X

X

X

X

X

Esercizio 1. Calcolare

$$\lim_{x\to 0} \frac{(\arctan(e^{2x}-1))^2}{\cos(\sin(x))-1}$$

$$\frac{[\operatorname{atan}(e^{2x}-1)]^2}{\cos(\sin x)-1} = \left[\frac{0}{0}\right]$$

$$\text{H\^o}: \frac{2(\operatorname{atan}(e^{2x}-1))}{1+(e^{2x}-1)^2} \cdot 2e^{2x} = \frac{4\operatorname{atan}(e^{2x}-1)\,e^{2x}}{e^{4x}-2e^{2x}+2}$$

$$D_d' = -\sin(\sin x)\cos x$$

$$\Rightarrow \lim_{x\to 0} \frac{\dfrac{4\operatorname{atan}(e^{2x}-1)e^{2x}}{e^{4x}-2e^{2x}+2}}{-\sin(\sin x)\cos(x)} = \frac{\overset{4}{4}\operatorname{atan}(e^{2x}-1)\overset{1}{e^{2x}}}{-\sin(\sin x)\cdot \underbrace{\cos x}_{1}\ \underbrace{(e^{4x}-2e^{2x}+2)}_{2-e}}$$

$$= \frac{\overset{4}{4e^{2x}}}{\underbrace{\cos x}_{1}\underbrace{(e^{4x}-2e^{2x}+2)}_{2-2+1=1}} \cdot \frac{\operatorname{atan}(e^{2x}-1)}{-\sin(\sin x)} = -4 \lim_{x\to 0} \frac{\operatorname{atan}(e^{2x}-1)}{\sin(\sin x)} = \left[\frac{0}{0}\right]$$

$$\Rightarrow \text{H\^o}: -4 \lim_{x\to 0} \frac{\dfrac{\overset{2}{2e^{2x}}}{1+(e^{2x}-1)^2}}{\underbrace{\cos(\sin x)}_{1}\cdot\underbrace{\cos(x)}_{1}} = \frac{\dfrac{\overset{2}{2e^{2x}}}{2+e^{4x}-2e^{2x}} \to 2+1-2=1}{\cancel{\cos(\sin x)\cos(x)} \to 1}$$

$$\Rightarrow \lim_{x\to 0} f(x) = -4\cdot \lim_{x\to 0} 2 = -4\cdot 2 = -8$$

Esercizio 2. Determinare la convergenza

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

1) Teorema di convergenza

$\lim_{n \to +\infty} \frac{n!}{n^n} \Rightarrow n! << n^n \to 0$ Potrebbe convergere

2) Criterio del rapporto

$$\lim_{n\to+\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \frac{\cancel{(n+1)} \cdot \cancel{n!}}{(n+1)^n \cdot \cancel{(n+1)}} \cdot \frac{n^n}{\cancel{n!}} = \frac{n^n}{(n+1)^n}$$

$$= \lim_{n\to+\infty} \left(\frac{n}{n+1}\right)^n = \text{pongo } t = n+1 \to n = t-1 \Rightarrow \lim_{t\to+\infty} \left(\frac{t-1}{t}\right)^{t-1} = \left(1-\frac{1}{t}\right)^{t-1} \Rightarrow \left(1-\frac{1}{t}\right)^{t \cdot \frac{t-1}{t}}$$

Se $n \to +\infty$, $t \to +\infty$

$$= \left[\left(1+\frac{-1}{t}\right)^t\right]^{\frac{t-1}{t}} \Rightarrow \lim_{n\to+\infty}\left(1+\frac{a}{n}\right)^n = e^a \Rightarrow \lim_{t\to+\infty}\left[\left(1+\frac{-1}{t}\right)^t\right]^{\lim_{t\to+\infty}\frac{t-1}{t}} = \left[\left(1+\frac{-1}{t}\right)^t\right]^1 = e^{-1} = \frac{1}{e}$$

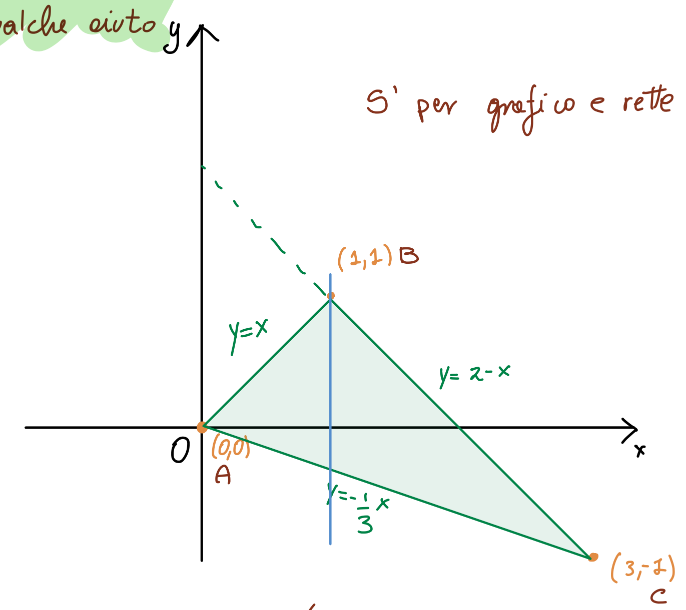
Converge

Tempo 8'30"
con qualche aiuto

Esercizio 3. Calcolare

$$\iint_D xy\,dxdy$$

Dove D è il triangolo di vertici $A = (0,0)$, $B = (1,1)$, $C = (3,-1)$



S' per grafico e rette

$\overline{AB} = x = y$ ✓

$\overline{BC} = \frac{x-1}{3-1} = \frac{y-1}{-1-1} = \frac{x-1}{2} = \frac{y-1}{-2}$

$\to x-1 = -y+1 \to x-2 = -y \to y = 2-x$ ✓

$\overline{AC}$ $\frac{x}{3} = \frac{y}{-1} \to \frac{x}{3} = -y \to y = -\frac{1}{3}x$

$D_1 : \{(x,y) \,/\, 0 < x < 1 ,\ -\frac{1}{3}x < y < x\} \cup$

$\cup\ D_2 : \{(x,y) \,/\, 1 < x < 3 ,\ -\frac{1}{3}x < y < 2-x\}$

$$\int_0^1 \int_{-\frac{1}{3}x}^{x} xy\,dy\,dx = \int_0^1 x \int_{-\frac{1}{3}x}^{x} y\,dy = \int_0^1 x \left[\frac{y^2}{2}\right]_{-\frac{1}{3}x}^{x} dx$$

$$= \frac{x^2}{2} - \frac{\frac{1}{9}x^2}{2} = \frac{9x^2 - x^2}{18} = \frac{8}{18}x^2$$

$$\to \frac{8}{18}\int_0^1 x^3\,dx = \frac{8}{18}\left[\frac{x^4}{4}\right]_0^1 = \frac{\cancel{8}^{\,2}}{18} \cdot \frac{1}{\cancel{4}} = \frac{2}{18} = \frac{1}{9}$$

$$\int_1^3 x \int_{-\frac{1}{3}x}^{2-x} y\,dy\,dx = \int_1^3 x\left[\frac{y^2}{2}\right]_{-\frac{1}{3}x}^{2-x} dx = \frac{(2-x)^2}{2} - \frac{\frac{1}{9}x^2}{2}$$

$$= \frac{9(2-x)^2 - x^2}{18} = \frac{9(4-4x+x^2)-x^2}{18} = \frac{36-36x+9x^2-x^2}{18} = \frac{8x^2-36x+36}{18}$$

$$\to \int_1^3 x \cdot \frac{8x^2-36x+36}{18}\,dx = \int_1^3 \frac{8x^3-36x^2+36x}{18} = \frac{4}{18}\int_1^3 2x^3 - 9x^2 + 9x\,dx$$

$$= \frac{2}{9}\left[\frac{x^4}{2} - 3x^3 + \frac{9}{2}x^2\right]_1^3 = \left[\frac{x^4}{9} - \frac{2}{3}x^3 + x^2\right]_1^3 = \left[\cancel{9-18+9}_{\,0}\right] - \left[\frac{1}{9} - \frac{2}{3} + 1\right] = \frac{1-6+9}{9} = -\frac{4}{9}$$

$$\Rightarrow \iint_D xy\,dxdy = \frac{1}{9} - \frac{4}{9} = -\frac{1}{3}$$

Esercizio 4. Per quali valori del parametro α la funzione $f:\mathbb{R}^2\to\mathbb{R}$ definita da

$$f(x,y)=\begin{cases}\frac{x|y|^\alpha}{x^2+y^2} & \text{se } (x,y)\neq(0,0),\\ 0 & \text{se } (x,y)=(0,0),\end{cases}$$

risulta continua?
Per quali valori di α risulta differenziabile?

$$\begin{cases}\dfrac{x|y|^\alpha}{x^2+y^2} & \text{Se } (x,y)\neq(0,0)\\ 0 & \text{Se } (x,y)=(0,0)\end{cases}$$

$\lim_{(x,y)\to(0,0)} f(x,y)=f(0,0) \Rightarrow \lim_{(x,y)\to(0,0)} f(x,y)=\emptyset$

calcolo il limite lungo il fascio di rette passanti per o (0,0)

in (0,0)

$\Rightarrow$ lungo $y-x_0=m(x-y_0) \to \boxed{y=mx}$

$\Rightarrow \lim_{x\to 0}\dfrac{x|mx|^\alpha}{x^2+m^2x^2}=\dfrac{x^{\alpha+1}m^\alpha}{x^2(m^2+1)}=\dfrac{x^{\alpha-1}m^\alpha}{m^2+1}$ il lim dipende da $x \Leftrightarrow \alpha\geq 2$

Considero inoltre $f(x,0)=\dfrac{x}{x^2}=\dfrac{1}{x}$ e $f(0,y)=\dfrac{|y|^\alpha}{y^2}=$
- $y^{\alpha-2}$ per $\alpha>2$
- 1 per $\alpha=2$
- $\frac{1}{y}$ per $\alpha=1$
- $\frac{1}{y^{\alpha+2}}$ per $\alpha<0$

Def differenziabilità:

$f(x,y)$ è Diff in $P_0(x_0,y_0)$

$\Leftrightarrow f(P_0)=\lim_{P\to P_0}\dfrac{f(P)-f(P_0)-f_x'(P_0)\cdot(x-x_0)-f_y'(P_0)(y-y_0)}{\overline{PP_0}}$

$\overline{PP_0}\to\sqrt{(x-x_0)^2+(y-y_0)^2}$

Ma prima

$f_x(P_0)=\dfrac{y^\alpha(x^2+y^2)-xy^\alpha(2x)}{(x^2+y^2)^2}=\left[\dfrac{0}{0}\right]\to\lim_{(x,y)\to(0,0)} f_x'(P_0)$ lungo $y=mx$

$\to \lim_{x\to 0}\dfrac{(mx)^\alpha(x^2+m^2x^2)-2mx^{\alpha+2}}{(x^2+m^2x^2)^2}=\dfrac{x^2(mx)^\alpha+\cancel{(mx)^{\alpha+2}}-\cancel{2}(mx)^{\alpha+2}}{(x^2(1+m^2))^2}=\dfrac{(mx)^\alpha(x^2-1)}{(mx)^4+2m^2x^4+x^4}$

$=\dfrac{(mx)^\alpha(x^2-1)}{x^4(m^4+2m^2+1)}=\dfrac{mx^{\alpha+2}-mx^\alpha}{x^4(m^4+2m^2+1)}=\dfrac{x^\alpha(mx^2-m)}{\cancel{x^4}(m^4+2m^2+1)}=\dfrac{x^{\alpha-4}(mx^2-m)}{(m^4+2m^2+1)}=\emptyset$

per $\alpha>4$

$f_y(P_0)=$ Immagino Sia 0 anche lei

$\Rightarrow \lim_{P\to P_0}\dfrac{\frac{x|y|^\alpha}{x^2+y^2}-\emptyset-0-0}{\sqrt{x^2+y^2}}=\dfrac{xy^\alpha}{(x^2+y^2)\sqrt{x^2+y^2}}$ lungo $y=mx$

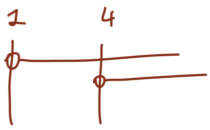
$\to\lim_{x\to 0}\dfrac{x\cdot(mx)^\alpha}{(x^2+m^2x^2)\sqrt{x^2+m^2x^2}}=\dfrac{m^\alpha x^{\alpha+1}}{x^2\sqrt{x^2+m^2x^2}+m^2x^2\sqrt{x^2+m^2x^2}}=\dfrac{m^\alpha x^{\alpha+1}}{\cancel{x^2}\sqrt{x^2+m^2x^2}(1+m^2)}$

$=\dfrac{m^\alpha x^{\alpha-1}}{\sqrt{x^2+m^2x^2}(1+m^2)}\cdot\dfrac{\sqrt{x^2+m^2x^2}}{\sqrt{x^2+m^2x^2}}=\dfrac{m^\alpha x^{\alpha-1}\sqrt{x^2+m^2x^2}}{|x^2+m^2x^2|(1+m^2)}$

$$= \frac{m^\alpha x^{\alpha-1}\sqrt{x^2+m^2x^2}}{x^2+m^2x^2+m^2x^2+m^4x^2} = \frac{m^\alpha x^{\alpha-1}\sqrt{x^2+m^2x^2}}{x^2(1+2m^2+m^4)} = \frac{m^\alpha x^{\alpha-3}\sqrt{x^2+m^2x^2}}{1+2m^2+m^4}$$

$$\lim_{x\to 0} m^\alpha x^{\alpha-3}\sqrt{x^2+m^2x^2} \to m^\alpha x^{\alpha-1}\sqrt{1+m^2}$$

- $\to$ per $\alpha < 0 \to +\infty$
- $\to$ per $\alpha = 1 \to m\sqrt{1+m^2}$
- $\to$ per $\alpha > 1 \to 0$



$\Rightarrow$ $f(x,y)$ è Differenziabile per $\alpha > 4$

Esempio continuità funzioni di 2 variabili

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{per } (x,y) \neq (0,0) \\ 0 & \text{per } (x,y) = (0,0) \end{cases}$$

1) deve risultare $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$ Allora è continua

2) Calcolo possiamo usare le coordinate polari o prendere la retta $y-0 = m(x-0)$ $\uparrow$ $y = mx$

$$\to \lim_{x\to 0} \frac{x\cdot mx}{x^2+(mx)^2} = \frac{mx^2}{x^2+m^2x^2} = \frac{\cancel{x^2}m^2}{\cancel{x^2}(1+m^2)} = \frac{m^2}{1+m^2}$$

Dipende solo da m $\to$ Non esiste

$\Rightarrow$ Se il lim non esiste la funzione non è continua in $O(0,0)$.

- $$\begin{cases} \frac{\ln(1+x^2+y^2)}{\sqrt{x^2+y^2}} & \text{per } (x,y) \neq (0,0) \\ 0 & \text{per } (x,y) = (0,0) \end{cases}$$

$\Rightarrow$ Calcolo lungo $y = mx$

Cordinate polari

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

$$\Rightarrow \lim_{\rho\to 0^+} \frac{\ln(1+\rho^2\cos^2\theta+\rho^2\sin^2\theta)}{\sqrt{\rho^2\cos^2\theta+\rho^2\sin^2\theta}} = \frac{\ln(1+\rho^2\underbrace{(\cancel{\cos^2\theta+\sin^2\theta})}_{1})}{\sqrt{\rho^2\underbrace{(\cancel{\cos^2\theta+\sin^2\theta})}_{1}}} = \frac{\ln(1+\rho^2)}{\rho}$$

$$\to \frac{\ln(1)}{0^+} \to \left[\frac{0}{0^+}\right]$$

Limite notevole

$$\lim_{\rho\to 0} \frac{\ln(1+\rho^2)}{\rho^2} = 1 \Rightarrow \lim_{\rho\to 0^+} \frac{\ln(1+\rho^2)\,\rho}{\rho^2} = \lim_{\rho\to 0^+} 1\cdot\rho = 0$$

$\Rightarrow$ coincide con $f(0,0) = 0$ $\to$ Continua.

- $$f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0$$

Considero lungo gli assi

$$\begin{cases} f(x,0) = \frac{x^2}{x^2} = 1 \\ f(0,y) = \frac{-y^2}{y^2} = -1 \end{cases}$$

Considerando le restrizioni $f(x,0)$ e $f(0,y)$ $\to$ $\lim_{x\to 0} f(x,0) = 1$ $\quad \lim_{y\to 0} f(0,y) \to -1$

Siccome $\lim_1 \neq \lim_2$, il lim in $(x,y) = (0,0)$ NON ESISTE $\Rightarrow$ NON CONTINUA

- $$f(x,y) = \begin{cases} \left(\frac{x^2y}{x^4+y^2}\right)^2 & \text{per } (x,y) \neq (0,0) \\ 0 & \text{per } (x,y) = (0,0) \end{cases}$$

Metodo della parametrizzazione, curva γ

scegliamo il cammino

$$\gamma: \begin{cases} x = t \\ y = t^2 \end{cases} \quad \lim_{t\to 0} f(t,t^2) = \left(\frac{t^2\,t^2}{t^4+t^4}\right)^2 = \left(\frac{\cancel{t^4}}{2\cancel{t^4}}\right)^2 = \frac{1}{4} \neq 0$$

Non è continua in $(0,0)$

Esercizio 6. Si risolva il seguente problema di Cauchy

$$\begin{cases} y' = \frac{e^y}{x^2+4x+5} \\ y(-2) = 0 \end{cases}$$

$$y' = \frac{e^y}{x^2+4x+5} \to y' \frac{1}{e^y} = (x^2+4x+5)^{-1} \to \int \frac{1}{e^y}\,dy = \underbrace{\int (x^2+4x+5)^{-1}\,dx}_{2)}$$

$$\to \int e^{-y}\,dy \quad t=-y \to dy = \frac{1}{-1}\,dt \to -\int e^t = -e^{-y} + c$$

Tempo ~ 15'

$$2) \int \frac{1}{x^2+4x+5}\,dx \to \Delta = 16-4\cdot 5 = -4 < 0 \to \text{NON RIDUCIBILE}$$

$$\Rightarrow \int \frac{1}{x^2+4x+4+1}\,dx = \int \frac{1}{(x+2)^2+1}\,dx \quad \text{pongo} \quad x+2 = t \to \int \frac{1}{t^2+1}\,dt$$

$$\int \frac{1}{a^2+f^2}\,dt = \frac{1}{a}\,\text{atan}\left(\frac{f}{a}\right) + c \Rightarrow \text{atan}(x+2) + c$$

$$\Rightarrow -e^{-y} = \text{atan}(x+2)+c \to e^{-y} = -\text{atan}(x+2)+c \to y = -\ln(-\text{atan}(x+2)+c)$$

$$y(-2) = -\ln(c - \cancel{\text{atan}}\,(0)) = 0 \to -\ln(c) = 0 \to -c = 0 \to c = 1$$

$$\Rightarrow y(x) = \text{atan}(x+2)+1$$

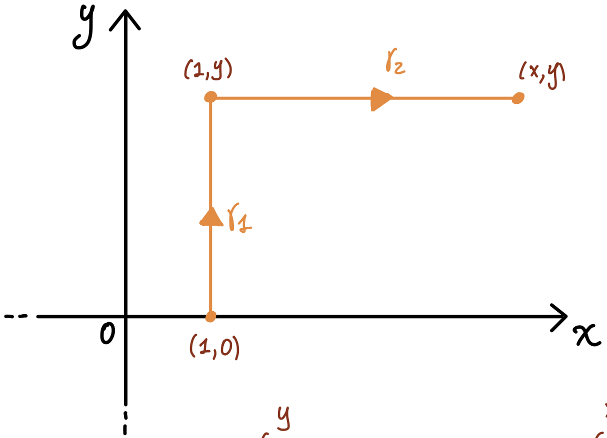
Esercizio 5. Si consideri la seguente forma differenziale

$$\omega = 2xy\,dx + \left(x^2 + y\sin(y)\right)dy$$

Dire se ω è chiusa.
E' anche esatta? Giustificare la risposta e, in caso affermativo, trovare una primitiva.

$$\omega = 2xy\,dx + x^2 + y\sin y\,dy$$

$$\left.\begin{array}{ll} X = 2xy & \to X'_y = 2x \\ Y = x^2 + y\sin y & \to Y'_x = 2x \end{array}\right\}\text{uguali} \to \text{F.D. Chiusa}, \quad A: \mathbb{R}^2 \Rightarrow \text{ESATTA}$$



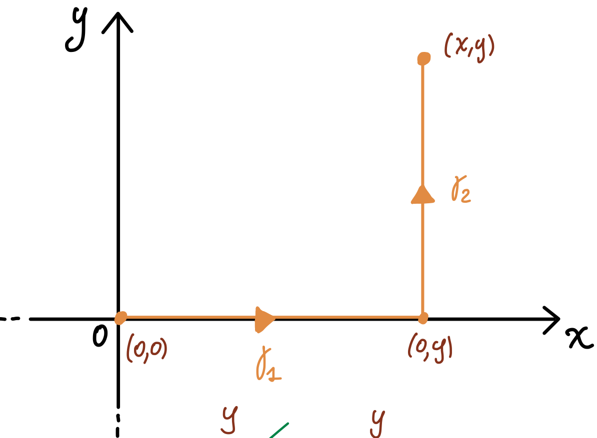
$$\gamma_1: \begin{cases} x = 1 \\ y = t \end{cases} \qquad 0 < t < y$$

$$\gamma_2: \begin{cases} x = t \\ y = y \end{cases} \qquad 1 < t < x$$

$$\Rightarrow F(x,y) = C + \int_0^y 1 + t\sin t\,dt + \int_1^x 2ty\,dt = C + \int_0^y dt + \int_0^y t\sin t\,dt + 2y\int_1^x t\,dt$$

$$= C + y + \left[-t\cos t + \int_0^y \cos t\,dt\right] + 2y\left[\frac{t^2}{2}\right]_1^x = C + y + \left[-t\cos t + \sin t\right]_0^y + yx^2 - y$$

$$= C + \cancel{y} - y\cos y + \sin y + yx^2 - \cancel{y} = C + \sin y - y\cos y + yx^2$$



$$\gamma_1: \begin{cases} x = t \\ y = 0 \end{cases} \qquad 0 < t < y$$

$$\gamma_2: \begin{cases} x = x \\ y = t \end{cases} \qquad 0 < t < y$$

$$F(x,y) = C + \int_0^y \cancel{0\,dt} + \int_0^y x^2 + t\sin t\,dt = C + x^2\int_0^y dt + \int_0^y t\sin t\,dt = C + x^2 y + \left[-t\cos t + \int_0^y \cos t\,dt\right]$$

$$= C + x^2 y + \left[-t\cos t + \sin t\right]_0^y = C + x^2 y - y\cos y + \sin y$$

Stesso risultato di prima

2 metodo

$w = 2xy\,dx + (x^2 + y\sin y)\,dy \qquad A: \mathbb{R}^2$

$X = 2xy \implies X'_y = 2x$

$Y = x^2 + y\sin y \qquad Y'_x = 2x$

$\Big\}$ UGUALI $\implies$ chiusa, in $\mathbb{R}^2 \implies$ anche ESATTA

$$\int 2xy\,dx = 2y\int x\,dx = 2y\,\frac{x^2}{2} + c(y) = \frac{1}{2}yx^2 + c(y)$$

$$\implies D_y = \frac{1}{2}x^2 + c'(y) = x^2 + y\sin y \implies c'(y) = x^2 + y\sin y - \frac{1}{2}x^2$$

$$c(y) = x^2\int dy + \int y\sin y - \frac{1}{2}x^2\int dy = \cancel{x^2y} - y\cos y + \sin y - \cancel{\frac{1}{2}x^2y} + K$$

$\frac{1}{2}x^2y$

$$= \frac{1}{2}x^2y - y\cos y + \sin y + K$$

$$\implies F(x,y) = \underbrace{\frac{1}{2}yx^2 + \frac{1}{2}x^2y}_{yx^2} - y\cos y + \sin y + K = yx^2 - y\cos y + \sin y + K$$

Bonus integro Y

$$x^2\int dy + \int y\sin y\,dy = x^2y - y\cos y + \sin y + c(x)$$

$$\implies D_x = \cancel{2xy} + c'(x) = \cancel{2xy} \implies c'(x) = 0 \implies c(x) = K$$

$$\implies F(x,y) = \underline{x^2y - y\cos y + \sin y + K}$$

Esercizio 4. Per quali valori del parametro α la funzione $f : \mathbb{R}^2 \to \mathbb{R}$ definita da

$$f(x,y) = \begin{cases} \frac{x|y|^\alpha}{x^2+y^2} & \text{se } (x,y) \neq (0,0), \\ 0 & \text{se } (x,y) = (0,0), \end{cases}$$

risulta continua?
Per quali valori di α risulta differenziabile?

Continuità:
Calcoliamo il valore della funzione lungo il fascio di rette passanti per 0: $y = mx$

$$\Rightarrow f(x,mx) = \frac{x|mx|^\alpha}{x^2+m^2x^2} \qquad \lim_{\substack{x \to 0 \\ \uparrow \\ x_0}} \frac{x|mx|^\alpha}{x^2+m^2x^2} = \begin{cases} \frac{x(mx)^\alpha}{x^2+m^2x^2} & \text{per } mx>0 \\ \frac{x(-mx)^\alpha}{x^2+m^2x^2} & \text{per } mx<0 \end{cases}$$

$$\to \frac{\cancel{x}(mx)^\alpha}{x^{\cancel{2}}(m^2+1)} = \frac{(mx)^\alpha}{x(m+1)} \Rightarrow$$

per $\alpha > 1$ $\lim \to 0$
per $\alpha = 1$ $\lim \to \frac{m}{m+1} \to$ Non continua
per $\alpha < 0$ $\lim \to \frac{1}{x(m+1)\cdot mx^\alpha} \to +\infty$

Quindi Affinché $f(x,y)$ sia continua in 0 $(0,0)$ $\Rightarrow$ $\underline{\alpha > 1}$

$\Rightarrow$ Dobbiamo dimostrare che f è continua:

$$|f(x,y)| \leq \frac{|x||y|^\alpha}{x^2+y^2} \leq \frac{|xy||y|^{\alpha-1}}{x^2+y^2} \leq \frac{1}{2}|y|^{\alpha-1}$$

$y^\alpha = y \cdot y^{\alpha-1}$

Altro metodo

$$\left.\begin{aligned} f(x,0) &= \frac{0}{x^2} = \frac{0}{0^+} \to 0 \\ f(0,y) &= \frac{0}{y^2} = \frac{0}{0^+} \to 0 \end{aligned}\right\} \quad f \text{ continua in } (x,y) = (0,0) \quad (\text{per } \alpha > 1)$$

lungo $y = mx \to \lim f(x,mx) \to 0$ per $\boxed{\alpha > 1}$ Come detto prima

Differenziabilità:

$$F(0,0) = \lim \frac{f(x,y) - \cancel{f(0,0)}^{0} - \cancel{f'_x(0,0)}^{0}(x-x_0) - \cancel{f'_y(0,0)}^{0}(y-y_0)}{\sqrt{(x-\underset{0}{x_0})^2+(y-\underset{0}{y_0})^2}} \Rightarrow F(0,0) = \lim_{(x,y)\to(0,0)} \frac{f(x,y)}{\sqrt{x^2+y^2}} = 0$$

Se l'eq è soddisfatta la F è Diff.

$$\lim_{(x,y)\to(0,0)} \frac{\frac{x|y|^\alpha}{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim_{x \to 0} \frac{\frac{x|mx|^\alpha}{x^2+m^2x^2}}{\sqrt{x^2+m^2x^2}} \begin{cases} \frac{\frac{m^\alpha x^{\alpha+1}}{x^2+m^2x^2}}{\sqrt{x^2+m^2x^2}} & mx>0 \\ \frac{\frac{-m^\alpha x^{\alpha+1}}{x^2+m^2x^2}}{\sqrt{x^2+m^2x^2}} & mx<0 \end{cases}$$

$y = mx$
$\Rightarrow$ Se $(x,y) \to (0,0)$
$\Rightarrow (x,mx) \to 0$

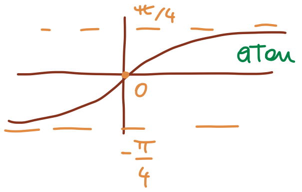
$$\Rightarrow \frac{\frac{m^\alpha x^{\alpha+1}}{x^2(1+m^2)}}{\sqrt{x^2(m^2+1)}} = \frac{\frac{m^\alpha x^{\alpha+\cancel{1}}}{\cancel{x^2}(1+m^2)}}{\cancel{x}\sqrt{1+m^2}} = \frac{m^\alpha x^\alpha}{(1+m^2)\sqrt{1+m^2}} \cdot \frac{\sqrt{1+m^2}}{\sqrt{1+m^2}} = \frac{m^\alpha x^\alpha \sqrt{1+m^2}}{(1+m^2)^2}$$

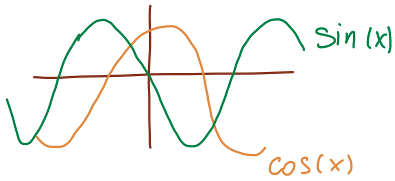
$\alpha > 0$ $\lim \to 0$
$\alpha < 0$ $\lim \to +\infty$

$$\text{lungo } x \to \lim_{(x,0)\to 0} \frac{\frac{x|0|^\alpha}{x^2}}{\sqrt{x^2}} = 0$$

Esercizio 1. Calcolare

$$\lim_{x\to 0} \frac{\left(\arctan(e^{2x}-1)\right)^2}{\cos(\sin(x))-1}$$





$$\lim_{x\to 0} \frac{\left[\operatorname{atan}(e^{2x}-1)\right]^2}{\cos(\sin(x))-1} = \left[\frac{0}{0}\right]$$

$$D\left[\operatorname{atan}(e^{2x}-1)\right]^2 = \frac{2\operatorname{atan}(e^{2x}-1)\cdot 2e^{2x}}{1+(e^{2x}-1)^2} = \frac{2e^{2x}\operatorname{atan}(e^{2x}-1)}{1+e^{4x}-2e^{2x}+1}$$

$$D\left[\cos(\sin(x))-1\right] = -\sin(\sin(x))\cdot\cos(x)$$

$$\overset{\text{Hô}}{\Longrightarrow} \lim_{x\to 0} \frac{\dfrac{2e^{2x}\operatorname{atan}(e^{2x}-1)}{e^{4x}-2e^{2x}+2}}{-\sin(\sin(x))\cos(x)} = -\frac{4e^{2x}\operatorname{atan}(e^{2x}-1)}{(e^{4x}-2e^{2x}+2)\sin(\sin(x))\cos(x)}$$

$$= -4\lim_{x\to 0} \frac{e^{2x}}{(e^{4x}-2e^{2x}+2)\cos x} \cdot \frac{\operatorname{atan}(e^{2x}-1)}{\sin(\sin(x))} \to \left[\frac{0}{0}\right]$$ Applico dinuovo De Hôpital

$$\frac{1}{(1-2+2)\cdot 1} \to 1$$

$$\Longrightarrow D_x\left[\operatorname{atan}(e^{2x}-1)\right] = \frac{2e^{2x}}{1+(e^{2x}-1)^2} = \frac{2e^{2x}}{1+e^{4x}-2e^{2x}+1} = \frac{2e^{2x}}{2+e^{4x}-2e^{2x}}$$

$$D_x\left[\sin(\sin(x))\right] = \cos(\sin(x))\cos(x)$$

$$\Longrightarrow -4\cdot\lim_{x\to 0} \frac{\dfrac{2e^{2x}}{2+e^{4x}-2e^{2x}}}{\cos(\sin(x))\cos(x)} = \frac{2e^{\cancel{2x}\to 1}}{(2+e^{4x}-2e^{2x})(\cos(\sin(x))\cos(x))}$$

$2+1-2 \qquad \cos(0)\cdot\cos(0) \to 1$

$$= -4\lim_{x\to 0} 2 = -8$$