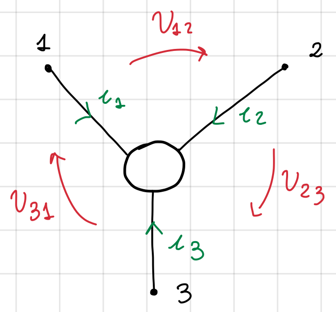
N - POLI

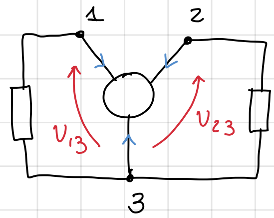


$$\begin{matrix} LKC: \\ LKT: \end{matrix} \begin{cases} i_1 + i_2 + i_3 = 0 \\ v_{12} + v_{23} + v_{31} = 0 \end{cases} \overset{ES.}{\Rightarrow} \begin{cases} i_1 = -(i_2 + i_3) \\ v_{12} = -(v_{23} + v_{31}) \end{cases}$$

INDIPENDENTI

(1) Solo N-1 tensioni e correnti sono indipendenti.

(2) Fissato un nodo, (ES: 3) le grandezze indip saranno



i_1, i_2, v_{13}, v_{23}

RELAZIONI CARATT

Possiamo esprimere le correnti con funz. di due variabili:

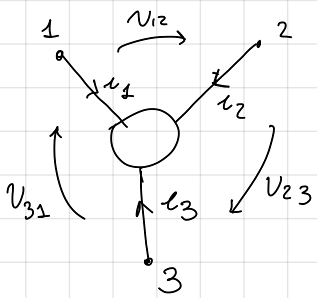
$$\begin{cases} i_1 = g_1(v_{13}, v_{23}) \\ i_2 = g_2(v_{13}, v_{23}) \end{cases}$$

CORRENTE

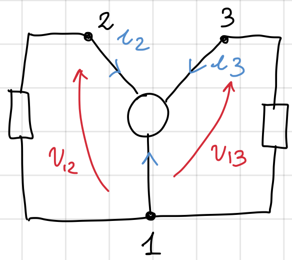
Ma abbiamo anche

$$\begin{cases} v_1 = r_1(i_1, i_2) \\ v_2 = r_2(i_1, i_2) \end{cases} \qquad \begin{cases} i_1 = h_1(i_1, v_2) \\ v_2 = h_2(i_1, v_2) \end{cases} \qquad \begin{cases} v_1 = h_1(v_1, i_2) \\ i_2 = h_2(v_1, i_2) \end{cases}$$

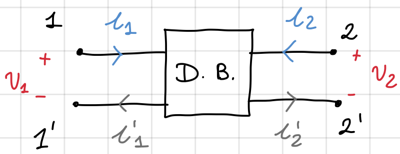
POTENZA ASSORBITA



Scelgo 1



$$\Rightarrow \quad P(t) = i_2(t) \cdot v_{12}(t) + i_3 \, v_{13}(t)$$



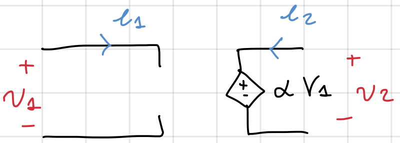
CONDIZIONI DI PORTA

$$\begin{cases} i_1 = -i_1' \\ i_2 = -i_2' \end{cases}$$

$$P(t) = v_{11'} \cdot i_1 + v_{22'} \, i_2 = v_1 i_1 + v_2 i_2$$

GENERATORI CONTROLLATI

G.T. Controllato in Tensione

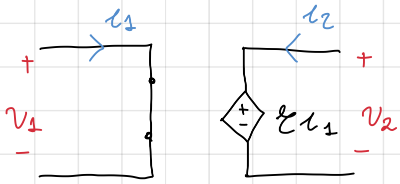


R.C. $\begin{cases} i_1 = 0 & \forall v_2 \\ v_2 = \alpha v_1 \end{cases}$ $\qquad [\alpha] = \frac{V}{V}$

$$\leadsto \begin{cases} i_1 = 0 v_1 + 0 i_2 \\ v_2 = \alpha v_1 + 0 i_2 \end{cases} \leadsto \begin{pmatrix} i_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ i_2 \end{pmatrix}$$

$\underline{\underline{H}}$ ibrida

G.T. Controllato in CORRENTE

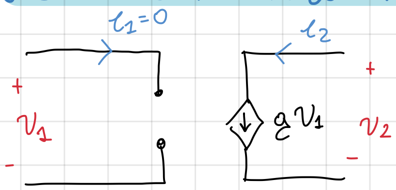


R.C. $\begin{cases} v_1 = \varnothing \\ v_2 = r i_1 \end{cases}$ $\qquad [r] = R$

$$\leadsto \begin{cases} v_1 = 0 i_1 + 0 i_2 \\ v_2 = r i_1 + 0 i_2 \end{cases} \leadsto \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ r & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \leadsto \underline{V} = \underline{\underline{R}} \cdot \underline{i}$$

$\underline{\underline{R}}$ Res

G.C. Controllato in Tensione

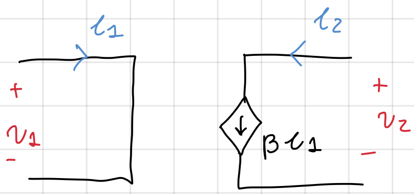


R.C. $\begin{cases} i_1 = 0 \\ i_2 = g v_1 \end{cases}$ $\qquad [g] = \frac{1}{R}$

$$\leadsto \begin{cases} i_1 = 0 v_1 + 0 v_2 \\ i_2 = g v_1 + 0 v_2 \end{cases} \leadsto \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ g & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \leadsto \underline{i} = \underline{\underline{G}} \cdot \underline{V}$$

$\underline{\underline{G}}$ cond

G.C. Controllato in CORRENTE

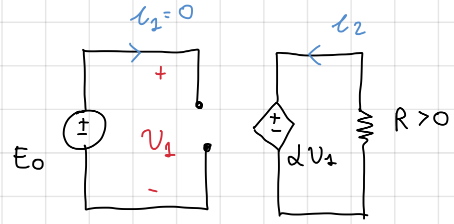


R.C $\begin{cases} v_1 = 0 \\ i_2 = \beta i_1 \end{cases}$ $\qquad [\beta] = \frac{A}{A}$

$$\leadsto \begin{cases} v_1 = 0 i_1 + 0 v_2 \\ i_2 = \beta i_1 + 0 v_2 \end{cases} \leadsto \begin{pmatrix} v_2 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \beta & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ i_2 \end{pmatrix}$$

$\underline{\underline{H}}$ ibrida

I G.C. SONO BIPOLI ATTIVI



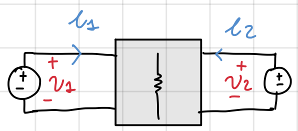
$$\begin{cases} i_1 = 0 \\ v_2 = \alpha v_1 \end{cases} \rightsquigarrow \begin{pmatrix} i_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ i_2 \end{pmatrix}$$

$$\text{ma} \begin{cases} v_2 = -i_2 R_2 \\ v_1 = E_0 \end{cases} \rightsquigarrow i_2 = \frac{v_2}{-R} = \frac{E_0}{-R}$$

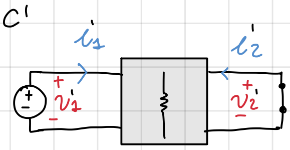
$$\Rightarrow P_a(t) = v_1 i_1 + v_2 i_2 = E_0 \cdot 0 + \alpha E_0 \cdot \frac{E_0}{-R} = \frac{\alpha E_0^2}{-R} \leq 0 \; ! \rightsquigarrow \underline{\text{D.B. ATTIVO}}$$

PROPRIETA' DI RECIPROCITA'

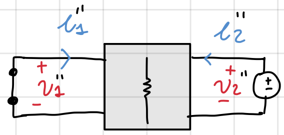
PRIMA FORMA (TENSIONE)



C'



C''



Sfrutto la cons. delle potenze virtuali $\sum^{e} i'(t) \cdot v''(t) = \sum^{e} i''(t) \cdot v'(t) = 0$

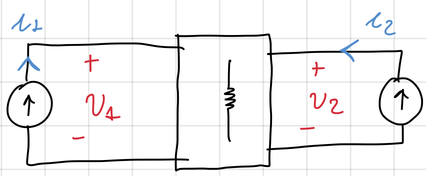
$$\begin{matrix} C' \\ C'' \end{matrix} \begin{cases} \overset{*}{\cancel{\sum^{e}_{(a)} i'_k v''_k}} - i'_1 \underset{}{\overset{v''_1 = 0}{\cancel{v''_1}}} - i'_2 v''_2 = 0 & (1) \\ \cancel{\sum^{e}_{(b)} i''_k v'_k} - i''_1 v'_1 - \underset{v'_2 = 0}{\cancel{i''_2 v'_2}} = 0 & (2) \end{cases}$$

$$* \underset{(a)}{\sum i'_k v''_k} = \sum \underbrace{i'_k R_k}_{v'_k} i''_k = \underset{(b)}{\sum v'_k \cdot i''_k} \quad \text{QED}$$

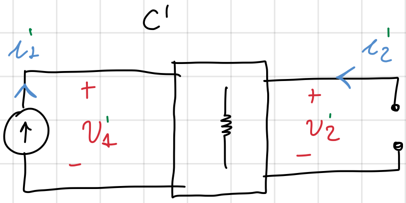
$$(1) - (2) \rightsquigarrow -i'_2 v''_2 + i''_1 v'_1 = 0 \quad \text{con } v'_1 = v''_2 = V \text{ generatori}$$

$$\Rightarrow \boxed{\frac{i'_2}{V} = \frac{i''_1}{V}}$$

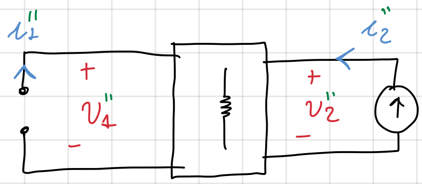
SECONDA FORMA Corrente



C'



C''

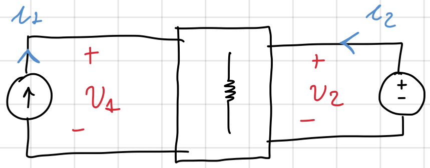


$$\begin{matrix} (1) \\ (2) \end{matrix} \begin{cases} \cancel{\sum^{e} i'_k v''_k} - i'_1 v''_1 - \overset{\emptyset}{\cancel{i'_2 v''_2}} \\ \cancel{\sum^{e} i''_k v'_k} - \underset{0}{\cancel{i''_1 v'_1}} - i''_2 v'_2 \end{cases}$$

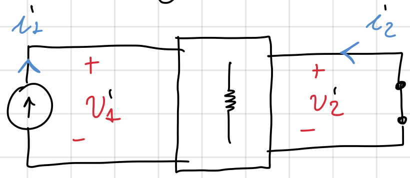
$$\rightsquigarrow (1) - (2) \longrightarrow -i'_1 v''_1 + i''_2 v'_2$$

$$i'_1 = i''_2 = I \rightsquigarrow \boxed{\frac{v''_1}{I} = \frac{v'_2}{I}}$$

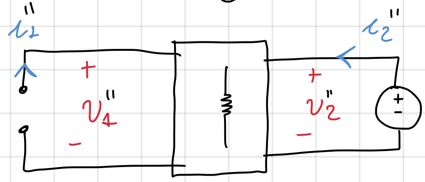
TERZA FORMA IBRIDA



C'



C''

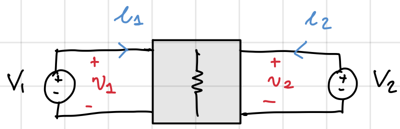


$$\begin{cases} (1) & \cancel{\sum^{e} i_k' v_k''} - i_1' v_1'' - i_2' v_2'' = 0 \\ (2) & \cancel{\sum^{e} i_k'' v_k'} - \cancel{i_1''}_{0} v_1' - \cancel{i_2''}_{0} v_2' = 0 \end{cases} \rightsquigarrow -i_1' v_1'' - i_2' v_2'' = 0 \quad \text{con } i_1' = I, \; v_2'' = V$$

$$\Rightarrow \boxed{\frac{v_1''}{I} = \overset{!}{(-)} \frac{i_2'}{V}}$$

MATRICE DELLE CONDUTTANZE

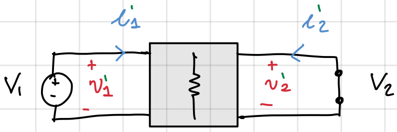
Controlliamo un D. B. in <u>Tensione</u>



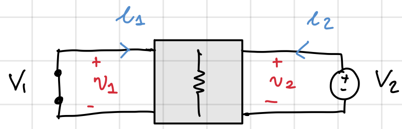
se controlliamo in Tensione → le incognite sono le i:

$$\begin{cases} i_1 = i_1' + i_1'' \\ i_2 = i_2' + i_2'' \end{cases}$$

C'



C''



$$\left[\begin{matrix} i_1' = \dfrac{V_1}{R_{11}} = G_{11} V_1 & i_1'' = G_{12} V_2 \\ i_2' = G_{21} V_1 & i_2'' = G_{22} V_2 \end{matrix}\right] \rightsquigarrow \underline{\underline{G}} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad i_k = \frac{v_k}{R_{ij}} \to G_{ij} = \frac{i_k}{v_k}$$

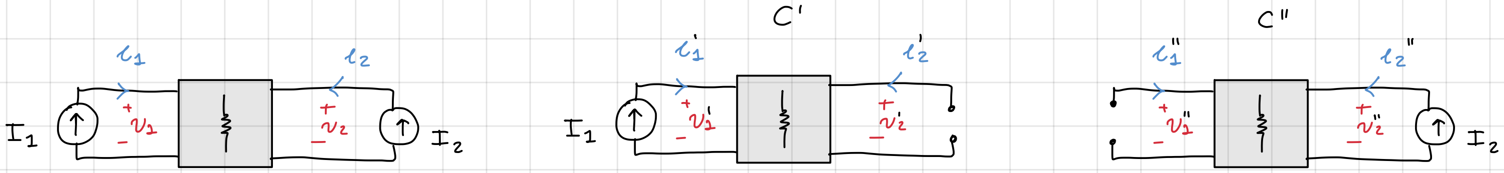
$$\boxed{\underline{\underline{G}} = \begin{pmatrix} G_{11} = \dfrac{i_1'}{v_1}\Big|_{v_2=0} & G_{12} = \dfrac{i_1''}{v_2}\Big|_{v_1=0} \\ G_{21} = \dfrac{i_2'}{v_1}\Big|_{v_2=0} & G_{22} = \dfrac{i_2''}{v_2}\Big|_{v_1=0} \end{pmatrix}}$$

POTENZA

$$\underline{i} = (i_1 \;\; i_2)^T \qquad \underline{v} = (v_1 \;\; v_2)^T \quad \to \boxed{\underline{i} = \underline{\underline{G}} \cdot \underline{v}}$$

$$P(t) = \underline{v}^T \cdot \underline{i} \Rightarrow \boxed{P(t) = \underline{v}^T \cdot \underline{\underline{G}} \cdot \underline{v}}$$

MATRICE RESISTENZE



$$\begin{cases} v_1 = v_1' + v_1'' \\ v_2 = v_2' + v_2'' \end{cases} \longrightarrow \begin{cases} v_1 = R_{11} i_1' + R_{12} i_1'' \\ v_2 = R_{21} i_2' + R_{22} i_2'' \end{cases} \Rightarrow$$

$$R_{11} = \left.\frac{v_1'}{i_1'}\right|_{i_2=0} \qquad R_{12} = \left.\frac{v_1''}{i_1''}\right|_{i_1=0}$$

$$R_{21} = \left.\frac{v_2'}{i_2'}\right|_{i_2=0} \qquad R_{22} = \left.\frac{v_2''}{i_2''}\right|_{i_1=0}$$

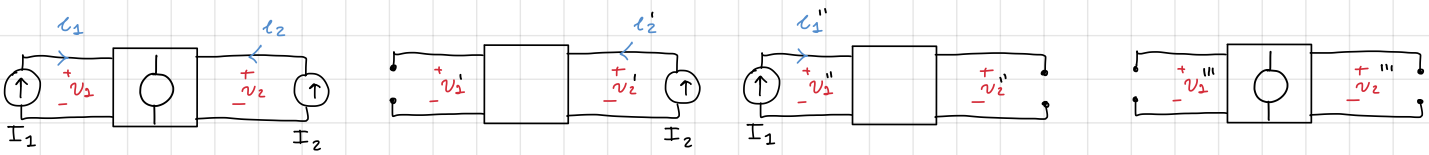
BIPOLI INERTI

Sono **inerti** quei bipoli la cui R.C. passa per 0; ovvero che non erogano potenza quando non sono Alimentati.

Sono **non inerti** quando almeno una delle Tensioni è $\neq 0$ quando le correnti sono 0.

THEVENIN PER I DB

Base corrent $\rightarrow$ $v_{1,2}$ incognite



$$\begin{cases} v_1 = v_1' + v_1'' + v_1''' \\ v_2 = v_2' + v_2'' + v_2''' \end{cases} = \begin{cases} v_1 = R_{11} i_1' + R_{12} i_1'' + \boxed{V_{10}} \\ v_2 = R_{21} i' + R_{22} i'' + \boxed{V_{20}} \end{cases}$$

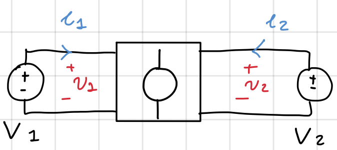
Tensioni A Vuoto

definisco $\underline{V_0} = (V_{10} \quad V_{20})^T$

$\Rightarrow$ $\boxed{\underline{V} = \underline{R}\,\underline{i} + \underline{V_0}}$ Thevenin

NORTON PER I D.B.

$\rightarrow$ Base Tensione $\rightarrow$ i_1, i_2 incognite



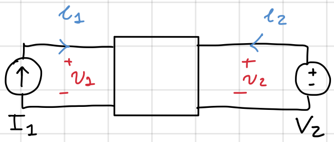
$$\begin{cases} i_1 = G_{11} V_1 + G_{12} V_1 + I_{01} \\ i_2 = G_{21} V_2 + G_{22} V_2 + I_{02} \end{cases}$$

con $\underline{I} = (I_{01} \quad I_{02})^T$

$\Rightarrow$ $\boxed{\underline{I} = \underline{G}\,\underline{V} + \underline{I_0}}$ NORTON

MATRICE IBRIDA

$\rightsquigarrow$ incognite: v_1, i_2

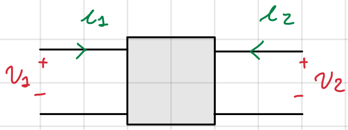


$$\begin{cases} v_1 = H_1(i_1, v_2) = H_{11} i_1 + H_{12} v_2 \\ i_2 = H_2(i_1, v_2) = H_{21} i_1 + H_{22} v_2 \end{cases}$$

$$H_{11} = \frac{v_1}{i_1}\Big|_{v_2=0} \qquad H_{12} = \frac{v_1}{v_2}\Big|_{v_2=0}$$

$$H_{21} = \frac{i_2}{i_1}\Big|_{i_1=0} \qquad H_{22} = \frac{i_2}{v_2}\Big|_{i_1=0}$$

MATRICE DI TRASMISSIONE

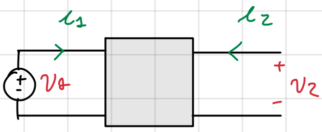


Ricariamo le grandezze della prima porta in funzione della seconda

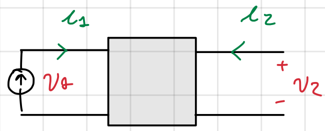
$$\begin{cases} v_1 = T_{11} v_2 + T_{12}(-i_2) \\ i_1 = T_{21} v_2 + T_{22}(-i_2) \end{cases}$$

definiti $V_0 = (v_1 \ \ i_1)^T$ e $V_1 = (v_2 \ \ -i_2)^T$ $\rightsquigarrow$ $V_0 = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \cdot V_1$

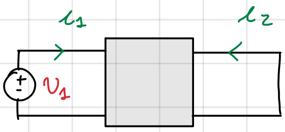
mat. di Tras.



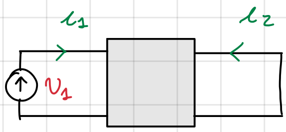
$$T_{11} = \frac{v_1}{v_2}\Big|_{i_2=0}$$



$$T_{21} = \frac{i_1}{v_2}\Big|_{i_2=0}$$



$$T_{12} = \frac{v_1}{-i_2}\Big|_{v_2=0}$$



$$T_{22} = \frac{i_1}{i_2}\Big|_{v_2=0}$$

ACCOPPIAMENTO MUTUO

$$\begin{cases} \phi_1 = L_1 i_1 + M_{12} i_2 \\ \phi_2 = L_2 i_2 + M_{12} i_1 \end{cases} \qquad \text{ma} \quad v = \frac{d\phi}{dt} \Rightarrow \quad \boxed{\begin{cases} v_1 = L_1 \dot{i}_1 + M \dot{i}_2 \\ v_2 = L_2 \dot{i}_2 + M \dot{i}_1 \end{cases}}$$

POTENZA DEL TRASFORM.

$$p^a(t) = v_1(t) \cdot i_1(t) + v_2(t) i_2(t) = L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}$$

Siccome $\frac{d}{dt}\left[\frac{di^2}{dt}\right] = 2i\frac{di}{dt} \rightarrow p^a = \frac{d}{dt}\left[\frac{1}{2} L_1 i_1^2\right] + M \underbrace{\left(i_1 \dot{i}_2 + i_2 \dot{i}_1\right)}_{\frac{d}{dt}\text{ prod}} + \frac{d}{dt}\left[\frac{1}{2} L_2 i_2^2\right]$

$$\rightarrow p^a = \frac{d}{dt}\left(\frac{1}{2} L_1 i_1^2\right) + M \frac{d}{dt}\left(i_1 \cdot i_2\right) + \frac{d}{dt}\left[\frac{1}{2} L_2 i_2^2\right]$$

Siccome $P = \frac{dW}{dt}$ ← LAVORO $\qquad W = \int P \, dt$

$$\Rightarrow W = \frac{1}{2} L_1 i_1^2 + M i_1 i_2 + \frac{1}{2} L_2 i_2^2$$

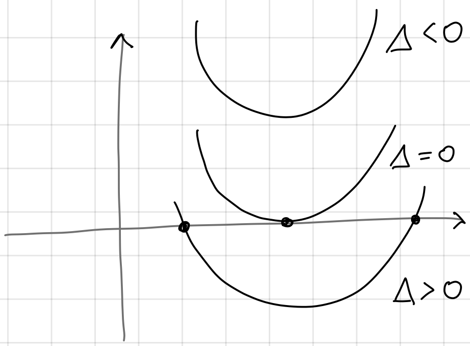
Energia immagazz.

IL TRAS. E' UN BIPOLO PASSIVO → $W_a \geq 0$

$$\frac{1}{2} L_1 i_1^2 + M i_1 i_2 + \frac{1}{2} L_2 i_2^2 \geq 0 \rightarrow \cancel{i_2^2}\left[\frac{1}{2} L_1 \left(\frac{i_1}{i_2}\right)^2 + \frac{1}{2} L_2 + M \frac{i_1}{i_2}\right] \geq 0$$

→ pongo $\frac{i_1}{i_2} = x \rightarrow \frac{1}{2} L_1 x^2 + M x + \frac{1}{2} L_2 \geq 0$

ANALISI



$$\Delta = b^2 - 4 \cdot a \cdot c = M^2 - 4 \cdot \frac{1}{2} \cdot \frac{1}{2} L_1 \cdot L_2 = M^2 - L_1 L_2$$

$$\rightsquigarrow \begin{array}{lll} \Delta \leq 0 & \rightarrow & M^2 \leq L_1 L_2 \\ \Delta = 0 & \rightarrow & M = L_1 L_2 \\ \Delta > 0 & \rightarrow & M \geq L_1 L_2 \end{array}$$

$$\Rightarrow K \triangleq \frac{M}{\sqrt{L_1 L_2}}$$

Coeff di Accoppiamento

TRASFORM. IN ACCOPPIAMENTO PERFETTO

$$\begin{cases} v_1 = L_1 i_1 + M i_2 = L_1\left(i_1 + \frac{M}{L_1} i_2\right) \\ v_2 = L_2 i_2 + M i_1 = M\left(i_1 + \frac{L_2}{M} i_2\right) \end{cases}$$

$$K = \frac{M^2}{L_1 L_2} = 1 \iff M^2 = L_1 L_2 \to \frac{M}{L_1} = \frac{L_2}{M} = m$$

con $n = \frac{L_1}{M}$

$$\begin{cases} v_1 = L_1(i_1 + n i_2) & (1) \\ v_2 = M(i_1 + n i_2) & (2) \end{cases} \to \frac{v_1}{v_2} = \overbrace{\frac{L_1}{M}}^{n} \cdot \frac{i_1 + \cancel{m i_2}}{\cancel{i_1} + m i_2} \to v_1 = n v_2$$

$$i_1 = -\frac{1}{n} i_2$$

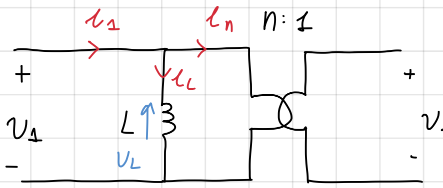
$$\left[\frac{V_1}{V_2} = \frac{n_1}{n_2}\right] \to n_1 V_1 = n_2 V_2 \quad \text{se } n_1 = 1 \to V_1 = n V_2$$

$$\left[\frac{i_1}{i_2} = -\frac{n_2}{n_1}\right] \to i_1 = -\frac{1}{n} i_2$$

RELAZIONI CARATTERISTICHE T.I.

$$\begin{cases} v_1 = n v_2 \\ i_2 = -n i_1 \end{cases} \to \begin{cases} v_2 = \frac{1}{n} v_1 \\ i_1 = -\frac{1}{n} i_2 \end{cases}$$

CIRCUITO EQUIVALENTE



Se le eq del circ sono quelle del T.I allora QED

R.C. $v_L = L i_L = L \frac{d}{dt}(i_1 - i_n)$

$i_L = i - i_n$

ma $i_n = -\frac{1}{n} i_2 \rightsquigarrow v_L = L \frac{d}{dt}\left(i_1 + \frac{1}{n} i_2\right) = \frac{L}{n} i_2 + L i_1$

$\frac{L_1}{M} = n \to \frac{L_1}{n} = M \rightsquigarrow v_L = M i_2 + L i_1$ Eq Trasf (1)

(2) devo ottenere $v_2 = L_2 i_2 + M i_1$

$v_1 = n v_2 \to n v_2 = v_L \rightsquigarrow n v_2 = L i_L \rightsquigarrow v_2 = \frac{L}{n} i_1 + L i_2$

Eq (2)