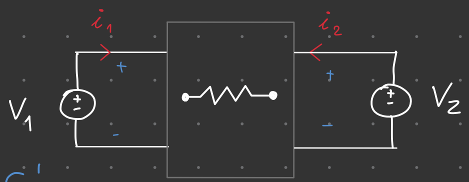
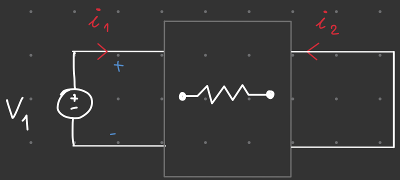
MATRICI DELLE CONDUTTANZE E RESISTENZE

MATRICE DELLE CONDUTTANZE



Per Sovrapposizione

C'



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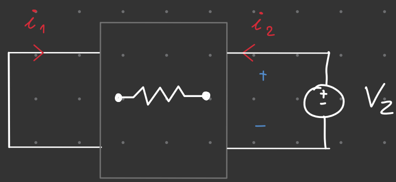


$$\begin{cases} i_1 = i_1' + i_1'' \\ i_2 = i_2' + i_2'' \end{cases} \qquad \begin{aligned} i_1' &= G_{11} \cdot V_1 \\ i_2' &= G_{21} V_1 \end{aligned} \qquad \begin{aligned} i_1'' &= G_{12} \cdot V_2 \\ i_2'' &= G_{22} V_2 \end{aligned}$$

$$\Rightarrow \quad \begin{cases} i_1 = G_{11} \cdot V_1 + G_{12} \cdot V_2 \\ i_2 = G_{21} V_1 + G_{22} V_2 \end{cases}$$

Siccome $V = R \cdot i \rightarrow \frac{1}{R} = \frac{i}{V} \Rightarrow G = \frac{i}{V}$

$$\Rightarrow \quad G_{11} = \left.\frac{i_1'}{V_1}\right|_{V_2 = \emptyset} \qquad G_{12} = \left.\frac{i_1''}{V_2}\right|_{V_1 = 0}$$

$$G_{21} = \left.\frac{i_2'}{V_1}\right|_{V_2 = 0} \qquad G_{22} = \left.\frac{i_2''}{V_2}\right|_{V_1 = 0}$$

Conduttanze proprie

Conduttanze MUTUE

Definisco

$\underline{i} = (i_1, i_2)^T \qquad \underline{V} = (v_1, v_2)^T$

$$\Rightarrow \quad \boxed{\underline{i} = \underline{\underline{G}} \cdot \underline{V}} \qquad \text{con} \quad \underline{\underline{G}} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

• Simmetrica

Potenza Assorbita

$$p = \underline{v}^T \cdot \underline{i} \Rightarrow \boxed{p = \underline{V}^T \cdot \underline{\underline{G}}\,\underline{V}}$$

Proprieta' della Matrice delle conduttanze

- $G_{12} = G_{21}$

proof dalla proprieta' di reciprocita' (I)

$$\underbrace{\frac{i_1''}{V_2}}_{G_{12}} = \underbrace{\frac{i_2'}{V_1}}_{G_{21}} \Rightarrow \quad G_{12} = G_{21} \quad \text{QED}$$

- Se i resistori sono passivi $G_{11} \geq 0$, $G_{22} \geq 0$

proof G_{11} e G_{22} sono conduttanze PROPRIE

- $G_{11} \geq 0$
  $G_{22} \geq 0$ perchè le Resistenze sono ≥ 0

- $|G_{21}| < G_{11}$, $|G_{12}| \leq G_{22}$

Siccome abbiamo resistori passivi ed un solo Bipolo Attivo
—▷ Non Ampl. delle correnti Cap 3.8

—▷ $\left|\frac{i_2}{V_1}\right| \leq \left|\frac{i_1}{V_1}\right|$

MATRICE DELLE RESISTENZE

$$\begin{cases} v_1 = R_{11}\, i'_1 + R_{12}\, i''_2 \\ v_2 = R_{21}\, i'_1 + R_{22}\, i''_2 \end{cases}$$

$$\boxed{\underline{V} = \underline{\underline{R}} \cdot \underline{i}}$$

con $\underline{\underline{R}} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$

Resistenze proprie

MUTUE

Potenza assorbita

$$\boxed{p = \underline{v} \cdot \underline{i}^T = \underline{i}^T \cdot \underline{\underline{R}}\, \underline{i}}$$

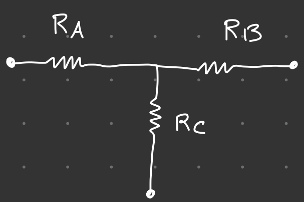
MATRICI IBRIDE E DI TRASMISSIONE

MATRICE IBRIDA

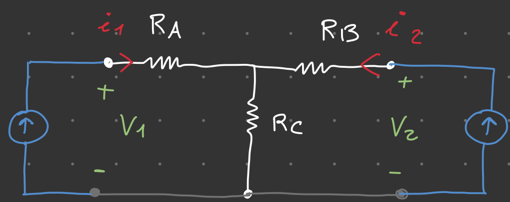
DOPPI BIPOLI Resistivi : "A T"

NOTA: Se ho due bipoli che hanno la stessa corrente e tensione sono sia in serie che in parallelo: sono in CASCATA

TRIPOLO

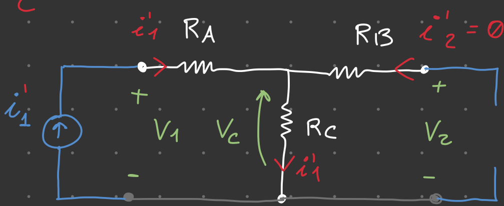


DOPPIO BIPOLO



-> Ricaviamo la matrice $\underline{\underline{R}}$ (caratterizzazione corrente)

C'



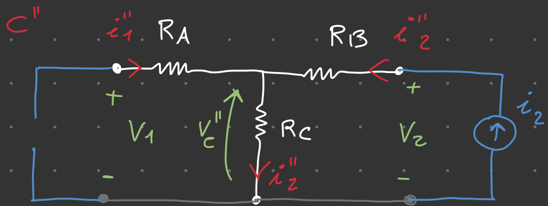
Devo trovare v_1 e v_2

$$\begin{cases} v_1 = R_{11}\, i'_1 + R_{12}\, i''_2 \\ v_2 = R_{21}\, i'_1 + R_{22}\, i''_2 \end{cases}$$

$$r_{11} = \frac{v_1}{i'_1} = \frac{\cancel{i_1}\,(R_A + R_C)}{\cancel{i_1}} = R_A + R_C$$

$$r_{21} = \frac{v_2}{i'_1} = \frac{V_C''}{i_1} = \frac{i_1 R_C}{i'_1} = R_C$$

C''



$$r_{22} = \frac{V_2}{i''_2} = \frac{i''_2\,(R_B + R_C)}{i''_2} = R_B + R_C$$

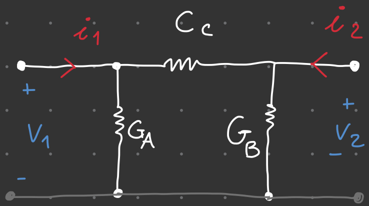
$$r_{12} = \frac{V_1}{i''_2} = \frac{V_C''}{i''_2} = \frac{i''_2 R_C}{i''_2} = R_C$$

Verifica

$$\rightarrow \underline{\underline{R}} = \begin{bmatrix} (R_A + R_C) & R_C \\ R_C & (R_B + R_C) \end{bmatrix}$$

Doppio Bipolo "A π"   Visto nella lez 23/22

Matrice conduttanze ⟶ Caratterizzazione su base Tensione



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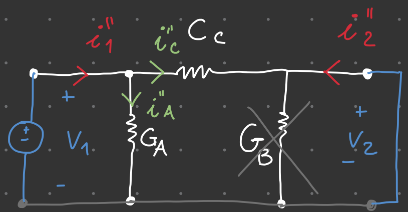


$$\begin{cases} i_1 = G_{11} V_1 + G_{12} V_2 \\ i_2 = G_{21} V_1 + G_{22} V_2 \end{cases}$$

$G_C V_C = G_C V_1$

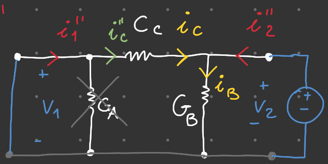
$G_A V_A = G_A V_1$

$$\Rightarrow \quad G_{11} = \left.\frac{i_1'}{V_1}\right|_{V_2=0} = \frac{i_c + i_A}{V_1} = \frac{V_1 (G_C + G_A)}{V_1} = G_A + G_C \quad G_{11}$$

$i_1' = i_A + i_c$

$$G_{12} = \left.\frac{i_2'}{V_1}\right|_{V_2=0} = \frac{-i_c}{V_1} = -\frac{G_c V_1}{V_1} = -G_c \quad G_{12}$$

C''



$$G_{22} = \frac{i_2''}{V_2} = \frac{i_B - i_c}{V_2} = \frac{(G_B V_B) - (G_c V_c)}{V_2} = \frac{V_2 (G_B - G_c)}{V_2} = G_B - G_c$$

$$G_{21} = \frac{i_1''}{V_2} = \frac{i_c''}{V_2''} = G_c$$

⟶ Se prendiamo i_c con verso opposto

$G_{22} = G_B + G_c \qquad G_{21} = -G_c$

$$\Rightarrow \quad \underline{\underline{G}} = \begin{bmatrix} (G_A + G_c) & (-G_c) \\ (-G_c) & (G_B + G_c) \end{bmatrix}$$

Sintesi - PROGETTAZIONE

Abbiamo $\underline{\underline{R}} = \begin{pmatrix} 20 & 6 \\ 6 & 18 \end{pmatrix}$

Q: Costruisci un <u>DB</u> avente $\underline{\underline{R}}$ come matrice

$$\begin{cases} R_A + R_C = 20 & \longrightarrow \quad R_A = 20 - 6 = 14 \quad R_A \\ R_C = 6 \quad R_C & \\ R_B + R_C = 18 & \longrightarrow \quad R_B = 18 - 6 = 12 \quad R_B \end{cases}$$