Funzioni Razionali Fratte

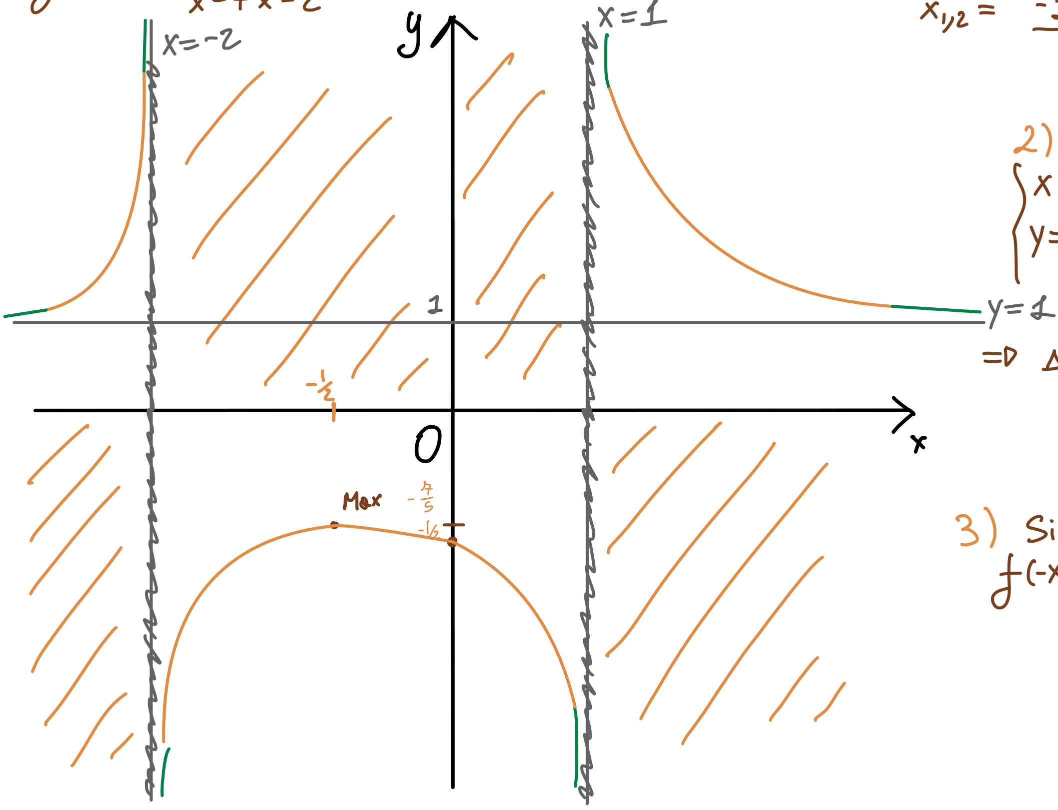
ES 32

$$f(x) = \frac{x^2+x+1}{x^2+x-2}$$

1) Dominio: $x^2+x-2 \neq 0$ $\Delta = 1-4\cdot 1\cdot(-2) = 9$

$x_{1,2} = \frac{-1 \pm 3}{2}$ $\begin{cases} x_1 = 1 \\ x_2 = -2 \end{cases}$

$\underline{x \neq 1 \text{ e } x \neq -2}$



2) Intersezioni

$\begin{cases} x=0 \\ y=-\frac{1}{2} \end{cases}$ $\begin{cases} y=0 \\ \text{per } x^2+x+1=0 \end{cases}$

$\Rightarrow \Delta = 1-4 < 0 \Rightarrow$ No int. con x

$\underline{P(0;-\frac{1}{2}) \in f(x)}$

3) Simmetrie

$f(-x) = \frac{x^2-x+1}{x^2-x-2}$ $\begin{cases} \neq f(x) \\ \neq -f(x) \end{cases}$ No Simm

4) Segno: $f(x) > 0$ per $\frac{x^2+x+1}{x^2+x-2} > 0$ N $x^2+x+1>0$ per $x<-2 \vee x>1$

D $x^2+x-2>0$ $\forall x \in \mathbb{R}$

$\Rightarrow \underline{f(x) > 0 \text{ per } x<-2 \vee x>1}$

5) Asintoti cerco in $x=-2$ e $x=1$

$\lim_{x\to -2^+} f(x) = \frac{4-2+1}{4^+-2^+-2} = \frac{3}{0^+} = +\infty$ $\lim_{x\to -2^+} f(x) = \frac{4-2+1}{4^- - 2^- - 2} = -\infty$

$\Rightarrow \underline{x=-2 \text{ Asintoto verticale}}$

$\lim_{x\to 1^\pm} f(x) = \frac{1+1+1}{1^+ + 1^+ - 2} = \frac{3}{0^\pm} = \pm\infty \Rightarrow \underline{x=1 \text{ Asintoto Verticale}}$

A. Orizz:

$\lim_{x\to +\infty} f(x) = \frac{\cancel{x^2}(1)}{\cancel{x^2}(1)} = 1 \Rightarrow \underline{y=1 \text{ Asintoto Orizzontale}}$

A. Obliquo

$\lim_{x\to\infty} \frac{f(x)}{x} = \frac{1}{x} = \emptyset \Rightarrow$ No A. obliquo

6) Derivate: Max-min

$$f'(x) = \frac{2x+1\,(x^2+x-2)-(x^2+x-1)\,2x+1}{(x^2+x-2)^2} = \frac{2x+1(\cancel{x^2}\cancel{+x}-2-\cancel{x^2}\cancel{-x}+1)}{(x^2+x-2)^2} = -\frac{2x+1}{(x^2+x-2)^2}$$

Studio la f': $f'(x) > 0$

$-\frac{2x+1}{(x^2+x-2)^2} > 0$ D>0 $\nexists x \in \mathbb{R}$

$\Rightarrow f'(x) > 0$ per $2x+1 < 0 \Rightarrow x < -\frac{1}{2}$

½

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↗ ↘

$\underline{x = \frac{1}{2} \text{ Max}}$

$f(\frac{1}{2}) = \frac{(\frac{1}{2})^2+(\frac{1}{2})+1}{(\frac{1}{2})^2+(\frac{1}{2})-2} = \frac{\frac{1}{4}+\frac{1}{2}+1}{\frac{1}{4}+\frac{1}{2}-2} = \frac{\frac{1+2+4}{4}}{\frac{1+2-8}{4}} = \frac{\frac{7}{4}}{-\frac{5}{4}} = \frac{7}{4}\cdot\left(-\frac{4}{5}\right) = -\frac{28}{20} = -\frac{7}{5}$

Deriv II° : $f''(x) = -\left[\frac{2(x^2+x-2)^2-(2x+1)\cdot[2(x^2+x-2)(2x+1)]}{(x^2+x-2)^4}\right] = -\left[\frac{2(x^2+x-2)^2-\underbrace{[2x^2+2x-4\cdot}_{2(2x+1)}\;\cdot(4x+2)]}{//}\right]$

$= -\left[\frac{2(x^2+x-2)^2-(2x+1)\cdot(8x^3+4x^2+8x^2+4x-16x-8)}{//}\right] = -\left[\frac{2(x^2+x-2)^2-(2x+1)(8x^3+12x^2-12x-8)}{(x^2+x-2)^4}\right]$

$= -\left[\frac{2(x^2+x-2)^2-(16x^4+24x^3-24x^2-16x+8x^3+12x^2-12x-8)}{//}\right] = -\left[\frac{2(x^2+x-2)^2-(16x^4+32x^3-12x^2-28x-8)}{(x^2+x-2)^4}\right]$

sempre pos

$\Rightarrow f''(x)>0$ per $16x^4+32x^3-12x^2-28x-8>0$

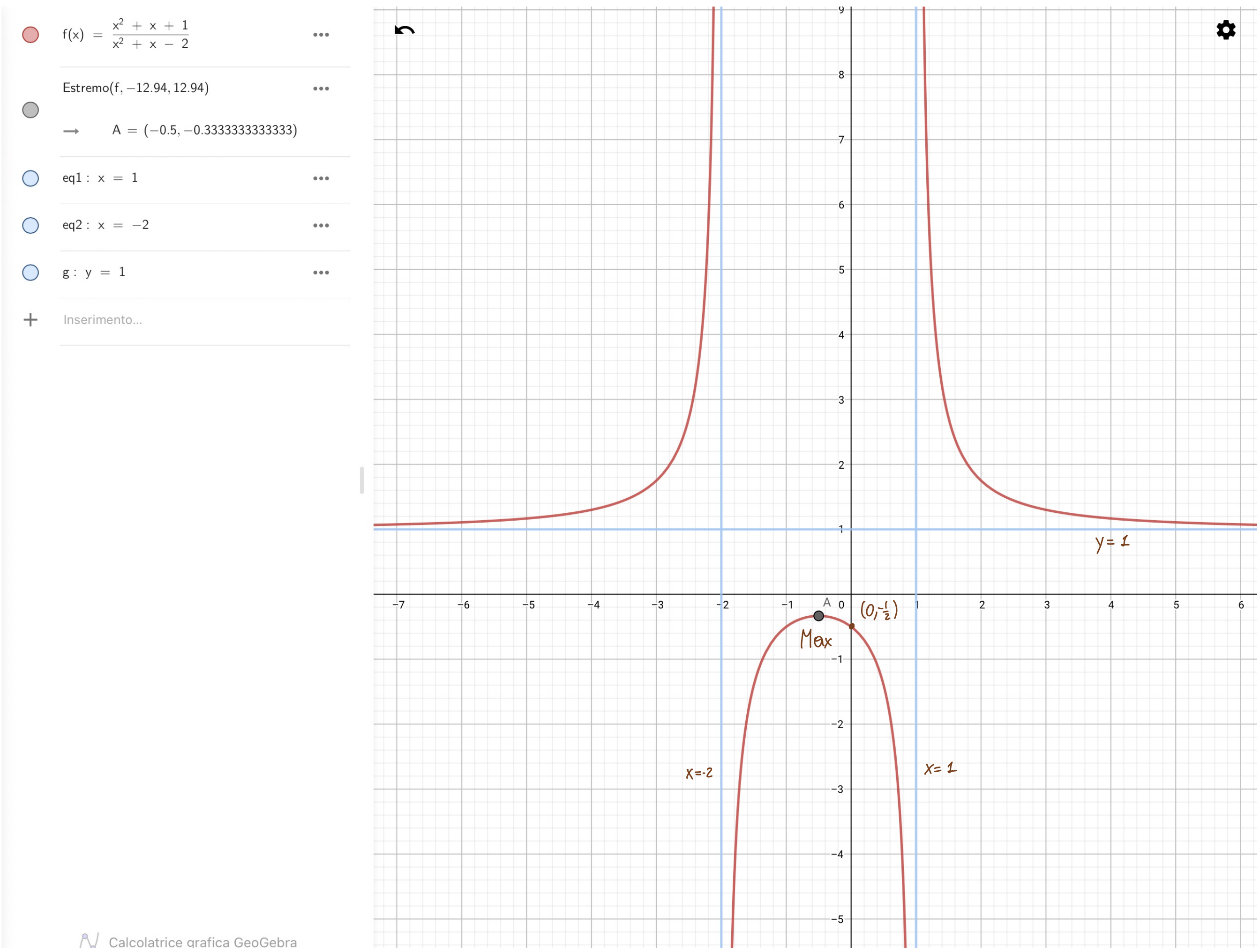
$\Rightarrow 8(2x^4+4x^3-1)-4(4x^2+7x)>0$

per a) $2x^4+4x^3-1>0$
b) $4x^2+7x<0$

a) $2x^3(x+2)>0$ per a.1) $2x^3>0 \rightarrow x>0$
a.2) $x+2>0 \rightarrow x>-2$

Deriv II sbagliata

b) $4x^2+7x<0$ per $x(4x+7)$ b.1) $x>0$
b.2) $\cancel{x>-\frac{7}{4}}$?



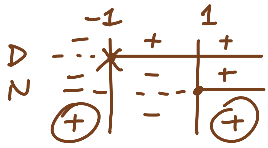
Funzioni irrazionali

ES: 61)

$$f(x)=\sqrt{\frac{x-1}{x+1}}$$

1) Dominio $\quad \frac{x-1}{x+1}\geq 0 \qquad N: x-1\geq 0$ per $x\geq 1$ $\quad D: x+1>0$ per $x>-1$

$D: \; x<-1 \cup x\geq 1$



2) Simmetrie

$$f(-x)=\sqrt{\frac{-x-1}{-x+1}} \begin{cases}\neq f(x)\\ \neq -f(x)\end{cases}$$

3) Intersezioni

$$\begin{cases}x=0\\ \sqrt{\frac{-1}{1}} \quad \nexists\, x\in\mathbb{R}\end{cases} \qquad \begin{cases}y=0\\ \text{per } x-1=0,\ \underline{x=1}\end{cases} \Rightarrow \underline{(1,0)\in f(x)}$$

4) Segno

$\underline{f(x)>0 \quad \forall x\in D}$

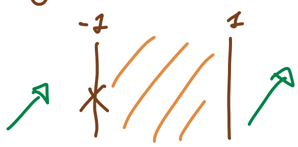
5) Studio crescenza: deriv I^a

$$f'(x)=D\left[\left(\frac{x-1}{x+1}\right)^{1/2}\right]=\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-1/2}\cdot D\left(\frac{x-1}{x+1}\right)=\frac{1(x+1)-(x-1)1}{(x+1)^2}=\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}=\frac{2}{(x+1)^2}$$

$$\Rightarrow f'(x)=\frac{1}{\cancel{2}}\frac{1}{\sqrt{\frac{x+1}{x-1}}}\cdot\frac{\cancel{2}}{(x+1)^2}=\frac{\sqrt{x-1}}{(x+1)^2\sqrt{x+1}}$$

sempre pos

$\Rightarrow \underline{f'(x)\geq 0 \quad \forall x\in\mathbb{R}}$



6) Asintoti

$$\lim_{x\to -1^-} f(x)=\sqrt{\frac{-2^-}{0^-}}^{+\infty}=+\infty \Rightarrow \underline{x=-1 \quad A.V.}$$

$$\lim_{x\to 1^+} f(x)=\sqrt{\frac{1^+-1}{1+1}}=\sqrt{\frac{0^+}{2}}=0 \quad \text{No A.V.}$$

$$\lim_{x\to +\infty} f(x)=\sqrt{\frac{\cancel{x}(1)}{\cancel{x}(1)}}=1 \Rightarrow \underline{y=1 \quad A.\,Orizz}$$

$$f'(x)=\frac{\overset{a}{\sqrt{x-1}}}{\underbrace{(x+1)^2}_{b}\underbrace{\sqrt{x+1}}_{d}}\ c$$

5.1) Deriv II

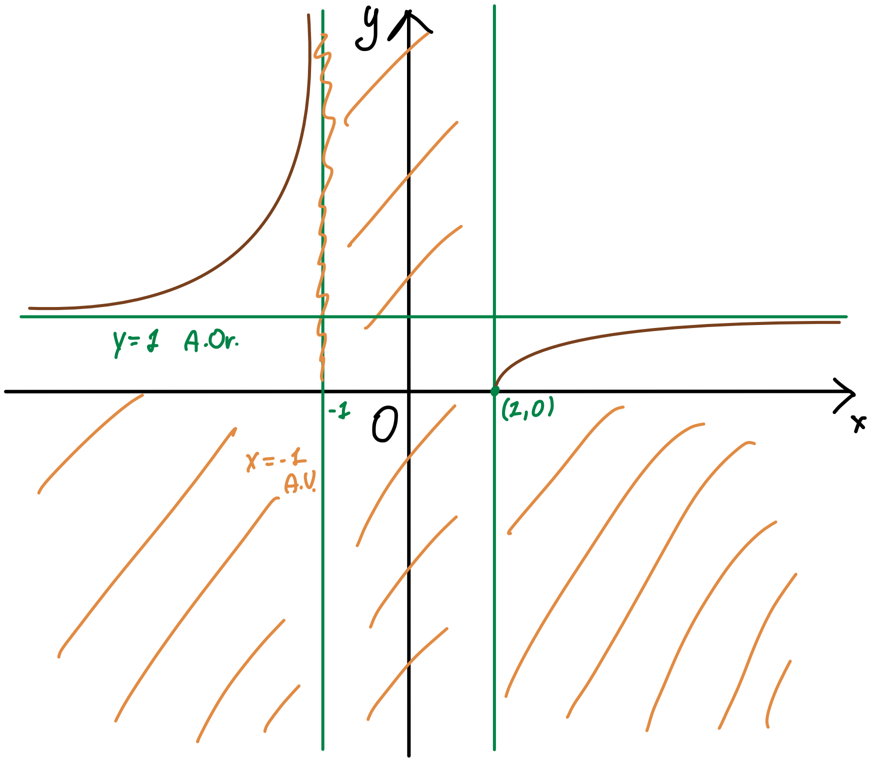
$f''(x)=$ a) $D(\sqrt{x-1})=-\frac{1}{2}(x-1)^{-1/2}=-\frac{1}{2\sqrt{x-1}}$ $\qquad$ c) $D(\sqrt{x+1})=\frac{1}{2\sqrt{x+1}}$

b) $D(x+1)^2=2(x+1)$

d) $D\left[(x+1)^2\sqrt{x+1}\right]=2(x+1)\sqrt{x+1}+(x+1)^2\left(\frac{1}{2\sqrt{x+1}}\right)=2x+2\sqrt{x+1}+x^2+2x+1\left(-\frac{1}{2\sqrt{x+1}}\right)$

$$=2x+2\sqrt{x+1}-\frac{x^2+2x+1}{2\sqrt{x+1}}=\frac{2x\sqrt{x+1}+4(x+1)-x^2+2x+1}{2\sqrt{x+1}}=\frac{2x\sqrt{x+2}+4x+4-x^2+2x+1}{2\sqrt{x+1}}$$

$= -x^2+6x+2x\sqrt{x+2}+5\geq 0$ per $x<0$ $\qquad 0$


y
x
y=1 A.Or.
-1
O
(1,0)
x=-1
A.V.

ES 62) $f(x)=\sqrt{\frac{1-x}{1+x}}$

1) Dominio $\frac{1-x}{1+x} \geq 0$ $\quad$ N $1-x \geq 0$ $\quad$ D $1+x > 0$

$\Rightarrow$ N: $x \leq 1$
D: $x > -1$

	-1		1	
$+$		$+$		$-$
$-$	$\times$	$+$	$\bullet$	$+$
$-$		$+$		$-$

f è definita per $-1 < x \leq 1$

2) Simmetrie

$f(-x)=\sqrt{\frac{1+x}{1-x}}$ $\neq f(x)$, $\neq -f(x)$

3) Intersezioni:

$\begin{cases} x=0 \\ \sqrt{\frac{1}{1}} \end{cases} \Rightarrow (0,1) \in f(x)$

$\begin{cases} y=0 \\ \text{per } x=1 \end{cases} \Rightarrow (1,0) \in f(x)$

4) Segno

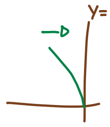
$f(x)>0 \quad \forall x \in \mathbb{D}$

5) limiti

$\lim_{x\to -1^+} f(x)=\sqrt{\frac{1+1}{0^+}} = +\infty \Rightarrow x=-1$ A.V.

$\lim_{x\to +\infty} f(x)=\sqrt{\frac{\cancel{x}(-1)}{\cancel{x}(1)}} = \sqrt{-1} \quad \nexists x \in \mathbb{R}$

$\lim_{x\to 1^-} f(x)=\sqrt{\frac{1-1^-}{1+1^-}} = \sqrt{\frac{0^+}{2^-}} = \sqrt{0^+} = 0$

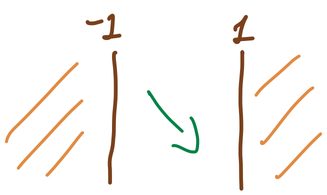


6) Derivate

$D\left(\sqrt{\frac{1-x}{1+x}}\right) = D\left(\frac{1-x}{1+x}\right) = \frac{-1-x-(1-x)}{(1+x)^2} = \frac{-1\cancel{-x}-1\cancel{+x}}{(1+x)^2} = -\frac{2}{(1+x)^2}$

$\Rightarrow D\left(\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\right) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \left(-\frac{2}{(1+x)^2}\right) = -\frac{\sqrt{1+x}}{\sqrt{1-x}\,(1+x)^2}$

$\Rightarrow f'(x)<0 \quad \forall x \in \mathbb{D}$



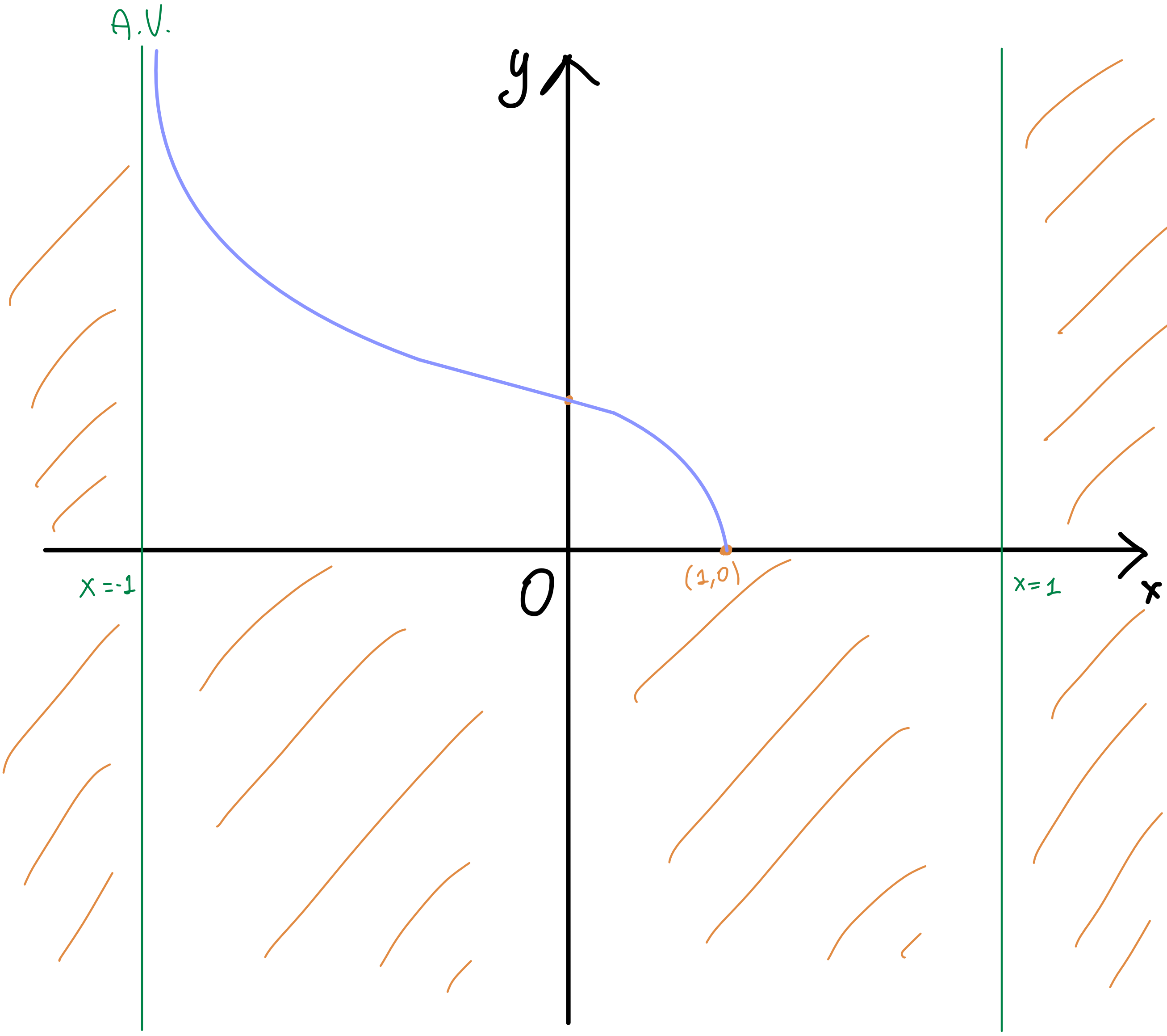
No min No max

$f''(x) =$ vedo $f'(x)$ come $-\sqrt{\frac{1+x}{1-x}} \cdot (1+x)^{-2} \Rightarrow$

$$f''(x) = -\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{2}{(1+x)^2} - \sqrt{\frac{1+x}{1-x}} \cdot (-2)(1+x)^{-3} = -\frac{1}{2} \frac{2}{\sqrt{\frac{1+x}{1-x}}(1+x)^2} - 2\sqrt{\frac{1+x}{1-x}}\,(1+x)^{-3}$$

$$= -\frac{1}{2} \frac{2}{\sqrt{\frac{1+x}{1-x}}(1+x)^2} - 2\sqrt{\frac{1+x}{1-x}} \cdot \frac{1}{(1+x)^3}$$

Boh



ES 64)

$f(x) = 2\sqrt{x} - x$

1) Dominio: $\underline{x \geq 0}$

Funzione Definita

2) Simmetrie: $f(-x) = 2\sqrt{-x} + x \ \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases}$

3) Intersezioni

$\begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow \underline{(0,0) \in f(x)}$

$\begin{cases} y = 0 \\ \text{per } 2\sqrt{x} - x = 0 \end{cases} \Rightarrow x = 2\sqrt{x} \Rightarrow x^2 = 4x, \ x^2 - 4x = 0$

per $x(x-4) = 0$, $x = 0 \cup x = 4$

$\Rightarrow \underline{(4,0) \in f(x)}$

4) Segno $f(x) > 0$ per $x^2 - 4x < 0$ $a > 0$, $eq < 0$

$\Rightarrow$ Soluzioni interne

$\Delta = 16$ $x_{1/2} = \dfrac{4 \pm 4}{2} \begin{cases} 4 \\ 0 \end{cases}$

$x > 0 \cup x < 4 \ \cap \ x \geq 0$

0 4

$+$ $+$

$+$ $-$

$+$ $-$

Segno

5) Asintoti

$\lim_{x \to 0^+} f(x) = 2\sqrt{0^+} - 0^+ = 0$ No A.U

$\lim_{x \to +\infty} f(x) = 2x^{-\frac{1}{2}} - x = x\left(\dfrac{2}{\sqrt{x}} - 1\right) = -\infty \Rightarrow$ No A.Oriz

$\lim_{x \to +\infty} \dfrac{f(x)}{x} = m = \dfrac{\cancel{x}\left(\frac{2}{\sqrt{x}} - 1\right)}{\cancel{x}} = -1$

$\lim_{x \to +\infty} f(x) - mx = 2\sqrt{x} \cancel{-x} \cancel{+x} = +\infty \Rightarrow$ No A.Ob.

6) Derivate

$f'(x) = D(2\sqrt{x} - x) = 2x^{\frac{1}{2}} - x = \dfrac{1}{\sqrt{x}} - 1 = \underbrace{\dfrac{1 - \sqrt{x}}{\sqrt{x}}}_{f'(x)} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{1\sqrt{x} - x}{x}$

$f'(x) \geq 0$ per $1 - \sqrt{x} > 0$, $\sqrt{x} \leq 1$

ovvero per $x \leq +1 \ \cap x \geq 0$

0 1

$+$ $-$

$\nearrow$ $\searrow$

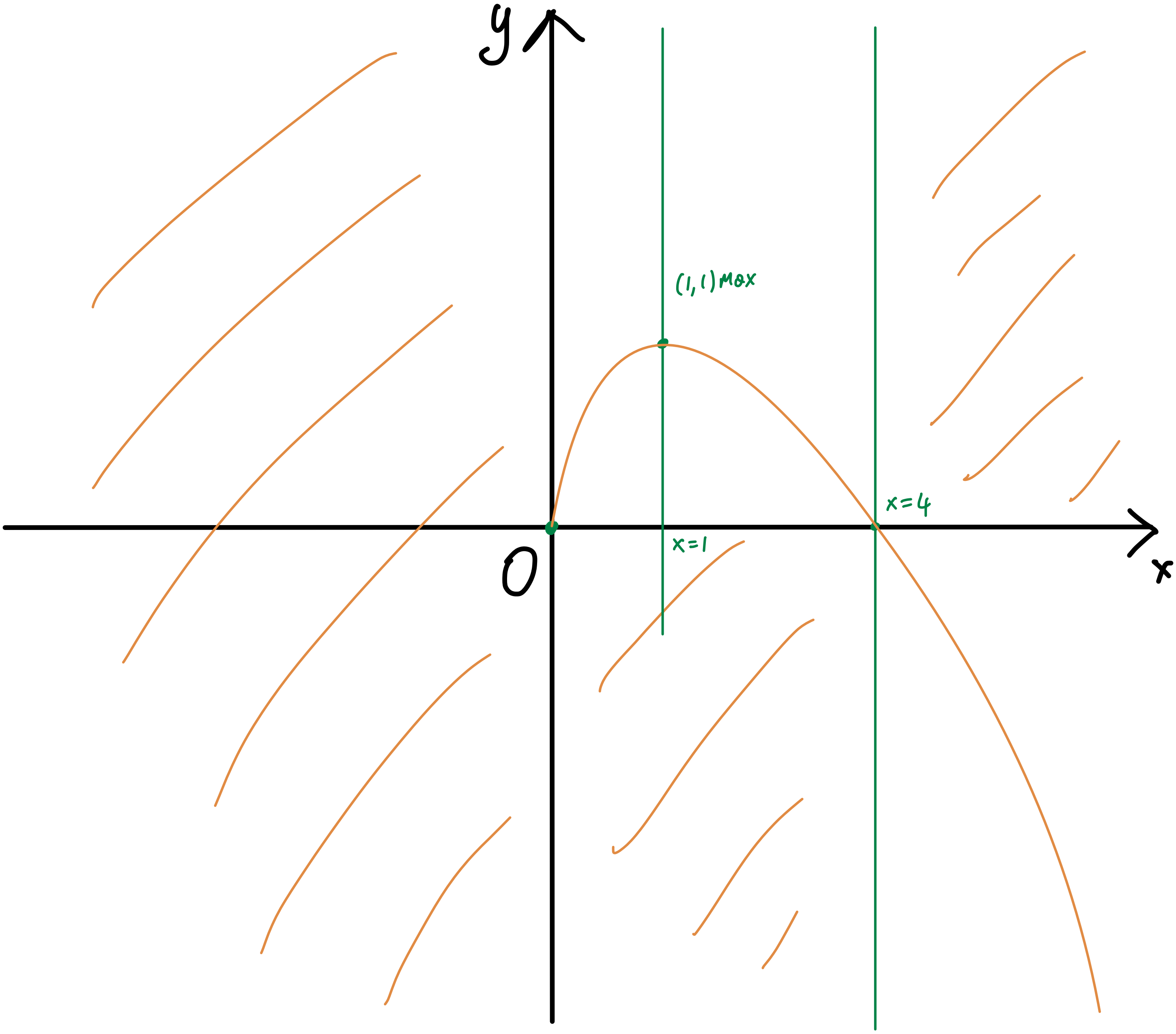
$x = 1$ max $f(1) = 2\sqrt{1} - 1 = 1 \Rightarrow \underline{(1,1) \text{ Max}}$

$\dfrac{1 - \sqrt{x}}{\sqrt{x}}$

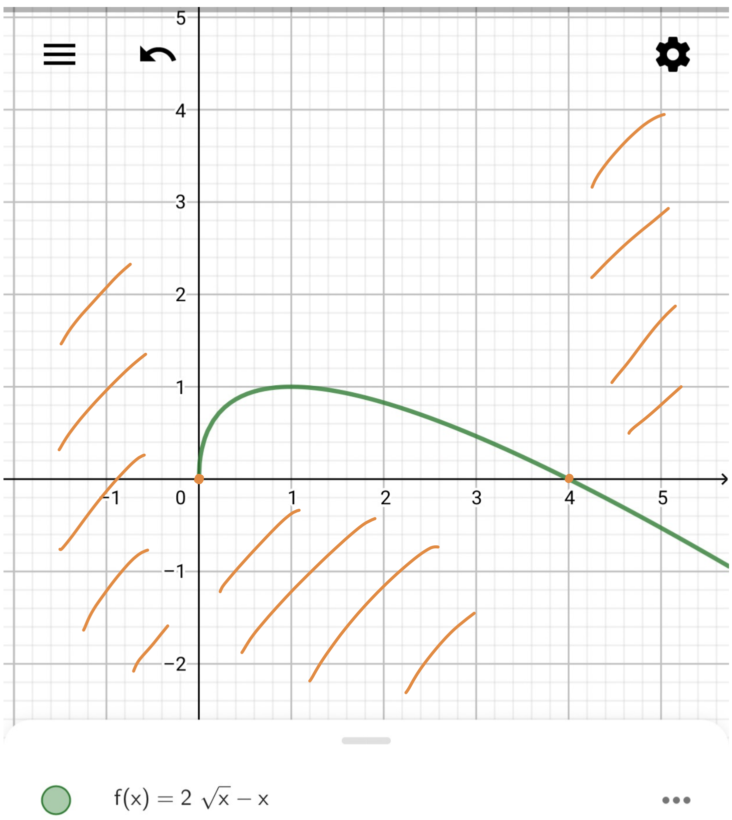
Deriv II $f''(x) = -\dfrac{1}{2}\dfrac{1}{\sqrt{x}} \dfrac{1}{\sqrt{x}} - \left[(1 - \sqrt{x}) \cdot \left(-\dfrac{1}{2}\dfrac{1}{\sqrt{x}}\right)\right]$

$= \dfrac{1}{2x} + \left[\dfrac{1 - \sqrt{x}}{2\sqrt{x}}\right] = \dfrac{\sqrt{x} + (1 - \sqrt{x})x}{2x\sqrt{x}} = \dfrac{\sqrt{x} + x - x\sqrt{x}}{2x\sqrt{x}} = \dfrac{\sqrt{x}(1 - x) + x}{2x\sqrt{x}}$

BOH



y
(1,1) max
x=4
x=1
0
x



5
4
3
2
1
0
-1
-2
1
2
3
4
5
-1
f(x) = 2 √x − x

$f(x) = \sqrt{8-x^3}$ $\quad D = \ 8-x^3 \geq 0 \quad$ per $x^3 \leq 8$; $\underline{x \leq \sqrt[3]{8} \leq 2}$

2) Simmetrie $\quad f(-x) = \sqrt{8+x}$ $\begin{cases} \neq f(x) \\ \neq -f(x) \end{cases}$

3) intersezioni

$\begin{cases} x = 0 \\ \sqrt{8} = 2\sqrt{2} \end{cases}$ $\quad \cancel{(0, 2\sqrt{2}) \in f(x)}$ $\notin D$ $\qquad \begin{cases} y = 0 \\ \sqrt{8-x^3} = 0 \end{cases}$ per $x = 2$ $\quad \underline{\Rightarrow (2,0) \in f(x)}$

4) Segno

$f(x) > 0 \quad \forall x \in D$

5) Asintoti

$\lim_{x \to 2^\pm} f(x) = \sqrt{8-8^\pm} = \sqrt{0^\pm} = \pm\infty$ $\quad$ No A.V

$\lim_{x \to +\infty} f(x) = \sqrt{8-\infty} = \nexists x \in \mathbb{R}$

$\lim_{x \to -\infty} f(x) = \sqrt{8+\infty} = +\infty$ $\quad$ Cerco A. Ob.

$\lim_{x \to -\infty} \frac{f(x)}{x} = \frac{\sqrt{x^2(\frac{1}{x^2} - x)}}{x} = \frac{\cancel{|x|}\sqrt{-x}}{\cancel{x}} = -\sqrt{+\infty} \Rightarrow \infty$ $\quad$ No A. Ob

6) Derivate

$D(\sqrt{8-x^3}) = \frac{1}{2} \frac{1}{\sqrt{8-x^3}} \cdot (-3x^2) = -\frac{3x^2}{2\sqrt{8-x^3}} \geq 0$ $\quad \nexists x \in D$ $\quad f(x) \leq 0 \quad \forall x \in D - \{x=0\}$

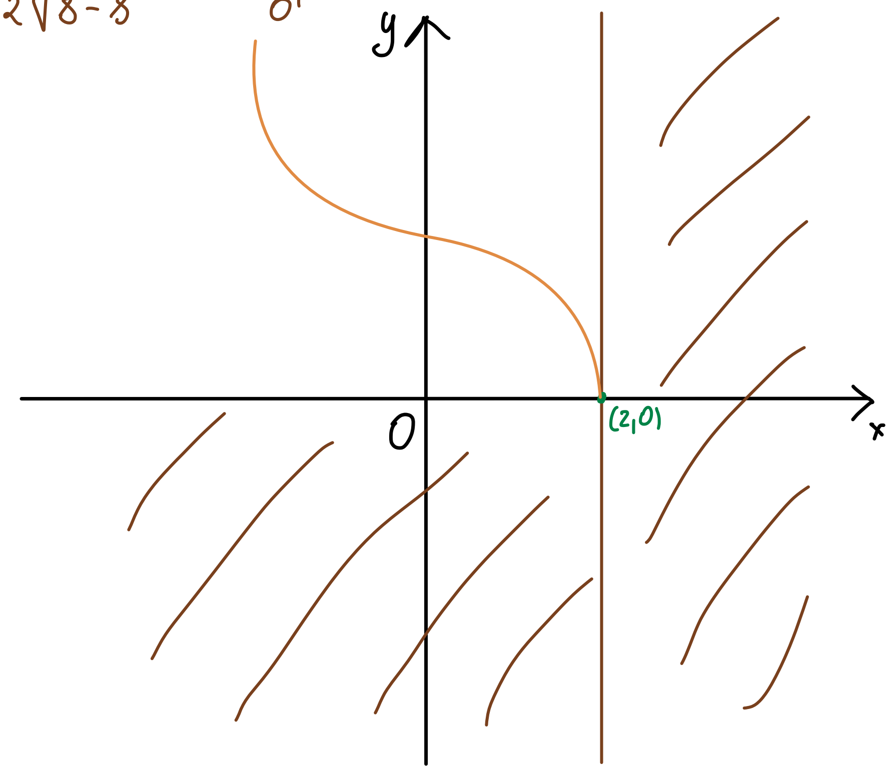
0

F

$D'\left(-\frac{3x^2}{2\sqrt{8-x^3}}\right) = -6x(2\sqrt{8-x^3}) - \left[(-3x^2)\left(\frac{1}{\sqrt{8-x^3}} \cdot (-3x^2)\right)\right]$

$= -6x \, 2\sqrt{8-x^3} - \left[-\frac{3x^2}{\sqrt{8-x^3}} \cdot (-3x^2)\right] = \ // \ - \left[- \frac{\text{BOH}}{\sqrt{8-x^3}}\right.$

$\lim_{x \to 2^-} f'(x) = -\frac{12}{2\sqrt{8-8^-}} = -\frac{12}{0^+} = -\infty$ $\quad x = 2$ Tangente verticale ?



ES: 70 $\quad f(x)=\dfrac{x+2}{\sqrt{x^2-x}}$

1) Dominio: $\quad x^2-x>0$ per $x(x-1)>0$

$a>0$, eq >0, sol esterne $\quad \to x>0$, $x>1$

$\Rightarrow f(x)\ \exists$ per $\quad x<0 \cup x>1$

2) Simmetrie:

$f(-x)=\dfrac{-x+2}{\sqrt{x^2+x}}\ \begin{cases}\neq f(x)\\ \neq -f(x)\end{cases} \Rightarrow$ No simm

3) Intersez:

$\begin{cases}x=0\\ \frac{2}{0}\ \nexists x\in\mathbb{R}\end{cases}$ $\qquad \begin{cases}y=0\\ x+2=0 \text{ per } x=-2\end{cases} \Rightarrow (-2,0)\in f(x)$

4) Segno $\quad f(x)\geq 0$ per $\quad x\geq -2$

$f(x)>0$ per $-2<x<0\ \cup\ x>1$

	-2		0		1	
$-$	•	$+$		$+$		$+$
$+$		$+$	×	$-$	×	$+$
		$\oplus$				$\oplus$

5) Asintoti

$\displaystyle\lim_{x\to 0^+} f(x)=\frac{2}{\sqrt{0^+-0^+}}=\frac{n}{0}=+\infty$

$\displaystyle\lim_{x\to 1^-} f(x)\doteq\frac{1+2}{\sqrt{1^+-1^-}}=\frac{3}{\sqrt{0^+}}=+\infty$

$\Big\}\ x=0,1$ Asintoti Verticali

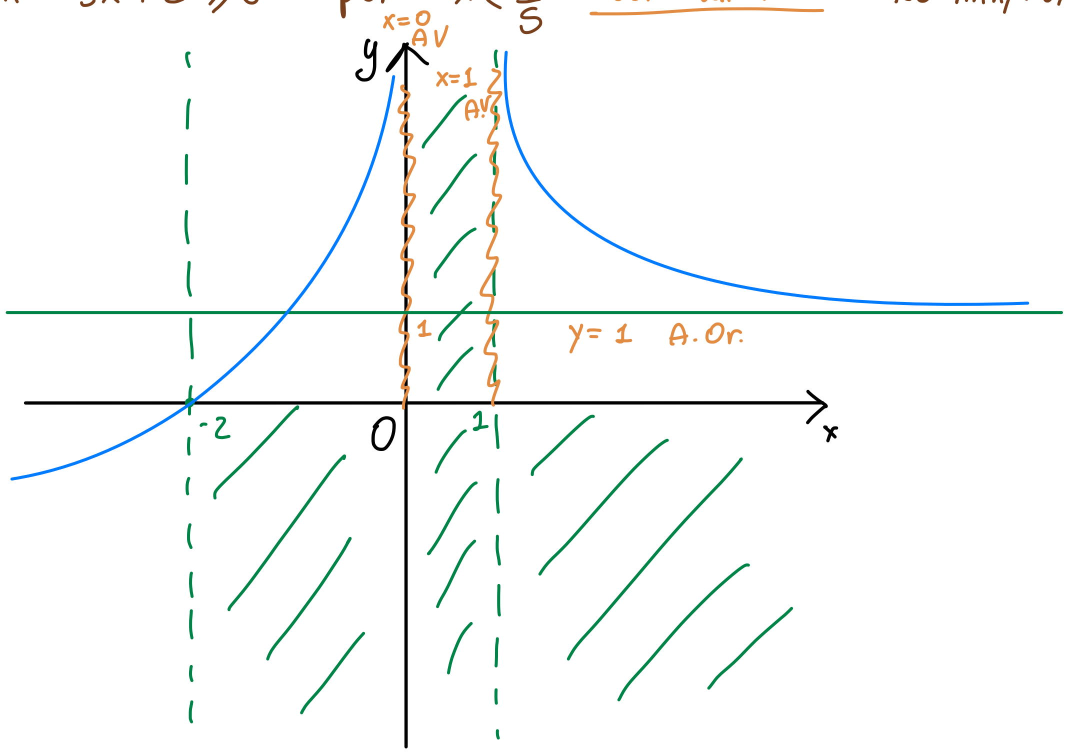
$\displaystyle\lim_{x\to\infty} f(x)=\frac{\cancel{x}(1+0)}{\cancel{x}\sqrt{(1-0)}}=1 \Rightarrow y=1$ Asintoto orizzontale

6) Derivata

$D\left(\dfrac{x+2}{\sqrt{x^2-x}}\right)=\sqrt{x^2-x}-\left[(x+2)\left(-\frac{1}{2}(x^2-x)^{-\frac{3}{2}}\cdot(2x-1)\right)\right]\cdot\dfrac{1}{(\sqrt{x^2-x})^2}$

$=\sqrt{x^2-x}-\left[x+2\cdot\dfrac{1}{2\sqrt{(x^2-x)}}\cdot(2x-1)\right]\cdot\ // = \sqrt{x^2-x}-\left[\dfrac{2x^2-x+4x-2}{2\sqrt{x^2-x}}\right]\cdot //$

$=\sqrt{x^2-x}-2x^2-3x+2\geq 0$ $\quad$ per $\quad x\leq\frac{2}{5}$ $\quad$ Fuori dal D $\qquad$ No min/Max



Inizio funzioni esponenziali

ES 1)

$$f(x) = \frac{e^{1-x}}{x^2-1}$$

1) Dominio $x^2-1 \neq 0$ per $\underline{x \neq \pm 1}$

2) Simmetrie

$$f(x) = \frac{e^{1+x}}{x^2-1} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases} \Rightarrow \text{No Simm}$$

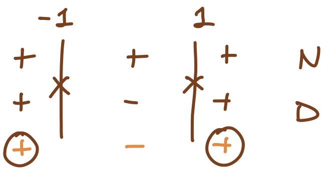
3) Intersezioni

$$\begin{cases} x=0 \\ \frac{e}{-1} = -e \end{cases} \Rightarrow \underline{(0,-e) \in f(x)}$$

$$\begin{cases} y=0 \\ \frac{e^{1-x}}{x^2-1} = 0 \end{cases}$$ per $e^{1-x}=0$ $\nexists x \in \mathbb{R} / e^{1-x}=0$

4) Segno $f(x)>0$ per N: $e^{1+x}>0$, $\forall x \in \mathbb{R}$

D: $x^2-1>0$, per $x > \pm 1$, $a>0, eq>0 \Rightarrow$ Soluzioni esterne $\Rightarrow x<-1 \cup x>1$



$\underline{f(x)>0 \text{ per } x<-1 \cup x>1}$

5) Asintoti

$$\lim_{x \to -1^{\pm}} f(x) = \frac{e^{1+1^{\pm}}}{1^{+}-1} = \frac{e^2}{0^+} = +\infty \Rightarrow \underline{x=-1 \text{ A. Vert}}$$

$$\lim_{x \to 1^{\pm}} f(x) = \frac{e^0}{1^{\pm}-1} = +\infty \leftarrow \underline{x=1 \text{ A. Vert}}$$

$$\lim_{x \to +\infty} f(x) \stackrel{\sim}{=} \frac{\cancel{e^{-x}} \to \emptyset}{x^2} = \emptyset \rightarrow \underline{y=0 \text{ A. Orizz}}$$

6) Crescenza

$$D\left(\frac{e^{1-x}}{x^2-1}\right) = \left[e^{1-x} \cdot (-1) \cdot (x^2-1)\right] - \left[e^{1-x} \cdot (2x)\right] \cdot \frac{1}{\cancel{(x^2-1)^2}} \to \text{Sempre pos}$$

$$= -x^2 e^{1-x} + e^{1-x} - \left[2x^2 e^{1-x}\right] \cdot // = -e^{1-x}(x^2-1) - 2x e^{1-x} = -\cancel{e^{1-x}}(x^2-1+2x) \geq 0$$

(sempre pos)

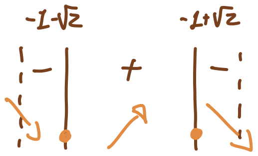
$f'(x)>0$ per $x^2+2x-1 \leq 0$ $\quad \Delta = 4-4(-1) = 8 \Rightarrow x_{1,2} = \frac{-2 \pm \sqrt{8}}{2}$ $\begin{cases} \frac{-2+2\sqrt{2}}{2} \\ \frac{-2-2\sqrt{2}}{2} \end{cases}$

$\Rightarrow a>0$, $eq \leq 0 \Rightarrow$ Soluzione interne

$x_1 = -1+\sqrt{2}$

$x_2 = -1-\sqrt{2}$

$\Rightarrow f'(x)>0$ per $-1-\sqrt{2} < x < -1+\sqrt{2}$



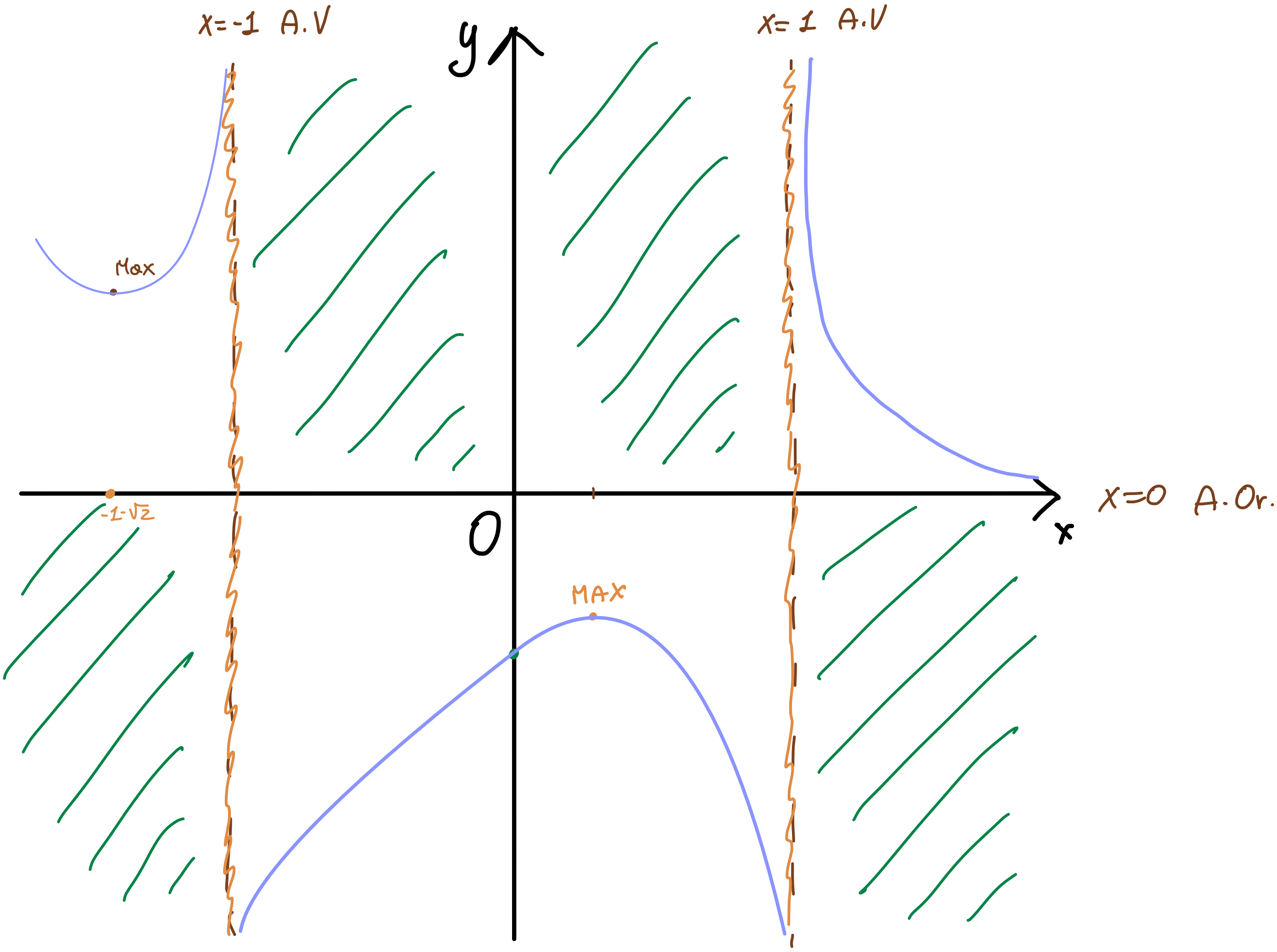
Trovo la y

$$f(-1+\sqrt{2}) = \frac{e^{1-(-1+\sqrt{2})}}{(-1+\sqrt{2})^2-1} = \frac{e^{2+1-\sqrt{2}}}{1-2\sqrt{2}+2-1} = \frac{e^{\text{"}}}{2-2\sqrt{2}} \cong \frac{4.8}{-0,82} = \cong 5,8$$

$$\frac{e^{1-(1+\sqrt{2})}}{(1+\sqrt{2})^2-1} = \frac{e^{1-\sqrt{2}}}{\cancel{1}+2\sqrt{2}+2-\cancel{1}} = \frac{e^{1-\sqrt{2}}}{2\sqrt{2}+2} = \frac{0,66}{4.8} = \underline{-0.13}$$

$\Rightarrow \underline{(-1-\sqrt{2}, 5,8) \text{ Min} \; ; \; (-1+\sqrt{2}, 0,13) \text{ MAX}}$

$\uparrow -2.4 \qquad \uparrow 0.4$



x=-1 A.V
x=1 A.V
y
Max
-1-√2
O
MAX
x
x=0 A.Or.

ES 2: $f(x) = (x^2-1)e^x$

1) Dominio $= \mathbb{R}$

2) Simmetrie $f(-x) = (x^2-1)e^{-x} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases} \Rightarrow$ No Simm

3) Intersezioni

$\begin{cases} x = 0 \\ y = -1 \end{cases} \Rightarrow (0,-1) \in f(x)$

$\begin{cases} y = 0 \\ (x^2-1)e^x = 0 \end{cases}$ per $x^2-1=0$, $x = \pm 1$ $\Rightarrow \boxed{\begin{matrix}(1,0) \in f(x) \\ (-1,0) \in f(x)\end{matrix}}$

4) Segno: $f(x) \geq 0$ per $(x^2-1) \geq 0$, $x^2 \geq 1$ per $x \geq \pm 1$

$a>0$, $eq>0 \Rightarrow$ Valori esterni

$f(x) \geq 0$ per $x \leq -1 \cup x \geq 1$

5) Asintoti

$\lim_{x \to \infty} f(x) = (\infty - 1)\cdot\infty = +\infty \Rightarrow$ No A. Oriz.

$\lim_{x \to \infty} \frac{f(x)}{x} = \frac{x^{\cancel{2}}(1)\cdot e^x}{\cancel{x}} = +\infty \Rightarrow$ NO A. ob.

$\lim_{x \to -\infty} f(x) = (+\infty - 1)e^{-\infty} = +\infty \cdot 0 \Rightarrow \sim \frac{\cancel{x^2} \to +\infty}{\cancel{e^{-x}} \to +\infty}$ ma $e^x >> x^2 \to 0$ → $y = 0$ A. Oriz

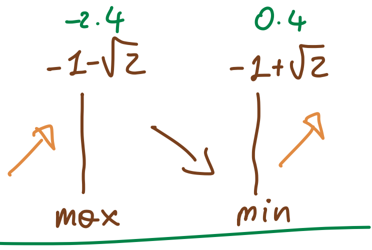
6) Derivate:

sempre pos

$D[(x^2-1)e^x] = 2xe^x + (x^2-1)e^x = 2xe^x + x^2e^x - e^x = \cancel{e^x}(2x + x^2 - 1) > 0$

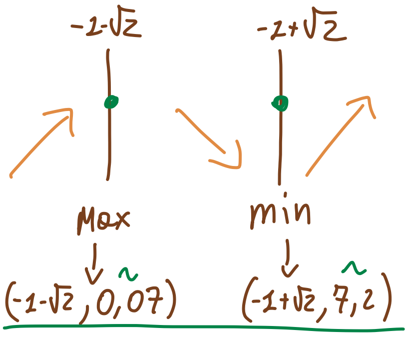
$f'(x) > 0$ per $x^2 + 2x - 1 > 0$, $\Delta = 4 - 4(-1) = 8$

$x_{1,2} = \frac{-2 \pm 2\sqrt{2}}{2} \begin{cases} \frac{-2+2\sqrt{2}}{2} /2 = -1+\sqrt{2} \sim 0,4 \\ \frac{-2-2\sqrt{2}}{2} /2 = -1-\sqrt{2} \sim -2.4 \end{cases}$



$a>0$, $eq>0 \Rightarrow$ Sol esterne

$f'(x) > 0$ per $x < -1-\sqrt{2} \cup x > -1+\sqrt{2}$



$f(-1-\sqrt{2}) = ((-1-\sqrt{2})^2 - 1)\cdot e^{-1-\sqrt{2}} = (\cancel{1} - 2\sqrt{2} + 2 - \cancel{1})e^{-1-\sqrt{2}}$

$= (-2\sqrt{2}+2)e^{-1-\sqrt{2}} = -0,8 \cdot 0,08 \simeq 0,07$

$f(-1+\sqrt{2}) = ((-1+\sqrt{2})^2 - 1)e^{-1+\sqrt{2}} = (\cancel{1} + 2\sqrt{2} + 2 - \cancel{1})e^{-1+\sqrt{2}}$

$= (2\sqrt{2}+2)e^{-1+\sqrt{2}} = 4.8 \cdot 1.51 \simeq 7,2$

Derivata II $D[e^x(2x + x^2 - 1)] = e^x(x^2+2x-1) + e^x(2x+2) = x^2e^x + 2xe^x - e^x + 2xe^x + 2e^x$

sempre pos

$= x^2e^x + 4xe^x + e^x = \cancel{e^x}(x^2+4x+1) \geq 0$

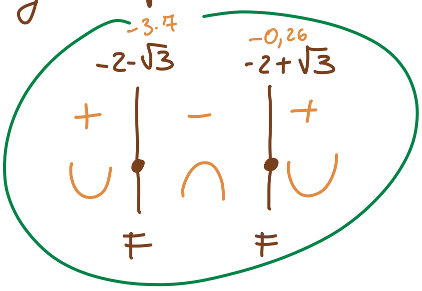
$x_{1,2} = \frac{-4 \pm 2\sqrt{3}}{2} \begin{cases} \frac{-4+2\sqrt{3}}{2} /2 = -2+\sqrt{3} \\ \frac{-4-2\sqrt{3}}{2} /2 = -2-\sqrt{3} \end{cases}$

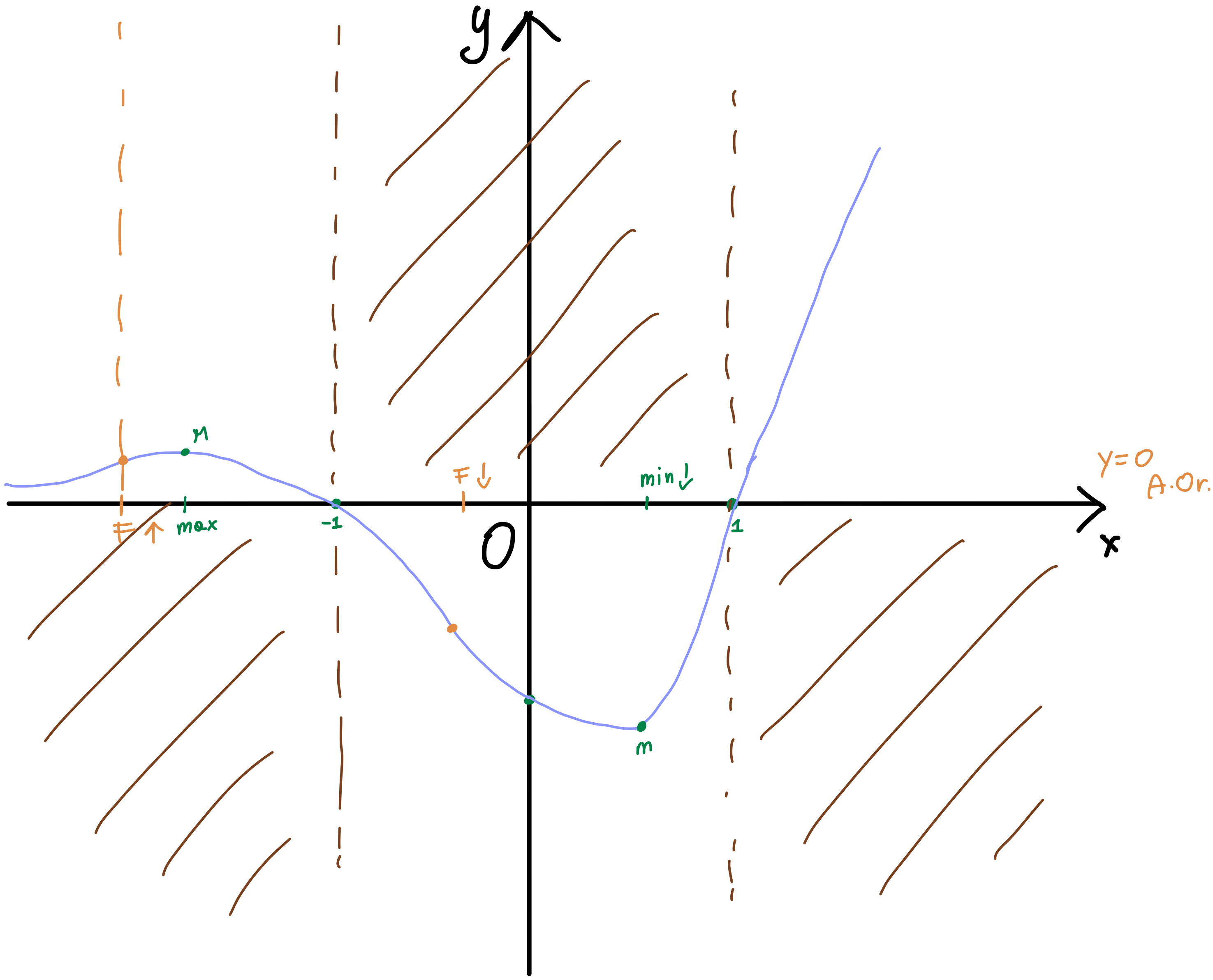
$f''(x) > 0$ per $x^2 + 4x + 1 > 0$

$\Delta = 16 - 4 = 12$

$a>0$, $eq>0$

Val esterni





y
x
O
M
max
-1
F
F
min
1
m
y=0
A.Or.

ES 5: $f(x) = x^2 e^{3x+5}$

1) Dominio $= \mathbb{R}$

2) Simmetrie: $f(-x) = x^2 e^{-3x+5}$ $\begin{cases} \neq -f(x) & \text{NO Dispari} \\ \neq f(x) & \text{No pari} \end{cases}$

3) Intersezioni:

$\begin{cases} x=0 \\ y=0 \end{cases} \Rightarrow (0,0) \in f(x)$

$\begin{cases} y=0 \\ x^2 e^{3x+5}=0 \end{cases}$ per $x^2=0 \Rightarrow x=0$

4) Segno $f(x)>0$

$x^2 e^{3x+5} > 0 \quad \forall x \in D$

5) Asintoti

$\lim_{x\to\infty} f(x) \simeq \underbrace{x^2}_{+\infty} \cdot \underbrace{e^x}_{+\infty} \Rightarrow +\infty \quad \Rightarrow$ NO A. Oriz.

$\lim_{x\to+\infty} \frac{f(x)}{x} \simeq \frac{x^{\cancel{2}} \cdot e^x}{\cancel{x}} = +\infty \quad \Rightarrow$ NO A. Ob.

$\lim_{x\to-\infty} f(x) = \infty \cdot 0 \simeq \underbrace{e^x}_{\to 0} \cdot \underbrace{x^2}_{\to+\infty} = 0$

$\Rightarrow \frac{1}{\frac{1}{e^x}} \cdot x^2 = \frac{x^2}{\underbrace{\frac{1}{e^x}}_{\to+\infty}}$ $\quad e^x \gg x^2 = 0$

lim $\to 0$

$\Rightarrow y = 0$ Asintoto Verticale

6) Derivata

$f'(x) = 2x\left(e^{3x+5}\right) + x^2 e^{3x+5} \cdot 3 = 2xe^{3x+5} + 3e^{3x+5}x^2 \Rightarrow xe^{3x+5}(2+3x)$

$f'(x) \geq 0$ per $\begin{cases} x>0 \\ 2+3x>0 \end{cases}$ $\begin{cases} x>0 \\ x>-\frac{2}{3} \end{cases}$

	$-\frac{2}{3}$		0	
$-$		$+$		$+$
$-$		$-$		$+$
$\oplus$		$-$		$\oplus$

$f'(x) \geq 0$ per $x \leq -\frac{2}{3} \cup x \geq 0$

$-\frac{2}{3}$ max, 0 min

$f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^2 \cdot e^{-2+5} = \frac{4}{9}e^3 \simeq 8.9 \Rightarrow \left(-\frac{2}{3}, \frac{4}{9}e^3\right)$ Max ~ 8.9

$f(0) = 0 \Rightarrow (0,0)$ min

Deriv II^a

$f''(x) = D\left(e^{3x+5}(3x^2+2x)\right) = \left[3e^{3x+5}(3x^2+2x)\right] + e^{3x+5} \cdot \left[6x+2\right] =$

$= e^{3x+5}(9x^2+6x) + e^{3x+5}(6x+2) = e^{3x+5}(9x^2+6x+6x+2) = e^{3x+5}(9x^2+12x+2)$

$f''(x) \geq 0$ per $9x^2+12x+2 \geq 0 \quad \Delta = 144 - 4 \cdot 9 \cdot 2 = 72$

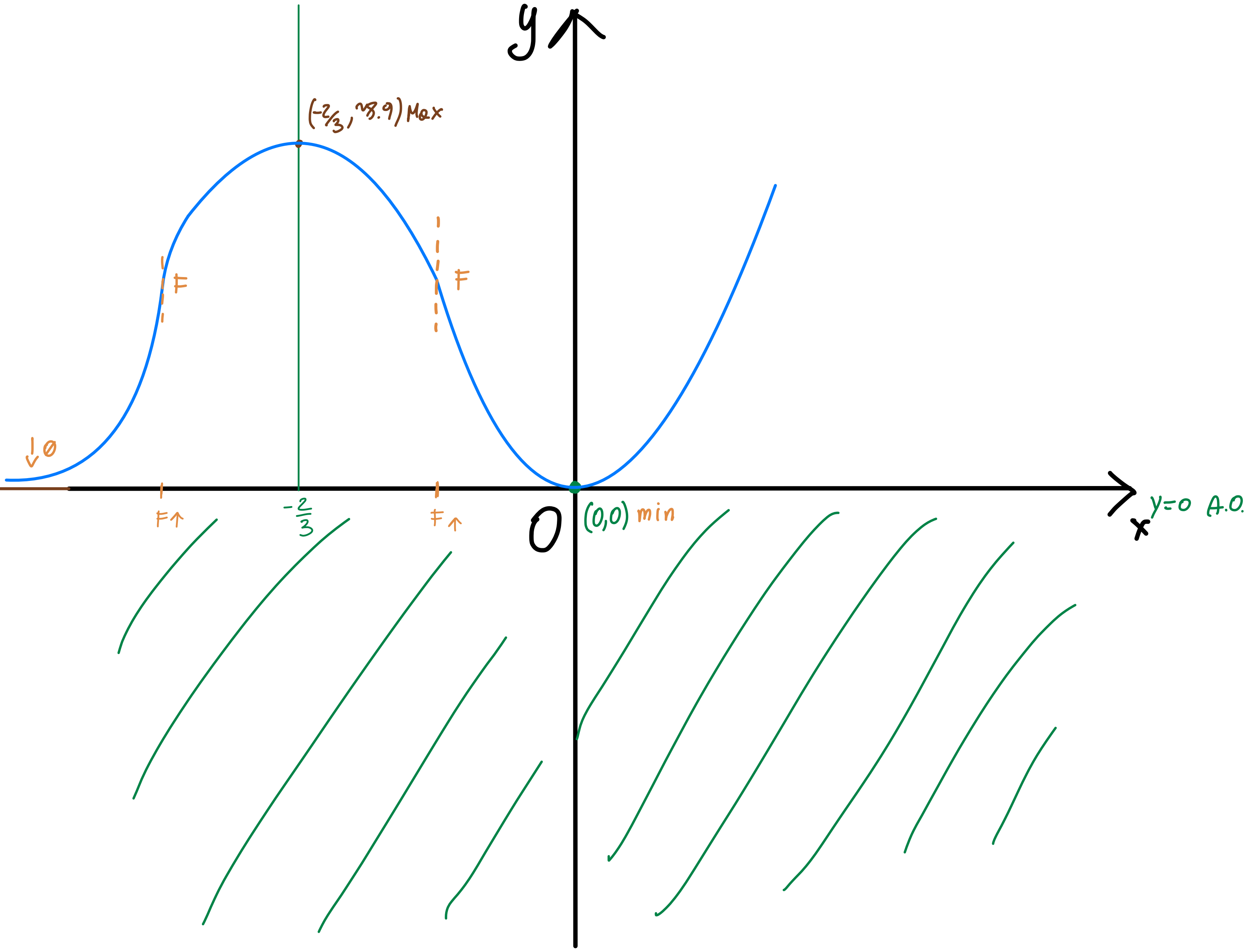
$\sqrt{72} = \sqrt{9 \cdot 8} = \sqrt{3^2 \cdot 2^2 \cdot 2} = 6\sqrt{2}$

$x_{1,2} = \frac{-12 \pm 6\sqrt{2}}{18}$ $\begin{cases} \frac{-12+6\sqrt{2}}{18} /6 = \frac{-2+\sqrt{2}}{3} \sim -0.1 \\ \frac{-12-6\sqrt{2}}{18} /6 = \frac{-2-\sqrt{2}}{3} \sim -1.1 \end{cases}$

$a>0$, eq >0

sol est

	-1.1		$-2/3$		-0.1		0
$+$		$-$				$+$	
$\cup$	F	$\cap$			F	$\cup$	



y
x
O
$(-\frac{2}{3}, 28.9)$ Max
F
F
↓0
F↑
$-\frac{2}{3}$
F↑
(0,0) min
y=0 A.O.

Es: $f(x) = \sqrt{1-e^x}$

1) Dominio $\quad 1-e^x \geq 0 \; ; \; e^x < 1 \; ; \; \ln|e^x| < \ln(1) \; ; \; x < 0$

$\Rightarrow D = (-\infty, 0]$

2) Simm $\quad f(-x) = \sqrt{1-e^{-x}} = \begin{cases} \neq -f(x) \\ \neq f(x) \end{cases}$

3) Segno $\quad f(x) > 0 \quad \forall x \in D$

4) Intersez

$\begin{cases} x = 0 \\ \sqrt{1-1} = 0 \end{cases} \Rightarrow (0,0) \in f(x)$

$\begin{cases} y = 0 \\ \sqrt{1-e^x} = 0 \end{cases}$ per $1 - e^x = 0 \; ; \; \ln(e^x) = \ln|1| \; ; \; x = 0$

5) Asintoti

$\lim_{x \to \infty} f(x) = \sqrt{1-(+\infty)} = \sqrt{-\infty} \quad \nexists x \in \mathbb{R}$

$\lim_{x \to -\infty} f(x) = \sqrt{1-\underbrace{e^x}_{0}} \to 1 \Rightarrow y = 1$ A. Orizz

6) Derivate

$D(\sqrt{1-e^x}) = D\left((1-e^x)^{\frac{1}{2}}\right) = \frac{1}{2} \frac{1}{\sqrt{1-e^x}} \cdot (-e^x) = \ominus \frac{e^x}{2\sqrt{1-e^x}} > 0$ per $\nexists x \in \mathbb{R}$

$e^x \rightarrow$ sempre pos

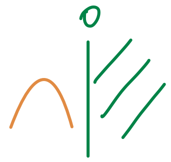
$2\sqrt{1-e^x} \rightarrow$ sempre

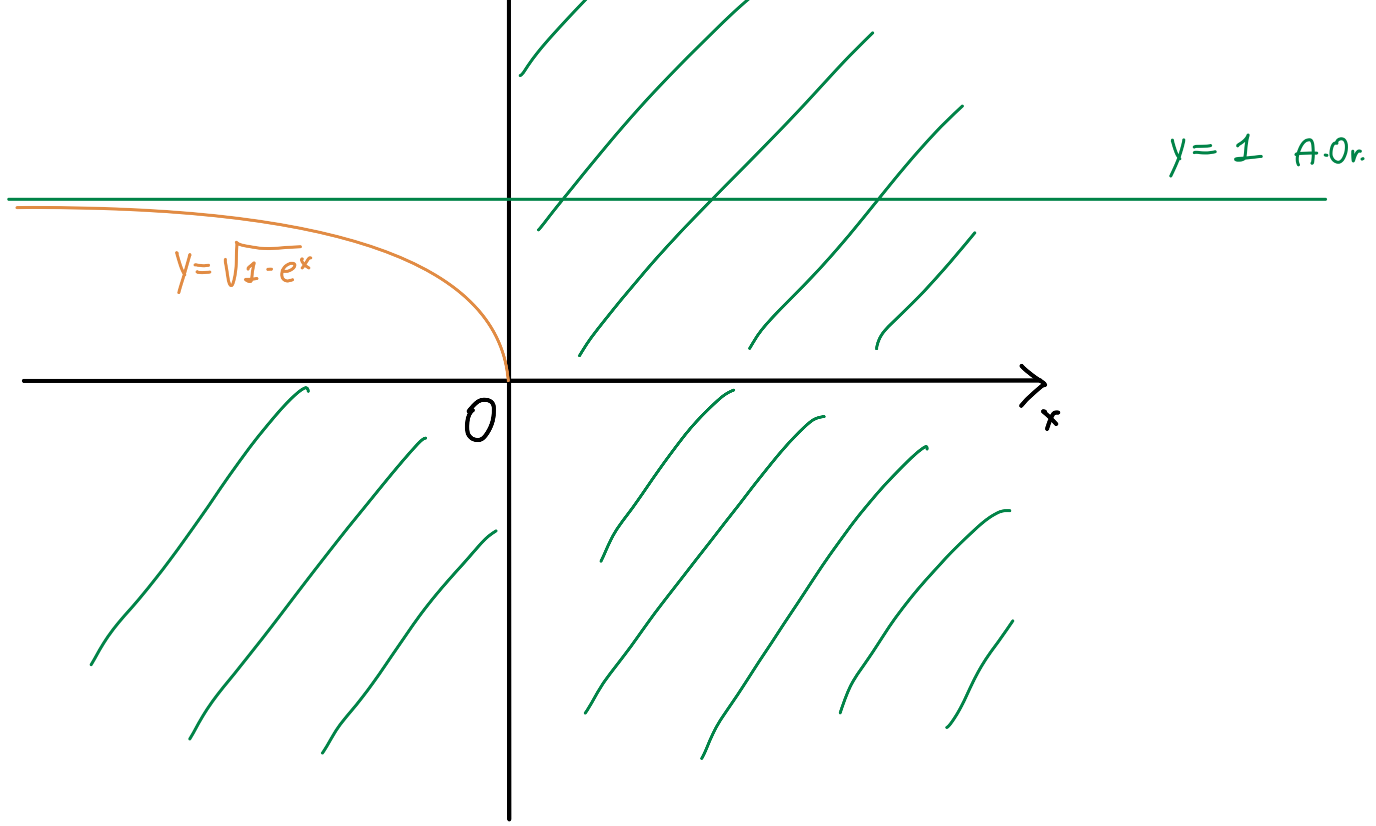
$$D''\left(-\frac{e^x}{2\sqrt{1-e^x}}\right) = -e^x\left(2\sqrt{1-e^x}\right) - \left[-e^x\left(-\frac{e^x}{\sqrt{1-e^x}}\right)\right] \cdot \frac{1}{4(1-e^x)} = -2e^x\sqrt{1-e^x} - \left[\frac{e^{2x}}{\sqrt{1-e^x}}\right] \cdot \frac{1}{//}$$

$$\frac{-2e^x(1-e^x) - e^{2x}}{\sqrt{1-e^x}} = \frac{-2e^x + \cancel{2e^{2x}} - \cancel{2e^{2x}}}{\sqrt{1-e^x}} \cdot \frac{1}{//} = \ominus \frac{2e^x}{\sqrt{1-e^x}}$$

$2e^x \rightarrow$ sempre pos

$\sqrt{1-e^x} \rightarrow$ sempre pos





ES 72)

$f(x) = e^{-x^2}$ 1) Dominio : $\mathbb{R}$

2) Simm $f(-x) = e^{-x^2} = f(x) \Rightarrow$ Pari

3) Intersezioni

$\begin{cases} x=0 \\ e^0 = 1 \end{cases} \Rightarrow (0,1) \in f(x)$

↳ Probabile max/min

$\begin{cases} y=0 \\ e^{-x^2} = 0 \end{cases} \quad \nexists x \in \mathbb{R}$

4) Segno

$f(x) > 0 \quad \forall x \in \mathbb{R}$

5) Derivata I°

$D(e^{-x^2}) = e^{-x^2} \cdot (-2x) = -2xe^{-x^2} > 0$ per $x < 0$

0

$f(0) = 1 \Rightarrow (0,1)$ Max

6) Limiti

$\lim_{x \to \pm\infty} f(x) = e^{-\infty} \to 0 \Rightarrow y = 0$ A. Orizz

7) Deriv II

$D(-2xe^{-x^2}) = -2e^{-x^2} - 2xe^{-x^2} \cdot (-2x) = -2e^{-x^2} + 4x^2e^{-x^2} = 2e^{-x^2}(-1+2x^2) > 0$

3 2 2

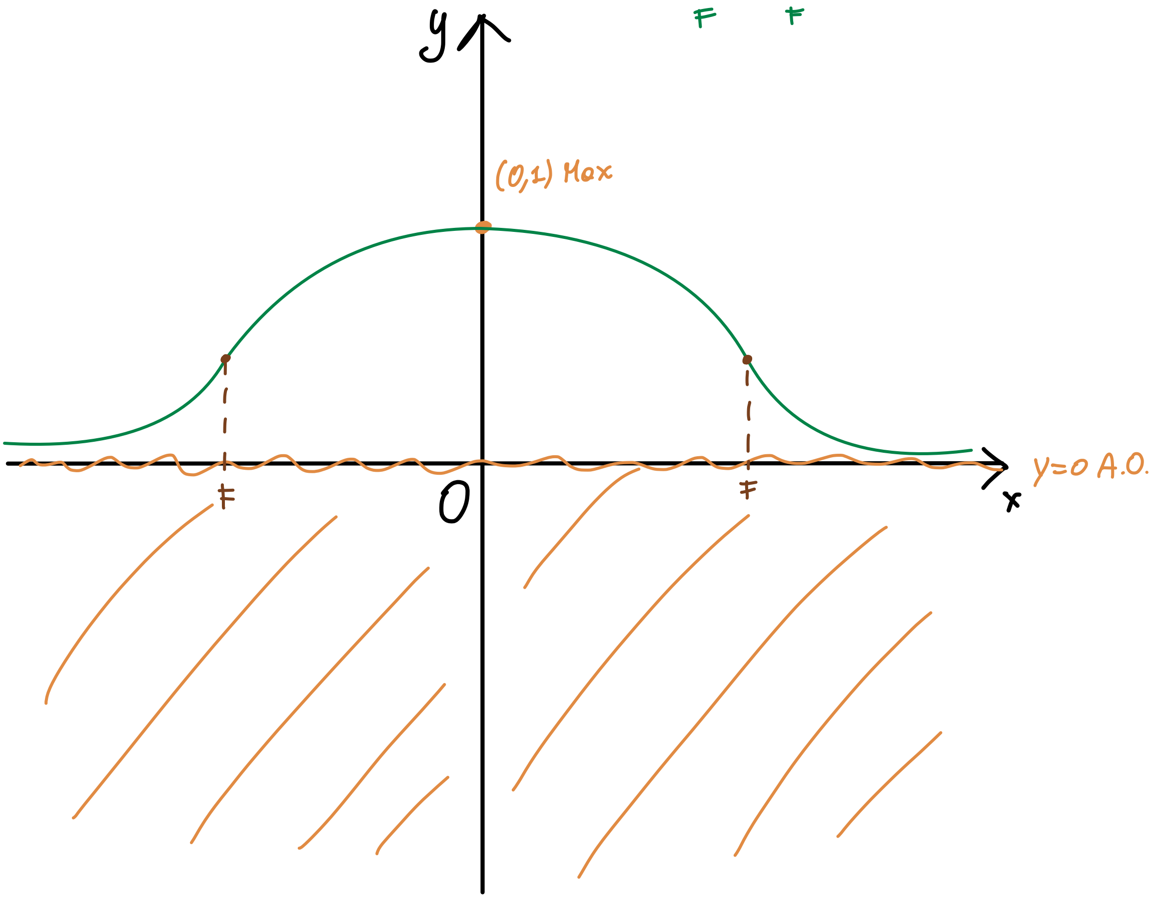
per $2x^2 - 1 > 0$; $x > \pm \frac{1}{2}$

$a > 0$, $eq > 0 \Rightarrow$ Val esterni

$-\frac{1}{2}$ $\frac{1}{2}$

+ − +

F F



ES 73)

$f(x) = e^x + 2e^{-x}$ 1) Dominio: $\mathbb{R}$

2) Simm $f(-x) = e^{-x} + 2e^x \Rightarrow$ No Simm

3) Intersezioni

$\begin{cases} x=0 \\ 1+2=3 \end{cases} \Rightarrow (0,3) \in f(x)$ $\begin{cases} y=0 \\ \nexists x \in \mathbb{R} \end{cases}$

4) Segno $f(x) > 0 \quad \forall x \in \mathbb{D}$

5) Asintoti

$\lim_{x\to+\infty} f(x) = \cancel{e^{x}}^{+\infty} + 2\cancel{e^{-x}}^{0} = +\infty$ $\lim_{x\to-\infty} \cancel{e^{x}}^{0} + \cancel{e^{-x}}^{+\infty} = +\infty$ No A.V.

$\lim_{x\to+\infty} \frac{f(x)}{x} = \frac{e^{+\infty} + e^{-\infty}}{+\infty} = \frac{e^x}{\cancel{x}} + \frac{e^{-x} \to 0}{\cancel{x} \to +\infty}$ $e^x >> x \Rightarrow +\infty \Rightarrow$ No A. Ob.

6) Derivate

$D(e^x + 2e^{-x}) = e^x + 2e^{-x} \cdot (-1) = e^x - 2e^{-x} > 0$ per $e^x > 2e^{-x} \Rightarrow e^x - \frac{2}{e^x} > 0$

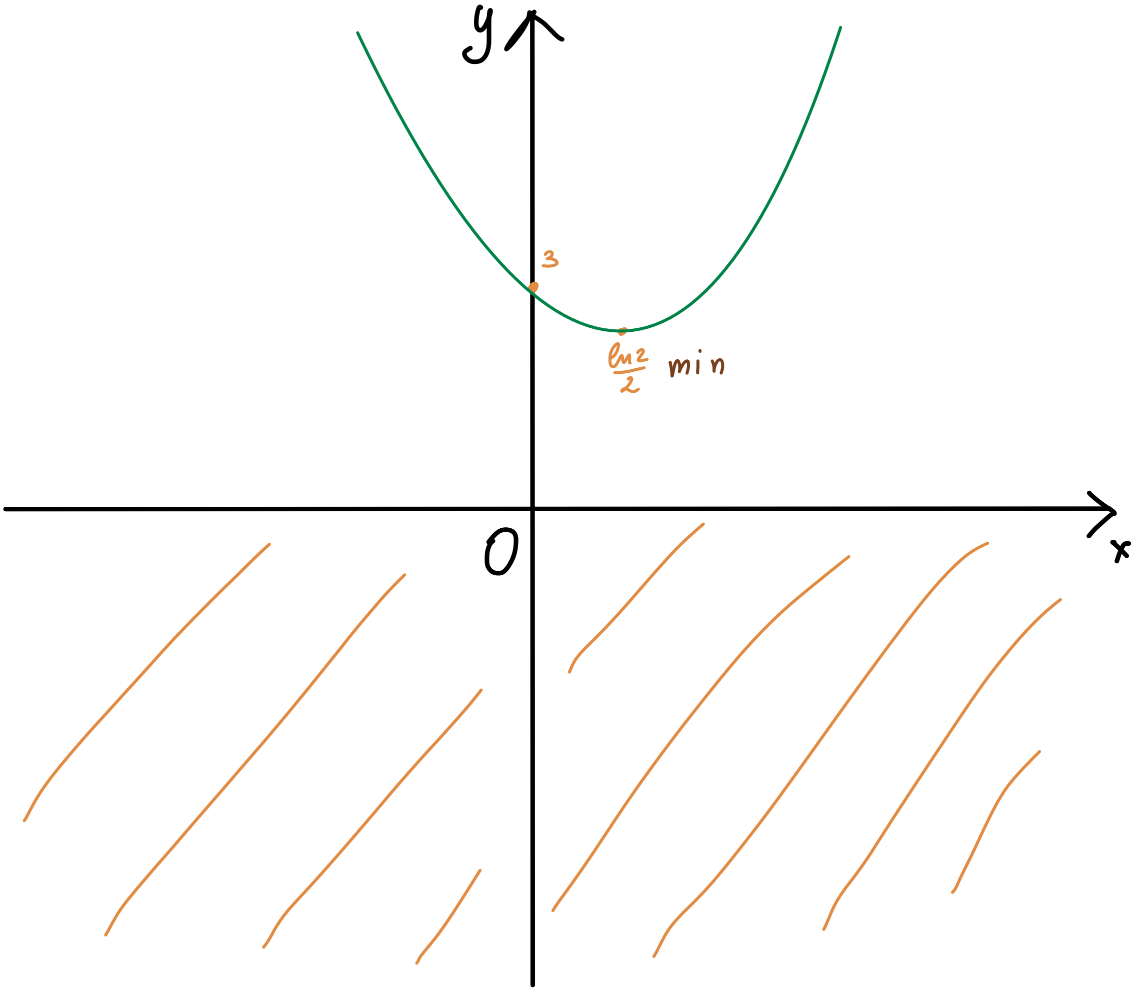
$\Rightarrow \frac{e^{2x} - 2}{e^x} > 0$ per $e^{2x} - 2 > 0$; $e^{2x} > 2 \Rightarrow 2\ln x > \ln|2| \Rightarrow x > \frac{1}{2}\ln 2 \sim 0{,}34$

$f(\frac{\ln 2}{2}) = 1{,}4 + 1{,}4 \cong 2{,}8$

$\frac{\ln 2}{2}$: $-$ $\searrow$ | $+$ $\nearrow$

Deriv II $D(e^x - 2e^{-x}) = e^x + 2e^{-x} > 0 \quad \forall x \in \mathbb{R}$

0: $\cup$ | $\cup$



ES 74)

$f(x) = \dfrac{e^x}{e^x-1}$ D: $e^x - 1 \neq 0$ per $e^x \neq 1$; $\ln(e^x) \neq \ln(1) \Rightarrow x \neq 0$

2) Simm: $f(-x) = \dfrac{e^{-x}}{e^{-x}-1} \Rightarrow$ NO Simm

3) Intersez.

$\begin{cases} x=0 \\ \dfrac{1}{0} \end{cases}$ $\nexists x \in \mathbb{R}$ $\begin{cases} y=0 \\ \dfrac{e^x}{e^x-1}=0 \end{cases}$ $\exists x \in \mathbb{R}$

4) Segno $f(x)>0$ per $e^x>1$, $\ln e^x > \ln 1 \Rightarrow x>0$

5) Asintoti

$\lim_{x\to 0^+} f(x) = \dfrac{1}{1^+-1} = \dfrac{1}{0^\pm} = \pm\infty \Rightarrow x=0$ A.V

$\lim_{x\to -\infty} f(x) = \emptyset$ $y=0$ A. Or.

$\lim_{x\to +\infty} f(x) = \dfrac{+\infty}{+\infty} = \dfrac{\cancel{e^x}}{\cancel{e^x}(1-0} = 1 \Rightarrow y=1$ A. Oriz

6) Deriv I $D\left(\dfrac{e^x}{e^x-1}\right) = e^x(e^x-1) - e^x(e^x) \cdot \dfrac{1}{(e^x-1)^2}$

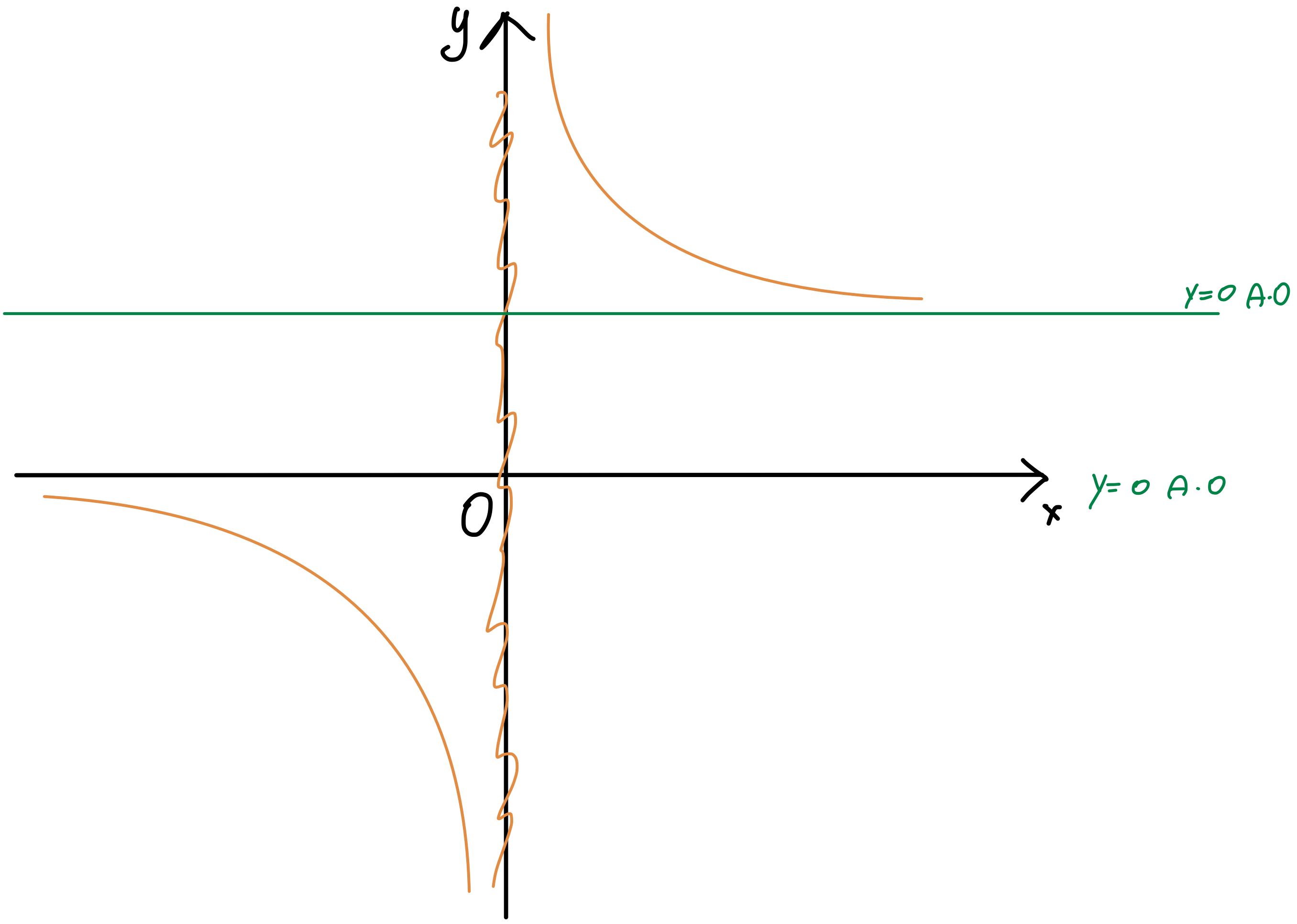
$= \cancel{e^{2x}} - e^x - \cancel{e^{2x}} \cdot \dfrac{1}{//} = \dfrac{-e^x}{(e^x-1)^2} > 0$ $\exists x \in \mathbb{R}$

0
↘ | ↘

Deriv II

$D\left(-\dfrac{e^x}{(e^x-1)^2}\right) = \dfrac{e^x(e^x+1)}{(e^x-1)^3} > 0$ per $e^x > 1 \Rightarrow x > \ln 1 \Rightarrow x > 0$

0
− | +
∩ x ∪



ES 75)

$$f(x)=\begin{cases} e^{-\frac{1}{x^2}} & \text{per } x \neq 0 \\ 0 & \text{per } x=0 \end{cases}$$

1) Dominio: $\underline{\mathbb{R}}$

2) Intersezioni

$\begin{cases} x=0 \\ y=0 \end{cases} \Rightarrow \underline{(0,0) \in \mathbb{R}}$ $\qquad \begin{cases} y=0 \\ e^{-\frac{1}{x^2}}=0 \end{cases} \quad \nexists x \in \mathbb{R}$

3) Simm

$f(-x)= e^{-\frac{1}{x^2}} = \underline{f(x) \Rightarrow \text{PARI}}$

4) Segno

$f(x)>0 \qquad e^{-1\cdot\frac{1}{x^2}} = \dfrac{1}{e^{\frac{1}{x^2}}} \underline{>0 \ \forall x \in \mathbb{R} - \{0\}}$

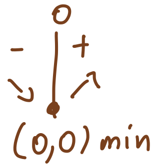
5) Asintoti

$\lim\limits_{x\to\pm\infty} f(x) = \dfrac{1}{e^{\frac{1}{x^2}}} \to \dfrac{1}{1} \to 1 \Rightarrow \underline{y=1 \text{ A. Orizz}}$

6) Deriv I

$D\left(-\frac{1}{x^2}\right) = D\left(-x^{-2}\right) \Rightarrow +2x^{-3} = \dfrac{2}{x^3}$

$D\left(e^{-\frac{1}{x^2}}\right) = \underbrace{e^{-\frac{1}{x^2}}}_{\text{Sempre pos}} \cdot \dfrac{\overset{\text{Sempre pos}}{2}}{x^3} >0$ per $x^3>0 \Rightarrow \underline{x>0}$

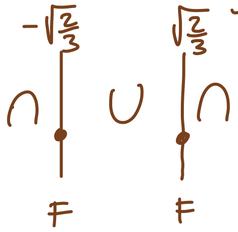


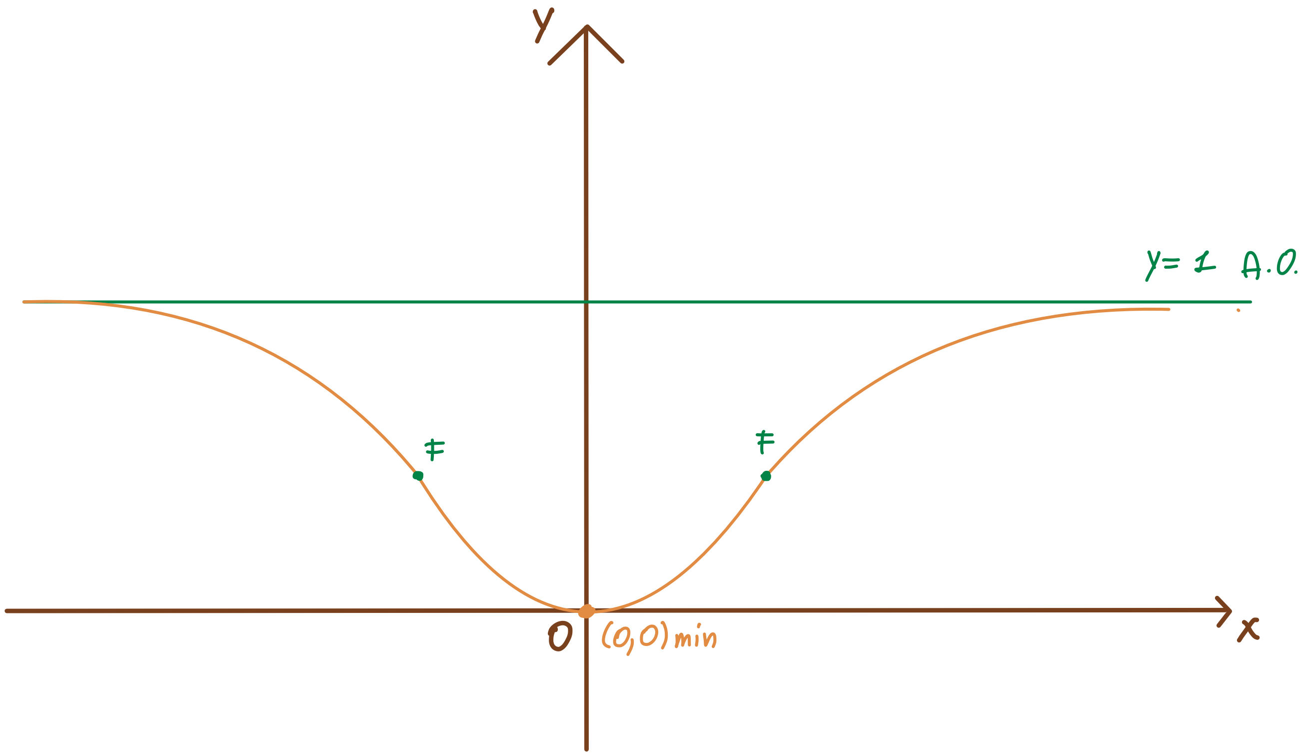
Deriv II: $D\left(\dfrac{2e^{-\frac{1}{x^2}}}{x^3}\right) \left[2e^{-\frac{1}{x^2}}\cdot\left(2\cancel{x^{-3}}\right)\right]\cdot \cancel{x^3} + \left[2e^{-\frac{1}{x^2}}\cdot 3x^2\right]$

$= 4e^{-\frac{1}{x^2}} + 6x^2 e^{-\frac{1}{x^2}} = 2\underbrace{\cancel{e}^{-\frac{1}{x^2}}}_{\text{Sempre pos}}\left(2-3x^2\right) >0$ per $2-3x^2>0 \quad x<\pm\sqrt{\frac{2}{3}}$

$a<0,\ eq>0 \Rightarrow$ Val interni

$f''(x)>0$ per $-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$





ES 76)

$f(x) = x^2 \cdot e^{-2x}$ 1) Dominio $\mathbb{R}$

2) Simm: $f(-x) = x^2 e^{2x} \Rightarrow$ No Simm

3) Intersezioni

$\begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow \underline{(0,0) \in f(x)}$ $\begin{cases} y = 0 \\ \text{per } x = 0 \end{cases}$

4) Segno $\underline{f(x) > 0 \quad \forall x \in \mathbb{R}}$

5) Asintoti

$\lim_{x \to +\infty} f(x) = +\infty \cdot \cancel{e^{-\infty}}^{\nearrow 0} \Rightarrow +\infty \cdot 0$; $\frac{\cancel{x^2}^{\to +\infty}}{\cancel{e^{2x}}_{\to +\infty}}$ $e^x >> x^2 \sim \frac{n}{+\infty} \to \emptyset$

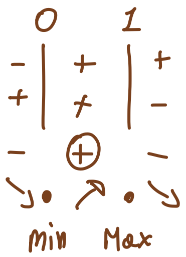
$\underline{y = 0 \quad A.\ Or}$

6) Derivata I°

$D(x^2 e^{-2x}) \Rightarrow \left(\frac{x^2}{e^{2x}}\right) = \frac{2x\, e^{2x} - x^2 \cdot e^{2x} \cdot 2}{(e^{2x})^2} = \frac{2xe^{2x} - 2x^2 e^{2x}}{(e^{2x})^2}$

$\Rightarrow \frac{2\cancel{e^{2x}}(1 - x^2)}{(e^{2x})^{\cancel{2}}} = \frac{2(x - x^2)}{e^{2x}} > 0$ per $x - x^2 > 0$, $x(1-x) > 0$ $\hookrightarrow \underline{x > 0}$

$\underline{x < 1}$



$f(0) = \emptyset \Rightarrow \underline{(0,0)\ min}$

$f(1) = e^{-2} \simeq 0.13 \Rightarrow \underline{(1, e^{-2})\ Max}$

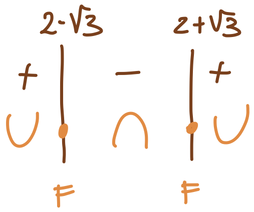
Deriv II: $D\left(\frac{2(x - x^2)}{e^{2x}}\right) = \frac{2x - 2x^2}{e^{2x}} = (2 - 4x) \cdot e^{2x} - \left[(2x - 2x^2) \cdot e^{2x} \cdot 2\right] \cdot \frac{1}{(e^{2x})^2}$

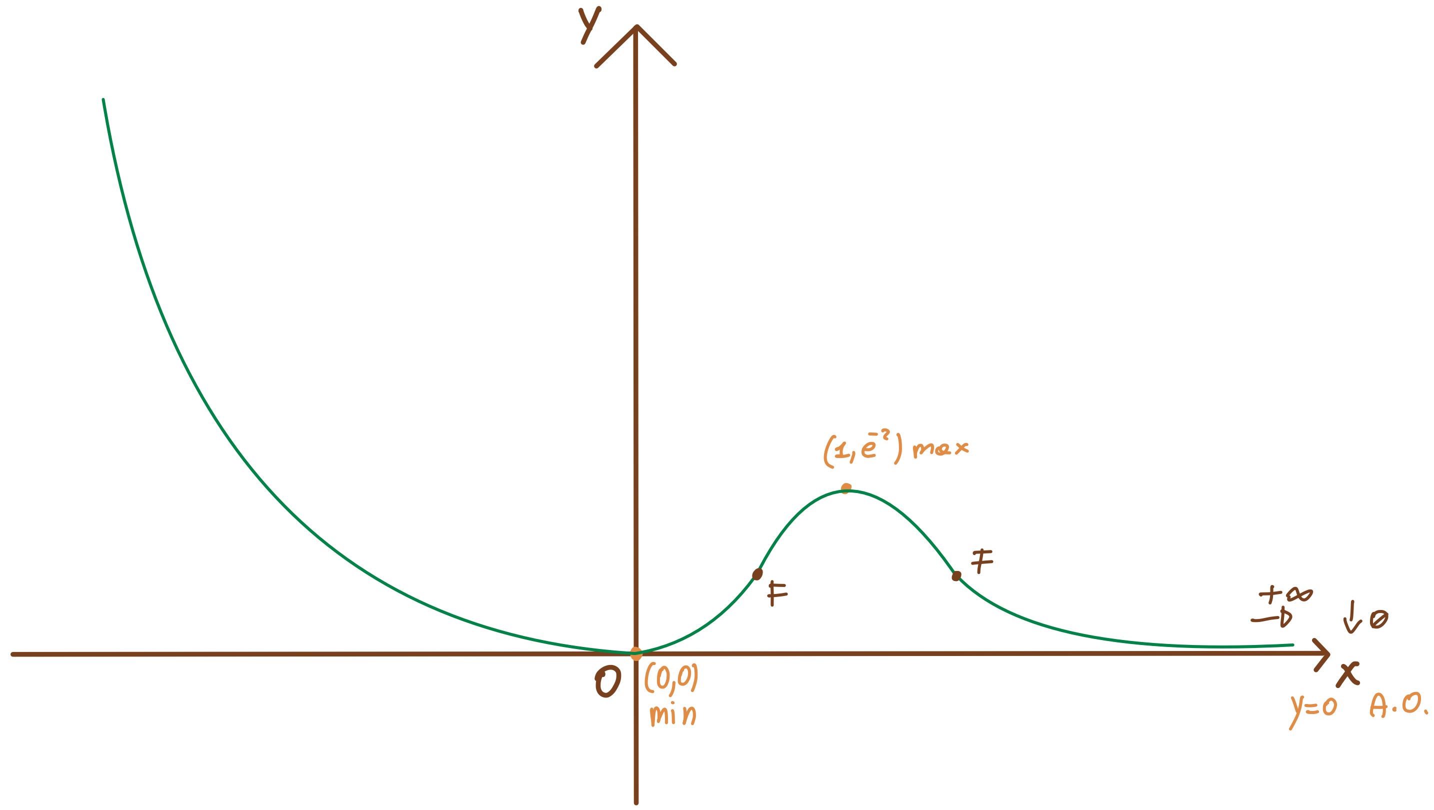
$= 2e^{2x} - 4xe^{2x} - \left[4xe^{2x} - 4x^2 e^{2x}\right] \cdot \frac{1}{\text{//}} = \frac{2e^{2x} - 8xe^{2x} + 4x^2 e^{2x}}{(e^{2x})^2}$

$= \frac{2\cancel{e^{2x}}(1 - 4x + x^2)}{(e^{2x})^{\cancel{2}}} = \frac{2(x^2 - 4x + 1)}{e^{2x}} > 0$ per $x^2 - 4x + 1 > 0$ $\Delta = 16 - 4 = 12$

$x_{1,2} = \frac{4 \pm 2\sqrt{3}}{2}$ $\begin{matrix} 2 + \sqrt{3} \\ 2 - \sqrt{3} \end{matrix}$

$a > 0$, eq $> 0 \Rightarrow \underline{\text{Val est}}$





y
x
O
$(1, e^{-2})$ max
F
F
$(0,0)$
min
$+\infty$
$\to$
$\downarrow 0$
$y=0$ A.O.

ES: 77) $f(x) = x^4 \cdot e^{-\frac{x^2}{2}} = \dfrac{x^4}{\sqrt{e^{x^2}}} \Rightarrow$ 1) Dominio $e^{x^2} > 0 \ \forall x \in \mathbb{R}$

$\Rightarrow D: \underline{\forall x \in \mathbb{R}}$

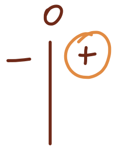
2) Simm:

$f(-x) = x^4 \cdot e^{-\frac{x^2}{2}} = f(x) \Rightarrow \underline{\text{PARI}}$

3) Intersezioni

$\begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow (0,0) \in f(x)$ $\qquad \begin{cases} y = 0 \\ \text{per } x = 0 \end{cases}$

4) Segno $f(x) > 0$ per $x > 0$



5) Asintoti

$\lim_{x \to \pm\infty} f(x) = +\infty \cdot 0 \Rightarrow \dfrac{x^4}{e^{\frac{x^2}{2}}} \sim \dfrac{x^n}{e^x} \quad e^x \gg x^n \Rightarrow \to 0 \qquad \underline{y = 0 \ \text{A.Or}}$

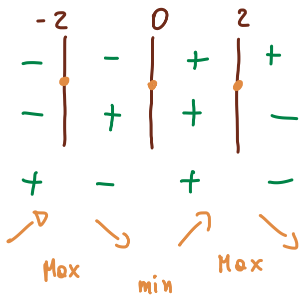
6) Derivate

$D(x^4 e^{-\frac{x^2}{2}}) = D\left(\dfrac{x^4}{e^{\frac{x^2}{2}}}\right) = \left(4x^3 e^{\frac{x^2}{2}}\right) - \left[x^4 \cdot \left(e^{\frac{x^2}{2}} \cdot x\right)\right] \cdot \dfrac{1}{\left(e^{\frac{x^2}{2}}\right)^2}$

$= \left(4x^3 e^{\frac{x^2}{2}}\right) - \left(x^5 e^{\frac{x^2}{2}}\right) \cdot \dfrac{1}{//} = x^3 e^{\frac{x^2}{2}}\left(4 - x^2\right) > 0$

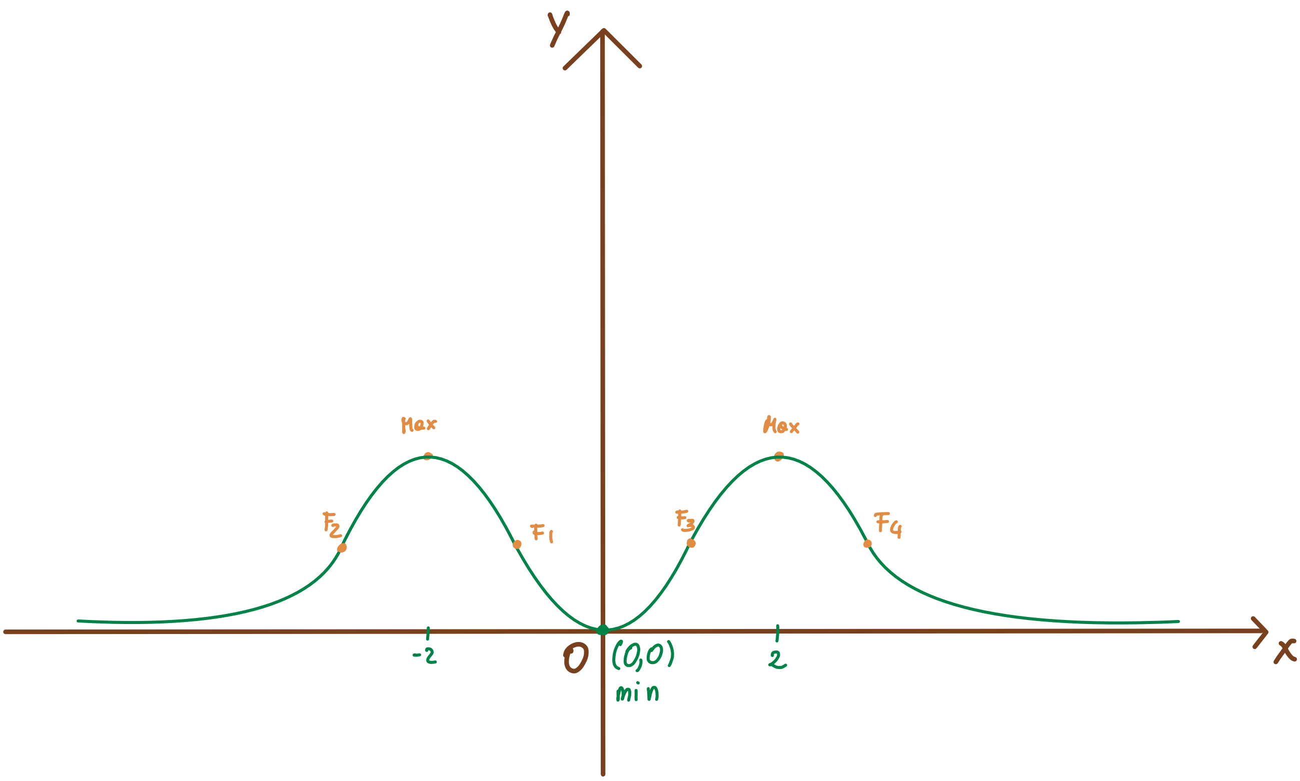
$\begin{cases} x^3 > 0 \text{ per } \underline{x > 0} \ \ 1 \\ 4 - x^2 > 0 \text{ per } x^2 < 4 \quad \text{per } \underline{x < \pm 2} \ \ 2 \end{cases}$

$a < 0$, eq $> 0 \to$ Val interni



$y(2) = 2^4 \cdot e^{-\frac{2^2}{2}} = 16 \cdot \dfrac{1}{e^2} \simeq 2.1 = \dfrac{16}{e^2}$

Deriv II: $D\left[x^3 e^{\frac{x^2}{2}}(4 - x^2) \cdot \dfrac{1}{\left(e^{\frac{x^2}{2}}\right)^2}\right] =$ FATELA TU



ES 78) $f(x)=\dfrac{e^x-2}{x}$ 1) Dominio: $D: \forall x\in\mathbb{R}-\{0\}$

2) Simm

$f(-x)=\dfrac{e^{-x}-2}{-x} \Rightarrow$ NO simm

3) Intersez

$\begin{cases} x=0 \\ \frac{1-2}{0} \quad \nexists x\in\mathbb{R} \end{cases}$ $\begin{cases} y=0 \\ \text{per } e^x=2 \Rightarrow x=\ln(2) \end{cases} \Rightarrow (\ln(2), 0)\in f(x)$ $\ln(2)\sim 0.7$

4) Segno $f(x)>0$ $\begin{cases} x>\ln(2) \\ x>0 \end{cases}$

	0		$\ln 2$	
$-$		$-$		$+$
$-$		$+$		$+$
$+$		$-$		$+$

$f(x)>0$ per $x<0 \cup x>\ln 2$

5) Asintoti

$\lim_{x\to 0^\pm} f(x) = \dfrac{1^+-2}{0^+} = -\dfrac{1}{0^+} = \pm\infty \Rightarrow$ NO A. Vert

$\lim_{x\to+\infty} f(x)= \dfrac{e^x-2}{x} \sim \dfrac{e^x}{x} \Rightarrow e^x >> x \Rightarrow +\infty$

$\lim_{x\to-\infty} f(x)= \sim \dfrac{e^x \to 0}{x} = 0 \Rightarrow$ $y=0$ A. Orizz

$\lim_{x\to+\infty} \dfrac{f(x)}{x} \sim \dfrac{e^x}{x^2} \to +\infty \Rightarrow$ NO A. Ob.

6) Deriv I: $D\left(\dfrac{e^x-2}{x}\right) = \dfrac{e^x\cdot x-(e^x-2)}{x^2} = \dfrac{xe^x-e^x+2}{\cancel{x^2}\to>0} > 0$ per $e^x(x-1)>-2$

Sempre pos

$\to e^x>-2 \quad \forall x\in\mathbb{R}$

$x-1>-2 ; x>-3$

Deriv II:

$D\left(\dfrac{xe^x-e^x+2}{x^2}\right) = \dfrac{-xe^x-(e^x+2)(2x)}{x^4} = \dfrac{-xe^x-[2xe^x+4x]}{x^4}$

$= -xe^x-2xe^x-4x = \dfrac{-3xe^x-4x}{//} = \dfrac{x(-3e^x-4)}{//} > 0$ $\begin{cases} -3e^x-4>0 \\ x>0 \to x<0 \end{cases}$

$\Rightarrow -3e^x>4 ; \quad e^x<-\frac{4}{3} ; \quad x<\ln-\frac{4}{3} \quad \nexists x\in\mathbb{R}$

Sempre neg $\Rightarrow$ x deve essere neg

Non Completo

2.37 Studiare le seguenti funzioni e disegnarne il grafico

(a) $f(x) = \dfrac{x^2-3}{x-2}$ (b) $f(x) = \dfrac{x^2}{1-x}$

Funzione (a) $f(x) = \dfrac{x^2-3}{x-2}$

1) Dominio $x - 2 \neq 0$ per $\underline{x \neq 2}$

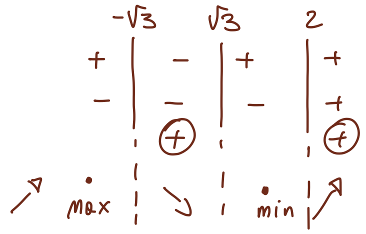
2) Simm:

$f(-x) = \dfrac{x^2-3}{-x-2} \Rightarrow$ NO Simm

3) Segno

$f(x) > 0$ per $x^2 - 3 > 0$; $x > \pm\sqrt{3}$ $\cup$ $x > 2$

$\underline{f(x) > 0 \text{ per } -\sqrt{3} \leq x \leq \sqrt{3} \ \cup \ x > 2}$



4) intersezioni

$\begin{cases} x = 0 \\ -\dfrac{3}{-2} = \dfrac{3}{2} \end{cases} \Rightarrow \underline{\left(0, \tfrac{3}{2}\right) \in f(x)}$ $\begin{cases} y = 0 \\ \text{per} \end{cases}$ $\underline{\begin{matrix} (\sqrt{3}, 0) \\ (-\sqrt{3}, 0) \end{matrix} \in f(x)}$

Asintoti: $\lim\limits_{x \to 2^\pm} f(x) = \dfrac{4-3}{2^\pm - 2} = \dfrac{1}{0^\pm} = \pm\infty \Rightarrow$ $\underline{x = 2 \text{ A. Vert Dx e Sx}}$

$\lim\limits_{x \to \pm\infty} f(x) \sim \dfrac{x^{\cancel{2}}}{\cancel{x}} = \pm\infty \Rightarrow$ NO A. Or.

$\lim\limits_{x \to \pm\infty} \dfrac{f(x)}{x} \sim \dfrac{x^2}{x^2} = 1 \Rightarrow m = 1$ $\lim\limits_{x \to +\infty} f(x) - mx = \dfrac{x^2-3}{x-2} - x \overset{\sim}{=} \dfrac{x^{\cancel{2}}}{\cancel{x}} - x = \infty - \infty$

$\dfrac{\cancel{x^2} - \cancel{x^2}}{x} = \dfrac{1}{x} = 0 \Rightarrow q = 0 \Rightarrow \underline{y = x = \text{A. Obl.}}$

6) Deriv I $D\left(\dfrac{x^2-3}{x-2}\right) = \dfrac{2x(x-2) - (x^2-3)}{(x-2)^2} = \dfrac{2x^2 - 4 - x^2 - 3}{//} = \dfrac{x^2 - 7}{//} > 0$ per $x^2 > 7$ per $x > \pm\sqrt{7}$

Deriv II: $D\left(\dfrac{x^2-7}{(x-2)^2}\right) = \dfrac{2x(x-2)^2 - (x^2-7)\cdot[2(x-2)]}{(x-2)^4} = \dfrac{2x[x^2-2x+4] - [(x^2-7)\cdot(2x-4)]}{//}$

$= \dfrac{\cancel{2x^3} - \cancel{4x^2} + 6x - [\cancel{2x^3} - \cancel{4x^2} - 14x + 28]}{//} = \dfrac{20x - 28}{(x-2)^2} > 0$ per $\cancel{20}x > \dfrac{28}{20}$ per $x > \dfrac{7}{5}$