Limiti - Esercizi

Verificare i limiti $\longrightarrow +\infty$

1) $\lim_{x \to 0^+} \sqrt{\frac{1}{x}} = \sqrt{\frac{1}{0^+}} = +\infty$ ✓

2) $\lim_{x \to 7} \frac{1}{(x-7)^2} = \begin{cases} \lim_{x \to 7^+} \frac{1}{(7^+-7)^2} = \frac{1}{(0^+)^2} = +\infty \\ \lim_{x \to 7^-} \frac{1}{(7^--7)^2} = \frac{1}{(0^-)^2} = +\infty \end{cases} \lim_{x \to 7} f(x) \; +\infty$ ✓

3) $\lim_{x \to 0^+} e^{\frac{2}{x}} = \lim_{x \to 0^+} \frac{2}{0^+} = +\infty \Rightarrow e^{+\infty} = +\infty$ ✓

193) $\lim_{x \to 0^+} (-\ln x) = -(-\infty) = +\infty$ ✓

194) $\lim_{x \to 0} \ln\left(\frac{1}{x^2}\right) = \lim_{x \to 0} \frac{1}{x^2} = \begin{cases} \lim_{x \to 0^+} \frac{1}{(0^+)^2} = +\infty \\ \lim_{x \to 0^-} \frac{1}{(0^-)^2} = +\infty \end{cases} = +\infty \Rightarrow \lim_{x \to 0} \ln(+\infty) = +\infty$ ✓

195) $\lim_{x \to \frac{1}{2}} \frac{1}{(2x-1)^2} = \begin{cases} \lim_{x \to \frac{1}{2}^+} \frac{1}{(2\frac{1}{2}^+-1)^2} = +\infty \\ \lim_{x \to \frac{1}{2}^-} \frac{1}{(2\frac{1}{2}^--1)^2} = +\infty \end{cases} = +\infty$ ✓

196) $\lim_{x \to 2^+} \frac{1}{x^2-4} = \frac{1}{4^+-4} = \frac{1}{0^+} = +\infty$

201) $\lim_{x \to 2^+} \frac{1}{x-2} + 1 = \frac{1}{0^+} + 1 = +\infty$

205) $\lim_{x \to 0^-} \frac{5+2x}{-x} = \frac{5+0^-}{-(0^-)} = \frac{5}{0^+} = +\infty$

210) $\lim_{x \to 1^+} \log\left(\frac{2}{x+1}\right) = \log\left(\frac{2}{2}\right) = \log e = +\infty$

Verificare i limiti $\longrightarrow -\infty$

212) $\lim_{x \to 0} -\frac{1}{3x^4} = \begin{cases} \lim_{0^-} -\frac{1}{3 0^+} = -\infty \\ \lim_{0^+} -\frac{1}{3 0^+} = -\infty \end{cases} = -\infty$ ✓

213) $\lim_{x \to \frac{3}{2}^-} \frac{1}{4x^2-9} = \frac{1}{4\left(\frac{3}{2}\right)^2-9} = +\infty$?

214) $\lim_{x \to -1} \frac{-1}{x^2+2x+1} = \begin{cases} \lim_{-1^+} \frac{-1}{1^+-2^++1} = \frac{-1}{0^+} = -\infty \\ \lim_{-1^-} \frac{-1}{1^--2^-+1} = -\frac{1}{0^-} = +\infty \end{cases}$?

215) $\lim_{x \to 1^-} \log_2(1-x) = \log(1-1^-) = \log(0^-) = -\infty$ ✓

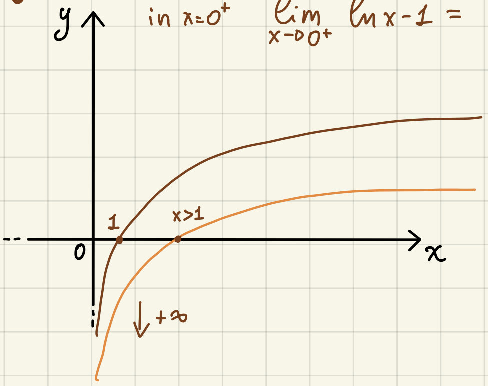
216) $\lim_{x \to 3^-} \frac{1}{x-3} = \frac{1}{3^--3} = -\infty$ ✓

217) $\lim_{x \to 0} \frac{-1}{|x|} = \begin{cases} \lim_{x \to 0^+} -\frac{1}{|0^+|} = -\infty \\ \lim_{x \to 0^-} -\frac{1}{|0^-|} = -\infty \end{cases} = -\infty$ ✓

Rappresentazione grafica

$f = \ln x - 1$

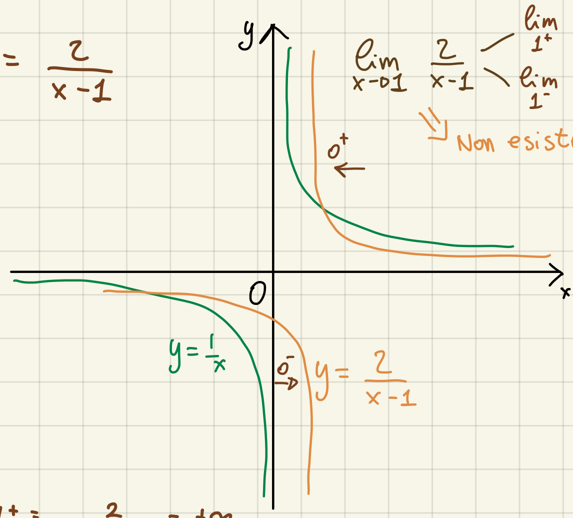
in $x = 0^+$ $\quad \lim_{x \to 0^+} \ln x - 1 = \ln(0^+) - 1 = -\infty$



$f = \dfrac{2}{x-1}$

$\lim_{x \to 1} \dfrac{2}{x-1} \begin{cases} \lim_{1^+} \dfrac{2}{0^+} = +\infty \\ \lim_{1^-} \dfrac{2}{0^-} = -\infty \end{cases}$

$\Rightarrow$ Non esiste



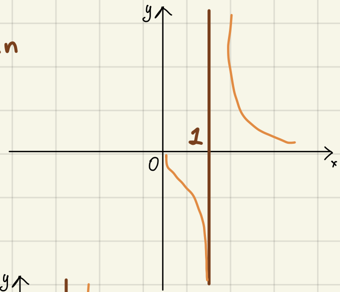
Asintoti verticali

249) $y = \dfrac{2}{(x-1)^2}$ in $x = 1$

$\lim_{x \to 1} \dfrac{2}{(x-1)^2} \begin{cases} 1^+ = \dfrac{2}{0^+} = +\infty \\ 1^- = \dfrac{2}{0^+} = +\infty \end{cases} \Rightarrow$ Asintoto Vert in $x = 1$

250) $y = \dfrac{1}{\ln x}$ in $x = 1$

$\lim_{x \to 1} \dfrac{1}{\ln x} = \begin{cases} 1^+ = \dfrac{1}{0^+} = +\infty \\ 1^- = \dfrac{1}{0^-} = -\infty \end{cases}$ Asintoto in $x = 1$

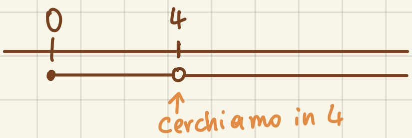


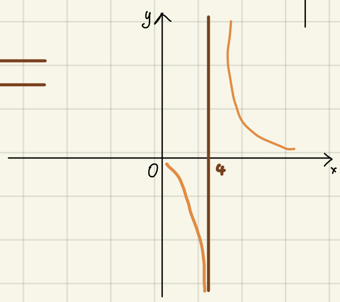
251) $y = \dfrac{2}{\sqrt{x} - 2}$ in $x = 4$

$D = \sqrt{x} \geq$; $x \geq 0$

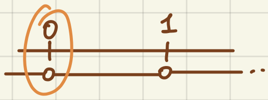
$\sqrt{x} - 2 \neq 0$ per $(\sqrt{x})^2 \neq 2^2 \Rightarrow x \neq 4$

$\lim_{x \to 4} f(x) \begin{cases} 4^+ = \dfrac{2}{0^+} = +\infty \\ 4^- = \dfrac{2}{0^-} = -\infty \end{cases}$ A.V. in $x = 4$





252) $y = \dfrac{x-3}{x^2 - x}$ $\quad D = x^2 - x \neq 0$ per $x(x-1) \neq 0$ $\Rightarrow \begin{cases} x \neq 0 \\ x \neq 1 \end{cases}$



$\lim_{x \to 0} f(x) \begin{cases} 0^+ = \dfrac{-3}{0^+} = -\infty \\ 0^- = \dfrac{-3}{0^-} = -\infty \end{cases}$

253) $y = 1 + e^{\frac{1}{x}}$ in $x = 0^+$ $\quad \lim_{x \to 0^+} 1 + e^{\frac{1}{x}} = 1 + e^{\frac{1}{0^+}} = +\infty$

Limiti tendenti ad e

264) $\lim_{x\to+\infty} \frac{2}{x+10} = 0 \quad \frac{2}{x(1+\cancel{\frac{1}{x}}_{\to 0})} = \frac{2}{\infty} = 0$ ✓

265) $\lim_{x\to+\infty} \frac{4x-1}{2x+1} = \frac{\cancel{x}(4-\cancel{\frac{1}{x}}^{\to 0})}{\cancel{x}(2+\cancel{\frac{1}{x}}_{\to 0})} = \frac{4}{2} = 2$ ✓

269) $\lim_{x\to+\infty} \frac{2^x-1}{2^x} = ?$

270) $\lim_{x\to+\infty} \frac{-3x}{|x|+1} = \frac{x(-3)}{x+1} = \frac{\cancel{x}(-3)}{\cancel{x}(1+\cancel{\frac{1}{x}}_{\to 0})} = \frac{-3}{1} = -3$

siccome siamo in $+\infty$

271) $\lim_{x\to+\infty} (\sqrt{x^2-1} - x = 0) = (x^2)^{\frac{1}{2}} - 1^{\frac{1}{2}} - x = x^{\frac{4+1}{2}} - x^{\frac{1}{2}} - x = x^{\frac{5}{2}} - x^{\frac{1}{2}} - x = x^{\frac{5}{2}-\frac{1}{2}-1} = \frac{5-1-2}{2} = x^{-2} = \frac{1}{x^2}$

$\Rightarrow \lim_{x\to+\infty} \frac{1}{x^2} = 0$ ✓

272) $\lim_{x\to\infty} \ln\left(\frac{x}{x-1}\right) = \lim_{x\to\infty} \frac{x}{x-1} = \frac{\cancel{x}}{\cancel{x}(1-\cancel{\frac{1}{x}}_{\to 0})} = \frac{1}{1} = 1 \Rightarrow \lim_{x\to\infty} \ln(1) = 0$ ✓

273) $\lim_{x\to\infty} \frac{x}{x^2-1} = \frac{\cancel{x}}{x^{\cancel{2}}(1-\cancel{\frac{1}{x^2}}_{\to 0})} = \frac{1}{+\infty} = 0$ ✓

274) $\lim_{x\to\infty} \left(\frac{1}{2}\right)^{2x}$ = All'aumentare dell'esponente aumenta il denominatore $\Rightarrow \frac{1}{\infty} = 0$ ✓

275) $\lim_{x\to+\infty} \left[\underbrace{\left(\frac{1}{3}\right)^{x+1}}_{\text{stesso di prima}} + 1\right] = 0 + 1 = 1$ ✓

277) $\lim_{x\to-\infty} \frac{2}{2x+1} = \frac{2}{-\infty} = 0$ ✓

278) $\lim_{x\to-\infty} \frac{x^3+1}{2x^3} = \frac{\cancel{x^3}(1+0)}{2} = \frac{1}{2}$ ✓

279) $\lim_{x\to-\infty} \frac{3x+1}{1-2x} = \frac{\cancel{x}(3+\cancel{\frac{1}{x}}^{\to 0})}{\cancel{x}(\cancel{\frac{1}{x}}_{\to 0}-2)} = -\frac{3}{2}$ ✓

283) $\lim_{x\to-\infty} \frac{-1}{e^{|x|}} = \frac{-1}{e^{-x}} = \frac{-1}{e^{+\infty}} = 0$ ✓

Siamo in $-\infty \Rightarrow |x| = -x$

284) $\lim_{x\to-\infty} 2e^{-4x^2} = \lim_{x\to-\infty} -4x^2 = +\infty \Rightarrow \lim_{x\to-\infty} 2e^{+\infty} =$

Limiti vari da Elia Bomb.

1) $\lim_{x\to 0} \frac{x^3}{tg\,x - \sin x}$

a) limiti notevoli

$$\frac{x^3}{tg\,x-\sin x} = \frac{x^3}{\sin x\left(\frac{1}{\cos x}-1\right)} = \frac{x^3}{\sin x \cdot \frac{1-\cos x}{\cos x}} = \frac{\cos x}{\underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\frac{1-\cos x}{x^2}}_{\to \frac{1}{2}}} = 2\cos x = 2$$

$tg\,x = \frac{\sin x}{\cos x}$

b) Sviluppi di taylor

- $tg\,x = x+\frac{x^3}{3}+o(x^3)$
- $\sin x = x-\frac{x^3}{6}+o(x^3)$

$$\Rightarrow \cancel{x}+\frac{x^3}{\cancel{3}}+o(x^3)-\left[\cancel{x}-\frac{x^3}{\cancel{6}\,2}+o(x^3)\right] = \frac{x^3}{2}+o(x^3) = \lim_{x\to 0}\frac{\cancel{x^3}}{\frac{\cancel{x^3}}{2}+o(x^3)} = 2$$

Serie di Taylor

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k \qquad a_k = \frac{f^{(k)}(x_0)}{k!}$$