5.49 Verificare che, se $p < 1$ e $b > 0$, allora

$$\int_0^b \frac{1}{x^p}\,dx = \frac{b^{1-p}}{1-p}$$

$\int_0^b \frac{1}{x^p}\,dx$ con $p<0 \Rightarrow 0 \leq x^p < x$

$\Rightarrow D\left(\frac{1}{x^p}\right) = x \neq 0 \Rightarrow \lim_{t\to 0}\int_t^b \frac{1}{x^p}$

$\Rightarrow \int x^{-p}\,dx = \left[\frac{x^{-p+1}}{p+1}\right]_t^b = \lim_{t\to 0^-}\frac{t^{-p+1}}{p+1} \Rightarrow \frac{b^{-p+1}}{p+1} - \frac{0}{p+1} = \frac{b^{1-p}}{1-p}$

5.50 Verificare che, se $p > 1$ e $b > 0$, allora l'integrale

$$\int_0^b \frac{1}{x^p}\,dx$$

è divergente.

$\int_0^b \frac{1}{x^p}\,dx$ con $p>1 \to x > x > +\infty$

$\Rightarrow D:\ x \in \mathbb{R} - \{0\}$

$\Rightarrow \lim_{t\to 0}\int_t^b \frac{1}{x^p}\,dx = \int x^{-p}\,dx = \left[\frac{x^{1-p}}{1-p}\right]_t^b \Rightarrow \frac{b^{1-p}}{1-p} - \left[\lim_{t\to 0}\frac{\underbrace{t^{1-p}}_{+\infty}}{1-p}\right] = \frac{b^{1-p}}{1-p} - \infty$ DIVERGE

$p>1$

5.51 Verificare che l'integrale

$$\int_0^b \frac{1}{x}\,dx$$

è divergente.

$\int_0^b \frac{1}{x}\,dx$ $D:\ x\in\mathbb{R}-\{0\}$

$\Rightarrow \lim_{t\to 0}\int_t^b \frac{1}{x}\,dx = \left[\ln|x|\right]_t^b = \ln|b| - \lim_{t\to 0}\ln|t|$

$= \ln|b| + \infty \to +\infty$ DIVERGE

5.52 Verificare che l'integrale

$$\int_0^2 \frac{1}{(x-2)^2}\,dx$$

è divergente.

$\int_0^2 \frac{1}{(x-2)^2}\,dx$ $D:\ x \neq 2$

$\Rightarrow \lim_{t\to 2}\int_0^t \frac{1}{(x-2)^2}\,dx \to \left[-\frac{1}{x-2}\right]_0^t$

$= \lim_{t\to 2}\underbrace{-\frac{1}{t-2}}_{-\infty} - \frac{1}{2} = -\infty$ DIVERGE

$$\int_1^2 \frac{1}{\sqrt{2-x}}\,dx = 2$$

$\int_1^2 \frac{1}{\sqrt{2-x}}\,dx$ $D:\ x>2$

$\Rightarrow \lim_{t\to 2}\int_1^t \frac{1}{\sqrt{2-x}}\,dx = (2-x)^{-\frac{1}{2}} = -2(2-x)^{\frac{1}{2}}$

$= \lim_{t\to 2^-}\left[2(2-x)^{\frac{1}{2}}\right]_1^t = \lim_{t\to 2^-}\left[-2\underbrace{(2-2^-)}_{0^+}^{\frac{1}{2}}\right] + \left[2\right] = 2$

5.54 Verificare che

$$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \frac{\pi}{2}$$

$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ $D=\ 4-x^2>0$ per $x<\pm 2$

eq>0, $a<0$

$\to$ val interni

$D:\ -2<x<2$

$\Rightarrow \lim_{t\to 2^-}\int_0^t \frac{1}{\sqrt{4-x^2}}\,dx = \int \frac{1}{\sqrt{2^2-x^2}} = \arcsin\left(\frac{x}{a}\right) \Rightarrow \lim\left[\arcsin\left(\frac{x}{2}\right)\right]_0^t$

$= \lim_{t\to 2^-}\underbrace{\arcsin\left(\frac{t}{2}\right)}_{\frac{\pi}{2}} - \underbrace{\arcsin(0)}_{0} = \frac{\pi}{2}$

5.55 I seguenti passaggi

$$\int_{-3}^{1} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_{-3}^{1} = -1 - \frac{1}{3} = -\frac{4}{3}$$

$\int_{-3}^{1} \frac{1}{x^2}\,dx \qquad \mathbb{D}: \mathbb{R}-\{0\}$

$\Rightarrow \lim_{t\to 0^-} \int_{-3}^{t} \frac{1}{x^2}\,dx + \lim_{t\to 0^+} \int_{t}^{1} \frac{1}{x^2}\,dx = \lim_{t\to 0^-}\left[-\frac{1}{x}\right]_{-3}^{t} + \lim_{t\to 0^+}\left[-\frac{1}{x}\right]_{t}^{1} = \lim_{t\to 0^-}\underbrace{\left(-\frac{1}{0^-}\right)}_{+\infty} - \frac{1}{3} + \left(-1 + \lim_{t\to 0^+}\underbrace{\left(-\frac{1}{0^-}\right)}_{+\infty}\right)$

$= +\infty - \frac{1}{3} - 1 + \infty \Rightarrow$ DIVERGE

5.56 Verificare che

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx = \pi$$

$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx \qquad \mathbb{D}: 1-x^2>0$ per $x<\pm 1$

$a<0$, $eq>0 \Rightarrow$ val interni

$\Rightarrow \mathbb{D}: -1<x<1$

$\Rightarrow \lim_{t\to -1^+} \int_{t}^{0} \frac{1}{\sqrt{1-x^2}}\,dx + \lim_{t\to 1^-} \int_{0}^{t} \frac{1}{\sqrt{1-x^2}}\,dx \overset{\lim_{t\to -1^+}}{=} \left[\arcsin x\right]_{t}^{0} + \lim_{t\to 1^-}\left[\arcsin x\right]_{0}^{t}$

$= 0 - \underbrace{\lim_{t\to -1^+} \arcsin t}_{+\pi/2} + \underbrace{\lim_{t\to 1^-} \arcsin t}_{\frac{\pi}{2}} - 0 \Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = \pi$

5.57 Verificare che

$$\int_{0}^{9} \frac{1}{\sqrt[3]{(x-1)^2}}\,dx = 9$$

[Si ha

$\int_{0}^{9} \frac{1}{\sqrt[3]{(x-1)^2}}\,dx \qquad \mathbb{D}: x\neq 1$

$\Rightarrow \lim_{t\to 1^-} \int_{0}^{t} \frac{1}{\sqrt[3]{(x-1)^2}}\,dx + \lim_{t\to 1^+} \int_{t}^{9} \frac{1}{\sqrt[3]{(x-1)^2}}\,dx$

pongo $(x-1)^2 = t \Rightarrow x^2-2x+1 = t$; $\Rightarrow x = \pm\sqrt{t}+1 \Rightarrow dx = \frac{1}{2\sqrt{t}}dt$

$\Rightarrow \int (t)^{-\frac{1}{3}} \cdot \frac{1}{2\sqrt{t}}\,dt = \frac{1}{2}\int t^{-\frac{1}{3}} \cdot t^{-\frac{1}{2}}\,dt = \frac{1}{2}\int t^{-\frac{5}{6}}\,dt = \frac{1}{2}\,\frac{t^{\frac{1}{6}}}{\frac{1}{6}} = 3\sqrt[6]{t}$

$\Rightarrow \quad \cancel{3} - 1 + 3\sqrt[6]{9} - \cancel{3} = 3\sqrt[6]{9} - 1$

Fatto di fretta
ho sbagliato qualcosa

Verificare che:

5.59 $$\int_2^3 \frac{x(x+1)}{\sqrt{9-x^2}}\,dx = 3 + \frac{9}{4}\pi$$

$\int \frac{x(x+1)}{\sqrt{9-x^2}}\,dx$ ho $\sqrt{3^2-x^2} \to$ Pongo $x = 3\sin t$

$\to \sin t = \frac{x}{3}$; $t = \operatorname{asin}\frac{x}{3}$

$\to dx = 3\cos t \to \int \frac{x(x+1)}{\sqrt{9-x^2}} \cdot 3\cos t\,dt$

$\to \int \frac{x(x+1)}{\sqrt{3^2-(3\sin t)^2}} \cdot 3\cos t\,dt$

$= \int \frac{x(x+1)}{\sqrt{3^2-3^2\sin t^2}} \cdot 3\cos t\,dt = \int \frac{x(x+1)}{3\sqrt{1-\sin^2}} \cdot 3\cos t\,dt = \int \frac{x(x+1)}{\cancel{\sqrt{\cos^2 t}}} \cdot \cancel{\cos t}\,dt$

$= \int 3\sin t\,(3\sin t+1)\,dt = \int 9\sin^2 t + 3\sin t\,dt = 9\int \sin^2 t + 3\int \sin t = -9\sin t\cos t + t -$
$- 3\cos t$

$\to \sqrt{9-x^2} > 0 \to x^2 < 9$; $x < \pm 3$ $\begin{matrix} a<0 \\ eq>0 \end{matrix}$ val interni $-3 < x < 3$

$\lim_{t\to 3^-} \left[-9\sin\left(\operatorname{asin}\frac{x}{3}\right) + \operatorname{asin}\frac{x}{3} - 3\cos\left(\operatorname{asin}\frac{x}{3}\right)\right]_2^t = -3 - \left[-\frac{18}{3} + \operatorname{asin}\frac{2}{3} - 3\cos\left(\operatorname{asin}\frac{2}{3}\right)\right]$

Boh

5.62 $$\int_0^{\pi/2} \frac{\operatorname{sen} x}{\cos^3 x \cdot e^{\operatorname{tg} x}}\,dx = 1$$

$\int_0^{\pi/2} \frac{\sin x}{\cos^3 x \cdot e^{\operatorname{tg} x}}\,dx$ pongo $\operatorname{tg} x = t$

$\Rightarrow x = \operatorname{arctg}(t) \to dx = \frac{1}{1+t^2}\,dt$

$\to \int \frac{\sin x}{\cos^3 x \cdot e^t} \cdot \frac{1}{1+t^2}\,dt =$

5.64 $$\int_0^1 \log x\,dx = -1$$

$\int_0^1 \ln x\,dx$ $\mathbb{D}: x > 0$

$\lim_{t\to 0^+} \int_t^1 \ln x\,dx$

$\int \ln x \cdot D[x]\,dx = \ln x \cdot x - \int x \frac{1}{x}\,dx = x\ln x - x$

$\to \lim_{t\to\infty} [x\ln x - x]_t^1 = (\ln 1 - 1) - [\lim_{t\to 0} t\ln t - \cancel{t}]$

$\lim \frac{\ln t}{\frac{1}{t}} \to 0$ $\ln t \ll t$ $\to 0$

$+\infty$

$\Rightarrow \ln(1) - 1 - 0 = -1$

5.66 Verificare che, se $p > 1$ ed $a > 0$, allora
$$\int_a^{+\infty} \frac{1}{x^p}\,dx = \frac{a^{1-p}}{p-1}$$

$\int_a^{+\infty} \frac{1}{x^p}\,dx$ con $p > 1$ e $a > 0$

$\int \frac{1}{x^p}\,dx = \int x^{-p}\,dx = \frac{x^{1-p}}{1-p}$

$\to \mathbb{D}\left(\frac{1}{x^p}\right): x \neq 0 \Rightarrow \lim_{t\to+\infty} \int_a^t \frac{1}{x^p}\,dt = \lim_{t\to+\infty} \left[\frac{x^{1-p}}{1-p}\right]_a^t = \lim \frac{t^{1-p}}{1-p} - \frac{a^{1-p}}{1-p}$

$t^{1-p} \to 0$, $1-p < 0$, $\sim \frac{0}{n}$

$0 - \frac{a^{1-p}}{1-p} = \frac{a^{1-p}}{p-1}$