TOPIC 8

Covering problems

Spatially-explicit optimisation

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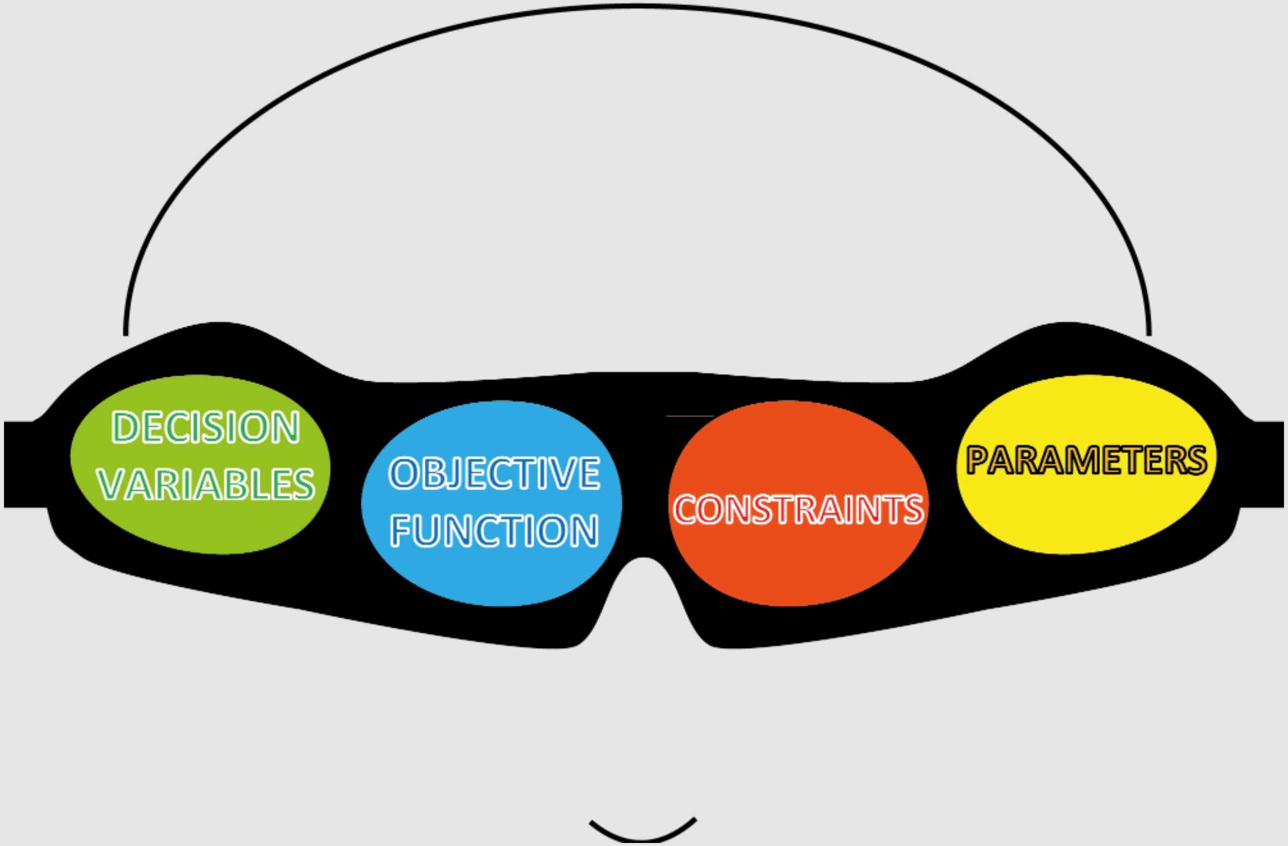
Put on your «optimizer glasses», now !



Learning goals

CONCEPTUALIZE AND REPRESENT

Learn to formulate optimization models that include…

… decision units that refer to points

… state variables which capture a spatial relationship

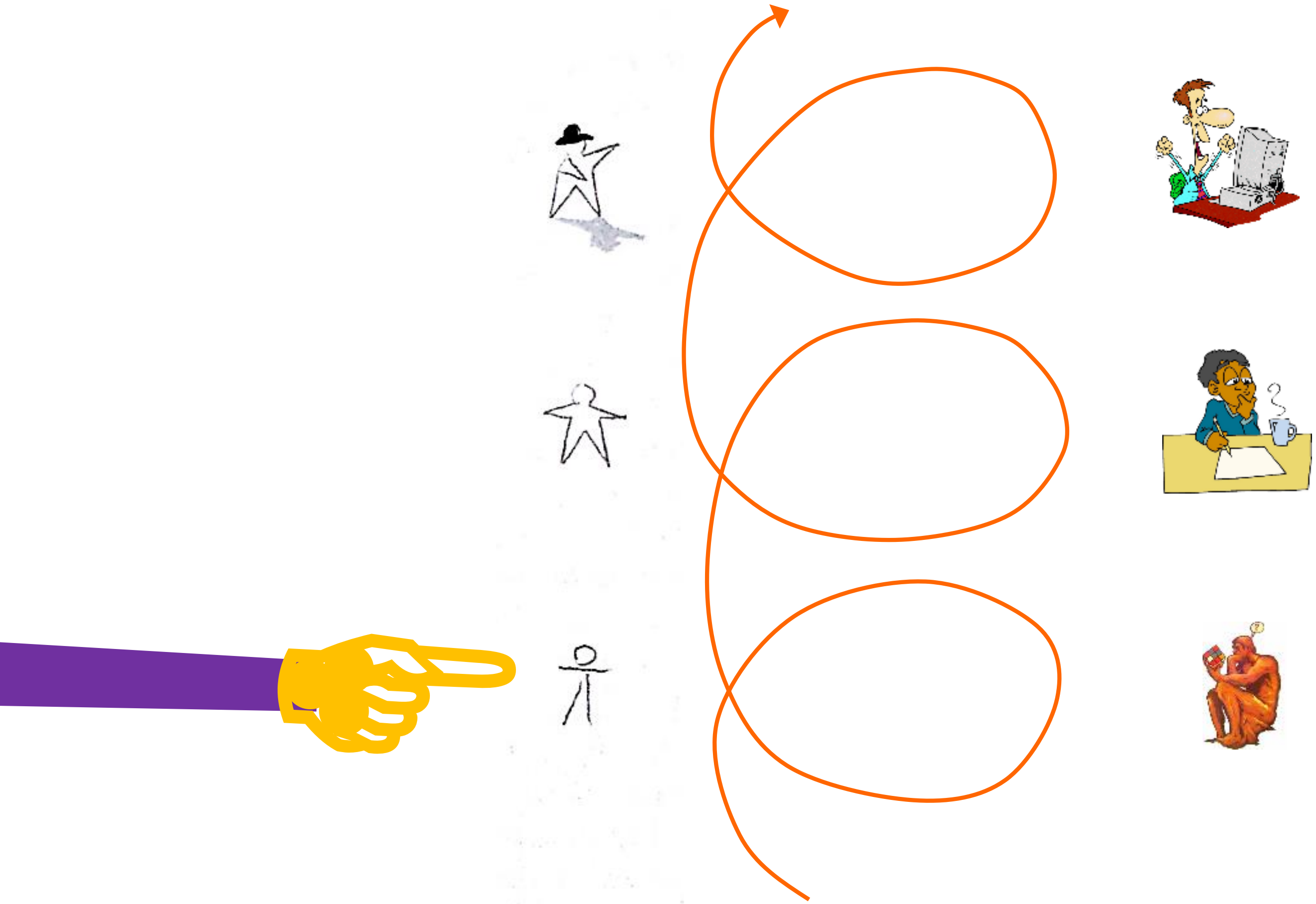
IMPLEMENT

- Automate the creation of large matrix notation optimisation models (MATLAB)
- Learn to extract the spatial information relevant to a coverage problem (ArcGIS)
- Learn to [1] import, [2] manipulate and [3] export shapefiles in MATLAB

Time table

10:15-11:00	**-A-**	Introduction to covering problems
	-B-	Mathematical formulation of covering problems
11:15-12:00	**-C-**	Implementation of a covering problem
13:15-15:00	**-D-**	Implement and solve problem *[computer lab]*

- A -
Introduction to covering problems

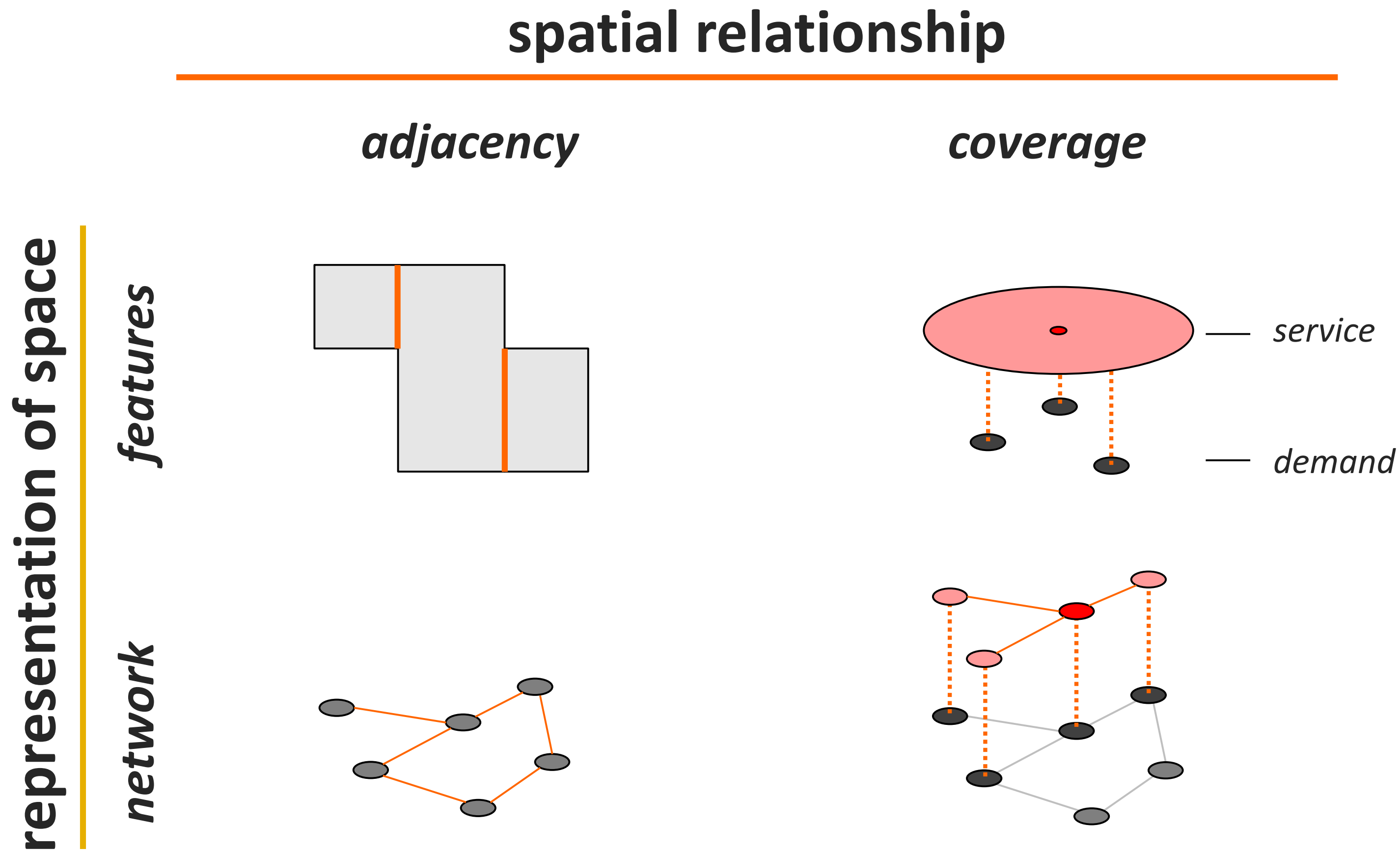


implement

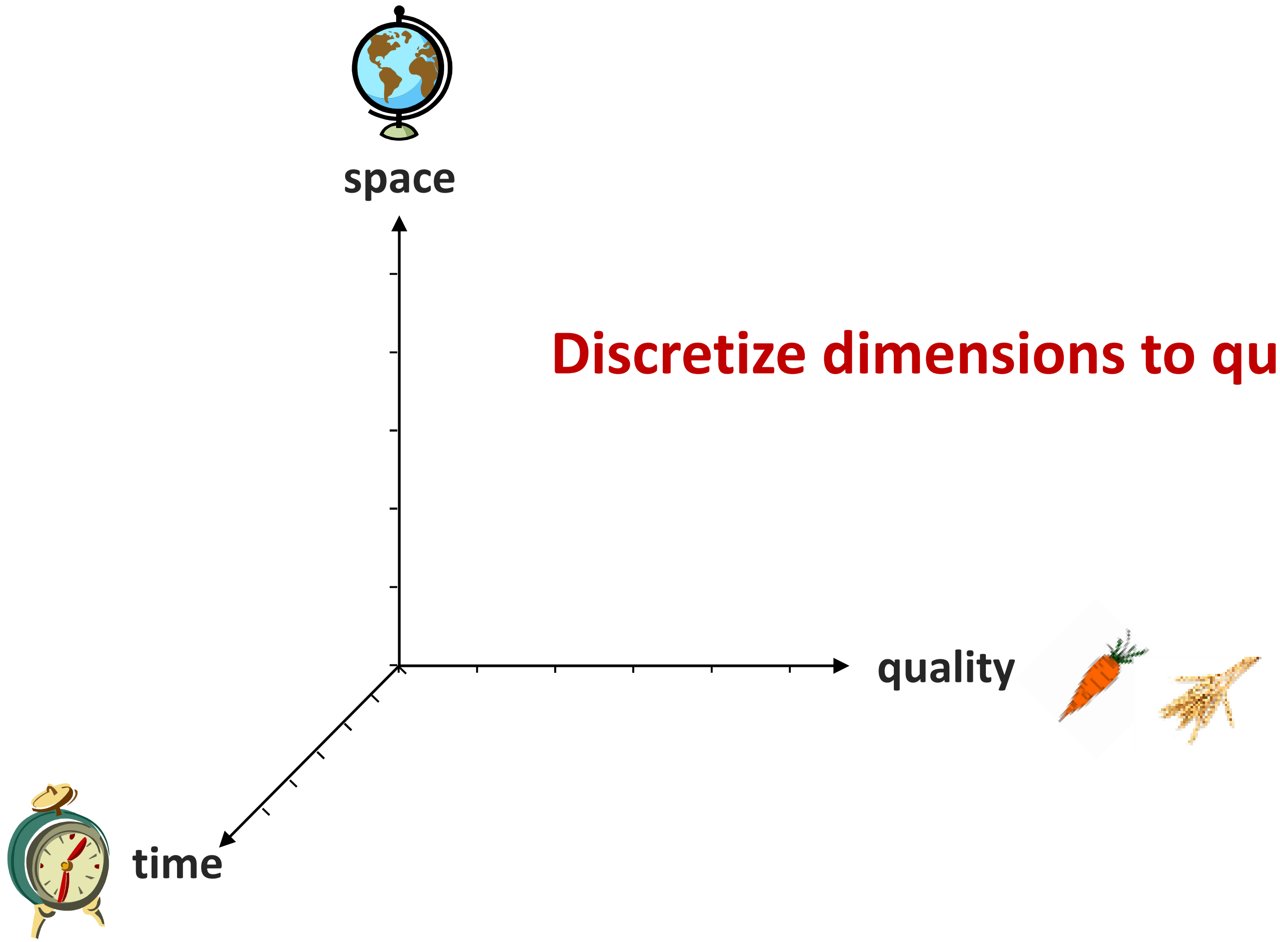
conceptualize & represent

have a problem...

Spatial relationships



Coverage can be defined for various dimensions

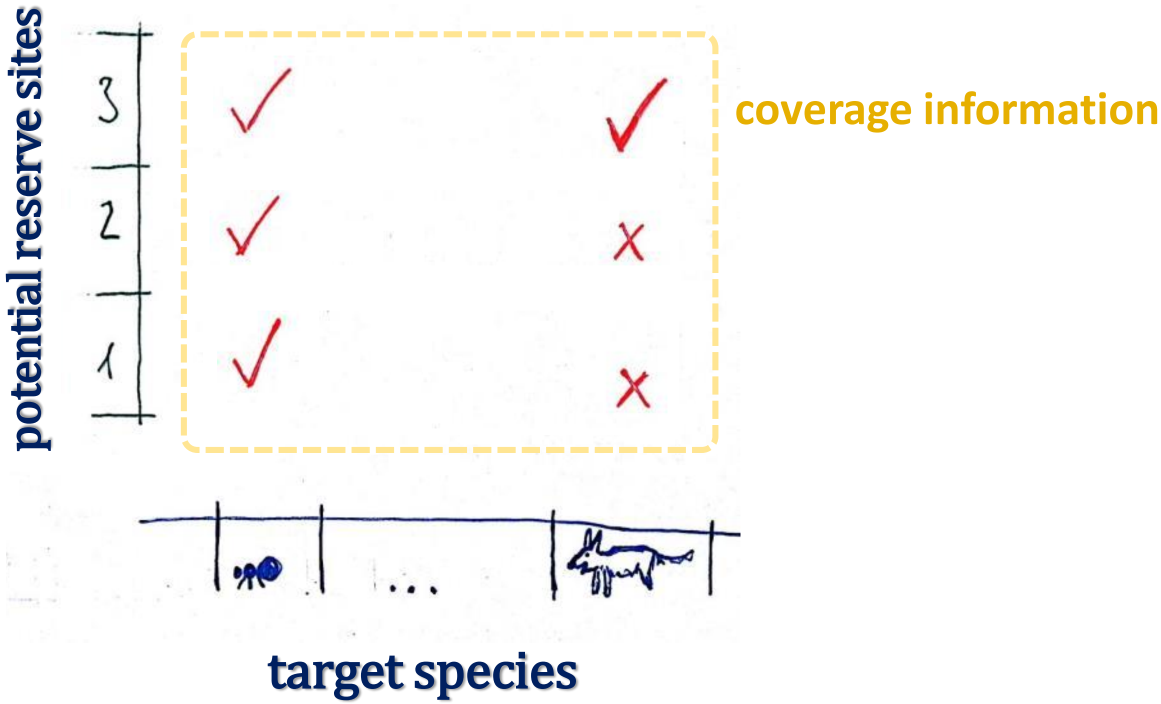


Discretize dimensions to quantify coverage!

Coverage by QUALITY

Select best combination from a set of potential reserve sites!

Maximize coverage of target species for a given count of reserve sites

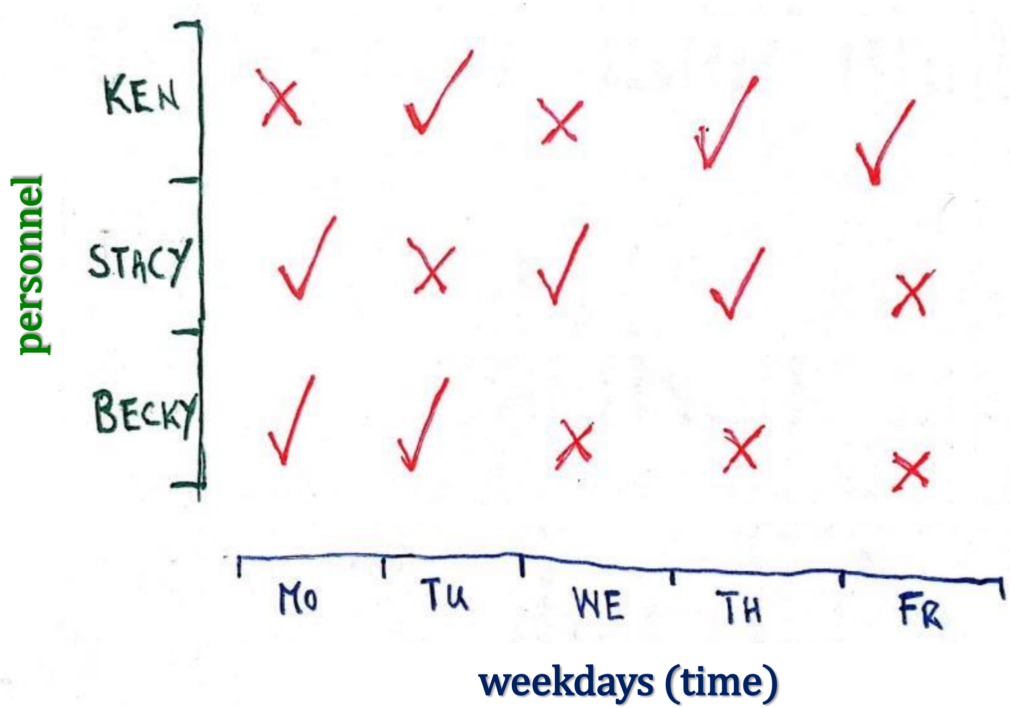


Church RL, Stoms DM, Davis FW (1996) Reserve selection as a maximal covering location problem. *Biological conservation* 76(2): 105-112.

Coverage by TIME

Select best combination of labor force

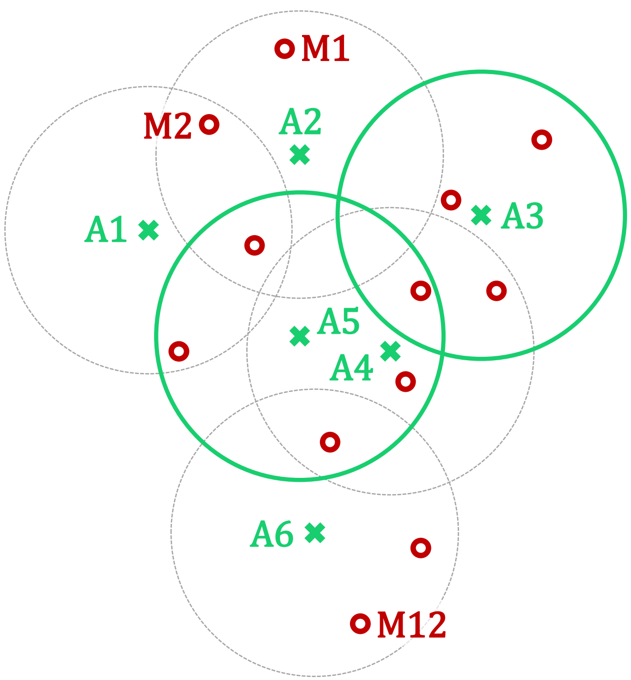
Minimize the number of personnel to cover all week days (e.g., hospital, bar, etc.)

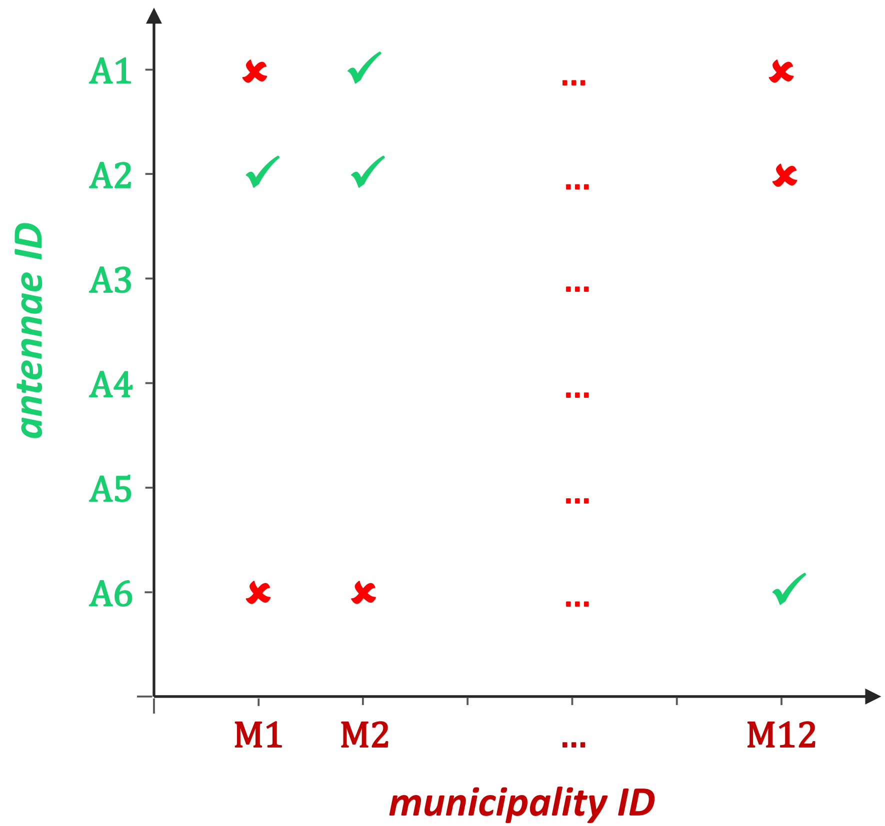


Coverage by SPACE

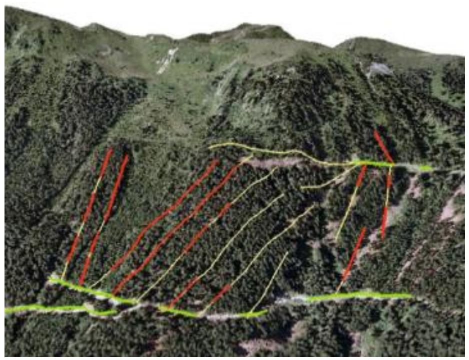
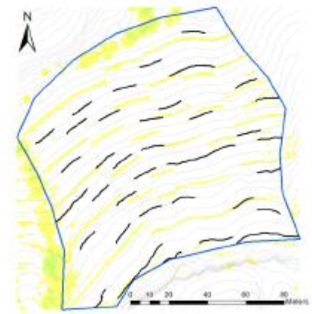
Select best combination of cellular antennae:

Maximize the municipalities covered for a given number of cellular antennae (e.g., 2)



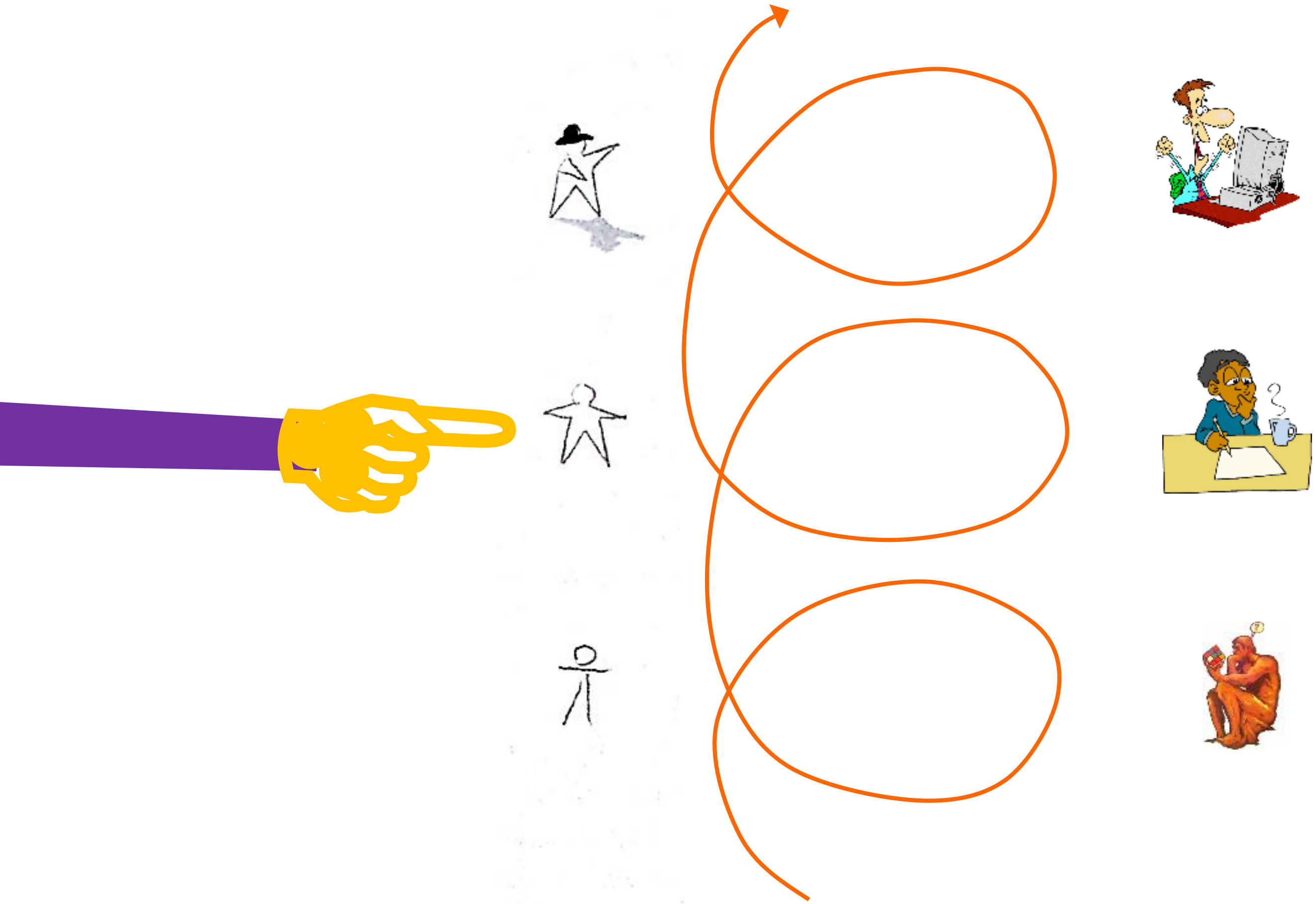


Coverage problems addressed in our professorship

		
demand	forested areas subject to harvest	areas suceptible to avalanches subject to protection
facility	**cable yarder lines**	**protection barriers**
coverage	perpendicular reach of a cable line	upslope area of a barrier
objective	identify layout which covers all forest at least cost	identify layout which covers all susceptible area at least cost
author	L. Bont (doctoral thesis)	A. Balicka (master thesis)

- B -

Mathematical formulation of covering problems



implement

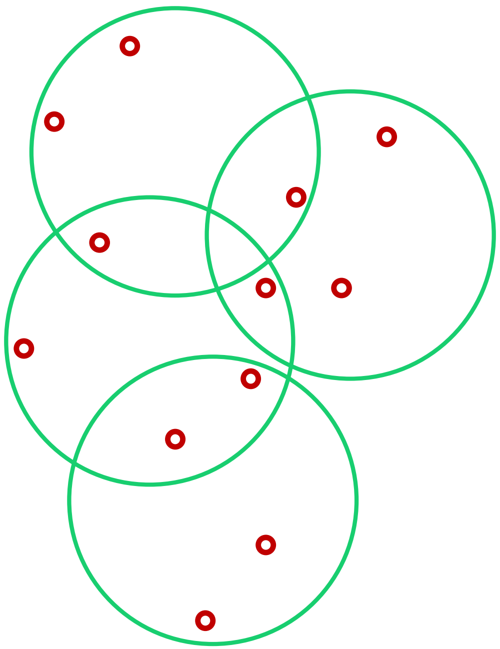
conceptualize & represent

have a problem...

Mathematical formulation of covering problems

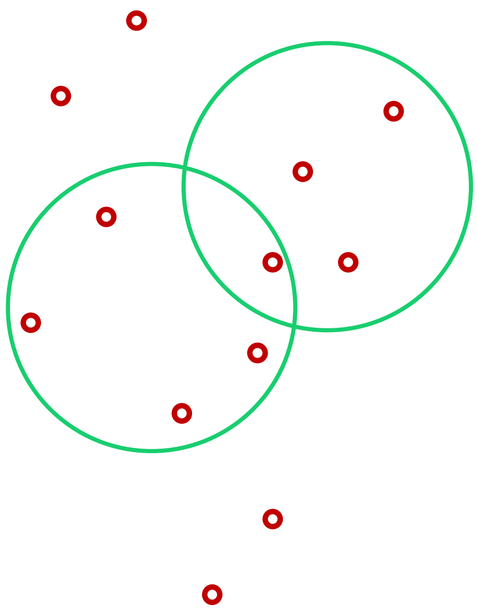
Set-Covering-Location-Problem [SCLP]

minimize the count of **service facilities** to cover all **demands**

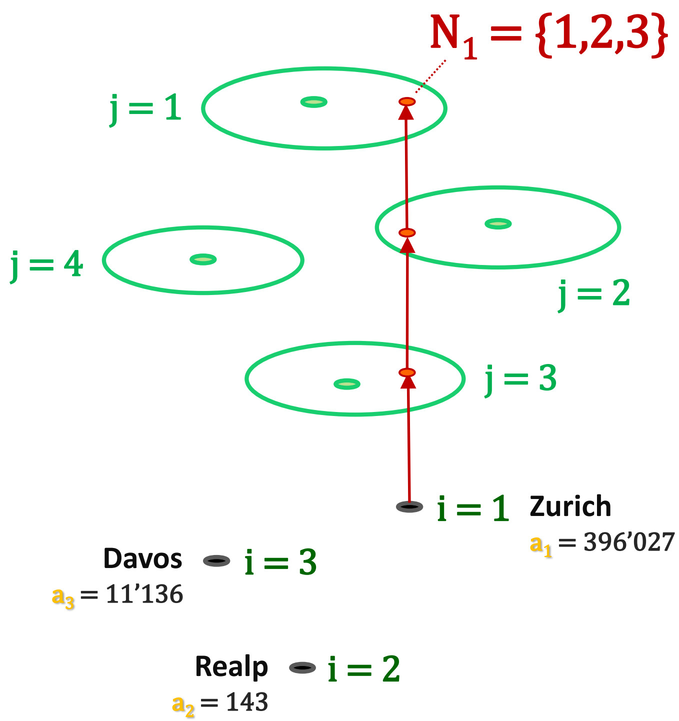


Maximal-Covering-Location-Problem [MCLP]

maximize the covered **demands** with a given count of **service facilities**



Variables



Decision variable

select facility j

$$x_j = \begin{cases} 1, \textit{select facility } j \\ 0, \textit{not selected} \end{cases}$$

$\mathbf{n}$: count of facility locations

State variable

coverage status of demand i

$$y_i = \begin{cases} 1, \textit{demand } i \textit{ is covered} \\ 0, \textit{not covered} \end{cases}$$

$\mathbf{m}$: count of demand points

Coverage relationship

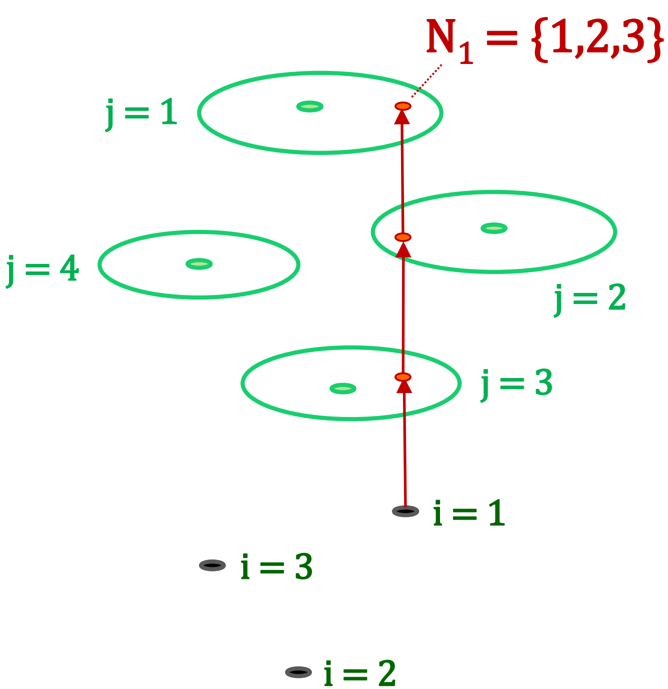
N_i : set of facilities j which cover demand i

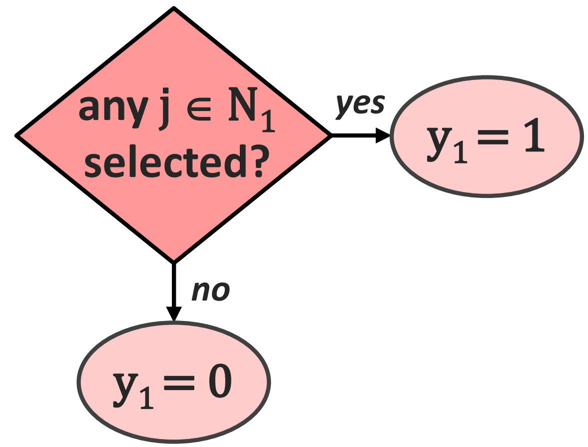
Demand information

a_i : weight (amount/importance)

***State variables** and the **coverage relationship** are the secret ingredients*

Control coverage status via constraint





translate to optimisation model

$$\text{MAX} \quad y_1$$

$$x_1 + x_2 + x_3 \geq y_1$$

Optimisation model for MCLP

maximize the covered **demands (y_i)** with a given count **p** of **service facilities (x_j)**

maximize coverage

$$\text{MAX} \quad y_1 + y_2 + y_3$$

(C1) control coverage

$$
\begin{aligned}
N_1 = \{1, 2, 3\} &\quad x_1 + x_2 + x_3 - y_1 \geq 0 \\
N_2 = \{1\} &\quad x_1 - y_2 \geq 0 \\
N_3 = \{1,4\} &\quad x_1 + x_4 - y_3 \geq 0
\end{aligned}
$$

(C2) control count of facilities

$$x_1 + x_2 + x_3 + x_4 = p$$

$$x_j, y_i \in \{0,1\}$$

Church R, ReVelle CS (1974). The maximal covering location problem. *Papers in regional science*, *32*(1), 101-118.

Optimisation model for SCLP

minimize the count of **service facilities (x_j)** to cover all **demands (y_i)**

all demands must be covered ⇨ $y_i = 1$

minimize count of facilities

$$\text{MIN} \quad x_1 + x_2 + x_3 + x_4$$

control coverage

$$N_1 = \{1, 2, 3\} \qquad x_1 + x_2 + x_3 \geq 1$$

$$N_2 = \{1\} \qquad x_1 \geq 1$$

$$N_3 = \{1,4\} \qquad x_1 + x_4 \geq 1$$

$$x_j \in \{0,1\}$$

Condensed notation of optimisation models

Set-Covering-Location-Problem [SCLP]

MIN $$\sum_{j=1}^{n} x_j$$

s.t. $$\sum_{j \in N_i} x_j \geq 1 \quad \text{, for all i=1,...,m}$$

$$x_j, y_i \in \{0,1\}$$

Maximal-Covering-Location-Problem [MCLP]

MAX $$\sum_{i=1}^{m} y_i$$

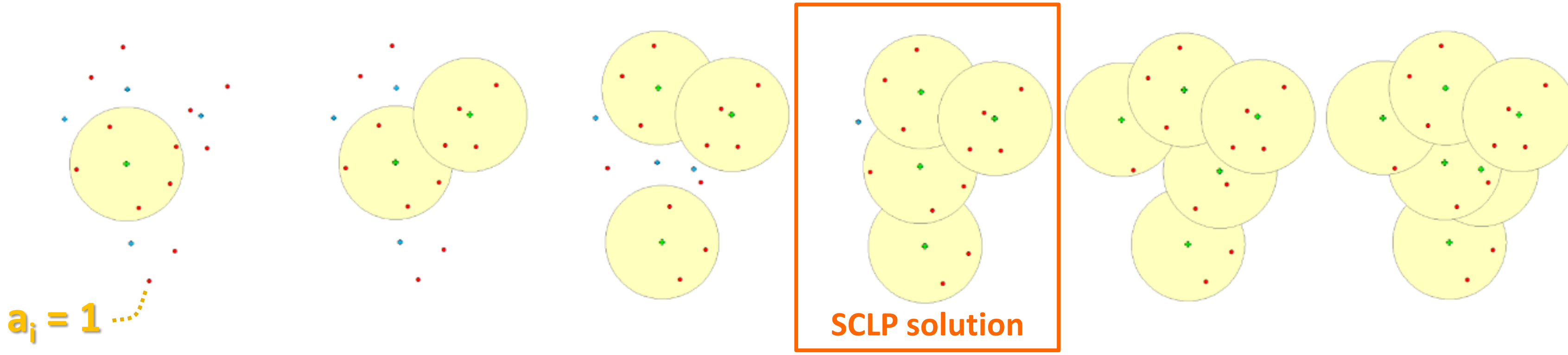
s.t. $$\sum_{j \in N_i} x_j - y_i \geq 0 \quad \text{, for all i=1,...,m}$$

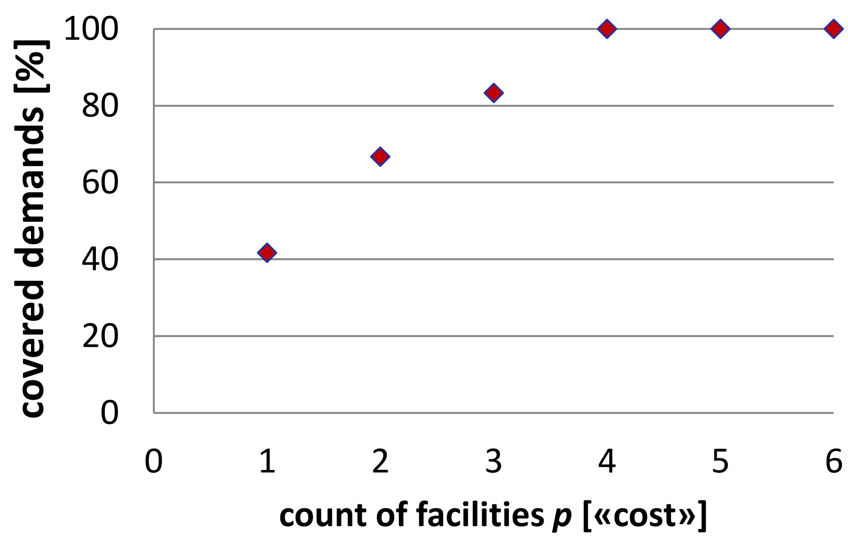
$$\sum_{j=1}^{n} x_j = p$$

$$x_j, y_i \in \{0,1\}$$

Exploring MCLP for the «cellular antennae location problem»

p **:**	1	2	3	4	5	6
f **:**	5	8	10	12	12	12



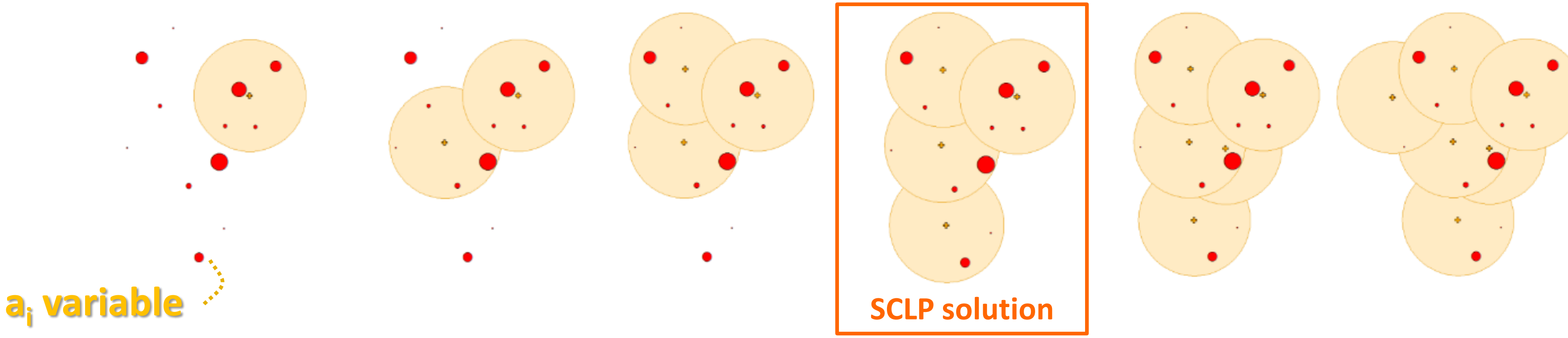


Trade-off

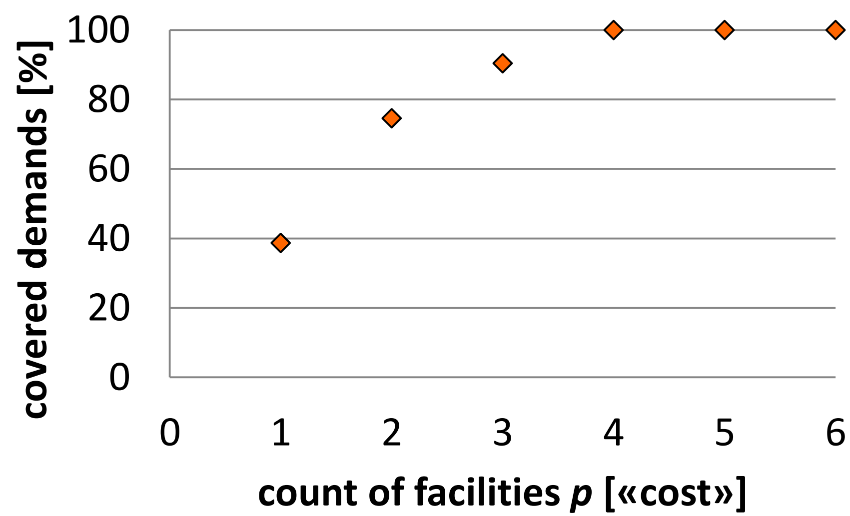
objectives cannot be fully achieved concurrently

Exploring MCLP for the «cellular antennae location problem»

p **:**	1	2	3	4	5	6
f **:**	125’000	241’000	292’000	323’000	323’000	323’000

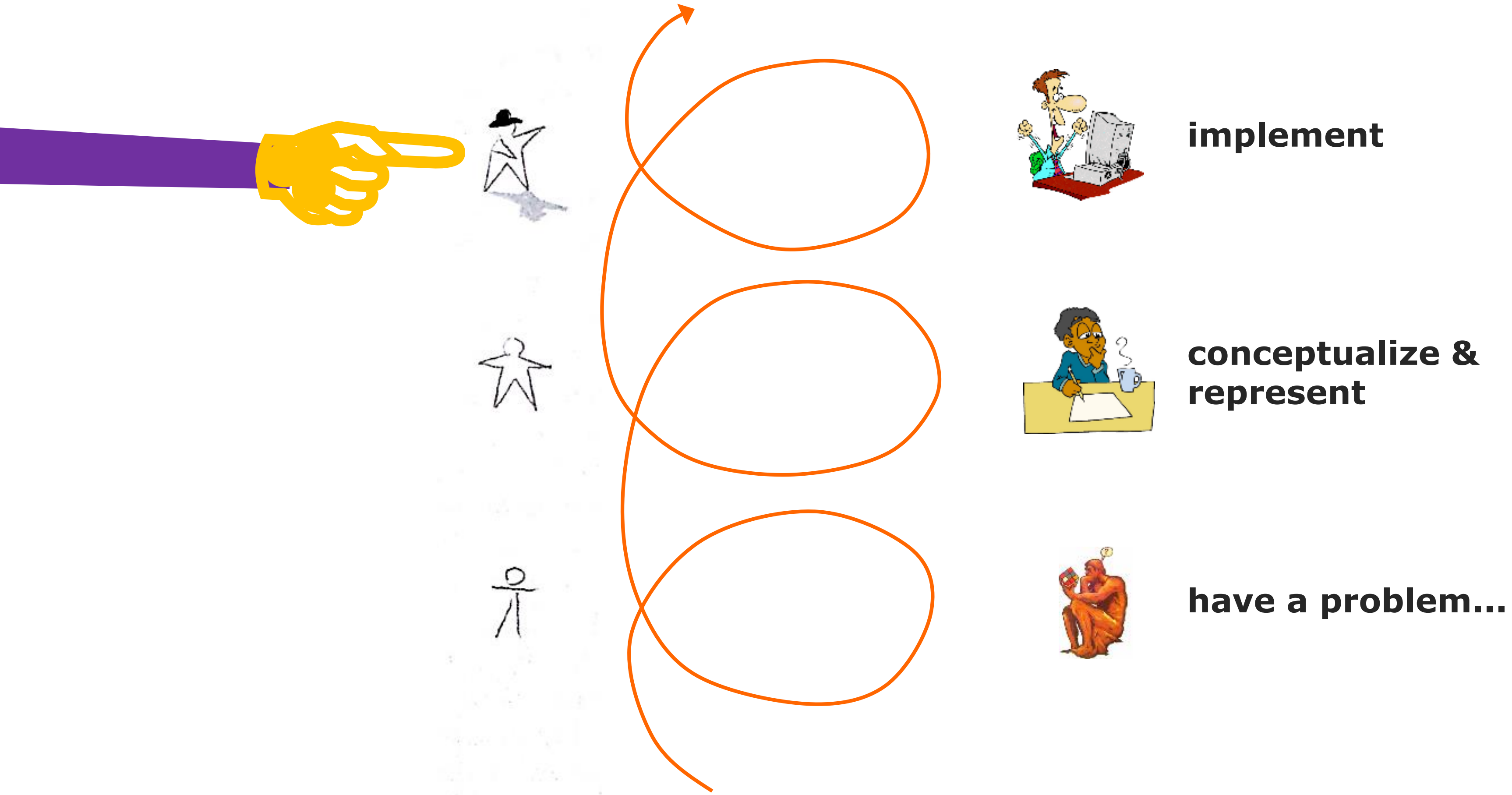


count of facilities	share of covered demand points
1	**39**
2	**75**
3	**90**
4	**100**
5	**100**
6	**100**



- C -

Implementation of a covering problem



implement
conceptualize &
represent
have a problem...

1st step of implementation – matrix notation model

maximize coverage

$$\text{MAX} \quad a_1y_1 + a_2y_2 + a_3y_3$$

control coverage

$$N1 = \{1, 2, 3\} \qquad x_1 + x_2 + x_3 - y_1 \geq 0$$

$$N2 = \{1\} \qquad x_1 - y_2 \geq 0$$

$$N3 = \{1,4\} \qquad x_1 + x_4 - y_3 \geq 0$$

control count of facilities

$$x_1 + x_2 + x_3 + x_4 = p$$

Transfer to matrix notation model

decision variables | state variables

$$f' - \begin{bmatrix} 0 & 0 & 0 & 0 & a_1 & a_2 & a_3 \end{bmatrix}$$

get information from spatial analysis!

$$A - \begin{bmatrix} 1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \geq \\ \geq \\ \geq \\ = \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ p \end{bmatrix} - b$$

Matrix notation model versions

V1 : Easy-to-read

use for planning of matrix notation model

$$
\begin{array}{r}
\textit{xType}' \; [\text{'B'} \quad \text{'B'} \quad \text{'B'} \quad \text{'B'} \quad \text{'B'} \quad \text{'B'} \quad \text{'B'}] \\
f' \; [0 \quad 0 \quad 0 \quad 0 \quad a_1 \quad a_2 \quad a_3]
\end{array}
$$

$$
A \begin{bmatrix} 1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} \geq \\ \geq \\ \geq \\ = \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \\ p \end{bmatrix} b
$$

sign

V2 : Implementation

Matlab

$$
f = \begin{pmatrix} 0 \\ \dots \\ a_3 \end{pmatrix} \qquad xType = \begin{pmatrix} 'B' \\ \dots \\ 'B' \end{pmatrix}
$$

$$
A = \begin{pmatrix} 1 & \dots & 0 \\ \dots & \ddots & \dots \\ 1 & \dots & 0 \end{pmatrix} \quad sign = \begin{pmatrix} 1 \\ \dots \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ \dots \\ p \end{pmatrix}
$$

Transpose
f, xType

Forest fire surveillance station location planning

The canton of Glarus has identified 16 potential sites for fire surveillance stations for a region of interest. They would like to select 8 out of them to maximize coverage of forested areas. Coverage should be assessed based on a digital surface model (DSM) to include shading by vegetation. A special emphasis should be given to southern slopes where fire susceptibility is supposed to be higher.

(1) Suggest the 8 stations which should be realized to maximize coverage!
(2) Provide a figure which characterizes the trade-off between the count of stations and the coverage. Is 8 stations really a good investment?

Geodata

- StationLocations *[point]*
- ForestCover *[polygon]*
- DSM *[raster]*
- DTM *[raster]*

Station parameters

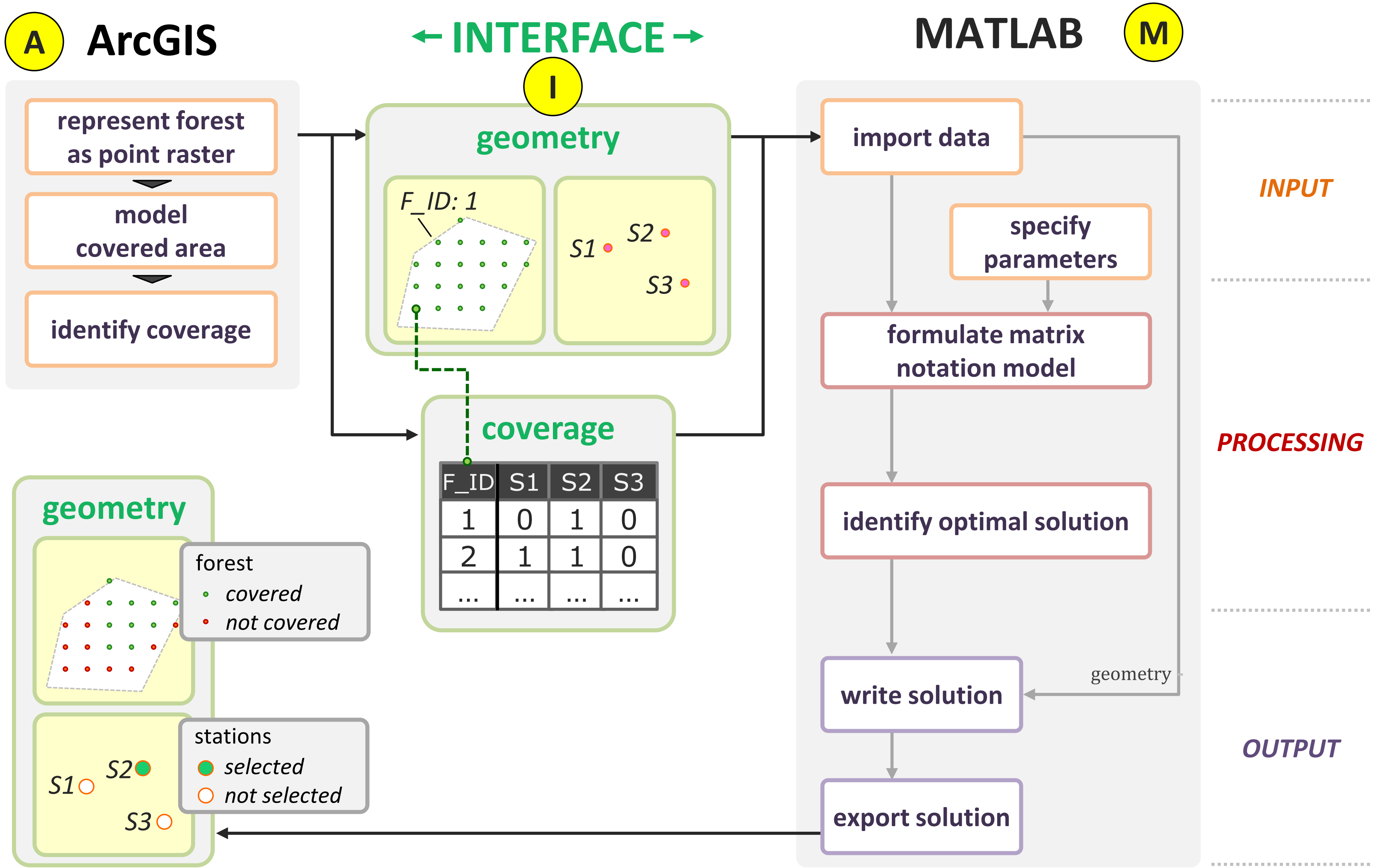
- station height: 40m
- cost are supposed to be equal for each station

forest susceptibility (weight)

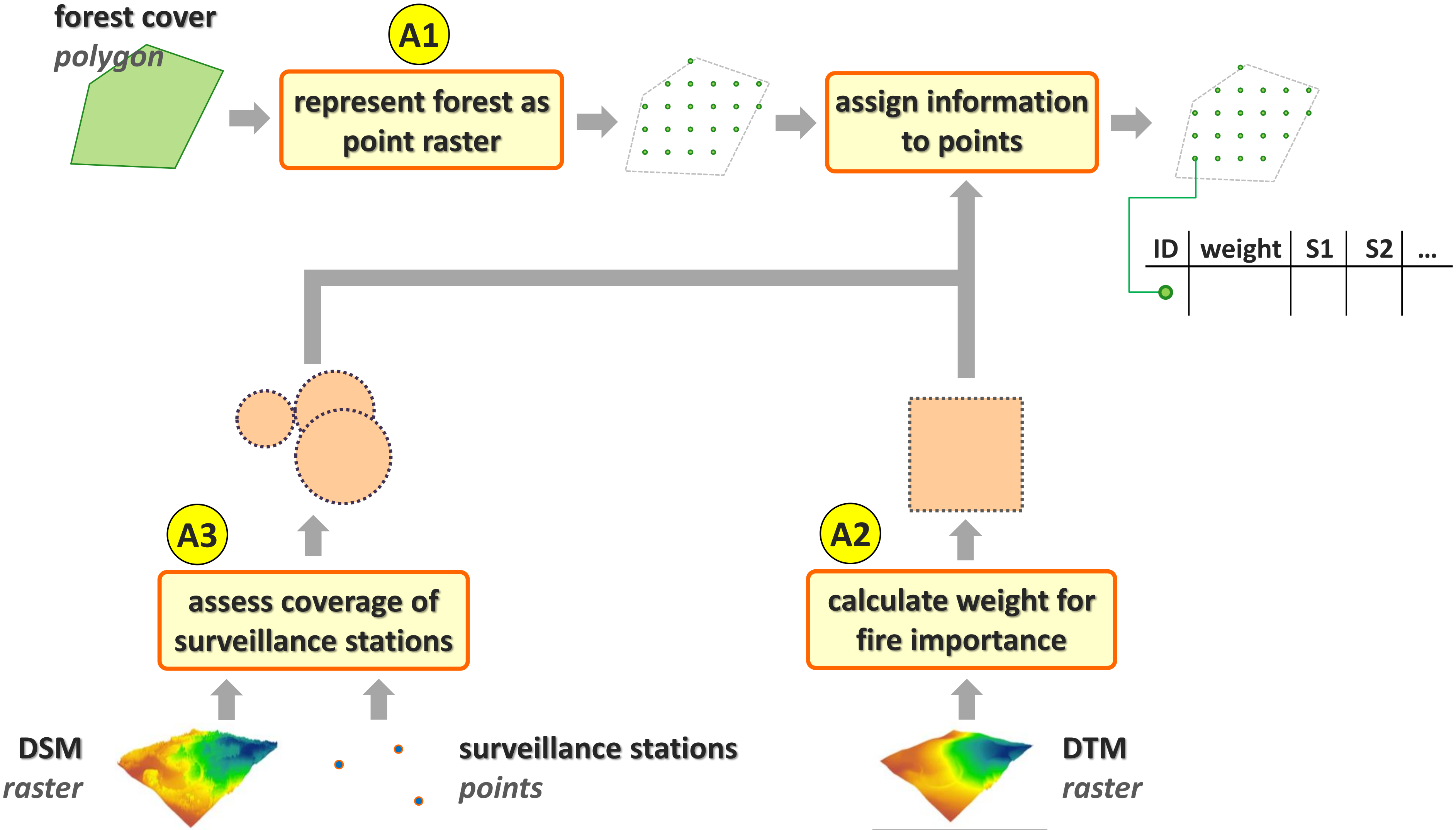
- south : **2**
- other : **1**

Bao S, Xiao N, Lai Z, Zhang H, Kim C (2015) Optimizing watchtower locations for forest fire monitoring using location models. *Fire Safety Journal 71:* 100-109.

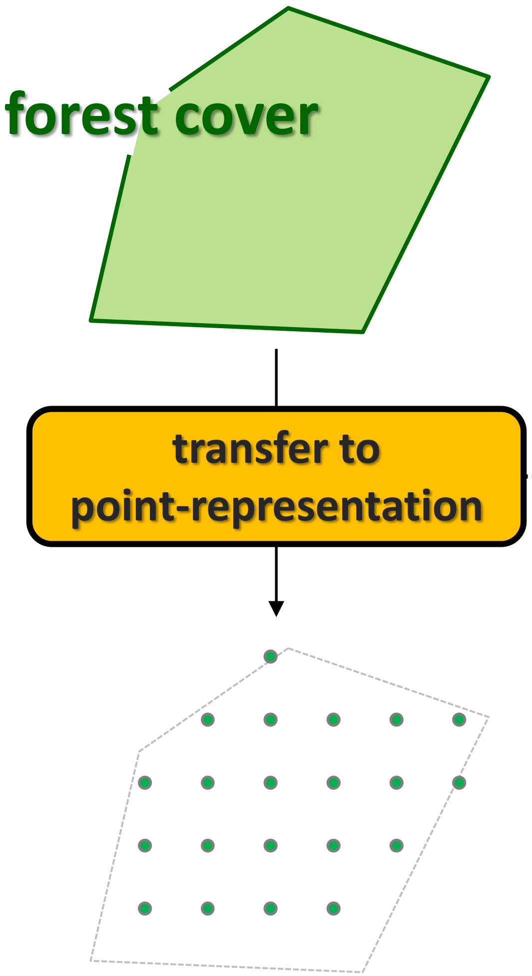
Conceptualisation ➲ work flow



(A) Work flow ArcGIS



(A1) Represent demand: forested area

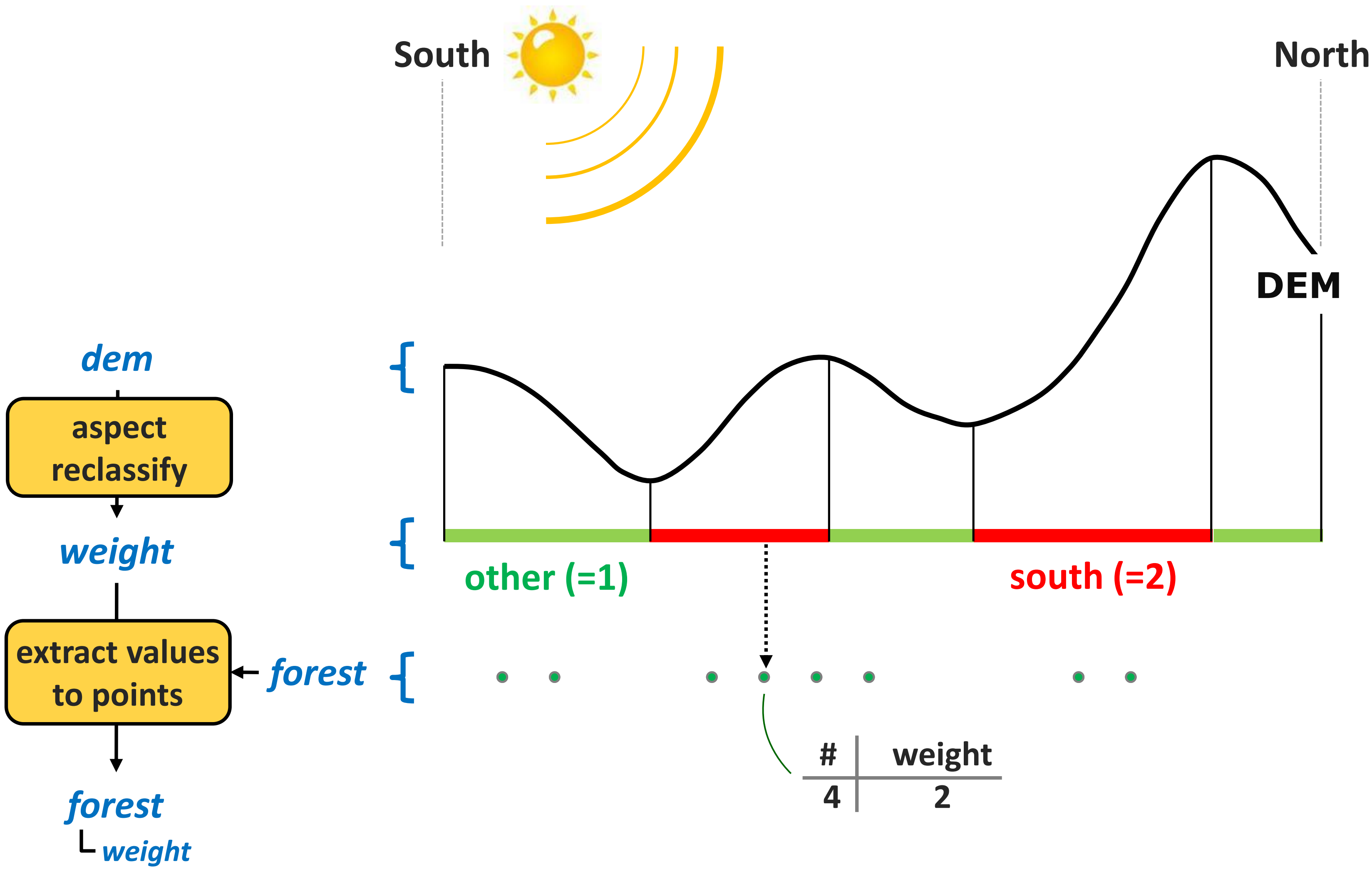


Resolution

- ➲ **adjust resolution depending on**
  (1) problem and
  (2) available data
- ➲ **affects computability of optimisation model**

Combine «create fishnet» with «clip» command in ArcGIS

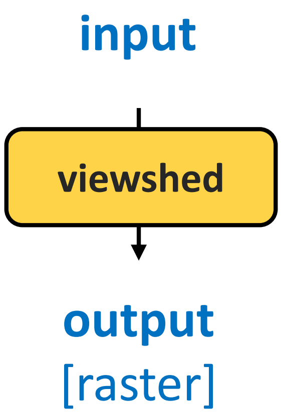
(A2) Assign «weight» to forest points

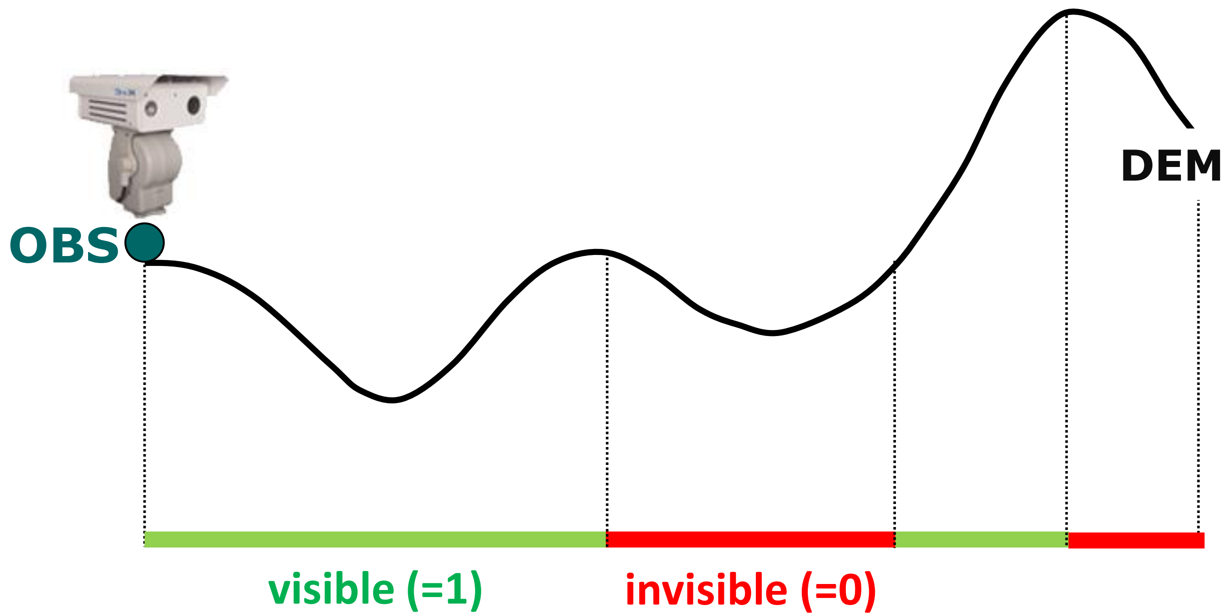


(A3) Model coverage – viewshed

Viewshed defines …

… all the points in the terrain (**DEM**) which are visible from one point (**OBS**)





(A3) Computing coverage for several observer points

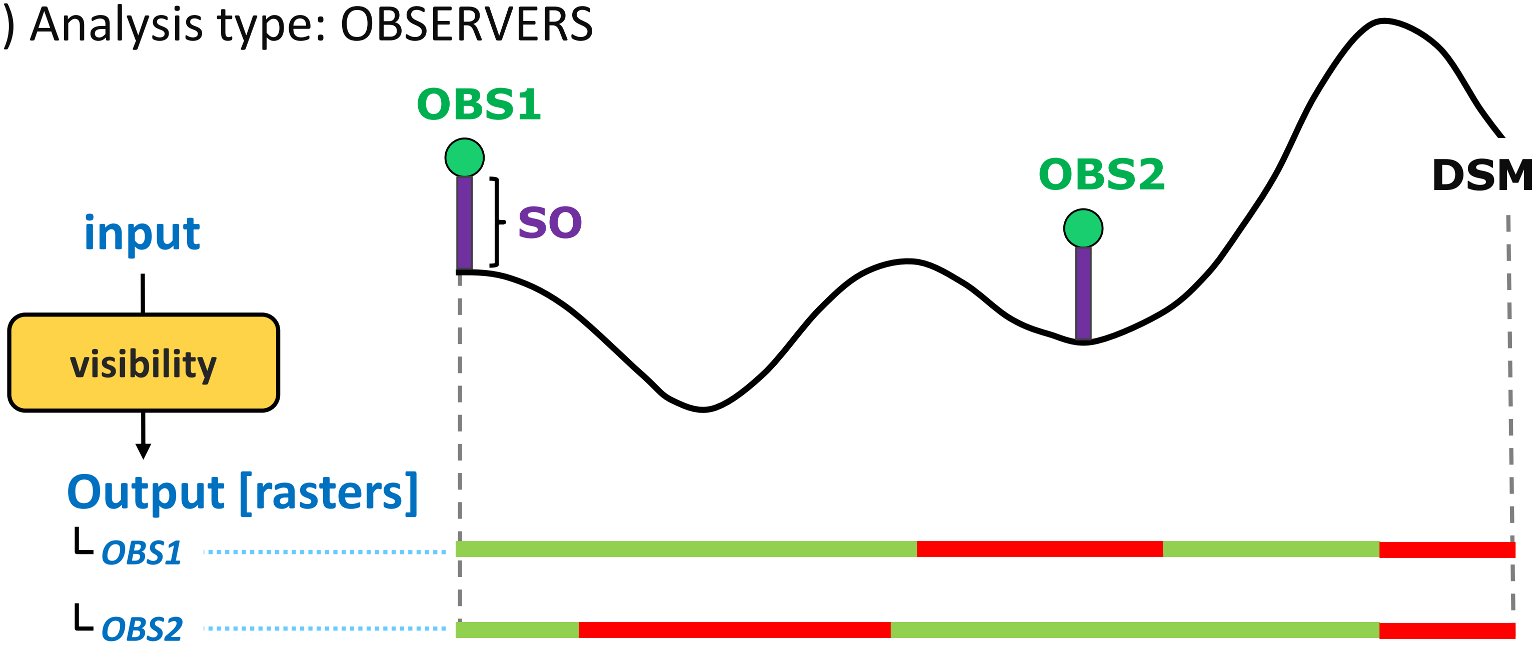
"Visibility" tool (ArcGIS) concurrently processes up to 16 observer points

(a) Digital SURFACE Model **DSM** [raster]

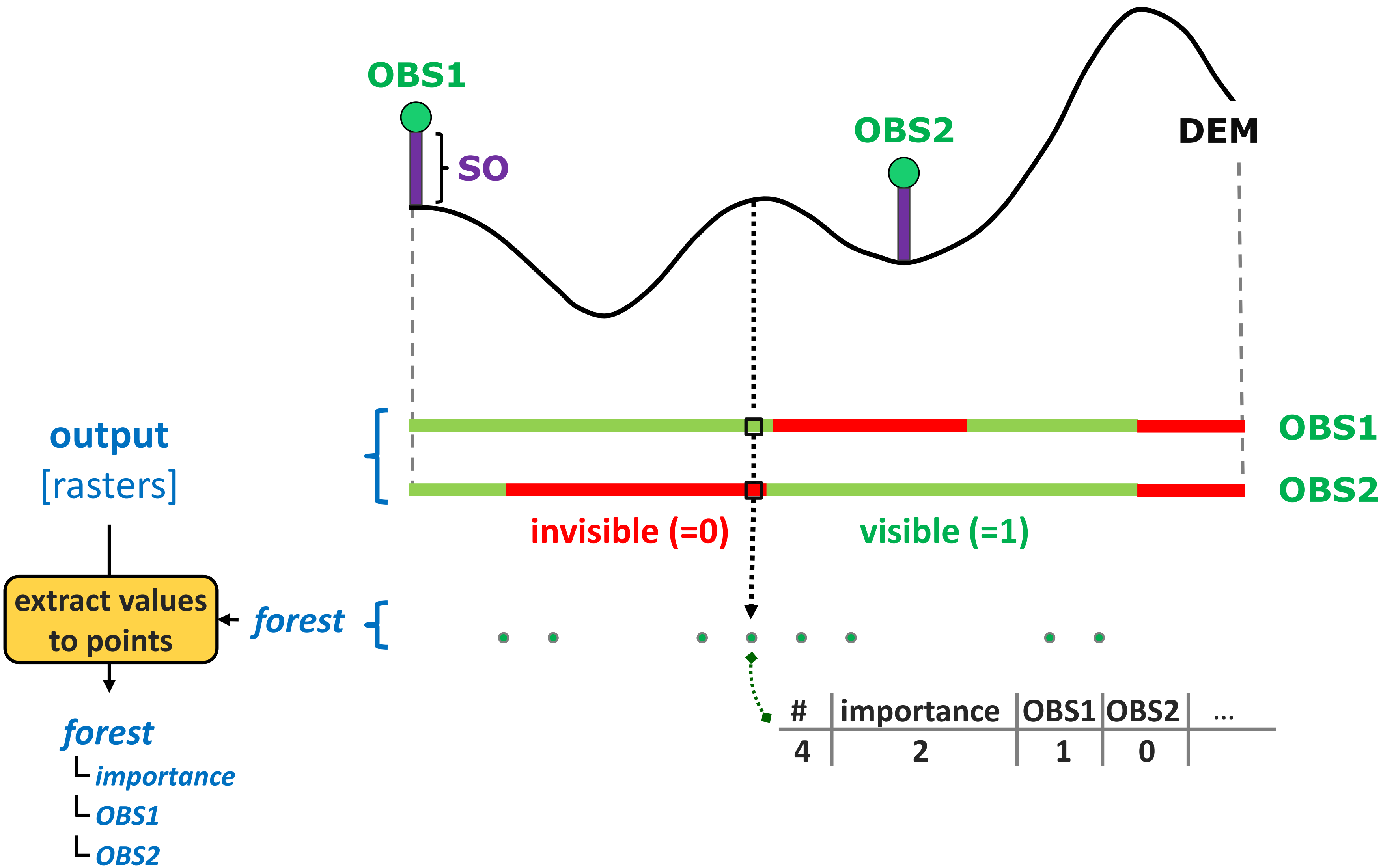
(b) Observer Points **OBS** [point feature]

(c) Observer offset **SO** [parameter in *m*]

(d) Analysis type: OBSERVERS

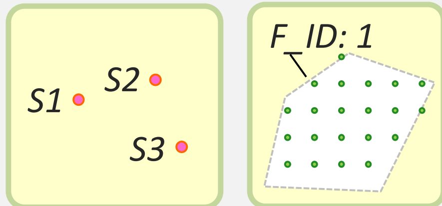


(A3) Identification of coverage table



(I) What do we have to pass to MATLAB?

geometry



coverage

#	OBS1	OBS2	...	weight
1	1	0		2
2	...	...		...

count of facilities

count of demands

importance

coverage

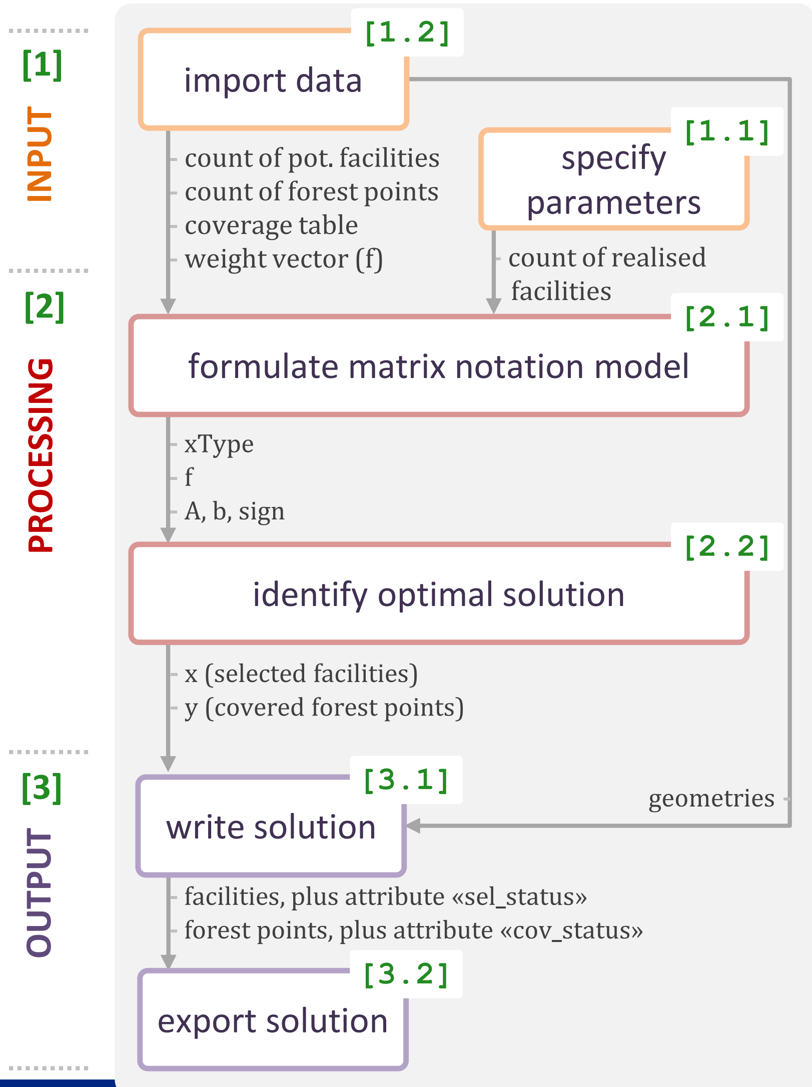
$$\boldsymbol{f} \text{ – } \begin{bmatrix} 0 & 0 & 0 & 0 & a_1 & a_2 & a_3 \end{bmatrix}$$

$$\boldsymbol{A} \text{ – } \begin{bmatrix} 1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \geq \\ \geq \\ \geq \\ = \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ p \end{bmatrix} \text{ – } \boldsymbol{b}$$

The necessary information to formulate the optimisation model!

(M) Work flow MATLAB

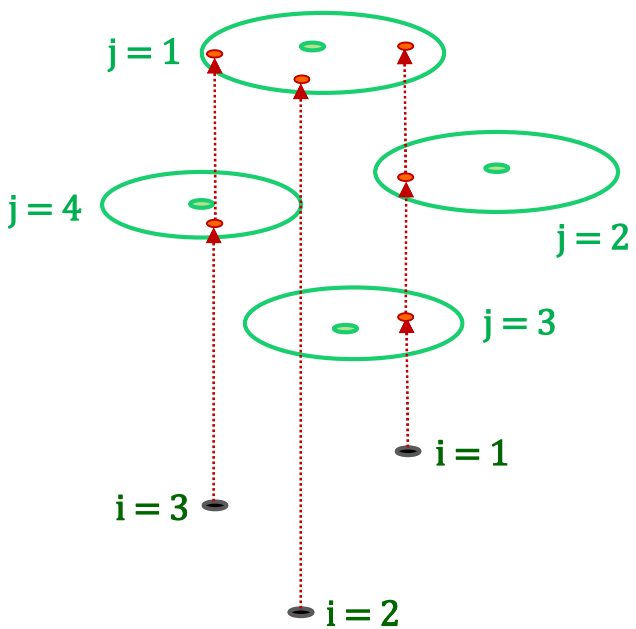
CONCEPT



IMPLEMENTATION

```
%% [1] Input

%  [1.1] Specify parameters

%  [1.2] Import data


%% [2] Processing

%  [2.1] Formulate model

%  [2.2] Identify optimal solution


%% [3] Output

%  [3.1] Write solution

%  [3.2] Export solution
```

(M) Coverage representation for computation



- **mathematical notation**

$$N_1 = \{1, 2, 3\}$$
$$N_2 = \{1\}$$
$$N_3 = \{1,4\}$$

- **computer-friendly notations**

«list»

i	*j*
1	1
1	2
1	3
2	1
3	1
3	4

«matrix»

$$\begin{array}{c|cccc} & & j & & \\ \hline i & 1 & 1 & 1 & 0 \\ & 1 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 1 \end{array}$$

1 : facility *j* covers demand *i*
0 : else

(M) Creation of the «coverage matrix» within the optimisation model

«list»

for

i	j
1	1
1	2
1	3
2	1
3	1
3	4

end

«matrix»

$$\begin{array}{c|cccc} & & & j \rightarrow & \\ \hline i \downarrow & 1 & 1 & 1 & 0 \\ & 1 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 1 \end{array}$$

Paste!

$$f - \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$A - \begin{bmatrix} 1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \geq \\ \geq \\ \geq \\ = \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - b$$

(M) Extract coverage information

facility-ID

j

demand-ID

i

all

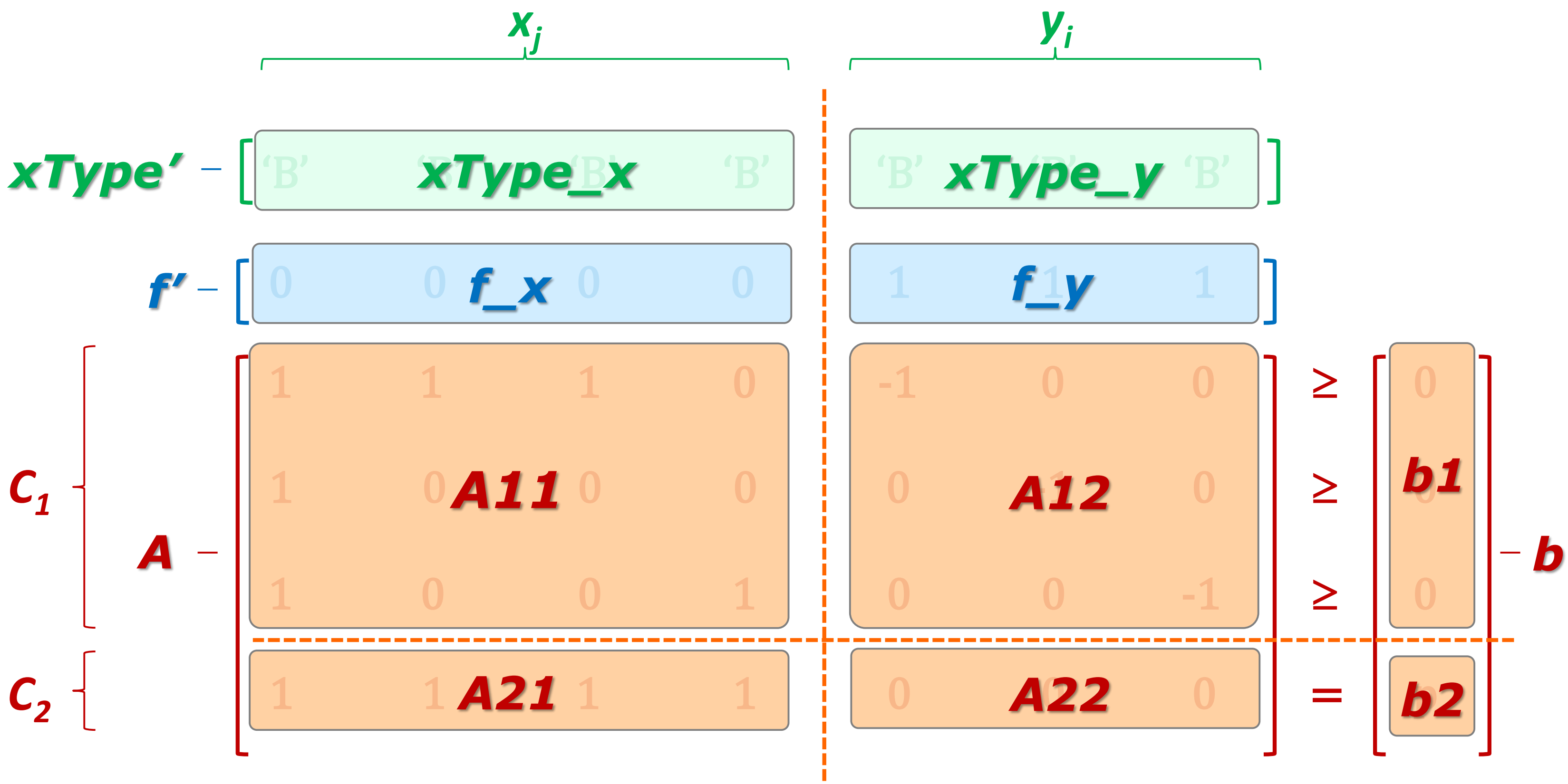
#	OBS1	OBS2	OBS3	OBS4	w
1	1	1	1	0	1
2	1	0	0	0	2
3	1	0	0	1	1

table

2 to 5

```
coverage_table = table(:,2:5);
f_weight = table(:,6);
```

(M) Naming of arrays constituting an optimisation model



(M) Write the solution to shapefiles

FacilityLocations

ID	SelStatus
1	1
2	0
3	0
4	0

ForestPointRaster

ID	Weight	CoverStatus
1	1	1
2	2	1
3	1	1

`x(1:n,1)`

`x((n+1):(m+n),1)`

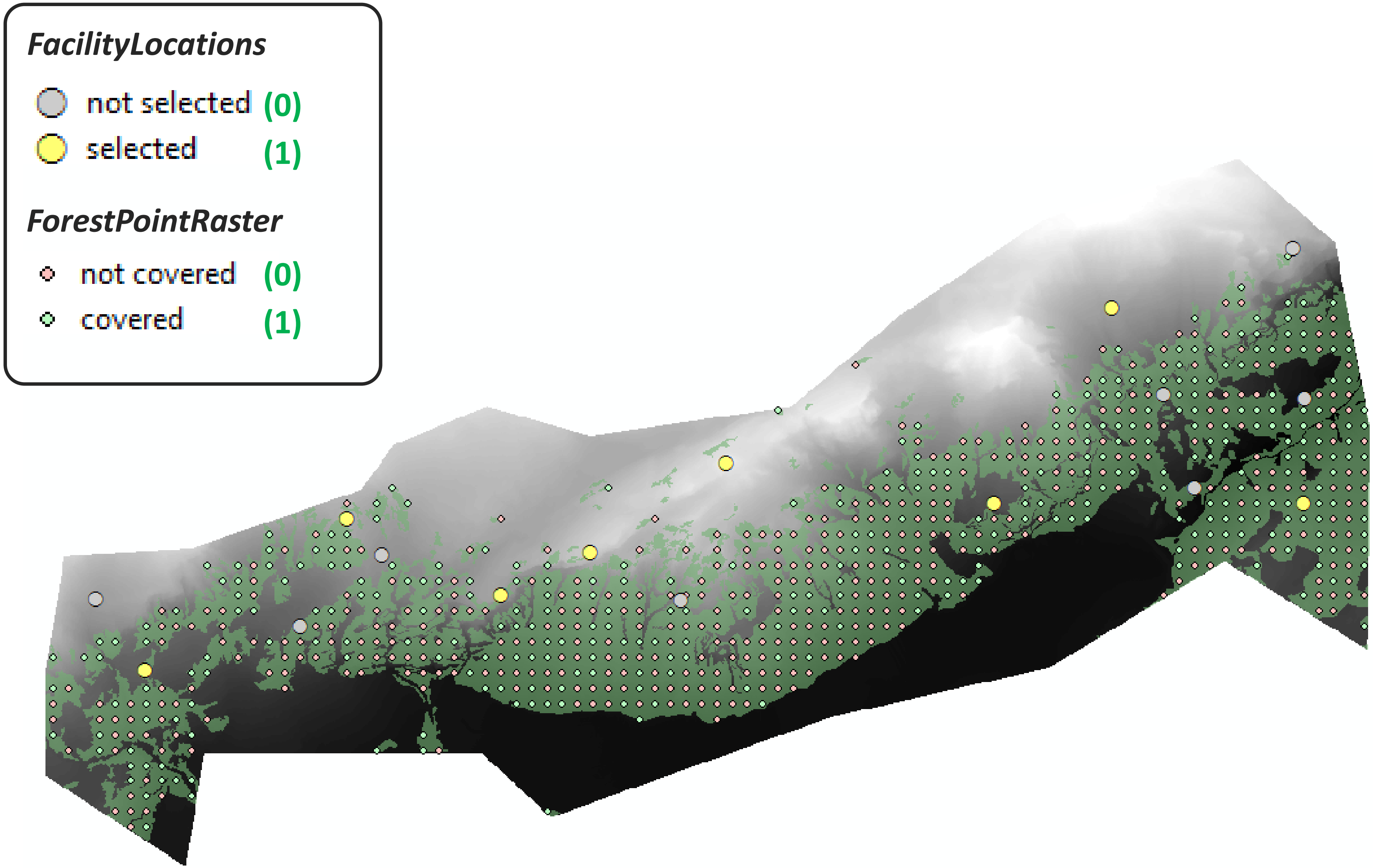
$$\begin{array}{cccccccc} & 1 & & & n & n+1 & & m+n \\ & \downarrow & & & \downarrow & \downarrow & & \downarrow \\ x' - & [\,1 & 0 & 0 & 0 & 1 & 1 & 1\,] \end{array}$$

n: count of facility locations
m: count of demand points

$$f' - [\,0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1\,]$$

$$A - \begin{bmatrix} 1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \geq \\ \geq \\ \geq \\ = \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - b$$

(A) Visualization of the results



(A) Trade-off curve

Run model for p = 1,…, 16