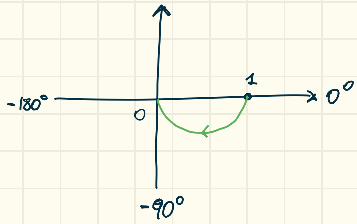
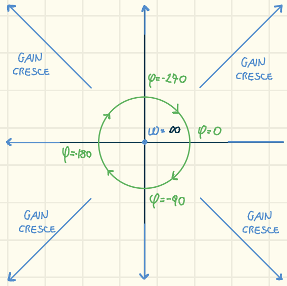
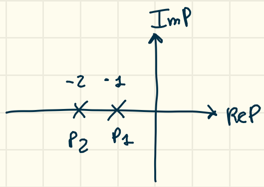
Esempi Basilari

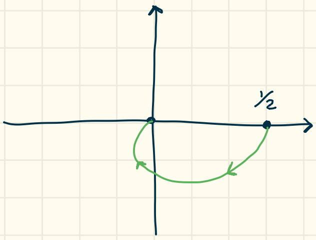
- 1 Polo $\quad G(s) = \dfrac{1}{s+1}$, $K=1$
  $ReP<0$



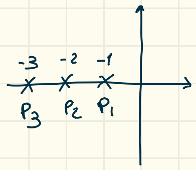


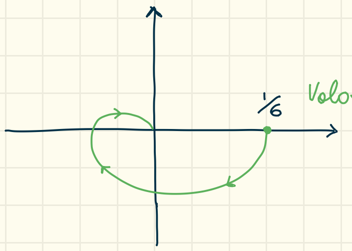
- 2 Poli $\quad G(s) = \dfrac{1}{(s+1)(s+2)}$
  $ReP<0$





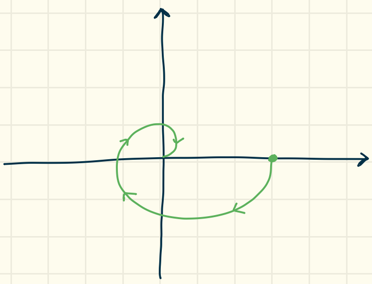
- 3 Poli $\quad G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$





Valore naturale $\quad \left.\frac{1}{6}\right|_{dB} = 20\log\left(\frac{1}{6}\right) \cong -15$

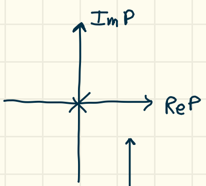
- 4 Poli $\quad G(s) = \dfrac{1}{(s+1)(s+2)(s+3)(s+4)}$

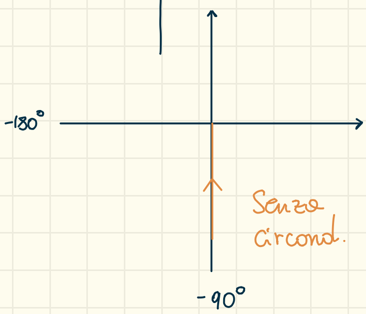


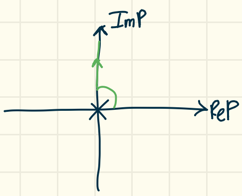
- 1 Polo in Origine $\quad G(s) = \dfrac{1}{s}$

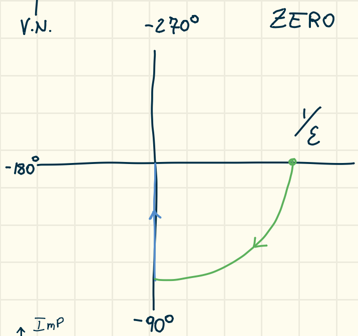
Fase iniziale $= +90° =$ F. Finale $\qquad$ LA FASE RIMANE COSTANTE

$M_{in} = M_{fin} = -20\,dB/dec \Rightarrow \left.M_{in}\right|_{V.N.} = +\infty$, $\left.M_{Fin}\right|_{V.N.} = 0$ $\qquad$ IL MODULO TENDE A ZERO DA $+\infty$

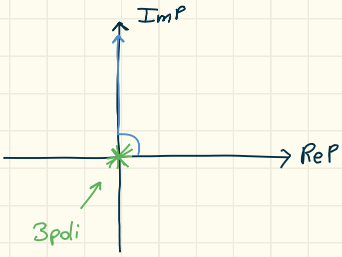


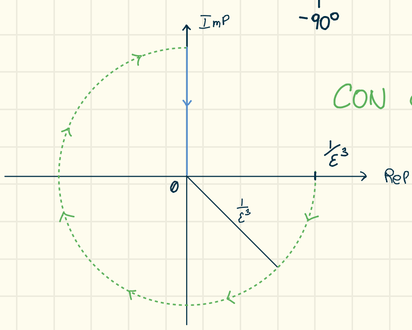






$G(s) = 1/s^3$

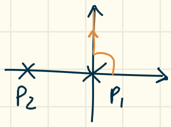


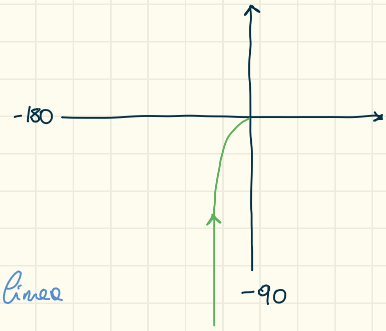


CON CIRCONDAMENTO

- 1 Polo in 0 ed uno a $ReP < 0$

$$G(S) = \frac{1}{S(S+1)}$$





- 20dB/dec iniziale ⇓ Viene da $+\infty$ ⟶ Disegnamo una linea PARALLELA

- 20dB/dec a $\omega_0 = 1$

$$\varphi = -90° - 90° = -180° \text{ TOT}$$

iniziale

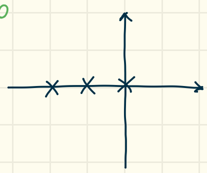
Contributo a $\omega_0 = 1$

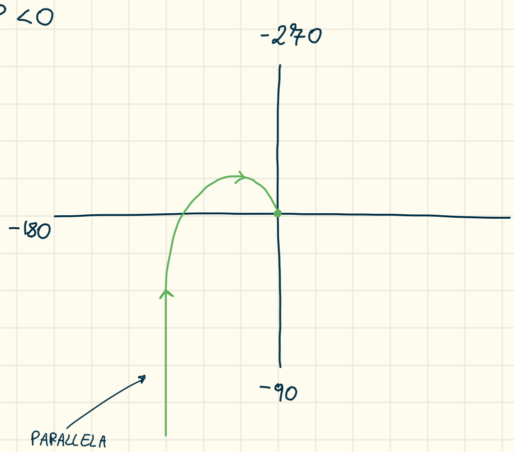
- 1 Polo in 0 e due a $ReP < 0$

$$G(S) = \frac{1}{S(S+1)(S+2)}$$

$\varphi_i = -90$

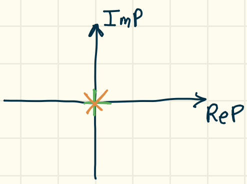
- 180 a ω_0 e ω_1

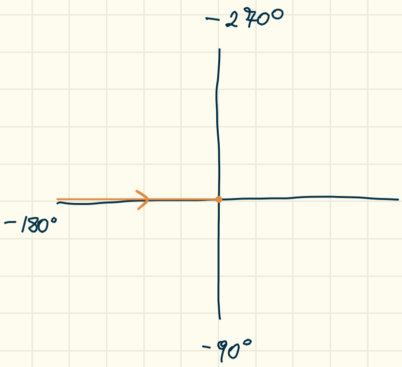




- 2 Poli in 0 (Tipo 2 ≡ g = 2 , ordine 2)

$$G(S) = \frac{1}{S^2}$$





$$\varphi_i = -90 = \varphi_f$$

ES 1

$$s^2+2s+s^3+2s^2 = s^3+3s^2+2s$$

$$G(s) = \frac{1}{s(1+s)(s+2)}$$

$Z = \text{NONE}$

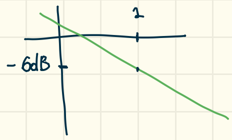
$P = 3 \rightarrow \begin{cases} P_1 = 0 \\ P_2 = -1 \\ P_3 = -2 \end{cases}$

Forma Bode: $\frac{1}{s \cdot (1+s) \cdot 2\,(1+\frac{1}{2}s)} = \frac{1}{2} \frac{1}{s(s+1)(1+\frac{1}{2}s)}$

gain

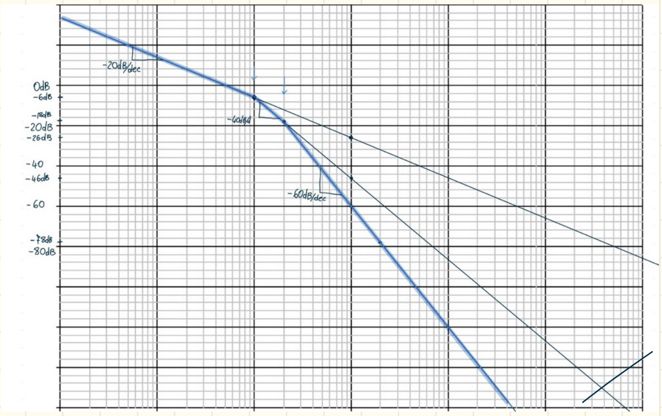
$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

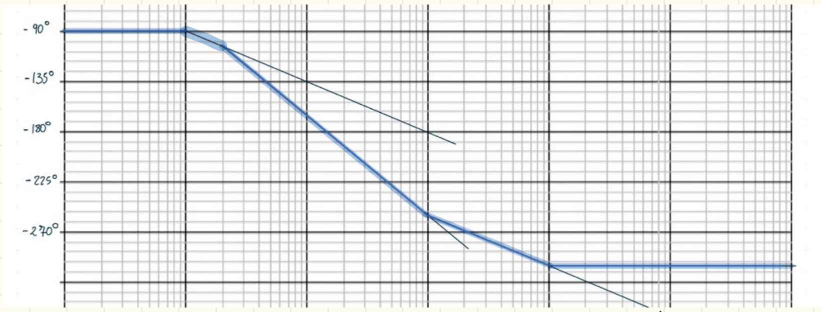
andamento iniziale $\quad 20\log_{10}\left(\frac{K}{\omega_0}\right) = 20\log_{10}\left(\frac{1/2}{1}\right) \cong -6\,dB$



FASE: 1 Polo in 0 $\Rightarrow -90°$ iniziale cost

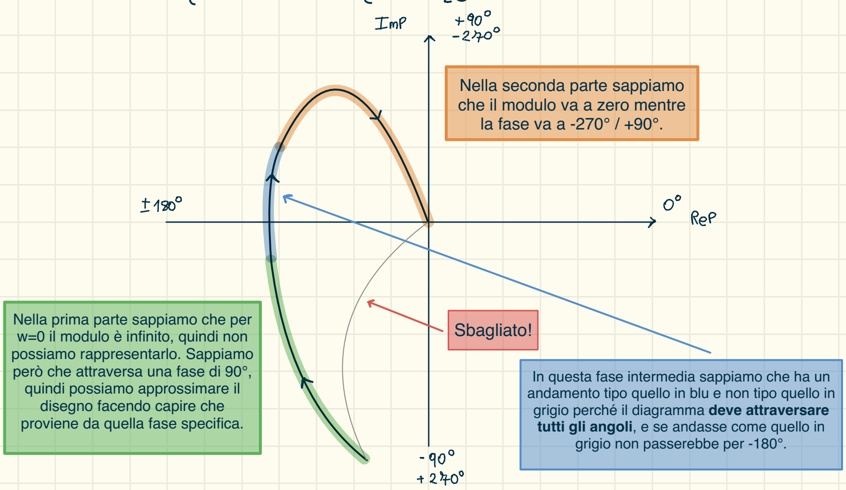
2 Poli a $Re<0 \Rightarrow -90°-90° = -180° - 90° = -270$ finale





$$20\log(a) = b \rightarrow \log(a) = \frac{b}{20} \rightarrow a = 10^{\frac{b}{20}}$$

	MODULO dB	MODULO V.N.		FASE
INIZIALE	$20\log(x) = \infty \Rightarrow$	$10^{\frac{\infty}{20}} = \infty$	$(\omega=0)$	-90
FINALE	$20\log(x) = -\infty \Rightarrow$	$10^{-\frac{\infty}{20}} = \emptyset$	$(\omega=\infty)$	-270



Disegnare il diagramma Senza Bode

Abbiamo ottenuto le informazioni che ci servivano usando un diagramma di Bode precedentemente costruito, ma possiamo trovare le informazioni anche andando a valutare modulo e fase per w=0 e w=inf

$$G(s) = \frac{1}{s(s+1)(s+2)} \rightarrow G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

$$\Rightarrow |G(j\omega)| = \frac{1}{\omega} \cdot \frac{1}{\sqrt{1+\omega^2}} \cdot \frac{1}{\sqrt{4+\omega^2}}$$

$\omega = 0 \rightarrow |\dots| = \infty$

$\omega = \infty \rightarrow |\dots| = 0°$

$$\angle G(j\omega) = \angle \overset{0}{1} - \angle \overset{90}{j\omega} - \angle \overset{atan(\omega)}{1+j\omega} - \angle \overset{atan(\frac{\omega}{2})}{2+j\omega} = -90 - atan(\omega) - atan\left(\frac{\omega}{2}\right)$$

$\omega = 0 \rightarrow -90°$

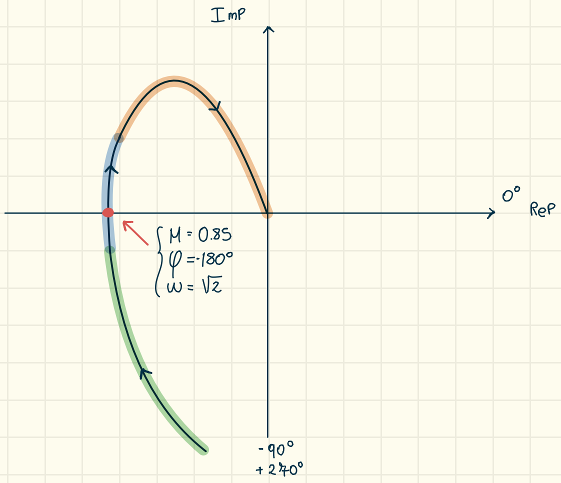
$\omega = \infty \rightarrow -90° - 90° - 90° = -270°$

Trovare il Valore del modulo in uno specifico punto

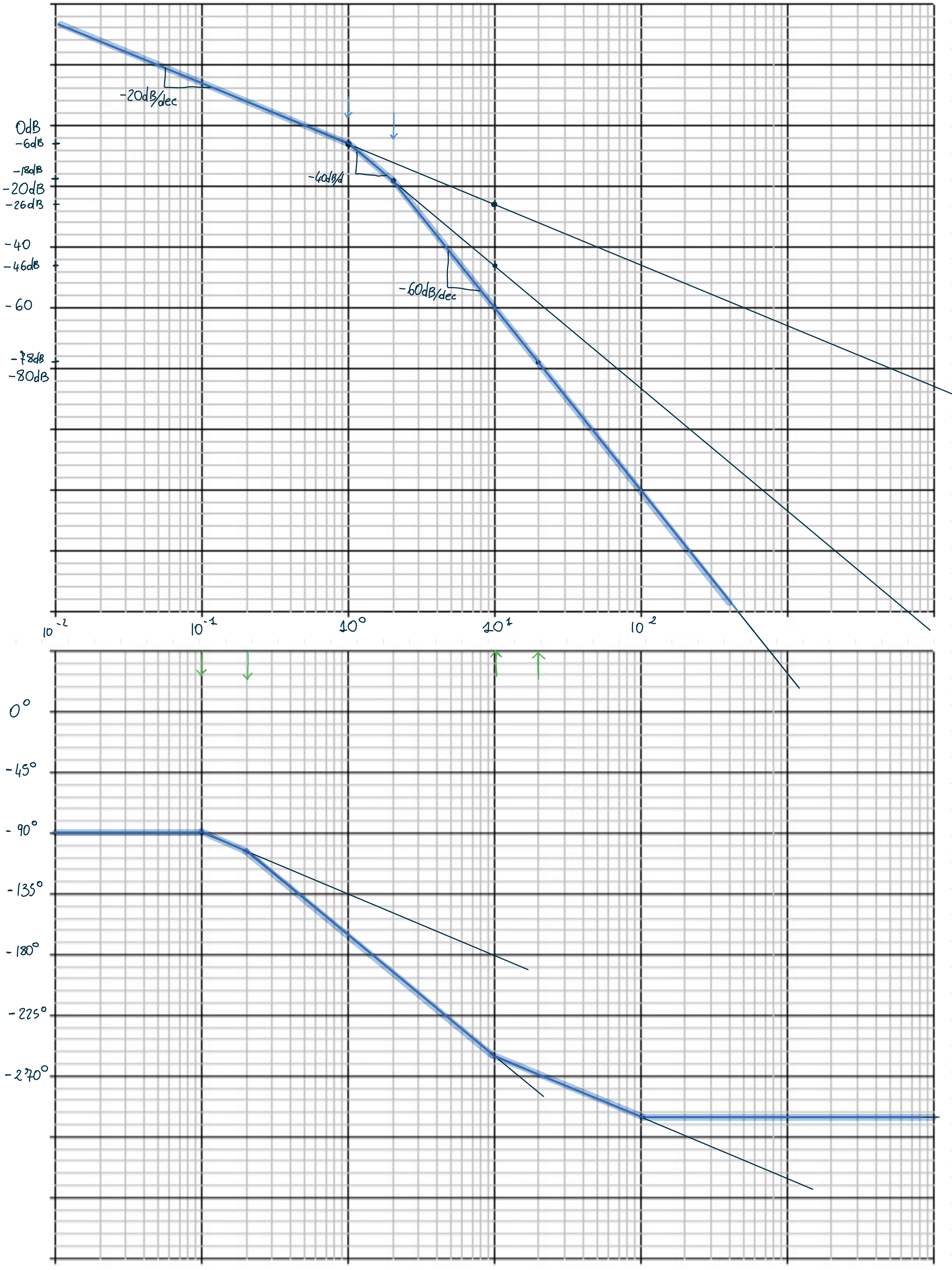
È difficile trovare il valore del modulo/fase in uno specifico punto. Quello che possiamo fare è trovare il valore del modulo in punti "strategici", come nell'intersezione con gli assi.
Nel caso precedente il diagramma interseca l'asse reale negativo, di cui conosciamo l'angolo: +-180°. Possiamo usare questo angolo per trovare la pulsazione a cui questo accade ed usare poi la pulsazione per trovare il modulo.

$$\varphi = -90 - \operatorname{atan}(\omega) - \operatorname{atan}\left(\frac{\omega}{2}\right) = 180° \iff \omega = \sqrt{2}$$

$$\Rightarrow M = \left|G(j\omega_0)\right|_{\omega_0 = \sqrt{2}} = 0.85$$



Matricola: ______________ Nome: ____________________

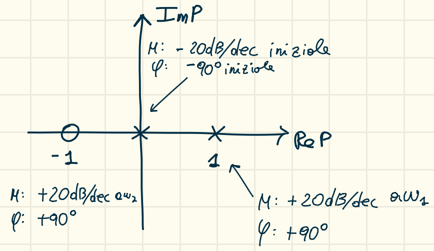


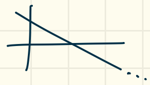
ES 2: Lezione 14: 18/04/24

$$G(s) = 2\,\frac{s+1}{s(s-1)} = 2\,\frac{1+s}{s\cdot(-1)(1-s)} = \underbrace{(-2)}_{K}\,\frac{s+1}{s(1-s)}$$

1 ReP>0 Polo

1 polo in 0





$$M: \frac{2\cdot\cancel{\sqrt{1+\omega^2}}}{\omega\cdot\cancel{\sqrt{1+\omega^2}}} = \frac{2}{\omega}$$

$\omega = 0 \rightarrow M = \infty$

$\omega = \infty \rightarrow M = 0$

Valori Naturali

$\varphi =$ 1 Polo in 0 $\Rightarrow$ $-90°$ iniziale

1 Zero a ReP<0 $\Rightarrow$ $+90°/2$dec

1 Polo a ReP>0 $\Rightarrow$ $90°/2$dec

$$\angle 2 + \angle j\omega+1 - \angle j\omega - \angle j\omega-1 = \operatorname{atan}(\omega) - \operatorname{atan}\left(\frac{\omega}{0}\right) - \operatorname{atan}(-\omega)$$

90°

$$= 2\operatorname{atan}(\omega) - 90°$$

$\omega = 0 \rightarrow 0 - 90° = -90°$

$\omega = \infty \rightarrow 90°$ Boh Non mi Trovo con la fase

Diciamo che

$$\begin{cases} \omega = 0 \rightarrow \varphi = -270° \equiv +90° \\ \omega = \infty \rightarrow \varphi = -90° \equiv +270° \end{cases}$$