Esercizio 6. Si risolva il seguente problema di Cauchy

$$\begin{cases} y' = \frac{2x}{x^2+1}y + x^2 \\ y(0) = 1 \end{cases}$$

$$y' = \frac{2x}{x^2+1}\,y + x^2$$

$$\Rightarrow \quad y' - \frac{2x}{x^2+1}\,y = \varnothing$$

$$\Rightarrow \frac{dy}{y} = \frac{2x}{x^2+1}\,dx \rightarrow \int \frac{dy}{y} = \int \frac{2x}{x^2+1}\,dx \rightarrow \ln(y) = \ln(x^2+1) + c$$

$$\Rightarrow \ln y - \ln(x^2+1) = c \Rightarrow \ln\left(\frac{y}{x^2+1}\right) = c \rightarrow \frac{y}{x^2+1} = e^c \Rightarrow y = (x^2+1)c$$

$$\underset{c}{\vee}$$

$$y' = (2x)c + (x^2+1)c'$$

$$\Rightarrow 2cx + c'x^2 + c' = \frac{2x}{\cancel{x^2+1}}\cancel{(x^2+1)}\,c + x^2 \rightarrow \cancel{2cx} + c'x^2 + c' = \cancel{2cx} + x^2$$

$$\rightarrow c'x^2 + c' = x^2 \rightarrow c' = \frac{x^2}{x^2+1}$$

$$\begin{array}{r|l} \cancel{x^2} & x^2+1 \\ \cancel{x^2}+1 & 1 \\ \hline -1 & \end{array} \Rightarrow 1 - \frac{1}{x^2+1}$$

$$\rightarrow c = \int dx - \int \frac{1}{x^2+1}\,dx \Rightarrow c = x - \operatorname{atan}(x) + k$$

$$\Rightarrow \text{Sol}: \ (x^2+1)(x - \operatorname{atan}(x) + k) = x^3 - x^2\operatorname{atan}(x) + kx^2 + x - \operatorname{atan}(x) + k$$

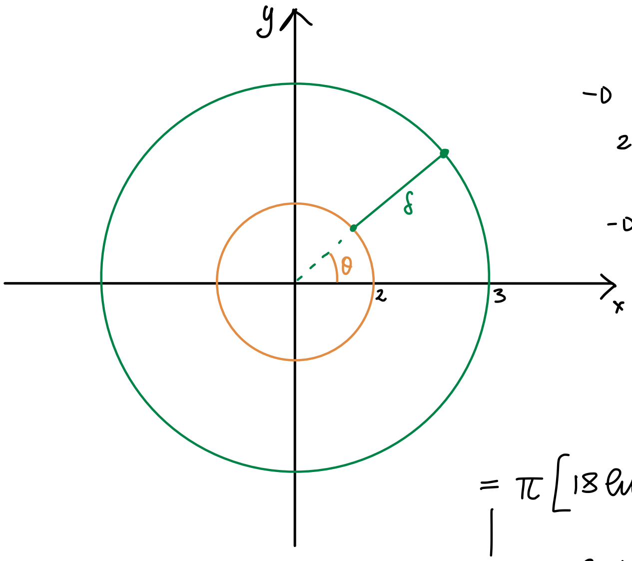
$$y(0) = \underset{0}{\cancel{x^3}} - \underset{0}{\cancel{x^2\operatorname{atan}(x)}} + \underset{0}{\cancel{kx^2}} + \underset{0}{\cancel{x}} - \underset{0}{\cancel{\operatorname{atan}(x)}} + k = 1 \Rightarrow k = 1$$

$$\text{Sol Cauchy}: \quad y = x^3 - x^2\operatorname{atan}(x) + x^2 + x - \operatorname{atan}(x) + 1$$

Esercizio 3. Calcolare

$$\iint_D \ln(x^2+y^2)\,dx\,dy$$

Dove D è la corona circolare di centro l'origine e raggio interno $r_i = 2$ e raggio esterno $r_e = 3$.



$\Rightarrow D' = \{(\delta,\theta) \,/\, 2<\delta<3 \;,\; 0<\theta<2\pi\}$

$$\Rightarrow \int_2^3 \delta \int_0^{2\pi} \ln(\delta^2 \underbrace{(\cos^2\theta+\sin^2\theta)}_{1})\,d\delta\,d\theta$$

$$\Rightarrow \int_2^3 \delta \ln(\delta^2)\,d\delta \int_0^{2\pi} d\theta = 2\pi \int_2^3 \delta \ln(\delta^2)\,d\delta = 4\pi \int_2^3 \underbrace{\delta \ln(\delta)\,d\delta}_{\text{Parti}}$$

$$\Rightarrow \frac{\delta^2}{2}\ln(\delta) - \int \frac{1}{\cancel{\delta}}\cdot\frac{\delta^{\cancel{2}}}{2}\,d\delta = \frac{\delta^2 \ln(\delta)}{2} - \frac{1}{2}\int \delta\,d\delta$$

$$= \cancel{4}\pi\left[\frac{\delta^2\ln(\delta)}{2} - \frac{\delta^2}{4}\right]_2^3 = \pi\left[2\delta^2\ln(\delta) - \delta^2\right]_2^3$$

$$= \pi\left[18\ln(3) - 9\right] - \pi\left[8\ln(2) - 4\right] = 18\pi\ln(3) - 9\pi - 8\pi\ln(2) + 4\pi$$

$$= 18\pi\ln(3) - 8\pi\ln(2) - 5\pi \simeq \underline{28{,}9}$$

Tempo $\sim 6'$

(lo avevo fatto di recente)

Esercizio 1. Calcolare

$$\lim_{x\to 0}\frac{2\cos(3(\ln(2x+1)))-2}{\sin(\sin(x^2))}.$$

$$\lim_{x\to 0}\frac{2\cos(3\ln(2x+1))-2}{\sin(\sin(x^2))}=\frac{2\cos(3\cdot\ln(1))-2}{\sin(\sin(0))}=\left[\frac{0}{0}\right]$$

Hôpital

$$D_N'=-2\sin(3\ln(2x+1))\cdot 3\,\frac{2}{(2x+1)}=\frac{-12\sin(3\ln(2x+1))}{(2x+1)}$$

$$D_D'=\cos(\sin(x^2))\cos(x^2)\,2x$$

$$\to\lim_{x\to 0}\frac{-\cancel{12}^{\,6}\sin(3\ln(2x+1))}{(2x+1)(\cos(\sin(x^2))\cos(x^2)\cdot\cancel{2}x}=-6\lim_{x\to 0}\frac{\sin(3\ln(2x+1))}{(2x+1)(\cos(\sin(x^2))\cos(x^2)\cdot x}$$

Raggruppo tutti gli elementi
che NON tendono a zero

$$\to -6\lim_{x\to 0}\underbrace{\frac{1}{(2x+1)(\cos(\sin(x^2))\cos(x^2)}}_{1}\cdot\frac{\sin(3\ln(2x+1))}{x}=-6\lim_{x\to 0}1\cdot\left[\frac{0}{0}\right]$$

$$\Rightarrow \text{Hô}\cdot\quad D_N'=\cos(3\ln(2x+1))\cdot 3\,\frac{2}{2x+1}=\frac{\cos(3\ln(2x+1))\cdot 6}{2x+1}$$

$$D_D'=1\qquad\Rightarrow -6\cdot 6\lim_{x\to 0}\frac{\cos(3\ln(2x+1))}{2x+1}=-36\lim_{x\to 0}1=-36$$