1. Determinare dominio, asintoti, intervalli di monotonia, massimi e minimi, e disegnare un grafico qualitativo delle seguenti funzioni:

a) $f(x) = \dfrac{x^3 - x}{x^2 - 4}$

Dominio: $x^2 - 4 \neq 0$ per $\underline{x \neq \pm 2}$

Asintoti: $\lim_{x \to -2^\pm} f(x) = \dfrac{-8+2}{4^+ - 4} = -\infty \Rightarrow \underline{x = -2 \text{ A.V.}}$

$\lim_{x \to 2^\pm} f(x) = \dfrac{8-2}{4^+ - 4} = +\infty \Rightarrow \underline{x = 2 \text{ A.V.}}$

$\lim_{x \to +\infty} f(x) = \sim \dfrac{x^{\cancel{3}}}{\cancel{x^2}} = \dfrac{x^2}{x} \to x^2 >> x \Rightarrow +\infty \Rightarrow$ NO A. Or.

$\lim_{x \to +\infty} \dfrac{f(x)}{x} \sim \dfrac{x^3}{x^3} = 1 \Rightarrow m = 1$, $\lim_{x \to +\infty} f(x) - x = \dfrac{x^2}{x} - x = \dfrac{x^2 - x^2}{x} = 0$

$\Rightarrow \underline{y = x \text{ A. Obliquo}}$

Segno: $f(x) > 0$ per $x^3 - x > 0 \to x(x^2 - 1) > 0$

$\hookrightarrow x > 0$

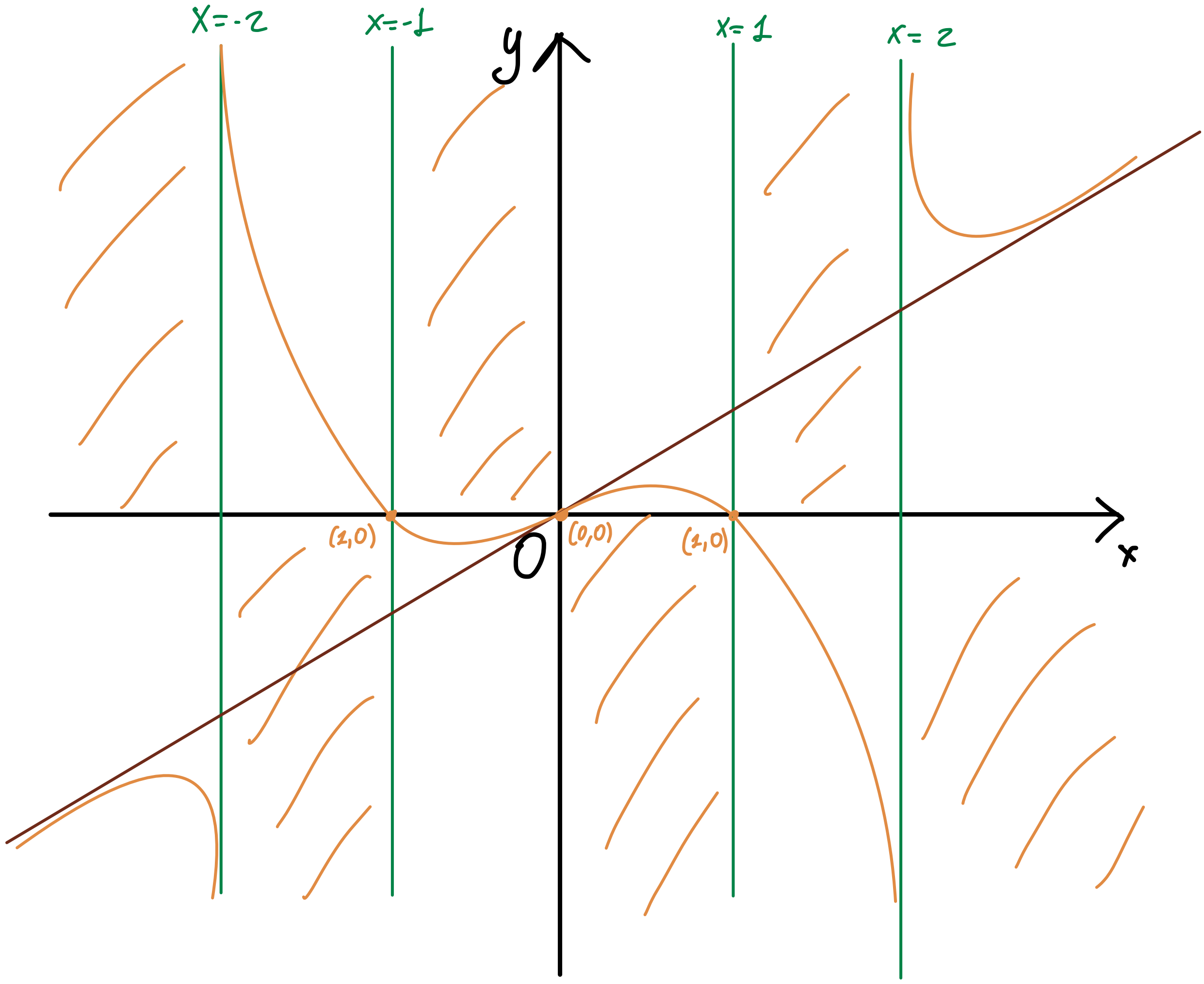
$x > \pm 1 \to$ Val esterni

$x^2 - 4 > 0$

$\hookrightarrow x > \pm 2$

Val est

-2		-1		0		1		2
-	×	-	| - |	+	| +	×	+	
+	| -	| - |	-	| -	| +			
+	| +	| - |	-	| +	| +			
-	+	-	+	-	+			



Deriv: $D\left(\frac{x^3-x}{x^2-4}\right) = \frac{(3x^2-1)(x^2-4)-[(x^3-x)(2x)]}{(x^2-4)^2} = \frac{\cancel{3x^4}-12x^2-\cancel{x^2}+4-\cancel{2}x^4+\cancel{2}x^2}{}$

$= \frac{-x^4-11x^2+4}{(x^2-4)^2} = -(x^4+11x^2-4) > 0$ pongo $t=x^2 \Rightarrow x^2+11t-4<0$ $\Delta = 121-4(-4) = 137$

$\Rightarrow t_{1,2} = \frac{-11 \pm \sqrt{137}}{2}$ $\frac{-11-\sqrt{137}}{2}$, $\frac{-11+\sqrt{137}}{2}$ $t = x^2 \Rightarrow x = \sqrt{t} \Rightarrow$

Intersezioni

$\begin{cases} x=0 \\ y=0 \end{cases} \Rightarrow (0,0) \in f(x)$ $\begin{cases} y=0 \\ x^3-x=0 \end{cases}$ per $x(x^2-1)=0$ $\Rightarrow x=0$ $x^2=1$ $\Rightarrow x=\pm 1$ $\Rightarrow$ $(-1,0)$ $(1,0)$ $\in f(x)$

b) $f(x) = \dfrac{\log x}{x}$ 1) Dominio $x>0$

2) Intersezioni

$\begin{cases} x=0 \\ \nexists x \in \mathbb{R} \end{cases}$ $\begin{cases} y=0 \\ \dfrac{\ln x}{x} = 0 \end{cases} \to \ln x = 0$ per $x=1$ $\Rightarrow (1,0) \in f(x)$

Simm: $f(-x) = \dfrac{\log(-x)}{-x} \Rightarrow$ No simm

3) Segno $f(x) > 0$ per $x > 1$

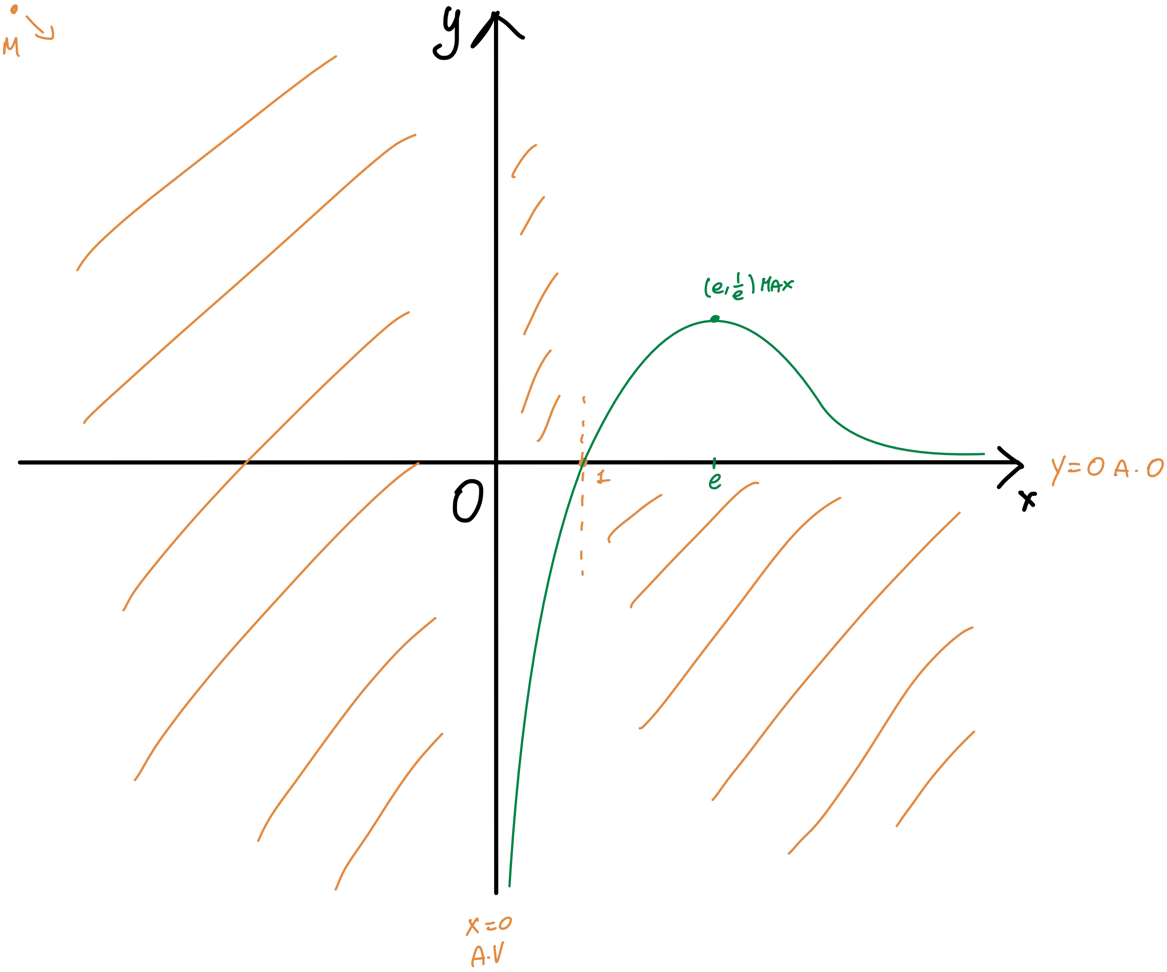
4) Asintoti: $\lim\limits_{x \to 0^+} f(x) = \dfrac{\ln(0^+)}{0^+} \Rightarrow x \gg \ln(x) \Rightarrow \sim \dfrac{n}{0} \to +\infty$ $x=0$ A.V. Dx

$\lim\limits_{x \to \infty} f(x) \sim \dfrac{\ln}{x} \to x \gg \ln \Rightarrow 0 \to y=0$ A.O.

5) Deriv I $D\left(\dfrac{\ln x}{x}\right) = \dfrac{1-[\ln x]}{x^2} = \dfrac{1-\ln x}{x^2} > 0$ per $\ln x < 1$ per $e^{\ln x} < e^1 \to$ $\Rightarrow x < e$

$f(e) = \dfrac{\ln e}{e} = \dfrac{1}{e} \Rightarrow \left(e, \dfrac{1}{e}\right)$ Max cde: $x > 0$

	0		e	
+		+		−
−		+		+
///		⊕ ↗	M	− ↘



c) $f(x) = 2x + \sqrt{x^2 - 1}$.

Dominio $x^2 \geq 1$ per $x \geq \pm 1$ Valori esterni

$f(x)$ definita per $x < -1 \cup x > 1$

2) Intersez.

$\begin{cases} x = 0 \\ \sqrt{-1} \quad \nexists x \in \mathbb{R} \end{cases}$

$\begin{cases} y = 0 \\ \nexists x \in \mathbb{R} \end{cases}$

No intersez

3) Segno $f(x) > 0$ per $2x + \sqrt{x^2 - 1} > 0$ $\sqrt{x^2-1} > -2x$ $\begin{cases} \text{per } -2x \geq 0 \\ \\ \text{per } -2x < 0 \end{cases}$