Simple Orbital Mechanical System Simulation (SOMSS)

Model Description And Sample Simulation Results

Mechanics

Folkenberg, Siro

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INTRODUCTION

The Earth is a geoid, but to make life less painful, we make mathematical equations involved less complex in our computations by approximating its shape as an oblate spheroid or an ellipsoid. Whichever of these shapes, that we choose to use in our approximations still reflects a figure of the Earth that is bulging at the equator and flattening at the poles. With an ellipsoid used in the approximation, the surface at the equator is units away from the center mass of the Earth on a semi-major axis(a) and the surface on each of the poles units away from the center mass of the Earth on the semi-minor axis(b). What is the meaning of this? In high school I was once told that the Earth's gravity is fairly constant allover the surface of the Earth. This is just an assumption for a perfectly spherical Earth and with perfectly uniform density. But this isn't the case, because the Earth is not a perfect sphere. The Earth fits an ellipsoid, an oblate spheroid, even much better as a geoid, but a geoid would be a pain in the ass in computations. At the equator, you are about 6378km(the magnitude of semi-major axis) away from the center mass of the Earth, and at one of the poles, 6357km(magnitude of semi-minor axis) away from the center mass. In accordance to the law of gravitation, the gravitational effect has an inverse square relationship to the distance between the center mass of the Earth and the center mass of the body under acceleration due to gravity. This body would thus weigh less at the equator and more at the poles.

It's a fact that the Earth spins on its axis counterclockwise from West to East in prograde motion, and a spot on the equator rotates at approximately 1669.8 $kmhr^{-1}$, faster than at the poles, in fact at the North pole(90^0 North) and South pole(90^0 South) the speed is effectively 0, since that spot rotates once in 24 hours at a very very slow speed. So why is it ideal to put a payload in orbit in the East coast of an equatorial region like Kenya? Well, I'll give you a few reasons. At the equator, the payload weighs less, the rotation rate is higher and the payload is subjected to the the slingshot effect during launch(some sudden natural push). In case something unexpected goes wrong so that there's hindrance during launch, that would prevent the body from getting into destined orbit, the payload and launch vehicle debris fall into the ocean, away from populated areas. These are just the initial steps to be gotten right for a successful placement of a payload in its defined orbit.

1. AN ORBIT

Once the body is in orbit, what does its orbit really look like? In order to know and understand what keeps a spaceborne vehicle like an artificial satellite in orbit, it's important to understand what an orbit really is or what it really looks like.
An orbit in a layman's language is a closed path around which a planetary body or an artificial satellite travels. The orbit of a spaceborne vehicle around the Earth seems to be elliptical, a 2-dimensional figure defined by a plane that fully intersects a cone. An elliptical orbit is not defined by a center like a circular orbit, instead it's defined by 2 focii, and the sum of the distance from focii is always a constant.

A spaceborne vehicle orbits the Earth on an orbit with one focus at the Earth's center mass, whereas the other focus happens to be imaginary or just an empty point.

1.1 Orbital elements

We define orbits using 6 Keplerian elements(eccentricity, semi-major axis, inclination, right ascension of the ascending node, argument of perigee, and true anomaly) and a time stamp usually referred to as the **epoch**, some of which describe the orientation of orbit, the size and shape of orbit, and the location of the spaceborne vehicle within the orbit.

1.1.1 Eccentricity

Represented by the symbol ***e*,** defines the roundness or shape of an orbit.
It can vary from $0 \le e < 1$ for closed orbits

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Where

- a is the semi-major axis.
- b is the semi-minor axis.

$e = 0$; means the orbit is circular.
$e \ge 1$; means the orbit is not closed(Used for interplanetary missions, so that the satellite never returns to the starting point).
$0<e<1$; means the orbit is elliptical.

1.1.2 Semi-major axis

Represented by the symbol ***a*,** defines the size of the orbit.
Knowing the semi-major axis, we can define the **apogee** and **perigee** of an orbit; the point in an orbit that is farthest and closest from the Earth respectively. We can also define the **apogee altitude** which is the distance between the surface of the Earth and the apogee, and the **perigee altitude** which is the distance between the perigee and the Earth's surface.

To define an orbital period ***T*,** the time taken to complete the orbit, we need to know the semi-major axis, so that;

$$T = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\mu}}$$

μ is a gravitational parameter.

1.2 Kepler's Laws

These laws govern planetary motion, or orbital mechanics of a planetary body.

1.2.1 Kepler's 1st Law

States that a spaceborne vehicle will orbit the Earth in an elliptical path, with the center mass of the Earth as one of its focii.

1.2.2 Kepler's 2nd Law

A straight line defined between the spaceborne vehicle and the Earth will sweep out equal areas during equal time periods anywhere along the orbit.
This law simply tells us that the speed of a spaceborne vehicle changes as the distance between it and the Earth changes.
The vehicle moves fastest at perigee and slowest at apogee.

1.2.3 Kepler's 3rd Law

That the orbital period of a spaceborne vehicle is related to semi-major axis of its orbit by

$$T^2 = \frac{4\ \pi^2\ a^3}{\mu}$$

1.3 Orbit Classification by Altitude

When we are launching a payload like a satellite to the outer space, we choose its orbit depending on the mission that we want it to achieve. The missions can be achieved at different altitudes. In Spaceborne Remote Sensing for instance, we can decide to put a satellite in a low earth orbit for Earth observation, or a geostationary orbit for climate change monitoring or weather forecasting and communication.

1.3.1 Sun-synchronous Orbit

Also known as low earth orbit. This orbit is relatively close to the Earth with a short orbital period of about 90 minutes. At this altitude, the satellite orbits the Earth in descending(North to South) and ascending modes(South to North). Data acquisition occurs in the descending mode. The path that the satellite traces on the ground during data acquisition is known as swath. At this orbit, satellites are commonly used for Earth observation involving agricultural activities, geothermal anomaly investigations et cetera.

1.3.2 Medium Earth Orbit

These orbits exist between Geostationary and sun-synchronous orbits.

1.3.3 Geostationary Orbit

This orbit exists at an altitude of about 36000km above the Earth's surface or 42164 km from the center mass of the Earth. The orbital period of a satellite in this orbit is equal to 24 hours. At this altitude, the satellite's orbital speed matches the Earth's rotation rate and so the satellite acquires or transmits data over one location on the Earth. This orbit is directly above the equator and so the inclination is exactly 0^0.

2. SLINGSHOT EFFECT AND ORBITAL ENERGY

The orbit of any payload that we send to the outer space is primarily dictated by the gravitational field strength of the closest single large primary central mass, say mass **m**, for instance the case of the Earth and its moon. This is the case if this primary central mass happens to be the only gravitational matter with a gravitational field that brings forth a vicinity within which the payload manifests. This spaceborne vehicle would thus trace an elliptical path around mass **m**, with constant angular momentum and orbital energy.

Orbital energy of two orbiting masses is considered the constant sum of their total kinetic energy(KE) and mutual potential energy(PE), divided by the reduced mass.

We can compute the kinetic energy of a given mass by;

$$KE=\frac{1}{2}\,mv^2 \quad ; \qquad \rightarrow \frac{KE}{m}=\frac{v^2}{2} \quad \text{.......(i)}$$

From Newton's law of gravitation; inverse square law,

$$F=G\,\frac{Mm}{R^2} \quad ;\text{ say} \quad \mu=GM \quad ;$$

$$F=m\;g \quad ;$$

In order to compute local gravitational acceleration, we would say;

$$g=\frac{\mu}{R^2}$$

We can compute the potential energy of a given mass using;

$$PE=-\,mgR \quad ; \quad \frac{PE}{m}=-\,Rg=\frac{-\,\mu}{R} \quad \text{....(ii)}$$

Combining equations (i) and (ii) gives us orbital energy.

$$E_{total}=\frac{v^2}{2}-\frac{\mu}{R}=\frac{-\,1}{2}\,\frac{\mu^2}{h^2}\left(1-e^2\right)=\frac{-\,\mu}{2\,a}$$

where;

- v is the relative orbital speed
- R is the orbital distance between the two bodies.
- h is the specific relative angular momentum.
- e is the orbital eccentricity.
- a is the semi-major axis.

What then would happen if this same space vehicle finds its way in the vicinity of a second massive body, say mass B, that happens to be orbiting the same primary central mass that it's orbiting? The moon for instance? Well, in this case the laws of Physics say that there will be exchange of both angular momentum and orbital energy between this space vehicle and mass B. The total orbital energy has to be conserved and this means whenever there is the exchange between the space vehicle and mass B, such that the space vehicle gains orbital energy, orbital energy of mass B has to decrease. And since there exist a direct proportionality between the orbital energy and the time taken to complete one orbit around the central mass, the **slingshot effect** on the spaceborne vehicle increases, so that it would appear like a slingshot throwing of this space vehicle into the orbit around the central mass A, the larger orbit, while at the same time the orbital energy and period of the massive body B decreases in this scenario. The effects of orbital energy exchange on the orbit and velocity are much felt on the space vehicle than mass B.

By the way, it is because of orbital energy exchange, that comets are often thrown into our solar system, by massive planets like Jupiter etc. Slingshot effect is also encountered during the launch of a space vehicle. The East coat of the equator happens to be a perfect launch site because of the slingshot effect too. It is from this region that the vehicle gets additional boost(natural push) during its initial ascent. This natural push comes from the Earth's rotation speed, which is much felt at the equator. In this case we have to launch the satellite Eastwards.

3. ESCAPE AND ORBITAL VELOCITIES

A satellite is taken to orbit, Geostationary orbit, sun-synchronous orbit or medium earth orbit by a rocket. In order for this rocket to get into outer space, it must be capable of escaping the Earth's gravitational field. Meaning this rocket must be capable of breaking free from the gravitational field of the Earth. It must be capable of increasing its acceleration to a minimum of 40320kmhr^{-1}.

3.1 Escape velocity(v_{esc})

This is the minimum velocity necessary for a body such as a rocket to break free from the Earth's gravitational field. For a fairly spherically symmetric massive object, the escape velocity can be computed using;

$$V_{esc} = \sqrt{\frac{2\ GM}{R}}$$

Where;

- G is the universal gravitational constant.
- M is the mass of the body whose gravitational field is to be escaped.
- R is the distance from the center mass of the massive object to the escaping object.

The Earth's escape velocity is approximately 11.186kms^{-1}. Once this mysterious velocity is achieved, no further impulse is necessary for the object to continue escaping. The object goes to a point of no return. At this velocity, the sum of kinetic energy and gravitational potential energy of the escaping object equal to zero. This object will move away from the Earth, slowing forever and approaching, but never attaining zero speed.

If this body has a velocity way less than escape velocity, it defines elliptical or circular orbit. If its velocity matches the escape velocity, it traces a parabolic trajectory. And if its velocity is greater than the escape velocity of the massive body, the parabolic trajectory graduates to a hyperbolic trajectory, and in this case, it has no other choice but to escape the massive body's gravitational field to a point of no return. The escape velocity of Earth is greater than the force that is needed to propel a satellite into orbit, and for this satellite to stay in orbit, it must not escape the Earth's gravity, and gravity must also not win to an extent of pulling it back to the atmosphere. The satellite must therefore balance acceleration due to the Earth's gravity with its inertia. If the satellite's velocity goes beyond the massive body's escape velocity, the satellite maneuvers towards a parabolic orbit and ultimately to a hyperbolic orbit, and then goes to a point of no return

3.2 Orbital Velocity

This velocity is needed in order for a satellite to achieve balance in a defined orbit or in order to keep this orbit stable. As once told by my former Spaceborne Remote Sensing lecturer, Dr. Capt.(rtd). S. Kanani, that on attaining the perfect orbital velocity for a particular orbit, the satellite 'falls forever'.

This velocity is complicated in that for a particular orbit, if the satellite's speed goes below this velocity, gravity 'takes charge' , the orbit becomes unstable and decays exponentially until the satellite finds its way into the atmosphere where it is subjected to burn upon reentry. If the satellite's speed is above the orbital velocity, its inertia wins and propels it farther into space, either way the orbit is not maintained.

4. PROGRAMMING ENVIRONMENT

SOMSS is entirely implemented in Python 3, an open source interpreted, object oriented high level scientific programming language with robust modules suitable for simulation.

4.1. Python Modules Used in SOMSS

In order to run SOMSS in your machine, you would need to install a Python 3 IDLE an the required modules:

4.1.1. numpy

The numpy(Numerical python) module provide mathematical functions that handle large dimensional arrays. It enhances performance and speeds up execution by adopting vectorization of mathematical functions.

4.1.2. matplotlib

This is for 2D data visualization(plotting of scientific data). Every aspect of a figure is customizable in this module.

4.1.3. tkinter

Tkinter is vital in implementation of Graphical User Interfaces(GUI). SOMSS GUI interface is implemented in Tkinter.

5. SAMPLE SIMULATION RESULTS

The initial conditions adopted in this simulation were computed for a geostationary orbit. The orbital stability of a geostationary orbit can only be achieved at 42164 km from the center of the Earth and directly above the equator.
At this altitude, SAT-X orbits the Earth at the same rate as the Earth's rotation rate. This results into an orbital period of about 1440 minutes. This value is equal to one sidereal day, the time the Earth takes for a completed rotation on its axis in a day. This also equates to an orbital velocity of about 3.07 kms^{-1}. A geostationary satellite thus has a stationary footprint on the ground.
The orbital period is directly related to the semi-major axis or orbital radius of geostationary orbit by;

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

where
a is the orbital radius
μ is a standard gravitational parameter.

The required velocity for achieving orbital stability in a geostationary orbit was computed from the following;

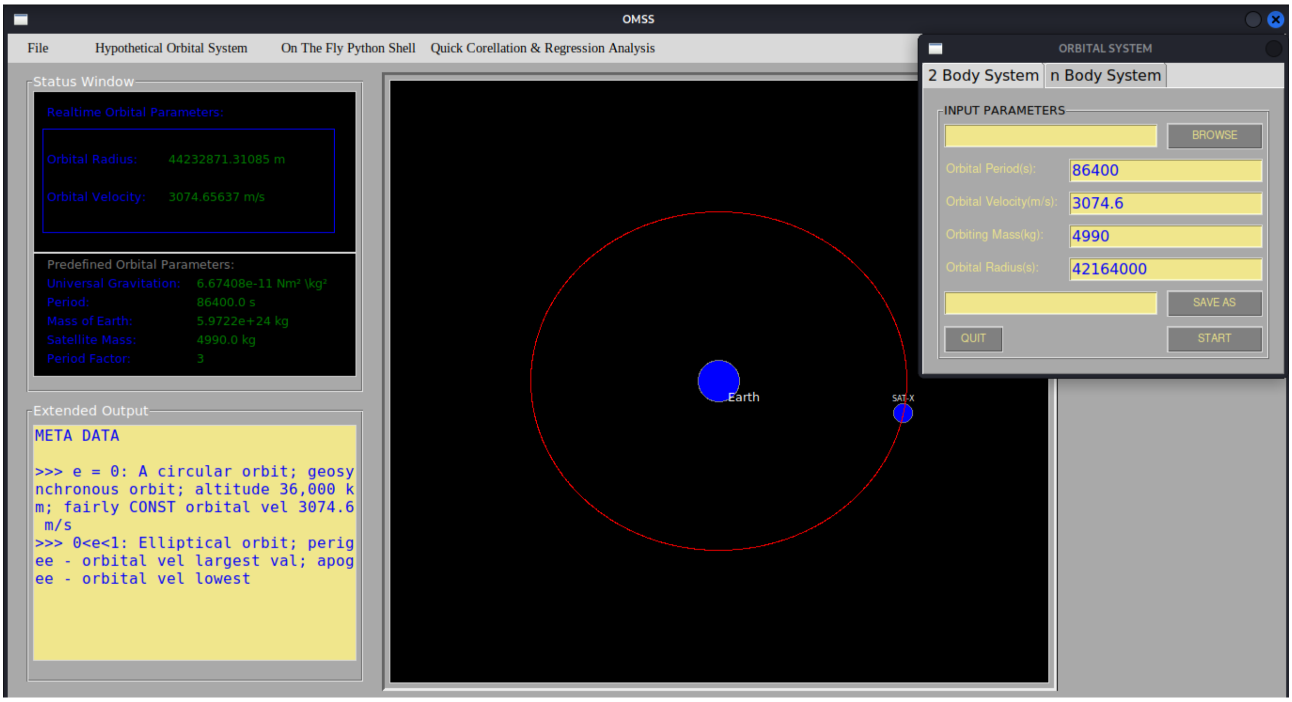
$$R_{orb} = \sqrt[3]{\frac{GMT^2}{4\pi^2}}\ ;\ \omega = \frac{2\pi}{T}$$

$$V_{orb} = \omega R_{orb} = \frac{2\pi}{T}\sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

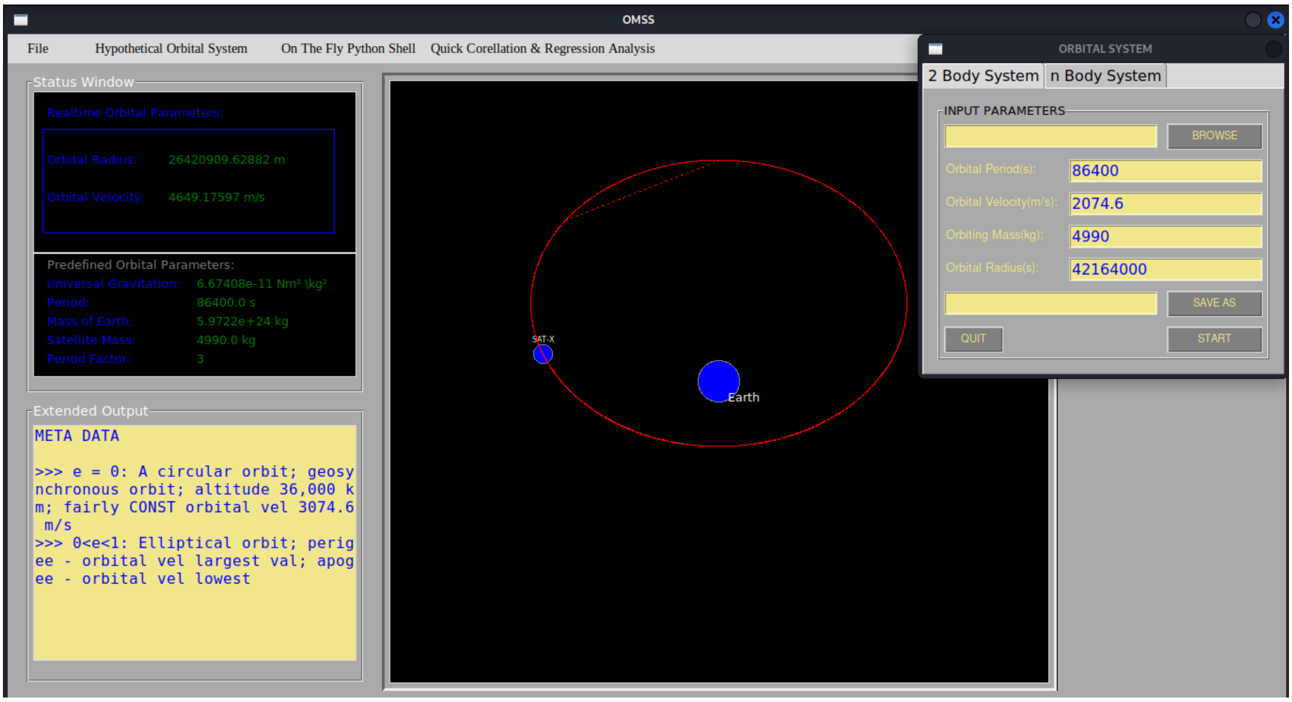
Where
$\boldsymbol{R_{orb}}$ is the orbital radius (42164km).
$\boldsymbol{V_{orb}}$ is the orbital velocity for a stable geostationary orbit(3.0746kms^{-1})
ω is the angular velocity, which is taken as that of the Earth.
The masses adopted in the simulation are as follows;
$\boldsymbol{M_E}$ = 5.9722 x 10^{24}kg; mass of the Earth(the central mass in the system).
$\mathbf{m_s}$ = 4990kg; the orbiting mass. This mass is equivalent to NOAA-GOES's(Geostationary Operational Environmental Satellite's) mass.

Using the defined initial conditions as the initial input data for SOMSS, the following self-explanatory results were obtained as shown on the attached screenshots.

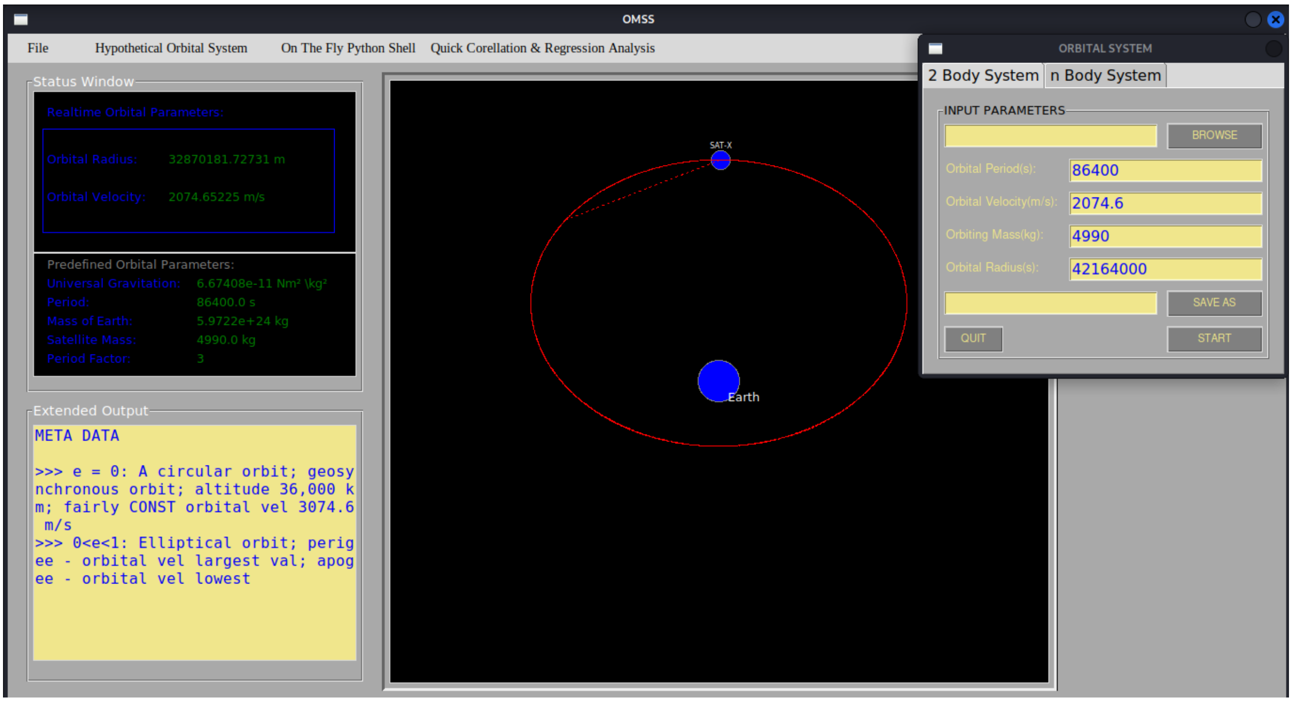
5.1. Geostationary Orbit



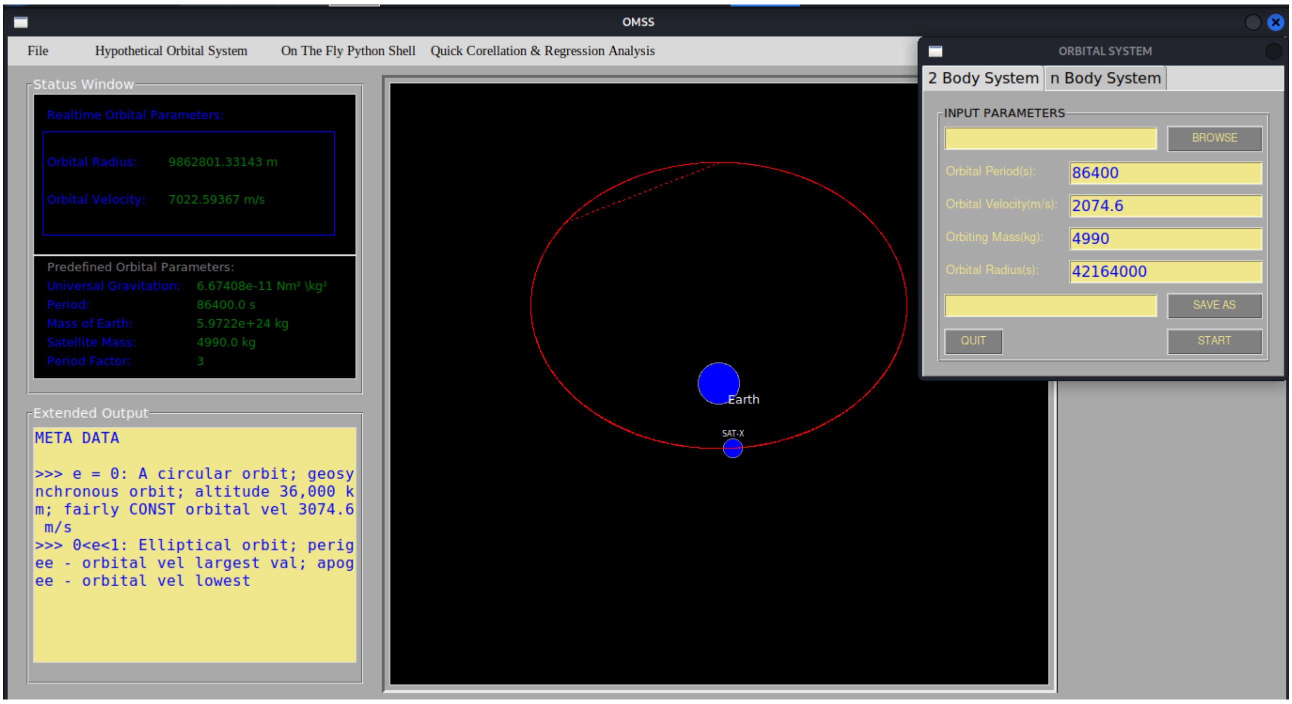
5.2. Low/Medium Earth Orbit



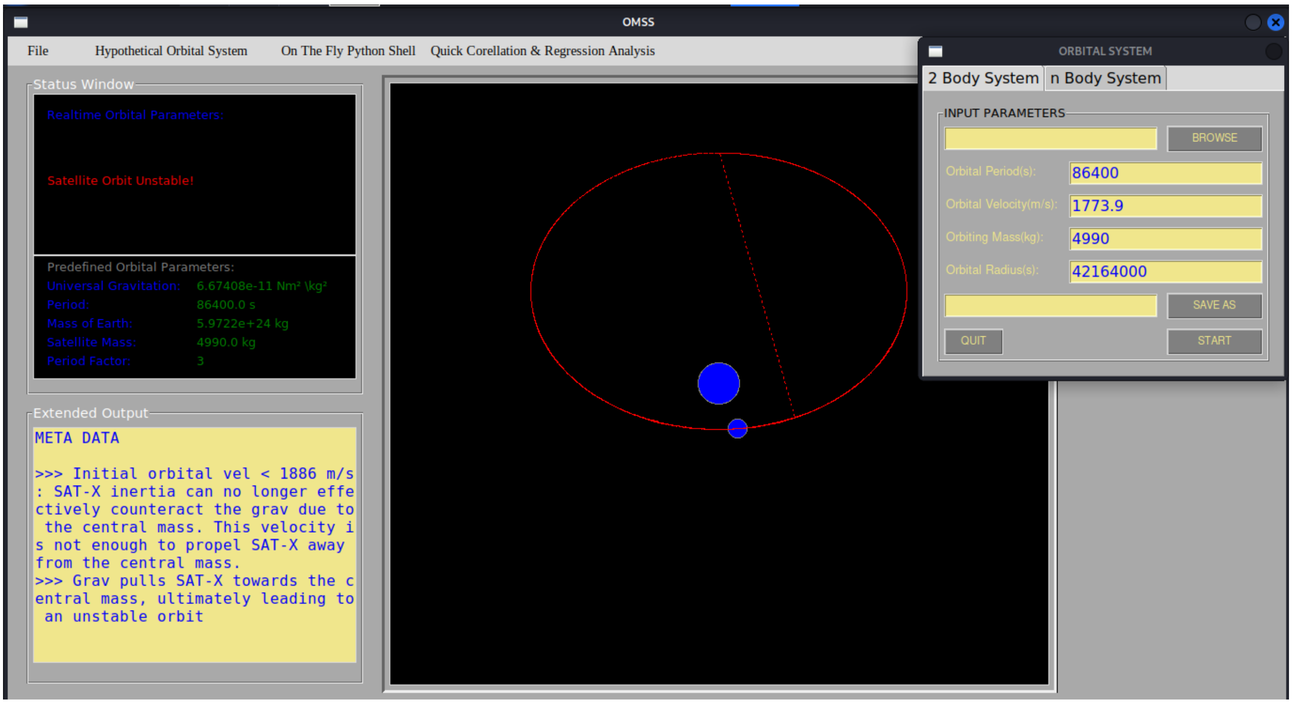
5.3. At Apogee



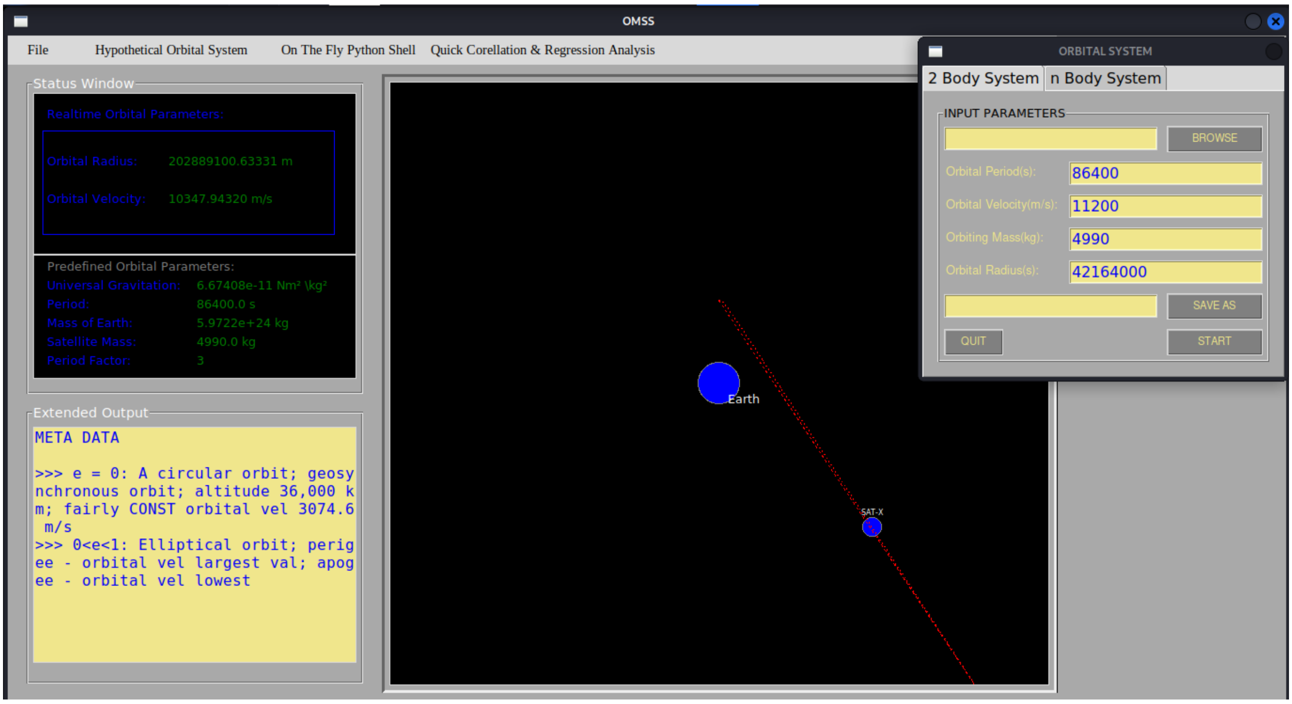
5.4. At Perigee



5.5. Unstable Orbit



5.6. Escape Velocity(11.186 kms^{-1})



In figure 5.1 the orbit has an eccentricity value approximately equivalent to 0, $\boldsymbol{e \approx 0}$, hence it's a circular orbit, which is typical for a geosynchronous orbit. The orbital velocity is fairly a constant value, 3074.6 ms^{-1} throughout the motion. This demonstrates a stable orbit for SAT-X at an orbital velocity of 3.0746kms^{-1}. And at an altitude of 36000km above the Earth's surface. SAT-X therefore stays in a geostationary orbit by maintaining this orbital velocity.

By slightly modifying the initial velocity of SAT-X to say 2074.6ms^{-1}, below the required orbital velocity for a geosynchronous orbit, the eccentricity is slightly changed, $0 < \boldsymbol{e} < 1$. The orbit is thus elliptical as shown in figure 5.2, resulting into manifestation of semi-minor, semi major axes, two focii, and consequently apogee and perigee as depicted in figure and figure 5.3 and 5.4 respectively. The center mass of the Earth in this case lies on one of the focii. Figure 5.3 shows SAT-X's behavior at the approximate apogee, the farthest point in the orbit from the focus where the center mass of Earth lies. As you can see from the simulation results in figure 5.3, SAT-X attained approximately the lowest velocity at the approximate apogee, the approximated farthest point. Figure 5.4 shows the behavior of SAT-X at perigee, the closest point in the orbit from the focus where the center mass of the Earth lies. The results show that SAT-X attained approximately the highest velocity at the approximate perigee, the approximated closest point. Figure 5.5 is a result of slight modification of velocity to 1773.9ms^{-1}. At this velocity, the satellite's inertia can no longer counteract gravity, and hence this gives gravity the chance to successfully accelerate SAT-X towards the ground and ultimately resulting into an unstable orbit. The simulator thus spits the message 'unstable orbit.

We can thus see that SAT-X's velocity increases as the satellite approaches close and closer to the central mass, Earth. This is how SAT-X's inertia compensates for the lost altitude to keep the satellite in orbit. For it to stay in orbit, its inertia must counteract gravitational pull. And for the satellite to maintain a particular allocated orbit, say a geosynchronous orbit, gravity must not 'win' against its inertia, to prevent it from approaching the reentry point. Similarly, its inertia must not 'win' against gravity to prevent it from propelling itself farther into space, being subject to orbital maneuver to a high and even higher orbit. SAT-X's inertia must thus counteract gravity in a manner that there exist balance between the two, so that the defined orbit remains stable.