Limiti - Esercizi

Verificare i limiti $\longrightarrow +\infty$

1) $\lim_{x\to 0^+} \sqrt{\frac{1}{x}} = \sqrt{\frac{1}{0^+}} = +\infty$ ✓

2) $\lim_{x\to 7} \frac{1}{(x-7)^2} = \left.\begin{array}{l} \lim_{x\to 7^+} \frac{1}{(7^+-7)^2} = \frac{1}{(0^+)^2} = +\infty \\ \lim_{x\to 7^-} \frac{1}{(7^--7)^2} = \frac{1}{(0^-)^2} = +\infty \end{array}\right\} \lim_{x\to 7} f(x)\ +\infty$ ✓

3) $\lim_{x\to 0^+} e^{\frac{2}{x}} = \lim_{x\to 0^+} \frac{2}{0^+} = +\infty \Rightarrow e^{+\infty} = +\infty$ ✓

193) $\lim_{x\to 0^+} (-\ln x) = -(-\infty) = +\infty$ ✓

194) $\lim_{x\to 0} \ln\left(\frac{1}{x^2}\right) = \lim_{x\to 0} \frac{1}{x^2} = \left.\begin{array}{l} \lim_{x\to 0^+} \frac{1}{(0^+)^2} = +\infty \\ \lim_{x\to 0^-} \frac{1}{(0^-)^2} = +\infty \end{array}\right\} = +\infty \Rightarrow \lim_{x\to 0} \ln(+\infty) = +\infty$ ✓

195) $\lim_{x\to \frac{1}{2}} \frac{1}{(2x-1)^2} = \left.\begin{array}{l} \lim_{x\to \frac{1}{2}^+} \frac{1}{(2\frac{1}{2}^+-1)^2} = +\infty \\ \lim_{x\to \frac{1}{2}^-} \frac{1}{(2\frac{1}{2}^--1)^2} = +\infty \end{array}\right\} = +\infty$ ✓

196) $\lim_{x\to 2^+} \frac{1}{x^2-4} = \frac{1}{4^+-4} = \frac{1}{0^+} = +\infty$

201) $\lim_{x\to 2^+} \frac{1}{x-2} + 1 = \frac{1}{0^+} + 1 = +\infty$

205) $\lim_{x\to 0^-} \frac{5+2x}{-x} = \frac{5+0^-}{-(0^-)} = \frac{5}{0^+} = +\infty$

210) $\lim_{x\to 1^+} \log\left(\frac{2}{x+1}\right) = \log\left(\frac{2}{2}\right) = \log e = +\infty$

Verificare i limiti $\longrightarrow -\infty$

212) $\lim_{x\to 0} -\frac{1}{3x^4} = \left.\begin{array}{l} \lim_{0^-} -\frac{1}{3\,0^+} = -\infty \\ \lim_{0^+} -\frac{1}{3\,0^+} = -\infty \end{array}\right\} = -\infty$ ✓

213) $\lim_{x\to \frac{3}{2}^-} \frac{1}{4x^2-9} = \frac{1}{4\left(\frac{3}{2}\right)^2-9} = +\infty$?

214) $\lim_{x\to -1} \frac{-1}{x^2+2x+1} = \begin{array}{l} \lim_{-1^+} \frac{-1}{1^+-2^++1} = \frac{-1}{0^+} = -\infty \\ \lim_{-1^-} \frac{-1}{1^--2^-+1} = -\frac{1}{0^-} = +\infty \text{ ?} \end{array}$

215) $\lim_{x\to 1^-} \log_2(1-x) = \log(1-1^-) = \log(0^-) = -\infty$ ✓

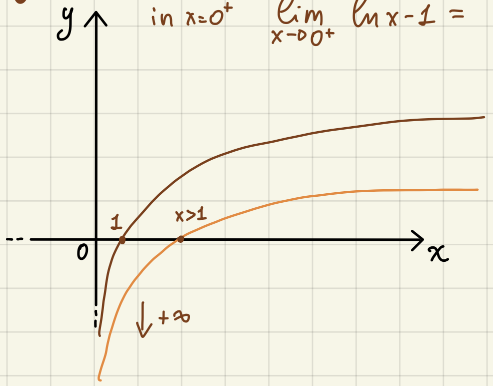
216) $\lim_{x\to 3^-} \frac{1}{x-3} = \frac{1}{3^--3} = -\infty$ ✓

217) $\lim_{x\to 0} \frac{-1}{|x|} = \left.\begin{array}{l} \lim_{x\to 0^+} -\frac{1}{|0^+|} = -\infty \\ \lim_{x\to 0^-} -\frac{1}{|0^-|} = -\infty \end{array}\right\} = -\infty$ ✓

Rappresentazione grafica

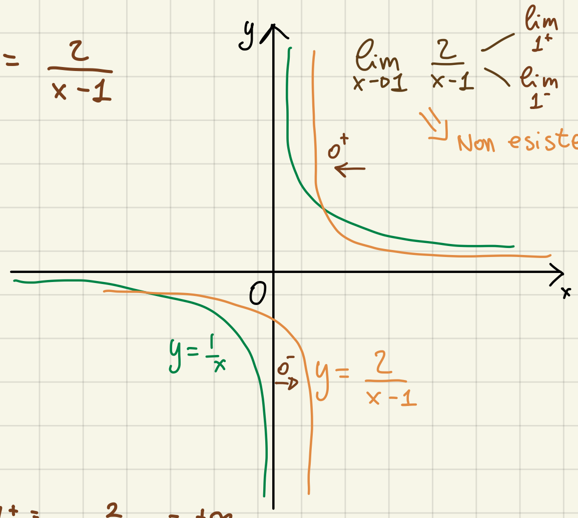
$f = \ln x - 1$

in $x = 0^+$ $\lim_{x \to 0^+} \ln x - 1 = \ln(0^+) - 1 = -\infty$



$f = \dfrac{2}{x-1}$

$\lim_{x \to 1} \dfrac{2}{x-1}$ $\begin{cases} \lim_{1^+} \dfrac{2}{0^+} = +\infty \\ \lim_{1^-} \dfrac{2}{0^-} = -\infty \end{cases}$ $\Rightarrow$ Non esiste



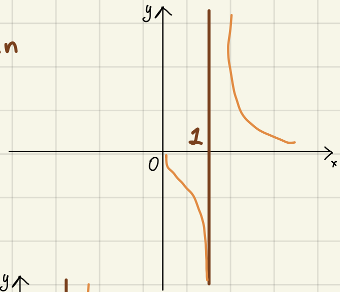
Asintoti verticali

249) $y = \dfrac{2}{(x-1)^2}$ in $x = 1$

$\lim_{x \to 1} \dfrac{2}{(x-1)^2}$ $\begin{cases} 1^+ = \dfrac{2}{0^+} = +\infty \\ 1^- = \dfrac{2}{0^+} = +\infty \end{cases}$ $\Rightarrow$ Asintoto Vert in $x = 1$

250) $y = \dfrac{1}{\ln x}$ in $x = 1$

$\lim_{x \to 1} \dfrac{1}{\ln x} = \begin{cases} 1^+ = \dfrac{1}{0^+} = +\infty \\ 1^- = \dfrac{1}{0^-} = -\infty \end{cases}$ Asintoto in $x = 1$

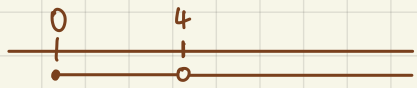


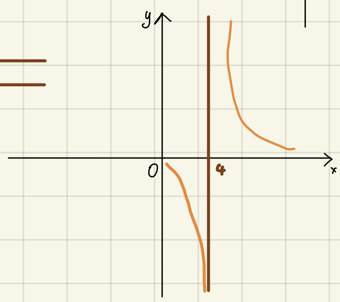
251) $y = \dfrac{2}{\sqrt{x} - 2}$ in $x = 4$

$\mathbb{D} = \sqrt{x} \geq 0$ $x \geq 0$

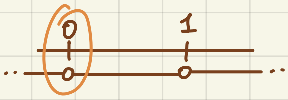
$\sqrt{x} - 2 \neq 0$ per $(\sqrt{x})^2 \neq 2^2 \Rightarrow x \neq 4$

$\lim_{x \to 4} f(x)$ $\begin{cases} 4^+ = \dfrac{2}{0^+} = +\infty \\ 4^- = \dfrac{2}{0^-} = -\infty \end{cases}$ A.V. in $x = 4$





252) $y = \dfrac{x-3}{x^2 - x}$ $\mathbb{D} = x^2 - x \neq 0$ per $x(x-1) \neq 0$ $\Rightarrow \begin{cases} x \neq 0 \\ x \neq 1 \end{cases}$



$\lim_{x \to 0} f(x)$ $\begin{cases} 0^+ = \dfrac{-3}{0^+} = -\infty \\ 0^- = \dfrac{-3}{0^-} = -\infty \end{cases}$

253) $y = 1 + e^{\frac{1}{x}}$ in $x = 0^+$ $\lim_{x \to 0^+} 1 + e^{\frac{1}{x}} = 1 + e^{\frac{1}{0^+}} = +\infty$

Limiti tendenti ad e

264) $\lim_{x\to+\infty} \frac{2}{x+10} = 0 \qquad \frac{2}{x(1+\cancel{\frac{1}{x}}_{\to 0})} = \frac{2}{\infty} = 0$ ✓

265) $\lim_{x\to+\infty} \frac{4x-1}{2x+1} = \frac{\cancel{x}(4-\cancel{\frac{1}{x}}^{\to 0})}{\cancel{x}(2+\cancel{\frac{1}{x}}_{\to 0})} = \frac{4}{2} = 2$ ✓

269) $\lim_{x\to+\infty} \frac{2^x-1}{2^x} = ?$

270) $\lim_{x\to+\infty} \frac{-3x}{|x|+1} = \frac{x(-3)}{x+1} = \frac{\cancel{x}(-3)}{\cancel{x}(1+\cancel{\frac{1}{x}}_{\to 0})} = \frac{-3}{1} = -3$

siccome siamo in $+\infty$

271) $\lim_{x\to+\infty} (\sqrt{x^2-1} - x = 0) = (x^2)^{\frac{1}{2}} - 1^{\frac{1}{2}} - x = x^{\frac{4+1}{2}} - x^{\frac{1}{2}} - x = x^{\frac{5}{2}} - x^{\frac{1}{2}} - x = x^{\frac{5}{2}-\frac{1}{2}-1} = \frac{5-1-2}{2} = x^{-2} = \frac{1}{x^2}$

$\Rightarrow \lim_{x\to+\infty} \frac{1}{x^2} = 0$ ✓

272) $\lim_{x\to\infty} \ln\left(\frac{x}{x-1}\right) = \lim_{x\to\infty} \frac{x}{x-1} = \frac{\cancel{x}}{\cancel{x}(1-\cancel{\frac{1}{x}}_{\to 0})} = \frac{1}{1} = 1 \Rightarrow \lim_{x\to\infty} \ln(1) = 0$ ✓

273) $\lim_{x\to\infty} \frac{x}{x^2-1} = \frac{\cancel{x}}{x^{\cancel{2}}(1-\cancel{\frac{1}{x^2}}_{\to 0})} = \frac{1}{+\infty} = 0$ ✓

274) $\lim_{x\to\infty} \left(\frac{1}{2}\right)^{2x} =$ All'aumentare dell'esponente aumenta il denominatore $\Rightarrow \frac{1}{\infty} = 0$ ✓

275) $\lim_{x\to+\infty} \left[\underbrace{\left(\frac{1}{3}\right)^{x+1}}_{\text{stesso di prima}} + 1\right] = 0 + 1 = 1$ ✓

277) $\lim_{x\to-\infty} \frac{2}{2x+1} = \frac{2}{-\infty} = 0$ ✓

278) $\lim_{x\to-\infty} \frac{x^3+1}{2x^3} = \frac{\cancel{x^3}(1+0)}{2} = \frac{1}{2}$ ✓

279) $\lim_{x\to-\infty} \frac{3x+1}{1-2x} = \frac{\cancel{x}(3+\cancel{\frac{1}{x}}^{\to 0})}{\cancel{x}(\cancel{\frac{1}{x}}_{\to 0}-2)} = -\frac{3}{2}$ ✓

283) $\lim_{x\to-\infty} \frac{-1}{e^{|x|}} = \frac{-1}{e^{-x}} = \frac{-1}{e^{+\infty}} = 0$ ✓

Siamo in $-\infty \Rightarrow |x| = -x$

284) $\lim_{x\to-\infty} 2e^{-4x^2} = \lim_{x\to-\infty} -4x^2 = +\infty \Rightarrow \lim_{x\to-\infty} 2e^{+\infty} =$

Limiti vari da Elia Bomb.

1) $\lim_{x\to 0} \frac{x^3}{tg\,x - \sin x}$

a) limiti notevoli

$$\frac{x^3}{tg\,x-\sin x} = \frac{x^3}{\sin x\left(\frac{1}{\cos x}-1\right)} = \frac{x^3}{\sin x \cdot \frac{1-\cos x}{\cos x}} = \frac{\cos x}{\cancel{\frac{\sin x}{x}} \cdot \cancel{\frac{1-\cos x}{x^2}}} = 2\cos x = 2$$

$\frac{\sin x}{x} \to 1$, $\frac{1-\cos x}{x^2} \to \frac{1}{2}$

$tg\,x = \frac{\sin x}{\cos x}$

b) Sviluppi di taylor

- $tg\,x = x + \frac{x^3}{3} + o(x^3)$
- $\sin x = x - \frac{x^3}{6} + o(x^3)$

$$\Rightarrow \cancel{x} + \frac{x^3}{\cancel{3}} + o(x^3) - \left[\cancel{x} - \frac{x^3}{\cancel{6}_{2}} + o(x^3)\right] = \frac{x^3}{2} + o(x^3) = \lim_{x\to 0} \frac{\cancel{x^3}}{\frac{\cancel{x^3}}{2} + o(x^3)} = 2$$

Esercizi serie numeriche

ES: $\sum_{n=1}^{+\infty}\left(-\frac{2}{3}\right)^n$; $\underbrace{-\frac{2}{3}=-1<-0,6<1}_{\text{Converge}}$ $\Rightarrow \sum_{n=1}^{+\infty}\left(-\frac{2}{3}\right)^n =$

Siccome n parte da 1 e non da 0, dobbiamo sottrarre un elemento dalla somma:

$$\Rightarrow \frac{1}{1-q}-q^0 = \frac{1}{1-\left(-\frac{2}{3}\right)} - \cancel{\left(-\frac{2}{3}\right)^0}\nearrow 0 = \frac{1}{\frac{5}{3}} - 1 = \frac{3}{5}-1 = \frac{3-5}{5} = -\frac{2}{5}$$

ES $\sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^{n+1}$ $q=\frac{1}{2} \Rightarrow -1<q<1 \Rightarrow$ Siccome l'esp= n+1 e non n, riscriviamo $=\frac{1}{2}\cdot\left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{2}\sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^n$

$$\Rightarrow Sum = \frac{1}{1-\left(\frac{1}{2}\right)} = \frac{1}{\frac{2-1}{2}} = \frac{1}{\frac{1}{2}} = 2 \Rightarrow \frac{1}{2}\sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^n = \frac{1}{2}\cdot 2 = 1$$

ES: $\sum_{n=0}^{+\infty}\left(\frac{4}{7}\right)^n$ $-1<q<1 \Rightarrow \frac{1}{1-\frac{4}{7}} = \frac{1}{\frac{7-4}{7}} = \frac{7}{3}$ Convergente

ES: $\sum_{n=0}^{+\infty}\left(\frac{2}{5}\right)^{2n} =$ $-1<q<1$ $= \left[\left(\frac{2}{5}\right)^2\right]^n = \left[\frac{4}{25}\right]^n \Rightarrow \frac{1}{1-\frac{4}{25}} = \frac{25}{21}$

ES: $\sum_{n=1}^{+\infty}\left(\frac{3}{4}\right)^n$ $-1<q<1 \Rightarrow \frac{1}{1-\frac{3}{4}} - \cancel{\left(\frac{3}{4}\right)^0}\nearrow 1 \Rightarrow \frac{4}{2}-1 = \frac{4-2}{2} = 1$

ES: $\sum_{n=1}^{+\infty}\left(\frac{3}{5}\right)^{n-1}$ $-1<q<1 \Rightarrow \left(\frac{3}{5}\right)^q\cdot\left(\frac{3}{5}\right)^{-1} \Rightarrow \left(\frac{3}{5}\right)^{-1}\sum_{n=1}^{+\infty}\left(\frac{3}{5}\right)^n = \left(\frac{3}{5}\right)^{-1}\cdot\frac{1}{1-\frac{3}{5}}-1 =$

$$= \frac{5}{2}\cdot\left(\frac{3}{5}\right)^{-1} - 1 = \frac{3^{-1}}{5^{-1}}\cdot\frac{5}{2} - 1 = \frac{19}{6}$$

ES: $\sum_{n=1}^{+\infty}(-1)^{n-1}\left(\frac{3}{5}\right)^{n-1} =$

Somme parziali

ES $\sum_{n=4}^{+\infty}\left(\frac{1}{n+1}-\frac{1}{n-3}\right)$

$S_4 = \frac{1}{5} - 1$

$S_5 = \frac{1}{6} - \frac{1}{2} + \frac{1}{5} - 1$

$S_6 = \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \frac{1}{2} + \frac{1}{5} - 1$

$S_7 = \frac{1}{8} - \frac{1}{4} + \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \frac{1}{2} + \frac{1}{5} - 1$

$S_8 = \frac{1}{9} - \cancel{\frac{1}{5}} + \frac{1}{8} - \frac{1}{4} + \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \frac{1}{2} + \cancel{\frac{1}{5}} - 1$

$S_9 = \frac{1}{10} - \cancel{\frac{1}{6}} + \frac{1}{9} - \cancel{\frac{1}{5}} + \frac{1}{8} - \frac{1}{4} + \frac{1}{7} - \frac{1}{3} + \cancel{\frac{1}{6}} - \frac{1}{2} + \cancel{\frac{1}{5}} - 1$

$S_{10} = \frac{1}{11} - \cancel{\frac{1}{7}} + \frac{1}{10} + \frac{1}{9} + \frac{1}{8} - \frac{1}{4} + \cancel{\frac{1}{7}} - \frac{1}{3} - \frac{1}{2} - 1$

$S_{11} = \frac{1}{12} - \cancel{\frac{1}{8}} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} + \cancel{\frac{1}{8}} - \frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1$

da qui in poi vengono eliminati

costanti: $-\frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1$

In ogni momento sono presenti

$$\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \frac{1}{n-3}$$

Quindi $S_n = \frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \frac{1}{n-3} - \frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1$

Per calcolare S_n con $n=\infty$ ci basta calcolare il limite per $x\to+\infty$ di S_n:

$$\lim_{n\to+\infty} \underset{\downarrow 0}{\cancel{\frac{1}{n+1}}} + \underset{\downarrow 0}{\cancel{\frac{1}{n}}} + \underset{\downarrow 0}{\cancel{\frac{1}{n-1}}} + \underset{\downarrow 0}{\cancel{\frac{1}{n-2}}} + \underset{\downarrow 0}{\cancel{\frac{1}{n-3}}} - \frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1$$

$$= \frac{-3-4-6-12}{12} = -\frac{25}{12}$$

ES 6.2 $a + ax + ax^2 + ax^3 + \dots = \sum_{n=1}^{+\infty} a x^{n-1}$

Se $x > 0 \Rightarrow \lim_{n \to +\infty} a x^{n-1} = +\infty$

$1 + 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} \dots$

1) $\sum_{n=1}^{+\infty} \frac{2}{n^2 + 2n} = \frac{2}{n(n+2)} \neq \frac{2}{n} - \frac{2}{(n+2)} \overset{\text{test}}{=} \frac{2(n+2) - 2n}{n(n+2)} = \frac{\cancel{2n} + 4 - \cancel{2n}}{n(n+2)} = \frac{4}{n(n+2)}$ Differisce di 2

oppure

$= \frac{1}{n} - \frac{1}{(n+2)} \overset{\text{TesT}}{=} \frac{\cancel{n}+2 - \cancel{n}}{n(n+2)} = \frac{2}{n(n+2)}$ ✓

Quindi: Calcoliamo le ridotte

$S_1 = 1 - \frac{1}{3}$

$S_2 = \frac{1}{2} - \frac{1}{4} + 1 - \frac{1}{3}$

$S_3 = \frac{1}{3} - \frac{1}{5} + \frac{1}{2} - \frac{1}{4} + 1 - \frac{1}{3}$

$S_4 = \frac{1}{4} - \frac{1}{6} + \cancel{\frac{1}{3}} - \frac{1}{5} + \frac{1}{2} - \frac{1}{4} + 1 - \cancel{\frac{1}{3}}$

$S_5 = \cancel{\frac{1}{5}} - \frac{1}{7} + \frac{1}{4} - \frac{1}{6} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \frac{1}{2} - \frac{1}{4} + 1 - \cancel{\frac{1}{3}}$

$S_6 = \cancel{\frac{1}{6}} - \frac{1}{8} - \frac{1}{7} + \frac{1}{4} - \cancel{\frac{1}{6}} + \frac{1}{2} - \frac{1}{4} + 1$

$S_7 = \cancel{\frac{1}{7}} - \frac{1}{9} - \frac{1}{8} - \cancel{\frac{1}{7}} + \frac{1}{4} + \frac{1}{2} - \frac{1}{4} + 1$

$S_8 = \underbrace{\frac{1}{8} - \frac{1}{10} - \frac{1}{9} - \frac{1}{8}}_{\text{Variano}} + \underbrace{\cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{4}} + 1}_{\text{CostanTi}}$

$S_n = -\frac{1}{n+2} - \frac{1}{n+1} + \frac{1}{2} + 1$

Per calcolare S_n per $n = \infty$ risolviamo il lim

$\lim_{n \to +\infty} S_n = -\underbrace{\cancel{\frac{1}{n+2}}}_{0} - \underbrace{\cancel{\frac{1}{n+1}}}_{0} + \frac{1}{2} + 1 = \frac{3}{2}$

ES 2: $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2 + n}} = \frac{\cancel{\sqrt{n+1}}}{\sqrt{n}\cancel{\sqrt{n+1}}} - \frac{\sqrt{n}}{\sqrt{n}\sqrt{n+1}} = \frac{1}{\sqrt{n}} - \frac{\sqrt{n}}{\sqrt{n}\sqrt{n+1}}$

$S_1 = 1 - \frac{1}{1 \cdot \sqrt{2}}$

$S_2 = \frac{1}{\sqrt{2}} - \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\sqrt{3}} = \cancel{\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{3}} + 1 - \cancel{\frac{1}{\sqrt{2}}}$

$S_3 = \frac{1}{\sqrt{3}} - \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\, 2} = \cancel{\frac{1}{\sqrt{3}}} - 2 - \cancel{\frac{1}{\sqrt{3}}} + 1$

$S_4 = \frac{1}{2} - \frac{\cancel{2}}{\cancel{2}\sqrt{5}} = \frac{1}{2} - \frac{1}{\sqrt{5}} - 2 + 1$

$S_5 = \frac{1}{\sqrt{5}} - \frac{\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}\sqrt{6}} = \cancel{\frac{1}{\sqrt{5}}} - \frac{1}{\sqrt{6}} + \frac{1}{2} - \cancel{\frac{1}{\sqrt{5}}} - 2 + 1$

$S_6 = \frac{1}{\sqrt{6}} - \frac{\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}\sqrt{7}} = \cancel{\frac{1}{\sqrt{6}}} - \frac{1}{\sqrt{7}} - \cancel{\frac{1}{\sqrt{6}}} + \frac{1}{2} - 2 + 1$

$S_n = -\frac{1}{\sqrt{n+1}} + 1$

$\lim_{n \to +\infty} -\underbrace{\cancel{\frac{1}{\sqrt{n+1}}}}_{0} + 1 = 1$ Converge

ES: $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{2n}} = \sum_{n=1}^{\infty} \frac{4^{n+1}}{9^n} = \frac{4^n}{9^n} \cdot 4 = 4\left(\frac{4}{9}\right)^n$

Ragione

$= 4 \cdot \left(\frac{1}{1-\frac{4}{9}} - 1\right) = 4\left(\frac{4}{5}\right) = \frac{16}{5}$

ES: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \overset{\text{Test}}{=} \frac{\cancel{n}+1-\cancel{n}}{n(n+1)}$ ✓

$S_1 = 1 - \frac{1}{2}$

$S_2 = \cancel{\frac{1}{2}} - \frac{1}{3} + 1 - \cancel{\frac{1}{2}}$

$S_3 = \cancel{\frac{1}{3}} - \underbrace{\frac{1}{4}}_{\text{Varia}} - \cancel{\frac{1}{3}} \underbrace{+1}_{\text{COST}}$

$S_n = \frac{1}{n+1} + 1$

$\lim_{n \to +\infty} \frac{1}{n+1} + 1 = 1$ ($\frac{1}{n+1} \to 0$)

~~6.6~~ Verificare che la serie

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \ldots$$

è divergente.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \sum_{n=1}^{\infty} \frac{1}{2n}$

Questa serie è di tipo Armonico, quindi Converge per $\alpha > 1$, Siccome n^{α} $1 \leq 1$ la serie diverge.

Verificare che la serie 1+2+3+...+n+... è divergente.

$1+2+3+\ldots n = \sum_{n=1}^{\infty} n \qquad \lim_{n \to \infty} n = +\infty$

Verificare che è indeterminata la serie:

$$\sum_{n=1}^{\infty} (-1)^n = -1+1-1+1-1+\ldots$$

Per n pari: $\sum (-1)^n = +1$

Per n dispari: $\sum (-1)^n = -1$

Non regolare $\Downarrow$ Indeterminata

Calcolare la somma delle seguenti serie geometriche

(a) $\sum_{n=0}^{\infty} \frac{1}{2^n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

(c) $\sum_{n=0}^{\infty} 3^n$ (d) $\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$

a) $\sum_{n=0}^{\infty} \frac{1}{2^n} =$