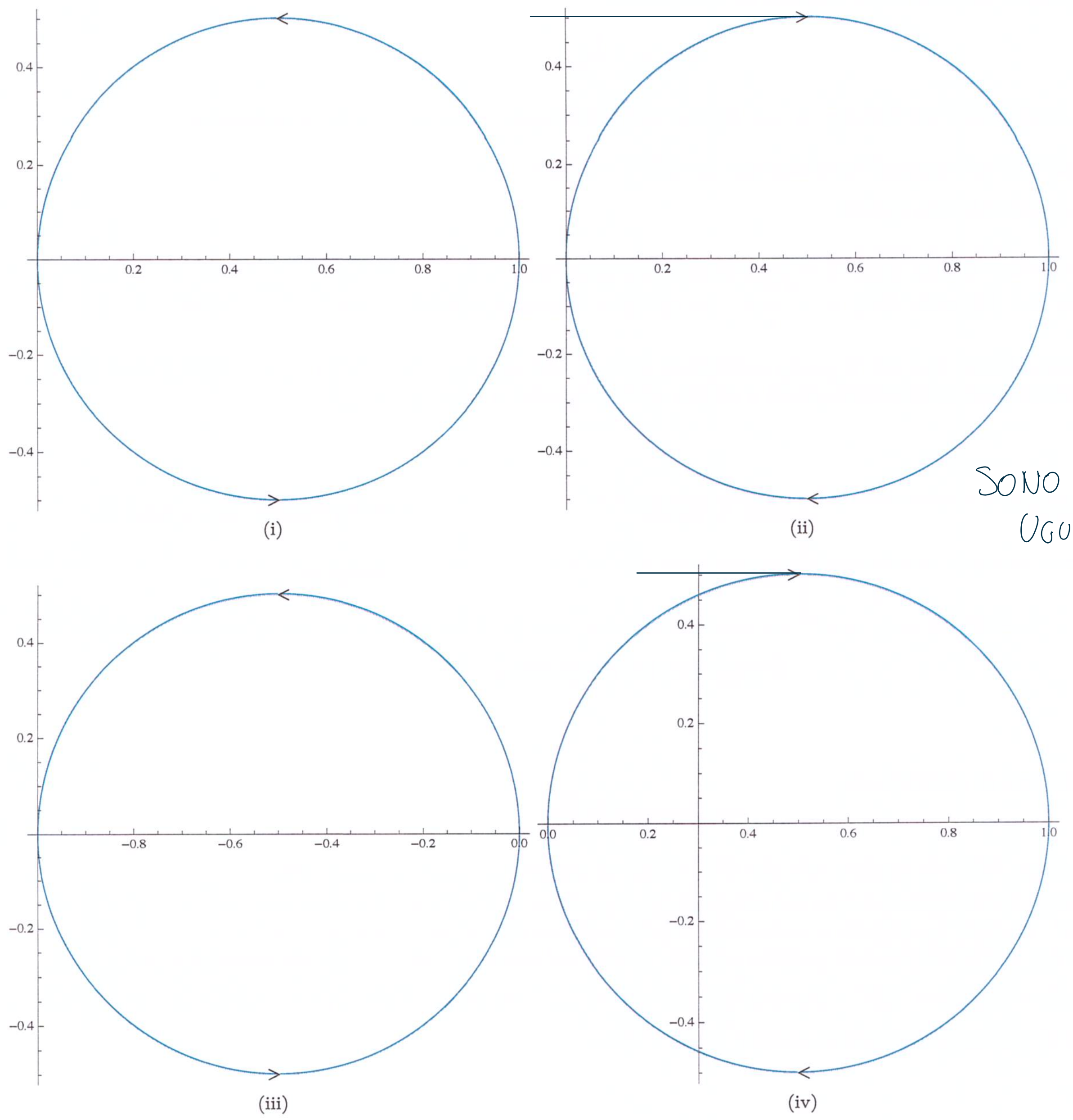


SONO LETTERALMENTE
UGUALI

ESERCIZIO 2.
Si abbinino le funzioni di trasferimento con i corrispondenti diagrammi di Nyquist riportati nelle figure:

5 punti

I   I $L(s) = \dfrac{s}{s-1}$ ✓   (A) Fig. (ii)

II   IV $L(s) = \dfrac{s}{s+1}$ ✗   (B) Fig. (iv)

III   III $L(s) = \dfrac{1}{s-1}$ ✓   (C) Fig. (iii)

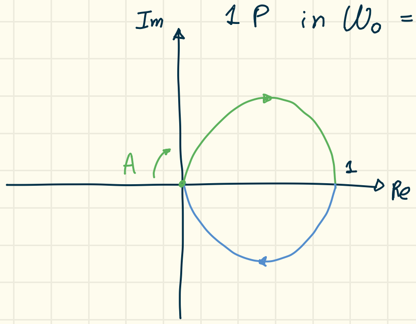
IV   II $L(s) = \dfrac{1}{s+1}$ ✗   (D) Fig. (i)

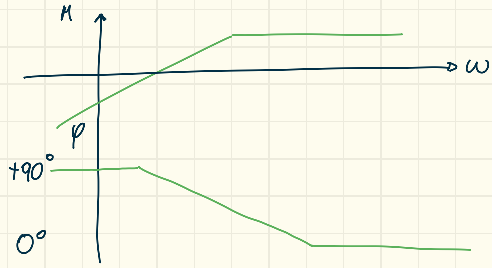
$\times$ $L(s) = \dfrac{s}{s+1}$

$\omega \to 0 \quad M = 0$

$1\ Z$ in $0 \Rightarrow M_0 = +20\,dB/dec \quad \varphi_0 = +90^\circ \qquad \mu_0 = 1 > 0 \Rightarrow \varphi_0 = +90^\circ$

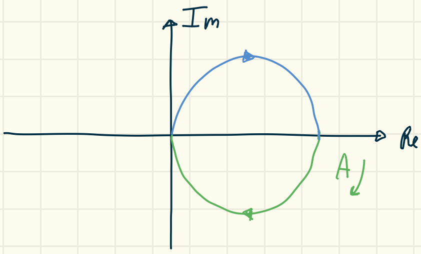
$1\ P$ in $\omega_0 \Rightarrow M_{\omega_0} = -20\,dB/dec \quad \varphi_{\omega_0} = -90^\circ$

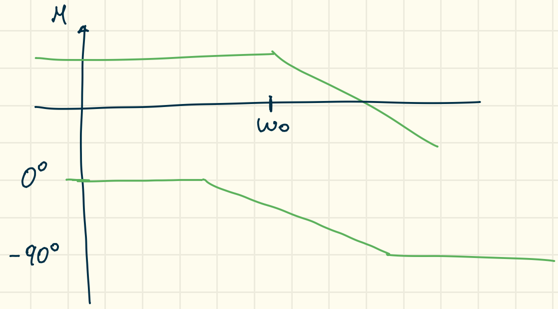




$\times$ $L(s) = \dfrac{1}{s+1}$

1 Polo in $\omega_0 \Rightarrow M_0 = (1)_{VN} = 0\,dB \quad M_{\omega_0} = -20\,dB/d \quad \varphi_{\omega_0} = -90^\circ$





$$L(s) = \frac{s^2+4s+9}{(s-1)(s-10)^2}$$

1) C.Trasf $\lim_{s \to \emptyset} s^g L(s) = \frac{9}{-1 \cdot 100} = -0.09 < 0 \to$ Inizio da sx

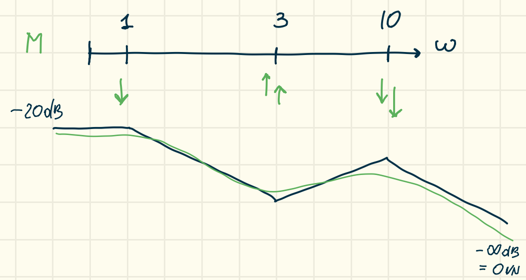
Zeri: $\begin{cases} \bar{z}_1 = -2-\sqrt{5}j \\ \bar{z}_2 = -2+\sqrt{5}j \end{cases}$ $\Rightarrow$ Re P $< \emptyset$ $\leadsto$ $\omega_n = \sqrt{9} = 3$ Rad/s $\Rightarrow$ $M_{\omega_n} = +40$dB/dec, $\varphi_{\omega_n} = +180°$ $(90°/\text{dec})$

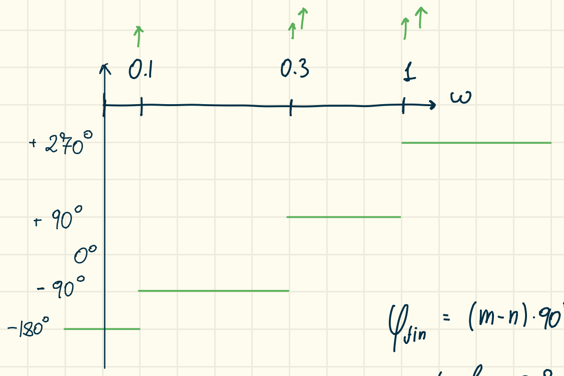
$M_0 = |-0.09|_{dB} \cong -20$dB

Poli: $\begin{cases} P_1 = 1 & \text{ReP} > 0 & \omega_1 = 1 \text{ Rad/s} \\ P_{2,3} = -10 & \text{ReP} > 0 & \omega_{2,3} = 10 \text{ Rad/s} \end{cases}$

M_{ω_1}: -20dB/dec $\quad \varphi_{\omega_1}$: $+90°$

M_{ω_2}: -40dB/dec $\quad \varphi_{\omega_{2,3}}$: $+180°$



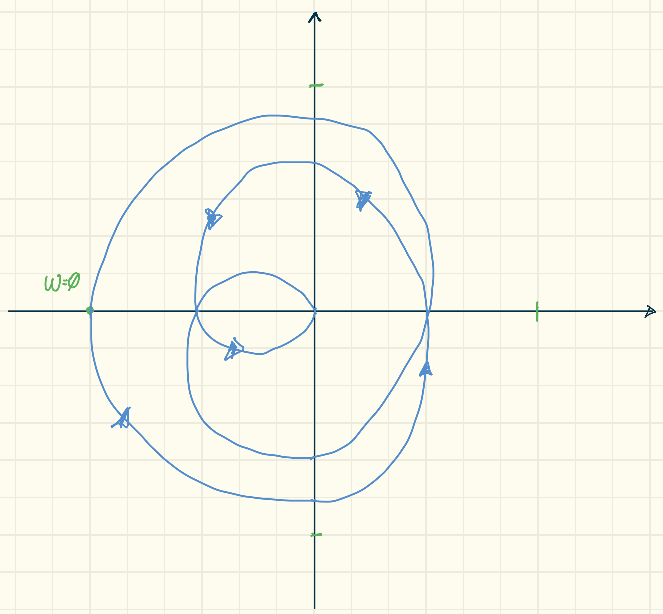


! Rispetto a -180° !

$\varphi_{fin} = (m-n) \cdot 90° = 5 \cdot 90° = 450°$

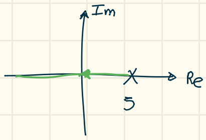
ma parto da $-180° \to -180+450 = 270°$

! Rispetto 0° !



Luogo delle Radici

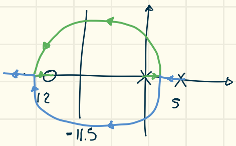
$G(S) = \frac{1}{S-5}$



- $Q: T_{a1} < 0.4s$
- $e_r < 10\%$ → Mi serve un integratore

$\frac{4.6}{\sigma} < 0.4 \rightarrow |\sigma| > \frac{4.6}{0.4} = 11.5$

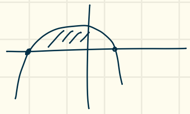
$C(S) = \frac{K(S+12)}{S} \Rightarrow C'(S) = \frac{K(S+12)}{S(S-5)}$

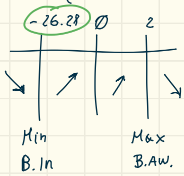


$\gamma(x) = -\frac{D(x)}{N(x)} = \frac{-(x^2-5x)}{x+12} \rightarrow \gamma'(x) = -(2x-5)(x+12)+(x^2-5x) > 0$

$-2x^2-24x+\cancel{5x}+60+x^2-\cancel{5x} > 0$

$-x^2-24x+60>0$ per $\begin{cases} x > -26.28 \\ x < 2.28 \end{cases}$





$|K| = \frac{D(s^*)}{N(s^*)} = \frac{-[(s^*)^2 - 5s^*]}{s^*+12} = 57.57$

- $\mu = \lim_{s\to 0} \cancel{s} \frac{K(S+12)}{\cancel{s}(S-5)} = K \cdot \frac{12}{-5} = -2.4K \Rightarrow e_r = \left|\frac{1}{\mu}\right| = \left|\frac{1}{-2.4K}\right| < 0.1 \rightarrow K > \frac{1}{0.1 \cdot 2.4} \Rightarrow K > 4.16$

$\Rightarrow$ Prendo $K = 58 \rightarrow C(S) = \frac{58(S+12)}{S}$ Ans

Perchè si sceglie il margine critico Negativo?

Progetto in frequenza

Esercizio 1.
Si consideri la funzione di trasferimento

$$G(s) = k \cdot \frac{s-9}{s(s+9)}$$

5 punti

e si scelga il guadagno $k \in \mathbb{R}$ in maniera tale che $G(s)$ abbia un margine di fase pari a 40°.

Per avere $\varphi_m = 40 \rightarrow 180 - |\varphi_c| = 40° \Rightarrow |\varphi_c| = 140° \rightarrow \varphi_c = \pm 140° \rightarrow \varphi_c = -140°$

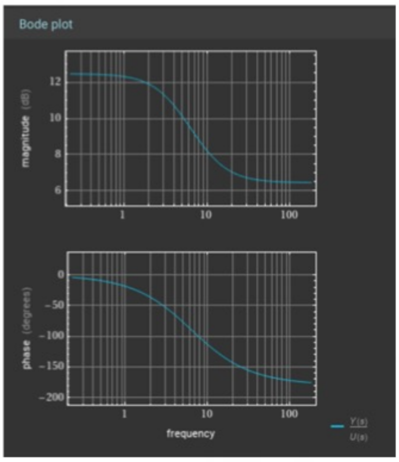
- ω_c

$|G(j\omega_c)| = 1$ per $\frac{K\sqrt{\omega^2+81}}{\omega\sqrt{\omega^2+81}} = 1 \rightarrow \omega_c = K$

- $\angle G(j\omega_c) = -140° \rightarrow \angle K + \angle j\omega_c - 9 - \angle j\omega_c - \angle j\omega_c + 9 = -\mathrm{aTan}\left(\frac{\omega_c}{9}\right) - 90° - \mathrm{atan}\left(\frac{\omega_c}{9}\right) = -140°$

$\rightarrow -2\,\mathrm{aTan}\left(\frac{\omega_c}{9}\right) = -50 \rightarrow \mathrm{aTan}\left(\frac{\omega_c}{9}\right) = 25° \rightarrow \omega_c = 9\,\mathrm{Tan}(25°) = 4.197$

$\Rightarrow |K| = \pm 4.2$ ma $\mu = \lim_{s\to 0} \cancel{s} \frac{K(S-9)}{\cancel{s}(S+9)} = -K$ ma voglio $\mu > 0 \Rightarrow K < 0 \Rightarrow K = -4.2$



2. MARGINE DI AMPIEZZA Con scelta di guadagno e zero

OPPOSTO AL POLO

$G(s) = k\frac{(s+z)}{(s+5)^2}$ scelgo $z = -5$ $\Rightarrow$ $G(s) = \frac{k(s-5)}{(s+5)^2}$ vogliamo $M_a = 6\,dB = 10^{\frac{6}{20}} = (2)_{dB}$

Valore Naturale

Siccome cerco il margine di ampiezza, devo trovare la ω_c per la quale $\angle G(j\omega_c) = -180°$

Dopodichè calcolo $|G(j\omega_c)|$ e vedo quanto differisce $(1)dB = 0dB$

1) Guadagno

$\mu = \lim_{s\to 0} s^{g} \cdot G(s) = \lim_{s\to 0} \frac{k(s-5)}{(s+5)^2} = -\frac{5}{25}k$ Vogliamo un guadagno $\mu > 0 \Rightarrow k<0$

2) Trovo ω_c

$\angle G(j\omega_c) = -180 = -\pi \;\Rightarrow\; \angle k(j\omega-5) - \angle (j\omega+5)^2 = -\pi \;\Rightarrow\; -\operatorname{atan}\left(-\frac{\omega}{5}\right) - 2\operatorname{atan}\left(\frac{\omega}{5}\right) = -\pi$

$\Rightarrow -3\operatorname{atan}\left(\frac{\omega}{5}\right) = -\pi \;\Rightarrow\; 3\operatorname{atan}\left(\frac{\omega_c}{5}\right) = \pi \;\Rightarrow\; \frac{\omega_c}{5} = \tan\left(\frac{\pi}{3}\right) \;\Rightarrow\; \omega_c = 5\tan\left(\frac{\pi}{3}\right) = 8.66\ Rad/s = 5\sqrt{3}\ Rad/s$

ω_c

3) Modulo in $\omega_c = 5\sqrt{3}\ R/s$

$|G(j\omega_c)| = \frac{|k(j\omega_c-5)|}{|(j\omega_c+5)^2|} = \frac{|k|\sqrt{\omega_c^2+5^2}}{\omega_c^2+25}$ per $k = \frac{2(\omega_c^2+25)}{\sqrt{\omega_c^2+5^2}} = 20$

Per trovare k: $\frac{1}{|G(j\omega_c)|} = (M_a)_{dB}$ oppure $\frac{1}{|G(j\omega_c)|_{dB}} = M_a\,dB$

$\Rightarrow \frac{1}{|G(j\omega_c)|} = \frac{\omega_c^2+25}{|k|\sqrt{\omega_c^2+5^2}} = 2$ per $|k| = \frac{\omega_c^2+25}{2\sqrt{\omega_c^2+25}} = \frac{75+25}{2\sqrt{100}} = 5$

Quindi $|k| \Rightarrow \begin{cases} k>0 \Rightarrow k=5 \\ k<0 \Rightarrow k=-5 \end{cases}$ Scelgo $k<0$ perchè mi serve $\mu>0$

$\Rightarrow G(s) = -\frac{5(s-5)}{(s+5)^2}$ Ans