Esercizio 1

Si calcolino gli errori di posizione, di velocità e di accelerazione che si ottengono sollecitando il sistema con funzione di trasferimento:

$$G(s) = \frac{s^2+2s+5}{s^3+5s^2+6s+4} \tag{1}$$

con i seguenti segnali:

1. un gradino $r(t) = 1(t)$
2. una rampa $r(t) = t \cdot 1(t)$
3. una parabola: $r(t) = t^2 \cdot 1(t)$
4. il segnale r(t) di Figura 1

$$G(s) = \frac{s^2+2s+5}{s^3+5s^2+6s+4} = L(s)$$

A ciclo Aperto

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

Routh: $P_c(s) = s^3 + 5s^2 + 6s + 4$

s^3	1	6
s^2	5	4
s^1	$\frac{26}{5}$	0
s^0	4	

A. STABILE

$$\frac{30-4}{5} = \frac{26}{5}$$

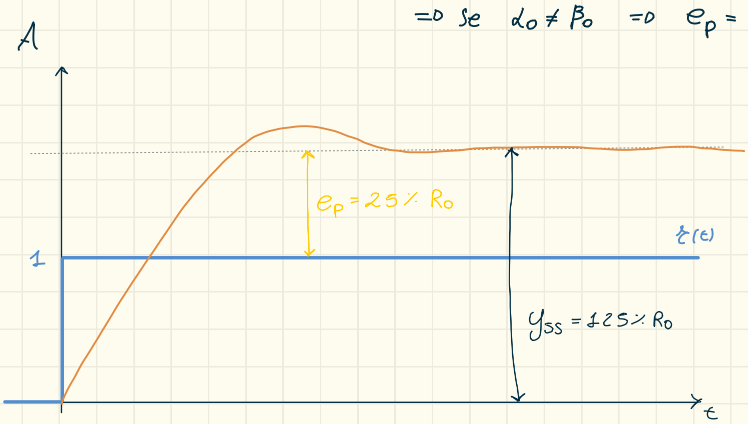
$$V_f = \lim_{s \to 0} s \cdot R(s) \cdot G(s) = \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{s^2+2s+5}{s^3+5s^2+6s+4} = \frac{5}{4}$$

$$\Rightarrow e_{ss} = V_f - R_0 = \frac{5}{4} - 1 = \frac{5-4}{4} = \frac{1}{4} = 25\% \quad \text{IN PIU'!}$$

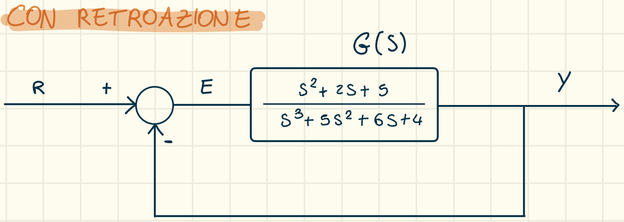
METODO CON LA TABELLA

Dato $G(s) = \dfrac{\beta_n s^n + \dots + \beta_1 s^1 + \beta_0}{\alpha_n s^n + \dots + \alpha_1 s^1 + \alpha_0}$ la nostra $G(s)$ ha $\begin{cases} \beta_0 = 5 \\ \alpha_0 = 4 \end{cases}$ con $\beta_0 \neq \alpha_0$

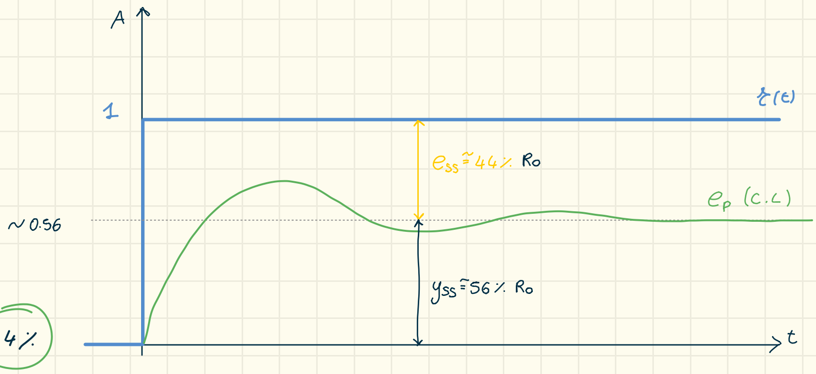
$\Rightarrow$ se $\alpha_0 \neq \beta_0$ $\Rightarrow$ $e_p = \left|\dfrac{\beta_0 - \alpha_0}{\alpha_0}\right| = \dfrac{5-4}{4} = \dfrac{1}{4} = 25\%$ di R_0 dove R_0 e' il gain del Riferimento (1)



CON RETROAZIONE



$$e_p = \frac{1}{1+\mu} \quad \text{con} \quad \mu = \lim_{s \to 0} G(s) = \frac{5}{4} \quad \Rightarrow \quad e_p = \frac{4}{9} \cong 44\%$$



$$G(s) = \frac{s^2+2s+5}{s^3+5s^2+6s+4} \quad \text{Con} \quad r(t) = t \cdot 1(t) \rightleftarrows R(s) = \frac{1}{s^2} \qquad \beta_0 \neq \alpha_0 \Rightarrow e_p = \infty$$

ES:

$$G(s) = \frac{s^2+2s+5}{s^3+5s^2+6s+5} \quad \text{con} \quad R(s) = \frac{1}{s} \rightarrow e_p = \emptyset \ \text{perche'} \ \beta_0 = \alpha_0$$

$\beta_1 \neq \alpha_0 \Rightarrow$ se $R(s) = \frac{1}{s^2} \rightarrow e_r = \left|\dfrac{2-6}{5}\right| = \dfrac{4}{5} = 0.8 = 80\%\ R_0$ errore negativo

Es:

$$G(s) = \frac{(s+1)(s+2)}{s(s+5) + s^2 - 2s + 2} = \frac{s^2+3s+2}{2s^2+3s+2}$$

$P_c(s) = 2s^2 + 3s + 2$

$$\begin{array}{c|cc} s^2 & 2 & 2 \\ s^1 & 3 & 0 \\ s^0 & 2 & \end{array}$$

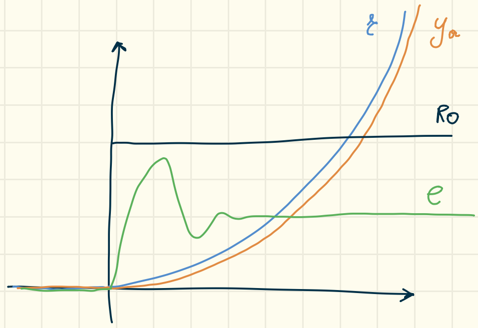
Dipende dall'ultimo

A.S.

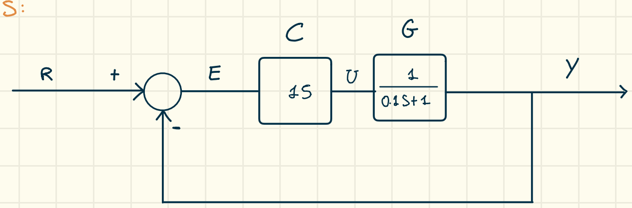
$\beta_0 = \alpha_0 \Rightarrow e_p = 0$

$\beta_1 = \alpha_1 \Rightarrow e_r = \emptyset$

$\beta_2 \neq \alpha_2 \Rightarrow e_a = \left|\frac{\beta_2 - \alpha_2}{\alpha_0}\right| = \left|\frac{1-2}{2}\right| = 0.5 = 50\%$

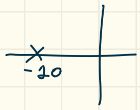


Es:



$C = 15$

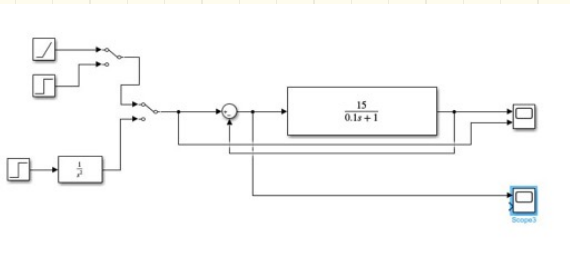
$G(s) = \frac{1}{0.1s+1}$ $\rightsquigarrow L(s) = \frac{15}{0.1s+1} \longrightarrow \bar{s} = -10$

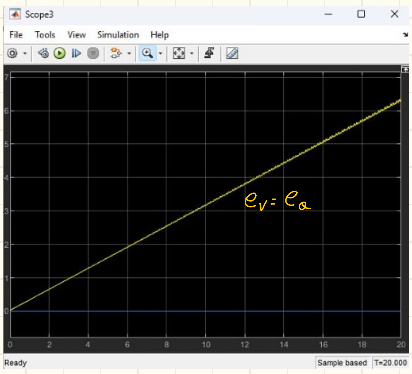


- $u = 1(t)$ $\quad U(s) = \frac{1}{s}$ $\quad \mu_p = \lim_{s \to 0} \frac{15}{0.1s+1} = 15 \Rightarrow e_p = \frac{1}{1+\mu_p} = \frac{1}{16} = 0.062 \approx 6\%$

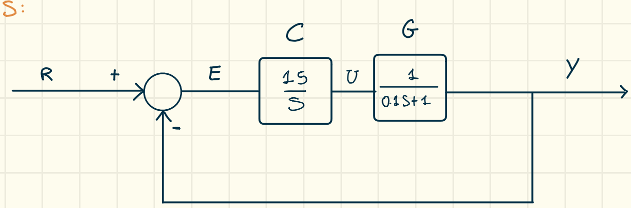
- $u = t \cdot 1(t)$ $\quad U(s) = \frac{1}{s^2}$ $\quad \mu_v = \lim_{s \to 0} s \cdot \frac{15}{0.1s+1} = \emptyset \Rightarrow \frac{1}{\mu_v} \rightarrow \infty$

Infatti non ci sono azioni integrali





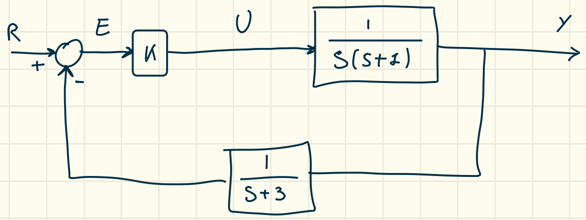
Es:



$\rightsquigarrow L(s) = \frac{15}{s\left(\frac{1}{10}s+1\right)}$ $\quad$ Tipo 1 $\Rightarrow e_p = 0$

$$\mu_v = \lim_{s \to 0} \cancel{s} \frac{15}{\cancel{s}\left(\frac{1}{10}s+1\right)} = 15 \Rightarrow e_v = \frac{1}{15} \simeq 0.067 \simeq 6.7\%$$

ES LEZ 10



Se $K=5$ calcolare e_p, e_r, e_a

$L(s)=\frac{5}{s(s+1)}$ $\quad H(s)=\frac{1}{s+3}$

$\Rightarrow F(s)=\frac{L(s)}{1+L(s)\cdot H(s)}=\frac{\frac{5}{s(s+1)}}{1+\frac{5}{s(s+1)(s+3)}}=\frac{5(s+3)}{s(s+1)(s+3)+5}$

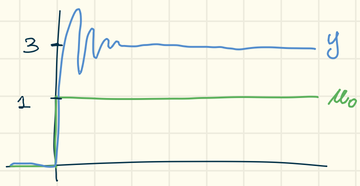
$=\frac{5s+15}{s^3+4s^2+3s+5}$ $\quad \beta_0 \neq \alpha_0$

$P_c(s)=s^3+3s^2+s^2+3s+5$

s^3	1	3
s^2	4	5
s^1	$7/4$	0
s^0	5	

✓ A.S.

$\Rightarrow$ Se $u_0(t)=1(t)$ $\Rightarrow e_p=\left|\frac{\beta_0-\alpha_0}{\alpha_0}\right|=\left|\frac{15-5}{5}\right|=2$ e_p



$\Rightarrow e_r=e_a=\infty$