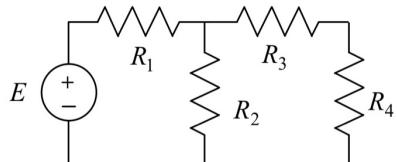


ES. 1.1 Calcolare la resistenza equivalente vista ai capi del generatore E.



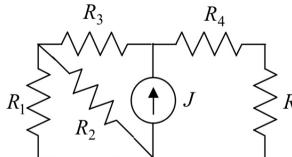
$$R_1 = 1 \Omega \quad R_2 = 4 \Omega \\ R_3 = 3 \Omega \quad R_4 = 2 \Omega$$

$$R_{34} = R_3 + R_4$$

$$R_{2-34} = R_2 \parallel R_{34}$$

$$R_{EQ} = R_1 + R_{2-34} = R_1 + \left[\frac{R_2 (R_3 + R_4)}{R_2 + R_3 + R_4} \right] = 3.22 \Omega \quad \text{Time } 40''$$

ES. 1.2 Calcolare la resistenza equivalente vista dal generatore J.



$$R_1 = R_4 = 5 \Omega \quad R_2 = 3 \Omega \\ R_3 = R_5 = 2 \Omega$$

$$R_{45} = R_4 + R_5 = 7 \Omega$$

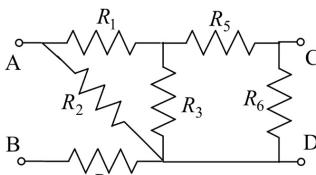
$$R_1 \text{ e } R_2 \text{ sono in parallelo} \rightarrow R_{12} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = 1.875 \Omega$$

$$R_{12+3} = R_{12} + R_3 = 3.875 \Omega$$

$$R_{12+3-45} = 2.49 \Omega$$

TIME 3'

ES. 1.3 - Calcolare la R_{eq} vista ai morsetti A-B e quella vista ai morsetti C-D.



$$R_1 = R_2 = 5 \Omega \quad R_3 = 10 \Omega \\ R_4 = 4 \Omega \quad R_5 = 3 \Omega \\ R_6 = 2 \Omega$$

Risultato: $R_{eqAB} = 7.125 \Omega$, $R_{eqCD} = 1.600 \Omega$.

• R_{AB}

$$R_{S6} = R_5 + R_6 \\ R_{3-S6} = R_3 \parallel R_{S6}$$

$$R_{1-3-S6} = R_x = R_1 + R_{3-S6}$$

$$R_{2x} = R_2 \parallel R_x$$

$$R_{EQ} = R_{2x} + R_4$$

$$\rightarrow R_{EQ} = R_4 + \left[\frac{R_2 \cdot R_x}{R_2 + R_x} \right], \quad R_x = R_1 + \left[\frac{R_3 \cdot (R_5 + R_6)}{R_3 + R_5 + R_6} \right]$$

$$= 8.3 \Omega$$

$$= 7.125 \Omega \quad \text{Ans 1}$$

$$Q_2: R_{CD}$$

R_4 viene eliminata

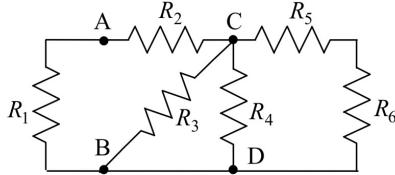
$$R_{12} = R_1 + R_2$$

$$R_{12-3} = R_3 \parallel R_{12}$$

$$R_{S-12-3} = R_S + R_{12-3} \quad \Rightarrow \quad R_X = R_S + \left[\frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3} \right] = 8 \Omega$$

$$R_{EQ} = \underbrace{R_{S-12-3}}_{R_X} \parallel R_6 \quad \Rightarrow \quad R_{EQ} = \frac{R_6 \cdot R_X}{R_6 + R_X} = \underline{\underline{1.600 \Omega}} \quad Ans 2$$

ES. 1.4 - Calcolare la R_{eq} vista ai morsetti A-B e quella vista ai morsetti C-D.



$$\begin{aligned} R_1 &= R_3 = 0.2 \Omega & R_2 &= 0.4 \Omega \\ R_4 &= R_5 = 1 \Omega & R_6 &= 3 \Omega \end{aligned}$$

Risultato: $R_{eqAB} = 0.147 \Omega$, $R_{eqCD} = 0.126 \Omega$.

• $Q_1: R_{AB}$

$$R_{S6} = R_S + R_6$$

$$R_X = R_3 \parallel R_{S6-4}$$

$$R_{S6-4} = R_4 \parallel R_{S6}$$

$$R_Y = R_2 + R_X$$

$$R_{EQ} = R_Y \parallel R_1 = R_1 \parallel \left\{ R_2 + R_3 \parallel \left[R_4 \parallel (R_5 + R_6) \right] \right\}$$

$\Rightarrow R_{EQ} = 0.146 \Omega$ Ans 1 (Accettabile)

• $Q_2: R_{CD}$

• $R_{S6} = R_S + R_6$

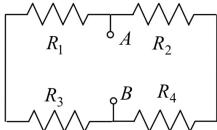
• $R_X = R_{S6} \parallel R_4$

• $R_{12} = R_1 + R_2$

• $R_{EQ} = R_X \parallel R_{12-3} = \left[R_4 \parallel (R_S + R_6) \right] \parallel \left[R_3 \parallel (R_1 + R_2) \right]$

$$= 0.126 \Omega \quad \underline{Ans 2}$$

ES. 1.5 - Calcolare il valore di R_4 tale che ai morsetti A-B si abbia $R_{eq} = R$.



$$R_1 = R_2 = R \quad R_3 = R/2$$

Risultato: $R_4 = 2R$.

$$\left. \begin{array}{l} R_{13} = R_1 + R_3 \\ R_{24} = R_2 + R_4 \end{array} \right\} R_{EQ} = R_{13} // R_{24} = \frac{(R_1 + R_3)(R_2 + R_4)}{R_1 + R_2 + R_3 + R_4}$$

$$\Rightarrow R_{EQ} = \frac{R_1 R_2 + R_1 R_4 + R_2 R_3 + R_3 R_4}{R_1 + R_2 + R_3 + R_4} = \frac{R_4 (R_1 + R_3) + R_1 R_2 + R_2 R_3}{R_1 + R_2 + R_3 + R_4}$$

$$\Rightarrow R_{EQ} (R_1 + R_2 + R_3 + R_4) = R_4 (R_1 + R_3)$$

$$\Rightarrow \underbrace{R_{EQ} R_3 + R_{EQ} R_1 + R_{EQ} R_2}_{R_1 + R_3} = R_4 - R_{EQ} \cdot R_4$$

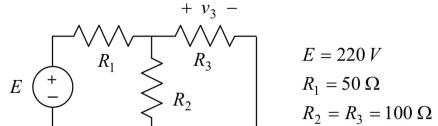
$$\Rightarrow R_4 (1 - R_{EQ}) = \frac{R_{EQ} (R_1 + R_2 + R_3)}{R_1 + R_3}$$

$$\Rightarrow R_4 = \frac{R_{EQ} (R_1 + R_2 + R_3)}{(R_1 + R_3) (1 - R_{EQ})}$$

$$\Rightarrow R_4 = \frac{R (R + R + \frac{1}{2}R)}{(R + \frac{1}{2}R) (1 - R)} = \frac{R (\frac{4R + R}{2})}{\frac{3}{2}R - \frac{3}{2}R}$$

$$= \frac{\frac{5}{2}R}{\frac{3 - 3R}{2}} = \text{Bohl}$$

ES. 1.7 - Calcolare la tensione v_3 usando il partitore di tensione.

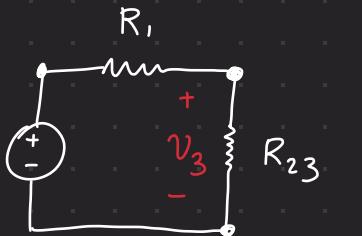


Part tens -> SERIE

$$LKT: \mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2$$

$$\begin{aligned} \text{ma } & \mathcal{V}_1 = R_1 \cdot i \quad (1) \\ \Rightarrow \mathcal{V} &= (R_1 + R_2) \cdot i = \mathcal{V} \quad (2) \quad | \cdot \frac{1}{R_1 + R_2} \\ \Rightarrow \mathcal{V}_1 &= \frac{R_1 \cdot \mathcal{V}}{R_1 + R_2} \end{aligned}$$

Partitore di tensione



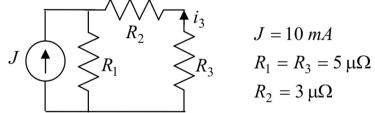
$$R_{23} = R_2 \parallel R_3 =$$

$$\frac{R_2 \cdot R_3}{R_2 + R_3} = 50 \Omega$$

$$\Rightarrow v_3 = \frac{50 \Omega \cdot 220 \text{ V}}{R_1 + 50} = 110 \text{ V} \quad \text{Ans}$$

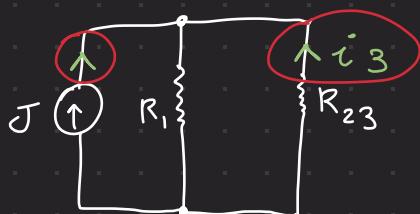
Partitore di corrente

ES. 1.8 - Calcolare la corrente i_3 usando il partitore di corrente.



PART CURR. -> PARALLELO

VERSO OPPOSTO -> i_3 è Negativo!



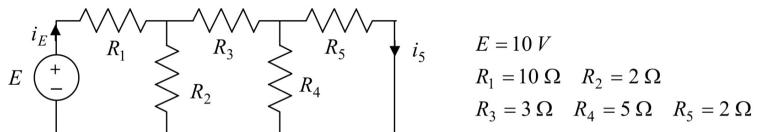
$$\left\{ \begin{array}{l} i_1 = G_1 \cdot \mathcal{V} \\ \mathcal{V} = \frac{i}{G_1 + G_2} \end{array} \right.$$

$$\Rightarrow i_1 = \frac{G_1}{G_1 + G_2} \cdot i \quad \begin{array}{l} \uparrow \text{Corrente in uscita} \\ \uparrow \text{Corrente in uscita} \end{array} \\ = \frac{R_2}{R_1 + R_2} i$$

$$i_3 = \frac{R_1}{R_1 + R_{23}} \cdot i = \frac{R_1}{R_1 + R_2 + R_3} (-J) = -384 \times 10^{-3} \text{ A} = -3.84 \text{ mA}$$

$$\left. \begin{array}{l} J = 10 \text{ mA} = 10 \times 10^{-3} \text{ A} = 0.01 \text{ A} \\ R_1 = R_3 = 5 \mu\Omega = 5 \times 10^{-6} \Omega \\ R_2 = 3 \mu\Omega = 3 \times 10^{-6} \Omega \end{array} \right\}$$

ES. 1.9 - Calcolare la potenza erogata dal generatore E e quella assorbita dal resistore R_5



$$Q_1: P_E^{\text{erog}} = E \cdot i_E$$

- Devo Trovare i_E \rightarrow Riduco il circuito

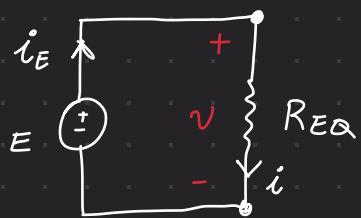
$$\bullet R_{4S} = R_4 // R_5$$

$$R_X = R_2 // R_{3-4S}$$

$$\bullet R_{3-4S} = R_3 + (R_4 // R_5)$$

$$R_{EQ} = R_1 + R_X = R_1 + \left[R_2 // \left(R_3 + \frac{R_4 R_5}{R_4 + R_5} \right) \right]$$

$$= 11.38 \Omega$$



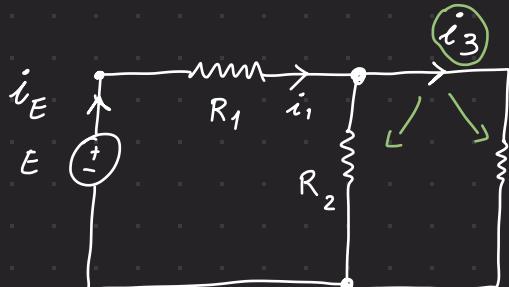
$$\bullet V = R \cdot i \Rightarrow i_E = \frac{V}{R} = \frac{10 \text{ V}}{11.38 \Omega} = 0.878 \text{ A}$$

$$\Rightarrow P_E^{\text{erog}} = E \cdot i_E = 10 \text{ V} \cdot 0.878 \text{ A} = \underline{\underline{8.78 \text{ W}}}$$

Ans 1

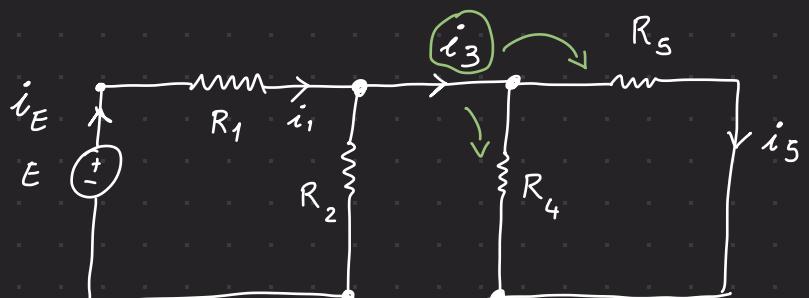
$$Q_2: P_{R_5}^{\text{ass}} = R_5 i_5^2$$

- Devo Trovare i_5 : Part Corr $\rightarrow i_3$; Part Corr $\rightarrow i_S$



$$R_B = R_4 // R_5 = \frac{R_4 R_5}{R_4 + R_5} = 1.428 \Omega \quad (A)$$

$$\text{PART}_{2-B} \quad i_3 = i_E \cdot \frac{R_2}{R_2 + R_B} = 0.513 \text{ A}$$



$$\text{PART}_{4-S} \rightarrow i_S = i_3 \cdot \frac{R_4}{R_4 + R_5}$$

$$= \underline{\underline{0.365 \text{ A}}} \quad \text{sugli es e } 0.19 \text{ A}$$

$$\Rightarrow P_{R_5}^{\text{ass}} = R_5 i_S^2 = 0.266 \text{ W} \quad ???$$

$$V = R \cdot i \quad (1)$$

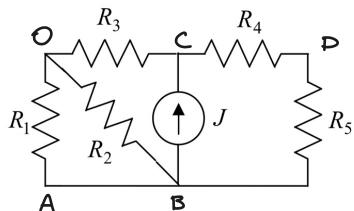
$$P_a = V \cdot i \quad (2)$$

Se non abbiamo V

$$\therefore P_a = R i^2 \quad (3)$$

Potenza assorbita da un resistore

ES. 1.10 - Calcolare la potenza erogata dal generatore J e quella assorbita dal resistore R_1 .



$$J = 5 \text{ A}$$

$$R_1 = R_4 = 5 \Omega \quad R_2 = 3 \Omega$$

$$R_3 = R_5 = 2 \Omega$$

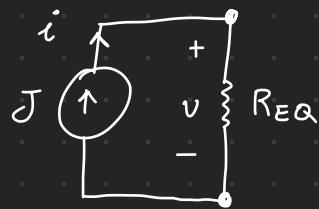
Risultato: $P_J^{erog} = 62.25 \text{ W}$, $P_{R_1} = 7.25 \text{ W}$.

$$R_{2S} = 2 + 5 = 7$$

$$R_x = R_{S3} + R_2 = 3.875$$

$$R_{S3} = 5/3 = 1.875$$

$$R_{EQ} = R_x \parallel R_{2S} = 2.49 \Omega$$

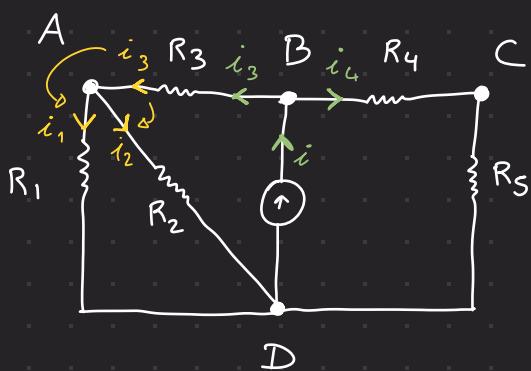


$$P_J^e = R \cdot i^2 = R_{EQ} \cdot J^2 = 2.49 \times 25 = \underline{\underline{62.36 \text{ W}}}$$

Ans 1

$$\bullet Q_2 \cdot P_{R_1}$$

$$\bullet i \text{ si ripartisce in } i_3 \text{ e } i_4$$



$$R_{45} = R_4 + R_5 = \underline{\underline{7 \Omega}}$$

$$R_{12} = R_1 \parallel R_2 = 1.875 \Omega$$

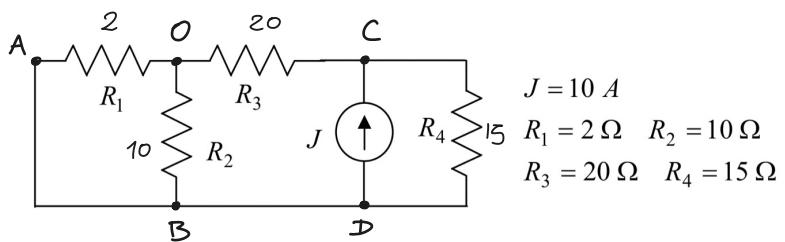
$$R_{3-12} = R_3 + R_{12} = \underline{\underline{3.875 \Omega}}$$

$$\Rightarrow i_3 = i \cdot \frac{R_{45}}{R_{3-12} + R_{45}} = 3.218 \text{ A}$$

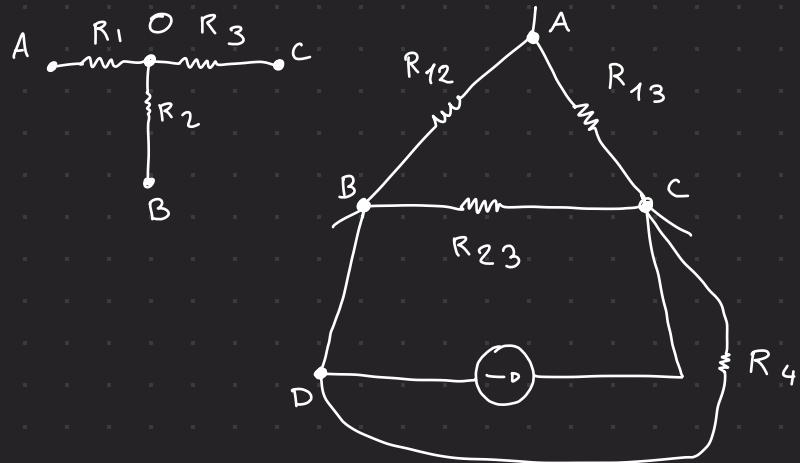
$$i_1 = i_3 \cdot \frac{R_2}{R_1 + R_2} = 1.206 \text{ A} \quad \rightarrow P_{R_1}^a = \underline{\underline{7.28 \text{ W}}}$$

Ans

ES. 1.11 - Calcolare la potenza erogata dal generatore e quella assorbita da ogni resistore.
Verificare la conservazione delle potenze.



Risultato: $P_J^{erog} = 0.886 \text{ kW}$, $P_{R_1} = 0.023 \text{ kW}$, $P_{R_2} = 0.004 \text{ kW}$, $P_{R_3} = 0.335 \text{ kW}$, $P_{R_4} = 0.524 \text{ kW}$.

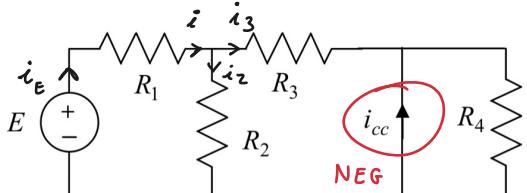


$$R_{12} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3} = 13 \Omega$$

$$R_{13} = 26 \Omega$$

$$R_{23} = 130 \Omega$$

ES. 1.12 - Calcolare la corrente i_{cc} che circola nel corto-circuito.



$$E = 220 \text{ V}$$

$$R_1 = 10 \Omega \quad R_2 = 0.1 \text{ k}\Omega$$

$$R_3 = 25 \Omega \quad R_4 = 2 \text{ k}\Omega$$

$$0.1 \times 10 = 100 \Omega$$

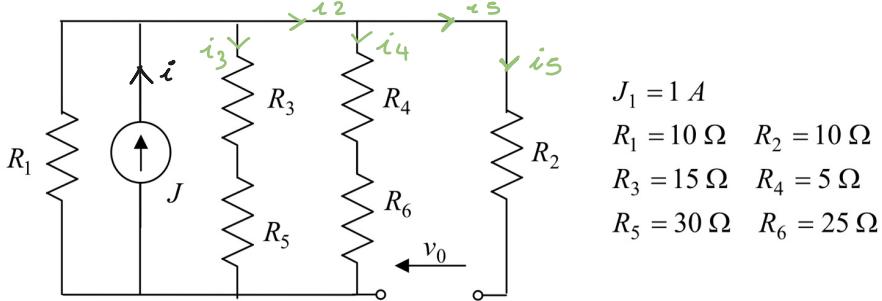
Corrente che circola nel corto-circuito

Risultato: $i_{cc} = -5.87 \text{ A}$.

$$\begin{aligned} \mathcal{V} = R \cdot i &= R_{23} = R_2 \parallel R_3 = 20 \Omega \\ R_{EQ} &= R_1 + R_{23} = 30 \Omega \end{aligned} \quad \left. \begin{array}{l} i = \frac{\mathcal{V}}{R} \\ i_E = \frac{E}{R_E} = 7.3 \text{ A} \end{array} \right\}$$

$$\Rightarrow i_3 = i_E \cdot \frac{R_2}{R_2 + R_3} = 5.86 \text{ A} \Rightarrow i_{cc} \approx -5.87 \text{ A}$$

ES. 1.13 - Calcolare la tensione v_0 sul circuito aperto in figura.



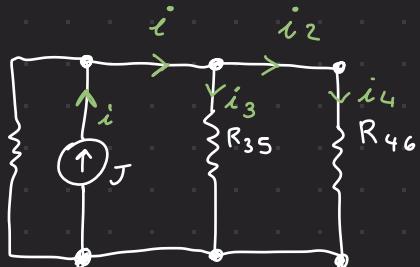
$$\begin{array}{ll} J_1 = 1 \text{ A} & \\ R_1 = 10 \Omega & R_2 = 10 \Omega \\ R_3 = 15 \Omega & R_4 = 5 \Omega \\ R_5 = 30 \Omega & R_6 = 25 \Omega \end{array}$$

Tensione sul circuito aperto

Risultato: $v_0 = -6.43 \text{ V}$.

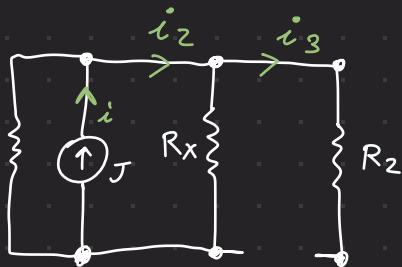
$$\begin{array}{l} R_x = R_3 + R_5 \\ R_y = R_4 + R_6 \end{array} \quad \left. \begin{array}{l} R_A = R_x \parallel R_y = 18 \Omega \\ R_{EQ} = R_A \parallel R_1 = 6.42 \Omega \end{array} \right\}$$

$$\mathcal{V} = R \cdot i \Rightarrow \mathcal{V}_1 = R_{EQ} \cdot J = 6.429 \text{ V}$$



$$i_2 = i \cdot \frac{R_{46}}{R_{46} + R_{35}} = 2.57 \text{ A}$$

BoH

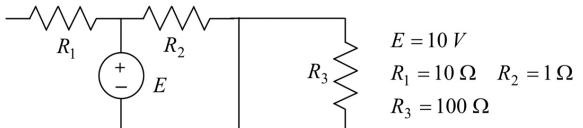


$$R_x = (R_3 + R_5) \parallel (R_4 + R_6) = 18 \Omega$$

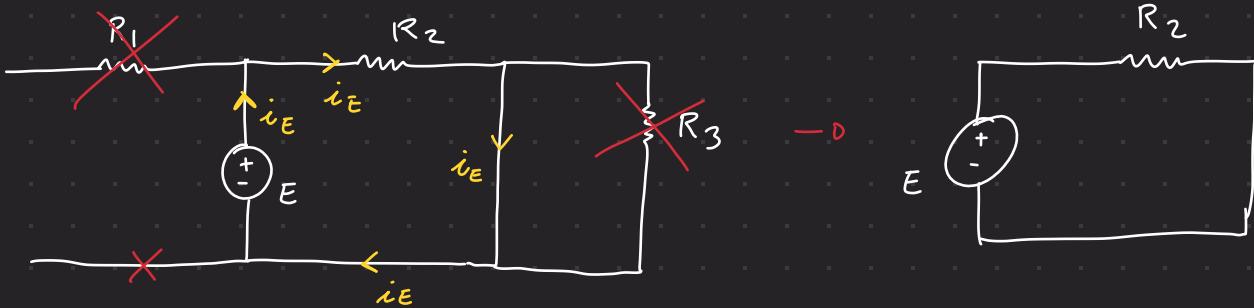
$$\Rightarrow i_3 = i_2 \cdot \frac{R_x}{R_x + R_2} = 1.65 \text{ A}$$

$$\mathcal{V} = R \cdot i = 0$$

ES. 1.14 - Valutare la potenza assorbita dai resistori della rete in figura.



Risultato: $P_{R_1} = P_{R_3} = 0$, $P_{R_2} = 100 \text{ W}$.



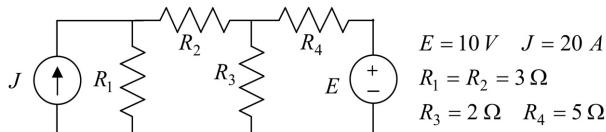
$$= 0 \quad V = R \cdot i \Rightarrow i_E = \frac{E}{R_2} = \underline{10 \text{ A}}$$

$$P_{R_2}^a = R_2 \cdot i_E^2 = \underline{100 \text{ W}} \quad \text{Ans 1}$$

$$P_{R_1}^a = P_{R_2}^a = R_1 \cdot 0 = R_2 \cdot 0 = \underline{0} \quad \text{Ans 2}$$

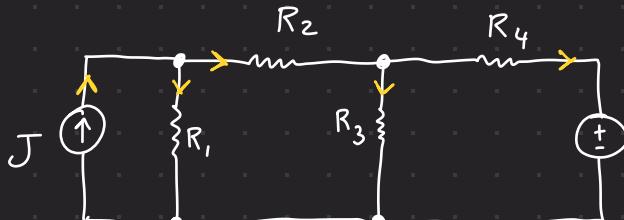
2. Sovrapposizione degli effetti

ES. 2.1 - Calcolare la potenza totale erogata dai generatori.

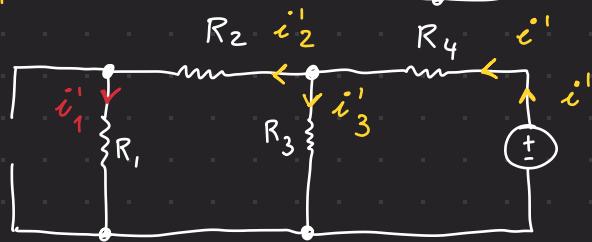


$$Q_1: P_E^e = E \cdot i_E$$

$$P_J^e = V_J \cdot J$$



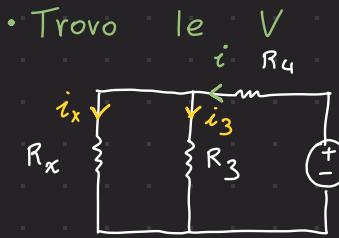
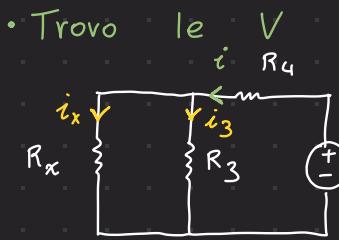
C'



• Trovo \mathcal{I}

$$R_{Ea} = R_4 + [(R_2 + R_1) // R_3] = 6.5 \Omega$$

$$V = R \cdot I \rightarrow i = \frac{E}{R_{Ea}} = 1.54 \text{ A}$$



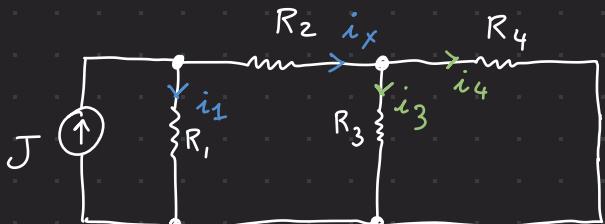
$$R_X = R_1 + R_2$$

Partitore di corrente

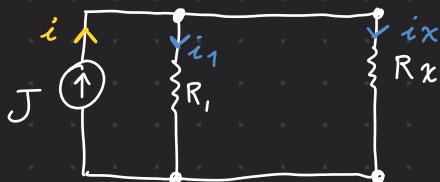
$$\rightarrow i_X = i \cdot \frac{R_3}{R_X + R_3}$$

$$\rightarrow i_X = 0.385 \text{ A} \quad \rightarrow \text{Ohm: } V = R \cdot i \rightarrow$$

$$\rightarrow V_X'' = i_X \cdot R_1 = 1.155 \text{ V}$$



C'''



• Trovo V

$$R_{Ea} = R_1 // [(R_3 // R_4) + R_2] = 1.78 \Omega$$

$$\rightarrow V = R_{Ea} \cdot J = 35.8 \text{ V}$$

$$R_X = [(R_3 // R_4)] + R_2 = 4.43 \Omega$$

$$\rightarrow i_X = i \cdot \frac{R_1}{R_X + R_1} = 8.07 \text{ A}$$

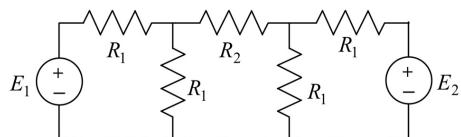
$$\rightarrow i_4 = i_X \cdot \frac{R_3}{R_3 + R_4} = 2.31 \text{ A}$$

$$\mathcal{I} = \mathcal{I}' + \mathcal{I}'' = 1.54 \text{ A} - 2.31 \text{ A} = -0.77 \text{ A}$$

$$\rightarrow P_E^e = E \cdot i_E = -7.70 \text{ W}$$

$$P_J^e = J \cdot V = 739 \text{ W}$$

ES. 2.2 - Calcolare la potenza totale erogata dai generatori.

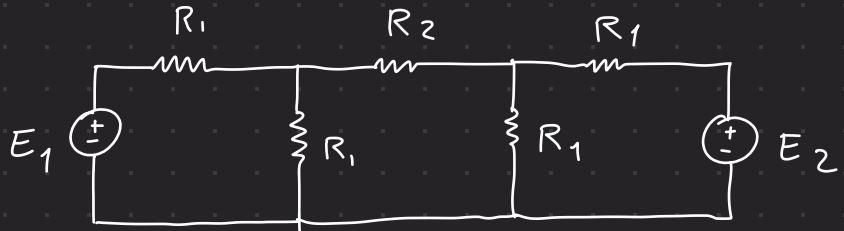


$$E_1 = 10 \text{ V}, E_2 = 20 \text{ V}$$

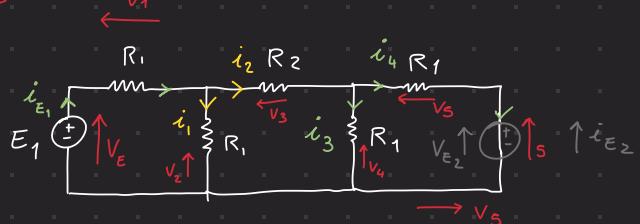
$$R_1 = 2 \Omega, R_2 = 1 \Omega$$

Risultato: $P_{E_1}^{erog} = 16.67 \text{ W}$, $P_{E_2}^{erog} = 0.12 \text{ kW}$.

$$\Rightarrow V_1 = \frac{R_1 \cdot V}{R_1 + R_2}$$



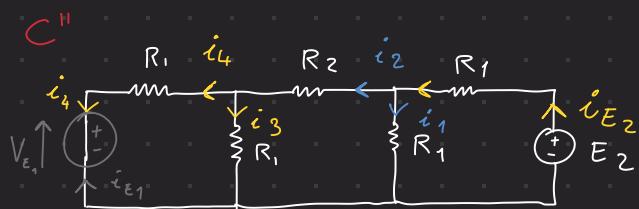
C'



$$\cdot I_{E_1}^I = \frac{E_1}{R_{eq}} = \left\{ \left[\frac{E_1}{(R_1 \parallel R_1) + R_2} \right] \parallel R_1 \right\} + R_1$$

$$= 3.3 \text{ A} \quad i_{E_1}^I$$

$$\cdot i_{E_2}^I = i_{E_1}^I \cdot \frac{R_1}{R_1 + R_2} = 1.6 \text{ A} \rightarrow i_{E_2}^I = -i_{E_1}^I = i_2 \cdot \frac{R_1}{R_1 + R_2} = -0.83 \text{ A} = 833 \text{ mA}$$



$$\cdot i_{E_2}^{\prime \prime} = \frac{E_2}{R_{eq}} = \left\{ \left[\frac{E_2}{(R_1 \parallel R_1) + R_2} \right] \parallel R_1 \right\} + R_1$$

$$= 6.67 \text{ A} \quad i_{E_2}^{\prime \prime}$$

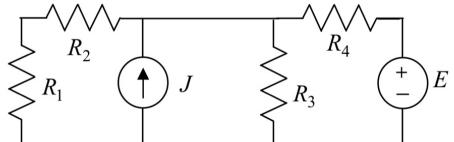
$$i_2 = i_{E_1} \cdot \frac{R_1}{R_1 + R_2} = 3.33 \text{ A}$$

$$i_{E_1}^{\prime \prime} = -i_4^{\prime \prime} = -i_2 \cdot \frac{R_1}{R_1 + R_2} = -1.66 \text{ A} \quad i_{E_1}^{\prime \prime}$$

$$\Rightarrow P_{E_1}^e = E_1 \cdot (i_{E_1}^I + i_{E_1}^{\prime \prime}) = 16.67 \text{ W} \quad Ans 1$$

$$P_{E_2}^e = E_2 \cdot (i_{E_2}^I + i_{E_2}^{\prime \prime}) = 116.8 = 0.12 \text{ kW} \quad Ans 2$$

ES. 2.3 - Calcolare la potenza totale erogata dai generatori.

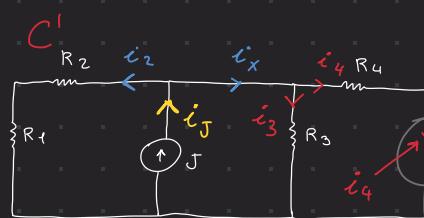
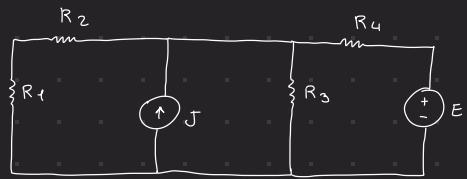


$$E = 50 \text{ V} \quad J = 20 \text{ A}$$

$$R_1 = 1 \Omega \quad R_2 = 5 \Omega$$

$$R_3 = R_4 = 10 \Omega$$

Risultato: $P_E^{erog} = -0.09 \text{ kW}$, $P_J^{erog} = 1.36 \text{ kW}$.



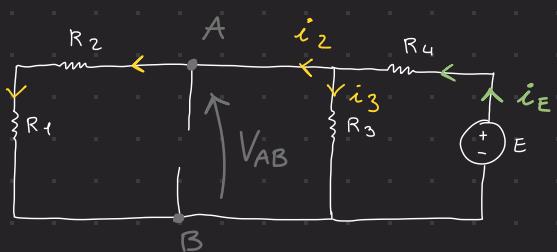
$$\bullet \quad V_J : \quad \text{Req} = \frac{(R_1 + R_2) \parallel (R_3 \parallel R_4)}{= 2.72 \Omega}$$

$$\Rightarrow V_J = \frac{J}{\text{Req}} = 54.54 \text{ V}$$

• i_4 :

$$i_x = i_J \cdot \frac{R_{12}}{R_{12} + R_x} = 10.91 \text{ A} \quad i_4 = i_x \cdot \frac{R_3}{R_3 + R_4} = \underline{5.45 \text{ A}} = -i_E$$

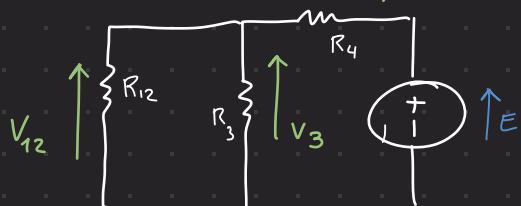
C''



• i_E^{\parallel} :

$$i_E^{\parallel} = -i_E^{\parallel} = \frac{E}{\text{Req}} = \frac{E}{[(R_1 + R_2) \parallel R_3] + R_4} = \underline{3.63 \text{ A}}$$

$$\bullet \quad V_{AB} = V_{R_{12}} \quad \Rightarrow \quad R_{12} = R_1 + R_2 = 6 \Omega$$



$$V_4 + V_3 = E \quad \text{ma} \quad V_4 = R_4 \cdot i_4 = 36.4 \text{ V}$$

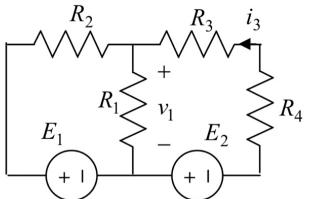
$$\Rightarrow V_3 = E - 36.4 = 13.64 \text{ V} \quad i_4 = i_4$$

$$V_{12} - V_3 = 0 \quad \Rightarrow \quad V_{12} = +13.64$$

$$\Rightarrow P_J^e = J \cdot (V' + V'') = 1361 \text{ W} = \underline{1.36 \text{ kW}}$$

$$\Rightarrow P_E^e = E \cdot (i' + i'') = -90.91 \text{ W} = \underline{-0.091 \text{ kW}}$$

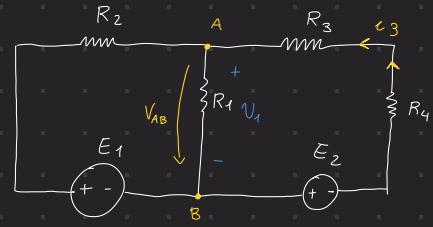
ES. 2.4 - Calcolare la tensione v_1 e la corrente i_3 .



$$R_1 = R_2 = R_3 = R_4 = 2 \Omega$$

$$E_1 = 5 \text{ V}, E_2 = 2 \text{ V}$$

Risultato: $v_1 = 1.60 \text{ V}$, $i_3 = -0.90 \text{ A}$.



$$V_2 = E_1 \cdot \frac{R_2}{R_2 + R_x}$$

$$\text{con } R_x = (R_3 + R_4) \parallel R_1 = 1.33 \Omega$$

$$\Rightarrow V_2 = 3 \text{ V}$$

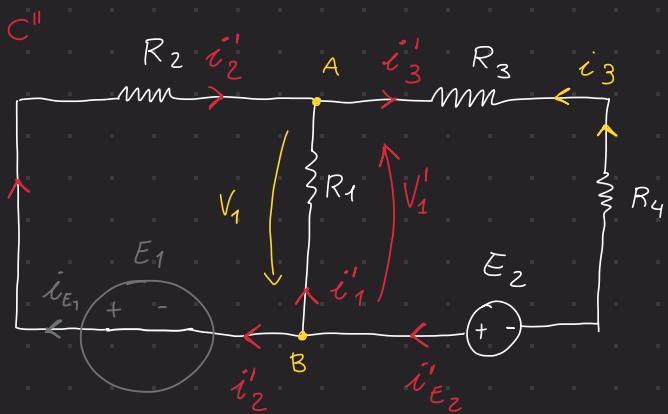
$$V_2 + V_{AB} = E_1 = 0 \quad V_{AB} = E_1 - V_2 = 2 \text{ V}$$

$$i_3 = -i'_3 = i_{E_1} \cdot \frac{R_2}{R_2 + R_L}$$

$$= -0.5 \text{ A}$$

$$\cdot i_{E_1} = \frac{E_1}{R_{\text{eq}}} = \left[(R_3 + R_4) \parallel R_1 \right] + R_2 \cdot E_1 = 1.5 \text{ A}$$

$$\cdot R_L = (R_3 + R_4) = 4 \Omega$$



$$\cdot -i_{E_2} = \frac{E_2}{R_{\text{eq}}} = -0.4 \text{ A} = i_3$$

$$V_{12} + V_3 + V_4 = E_2$$

$$\Rightarrow V_{12} = E_2 - [i_{E_2} \cdot (R_3 + R_4)] = 0.4 \text{ V}$$

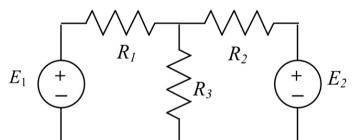
$$V_1'' = V_2'' = V_{12}''$$

$$\text{ma } V_1 = -V_1''$$

$$\Rightarrow V_1 = (V_1' - V_1'') = 1.6 \text{ V} \quad \text{Ans 1}$$

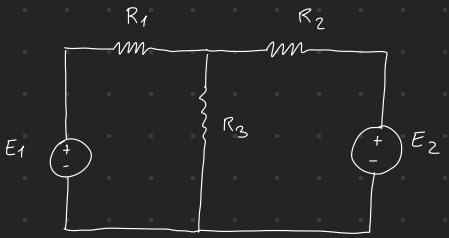
$$i_3 = (-i'_3 - i''_3) = -0.9 \text{ A} \quad \text{Ans 2}$$

ES. 2.6 - Determinare la potenza erogata dal generatore E_1 .

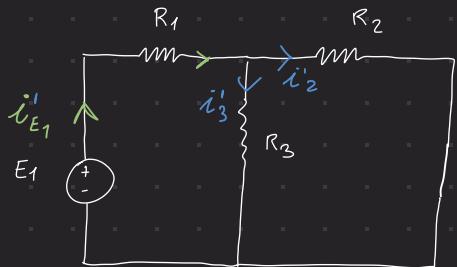


$$E_1 = 5 \text{ V}, E_2 = 12 \text{ V}, R_1 = 3.5 \Omega, R_2 = 2.3 \Omega, R_3 = 3.2 \Omega.$$

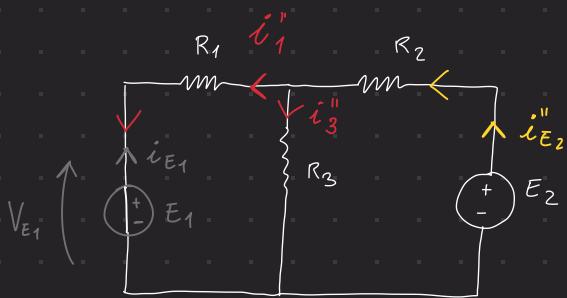
Risultato: $P_{E_1}^{erog} = -2.05 \text{ W}$.



C'



$$\vec{i}_{E_1}' = \frac{E_1}{R_{\text{req}}} = 1.03 \text{ A}$$



$$\vec{i}_{E_2}'' = \frac{E_2}{R_{\text{req}}} = 3.02 \text{ A}$$

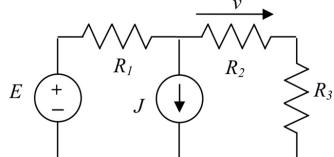
$$\therefore \vec{i}_{E_1}'' = -\vec{i}_1'' = -i_{E_2}'' \cdot \frac{R_3}{R_3 + R_1} = -1.44 \text{ A}$$

$$\therefore \vec{i}_{E_1} = \vec{i}' + \vec{i}'' = 1.03 - 1.44 = -0.413 \text{ A}$$

$$\therefore P_{E_1}^e = E_1 \cdot i_{E_1} = -2.065 \text{ W} \quad \text{Ans}$$

Time 11'

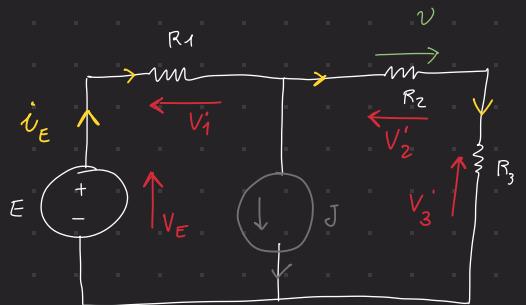
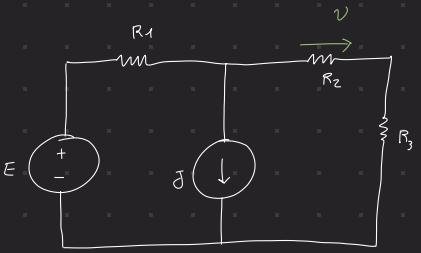
ES. 2.7 - Utilizzando il principio di sovrapposizione degli effetti, determinare la tensione v .



$$E = 5 \text{ V}, J = 2 \text{ mA}$$

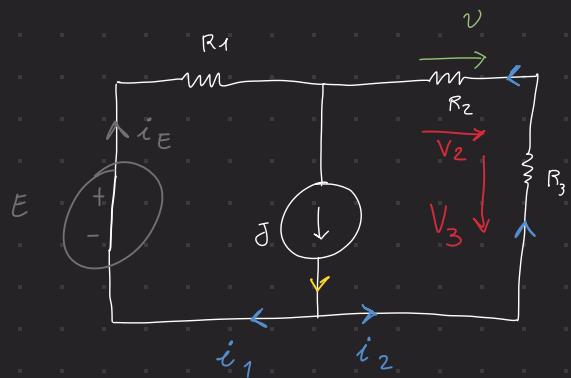
$$R_1 = 3 \text{ k}\Omega, R_2 = 2.4 \text{ k}\Omega, R_3 = 3.2 \text{ k}\Omega$$

Risultato: $v = 0.28 \text{ V}$.



$$\dot{i}_E = \frac{E}{R_{\text{req}}} = 0.58 \text{ mA} = 581 \mu\text{A}$$

$$V_2' = -V_2 = -R_2 \cdot \dot{i}_E = -1.39 \text{ V}$$



$$\dot{i}_2'' = J \cdot \frac{R_1}{R_1 + R_L} = 2 \times 10^{-3} \cdot \frac{3 \times 10^3}{(3 + 5.6) \times 10^3} =$$

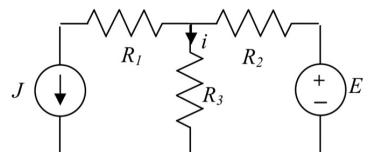
$$= 0.697 \times 10^{-3} = 697 \mu\text{A}$$

$$= V_2'' = R_2 \cdot \dot{i}_2 = 1.67 \text{ V}$$

$$= V_2 = V_2' + V_2'' = 0.28 \text{ V}$$

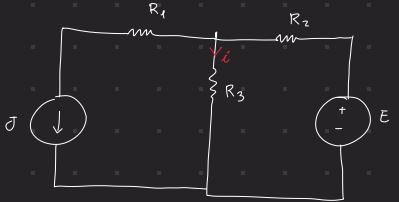
Time 22
(ho fatto casino)

ES. 2.8 - Utilizzando il principio di sovrapposizione degli effetti, determinare la corrente i e la potenza assorbita da R_3



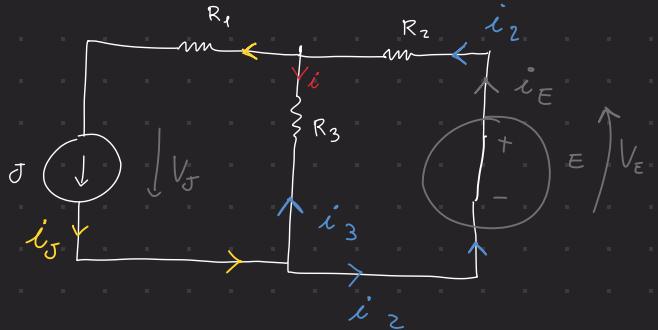
$$E = 10 \text{ V}, J = 1 \text{ mA}$$

$$R_1 = 3.2 \text{ k}\Omega, R_2 = 2.2 \text{ k}\Omega, R_3 = 3.5 \text{ k}\Omega$$



Risultato: $i = 1.37 \text{ mA}$, $P = 6.57 \text{ mW}$.

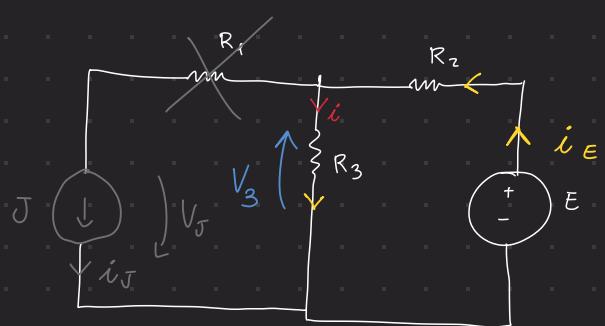
C^I



$$\dot{i}_3' = -\dot{i}_3'' = -J \cdot \frac{R_2}{R_2 + R_3} = -0.38 \times 10^{-3} \text{ A}$$

$$= -386 \mu\text{A}$$

C^{II}

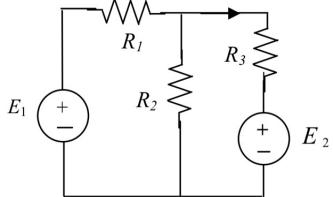


$$\dot{i}_3'' = \dot{i}_E'' = \frac{E}{R_{\text{eq}}} = 1.75 \text{ mA}$$

$$\Rightarrow i = 1.75 \text{ mA} - 0.38 \text{ mA} = 1.37 \text{ mA} \quad \text{Ans}$$

$$\Rightarrow P_{R_3}^a = -P_{R_3}^e = R_3 \cdot i^2 = 6.57 \text{ mW}$$

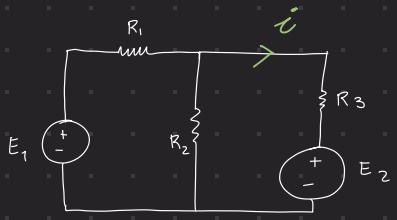
ES. 2.9 - Valutare la corrente i e la potenza erogata dal generatore E_1 .



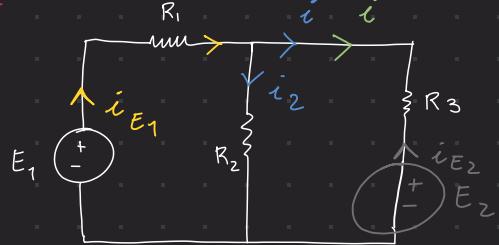
$$E_1 = 10 \text{ V}, \quad E_2 = 20 \text{ V}$$

$$R_1 = 5 \Omega, \quad R_2 = 20 \Omega, \quad R_3 = 10 \Omega$$

Risultato: $i = -0.86 \text{ A}$, $P_{E_1}^{\text{erog}} = -2.86 \text{ W}$.



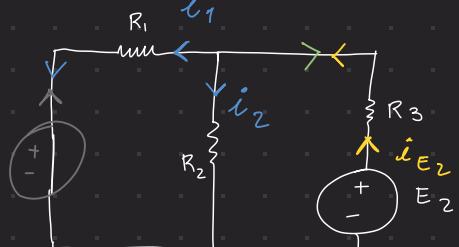
C'



$$\dot{i}_{E_1} = \frac{E_1}{R_{\text{req}}} = 0.857 \text{ A}$$

$$\Rightarrow i' = \dot{i}_{E_1} \cdot \frac{R_2}{R_2 + R_3} = 0.57 \text{ A}$$

C''



$$\dot{i}_{E_2}'' = \frac{E_2}{R_{\text{req}}} = 1.43 \text{ A} = -i$$

$$\Rightarrow i'' = -1.43 \text{ A}$$

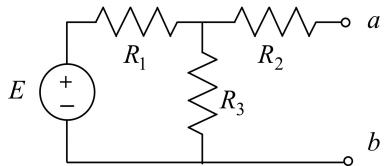
$$\dot{i}_{E_1}'' = -\dot{i}_1'' = i \cdot \frac{R_2}{R_2 + R_1} = -1.144 \text{ A}$$

$$\Rightarrow i = -0.86 \text{ A}$$

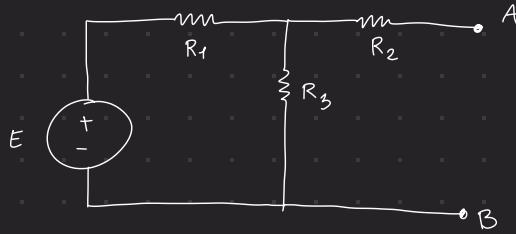
$$\Rightarrow \dot{i}_{E_1} = 1.81 \text{ A} \quad \Rightarrow P_{E_1}^{\text{erog}} = E_1 \cdot \dot{i}_{E_1} = 0.287 \cdot 10 = -2.87 \text{ W}$$

3. Generatori equivalenti di Thevenin e norton

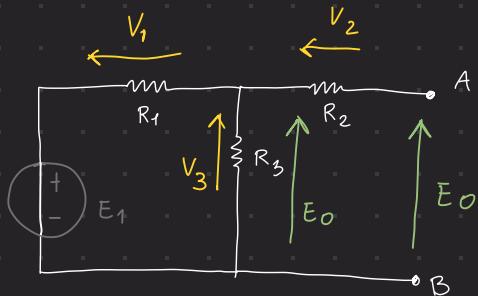
ES. 3.1 - Calcolare l'equivalente di Thévenin visto ai capi dei morsetti a-b.



Equivalente di
Thevenin



Spengo l'unico
generatore



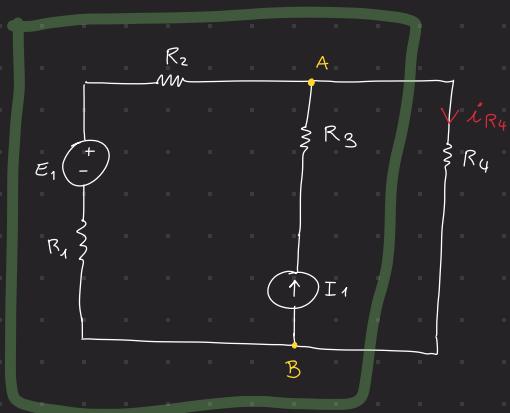
La Tensione a Vuoto E_o si ottiene con V_{AB} , siccome non c'è corrente $V_2 = V_3 = E_o$

\Rightarrow Part Tens \rightarrow

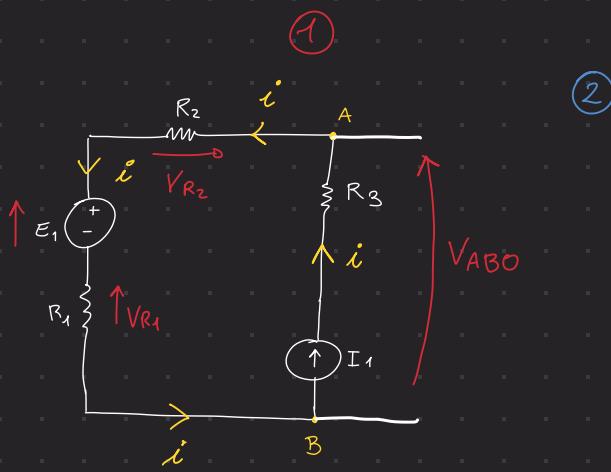
$$E_o = E \cdot \frac{R_3}{R_3 + R_1}$$

L'esercizio ci chiede di trovare I_{R4} , quindi lo scopo del gioco è di semplificare tutta la parte del circuito cerchiata in verde con un **generatore ideale ed una resistenza in parallelo (Norton)**.

La resistenza in parallelo si calcola andando a **spegner tutti i generatori** e calcolando così la resistenza equivalente.



1. Isoliamo il circuito cerchiato
2. Calcoliamo la **tensione a vuoto**
3. Resistenza equivalente —> sarà la resistenza in || del nuovo gen
4. Calcolo la corrente del nuovo generatore In



(2) $\rightarrow V_{AB0} = | V_{R1} + E_1 + V_{R2} | = I_1 (R_1 + R_2) + E_1 = 105 \text{ V}$

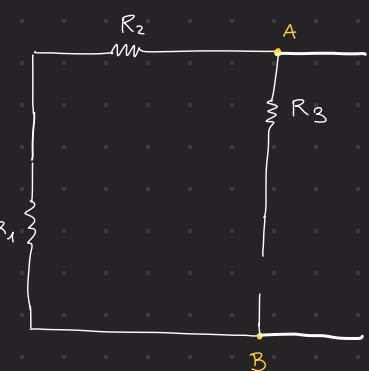
(3) Resistenza in || del nuovo generatore

$$R_n = R_{eq} = R_1 + R_2 = 150 \Omega$$

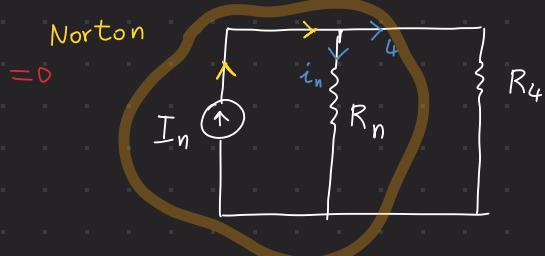
(4) Corrente del nuovo generatore $I_n = \frac{V_{AB0}}{R_{eq}} = 0.7 \text{ A}$

Ans

$$I_{R4} = I_n \cdot \frac{R_n}{R_n + R_4} = 0.62 \text{ A}$$



NUOVO CIRCUITO

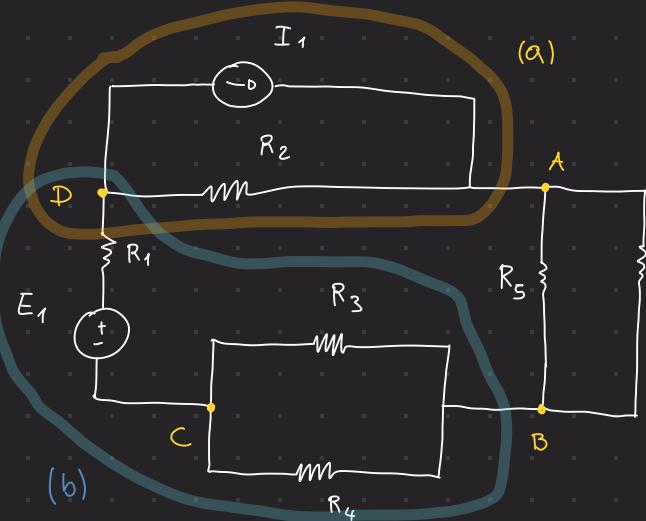


Esercizio

Video

Elisabetta

Thevenin



Q: $I R_6 \rightarrow$ Serve V_{AB}

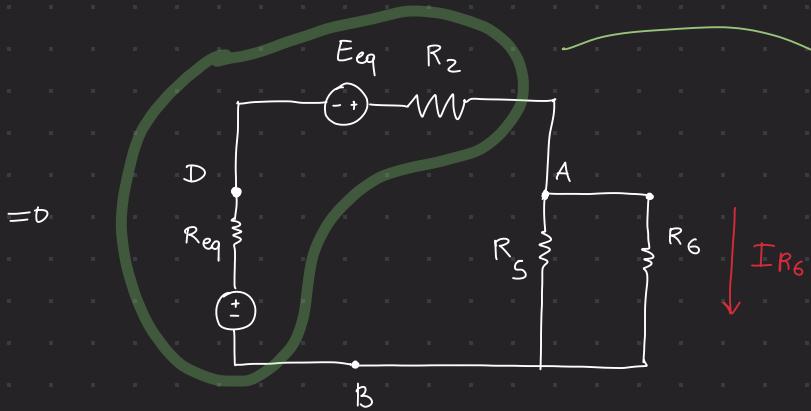
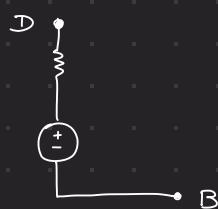
$I R_6$

(a) : gen curr \rightarrow gen tens

$$V = R \cdot I = 0 \quad E_{eq} = R_2 \cdot I_1 = 4.8V$$



$$(b) \quad R_{eq} = (R_3 \parallel R_4) + R_1 = 40\Omega$$



Quando ho due generatori in serie questi si possono sempre sommare. Quando hanno lo stesso "verso" allora li sommiamo, quando invece sono discordi uno dei due verrà sottratto.

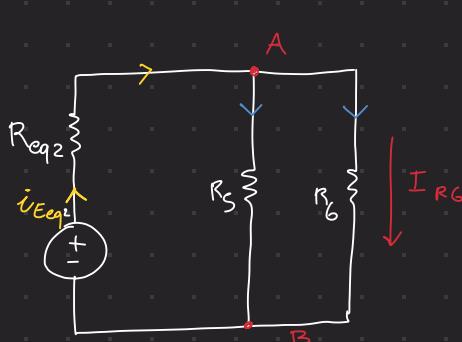
Le due resistenze vengono messe in serie e ne viene calcolata la somma.

$$28.8V$$

$$\dot{i}_{E_{eq}} = \frac{E_{eq2}}{R_{eq}} = \frac{E_1 + E_{eq}}{R_{eq} + (R_S \parallel R_6)} = 0.373A$$

$$77.14\Omega$$

$$\Rightarrow i_6 = \dot{i}_{E_{eq}} \cdot \frac{R_S}{R_S + R_6} = 0.16A \quad \text{Ans}$$



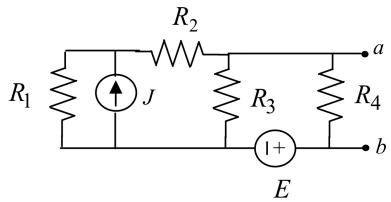
$i_{E_{eq}}$

R_{req2}

Processo Alternativo: Diverso dal punto (x) in poi

$$V_{AB} = E_{eq2} \cdot \frac{R_{S6}}{R_{S6} + R_{req2}} = 6.4V \quad \Rightarrow \quad I_6 = \frac{V_{AB}}{R_6} = 0.16A \quad \text{Ans}$$

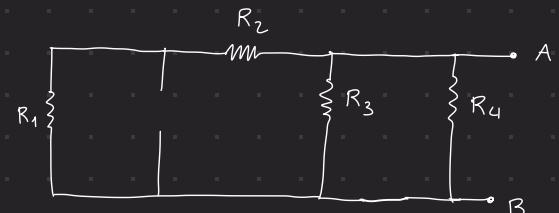
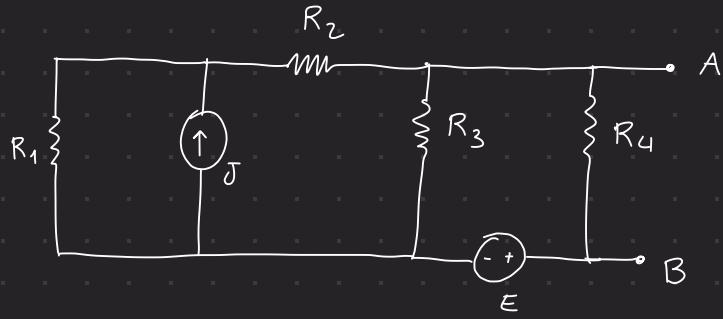
ES. 3.2 - Calcolare l'equivalente di Norton visto ai capi dei morsetti a-b.



$$J = 20 \text{ A} \quad E = 10 \text{ V}$$

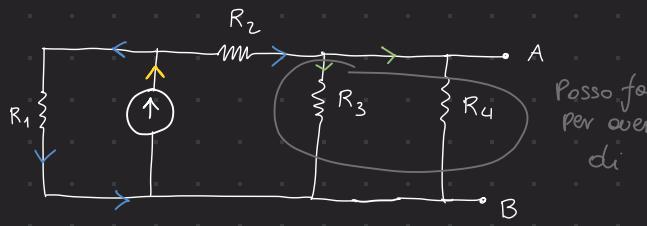
$$R_1 = R_2 = 2 \Omega$$

$$R_3 = R_4 = 4 \Omega$$

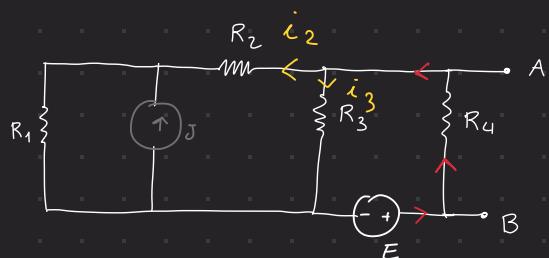


$$\bullet \quad R_{\text{eq}} = [(R_1 + R_2) // R_3] // R_4 = 1.3 \Omega$$

C'



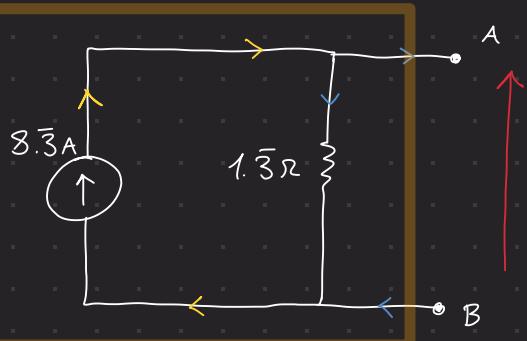
$$\Rightarrow I_{\text{cc}}' = J \cdot \frac{R_1}{R_1 + R_2} = 10 \text{ A}$$



$$I_{\text{cc}}'' = -i_E = \frac{E}{R_{\text{eq}}} = \frac{E}{[(R_1 + R_2) // R_3]} = 5 \text{ A}$$

R₄ in Serie NON viene inclusa

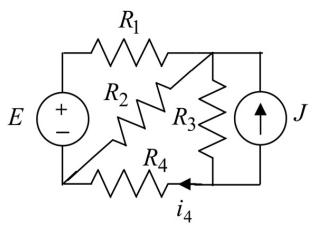
$$\Rightarrow I_{\text{cc}} = I_{\text{cc}}' + I_{\text{cc}}'' = 5 \text{ A} \quad \text{Ans}$$



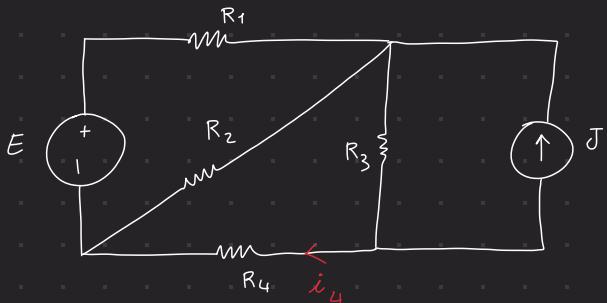
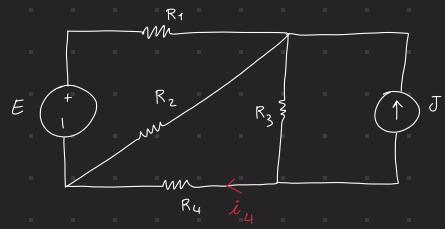
$$\Rightarrow V_{AB} = 5 \text{ A} \cdot 1.3 \Omega = 6.6 \text{ V}$$

Circuito equivalente di norton

ES. 3.3 - Utilizzando l'equivalente di Norton calcolare la corrente che circola in R_4 .

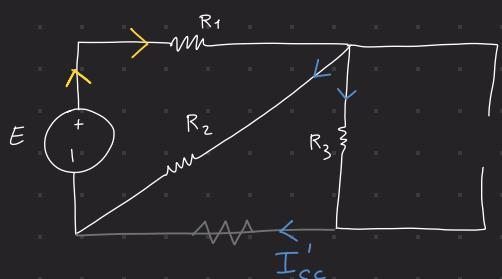


$$\begin{aligned} E &= 54 \text{ V} \\ J &= 10 \text{ A} \\ R_1 &= 6 \Omega \quad R_2 = R_3 = 4 \Omega \\ R_4 &= 12 \Omega \end{aligned}$$



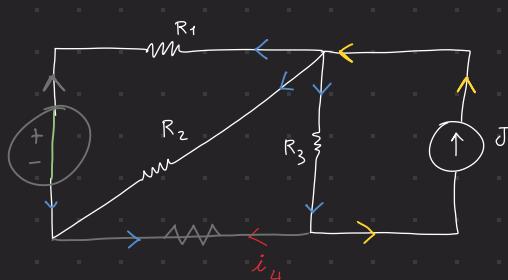
$$(1) \text{ Req} : (R_1 \parallel R_2) + R_3 = 6.4 \Omega$$

C'



$$i_E = \frac{E}{\text{Req}} = 6.75$$

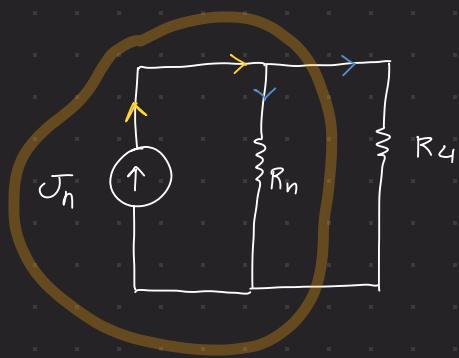
$$\Rightarrow I_{cc'} = i_E \cdot \frac{R_2}{R_2 + R_3} = 3.375 \text{ A}$$



$$I_{cc''} = -i_{12} = J \cdot \frac{R_3}{R_3 + R_{12}} = -6.25 \text{ A}$$

$$\Rightarrow I_N = I' + I'' = -2.875 \text{ A}$$

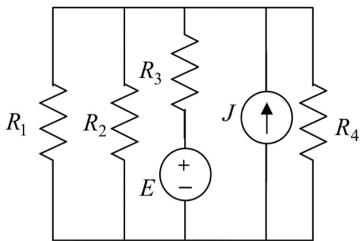
$$R_N = 6.4 \Omega$$



NORTON

$$i_4 = J_n \cdot \frac{R_n}{R_n + R_4} = -1 \text{ A} \quad \text{Ans}$$

ES. 3.4 - Utilizzando il teorema di Thévenin calcolare la potenza assorbita dal resistore R_2 .

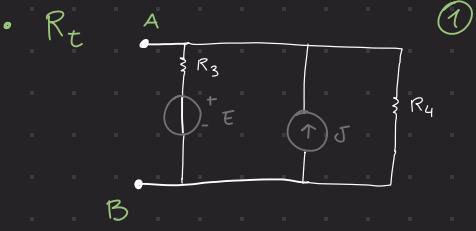
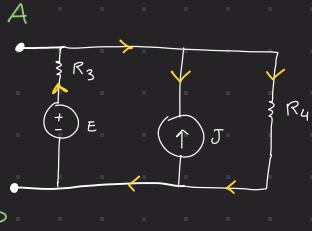
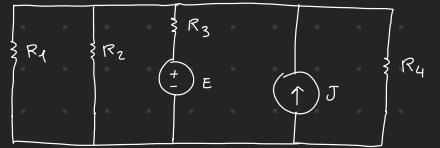


$$E = 1 \text{ V} \quad J = 2 \text{ mA}$$

$$R_1 = R_2 = 1 \text{ k}\Omega \quad R_3 = 2 \text{ k}\Omega$$

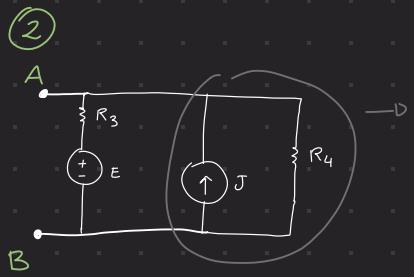
$$R_4 = 5 \text{ k}\Omega$$

Risultato: $P_{R_2} = 0.85 \text{ mW}$.



$$R_{th} = R_{AB} = R_3 // R_4 = 14.2857 \text{ }\Omega$$

$$R_{th}$$

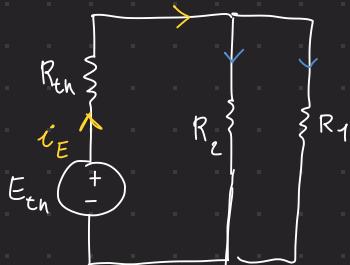


$$R_4 \quad \rightarrow \quad E_2 = R_4 \cdot J = 10 \text{ V}$$

$$-E + V_3 + V_4 + E_2 = 0 \rightarrow i_E (R_3 + R_4) = E - E_2$$

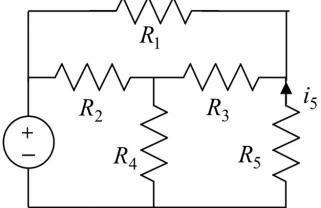
$$-E = \frac{E - E_2}{R_3 + R_4} = -1.28 \text{ mA}$$

$$V_{AB} = 3.6 \text{ V} \quad \rightarrow$$



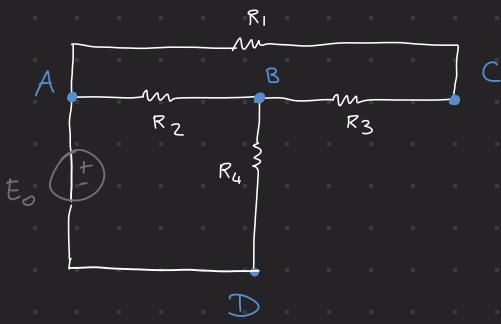
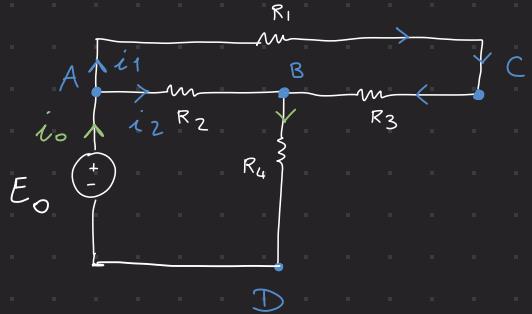
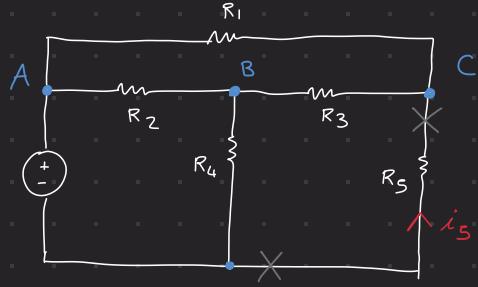
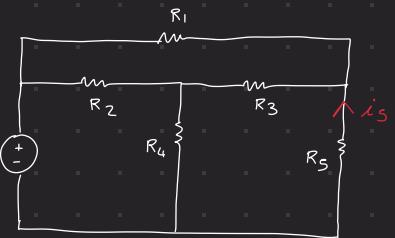
$$\rightarrow i_2 = i_E \cdot \frac{R_1}{R_1 + R_2} = -1.28 \cdot \frac{1\text{k}}{1\text{k} + 1\text{k}} = \frac{1.28}{2} = 0.64 \text{ mV}$$

ES. 3.5 - Utilizzando il teorema di Thévenin calcolare la corrente i_5 .



$$\begin{aligned} E &= 12 \text{ V} \\ R_1 &= R_3 = 0.2 \text{ k}\Omega \\ R_2 &= 0.6 \text{ k}\Omega \\ R_4 &= R_5 = 0.4 \text{ k}\Omega \end{aligned}$$

Risultato: $i_5 = -18 \text{ mA}$.



$A \equiv D$

$$R_{thDC} = [(R_2 \parallel R_4) + R_3] \parallel R_1 = 137.5 \Omega$$

R_{th}

$$R_{eq} = [(R_1 + R_3) \parallel R_2] + R_4 = 640 \Omega$$

$$\therefore i_o = \frac{E_0}{R_{eq}} = 0.01875 A = 18.75 \text{ mA}$$

$$LKC_A: -i_o + i_2 + i_1 = 0 \quad \therefore i_2 = i_o - i_1 = 7.5 \text{ mA}$$

$$i_1 = i_o \cdot \frac{R_2}{R_2 + (R_1 + R_3)} = 11.25 \text{ mA}$$

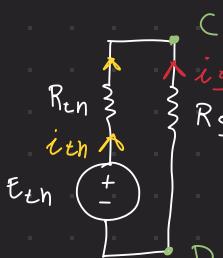
$$LKC_B: -i_2 + i_4 - i_3 = 0 \quad \therefore i_4 = i_1 + i_2 = 18.75 \text{ mA}$$

$$i_3 = i_1$$

$$\therefore LKT_{M_1}: -E_0 + V_2 - V_3 - V_5 = 0 \quad \therefore V_5 = V_2 - V_3 - E_0$$

$$\therefore V_5 = (i_2 R_2) - (i_1 R_3) - E_0 = -9.75 \text{ V}$$

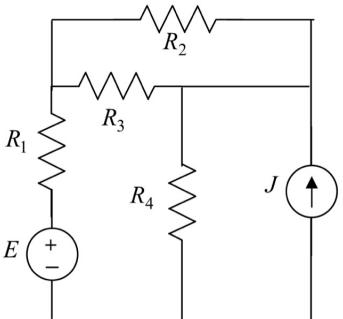
E_{thDC}



$$\therefore i_{th} = \frac{E_{th}}{R_{th} + R_5} = -0.018 = -18.34 \text{ mA}$$

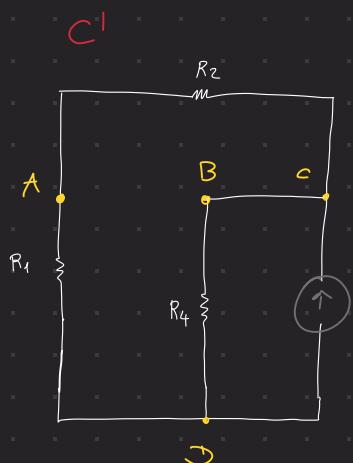
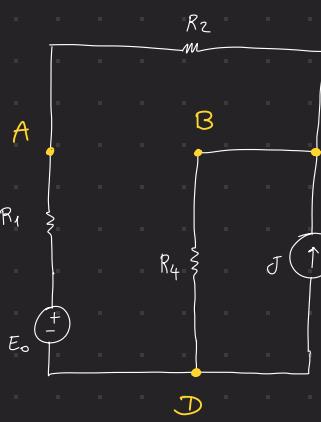
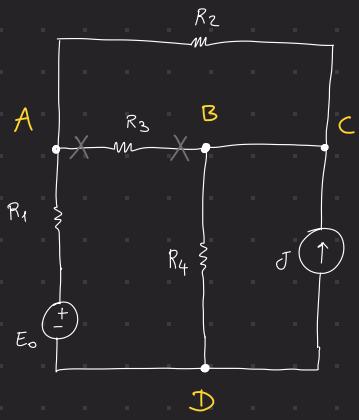
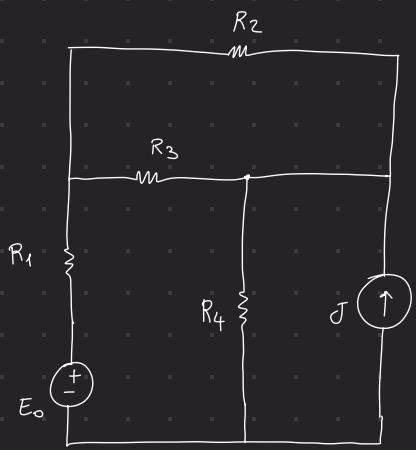
Ans

ES. 3.6 - Utilizzando il teorema di Norton calcolare la potenza assorbita dal resistore R_3 .



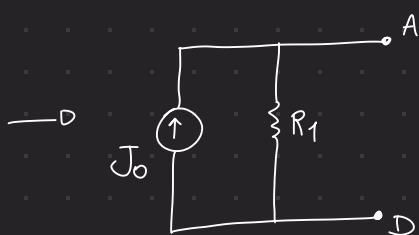
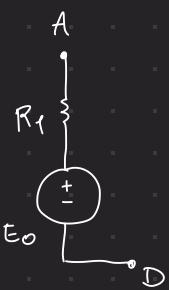
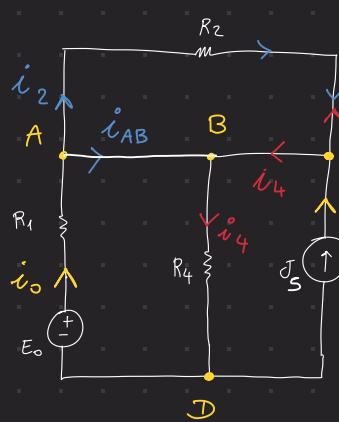
$$\begin{aligned} E &= 5 \text{ V} \\ J &= 1 \mu\text{A} \\ R_1 &= R_3 = 2 \text{ M}\Omega \\ R_2 &= 800 \text{ k}\Omega \\ R_4 &= 300 \text{ k}\Omega \end{aligned}$$

Risultato: $P_{R_3} = 0.43 \mu\text{W}$.

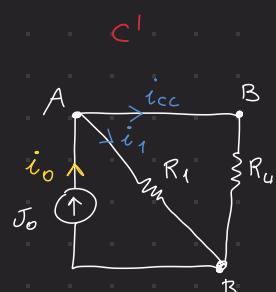
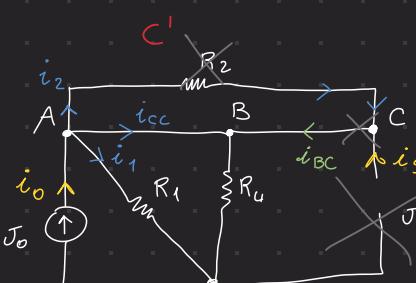
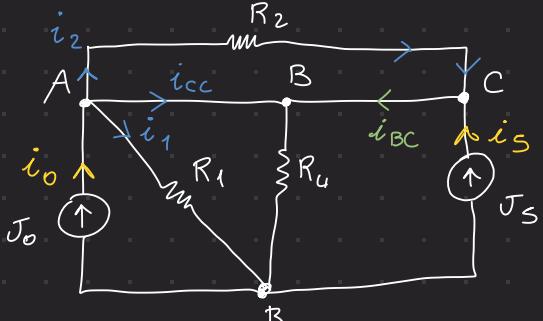


$$\begin{aligned} R_{NAB} &= \\ &= (R_1 + R_4) \parallel R_2 \\ &= 593.54 \text{ k}\Omega \end{aligned}$$

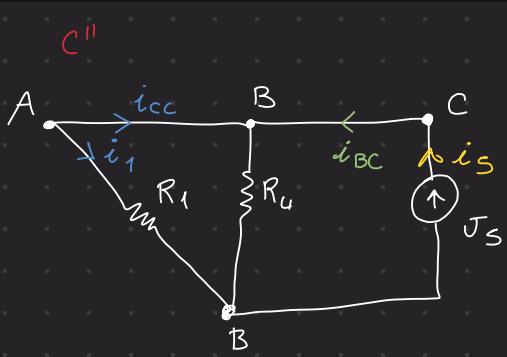
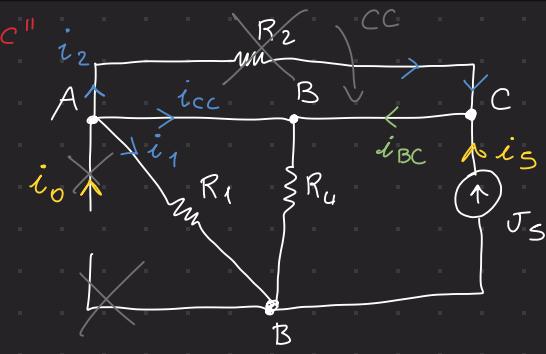
C''



$$\begin{aligned} \text{con } J_o &= \frac{E_o}{R_1} - \\ i_o &= 2.5 \mu\text{A} \end{aligned}$$



$$i_{cc}' = J_o \cdot \frac{R_1}{R_1 + R_4} = 2.174 \mu\text{A}$$



$$i_{cc}''' = -i_{cc} = -J_S \cdot \frac{R_4}{R_4 + R_1} = -0.130 \mu A$$

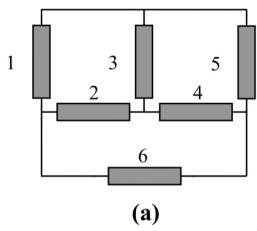
$$\Rightarrow i_{cc} = 2.043 \mu A$$

J_N

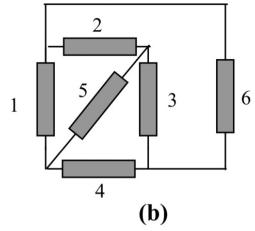
$$I_3 = J_N \cdot \frac{R_N}{R_N + R_3} = 0.467 \mu A$$

$$\rightarrow P_{R_3} = R_3 \cdot I_3^2 = 0.43 \mu W$$

ES. 4.1 - Date le seguenti reti di bipoli, scrivere un sistema completo di equazioni di Kirchhoff indipendenti.

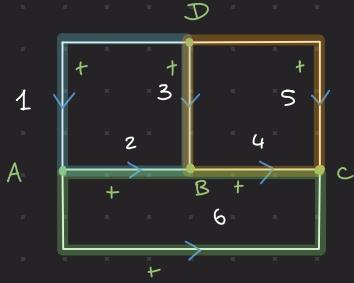


(a)



(b)

(a)

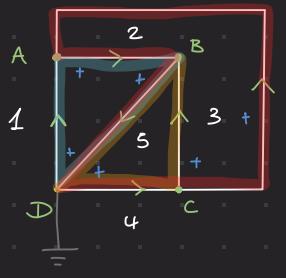


$$\text{LKC} : \begin{cases} -i_1 + i_2 + i_6 = 0 \\ -i_2 - i_3 + i_4 = 0 \\ -i_4 - i_5 - i_6 = 0 \end{cases}$$

(b)

$$\text{LKT} \begin{cases} -V_1 + V_3 - V_2 = 0 \\ -V_3 + V_5 - V_4 = 0 \\ +V_2 + V_4 - V_6 = 0 \end{cases}$$

C.U. - Orario

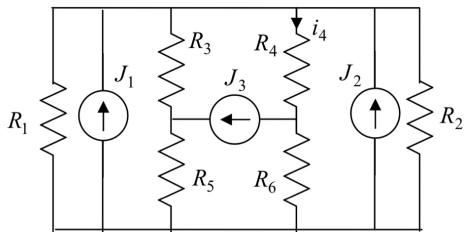


$$\text{LKC} \begin{cases} -i_1 + i_2 - i_6 = 0 & \text{A} \\ -i_2 - i_3 + i_5 = 0 & \text{B} \\ -i_4 + i_3 + i_6 = 0 & \text{C} \end{cases}$$

LKT

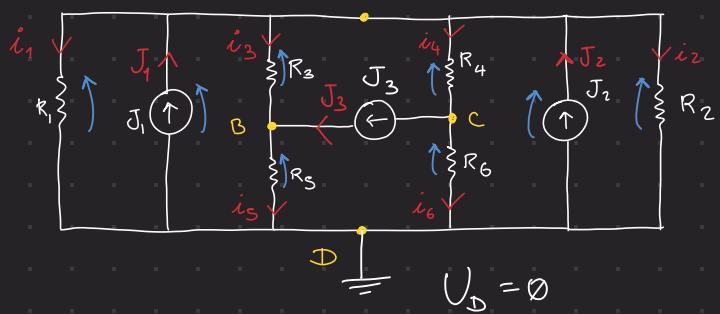
$$\begin{cases} M_1 \quad V_1 + V_2 + V_5 = 0 \\ M_2 \quad -V_5 - V_3 - V_4 = 0 \\ M_3 \quad -V_6 - V_4 - V_5 - V_2 = 0 \end{cases}$$

ES. 4.2 - Utilizzando il metodo dei potenziali nodali calcolare la corrente nel resistore R_4 .



$$\begin{aligned} J_1 &= J_2 = 1 \text{ A} & J_3 &= 3 \text{ A} \\ R_1 &= 30 \Omega & R_2 &= 10 \Omega \\ R_3 &= 25 \Omega & R_4 &= 5 \Omega \\ R_5 &= 35 \Omega & R_6 &= 15 \Omega \end{aligned}$$

(1) Potenziale a zero



$$(2) LKC : \begin{aligned} A: \quad & i_3 + i_4 + i_1 + i_2 = J_1 + J_2 \\ B: \quad & -i_3 + i_5 = J_3 \end{aligned}$$

$$C: -i_4 + i_6 = -J_3$$

(3) Correnti di lato

$$V_1 = U_A - U_D = U_A$$

$$V_2 = U_A$$

$$V_3 = U_A - U_B$$

$$V_4 = U_A - U_C$$

$$V_5 = U_B$$

$$V_6 = U_C$$

$$\begin{aligned} \Rightarrow i_1 &= \frac{U_A}{R_1} & i_4 &= \frac{U_A - U_C}{R_4} \\ i_2 &= \frac{U_A}{R_2} & i_5 &= \frac{U_B}{R_5} \\ i_3 &= \frac{U_A - U_B}{R_3} & i_6 &= \frac{U_C}{R_6} \end{aligned}$$

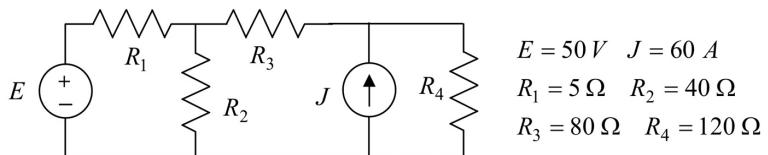
(4) Uniscasi

$$\left\{ \begin{array}{l} \frac{U_A - U_B}{R_3} + \frac{U_A - U_C}{R_4} + \frac{U_A}{R_1} + \frac{U_A}{R_2} = J_1 + J_2 \\ \frac{U_B - U_A}{R_3} + \frac{U_B}{R_5} = J_3 \\ \frac{U_C - U_A}{R_4} + \frac{U_C}{R_6} = -J_3 \end{array} \right.$$

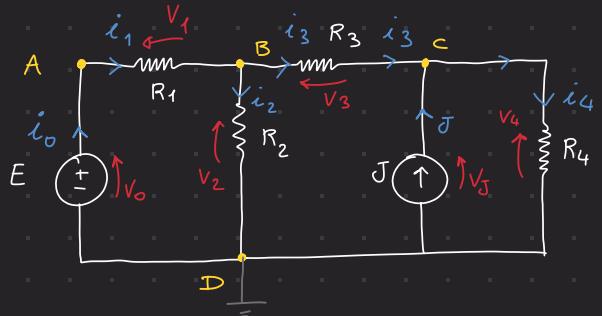
$$\Rightarrow \left\{ \begin{array}{l} G_3(U_A - U_B) + G_4(U_A - U_C) + G_1 U_A + G_2 U_A = J_1 + J_2 \\ G_3(U_B - U_A) + G_5 U_B = J_3 \\ G_4(U_C - U_A) + G_6 U_C = -J_3 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} U_A(G_3 + G_4 + G_1 + G_2) + U_B(-G_3) + U_C(-G_4) = J_1 + J_2 \\ U_A(-G_3) + U_B(G_3 + G_5) = J_3 \\ U_A(-G_4) + U_C(G_6 + G_4) = -J_3 \end{array} \right.$$

ES. 4.3 - Utilizzando il metodo dei potenziali nodali modificato calcolare la potenza erogata dai due generatori e la potenza assorbita dai resistori (verificare la conservazione delle potenze).



La differenza con il metodo "base" è che in questo caso i generatori di tensione vengono considerati **direttamente** nei calcoli (non si convertono in corrente)



$$(1) LKC : \quad B : \begin{cases} -i_1 + i_3 + i_2 = 0 \\ i_3 - i_4 = J \end{cases}$$

Siccome $U_A = E$

(2) Potenziali

$$V_1 = U_A - U_B = E - U_B \quad \rightarrow \quad i_1 = G_1(E - U_B)$$

$$V_2 = U_B, \quad V_3 = U_B - U_C \quad i_2 = G_2 U_B \quad i_3 = G_3(U_B - U_C)$$

$$V_4 = U_C \quad i_4 = G_4 U_C$$

$$\Rightarrow \begin{cases} G_1 U_B - G_1 E + G_3 U_B - G_3 U_C + G_2 U_B = 0 \\ G_3 U_C - G_3 U_B + G_4 U_C = J \end{cases} \Rightarrow \begin{cases} U_B (G_1 + G_3 + G_2) - U_C (G_3) = G_1 E \\ U_B (-G_3) + U_C (G_3 + G_4) = J \end{cases}$$

$$\begin{bmatrix} (G_1 + G_2 + G_3) & (-G_3) \\ (-G_3) & (G_3 + G_4) \end{bmatrix} \begin{bmatrix} U_B \\ U_C \end{bmatrix} = \begin{bmatrix} G_1 E \\ J \end{bmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} U_B \\ U_C \end{pmatrix} = \begin{pmatrix} E \\ F \end{pmatrix} \quad \Rightarrow \quad U_B = 200 \text{ V} = 0.2 \text{ kV}$$

$$U_C = 3 \text{ kV}$$

$$\Rightarrow i_1 = -30 \text{ A} \quad i_3 = -35 \text{ A}$$

$$i_2 = 5 \text{ A} \quad i_4 = 25 \text{ A}$$

Ans

$$\Rightarrow P_E^e = E \cdot i_0 = E \cdot i_1 = -1.5 \text{ kW}$$

$$P_J^e = J \cdot V_{CD} = J \cdot V_4 = 180 \text{ kW}$$

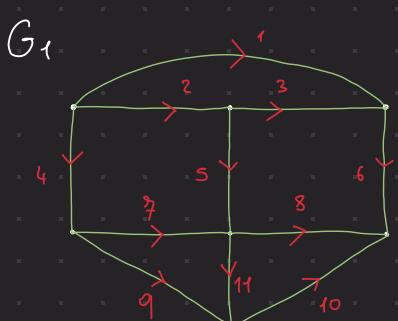
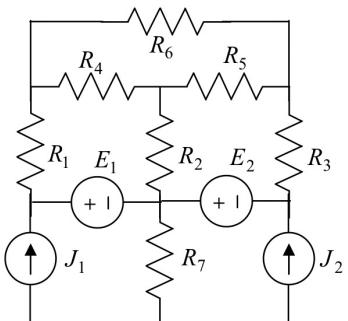
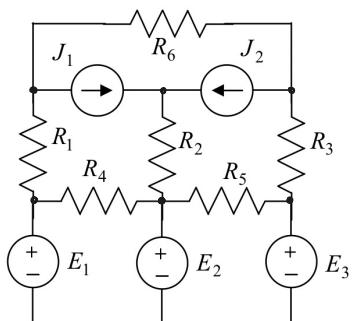
$$P_{R1} = R_1 \cdot i_1^2 = 4.5 \text{ kW}$$

$$P_{R2} = 1 \text{ kW}$$

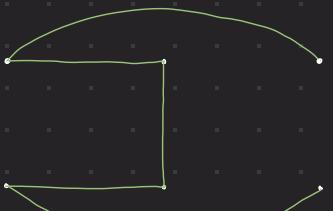
$$P_{R3} = 98 \text{ kW}, \quad P_{R4} = 75 \text{ kW}$$

ES. 4.4 - Con riferimento alla seguenti reti:

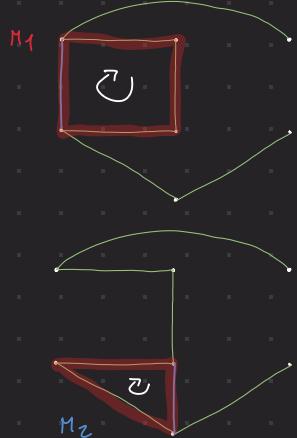
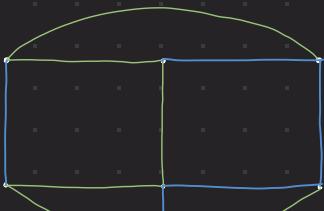
- scrivere il sistema completo delle equazioni di Kirchhoff e delle equazioni caratteristiche (utilizzare grafo, albero e co-albero).
- scrivere il suddetto sistema in forma matriciale, individuando le matrici di incidenza ridotta e di maglia fondamentale.



Albero

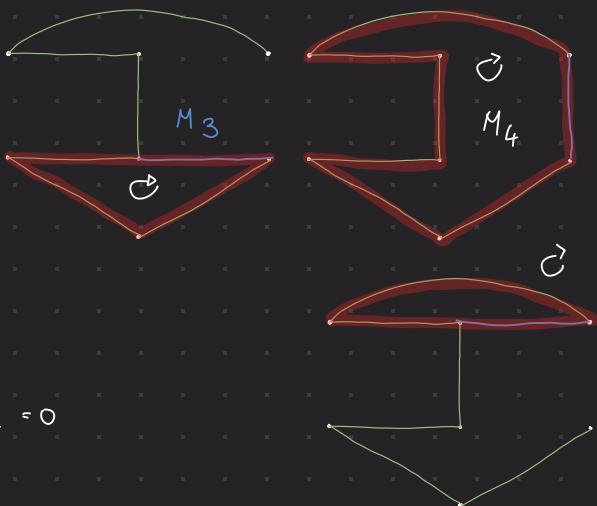


Albero + Coalbero

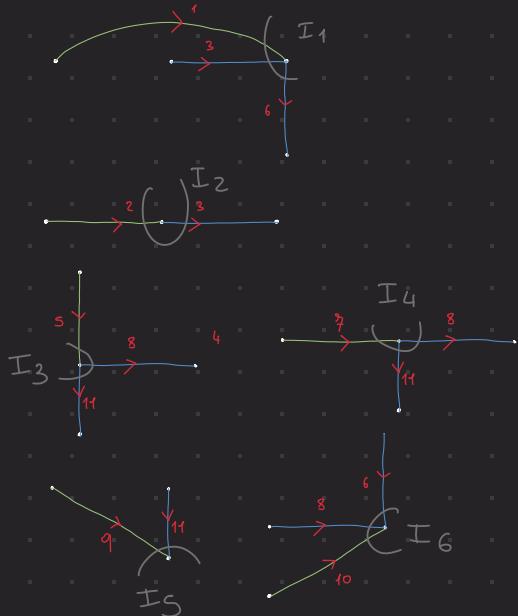


LKT Alle Maglie fondamentali
(Albero + 1 lato di coalbero)

$$\begin{aligned} M_1 : V_2 + V_5 - V_7 - V_4 &= 0 \\ M_2 : V_7 + V_{11} - V_9 &= 0 \\ M_3 : V_7 + V_8 - V_{10} - V_9 &= 0 \\ M_4 : V_1 + V_6 - V_{10} - V_9 + V_7 - V_5 - V_2 &= 0 \\ M_5 : V_1 - V_3 - V_2 &= 0 \end{aligned}$$



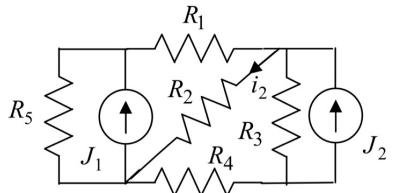
LKC agli Idt fondamentali (1 lato Albero + n del coalbero)



LKC Idt :

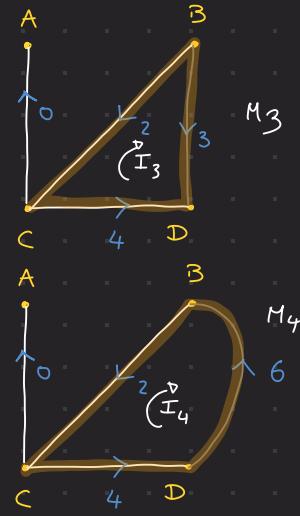
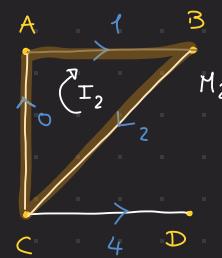
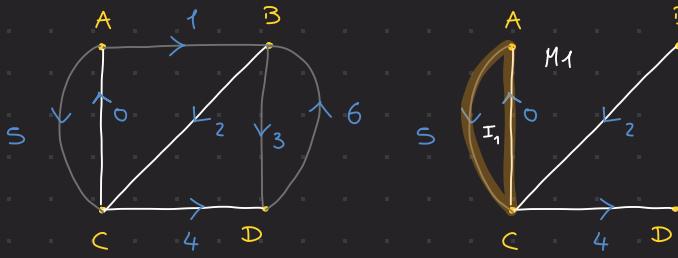
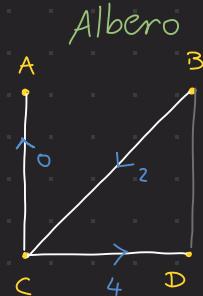
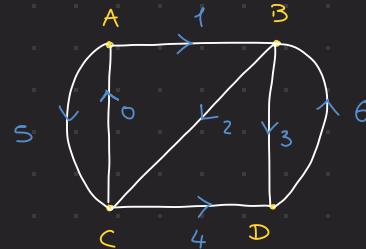
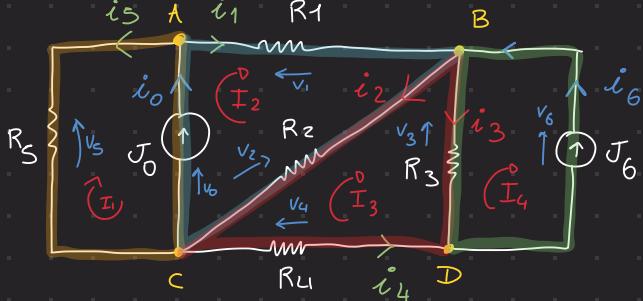
$$\begin{cases} i_1 - i_3 + i_6 = 0 \\ -i_2 + i_3 = 0 \\ -i_5 + i_8 + i_{11} = 0 \\ -i_7 + i_3 + i_{11} = 0 \\ -i_9 - i_{11} = 0 \\ -i_8 - i_{10} - i_6 = 0 \end{cases}$$

ES. 4.5 - Utilizzando il metodo delle correnti di maglia calcolare la corrente in R_2 .



$$\begin{aligned} J_1 &= 10 \text{ A} \\ J_2 &= 5 \text{ A} \\ R_1 &= 2 \Omega \quad R_2 = R_3 = 3 \Omega \\ R_4 &= R_5 = 5 \Omega \end{aligned}$$

Risultato: $i_2 = 5 \text{ A}$.



LKT Maglie orario - C.M.

$$\begin{aligned} M_1: \quad & -V_S - V_0 = 0 \\ M_2: \quad & V_0 + V_1 + V_2 = 0 \\ M_3: \quad & -V_2 + V_3 - V_4 = 0 \\ M_4: \quad & -V_2 - V_6 - V_4 = 0 \end{aligned}$$

$$\begin{aligned} \text{Req}_{AC} &= \left\{ \left[(R_3 + R_4) // R_2 \right] + R_1 \right\} // R_5 \\ &= 2.277 \Omega \end{aligned}$$

$$\text{Req}_{BD} = \left\{ \left[(R_S + R_1) // R_2 \right] + R_4 \right\} // R_3 = 2.11 \Omega$$

Correnti di Maglia

$$\begin{aligned} i_0 &= -I_1 + I_2 & i_4 &= -I_3 \\ i_1 &= I_2 & i_5 &= -I_1 \\ i_2 &= I_2 & i_6 &= -I_4 \\ i_3 &= I_3 \end{aligned}$$

$$\left\{ \begin{array}{l} V_0 = i_0 \cdot \text{Req}_{AC} \\ V_1 = R_1 i_1 = R_1 \cdot I_2 \\ V_2 = R_2 i_2 = R_2 \cdot I_2 \\ V_3 = R_3 i_3 = R_3 \cdot I_3 \\ V_4 = R_4 i_4 = -R_4 \cdot I_4 \\ V_5 = R_5 i_5 = -R_5 \cdot I_1 \\ V_6 = i_6 \cdot \text{Req}_{BD} \end{array} \right.$$

=> Sostituisco

$$\begin{cases} R_S I_1 = i_0 \text{Req}_{AC} \\ R_1 I_2 + R_2 I_2 = -i_0 \text{Req}_{AC} \\ -R_2 I_2 + R_3 I_3 + R_4 I_4 = 0 \\ -R_2 I_2 + R_4 I_4 = i_6 \text{Req}_{BD} \end{cases}$$

=>

$$\begin{bmatrix} (R_S) & 0 & 0 & 0 \\ 0 & (R_1 + R_2) & 0 & 0 \\ 0 & (-R_2) (R_3) (R_4) & 0 & 0 \\ 0 & (-R_2) & 0 & (R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} i_0 \text{Req}_{AC} \\ -i_0 \text{Req}_{AC} \\ 0 \\ i_6 \text{Req}_{BD} \end{bmatrix}$$

$$\begin{cases} I_1 = 4.55 A \\ I_2 = -4.55 A \\ I_3 = -3.51 A \\ I_4 = 3.33 A \\ I_5 = -0.62 A \end{cases}$$

$$I_2 = I_2 = -4.55 A \dots$$

Boh