Limiti - Esercizi

1) $\lim\limits_{x \to 0^+} \sqrt{\dfrac{1}{x}} = \sqrt{\dfrac{1}{0^+}} = +\infty$ ✓

2) $\lim\limits_{x \to 7} \dfrac{1}{(x-7)^2} = \begin{cases} \lim\limits_{x \to 7^+} \dfrac{1}{(7^+-7)^2} = \dfrac{1}{(0^+)^2} = +\infty \\ \lim\limits_{x \to 7^-} \dfrac{1}{(7^--7)^2} = \dfrac{1}{(0^-)^2} = +\infty \end{cases}$ $\Bigg\} \lim\limits_{x \to 7} f(x) +\infty$ ✓

3) $\lim\limits_{x \to 0^+} e^{\frac{2}{x}} = \lim\limits_{x \to 0^+} \dfrac{2}{0^+} = +\infty \Rightarrow e^{+\infty} = +\infty$ ✓

193) $\lim\limits_{x \to 0^+} (-\ln x) = -(-\infty) = +\infty$ ✓

194) $\lim\limits_{x \to 0} \ln\left(\dfrac{1}{x^2}\right) = \lim\limits_{x \to 0} \dfrac{1}{x^2} = \begin{cases} \lim\limits_{x \to 0^+} \dfrac{1}{(0^+)^2} = +\infty \\ \lim\limits_{x \to 0^-} \dfrac{1}{(0^-)^2} = +\infty \end{cases} \Bigg\} = +\infty \Rightarrow \lim\limits_{x \to 0} \ln(+\infty) = +\infty$ ✓

195) $\lim\limits_{x \to \frac{1}{2}} \dfrac{1}{(2x-1)^2} = \begin{cases} \lim\limits_{x \to \frac{1}{2}^+} \dfrac{1}{(2\frac{1}{2}^+-1)^2} = +\infty \\ \lim\limits_{x \to \frac{1}{2}^-} \dfrac{1}{(2\frac{1}{2}^--1)^2} = +\infty \end{cases} \Bigg\} = +\infty$ ✓

196) $\lim\limits_{x \to 2^+} \dfrac{1}{x^2-4} = \dfrac{1}{4^+-4} = \dfrac{1}{0^+} = +\infty$

201) $\lim\limits_{x \to 2^+} \dfrac{1}{x-2} + 1 = \dfrac{1}{0^+} + 1 = +\infty$

205) $\lim\limits_{x \to 0^-} \dfrac{5+2x}{-x} = \dfrac{5+0^-}{-(0^-)} = \dfrac{5}{0^+} = +\infty$

210) $\lim\limits_{x \to 1^+} \log\left(\dfrac{2}{x+1}\right) = \log\left(\dfrac{2}{2}\right) = \log e = +\infty$

212) $\lim\limits_{x \to 0} -\dfrac{1}{3x^4} = \begin{cases} \lim\limits_{0^-} -\dfrac{1}{30^+} = -\infty \\ \lim\limits_{0^+} -\dfrac{1}{30^+} = -\infty \end{cases} \Bigg\} = -\infty$ ✓

213) $\lim\limits_{x \to \frac{3}{2}^-} \dfrac{1}{4x^2-9} = \dfrac{1}{4\left(\frac{3}{2}^-\right)^2-9} = +\infty$?

214) $\lim\limits_{x \to -1} \dfrac{-1}{x^2+2x+1} = \begin{cases} \lim\limits_{-1^+} \dfrac{-1}{1^+-2^++1} = \dfrac{-1}{0^+} = -\infty \\ \lim\limits_{-1^-} \dfrac{-1}{1^--2^-+1} = -\dfrac{1}{0^-} = +\infty \end{cases}$?

215) $\lim\limits_{x \to 1^-} \log_2(1-x) = \log(1-1^-) = \log(0^-) = -\infty$ ✓

216) $\lim\limits_{x \to 3^-} \dfrac{1}{x-3} = \dfrac{1}{3^--3} = -\infty$ ✓

217) $\lim\limits_{x \to 0} \dfrac{-1}{|x|} = \begin{cases} \lim\limits_{x \to 0^+} -\dfrac{1}{|0^+|} = -\infty \\ \lim\limits_{x \to 0^-} -\dfrac{1}{|0^-|} = -\infty \end{cases} \Bigg\} = -\infty$ ✓

Rappresentazione grafica

$f = \ln x - 1$

in $x = 0^+$ $\lim\limits_{x \to 0^+} \ln x - 1 = \ln(0^+) - 1 = -\infty$

$\downarrow +\infty$

$f = \dfrac{2}{x-1}$

$\lim\limits_{x \to 1} \dfrac{2}{x-1} \begin{cases} \lim\limits_{1^+} \dfrac{2}{0^+} = +\infty \\ \lim\limits_{1^-} \dfrac{2}{0^-} = -\infty \end{cases}$

$\Downarrow$ Non esiste

$0^+ \leftarrow$
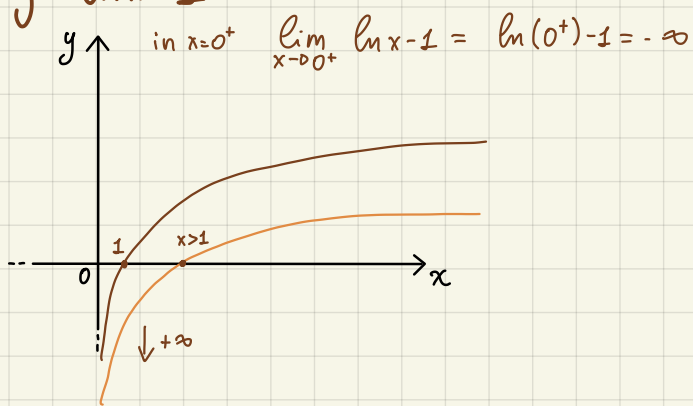
$y = \dfrac{1}{x}$ $\overset{0^-}{\to} \quad y = \dfrac{2}{x-1}$
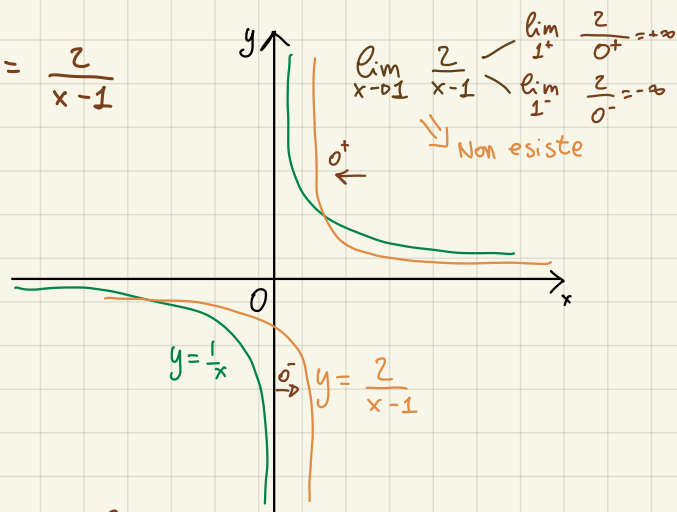
Asintoti verticali

249) $y = \dfrac{2}{(x-1)^2}$ in $x = 1$ $\lim\limits_{x \to 1} \dfrac{2}{(x-1)^2}$
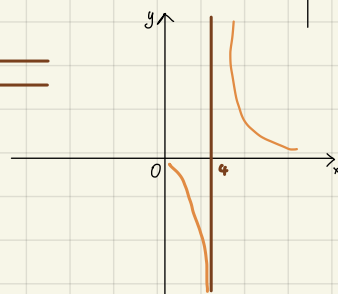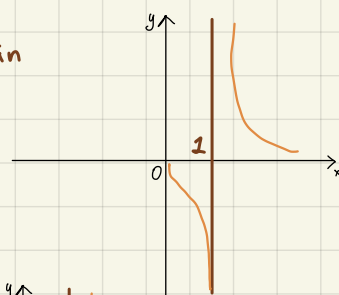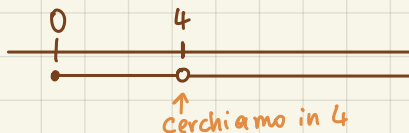$\begin{cases} 1^+ = \dfrac{2}{0^+} = +\infty \\ 1^- = \dfrac{2}{0^+} = +\infty \end{cases} \Rightarrow$ Asintoto Vert in $x = 1$

250) $y = \dfrac{1}{\ln x}$ in $x = 1$ $\lim\limits_{x \to 1} \dfrac{1}{\ln x} = \begin{cases} 1^+ = \dfrac{1}{0^+} = +\infty \\ 1^- = \dfrac{1}{0^-} = -\infty \end{cases}$ Asintoto in $x = 1$

251) $y = \dfrac{2}{\sqrt{x} - 2}$ in $x = 4$ $\mathbb{D} = \boxed{x \geq 0} \quad \sqrt{x}\geq$

$\sqrt{x} - 2 \neq 0$ per $\left(\sqrt{x}\right)^2 \neq 2^2 \Rightarrow \boxed{x \neq 4}$

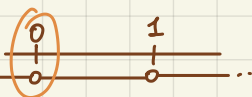$\lim\limits_{x \to 4} f(x) \begin{cases} 4^+ = \dfrac{2}{0^+} = +\infty \\ 4^- = \dfrac{2}{0^-} = -\infty \end{cases}$ A.V. in $x = 4$

cerchiamo in 4

252) $y = \dfrac{x-3}{x^2 - x}$ $\mathbb{D} = x^2 - x \neq 0$ per $x(x-1) \neq 0$
$\Rightarrow \begin{cases} x \neq 0 \\ x \neq 1 \end{cases}$

$\lim\limits_{x \to 0} f(x) \begin{cases} 0^+ = \dfrac{-3}{0^+} = -\infty \\ 0^- = \dfrac{-3}{0^-} = -\infty \end{cases}$

253) $y = 1 + e^{\frac{1}{x}}$ in $x = 0^+$ $\lim\limits_{x \to 0^+} 1 + e^{\frac{1}{x}} = 1 + e^{\frac{1}{0^+}} = +\infty$

Limiti tendenti ad e

264) $\lim\limits_{x \to +\infty} \dfrac{2}{x+10} = 0$ $\quad \dfrac{2}{x\left(1+\frac{1}{x}\right)^{\nearrow 0}} = \dfrac{2}{\infty} = 0$ ✓

265) $\lim\limits_{x \to +\infty} \dfrac{4x-1}{2x+1} = \dfrac{x\left(4-\frac{1}{x}\right)^{\nearrow 0}}{x\left(2+\frac{1}{x}\right)_{\searrow 0}} = \dfrac{4}{2} = 2$ ✓

269) $\lim\limits_{x \to +\infty} \dfrac{2^x-1}{2^x} = ?$

270) $\lim\limits_{x \to +\infty} \dfrac{-3x}{|x|+1} = \dfrac{x(-3)}{\underset{\text{siccome siamo in }+\infty}{\overset{\curvearrowright}{x}+1}} = \dfrac{x(-3)}{x\left(1+\frac{1}{x}\right)_{\searrow 0}} = \dfrac{-3}{1} = -3$

271) $\lim\limits_{x \to +\infty} \left(\sqrt{x^2-1} - x = 0\right) = \left(x^2\right)^{\frac{1}{2}} - 1^{\frac{1}{2}} \sim x = x^{\frac{4+1}{2}} - x^{\frac{1}{2}} - x = x^{\frac{5}{2}} - x^{\frac{1}{2}} - x = x^{\frac{5}{2}-\frac{1}{2}-1} = x^{\frac{5-1-2}{2}} = x^{-2} = \dfrac{1}{x^2}$

$= 0 \quad \lim\limits_{x \to +\infty} \dfrac{1}{x^2} = 0$ ✓

272) $\lim\limits_{x \to \infty} \ln\left(\dfrac{x}{x-1}\right) = \lim\limits_{x \to \infty} \dfrac{x}{x-1} = \dfrac{x}{x\left(1 \cdot \frac{1}{x}\right)_{\searrow 0}} = \dfrac{1}{1} = 1 \implies \lim\limits_{x \to \infty} \ln(1) = 0$ ✓

273) $\lim\limits_{x \to \infty} \dfrac{x}{x^2-1} = \dfrac{x}{x^2\left(1-\frac{1}{x^2}\right)_{\searrow 0}} = \dfrac{1}{+\infty} = 0$ ✓

274) $\lim\limits_{x \to \infty} \left(\dfrac{1}{2}\right)^{2x} = $ All'aumentare dell'esponente $\implies$ $\dfrac{1}{\infty} = 0$ ✓
aumenta il denominatore

275) $\lim\limits_{x \to +\infty} \left[\left(\dfrac{1}{3}\right)^{x+1} + 1\right] = 0 + 1 = 1$ ✓
$\underbrace{\qquad}$ Stesso di prima

277) $\lim\limits_{x \to -\infty} \dfrac{2}{2x+1} = \dfrac{2}{-\infty} = 0$ ✓

278) $\lim\limits_{x \to -\infty} \dfrac{x^3+1}{2x^3} = \dfrac{x^3(1+0)}{2} = \dfrac{1}{2}$ ✓

279) $\lim\limits_{x \to -\infty} \dfrac{3x+1}{1-2x} = \dfrac{x\left(3+\frac{1}{x}\right)^{\nearrow 0}}{x\left(\frac{1}{x}-2\right)_{\searrow 0}} = -\dfrac{3}{2}$ ✓

283) $\lim\limits_{x \to -\infty} \dfrac{-1}{e^{|x|}} = \dfrac{-1}{e^{-x}} = \dfrac{-1}{e^{+\infty}} = 0$ ✓
Siamo in $-\infty \implies |x|=-x$

284) $\lim\limits_{x \to -\infty} 2e^{-4x^2} = \lim\limits_{x \to -\infty} -4x^2 = +\infty \implies \lim\limits_{x \to -\infty} 2e^{+\infty} =$