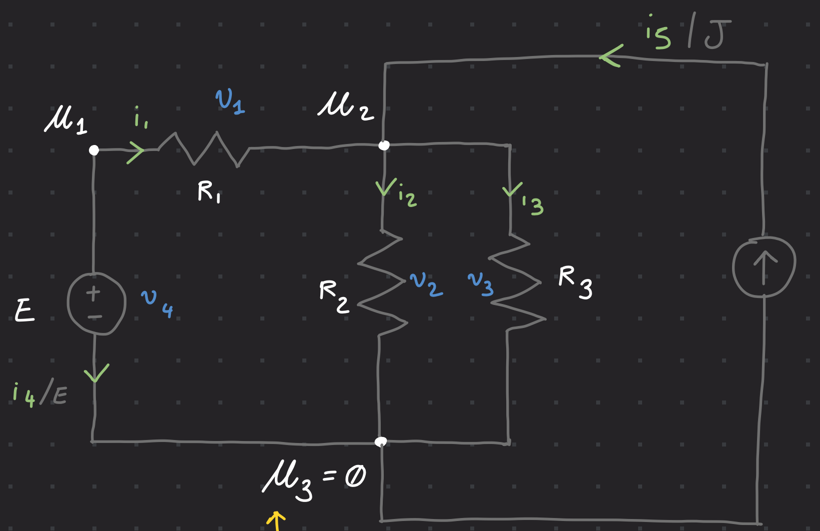
Metodo di risoluzione PER ISPEZIONE



NODO DI RIFERIMENTO

$E_1 = 10\ V$
$R_1 = 1\ \Omega$
$R_2 = 1\ \Omega$
$R_3 = 2\ \Omega$
$J = 1\ A$

CORRENTI (LKC)

$$\begin{aligned} \mu_1 &\rightarrow \\ \mu_2 &\rightarrow \end{aligned} \begin{cases} i_1 + i_4 = \emptyset & (i.1) \\ i_2 + i_3 - i_5 - i_1 = \emptyset & (i.2) \end{cases}$$

TENSIONI (LKT)

$$\begin{aligned} v_1 &= \mu_1 - \mu_2 \\ v_2 &= \mu_2 - \mu_3 = \mu_2 \\ v_3 &= \mu_2 - \mu_3 = \mu_2 \\ v_4 &= \mu_1 - \mu_3 = \mu_1 \\ v_5 &= \mu_2 - \mu_3 = \mu_2 \end{aligned} \rightarrow$$

Siccome $v_{R_k} = R_k \cdot i_k$ $\rightarrow$

EQ CARATT

$$\begin{cases} v_1 = R_1 \cdot i_1 \\ v_2 = R_2 \cdot i_2 \\ v_3 = R_3 \cdot i_3 \\ v_4 = E \\ v_5 = J \end{cases} \rightarrow \begin{cases} v_1 - R_1 i_1 = 0 \\ v_2 - R_2 i_2 = 0 \\ v_3 - R_3 i_3 = 0 \\ v_4 = E \\ v_5 = J \end{cases}$$

QUINDI

$$\begin{cases} i_1 + i_4 = \emptyset \\ i_2 + i_3 - i_5 - i_1 = \emptyset \\ v_1 = \mu_1 - \mu_2 \\ v_2 = \mu_2 \\ v_3 = \mu_2 \\ v_4 = \mu_1 \\ v_5 = \mu_2 \\ v_1 - R_1 i_1 = 0 \\ v_2 - R_2 i_2 = 0 \\ v_3 - R_3 i_3 = 0 \\ v_4 = E \\ v_5 = J \end{cases}$$

Sistema di 12 equazioni in 12 incognite

$\rightarrow$ Metodo di risoluzione OTTIMALE

(1) μ_1 è nota, infatti $\mu_1 = E$

(2) Esprimiamo l'intensità di corrente in funzione della Tensione

Siccome $v = R \cdot i \rightarrow i = \frac{v}{R}$

ma siccome $v_1 = \mu_1 - \mu_2$ e $\mu_1 = E$

$$\Rightarrow \underset{(2.a)}{i_1 = \frac{E - \mu_2}{R_1}}, \quad \underset{(2.b)}{i_2 = \frac{\mu_2}{R_2}}, \quad \overset{(2.c)}{i_3 = \frac{\mu_2}{R_3}}$$

(3) Imponiamo la LKC al nodo 2

$\rightarrow i_2 + i_3 - i_1 - J = 0$ $\rightarrow$ Sostituisco dalle 2 a, 2.b, 2.c

$$\frac{u_2}{R_2} + \frac{u_2}{R_3} - \frac{E - u_2}{R_1} - J = 0$$

Isoliamo u_2 (incognita) $\leftarrow$

$$\frac{u_2}{R_2} + \frac{u_2}{R_3} + \frac{u_2}{R_1} = J + \frac{E}{R_1} \rightarrow u_2 \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_1} \right) = J + E/R_1$$

(3.a)

$$\rightarrow \boxed{u_2 = \frac{J + \frac{E}{R_1}}{1/R_2 + 1/R_3 + 1/R_1}}$$

Ci basta sostituire i valori numerici nella (3.a) per ottenere:

$$u_2 = \frac{1\,A + \frac{10\,V}{1\,\Omega}}{\frac{1}{1\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}} = \frac{1\,A + \frac{10\,V}{1\,\Omega}}{1 + \frac{1}{2} + 1} = 4.4\,V$$

u_2 Ans 1

4) Sostituisco Ans 1 nelle 2.a, 2.b, 2.c per trovare le correnti

- $i_1 = \frac{E - u_2}{R_1} = \frac{10\,V - 4.4\,V}{1\,\Omega} = 5.6\,A$
- $i_2 = \frac{u_2}{R_2} = \frac{4.4\,V}{1\,\Omega} = 4.4\,A$
- $i_3 = \frac{u_3}{R_3} = \frac{4.4\,V}{2\,\Omega} = 2.2\,A$

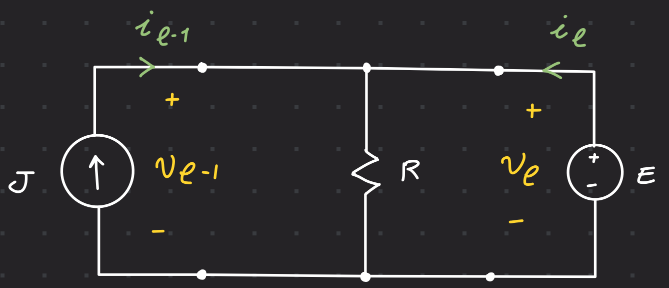
Ans 2

Inoltre dalla i.1 . $i_1 + i_4 = 0 \rightarrow i_4 = -i_1 \Rightarrow i_4 = -5.6\,A$

Sovrapposizione degli effetti

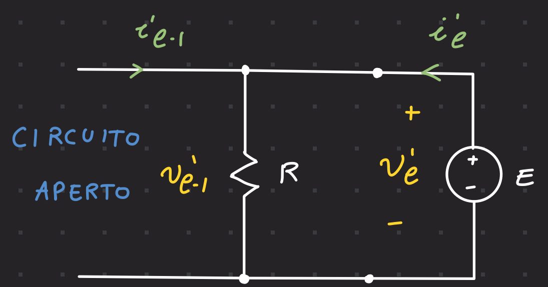
Enunciato: L'intensità di *corrente* e la *tensione* di un qualsiasi dipolo di un circuito resistivo lineare (con più generatori ideali) sono, rispettivamente, uguali alla somma delle intensità di corrente e delle tensioni che ciascuno dei generatori ideali produrrebbe se agisse da solo con tutti gli altri generatori ideali spenti.

Dimostrazione



$$A i = \emptyset$$
$$B v = \emptyset$$
$$v_K - R_K i_K = \emptyset$$
$$v_{e-1} = E$$
$$i_e = J$$

Circuito di partenza ci riferimo ad esso senza apici



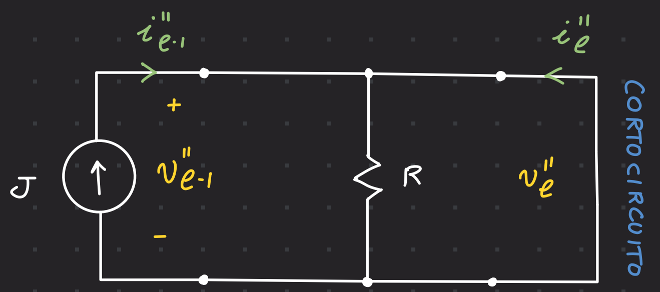
Circuito C'

$$A i' = \emptyset$$
$$B v' = \emptyset$$
$$v'_K - R_K i'_K = \emptyset$$
$$v'_{e-1} = E$$
$$i'_e = \emptyset$$

ottenuto spegnendo il gen di corrente

→ circuito Aperto

→ singolo Apice



Circuito C''

$$A i'' = \emptyset$$
$$B v'' = \emptyset$$
$$v''_K - R_K i''_K = \emptyset$$
$$v''_{e-1} = \emptyset$$
$$i''_e = J$$

ottenuto spegnendo il gen di Tensione

→ cortocircuito

→ Doppio Apice

Tesi

L'enunciato sostiene che:

$$\boxed{\begin{cases} i = i' + i'' \\ v = v' + v'' \end{cases}} \quad \begin{matrix} (a) \\ (b) \end{matrix}$$

- verifico (a) $A i = \emptyset \rightarrow$ se $i = i' + i'' \rightarrow i = \underbrace{A i'}_{0} + \underbrace{A i''}_{0} = 0$ ✓
- Verifico (b) $B v = B v' + B v'' = 0$ ✓

- verifico i resistori $v_K - R_K i_K = 0 \rightarrow$ data la Tesi

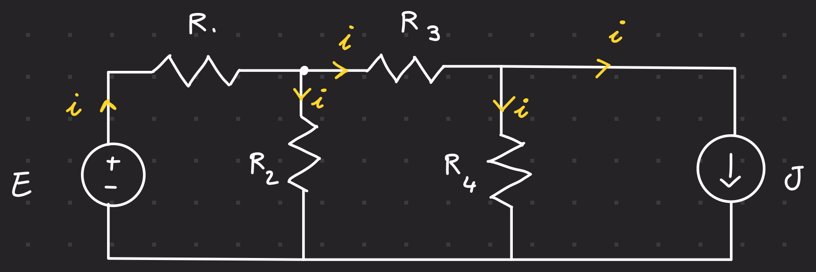
$$\rightarrow v_K - R_K i_K = (v'_K + v''_K) - (R_K i'_K + R_K i''_K) = v'_K + v''_K - R_K i'_K - R_K i''_K$$
$$= \underbrace{(v'_K - R_K i'_K)}_{0} + \underbrace{(v''_K - R_K i''_K)}_{0} = \emptyset \quad ✓$$

- Verifico i generatori $v_{e-1} = E \rightarrow \overset{E}{v'_{e-1}} + \overset{\emptyset}{v''_{e-1}} = E$ ✓

$$i_e = \emptyset \rightarrow \overset{\emptyset}{i'_e} + \overset{J}{i''_e} = J \quad ✓$$

QED

ESEMPIO - Soluz. per Sovrapp. pg 188



$R_1 = 1\ \Omega$
$R_2 = 1\ \Omega$
$R_3 = .5\ \Omega$
$R_4 = .5\ \Omega$

$J = 50\ A$
$E = 15\ V$

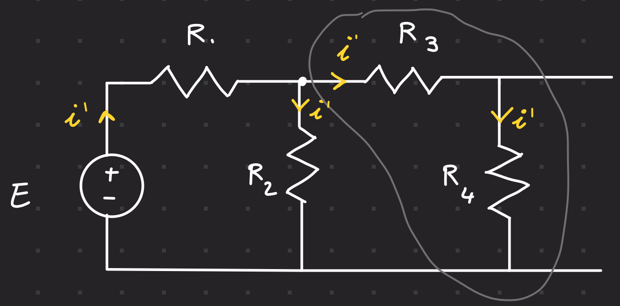
Q_1 Potenza elettrica Assorbita da R_1 $\rightarrow$ $P_1 = R_1 i_1^2 = i_1^2$ (a)

???

TROVIAMO i_1

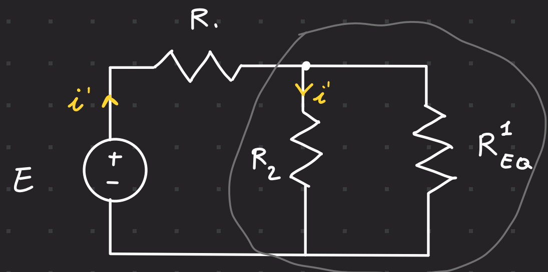
$\rightarrow$ Dalla Sovrapposizione : $i_1 = i_1' + i_1''$

(1) Rimuovo J $\rightarrow$ C'



R3 ed R4 Sono in serie $\rightarrow$

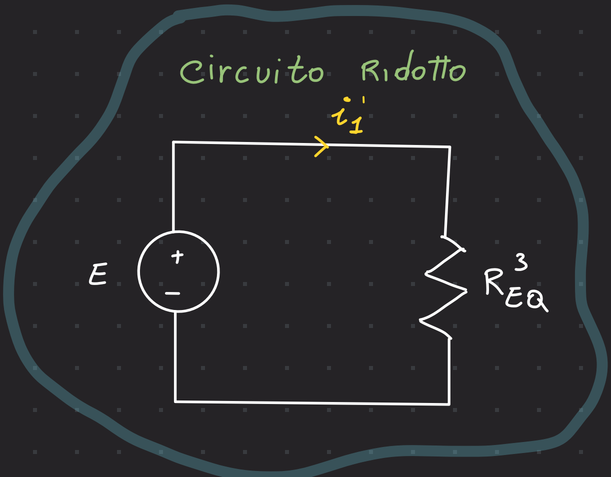
$R_{EQ}^{1} = R_3 + R_4$



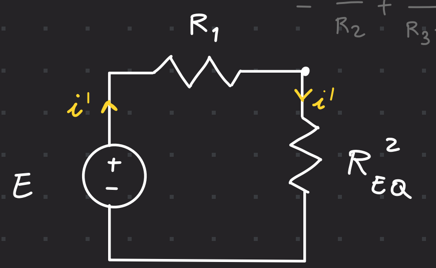
R2 ed REQ Sono in parallelo

$\frac{1}{R_{EQ}^2} = \frac{1}{R_2} + \frac{1}{R_{EQ}^1} = \frac{1}{R_2} + \frac{1}{R_3+R_4}$

$= \frac{R_3+R_4+R_2}{R_2(R_3+R_4)}^{-1} \rightarrow \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$ $R_{EQ}^{(2)}$



R_1 ed R_{EQ}^2 Sono in Serie

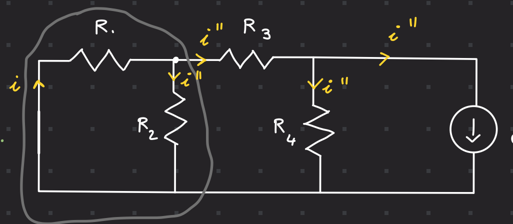


$R_{EQ}^3 = R_1 + R_{EQ}^{(2)}$

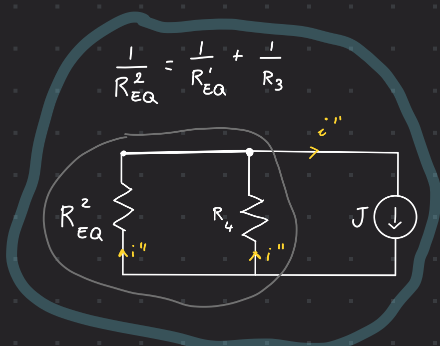
$= R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} = 1 + \frac{1(.5+.5)}{1+.5+.5} = \frac{3}{2}$ $R_{EQ}^{(3)}$

(2) Rimuovo E $\rightarrow$ C''

C.C.



$\frac{1}{R_{EQ}^2} = \frac{1}{R_{EQ}'} + \frac{1}{R_3}$



$R_{EQ}^{(1)} = R_1 + R_2$, $R_{EQ}^{(2)} = \frac{1}{R_{EQ}^{(1)}} + \frac{1}{R_3} = \frac{1}{R_1+R_2} + \frac{1}{R_3} = \frac{R_3+R_1+R_2}{(R_1+R_2)R_3} \Rightarrow R_{EQ}^{(3)} = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$

Dalla (a) $p_1 = R_1 i_1^2 = R_1 (i_1' + i_1'')^2 = R_1 i_1'^2 + R_1 i_1''^2 + 2 R_1 i_1' i_1''$

$i_1 = i_1' + i_1''$

(3) Determiniamo le i

Siccome $v = R \cdot i \rightarrow i = \frac{v}{R}$

$\Rightarrow i_1' = \frac{v}{R^3_{EQ}}$ ma $v = E \rightarrow i_1' = \frac{E}{R^3_{EQ}} = \frac{15\ V}{3\ \Omega} \cdot 2 = 10\ A$

i_1'' si calcola con la regola del partitore di corrente

Sappiamo che: $\begin{cases} E = R_2 \cdot i_2 \\ E = R^3_{EQ} \cdot i \end{cases}$ ma $R_{EQ} = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3}$

$\Rightarrow R_2 \cdot i_2 = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} \cdot i \rightarrow i_2 = \frac{R_3 (R_1 + R_2)}{(R_1 + R_2 + R_3) R_2} = ???$

BOH!

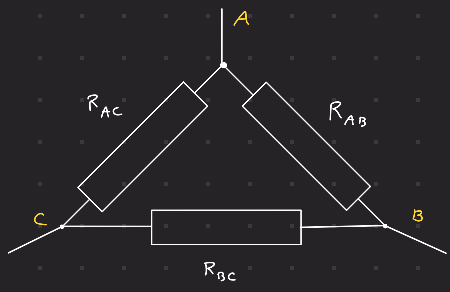
EQUIVALENZA STELLA - TRIANGOLO

- TRIANGOLO

$$\frac{1}{R_{AB}^{(1)}} = \frac{1}{R_{AB}} + \left(\frac{1}{R_{CA}+R_{BC}}\right) = \frac{R_{AB}+R_{BC}+R_{CA}}{R_{AB}(R_{CA}+R_{BC})} \Rightarrow R_{AB}^{(1)} = \frac{R_{AB}(R_{CA}+R_{BC})}{R_{AB}+R_{BC}+R_{CA}}$$

$$\Rightarrow R_{BC}^{(1)} = \frac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}}$$

$$\Rightarrow R_{CA}^{(1)} = \frac{R_{CA}(R_{BC}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}}$$

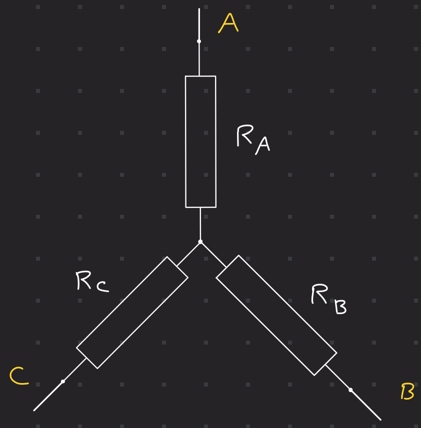


- STELLA

$$R_{AB}^{(11)} = R_A + R_B$$

$$R_{AC}^{(11)} = R_A + R_C$$

$$R_{BC}^{(11)} = R_B + R_C$$



$\rightarrow$ Per avere l'equivalenza, Tutte le resistenze DEVONO essere uguali!

$$R_{AB}^{(1)} = R_{AB}^{(11)}, \quad R_{BC}^{(1)} = R_{BC}^{(11)}, \quad R_{AC}^{(1)} = R_{AC}^{(11)}$$

OVVERO

$$\begin{cases} \dfrac{R_{AB}(R_{CA}+R_{BC})}{R_{AB}+R_{BC}+R_{CA}} = R_A + R_B \\[2ex] \dfrac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} = R_A + R_C \\[2ex] \dfrac{R_{CA}(R_{BC}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} = R_B + R_C \end{cases}$$

Stella $\rightarrow$ Triangolo

$$R_{AB} = \frac{R_A \cdot R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_{BC} = \frac{R_A \cdot R_B + R_B R_C + R_C R_A}{R_A}$$

- $$R_{CA} = \frac{R_A \cdot R_B + R_B R_C + R_C R_A}{R_B}$$

Triangolo $\rightarrow$ Stella

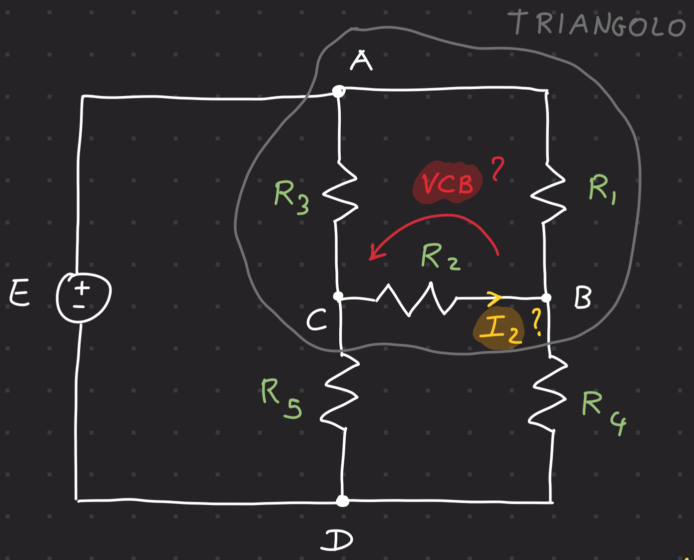
- $$R_A = \frac{R_{AB}\ R_{AC}}{R_{AB}+R_{AC}+R_{CA}}$$

$$R_B = \frac{R_{AB}\ R_{BC}}{R_{AB}+R_{AC}+R_{CA}}$$

$$R_C = \frac{R_{AC}\ R_{BC}}{R_{AB}+R_{AC}+R_{CA}}$$

ESERCIZIO $\triangle \rightarrow \curlywedge$ Link

Scopo del gioco: TROVARE I_2, V_{CB}



DATI

$E = 24\ V$

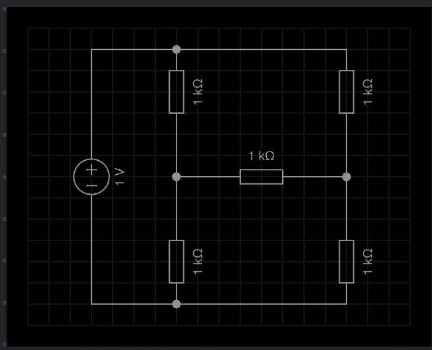
$R_1 = 20\ \Omega$

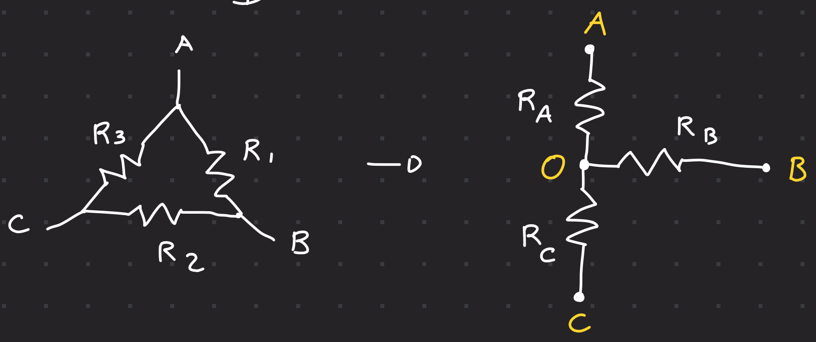
$R_2 = 30\ \Omega$

$R_3 = 10\ \Omega$

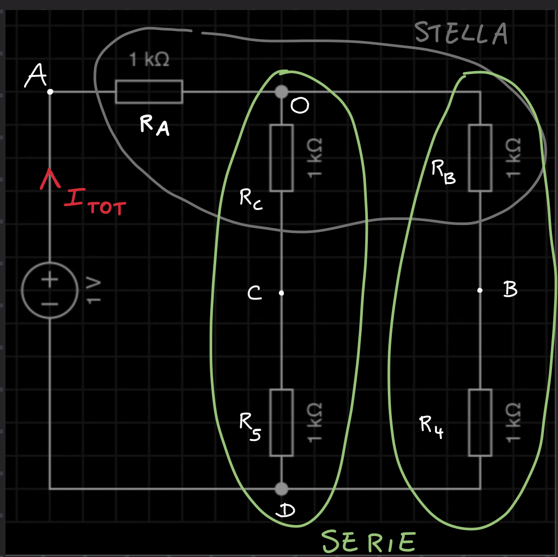
$R_4 = 40\ \Omega$

$R_5 = 50\ \Omega$





$\rightarrow$ Riscrivo il circuito



(1) Ricavo le resistenze

Res con il nodo A (Nel Triangolo) in comune

$$R_A = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3} = 3.\bar{3}\ \Omega$$

$$R_B = \frac{R_1\ R_2}{R_1 + R_2 + R_3} = 10\ \Omega$$

$$R_C = \frac{R_2 \cdot R_3}{R_1 + R_2 + R_3} = 5\ \Omega$$

$\rightarrow$ $R_{C5} = R_C + R_5 = 50\ \Omega + 5\ \Omega = \underline{55\ \Omega}$

$R_{B4} = R_B + R_4 = 10\ \Omega + 40 = \underline{50\ \Omega}$

SERIE

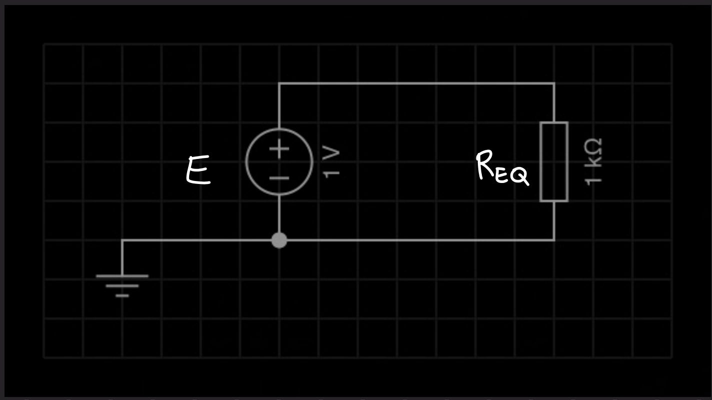
$$R_{C5B4} = R_{C5} \,//\, R_{B4} = 55 \,\|\, 50 \Rightarrow \frac{1}{R_{C5B4}} = \frac{1}{R_{C5}} + \frac{1}{R_{B4}} = \frac{1}{55} + \frac{1}{50} = \frac{21}{550}$$

$$\rightarrow R_{C5B4} = \frac{550}{21} = \underline{26.2\ \Omega}$$

PARALLELO

oppure $R_{C5B4} = \dfrac{R_{C5}\ R_{B4}}{R_{C5} + R_{B4}} = 26.2\ \Omega$

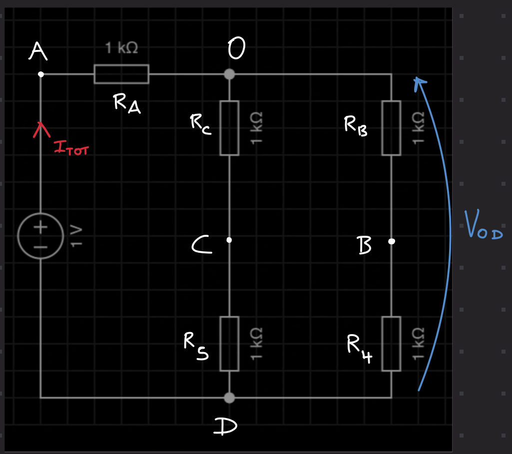
$\Rightarrow R_{EQ} = R_A + R_{CSB4} = 29.5\ \Omega$

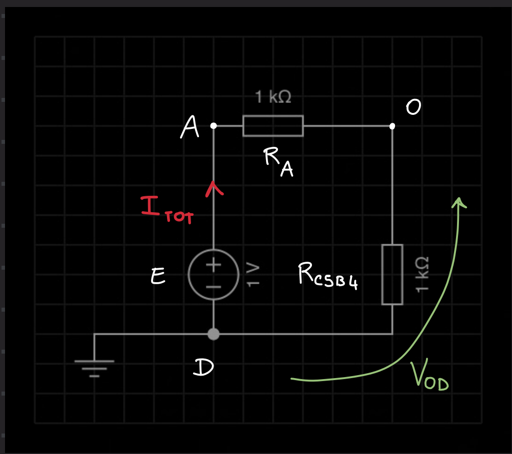


(2) Trovo la corrente

siccome $v = R \cdot i \rightarrow I_{TOT} = \frac{v}{R_{EQ}} = \frac{E}{R_{EQ}} = \frac{24\,V}{29.5\ \Omega} = 0.8\ A$

(3) Trovo V_{OD}



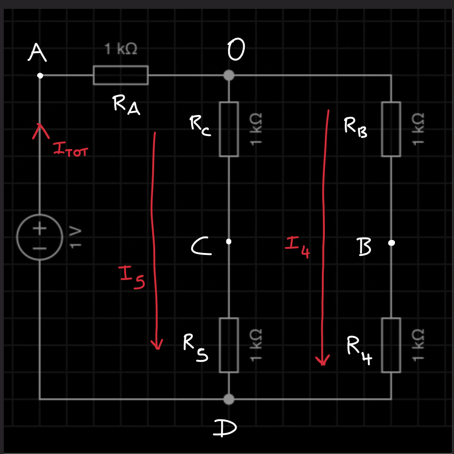


$\Rightarrow V_{OD} = R_{CSB4} \cdot I_{TOT}$

$= 26.2 \cdot 0.8 =$

$= 20.96\ V$

(4) Trovo le correnti

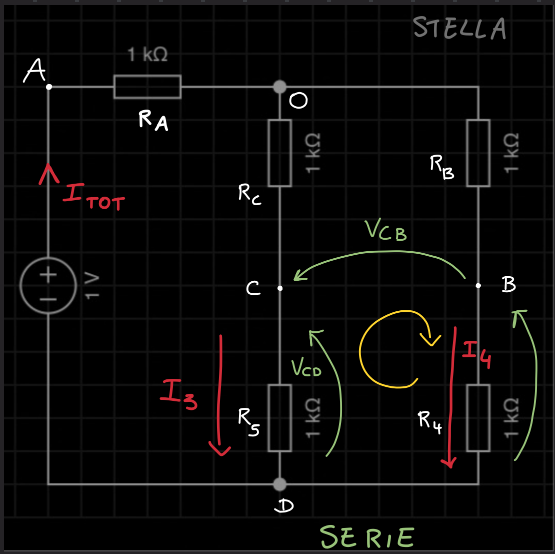


$I_5 = \frac{V}{R_{C5}} = \frac{V_{OD}}{(R_C + R_5)} = \frac{20.96\,V}{5\Omega + 50\Omega} = 0.38\ A$

$I_4 = \frac{V_{OD}}{R_B + R_4} = 0.42\ A$

(5) TROVO V_{CB}

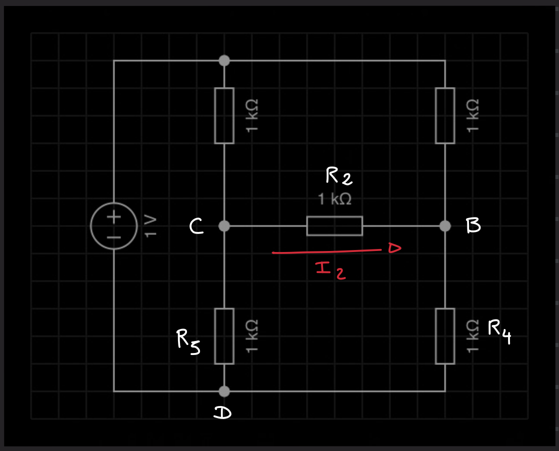
Per trovare I_2 ci serve la Tensione Tra C e B quindi



Da Kirchhoff: $V_{CD} - V_{CB} - V_{BD} = 0$

$$\Rightarrow V_{CB} = V_{CD} - V_{BD} = (R_5 \cdot I_5) - (R_4 \cdot I_4)$$

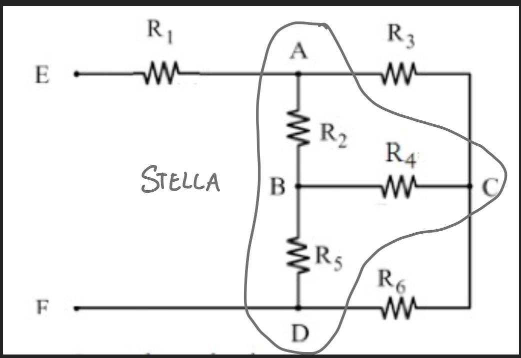
$$= (50 \cdot 0.38) - (40 \cdot 0.42) = 2.2\text{ V} \quad V_{CB}$$



$$\Rightarrow V = R \cdot I \Rightarrow I_2 = \frac{V_{CB}}{R_2} \cong 0.073\text{ A} = 73\text{ mA}$$

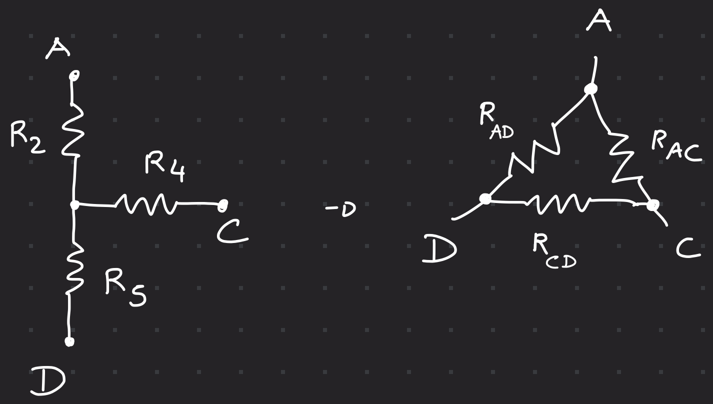
Ans

Esercizio al link



DATI

$R_1 = 5\,\Omega$
$R_2 = 20\,\Omega$
$R_3 = 12\,\Omega$
$R_4 = 16\,\Omega$
$R_5 = 25\,\Omega$
$R_6 = 30\,\Omega$

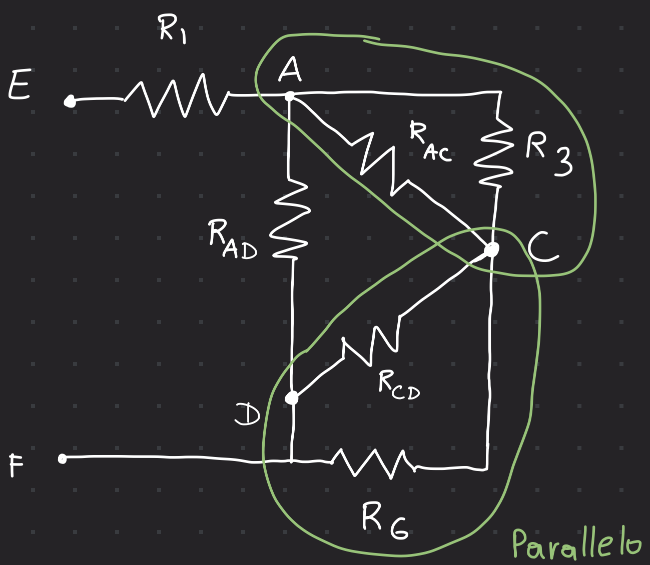


(1) Resistenze del Triangolo

$$R_{AC} = \frac{(R_2 \cdot R_4)+(R_5 \cdot R_4)+(R_2 \cdot R_5)}{R_5} = 48.8\,\Omega$$

$$R_{AD} = \frac{(R_2 \cdot R_4)+(R_5 \cdot R_4)+(R_2 \cdot R_5)}{R_4} = 76.25\,\Omega$$

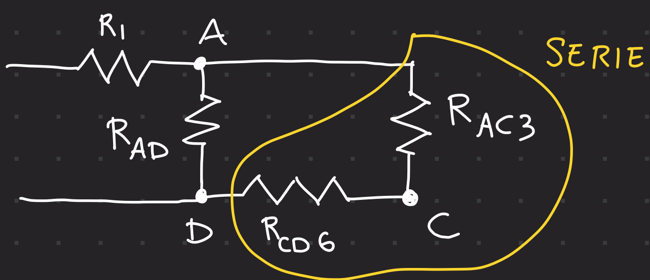
$$R_{CD} = \frac{(R_2 \cdot R_4)+(R_5 \cdot R_4)+(R_2 \cdot R_5)}{R_2} = 61\,\Omega$$



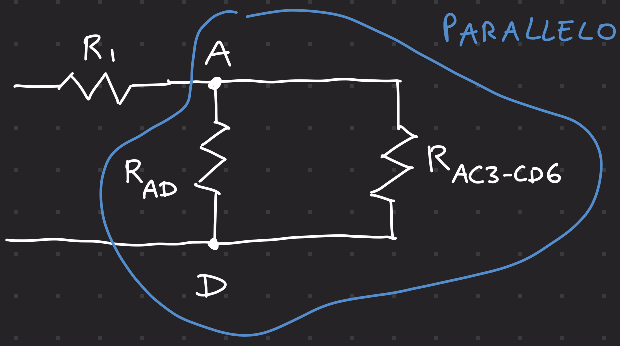
(2) Res in Parallelo

$$R_{CD6} = R_{CD} \parallel R_6 \rightarrow \frac{1}{61\,\Omega} + \frac{1}{30\,\Omega} \rightarrow R_{CD6} = 20.1\,\Omega$$

$$R_{AC3} = R_{AC} \parallel R_3 \rightarrow \frac{1}{48.8\,\Omega} + \frac{1}{12\,\Omega} \rightarrow R_{AC3} = 9.6\,\Omega$$



$$R_{AC3-CD6} = R_{AC3} + R_{CD6} = 29.73\,\Omega$$



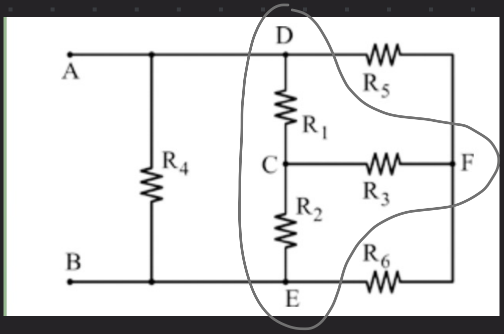
$$\frac{1}{R_{PAR}} = \frac{1}{R_{AC3-\dots}} + \frac{1}{R_{AD}} \Rightarrow R_{PAR} = 21.4\ \Omega$$

$$\Rightarrow R_{EQ} = R_1 + R_{PAR} = \underbrace{26.4\ \Omega}_{R_{EQ}}$$

CIRCUITO RISOLTO ✓

ESERCIZIO

Link



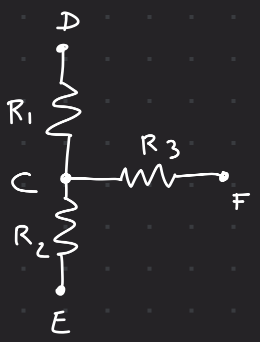
DATI

$R_1 = R_2 = R_3 = 30\ \Omega$

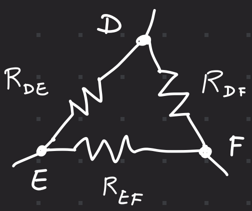
$R_4 = R_5 = R_6 = 150\ \Omega$

Q : R_{AB}

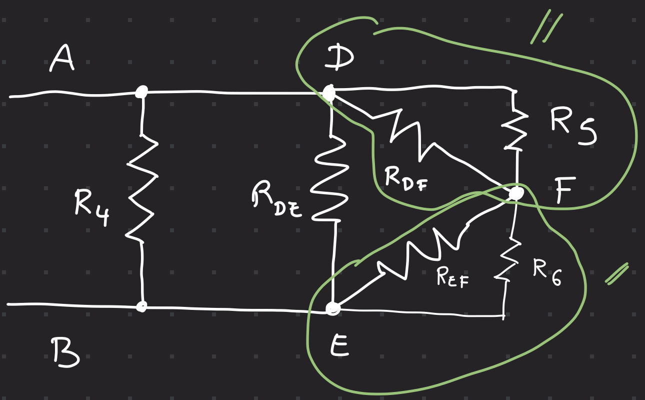
TRIANGOLO



$\rightarrow$



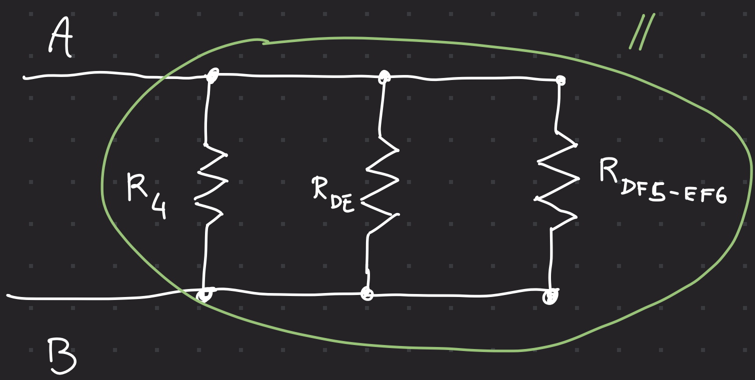
$$R_{DE} = \frac{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}{R_3} = R_{DF} = R_{EF} = 90\ \Omega$$



$$\frac{1}{R_{DF5}} = \frac{1}{R_{DF}} + \frac{1}{R_5} \rightarrow R_{DF5} = 56.25\ \Omega$$

$$\frac{1}{R_{EF6}} = \frac{1}{R_{EF}} + \frac{1}{R_6} = R_{EF6} = 56.25\ \Omega$$

$$R_{DF5-EF6} = R_{DF5} + R_{EF6} = 112.5\ \Omega$$



$$\Rightarrow \frac{1}{R_{AB}} = \frac{1}{R_4} + \frac{1}{R_{DE}} + \frac{1}{R_{DF5-...}}$$

$$\Rightarrow R_{AB} = 37.5\ \Omega$$

Ans

Esercizio 13

Nel circuito seguente con

R_1=1kΩ

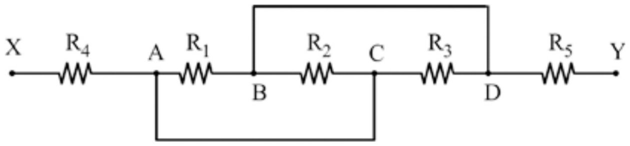
R_2=2kΩ

R_3=3kΩ

R_4=400Ω

R_5=500Ω

Calcola la resistenza R_{XY} fra i morsetti X e Y.



Considerando che R_1=R_2=R_3 =30Ω e poi R_4=R_5=R_6=150Ω.

[Risp.: R_{AB}=1,445kΩ]

I cortocircuiti ci fanno dire

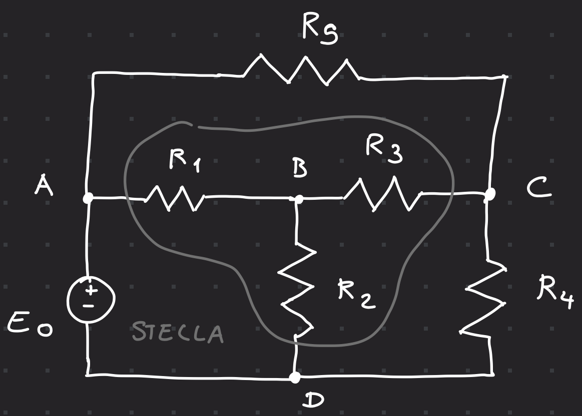
$A = C$ e $B = D$

-o colleghiamo i dipoli di A anche a C e viceversa

BOH

ESERCIZIO

DA COMPLETARE !!



DATI

$E_0 = 100\ V$

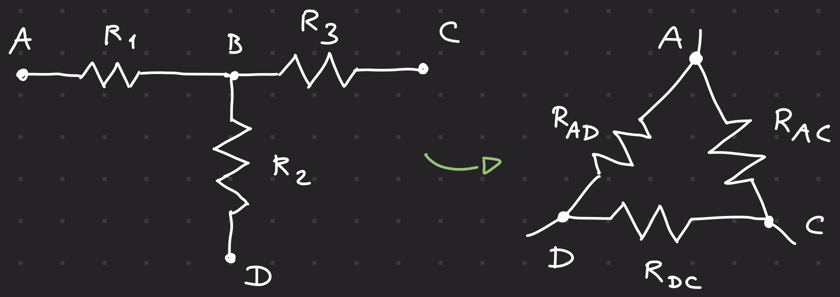
$R_1 = 20\ \Omega$

$R_2 = 30\ \Omega$

$R_3 = 15\ \Omega$

$R_4 = 24\ \Omega$

$R_5 = 60\ \Omega$



$$R_{AD} = \frac{R_A \cdot R_D + R_A R_C + R_D \cdot R_C}{R_C}$$

OVVERO $$\frac{R_1 R_2 + R_1 R_3 + R_2 \cdot R_3}{R_3}$$

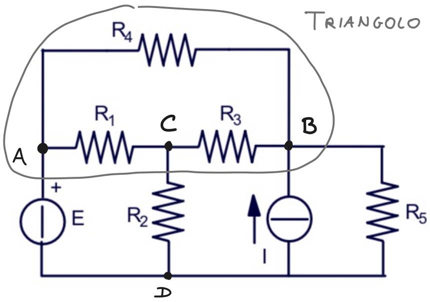
$$\Rightarrow \quad R_{AD} = \frac{1350}{15} = 90\ \Omega$$

$$R_{AC} = \frac{1350}{30} = 45\ \Omega$$

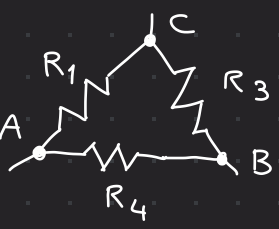
$$R_{DC} = \frac{1350}{20} = 67.5\ \Omega$$

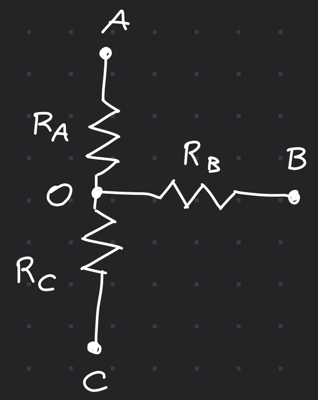
Link

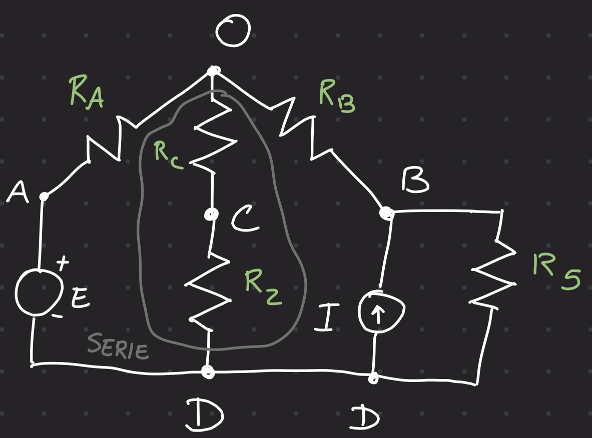
Calcolare la potenza assorbita da ogni resistore presente nel circuito, tensioni e correnti in ogni ramo.



$E = 30\ V$
$I = 2\ A$
$R_1 = 40\ \Omega$
$R_2 = 25\ \Omega$
$R_3 = 20\ \Omega$
$R_4 = 400\ \Omega$
$R_5 = 100\ \Omega$







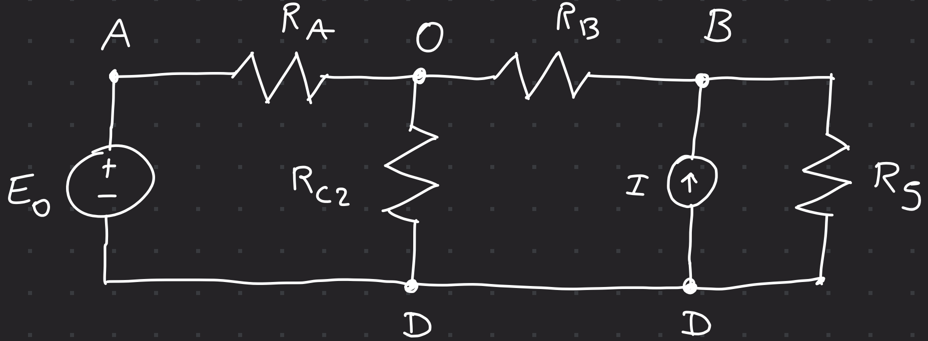
$$R_A = \frac{R_1 \cdot R_4}{R_1 + R_3 + R_4} = 34.78\ \Omega$$

$$R_B = \frac{R_3 \cdot R_4}{R_1 + R_3 + R_4} = 17.39\ \Omega$$

$$R_C = \frac{R_1 \cdot R_3}{R_1 + R_3 + R_4} = 1.74\ \Omega$$

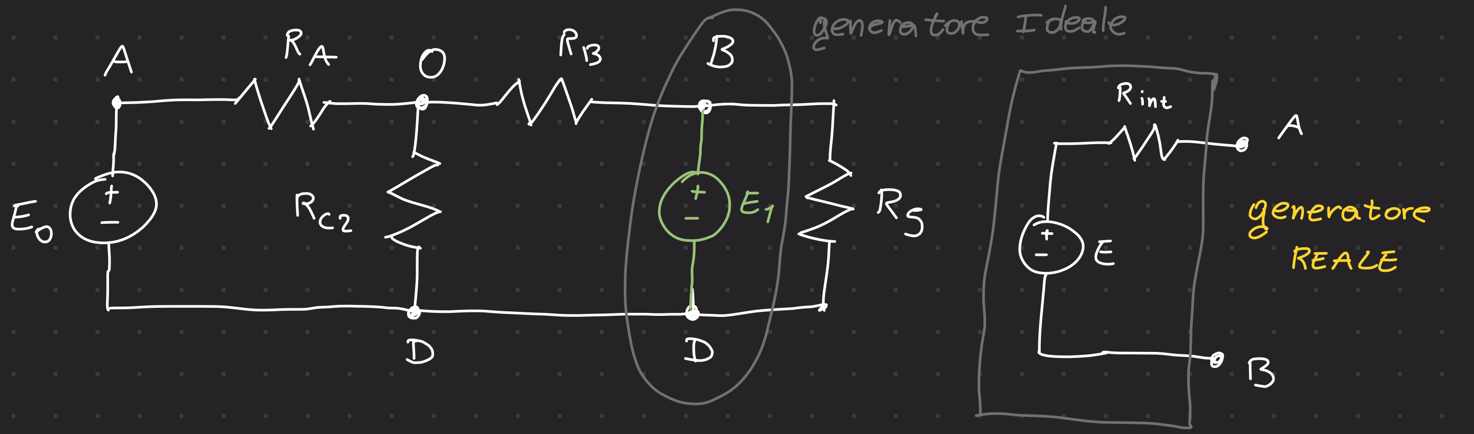
$$R_{C2} = R_C + R_2 = 1.74 + 25 = \underline{26.74\ \Omega}$$

OTTENIAMO

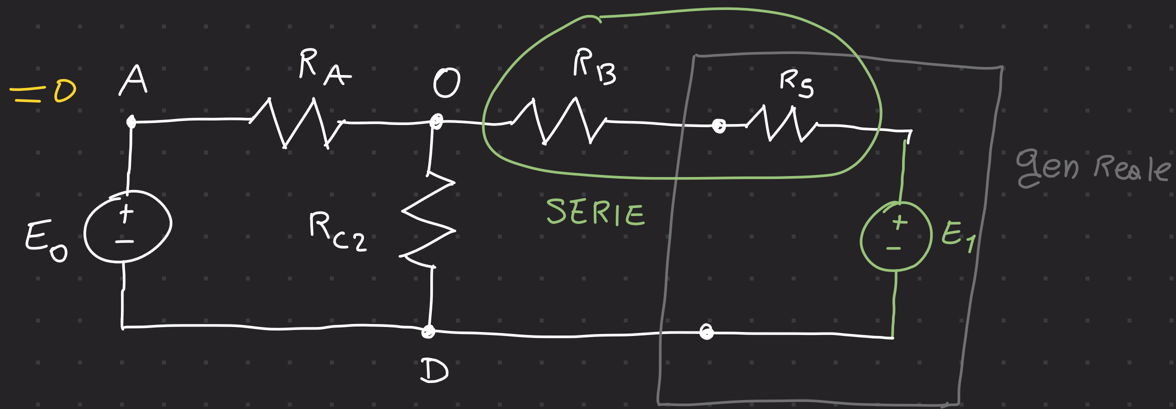


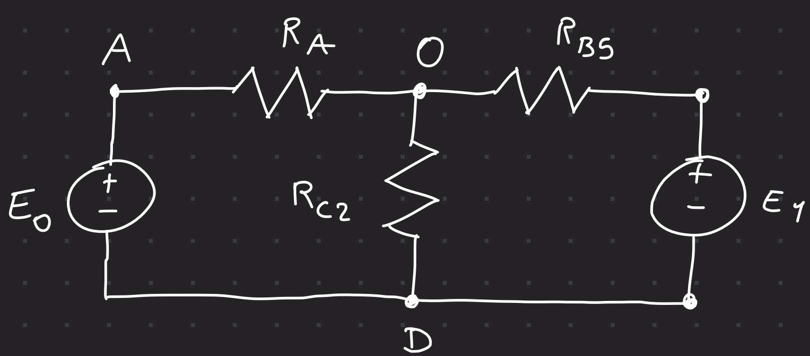
Possiamo Trasformare un generatore di corrente in uno di Tensione grazie alla legge di Ohm: Link generatore di corrente —> tensione

$$V = R \cdot I \Rightarrow E_1 = I \cdot R_S = 2 \cdot 100 = \underline{200\,V}$$



⇒





$$R_{BS} = R_B + R_S = 17.39 + 100 = \underline{117.39\,\Omega}$$