1. Determinare dominio, asintoti, intervalli di monotonia, massimi e minimi, e disegnare un grafico qualitativo delle seguenti funzioni:

a) $f(x) = \dfrac{x^3 - x}{x^2 - 4}$

Dominio: $x^2-4 \neq 0$ per $\underline{x \neq \pm 2}$

Asintoti: $\lim_{x\to -2^\pm} f(x) = \dfrac{-8+2}{4^+-4} = -\infty \Rightarrow \underline{x=-2 \text{ A.V.}}$

$\lim_{x\to 2^\pm} f(x) = \dfrac{8-2}{4^+-4} = +\infty \Rightarrow \underline{x = 2 \text{ A.V.}}$

$\lim_{x\to +\infty} f(x) = \sim \dfrac{x^{\cancel{3}}}{x^{\cancel{2}}} = \dfrac{x^2}{x} \to x^2 >> x \Rightarrow +\infty \Rightarrow$ NO A. Or.

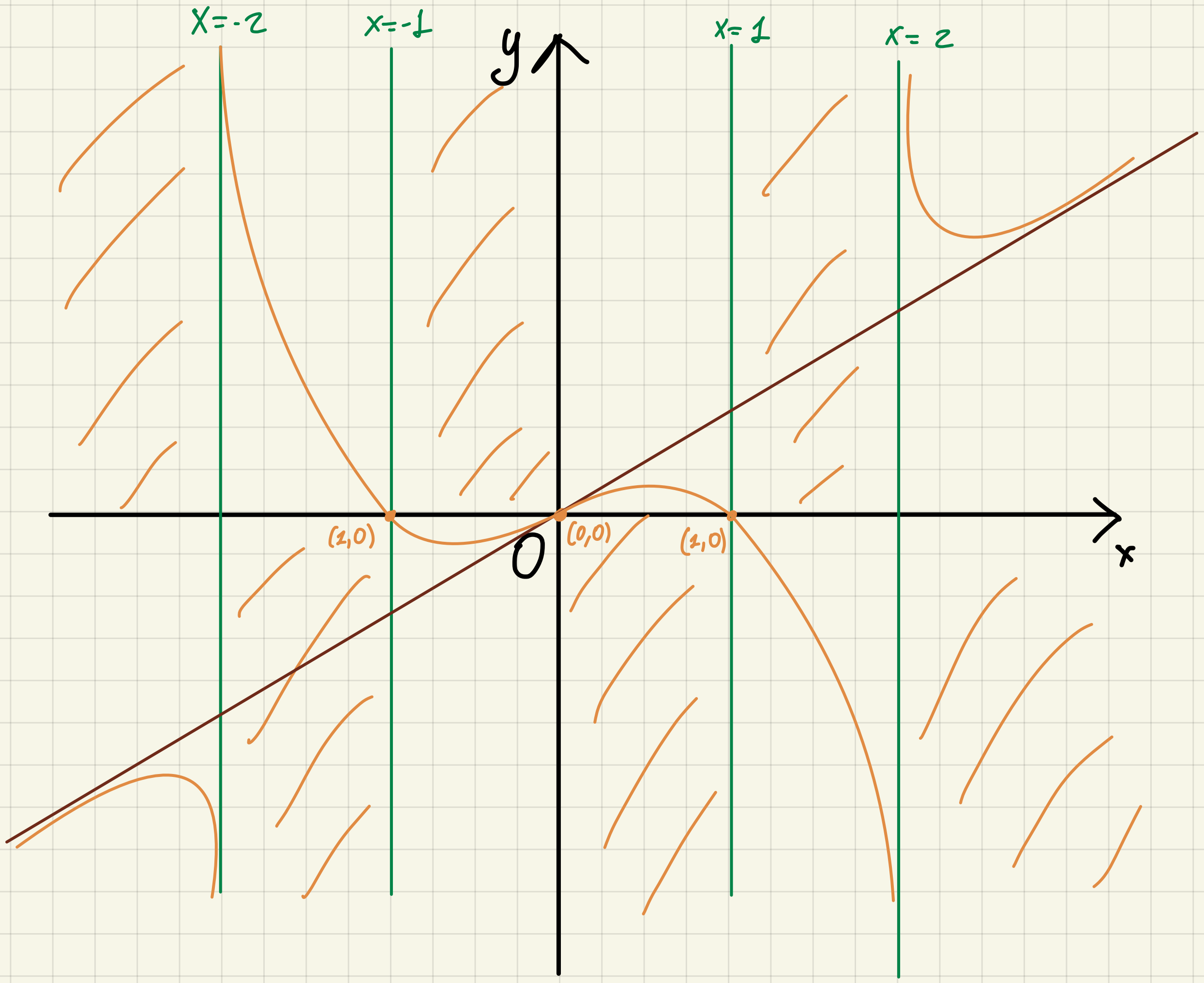
$\lim_{x\to +\infty} \dfrac{f(x)}{x} \sim \dfrac{x^3}{x^3} = 1 \Rightarrow m = 1$, $\lim_{x\to +\infty} f(x) - x = \dfrac{x^2}{x} - x = \dfrac{x^2-x^2}{x} = 0$

$\Rightarrow \underline{y = x \text{ A. Obliquo}}$

Segno: $f(x) > 0$ per $x^3 - x > 0 \to x(x^2-1) > 0$
↳ $x > 0$
$x > \pm 1 \to$ Val esterni

$x^2 - 4 > 0$
↳ $x > \pm 2$
Val est

	-2	-1	0	1	2
-	-	-	+	+	+
+	-	-	-	-	+
+	+	-	-	+	+
-	+	-	+	-	+



Deriv: $D\left(\frac{x^3-x}{x^2-4}\right) = \frac{(3x^2-1)(x^2-4)-[(x^3-x)(2x)]}{(x^2-4)^2} = \frac{\cancel{3}x^4 - 12x^2 - \cancel{x^2} + 4 - \cancel{2}x^4 + \cancel{2}x^2}{//}$

$= \frac{-x^4-11x^2+4}{(x^2-4)^2} = -(x^4+11x^2-4) > 0$ pongo $t = x^2 \rightarrow x^2+11t-4<0$ $\quad \Delta = 121-4(-4) \overset{!}{=} 137$

$\Rightarrow t_{1,2} = \frac{-11\pm\sqrt{137}}{2} \begin{cases} \frac{-11-\sqrt{137}}{2} \\ \frac{-11+\sqrt{137}}{2} \end{cases} t = x^2 \Rightarrow x = \sqrt{t} \Rightarrow$

Intersezioni

$\begin{cases} x=0 \\ y=0 \end{cases} \Rightarrow (0,0) \in f(x)$ $\qquad \begin{cases} y=0 \\ x^3-x=0 \end{cases}$ per $x(x^2-1)=0$ $\quad x^2 = 1$ $\Rightarrow \begin{matrix} (-1,0) \\ (1,0) \end{matrix} \in f(x)$

$\hookrightarrow x=0$ $\qquad \Rightarrow x = \pm 1$

$b)\ f(x) = \dfrac{\log x}{x}$

1) Dominio $x > 0$

2) Intersezioni

$\begin{cases} x=0 \\ \nexists x \in \mathbb{R} \end{cases}$ $\begin{cases} y=0 \\ \frac{\ln x}{x} = 0 \rightarrow \ln x = 0 \text{ per } x=1 \end{cases} \Rightarrow (1,0) \in f(x)$

Simm: $f(-x) = \dfrac{\log(-x)}{-x} \Rightarrow$ NO simm

3) Segno $f(x) > 0$ per $x > 1$

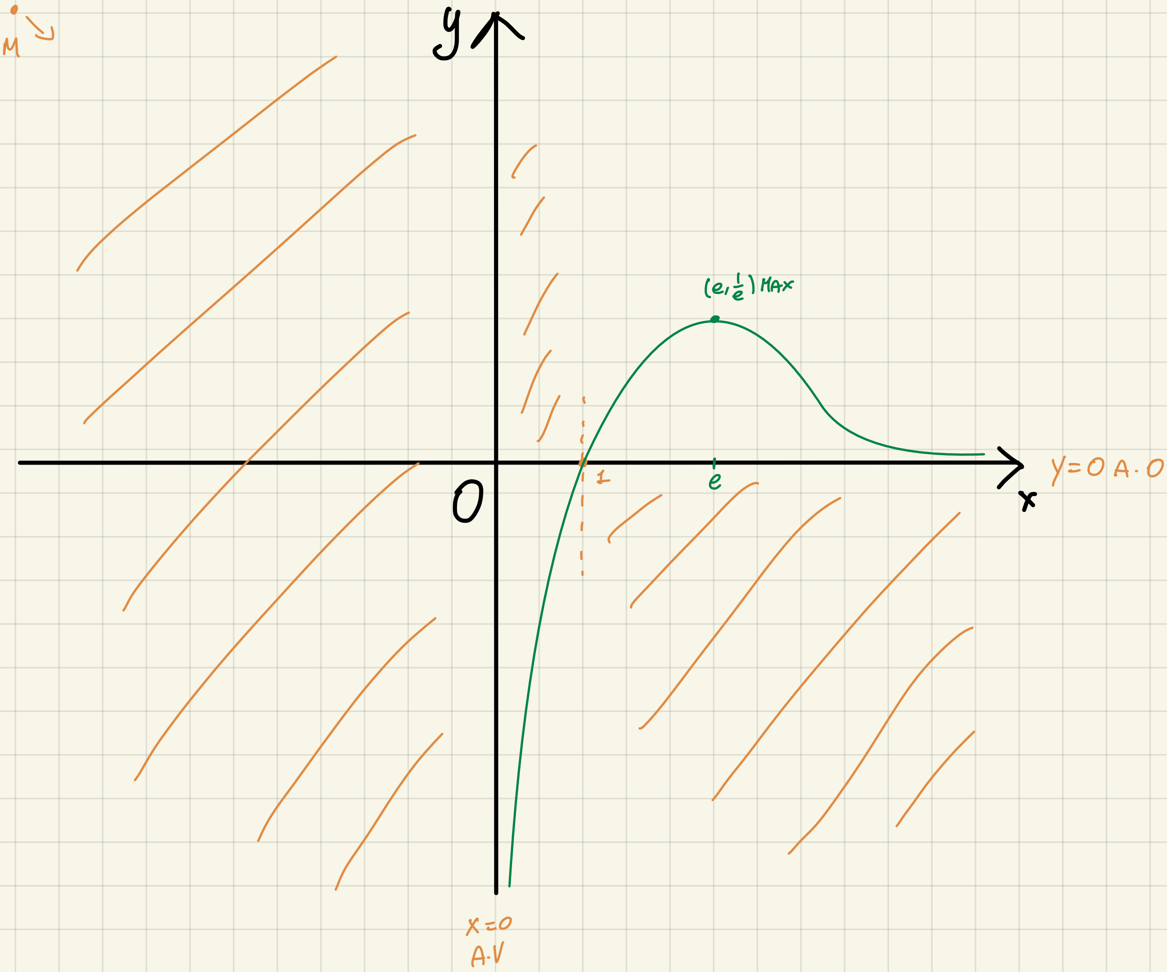
4) Asintoti: $\lim_{x \to 0^+} f(x) = \dfrac{\ln(0^+)}{0^+} \Rightarrow x \gg \ln(x) \Rightarrow \sim \dfrac{n}{0} \rightarrow +\infty$ $x=0$ A.V. Dx

$\lim_{x \to \infty} f(x) \sim \dfrac{\ln}{x} \rightarrow x \gg \ln \Rightarrow \emptyset \rightarrow y=0$ A.O.

5) Deriv I $D\left(\dfrac{\ln x}{x}\right) = \dfrac{1-[\ln x]}{x^2} = \dfrac{1-\ln x}{x^2} > 0$ per $\ln x < 1$ per $e^{\ln x} < e^1 \rightarrow$
$\Rightarrow x < e$

$f(e) = \dfrac{\ln e}{e} = \dfrac{1}{e} \Rightarrow \left(e, \dfrac{1}{e}\right)$ Max cde: $x > 0$

0		e	
+	+		−
−	+		+
///	⊕ ↗	M	− ↘



c) $f(x) = 2x + \sqrt{x^2 - 1}.$

Dominio $x^2 \geq 1$ per $x \geq \pm 1$ Valori esterni

$f(x)$ definita per $x < -1 \cup x > 1$

2) Intersez.

$\begin{cases} x = 0 \\ \sqrt{-1} \quad \nexists x \in \mathbb{R} \end{cases}$

$\begin{cases} y = 0 \\ \nexists x \in \mathbb{R} \end{cases}$

No intersez

3) Segno $f(x) > 0$ per $2x + \sqrt{x^2 - 1} > 0$ $\quad \sqrt{x^2 - 1} > -2x$ $\begin{cases} \text{per } -2x \geq 0 \\ \text{per } -2x < 0 \end{cases}$