

Limiti - Esercizi

Verificare i limiti $\rightarrow +\infty$

$$1) \lim_{x \rightarrow 0^+} \sqrt{\frac{1}{x}} = \sqrt{\frac{1}{0^+}} = +\infty \quad \checkmark$$

$$2) \lim_{x \rightarrow 7^+} \frac{1}{(x-7)^2} = \left\{ \begin{array}{l} \lim_{x \rightarrow 7^+} \frac{1}{(7^+-7)^2} = \frac{1}{(0^+)^2} = +\infty \\ \lim_{x \rightarrow 7^-} \frac{1}{(7^--7)^2} = \frac{1}{(0^-)^2} = +\infty \end{array} \right\} \lim_{x \rightarrow 7^+} f(x) = +\infty \quad \checkmark$$

$$3) \lim_{x \rightarrow 0^+} e^{\frac{2}{x}} = \lim_{x \rightarrow 0^+} \frac{2}{0^+} = +\infty \Rightarrow e^{+\infty} = +\infty \quad \checkmark$$

$$193) \lim_{x \rightarrow 0^+} (-\ln x) = -(-\infty) = +\infty \quad \checkmark$$

$$194) \lim_{x \rightarrow 0} \ln\left(\frac{1}{x^2}\right) = \lim_{x \rightarrow 0} \frac{1}{x^2} = \left\{ \begin{array}{l} \lim_{x \rightarrow 0^+} \frac{1}{(0^+)^2} = +\infty \\ \lim_{x \rightarrow 0^-} \frac{1}{(0^-)^2} = +\infty \end{array} \right\} = +\infty \Rightarrow \lim_{x \rightarrow 0} \ln(+\infty) = +\infty \quad \checkmark$$

$$195) \lim_{x \rightarrow \frac{1}{2}} \frac{1}{(2x-1)^2} = \left\{ \begin{array}{l} \lim_{x \rightarrow \frac{1}{2}^+} \frac{1}{(2\frac{1}{2}^+-1)^2} = +\infty \\ \lim_{x \rightarrow \frac{1}{2}^-} \frac{1}{(2\frac{1}{2}^--1)^2} = +\infty \end{array} \right\} = +\infty \quad \checkmark$$

$$196) \lim_{x \rightarrow 2^+} \frac{1}{x^2-4} = \frac{1}{4^+-4} = \frac{1}{0^+} = +\infty$$

$$201) \lim_{x \rightarrow 2^+} \frac{1}{x-2} + 1 = \frac{1}{0^+} + 1 = +\infty$$

$$205) \lim_{x \rightarrow 0^-} \frac{5+2x}{-x} = \frac{5+0^-}{-(0^-)} = \frac{5}{0^+} = +\infty$$

$$210) \lim_{x \rightarrow 1^+} \log\left(\frac{2}{x+1}\right) = \log\left(\frac{2}{2}\right) = \log 1 = +\infty$$

Verificare i limiti $\rightarrow -\infty$

$$212) \lim_{x \rightarrow 0} -\frac{1}{3x^4} = \left\{ \begin{array}{l} \lim_{0^-} -\frac{1}{3 \cdot 0^+} = -\infty \\ \lim_{0^+} -\frac{1}{3 \cdot 0^+} = -\infty \end{array} \right\} = -\infty \quad \checkmark$$

$$213) \lim_{x \rightarrow \frac{3}{2}^-} \frac{1}{4x^2-9} = \frac{1}{4\left(\frac{3}{2}\right)^2-9} = +\infty ?$$

$$214) \lim_{x \rightarrow -1} \frac{-1}{x^2+2x+1} = \left\{ \begin{array}{l} \lim_{-1^+} \frac{-1}{1^+-2^++1} = -\frac{1}{0^+} = -\infty \\ \lim_{-1^-} \frac{-1}{1^--2^-+1} = -\frac{1}{0^-} = +\infty ? \end{array} \right.$$

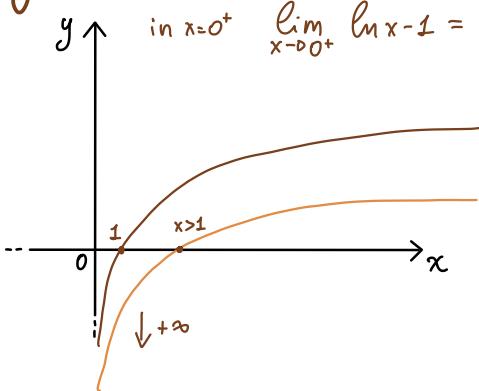
$$215) \lim_{x \rightarrow 1^-} \log_2(1-x) = \log_2(1-1^-) = \log_2(0^-) = -\infty \quad \checkmark$$

$$216) \lim_{x \rightarrow 3^-} \frac{1}{x-3} = \frac{1}{3^-3} = -\infty \quad \checkmark$$

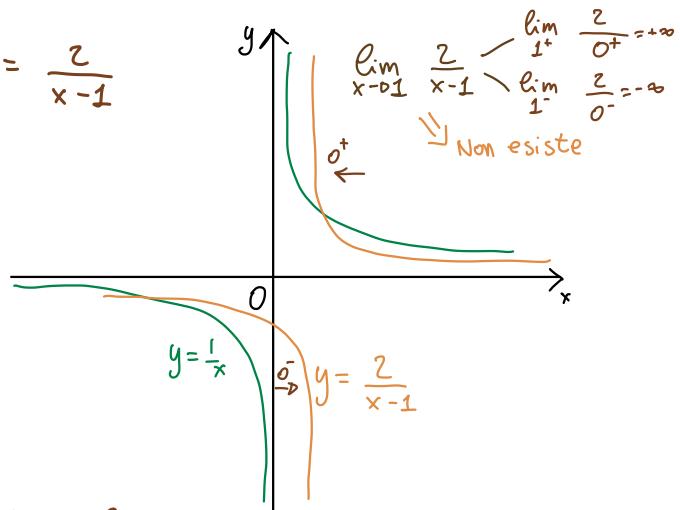
$$217) \lim_{x \rightarrow 0} \frac{-1}{|x|} = \left\{ \begin{array}{l} \lim_{x \rightarrow 0^+} -\frac{1}{|0^+|} = -\infty \\ \lim_{x \rightarrow 0^-} -\frac{1}{|0^-|} = -\infty \end{array} \right\} = -\infty \quad \checkmark$$

Rappresentazione grafica

$$f = \ln x - 1$$



$$f = \frac{2}{x-1}$$



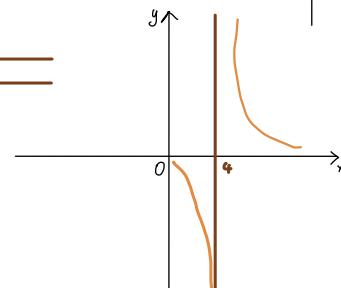
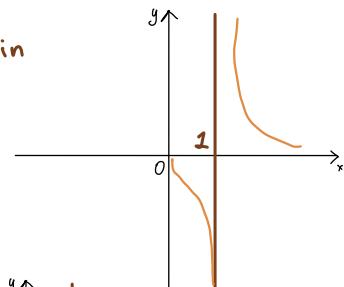
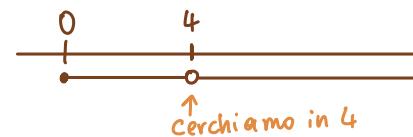
Asintoti verticali

$$249) \quad y = \frac{2}{(x-1)^2} \quad \text{in } x=1 \quad \lim_{x \rightarrow 1} \frac{2}{(x-1)^2} \quad \left. \begin{array}{l} 1^+ = \frac{2}{0^+} = +\infty \\ 1^- = \frac{2}{0^+} = +\infty \end{array} \right\} \Rightarrow \text{Asintoto Vert in } x=1$$

$$250) \quad y = \frac{1}{\ln x} \quad \text{in } x=1 \quad \lim_{x \rightarrow 1} \frac{1}{\ln x} = \left. \begin{array}{l} 1^+ = \frac{1}{0^+} = +\infty \\ 1^- = \frac{1}{0^-} = -\infty \end{array} \right\} \text{Asintoto in } x=1$$

$$251) \quad y = \frac{2}{\sqrt{x}-2} \quad \text{in } x=4 \quad \mathbb{D} = \begin{array}{l} \sqrt{x} \\ x \geq 0 \end{array} \quad \left. \begin{array}{l} \sqrt{x}-2 \neq 0 \text{ per } (\sqrt{x})^2 \neq 2^2 = 0 \\ x \neq 4 \end{array} \right\}$$

$$\lim_{x \rightarrow 4} f(x) \quad \left. \begin{array}{l} 4^+ = \frac{2}{0^+} = +\infty \\ 4^- = \frac{2}{0^-} = -\infty \end{array} \right\} \text{A.V. in } x=4$$



$$252) \quad y = \frac{x-3}{x^2-x} \quad \mathbb{D} = x^2-x \neq 0 \quad \text{per } x(x-1) \neq 0 \quad \left. \begin{array}{l} \hookrightarrow x \neq 0 \\ \hookrightarrow x \neq 1 \end{array} \right\} \dots$$

$$\lim_{x \rightarrow 0} f(x) \quad \left. \begin{array}{l} 0^+ = \frac{-3}{0^+} = -\infty \\ 0^- = \frac{-3}{0^-} = +\infty \end{array} \right.$$



$$253) \quad y = 1 + e^{\frac{1}{x}} \quad \text{in } x=0^+ \quad \lim_{x \rightarrow 0^+} 1 + e^{\frac{1}{x}} = 1 + e^{\frac{1}{0^+}} = +\infty$$

Limiti tendenti ad e

$$264) \lim_{x \rightarrow +\infty} \frac{2}{x+10} = 0 \quad \frac{\frac{2}{x(1+\frac{10}{x})}}{\downarrow 0} = \frac{2}{\infty} = 0 \checkmark$$

$$265) \lim_{x \rightarrow +\infty} \frac{4x-1}{2x+1} = \frac{x(4-\frac{1}{x})}{x(2+\frac{1}{x})} \stackrel{\downarrow 0}{=} \frac{4}{2} = 2 \checkmark$$

$$269) \lim_{x \rightarrow +\infty} \frac{2^x - 1}{2^x} = ?$$

$$270) \lim_{x \rightarrow +\infty} \frac{-3x}{|x|+1} = \frac{x(-3)}{\cancel{x}+1} = \frac{\cancel{x}(-3)}{\cancel{x}(1+\frac{1}{x})} \stackrel{\downarrow 0}{=} \frac{-3}{1} = -3$$

$$271) \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 1} - x) = \text{Siccome Siamo in } +\infty \quad \stackrel{\downarrow 0}{=} (x^2)^{\frac{1}{2}} - 1^{\frac{1}{2}} - x = x^{\frac{1}{2}} - x^{\frac{1}{2}} - x = x^{\frac{1}{2}} - x^{\frac{1}{2}} - x = x^{\frac{1}{2} - \frac{1}{2} - 1} = \frac{x^{-1/2}}{2} = x^{-2} = \frac{1}{x^2}$$

$$272) \lim_{x \rightarrow \infty} \ln\left(\frac{x}{x-1}\right) = \lim_{x \rightarrow \infty} \frac{x}{x-1} = \frac{\cancel{x}}{\cancel{x}(1-\frac{1}{x})} \stackrel{\downarrow 0}{=} \frac{1}{1} = 1 \Rightarrow \lim_{x \rightarrow \infty} \ln(1) = 0 \checkmark$$

$$273) \lim_{x \rightarrow \infty} \frac{x}{x^2 - 1} = \frac{x}{x^2(1 - \frac{1}{x^2})} \stackrel{\downarrow 0}{=} \frac{1}{+\infty} = 0 \checkmark$$

$$274) \lim_{x \rightarrow \infty} \left(\frac{1}{2}\right)^{2x} = \text{All'aumentare dell'esponente aumenta il denominatore} \Rightarrow \frac{1}{\infty} = 0 \checkmark$$

$$275) \lim_{x \rightarrow +\infty} \left[\left(\frac{1}{3} \right)^{x+1} + 1 \right] = 0 + 1 = 1 \checkmark$$

Stesso di prima

$$277) \lim_{x \rightarrow -\infty} \frac{2}{2x+1} = -\frac{2}{\infty} = 0 \checkmark$$

$$278) \lim_{x \rightarrow -\infty} \frac{x^3 + 1}{2x^3} = \frac{x^3(1+0)}{2} \stackrel{\downarrow 0}{=} \frac{1}{2} \checkmark$$

$$279) \lim_{x \rightarrow -\infty} \frac{3x+1}{1-2x} = \frac{x(3+\frac{1}{x})}{x(\frac{1}{x}-2)} \stackrel{\downarrow 0}{=} -\frac{3}{2} \checkmark$$

$$283) \lim_{x \rightarrow -\infty} \frac{-1}{e^{|x|}} = \frac{-1}{e^{-x}} = \frac{-1}{e^{+\infty}} = 0 \checkmark$$

$$284) \lim_{x \rightarrow -\infty} 2e^{-4x^2} = \lim_{x \rightarrow -\infty} -4x^2 = +\infty \Rightarrow \lim_{x \rightarrow -\infty} 2e^{+\infty} =$$

Siamo in $-\infty \Rightarrow |x| = -x$

Limiti vari da Elia Bomb.

1) $\lim_{x \rightarrow 0} \frac{x^3}{\tan x - \sin x}$

a) limiti notevoli

$$\frac{x^3}{\tan x - \sin x} = \frac{x^3}{\sin x \left(\frac{1}{\cos x} - 1 \right)} = \frac{x^3}{\sin x \cdot \frac{1 - \cos x}{\cos x}} = \frac{\cos x}{\frac{\sin x}{x} \cdot \frac{1 - \cos x}{x^2}} = 2 \cos x = 2$$

$\tan x = \frac{\sin x}{\cos x}$

b) Sviluppi di Taylor

$$\bullet \tan x = x + \frac{x^3}{3} + o(x^3) \quad \bullet \sin x = x - \frac{x^3}{6} + o(x^3)$$

$$\Rightarrow x + \cancel{\frac{x^3}{3}} + o(x^3) - \left[x - \cancel{\frac{x^3}{6}} + o(x^3) \right] = \frac{x^3}{2} + o(x^3) = \lim_{x \rightarrow 0} \frac{\frac{x^3}{2} + o(x^3)}{\frac{x^3}{2} + o(x^3)} = 2$$

Esercizi serie numeriche

ES: $\sum_{n=1}^{+\infty} \left(-\frac{2}{3}\right)^n$; $-\frac{2}{3} = -1 < -0,6 < 1 \Rightarrow$ converge $\Rightarrow \sum_{n=1}^{+\infty} \left(-\frac{2}{3}\right)^n =$ Siccome n parte da 1 e non da 0, dobbiamo sottrarre un elemento dalla somma:
 $\Rightarrow \frac{1}{1-q} - q^0 = \frac{1}{1-\left(-\frac{2}{3}\right)} - \left(-\frac{2}{3}\right)^0 = -\frac{1}{\frac{5}{3}} - 1 = \frac{3}{5} - 1 = \frac{3-5}{5} = -\frac{2}{5}$

ES $\sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^{n+1}$ $q = \frac{1}{2} \Rightarrow -1 < q < 1 \Rightarrow$ Siccome l'esp = $n+1$ e non n , riscriviamo $= \frac{1}{2} \cdot \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{2} \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n$
 $\Rightarrow \text{Sum} = \frac{1}{2 \cdot \left(\frac{1}{2}\right)} = \frac{1}{\frac{2-1}{2}} = \frac{1}{\frac{1}{2}} = 2 \Rightarrow \frac{1}{2} \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} \cdot 2 = 1$

ES: $\sum_{n=0}^{+\infty} \left(\frac{4}{7}\right)^n$ $-1 < q < 1 \Rightarrow \frac{1}{1-\frac{4}{7}} = \frac{1}{\frac{3}{7}} = \frac{7}{3}$ convergente

ES: $\sum_{n=0}^{+\infty} \left(\frac{2}{5}\right)^{2n} = -1 < q < 1 = \left[\left(\frac{2}{5}\right)^2\right]^n = \left[\frac{4}{25}\right]^n \Rightarrow \frac{1}{1-\frac{4}{25}} = \frac{25}{21}$

ES: $\sum_{n=1}^{+\infty} \left(\frac{3}{4}\right)^n$ $-1 < q < 1 \Rightarrow \frac{1}{1-\frac{3}{4}} - \left(\frac{3}{4}\right)^0 = \frac{4}{2} - 1 = \frac{4-2}{2} = 1$

ES: $\sum_{n=1}^{+\infty} \left(\frac{3}{5}\right)^{n-1}$ $-1 < q < 1 \Rightarrow \left(\frac{3}{5}\right)^0 \cdot \left(\frac{3}{5}\right)^{-1} \Rightarrow \left(\frac{3}{5}\right)^{-1} \sum_{n=1}^{+\infty} \left(\frac{3}{5}\right)^n = \left(\frac{3}{5}\right)^{-1} \cdot \frac{1}{1-\frac{3}{5}} - 1 =$
 $= \frac{5}{2} \cdot \left(\frac{3}{5}\right)^{-1} - 1 = \frac{3^{-1}}{5^{-1}} \cdot \frac{5}{2} - 1 = \frac{19}{6}$

ES: $\sum_{n=1}^{+\infty} (-1)^{n-1} \left(\frac{3}{5}\right)^{n-1} =$

Somme parziali

ES $\sum_{n=4}^{+\infty} \left(\frac{1}{n+1} - \frac{1}{n-3}\right)$

$S_4 = \frac{1}{5} - 1$

$S_5 = \frac{1}{6} - \frac{1}{2} + \frac{1}{5} - 1$

$S_6 = \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \frac{1}{2} + \frac{1}{5} - 1$

$S_7 = \frac{1}{8} - \frac{1}{4} + \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \frac{1}{2} + \frac{1}{5} - 1$

$S_8 = \frac{1}{9} - \cancel{\frac{1}{5}} + \frac{1}{8} - \frac{1}{4} + \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{5}} - 1$

$S_9 = \frac{1}{10} - \cancel{\frac{1}{6}} + \frac{1}{9} - \cancel{\frac{1}{5}} + \frac{1}{8} - \frac{1}{4} + \frac{1}{7} - \frac{1}{3} + \frac{1}{6} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{5}} - 1$

$S_{10} = \frac{1}{11} - \cancel{\frac{1}{7}} + \frac{1}{10} + \frac{1}{9} + \frac{1}{8} - \frac{1}{4} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{3}} - \cancel{\frac{1}{2}} - 1$

$S_{11} = \frac{1}{12} - \cancel{\frac{1}{8}} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{4}} - \cancel{\frac{1}{3}} - \cancel{\frac{1}{2}} - 1$

costanti: $-\frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1$

In ogni momento sono presenti

$$\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \frac{1}{n-3}$$

Quindi $S_n = \frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \frac{1}{n-3} - \frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1$

Per calcolare S_n con $n=\infty$ ci basta calcolare il limite per $x \rightarrow +\infty$ di S_n :

$$\lim_{n \rightarrow +\infty} \frac{\cancel{\frac{1}{n+1}} + \cancel{\frac{1}{n}} + \cancel{\frac{1}{n-1}} + \cancel{\frac{1}{n-2}} + \cancel{\frac{1}{n-3}} - \frac{1}{4} - \frac{1}{3} - \frac{1}{2} - 1}{0 + 0 + 0 + 0 + 0} = \frac{-3-4-6-12}{12} = -\frac{25}{12}$$

da qui in poi vengono eliminati

$$ES\ 6.2 \quad a + ax + ax^2 + ax^3 + \dots = \sum_{n=1}^{+\infty} a x^{n-1}$$

Se $x > 0 \Rightarrow \lim_{n \rightarrow +\infty} a x^{n-1} = +\infty$

$$1 + 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} \dots$$

Differisce di 2

$$1) \sum_{n=1}^{+\infty} \frac{2}{n^2 + 2n} = \frac{2}{n(n+2)} \stackrel{\text{oppure}}{=} \frac{2}{n} - \frac{2}{(n+2)} \stackrel{\text{test}}{=} \frac{2(n+2) - 2n}{n(n+2)} = \frac{2n+4 - 2n}{n(n+2)} = \frac{4}{n(n+2)}$$

Quindi: Calcoliamo le ridotte

$$S_1 = 1 - \frac{1}{3}$$

$$S_2 = \frac{1}{2} - \frac{1}{4} + 1 - \frac{1}{3}$$

$$S_3 = \frac{1}{3} - \frac{1}{5} + \frac{1}{2} - \frac{1}{4} + 1 - \frac{1}{3}$$

$$S_4 = \frac{1}{4} - \frac{1}{6} + \cancel{\frac{1}{3}} - \frac{1}{5} + \frac{1}{2} - \frac{1}{4} + 1 - \cancel{\frac{1}{3}}$$

$$S_5 = \cancel{\frac{1}{5}} - \frac{1}{7} + \frac{1}{4} - \frac{1}{6} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \frac{1}{2} - \frac{1}{4} + 1 - \cancel{\frac{1}{3}}$$

$$S_6 = \cancel{\frac{1}{6}} - \frac{1}{8} - \frac{1}{7} + \frac{1}{4} - \cancel{\frac{1}{6}} + \frac{1}{2} - \frac{1}{4} + 1$$

$$S_7 = \cancel{\frac{1}{7}} - \frac{1}{9} - \frac{1}{8} - \cancel{\frac{1}{7}} + \frac{1}{4} + \frac{1}{2} - \frac{1}{4} + 1$$

$$S_8 = \frac{1}{8} - \frac{1}{10} - \frac{1}{9} - \frac{1}{8} + \cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{4}} + 1$$

Variano

Costanti

$$S_n = -\frac{1}{n+2} - \frac{1}{n+1} + \frac{1}{2} + 1$$

Per calcolare S_n per $n = \infty$ risolviamo il lim

$$\lim_{n \rightarrow +\infty} S_n = \cancel{-\frac{1}{n+2}} - \cancel{-\frac{1}{n+1}} + \frac{1}{2} + 1 = \frac{3}{2}$$

$$ES\ 2: \sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2 + n}} = \frac{\sqrt{n+1}}{\sqrt{n} \sqrt{n+1}} - \frac{\sqrt{n}}{\sqrt{n} \sqrt{n+1}} = \frac{1}{\sqrt{n}} - \frac{\sqrt{n}}{\sqrt{n} \sqrt{n+1}}$$

$$S_1 = 1 - \frac{1}{1 \cdot \sqrt{2}}$$

$$S_2 = \frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2} \sqrt{3}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + 1 - \cancel{\frac{1}{\sqrt{2}}}$$

$$S_3 = \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{3} \sqrt{2}} = \frac{1}{\sqrt{3}} - 2 - \cancel{\frac{1}{\sqrt{3}}} + 1$$

$$S_4 = \frac{1}{2} - \frac{2}{2\sqrt{5}} = \frac{1}{2} - \frac{1}{\sqrt{5}} - 2 + 1$$

$$S_5 = \frac{1}{\sqrt{5}} - \frac{\sqrt{5}}{\sqrt{5} \sqrt{6}} = \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}} + \frac{1}{2} - \cancel{\frac{1}{\sqrt{5}}} - 2 + 1$$

$$S_6 = \frac{1}{\sqrt{6}} - \frac{\sqrt{6}}{\sqrt{6} \sqrt{7}} = \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{7}} - \cancel{\frac{1}{\sqrt{6}}} + \frac{1}{2} - 2 + 1$$

$$S_n = -\frac{1}{\sqrt{n+1}} + 1$$

$$\lim_{n \rightarrow +\infty} -\frac{1}{\sqrt{n+1}} + 1 = 1 \quad \text{Converge}$$

$$\text{ES: } \sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{2n}} = \sum_{n=1}^{\infty} \frac{4^{n+1}}{9^n} = \frac{4^n}{9^n} \cdot 4 = 4 \left(\frac{4}{9} \right)^n$$

Ragione

$$= 4 \cdot \left(\frac{1}{1 - \frac{4}{9}} - 1 \right) = 4 \left(\frac{4}{5} \right) = \frac{16}{5}$$

$$\text{ES: } \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad \text{Test: } \frac{n+1-n}{n(n+1)} \quad \checkmark$$

$$\begin{aligned} S_1 &= 1 - \frac{1}{2} \\ S_2 &= \cancel{\frac{1}{2}} - \frac{1}{3} + 1 - \cancel{\frac{1}{2}} \\ S_3 &= \cancel{\frac{1}{3}} - \frac{1}{4} - \cancel{\frac{1}{3}} + 1 \quad \text{cost} \\ &\quad \text{Variaz.} \end{aligned}$$

$$S_n = \frac{1}{n+1} + 1$$

$$\lim_{n \rightarrow +\infty} \frac{1}{n+1} + 1 = 1$$

~~6~~ Verificare che la serie $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots$ è divergente.

Verificare che la serie $1+2+3+\dots+n+\dots$ è divergente.

Verificare che è indeterminata la serie:

$$-\sum_{n=1}^{\infty} (-1)^n = -1 + 1 - 1 + 1 - 1 + \dots$$

Calcolare la somma delle seguenti serie geometriche

(a) $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

(c) $\sum_{n=0}^{\infty} 3^n$

(d) $\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \sum_{n=1}^{\infty} \frac{1}{2n}$$

Questa serie è di tipo Armonico, quindi converge per $d > 1$, siccome $n^{\frac{1}{2n}} \rightarrow 1 \leq 1$ la serie diverge.

$$1+2+3+\dots+n = \sum_{n=1}^{\infty} n \quad \lim_{n \rightarrow \infty} n = +\infty$$

Per n pari: $\sum (-1)^n = +1$
 Per n dispari: $\sum (-1)^n = -1$

\downarrow
Indeterminata

a) $\sum_{n=0}^{\infty} \frac{1}{2^n} =$

