7.12 Calcolare il limite: $\lim_{n\to+\infty} \dfrac{3n-1}{n+3}$.

$$\lim_{n\to\infty} \frac{\cancel{n}(3-\frac{1}{n})}{\cancel{n}(1+\frac{3}{n})} = 3$$

7.13 Calcolare i limiti per $n \to +\infty$ delle successioni

(a) $\dfrac{n+1}{n^2+1}$ (b) $\dfrac{n^4+5}{n^5+7n-1}$

(c) $\dfrac{n^3+1}{2n-1}$ (d) $\dfrac{1-n^2}{(n+2)^2}$

a) $\lim \dfrac{\cancel{n}(1+0)}{n^{\cancel{2}}(1+0)} = 0$

b) $\dfrac{\cancel{n^4}(1+0)}{n^8(1+\frac{7}{n^4}-0)} = 0$

c) $\dfrac{n^{\cancel{3}\,2}(1+0)}{2\cancel{n}(1-0)} = \dfrac{n^2}{2} = +\infty$

d) $\dfrac{\cancel{n^2}(0-1)}{\cancel{n^2}(1+\frac{2n}{n^2}+\frac{4}{n^2})} = \dfrac{-1}{1} = -1$

7.14 Calcolare il limite: $\lim_{n\to+\infty} \dfrac{1-n}{\sqrt{n}+1}$.

$$\lim \frac{n(-1)}{\sqrt{n}(1)} \simeq -\frac{n}{\sqrt{n}} \to n \gg \sqrt{n} \to -\infty$$

7.15 Calcolare il limite: $\lim_{n\to+\infty} \dfrac{n+(-1)^n}{n-(-1)^n}$.

$(-1)^n$ é limitata

$+1, -1, +1, \dots$

$$\lim_n \frac{\cancel{n}(\frac{\cancel{a}}{n}^{\to 0}+1)}{\cancel{n}(-\frac{\cancel{a}}{n}^{0}+1)} = 1$$

7.16 Calcolare il limite: $\lim_{n\to+\infty} (\sqrt{n+2}-\sqrt{n-1})$.

$$\lim_n \frac{(\sqrt{n+2}-\sqrt{n-1})(\sqrt{n+2}+\sqrt{n-1})}{\sqrt{n+2}+\sqrt{n-1}} = \frac{n+2+\cancel{\sqrt{n+2}\sqrt{n-1}}-\cancel{\sqrt{n+2}\sqrt{n-1}}+n-1}{\text{"}}$$

$$= \frac{(n+2)-(n-1)}{\sqrt{n+2}+\sqrt{n-1}} = \frac{3}{\sqrt{\text{"}}+\sqrt{\text{"}}} \simeq\!\!\Rightarrow \frac{3}{+\infty} = 0$$

7.17 Calcolare il limite: $\lim_{n\to+\infty} (\sqrt{n^2+1}-\sqrt{n})$.

$$\sqrt{n^2+1}-\sqrt{n} \cdot \frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt{n^2+1}+\sqrt{n}} = \frac{(n^2+1)+\cancel{\sqrt{n^2+1}\cdot\sqrt{n}}-\cancel{\sqrt{n^2+1}\cdot\sqrt{n}}-n}{\text{"}}$$

$$= \frac{n^2+1-n}{\sqrt{n^2+1}+\sqrt{n}} = \frac{n^2(0-0)}{\text{ord } \sqrt{n}} \qquad n^2 \gg \sqrt{n} \to +\infty$$

7.18 Calcolare i limiti di successione

(a) $\lim_{n\to+\infty} (\sqrt{n+1}-n)$ (b) $\lim_{n\to+\infty} n\sqrt{\dfrac{1}{n+1}}$

a) $$\sqrt{n+1}-n \cdot \frac{\sqrt{n+1}+n}{\sqrt{n+1}+n}$$

$$= \frac{(n+1)\cancel{\sqrt{n+1}\,n}-\cancel{n\sqrt{n+1}}-n^2}{\sqrt{n+1}+n}$$

$$\Rightarrow \frac{n+1-n^2}{\sim n} \to \frac{n^{\cancel{2}}(0+0-1)}{\sim \cancel{n}} \to -\infty$$

b) $n\sqrt{\dfrac{1}{n+1}}$ · boh

Limiti notevoli

$$\lim_{n\to\infty} a^n \begin{cases} a>0 \to +\infty \\ a=1 \to 1 \\ -1<a<1 \to 0 \end{cases}$$

Non esiste se $a \leq -1$

$$\lim_{n\to\infty} n^b = \begin{cases} b>0 \to +\infty \\ b=0 \to 1 \\ b<0 \to 0 \end{cases}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

generalmente: $\lim_{n\to+\infty}\left(1+\frac{x}{n}\right)^n = e^x$

Utile per la trigonometria:

Quando $a_n \to 0 \Rightarrow \frac{\sin(a_n)}{a_n} \to 1$

7.32 Calcolare i limiti

$$(a)\quad \lim_{n\to+\infty} e^n \qquad (b)\quad \lim_{n\to+\infty}\left(\frac{1}{2}\right)^n$$

a) $\lim_{\infty} e^n \qquad e>0,\ n\to 0$

$\Rightarrow \lim e^n \to +\infty$

$\lim_{n\to+\infty}\left(\frac{1}{2}\right)^n$; $\frac{1}{2}<0 \to \lim \to 0$

$+\infty$

7.33 Calcolare i limiti

$$(a)\quad \lim_{n\to+\infty}(e^n-2^n) \qquad (b)\quad \lim_{n\to+\infty}(3^n+4^n-5^n)$$

a) $\lim_{n\to+\infty}(\cancel{e^n}-\cancel{2^n}) \Rightarrow \qquad e^n\left[1-\left(\frac{2}{e}\right)^n\right] \sim e^n \to +\infty$

$+\infty \quad +\infty$

$-1<a<1 \to 0$

b) $3^n+4^n-5^n \to \lim_{n\to+\infty} \cancel{5^n}\left(\left(\cancel{\frac{3}{5}}\right)^n+\left(\cancel{\frac{4}{5}}\right)^n-1\right) \to -\infty$

$+\infty \quad 0 \quad 0$

7.34 Calcolare i limiti

$$(a)\quad \lim_{n\to+\infty}\frac{2^{n+1}-4^{n-1}}{3^n} \qquad (b)\quad \lim_{n\to+\infty}\frac{2^{n+1}+1}{3^n+1}$$

$$\left[(a)\ \frac{2^{n+1}-4^{n-1}}{3^n} = 2\left(\frac{2}{3}\right)^n - \frac{1}{4}\left(\frac{4}{3}\right)^n \to -\infty;\ (b)\ 0\right]$$

a) $\frac{2^{n+1}-4^{n-1}}{3^n} \to a^{n+1} = a^n \cdot a\ ,\ a^{n-1} = \frac{a^n}{a}$

$\Rightarrow 2\left(\cancel{\frac{2}{3}}\right)^n - \frac{1}{4}\left(\cancel{\frac{4}{3}}\right)^n \to -\infty$

$0 \quad +\infty$

b) $\frac{2^{n+1}+1}{3^n+1} = 2\left(\cancel{\frac{2}{3}}\right)^n + \frac{1}{\cancel{3^n}} \to 0$

$0 \quad 0$

7.35 Calcolare i limiti di successione

(a) $\lim_{n\to+\infty} n^{\sqrt{2}}$ (b) $\lim_{n\to+\infty} n^{-e}$

a) $n^{\sqrt{2}} = n^{2^{\frac{1}{2}}}$ $2^{\frac{1}{2}} > 0$ $\Rightarrow +\infty$

b) n^{-e} $-e<0$ $\to 0$

7.36 Calcolare i limiti

(a) $\lim_{n\to+\infty} \sqrt[n]{\pi}$ (b) $\lim_{n\to+\infty} \frac{1}{\sqrt[n]{5}}$

a) $\pi^{\frac{1}{n}} \to 1$ $\frac{1}{n}\to 0$ $\pi > 0$

b) $\frac{1}{5^{\frac{1}{n}}} \to 1$ $\frac{1}{n}\to 0$ $5^{\frac{1}{n}} \to 1$

7.37 Calcolare i limiti di successione

(a) $\lim_{n\to+\infty} \sqrt[n]{n^2}$ (b) $\lim_{n\to+\infty} \sqrt[n]{2n}$

a) $(n^2)^{\frac{1}{n}} = n^{\frac{2}{n}} \to 1$ $\frac{2}{n}\to 0$

b) $\sqrt[n]{2n} = \sqrt[n]{2} \cdot \sqrt[n]{n} = 2^{\frac{1}{n}} \cdot 1 \to 1$ $\sqrt[n]{n}\to 1$ $\frac{1}{n}\to 0$

7.38 Calcolare $\lim_{n\to+\infty}(n - \log n)$.

$n - \log n \to$ $n\left(1 - \frac{\log n}{n}\right)$ $\frac{\log n}{n}\to 0$ $n \gg \log n \to +\infty$

7.39 Calcolare $\lim_{n\to+\infty}(2^n - n^2)$.

$2^n - n^2 \Rightarrow 2^n\left(1 - \cancel{\frac{n^2}{2^n}}\right)$ $\to 0$ $2^n \gg n^2 \to +\infty$

7.40 Calcolare $\lim_{n\to+\infty}(2^n - n!)$.

$n!\left(\cancel{\frac{2^n}{n!}} - 1\right) \to -\infty$ $\frac{2^n}{n!}\to 0$

7.42 Calcolare, per $n \to +\infty$, i limiti delle successioni

(a) $\frac{2^n - 4^n}{3^n - n!}$ (b) $\frac{(n3^{n+1} + n^5 + 1)n!}{(3^n + 2^n)(n+1)!}$

a) $\frac{2^n - 4^n}{3^n - n!} = \frac{4^n\left(\cancel{\left(\frac{2}{4}\right)^n}\right)}{n!\left(\cancel{\frac{3^n}{n!}} - 1\right)}$ $\left(\frac{2}{4}\right)^n\to 0$ $\frac{3^n}{n!}\to 0$ $a^n \ll n! \to 0$

b) $\frac{(n3^{n+1} + n^5 + 1)\cdot n!}{(3^n + 2^n)(n+1)!} =$ $n3^{n+1} = 3n3^n \Rightarrow$ $(3n3^n + n^5 + 1)n! = 3^n\left(3n + \cancel{\frac{n^5}{3^n}} + \cancel{\frac{1}{3^n}}\right)n!$

$(n+1)! = (n+1)n! \Rightarrow \frac{3^n \cdot 3n \cdot \cancel{n!}}{[3^n + n3^n + n2^n + 2^n]\cdot \cancel{n!}} = \frac{\cancel{3^n}\cdot 3n}{\cancel{3^n}(1 + n + 1 + 1)}$

$= \frac{3n}{n+3} = \frac{3\cancel{n}}{\cancel{n}(1+0)} \to 3$

7.43 Calcolare i limiti

$$(a)\quad \lim_{n\to+\infty} \frac{\log^2 n}{n} \qquad (b)\quad \lim_{n\to+\infty} \frac{1}{1+\log^3 n}$$

a) $\frac{\ln n}{n}$ $n >> \ln n \to 0$

b) $\frac{1}{1+\ln^3 n} \sim \frac{1}{\ln^3(n)(0+1)} \to \frac{1}{+\infty} \to 0$?

Sul libro risulta $+\infty$

7.44 Calcolare i limiti

$$(a)\quad \lim_{n\to+\infty} \frac{5^n-n^5}{4^n+n^6} \qquad (b)\quad \lim_{n\to+\infty} \frac{(n^2+1)\log n}{n^3}$$

a) $\frac{5^n-n^5}{4^n+n^6} = \frac{5^n(1-0)}{4^n(1+0)} \sim \left(\frac{5}{4}\right)^n$ $a>0 \to +\infty$

b) $\frac{(n^2+1)\ln n}{n^3} \Rightarrow \frac{n^2\ln n+\ln n}{n^3} = \frac{\cancel{n^2}(\ln n+0)}{n^{\cancel{3}}} = \frac{+\infty}{n}$ ma $\sim \frac{\ln n}{n}$ $n>>\ln n \to 0$

7.45 Calcolare i limiti, per $n\to+\infty$, delle successioni

$$(a)\quad \frac{n!+2^n}{(n+1)!} \qquad (b)\quad \frac{n!-(n+1)!}{n^2e^n}$$

a) $\frac{\cancel{n!}(1+\frac{2^n}{n!}\to 0)}{(n+1)\cancel{n!}} = \frac{1}{n+1} \to 0$

b) $\frac{n!-(n+1)!}{n^2e^n} = \frac{n!-(n+1)n!}{n^2e^n} =$

$e^n >> n^2$

7.47 Calcolare il limite $\lim_{n\to+\infty} \left(\frac{n+1}{n}\right)^{2n}$.

$\left(1+\frac{a}{n}\right)^{bn} = e^{ba} \Rightarrow \left(\frac{n+1}{n}\right)^{2n} = \left(\frac{n}{n}+\frac{1}{n}\right)^{2n} = \left(1+\frac{1}{n}\right)^{2n} =$

$\to e^2$

7.48 Calcolare i limiti

$$(a)\quad \lim_{n\to+\infty} \left(1+\frac{1}{2n}\right)^n \qquad (b)\quad \lim_{n\to+\infty} \left(1-\frac{1}{n}\right)^n$$

a) $\left(1+\frac{1}{2n}\right)^n = e^{\frac{1}{2}} \to \sqrt{e}$

b) $\left(1+\frac{1}{-n}\right)^n \to e^{-1} = \frac{1}{e}$

7.49 Calcolare i limiti di successione

$$(a)\quad \lim_{n\to+\infty} \left(1+\frac{2}{\sqrt{n}}\right)^{\sqrt{n}} \qquad (b)\quad \lim_{n\to+\infty} \left(\frac{n}{n-1}\right)^{n+1}$$

a) $\left(1+\frac{2}{\sqrt{n}}\right)^{\sqrt{n}} \to e^2$

b) $\left(\frac{n}{n-1}\right)^{n+1}$ = pongo $t=\frac{n}{n-1} \Rightarrow (t)^{n+1} = t^n \cdot t \Rightarrow \left(\frac{n}{n-1}\right)^{n-1} \cdot \left(\frac{n}{n-1}\right)^2$ Boh

7.50 Calcolare, per $n\to+\infty$, i limiti delle successioni

$$(a)\quad \left(\frac{n^2+n}{n^2-n+2}\right)^n \qquad (b)\quad \left(\frac{n^2+n}{n^2+n+1}\right)^{n^2}$$

a) $\left(\frac{n^2+n}{n^2-n+2}\right)^n = \left(\frac{n^2}{n^2-n+2}+\frac{n}{n^2-n+2}\right)^n$

$\left(\frac{\cancel{n^2}}{\cancel{n^2}(1-\frac{1}{n}+\frac{1}{n})}+\frac{\cancel{n}}{\cancel{n}(n-1+0)}\right)^n$

$\downarrow$

$\left(1+\frac{1}{n}\right)^n = e$

$\left(1+\frac{1}{n}\right)^n = e$ Lim not

b) $\left(\frac{n^2+n}{n^2+n+1}\right)^{n^2} = \left(\frac{\cancel{n^2}}{\cancel{n^2}(+1)}+\frac{\cancel{n}}{\cancel{n}(n+1)}\right)^{n^2} =$

$\left(1+\frac{1}{n+1}\right)^n \cdot$

Sbagliato

Nuovi ESERCIZI

T. Confronto

ES: $\lim_{x\to-\infty} e^{2x}\cdot \sin x \qquad -1\leq \sin x\leq 1 \quad \overset{\cdot e^{2x}}{\Longrightarrow} \quad \underset{\downarrow\ 0}{\overset{h(x)}{-e^{2x}}} \leq \underset{?}{\overset{f(x)}{e^{2x}\sin x}} \leq \underset{\downarrow\ 0}{\overset{g(x)}{e^{2x}}} \;\longrightarrow\; \lim_{x\to-\infty} e^{2x}\sin x \to 0$

ES: $\lim_{x\to+\infty} \frac{1+\cos x}{x^2} \qquad -1\leq\cos x\leq 1 \;\longrightarrow\; \underset{\downarrow\ 0}{\frac{1-1}{x^2}} \leq \underset{\Downarrow\ 0}{\frac{1+\cos x}{x^2}} \leq \underset{\downarrow\ 0}{\frac{1+1}{x^2}}$

ES: $\lim_{x\to+\infty} 3^x(\sin x+2) \qquad -1\leq \sin x\leq 1 \;\longrightarrow\; \underset{\downarrow\ +\infty}{3^x} \leq \underset{\Downarrow\ +\infty}{3^x(\sin x+2)} \leq \underset{\downarrow\ +\infty}{3^x(3)}$

ES: $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) \;\longrightarrow\; -1\leq \sin\left(\frac{1}{x}\right)\leq 1 \qquad \underset{\downarrow\ 0}{-x} \leq \underset{\Downarrow\ 0}{x\sin\frac{1}{x}} \leq \underset{\downarrow\ 0}{x}$

ES: $\lim_{x\to+\infty} \frac{\cos x}{x^2+1} \qquad -1\leq\cos x\leq 1 \;\longrightarrow\; \underset{\downarrow\ 0}{-\frac{1}{x^2+1}} \leq \underset{\Downarrow\ 0}{\frac{\cos x}{x^2+1}} \leq \underset{\downarrow\ 0}{\frac{1}{x^2+1}}$

ES $\lim_{n\to+\infty} \frac{2n^4-5n}{1-n} = \frac{n^{\cancel{4}\,3}(2-0)}{\cancel{n}(0-1)} = -2n^3 \longrightarrow -\infty$

ES: $\lim_{n\to+\infty} \frac{-2n^2+n+4}{3n^2-5} = \frac{\cancel{n^2}(-2+\cancel{\frac{1}{n}}^{\to 0}+\cancel{\frac{4}{n^3}}^{\to 0})}{\cancel{n^2}(3-\cancel{\frac{5}{n^2}}^{\to 0})} = -\frac{2}{3}$ CONVERGE

ES: $\lim_{n\to+\infty} \sqrt{n+1}-\sqrt{n} = \underbrace{(n+1)^{\frac{1}{2}}}_{+\infty} - \underbrace{n^{\frac{1}{2}}}_{+\infty} = +\infty-\infty \Longrightarrow \sqrt{n+1}-\sqrt{n}\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} =$

$= \frac{(n+1)(\cancel{\sqrt{n+1}\cdot\sqrt{n}})-(\cancel{\sqrt{n}\sqrt{n+1}})-n}{//} = \frac{n+1-n}{//} = \frac{1}{\underbrace{\sqrt{n+1}+\sqrt{n}}_{\to+\infty}} \longrightarrow 0$

ES: $\lim_{n\to+\infty} \frac{\cos n}{n^2} \qquad -1\leq\cos n\leq 1 \;\longrightarrow\; -\frac{1}{\cancel{n}}^{\to 0} \leq \frac{\cos n}{n^2} \leq \frac{1}{\cancel{n}}^{\to 0} \Longrightarrow 0$ CONV

ES: $\lim_{n\to+\infty} \frac{7^n(1-n)}{1+n^2}$ ① la succ e' crescente? $\quad a_{n+1}>a_n$? $\quad a_{n+1} = \frac{7^{n+1}(\cancel{1}-n-\cancel{1})}{1+(n+1)^2} = -\frac{7^n\cdot 7\cdot n}{n^2+2n+2}$

$\Longrightarrow -\frac{7^n 7n}{\underbrace{n^2+2n+2}_{}} > \frac{7^n(1-n)}{\underbrace{1+n^2}_{}}$ Positivi $\forall x$ $\Longrightarrow 7^n 7n > -7^n(1-n)$? $\longrightarrow -7^n(1-n)-7^n7n>0 \longrightarrow -7^n+7n-7^n7n>0$

$\longrightarrow -7^n(1+n-7n)>0$ MAI $\Longrightarrow a_{n+1}<a_n \longrightarrow$ la succ e' DECRESCENTE

Calcolare $\lim\limits_{n\to+\infty} a_n$ di

a) $n^4 - 10n^3 = n^4(1-0) = +\infty$

$-3n^3+2n^2+n+\frac{1}{n} = n^3(-3+0+0+0) = -\infty$

$n^5 - 2n^4 - 5n - \left(\frac{1}{2}\right)^n = \underbrace{n^5}_{+\infty}(1-0-0) - \underbrace{\left(\frac{1}{2}\right)^n}_{0} = +\infty$

4) $\frac{3n^2-7n+1}{2n^3+5n} = \frac{\cancel{n^2}(3-0+0)}{n^{\cancel{3}}(2+0)} \sim \frac{1}{n} = +\infty$

5) $\frac{4n^4-7n^2+1}{-3n^3+n^2-n} = \frac{n^{\cancel{4}}(4-0+0)}{\cancel{n^3}(-3+0-0)} \sim \frac{n}{-1} = -\infty$

6) $\frac{4n^4-3n^3}{2n^4+7n^3-5} = \frac{\cancel{n^4}(4-0)}{\cancel{n^4}(2+0-0)} = 2$ $\quad$ <u>CONV</u>

7) $\sqrt{n^2+n} - \sqrt{n^2+4} \cdot \frac{\sqrt{n^2+n}+\sqrt{n^2+4}}{//} = \frac{(n^2+n)+\left(\cancel{\sqrt{n^2+n}+\sqrt{n^2+4}}\right)-\left(\cancel{\sqrt{n^2+n}+\sqrt{n^2+4}}\right)-(n^2+4)}{//}$

$= \frac{\cancel{n^2}+n-\cancel{n^2}-4}{\sqrt{n^2+n}\ \sqrt{n^2+4}} = \frac{n-4}{\sqrt{n^2+n}+\sqrt{n^2+4}}$ $\quad \frac{n\left(1-\frac{4}{x}\right)}{\sqrt{n^2\left(1+\frac{1}{n}\right)}+\sqrt{n^2\left(1+\frac{4}{n^2}\right)}} = \frac{n\left(1-\frac{4}{x}\right)}{n\sqrt{1}+n\sqrt{1}} \sim \frac{n}{2n} = \frac{1}{2}$

8) $2^{3n+1} - 3^{2n} = 2^{3n}\cdot 2 - 3^{2n} = 2^{3n}\Big(2 - \underbrace{\frac{3^{2n\,a}}{2^{3n\,b}}}_{b<a\to 0}\Big) \to 0 \quad +\infty$?