X

X

X

1. Studiare la seguente funzione e disegnarne il grafico: $y = \ln(5x^2 + 4x + 4)$.

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1) Dominio: $5x^2+4x+4>0$ $\quad \Delta = 16-4\cdot 5 = -4 < 0$ NON RIDUCIBILE

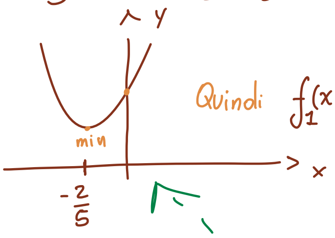
Facciamo ad "occhio": eq di 2^o grado $\to$ $\cup$

Dominio: $\forall x \in \mathbb{R}$ $\quad$ inters. $\begin{cases} 5x^2+4x+4=y \to y=4 \\ x=0 \end{cases} \Rightarrow (0,4) \in f(x)$ $\quad \begin{cases} 5x^2+4x+4=y \to \nexists x \in \mathbb{R} \\ y=0 \end{cases}$

$f_1'(x) = 10x+4 > 0 \to x > -\frac{2}{5}$ $\quad$ $-\frac{2}{5}$: $-$ $\searrow$ | $+$ $\nearrow$ $\quad$ $f_2(-\frac{2}{5}) = \cancel{5}\cdot\frac{4}{\cancel{25}_5} - 4\cdot\frac{2}{5}+4 = \frac{4}{5}-\frac{8}{5}+4 = \frac{4-8+20}{5} = \frac{16}{5}$

$\Rightarrow \left(-\frac{2}{5}, \frac{16}{5}\right)$ Minimo

$-\frac{2}{5}$ (min) $\quad$ 0 (ints.) $\quad f_1(x) \Rightarrow$



Quindi $f_1(x) > 0 \ \forall x \in \mathbb{R}$

$\Rightarrow D f(x) = \{\forall x \in \mathbb{R}\}$ ✓

2) Inters. $\begin{cases} y = f(x) \to y = \ln(4) \\ x=0 \end{cases} \Rightarrow (0, \ln(4)) \in f(x)$ Int $\quad$ 1,4

$\begin{cases} y=f(x) \\ y=0 \end{cases}$ $\quad \ln(5x^2+4x+4)=0$

per $5x^2+4x+4=0 \to \nexists x\in\mathbb{R} \to$ NO int con x

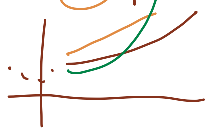
3) Simmetrie $\quad f(-x) = \ln(5x^2-4x+4)$ $\begin{matrix} \neq f(x) \\ \neq -f(x) \end{matrix}$

4) Segno $\quad f(x)>0$ per $\ln(5x^2+4x+4)>0$ $\quad \forall x \in \mathbb{R}$ ✓

5) Asintoti

$D: \forall x\in\mathbb{R} \to$ NO Asintoti Vert $\to \lim\limits_{x\to+\infty} \ln(5x^2+4x+4) = +\infty \Rightarrow$ NO A.O. ma la f cresce a $+\infty$ per $x\to+\infty$

$\lim\limits_{x\to-\infty} \ln(\underset{+\infty}{5x^2}+\underset{-\infty}{4x}+4) \to \ln\left(\underset{+\infty}{x^2}\left(5+\underset{0}{\frac{4}{x}}+\underset{0}{\frac{4}{x^2}}\right)\right) \to +\infty$



6) Max/min $\quad f'(x) = \frac{10x+4}{\cancel{5x^2+4x+4}>0} > 0$ per $10x+4>0 \to x > -\frac{2}{5}$ $\quad \forall x \in \mathbb{R}$

$-\frac{2}{5}$: $-$ $\searrow$ | $+$ $\nearrow$ min

$f(-\frac{2}{5}) = \ln\left(\frac{16}{5}\right) \Rightarrow \left(-\frac{2}{5}, \ln\left(\frac{16}{5}\right)\right) \in f(x)$ minimo $\quad$ 0,4 $\quad$ 1,16

No MAX

7) Flessi $\quad f''(x) = \frac{10(5x^2+4x+4)-(10x+4)(10x+4)}{\cancel{(5x^2+4x+4)^2}>0} = 50x^2+40x+40-100x^2-80x-16>0$

$\to -50x^2-40x+24>0$

$\Delta = 1600-4(-50)(24) = 6400$

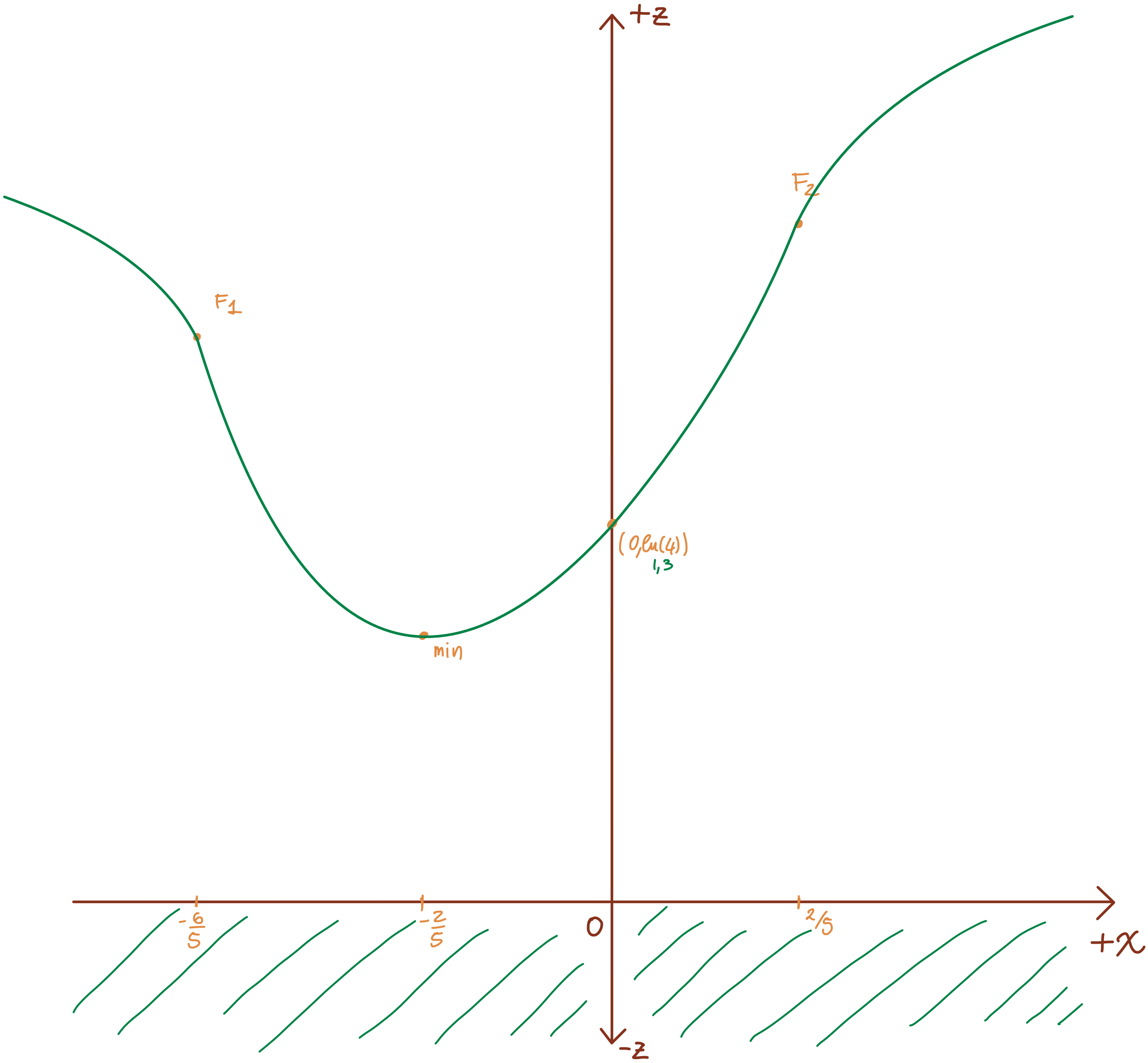
$\Rightarrow x_{1,2} = \frac{40\pm 80}{-100}$ $\begin{matrix} -\frac{6}{5} \\ \frac{2}{5} \end{matrix}$ $a<0, eq>0$ Val interni

$-6/5$ (F_1) $\quad 0 \quad$ $2/5$ (F_2): $\cap$ | $\cup$ | $\cap$

$f(-\frac{6}{5}) = \ln\left(\cancel{5}\cdot\frac{36}{\cancel{25}_5} - 4\,\frac{6}{5}+4\right) = \ln\left(\frac{36-24+4}{5}\right) = \frac{16}{5}$

$\Rightarrow \left(-\frac{6}{5}, \frac{16}{5}\right) \in f(x)$ Flesso $\quad$ -1,2 $\quad$ 3,2

$f(\frac{2}{5}) = \ln\left(5\,\frac{4}{25}+4\,\frac{2}{5}+4\right) = \ln\left(\frac{4+8+4}{5}\right) = \ln\left(\frac{24}{5}\right) \Rightarrow \left(\frac{2}{5}, \frac{24}{5}\right) \in f(x)$ Flesso $\quad$ 0,4 $\quad$ 4,8



+z
-z
+x
F1
F2
(0,ln(4))
1,3
min
-6/5
-2/5
0
2/5

3. Calcolare il seguente integrale: $\int \frac{\arcsin \sqrt{x}}{\sqrt{x}} dx$.

$\int \frac{\operatorname{asin}(\sqrt{x})}{\sqrt{x}} dx$ pongo $t = \sqrt{x} \Rightarrow dx = \frac{1}{\frac{1}{2\sqrt{x}}} dt$

Tempo: 5'30"

$\Rightarrow 2\int \frac{\operatorname{asin}(\sqrt{x})}{\cancel{\sqrt{x}}} \cdot \cancel{\sqrt{x}}\, dt = 2\int \operatorname{asin}(t)\, dt \overset{\text{PARTI}}{=} 2\left[t \arcsin(t) - \int \frac{t}{\sqrt{1-t^2}} dt\right]$

$\int \operatorname{asin}(t) \cdot D[t]\, dt$

$\to \int \frac{t}{\sqrt{1-t^2}} dt$ pongo $z = \sqrt{1-t^2} \Rightarrow dz = \frac{1}{\frac{-2t}{\cancel{2}\sqrt{1-t^2}}} = -\frac{\sqrt{1-t^2}}{t} dt \Rightarrow -\int \frac{\cancel{t}}{\cancel{\sqrt{1-t^2}}} \cdot \frac{\cancel{\sqrt{1-t^2}}}{\cancel{t}} dz = -\int dz$

$\Rightarrow \int \frac{t}{\sqrt{1-t^2}} dt = -\sqrt{1-t^2} + c \to 2\int \operatorname{asin}(t) = 2\left[t \operatorname{asin}(t) + \sqrt{1-t^2}\right] + c$

$\Rightarrow \int \frac{\operatorname{asin}\sqrt{x}}{\sqrt{x}} dx = 2\left[\sqrt{x} \operatorname{asin}\sqrt{x} + \sqrt{1-x}\right] + c$

4. Studiare la seguente serie di potenze: $\sum_{n=1}^{+\infty} \frac{2^n+1}{n5^n}(x+1)^n$.

$\lim_{n \to +\infty} \frac{2^n+1}{n5^n}(x+1)^n = \frac{2^n(\frac{1}{+\infty})}{5^n n} \cdot (x+1)^n$

$\to +\infty$ $\quad +\infty$ $\quad +\infty$

$= \frac{2^n(x+1)^n}{5^n n} + \frac{(x+1)^n}{5^n n} = \left(\frac{2}{5}\right)^n \cdot \frac{(x+1)^n}{n} + \frac{(x+1)}{5^n n} = \left(\frac{2}{5}\right)^n \cdot \frac{5^n(x+1)^n + (x+1)^n}{5^n n}$

$= \left(\frac{2}{5}\right)^n \cdot \frac{(x+1)^n(5^n+1)}{5^n n} =$

$\frac{2^n(x+1)^n + (x+1)^n}{n5^n}$

$D[N] = n2^{n-1}(x+1)^n + n2^n(x+1)^{n-1}$

$\frac{n2^n}{2}$ $\quad \frac{(x+1)^n}{x+1}$

$D[D] = n^2 5^{n-1}$

$\lim_{n \to +\infty} \frac{D[N]}{D[D]} = \frac{n2^n\left(\frac{1}{2}(x+1)^n + (x+1)^{n-1}\right)}{\frac{n^2 5^n}{5}}$

$\Rightarrow \left(\frac{\cancel{n}2^n}{\cancel{n}5^n}\right) \cdot \frac{\frac{1}{2}(x+1)^n + (x+1)^{n-1}}{n\frac{1}{5}}$

$\left(\frac{2}{5}\right)^n \to 0$

$\frac{\frac{1}{2}(x+1)^n + (x+1)^{n-1}}{n\frac{1}{5}} = \frac{\frac{(x+1)^n + 2(x+1)^{n-1}}{2}}{\frac{n}{5}} = \frac{(x+1)^n + 2(x+1)^{n-1}}{2} \cdot \frac{5}{n}$

$\frac{(x+1)^n}{x+1}$

$= \frac{5(x+1)^n + 10(x+1)^{n-1}}{2n} = \frac{(x+1)^n}{2n}\left[5 + \cancel{\frac{10}{(x+1)}}\right] \to +\infty \Rightarrow [0 \cdot +\infty]\ ?$

$\frac{2^n}{5^n} \cdot \frac{5}{2} \cdot \frac{(x+1)^n + 2(x+1)^{n-1}}{2}$

5. Calcolare l'integrale del seguente problema di Cauchy: $\begin{cases} y'' - 4y' + 3y = e^{-x}, \\ y(0) = 0, \quad y'(0) = 0. \end{cases}$

$\lambda^2 - 4\lambda + 3 = 0 \quad \Delta = 16 - 4 \cdot 3 = 4 > 0$

$\lambda_{1,2} = \frac{+4 \pm 2}{2} \begin{cases} 3 \\ 1 \end{cases}$

$\Rightarrow y_0(x) = c_1 e^{3x} + c_2 e^{x}$

$f(x) = e^{-x} \Rightarrow \gamma = -1$ NO Sol $\Rightarrow h = \emptyset$

$\Rightarrow y_p(x) = e^{-x}(A) \Rightarrow y_p'(x) = -Ae^{-x} \quad y_p''(x) = Ae^{-x}$

$\Rightarrow Ae^{-x} + 4Ae^{-x} + 3Ae^{-x} = e^{-x} \Rightarrow e^{-x}(A + 4A + 3A) = e^{-x} \Rightarrow A + 4A + 3A = 1 \rightarrow A = 1/8$

$\Rightarrow y_p(x) = \frac{1}{8} e^{-x} \Rightarrow y(x) = \frac{1}{8} e^{-x} + c_1 e^{3x} + c_2 e^{x}$

Tempo 7'

$y(0) = \frac{1}{8} \underbrace{\cancel{e^{-x}}}_{1} + c_1 \underbrace{\cancel{e^{3x}}}_{1} + c_2 \underbrace{\cancel{e^{x}}}_{1} = 0 \rightarrow \frac{1}{8} + c_1 + c_2 = 0 \begin{cases} c_1 = -c_2 - \frac{1}{8} \\ c_2 = -c_1 - \frac{1}{8} \end{cases}$

$y'(x) = -\frac{1}{8} e^{-x} + 3c_1 e^{3x} + c_2 e^{x} \Rightarrow y'(0) = -\frac{1}{8} \underbrace{\cancel{e^{-x}}}_{1} + 3c_1 \underbrace{\cancel{e^{3x}}}_{1} + c_2 \underbrace{\cancel{e^{x}}}_{1} = 0 \rightarrow -\frac{1}{8} + 3c_1 + c_2 = 0$

$= -\frac{1}{8} + 3c_1 - c_1 - \frac{1}{8} = 0 \rightarrow 2c_1 = \frac{1}{4} \rightarrow c_1 = \frac{1}{8}$

$c_2 = -c_1 - \frac{1}{8}$

$\Rightarrow c_2 = -\frac{1}{8} - \frac{1}{8} = -\frac{1}{4}$

$\Rightarrow$ SOL: $y(x) = \frac{1}{8} e^{-x} + \frac{1}{8} e^{3x} - \frac{1}{4} e^{x}$

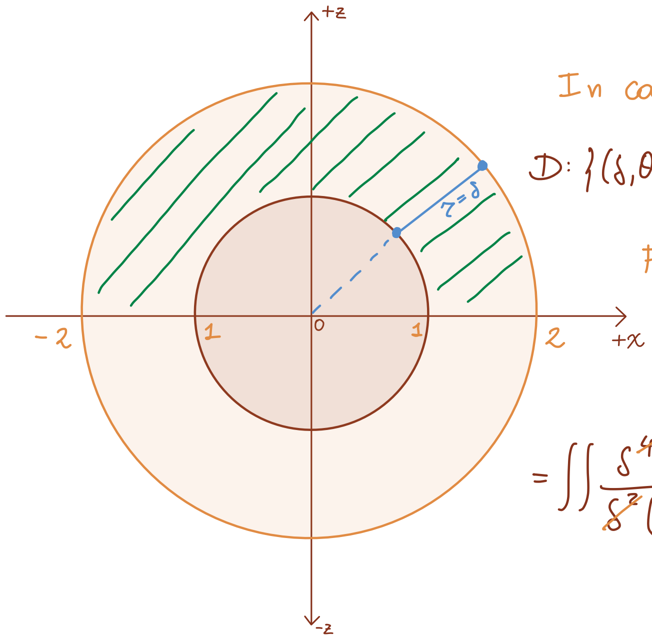
6. Calcolare l'integrale doppio $\displaystyle\iint_D \frac{x^2 y}{x^2+y^2}\,dx\,dy$, dove

$$D = \{(x,y) : 1 \le x^2+y^2 \le 4,\ y \ge 0\}.$$

$D: \{(x,y) / 1 \le x^2+y^2 \le 4,\ y \ge 0\}$

$C_1:\quad x^2+y^2 = 1 \ \rightarrow\ c = 0,0,\ r = 1$

$C_2:\quad x^2+y^2 = 4 \qquad c = 0,0,\ r = 2$



In coordinate Polari

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

$D: \{(\rho,\theta) / 1 < \rho < 2,\ 0 < \theta < \pi\}$

Formule di riduzione:

$$\int_1^2\int_0^\pi \frac{\rho^2\cos\theta\cdot\rho\sin\theta}{\rho^2\cos^2\theta+\rho^2\sin^2\theta}\cdot\rho\,d\theta\,d\rho$$

$$= \iint \frac{\rho^{\cancel{4}\,2}\cos\theta\sin\theta}{\cancel{\rho^2}\underbrace{(\cos^2\theta+\sin^2\theta)}_{1}}\,d\theta\,d\rho = \int_1^2 \rho^2 \int_0^\pi \cos\theta\sin\theta\,d\theta\,d\rho$$

Tempo 7'40"

$$\rightarrow \int \cos\theta\sin\theta\,d\theta \rightarrow t = \sin\theta \rightarrow d\theta = \frac{1}{\cos\theta}\,dt \rightarrow \int \cancel{\cos\theta}\cdot\sin\theta\cdot\frac{1}{\cancel{\cos\theta}}\,dt$$

$$\rightarrow \int t\,dt = \frac{t^2}{2} + c = \frac{\sin^2\theta}{2}$$

$$\Rightarrow \int_1^2 \rho^2\cdot\Big[\underbrace{\frac{\sin\theta}{2}}_{[0-0]}\Big]_0^\pi\,d\rho = \int_1^2 \rho^2\,d\rho = \Big[\frac{\rho^3}{3}\Big]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$