

Esercizi da Matematica Blu

$$f(x) = \frac{x^2 - 2}{x + 1} \quad \text{in } c = -2$$

$$= \frac{2x \cdot (x+1) - (x^2 - 2) \cdot 1}{(x+1)^2} = \frac{2x^2 + 2x - x^2 - 2}{(x+1)^2}$$

$$= \frac{x^2 + 2x - 2}{(x+1)^2}$$

$$f(-2) = \frac{(-2)^2 + 2(-2) - 2}{(-2+1)^2}$$

$$= \frac{4 + 4 - 2}{1} = 6$$

33) $f(x) = x^3 + 4x + 1$ con $c = 1$

$$f'(x) = 3x^2 + 4 \quad f'(c) = 3 + 4 = 7$$

$$f(x) = -\frac{5}{x} \quad \text{con } c = 2$$

$$f'(x) = -5 \cdot x^{-1} = -5 \cdot (-1) x^{-2} = \frac{5}{x^2}$$

$$f(c) = \frac{5}{2^2} = \frac{5}{4}$$

34) $f(x) = 1 + \sqrt{x}$ in $c = 4$

$$1 + \sqrt{x} = 1 + x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f(c) = 4$$

$$f(x) = 2x^3 - x \quad \text{in } c = 0$$

$$f'(x) = 6x^2 - 1 \quad f(c) = -1$$

35) $f = x^2 - 1$ in $c = 3$

$$f' = 2x \quad f(3) = 6$$

$$f = \frac{x+3}{x-4} \quad \text{in } c = -1$$

$$f' = \frac{x-4 - x-3}{(x-4)^2} = \frac{-7}{(x-4)^2}$$

$$f^c = \frac{-7}{(-1-4)^2} = \frac{-7}{25} = -\frac{7}{25}$$

36)

$$f = \frac{x^2-1}{2-x} \quad \text{in } c = 1$$

$$f' = \frac{2x(2-x) - (x^2-1) \cdot (-1)}{(2-x)^2} = \frac{4x - 2x^2 - (-x^2+1)}{(2-x)^2}$$

$$= \frac{4x - 2x^2 + x^2 + 1}{(2-x)^2} = \frac{-x^2 + 4x + 1}{(2-x)^2} \quad f^c = \frac{1+4+1}{1} = 6$$

$$f = \frac{1}{2}x^2 - 2x \quad \text{in } c = -3$$

$$f' = x - 2 \quad f^c = -5$$

37) $f = 2x - 1 \quad \text{in } c = 6 \quad f' = 2$

$$f = \frac{3}{x-1} \quad \text{in } c = 4 \quad f^c = -\frac{3}{(x-1)^2} \quad f^4 = -\frac{3}{9} = -\frac{1}{3}$$

38) $f = \frac{1}{1-x^2} \quad \text{in } c = -2$

$$f' = \frac{-(-2x)}{(1-x^2)^2} = \frac{2x}{(1-x^2)^2} \quad f^{-2} = \frac{-4}{9} = -\frac{4}{9}$$

$$f = \frac{1}{\sqrt{x-1}} \quad \text{in } c = 5$$

$$f = \frac{1}{(x-1)^{\frac{1}{2}}} = (x-1)^{-\frac{1}{2}}$$

$$f' = -\frac{1}{2} (x-1)^{-\frac{1}{2}-1} = -\frac{1}{2} (x-1)^{-\frac{3}{2}}$$

$$= -\frac{1}{2} (x-1)^{-\frac{3}{2}} = -\frac{1}{2 (x-1)^{\frac{3}{2}}}$$

$$\Rightarrow f(5) = \frac{1}{2 \sqrt{(5-1)^3}} = \frac{1}{16}$$

$$39) f = \frac{x-1}{x} \quad f' = \frac{x - x-1}{x^2} = -\frac{1}{x^2}$$

$$\text{in } c=2 \Rightarrow f' = -\frac{1}{4}$$

$$f = \frac{2-x^2}{1-x^2} \quad \text{in } c=0 \quad f' = \frac{-2x(1-x^2) - (2-x^2) \cdot -2x}{(1-x^2)^2}$$

$$= \frac{-2x + \cancel{2x^3} + 4x - \cancel{2x^3}}{(1-x^2)^2} = \frac{2x}{(1-x^2)^2} \quad f' = \frac{0}{1} = 0$$

$$40) f = -\frac{1}{\sqrt{x}} = -x^{-\frac{1}{2}}$$

$$\frac{d}{dx} \left(\frac{1}{f} \right) = -\frac{\frac{d}{dx}(f)}{f^2}$$

$$f' = \frac{1}{2} \cdot x^{-\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x^3}} \quad f'' = \frac{1}{54}$$

$$f = \frac{4-x^2}{x^2-2x+2} \quad \text{in } c=-2$$

$$f' = \frac{-2x(x^2-2x+2) - (4-x^2)(2x-2)}{(x^2-2x+2)^2}$$

$$= \frac{-\cancel{2x^3} + \cancel{4x^2} - 4x - 8x + 8 + \cancel{2x^3} - \cancel{2x^2}}{(x^2-2x+2)^2}$$

$$= \frac{2x^2 - 12x + 8}{(x^2-2x+2)^2}$$

$$f' = \frac{2(-2)^2 - 12(-2) + 8}{((-2)^2 - 2(-2) + 2)^2}$$

$$= \frac{8 + 24 + 8}{(4 + 4 + 2)^2} = \frac{40}{100} = \frac{2}{5}$$

$$41) f(x) = -2 \ln x \quad f' = -\frac{2}{x} \quad f(1) = -2$$

$$f = e^{x-1} \quad inc = 1 \quad f' = e^{x-1} \cdot 1 = e^{x-1} \quad \text{Funzione composta}$$

$$f(1) = e^0 = 1$$

$$42) f = \tan x \quad f' = \sec^2 x \quad f'(0) =$$

$$f(x) = 2^{-2x} \quad \frac{d}{dx} a^x = \ln(a) \cdot a^x \Rightarrow \ln(2) \cdot 2^{-2x} \cdot \frac{d}{dx} (-2x)$$

$$= \ln(2) \cdot 2^{-2x} \cdot (-2) = -2 \ln(2) \cdot 2^{-2x}$$

$$f'(0) = -2 \ln 2 \cdot 1 = -2 \ln 2$$

Calcolare la derivata in un punto generico c

$$f = \frac{1}{2}x^2 - 4x \quad f' = x - 4$$

$$50) f = \frac{2}{x} = 2 \cdot x^{-1} \Rightarrow f' = -\frac{2}{x^2} \checkmark$$

$$51) f = 4x - 9 \quad f' = 4 \checkmark$$

$$52) f = \frac{1}{2}\sqrt{x} = \frac{1}{2}x^{\frac{1}{2}} \quad f' = \left(\frac{1}{2}\right)^2 \cdot x^{-\frac{1}{2}} = \frac{1}{4\sqrt{x}} \checkmark$$

$$53) f = 2x^3 - x = 6x^2 - 1 \checkmark$$

$$54) f = \frac{1}{x^2 - 1} \quad \frac{d}{dx} \frac{1}{f} = -\frac{f'}{f^2}$$

$$\Rightarrow f' = -\frac{2x}{(x^2 - 1)^2} \checkmark$$

$$55) f = -x^2 + 4x \quad f' = -2x + 4 \checkmark$$

$$56) f = \frac{x+1}{x} \quad f' = \frac{1(x) - (x+1) \cdot 1}{x^2} = \frac{x - x - 1}{x^2} = -\frac{1}{x^2} \checkmark$$

$$57) f = \frac{1}{\sqrt{x+2}} = \frac{1}{(x+2)^{\frac{1}{2}}} = (x+2)^{-\frac{1}{2}}$$

$$\Rightarrow f' = -\frac{1}{2}(x+2)^{-\frac{1}{2}-1} = -\frac{1}{2}(x+2)^{-\frac{3}{2}} = -\frac{1}{2\sqrt{(x+2)^3}} = -\frac{1}{2(x+2)\sqrt{x+2}} \checkmark$$

$$58) f(x) = \sin(-x) \quad \text{funzione composta}$$

$$\Rightarrow f' = \cos(-x) \cdot -1 = -\cos(-x) \checkmark$$

$$59) f = -e^{1+x} \quad f' = -e^{1+x} \cdot 1 = -e^{1+x}$$

$$60) f = \sqrt{3x} = (3x)^{\frac{1}{2}} \quad f' = \frac{1}{2}(3x)^{\frac{1}{2}-1} \cdot 3$$

$$= \frac{(3x)^{-\frac{1}{2}} \cdot 3}{2} = \frac{3}{2\sqrt{3x}} \checkmark$$

$$61) f = 3 \ln x \quad \frac{3}{x} \checkmark$$

$$62) f = x^2 - 8x \quad f' = 2x - 8 \checkmark$$

$$63) f = 2\sqrt{x} = 2x^{\frac{1}{2}} \quad f' = 2x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}} \checkmark$$

$$64) f = \frac{1}{\sqrt{x}-1} = \frac{1}{x^{\frac{1}{2}}-1} = \frac{1}{-1} \cdot \frac{1}{x^{\frac{1}{2}}} = -x^{-\frac{1}{2}}$$

$$f' = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}} \quad (?)$$

$$65) f = \frac{\sqrt{x}-2}{x^{\frac{1}{2}}-2} \quad f' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad \checkmark$$

$$66) f = \frac{x}{x-5}$$

$$f' = \frac{(x-5) - x}{(x-5)^2} = -\frac{5}{(x-5)^2} \quad \checkmark$$

$$67) f = \frac{9-x}{x^2-1} \quad f' = \frac{-(x^2-1) - (9-x) \cdot 2x}{(x^2-1)^2}$$

$$= \frac{\cancel{x^2} + 1 - 18x + 2x^2}{(x^2-1)^2} = \frac{x^2 - 18x + 1}{(x^2-1)^2} \quad \checkmark$$

