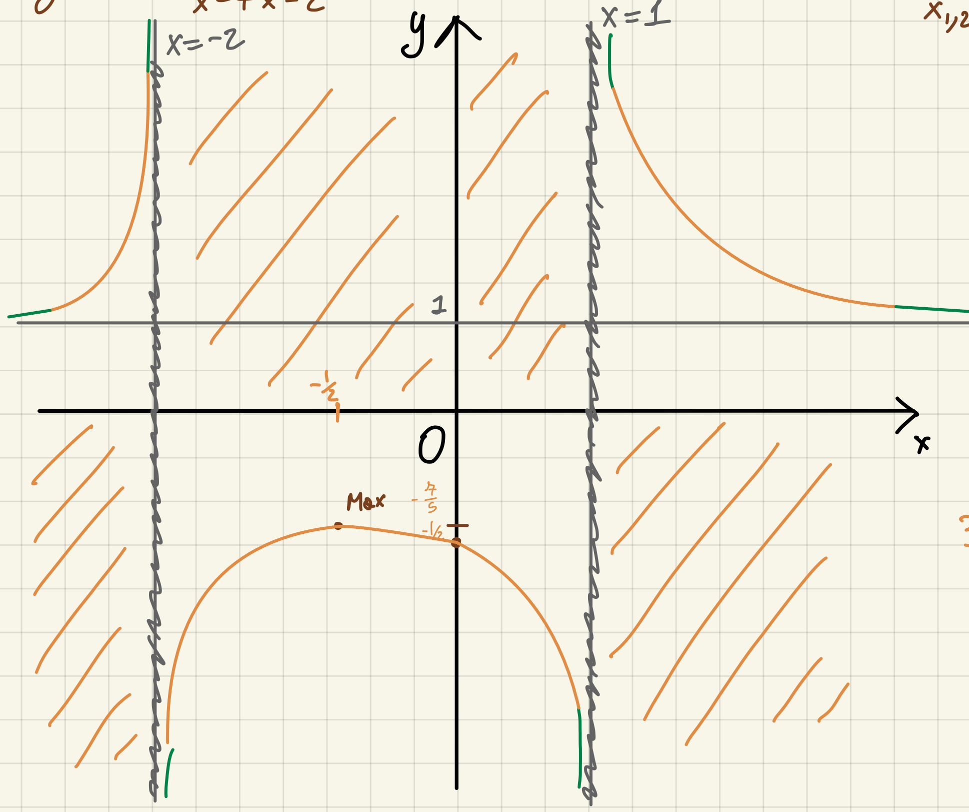
Funzioni Razionali Fratte

ES 32

$$f(x) = \frac{x^2+x+1}{x^2+x-2}$$



1) Dominio: $x^2+x-2 \neq 0$ $\quad \Delta = 1-4\cdot 1\cdot(-2) = 9$

$$x_{1,2} = \frac{-1 \pm 3}{2} \begin{cases} x_1 = 1 \\ x_2 = -2 \end{cases}$$

$x \neq 1$ e $x \neq -2$

2) Intersezioni

$$\begin{cases} x=0 \\ y=-\frac{1}{2} \end{cases} \qquad \begin{cases} y=0 \\ \text{per } x^2+x+1=0 \end{cases}$$

$\Rightarrow \Delta = 1-4 < 0 \Rightarrow$ No int. con x

$P(0;-\frac{1}{2}) \in f(x)$

3) Simmetrie

$$f(-x) = \frac{x^2-x+1}{x^2-x-2} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases} \text{ No Simm}$$

4) Segno: $f(x) > 0$ per $\frac{x^2+x+1}{x^2+x-2} > 0$ $\quad$ N $x^2+x+1>0$ per $x<-2 \vee x>1$

D $x^2+x-2>0$ $\quad \forall x \in \mathbb{R}$

$\Rightarrow f(x) > 0$ per $x<-2 \vee x>1$

5) Asintoti cerco in $x=-2$ e $x=1$

$$\lim_{x\to -2^+} f(x) = \frac{4-2+1}{4^+-2^+-2} = \frac{3}{0^+} = +\infty \qquad \lim_{x\to -2^+} f(x) = \frac{4-2+1}{4^- - 2^- -2} = -\infty$$

$\Rightarrow x=-2$ Asintoto verticale

$$\lim_{x\to 1^\pm} f(x) = \frac{1+1+1}{1^+ + 1^+ -2} = \frac{3}{0^\pm} = \pm\infty$$ $\Rightarrow x=1$ Asintoto Verticale

A. Orizz:

$$\lim_{x\to+\infty} f(x) = \frac{\cancel{x^2}(1)}{\cancel{x^2}(1)} = 1$$ $\Rightarrow y=1$ Asintoto Orizzontale

A. Obliquo

$$\lim_{x\to\infty} \frac{f(x)}{x} = \frac{1}{x} = 0$$ $\Rightarrow$ No A. Obliquo

6) Derivate: Max-min

$$f'(x) = \frac{2x+1\,(x^2+x-2) - (x^2+x-1)\,2x+1}{(x^2+x-2)^2} = \frac{2x+1(\cancel{x^2}\cancel{+x}-2-\cancel{x^2}\cancel{-x}+1)}{(x^2+x-2)^2} = -\frac{2x+1}{(x^2+x-2)^2}$$

Studio la f': $f'(x) > 0$

$$-\frac{2x+1}{(x^2+x-2)^2} > 0 \qquad D>0 \;\; \nexists x \in \mathbb{R}$$

$\Rightarrow f'(x) > 0$ per $2x+1<0 \Rightarrow x<-\frac{1}{2}$

½	
+	−
↗	↘

$x = \frac{1}{2}$ Max

$$f\left(\tfrac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)+1}{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)-2} = \frac{\frac{1}{4}+\frac{1}{2}+1}{\frac{1}{4}+\frac{1}{2}-2} = \frac{\frac{1+2+4}{4}}{\frac{1+2-8}{4}} = \frac{\frac{7}{4}}{-\frac{5}{4}} = \frac{7}{4}\cdot\left(-\frac{4}{5}\right) = -\frac{28}{20} = -\frac{7}{5}$$

Deriv II° :

$$f''(x) = -\left[\frac{2(x^2+x-2)^2-(2x+1)\cdot[2(x^2+x-2)(2x+1)]}{(x^2+x-2)^4}\right] = -\left[\frac{2(x^2+x-2)^2-(2x+1)[2x^2+2x-4\cdot(4x+2)]}{//}\right]$$

$$= -\left[\frac{2(x^2+x-2)^2-(2x+1)\cdot(8x^3+4x^2+8x^2+4x-16x-8)}{//}\right] = -\left[\frac{2(x^2+x-2)^2-(2x+1)(8x^3+12x^2-12x-8)}{(x^2+x-2)^4}\right]$$

$$= -\left[\frac{2(x^2+x-2)^2-(16x^4+24x^3-24x^2-16x+8x^3+12x^2-12x-8)}{//}\right] = -\left[\frac{2(x^2+x-2)^2-(16x^4+32x^3-12x^2-28x-8)}{(x^2+x-2)^4}\right]$$

sempre pos

$\Rightarrow f''(x) > 0$ per $16x^4+32x^3-12x^2-28x-8>0$

$\Rightarrow 8(2x^4+4x^3-1)-4(4x^2+7x)>0$ per a) $2x^4+4x^3-1>0$ b) $4x^2+7x<0$

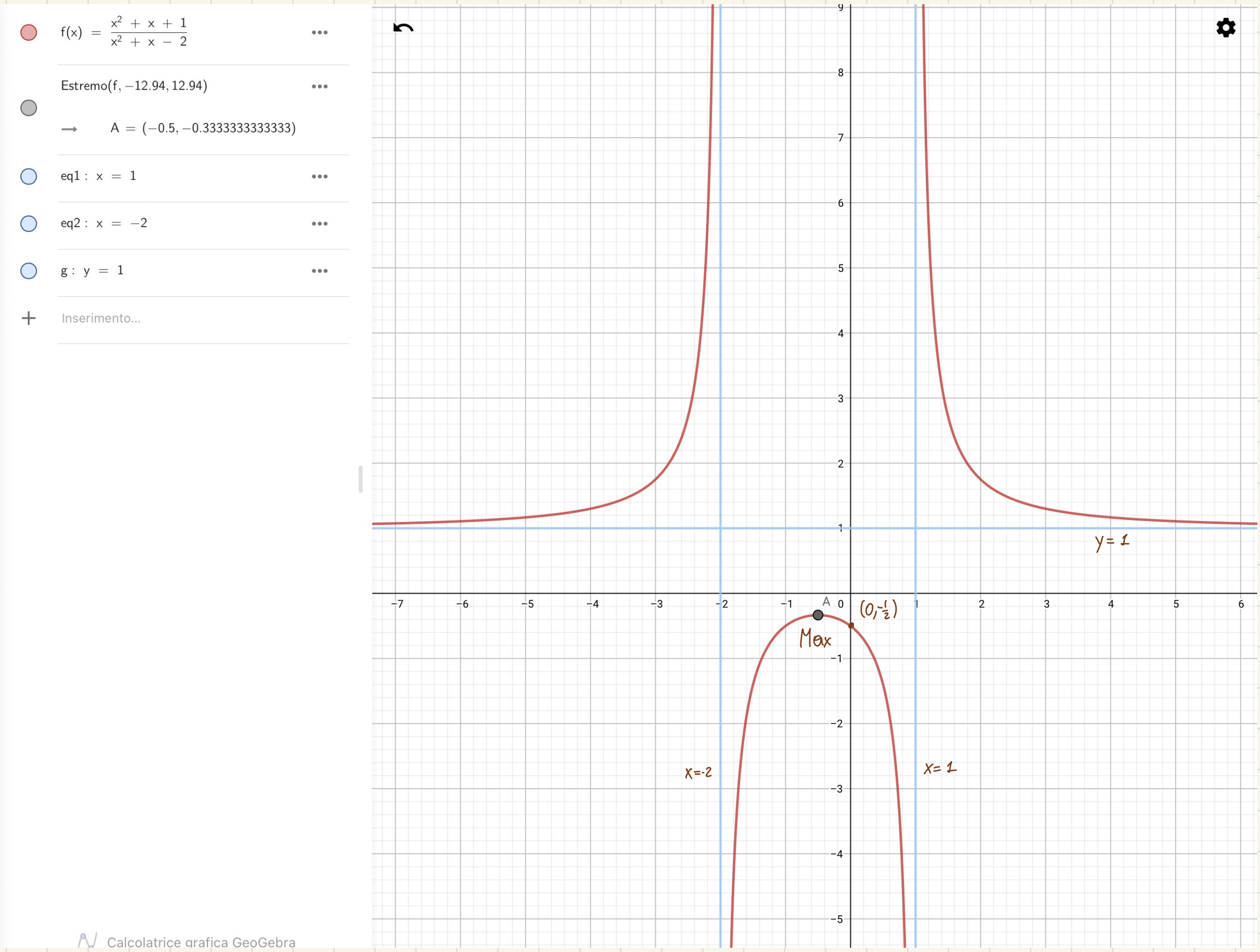
a) $2x^3(x+2)>0$ per a.1) $2x^3>0 \rightarrow x>0$

a.2) $x+2>0 \rightarrow x>-2$

Deriv II sbagliata

b) $4x^2+7x<0$ per $x(4x+7)$ b.1) $x>0$

b.2) $\cancel{x>-\frac{7}{4}}$?



Funzioni irrazionali

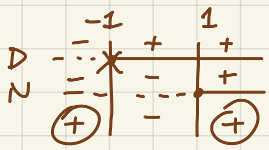
ES: 61)

$$f(x)=\sqrt{\frac{x-1}{x+1}}$$

1) Dominio $\frac{x-1}{x+1}\geq 0$ N: $x-1\geq 0$ per $x\geq 1$

D: $x+1>0$ per $x>-1$

D: $x<-1 \cup x\geq 1$



2) SimmeTrie

$$f(-x)=\sqrt{\frac{-x-1}{-x+1}} \quad \neq f(x) \quad \neq -f(x)$$

3) InTersezioni

$$\begin{cases} x=0 \\ \sqrt{\frac{-1}{1}} \end{cases} \nexists x\in\mathbb{R} \qquad \begin{cases} y=0 \\ \text{per } x-1=0,\ \underline{x=1} \end{cases} \Rightarrow \underline{(1,0)\in f(x)}$$

4) Segno

$\underline{f(x)>0 \quad \forall x\in D}$

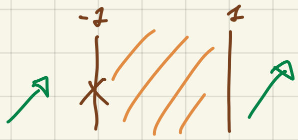
5) Studio crescenza: deriv I°

$$f'(x)=D\left[\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}\right]=\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot D\left(\frac{x-1}{x+1}\right)=\frac{1(x+1)-(x-1)1}{(x+1)^2}=\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}=\frac{2}{(x+1)^2}$$

$$\Rightarrow f'(x)=\frac{1}{\cancel{2}}\frac{1}{\sqrt{\frac{x+1}{x-1}}}\cdot\frac{\cancel{2}}{(x+1)^2}=\frac{\sqrt{x-1}}{(x+1)^2\sqrt{x+1}}$$

sempre pos

$\Rightarrow \underline{f'(x)\geq 0 \quad \forall x\in\mathbb{R}}$



6) AsinToti

$$\lim_{x\to -1^-} f(x)=\sqrt{\frac{-2^-}{0^-}}=+\infty \Rightarrow \underline{x=-1 \quad A.V.}$$

$$\lim_{x\to 1^+} f(x)=\sqrt{\frac{1^+-1}{1+1}}=\sqrt{\frac{0^+}{2}}=0 \quad \text{No A.V.}$$

$$\lim_{x\to +\infty} f(x)=\sqrt{\frac{\cancel{x}(1)}{\cancel{x}(1)}}=1 \Rightarrow \underline{y=1 \quad A.\ Orizz}$$

$$f'(x)=\frac{\overset{a}{\sqrt{x-1}}}{\underbrace{(x+1)^2}_{b}\underbrace{\sqrt{x+1}}_{d}}\ c$$

5.1) Deriv II

$f''(x)=$ a) $D(\sqrt{x-1})=-\frac{1}{2}(x-1)^{-\frac{1}{2}}=-\frac{1}{2\sqrt{x-1}}$

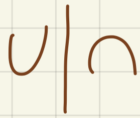
c) $D(\sqrt{x+1})=\frac{1}{2\sqrt{x+1}}$

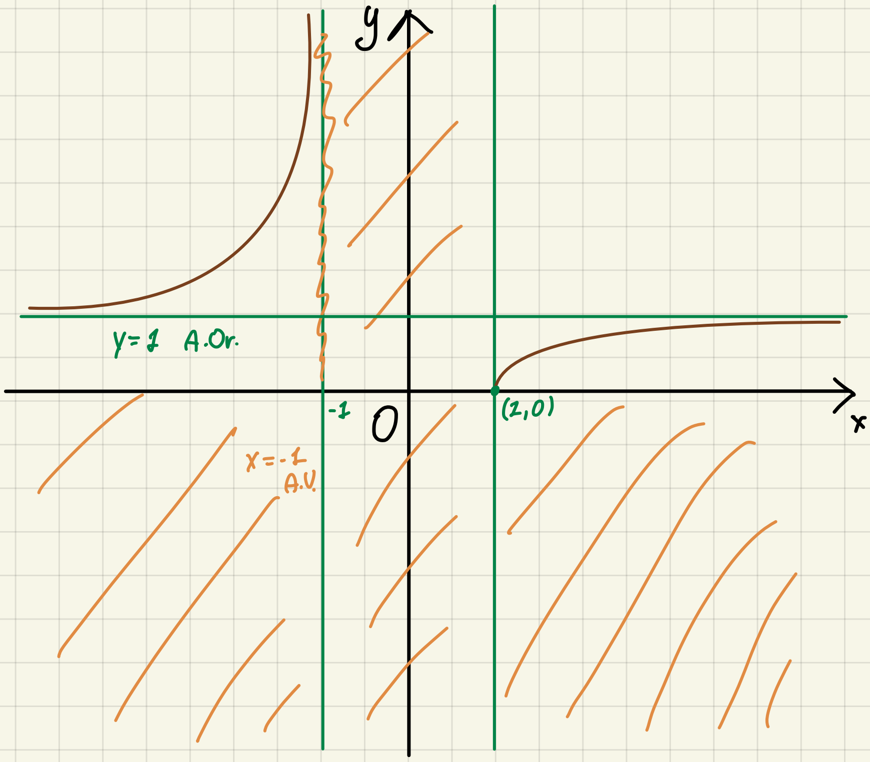
b) $D(x+1)^2=2(x+1)$

d) $D\left[(x+1)^2\sqrt{x+1}\right]=2(x+1)\sqrt{x+1}+(x+1)^2\left(-\frac{1}{2\sqrt{x+1}}\right)=2x+2\sqrt{x+1}+x^2+2x+1\left(-\frac{1}{2\sqrt{x+1}}\right)$

$$=2x+2\sqrt{x+1}-\frac{x^2+2x+1}{2\sqrt{x+1}}=\frac{2x\sqrt{x+1}+4(x+1)-x^2+2x+1}{2\sqrt{x+1}}=\frac{2x\sqrt{x+1}+4x+4-x^2+2x+1}{2\sqrt{x+1}}$$

$=-x^2+6x+2x\sqrt{x+1}+5\geq 0$ per $x<0$ 0

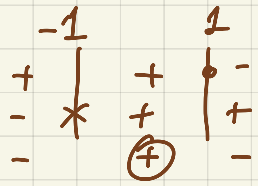



y
y=1 A.Or.
-1
0
(1,0)
x
x=-1
A.V.

ES 62) $f(x)=\sqrt{\frac{1-x}{1+x}}$

1) Dominio $\frac{1-x}{1+x}\geq 0$ $\quad N\ 1-x\geq 0$ $\quad D\ 1+x>0$

$\Rightarrow N:\ x\leq 1$
$D:\ x>-1$



f e' definita per $-1<x\leq 1$

2) Simmetrie

$f(-x)=\sqrt{\frac{1+x}{1-x}} < \begin{matrix}\neq f(x)\\ \neq -f(x)\end{matrix}$

3) Intersezioni:

$\begin{cases} x=0 \\ \sqrt{\frac{1}{1}} \end{cases} \Rightarrow (0,1)\in f(x)$ $\qquad \begin{cases} y=0 \\ \text{per } x=1 \end{cases} \Rightarrow (1,0)\in f(x)$

4) Segno

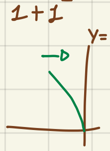
$f(x)>0\quad \forall x\in D$

5) limiti

$\lim_{x\to -1^+} f(x)=\sqrt{\frac{1+1}{0^+}}=+\infty \Rightarrow x=-1$ A.V.

$\lim_{x\to +\infty} f(x)=\sqrt{\frac{\cancel{x}(-1)}{\cancel{x}(1)}}=\sqrt{-1}\ \nexists x\in\mathbb{R}$

$\lim_{x\to 1^-} f(x)=\sqrt{\frac{1-1^-}{1+1^-}}=\sqrt{\frac{0^+}{2^-}}=\sqrt{0^+}=0$

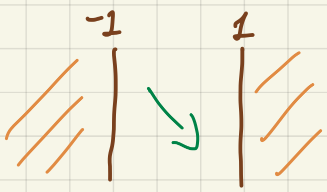


6) Derivate

$D\left(\sqrt{\frac{1-x}{1+x}}\right)=D\left(\frac{1-x}{1+x}\right)=\frac{-1-x-(1-x)}{(1+x)^2}=\frac{-1\cancel{-x}-1\cancel{+x}}{(1+x)^2}=-\frac{2}{(1+x)^2}$

$\Rightarrow D\left(\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\right)=\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\left(-\frac{2}{(1+x)^2}\right)=-\frac{\sqrt{1+x}}{\sqrt{1-x}\,(1+x)^2}$

$\Rightarrow f'(x)<0\quad\forall x\in D$



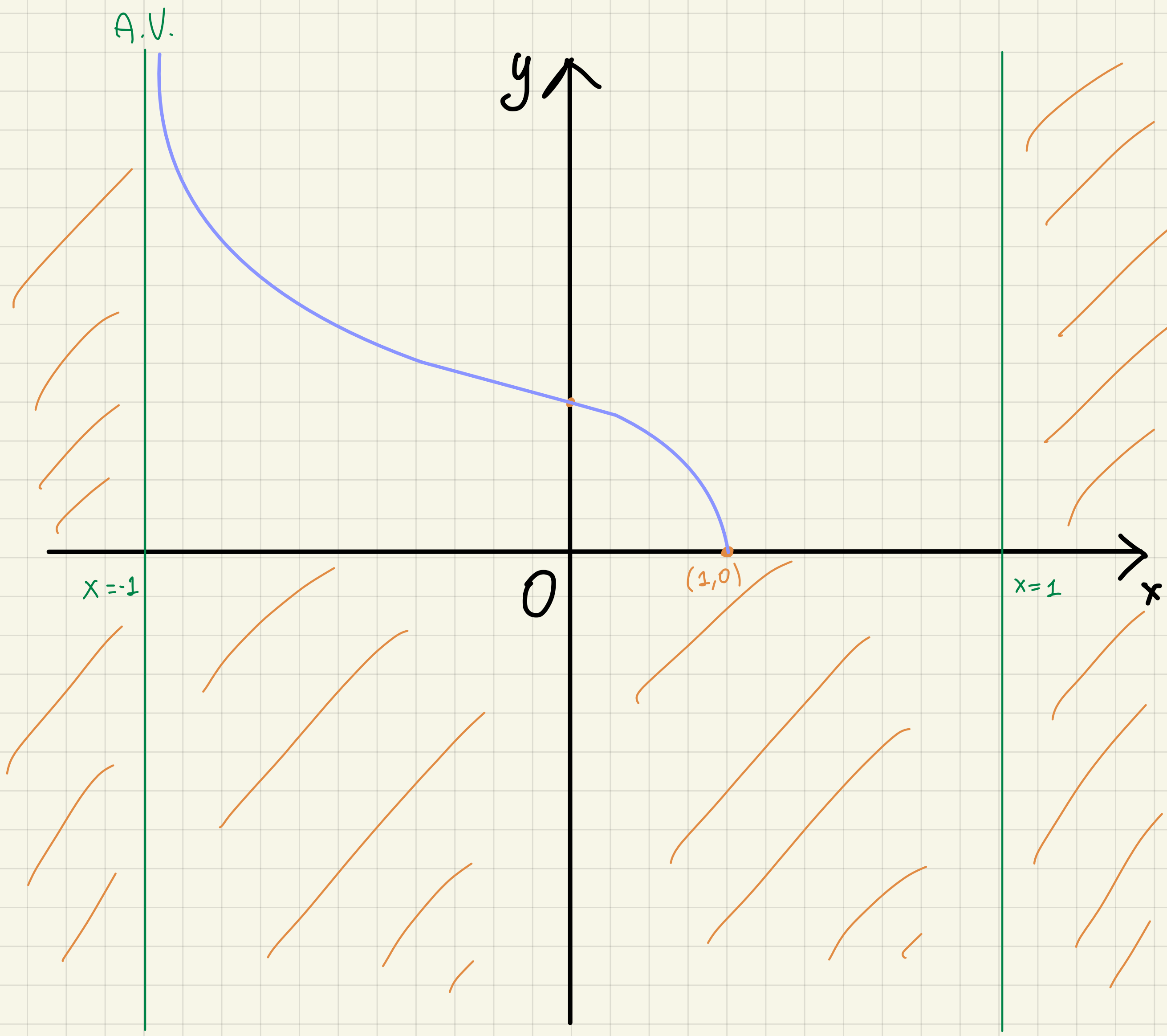
No min No max

$f''(x) =$ vedo $f'(x)$ come $-\sqrt{\frac{1+x}{1-x}} \cdot (1+x)^{-2} \Rightarrow$

$$f''(x) = -\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{2}{(1+x)^2} - \sqrt{\frac{1+x}{1-x}} \cdot (-2)(1+x)^{-3} = -\frac{1}{2}\,\frac{2}{\sqrt{\frac{1+x}{1-x}}\,(1+x)^2} - 2\sqrt{\frac{1+x}{1-x}}\,(1+x)^{-3}$$

$$= -\frac{1}{2}\,\frac{2}{\sqrt{\frac{1+x}{1-x}}\,(1+x)^2} - 2\sqrt{\frac{1+x}{1-x}} \cdot \frac{2}{(1+x)^3}$$

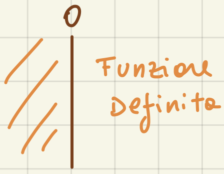
Boh



ES 64)

$f(x) = 2\sqrt{x} - x$

1) Dominio: $\underline{x \geq 0}$



2) Simmetrie: $f(-x) = 2\sqrt{-x} + x \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases}$

3) Intersezioni

$\begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow \underline{(0,0) \in f(x)}$

$\begin{cases} y = 0 \\ \text{per } 2\sqrt{x} - x = 0 \end{cases} \Rightarrow x = 2\sqrt{x} \Rightarrow x^2 = 4x, \quad x^2 - 4x = 0$

per $x(x-4) = 0, \quad x = 0 \cup x = 4$

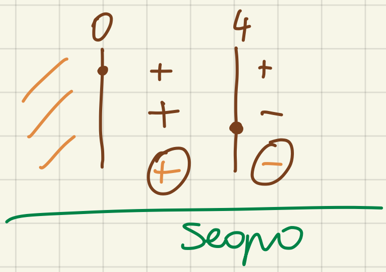
$\Rightarrow \underline{(4,0) \in f(x)}$

4) Segno $f(x) > 0$ per $x^2 - 4x < 0$ $\quad a > 0$, eq < 0

$\Rightarrow$ Soluzioni interne

$\Delta = 16 \qquad x_{1/2} = \dfrac{4 \pm 4}{2} \begin{cases} 4 \\ 0 \end{cases}$

$x > 0 \cup x < 4 \ \wedge \ x \geq 0$



5) Asintoti

$\lim_{x \to 0^+} f(x) = 2\sqrt{\cancel{0^+}}^{\,0} - 0^+ = 0$ No A.U

$\lim_{x \to +\infty} f(x) = 2x^{\frac{1}{2}} - x = x\left(\cancel{\dfrac{2}{\sqrt{x}}}^{\,0} - 1\right) = -\infty \Rightarrow$ No A.Oriz

$\lim_{x \to +\infty} \dfrac{f(x)}{x} = m = \dfrac{\cancel{x}\left(\cancel{\frac{2}{\sqrt{x}}}^{\,0} - 1\right)}{\cancel{x}} = -1$

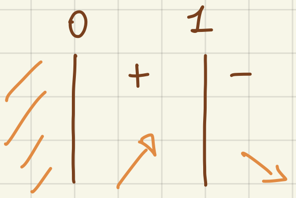
$\lim_{x \to +\infty} f(x) - mx = 2\sqrt{x} \cancel{-x} \cancel{+x} = +\infty \Rightarrow$ No A. Ob.

6) Derivate

$f'(x) = D(2\sqrt{x} - x) = 2x^{\frac{1}{2}} - x = \dfrac{1}{\sqrt{x}} - 1 = \underbrace{\dfrac{1 - \sqrt{x}}{\sqrt{x}}}_{f'(x)} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{1\sqrt{x} - x}{x}$

$f'(x) \geq 0$ per $1 - \sqrt{x} > 0, \quad \sqrt{x} \leq 1$

ovvero per $x \leq +1 \ \cap x \geq 0$



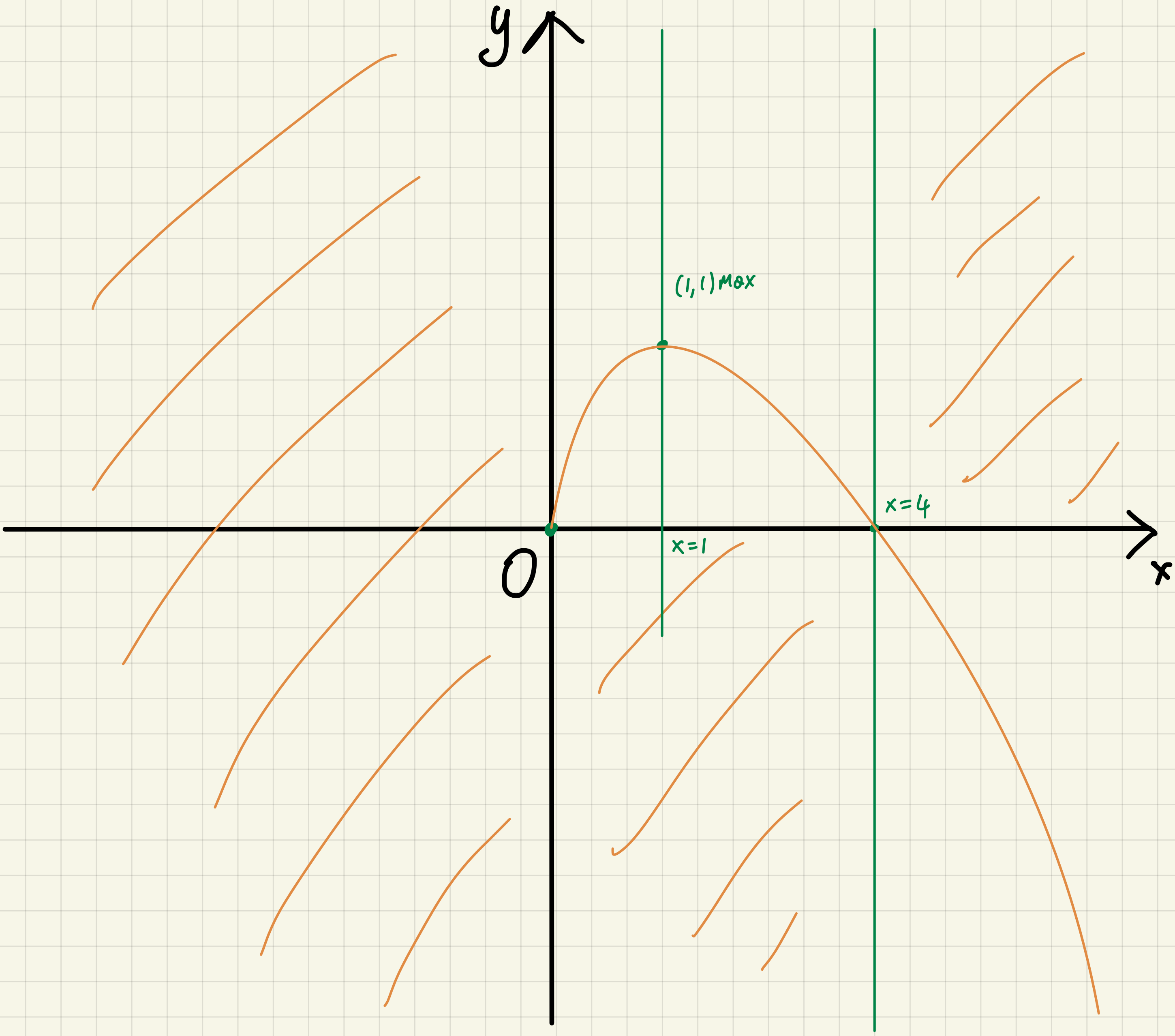
$x = 1$ max $\quad f(1) = 2\sqrt{1} - 1 = 1 \Rightarrow \underline{(1,1) \text{ Max}}$

$\dfrac{1 - \sqrt{x}}{\sqrt{x}}$

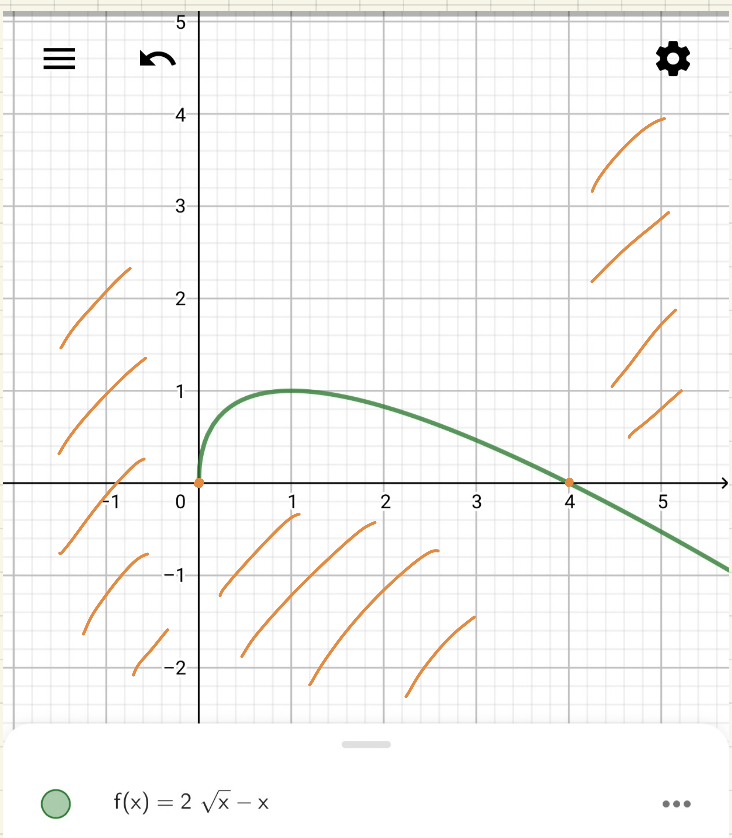
Deriv II $\quad f''(x) = -\dfrac{1}{2}\dfrac{1}{\sqrt{x}} \dfrac{1}{\sqrt{x}} - \left[(1 - \sqrt{x}) \cdot \left(-\dfrac{1}{2}\dfrac{1}{\sqrt{x}}\right)\right]$

$= \dfrac{1}{2x} + \left[\dfrac{1 - \sqrt{x}}{2\sqrt{x}}\right] = \dfrac{\sqrt{x} + (1 - \sqrt{x})x}{2x\sqrt{x}} = \dfrac{\sqrt{x} + x - x\sqrt{x}}{2x\sqrt{x}} = \dfrac{\sqrt{x}(1-x) + x}{2x\sqrt{x}}$

BOH



y
(1,1) max
x=4
x=1
O
x



f(x) = 2 √x − x

$f(x) = \sqrt{8-x^3}$ $\quad D = \; 8-x^3 \geq 0 \quad$ per $\; x^3 \leq 8 \;;\; x \leq \sqrt[3]{8} \leq 2$

2) Simmetrie $\quad f(-x) = \sqrt{8+x} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases}$

3) intersezioni

$\begin{cases} x = 0 \\ \sqrt{8} = 2\sqrt{2} \end{cases} \quad \cancel{(0, 2\sqrt{2}) \in f(x)} \; \notin D \qquad \begin{cases} y = 0 \\ \sqrt{8-x^3} = 0 \end{cases} \quad$ per $x = 2 \quad \Rightarrow (2,0) \in f(x)$

4) Segno

$f(x) > 0 \quad \forall x \in D$

5) Asintoti

$\lim_{x \to 2^\pm} f(x) = \sqrt{8 - 8^\pm} = \sqrt{0^\pm} = \pm\infty$ No A.V

$\lim_{x \to +\infty} f(x) = \sqrt{8-\infty} = \nexists x \in \mathbb{R}$

$\lim_{x \to -\infty} f(x) = \sqrt{8+\infty} = +\infty$ Cerco A. Ob.

$\lim_{x \to -\infty} \frac{f(x)}{x} = \frac{\sqrt{x^2(\frac{1}{x^2} - x)}}{x} = \frac{\cancel{|x|}\sqrt{-x}}{\cancel{x}} = -\sqrt{+\infty} \Rightarrow \infty$ No A. Ob

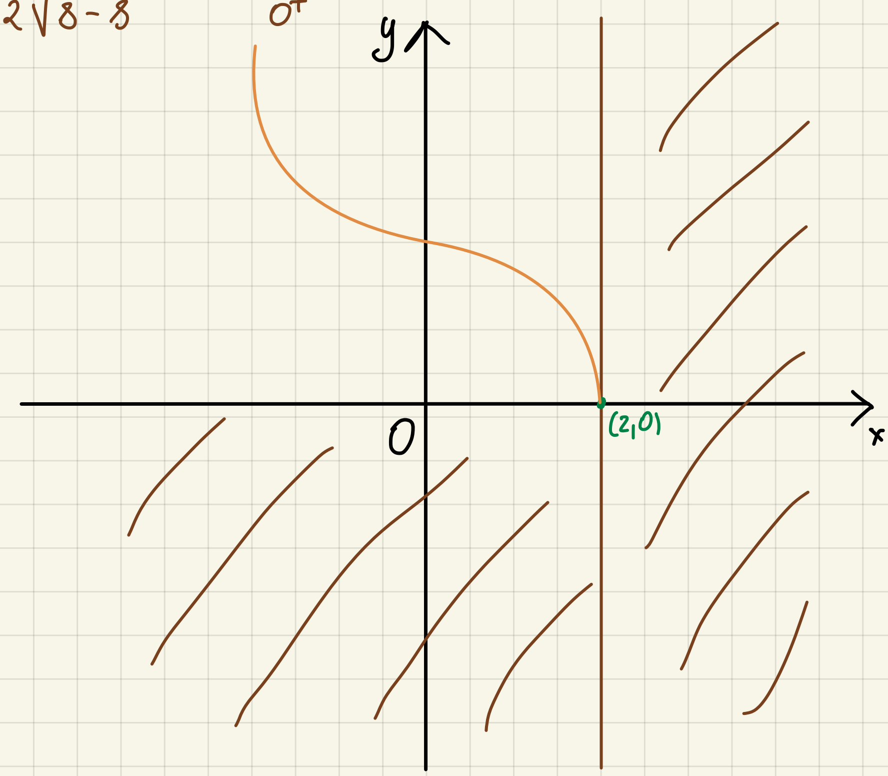
6) Derivate

$D(\sqrt{8-x^3}) = \frac{1}{2} \frac{1}{\sqrt{8-x^3}} \cdot (-3x^2) = -\frac{3x^2}{2\sqrt{8-x^3}} \geq 0 \quad \nexists x \in D \quad f(x) \leq 0 \quad \forall x \in D - \{x = 0\}$

$D'\left(-\frac{3x^2}{2\sqrt{8-x^3}}\right) = -6x(2\sqrt{8-x^3}) - \left[(-3x^2)\left(\frac{1}{\sqrt{8-x^3}} \cdot (-3x^2)\right)\right]$

$= -6x\, 2\sqrt{8-x^3} - \left[-\frac{3x^2}{\sqrt{8-x^3}} \cdot (-3x^2)\right] = \; // \; - \left[- \frac{\text{BOH}}{\sqrt{8-x^3}}\right.$

$\lim_{x \to 2^-} f'(x) = \frac{-12}{2\sqrt{8-8^-}} = -\frac{12}{0^+} = -\infty \quad x = 2$ Tangente verticale ?



ES: 70 $\quad f(x) = \dfrac{x+2}{\sqrt{x^2-x}}$

1) Dominio: $x^2 - x > 0$ per $x(x-1) > 0$ $\to$ $x > 0$, $x > 1$

$a > 0$, eq > 0, sol esterne

$\Rightarrow f(x) \,\exists$ per $\underline{x < 0 \cup x > 1}$

2) Simmetrie:

$$f(-x) = \frac{-x+2}{\sqrt{x^2+x}} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases} \Rightarrow \text{No simm}$$

3) Intersez:

$$\begin{cases} x = 0 \\ \frac{2}{0} \;\nexists x \in \mathbb{R} \end{cases} \qquad \begin{cases} y = 0 \\ x+2 = 0 \text{ per } x = -2 \end{cases} \Rightarrow \underline{(-2,0) \in f(x)}$$

4) Segno $\quad f(x) \geq 0$ per $x \geq -2$

$$\begin{array}{c|c|c|c} & -2 & 0 & 1 \\ - & + & + & + \\ + & + & - & + \\ & \oplus & & \oplus \end{array}$$

$\underline{f(x) > 0 \text{ per } -2 < x < 0 \cup x > 1}$

5) Asintoti

$$\lim_{x \to 0^+} f(x) = \frac{2}{\sqrt{0^+ - 0^+}} = \frac{n}{0} = +\infty$$

$$\lim_{x \to 1^-} f(x) = \frac{1+2}{\sqrt{1^+ - 1^-}} = \frac{3}{\sqrt{0^+}} = +\infty$$

$\underline{x = 0, 1 \text{ Asintoti Verticali}}$

$$\lim_{x \to \infty} f(x) = \frac{\cancel{x}(1+0)}{\cancel{x}\sqrt{(1-0)}} = 1 \Rightarrow \underline{y = 1 \text{ Asintoto orizzontale}}$$

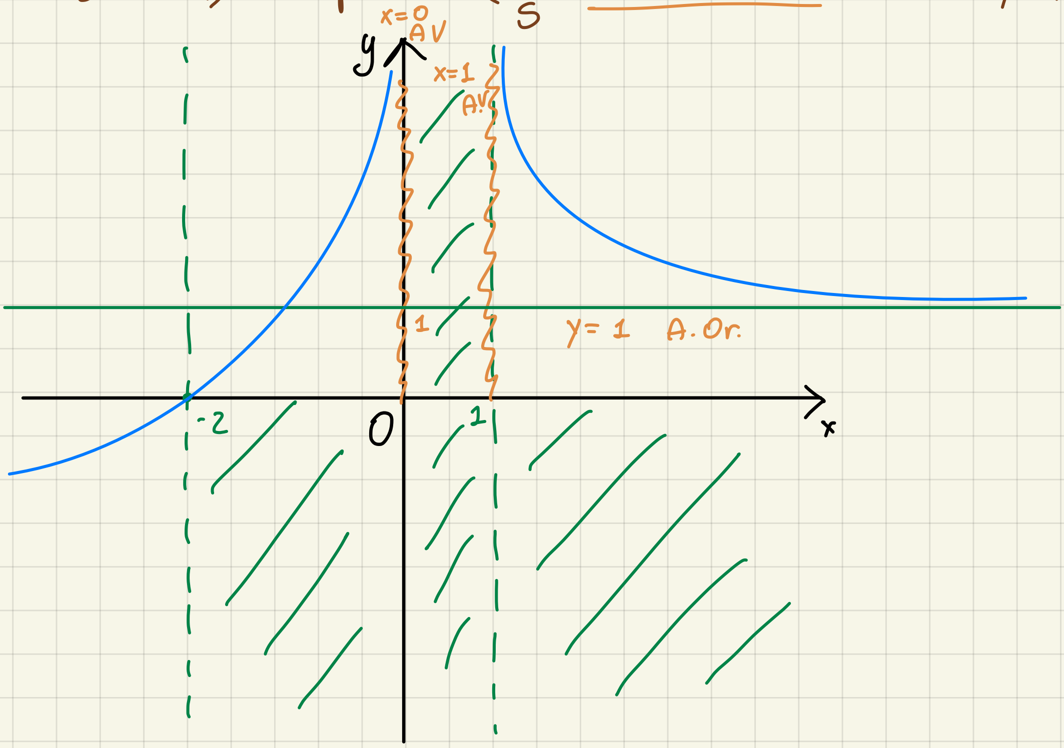
6) Derivata

$$D\left(\frac{x+2}{\sqrt{x^2-x}}\right) = \sqrt{x^2-x} - \left[(x+2)\left(-\frac{1}{2}(x^2-x)^{-\frac{3}{2}} \cdot (2x-1)\right)\right] \cdot \frac{1}{(\sqrt{x^2-x})^2}$$

$$= \sqrt{x^2-x} - \left[x+2 \cdot \frac{1}{2\sqrt{(x^2-x)}} \cdot (2x-1)\right] \cdot // = \sqrt{x^2-x} - \left[\frac{2x^2 - x + 4x - 2}{2\sqrt{x^2-x}}\right] \cdot //$$

$$= \sqrt{x^2-x} - 2x^2 - 3x + 2 \geq 0 \quad \text{per} \quad x \leq \frac{2}{5}$$

$\underline{\text{Fuori dal D}}$ $\qquad$ No min/Max



Inizio funzioni esponenziali

ES 1)

$$f(x) = \frac{e^{1-x}}{x^2-1}$$

1) Dominio $x^2-1 \neq 0$ per $\underline{x \neq \pm 1}$

2) SimmeTrie

$$f(x) = \frac{e^{1+x}}{x^2-1} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases} \Rightarrow \text{No Simm}$$

3) InTersezioni

$$\begin{cases} x=0 \\ \frac{e}{-1} = -e \end{cases} \Rightarrow \underline{(0,-e) \in f(x)}$$

$$\begin{cases} y=0 \\ \frac{e^{1-x}}{x^2-1} = 0 \end{cases} \quad \text{per } e^{1-x}=0 \quad \nexists x \in \mathbb{R} / \ e^{1-x}=0$$

4) Segno $f(x)>0$ per N: $e^{1+x}>0$, $\forall x \in \mathbb{R}$

D: $x^2-1>0$, per $x>\pm 1$, $a>0$, $eq>0 \Rightarrow$ soluzioni esterne $\Rightarrow x<-1 \cup x>1$

		-1		1		
	$+$		$+$		$+$	N
	$+$	$\times$	$-$	$\times$	$+$	D
	$\oplus$		$-$		$\oplus$	

$\underline{f(x)>0 \text{ per } x<-1 \cup x>1}$

5) AsinToti

$$\lim_{x \to -1^{\pm}} f(x) = \frac{e^{1+1^{\pm}}}{1^{+}-1} = \frac{e^2}{0^+} = +\infty \Rightarrow \underline{x=-1 \text{ A. VerT}}$$

$$\lim_{x \to 1^{\pm}} f(x) = \frac{e^0}{1^{\pm}-1} = +\infty \leftarrow \underline{x=1 \text{ A. VerT}}$$

$$\lim_{x \to +\infty} f(x) \stackrel{\sim}{=} \frac{\cancel{e^{-x}} \to \emptyset}{x^2} = \emptyset \to \underline{y=0 \text{ A. Orizz}}$$

6) Crescenza

$$D\left(\frac{e^{1-x}}{x^2-1}\right) = \left[e^{1-x} \cdot (-1) \cdot (x^2-1)\right] - \left[e^{1-x} \cdot (2x)\right] \cdot \frac{1}{\cancel{(x^2-1)^2}} \to \text{sempre pos}$$

$$= -x^2e^{1-x} + e^{1-x} - \left[2x^2e^{1-x}\right] \cdot // = -e^{1-x}(x^2-1) - 2xe^{1-x} = -\cancel{e^{1-x}}(x^2-1+2x) \geq 0$$

sempre pos

$f'(x)>0$ per $x^2+2x-1 \leq 0$ $\quad \Delta = 4-4(-1) = 8 \quad \Rightarrow x_{1,2} = \frac{-2 \pm \sqrt{8}}{2} \begin{cases} \frac{-2+2\sqrt{2}}{2} \\ \frac{-2-2\sqrt{2}}{2} \end{cases}$

$\Rightarrow a>0$, $eq \leq 0 \Rightarrow$ soluzione interne

$x_1 = -1+\sqrt{2}$

$x_2 = -1-\sqrt{2}$

$\Rightarrow f'(x)>0$ per $-1-\sqrt{2} < x < -1+\sqrt{2}$

	$-1-\sqrt{2}$		$-1+\sqrt{2}$	
$-$ ↘		$+$ ↗		$-$ ↘

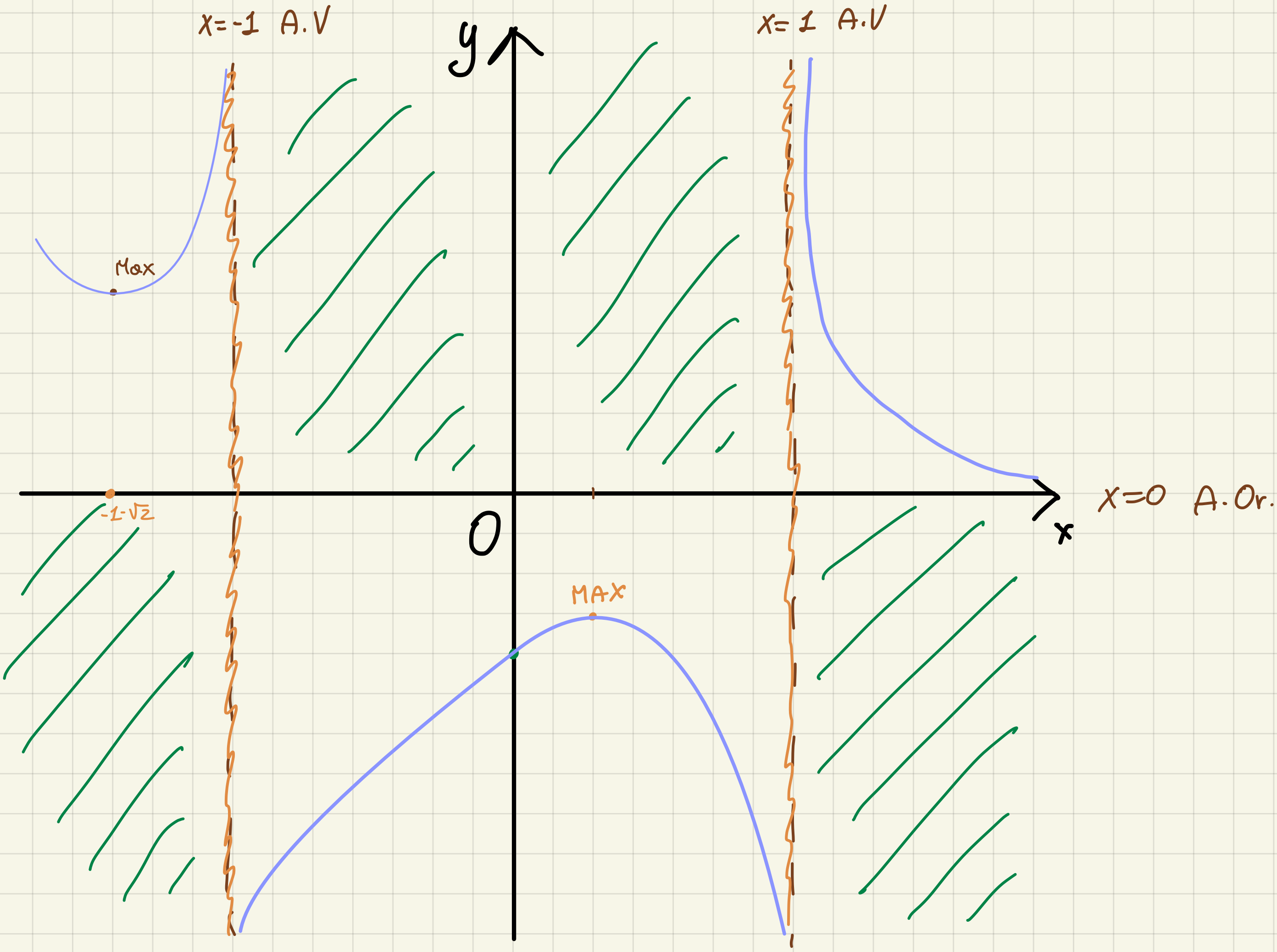
Trovo la y

$$f(-1+\sqrt{2}) = \frac{e^{1-(-1+\sqrt{2})}}{(-1+\sqrt{2})^2-1} = \frac{e^{2+1-\sqrt{2}}}{1-2\sqrt{2}+2-1} = \frac{e^{''}}{2-2\sqrt{2}} \stackrel{\sim}{=} \frac{4.8}{-0,82} = \stackrel{\sim}{=} 5,8$$

$$\frac{e^{1-(1+\sqrt{2})}}{(1+\sqrt{2})^2-1} = \frac{e^{1-\sqrt{2}}}{\cancel{1}+2\sqrt{2}+2-\cancel{1}} = \frac{e^{1-\sqrt{2}}}{2\sqrt{2}+2} = \frac{0,66}{48} = -0.13$$

$\Rightarrow \underline{(-1-\sqrt{2}, 5,8) \text{ Min} \ ; \ (-1+\sqrt{2}, 0,13) \text{ MAX}}$

$\uparrow -2.4 \qquad \uparrow 0.4$



x=-1 A.V
y
x=1 A.V
Max
-1-√2
x=0 A.Or.
x
O
MAX

ES 2: $f(x) = (x^2-1)e^x$

1) Dominio $= \mathbb{R}$

2) Simmetrie $f(-x) = (x^2-1)e^{-x} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases} \Rightarrow$ No Simm

3) Intersezioni

$\begin{cases} x = 0 \\ y = -1 \end{cases} \Rightarrow (0,-1) \in f(x)$

$\begin{cases} y = 0 \\ (x^2-1)e^x = 0 \end{cases}$ per $x^2-1=0$, $x = \pm 1$ $\Rightarrow \boxed{\begin{matrix}(1,0) \in f(x) \\ (-1,0) \in f(x)\end{matrix}}$

4) Segno: $f(x) \geq 0$ per $(x^2-1) \geq 0$, $x^2 \geq 1$ per $x \geq \pm 1$

$a>0$, $eq>0 \Rightarrow$ Valori esterni

$f(x) \geq 0$ per $x \leq -1 \cup x \geq 1$

5) Asintoti

$\lim_{x \to \infty} f(x) = (\infty - 1) \cdot \infty = +\infty \Rightarrow$ No A. Oriz.

$\lim_{x \to \infty} \frac{f(x)}{x} = \frac{x^{\cancel{2}}(1) \cdot e^x}{\cancel{x}} = +\infty \Rightarrow$ NO A. ob.

$\lim_{x \to -\infty} f(x) = (+\infty - 1) e^{-\infty} = +\infty \cdot 0 \Rightarrow \sim \frac{\cancel{x^2} \to +\infty}{\cancel{e^{-x}} \to +\infty}$ ma $e^x >> x^2 \to 0$ — $y = 0$ A. Oriz

6) Derivate:

$D[(x^2-1)e^x] = 2xe^x + (x^2-1)e^x = 2xe^x + x^2e^x - e^x = \overset{\text{sempre pos}}{\cancel{e^x}}(2x + x^2 - 1) > 0$

$f'(x) > 0$ per $x^2 + 2x - 1 > 0$, $\Delta = 4 - 4(-1) = 8$

$x_{1,2} = \frac{-2 \pm 2\sqrt{2}}{2} \begin{cases} \frac{-2+2\sqrt{2}}{2} / 2 = -1 + \sqrt{2} \sim 0,4 \\ \frac{-2-2\sqrt{2}}{2} / 2 = -1 - \sqrt{2} \sim -2.4 \end{cases}$

-2.4 0.4

$-1-\sqrt{2}$ $-1+\sqrt{2}$

max min

$a>0$, $eq>0 \Rightarrow$ Sol esterne

$f'(x) > 0$ per $x < -1-\sqrt{2} \cup x > -1+\sqrt{2}$

$-1-\sqrt{2}$ $-1+\sqrt{2}$

Max min

$(-1-\sqrt{2}, \tilde{0,07})$ $(-1+\sqrt{2}, \tilde{7,2})$

$f(-1-\sqrt{2}) = ((-1-\sqrt{2})^2 - 1) \cdot e^{-1-\sqrt{2}} = (\cancel{1} - 2\sqrt{2} + 2 - \cancel{1}) e^{-1-\sqrt{2}}$

$= (-2\sqrt{2} + 2) e^{-1-\sqrt{2}} = -0,8 \cdot 0,08 \simeq 0,07$

$f(-1+\sqrt{2}) = ((-1+\sqrt{2})^2 - 1) e^{-1+\sqrt{2}} = (\cancel{1} + 2\sqrt{2} + 2 - \cancel{1}) e^{-1+\sqrt{2}}$

$= (2\sqrt{2} + 2) e^{-1+\sqrt{2}} = 4.8 \cdot 1.51 \simeq 7,2$

Derivata II $D[e^x(2x + x^2 - 1)] = e^x(x^2 + 2x - 1) + e^x(2x + 2) = x^2e^x + 2xe^x - e^x + 2xe^x + 2e^x$

$= x^2e^x + 4xe^x + e^x = \overset{\text{sempre pos}}{\cancel{e^x}}(x^2 + 4x + 1) \geq 0$

$x_{1,2} = \frac{-4 \pm 2\sqrt{3}}{2} \begin{cases} \frac{-4+2\sqrt{3}}{2} / 2 = -2 + \sqrt{3} \\ -\frac{4-2\sqrt{3}}{2} / 2 = -2 - \sqrt{3} \end{cases}$

$f''(x) > 0$ per $x^2 + 4x + 1 > 0$

$\Delta = 16 - 4 = 12$

$a>0$, $eq>0$

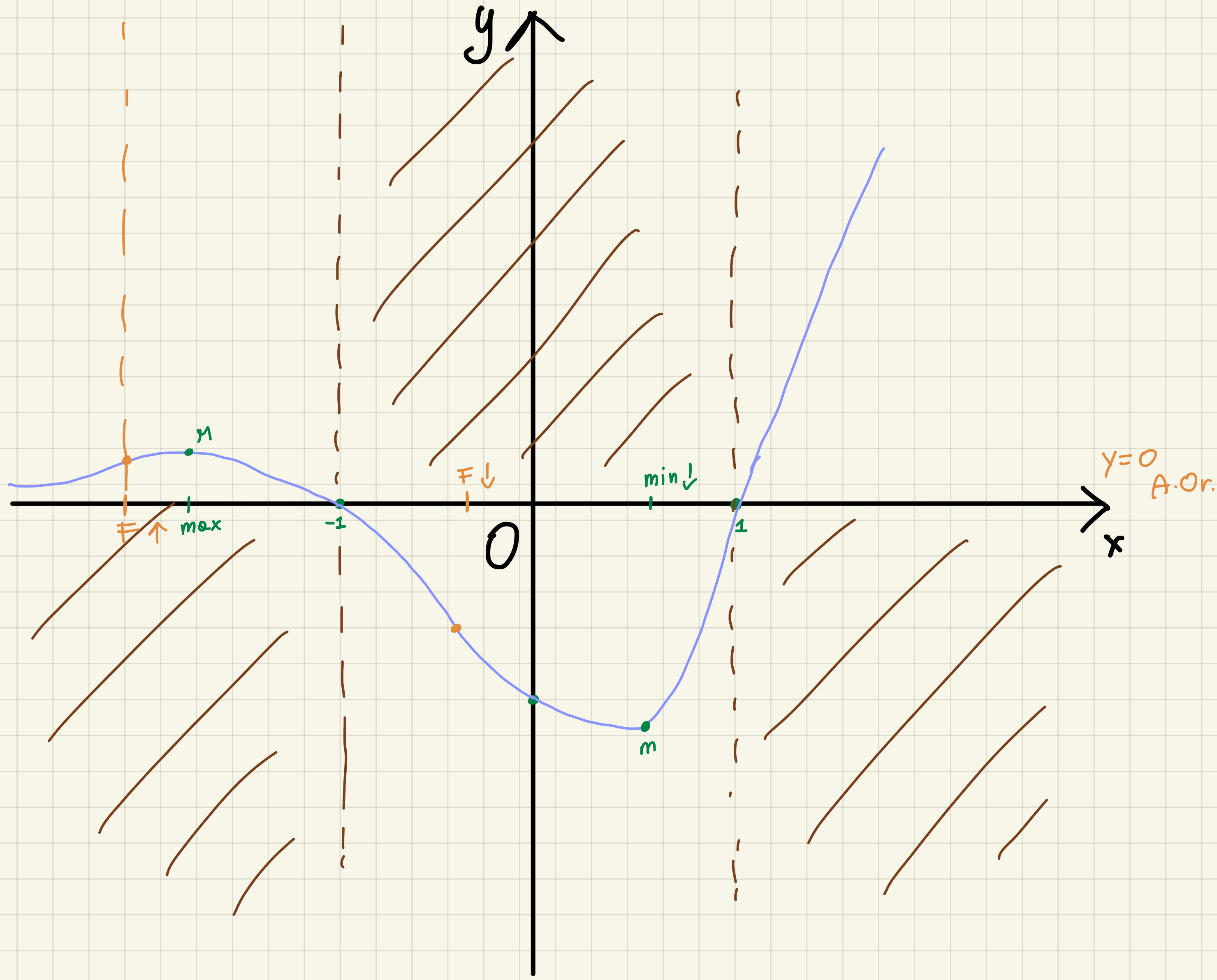
Val esterni

-3.7 $-0,26$

$-2-\sqrt{3}$ $-2+\sqrt{3}$

$+$ $-$ $+$

F F



y
x
O
M
max
F
-1
F
min
1
m
y=0
A.Or.

ES 5: $f(x) = x^2 e^{3x+5}$

1) Dominio $= \mathbb{R}$

2) Simmetrie: $f(-x) = x^2 e^{-3x+5}$ $\neq -f(x)$ NO Dispari; $\neq f(x)$ NO pari

3) Intersezioni:

$\begin{cases} x=0 \\ y=0 \end{cases} \Rightarrow (0,0) \in f(x)$

$\begin{cases} y=0 \\ x^2 e^{3x+5}=0 \end{cases}$ per $x^2=0 \Rightarrow x=0$

4) Segno $f(x)>0$

$x^2 e^{3x+5} > 0 \quad \forall x \in \mathbb{D}$

5) Asintoti

$\lim_{x\to\infty} f(x) \simeq \underset{\nearrow}{x^2} \cdot \underset{\nearrow}{e^x} \Rightarrow +\infty$ $\Rightarrow$ NO A. Oriz.

$\lim_{x\to+\infty} \frac{f(x)}{x} \simeq \frac{x^2 \cdot e^x}{\cancel{x}} = +\infty$ $\Rightarrow$ NO A. Ob.

$\lim_{x\to-\infty} f(x) = \infty \cdot 0 \simeq e^x \cdot x^2 = 0$

$\Rightarrow \frac{1}{\frac{1}{e^x}} \cdot x^2 = \frac{x^2}{\frac{1}{e^x}}$ $\quad e^x >> x^2 = 0$

$\frac{1}{e^x} \to +\infty$

$\lim \to 0$

$\Rightarrow y = 0$ Asintoto Verticale

6) Derivata

$f'(x) = 2x\left(e^{3x+5}\right) + x^2 e^{3x+5} \cdot 3 = 2xe^{3x+5} + 3e^{3x+5}x^2 \Rightarrow xe^{3x+5}(2+3x)$

$f'(x) \geq 0$ per $\begin{cases} x>0 \\ 2+3x>0 \end{cases}$ $\begin{cases} x>0 \\ x>-\frac{2}{3} \end{cases}$

	$-\frac{2}{3}$		0	
$-$		$+$		$+$
$-$		$-$		$+$
$\oplus$		$-$		$\oplus$

$f'(x) \geq 0$ per $x \leq -\frac{2}{3} \cup x \geq 0$

$-\frac{2}{3}$: max $\quad$ 0: min

$f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^2 \cdot e^{-2+5} = \frac{4}{9}e^3 \simeq 8.9 \Rightarrow \left(-\frac{2}{3}, \frac{4}{9}e^3\right)$ Max ~ 8.9

$f(0) = 0 \Rightarrow (0,0)$ min

Deriv II^a

$f''(x) = D\left(e^{3x+5}(3x^2+2x)\right) = \left[3e^{3x+5}(3x^2+2x)\right] + e^{3x+5} \cdot \left[6x+2\right] =$

$= e^{3x+5}(9x^2+6x) + e^{3x+5}(6x+2) = e^{3x+5}(9x^2+6x+6x+2) = e^{3x+5}(9x^2+12x+2)$

$f''(x) \geq 0$ per $9x^2+12x+2 \geq 0$ $\quad \Delta = 144 - 4 \cdot 9 \cdot 2 = 72$

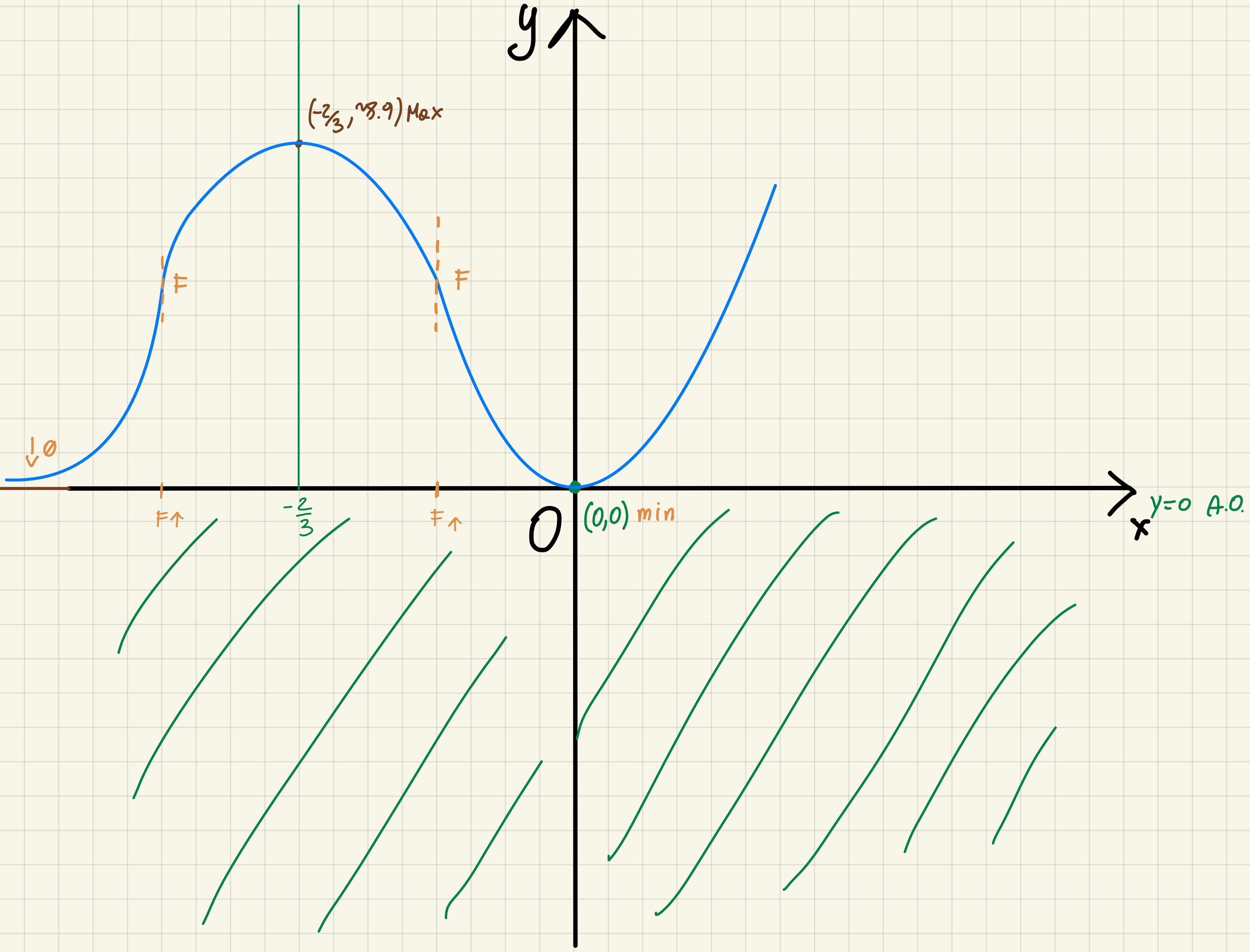
$\sqrt{72} = \sqrt{9 \cdot 8} = \sqrt{3^2 \cdot 2^2 \cdot 2} = 6\sqrt{2}$

$x_{1,2} = \frac{-12 \pm 6\sqrt{2}}{18}$ $\quad \frac{-12+6\sqrt{2}}{18} / 6 = \frac{-2+\sqrt{2}}{3} \sim -0.1$ $\quad \frac{-12-6\sqrt{2}}{18} / 6 = \frac{-2-\sqrt{2}}{3} \sim -1.1$

$a>0$, $eq>0$

sol est

	-1.1		$-2/3$		-0.1		0
$+$ $\cup$	F	$-$ $\cap$			F	$+$ $\cup$	



y
x
(-2/3, 78.9) Max
F
F
↓0
F↑
-2/3
F↑
0
(0,0) min
y=0 A.O.

Es: $f(x) = \sqrt{1-e^x}$ 1) Dominio $1-e^x \geq 0$; $e^x < 1$; $\ln|e^x| < \ln(1)$; $x < 0$

$\Rightarrow D = (-\infty, 0]$

2) Simm $f(-x) = \sqrt{1-e^{-x}} = \begin{cases} \neq -f(x) \\ \neq f(x) \end{cases}$

3) Segno $f(x) > 0$ $\forall x \in D$

4) Intersez

$\begin{cases} x=0 \\ \sqrt{1-1}=0 \end{cases} \Rightarrow (0,0) \in f(x)$ $\begin{cases} y=0 \\ \sqrt{1-e^x}=0 \end{cases}$ per $1-e^x=0$; $\ln(e^x) = \ln|1|$; $x = 0$

5) Asintoti

$\lim_{x \to \infty} f(x) = \sqrt{1-(+\infty)} = \sqrt{-\infty}$ $\nexists x \in \mathbb{R}$

$\lim_{x \to -\infty} f(x) = \sqrt{1-\underbrace{e^x}_{0}} \to 1 \Rightarrow y = 1$ A. Orizz

6) Derivate

$D(\sqrt{1-e^x}) = D\left((1-e^x)^{\frac{1}{2}}\right) = \frac{1}{2} \frac{1}{\sqrt{1-e^x}} \cdot (-e^x) = -\frac{e^x}{2\sqrt{1-e^x}} > 0$ per $\nexists x \in \mathbb{R}$

sempre pos

sempre

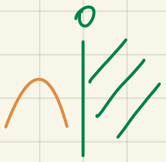
0

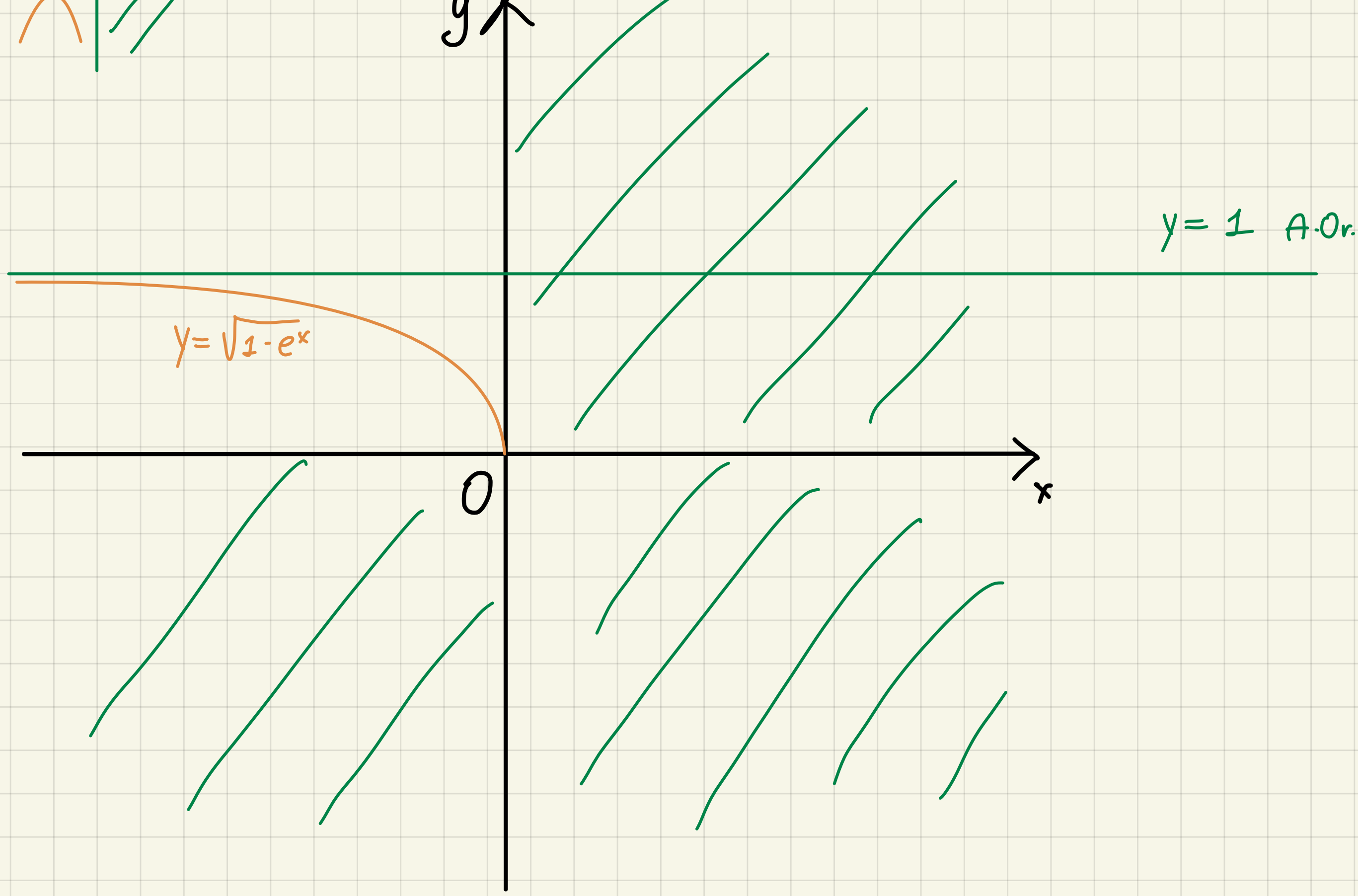
$D''\left(-\frac{e^x}{2\sqrt{1-e^x}}\right) = -e^x(2\sqrt{1-e^x}) - \left[-e^x\left(-\frac{e^x}{\sqrt{1-e^x}}\right)\right] \cdot \frac{1}{4(1-e^x)} = -2e^x\sqrt{1-e^x} - \left[\frac{e^{2x}}{\sqrt{1-e^x}}\right] \cdot \frac{1}{//}$

$\frac{-2e^x(1-e^x) - e^{2x}}{\sqrt{1-e^x}} = \frac{-2e^x + \cancel{2e^{2x}} - \cancel{2e^{2x}}}{\sqrt{1-e^x}} \cdot \frac{1}{//} = -\frac{2e^x}{\sqrt{1-e^x}}$

sempre pos

sempre pos





ES 72)

$f(x)=e^{-x^2}$

1) Dominio : $\mathbb{R}$

2) Simm $f(-x)=e^{-x^2}=f(x) \Rightarrow$ Pari

3) Intersezioni

$\begin{cases} x=0 \\ e^0=1 \end{cases} \Rightarrow (0,1)\in f(x)$

↳ Probabile max/min

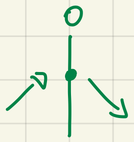
$\begin{cases} y=0 \\ e^{-x^2}=0 \end{cases}$ $\nexists x\in\mathbb{R}$

4) Segno

$f(x)>0 \quad \forall x\in\mathbb{R}$

5) Derivata I^a

$D(e^{-x^2}) = e^{-x^2}\cdot(-2x) = -2xe^{-x^2} > 0$ per $x<0$



$f(0)=1 \Rightarrow (0,1)$ Max

6) Limiti

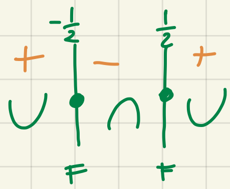
$\lim_{x\to\pm\infty} f(x) = e^{-\infty} \to 0 \Rightarrow y=0$ A. Orizz

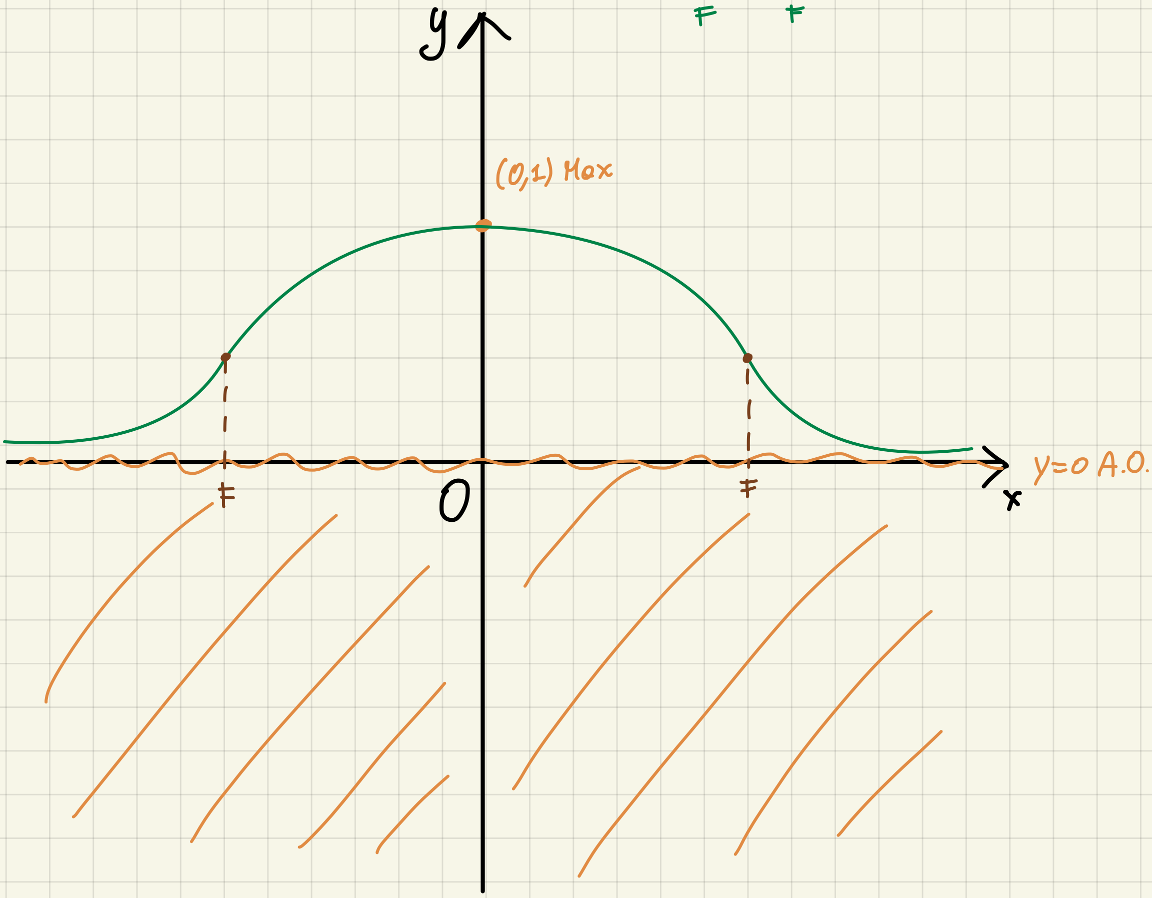
7) Deriv II

$D(-2xe^{-x^2}) = -2e^{-x^2}-2xe^{-x^2}\cdot(-2x) = -2e^{-x^2}+4x^2e^{-x^2} = 2e^{-x^2}(-1+2x^2)>0$

per $2x^2-1>0$; $x>\pm\frac{1}{2}$

a>0 , eq>0 ⇒ Val esterni





ES 73)

$f(x) = e^x + 2e^{-x}$ 1) Dominio: $\mathbb{R}$

2) Simm $f(-x) = e^{-x} + 2e^{x} \Rightarrow$ No Simm

3) Intersezioni

$\begin{cases} x=0 \\ 1+2=3 \end{cases} \Rightarrow (0,3) \in f(x)$ $\begin{cases} y=0 \\ \nexists x \in \mathbb{R} \end{cases}$

4) Segno $f(x)>0 \quad \forall x \in D$

5) Asintoti

$\lim_{x \to +\infty} f(x) = \cancel{e^{x}}^{+\infty} + 2\cancel{e^{-x}}^{0} = +\infty$ $\lim_{x \to -\infty} \cancel{e^{x}}^{0} + \cancel{e^{-x}}^{+\infty} = +\infty$ NO A.V.

$\lim_{x \to +\infty} \frac{f(x)}{x} = \frac{e^{+\infty} + e^{-\infty}}{+\infty} = \frac{e^x}{x} + \frac{e^{-x} \to 0}{x \to +\infty}$ $e^x >> x \Rightarrow +\infty \Rightarrow$ NO A.Ob.

6) Derivate

$D(e^x + 2e^{-x}) = e^x + 2e^{-x} \cdot (-1) = e^x - 2e^{-x} > 0$ per $e^x > 2e^{-x} \Rightarrow e^x - \frac{2}{e^x} > 0$

$\Rightarrow \frac{e^{2x} - 2}{e^x} > 0$ per $e^{2x} - 2 > 0$; $e^{2x} > 2 \Rightarrow 2\ln x > \ln|2| \Rightarrow x > \frac{1}{2}\ln 2 \simeq 0{,}34$

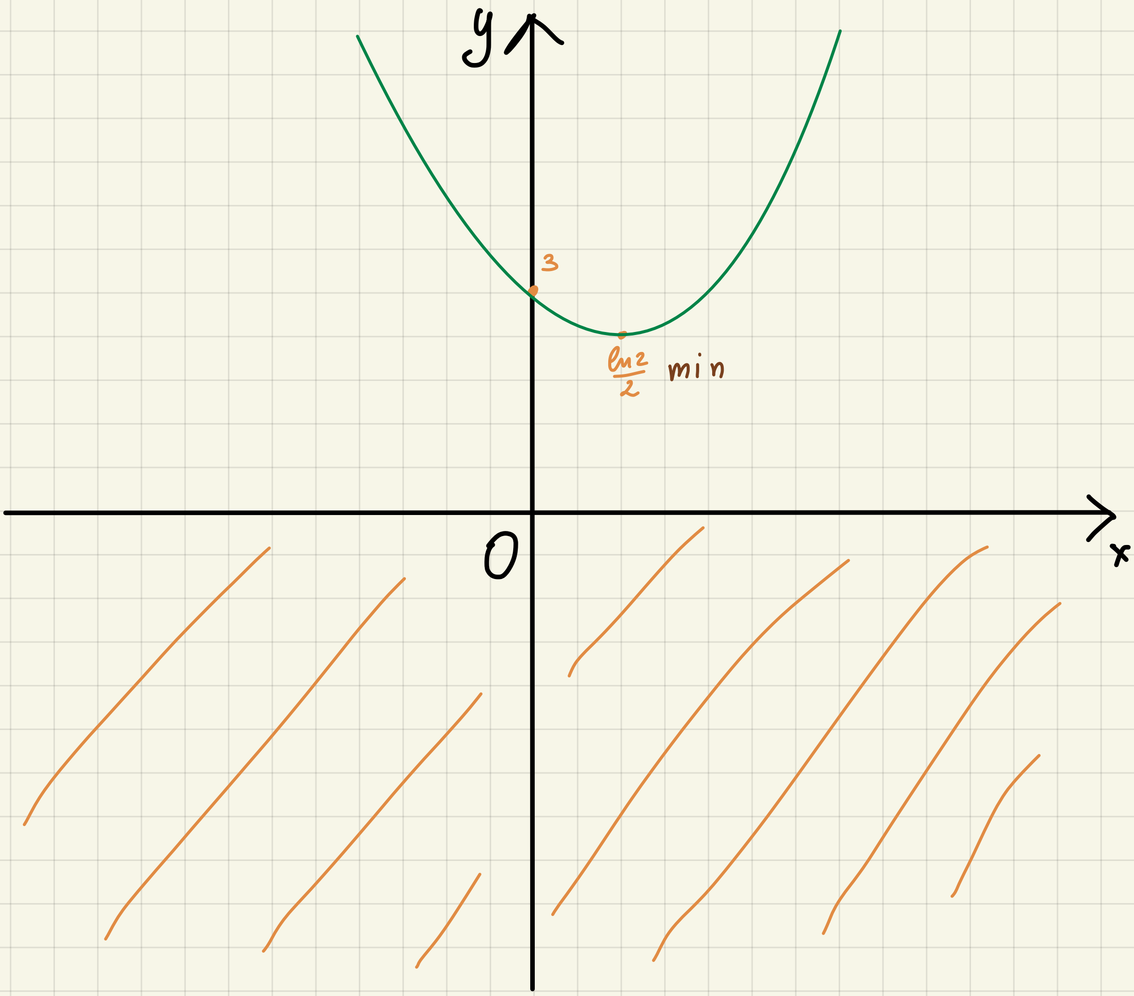
$f\left(\frac{\ln 2}{2}\right) = 1{,}4 + 1{,}4 \cong 2{,}8$

$-\ \frac{\ln 2}{2}\ +$ ↘ ↗

Deriv II $D(e^x - 2e^{-x}) = e^x + 2e^{-x} > 0 \quad \forall x \in \mathbb{R}$

0

∪ | ∪



ES 74)

$f(x) = \frac{e^x}{e^x - 1}$ D: $e^x - 1 \neq 0$ per $e^x \neq 1$; $\ln(e^x) \neq \ln(1) \Rightarrow x \neq 0$

2) Simm: $f(-x) = \frac{e^{-x}}{e^{-x} - 1} \Rightarrow$ NO Simm

3) Intersez.

$\begin{cases} x = 0 \\ \frac{1}{0} \end{cases}$ $\nexists x \in \mathbb{R}$ $\begin{cases} y = 0 \\ \frac{e^x}{e^x - 1} = 0 \end{cases}$ $\exists x \in \mathbb{R}$

4) Segno $f(x) > 0$ per $e^x > 1$, $\ln e^x > \ln 1 \Rightarrow x > 0$

5) Asintoti

$\lim_{x \to 0^+} f(x) = \frac{1}{1^+ - 1} = \frac{1}{0^\pm} = \pm\infty \Rightarrow x = 0$ A.V

$\lim_{x \to -\infty} f(x) = \emptyset$ $y = 0$ A. Or.

$\lim_{x \to +\infty} f(x) = \frac{+\infty}{+\infty} = \frac{\cancel{e^x}}{\cancel{e^x}(1 - 0)} = 1 \Rightarrow y = 1$ A. Oriz

6) Deriv I $D\left(\frac{e^x}{e^x - 1}\right) = e^x(e^x - 1) - e^x(e^x) \cdot \frac{1}{(e^x - 1)^2}$

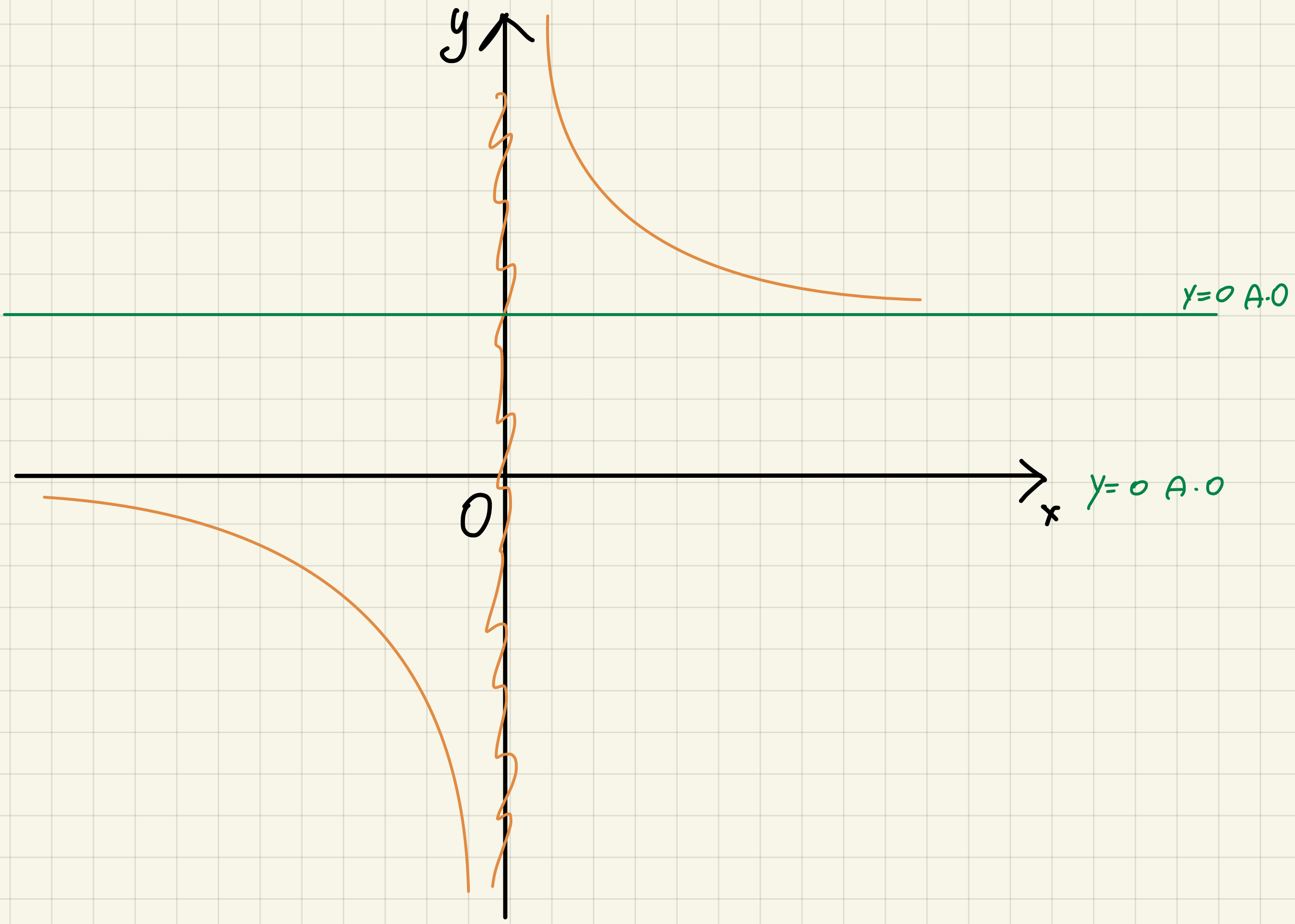
$= \cancel{e^{2x}} - e^x - \cancel{e^{2x}} \cdot \frac{1}{//} = \frac{-e^x}{(e^x - 1)^2} > 0$ $\nexists x \in \mathbb{R}$

0
↘ | ↘

Deriv II

$D\left(-\frac{e^x}{(e^x - 1)^2}\right) = \frac{e^x(e^x + 1)}{(e^x - 1)^3} > 0$ per $e^x > 1 \Rightarrow x > \ln 1 \Rightarrow x > 0$

0
$-$ | $+$
$\cap$ x $\cup$



ES 75)

$$f(x)=\begin{cases} e^{-\frac{1}{x^2}} & \text{per } x \neq 0 \\ 0 & \text{per } x=0 \end{cases}$$

1) Dominio: $\mathbb{R}$

2) Intersezioni

$\begin{cases} x=0 \\ y=0 \end{cases} \Rightarrow (0,0) \in \mathbb{R}$ $\qquad \begin{cases} y=0 \\ e^{-\frac{1}{x^2}}=0 \end{cases} \quad \nexists x \in \mathbb{R}$

3) Simm

$f(-x) = e^{-\frac{1}{x^2}} = f(x) \Rightarrow$ PARI

4) Segno

$f(x)>0 \qquad e^{-1 \cdot \frac{1}{x^2}} = \dfrac{1}{e^{\frac{1}{x^2}}} > 0 \ \forall x \in \mathbb{R} - \{0\}$

5) Asintoti

$\lim_{x \to \pm\infty} f(x) = \dfrac{1}{e^{\frac{1}{x^2}}} \to 1 \quad \Rightarrow \quad y=1$ A. Orizz

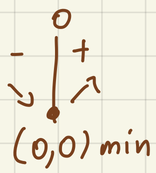
6) Deriv I

$D\left(-\frac{1}{x^2}\right) = D\left(-x^{-2}\right) \Rightarrow +2x^{-3} = \dfrac{2}{x^3}$

$D\left(e^{-\frac{1}{x^2}}\right) = e^{-\frac{1}{x^2}} \cdot \dfrac{2}{x^3} > 0$ per $x^3>0 \Rightarrow x>0$

Sempre pos

Sempre pos



(0,0) min

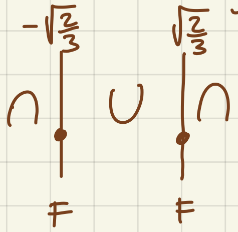
Deriv II: $D\left(\dfrac{2e^{-\frac{1}{x^2}}}{x^3}\right) \ \left[2e^{-\frac{1}{x^2}} \cdot \left(2\cancel{x^{-3}}\right)\right] \cdot \cancel{x^3} + \left[2e^{-\frac{1}{x^2}} \cdot 3x^2\right]$

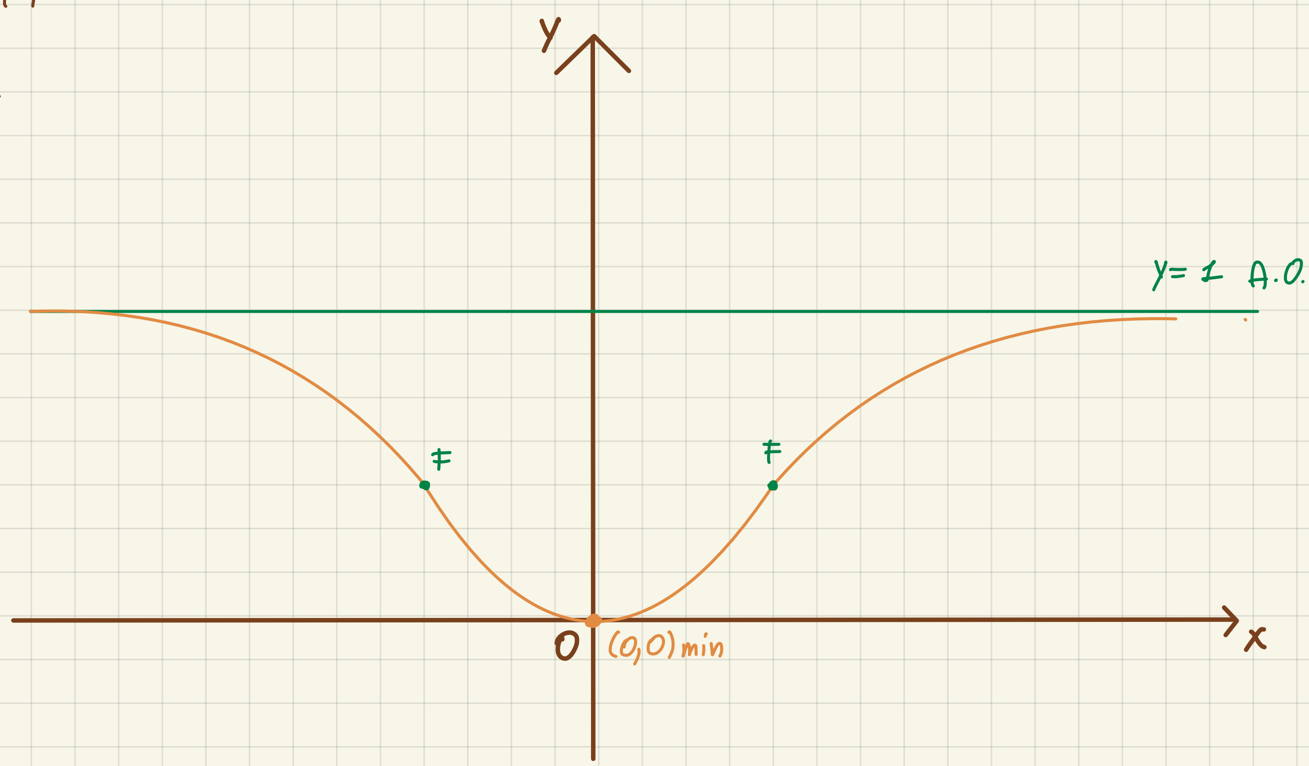
$= 4e^{-\frac{1}{x^2}} + 6x^2 e^{-\frac{1}{x^2}} = 2e^{-\frac{1}{x^2}}\left(2-3x^2\right) > 0$ per $2-3x^2>0 \quad x < \pm\sqrt{\frac{2}{3}}$

Sempre pos

$a<0$, $eq>0 \Rightarrow$ Val interm

$f''(x)>0$ per $-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$





ES 76) $f(x) = x^2 \cdot e^{-2x}$ 1) Dominio $\mathbb{R}$

2) Simm: $f(-x) = x^2 e^{2x} \Rightarrow$ No Simm

3) Intersezioni

$\begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow (0,0) \in f(x)$ $\begin{cases} y = 0 \\ \text{per } x = 0 \end{cases}$

4) Segno $f(x) > 0 \quad \forall x \in \mathbb{R}$

5) Asintoti

$\lim_{x \to +\infty} f(x) = +\infty \cdot \cancel{e^{-\infty}} \xrightarrow{} 0 \Rightarrow +\infty \cdot 0$; $\frac{\cancel{x^2} \to +\infty}{\cancel{e^{2x}} \to +\infty}$ $e^x >> x^2 \sim \frac{n}{+\infty} \to 0$

$y = 0$ A. Or

6) Derivata I°

$$D(x^2 e^{-2x}) \Rightarrow \left(\frac{x^2}{e^{2x}}\right) = \frac{2x\, e^{2x} - x^2 \cdot e^{2x} \cdot 2}{(e^{2x})^2} = \frac{2x e^{2x} - 2x^2 e^{2x}}{(e^{2x})^2}$$

$$\Rightarrow \frac{2\cancel{e^{2x}}(1 - x^2)}{(e^{2x})^{\cancel{2}}} = \frac{2(x - x^2)}{e^{2x}} > 0 \text{ per } x - x^2 > 0,\ x(1-x) > 0$$

$\hookrightarrow x > 0$

$x < 1$

	0		1	
	−	+	+	
	+	+	−	
	−	⊕	−	
	↘	↗	↘	
	min		Max	

$f(0) = 0 \Rightarrow (0,0)$ min

$f(1) = e^{-2} \simeq 0.13 \Rightarrow (1, e^{-2})$ Max

Deriv II: $D\left(\frac{2(x - x^2)}{e^{2x}}\right) = \frac{2x - 2x^2}{e^{2x}} = (2 - 4x) \cdot e^{2x} - [(2x - 2x^2) \cdot e^{2x} \cdot 2] \cdot \frac{1}{(e^{2x})^2}$

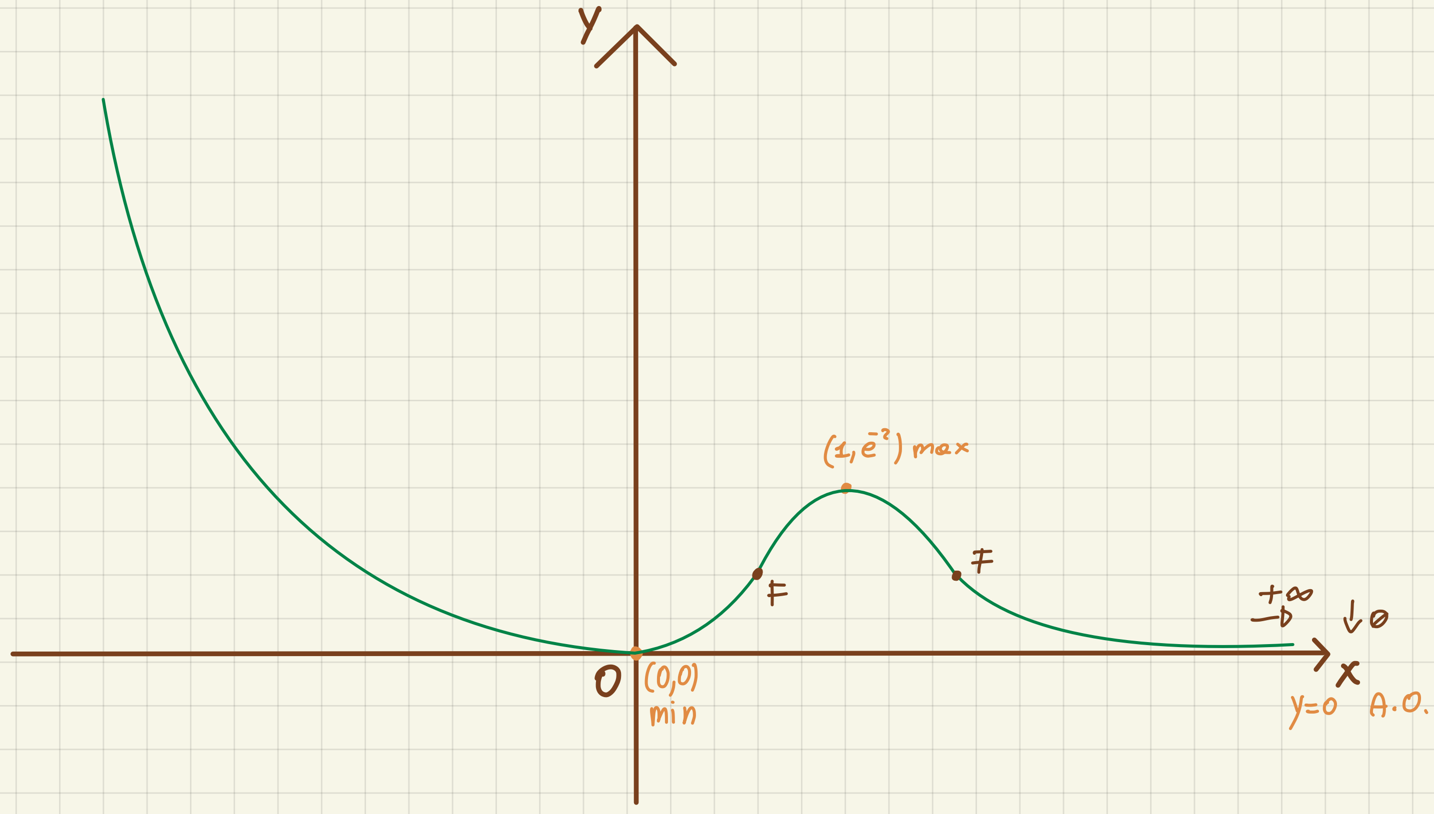
$$= 2e^{2x} - 4xe^{2x} - [4xe^{2x} - 4x^2 e^{2x}] \cdot \frac{1}{//} = \frac{2e^{2x} - 8xe^{2x} + 4x^2 e^{2x}}{(e^{2x})^2}$$

$$= \frac{2\cancel{e^{2x}}(1 - 4x + x^2)}{(e^{2x})^{\cancel{2}}} = \frac{2(x^2 - 4x + 1)}{e^{2x}} > 0 \text{ per } x^2 - 4x + 1 > 0 \quad \Delta = 16 - 4 = 12$$

$x_{1,2} = \frac{4 \pm 2\sqrt{3}}{2}$ → $2 + \sqrt{3}$, $2 - \sqrt{3}$

$a > 0$, eq $> 0 \Rightarrow$ Val est

	$2-\sqrt{3}$		$2+\sqrt{3}$	
+		−		+
∪	F	∩	F	∪



y
x
O
$(1, e^{-2})$ max
F
F
$(0,0)$
min
$+\infty$
$\to$
$\downarrow 0$
$y=0$ A.O.

ES: 77) $f(x) = x^4 \cdot e^{-\frac{x^2}{2}} = \dfrac{x^4}{\sqrt{e^{x^2}}} \Rightarrow$ 1) Dominio $e^{x^2} > 0 \;\; \forall x \in \mathbb{R}$

$\Rightarrow D: \underline{\forall x \in \mathbb{R}}$

2) Simm:
$f(-x) = x^4 \cdot e^{-\frac{x^2}{2}} = f(x) \Rightarrow \underline{\text{PARI}}$

3) Intersezioni
$\begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow (0,0) \in f(x)$ $\quad \begin{cases} y = 0 \\ \text{per } x = 0 \end{cases}$

4) Segno $f(x) > 0$ per $x > 0$

$$\begin{array}{c|c} & 0 \\ - & (+) \end{array}$$

5) Asintoti
$\lim_{x \to \pm\infty} f(x) = +\infty \cdot 0 \Rightarrow \dfrac{x^4}{e^{\frac{x^2}{2}}} \sim \dfrac{x^n}{e^x}$ $\quad e^x \gg x^n \Rightarrow \to 0$ $\quad \underline{y = 0 \text{ A.Or}}$

6) Derivate
$D(x^4 e^{-\frac{x^2}{2}}) = D\left(\dfrac{x^4}{e^{\frac{x^2}{2}}}\right) = \left(4x^3 e^{\frac{x^2}{2}}\right) - \left[x^4 \cdot \left(e^{\frac{x^2}{2}} \cdot x\right)\right] \cdot \dfrac{1}{\left(e^{\frac{x^2}{2}}\right)^2}$

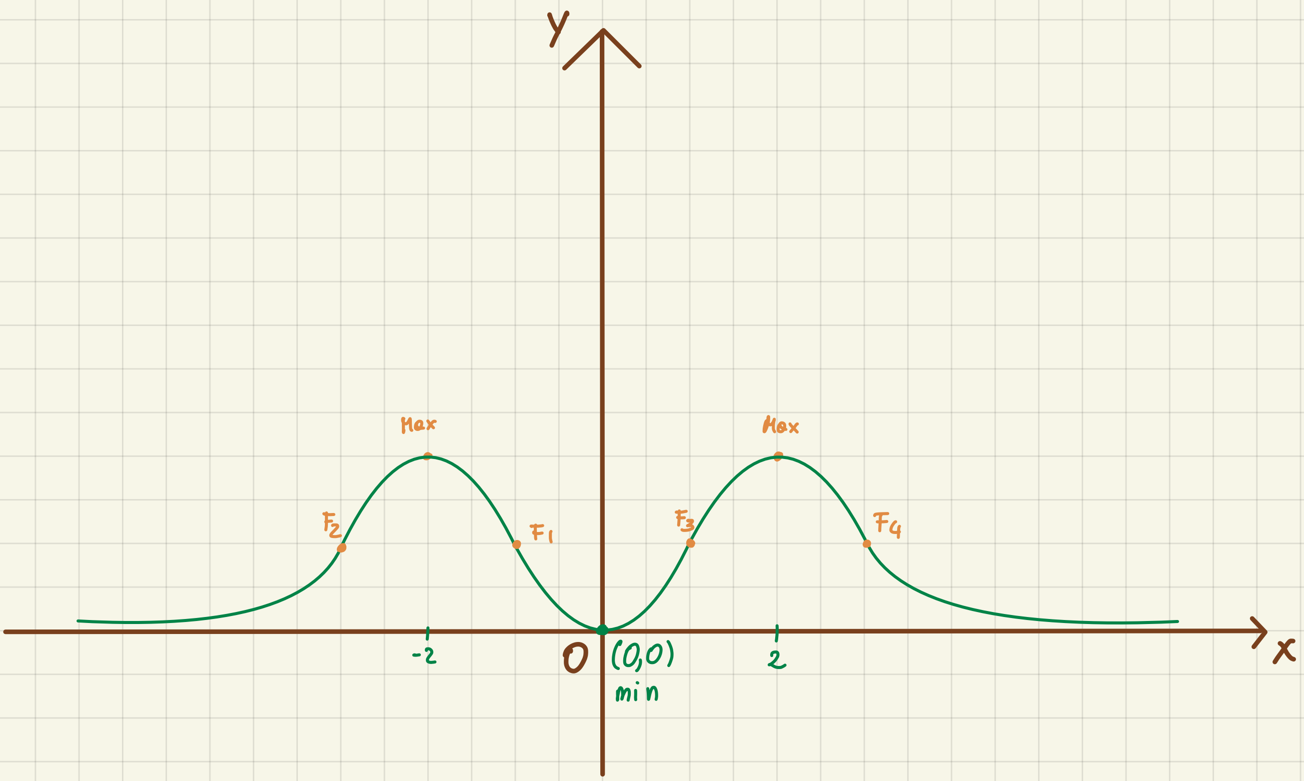
$= \left(4x^3 e^{\frac{x^2}{2}}\right) - \left(x^5 e^{\frac{x^2}{2}}\right) \cdot \dfrac{1}{//} = x^3 e^{\frac{x^2}{2}} \left(4 - x^2\right) > 0$

$\begin{cases} x^3 > 0 \text{ per } \underline{x > 0} \;\; 1 \\ 4 - x^2 > 0 \text{ per } x^2 < 4 \quad \text{per } \underline{x < \pm 2} \;\; 2 \end{cases}$
$a < 0, eq > 0 \to$ Val interni

	-2		0		2	
1	−		−		+	+
2	−		+		+	−
	+		−		+	−
	↗	Max	↘	min	↗ Max	↘

$y(2) = 2^4 \cdot e^{-\frac{2^2}{2}} = 16 \cdot \dfrac{1}{e^2} \simeq 2.1 = \dfrac{16}{e^2}$

Deriv II: $D\left[x^3 e^{\frac{x^2}{2}} (4 - x^2) \cdot \dfrac{1}{\left(e^{\frac{x^2}{2}}\right)^2}\right] =$ FATICL TU



ES 78) $f(x)=\dfrac{e^x-2}{x}$ 1) Dominio: $D: \forall x\in\mathbb{R}-\{0\}$

2) Simm

$f(-x)=\dfrac{e^{-x}-2}{-x} \Rightarrow$ NO simm

3) InTersez

$\begin{cases} x=0 \\ \dfrac{1-2}{0} \; \nexists x\in\mathbb{R}\end{cases}$ $\begin{cases} y=0 \\ \text{per } e^x=2 \Rightarrow x=\ln(2)\end{cases} \Rightarrow (\ln(2), 0)\in f(x)$ ~ 0.7

4) Segno $f(x)>0$ $\begin{cases} x>\ln(2) \\ x>0\end{cases}$

	0		$\ln 2$	
−		−		+
−		+		+
+		−		+

$f(x)>0$ per $x<0 \cup x>\ln 2$

5) AsinToTi

$\lim_{x\to 0^\pm} f(x)=\dfrac{1^+-2}{0^+}=-\dfrac{1}{0^+}=\pm\infty \Rightarrow$ NO A. VerT

$\lim_{x\to+\infty} f(x)=\dfrac{e^x-2}{x}\sim\dfrac{e^x}{x}\Rightarrow e^x>>x\Rightarrow+\infty$

$\lim_{x\to-\infty} f(x)=\sim\dfrac{\cancel{e^x}\to 0}{x}=0\Rightarrow$ $y=0$ A. Orizz

$\lim_{x\to+\infty}\dfrac{f(x)}{x}\sim\dfrac{e^x}{x^2}\to+\infty\Rightarrow$ NO A. Ob.

6) Deriv I: $D\left(\dfrac{e^x-2}{x}\right)=\dfrac{e^x\cdot x-(e^x-2)}{x^2}=\dfrac{xe^x-e^x+2}{\cancel{x^2}>0}>0$ per $e^x(x-1)>-2$

Sempre pos

$\to e^x>-2 \;\forall x\in\mathbb{R}$

$x-1>-2;\; x>-3$

Deriv II:

$D\left(\dfrac{xe^x-e^x+2}{x^2}\right)=\dfrac{-xe^x-(e^x+2)(2x)}{x^4}=\dfrac{-xe^x-[2xe^x+4x]}{x^4}$

$=-xe^x-2xe^x-4x=\dfrac{-3xe^x-4x}{//}=\dfrac{x(-3e^x-4)}{//}>0$ $\begin{cases}-3e^x-4>0\\ x>0 \to x<0\end{cases}$

$\Rightarrow -3e^x>4;\; e^x<-\dfrac{4}{3};\; x<\ln-\dfrac{4}{3}\;\nexists x\in\mathbb{R}$

Sempre neg $\Rightarrow$ x deve essere neg

Non CompleTo

2.37 Studiare le seguenti funzioni e disegnarne il grafico

(a) $f(x) = \dfrac{x^2-3}{x-2}$ (b) $f(x) = \dfrac{x^2}{1-x}$

Funzione (a) $f(x) = \dfrac{x^2-3}{x-2}$

1) Dominio $x-2 \neq 0$ per $x \neq 2$

2) Simm:

$f(-x) = \dfrac{x^2-3}{-x-2} \Rightarrow$ NO Simm

3) Segno

$f(x)>0$ per $x^2-3>0$; $x>\pm\sqrt{3}$ $\cup$ $x>2$

$f(x)>0$ per $-\sqrt{3}\leq x\leq\sqrt{3}$ $\cup$ $x>2$

$-\sqrt{3}$		$\sqrt{3}$		2	
+	−		+		+
−	−		−		+
	+				+
↗ max	↘		min	↗	

4) intersezioni

$\begin{cases} x=0 \\ -\dfrac{3}{-2} = \dfrac{3}{2} \end{cases} \Rightarrow \left(0,\frac{3}{2}\right) \in f(x)$

$\begin{cases} y=0 \\ \text{per} \end{cases}$ $(\sqrt{3},0)$ $(-\sqrt{3},0)$ $\in f(x)$

Asintoti: $\lim_{x\to 2^\pm} f(x) = \dfrac{4-3}{2^\pm-2} = \dfrac{1}{0^\pm} = \pm\infty \Rightarrow$ $x=2$ A. Vert Dx e Sx

$\lim_{x\to\pm\infty} f(x) \sim \dfrac{x^{\cancel{2}}}{\cancel{x}} = \pm\infty \Rightarrow$ NO A. Or.

$\lim_{x\to\pm\infty} \dfrac{f(x)}{x} \sim \dfrac{x^2}{x^2} = 1 \Rightarrow m=1$ $\quad\lim_{x\to\pm\infty} f(x)-mx = \dfrac{x^2-3}{x-2} - x \simeq \dfrac{x^{\cancel{2}}}{\cancel{x}} - x = \infty-\infty$

$\dfrac{\cancel{x^2}-\cancel{x^2}}{x} = \dfrac{1}{x} = 0 \Rightarrow q=0 \Rightarrow y=x =$ A. Obl.

6) Deriv I $\quad D\left(\dfrac{x^2-3}{x-2}\right) = \dfrac{2x(x-2)-(x^2-3)}{(x-2)^2} = \dfrac{2x^2-4-x^2-3}{//} = \dfrac{x^2-7}{//} > 0$ per $x^2>7$ per $x>\pm\sqrt{7}$

$-\sqrt{7}$		$\sqrt{7}$	
+	−		+
↗	↘		↗

Deriv II: $D\left(\dfrac{x^2-7}{(x-2)^2}\right) = \dfrac{2x(x-2)^2-(x^2-7)\cdot[2(x-2)]}{(x-2)^4} = \dfrac{2x[x^2-2x+4]-[(x^2-7)\cdot(2x-4)]}{//}$

$= \dfrac{\cancel{2x^3}-\cancel{4x^2}+6x-[\cancel{2x^3}-\cancel{4x^2}-14x+28]}{//} = \dfrac{20x-28}{(x-2)^2} > 0$ per $\cancel{20}x>\dfrac{28}{20}$ per $x>\dfrac{7}{5}$

	$7/5$	
−		+
∩		∪