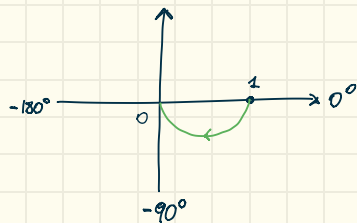
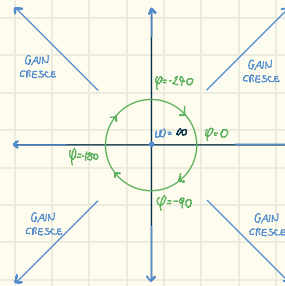
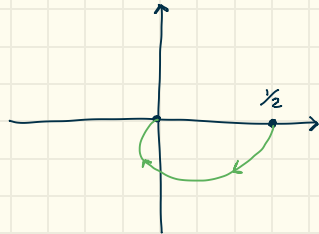
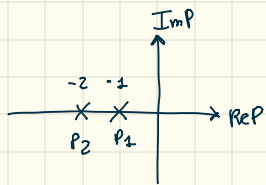


Esempi Basilari

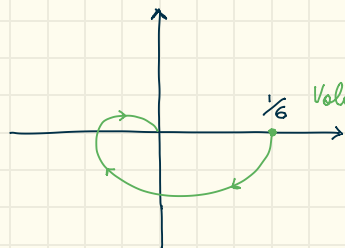
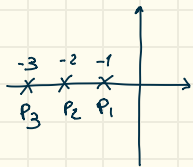
- 1 Polo $\text{Re}P < 0$ $G(s) = \frac{1}{s+1}, K=1$



- 2 Poli $\text{Re}P < 0$ $G(s) = \frac{1}{(s+1)(s+2)}$



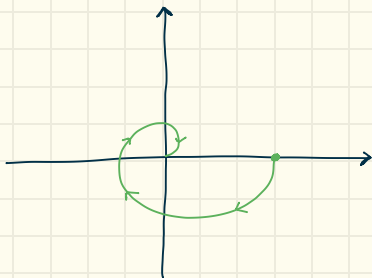
- 3 Poli $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$



Valore naturale

$$\left| \frac{1}{6} \right| = 20 \log_{10} \left(\frac{1}{6} \right) \approx -15 \text{ dB}$$

- 4 Poli $G(s) = \frac{1}{(s+1)(s+2)(s+3)(s+4)}$



- 1 Polo in Origine $G(s) = \frac{1}{s}$

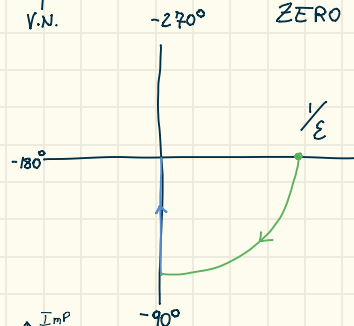
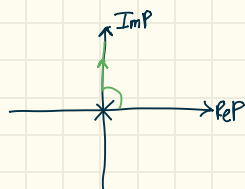
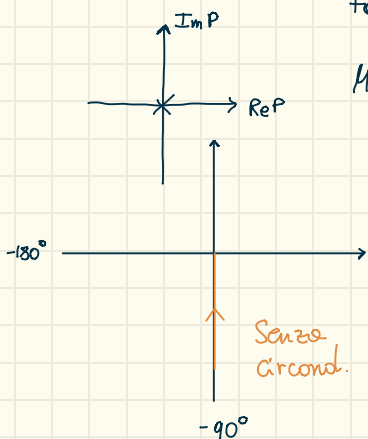
Fase iniziale = $+90^\circ$ = F. Finale

LA FASE RIMANE COSTANTE

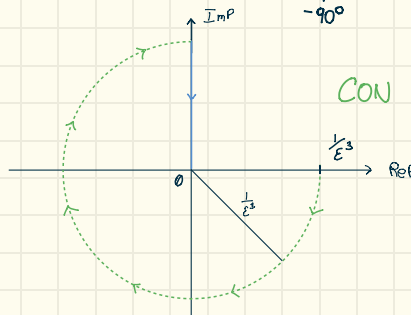
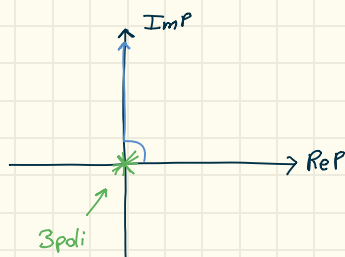
$$M_{in} = M_{fin} = -20 \text{ dB/dec} \Rightarrow M_{in} = +\infty, M_{fin} = 0$$

V.N. V.N.

IL MODULO TENDE A ZERO DA $+\infty$



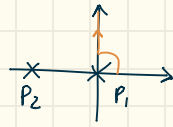
$$G(s) = \frac{1}{s^3}$$



CON CIRCONDAIMENTO

- 1 Polo in 0 ed uno a $\text{Re}P < 0$

$$G(s) = \frac{1}{s(s+1)}$$



-20dB/dec
iniziale

-20dB/dec
a $\omega_0 = 1$

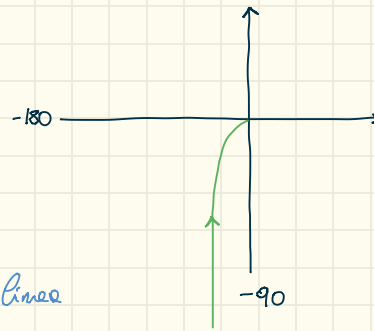
viene da
 $+\infty$

Disegnare una linea
PARALLELA

$$\varphi = -90^\circ - 90^\circ = -180^\circ \text{ Tot}$$

iniziale

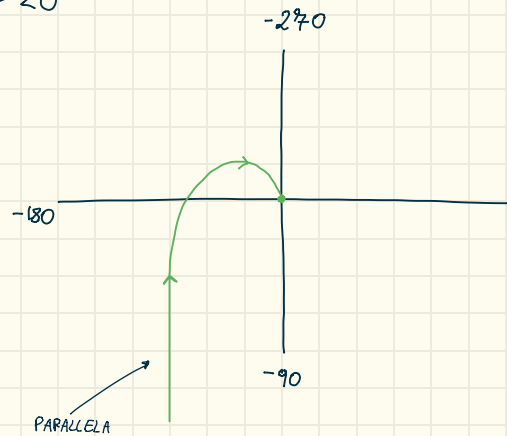
Contributo
a $\omega_0 = 1$



- 1 Polo in 0 e due a $\text{Re}P < 0$

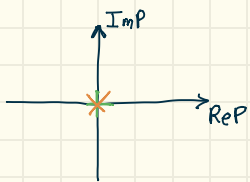
$$G(s) = \frac{1}{s(s+1)(s+2)}$$

-180 a $\omega_0 = \omega_1$
 $\varphi_i = -90$

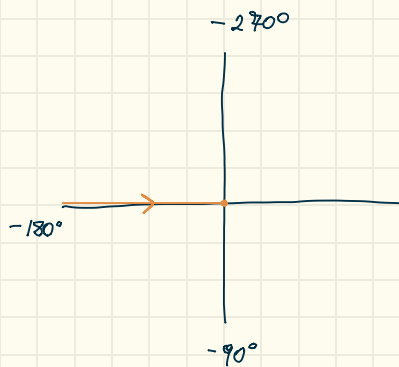


- 2 Poli in 0 (Tipo 2 = $g=2$, ordine 2)

$$G(s) = \frac{1}{s^2}$$



$$\varphi_i = -90 = \varphi_f$$



ES 1

$$s^2 + 2s + s^3 + 2s^2 = s^3 + 3s^2 + 2s$$

$$G(s) = \frac{1}{s(1+s)(s+2)}$$

$$\begin{aligned} Z &= \text{NONE} \\ P &= 3 \rightarrow \begin{cases} P_1 = 0 \\ P_2 = -1 \\ P_3 = -2 \end{cases} \end{aligned}$$

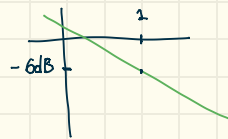
Forma Bode:

$$\frac{1}{s \cdot (1+s) \cdot 2 \cdot (1+\frac{1}{2}s)} = \frac{1}{2} \frac{1}{s(1+s)(1+\frac{1}{2}s)}$$

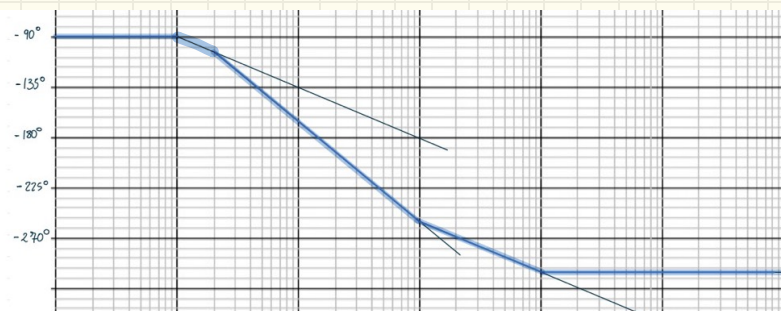
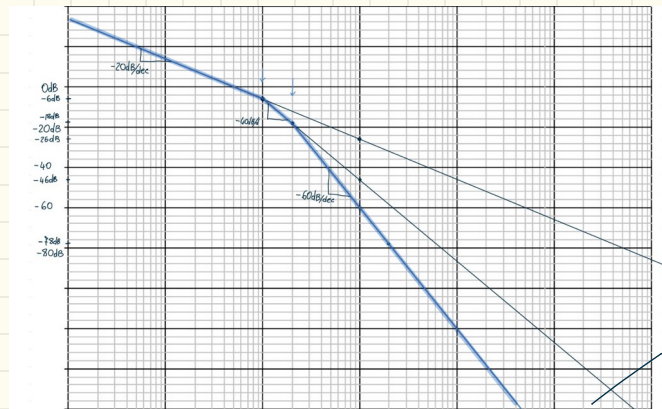
$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

Andamento iniziale

$$20 \log_{10} \left(\frac{K}{\omega_0} \right) = 20 \log_{10} \left(\frac{\frac{1}{2}}{1} \right) \approx -6 \text{ dB}$$



FASE: 1 Polo in 0 $\Rightarrow -90^\circ$ iniziale cost
2 Poli a $\text{Re}P < 0 \Rightarrow -90^\circ - 90^\circ = -180^\circ - 90^\circ = -270^\circ$ finale



$$20 \log_{10}(a) - b \rightarrow \log_{10}(a) = \frac{b}{20} \rightarrow a = 10^{\frac{b}{20}}$$

MODULO

dB V.N.

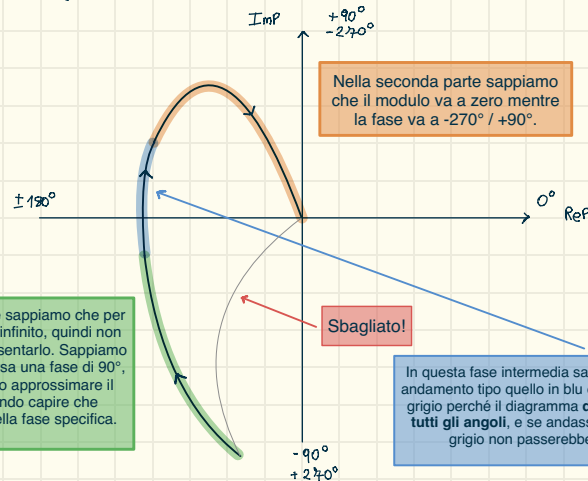
INIZIALE $20 \log_{10}(x) = \infty \Rightarrow 10^{\frac{\infty}{20}} = \infty \quad (\omega=0)$

FINALE $20 \log_{10}(x) = -\infty \Rightarrow 10^{\frac{-\infty}{20}} = 0 \quad (\omega=\infty)$

FASE

-90

-270



Nella seconda parte sappiamo che il modulo va a zero mentre la fase va a $-270^\circ / +90^\circ$.

In questa fase intermedia sappiamo che ha un andamento tipo quello in blu e non tipo quello in grigio perché il diagramma **deve attraversare tutti gli angoli**, e se andasse come quello in grigio non passerebbe per -180° .

Nella prima parte sappiamo che per $w=0$ il modulo è infinito, quindi non possiamo rappresentarlo. Sappiamo però che attraversa una fase di 90° , quindi possiamo approssimare il disegno facendo capire che proviene da quella fase specifica.

Disegnare il diagramma Senza Bode

Abbiamo ottenuto le informazioni che ci servivano usando un diagramma di Bode precedentemente costruito, ma possiamo trovare le informazioni anche andando a valutare modulo e fase per $w=0$ e $w=\infty$

$$G(s) = \frac{1}{s(s+1)(s+2)} \rightarrow G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

$$\Rightarrow |G(j\omega)| = \frac{1}{\omega} \cdot \frac{1}{\sqrt{1+\omega^2}} \cdot \frac{1}{\sqrt{4+\omega^2}}$$

$$\angle G(j\omega) = \angle \frac{1}{j\omega} - \angle \frac{1}{1+j\omega} - \angle \frac{1}{2+j\omega} = -90^\circ - \arctan(\omega) - \arctan\left(\frac{\omega}{2}\right)$$

$$\omega=0 \rightarrow -90^\circ$$

$$\omega=\infty \rightarrow -90^\circ - 90^\circ - 90^\circ = -270^\circ$$

$$\omega=0 \rightarrow |...| = \infty$$

$$\omega=\infty \rightarrow |...| = 0$$

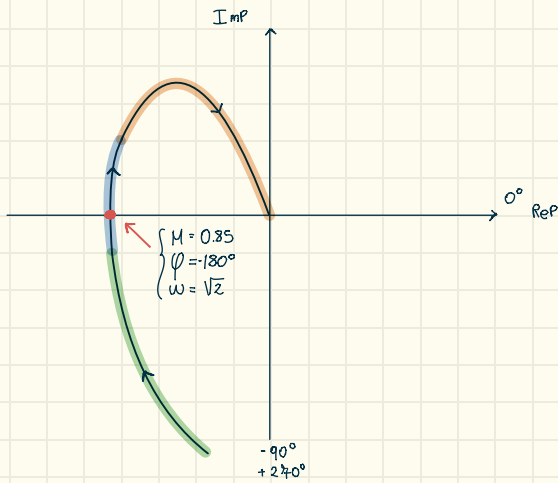
Trovare il valore del modulo in uno specifico punto

È difficile trovare il valore del modulo/fase in uno specifico punto. Quello che possiamo fare è trovare il valore del modulo in punti "strategici", come nell'intersezione con gli assi.

Nel caso precedente il diagramma interseca l'asse reale negativo, di cui conosciamo l'angolo: $+180^\circ$. Possiamo usare questo angolo per trovare la pulsazione a cui questo accade ed usare poi la pulsazione per trovare il modulo.

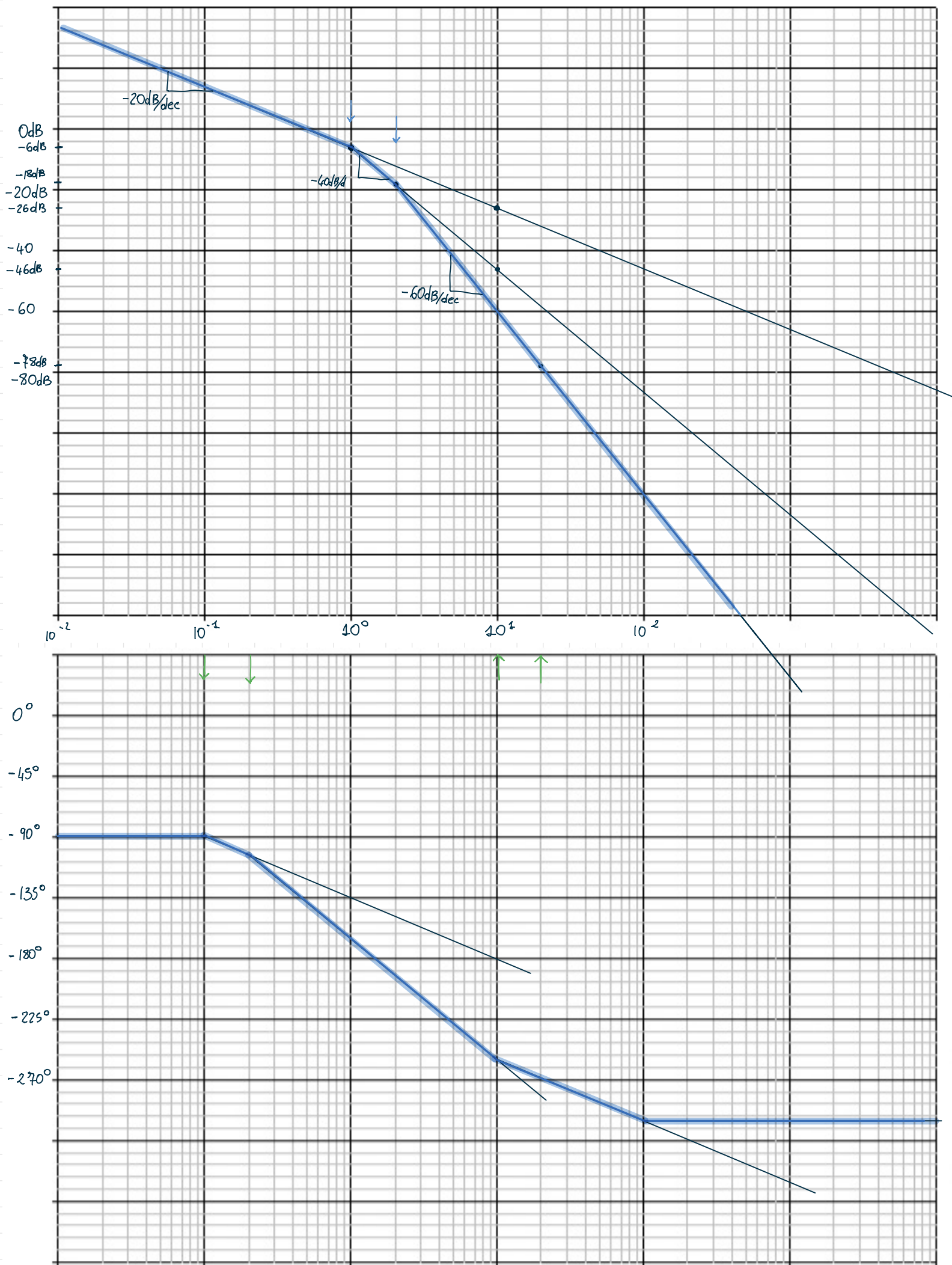
$$\varphi = -90 - \arctan(\omega) \cdot \arctan\left(\frac{\omega}{2}\right) = 180^\circ \Leftrightarrow \omega = \sqrt{2}$$

$$\Rightarrow M = \left| G(j\omega_0) \right| = 0.85$$
$$\omega_0 = \sqrt{2}$$



Matricola: _____

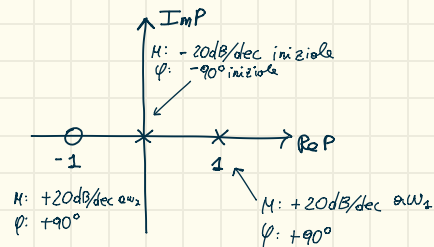
Nome: _____



ES 2: lezione 14: 18/04/24

$$G(s) = 2 \frac{s+1}{s(s-1)} = 2 \frac{1+s}{s \cdot (-1)(1-s)} = \underbrace{-2}_{K} \frac{s+1}{s(1-s)} \quad \leftarrow 1 \text{ ReP} > 0 \text{ polo}$$

\uparrow polo in 0



$$M: \frac{2 \cdot \sqrt{1+\omega^2}}{\omega \cdot \sqrt{1+\omega^2}} = \frac{2}{\omega}$$

$\omega=0 \rightarrow M = \infty$
 $\omega=\infty \rightarrow M = 0$

} Valori Naturali

$\varphi =$
 1 Polo in 0 $\Rightarrow -90^\circ$ iniziale
 1 Zero a $\text{ReP} < 0 \Rightarrow +90^\circ/2\text{dec}$
 1 Polo a $\text{ReP} > 0 \Rightarrow 90^\circ/2\text{dec}$

$$\angle 2 + \angle \frac{1}{j\omega+1} - \angle j\omega - \angle \frac{1}{j\omega-1} = \arctan(\omega) - \arctan\left(\frac{\omega}{1}\right) - \arctan(-\omega)$$

$$= 2\arctan(\omega) - 90^\circ$$

$\omega=0 \Rightarrow 0 - 90^\circ = -90^\circ$
 $\omega=\infty \Rightarrow 90^\circ$

Both Now mi Trovo con la fase

Diciamo che

$$\begin{cases} \omega=0 \rightarrow \varphi = -270^\circ \equiv +90^\circ \\ \omega=\infty \rightarrow \varphi = -90^\circ \equiv +270^\circ \end{cases}$$

