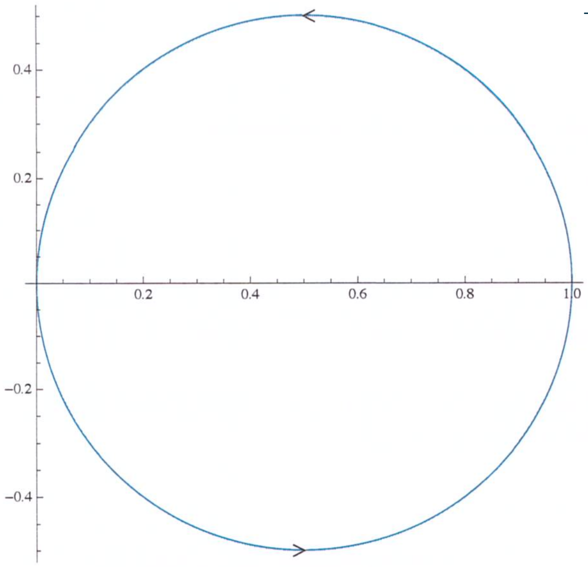
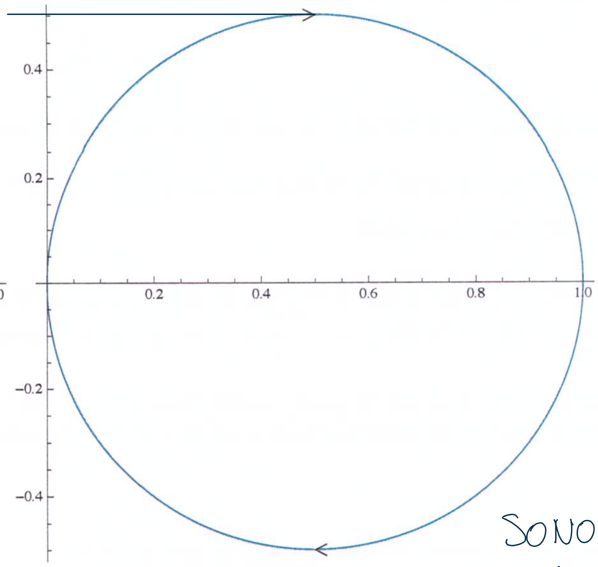
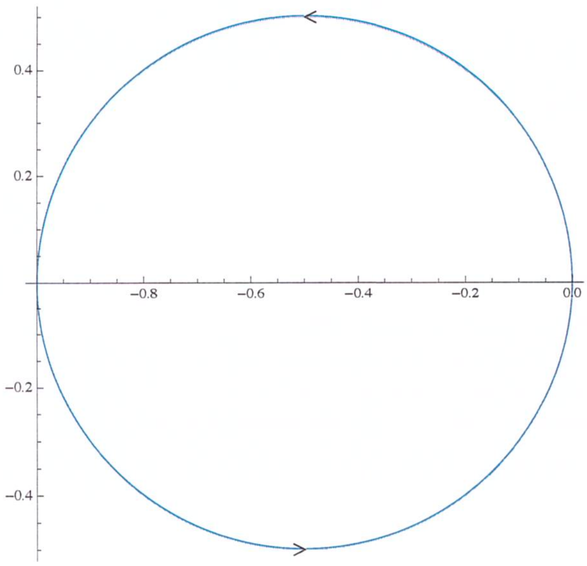


(i)

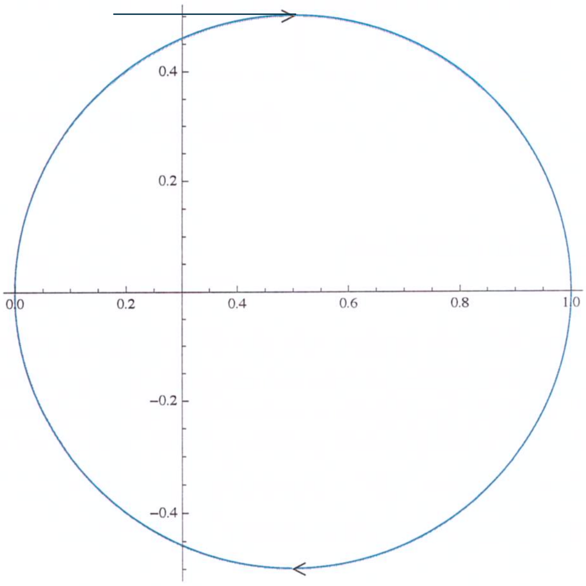


(ii)

SONO LETTERALMENTE
UGUALI



(iii)



(iv)

ESERCIZIO 2.

Si abbinino le funzioni di trasferimento con i corrispondenti diagrammi di Nyquist riportati nelle figure:

5 punti

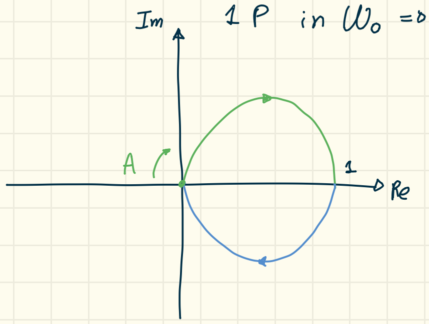
I	I $L(s) = \frac{s}{s-1}$ ✓	(A)	Fig. (ii)
II	IV $L(s) = \frac{s}{s+1}$ ✗	(B)	Fig. (iv)
III	III $L(s) = \frac{1}{s-1}$ ✓	(C)	Fig. (iii)
IV	II $L(s) = \frac{1}{s+1}$ ✗	(D)	Fig. (i)

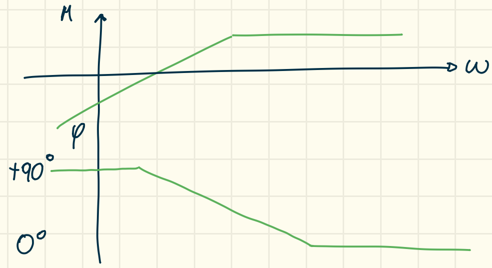
$L(s) = \frac{s}{s+1}$

$\omega \to 0 \quad M = 0$

$1\ Z$ in $0 \Rightarrow M_0 = +20\,dB/dec \quad \varphi_0 = +90^\circ \qquad \mu_0 = 1 > 0 \Rightarrow \varphi_0 = +90^\circ$

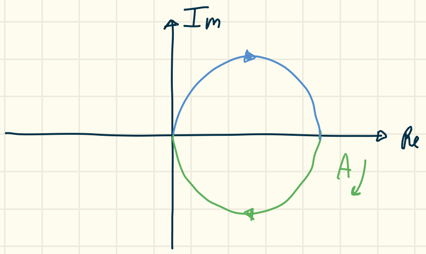
$1\ P$ in $\omega_0 \Rightarrow M_{\omega_0} = -20\,dB/dec \quad \varphi_{\omega_0} = -90^\circ$

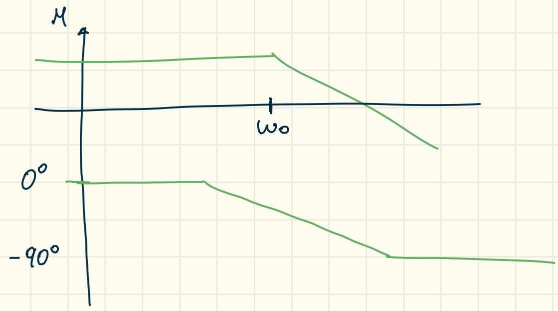




$L(s) = \frac{1}{s+1}$

1 Polo in $\omega_0 \Rightarrow M_0 = (1)_{VN} = 0\,dB \quad M_{\omega_0} = -20\,dB/d \quad \varphi_{\omega_0} = -90^\circ$





$$L(s) = \frac{s^2+4s+9}{(s-1)(s-10)^2}$$

1) C.Trasf $\lim_{s \to 0} s^g L(s) = \frac{9}{-1 \cdot 100} = -0.09 < 0 \rightarrow$ Inizio da sx

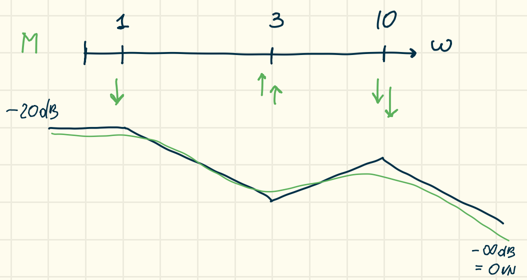
Zeri: $\begin{cases} \bar{z}_1 = -2-\sqrt{5}j \\ \bar{z}_2 = -2+\sqrt{5}j \end{cases}$ $\Rightarrow$ Re P < 0 $\rightsquigarrow$ $\omega_n = \sqrt{9} = 3$ Rad/s $\Rightarrow$ $M_{\omega_n} = +40$dB/dec, $\varphi_{\omega_n} = +180°$ $(90°/\text{dec})$

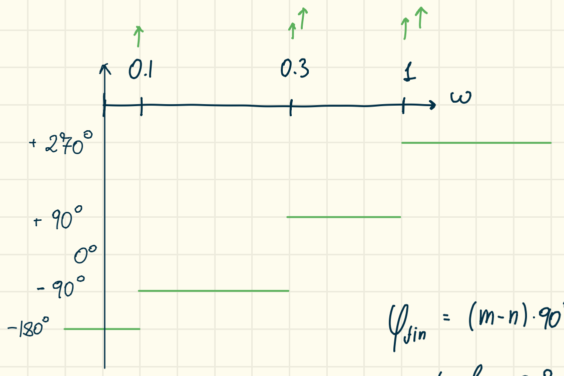
$M_0 = |-0.09|_{dB} \cong -20$dB

Poli: $\begin{cases} P_1 = 1 & \text{Re}P > 0 & \omega_1 = 1 \text{ Rad/s} \\ P_{2,3} = -10 & \text{Re}P > 0 & \omega_{2,3} = 10 \text{ Rad/s} \end{cases}$

M_{ω_1}: -20dB/dec $\quad \varphi_{\omega_1}$: $+90°$

M_{ω_2}: -40dB/dec $\quad \varphi_{\omega_{2,3}}$: $+180°$



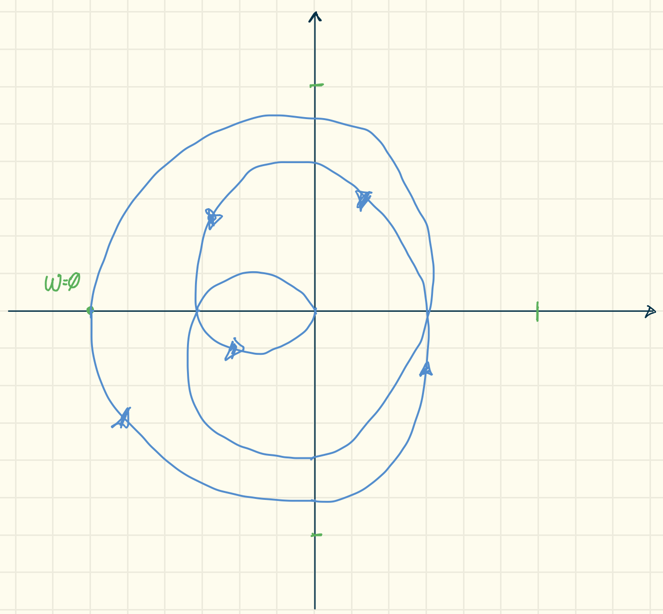


! Rispetto a -180° !

$\varphi_{fin} = (m-n) \cdot 90° = 5 \cdot 90° = 450°$

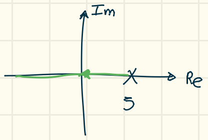
ma parto da $-180° \rightarrow -180 + 450° = 270°$

! Rispetto 0° !



• Luogo delle Radici

$G(S) = \frac{1}{S-5}$

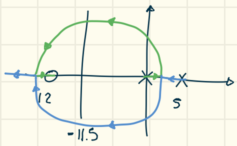


• Q: $T_{a1} < 0.4s$

• $e_r < 10\%$ ⟶ Mi serve un integratore

$\frac{4.6}{\sigma} < 0.4 \to |\sigma| > \frac{4.6}{0.4} = 11.5$

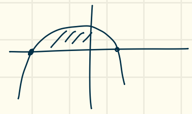
$C(S) = \frac{K(S+12)}{S} \Rightarrow C'(S) = \frac{K(S+12)}{S(S-5)}$

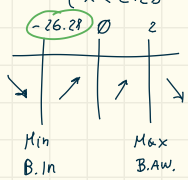


$\gamma(x) = -\frac{D(x)}{N(x)} = \frac{-(x^2-5x)}{x+12} \to \gamma'(x) = -(2x-5)(x+12)+(x^2-5x) > 0$

$-2x^2-24x+\cancel{5x}+60+x^2-\cancel{5x} > 0$

$-x^2-24x+60>0$ per $\begin{cases} x > -26.28 \\ x < 2.28 \end{cases}$





$|K| = \frac{D(s^*)}{N(s^*)} = \frac{-[(s^*)^2 - 5s^*]}{s^*+12} = 57.57$

• $\mu = \lim_{S\to 0} \cancel{S}\, \frac{K(S+12)}{\cancel{S}(S-5)} = K \cdot \frac{12}{-5} = -2.4K \Rightarrow e_r = \left|\frac{1}{\mu}\right| = \left|\frac{1}{-2.4K}\right| < 0.1 \to K > \frac{1}{0.1 \cdot 2.4} \Rightarrow K > 4.16$

$\Rightarrow$ Prendo $K = 58 \to C(S) = \frac{58(S+12)}{S}$ Ans

• Perché si sceglie il margine critico Negativo?

Progetto in frequenza

Esercizio 1.
Si consideri la funzione di trasferimento

$$G(s) = k \cdot \frac{s-9}{s(s+9)}$$

5 punti

e si scelga il guadagno $k \in \mathbb{R}$ in maniera tale che $G(s)$ abbia un margine di fase pari a 40°.

Per avere $\varphi_m = 40 \to 180 - |\varphi_c| = 40° \Rightarrow |\varphi_c| = 140° \to \varphi_c = \pm 140° \to \varphi_c = -140°$

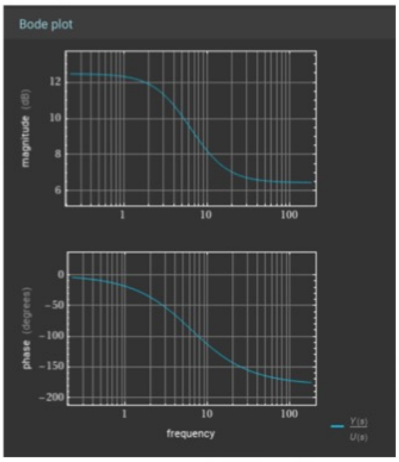
• ω_c

$|G(j\omega_c)| = 1$ per $\frac{K\sqrt{\omega^2+81}}{\omega\sqrt{\omega^2+81}} = 1 \to \omega_c = K$

• $\angle G(j\omega_c) = -140° \to \angle K + \angle j\omega_c - 9 - \angle j\omega_c - \angle j\omega_c + 9 = -a\tan\left(\frac{\omega_c}{9}\right) - 90° - a\tan\left(\frac{\omega_c}{9}\right) = -140°$

$\to -2a\tan\left(\frac{\omega_c}{9}\right) = -50 \to a\tan\left(\frac{\omega_c}{9}\right) = 25° \to \omega_c = 9\,\tan(25°) = 4.197$

$\Rightarrow |K| = \pm 4.2$ ma $\mu = \lim_{S\to 0} \cancel{S}\, \frac{K(S-9)}{\cancel{S}(S+9)} = -K$ ma voglio $\mu > 0 \Rightarrow K < 0 \Rightarrow K = -4.2$



2. MARGINE DI AMPIEZZA Con scelta di guadagno e zero

$$G(s) = k \frac{(s+z)}{(s+5)^2} \quad \text{scelgo } z = -5 \text{ (OPPOSTO AL POLO)} \Rightarrow G(s) = \frac{k(s-5)}{(s+5)^2}$$

vogliamo $M_a = 6\,dB = 10^{\frac{6}{20}} = (2)_{dB}$ ← valore Naturale

Siccome cerco il margine di ampiezza, devo trovare la ω_c per la quale $\angle G(j\omega_c) = -180°$

Dopodiché calcolo $|G(j\omega_c)|$ e vedo quanto differisce $(1)dB = 0dB$

1) Guadagno

$$\mu = \lim_{s\to 0} s^g \cdot G(s) = \lim_{s\to 0} \frac{k(s-5)}{(s+5)^2} = -\frac{5}{25}k$$

Vogliamo un guadagno $\mu > 0 \Rightarrow k<0$

2) Trovo ω_c

$$\angle G(j\omega_c) = -180 = -\pi \rightarrow \angle k(j\omega-5) - \angle (j\omega+5)^2 = -\pi \rightarrow -\text{atan}\left(-\frac{\omega}{5}\right) - 2\,\text{atan}\left(\frac{\omega}{5}\right) = -\pi$$

$$\rightarrow -3\,\text{atan}\left(\frac{\omega}{5}\right) = -\pi \Rightarrow 3\,\text{atan}\left(\frac{\omega_c}{5}\right) = \pi \rightarrow \frac{\omega_c}{5} = \text{Tan}\left(\frac{\pi}{3}\right) \rightarrow \omega_c = 5\,\text{Tan}\left(\frac{\pi}{3}\right) = 8.66\ Rad/s = 5\sqrt{3}\ Rad/s$$

ω_c

3) Modulo in $\omega_c = 5\sqrt{3}\ R/s$

$$|G(j\omega_c)| = \frac{|k(j\omega_c-5)|}{|(j\omega_c+5)^2|} = \frac{|k|\sqrt{\omega_c^2+5^2}}{\omega_c^2+25} \quad \text{per} \quad k = \frac{2(\omega_c^2+25)}{\sqrt{\omega_c^2+5^2}} = 20$$

Per trovare k: $\frac{1}{|G(j\omega_c)|} = (M_a)_{dB}$ oppure $\frac{1}{|G(j\omega_c)|_{dB}} = M_a\,dB$

$$\Rightarrow \frac{1}{|G(j\omega_c)|} = \frac{\omega_c^2+25}{|k|\sqrt{\omega_c^2+5^2}} = 2 \quad \text{per} \quad |k| = \frac{\omega_c^2+25}{2\sqrt{\omega_c^2+25}} = \frac{75+25}{2\sqrt{100}} = 5$$

$$\text{Quindi} \quad |k| \rightarrow \begin{cases} k>0 \rightarrow k = 5 \\ k<0 \rightarrow k = -5 \end{cases}$$

Scelgo $k<0$ perché mi serve $\mu>0$

$$\Rightarrow G(s) = -\frac{5(s-5)}{(s+5)^2}$$

Ans

NEW ↓

• Posso usare due controllori in cascata in modo da lavorare sul luogo diretto

Luogo delle radici

Esercizio 1.

Si consideri l'impianto descritto dalla funzione di trasferimento

$$G(s) = \frac{1}{2-s}$$

5 punti

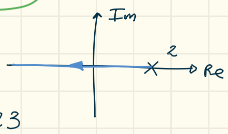
e si progetti un controllore $C(s)$ tale che, in uno schema con retroazione negativa unitaria, si garantisca:

- risposta indiciale con tempo di assestamento T_a all'1% inferiore a 0.2s;
- errore di velocità inferiore al 5%.

$C_1(S) = -1 \Rightarrow L(S) = \frac{1}{S-2}$ Luogo diretto

$G(S) = \frac{1}{2-S} = - \frac{1}{S-2}$?

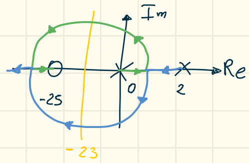
$2 - S = 0 \rightarrow S = 2$



Q_1: $T_{a1} < 0.2 \Rightarrow \frac{4.6}{\sigma} < 0.2 \rightarrow |\sigma| > \frac{4.6}{0.2} = 23$

Q_2: $e_r < 5\% \Rightarrow g = 1$

$\Rightarrow C_2(S) \triangleq \frac{K(S+Z)}{S}$ Scelgo $Z = 25 \rightarrow$



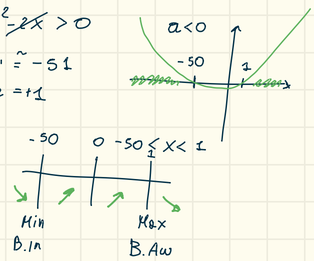
$L'(S) = \frac{K(S+25)}{S(S-2)} \rightarrow \gamma(x) = -\frac{x^2-2x}{x+25} \rightarrow \gamma' = -\frac{(2x-2)(x+25)+(x^2-2x)}{(\dots)^2} > 0$ per

$-2x^2 - 50x + \cancel{2x} + 50 + x^2 - \cancel{2x} > 0$

$-x^2 - 50x + 50 > 0$

$x_1 \simeq -51$

$x_2 = +1$



Scelgo $S^* = -50$

$\Rightarrow |\rho| = \left|\frac{(S^*)^2 - 2S^*}{S^* + 25}\right| = |104| \Rightarrow$ Scelgo $K = 105$

OPPURE SCEGLIERE $K < 0$?

• $e_v < 5\%$ Vuol dire $\left|\frac{1}{\mu_v}\right| < 0.05 \rightarrow L'(S) = \frac{K(S+25)}{S(S-2)} \rightarrow \mu_v = \lim_{S \to 0} \cancel{S} \frac{K(S+25)}{\cancel{S}(S-2)} = -\frac{25K}{2}$

$\rightarrow \left|-\frac{2}{25K}\right| < 0.05 \rightarrow \frac{2 \cdot 20}{25} < K \rightarrow K > 1.6 \Rightarrow K = 105$ è più che sufficiente

$\Rightarrow C(S) = - \frac{105(S+25)}{S}$ Ans

$C_1(S)$

---

errata, viene sottratto un quarto del suddetto punteggio. Se non è indicata alcuna risposta, vengono assegnati zero punti.

$S^2 + 2\xi\omega_n S + \omega_n^2$ se $\xi < -1$ $\quad \Delta = \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2} = 2\omega_n\sqrt{\xi^2 - 1} \rightarrow x_{1,2} = \frac{\overbrace{(-2\xi\omega_n)}^{>0} \pm \overbrace{(2\omega_n\sqrt{\xi^2-1})}^{>0}}{2}$ ma $2\xi\omega_n \overset{\text{Sicuramente}}{>} 2\omega_n\sqrt{\xi^2-1} \Rightarrow$ Poli a $ReP > 0$

1. *(1 punto)* Un polinomio caratteristico del secondo ordine con coefficiente di smorzamento $\xi < -1$ presenta:

(a) Due poli reali e distinti entrambi negativi

~~(b)~~ Due poli reali e distinti entrambi positivi

(c) Due poli reali coincidenti

(d) Due poli complessi e coniugati a parte reale negativa

(e) Due poli complessi e coniugati a parte reale positiva

3. *(2 punti)* Si consideri un sistema di controllo con retroazione negativa unitaria dove la funzione di anello è $L(s) = \frac{380}{(s+19)(s+4)(s+5)}$. Si determini il margine di fase.

(a) $-38°$

(b) $38°$

(c) $180°$

(d) ∞

(e) 0

Esercizio 2. Si tracci il diagramma di Nyquist della f.d.t.

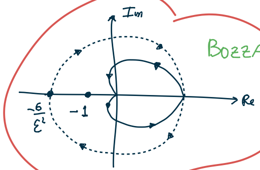
$$L(s) = 2\frac{s+3}{s^2(s-1)}.$$

5 punti

Si dica se il corrispondente sistema a ciclo chiuso con retroazione negativa unitaria è asintoticamente stabile o meno.

$2\,\dfrac{(s+3)}{s^2(s-1)}$ $\quad \mu = -6 \;\Rightarrow\; \varphi_{in} = -180°$

INIZIO



BOZZA

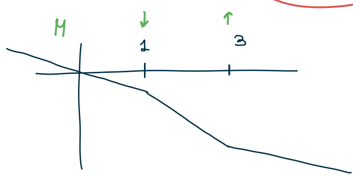
$P_c = s^3 - s^2 + 2s + 6$

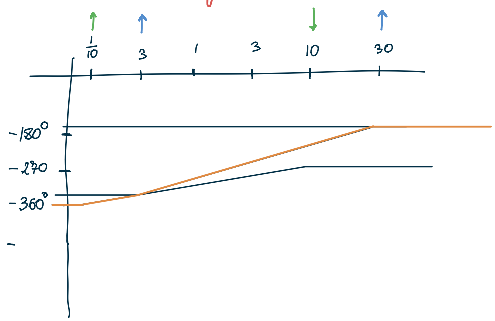
$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & -1 & 6 \\ s^1 & & \\ s^0 & & \end{array}$$

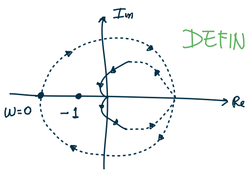
A.I.

Basta così oppure devo fare i tratteggi?

Provo a farlo più accurato

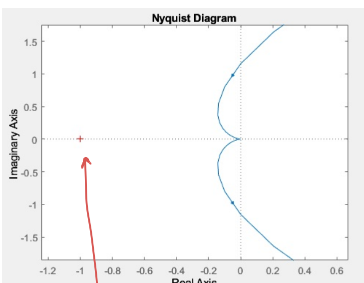






DEFINITIVO

1 Polo a $Re P > 0$ $\Rightarrow$ voglio 1 circondamento ma ne ho zero! $\Rightarrow$ A.I.



Nessun Circondamento