Esercizi da Matematica Blu

Esercizi Derivate

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$f(x) = \frac{x^2-2}{x+1}$ in $c=-2$

$= \frac{2x\cdot(x+1)-(x^2-2)\cdot 1}{(x+1)^2} = \frac{2x^2+2x-x^2-2}{(x+1)^2}$

$= \frac{x^2+2x-2}{(x+1)^2}$

$f(-2) = \frac{(-2)^2+2(\cdot 2)-2}{(-2+1)^2}$

$= \frac{4+4-2}{1} = 6$

33) $f(x) = x^3+4x+1$ con $c=1$

$f'(x) = 3x^2+4$ $f'(c) = 3+4 = 7$

$f(x) = -\frac{5}{x}$ con $c=2$

$f'(x) = -5\cdot x^{-1} = -5\cdot(-1)x^{-2} = \frac{5}{x^2}$

$f(c) = \frac{5}{2^2} = \frac{5}{4}$

34) $f(x) = 1+\sqrt{x}$ in $c=4$

$1+\sqrt{x} = 1+x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(c) = 4$

$f(x) = 2x^3-x$ in $c=0$

$f'(x) = 6x^2-1$ $f(c) = -1$

35) $f = x^2-1$ in $c=3$

$f' = 2x$ $f3 = 6$

$$f = \frac{x+3}{x-4} \qquad \text{in} \quad c=-1$$

$$f' = \frac{\cancel{x}-4 - \cancel{x}-3}{(x-4)^2} = \frac{-7}{(x-4)^2}$$

$$f\,c = \frac{-7}{(-1-4)^2} = \frac{-7}{25} = -\frac{7}{25}$$

36)

$$f = \frac{x^2-1}{2-x} \qquad \text{in} \quad c=1$$

$$f' = \frac{2x(2-x) - (x^2-1)\cdot(-1)}{(2-x)^2} = \frac{4x - 2x^2 - (-x^2+1)}{(2-x)^2}$$

$$= \frac{4x - \cancel{2}x^2 + \cancel{x^2} + 1}{(2-x)^2} = \frac{-x^2+4x+1}{(2-x)^2} \qquad f\,c = \frac{1+4+1}{1} = 6$$

$$f = \frac{1}{2}x^2 - 2x \qquad \text{in} \quad c=-3$$

$$f' = x-2 \qquad f\,{-3} = -5$$

37) $f = 2x-1$ in $c=6$ $\qquad f' = 2$

$$f = \frac{3}{x-1} \qquad \text{in} \quad c=4 \qquad \frac{-3}{(x-1)^2} \qquad f\,4 = -\frac{\cancel{3}^{1}}{\cancel{9}_{3}} = -\frac{1}{3}$$

38) in $c=-2$

$$f = \frac{1}{1-x^2} \qquad f' = \frac{-(-2x)}{(1-x^2)^2} = \frac{2x}{(1-x^2)^2} \qquad f\,{-2} = \frac{-4}{9} = -\frac{4}{9}$$

$$f = \frac{1}{\sqrt{x-1}} \qquad \text{in} \quad c=5$$

$$f = \frac{1}{(x-1)^{\frac{1}{2}}} = (x-1)^{-\frac{1}{2}}$$

$$f' = -\frac{1}{2}(x-1)^{-\frac{1}{2}-1} = -\frac{1}{2}(x-1)^{\frac{-1-2}{2}}$$

$$= -\frac{1}{2}(x-1)^{-\frac{3}{2}} = -\frac{1}{2(x-1)^{\frac{3}{2}}}$$

$$= -\frac{1}{2\sqrt[3]{(x-1)^2}}$$

$$\Rightarrow f(5) = \frac{1}{2\sqrt{(5-1)^3}} = \frac{1}{16}$$

39) $f = \dfrac{x-1}{x}$ $\qquad f' = \dfrac{x - x - 1}{x^2} = -\dfrac{1}{x^2}$

in $c = 2 \Rightarrow f c = -\dfrac{1}{4}$

$f = \dfrac{2-x^2}{1-x^2}$ in $c=0$ $\qquad f' = \dfrac{-2x(1-x^2)-(2-x^2)\cdot -2x}{(1-x^2)^2}$

$= \dfrac{-2x + \cancel{2x^3} + 4x - \cancel{2x^3}}{(1-x^2)^2} = \dfrac{2x}{(1-x^2)^2}$ $\quad f0 = \dfrac{0}{n} = 0$

40) $f = -\dfrac{1}{\sqrt{x}} = -x^{-\frac{1}{2}}$

$$\frac{d}{dx}\left(\frac{1}{f}\right) = -\frac{\frac{d}{dx}(f)}{f^2}$$

$f' = \dfrac{1}{2} \cdot x^{-\frac{1}{2}-1} = \dfrac{1}{2} x^{-\frac{3}{2}} = \dfrac{1}{2\sqrt{x^3}}$ $\quad f9 = \dfrac{1}{54}$

$f = \dfrac{4-x^2}{x^2-2x+2}$ in $c = -2$

$f' = \dfrac{-2x(x^2-2x+2)-(4-x^2)(2x-2)}{(x^2-2x+2)^2}$

$= \dfrac{-\cancel{2x^3} + \cancel{4}^{2}x^2 - 4x - 8x + 8 + \cancel{2x^3} - \cancel{2}x^2}{(x^2-2x+2)^2}$

$= \dfrac{2x^2 - 12x + 8}{(x^2-2x+2)^2}$ $\qquad fc = \dfrac{2(-2)^2 - 12(-2)+8}{((-2)^2 - 2(-2)+2)^2}$

$= \dfrac{8+24+8}{(4+4+2)^2} = \dfrac{\cancel{40}\ \cancel{20}\ \cancel{10}\ 2}{\cancel{100}\ \cancel{50}\ \cancel{25}\ 5} = \dfrac{2}{5}$

41) $f(x) = -2 \ln x$ $\quad f' = -\dfrac{2}{x}$ $\quad f(1) = -2$

$f = e^{x-1}$ $\quad$ inc $= 1$ $\quad f' = e^{x-1} \cdot 1 = e^{x-1}$ Funzione composta

$f c = e^{0} = 1$

42) $f = tg\, x$ $\quad f' = \sec^2 x$ $\quad f'(0) =$

$f(x) = 2^{-2x}$ $\quad \dfrac{d}{dx} a^x = \ln(a) \cdot a^x$ $\Rightarrow \ln(2) \cdot 2^{-2x} \cdot \dfrac{d}{dx}(-2x)$

$= \ln(2) \cdot 2^{-2x} \cdot (-2) = -2 \ln(2) \cdot 2^{-2x}$

$f'(0) = -2 \ln 2 \cdot 1 = -2 \ln 2$

Calcolare la derivata in un punto generico c

$f = \frac{1}{2}x^2 - 4x \qquad f' = x - 4$

50) $f = \frac{2}{x} = 2 \cdot x^{-1} \Rightarrow f' = \frac{-2}{x^2}$ ✓

51) $f = 4x - 9 \qquad f' = 4$ ✓

52) $f = \frac{1}{2}\sqrt{x} = \frac{1}{2} x^{\frac{1}{2}} \qquad f' = \left(\frac{1}{2}\right)^2 \cdot x^{-\frac{1}{2}} = \frac{1}{4\sqrt{x}}$ ✓

53) $f = 2x^3 - x = 6x^2 - 1$ ✓

54) $f = \frac{1}{x^2 - 1} \qquad \frac{d}{dx}\frac{1}{f} = -\frac{f'}{f^2}$

$\Rightarrow f' = -\frac{2x}{(x^2-1)^2}$ ✓

55) $f = -x^2 + 4x \qquad f' = -2x + 4$ ✓

56) $f = \frac{x+1}{x} \quad f' = \frac{1(x) - (x+1)\cdot 1}{x^2} = \frac{x - x - 1}{x^2} = -\frac{1}{x^2}$ ✓

57) $f = \frac{1}{\sqrt{x+2}} = \frac{1}{(x+2)^{\frac{1}{2}}} = (x+2)^{-\frac{1}{2}}$

$\Rightarrow f' = -\frac{1}{2}(x+2)^{-\frac{1}{2}-1} = -\frac{1}{2}(x+2)^{-\frac{3}{2}} = -\frac{1}{2\sqrt{(x+2)^3}}$

$= -\frac{1}{2(x+2)\sqrt{x+2}}$ ✓

58) $f(x) = \operatorname{sen}(-x)$ funzione composta

$\Rightarrow f' = \cos(-x) \cdot -1 = -\cos(-x)$ ✓

59) $f = -e^{1+x} \qquad f' = -e^{1+x} \cdot 1 = -e^{1+x}$

60) $f = \sqrt{3x} = (3x)^{\frac{1}{2}} \qquad f' = \frac{1}{2}(3x)^{\frac{1}{2}-1} \cdot 3$

$= \frac{(3x)^{-\frac{1}{2}}\, 3}{2} = \frac{3}{2\sqrt{3x}}$ ✓

61) $f = 3\ln x \qquad \frac{3}{x}$ ✓

62) $f = x^2 - 8x \qquad f' = 2x - 8$ ✓

63) $f = 2\sqrt{x} = 2x^{\frac{1}{2}} \qquad f' = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$ ✓

64) $f = \frac{1}{\sqrt{x}-1} = \frac{1}{x^{\frac{1}{2}}-1} = \frac{1}{-1} \cdot \frac{1}{x^{\frac{1}{2}}} = -x^{-\frac{1}{2}}$

$f' = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}}$?

65) $f = \sqrt{x} - 2 \stackrel{!}{=} x^{\frac{1}{2}} - 2$ $\quad f' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ ✓

66) $f = \frac{x}{x-5}$

$f' = \frac{(\cancel{x}-5)-\cancel{x}}{(x-5)^2} = -\frac{5}{(x-5)^2}$ ✓

67) $f = \frac{9-x}{x^2-1}$ $\quad f' = \frac{-(x^2-1)-(9-x)\cdot 2x}{(x^2-1)^2}$

$= \frac{-\cancel{x^2}+1-18x+\cancel{2}x^2}{(x^2-1)^2} = \frac{x^2-18x+1}{(x^2-1)^2}$ ✓