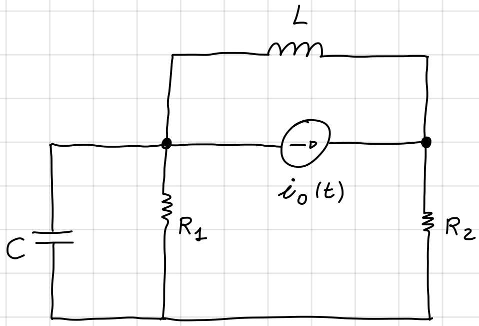
La rete in figura è in regime sinusoidale per t < 0. All'istante t = 0 il generatore diventa stazionario. Determinare la corrente nell'induttore L per ogni t



DATI

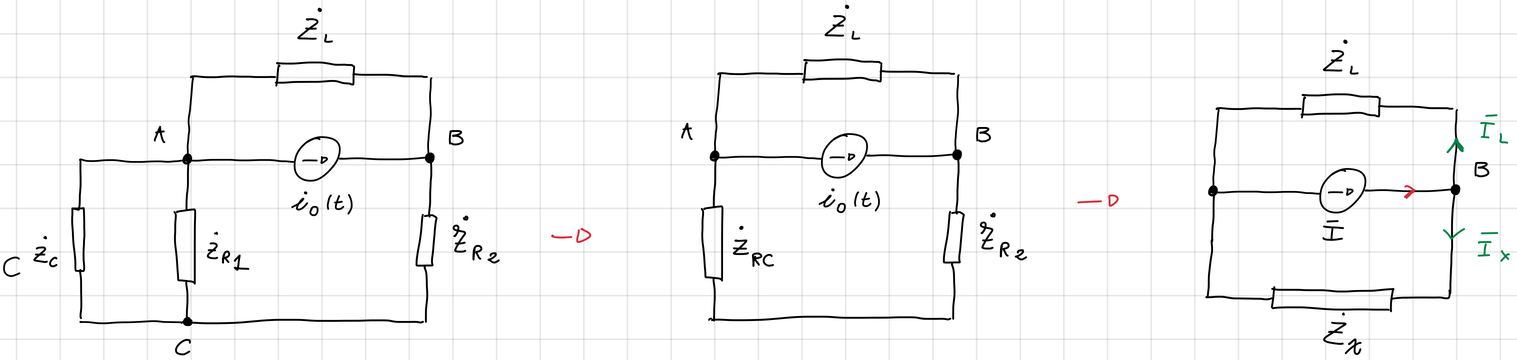
$R_1 = 170\,\Omega$ $\quad R_2 = 340\,\Omega$

$L = 0.4\,H$ $\quad C = 33\,\mu F$

$$i_0(t) = \begin{cases} 0.8\cos(100t) & t<0 \\ 0.8\ A & t>0 \end{cases}$$

(1) METODO DEI FASORI $\quad \boxed{t<0}$ Regime

$\dot{Z}_{R_1} = 170\,\Omega$ A $\quad \dot{Z}_{R_2} = 340\,\Omega$ B $\quad \dot{Z}_L = j\overset{100}{\omega}(0.4) = 40j$ D $\quad \dot{Z}_C = -\dfrac{j}{\omega C} = -303.03j$ C

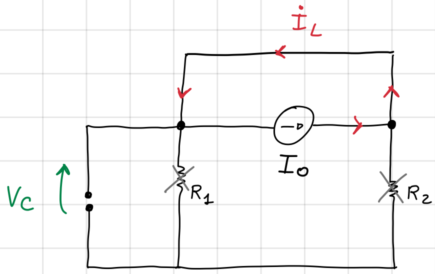


$i_0(t) = 0.8\cos(100t) \rightleftarrows \bar{I} = 0.8\ A$ $\quad \dot{Z}_x = (\dot{Z}_C \parallel \dot{Z}_{R_1}) + \dot{Z}_{R_2} = 469.3 - 72.5j$

$$\Rightarrow \bar{I}_L = \bar{I}\,\frac{\dot{Z}_x}{\dot{Z}_x + \dot{Z}_L} = 0.81 - 0.68j\ A$$

(2) $\boxed{t \to \infty}$ Regime

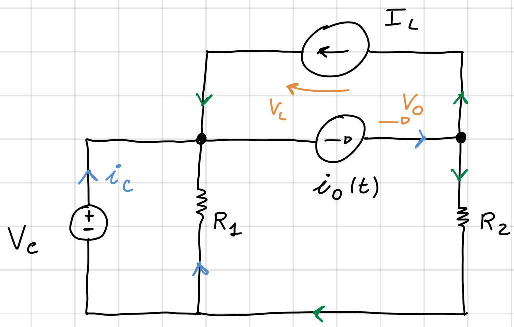
$\begin{cases} \text{Condensatore} \to \text{Aperto} \\ \text{Induttore} \to \text{Chiuso} \end{cases}$



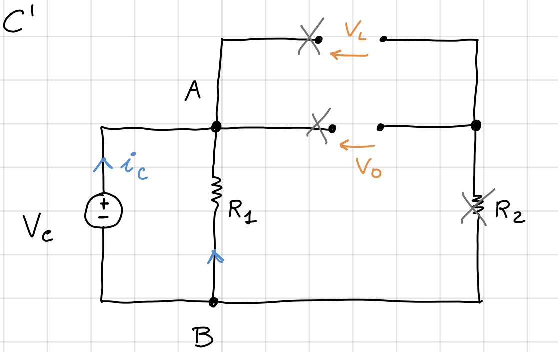
$\begin{cases} V_C = \emptyset \\ i_L = I_0 \end{cases}$

(3) $\boxed{t>0}$ Transitorio

(3.1) Circuito Resistivo



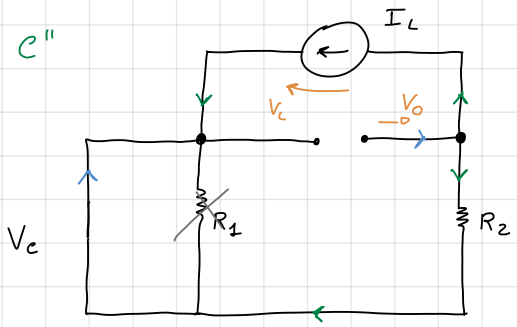
C'

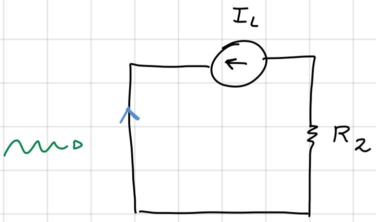


$$i_c' = \underbrace{\frac{V_c}{R_1}}_{i_c'}$$

$$V_L' = V_0' = V_{AB}' = i_c \cdot R_1 = \frac{V_c}{R_1} R_1 = \underbrace{V_c}_{V_L'}$$

C''

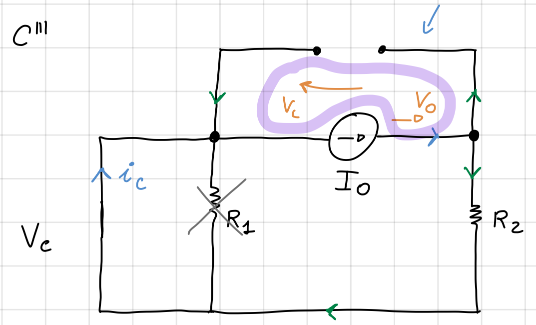




$$\begin{cases} V_L'' = R_2 \cdot I_L \\ i_c'' = -I_L \end{cases}$$

C'''

Hanno verso opposto!



$$V_0''' = I_0 \cdot R_2$$

$$V_L''' = -(I_0 \cdot R_2)$$

$$i_c''' = I_0$$

$$\Rightarrow \begin{cases} i_c = I_0 - I_L + \frac{V_c}{R_1} \\ V_L = V_c + R_2 I_L - I_0 R_2 = R_2 (I_L - I_0) + V_c \end{cases}$$

Rel Car
$$\begin{cases} V_L = L \cdot \frac{d i_L}{dt} = L \dot{i}_L \\ i_c = C \cdot \frac{d V_c}{dt} = C \dot{V}_c \end{cases}$$

$$\Rightarrow \begin{cases} C \dot{V}_c = I_0 - I_L + \frac{V_c}{R_1} \\ L \dot{i}_L = V_c + R_2 (I_L - I_0) + V_c \end{cases} \Rightarrow \begin{cases} \dot{V}_c = \frac{1}{C} I_0 - \frac{1}{C} i_L + \frac{V_c}{C R_1} & (1) \\ \dot{i}_L = \frac{V_c}{L} + \frac{R_2}{L} i_L - \frac{R_2}{L} I_0 & (2) \end{cases}$$

$$\begin{cases} \dot{V}_c = -\frac{V_c}{R_1 C} - \frac{i_L}{C} + \frac{I_0}{C} & (1) \\ \dot{i}_L = \frac{V_c}{L} - \frac{R_2 i_L}{L} + \frac{R_2 I_0}{L} & (2) \end{cases}$$

Dagli esercizi fotocopie

Posso trovare il polinomio caratteristico con

$\det(\lambda I - A)$ scrivendo

$$\begin{pmatrix} \dot{V}_c \\ \dot{i}_L \end{pmatrix} = \begin{pmatrix} -\frac{1}{R_1 C} - \lambda & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_2}{L} - \lambda \end{pmatrix} \begin{pmatrix} V_c \\ i_L \end{pmatrix}$$

ovvero
$$\boxed{\begin{pmatrix} -178-\lambda & -30303.3 \\ 2.5 & -850-\lambda \end{pmatrix}}$$

A

$\Rightarrow \det(\lambda I - A) \rightarrow A$ noi serve "$-A$" $\Rightarrow (\lambda I - A) = \begin{pmatrix} 178+\lambda & 30303.03 \\ -2.5 & 850\lambda \end{pmatrix}$

$\Rightarrow \det(\lambda I - A) = (178+\lambda)(850+\lambda) - 30303.03 \cdot (-2.5) = 151300 + 178\lambda + 850\lambda + \lambda^2 + 75757.57$

$= \lambda^2 + 1028\lambda + 227057.57 = 0$

$\Rightarrow \lambda_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \begin{cases} -706.71 & \lambda_1 \\ -321.28 & \lambda_2 \end{cases}$ $\quad > 0$

$\Rightarrow y_0(t) = C_1 e^{-707t} + C_2 e^{-321t} + y_P$

RISOLVERE IL PROBLEMA DI CAUCHY

Per trovare propri0 la corrente che serve a noi, dobbiamo risolvere il problema di Cauchy. Il problema di Cauchy necessità delle condizioni iniziali; queste condizioni sono proprio lo stato del circuito poco prima che iniziasse questo intervallo di tempo. Nel nostro caso le condizioni iniziali di t>0 sono quelle del sistema per t<0:

$I(0) = |\bar{I}_L| \cos(\omega t + \varphi) = |\bar{I}_L| \cos(100 \cdot 0 + \angle \bar{I}_L) = 0.804\ A$ $\quad i_L(0)$?

$\bar{I}_L = 0.81 - 0.68j\ A \Rightarrow |\bar{I}_L| = 0.807$

$$\begin{cases} i_0(t) = C_1 e^{-707t} + C_2 e^{-321t} + i_p \\ i(0) = 0.804\ A \\ \dot{i}(0) = \dfrac{V_C}{L} - \dfrac{R_2}{L} i_L + \dfrac{R_2}{L} I_0 = 1074 \end{cases}$$

7.375

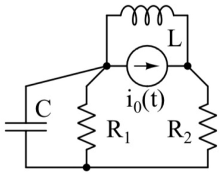
ci serve anche la ci della tensione

$\rightarrow V_C = |\bar{V}_C| \cos(\angle V_C) = 4.31$?

***Esercizio 1** - obbligatorio per tutti*

La rete in figura è in regime sinusoidale per $t < 0$. All'istante $t = 0$ il generatore diventa stazionario. Determinare la corrente nell'induttore L per ogni t. Dati: $R_1 = 170\Omega$, $R_2 = 340\Omega$, $L = 0.4$H, $C = 33\mu$F,

$$i_0(t) = \begin{cases} 0.8\cos(100t)\ A & t<0 \\ 0.8\ A & t>0 \end{cases}$$



(Lez 25 Novembre)

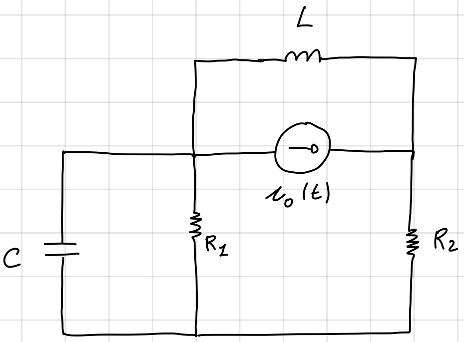
DATI

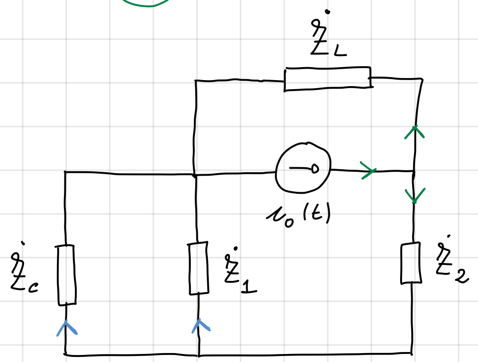
A $R_1 = 170\ \Omega$ B $R_2 = 340\ \Omega$

$L = 0.4\ H$ D $C = 33\ \mu F$ C

$$i_0(t) = \begin{cases} 0.8\ \cos(100t)\ A & t<0 \\ 0.8\ A & t>0 \end{cases}$$

$\boxed{t<0}$ Sinusoidale → Statico T_1





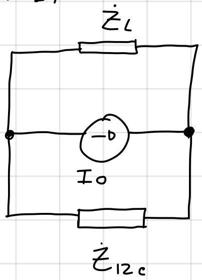
G $\dot{Z}_1 = R_1 = 170\ \Omega$

H $\dot{Z}_2 = R_2 = 340\ \Omega$

J $\dot{Z}_L = j\omega L = 40j$

I $\dot{Z}_c = -\dfrac{j}{\omega c} = -303.03j$

$$i_0(T_1) = 0.8\ \cos(100t) \rightleftarrows \bar{I}_0(T_1) = 0.8 \cdot \cancel{e^{j\cdot\varphi t}}^{\,1} = 0.8\ A \quad E$$



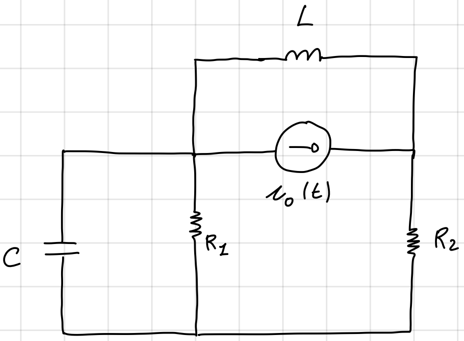
con $\dot{Z}_{12c} = (\dot{Z}_c /\!/ \dot{Z}_1) + \dot{Z}_2 = \underline{469.31 - 82.54j}$ Z_{12c}

I G H

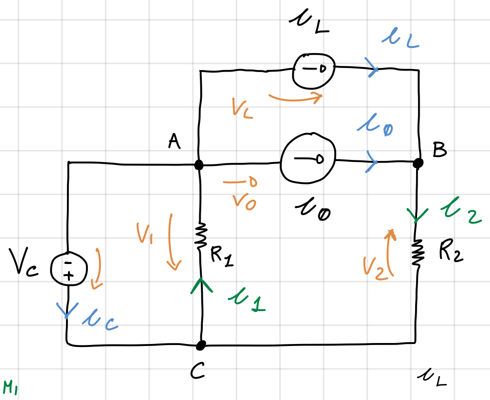
$$\Rightarrow \bar{I}_L = \bar{I}_0 \cdot \frac{\dot{Z}_{12c}}{\dot{Z}_{12c} \cdot \dot{Z}_L} = 0.81 - 0.068j\ A \quad \text{Ans 1}$$

$\bar{I}_L$

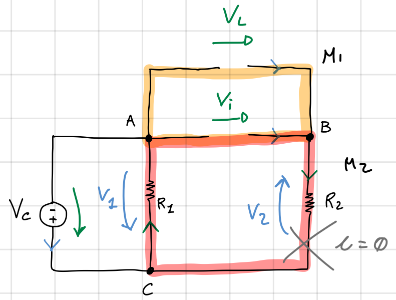
$\boxed{t>0}$ Regime Stazionario → Transitorio

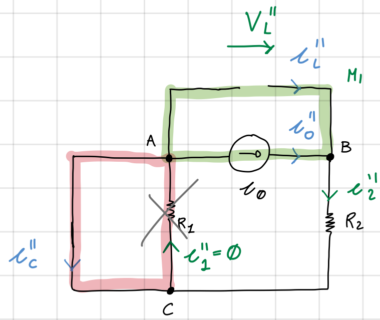


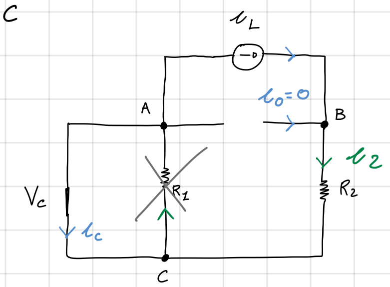
→ C.R.A.



C'







$-V_L' + V_i' = 0 \rightarrow V_i' = V_L'$

$V_1 - V_i + V_2 = 0 \rightarrow V_i' = V_1 + \cancel{V_2}^{\,0}$

$\rightarrow V_1 = V_c \rightarrow V_i' = V_L' = V_c$

$$i_c' = \frac{V_c}{R_1}$$

$LKT_{M_1}: -V_L'' + V_0 = 0$

$\rightarrow V_L'' = V_0 = R_2 i_0$

$i_c'' = -i_0$

$V_L''' = i_L \cdot R_2$

$i_c''' = -i_L$

$$\Rightarrow \begin{cases} V_L = V_C + R_2 i_0 + i_L R_2 \\ i_C = \dfrac{V_C}{R_1} - i_0 - i_L \end{cases} \xrightarrow{SUB} \begin{cases} v_L = -L \dot{i}_L \\ i_C = -C \dot{v}_C \end{cases} \rightarrow \begin{cases} -L \dot{i}_L = V_C + R_2 i_0 + R_2 i_L \\ -C \dot{v}_C = \dfrac{V_C}{R_1} - i_0 - i_L \end{cases}$$

Induttore e condens NON sono generatori... Andrebbero presi con la c.u.

$$\rightarrow \begin{cases} \dot{v}_C = -\dfrac{1}{R_1 C} v_C + \dfrac{1}{C} i_L + \dfrac{1}{L} i_0 \\ \dot{i}_L = -\dfrac{1}{L} V_C - \dfrac{R_2}{L} i_L - \dfrac{R_2}{L} i_0 \end{cases}$$

$$\Rightarrow (\lambda I - A) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} -\frac{1}{R_1 C} & +\frac{1}{L} \\ -\frac{1}{L} & -\frac{R_2}{L} \end{pmatrix} = \begin{pmatrix} \lambda + \frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & \lambda + \frac{R_2}{L} \end{pmatrix}$$

$$\rightarrow P_A = \det(\lambda I - A) = \begin{vmatrix} \lambda + 178 & -30303 \\ 2.5 & \lambda + 850 \end{vmatrix} = \lambda^2 + 850\lambda + 178\lambda + 151300 + 75757.57$$

$$= \lambda^2 + 1028\lambda + 227057.57$$

$$\Rightarrow \boxed{\begin{matrix} \lambda_1 = -321.2 \\ \lambda_2 = -706.7 \end{matrix}} \Rightarrow y_0(t) = K_1 e^{-321.2 t} + K_2 e^{-706.7 t}$$

Ne' $v(t)$ Ne' $i(t)$!

N.B.: MUSICA, SI! RUTTI, NO!!!

TROVO LE CONDIZIONI INIZIALI

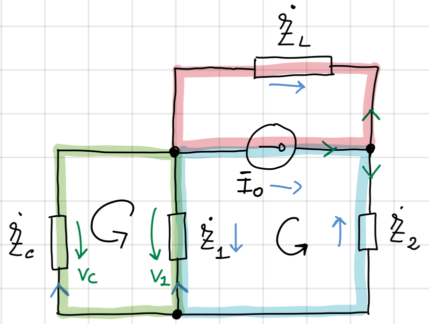
$$\bar{I}_L = 0.804 - 0.0679j \Rightarrow |\bar{I}_L| \cdot \cos(\omega t + \angle I_L)$$

$$\rightarrow i(t) = 0.8068 \cdot \cos(100t - 0.084) \Rightarrow \boxed{i_L(0^-) = 0.804} \; i_L(0^+)$$

$$\dot{i}_L(0) = \text{Dalla } (2) = \left[-\frac{1}{L} v_C - \frac{R_2}{L} i_L - \frac{R_2}{L} i_0 \right]_{0^+}$$

ci serve $v_C(0)$

Trovo $v_C(0)$ dall'intervallo $\boxed{t < 0}$



$$\begin{cases} V_0 - V_1 - V_2 = 0 \\ V_1 - V_C = 0 \\ V_L - V_0 = 0 \end{cases} \rightarrow \begin{cases} V_0 - V_1 - V_2 = 0 \\ V_1 = V_C \\ V_L = V_0 \end{cases} \rightarrow V_0 - V_C - V_2 = 0$$

$$V_C = V_0 - V_2$$

$$\bar{V}_2 = \bar{I}_2 \cdot \dot{Z}_2 \quad \text{ma} \quad \bar{I}_2 = I_0 \cdot \frac{\dot{Z}_L}{\dot{Z}_L + [\dot{Z}_2 + (\dot{Z}_1 \| \dot{Z}_C)]} = \ldots$$

G $\dot{Z}_1$
H $\dot{Z}_2$
J $\dot{Z}_L$
I $\dot{Z}_C$

$$\rightarrow V_2 = -1.6 + 23.07j \checkmark$$

$$\bar{V}_0 = \bar{I}_0 \cdot \left[(\underset{I}{\dot{Z}_C} \| \underset{G}{\dot{Z}_1}) + \underset{H}{\dot{Z}_2} \right] \| \underset{J}{\dot{Z}_L} = 2.71 + 32.18j$$

$$\Rightarrow V_C = 4.31 + 9.12j \checkmark$$

$$\rightarrow \boxed{v(0^+) = 4.31} \quad v(t) = |\bar{V}_c| \cos(\omega t + \angle \bar{V}_c)$$

... $\dot{i}_L(0)$ = Dalla (2) = $\left[-\frac{1}{\underset{D}{L}}(4.31) - \frac{\overset{B}{R_2}}{\underset{D}{L}}(0.804) - \frac{\overset{B}{R_2}}{\underset{D}{L}}(0.8)\right] = -1374.175\ A$

Cauchy

$$\begin{cases} i_L(t) = \underbrace{K_1 e^{-321.2t} + K_2 e^{-706.7t}}_{y_0} + y_P \\ i_L(0) = 0.804\ A \\ \dot{i}_L(0) = -1374.175\ A \end{cases}$$

FORZAMENTO ← I_0

$\Rightarrow 0.804 = K_1 + K_2 + y_P$

$0.804 = K_1 + K_2 + 0.8$ (A)

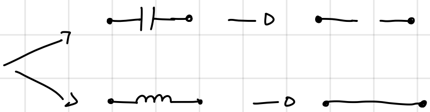
$$\dot{i}(t) = -321.2\,K_1 e^{-321.2t} - 706.7\,K_2 e^{-706.7t} \Rightarrow \dot{i}(0) = -1374.175 = -321.2\,K_1 - 706.7\,K_2 \quad (B)$$

$$\begin{cases} 0.804 = K_1 + K_2 + 0.8 & (A) \\ -1374.175 = -321.2\,K_1 - 706.7\,K_2 \end{cases} \rightarrow \begin{cases} K_1 + K_2 = -4 \times 10^{-3} \\ 321.2\,K_1 + 706.7\,K_2 = 1374.18 \end{cases}$$

$$\Rightarrow \boxed{\begin{matrix} K_1 = -3.57 \\ K_2 = 3.568 \end{matrix}}$$

$\boxed{t \to \infty}$

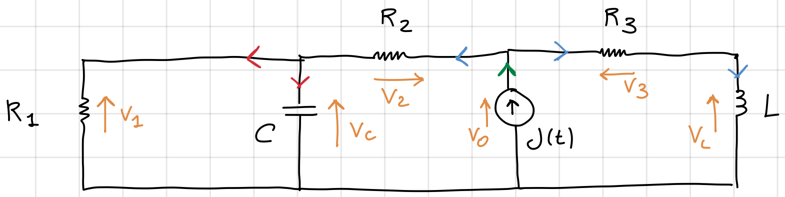
→ Nel tempo



$$\Rightarrow \begin{cases} V_C(\infty) = \emptyset \\ i(\infty) = I_0 = 0.8\ A \end{cases}$$

Esame 04/06/2021

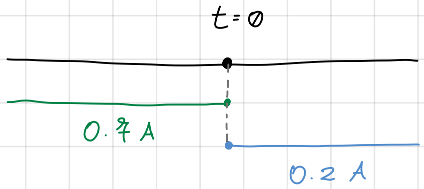
ESERCIZIO 2



DATI

A: $R_1 = 80\,\Omega$ B: $R_2 = 120\,\Omega$ C: $R_3 = 60\,\Omega$

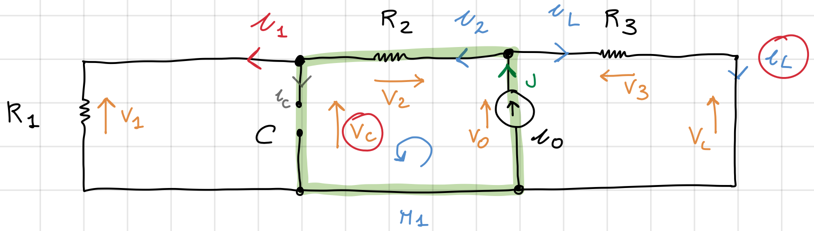
D: $L = 1\,H$ E: $C = 33\,\mu F$

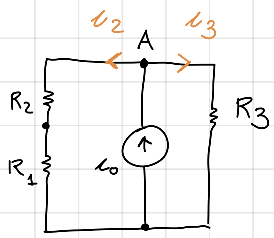


$$J(t) = \begin{cases} 0.7\,A & t<0 \\ 0.2\,A & t>0 \end{cases}$$

$t < 0$

Stazionario $\rightarrow$ CRA





$$\begin{cases} i_L = i_3 \\ -V_0 + V_2 + V_C = 0 \rightarrow V_C = V_0 - V_2 \end{cases}$$

$R_{eq} = (R_2 + R_1) \,//\, R_3 = 46.15\,\Omega$ (B A C)

$\Rightarrow V_0 = R_{eq} \cdot i_0 = \underline{32.31\,V}$

$$i_2 = i_0 \cdot \frac{R_3}{R_3 + R_1 + R_2} = 0.162\,A \quad \Rightarrow \quad V_2 = i_2 \cdot R_2 = \underline{19.38\,V}$$

$\Rightarrow V_C = V_0 - V_2 = 12.93\,V$ (V_C)

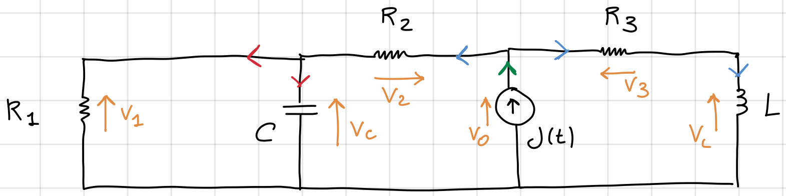
$LKC_A: \; -i_0 + i_2 + i_3 = 0 \rightarrow i_3 = i_0 - i_2 = 0.7 - 0.162 = 0.538\,A$ (i_L)

$\Rightarrow$ Condizioni iniziali per $\underline{T_1 = t > 0}$

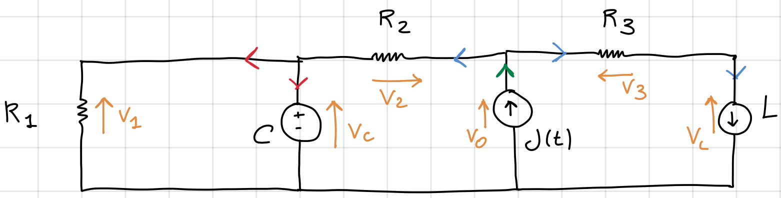
$$\begin{cases} V_C(0^+) = V_C(0^-) = 12.93\,V & (1) \\ i_L(0^+) = i_L(0^-) = 0.538\,A & (2) \end{cases}$$

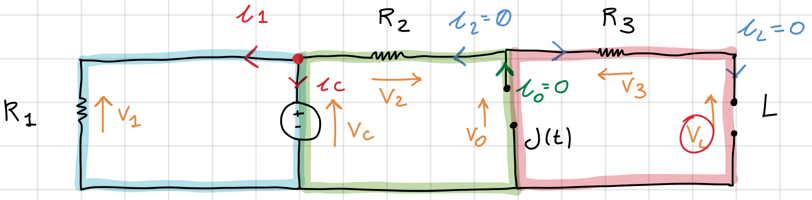
$t > 0$

$i(T_1) = 0.2\,A$



otteniamo il CRA

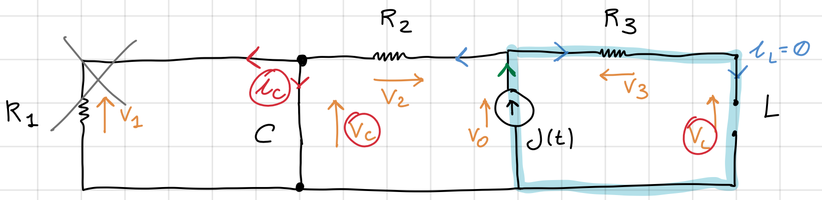




$$i_C' = -\frac{V_C}{R_1}$$

$$-V_C - V_2 + V_3 + V_L = 0 \Rightarrow V_L = V_C + \overset{0}{\cancel{V_2}} - \overset{0}{\cancel{V_3}}$$

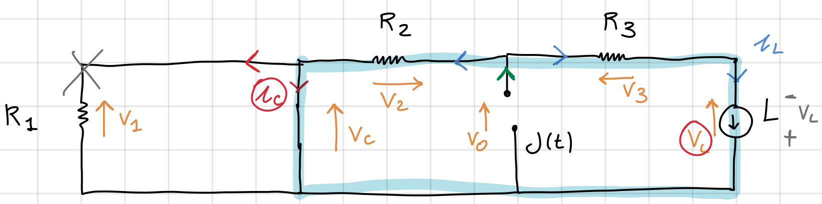
$$\Rightarrow V_L' = V_C$$



$$-V_0 + \underset{0}{\cancel{V_3}} + V_L = 0 \Rightarrow V_L'' = V_0$$

ma $V_0 = i_1 \cdot R_2 \Rightarrow V_L'' = i_1 R_2$

$$i_C'' = i_1$$



$$-\overset{0}{\cancel{V_C}} - V_2 + V_3 + V_L = 0 \Rightarrow V_L = V_2 - V_3$$

$$\Rightarrow V_L = -i_L(R_2+R_3)$$

$$i_C = -i_L$$

$$\Rightarrow \begin{cases} v_L = V_C - i_L(R_2+R_3) + i_1 R_2 \\ i_C = -\dfrac{V_C}{R_1} - i_L + i_1 \end{cases} \Rightarrow \begin{cases} i_C(t) = C \cdot \dot{v}_C \\ v_L(t) = L\,\dot{i}_L \end{cases} \Rightarrow \begin{cases} (3)\ \dot{i}_L = \dfrac{1}{L} v_C - \left(\dfrac{R_2+R_3}{L}\right) i_L + \dfrac{R_2}{L} i_1 \\ (4)\ \dot{v}_C = -\dfrac{1}{R_1 C} v_C - \dfrac{1}{C} i_L - \dfrac{1}{C} i_1 \end{cases}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda + \frac{1}{R_1 C} & \frac{1}{C} \\ -\frac{1}{L} & \lambda + \frac{R_2+R_3}{L} \end{vmatrix} = 0 \Rightarrow \lambda^2 + \frac{R_2+R_3}{L}\lambda + \frac{1}{R_1 C}\lambda + \frac{R_2+R_3}{L R_1 C} + \frac{1}{CL} = 0$$

DATI

A	B	C
$R_1 = 80\,\Omega$	$R_2 = 120\,\Omega$	$R_3 = 60\,\Omega$
D: $L = 1\,H$	E: $C = 33\,\mu F$	

$\Rightarrow$

$$\Rightarrow \lambda^2 + 558\lambda + 98484 \Rightarrow \lambda_{1,2} = -\overset{\alpha}{279.3} \pm \overset{\omega_d}{142.9}$$

$\lambda_{1,2}$ cmpx e coniugate

$\Rightarrow$ Soluzione del tipo: $y_0(t) = e^{-\alpha t}\left[K_1 \cos(\omega_d t) + K_2 \sin(\omega_d t)\right]$

$$= K_1 e^{-279.3t} \cos(\underbrace{142.9}_{\omega_d} t) + K_2 e^{-279.3} \sin(\underbrace{142.9}_{\omega_d} t)$$

CORRENTE INDUTTORE

$$\begin{cases} i_L(t) = K_1 e^{-279.3t} \cos(142.9t) + K_2 e^{-279.3} \sin(142.9t) \\ i_L(0^+) = 0.538\,A \quad \text{Dalla (2)} \uparrow \\ \dot{i}_L(0^+) = \dots ? \end{cases}$$

Dalla (3)

$$\dot{i}_L = \frac{1}{L} v_C - \left(\frac{R_2+R_3}{L}\right) i_L + \frac{R_2}{L} i_1 = 12.93 - 180 \cdot 0.538 + 120 \cdot 0.2 = -59.91\ A$$

$\dot{i}_L(0)$

$$\begin{cases} v_C(0) = 12.93\ V \\ i_L(0) = 0.538 \end{cases}$$

$$\begin{cases} i_L(t) = K_1 e^{-279.3t} \cos(142.9t) + K_2 e^{-279.3} \sin(142.9t) \\ i_L(0^+) = 0.538\ A \quad \text{Dalla (2)} \uparrow \\ \dot{i}_L(0^+) = -59.91\ A \end{cases}$$

$$[f(x) \cdot g(x)]' = f'(x) g(x) + f(x) g'(x)$$

$$\dot{i}_L(t) = -279.3 K_1 e^{-279t} \cos(142.9t) - K_1 e^{-279.3t} \cdot 142.9 \sin(142.9t) -$$
$$-279.3 K_2 e^{-279t} \sin(142.9t) + K_2 e^{-279.3t} \cdot 142.9 \cos(142.9t)$$

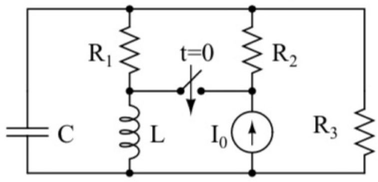
$$\Rightarrow \begin{cases} K_1 + K_2 = 0.538 \\ -279.3 K_1 + K_2 \cdot 142.9 = -59.91 \end{cases} \Rightarrow \boxed{\begin{matrix} K_1 = 0.324 \\ K_2 = 0.214 \end{matrix}}$$

SOLUZIONE

$$i_L(t) = 0.324\, e^{-279.3t} \cos(142.9t) + 0.214\, e^{-279.3} \sin(142.9t)$$

Esercizio 2 (PROVA D'ESAME 26/2/20)

Nel circuito in figura, l'interruttore **si chiude** all'istante t=0. Determinare la tensione ai capi del condensatore in ogni istante di tempo. *Dati:* $R_1 = 80\Omega$, $R_2 = 90\Omega$, $R_3 = 60\Omega$, $L = 0.05$H, $C = 4.7$mF, $I_0 = 0.6$A

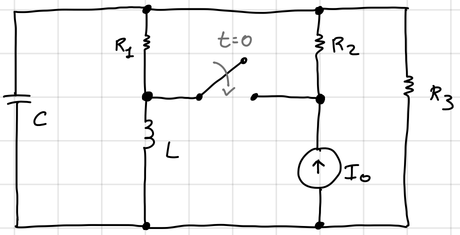


DATI

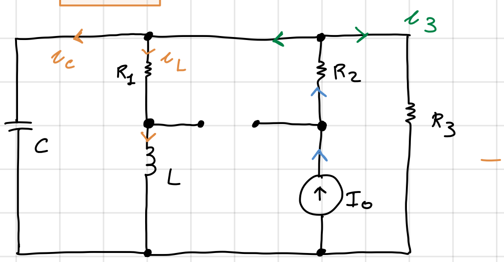
A $R_1 = 80\,\Omega$ B $R_2 = 90\,\Omega$ C $R_3 = 60\,\Omega$

$L = 0.05\,H$, $C = 4.7\,mF$ $I_0 = 0.6\,A$

L K J



t < 0



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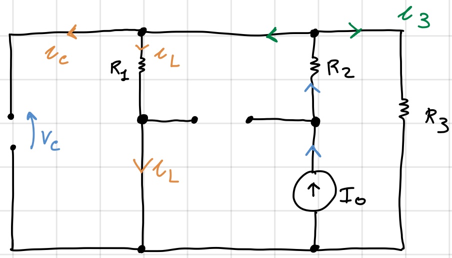

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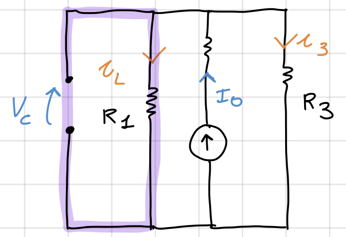
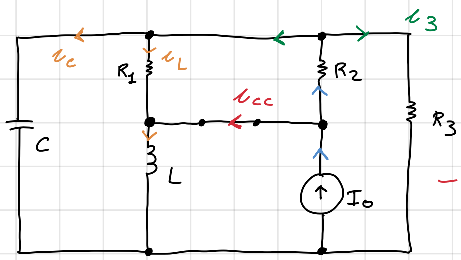


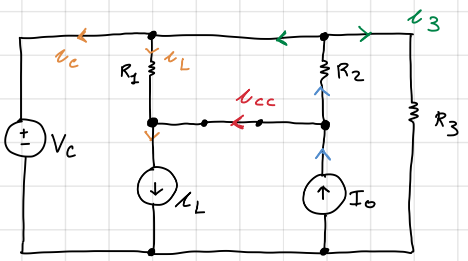
Siccome $i_L = i_1 \to i_L = I_0 \cdot \frac{R_3}{R_3 + R_1} = 0.257\,A$ $i_L(0^+)$

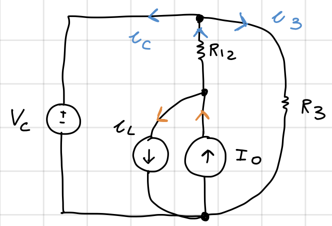
LKT_{M_1}: $-V_c + V_1 = 0 \to V_c = i_L \cdot R_1 = 20.57\,V$ V_c (1)   C.I. ↓

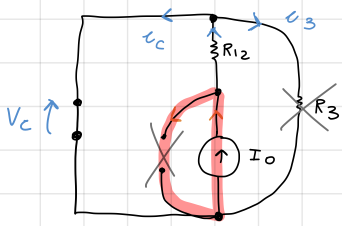
t > 0   l'interruttore si apre...



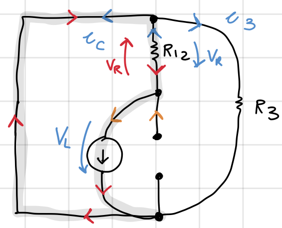
—o C.R.A. —o

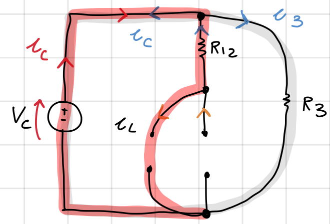






* $R_{12} = R_1 \,//\, R_2 = 42.35\,\Omega$





$$\begin{cases} V_L' = V_0 = I_0 \cdot R_{12} \\ i_c' = I_0 \end{cases} \quad V_L$$

$$\begin{cases} i_c'' = -i_L \\ V_L'' = -R_{12} \cdot i_L \end{cases}$$

$$i_c''' = -\frac{V_c}{R_3}$$

$\to -V_c + V_R + V_L = 0$

$\Rightarrow V_L = V_R + V_c = \cancel{R_{12}\, i_{12}} + V_c$ $i_{12} = 0$

$\Rightarrow V_L''' = V_c$

$$\Rightarrow (A) \begin{cases} i_c = I_0 - i_L - \frac{V_c}{R_3} \\ v_L = I_0 R_{12} - R_{12}\, i_L + v_c \end{cases} \quad \text{ma} \quad \begin{cases} i_c = C \cdot \dot{v}_c \\ v_L = L\, \dot{i}_L \end{cases} \ (C)$$

$$(B) \begin{cases} \dot{v}_c = -\frac{1}{R_3 C} v_c - \frac{1}{C} i_L + \frac{1}{C} I_0 \\ \dot{i}_L = \frac{1}{L} v_c - \frac{R_{12}}{L} i_L + \frac{R_{12}}{L} I_0 \end{cases}$$

$$\begin{cases} \dot{v}_C = -\dfrac{1}{R_3 C} v_C - \dfrac{1}{C} i_L + \dfrac{1}{C} I_0 & (2) \downarrow \\ \dot{i}_L = \dfrac{1}{L} v_C - \dfrac{R_{12}}{L} i_L + \dfrac{R_{12}}{L} I_0 \end{cases}$$

$$\Rightarrow \det(\lambda I - A) = \begin{vmatrix} \left(\lambda + \frac{1}{R_3 C}\right) & \left(\frac{1}{C}\right) \\ \left(-\frac{1}{L}\right) & \left(\lambda + \frac{R_{12}}{L}\right) \end{vmatrix} = \begin{vmatrix} (\lambda + 3.55) & (212.77) \\ (-20) & (\lambda + 847) \end{vmatrix} = \lambda^2 + 847\lambda + 3.54\lambda + 3006 + 4255$$

$$= \lambda^2 + 850.54\lambda + 7262.25$$

$$\Rightarrow \lambda_{1,2} = \begin{cases} \lambda_1 = -8.63 \\ \lambda_2 = -841.91 \end{cases}$$

Sappiamo che $v_C(t) \propto K_1 e^{\lambda_1 t} + K_2 e^{\lambda_2 t} \Rightarrow v_C(t) = K_1 e^{-863t} + K_2 e^{-842t}$

Per trovare K_1 e K_2 :

$$\begin{cases} v_C(t) = K_1 e^{-863t} + K_2 e^{-842t} \\ v_C(0^+) = 20.57 \ (1) \\ \dot{v}_C(0^+) = -\cancel{\frac{1}{C R_3} v_C(0^+)} - \frac{1}{C} i_L(0^+) + \frac{1}{C} I_0 = -\frac{1}{C} \cdot 0.257 + \frac{1}{C} \cdot 0.6 = 72.98 \ V \end{cases}$$

$\rightarrow$ Ci serve l'eq diff di II° ordine

$$(A) \begin{cases} i_C = I_0 - i_L - \dfrac{v_C}{R_3} \\ v_L = I_0 R_{12} - R_{12} i_L + v_C \end{cases} \rightarrow i_L = -i_C - \frac{v_C}{R_3} + I_0 \rightarrow \dot{i}_L = -\frac{\dot{v}_C}{R_3} - \dot{i}_C$$

$$(B) \begin{cases} \dot{v}_C = -\dfrac{1}{R_3 C} v_C - \dfrac{1}{C} i_L + \dfrac{1}{C} I_0 \\ \dot{i}_L = \dfrac{1}{L} v_C - \dfrac{R_{12}}{L} i_L + \dfrac{R_{12}}{L} I_0 \quad (2) \end{cases}$$

$\Rightarrow$ nella (2)

$$\rightarrow -\frac{\dot{v}_C}{R_3} - \dot{i}_C = \frac{1}{L} v_C - \frac{R_{12}}{L}\left(-i_C - \frac{v_C}{R_3} + I_0\right) + \frac{R_{12}}{L} I_0$$

$$\rightarrow -\frac{\dot{v}_C}{R_3} - \dot{i}_C = \frac{1}{L} v_C + \frac{R_{12}}{L} i_C + \frac{R_{12}}{L R_3} v_C - \cancel{\frac{R_{12}}{L} I_0} + \cancel{\frac{R_{12}}{L} I_0}$$

$$\dot{i}_C = C \ddot{v}_C$$

$$\frac{\dot{v}_C}{R_3} + C \ddot{v}_C \quad \frac{1}{L} v_C + \frac{R_{12} C}{L} \dot{v}_C + \frac{R_{12}}{L R_3} v_C = 0$$

$$\boxed{\begin{cases} i_C = C \cdot \dot{v}_C \\ v_L = L \ \dot{i}_L \end{cases} (C)}$$

$$\hookrightarrow C \ddot{v}_C + \dot{v}_C \left(\frac{1}{R_3} + \frac{R_{12} C}{L}\right) + v_C \left(\frac{1}{L} + \frac{R_{12}}{L R_3}\right) = 0$$

Siamo pronti !

$$\begin{cases} C\ddot{v}_c + \dot{v}_c\left(\frac{1}{R_3} + \frac{R_{12}C}{L}\right) + v_c\left(\frac{1}{L} + \frac{R_{12}}{LR_3}\right) = \emptyset \\ v_c(0^+) = 20.57 \ (1) \\ \dot{v}_c(0^+) = 72.98 \ V \end{cases}$$

$$\rightarrow \quad \ddot{v}_c + 72.98\left(\frac{1}{CR_3} + \frac{R_{12}C}{CL}\right) + 20.57\left(\frac{1}{CL} + \frac{R_{12}}{CLR_3}\right) = \emptyset$$

ALTRO METODO

$$v_c(t) = K_1 e^{-8.63\cdot t} + K_2 e^{-842\cdot t} \rightarrow \dot{v}_c(t) = -8.63\,K_1 e^{-8.63\cdot t} - 842\cdot K_2 e^{-842\cdot t}$$

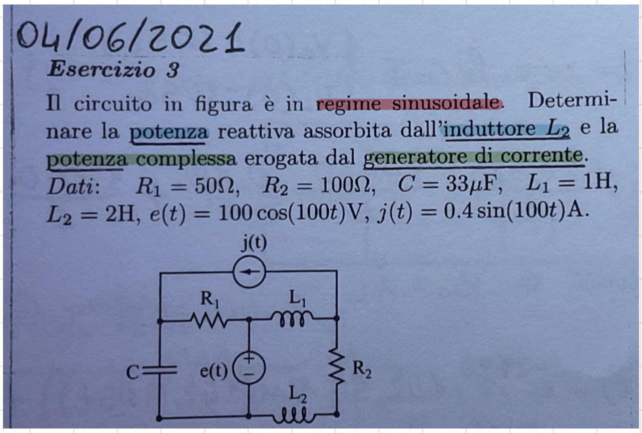
Cauchy ..

$$\begin{cases} v_c(t) = K_1 e^{-8.63t} + K_2 e^{-842t} \\ v_c(0^+) = 20.57 \\ \dot{v}_c(0^+) = 72.98 \end{cases} \Rightarrow \begin{cases} v_c(0^+) = K_1 \cancel{e^{-8.63\cdot\emptyset}} + K_2 \cancel{e^{-842\cdot\emptyset}} = 20.57 \rightarrow K_1 + K_2 = 20.57 \\ \dot{v}_c(0^+) = -8.63\,K_1 - 842\,K_2 = 72.98 \end{cases}$$

$$\Rightarrow \begin{cases} K_1 = 20.87 \\ K_2 = -0.3 \end{cases}$$

$$\Rightarrow \text{SOLUZIONE} \Rightarrow \boxed{v_c(t) = 20.87\,e^{-8.63\cdot t} - 0.3\,e^{-842\cdot t}}$$

Esame 04/06/21


04/06/2021

Esercizio 3

Il circuito in figura è in regime sinusoidale. Determinare la potenza reattiva assorbita dall'induttore L_2 e la potenza complessa erogata dal generatore di corrente.

Dati: $R_1 = 50\Omega$, $R_2 = 100\Omega$, $C = 33\mu F$, $L_1 = 1H$, $L_2 = 2H$, $e(t) = 100\cos(100t)V$, $j(t) = 0.4\sin(100t)A$.

DATI

$R_1 = 50\ \Omega$

$R_2 = 100\ \Omega$

$C = 33\ \mu F$

$L_1 = 1\ H$

$L_2 = 2\ H$

$e(t) = 100\cos(100t)$

$j(t) = 0.4\sin(100t)$

$= 0.4\cos(100t - \frac{\pi}{2}) \rightleftharpoons \bar{I} = 0.4\, e^{-\frac{\pi}{2}} = -0.4j$

A $\dot{Z}_1 = 50\ \Omega$

B $\dot{Z}_2 = 100\ \Omega$

C $\dot{Z}_C = -\frac{j}{100 \cdot 33 \times 10^{-6}} = -303.03j$

D $\dot{Z}_{L_1} = j100 \cdot 1 = 100j$

E $\dot{Z}_{L2} = j100 \cdot 2 = 200j$

$\bar{E} = 100\ V$

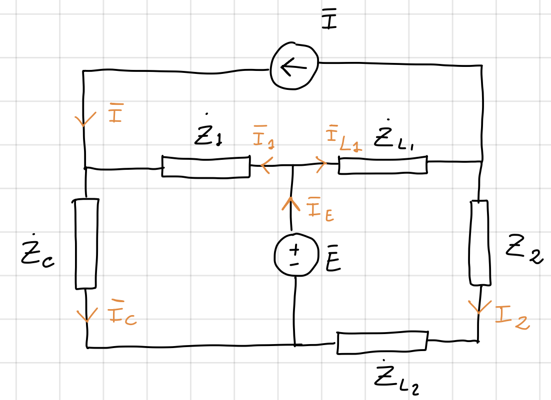
proof $e^{-\frac{\pi}{2}} = -j$

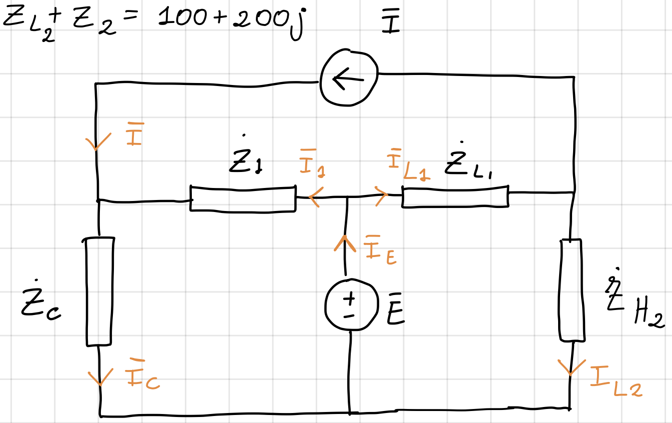
$\to e^{-\frac{\pi}{2}} = \cos(\frac{\pi}{2}) - j\sin(\frac{\pi}{2})$

$= \varnothing - j \cdot 1 = -j$ QED

STEP 1: METODO DEI FASORI

$\dot{Z}_{H2} = \dot{Z}_{L_2} + \dot{Z}_2 = 100 + 200j$ (E, B, F)





POTENZA GENERATORE

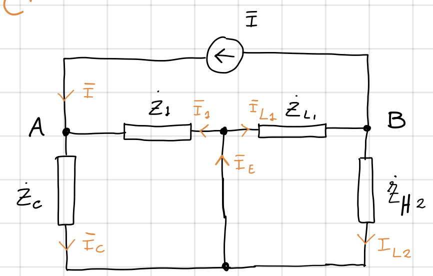
$P_J = \frac{1}{2} V_J \cdot I^*$

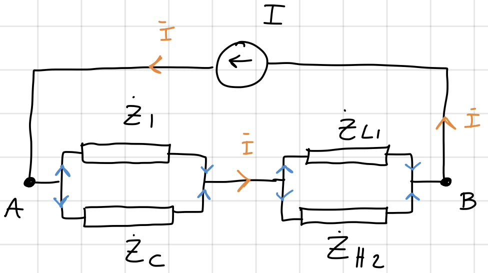
POTENZA INDUTTORE

$P_H = \frac{1}{2}(\bar{I}_H \cdot \dot{Z}_H) \cdot \bar{I}_H^*$

DA TROVARE

C'





$V_J' = V_{AB} = \dot{Z}_{eq} \cdot \bar{I}$

$= [(Z_1 /\!/ Z_C) + (Z_{L_1} /\!/ Z_{H_2})] \cdot \bar{I}$ (A, C, D, F)

$= (58.67 + 61.96j) \cdot \bar{I}$

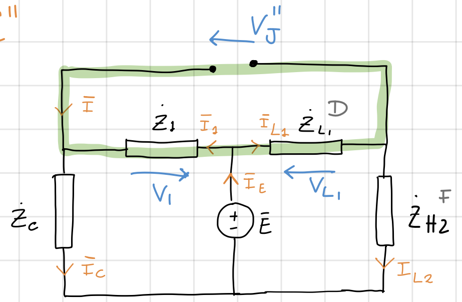
$= 24.79 - 23.47j\ V$ $\bar{V}_J'$

$I_{H_2} = \bar{I} \cdot \frac{\dot{Z}_{L_1}}{\dot{Z}_{L_1} + \dot{Z}_{H_2}} = 0.04 - 0.12j\ A$ $\bar{I}'_{L_2}$ (D, D, F)

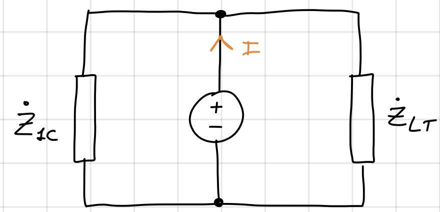
$\dot{Z}_{eq_1} = (Z_1 + Z_C) /\!/ (Z_{L_1} + Z_{H_2})$ (A, C, D, F)

$= 641.19 - 89.07j$

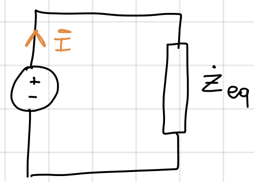
C''



$\to$



$\to$



$\bar{I}'' = \frac{\bar{E}}{\dot{Z}_{eq}} = 0.153 + 0.021j\ A$ (L)

$\to \bar{I}_{LT} = \bar{I}'' \cdot \frac{\dot{Z}_{1c}}{\dot{Z}_{1c} + \dot{Z}_{LT}} = 0.1 - 0.3j\ A$ $\bar{I}''_{L_2}$ (G, G, H, K)

$$\overset{\overline{V_J}''}{}$$

$$V_J - V_{L_1} + V_1 = 0 \quad \rightarrow \quad \bar{V}_J = \bar{V}_{L_1} - \bar{V}_1 = \overset{D}{\dot{Z}_{L_1}} \bar{I}_1^{J} - \overset{A}{\dot{Z}_1} \overset{K}{\bar{I}_{L_2}} = \boxed{-37.12 + 20.3j\ V}$$

$$J$$

$$-\bar{I}_E + \bar{I}_{L_1} + \bar{I}_1 = 0 \underset{I_{L_1} = I_{H_2} = I_{L_2}}{\rightarrow} \bar{I}_1 = \bar{I}_E - \bar{I}_{L_2} = 0.053 + 0.32j\ A$$

Applico la Sorrapposizione

$$\bar{I}_{L_2} = (0.04 - 0.12j) + (0.1 - 0.3j) = 0.14 - 0.42j\ A \quad H$$

$$\bar{V}_J = (24.79 - 23.47j) + (-37.12 + 20.3j) = -12.33 - 3.17j\ V \quad G$$

Ans 1

$$\Rightarrow P_J = \frac{1}{2} \underset{G}{\bar{V}_J} \cdot \underset{I}{\bar{I}}^* = 0.634 - 2.47j \quad W$$

Ans 2

$$P_H = \frac{1}{2} \underbrace{(\overset{H}{\bar{I}_{L_2}} \cdot \overset{E}{\dot{Z}_{L_2}})}_{\bar{V}_{L_2}} \cdot \overset{K^*}{\bar{I}^*_{L_2}} = 14j \quad W$$

L'esercizio chiedeva:

(P) - Media $\quad$ Q - Reattiva

Potenza REATTIVA di L_2 $\rightarrow$ $\dot{S} = \frac{1}{2} V \cdot I^* = \frac{1}{2} V_m I_m \left[\cos(\alpha - \beta) + j \sin(\alpha - \beta) \right]$

$\rightarrow$ $Q_{L_2} = \frac{1}{2} V_m I_m j \sin(\alpha - \beta)$ dove α, β sono le fasi di tensione e corrente

$$E = 100 e^{\alpha t} = 100 \quad \text{con } \alpha = 0$$

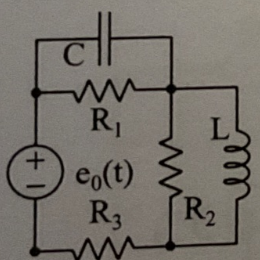
$$J = 0.4 e^{\beta t} = -0.4j \quad \text{con } \beta = -\frac{\pi}{2}$$

$$\Rightarrow \sin(0 + \frac{\pi}{2}) = 1 \Rightarrow Q_{L_2} = \frac{1}{2} V_m I_m j$$

7/04/2021

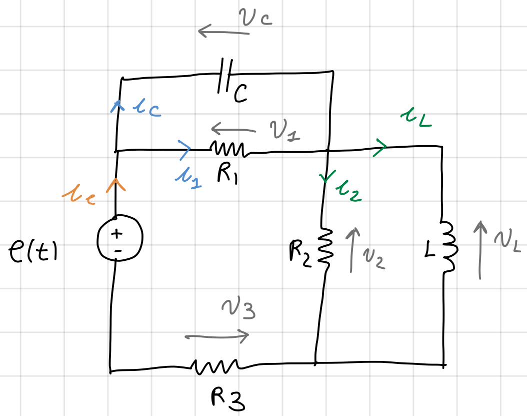
Esercizio 2

Il circuito è in regime stazionario per $t<0$. Determinare la corrente che scorre nell'induttore in ogni istante di tempo. *Dati:* $R_1 = 7\Omega$, $R_2 = 5\Omega$, $R_3 = 8\Omega$, $L = 0.01$H, $C = 47\mu$F. $e_0(t) = \begin{cases} 5V & t<0 \\ 7V & t>0 \end{cases}$



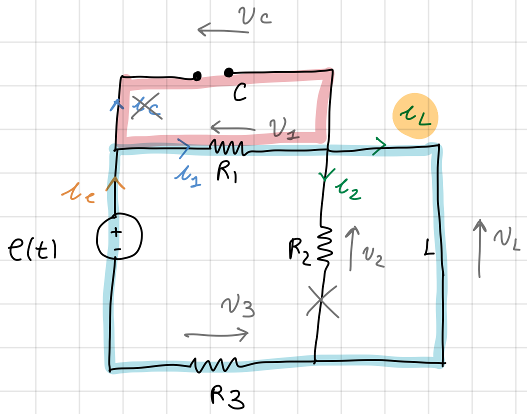
A $R_1 = 7\,\Omega$ C $R_3 = 8\,\Omega$ E $C = 47\mu F$

B $R_2 = 5\,\Omega$ D $L = 0.01\,H$



$$e_0(t) = \begin{cases} 5V & t<0 \\ 7V & t>0 \end{cases}$$

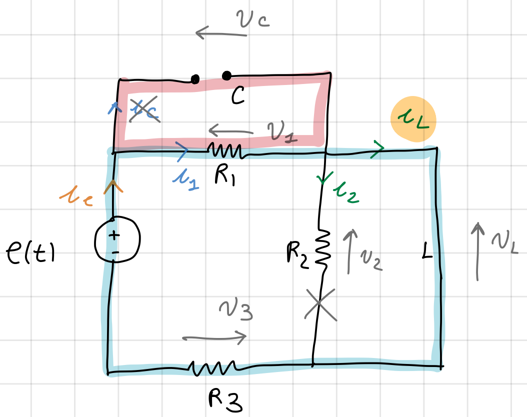
$\boxed{t<0}$ Stazionario $E_0 = 5V$



$$i_e = \frac{E_0}{R_{eq}} = \frac{E_0}{R_1+R_3} = 0.33\,A \quad i_L \quad \text{per } t<0$$

$$v_c + v_1 = 0 \to V_c = -v_1 = -i_e \cdot R_1 = -2.33\,V \quad V_c \; t<0$$

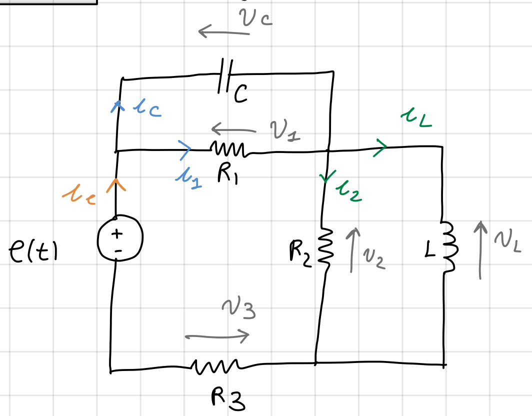
$\boxed{t\to\infty}$ Stazionario $E_1 = 7V$

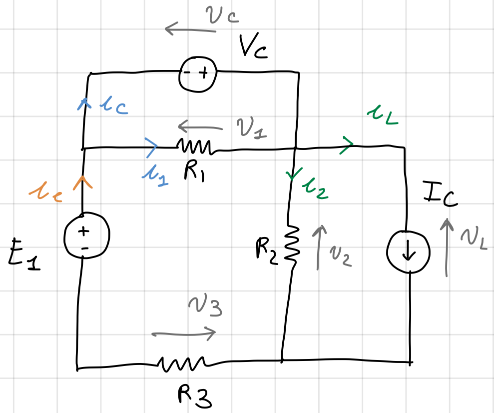


$$i_e = \frac{E_1}{R_{eq}} = \frac{E_1}{R_1+R_3} = 0.467\,A \quad i_L \quad t\to\infty$$

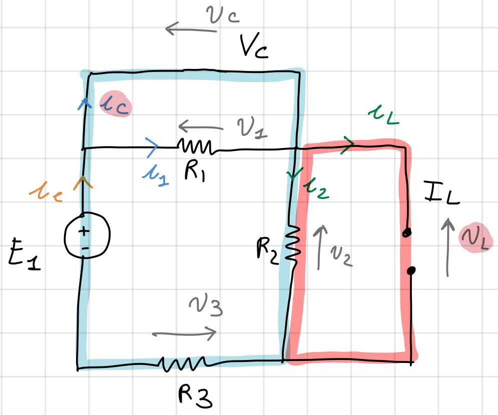
$$V_c = -3.267\,V \quad V_c \quad t\to\infty$$

$\boxed{t>0}$ $E_1 = 5V$





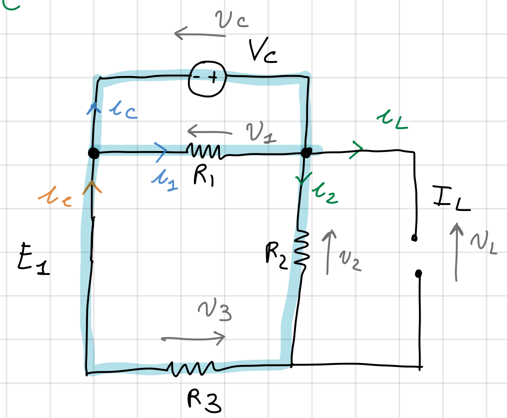
C'



$$i_C' = \frac{E_1}{R_3+R_2} \qquad i_C'$$

$$v_L - v_2 = 0 \;\to\; v_L = v_2 = i_C' \cdot R_2 = \frac{E_1}{R_3+R_2} \cdot R_2 = \frac{R_2}{R_2+R_3} E_1 \qquad v_L'$$

C''

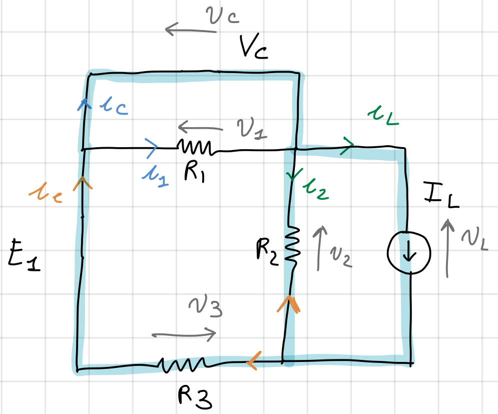


$$i_C'' = -\frac{v_C}{R_{eq}} = -\frac{v_C}{(R_2+R_3)//R_1} = -\frac{v_C}{\frac{(R_2+R_3)R_1}{R_2+R_3+R_1}} = -\frac{R_2+R_3+R_1}{R_1(R_2+R_3)} v_C \qquad i_C''$$

$$v_L = v_2 = v_C \cdot \frac{R_2}{R_2+R_3} \qquad v_L''$$

(part Tens)

C'''



$$v_L''' = I_L \cdot R_{eq} = i_L \cdot R_2//R_3 = -i_L \frac{R_2 \cdot R_3}{R_2+R_3} \qquad v_L$$

Verso opposto a v_L

$$i_C''' = i_L \cdot \frac{R_2}{R_2+R_3} \qquad i_C'''$$

$$\Rightarrow \begin{cases} v_L(t) = \frac{R_2}{R_2+R_3} E_1 + \frac{R_2}{R_2+R_3} v_C - \frac{R_2 R_3}{R_2+R_3} i_L \\ i_C(t) = \frac{1}{R_2+R_3} E_1 - \frac{R_1+R_2+R_3}{R_1(R_2+R_3)} v_C + \frac{R_2}{R_2+R_3} i_L \end{cases}$$

Con
$$\begin{cases} v_L(t) = L \dot{i}_L \\ i_C(t) = C \dot{v}_C \end{cases}$$

Sostituisco le R.C.

$$\begin{cases} C \dot{v}_C = \frac{1}{R_2+R_3} E_1 - \frac{R_1+R_2+R_3}{R_1(R_2+R_3)} v_C + \frac{R_2}{R_2+R_3} i_L \\ L \dot{i}_L = \frac{R_2}{R_2+R_3} E_1 + \frac{R_2}{R_2+R_3} v_C - \frac{R_2 R_3}{R_2+R_3} i_L \end{cases}$$

Riordino

$$\begin{cases} \dot{v} = M v + N i + O \\ \dot{i} = P v + Q i + R \end{cases}$$

$$\begin{cases} \dot{v}_C = \overbrace{-\frac{R_1+R_2+R_3}{C R_1(R_2+R_3)}}^{M} v_C + \overbrace{\frac{R_2}{C(R_2+R_3)}}^{N} i_L + \overbrace{\frac{1}{C(R_2+R_3)}}^{O} E_1 & (1) \\ \dot{i}_L = \underbrace{\frac{R_2}{L(R_2+R_3)}}_{P} v_C - \underbrace{\frac{R_2 R_3}{L(R_2+R_3)}}_{Q} i_L + \underbrace{\frac{R_2}{L(R_2+R_3)}}_{R} E_1 & (2) \end{cases}$$

$$\text{Polinomio}: \quad \det(\lambda I - A) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} (-4676) & (8183) \\ (38) & (-307) \end{pmatrix} = \begin{array}{cc} M & N \\ \left(\begin{array}{cc} \lambda + 4676 & -8183 \\ -38 & \lambda + 307 \end{array}\right) & \\ P & Q \end{array}$$

$$\to \lambda^2 + \lambda Q + M\lambda + MQ - NP = 0 \to \begin{cases} \lambda_1 = -236.7 \\ \lambda_2 = -4747 \end{cases}$$

$$\Rightarrow \underset{\substack{\uparrow \\ V(t) \text{ o } i(t)}}{y(t)} = C_1 e^{-236.7t} + C_2 e^{-4747t} + \underset{\substack{\uparrow \\ \text{Soluzione per } t \to \infty}}{y_P(t)}$$

$$\Rightarrow \begin{cases} i(t) = C_1 e^{-236.7t} + C_2 e^{-4747t} + 0.467 \\ i(0^+) = 0.33 \text{ A} \\ \dot{i}(0^+) = +P \cdot V_c(0) + Q\, i_L(0) + R = P \cdot (-2.33) + Q \cdot (0.33) + R = 460.39 \text{ A} \end{cases}$$

dalla (2)

$$\Rightarrow \dot{i}(t) = -236.7\, C_1 e^{-236.7t} - 4747\, C_2 e^{-4747t}$$

$$\Rightarrow \begin{cases} \dot{i}(0^+) = -236.7\, C_1 \cancel{e^{-236.7t}}^{1} - 4747\, C_2 \cancel{e^{-4747t}}^{1} = 460.39 \text{ A} \\ i(0^+) = C_1 \cancel{e^{-236.7t}}\ 1 + C_2 \cancel{e^{-4747t}}\ 1 + 0.467 = 0.33 \text{ A} \end{cases}$$

$$\to \begin{cases} -236.7\, C_1 - 4747\, C_2 = 460.39 \\ C_1 + C_2 = 0.33 - 0.467 \end{cases} \Rightarrow \begin{cases} C_1 = 0.421 \\ C_2 = 0.095 \end{cases}$$

$$i(t) = 0.421 \cdot e^{-236.7t} + 0.095 \cdot e^{-4747t} + 0.467$$

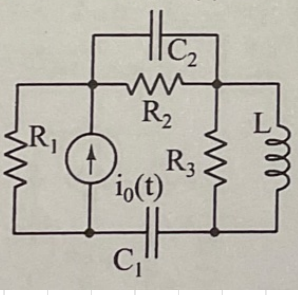
Per $t > 0$

G

28/06/2021

Esercizio 3

Determinare la potenza reattiva assorbita da C_2, del circuito in figura in regime sinusoidale. *Dati:* $R_1 = 20\Omega$, $R_2 = 15\Omega$, $R_3 = 16\Omega$, $C_1 = 5$ mF, $C_2 = 10$ mF, $L = 0.18$ H, $i_0(t) = 0.5\cos(10t)$ A.



$R_1 = 20\,\Omega \rightleftharpoons$ A $\dot Z_{R_1} = 20\,\Omega$

$R_2 = 15\,\Omega \rightleftharpoons$ B $\dot Z_{R_2} = 15\,\Omega$

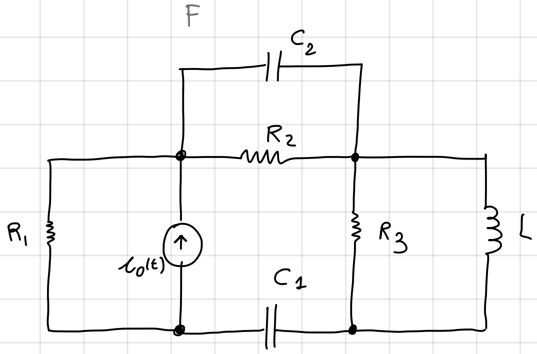
$R_3 = 16\,\Omega \rightleftharpoons$ C $\dot Z_{R_3} = 16\,\Omega$

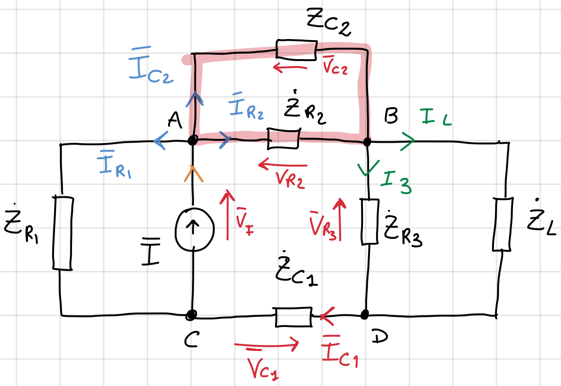
$C_1 = 5\,mF \rightleftharpoons$ D $\dot Z_{C_1} = -\dfrac{j}{10 \cdot 5\times10^{-3}} = -20j$

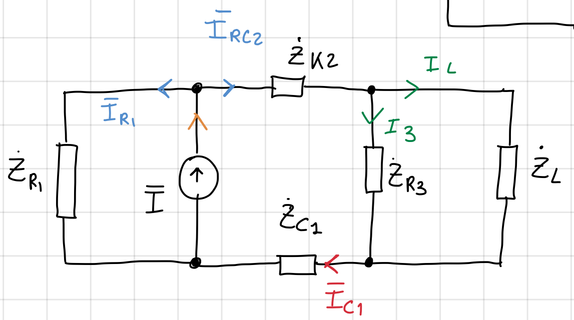
$C_2 = 10\,mF \rightleftharpoons$ E $\dot Z_{C_2} = -10j$

$i_0(t) = 0.5\cos(10t)\,A \rightleftharpoons \bar I = 0.5\,A$

$L = 0.18\,H \rightleftharpoons \dot Z_L = j10 \cdot 0.18 = 1.8j$ G (F)



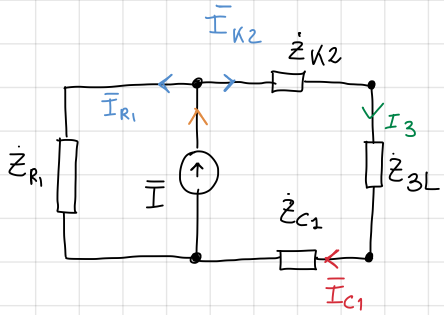


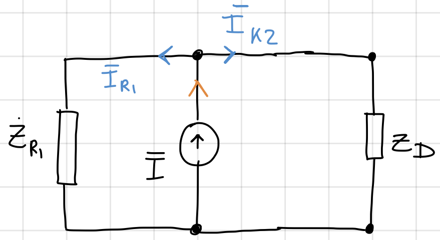


OBBIETTIVO

$\bar V_{C_2} = V_{AB} = ?$

$\bar I_{C_2} = ?$

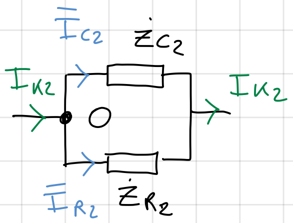




Con $\dot Z_{K2} = \dot Z_{C2} \parallel \dot Z_{R_2}$ (G; E, B)

$\dot Z_{3L} = \dot Z_{R_3} \parallel \dot Z_L$ (H; C, F)

$\dot Z_D = \dot Z_{K2} + \dot Z_{3L} + \dot Z_{C_1}$ (I; G, H, D)



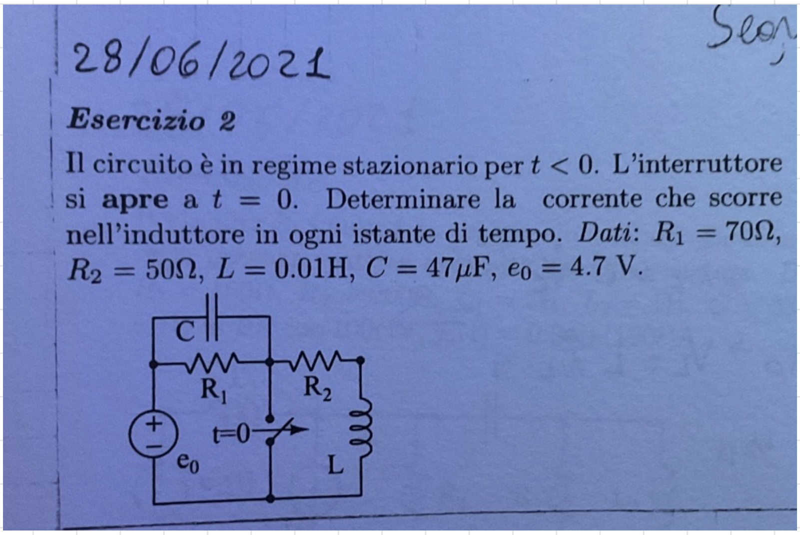
$$\bar I_{K2} = \bar I \cdot \frac{\dot Z_{R_1}}{\dot Z_{R_1} + \dot Z_D} = 0.198 + 0.2j\ A$$ (J; A, A, I)

$$\rightarrow \bar I_{C_2} = \bar I_{K2} \cdot \frac{\dot Z_{R_2}}{\dot Z_{R_2} + \dot Z_{C2}} = 0.45 + 0.23j\ A$$ (K; J, B, B, E)

$$\rightarrow \bar I_{R_2} = \bar I_{K2} - \bar I_{C2} = 0.15 - 0.03j\ A$$ (L; J, K)

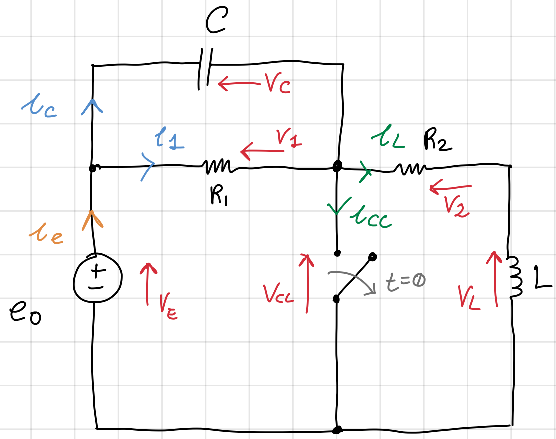
$$\text{LKT}_{M_1}:\quad \bar V_{C2} = \bar V_{R_2} = \bar I_{R_2} \cdot \dot Z_{R_2} = 2.31 - 0.45j\ V$$ (O; K, B)

$$\Rightarrow P_{C_2} = \frac{1}{2}\bar V_{C_2} \cdot \bar I^*_{C_2} = -0.247j\ W$$ (O, K*)


28/06/2021

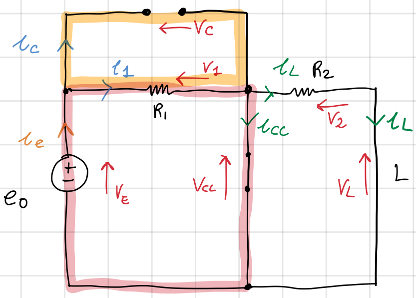
Esercizio 2

Il circuito è in regime stazionario per $t < 0$. L'interruttore si **apre** a $t = 0$. Determinare la corrente che scorre nell'induttore in ogni istante di tempo. *Dati:* $R_1 = 70\Omega$, $R_2 = 50\Omega$, $L = 0.01\text{H}$, $C = 47\mu\text{F}$, $e_0 = 4.7$ V.



A $R_1 = 70\,\Omega$ B $R_2 = 50\,\Omega$ C $L = 0.01$ H D $C = 47\,\mu F$ E $e_0 = 4.7$ V

$\boxed{t<0}$ Stazionario



C.I. v_C

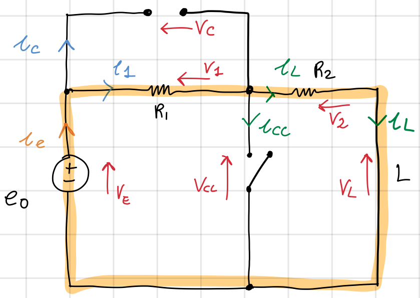
$v_C = V_1 \rightarrow$ $v_C = 4.7\,V$

$-V_E + V_1 = 0 \rightarrow V_E = V_1$

C.I i_L

$i_L = 0$

$\boxed{t \to \infty}$



i_L $t \to \infty$

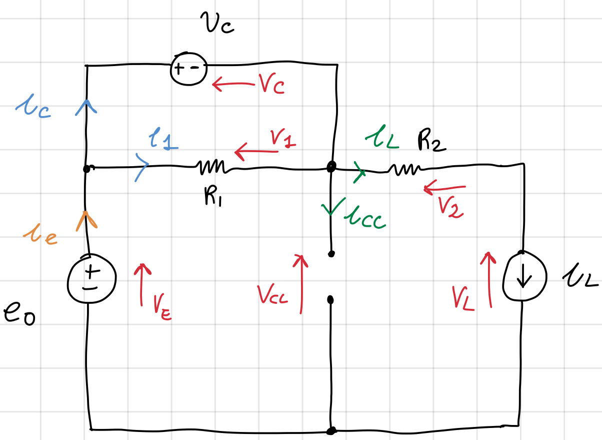
$$i_e = \frac{e_0}{R_1 + R_2} = 0.039 \text{ A}$$

Soluzioni particolari

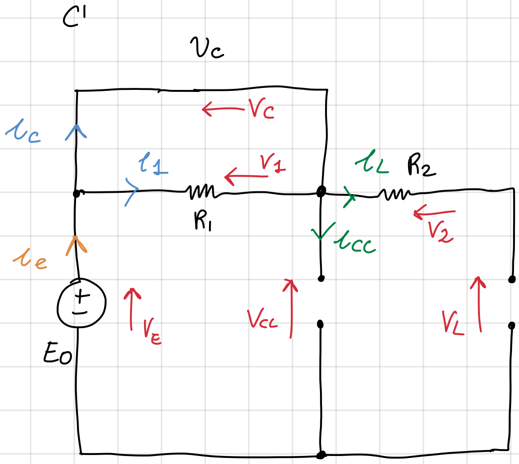
v_L $t \to \infty$

$$v_C = V_1 = R_1 \cdot i_e = 2.742\,V$$

$\boxed{t>0}$

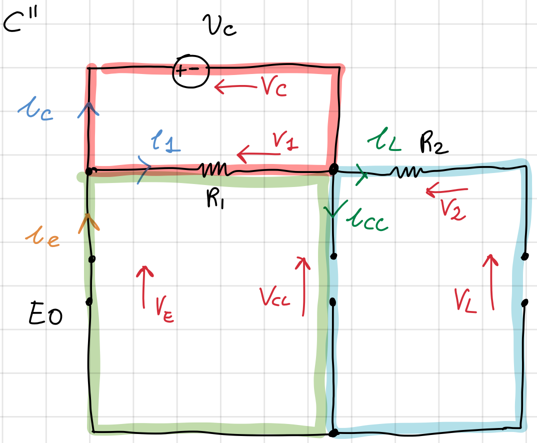


C'

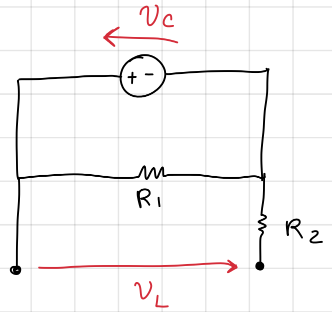


$$\begin{cases} i_C' = 0 \\ V_L' = E_0 \end{cases}$$

C''

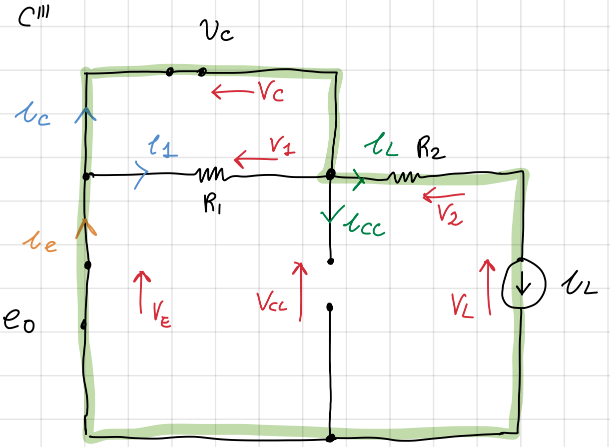


$$i_c'' = -\frac{v_c}{R_1}$$

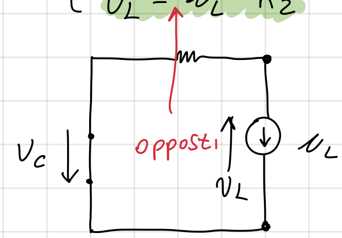


$\to$ $v_L'' = -v_c$

C'''



$$\begin{cases} i_c''' = i_L \\ v_L''' = -i_L \cdot R_2 \end{cases}$$



$$\begin{cases} i_c = -\dfrac{v_c}{R_1} + i_L \\ v_L = E_0 - v_c - R_2 i_L \end{cases} \quad \text{con} \quad \begin{cases} i_c = C\,\dot{v}_c \\ v_L = L\,\dot{i}_L \end{cases} \to \begin{cases} (1)\ \dot{v}_c = -\dfrac{1}{R_1 C} v_c + \dfrac{1}{C} i_L \\ (2)\ \dot{i}_L = -\dfrac{1}{L} v_c - \dfrac{R_2}{L} i_L + \dfrac{E_0}{L} \end{cases}$$

Polinomio: $\det(\lambda I - A) = \begin{vmatrix} \lambda + \frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & \lambda + \frac{R_2}{L} \end{vmatrix} = \begin{vmatrix} (\lambda + 304) & (-212\,77) \\ (100) & (\lambda + 5000) \end{vmatrix}$

$$\lambda^2 + \lambda 5000 + \lambda 304 + 1520000 + 2127700 \to \lambda^2 + 5304\lambda + 3647700$$

$$\Rightarrow \lambda_1 = -812 \qquad \lambda_2 = -4491$$

$$\Rightarrow y(t) = C_1 e^{-812\,t} + C_2 e^{-4491\,t} + y_p$$

$\to$ ci serve $i_L(t) = C_1 e^{-812\,t} + C_2 e^{-4491\,t} + 0.039$

Determiniamo c_1 e c_2 $\to$ Cauchy

$$\begin{cases} i_L(t) = C_1 e^{-812\,t} + C_2 e^{-4491\,t} + 0.039 \\ i_L(0^+) = \emptyset \\ (2)\ \dot{i}_L(0^+) = -\dfrac{1}{L}\cancel{v_c(0)} - \dfrac{R_2}{L}\cancel{i_L(0^+)} + \dfrac{E_0}{L} = \emptyset \end{cases} \to \begin{cases} \dot{i}_L(0^+) = -812\,C_1 - 4491\,C_2 = \emptyset \\ i_L(0^+) = C_1 + C_2 + 0.039 = \emptyset \end{cases}$$

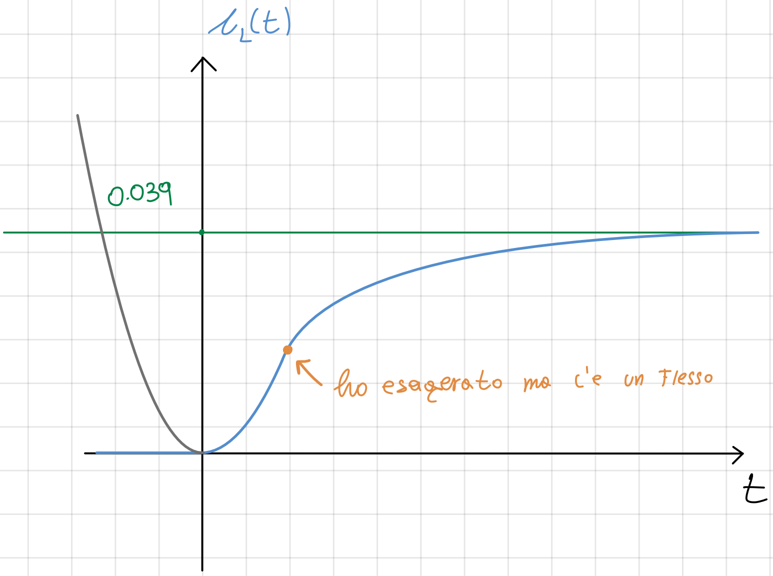
C 4.7 $\emptyset$ 47 C

$\Rightarrow C_1 = -0.039 - C_2 \quad \Rightarrow \quad +812 \cdot 0.039 + 821 C_2 - 4491 C_2 = 0$

$\Rightarrow C_2 = -\frac{31.668}{3679} = 8.6 \times 10^{-03} = \underline{0.008}$ c_2

$\Rightarrow C_1 = -0.039 - 0.008 = \underline{-0.048}$ c_1

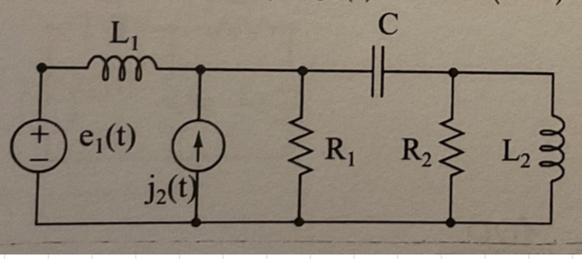
$$i_L(t) = -0.048\, e^{-812\, t} + 0.008\, e^{-4491\, t} + 0.039$$

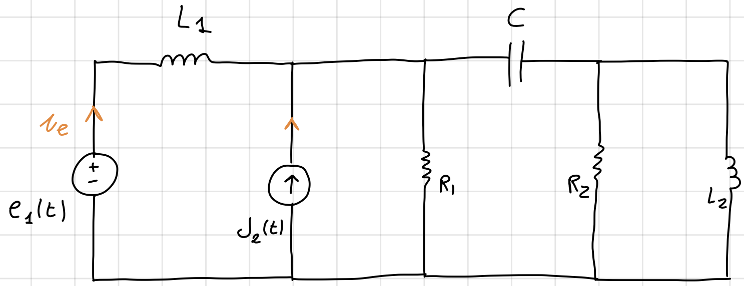


28/06/2021

Esercizio 3

Il circuito in figura è in regime sinusoidale. Determinare la potenza reattiva assorbita dal condensatore. *Dati:* $R_1 = 100\Omega$, $R_2 = 200\Omega$, $L_1 = 2\text{H}$, $L_2 = 1\text{H}$, $C = 33\mu\text{F}$, $e_1(t) = 100\cos(100t)\text{V}$, $j_2(t) = 0.3\sin(100t)\text{A}$





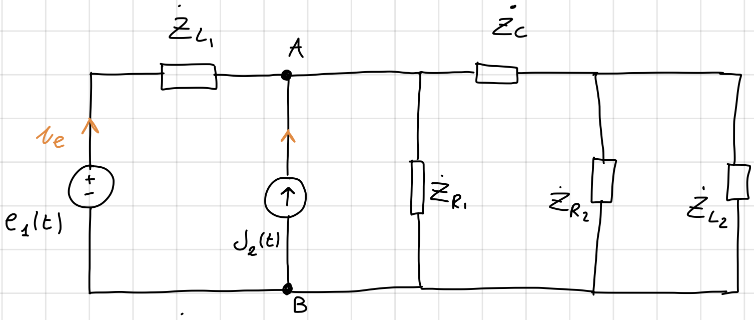
$R_1 = 1000\ \Omega \rightleftharpoons$ A $\dot Z_1 = 1000$

$R_2 = 200\ \Omega \rightleftharpoons$ B $\dot Z_2 = 200$

$L_1 = 2\,H \rightleftharpoons$ C $\dot Z_{L_1} = j\,100\,2 = 200j$

$L_2 = 1\,H \rightleftharpoons$ D $\dot Z_{L_2} = j\,100\,1 = 100j$

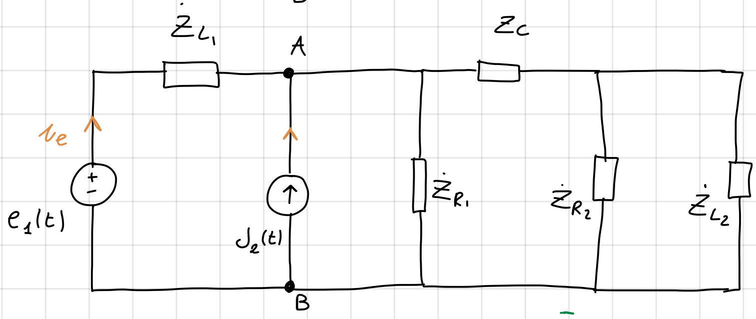
$C = 33\mu F \rightleftharpoons$ E $\dot Z_c = -\frac{j}{100\,33\mu} = -303.03j$



$e_1(t) = 100\cos(100t) \rightleftharpoons \bar E_1 = 100$

$j_2(t) = 0.3\sin(100t) = 0.3\cos(100t - \frac{\pi}{2})$

$\rightleftharpoons \bar J_2 = 0.3\, e^{-\frac{\pi}{2}} = -0.3j \quad \bar J_2$



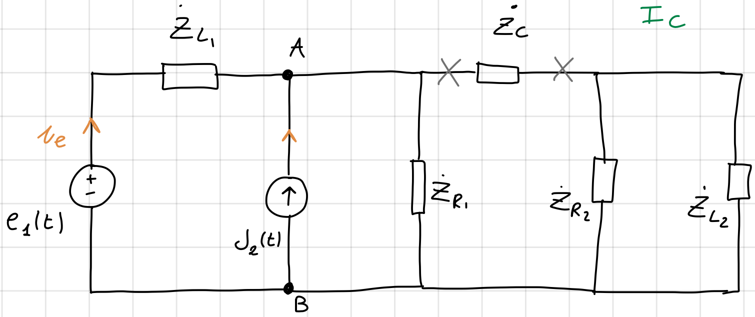
$e_1(t) = 100\cos(100t) \rightleftharpoons \bar E_1 = 100$

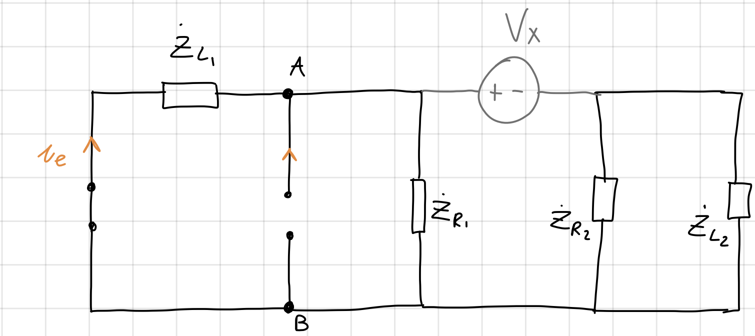
$j_2(t) = 0.3\sin(100t) = 0.3\cos(100t - \frac{\pi}{2})$

$\rightleftharpoons \bar J_2 = 0.3\, e^{-\frac{\pi}{2}} = -0.3j \quad \bar J_2$

Pow Reattiva: $\hat P = \frac{1}{2}\big(\underbrace{\dot Z_c \cdot \bar I_c}_{\bar V_c}\big) \cdot \bar I_c^*$

ci basta I_c

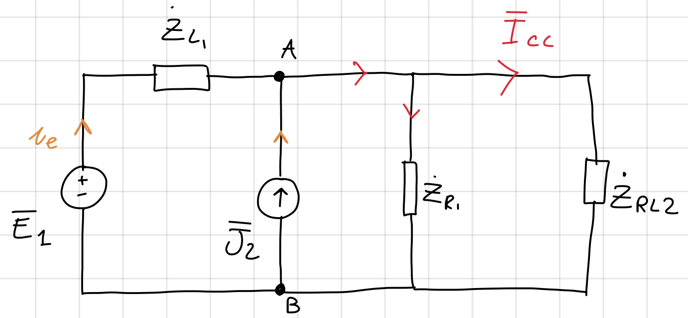




$\overset{F}{\dot Z_N} = (\overset{B}{\dot Z_{R_2}} \,\|\, \overset{D}{\dot Z_{L_2}}) + (\overset{C}{\dot Z_{L_1}} \,\|\, \overset{A}{\dot Z_{R_1}})$

$\overset{!}{=} 78.46 + 272.31j$

$$\text{Con } \overset{G}{\dot{Z}_{RL2}} = \overset{B}{\dot{Z}_{R2}} \,||\, \overset{D}{\dot{Z}_{L2}} = 40+80j$$

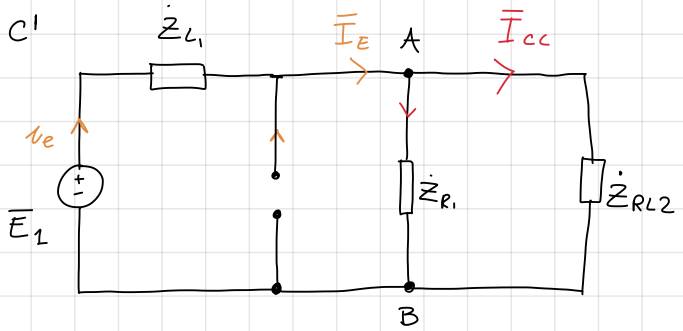


$\bar{E}_1 = 100$

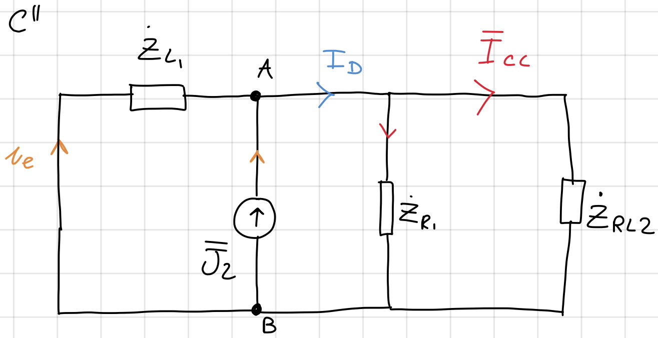
$\bar{J}_2 = -0.3j$

A $\dot{Z}_{R_1}$
B $\dot{Z}_{R_2}$
C $\dot{Z}_{L_1}$
D $\dot{Z}_{L_2}$
E $\dot{Z}_c$

C'



C''



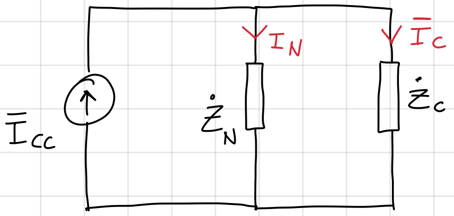
$$\bar{I}_E = \frac{\bar{E}_1}{Z_{eq}} = \frac{\bar{E}_1}{(\underset{A}{Z_{R_1}} \,||\, \underset{G}{Z_{RL2}}) + \underset{C}{Z_{L_1}}} = 0.057 - 0.035$$

$$\overset{I}{I_D} = \dot{J}_2 \cdot \frac{\overset{C}{\dot{Z}_{L_1}}}{\underset{C}{\dot{Z}_{L_1}} + (\underset{A}{\dot{Z}_{R_1}} \,||\, \underset{G}{\dot{Z}_{RL2}})} = 0.034 - 0.21j \text{ A}$$

$$\Rightarrow \underset{H}{\bar{I}'_{cc}} = \underset{I}{\bar{I}_E} \cdot \frac{\dot{Z}_{R_1}^{\,A}}{\underset{A}{\dot{Z}_{R_1}} + \underset{G}{\dot{Z}_{RL2}}} = 0.029 - 0.34j \text{ A}$$

$$\rightarrow \overset{I}{\bar{I}''_{cc}} = \underset{I}{I_D} \cdot \frac{\dot{Z}_{R_1}^{\,A}}{\underset{A}{\dot{Z}_{R_1}} + \underset{G}{\dot{Z}_{RL2}}} = 0.017 - 0.21j \text{ A}$$

$$\Rightarrow \bar{I}_{cc} = \overset{H}{\bar{I}'_{cc}} + \overset{I}{\bar{I}''_{cc}} = 0.0459 - 0.552j \text{ A}$$

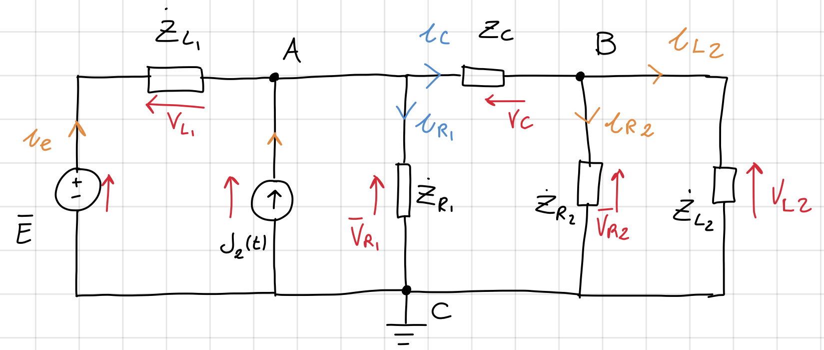


$$\begin{cases} \overset{H}{\bar{I}_{cc}} = 0.0459 - 0.552j \text{ A} \\ \overset{F}{\dot{Z}_N} = 78.46 + 272.31j \\ \overset{E}{\dot{Z}_c} = -303.03j \end{cases}$$

$$\overset{I}{\bar{I}_c} = \underset{H}{\bar{I}_{cc}} \cdot \frac{\overset{F}{\dot{Z}_N}}{\underset{F}{\dot{Z}_N} + \underset{E}{\dot{Z}_c}} = 1.83 + 0.33j \text{ A}$$

$$\Rightarrow \hat{P} = \frac{1}{2} (\underset{E}{\bar{Z}_c} \cdot \underset{I}{\bar{I}_c}) \cdot \underset{I^*}{\bar{I}_c^*} = -525.29j$$

Risoluzione con potenziali di Nodo



Pongo $U_C = 0$

$\bar E_1 = 100$

$\bar J_2 = -0.3j$

A $\dot Z_{R1}$
B $\dot Z_{R_2}$
C $\dot Z_{L_1}$
D $\dot Z_{L2}$
E $\dot Z_c$

(1) LKC

$$\begin{cases} A: -\bar I_E - J_2 + i_C + i_{R_1} = 0 \\ B: -i_C + i_{L2} + i_{R2} = 0 \end{cases}$$

(2) Potenziali

$V_{L1} = U_E - U_A$

$V_{R_1} = U_A$

$V_C = U_A - U_B$

$V_{R2} = U_B$

$V_{L2} = U_B$

(3) Sostituisco

$$\begin{cases} \bar I_E = -\bar J_2 + \dfrac{\bar U_A}{\dot Z_C} - \dfrac{\bar U_B}{\dot Z_C} + \dfrac{\bar U_A}{\dot Z_{R_1}} \\ \bar I_C = \dfrac{\bar U_B}{\dot Z_{L2}} + \dfrac{\bar U_B}{\dot Z_{R_2}} \end{cases}$$

ma

$$\begin{cases} (1)\ \bar I_E = \bar I_{L1} = \dfrac{\bar V_{L_1}}{\dot Z_{L_1}} = \dfrac{\bar U_E - \bar U_A}{\dot Z_{L1}} \\ (2)\ \bar I_C = \dfrac{\bar V_C}{\dot Z_C} = \dfrac{\bar U_A - \bar U_B}{\dot Z_C} \end{cases}$$

$$\Rightarrow \begin{cases} -\dfrac{\bar U_A}{\dot Z_{L_1}} - \dfrac{\bar U_A}{\dot Z_C} + \dfrac{\bar U_B}{\dot Z_C} - \dfrac{\bar U_A}{\dot Z_{R_1}} = -\bar J_2 - \dfrac{\bar U_E}{\dot Z_{L_1}} \\ \dfrac{\bar U_A}{\dot Z_C} - \dfrac{\bar U_B}{\dot Z_C} - \dfrac{\bar U_B}{\dot Z_{L2}} - \dfrac{\bar U_B}{\dot Z_{R_2}} = 0 \end{cases} = \begin{cases} U_A\left(-\underset{C}{\dfrac{1}{\dot Z_{L_1}}} - \underset{E}{\dfrac{1}{\dot Z_C}} - \underset{A}{\dfrac{1}{\dot Z_{R_1}}}\right) + U_B\left(\underset{E}{\dfrac{1}{\dot Z_C}}\right) = -J_2 - \dfrac{U_E}{\dot Z_{L_1}} \\ U_A\left(\underset{E}{\dfrac{1}{\dot Z_C}}\right) + U_B\left(-\underset{E}{\dfrac{1}{\dot Z_C}} - \underset{D}{\dfrac{1}{\dot Z_{L2}}} - \underset{B}{\dfrac{1}{\dot Z_{R2}}}\right) = 0 \end{cases}$$

$$\begin{cases} U_A\left[10^{-3}(-1+1.7j)\right] + U_B\left[10^{-3}(3.3j)\right] = 0.2j \\ U_A\left[10^{-3}(3.3j)\right] + U_B\left[-\frac{1}{200} + 6.7\times 10^{-3} j\right] = 0 \end{cases} \Rightarrow \begin{cases} M U_A + N U_B = O \\ P U_A + Q U_B = \emptyset = R \end{cases}$$

$$\Rightarrow M U_A + N U_B = O \Rightarrow U_A = \frac{O - N U_B}{M}$$

$$\Rightarrow \frac{PO}{M} - \frac{NP\, U_B}{M} + Q U_B = 0 \Rightarrow \overset{U}{U_B} = -\frac{PO}{M} \cdot \frac{1}{Q - \frac{NP}{M}} = 11.82 + 39.92j \text{ V}$$

$$\Rightarrow \overset{V}{U_A} = 36.49 - 98.96j \text{ V}$$

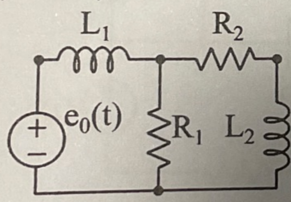
dalla (2) $\overset{W}{\bar I_C} = \dfrac{\overset{V}{\bar U_A} - \overset{U}{\bar U_B}}{\underset{E}{\dot Z_C}} = 0.46 + 0.081j$ A

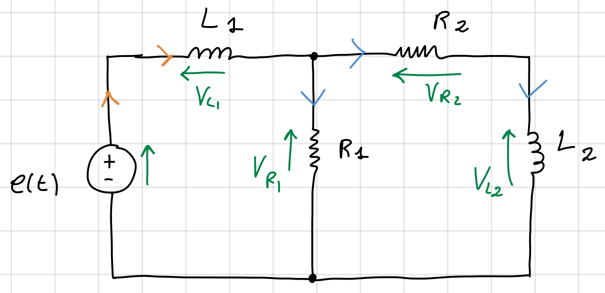
$$\Rightarrow \hat P = \frac{1}{2} \overset{E}{\dot Z_C} \cdot \overset{W}{\bar I_C} \cdot \overset{W^*}{\bar I_C^*} = -32.83j$$

24/02/2022

Esercizio 2

Il circuito è in regime stazionario per $\boxed{t < 0.}$ Il generatore si **spegne** all'istante $t = 0$ e il circuito va in evoluzione libera. Determinare la **corrente nell'induttore L_2 in ogni istante di tempo**. Dati: $R_1 = 40\Omega$, $R_2 = 80\Omega$, $L_1 = 1$ mH, $L_2 = 2$ mH, $e_0(t) = 3$ V per $t < 0$.

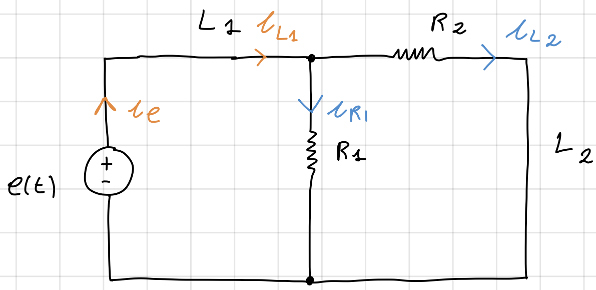




A $R_1 = 40\,\Omega$ B $R_2 = 80\,\Omega$ C $L_1 = 1mH$

D $L_2 = 2mH$ E $e_0(t) = 3V \quad t<0$

$\boxed{t<0}$

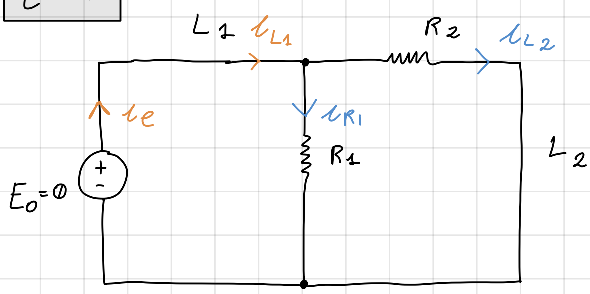


$$i_e = i_{L_1} = \frac{E_0}{R_1 // R_2} = 0.1125\,A \quad i_{L1} \quad t<0$$

C.I.

$$i_{L2} = i_e \cdot \frac{R_1}{R_1+R_2} = 0.0375\,A \quad i_{L2} \quad t<0$$

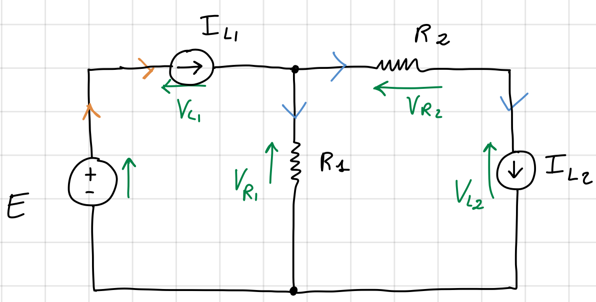
$\boxed{t \to \infty}$



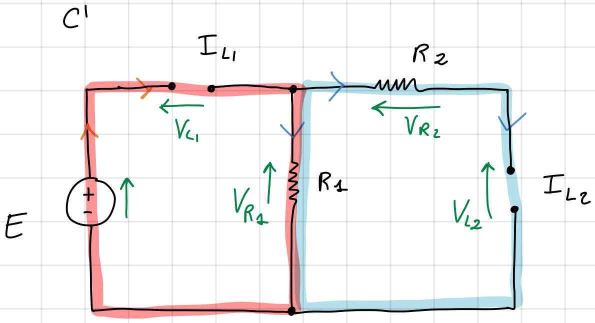
$$i_{L_1} = i_{L_2} = \emptyset \quad t \to \infty$$

C.P.

$\boxed{t>0}$



C'

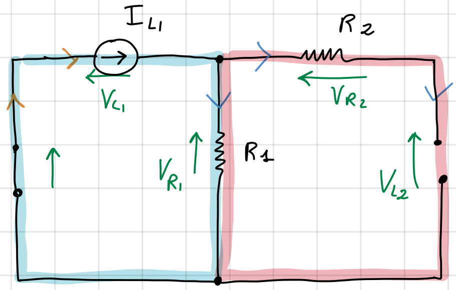


$$-E + v_{L_1} + \cancel{v_{R_1}} = 0 \to v'_{L_1} = E$$

$i_1 = 0$

$$-v_{R_2} + v_{L_2} - \overset{\emptyset}{\cancel{v_{R_1}}} = 0 \to v'_{L_2} = v_{R_2} = \emptyset$$

C''

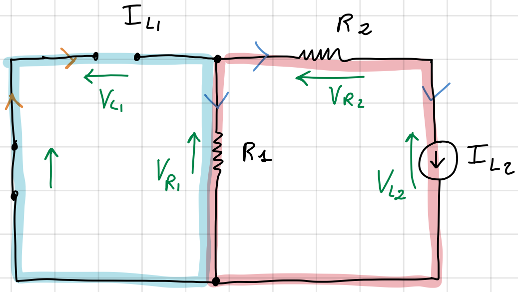


$$V''_{L1} = -i_{L1} \cdot R_1$$

$i_2 = 0$

$$\cancel{V_{R2}} + V_{L2} - V_{R_1} = 0 \to V''_{L2} = V_{R1} = i_{L1} R_1$$

C'''



$$V'''_{L2} = -i_{L2} \cdot (R_1 + R_2)$$

$$V_{L1} + V_{R_1} = 0 \to V'''_{L1} = -V_{R1} = -i_{L2} R_1$$

$$\Rightarrow \begin{cases} V_{L1} = E - i_{L1}R_1 - i_{L2}R_1 \\ V_{L2} = i_{L1}R_1 - i_{L2}(R_1+R_2) \end{cases} \quad \text{con} \quad V_L = L\dot{i}_L \to \begin{cases} L\dot{i}_{L1} = E - i_{L1}R_1 - i_{L2}R_1 \\ L\dot{i}_{L2} = i_{L1}R_1 - i_{L2}(R_1+R_2) \end{cases}$$

$$\begin{cases} L_1\dot{i}_{L1} = -i_{L1}R_1 - i_{L2}R_1 + E \\ L_2\dot{i}_{L2} = +i_{L1}R_1 - i_{L2}(R_1+R_2) \end{cases} \to \begin{cases} (1)\ \dot{i}_{L1} = -\frac{R_1}{L_1} i_{L1} - \frac{R_1}{L_1} i_{L2} + E \\ (2)\ \dot{i}_{L2} = \frac{R_1}{L_2} i_{L1} - \frac{R_1+R_2}{L_2} i_{L2} \end{cases}$$

$$\text{Polinomio} = \det(\lambda I - A) = \begin{vmatrix} \left(\lambda + \overset{M}{\frac{R_1}{L_1}}\right) & \overset{N}{\frac{R_1}{L_1}} \\ \overset{P}{-\frac{R_1}{L_2}} & \left(\lambda + \overset{Q}{\frac{R_1+R_2}{L_2}}\right) \end{vmatrix} = \lambda^2 + \lambda Q + \lambda M + MQ + NP =$$

$$= \lambda^2 + \lambda(Q+M) + MQ + NP$$

$$\to \lambda^2 + 100000\lambda + 32 \times 10^8 = 0$$

$\to$

$$\lambda_{1,2} = -50000 \pm 26457j$$

$$\Rightarrow y(t) = C_1 e^{Re(\lambda^+)} \cos[Imm(\lambda^+)t] + C_2 e^{Re(\lambda^-)} \sin[Imm(\lambda^-)t] = C_1 e + y_p(t)$$

$$\to Q: \begin{cases} i_{L2}(t) = C_1 e^{-50000t} \cos(26457t) + C_2 e^{-50000t} \sin(26457t) + 0 \\ i_{L2}(0^+) = 0.0375\ A \\ \dot{i}_{L2}(0^+) = 0 \end{cases}$$

da $t<0$

dalla (2)

$$g(x) \cdot f(x) = g'f + gf'$$

$$\Rightarrow \begin{cases} C_1 = 0.0375 \\ -50k\, C_1 \cancel{e^{-50kt}} \cancel{\cos(26k t)} - C_1 \cancel{e^{-50k} \sin(26k \cdot t)} - 50k\, C_2 \cancel{e^{-50kt}} \sin(26k \cdot t) + C_2 \cancel{e^{-50kt}} \cos(26k\, t) \end{cases}$$

??

$$\to -50k\, C_1 + C_2 = 0$$

$\Rightarrow \quad C_1 = \frac{C_2}{50k} \rightarrow \frac{C_2}{50k} = 0.0375 \Rightarrow C_2 = 50k \cdot 0.0375 = 1875 \quad C_2$

$C_1 = 50k \quad C_1$??

BOH c'è qualcosa che non va ¡¡

$\rightarrow i_{L_2}(t) = 50k \, e^{-50000t} \cos(26457t) + 1875 \, e^{-50'000t} \sin(26457t) + 0$

? ?

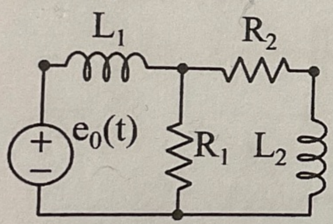
Sbagliato

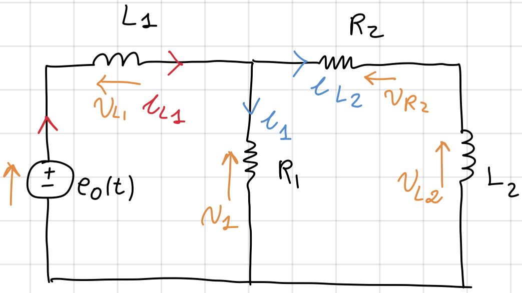
vedi giù

24/02/2022 giusto ↑

Esercizio 2

Il circuito è in regime stazionario per $t < 0$. Il generatore si spegne all'istante $t = 0$ e il circuito va in evoluzione libera. Determinare la **corrente nell'induttore L_2 in ogni istante di tempo.** Dati: $R_1 = 40\Omega$, $R_2 = 80\Omega$, $L_1 = 1$ mH, $L_2 = 2$ mH, $e_0(t) = 3$ V per $t < 0$.



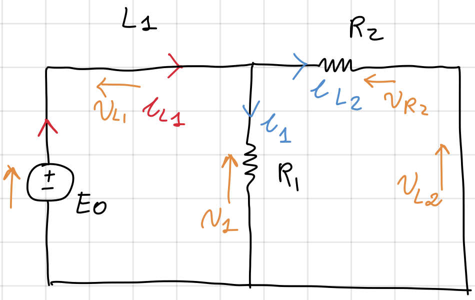


$R_1 = 40\,\Omega$ (A) $\quad R_2 = 80\,\Omega$ (B)

$L_1 = 1\,mH$ (C) $\quad L_2 = 2\,mH$ (D)

$$e_0(t) = \begin{cases} 3\,V_E & t < 0 \\ 0 & t > 0 \end{cases}$$

$\boxed{t < 0}$ Stazionario $\quad E_0 = 3V$



$$i_e = \frac{E_0}{R_1 \| R_2} = 0.1125\,A \quad i_{L1}$$

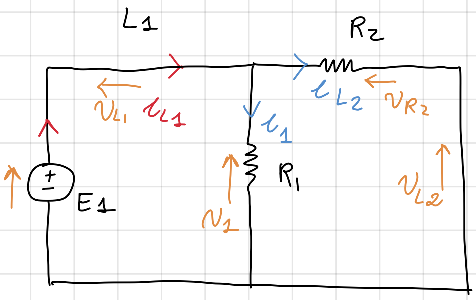
C.I.

$$i_{L2} = i_{L1} \cdot \frac{R_1^{\,A}}{R_1 + R_2} = 0.0375\,A \quad i_{L2}$$

(Ans; A, B)

per la continuità delle grandezze di stato $i_{L_2}(0^+) = i_{L_2}(0^-)$

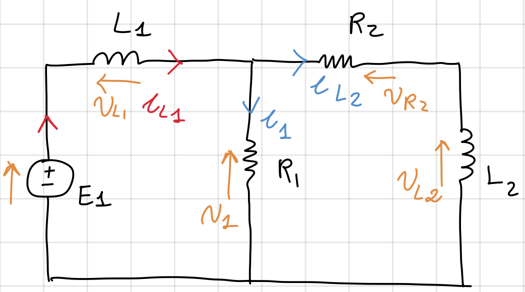
$\boxed{t \to \infty}$ $\quad E_1 = \emptyset$



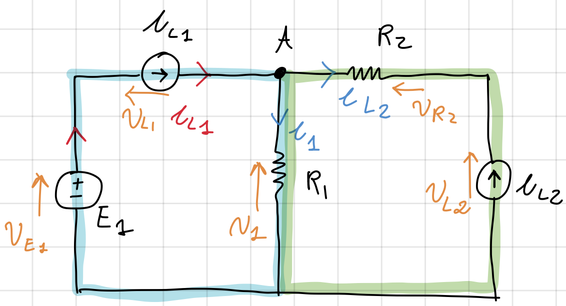
S. P.

$$E_1 = 0 \Rightarrow i_{L1}(\infty) = i_{L2}(\infty) = \emptyset$$

$\boxed{t > 0}$



C.R.A.



R.C. : $v_L = L\,\dot{i}_L$

$\to v_{L1} + v_1 - v_E = 0 \to v_{L1} + i_1 R_1 - E_1 = 0$

LKC$_{N_1}$: $-i_{L1} + i_1 + i_{L2} = \emptyset$

$\to v_{L2} - v_1 + v_2 = 0 \to v_{L2} - i_1 R_1 + i_{L2} R_2 = \emptyset$

$\to i_1 = i_{L1} - i_{L2}$

$$\Rightarrow \begin{cases} v_{L1} + i_{L1} R_1 - i_{L2} R_1 - E_1 = 0 \\ v_{L2} - i_{L1} R_1 + i_{L2} R_1 + i_{L2} R_2 = \emptyset \end{cases} \to \begin{cases} v_{L1} = -R_1 i_{L1} + R_1 i_{L2} + \cancel{E_1} \\ v_{L2} = R_1 i_{L1} - (R_1 + R_2) i_{L2} \end{cases}$$

$E_1 = \emptyset$

$$\begin{cases} \dot{i}_{L1} = -\frac{R_1}{L_1} i_{L1} + \frac{R_1}{L_1} i_{L2} \\ \dot{i}_{L2} = \frac{R_1}{L_2} i_{L1} - \frac{R_1+R_2}{L_2} i_{L2} \end{cases} \rightarrow \det(\lambda I - A) = \begin{vmatrix} \left(\lambda + \frac{R_1}{L_1}\right) & -\frac{R_1}{L_1} \\ -\frac{R_1}{L_2} & \left(\lambda + \frac{R_1+R_2}{L_2}\right) \end{vmatrix} = 0$$

$$\rightarrow \underset{M}{\lambda^2} + \underset{N}{\lambda\left(\frac{R_1}{L_1} + \frac{R_1+R_2}{L_2}\right)} + \underset{O}{\left(\frac{R_1}{L_1}\cdot\frac{R_1+R_2}{L_2} - \frac{R_1}{L_1}\cdot\frac{R_1}{L_2}\right)} = 0$$

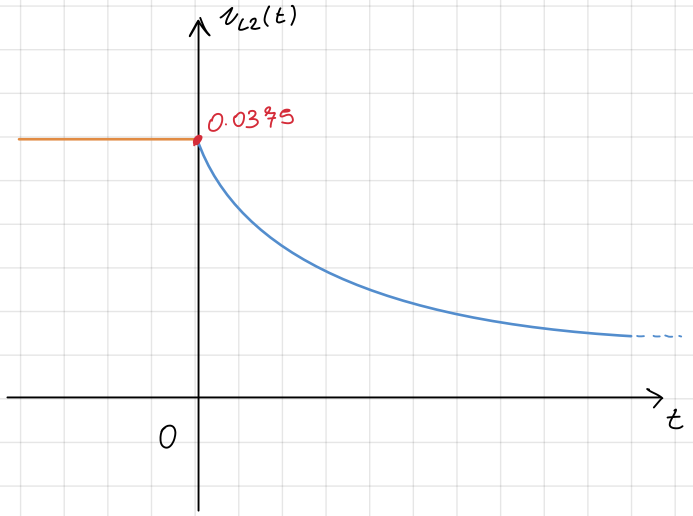
$$\rightarrow \begin{cases} \lambda_1 = -2000 \\ \lambda_2 = -8000 \end{cases} \Rightarrow y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \cancel{y_p(t)}^{\,0}$$

$$Q: \begin{cases} i_{L2}(t) = C_1 e^{-2000t} + C_2 e^{-8000t} \\ i_{L2}(0^+) = 0.0375\ A \\ \dot{i}_{L2}(0^+) = \frac{R_1}{L_2}\underbrace{i_{L1}(0^+)}_{0.1125} - \frac{R_1+R_2}{L_2}\underbrace{i_{L2}(0^+)}_{0.0375} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_1 + C_2 = 0.0375 \\ C_1\cdot\lambda_1 + C_2\lambda_2 = -2000C_1 - 8000C_2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_1 = 0.05 \\ C_2 = -0.012 \end{cases}$$

$$i_{L2}(t) = 0.05\, e^{-2000t} - 0.012\, e^{-8000t} \qquad t > 0$$

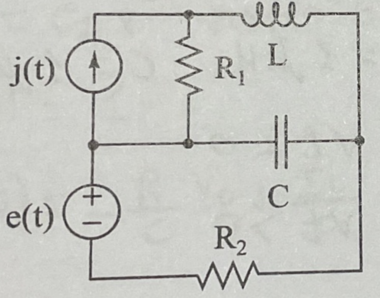


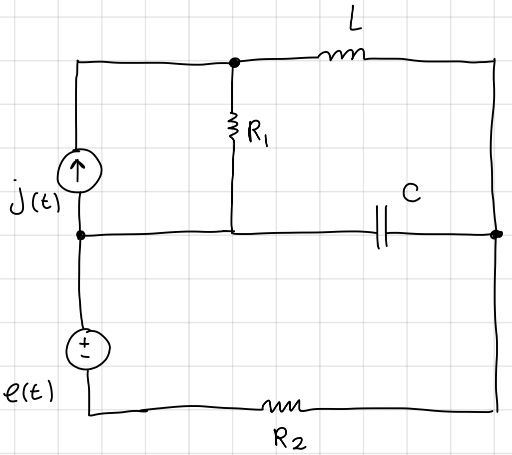
~30'

24/02/2022

Esercizio 3

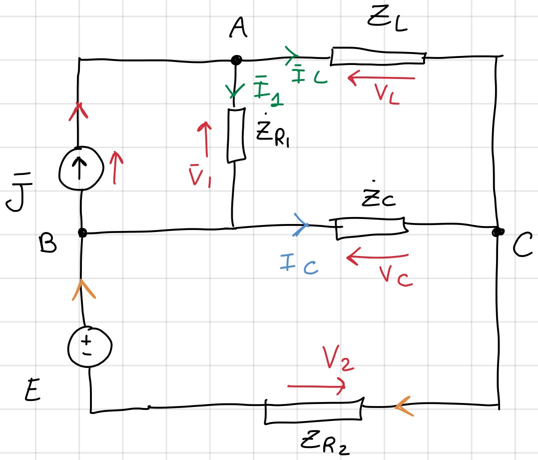
Il circuito in figura è in regime sinusoidale (*dati*: $e(t) = 20\cos(20t)$ V, $j(t) = 3\sin(20t)$ A, $R_1 = 40\Omega$, $R_2 = 5\Omega$, $L = 0.5$H e $C = 10m$F). Determinare la **potenza attiva erogata dal generatore di tensione**.





$e(t) = 20\cos(20t) \rightleftharpoons E = 20$

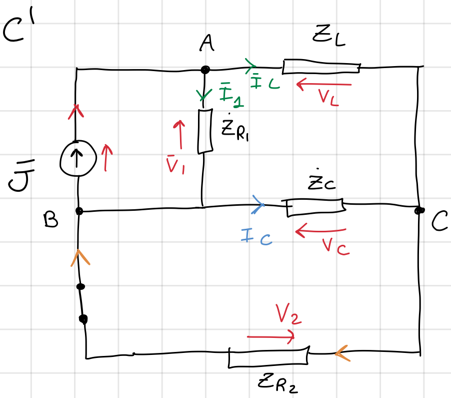
$j(t) = 3\sin(20t) \rightleftharpoons \bar{J} = 3\,e^{-\frac{\pi}{2}} = -3j$

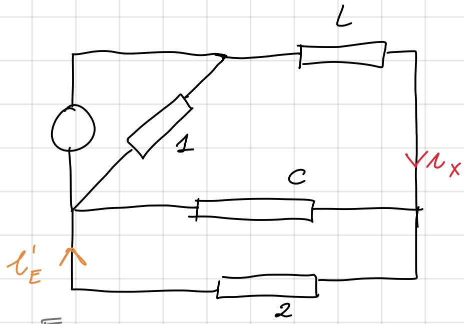


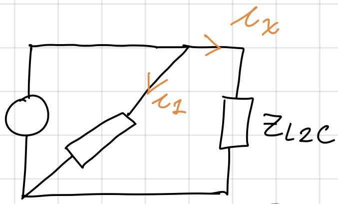
Q: $P_E = \frac{1}{2}\, E \cdot I_E^*$?

$$\begin{cases} R_1 = 40\,\Omega \\ R_2 = 5\,\Omega \\ L = 0.5H \\ C = 10mF \end{cases} \rightleftharpoons \begin{cases} \dot{Z}_1 = 40 & A \\ \dot{Z}_2 = 50 & B \\ \dot{Z}_L = 10j & D \\ \dot{Z}_c = -5j & C \end{cases}$$

C'



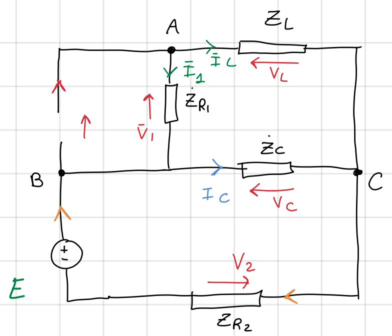


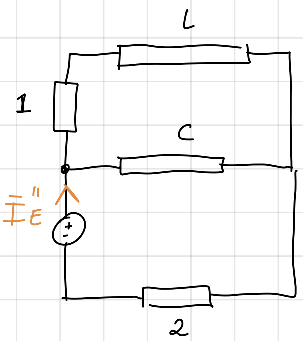


$$\dot{Z}^E_{L2C} = (\dot{Z}_C \parallel \dot{Z}_2) + \dot{Z}_L = 2.5 + 7.5j$$

$$i_x = J \cdot \frac{Z_1}{Z_1 + Z_x} = -0.48 - 2.74j$$

$$\rightarrow i_E = i_x \cdot \frac{Z_C}{Z_C + Z_2} = -1.61 - 1.13j \text{ A} \quad I'_E$$





$$Z_{eq} = \left[(\dot{Z}_1 + \dot{Z}_L) \parallel \dot{Z}_C\right] + \dot{Z}_2 = 5.62 - 5.08j$$

$$\Rightarrow I''_E = \frac{E}{Req} = 1.96 + 1.77j$$

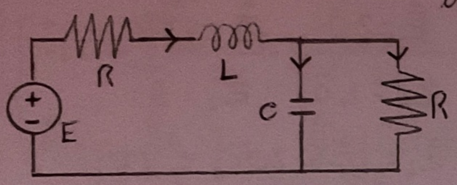
$$\Rightarrow I_E = I'_E + I''_E = 0.35 + 0.64j \quad I_E$$

Time 25'

$$\Rightarrow P = \frac{1}{2}\, E \cdot I^* = 3.49 - 6.44j$$

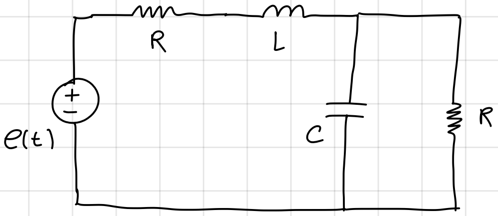
Maffucci - Dinamica 2.1

Calcolare la tensione ai capi del condensatore in ogni istante di tempo

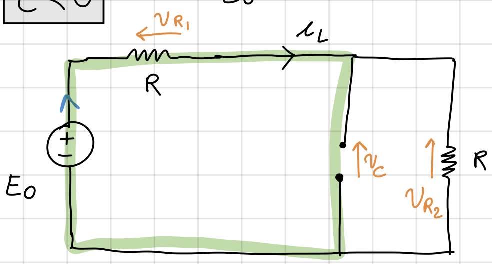


A $R = 1\,\Omega$ B $L = 1\,\mu H$ C $C = 1\,\mu F$

$$E(t) = \begin{cases} 2V & \forall t < 0 \\ -2V & \forall t > 0 \end{cases}$$



$\boxed{t<0}$ $E_0 = 2V$

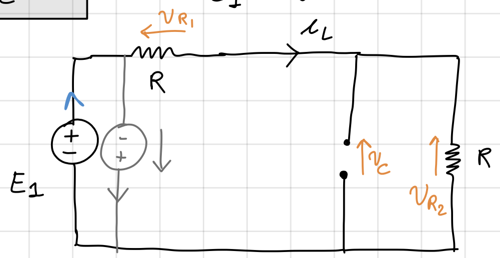


C. I.

$$i_E = \frac{E_0}{2R} = \frac{2}{2} = 1\,A \quad i_L$$

$$v_C = E_0 - v_{R_1} = E_0 - E_0 \cdot \frac{\cancel{R}}{\cancel{R+R}} = \frac{E_0}{2} = 1\,V \quad v_C$$

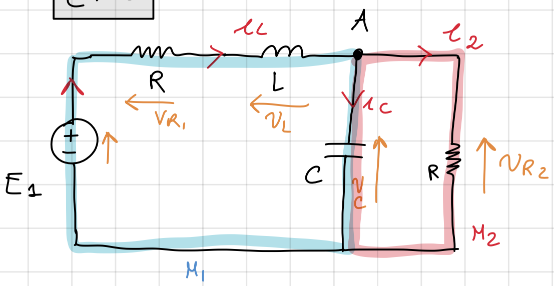
$\boxed{t\to\infty}$ $E_1 = -2V$



$$i_E = -1\,A$$
$$v_C = -1\,V$$

S. P.

$\boxed{t>0}$



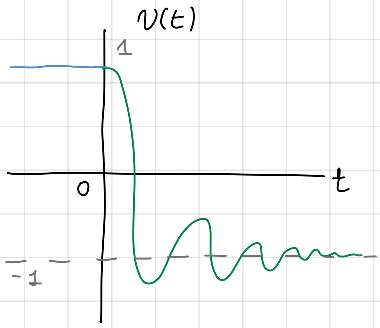
$$R.C.:\ \begin{cases} i_C = C\,\dot{v}_C \\ v_L = L\,\dot{i}_L \end{cases} \to \text{Trovo } i_C \text{ con LKC e } v_L \text{ con LKT}$$

$$LKC_A: -i_L + i_2 + i_C = 0 \to i_C = i_L - i_2 = i_L - \frac{v_C}{R}$$

$$LKT_{M_1}: v_L + v_C - E_1 - v_{R_1} = 0 \to v_L = E_1 + v_{R_1} - v_C = E_1 - R\cdot i_L - v_C$$

$$LKT_{M_2}: v_{R_2} - v_C = 0 \to v_{R_2} = v_C \to R\cdot i_2 = v_C \Rightarrow i_2 = \frac{v_C}{R}$$

$$\begin{cases} i_C = i_L - \frac{v_C}{R} \\ v_L = E_1 - R_1 i_L - v_C \end{cases} \to \boxed{\begin{cases} \dot{v}_C = -\frac{1}{RC} v_C + \frac{1}{C} i_L \\ \dot{i}_L = -\frac{1}{L} v_C - \frac{R}{L} i_L + \frac{E_1}{L} \end{cases}}$$



$$\det(\lambda I - A) = \begin{vmatrix} \lambda + \frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & \lambda + \frac{R}{L} \end{vmatrix} = \underset{1}{\lambda^2} + \lambda\left(\underset{AC}{\frac{1}{RC}} + \underset{B}{\frac{\overset{A}{R}}{L}}\right) + \underset{C}{\frac{1}{\cancel{R}C}} \cdot \underset{B}{\frac{\cancel{R}}{L}} + \underset{BC}{\frac{1}{LC}}$$

$$\to \lambda_{1,2} = 10^6(-1 \pm j) \rightsquigarrow y(t) = e^{\operatorname{Re} t}\left[C_1 \cos(\operatorname{Imm} \cdot t) + C_2 \sin(\operatorname{Imm} \cdot t)\right]$$

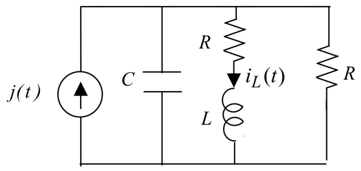
$$Q: \begin{cases} v_C(t) = e^{-10^6 t}\left[C_1 \cos(10^6 t) + C_2 \sin(10^6 t)\right] - 1V \\ v_C(0^+) = 1 \\ \dot{v}_C(0^+) = -\frac{1}{RC} \underbrace{v_C(0^+)}_{1} + \frac{1}{C} \underbrace{i_L(0^+)}_{1} = -10^6 + 10^6 = 0 \end{cases} \Rightarrow \begin{cases} C_1 - 1 = 1 \Rightarrow C_1 = 2 \\ -10^6 C_1 + 10^6 C_2 = 0 \Rightarrow C_2 = 2 \end{cases}$$

ESERCIZIO 11.1

Il seguente circuito è in regime stazionario fino a $t = 0$, quando il generatore si spegne. Calcolare:

a) il valore delle grandezze di stato all'istante $t = 0^+$

b) la corrente $i_L(t)$ per $t > 0$

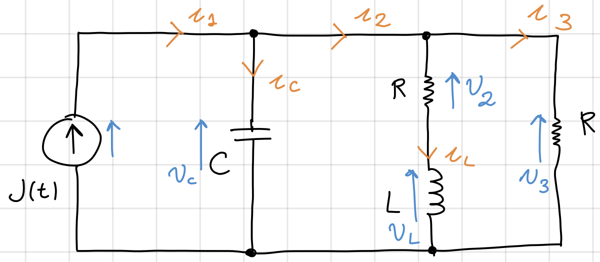


$$j(t) = \begin{cases} 20 \ A & t < 0 \\ 0 \ A & t > 0 \end{cases}$$

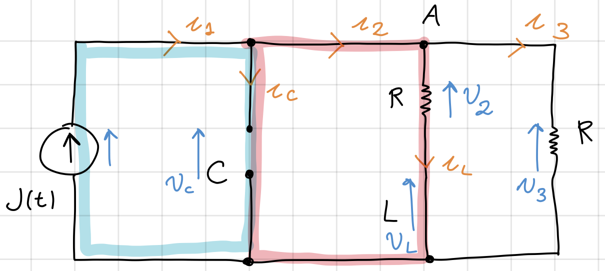
A $R = 2\,\Omega$

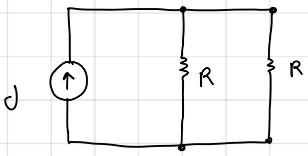
B $L = 10\,\mu H$

C $C = 5\,\mu F$



$\boxed{t < 0}$





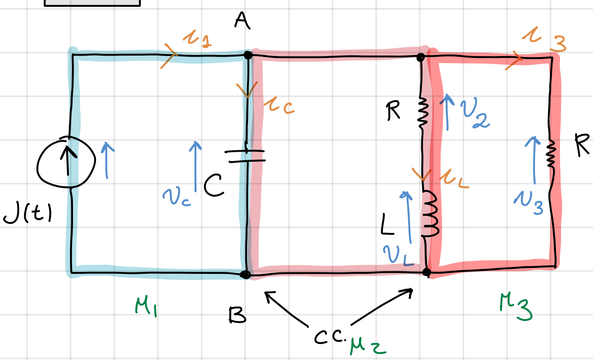
$\rightarrow \quad v_J = J \cdot R /\!/ R = 20\,V$

$$\begin{cases} -v_J + v_C = 0 \\ -v_C + v_2 + v_L = 0 \end{cases} \rightarrow \quad v_J = v_C \quad \Rightarrow \quad v_C(0^-) = 20\,V \quad \Rightarrow \quad v_C(0^+) = 20\,V \quad C.I.$$

* Per la continuità delle variabili

Part Corr. $\quad i_L = J \cdot \frac{\cancel{R}}{2\cancel{R}} \quad \Rightarrow \quad i_L(0^-) = 10\,A \quad \Rightarrow \quad i_L(0^+) = 10\,A \quad C.I$

$\boxed{t > 0}$



$$\begin{array}{ll} M_1: & -v_J + v_C = 0 \quad \rightarrow \quad v_C = v_J \\ M_2: & -v_C + v_2 + v_L = 0 \\ M_3: & -v_L - v_2 + v_3 = 0 \end{array}$$

$$\begin{cases} i_C = C\,\dot{v}_C \\ v_L = L\,\dot{i}_L \end{cases}$$

$$\begin{array}{ll} A: & -i_1 + i_C + i_3 + i_L = 0 \\ B: & \cancel{-i_C - i_L - i_3 + i_1 = 0} \end{array}$$

Non serve

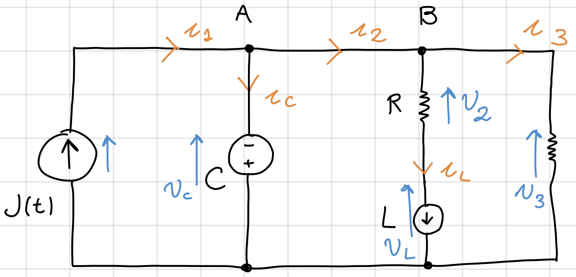
$\rightarrow \quad i_3 = i_1 - i_C - i_L$

$$v_L + v_2 - v_3 = 0 \quad \rightarrow \quad L\,\dot{i}_L + i_L R - v_3 = 0$$

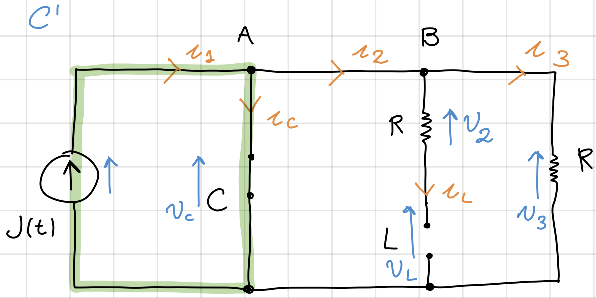
ma $\quad -v_C + v_3 = 0 \quad \rightarrow \quad v_3 = v_C \quad \Rightarrow \quad L\,\dot{i}_L + R\,i_L - v_C = 0$

NON RIESCO A
TROVARE LA II eq

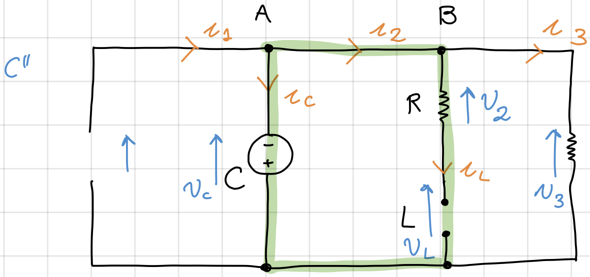
Metodo Alternativo : CRA



C'

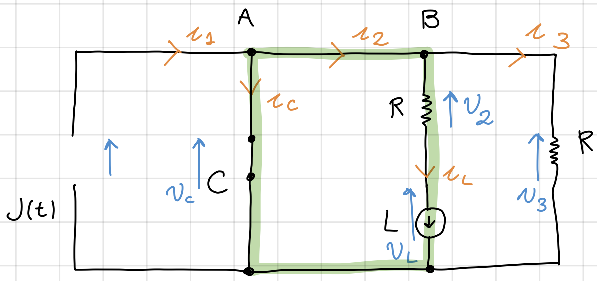


C''



$$i_C'' = -\frac{v_C}{R}$$

$$-v_C + \cancel{v_2}^{\emptyset} + v_L = 0 \to v_L'' = v_C$$



$$v_2 + v_L''' = 0 \to v_L''' = -R\, i_L$$

$$i_C = -i_L$$

$$L\dot{i}_L + R i_L - v_C = 0 \equiv v_C - L\dot{i}_L - R i_L = 0$$

$$\begin{cases} v_L = -R i_L + v_C \\ i_C = -\frac{v_C}{R} - i_L \end{cases} \to \begin{cases} i_C = C\dot{v}_C \\ v_L = L\dot{i}_L \end{cases} \to \begin{cases} L\dot{i}_L = -R i_L + v_C \\ C\dot{v}_C = -\frac{1}{R} v_C - i_L \to C\dot{v}_C + \frac{1}{R} v_C + i_L = \emptyset \end{cases}$$

$$\begin{cases} i_L + C\dot{v}_C + \frac{1}{R} v_C = 0 \\ v_C - R i_L - L\dot{i}_L = 0 \end{cases}$$

$$\to \dot{i}_L = -\frac{R}{L} i_L + \frac{1}{L} v_C = \frac{1}{L}\left(v_C - R i_L\right) \quad (A)$$

$$\to v_C = R i_L + L\dot{i}_L \to \dot{v}_C = R\dot{i}_L + L\ddot{i}_L$$

$$\to i_L + CR\dot{i}_L + CL\ddot{i}_L + i_L + \frac{L}{R}\dot{i}_L = \emptyset$$

$$\to CL\ddot{i}_L + \left(CR + \frac{L}{R}\right)\dot{i}_L + 2 i_L = \emptyset \to$$

eq DiFF

$$\boxed{\ddot{i}_L + \left(\frac{R}{L} + \frac{1}{RC}\right)\dot{i}_L + \frac{2}{LC} i_L = \emptyset}$$

A R
B L
C C

$$\lambda^2 + \left(\frac{R}{L} + \frac{1}{RC}\right)\lambda + \frac{2}{LC} = 0$$

$$\to \lambda = 10^5\left(1.5 \pm 1.3j\right)$$

$$\Rightarrow \quad i(t) = e^{\alpha t}\left[K_1 \cos(\beta t) + K_2 \sin(\beta t)\right]$$

Determino le costanti K_1 e K_2

$$\text{dalla (A)} \begin{cases} i(t) = e^{-10^5 \cdot 1.5\, t}\left[K_1 \cos(1.3\times10^5 t) + K_2 \sin(1.3\times10^5 t)\right] \\ i_L(0^+) = 10A \\ \dot{i}_L(0^+) = \left[\frac{1}{L}(v_C - R i_L)\right]\Big|_{t=0} = \frac{1}{L}\left(v_C(0^+) - R i_L(0^+)\right) = \frac{1}{L}(20 - 2\cdot 10) = \emptyset \end{cases}$$

$$\Rightarrow \quad i(0) = K_1 \cos(0) + K_2 \cancel{\sin(0)}^{\,\emptyset} = K_1 = 10 \qquad K_1$$

$$\Rightarrow \quad \dot{i}(0) = 1.5\times10^5\, \cancel{e^{\dots t}}^{\,1}\left[K_1 \cos(\dots t) + \cancel{K_2 \sin(\dots t)}^{\,\emptyset}\right] + \cancel{e^{\dots t}}^{\,1}\left[\cancel{-1.3\times10^5 K_1 \sin(\dots t)}^{\,\emptyset} + 1.3\times10^5 K_2 \cos(\dots t)\right]$$

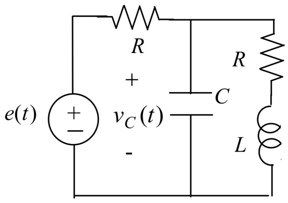
$$= 1.5\times10^5 K_1 + 1.3\times10^5 K_2 \approx \alpha K_1 + \beta K_2 = \emptyset$$

ma $K_1 = 10 \Rightarrow \alpha \cdot 10 + \beta K_2 = 0 \Rightarrow K_2 = -\frac{10\,\alpha}{\beta} = 11.54 \quad K_2$ $\qquad (\alpha \to -1.5\times10^5)$

$$\Rightarrow \quad i_L(t) = e^{-1.5\times10^5 t}\left[10 \cos(1.3\times10^5 t) + 11.54 \sin(1.3\times10^5 t)\right] \qquad \text{per } t > 0$$

ESERCIZIO 11.2

Con riferimento al seguente circuito, calcolare la tensione $v_C(t)$ in ogni istante.

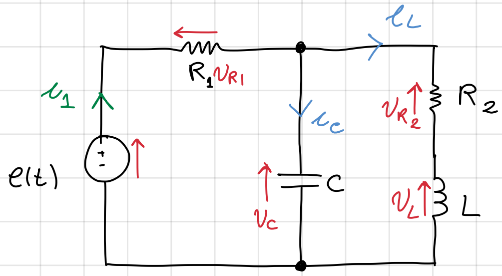


$$e(t) = \begin{cases} 20\ V & t<0 \\ -20\ V & t>0 \end{cases}$$

$R = 1\,\Omega$

$L = 5\,\mu H$

$C = 5\,\mu F$

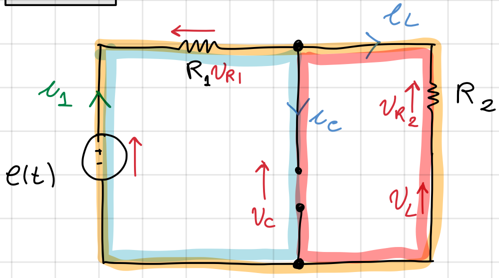


$$e(t) = \begin{cases} 20V & t<0 \\ -20V & t>0 \end{cases}$$

$R_1 = R_2 = 1\,\Omega \qquad L = 5\,\mu H \qquad C = 5\,\mu F$

$t<0$

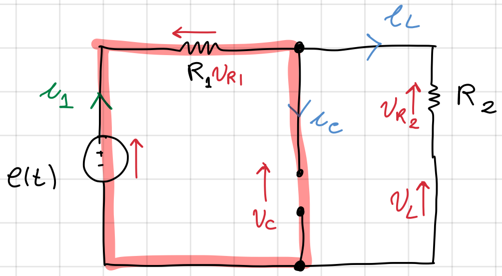
Stazionario $\qquad E_0 = 20V$



$$i_L = i_1 = \frac{E_0}{R_1+R_2} = 10A$$ C.I.

$$v_1 = E\cdot\frac{R_1}{R_1+R_2} = \frac{E_0}{2} = 10\,V$$ C.I.

$t \to 0^-$

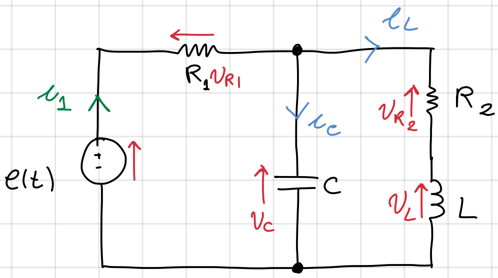


$$i_L = \frac{E_1}{R_1+R_2} = 10A \qquad v_c = E_1 - v_1 \to v_c = -20+10 = -10V$$ S.P.

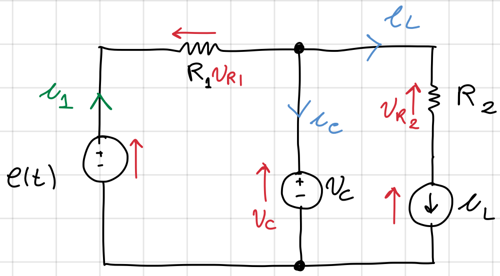
$$\text{ma } v_1 = E\cdot\frac{R_1}{R_1+R_2} = -10$$

$t>0$

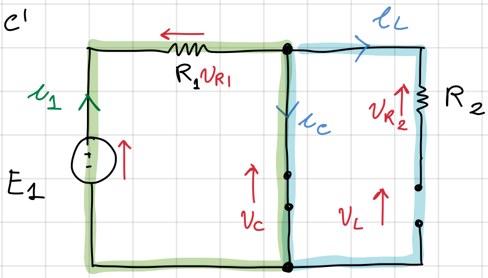
$E_1 = -20V$



CRA



c'

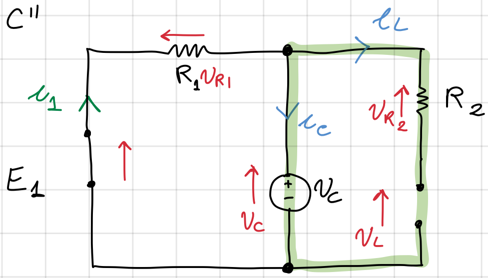


$$i_c' = i_1 = \frac{E_1}{R_1} \qquad i_c'$$

$$v_L' - v_c + \cancel{v_2} = 0 \to v_L' = v_c = E_1 - \frac{E_1}{R_1}\cdot R_1 = 0 \qquad v_L'$$

$i_2 = 0$

c''

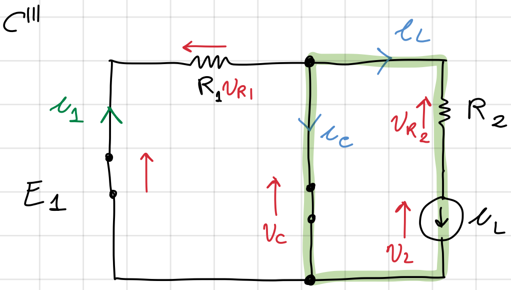


$$i_c'' = -\frac{v_c}{R_1} \qquad i_c''$$

$$v_L'' = v_c - \cancel{v_2} = v_c \qquad v_L''$$

0

C'''



i_C''': $i_C''' = -i_L$

v_L''': $v_L''' = -R_2 \cdot i_L$

$$\rightarrow \begin{cases} i_C = \dfrac{E_1}{R_1} - \dfrac{v_C}{R_1} - i_L \\ v_L = v_C + R_2 i_L \end{cases} \rightarrow \begin{cases} i_C = C\dot{v}_C \\ v_L = L\dot{i}_L \end{cases} \rightarrow \begin{cases} \dot{v}_C = -\dfrac{1}{R_1 C} v_C - \dfrac{1}{C} i_L + \dfrac{E_1}{R_1 C} \\ \dot{i}_L = \dfrac{1}{L} v_C - \dfrac{R_2}{L} i_L \end{cases}$$

$$\text{Polinomio} \equiv \det(\lambda I - A) = \begin{array}{cc} M & N \\ \end{array} \begin{vmatrix} \lambda + \dfrac{1}{R_1 C} & \dfrac{1}{C} \\ -\dfrac{1}{L} & \lambda + \dfrac{R_2}{L} \end{vmatrix}_{P,\ Q} = \lambda^2 + M\lambda + Q\lambda + MQ - (N \cdot P)$$

$$\rightarrow \lambda_{1,2} = 10^5(-2 \pm 2j)$$

$$\Rightarrow y(t) = e^{2\cdot 10^5 t}\left[c_1 \cos(2t) + c_2 \sin(2t)\right] + y_p(t)$$

$\rightarrow$ ci serve $\quad v_C(t) = e^{2\cdot 10^5 t}\left[c_1 \cos(2t) + c_2 \sin(2t)\right] + 10$

Determino c_1 e c_2

$$\begin{cases} v_C(t) = e^{2\cdot 10^5 t}\left[c_1 \cos(2t) + c_2 \sin(2t)\right] - 10 \\ v_C(0^+) = 10\ V \\ \dot{v}_C(0^+) = -\dfrac{1}{R_1 C} \underbrace{v_C(0^+)}_{10} - \dfrac{1}{C} \underbrace{i_L(0^+)}_{10} + \dfrac{E_1}{R_1 C} = -4 \times 10^6\ V \end{cases}$$

è una costante
non si conta

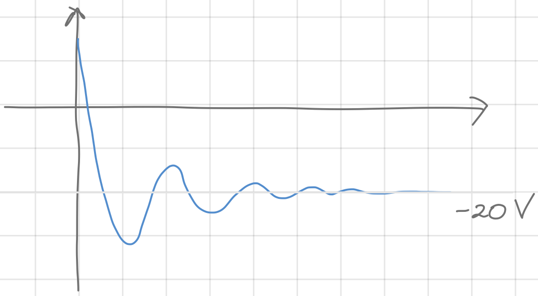
nella soluzione esce zero

C_2

$$\Rightarrow v_C(0^+) = C_1 - 10 = 10 \rightarrow C_1 = 20$$

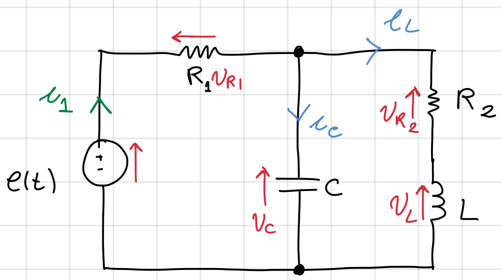
$$\dot{v}_C(0^+) = 2\cdot 10^5 C_1 + 2\cdot 10^5 C_2 = -4\times 10^6 \rightarrow C_2 = -\frac{4\times 10^6 + 4\cdot 10^6}{2\cdot 10^5} = -40$$

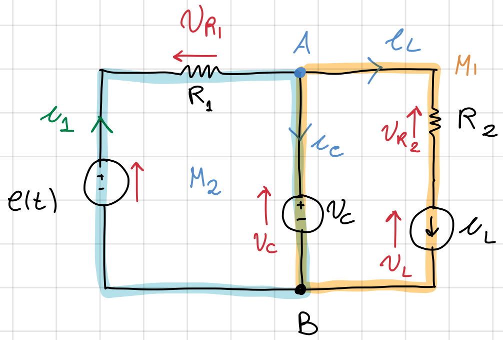
$$\rightarrow v_C(t) = e^{2\times 10^5 t}\left[20\cos(2\times 10^5 t) - 40\sin(2\times 10^5 t)\right] - 10$$



$\boxed{t>0}$ $E_1 = -20V$

Alternativa: uso le LKT e LKC con R.C.





CRA

OBBIETTIVO: Trovare i_c e v_L per sostituire le R.C.: $\overset{(3)}{\begin{cases} i_c = C\,\dot{v}_c \\ v_L = L\,\dot{i}_L \end{cases}}$

(1) (2)

Passaggi: Scrivo (i_c e v_L) $\to$ Trovo le (LKT) $\to$ Sostituisco le incognite da (2) in (1) $\to$ sostituisco (3)

LKC_A: $-i_1 + i_c + i_L = 0 \to i_c = i_1 - i_L$ (a)

(1)

LKT_{M_1} $v_L - v_c + v_2 = 0 \to v_L = v_c - v_2$ (b)

$$LKT \begin{cases} v_c - E + v_1 = 0 \\ v_L - v_c + v_2 = 0 \end{cases} \to \begin{cases} v_1 = E - v_c \Rightarrow i_1 = \dfrac{v_1}{R_1} = \dfrac{E - v_c}{R_1} \to (a) \\ // \quad \text{NO imp} \end{cases}$$

$\to i_c = \dfrac{E - v_c}{R_1} - i_L$ (2)

$\to v_2 = i_L \cdot R_2 \to$ $\to v_L = v_c - R_2 i_L$

Ora basta sostituire (3)

Stessa eq di prima

$$\begin{cases} i_c = \dfrac{E - v_c}{R_1} - i_L \\ v_L = v_c - R_2 i_L \end{cases} \to \begin{cases} i_c = C \cdot \dot{v}_c \\ v_L = L\,\dot{i}_L \end{cases} \to \boxed{\begin{cases} \dot{v}_c = \dfrac{E - v_c}{CR_1} - \dfrac{1}{C} i_L \\ \dot{i}_L = \dfrac{1}{L} v_c - \dfrac{R_2}{L} i_L \end{cases}}$$

COME OTTENERE L'EQ DI II ORDINE (per ottenere $\ddot{v}(t)$)

$$\begin{cases} \dot{v}_c = \dfrac{E - v_c}{CR_1} - \dfrac{1}{C} i_L \\ \dot{i}_L = \dfrac{1}{L} v_c - \dfrac{R_2}{L} i_L \end{cases} \to \frac{1}{C} i_L = \frac{E - v_c}{CR_1} - \dot{v}_c \to \underbrace{i_L = \frac{E - v_c}{R_1} - C\dot{v}_c}$$

ERRORE! VEDI 11. Ricav. le eq. di Stato..

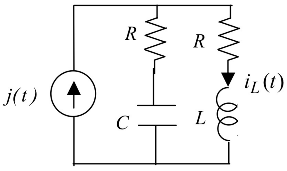
$$\to \dot{i}_L = \frac{1}{L} v_c - \frac{R_2}{L}\left(\frac{E - v_c}{R_1} - C\dot{v}_c\right) = \frac{1}{L} v_c - \frac{R_2 E}{LR_1} + \frac{R_2}{LR_1} v_c + \frac{R_2 C}{L} \dot{v}_c$$

ma $i_c = C\,\dot{v}_c \Rightarrow \dot{i}_c = C\,\ddot{v}_c \to \cancel{C}\,\ddot{v}_c = \dfrac{1 + R_2}{CLR_1} v_c + \dfrac{R_2 \cancel{C}}{L} \dot{v}_c$

$$\to \ddot{v}_c = \frac{R_2}{L} \dot{v}_c + \frac{1 + R_2}{CLR_1} v_c$$

ESERCIZIO 11.3

Il seguente circuito è in regime sinusoidale fino $t=0$, istante in cui il generatore si spegne. Calcolare la corrente $i_L(t)$ in ogni istante.

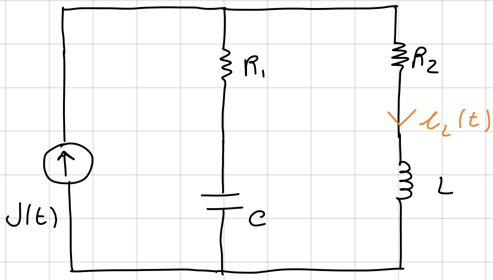


$$j(t)=\begin{cases}10\cos(100t)\ A & t<0\\ 0\ A & t>0\end{cases}$$

$R = 0.5\,\Omega$

$L = 10\ mH$

$C = 50\ mF$



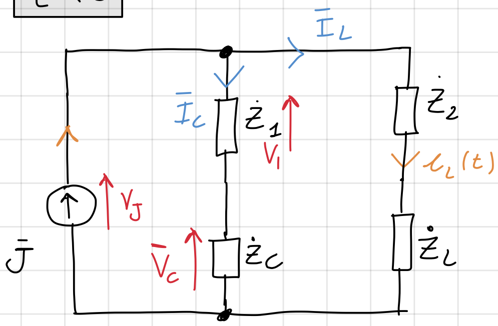
$$J(t)=\begin{cases}10\cos(100t)\ A & t<0\\ 0\quad A & t>0\end{cases}$$

1. Metodo dei fasori

$j(T_1) = 10\cos(100t) \rightleftharpoons \bar{J} = 10$ $\qquad R_1 = R_2 = \frac{1}{2}\,\Omega \rightleftharpoons \underset{A}{\dot{Z}_1} = \underset{B}{\dot{Z}_2} = \frac{1}{2}$

$L = 10\,mH \rightleftharpoons \underset{D}{\dot{Z}_L} = j\,100\cdot 10m = j$ $\qquad C = 50\,mF \rightleftharpoons \underset{C}{\dot{Z}_C} = -\frac{j}{100\cdot 5m} = -2j$

$\boxed{t<0}$ Sinusoidale



CONDIZIONI INIZIALI

$$\underset{E}{\bar{I}_L} = \bar{J}\cdot\frac{(\underset{A}{Z_1}+\underset{C}{Z_C})}{(\underset{A}{Z_1}+\underset{C}{Z_C})+(\underset{B}{Z_2}+\underset{D}{Z_L})} = 12.5-7.5j = 14.58\angle{-0.54}$$

$$\Rightarrow i_L(t) = 14.58\,e^{-0.54t} = \underbrace{14.58\cos(100t-0.54)}_{i_L(t)\quad t<0}\quad C.I.\ \text{-m}\quad Ans_1$$

$$V_C - V_J + V_1 = 0 \Rightarrow V_C = V_J - V_1 = \underset{G}{\bar{J}\cdot\left[(Z_1+Z_C)\,//\,(Z_2+Z_L)\right]} - \underset{F}{I_C}\cdot\underset{C}{Z_C}$$

$= -1.25+3.75j$

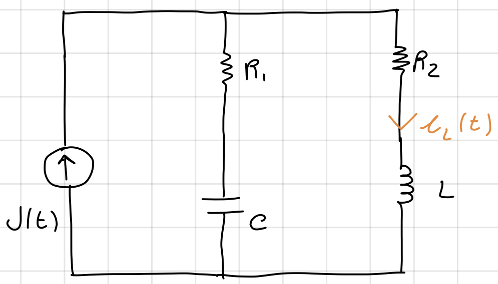
$LKC_A:\ -J+I_C+I_L=0$

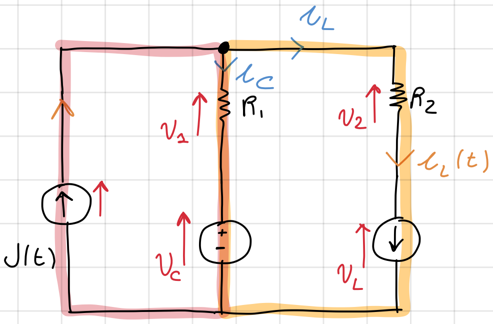
$\Rightarrow I_C = \underset{10}{J}-\underset{E}{I_L} = -2.5+7.5j$

$$\rightleftharpoons \underline{v_C(t) = 3.95\cos(100t+1.89)}\quad C.I.\ \text{-||-}$$

$\boxed{t\to\infty}$ $\qquad i_L(\infty)=\emptyset$ S.P. , $\quad v_C(\infty)=\emptyset$ S.P

$\boxed{t>0}$ DINAMICO





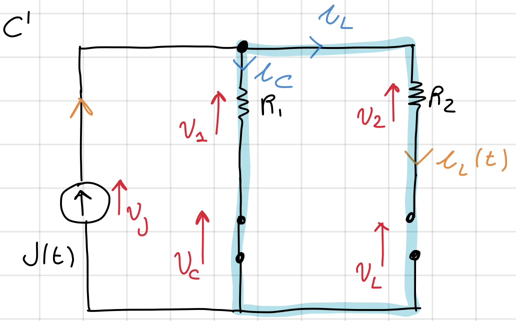
R.C. : $\begin{cases} i_C = C\,\dot{v}_C\\ v_L = L\,\dot{i}_L\end{cases}$

$v_L - v_J + v_2 = 0 \Rightarrow L\dot{i}_L - v_J + i_L R_2 = 0$ $\qquad v_C = v_L + v_2 - v_1$

$v_J - v_C - v_1 = 0 \Rightarrow v_J = v_C + v_1 = v_C + i_C R_1 = v_C + R_1 C\dot{v}_C$

$\Rightarrow L\dot{i}_L - v_C + RC\dot{v}_C + i_L R = \emptyset$

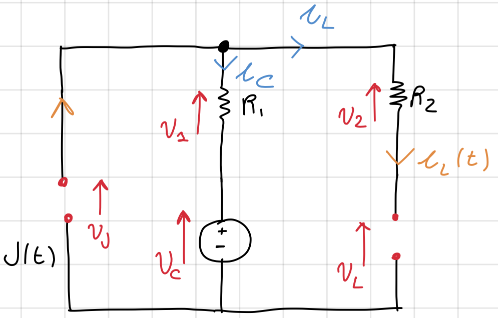
C'



$$\boxed{i_C' = J}$$

$$v_L - \cancel{v_C} - v_1 + \cancel{v_2} = 0 \rightarrow v_L = v_1 \rightarrow \boxed{v_L' = J \cdot R_1}$$

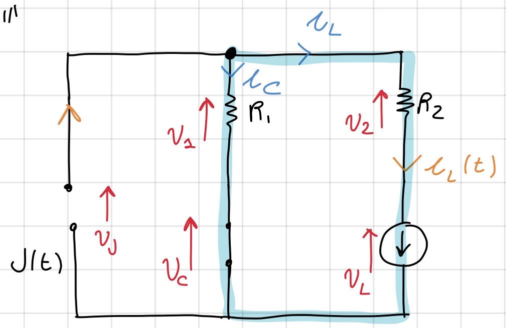
C''



$$i_C'' = 0$$

$$v_L'' = v_C$$

C'''



$$\begin{cases} i_C''' = -i_L \\ v_L = -i_L(R_1+R_2) \end{cases}$$

$$\rightarrow \begin{cases} i_C = J - i_L \\ v_L = J \cdot R_1 + v_C - i_L(R_1+R_2) \end{cases} \rightarrow \begin{cases} i_C = C\dot{v}_C \\ v_L = L\dot{i}_L \end{cases} \rightarrow \begin{cases} \dot{v}_C = -\frac{1}{C} i_L + \frac{J}{C} \\ \dot{i}_L = \frac{1}{L} v_C - \frac{R_1+R_2}{L} i_L + \frac{R_1}{L} J \end{cases}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda & \frac{1}{C} \\ -\frac{1}{L} & \lambda + \frac{R_1+R_2}{L} \end{vmatrix} = \lambda^2 + \lambda\left(\frac{R_1+R_2}{L}\right) + \frac{1}{CL} \Rightarrow \begin{matrix} \lambda_1 = -27.63 \\ \lambda_2 = -72.36 \end{matrix}$$

$$\Rightarrow y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + y_P$$

$$\rightarrow i_L(t) = C_1 e^{-27.63\,t} + C_2 e^{-72.36} + 0$$

Trovo C_1 e C_2

$$\underbrace{14.58\cos(100t - 0.54)}_{i_L(t) \quad t<0}$$

C.I. Ans1

$$v_C(t) = 3.95\cos(100t + 1.89)$$

C.I.

$$\begin{cases} i_L(t) = C_1 e^{-27.63\,t} + C_2 e^{-72.36} \\ i_L(0^+) = 14.58\cos(100\cdot 0 - 0.54) = 12.5 \\ \dot{i}_L(0^+) = \frac{1}{L} v_C(0^+) - \frac{R_1+R_2}{L} i_L(0^+) + \frac{R_1}{L} J = -874.5 \end{cases}$$

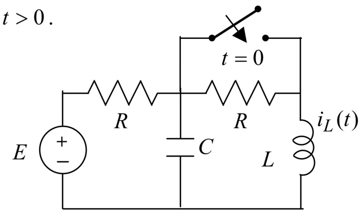
$3.95\cos(100\cdot 0 + 1.89)$ $\qquad$ $14.58\cos(100\cdot 0 - 0.54)$

$$\Rightarrow \begin{cases} i_L(0^+) = C_1 + C_2 = 12.5 \\ \dot{i}_L(0^+) = -27.63\, C_1 - 72.36\, C_2 = -874.5 \end{cases} \Rightarrow \begin{array}{l} C_1 = 0.67 \\ C_2 = 11.83 \end{array}$$

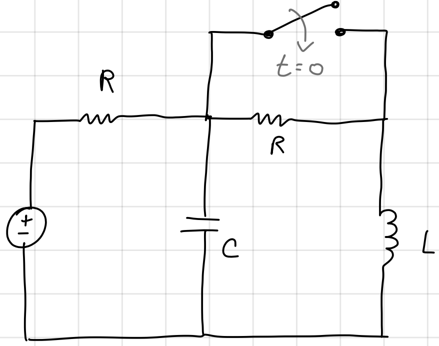
$$\Rightarrow i_L(t) = 0.67\, e^{-27.63\, t} + 11.83\, e^{-72.36 t} \text{ A} \qquad \text{per } t > 0$$

ESERCIZIO 11.4

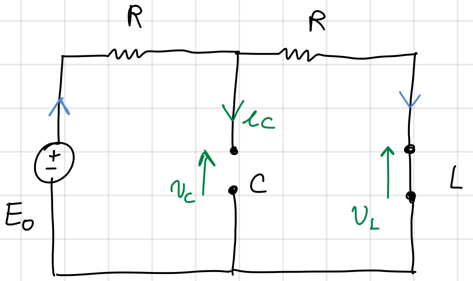
La rete in figura è in regime stazionario fino $t = 0$, istante in cui si chiude l'interruttore. Calcolare la corrente $i_L(t)$ per $t > 0$.



$E = 2\,V$
$R = 1/3\,\Omega$ A
$L = 1\,mH$ B
$C = 2\,mF$ C



$\boxed{t<0}$ Stazionario $E_0 = 2V$

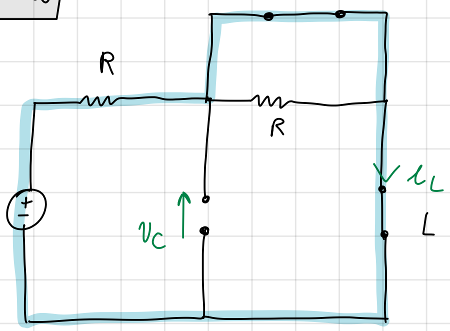


$i_e = \frac{E_0}{2R} = 3A$ $\quad i_L(0^\pm)$

C.I

$v_C = E_0 - V_1 = E_0 - \frac{E_0}{2\cancel{R}}\cdot\cancel{R} = \frac{E_0}{2} = 1V$ $\quad v_C(0^\pm)$

$\boxed{t \to \infty}$

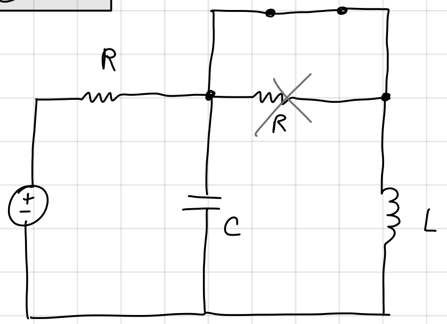


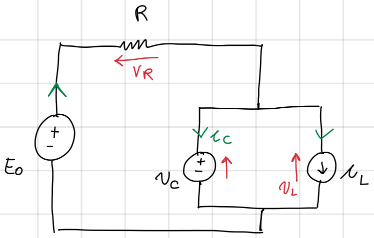
$i_L = \frac{E_0}{R} = 6A$

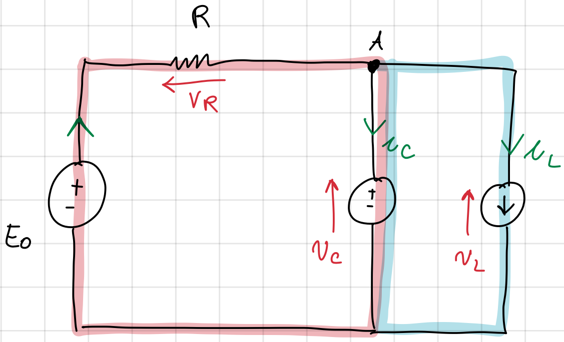
$v_C = E_0 - V_1 = 1V$

$$\begin{cases} i_C = C\dot{v}_C \\ v_L = L\,\dot{i}_L \end{cases}$$

$\boxed{t \to \infty}$







$$\begin{cases} v_C - E_0 + v_R = 0 \\ v_L + v_C = 0 \\ -i + i_C + i_L = 0 \end{cases} \to \quad v_R = E_0 - v_C \Rightarrow \quad i_R = \frac{E_0 - v_C}{R}$$

$$\begin{cases} i_C = i - i_L \\ v_L = v_C \end{cases} \to \begin{cases} i_C = i - i_L \\ L\,\dot{i}_L = v_C \end{cases} \to \begin{cases} C\dot{v}_C = \frac{E_0 - v_C}{R} - i_L \\ L\,\dot{i}_L = v_C \end{cases} \to \begin{cases} \dot{v}_C = \frac{E_0}{RC} - \frac{1}{RC}v_C - \frac{1}{C}i_L \\ \dot{i}_L = \frac{1}{L}v_C \end{cases}$$

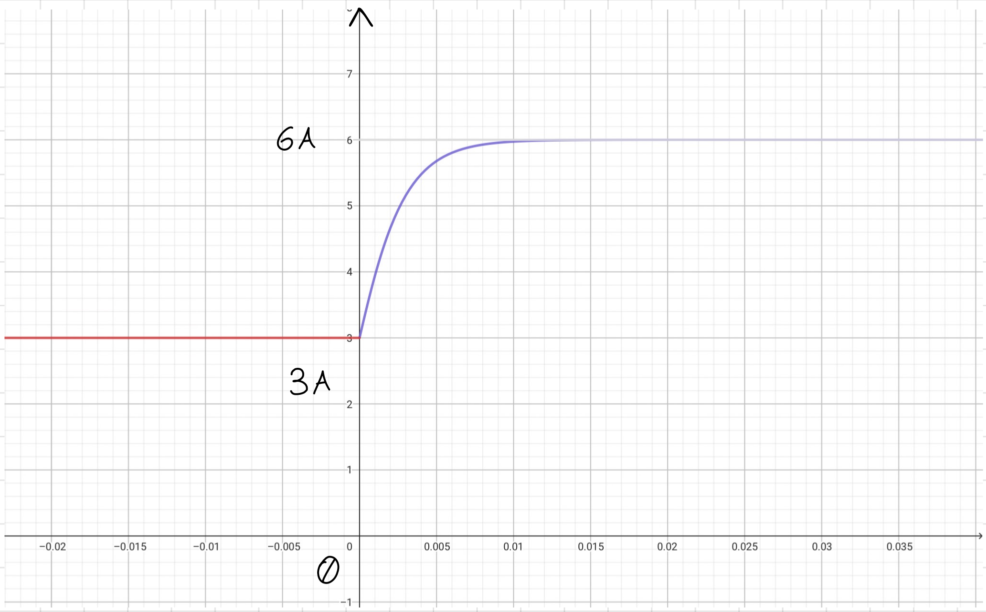
$$\det(\lambda I - A) = \begin{vmatrix} \lambda + \frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & \lambda + 0 \end{vmatrix} = \lambda^2 + \frac{1}{RC}\lambda + \frac{1}{LC} = 0 \to \begin{matrix} \lambda_1 = -500 \\ \lambda_2 = -1000 \end{matrix}$$

$$\to i_L(t) = C_1 e^{-500t} + C_2 e^{-1000t} + 6$$

$$\begin{cases} i_L(t) = C_1 e^{-500t} + C_2 e^{-1000\,t} + 6 \\ i_L(0^+) = 3 \\ \dot{i}_L(0^+) = \frac{1}{L} V_c(0^+) = \frac{1}{L} \cdot 1 = 1 \times 10^3 \end{cases} \longrightarrow \begin{cases} C_1 + C_2 + 6 = 3 \\ -500\, C_1 - 1000\, C_2 = 10^3 \end{cases}$$

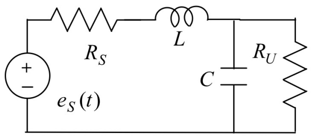
$$\longrightarrow \quad C_1 = -4 \qquad C_2 = 1$$

$$i_L(t) = -4\, e^{-500t} + e^{-1000\,t} + 6$$

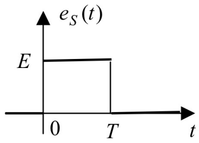


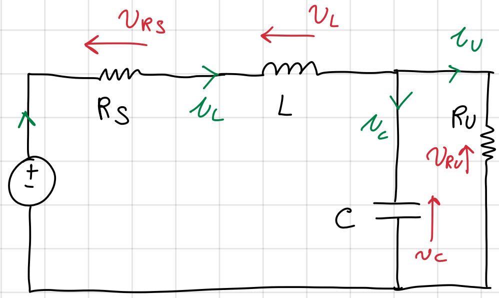
ESERCIZIO 11.5

Il seguente circuito rappresenta lo schema equivalente di un sistema digitale *trasmettitore-canale-ricevitore*. Calcolare la tensione sul ricevitore (R_U) in ogni istante.



$E = 6\,V$, $T = 1ns$
$R_S = R_U = 50\,\Omega$
$L = 2\,nH$, $C = 10\,pF$

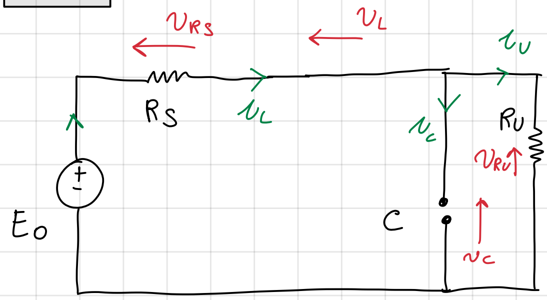




Risultato: $v(t) = 0\,V$ per $t < 0$; $v(t) = -3.74e^{-4.45\cdot 10^9 t} + 0.74e^{-22.55\cdot 10^9 t} + 3\,V$ per $0 < t < T$;
$v(t) = 320e^{-4.45\cdot 10^9 t} - 4.6\cdot 10^9 e^{-22.55\cdot 10^9 t}$ per $t > T$.

$$e_S(t) = \begin{cases} 0\ V & t<0 \\ 6\ V & 0<t<T \\ 0\ V & t>T \end{cases}$$

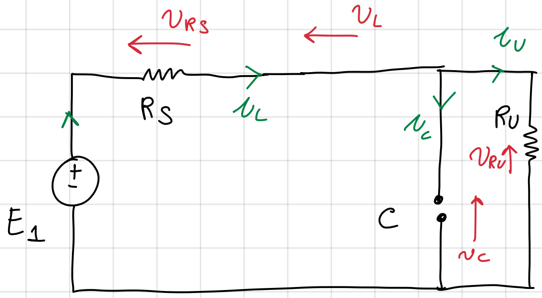
$\boxed{t<0}$



$$E_0 = 0 \Rightarrow \begin{cases} i_L(0^-) = 0 \\ v_C(0^-) = 0 \end{cases}$$

C.I.

$\boxed{t \to \infty}$

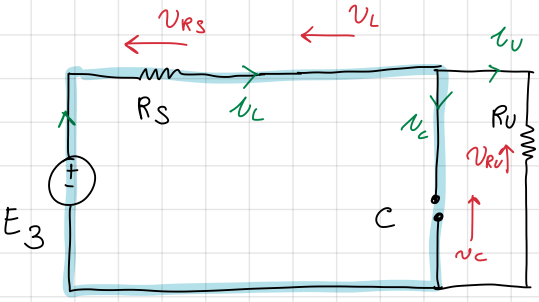


$$E_1 = 0 \Rightarrow \begin{cases} i_L(\infty) = 0 \\ v_C(\infty) = 0 \end{cases}$$

S. P.

$\boxed{y_p(t) \text{ per } t>0}$

$E_3 = 6V$



$$i_L = \frac{E_3}{2R} = 6 \times 10^{-2}\ A$$

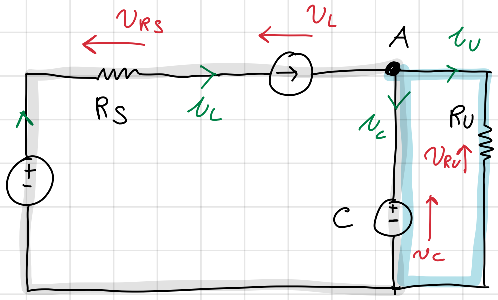
C.F.
$t>0$

$$v_C - E_3 + v_{RS} = 0 \rightarrow v_C = E_3 - i_L R = 3\,V$$

$0 < t < T$

$E_2 = 6V$

C R A



A $R_S \overset{R}{=} R_U = 50\,\Omega$

B $L = 2\,nH$

C $C = 10\,pF$

$$\begin{cases} i_C = C\dot{v}_C \\ v_L = L\,\dot{i}_L \end{cases}$$

$$-i_L + i_C + i_U = 0 \rightarrow i_C = i_L - i_U \rightarrow \underline{i_C = i_L - \frac{1}{R}v_C}$$

$$v_C - E_2 + v_{RS} + v_L = 0 \rightarrow v_L = E_2 - v_C - v_{RS} \Rightarrow \underline{v_L = E_2 - v_C - R\,i_L}$$

$$v_{RU} - v_C = 0 \rightarrow v_{RU} = v_C \leftrightarrow R\,i_U = v_C \rightarrow i_U = \frac{v_C}{R}$$

$$\begin{cases} \dot{v}_C = -\frac{1}{RC}v_C + \frac{1}{C}i_L \\ \dot{i}_L = -\frac{1}{L}v_C - \frac{R}{L}i_L + E_2 \end{cases}$$

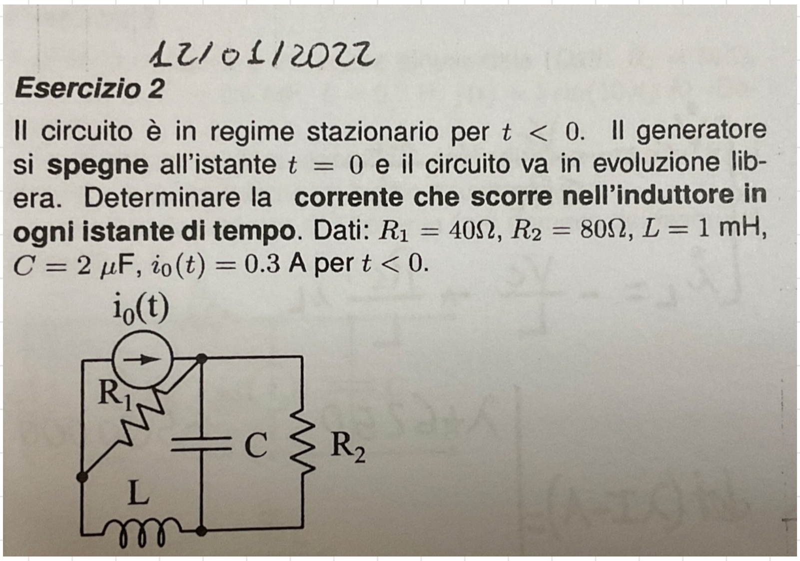
$$\det(\lambda I - A) = \begin{vmatrix} \lambda + \frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & \lambda + \frac{R}{L} \end{vmatrix} = \lambda^2 + \lambda\left(\frac{1}{RC} + \frac{R}{L}\right) + \frac{\cancel{R}}{\cancel{R}CL} + \frac{1}{CL}$$

$$\rightarrow \begin{cases} \lambda_1 = -4.43 \times 10^9 \\ \lambda_2 = -2.26 \times 10^{10} \end{cases} \rightarrow y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + y_P(t)$$

$\rightsquigarrow$ Q: v_{RU} ma $v_{RU} = v_C \Rightarrow v_C(t) = C_1 e^{-4.43\times 10^9 t} + C_2 e^{-2.26\times 10^{10} t} + 3$

$$\begin{cases} v_C(t) = C_1 e^{-4.43\times 10^9 t} + C_2 e^{-2.26\times 10^{10} t} + 3 \\ v_C(0^+) = 0\,V \\ \dot{v}_C(0^+) = -\frac{1}{RC}\underbrace{v_C(0^+)}_{0} - \frac{R}{L}\underbrace{i_L(0^+)}_{0} + \underbrace{E_2}_{0} = 0 \end{cases}$$

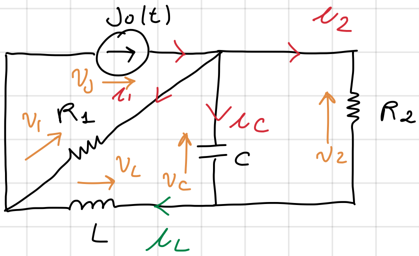
$$\begin{cases} C_1 + C_2 + 3 = 0 \\ \lambda_1 C_1 + \lambda_2 C_2 = 0 \end{cases} \Rightarrow C_1 = 3.124,\ C_2 = -6.124$$



12/01/2022

Esercizio 2

Il circuito è in regime stazionario per $t < 0$. Il generatore si **spegne** all'istante $t = 0$ e il circuito va in evoluzione libera. Determinare la **corrente che scorre nell'induttore in ogni istante di tempo**. Dati: $R_1 = 40\Omega$, $R_2 = 80\Omega$, $L = 1$ mH, $C = 2\ \mu$F, $i_0(t) = 0.3$ A per $t < 0$.



A $R_1 = 40\ \Omega$

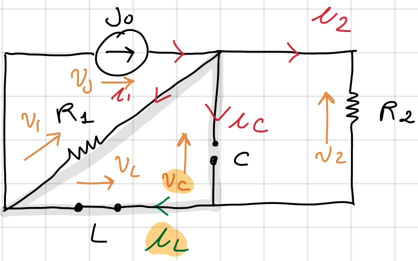
B $R_2 = 80\ \Omega$

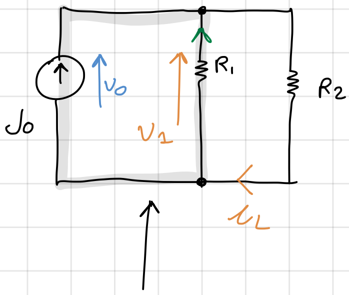
D $L = 1\,mH$

C $C = 2\,\mu F$

$$i_0(t) = \begin{cases} 0.3\ A & t<0 \\ 0\ A & t>0 \end{cases}$$

$\boxed{t<0}$

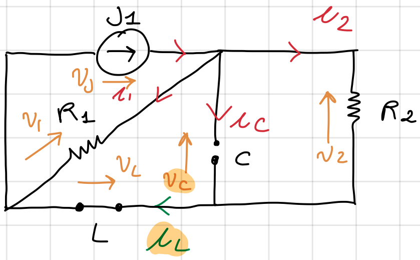




$$i_L = J_0 \cdot \frac{R_1}{R_1+R_2} = 1\times 10^{-1}\ A \quad i_L(0^-)$$

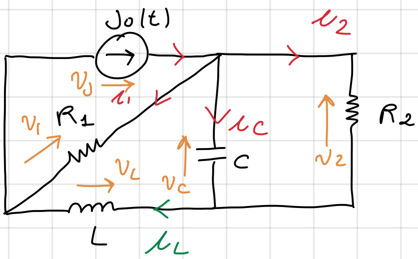
$$V_C + \cancel{V_L}_{0} - V_1 = 0 \Rightarrow V_C = V_1 \Rightarrow V_1 = V_0 \Rightarrow V_1 = J_0 \cdot (R_1 /\!/ R_2) = 8\,V \quad V_C(0^-)$$

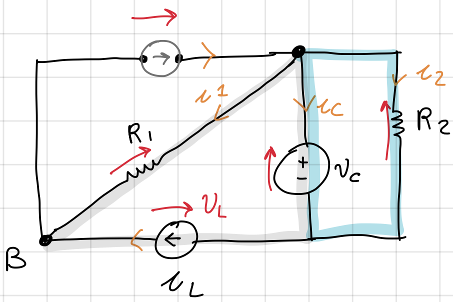
$\boxed{t \to \infty}$



$$J_1 = 0 \Rightarrow \begin{cases} i_L(\infty) = 0 \\ V_C(\infty) = 0 \end{cases}$$

$\boxed{t>0}$





$$R.C.: \begin{cases} i_C = C\,\dot{V}_C \\ V_L = L\,\dot{i}_L \end{cases}$$

$J_0 = 0$ per $t>0$

$$i_C + i_1 + i_2 = 0 \rightarrow i_C = -i_1 - i_2$$

$$LKT_{M_2}: \quad R_2\, i_2 = V_C \rightarrow i_2 = \frac{V_C}{R_2}$$

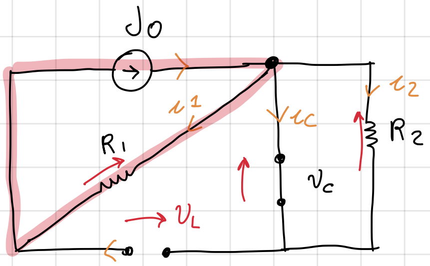
$$\hookrightarrow i_C = -i_1 - \frac{V_C}{R_2}$$

$$V_L - V_1 + V_C = 0 \rightarrow V_L = V_1 - V_C$$

$$\rightarrow LKC_B: \ -i_L + i_1 = 0 \Rightarrow i_1 = i_L \Rightarrow \quad i_C = -\frac{1}{R_2} V_C - i_L$$

$$V_L = R_1 i_L - V_C$$

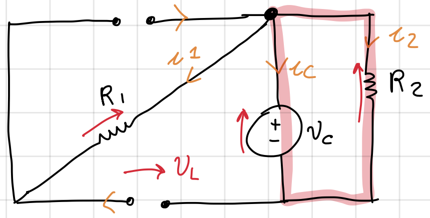
C'



$$\begin{cases} i_c' = \emptyset \\ v_L' = v_1 = J \cdot R_1 \quad \text{ma} \quad J = 0 \quad \text{per} \quad t > 0 \end{cases}$$

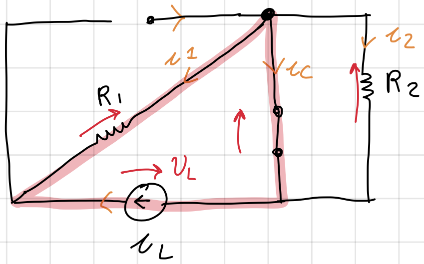
$$\Rightarrow v_L' = \emptyset$$

C''



$$\begin{cases} i_c'' = - \dfrac{v_c}{R_2} \\ v_L'' = -v_c \end{cases}$$

C'''



$$\begin{cases} i_c''' = i_L \\ v_L = - v_{R_1} = - i_L R_1 \end{cases}$$

$$\begin{cases} i_c = -\dfrac{1}{R_2} v_c + i_L \\ v_L = -v_c - R_1 i_L \end{cases} \Rightarrow \begin{cases} \dot{v}_c = -\dfrac{1}{R_2 C} v_c + \dfrac{1}{C} i_L \\ \dot{i}_L = -\dfrac{1}{L} v_c - \dfrac{R_1}{L} i_L \end{cases} \Rightarrow \det(\lambda I - A) = \begin{vmatrix} \lambda + \dfrac{1}{R_2 C} & -\dfrac{1}{C} \\ \dfrac{1}{L} & \lambda + \dfrac{R_1}{L} \end{vmatrix}$$

$$\Rightarrow \lambda^2 + \lambda \underbrace{\left(\frac{1}{R_2 C} + \frac{R_1}{L} \right)}^{6.75 \times 10^3} + \overbrace{\frac{R_1}{R_2 C L} + \frac{1}{LC}}^{7.5 \times 10^8} = 0 \qquad \lambda_{1,2} = -2.31 \times 10^4 \pm 1.47\, j \times 10^4$$

$$\alpha = G \qquad \beta = H$$

$$\Rightarrow y(t) = e^{\alpha t} \left[C_1 \cos(\beta t) + C_2 \sin(\beta t) \right]$$

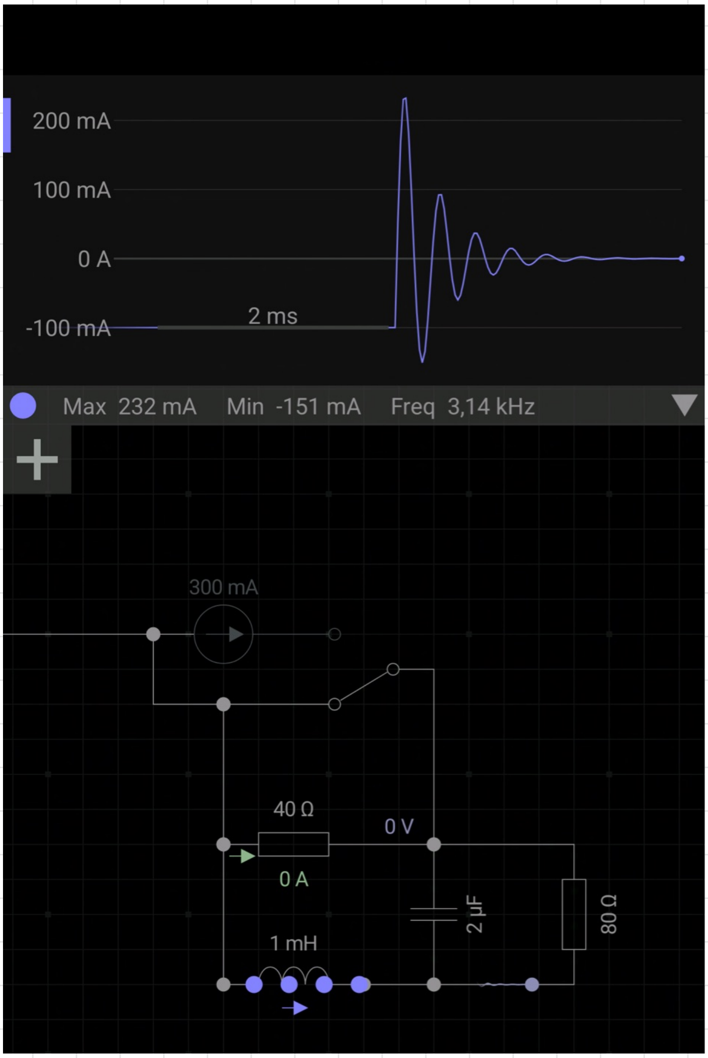
$$\Rightarrow i_L(t) = e^{-2.31 \times 10^3 t} \left[C_1 \cos(1.47 \times 10^4 t) + C_2 \sin(1.47 \times 10^4 t) \right]$$

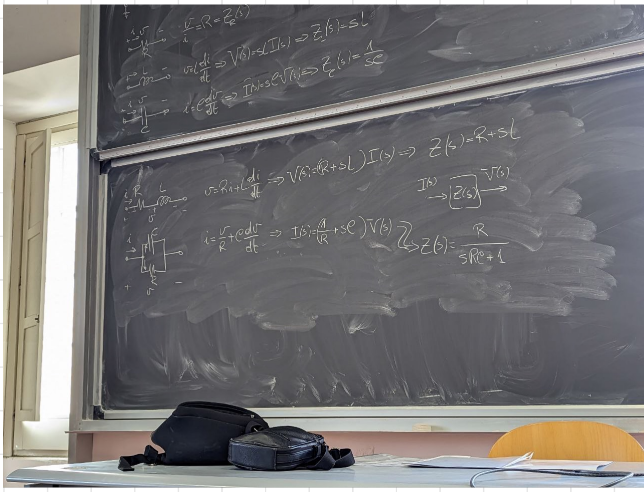
determino C_1 e C_2

$$\begin{cases} i_L(t) = e^{-3.38 \times 10^3 t} \left[C_1 \cos(2.72 \times 10^4 t) + C_2 \sin(2.72 \times 10^4 t) \right] \\ i_L(0^+) = 10^{-1} A \\ \dot{i}_L(0^+) = -\dfrac{1}{L} \underbrace{v_c(0^-)}_{8} - \dfrac{R_1}{L} \underbrace{i_L(0^-)}_{10^{-1}} = -1.2 \times 10^4 \end{cases}$$

$$\Rightarrow \begin{cases} C_1 = 10^{-1} \\ C_1 \alpha + C_2 \beta = -1.2 \times 10^4 \end{cases} \Rightarrow \begin{cases} C_1 = 0.1 \\ C_2 = -0.659 \end{cases}$$

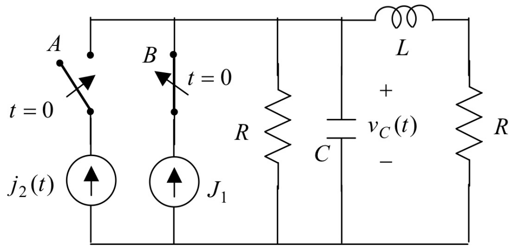
$$\Rightarrow i_L(t) = e^{-2.31 \times 10^3 t} \left[0.1 \cos(1.47 \times 10^4 t) - 0.659 \sin(1.47 \times 10^4 t) \right] + \emptyset$$





ESERCIZIO 11.6

All'istante $t=0$ si chiude l'interruttore A e si apre l'interruttore B. Calcolare la tensione sul condensatore per ogni istante di tempo.



$J_1 = 2\ A$

$j_2(t) = 2\sin(\omega t)\ A$

$\omega = 10^6\ rad/s$

$R = 1\,\Omega, \quad L = 1\ mH$

$C = 1\ mF$

Risultato: $v_C(t) = 1\,V$ per $t<0$; $\quad v_C(t) = 2.28e^{-10^6 t}\cos(10^6 t + 0.90) + 1.26\cos(10^6 t - 0.32)\ V$ per $t>0$.

$J_1 = 2A \qquad R = 1\,\Omega \qquad L = 1mH$

$C = 1\ mF$

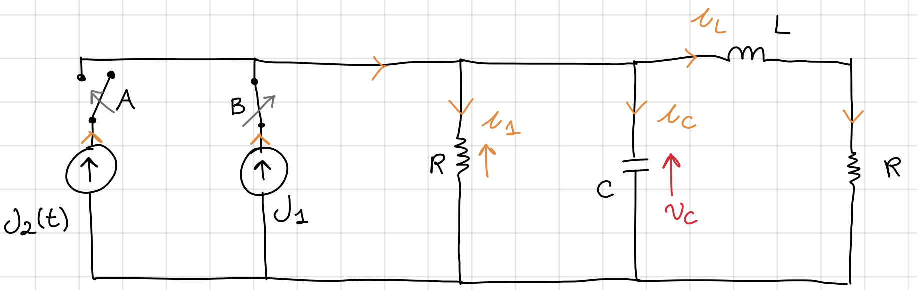
$J_2(t) = 2\sin(\omega t)\ A$

con $\omega = 10^6\ rad/s$

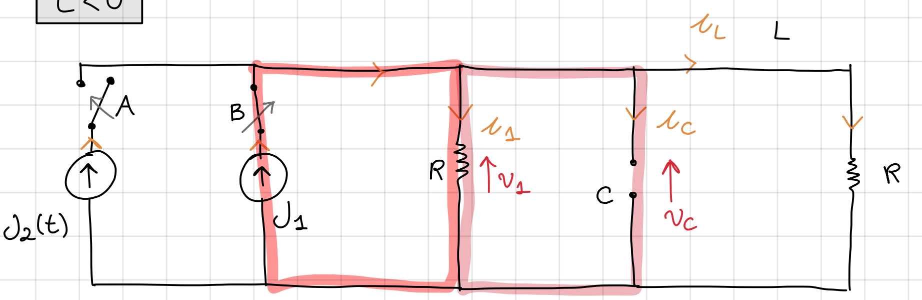
$\rightarrow \bar{J}_2 = 2\ A$

$\dot{Z}_R = 1 \qquad \dot{Z}_L = \omega \cdot 1m\, j = 10^3 j$

$\dot{Z}_C = -\dfrac{j}{1m\,\omega} = -10^{-3} j$



$\boxed{t<0}$



$i_L \ t<0$

$i_L = J_1 \cdot \dfrac{\cancel{R}^{\,1/2}}{R+\cancel{R}} = \dfrac{J_1}{2} = 1\ A$

$V_C - V_1 = 0 \rightarrow V_C = V_1$

V_C

$V_1 = V_{J1} = J_1 \cdot R /\!/ R = \dfrac{J}{2} = 1\ V$

$\boxed{t \rightarrow \infty}$