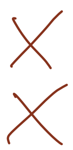
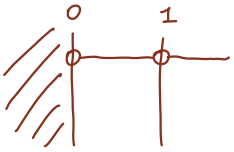


1) $f(x) = x\, e^{\frac{1}{\ln x}}$ $\quad \mathbb{D}: \; e^{\frac{1}{\ln x}} \Rightarrow \ln x \neq 0, \; x > 0$

$e^x \neq e^0 \Rightarrow x \neq 1$



• Segno $\quad f(x) > 0 \Rightarrow x\, \underbrace{e^{\frac{1}{\ln x}}}_{>0\, \forall x \in \mathbb{R}} > 0 \Rightarrow f(x) > 0$ per $x > 0$

• Intersezioni

$\begin{cases} y = x\, e^{\frac{1}{\ln x}} \Rightarrow y = 0 \\ x = 0 \end{cases} \Rightarrow P(0,0) \in f(x)$ $\qquad \begin{cases} y = x\, e^{\frac{1}{\ln x}} \Rightarrow x\, e^{\frac{1}{\ln x}} = 0 \text{ per } x = 0 \\ y = 0 \end{cases}$

• Asintoti

$\lim_{x \to 0^+} x\, e^{\frac{1}{\ln x}} = 0$ $\quad$ ($x \to 0$, $\frac{1}{\ln x} \to 0$, $\ln x \to -\infty$, $e^{\ldots} \to 1$) $\quad x = 0$ NON è A.V.

$\lim_{x \to 1^+} x\, e^{\frac{1}{\ln x}} = +\infty \Rightarrow x = 1^+$ A.V. Dx $\quad$ ($x \to 1$, $\ln x \to 0$, $\frac{1}{\ln x} \to +\infty$)

$\lim_{x \to 1^-} x\, e^{\frac{1}{\ln x}} = 0$ $\quad x = 1^-$ NO A.V $\quad$ ($\frac{1}{\ln x} \to -\infty$, $e^{\ldots} \to 0$; 1^- $\left(1^- < 1 = \frac{1}{n}\right)$)

$\lim_{x \to +\infty} \frac{f(x)}{x} = \frac{\cancel{x}\, e^{\frac{1}{\ln x}}}{\cancel{x}} = 1 = m$ $\quad$ ($\frac{1}{\ln x} \to 0$, $e^{\ldots} \to 1$)

$\lim_{x \to \infty} f(x) - mx = x\, e^{\frac{1}{\ln x}} - x = x\left(e^{\frac{1}{\ln x}} - 1\right) = 0 \cdot \infty$

$\Rightarrow x\, e^{\frac{1}{\ln x}} - x = \dfrac{e^{\frac{1}{\ln x}}}{\frac{1}{x}} - x = \infty - \infty$ $\quad$ ($e^{\ldots} \to 1$, $\frac{1}{x} \to 0$) $\quad$ BOH

• Max e Min $\quad D\left[x\, e^{\frac{1}{\ln x}}\right]$

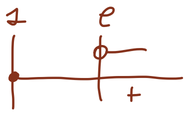
$D\left[\frac{1}{\ln x}\right] = \dfrac{-\frac{1}{x}}{\ln^2 x} = -\dfrac{1}{x \ln^2 x} \Rightarrow f'(x) = e^{\frac{1}{\ln x}} + \cancel{x}\, e^{\frac{1}{\ln x}} \cdot \left(-\dfrac{1}{\cancel{x} \ln^2 x}\right) = e^{\frac{1}{\ln x}} - \dfrac{e^{\frac{1}{\ln x}}}{\ln^2 x} = \dfrac{\ln^2 x\, e^{\frac{1}{\ln x}} - e^{\frac{1}{\ln x}}}{\ln^2 x}$

$= \dfrac{\overbrace{e^{\frac{1}{\ln x}}}^{>0\, \forall x \in \mathbb{R}} (\ln^2 x - 1)}{\underbrace{\ln^2 x}_{>0\, \forall x \in \mathbb{R}}} > 0 \qquad \ln^2 x > 1 \Rightarrow |\ln x| > 1$

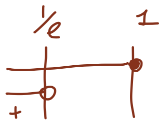
$\ln x > 1, \; \ln x > 0 \Rightarrow x > e, \; x > 1$

$-\ln x > 1, \; \ln x < 0 \Rightarrow x < \frac{1}{e}, \; x < 1$

$x > e \; \wedge x > 1$

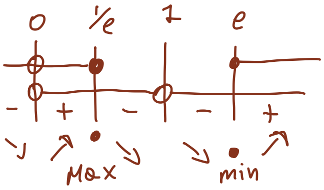


$\Rightarrow x > e$ $\qquad x < \frac{1}{e} \; \wedge \; x < 1$



$\Rightarrow x < \frac{1}{e}$

$x > e \;\cup\; x < \frac{1}{e} \Rightarrow I = \{x \,/\, x < 1/e \;\cup\; x > e\}$ $\quad I \cap \mathbb{D} =$



$\text{Max} = f(1/e) = \frac{1}{e} \cdot e^{\frac{1}{\ln|\frac{1}{e}|}} = \frac{1}{e} \cdot e^{-1} = \frac{1}{e^2} \sim 0.13$

$\Rightarrow \left(\frac{1}{e}, \frac{1}{e^2}\right)$ Max

$\text{Min} = f(e) = e \cdot e^{\frac{1}{\ln e}} = e \cdot e = e^2 \sim 7.3 \Rightarrow (e, e^2)$ Min

• Concavità $\quad D\left[e^{\frac{1}{\ln x}}\right] = -e^{\frac{1}{\ln x}} \cdot \dfrac{1}{x \ln^2 x}$ $\qquad D\left[\ln^2 x\right] = \dfrac{2 \ln x}{x}$

$\Rightarrow f''(x) = \left[-\dfrac{e^{\frac{1}{\ln x}}}{x \ln^2 x}(\ln^2 x - 1) + e^{\frac{1}{\ln x}}\left(\dfrac{2 \ln x}{x}\right)\right] \cdot \ln^2 x - \left\{\left[e^{\frac{1}{\ln x}}(\ln^2 x - 1)\right] \cdot \dfrac{2 \ln x}{x}\right\}$

$= \left[-\dfrac{e^{\frac{1}{\ln x}}}{x} + \dfrac{e^{\frac{1}{\ln x}}}{x \ln^2 x} + \dfrac{2 e^{\frac{1}{\ln x}} \ln x}{x}\right] \cdot \ln^2 x - \left\{\left[e^{\frac{1}{\ln x}} \ln^2 x - e^{\frac{1}{\ln x}}\right] \cdot \dfrac{2 \ln x}{x}\right\}$

$= \left[\dfrac{-\ln x\, e^{\frac{1}{\ln x}} + e^{\frac{1}{\ln x}} + 2 e^{\frac{1}{\ln x}} \ln^2 x}{x \cancel{\ln x}}\right] \cdot \ln^{\cancel{2}} x - \left\{\dfrac{2 e^{\frac{1}{\ln x}} \ln^3 x}{x} - \dfrac{2 e^{\frac{1}{\ln x}} \ln x}{x}\right\}$

$= \dfrac{\ln x\, e^{\frac{1}{\ln x}}}{x}\left(-\ln x + 1 + 2 \ln^2 x\right) - \left\{\dfrac{2 e^{\frac{1}{\ln x}} \ln^3 x - 2 e^{\frac{1}{\ln x}} \ln x}{x}\right\} = \dfrac{e^{\frac{1}{\ln x}}}{x}\left(-\ln^2 x + \cancel{\ln x} + \cancel{2 \ln^3 x}\right) - \dfrac{e^{\frac{1}{\ln x}}}{x}\left(\cancel{2 \ln^3 x} - \cancel{2 \ln x}\right)$ $\quad -\ln x$

$$= \frac{e^{\frac{1}{\ln x}}}{x}\left(-\ln^2 x - \ln x\right) > 0 \quad , \quad -\ln^2 x - \ln x > 0 \quad , \quad \ln^2 x + \ln x < 0 \quad , \quad \text{Pongo } t = \ln x$$

$e^{\frac{1}{\ln x}} > 0 \ \forall x \in \mathbb{R}$

$x \Rightarrow x > 0$

$$\Rightarrow t^2 + t < 0 \quad \Rightarrow \Delta = 1 - 4 \cdot 1 \cdot \cancel{0} = 0 \Rightarrow t_1 = \frac{-1}{2} = -\frac{1}{2}$$

$$t = -\frac{1}{2} \underset{t=\ln x}{=} \ln x = -\frac{1}{2} \Rightarrow x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \Rightarrow f''(x) < 0 \text{ per } x < \frac{1}{\sqrt{e}}$$

$1/\sqrt{e}$

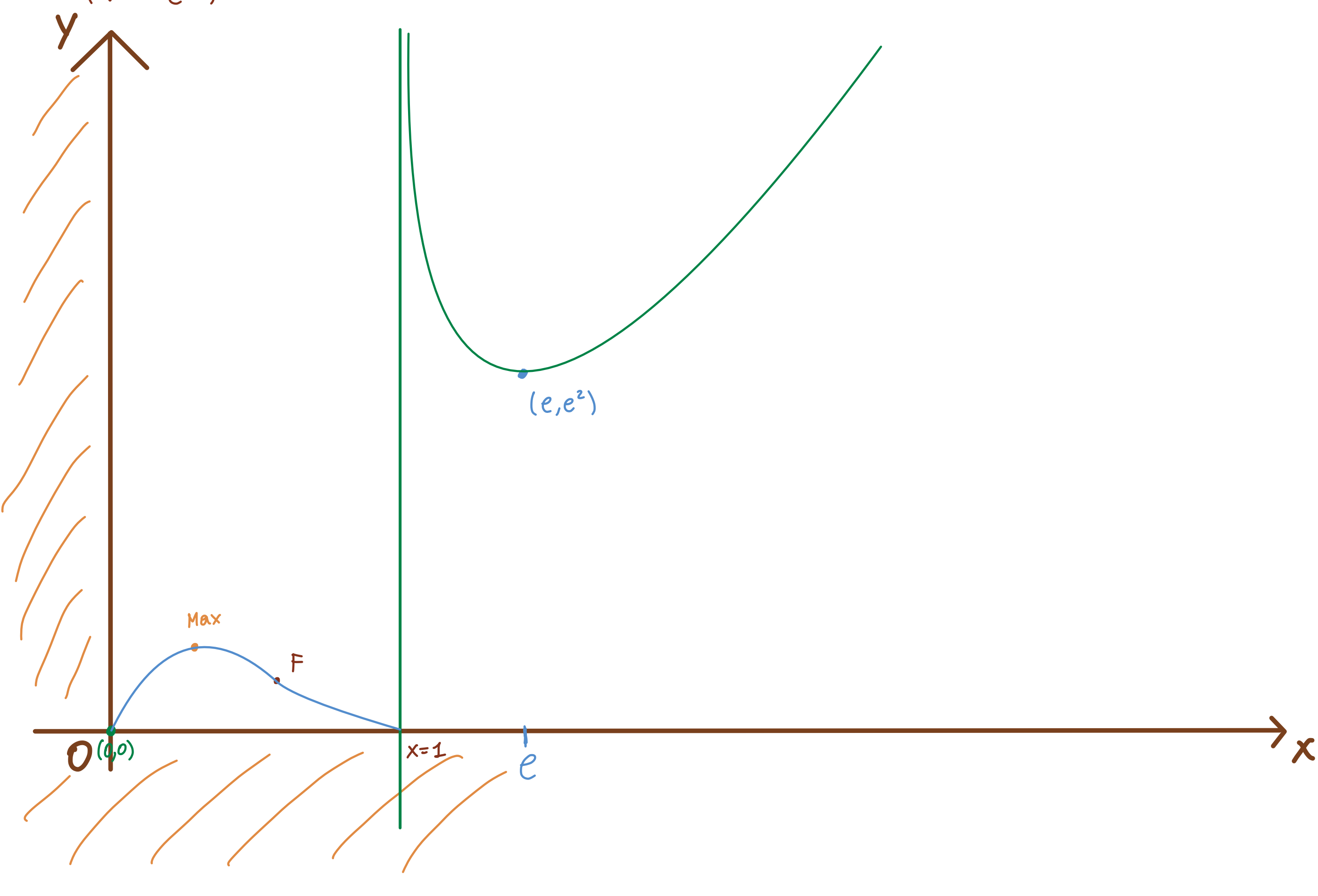
$+ \ | \ -$

$\cup \ | \ \cap$

$$f\left(\frac{1}{\sqrt{e}}\right) = \frac{1}{\sqrt{e}}\, e^{\frac{1}{\ln\left(\frac{1}{\sqrt{e}}\right)}} = \left[\frac{1}{\ln\left(\frac{1}{\sqrt{e}}\right)}\right] = \frac{1}{\ln\left(e^{-\frac{1}{2}}\right)} \underset{\ln(e^a)=a}{=} \frac{1}{-\frac{1}{2}} = -2$$

$$x = 1/\sqrt{e} \in D \Rightarrow x = \frac{1}{\sqrt{e}} \in f(x) \text{ , FLESSO} \qquad = \frac{1}{\sqrt{e}}\, e^{-2} = \frac{1}{\sqrt{e}\, e^2} = \frac{\sqrt{e}}{e^3}$$

$$\Rightarrow \left(\frac{1}{\sqrt{e}}, \frac{\sqrt{e}}{e^3}\right) \text{ FLESSO}$$



3) $\int_{-\pi}^{\pi} \cos(3x)\cos(4x)\,dx$ $\to \int \cos(4x)\,dx$ $\quad t=4x \to dx=\frac{1}{4}dt \to \frac{1}{4}\int \cos(t)\,dt = \frac{1}{4}\sin(4x)$

$\int \sin(4x)\,dx \quad t=4x \to dx=\frac{1}{4}dt \to \frac{1}{4}\int \sin(t)\,dt = -\frac{1}{4}\cos(4x)$

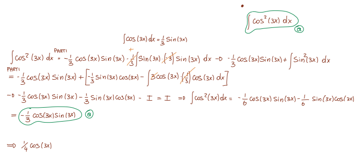
$D[\cos(3x)] = -3\sin(3x)$

$D[\sin(3x)] = 3\cos(3x)$

PARTI

$= \frac{1}{4}\cos(3x)\sin(4x) + \frac{3}{4}\int \sin(3x)\sin(4x)\,dx = \frac{1}{4}\cos(3x)\sin(4x) + \frac{3}{4}\left[-\frac{1}{4}\sin(3x)\cos(4x) + \frac{3}{4}\int \cos(4x)\cos(3x)\,dx\right]$

$\int \sin(3x)\,dx \quad t=3x \to dx=\frac{1}{3}dt \to \frac{1}{3}\int \sin(t)\,dt = -\frac{1}{3}\cos(3x)$



$= \frac{1}{4}\cos(3x)\sin(4x) - \frac{3}{16}\sin(3x)\cos(4x) + \frac{9}{16}\int \cos(4x)\cos(3x)\,dx$

$\int \cos(3x)\cos(4x)\,dx = I$

$\Rightarrow I = \frac{1}{4}\cos(3x)\sin(4x) - \frac{3}{16}\sin(3x)\cos(4x) + \frac{9}{16}I$

$\Rightarrow I - \frac{9}{16}I = //, \quad I\left(1-\frac{9}{16}\right) = // \to I\left(\frac{7}{16}\right) = //$

$\to I = \frac{4}{7}\cos(3x)\sin(4x) - 21\sin(3x)\cos(4x)$ $\quad \left[\frac{4}{7}\cos(3x)\sin(4x) - 21\sin(3x)\cos(4x)\right]_{-\pi}^{\pi}$

$= \left[\frac{4}{7}\cancel{\cos(3\pi)}^{-1}\sin(4\pi) - 21\cancel{\sin(3\pi)}\cos(4\pi)^{1}\right] - \left[\frac{4}{7}\cancel{\cos(-3\pi)}\sin(-4\pi) - 21\cancel{\sin(-3\pi)}\cos(-4\pi)\right] = 0?$

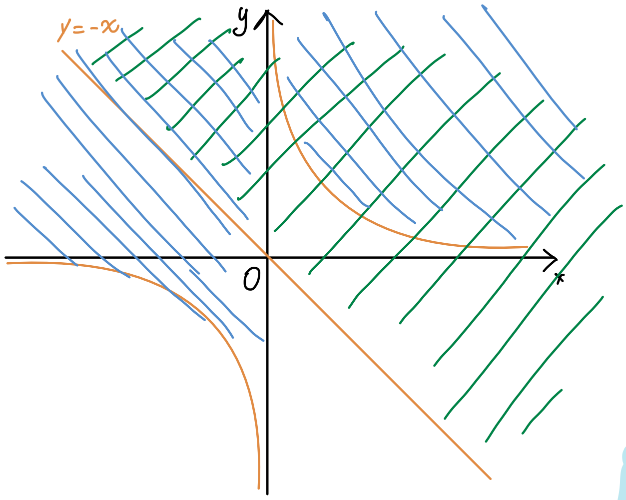
CORRETTO ✓

5. Calcolare il dominio della funzione: $f(x,y) = \ln(x+y) - \sqrt{xy-1}$;

$f(x,y) = \ln(x+y) - \sqrt{xy-1}$ (a) (b)

a) $x+y>0 \to y>-x$

b) $xy-1 \geq 0 \to xy \geq 1 \to y \geq x^{-1}$

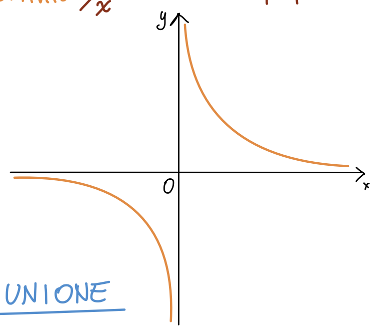


Mini studio $y=\frac{1}{x}$

Segno $\frac{1}{x}>0$ per $x>0$

Dominio $1/x$ $\forall x \in \mathbb{R}-\{0\}$

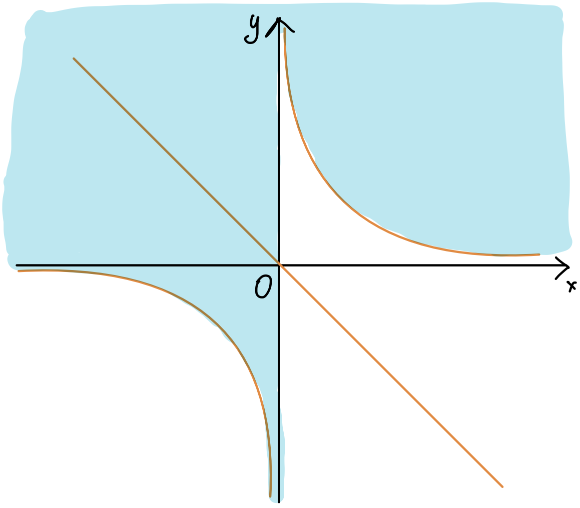
x	y
1	1
2	½
-1	-1
-2	-½
½	2
-½	-2



Si FA L'UNIONE

NO INTERSEZIONE!

Dominio Finale



6. Calcolare l'integrale del seguente problema di Cauchy: $\begin{cases} y'' - 2y = x + \sin x \\ y(0) = 0,\ y'(0) = 1 \end{cases}$;

$y'' - 2y = x + \sin x \to \lambda^2 - 2 = 0 \to \lambda^2 = 2 \to \lambda_{1,2} = \pm\sqrt{2}$

$y_0(x) = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$

Soluzione particolare per $\sin x$ $\quad y_p(x) = A\cos x + B\sin x$, $\quad y_p'(x) = -A\sin x + B\cos x$

$y_p''(x) = -A\cos x - B\sin x$

$\to -A\cos x - B\sin x - 2A\cos x - 2B\sin x = \sin x$

$\to \begin{cases} -A - 2A = 0 \to A = 0 \\ -B - 2B = 1 \to B = -\frac{1}{3} \end{cases} \Rightarrow y_0(x) = -\frac{1}{3}\sin x$

Sol part per x $\to y_p(x) = ax + b \to y'(x) = a \quad y''(x) = 0$

$\to -2ax + 2b = x \to \begin{cases} -2a = 1 \to a = -\frac{1}{2} \\ 2b = 0 \to b = 0 \end{cases} \quad y_0(x) = -\frac{1}{2}x$

$\Rightarrow y(x) = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} - \frac{1}{3}\sin x - \frac{1}{2}x$ $\quad y' = c_1\sqrt{2}e^{\sqrt{2}x} - c_2\sqrt{2}e^{-\sqrt{2}x} - \frac{1}{3}\cos x - \frac{1}{2}$

Cauchy $\quad y'' = c_1 2 e^{\sqrt{2}x} + c_2 2 e^{-\sqrt{2}x} + \frac{1}{3}\sin x \leftarrow$ Non serviva

$y(0) = c_1 + c_2 = 0 \to \begin{matrix} c_1 = -c_2 \\ c_2 = -c_1 \end{matrix} \quad y'(0) = c_1\sqrt{2} - c_2\sqrt{2} \underbrace{-\frac{1}{3} - \frac{1}{2}}_{-\frac{5}{6}} = 1$

$\underset{c_2 = -c_1}{=} c_1\sqrt{2} + c_1\sqrt{2} = \frac{11}{6} \to 2\sqrt{2}c_1 = \frac{11}{6} \to c_1 = \frac{11}{12\sqrt{2}}, \quad c_2 = -\frac{11}{12\sqrt{2}}$

$\Rightarrow$ Soluzione $\to y = \frac{11}{12\sqrt{2}} e^{\sqrt{2}x} - \frac{11}{12\sqrt{2}} e^{\sqrt{2}x} - \frac{1}{3}\sin x - \frac{1}{2}x$

2. Calcolare il seguente limite: $\lim_{x\to 0} \frac{\ln(1+x) - \ln(1-\sin x)}{x + \sin x}$; $\quad \lim_{x\to 0} \frac{\ln(1^+) - \ln(1^+)}{0} = \left[\frac{0}{0}\right]$ Candidato Hôpital

1) $g(x)$ e $f(x)$ continue $\quad z(x) = \ln(1+x) - \ln(1-\sin x) \to$ continua in $I(x_0) = I(0)$? e Deriv.

$\frac{1}{1+x} + \frac{\cos x}{1-\sin x} = f'(x)$, $\quad f'(x_0) = f'(0) = \frac{1}{1} + \frac{\cos(0)}{1-0} = 1 + 1 = 2 \neq 0$ continua e Deriv in $x_0 = 0$

2) $g'(x) \neq 0$ in $I(x_0) \to g'(x) = 1 + \cos x \neq 0$ per $\cos x \neq -1 \to x \neq a\cos(-1) \to x \neq \pi$

$\Rightarrow \pi \neq 0 \Rightarrow g'(x) \neq I(x_0)$

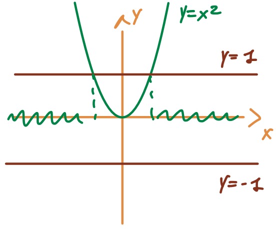
$\Rightarrow \lim_{x\to 0} \frac{\overset{\to 1}{\frac{1}{1+x}} + \overset{\to 1}{\frac{\cos x}{1-\sin x}}}{\underset{\to 2}{1 + \underset{1}{\cos x}}} = \frac{1+1}{1+1} = 1$ $\exists$ Finito

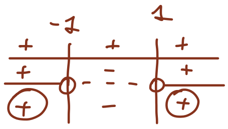
$\Rightarrow \lim_{x\to 0} f(x) = 1$

4. Studiare la seguente serie di potenze: $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{(n+1)!}\left(x^2-3x+1\right)^n$;

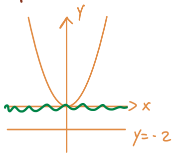
$f(x)=\sqrt{\dfrac{x^2+2}{x^2-1}}$ 1) D = $x^2-1\neq 0 \;\wedge\; \dfrac{x^2+2}{x^2-1}\geq 0$ $\begin{cases} x^2+2\geq 0 \rightarrow \forall x\in\mathbb{R} \\ x^2-1>0 \rightarrow x^2>1 \rightarrow x>\pm 1 \end{cases}$

$a>0$ eq $>0 \rightarrow$ Val est





$\Rightarrow D: \{ x \,/\, x<-1 \cup x>1\}$



② Simmetrie $f(-x)=\sqrt{\dfrac{x^2+2}{x^2-1}}=f(x)$ SIMMETRICA a y

BONUS, Non compreso nell'esame

③ Intersez

$\begin{cases} y=\sqrt{\dfrac{x^2+2}{x^2-1}} \\ y=0 \end{cases}$ int Asse x $\rightarrow \sqrt{\dfrac{x^2+2}{x^2-1}}=0 \rightarrow x^2+2=0 \rightarrow x^2=-2$ $\nexists x\in\mathbb{R}$

$\begin{cases} y=\sqrt{\dfrac{x^2+2}{x^2-1}} \\ x=0 \end{cases}$ int Asse y $\rightarrow y=\sqrt{\dfrac{2}{-1}}$ $\nexists x\in\mathbb{R}$

④ Segno $f(x)>0$ $\sqrt{\dfrac{x^2+2}{x^2-1}}>0$ $\forall x\in D$

⑤ Asintoti

$\lim_{x\to -1^-}\sqrt{\dfrac{x^2+2}{x^2-1}}=\sqrt{\dfrac{1+2}{1^+-1}}=\sqrt{\dfrac{3}{0^+}}=+\infty$ $\rightarrow$ Asintoto ✓ $x=-1$

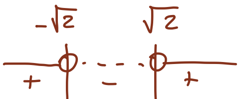
$\lim_{x\to 1^+}\sqrt{\dfrac{x^2+2}{x^2-1}}=\sqrt{\dfrac{1+2}{1^+-1}}=\sqrt{\dfrac{3}{0^+}}=+\infty \Rightarrow x=1$ A.V.

$\lim_{x\to +\infty}\sqrt{\dfrac{x^2+2}{x^2-1}}=\sqrt{\dfrac{+\infty}{+\infty}} \Rightarrow D[x^2+2]=2x$, $D[x^2-1]=2x$ $\Rightarrow 2x=0$ per $x=0$ $\nexists\, I(\infty)$ //

$\Rightarrow \lim_{x\to +\infty}\dfrac{2x}{2x}=1$ $\exists$ FINITO $\Rightarrow y=1$ A. Or. per SIMMETRIA Anche per $x<0$ ci sarà un asintoto su $y=1$

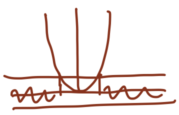
(4.1) Segno (bonus)

$f(x)>1 \rightarrow \sqrt{\dfrac{x^2+2}{x^2-1}}>1$ per $\begin{matrix}\sqrt{x^2+2}>1 \\ \sqrt{x^2-1}>1\end{matrix} \rightarrow \begin{matrix} x^2+2>1 \rightarrow x^2>-1 \;\forall x\in\mathbb{R} \\ x^2-1>0 \rightarrow x^2>1 \text{ per } x>\pm 1\end{matrix}$



$x<-1 \;\cup\; x>1$

La Funzione è al di sopra di $y=2$



⑥ Derivata

$f'(x)=\sqrt{\dfrac{x^2+2}{x^2-1}}=\dfrac{\sqrt{x^2+2}}{\sqrt{x^2-1}}$

$D\left[(x^2+2)^{\frac{1}{2}}\right]=\frac{1}{2}(x^2+2)^{-\frac{1}{2}}\cdot 2x=\dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+2}}=\dfrac{x}{\sqrt{x^2+2}}$

$D\left[(x^2-1)^{\frac{1}{2}}\right]=\frac{1}{2}(x^2-1)^{-\frac{1}{2}}\cdot 2x=\dfrac{x}{\sqrt{x^2-1}}$

$\Rightarrow f'(x)=\dfrac{\left[\dfrac{x}{\sqrt{x^2+2}}\sqrt{x^2-1}-\left(\sqrt{x^2+2}\;\dfrac{x}{\sqrt{x^2-1}}\right)\right]}{x^2-1}=\dfrac{\dfrac{x\sqrt{x^2-1}\sqrt{x^2+2}}{x^2+2}-\dfrac{x\sqrt{x^2+2}\sqrt{x^2-1}}{x^2-1}}{x^2-1}$

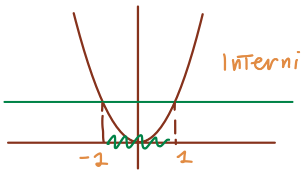
$=\dfrac{\dfrac{x\sqrt{x^2-1}\sqrt{x^2+2}\,(x^2-1)-(x^2+2)\sqrt{x^2+2}\sqrt{x^2-1}\,x}{(x^2+2)\;(x^2-1)}}{x^2-1}$

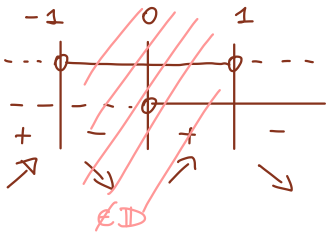
$$= \frac{\dfrac{\cancel{x^3\sqrt{x^2-1}\sqrt{x^2+2}} - \cancel{x\sqrt{x^2-1}\sqrt{x^2+2}} - \cancel{x^3\sqrt{x^2+2}\sqrt{x^2-1}} - \overset{3x}{\cancel{2x}}\sqrt{x^2+2}\sqrt{x^2-1}}{(x^2+2)(x^2-1)}}{x^2-1}$$

$$= \frac{-3x\sqrt{x^2+2}\sqrt{x^2-1}\,(x^2-1)}{(x^2+2)(x^2-1)} = \frac{-3x^3\sqrt{x^2+2}\sqrt{x^2-1} + 3x\sqrt{x^2+2}\sqrt{x^2-1}}{(x^2+2)(x^2-1)}$$

$$= 3x\sqrt{x^2+2}\sqrt{x^2-1}\,(1-x^2) > 0 \qquad 1 - x^2 > 0 \rightarrow x^2 < 1 \rightarrow x < \pm 1$$

$$\rightarrow 3x \underbrace{\cancel{\sqrt{x^2+2}}}_{>0}\ \underbrace{\cancel{\sqrt{x^2-1}}}_{>0} > 0 \rightarrow 3x > 0 \text{ per } x > 0$$





No Min NO Max

$f''(x) =$ Te la fai tu