate model
%  [2.2] Identify optimal solution

%% [3] Output
%  [3.1] Write solution
%  [3.2] Export solution
```

Plan the matrix notation model – RUDIMENTARY LAYOUT
for implementation in Matlab

Decision variables

xType

f

Constraints

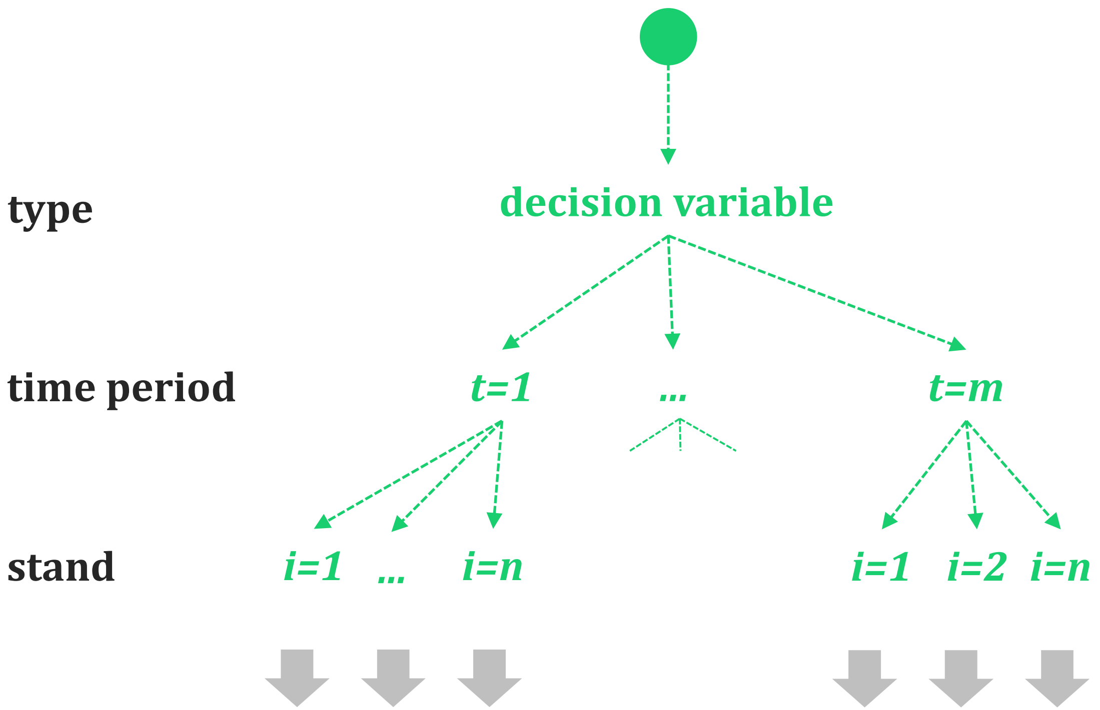
A

sign

b

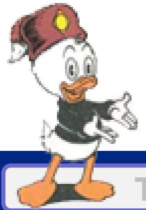
Plan the matrix notation model – variables

Hierarchical Order



Implementation

<table>
<tr><td colspan="3">period 1</td><td>period ...</td><td colspan="3">period m</td></tr>
<tr><td>$x_{1,1}$</td><td>...</td><td>$x_{1,n}$</td><td>...</td><td>$x_{m,1}$</td><td>...</td><td>$x_{m,n}$</td></tr>
</table>

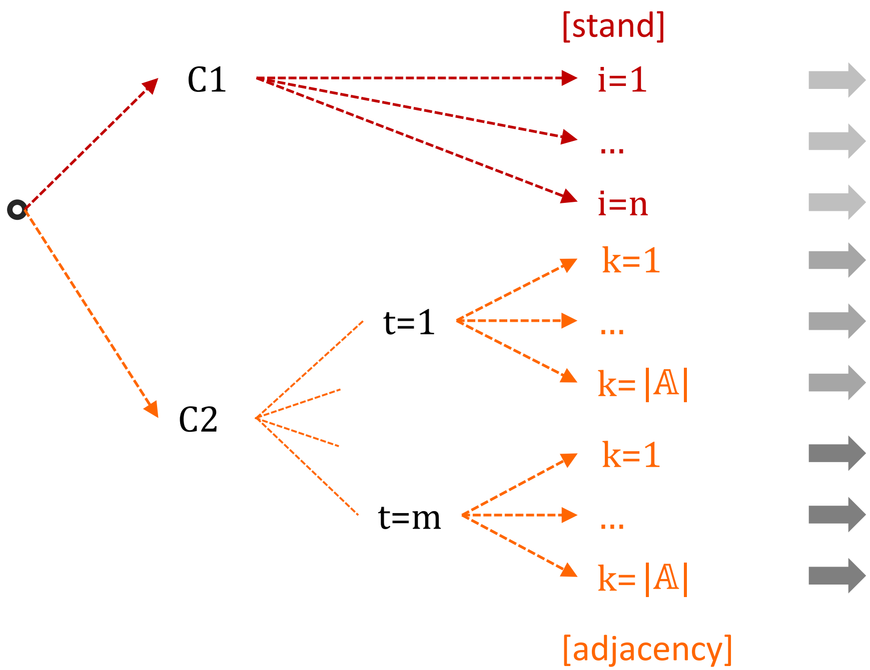


Writing the entire model is cumbersome! Make use of placeholders «...»

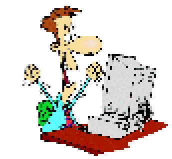
Plan the matrix notation model – constraints

Hierarchical order

constraint type *time period* *unit*

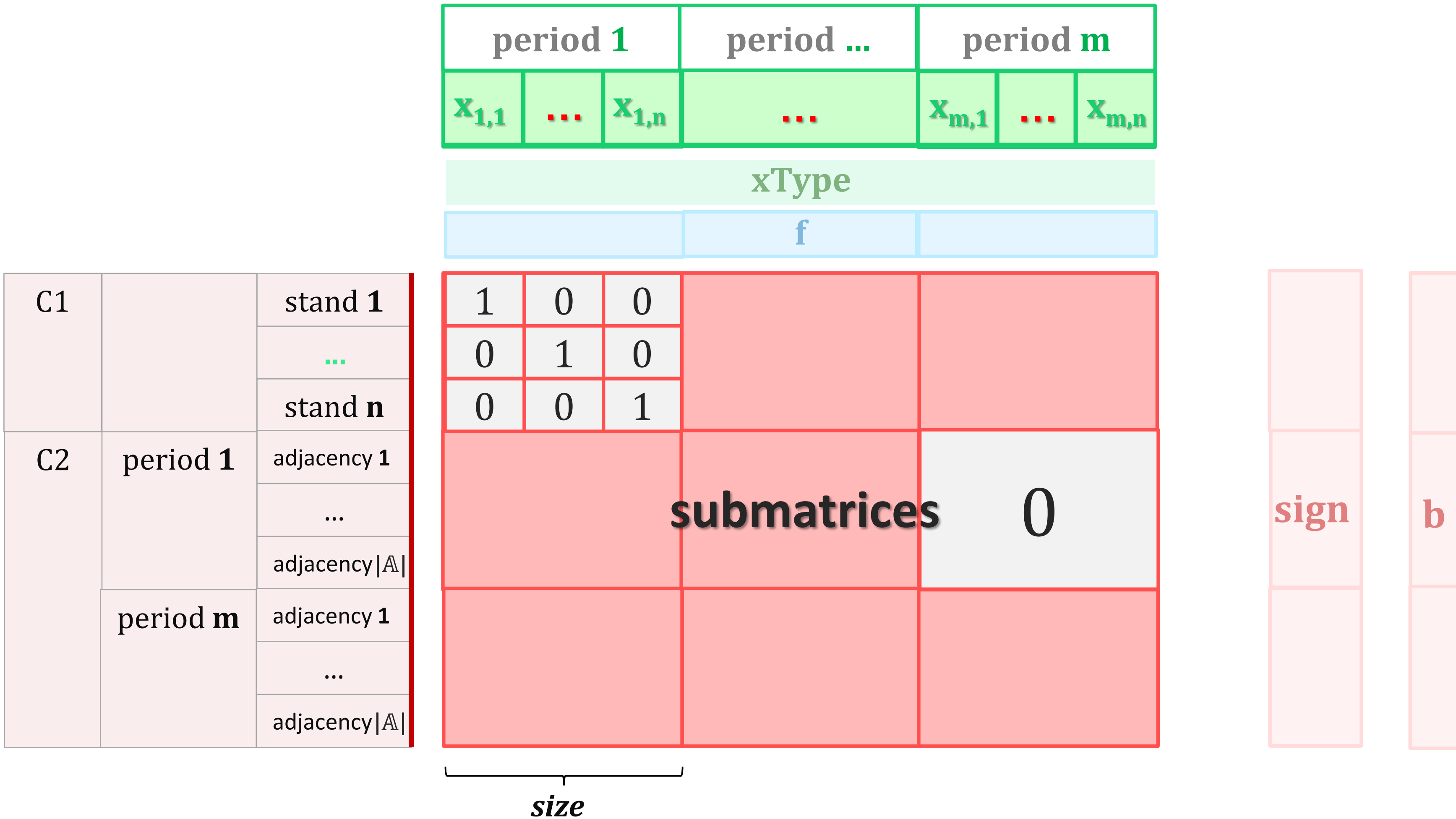


Implementation



constraint type	*time period*	*unit*
C1		stand **1**
		...
		stand **n**
C2	period **1**	adjacency **1**
		...
		adjacency $\lvert\mathbb{A}\rvert$
	period **m**	adjacency **1**
		...
		adjacency $\lvert\mathbb{A}\rvert$

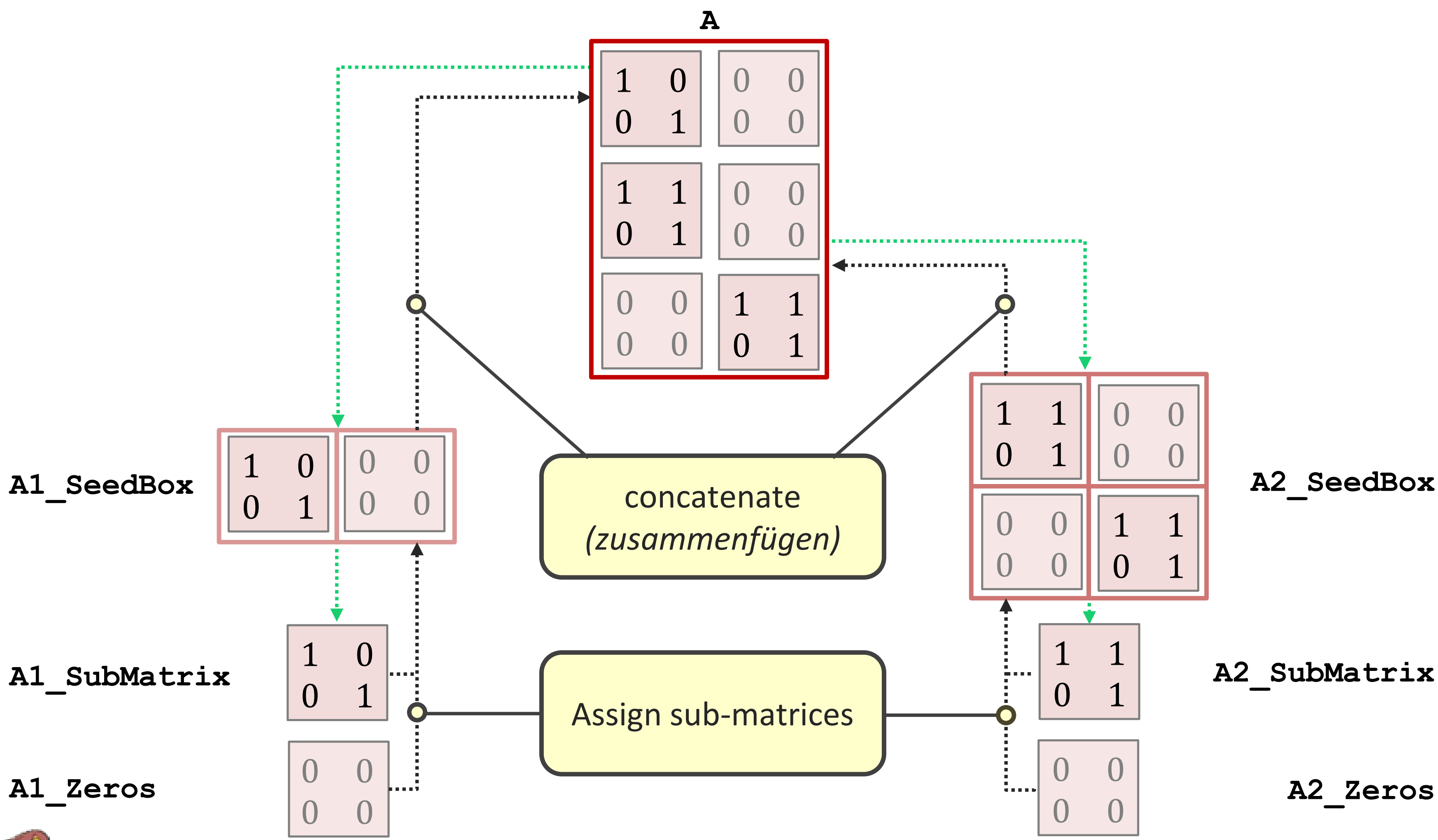
Plan the matrix notation model – A-Matrix



(1) Identify those submatrices which contain non-zero values!

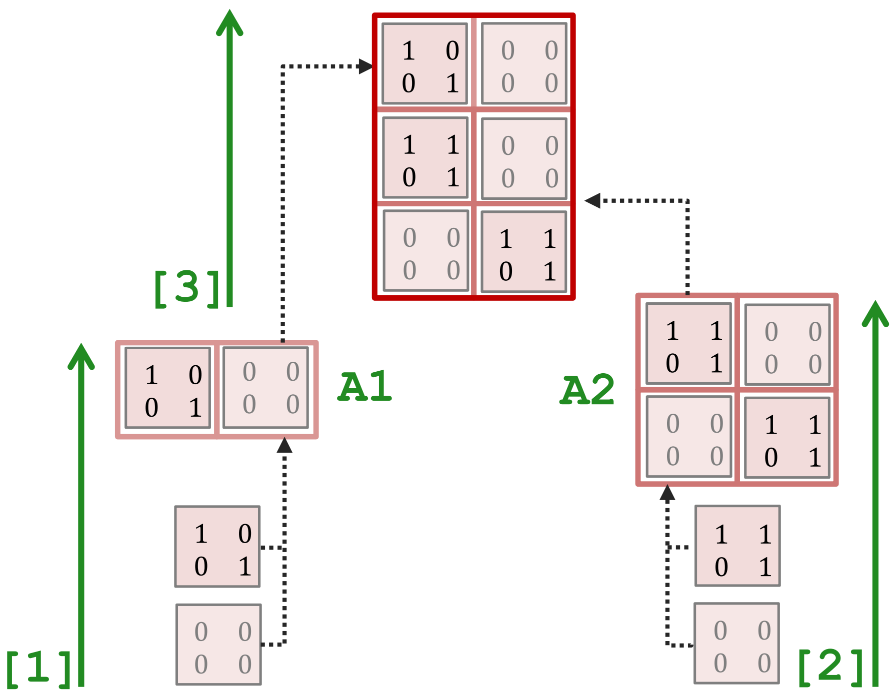
(2) Characterize the content of those sub-matrices, and specify their size!

Automate creation of A-matrix : Concept



Divide and conquer!

Automate creation of A-matrix : Work flow




```
%% Compute A-matrix

%% [1] Create A1-matrix
    …    % create A1-sub matrix
    …    % create A1-zeros
    …    % create A1 seed box
    …    % assign sub matrices to A1

%% [2] Create A2-matrix
    …    % create A2-sub matrix
    …    % create A2-zeros
    …    % create A2 seed box
    …    % assign sub matrices to A2

%% [3] Concatenate A1 and A2
    …
```

Matrix notation model versions

V1 : Easy-to-read

use for planning of matrix notation model

$$xType' \; \begin{bmatrix} 'B' & 'B' & 'B' & 'B' & 'B' & 'B' & 'B' \end{bmatrix}$$

$$f' \; \begin{bmatrix} 0 & 0 & 0 & 0 & a_1 & a_2 & a_3 \end{bmatrix}$$

$$A \begin{bmatrix} 1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \underset{sign}{\begin{matrix} \geq \\ \geq \\ \geq \\ = \end{matrix}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ p \end{bmatrix} b$$

V2 : Implementation

Matlab

$$f = \begin{pmatrix} 0 \\ \dots \\ a_3 \end{pmatrix} \qquad xType = \begin{pmatrix} 'B' \\ \dots \\ 'B' \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & \dots & 0 \\ \dots & \ddots & \dots \\ 1 & \dots & 0 \end{pmatrix} \quad sign = \begin{pmatrix} 1 \\ \dots \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ \dots \\ p \end{pmatrix}$$

Transpose
f, xType

Exercise 7

Task A
plan matrix notation model

Task B
plan work flow

Task C
create code

Task D
represent results

by hand | **MATLAB** | **ArcGIS**