Esercizio 1

Si calcolino gli errori di posizione, di velocità e di accelerazione che si ottengono sollecitando il sistema con funzione di trasferimento:

$$G(s) = \frac{s^2 + 2s + 5}{s^3 + 5s^2 + 6s + 4} \quad (1)$$

con i seguenti segnali:

1. un gradino $r(t) = 1(t)$
2. una rampa $r(t) = t \cdot 1(t)$
3. una parabola: $r(t) = t^2 \cdot 1(t)$
4. il segnale r(t) di Figura 1

$$G(S) = \frac{S^2+2S+5}{S^3+5S^2+6S+4} = L(S)$$ A ciclo aperto

$$V_f = \lim_{S\to 0} S \cdot R(S)\cdot G(S) = S \cdot \frac{1}{S} \cdot \frac{S^2+2S+5}{S^3+5S^2+6S+4} = \frac{5}{4}$$

Routh: $P_c(S) = S^3+5S^2+6S+4$

S^3	1	6
S^2	5	4
S^1	$\frac{26}{5}$	0
S^0	4	A. STABILE

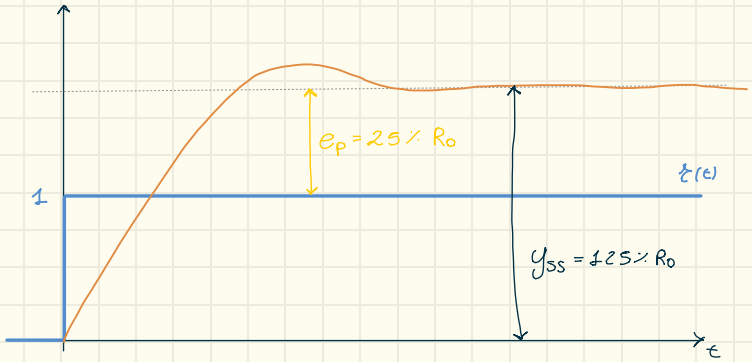
$\frac{30-4}{5} = \frac{26}{5}$

$\Rightarrow e_{SS} = V_f - R_0 = \frac{5}{4} - 1 = \frac{5-4}{4} = \frac{1}{4} = 25\%$ IN PIU'!

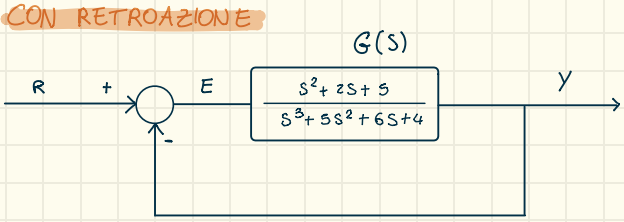
METODO CON LA TABELLA

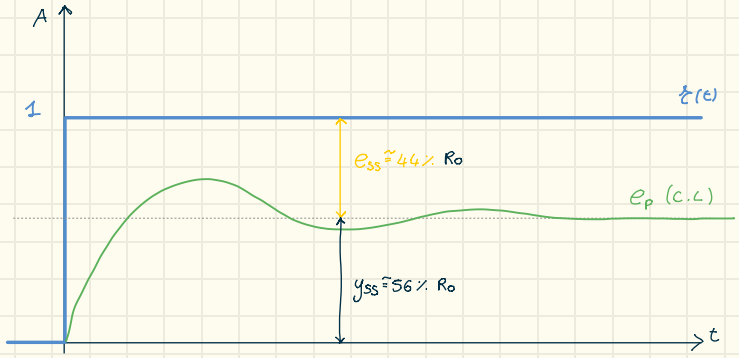
Dato $G(S) = \frac{\beta_n S^n + \dots + \beta_1 S^1 + \beta_0}{\alpha_n S^n + \dots + \alpha_1 S^1 + \alpha_0}$ la nostra G(S) ha $\begin{cases} \beta_0 = 5 \\ \alpha_0 = 4 \end{cases}$ con $\beta_0 \neq \alpha_0$

$\Rightarrow$ se $\alpha_0 \neq \beta_0 \Rightarrow e_p = \left|\frac{\beta_0 - \alpha_0}{\alpha_0}\right| = \frac{5-4}{4} = \frac{1}{4} = 25\%$ di R_0 dove R_0 è il gain del Riferimento (r)



CON RETROAZIONE





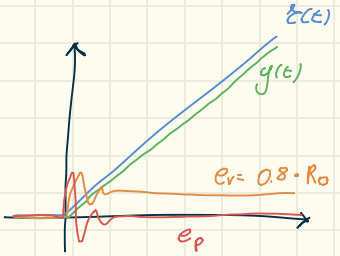
$e_p = \frac{1}{1+\mu}$ con $\mu = \lim_{S\to 0} G(S) = \frac{5}{4} \Rightarrow e_p = \frac{4}{9} \simeq 44\%$

$G(S) = \frac{S^2+2S+5}{S^3+5S^2+6S+4}$ Con $r(t) = t\cdot 1(t) \rightleftharpoons R(S) = \frac{1}{S^2}$ $\beta_0 \neq \alpha_0 \Rightarrow e_p = \infty$

ES:

$G(S) = \frac{S^2+2S+5}{S^3+5S^2+6S+5}$ con $R(S) = \frac{1}{S} \Rightarrow e_p = 0$ perché $\beta_0 = \alpha_0$

$\beta_1 \neq \alpha_0 \Rightarrow$ se $R(S) = \frac{1}{S^2} \Rightarrow e_r = \left|\frac{2-6}{5}\right| = \frac{4}{5} = 0.8 = 80\% R_0$ errore negativo



Es:

$$G(s) = \frac{(s+1)(s+2)}{s(s+5)+s^2-2s+2} \quad P_c(s) = 2s^2+3s+2$$

$$= \frac{s^2+3s+2}{2s^2+3s+2}$$

s^2	2	②
s^1	3	0
s^0	2	

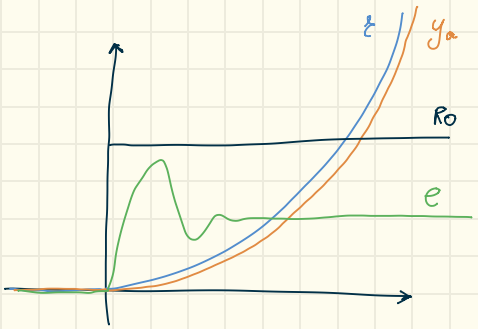
Dipende dall'ultimo

$\beta_0 = \alpha_0 \Rightarrow e_p = 0$

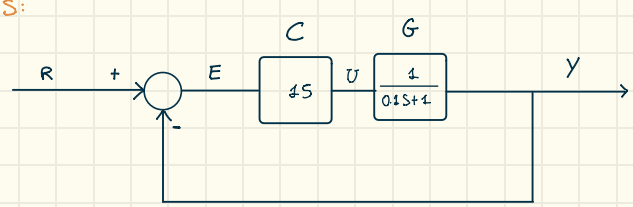
$\beta_1 = \alpha_1 \Rightarrow e_v = \emptyset$

$\beta_2 \neq \alpha_2 \Rightarrow e_a = \left|\frac{\beta_2 - \alpha_2}{\alpha_0}\right| = \left|\frac{1-2}{2}\right| = 0.5 = 50\%$

A.S.



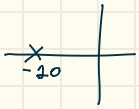
Es:



$C = 15$

$G(s) = \frac{1}{0.1s+1}$

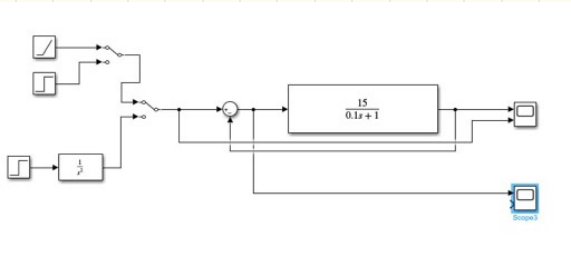
$\Rightarrow L(s) = \frac{15}{0.1s+1} \Rightarrow \bar{s} = -10$

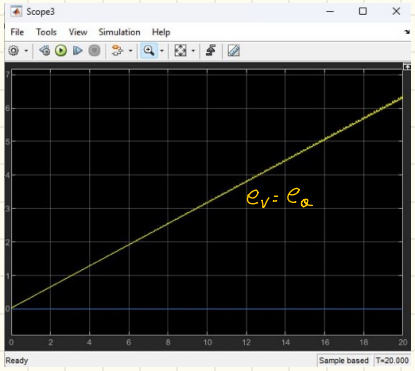


- $u = 1(t) \quad U(s) = \frac{1}{s} \quad \mu_p = \lim_{s\to 0} \frac{15}{0.1s+1} = 15 \Rightarrow e_p = \frac{1}{1+\mu_p} = \frac{1}{16} = 0.062 \approx 6\%$

- $u = t\cdot 1(t) \quad U(s) = \frac{1}{s^2} \quad \mu_v = \lim_{s\to 0} s\cdot\frac{15}{0.1s+1} = \emptyset \Rightarrow \frac{1}{\mu_v} \to \infty$

Infatti non ci sono abbastanza integrali





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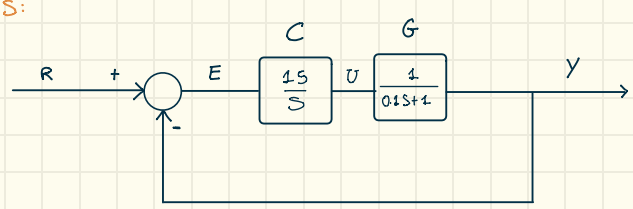
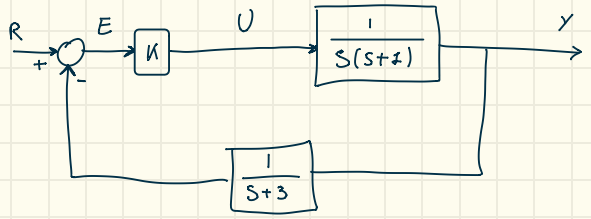


$\rightsquigarrow L(s) = \frac{15}{s(\frac{1}{10}s+1)}$ Tipo 1 $\Rightarrow e_p = 0$

$\mu_v = \lim_{s\to 0} s\cdot\frac{15}{s(\frac{1}{10}s+1)} = 15 \Rightarrow e_v = \frac{1}{15} \simeq 0.067 \simeq 6.7\%$

ES LEZ 10



Se $K=5$ calcolare e_p, e_r, e_a

$L(S) = \frac{5}{S(S+1)}$ $\quad H(S) = \frac{1}{S+3}$

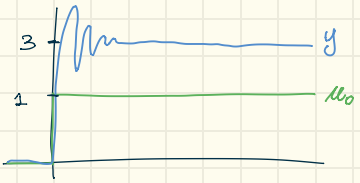
$\Rightarrow F(S) = \frac{L(S)}{1+L(S)\cdot H(S)} = \frac{\frac{5}{S(S+1)}}{1+\frac{5}{S(S+1)(S+3)}} = \frac{5(S+3)}{S(S+1)(S+3)+5}$

$P_c(S) = S^3+3S^2+S^2+3S+5$

S^3	1	3
S^2	4	5
S^1	7/4	0
S^0	5	✓ A.S.

$= \frac{5S+15}{S^3+4S^2+3S+5}$ $\quad \beta_0 \neq \alpha_0$

$\Rightarrow$ Se $u_0(t) = 1(t) \Rightarrow e_p = \left|\frac{\beta_0-\alpha_0}{\alpha_0}\right| = \left|\frac{15-5}{5}\right| = 2$ e_p



$\Rightarrow e_r = e_a = \infty$