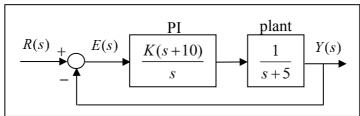
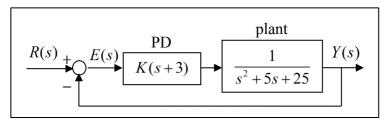
Introductory Control Systems Exercises #10 – Steady-State Error

1. A *proportional-integral* ("PI") controller is used to control a 1st order plant as shown. The system has input R(s), output Y(s), and error E(s). Find $\frac{E}{R}(s)$ the *error transfer function*, and then find the range of values for the parameter K so the system has a steady-state error e_{ss} less than 0.01 for a unit ramp input $(R(s) = 1/s^2)$. Assume K > 0.



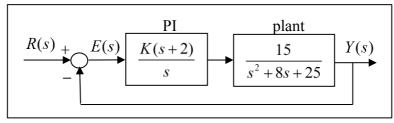
 $\left| \frac{E}{R}(s) = \frac{s(s+5)}{s(s+5) + K(s+10)} \right|; K > 50$

2. A proportional-derivative ("PD") controller is used to control a 2nd order plant as shown. The system has input R(s), output Y(s), and error E(s). Find $\frac{E}{R}(s)$ the **error transfer function**, and find e_{ss} the **steady**state error associated with a unit step input in terms of the parameter K.



Answers:
$$\left[\frac{E}{R}(s) = \frac{s^2 + 5s + 25}{s^2 + 5s + 25 + K(s+3)}\right]$$
; $\left[e_{ss} = \frac{25}{25 + 3K}\right]$

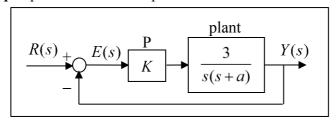
3. A *proportional-integral* ("PI") controller is used to control a 2nd order plant as shown. The system has input R(s), output Y(s), and error E(s). Find $\frac{E}{R}(s)$ the *error transfer function*, and then find the range of values for the parameter K so the system has a steady-state error less than 0.1 to a **unit ramp** input $(R(s) = 1/s^2)$. Assume K > 0.



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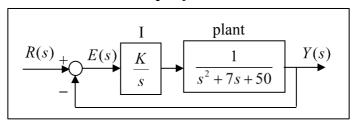
Answers:
$$\left[\frac{E}{R}(s) = \frac{s(s^2 + 8s + 25)}{s(s^2 + 8s + 25) + 15K(s + 2)} \right]$$
; $K > 8.33$

4. A *proportional* ("P") controller is used to control a 2^{nd} order plant as shown. The system has input R(s), output Y(s), and error E(s). Find $\frac{E}{R}(s)$ the *error transfer function*, and then find e_{ss} the *steady-state error* associated with a *unit ramp* input in terms of the parameters a and K.



Answers:
$$\left[\frac{E}{R}(s) = \frac{s(s+a)}{s^2 + as + 3K}\right], \left[e_{ss} = \frac{a}{3K}\right]$$

5. An *integral* ("I") controller is used to control a 2^{nd} order plant as shown. The system has input R(s), output Y(s), and error E(s). Find $\frac{E}{R}(s)$ the *error transfer function*, and then find the range of values for the gain K so e_{ss} the steady-state error due to a *unit ramp* input is less than 1.0. Assume K > 0.



Answers:
$$\left| \frac{E}{R}(s) = \frac{s(s^2 + 7s + 50)}{s^3 + 7s^2 + 50s + K} \right|$$
; $\left| \frac{K > 50}{s^3 + 7s^2 + 50s + K} \right|$

6. A *proportional-integral* ("PI") controller is used to control a 2nd order plant as shown. The system has input R(s), output Y(s), and error E(s). Find $\frac{E}{R}(s)$ the *error transfer function*, and then find the range of values for the parameter K so the steady-state error to a *unit ramp* input is less than 0.1. Assume K > 0.

Answers:
$$\frac{E}{R}(s) = \frac{s(s^2 + 10s + 125)}{s(s^2 + 10s + 125) + 100K(s + 1)}$$
; $K > 12.5$

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ES 1

$$R(S) + E(S) \times (S+LO) \times (S+S) \times (S+LO) \times (S+LO)$$

Arocesso

(U(s)
$$\frac{1}{s+5}$$
 $\frac{E(s)}{R(s)} = T_{e+e} = S(s) = \frac{1}{1 + C(s)G(s)} = \frac{1}{1 +$

2)
$$e_{SS}$$
 per RAMPA UNITARIA $\mathcal{E}(t) = t \cdot \mathcal{I}(t) \rightleftharpoons R(S) = \frac{1}{S^2}$ $(\kappa > 0) = \rho e_{\nu} < 1\% \equiv 0.01$

F. Anello
$$C(S) \cdot G(S) = L(S) = \frac{K(S+40)}{\Im(S+5)}$$
 $e_{\rho} = 0$, $e_{\nu} = \frac{1}{\mu}$ con $\mu = \text{avadaano oli } L(S)$

$$e_{\rho} = 0$$
, $e_{\nu} = \frac{1}{\mu}$ con $\mu = avadagno di L(s)$

$$\mu = \lim_{S \to 0} \frac{K(\$+10)}{\$(\$+5)} \cdot \$ = \frac{100}{5} = 2K = 0 \quad \text{ev} = \frac{1}{2K} < 0.01 - 0 \quad 2K > 100 = 0 \quad K > 50$$

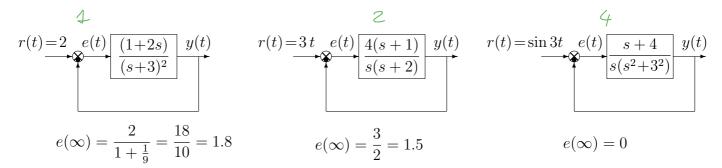
Procedimento alternativo

$$\frac{E(S)}{E(S)} = \frac{S(S+5)}{S(S+5)+K(S+10)} = 0 \quad E(S) = \frac{S(S+5)}{S(S+5)+K(S+10)} \cdot R(S) = 0 \quad e_{SS}(t) = \lim_{t \to \infty} e(t) = \lim_{t \to \infty} S \cdot E(S)$$

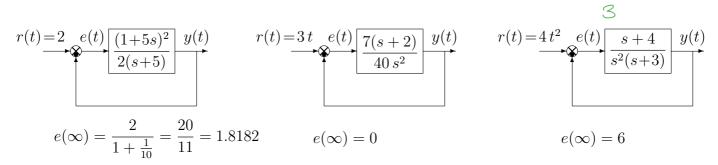
$$= P e_{SS} = \lim_{S \to 0} S \cdot \frac{S(S+5)}{S(S+5) + K(S+10)} \cdot \frac{1}{S^{2}} = \frac{5}{[5 + K(S+10)]S} =$$

Esempi

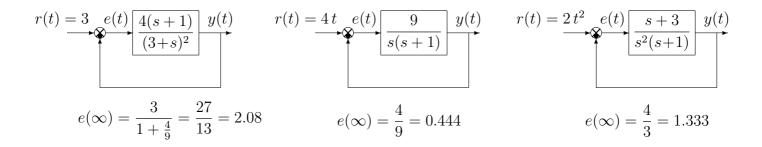
ullet Calcolare l'errore a regime $e(\infty)$ per i seguenti sistemi retroazionati:

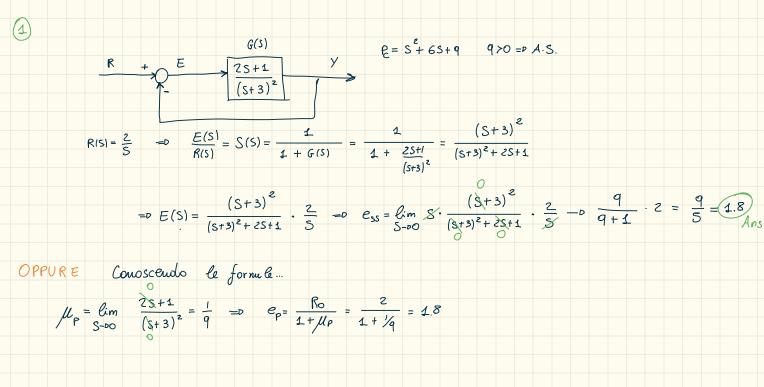


ullet Calcolare l'errore a regime $e(\infty)$ per i seguenti sistemi retroazionati:



 \bullet Calcolare l'errore a regime $e(\infty)$ per i seguenti sistemi retroazionati:





$$\begin{array}{c|c}
\hline
R(S) & E(S) \\
+ & & \\
\hline
 &$$

$$\xi(t) = 3t \rightleftharpoons R(S) = \frac{3}{S^2}$$
 =D $e(\infty) = e_{Velocite}$

$$=0 \quad \mu_{v} = \lim_{S \to 0} S \cdot G(S) = \lim_{S \to 0} S \cdot \frac{4(S+1)}{5(S+2)} = \frac{4}{2} = 2\mu_{v} \quad \sim 0 \quad e_{v} = \frac{R_{o}}{\mu_{v}} = \frac{3}{2} = 1.5$$

$$2(t) = 4t^2$$
 Siccome $Z[R_0 \cdot t \cdot 11(t)] = \frac{R_0 \cdot n!}{t^{n+1}}$
= $P(S) = \frac{4 \cdot 2}{t^3} = \frac{8}{t^3} R_0$

$$= 0 \quad \mu_{\alpha} = \lim_{S \to 0} s^{2} \frac{s_{14}}{s_{13}} - 0 \frac{4}{3} \mu_{\alpha} = 0 \quad e_{\rho} = e_{\gamma} = 0 \quad e_{\alpha} = \frac{R_{0}}{\mu_{\alpha}} = \frac{8}{4/3} = 6 \quad e_{\alpha}$$
Ans

$$\xi(S) = \sin 3t = \Re(S) = \frac{3}{S^2 + 9} = 0 \quad S_{yz} = \pm 13$$

$$\overline{S}_{1} = 0$$
, $S^{2} + 9 - 0$ $S_{1/2} = \pm \sqrt{3}$ $\sqrt{-p}$ $e_{SS} = \emptyset$ Ans

