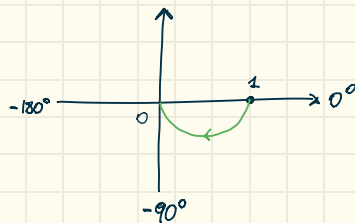
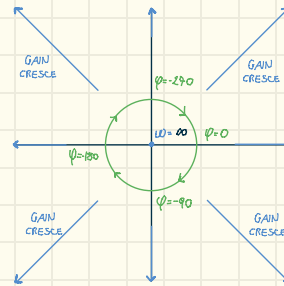
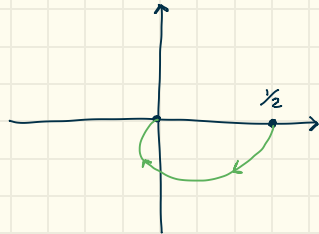
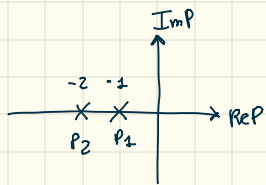


## Esempi Basilari

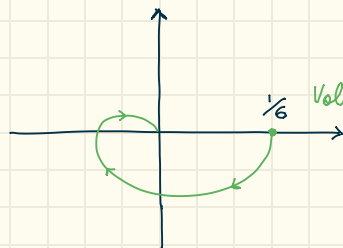
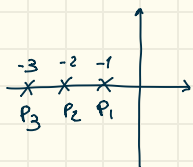
- 1 Polo  $\text{Re}P < 0$   $G(s) = \frac{1}{s+1}, K=1$



- 2 Poli  $\text{Re}P < 0$   $G(s) = \frac{1}{(s+1)(s+2)}$



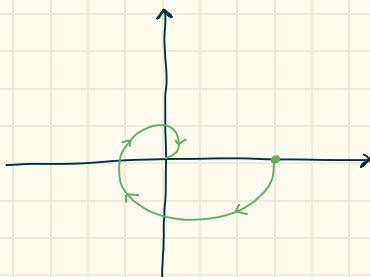
- 3 Poli  $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$



Valore naturale

$$\left| \frac{1}{6} \right| = 20 \log_{10} \left( \frac{1}{6} \right) \approx -15 \text{ dB}$$

- 4 Poli  $G(s) = \frac{1}{(s+1)(s+2)(s+3)(s+4)}$



- 1 Polo in Origine  $G(s) = \frac{1}{s}$

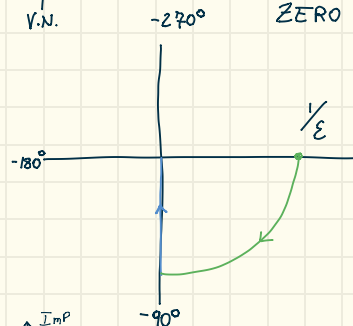
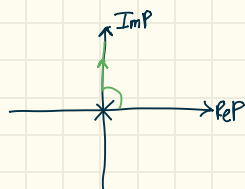
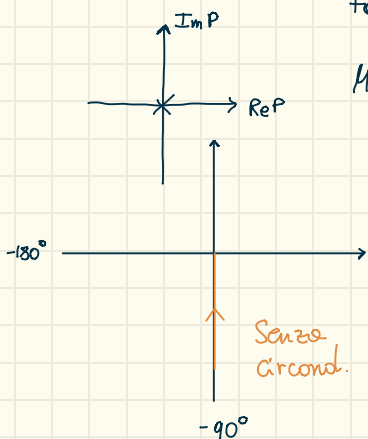
Fase iniziale =  $+90^\circ$  = F. Finale

LA FASE RIMANE COSTANTE

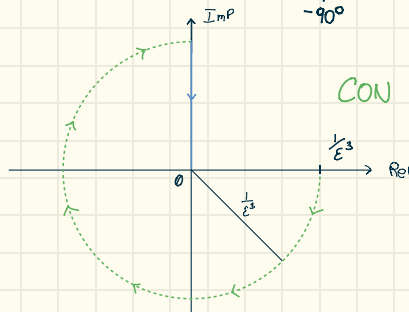
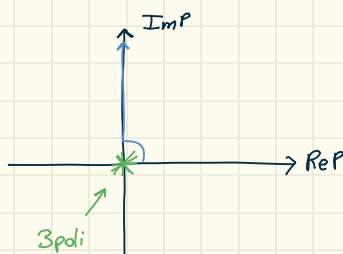
$$M_{in} = M_{fin} = -20 \text{ dB/dec} \Rightarrow M_{in} = +\infty, M_{fin} = 0$$

V.N.                      V.N.

IL MODULO TENDE A ZERO DA  $+\infty$



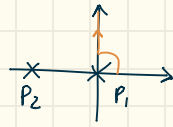
$$G(s) = \frac{1}{s^3}$$



CON CIRCONDAIMENTO

- 1 Polo in 0 ed uno a  $\text{Re}P < 0$

$$G(s) = \frac{1}{s(s+1)}$$



-20dB/dec  
iniziale

-20dB/dec  
a  $\omega_0 = 1$

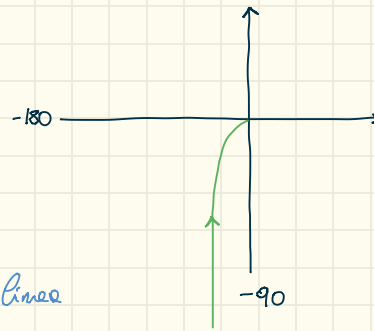
viene da  
 $+\infty$

Disegnare una linea  
PARALLELA

$$\varphi = -90^\circ - 90^\circ = -180^\circ \text{ Tot}$$

iniziale

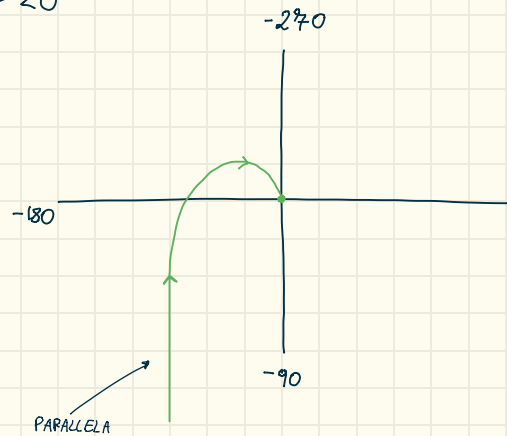
Contributo  
a  $\omega_0 = 1$



- 1 Polo in 0 e due a  $\text{Re}P < 0$

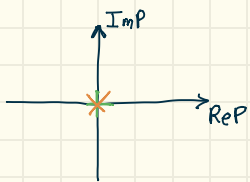
$$G(s) = \frac{1}{s(s+1)(s+2)}$$

-180 a  $\omega_0 = \omega_1$   
 $\varphi_i = -90$

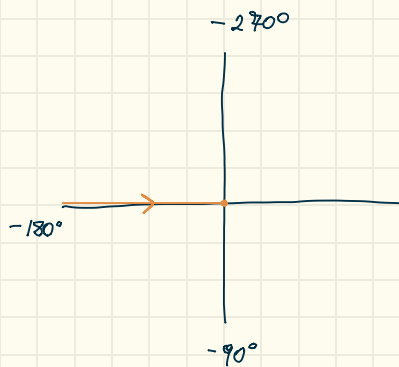


- 2 Poli in 0 (Tipo 2 =  $g=2$ , ordine 2)

$$G(s) = \frac{1}{s^2}$$



$$\varphi_i = -90 = \varphi_f$$



ES 1

$$s^2 + 2s + s^3 + 2s^2 = s^3 + 3s^2 + 2s$$

$$G(s) = \frac{1}{s(1+s)(s+2)}$$

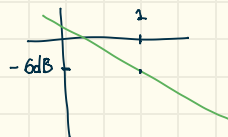
$$\begin{aligned} Z &= \text{NONE} \\ P &= 3 \rightarrow \begin{cases} P_1 = 0 \\ P_2 = -1 \\ P_3 = -2 \end{cases} \end{aligned}$$

Forma Bode:  $\frac{1}{s \cdot (1+s) \cdot 2(1+\frac{1}{2}s)} = \frac{1}{2} \frac{1}{s(1+s)(1+\frac{1}{2}s)}$

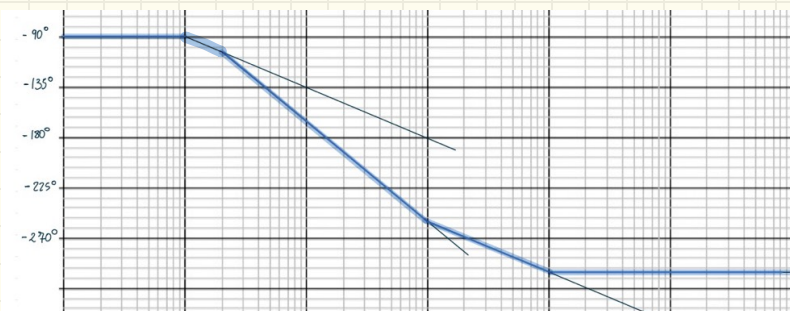
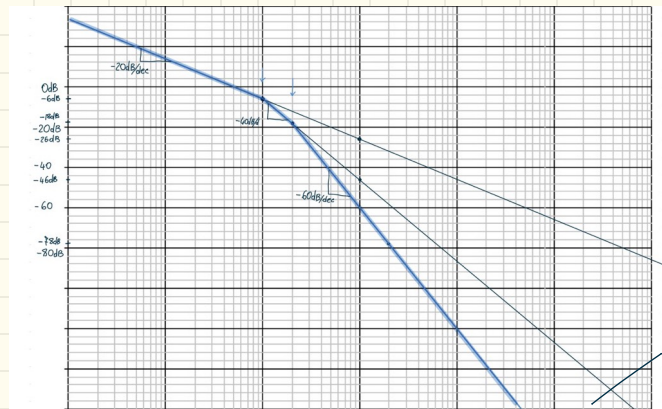
$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

Andamento iniziale

$$20 \log_{10} \left( \frac{K}{\omega_0} \right) = 20 \log_{10} \left( \frac{\frac{1}{2}}{1} \right) \approx -6 \text{ dB}$$



FASE: 1 Polo in 0  $\Rightarrow -90^\circ$  iniziale cost  
2 Poli a  $\text{Re}P < 0 \Rightarrow -90^\circ - 90^\circ = -180^\circ - 90^\circ = -270^\circ$  finale



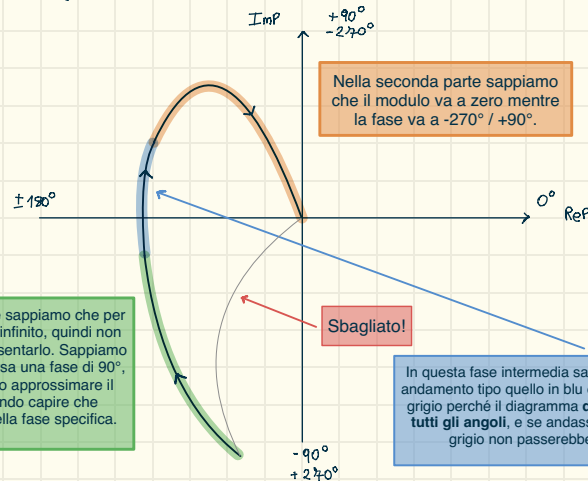
$$20 \log_{10}(a) - b \rightarrow \log_{10}(a) = \frac{b}{20} \rightarrow a = 10^{\frac{b}{20}}$$

MODULO

FASE

INIZIALE  $20 \log_{10}(x) = \infty \Rightarrow 10^{\frac{\infty}{20}} = \infty$  ( $\omega = 0$ )

FINALE  $20 \log_{10}(x) = -\infty \Rightarrow 10^{\frac{-\infty}{20}} = 0$  ( $\omega = \infty$ )



Disegnare il diagramma Senza Bode

Abbiamo ottenuto le informazioni che ci servivano usando un diagramma di Bode precedentemente costruito, ma possiamo trovare le informazioni anche andando a valutare modulo e fase per  $\omega=0$  e  $\omega=\infty$

$$G(s) = \frac{1}{s(s+1)(s+2)} \rightarrow G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

$$\Rightarrow |G(j\omega)| = \frac{1}{\omega} \cdot \frac{1}{\sqrt{1+\omega^2}} \cdot \frac{1}{\sqrt{4+\omega^2}}$$

$\omega=0 \rightarrow |...| = \infty$   
 $\omega=\infty \rightarrow |...| = 0$

$$\angle G(j\omega) = \angle \frac{1}{j\omega} - \angle \frac{1}{1+j\omega} - \angle \frac{1}{2+j\omega} = -90^\circ - \arctan(\omega) - \arctan\left(\frac{\omega}{2}\right)$$

$\omega=0 \rightarrow -90^\circ$   
 $\omega=\infty \rightarrow -90^\circ - 90^\circ - 90^\circ = -270^\circ$

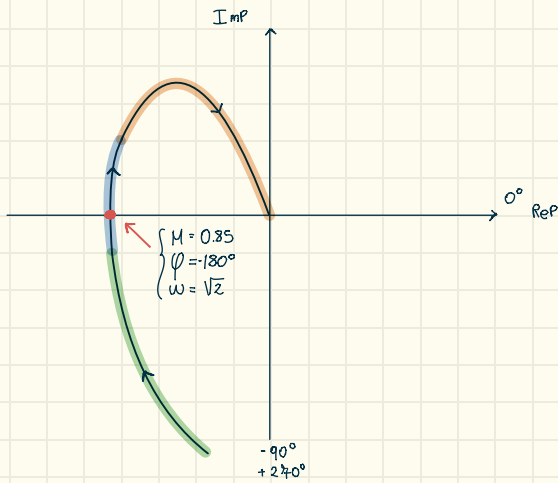
# Trovare il valore del modulo in uno specifico punto

È difficile trovare il valore del modulo/fase in uno specifico punto. Quello che possiamo fare è trovare il valore del modulo in punti "strategici", come nell'intersezione con gli assi.

Nel caso precedente il diagramma interseca l'asse reale negativo, di cui conosciamo l'angolo:  $+180^\circ$ . Possiamo usare questo angolo per trovare la pulsazione a cui questo accade ed usare poi la pulsazione per trovare il modulo.

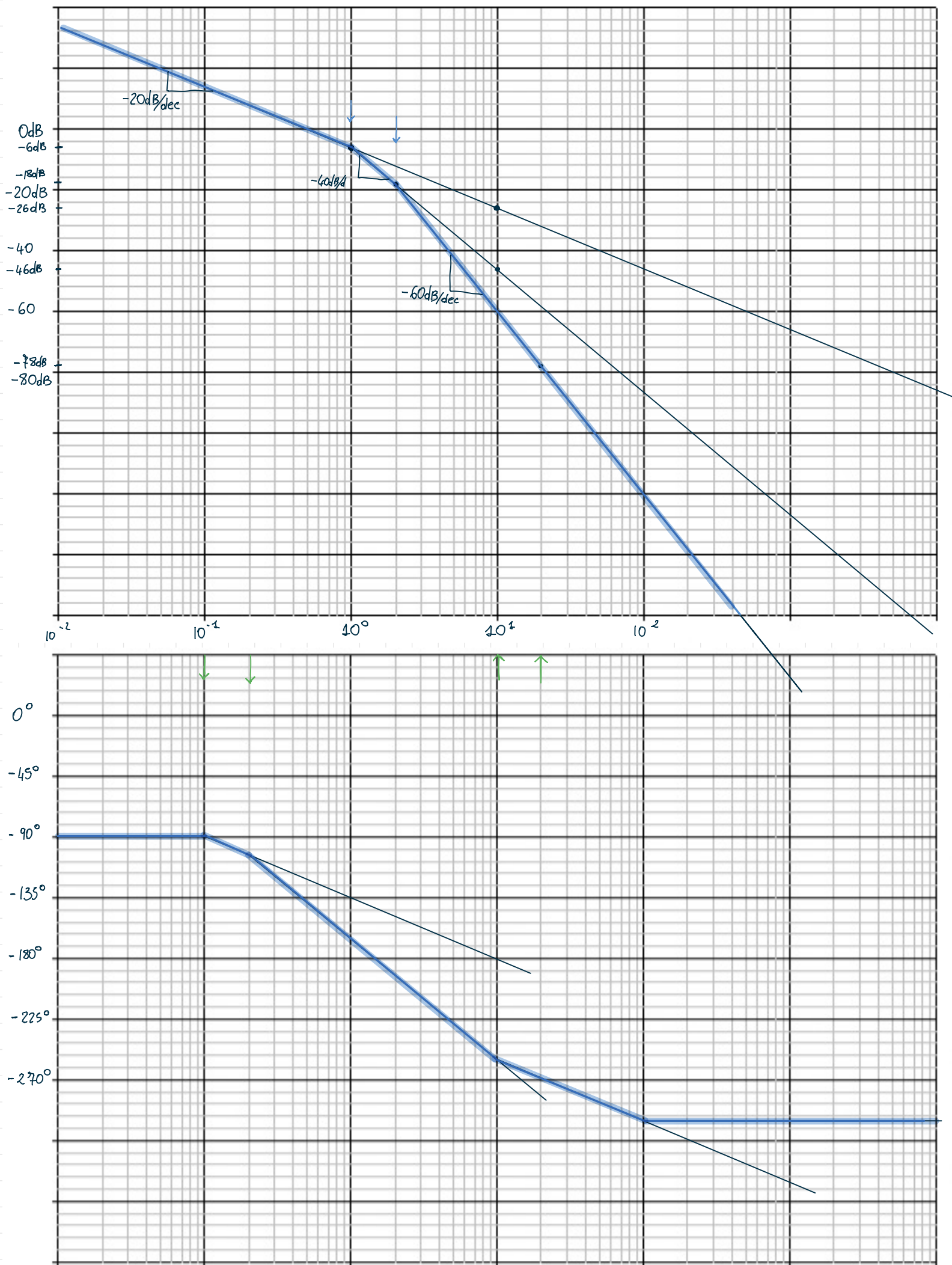
$$\varphi = -90 - \arctan(\omega) \cdot \arctan\left(\frac{\omega}{2}\right) = 180^\circ \Leftrightarrow \omega = \sqrt{2}$$

$$\Rightarrow M = \left| G(j\omega_0) \right| = 0.85$$
$$\omega_0 = \sqrt{2}$$



Matricola: \_\_\_\_\_

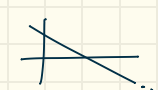
Nome: \_\_\_\_\_

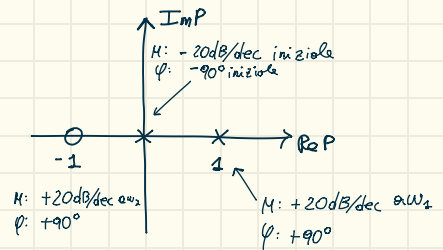


ES 2: lezione 14: 18/04/24

$$G(s) = 2 \frac{s+1}{s(s-1)} = 2 \frac{1+s}{s \cdot (-1)(1-s)} = \underbrace{-2}_{K} \frac{s+1}{s(1-s)} \quad \leftarrow 1 \text{ ReP} > 0 \text{ polo}$$

↑ polo in 0





$$M: \frac{2 \cdot \sqrt{1+\omega^2}}{\omega \cdot \sqrt{1+\omega^2}} = \frac{2}{\omega}$$

$\omega=0 \rightarrow M = \infty$   
 $\omega=\infty \rightarrow M = 0$

} Valori Naturali

$\varphi =$   
 1 Polo in 0  $\Rightarrow -90^\circ$  iniziale  
 1 Zero a  $\text{ReP} < 0 \Rightarrow +90^\circ/2\text{dec}$   
 1 Polo a  $\text{ReP} > 0 \Rightarrow 90^\circ/2\text{dec}$

$$\angle 2 + \angle \frac{1}{j\omega+1} - \angle j\omega - \angle \frac{1}{j\omega-1} = \arctan(\omega) - \arctan\left(\frac{\omega}{1}\right) - \arctan(-\omega)$$

$$= 2\arctan(\omega) - 90^\circ$$

$\omega=0 \Rightarrow 0 - 90^\circ = -90^\circ$   
 $\omega=\infty \Rightarrow 90^\circ$

Both Now mi Trovo con la fase

Diciamo che

$$\begin{cases} \omega=0 \rightarrow \varphi = -270^\circ \equiv +90^\circ \\ \omega=\infty \rightarrow \varphi = -90^\circ \equiv +270^\circ \end{cases}$$

