

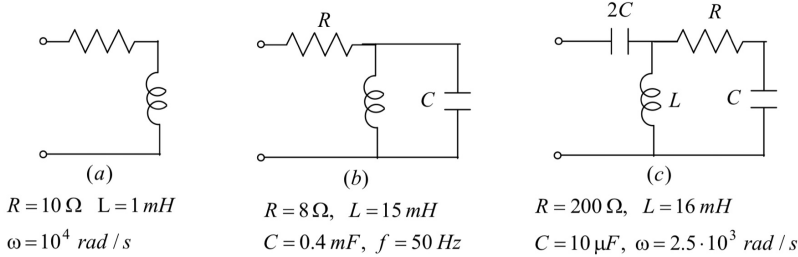
ES. 1.1 Esprimere la corrente $i(t)$ in termini di fasore nei seguenti tre casi:

a) $i(t) = 4 \sin(\omega t - 1.14)$ b) $i(t) = 10 \sin(\omega t - \pi)$ c) $i(t) = 8 \sin(\omega t + \pi/2)$

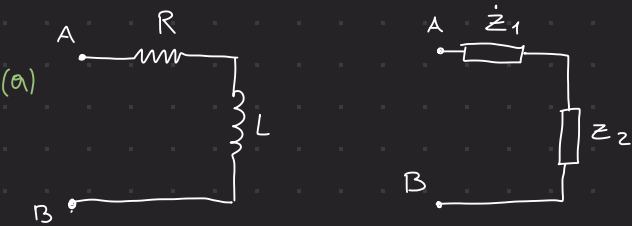
Risultato: a) $\bar{I} = 4 \exp(-j1.14)$; b) $\bar{I} = -10$; c) $\bar{I} = 8j$.

(a) $\bar{I} = 4 e^{-j1.14}$
 (b) $\bar{I}_2 = 10 e^{-j\pi} = -10$
 (c) $\bar{I} = 8 e^{j\frac{\pi}{2}} = 8 \left[\cos\left(\frac{\pi}{2}\right) + j \sin\left(\frac{\pi}{2}\right) \right] = 8j$

ES. 1.2 Valutare (in coordinate cartesiane e polari) le impedenze viste ai capi dei morsetti:



Risultato: a) $\dot{Z} = 10 + 10j = 10\sqrt{2} \exp(j\pi/4) \Omega$; b) $\dot{Z} = 8 + 11.54j = 14 \exp(j0.965) \Omega$; c) $\dot{Z} = 8 + 20j = 21.5 \exp(j1.19) \Omega$;



$\dot{Z}_1 = 10 \Omega$
 $\dot{Z}_2 = j \cdot 10 \cdot 1 \times 10^{-3} = 10j$

$\Rightarrow \dot{Z}_{eq} = \dot{Z}_1 + \dot{Z}_2 = 10 + 10j$

$\Rightarrow \text{Exp Form} = |\dot{Z}| e^{j\angle \dot{Z}}$

$\Rightarrow |\dot{Z}_{eq}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

$\angle \dot{Z} = \arctan\left(\frac{10}{10}\right) = \frac{1}{4} \pi$

$\Rightarrow \text{Exp } \dot{Z}_{eq} = 10\sqrt{2} e^{j\frac{1}{4}\pi}$

↑
 Sulla calcolatrice
 basta convertire in
 $r \angle \theta$

(b) $\omega = 2\pi f = 100\pi \text{ rad/s}$

$\dot{Z}_R = 8 \Omega$

$\dot{Z}_L = j \cdot 10^2 \pi \cdot 15 \times 10^{-3} = \frac{j\pi \cdot 15}{10} = 1.5\pi j$

$\dot{Z}_C = -\frac{j}{10^2 \pi \cdot \frac{2}{5} \cdot 10^{-6}} = -\frac{5j}{\pi \cdot 2 \cdot 10^{-4}} = -\frac{25}{\pi} j$

$\Rightarrow \dot{Z}_{eq} = (\dot{Z}_C \parallel \dot{Z}_L) + \dot{Z}_R = \text{Ans } 8 + 11.55j \Rightarrow \dot{Z}_{eq} = r e^{j\angle \theta} = \text{Ans } 14 e^{j0.965}$

(c)

$$\dot{Z}_R = 200 \Omega$$

$$\dot{Z}_L = j \cdot 2.5 \times 10^{-3} \cdot 16 \times 10^{-3} = 40j$$

$$\dot{Z}_C = -\frac{j}{10 \cdot 10^{-6} \cdot 2.5 \times 10^{-3}} = -40j$$

$$\Rightarrow \dot{Z}_{eq} = \left[(\dot{Z}_R + \dot{Z}_C) \parallel \dot{Z}_L \right] + 2 \dot{Z}_C = 8 - 40j = 8\sqrt{26} e^{-1.37j}$$

Boh!



ES. 1.3 Le seguenti coppie di fasori esprimono tensione e corrente relative ad un dato bipolo. Dire, nei tre casi, se si tratta di un resistore, un condensatore o un induttore e valutare il valore dei parametri corrispondenti R , C o L

a) $v(t) = 15 \cos(400t + 1.2)$, $i(t) = 3 \sin(400t + 1.2)$;

b) $v(t) = 8 \cos(900t - \pi/3)$, $i(t) = 2 \sin(900t + 2\pi/3)$;

c) $v(t) = 20 \cos(250t + \pi/3)$, $i(t) = 5 \sin(250t + 5\pi/6)$;

(a)

$$\bar{V} = 15 e^{1.2j} \quad i(t) = 3 \sin(400t + 1.2) = 3 \cos(400t + 1.2 - \frac{\pi}{2})$$

$$\Rightarrow \bar{I} = 3 e^{(1.2 - \frac{\pi}{2})j}$$

Per capire se si tratta di un resistore, induttore o condensatore dobbiamo guardare la **fase tra tensione e corrente**:

• **Resistore**: 0 o $\pi/2$ (in fase) $\alpha = \beta$

• **Induttore**: la tensione è in anticipo rispetto alla corrente di $\pi/2$ $\alpha = \beta + \frac{\pi}{2}$ $\beta = \alpha - \frac{\pi}{2}$

• **Condensatore**: la corrente è in anticipo rispetto alla tensione di $\pi/2$ (la corrente è in ritardo)

$$\alpha = \beta - \frac{\pi}{2} \quad \beta = \alpha + \frac{\pi}{2}$$

Controlla la fas , o arg

$$\angle \dot{Z} = \underbrace{\angle \bar{V}}_{\alpha} - \underbrace{\angle \bar{I}}_{\beta} = 1.2 - \left(1.2 + \frac{\pi}{2}\right) = -\frac{\pi}{2} \Rightarrow \alpha = \beta - \frac{\pi}{2} \Rightarrow \text{INDUTTORE}$$

$$\Rightarrow \dot{Z}_L = j\omega L \quad (1) \quad \text{INDUTTORE}$$

$$\bar{V} = \dot{Z} \bar{I} \quad (2)$$

$$\Rightarrow \bar{V} = j\omega L \bar{I} \Rightarrow L = \frac{|\bar{V}|}{\omega |\bar{I}|} = \frac{15}{\omega 3} = 0.0125 \text{ H} = 12.5 \text{ mH}$$

$$(b) \quad \bar{V} = 8 e^{-\frac{\pi}{3}j} \quad i(t) = 2 \sin(900t + \frac{2\pi}{3}) = 2 \cos(900t + \frac{2}{3}\pi - \frac{\pi}{2})$$

$$= 2 \cos(900t + \frac{1}{6}\pi)$$

$$\Rightarrow \bar{I} = 2 e^{\frac{1}{6}\pi j}$$

$$\Rightarrow \angle \dot{Z} = \angle \bar{V} - \angle \bar{I} = -\frac{\pi}{3} - \frac{1}{6}\pi = -\frac{1}{2}\pi \Rightarrow \alpha = \beta - \frac{\pi}{2} \Rightarrow \text{CONDENSATORE}$$

$$\Rightarrow \dot{Z}_C = -\frac{j}{\omega C} \Rightarrow \bar{V} = \dot{Z}_C \bar{I} = -\frac{j\bar{I}}{\omega C} \Rightarrow C = \frac{|\bar{I}|}{\omega |\bar{V}|} = 0.28 \text{ mF}$$

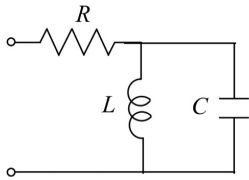
Ans

$$(c) \quad \bar{V} = 20 e^{\frac{\pi}{3}j} \quad \bar{I} = 5 e^{\frac{1}{3}\pi j}$$

$$\angle \dot{Z} = \angle \bar{V} - \angle \bar{I} = \frac{\pi}{3} - \frac{\pi}{3} = 0 \Rightarrow \alpha - \beta = 0 \Rightarrow \text{RESISTORE}$$

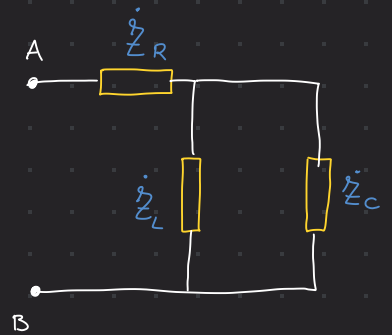
$$\dot{Z} = R \Rightarrow \bar{V} = R \cdot \bar{I} \Rightarrow R = \frac{|\bar{I}|}{|\bar{V}|} = \frac{20}{5} = 4 \Omega \quad \text{Ans}$$

ES. 1.4 - Si consideri il circuito in figura, determinando L tale che la parte immaginaria dell'impedenza vista ai capi dei morsetti risulti $\text{Im}\{\dot{Z}\} = 100 \Omega$.



$$C = 10 \mu\text{F}$$

$$f = 1 \text{ kHz}$$



$$L / \text{Im}\{\dot{Z}_{eq}\} = 100 \Omega \quad \text{ovvero} \quad \dot{Z}_{eq} = R + 100j$$

$$\Omega = 2\pi f = 2\pi \times 10^3$$

$$\left. \begin{aligned} \dot{Z}_L &= j\omega L \\ \dot{Z}_C &= -\frac{j}{\omega C} \end{aligned} \right\} \quad \dot{Z}_{eq} = \left(\dot{Z}_L \parallel \dot{Z}_C \right) + \dot{Z}_R = \frac{j\omega L \cdot \frac{j}{\omega C}}{j(\omega L - \frac{1}{\omega C})} + R$$

$$= R + j \frac{\frac{L}{C}}{\frac{\omega^2 LC - 1}{\omega C}} = R + j \frac{\omega L}{1 - \omega^2 LC}$$

$$\Rightarrow \text{Im}\{\dot{Z}_{eq}\} = \frac{\omega L}{1 - \omega^2 LC} = 100$$

$$\omega L = 100 - 100\omega^2 LC \Rightarrow \omega L + 100\omega^2 LC = 100 \Rightarrow L(\omega + 100\omega^2 C) = 100$$

$$\Rightarrow L = \frac{100}{\omega + 100\omega^2 C} = 2.185 \times 10^{-3} \text{ H} = 2.185 \text{ mH}$$

ES. 1.5 - A quale di queste impedenze corrisponde la fase $\varphi = -\pi/4$?

1: R-L serie	2: R-C serie	3: R-C parallelo	4: L-C serie
$R = 10 \Omega$	$R = 10 \Omega$	$R = 0.5 \Omega$	$C = 1 F$
$L = 10 mH$	$C = 10 mF$	$C = 0.2 F$	$L = 1 H$
$\omega = 100 rad/s$	$\omega = 100 rad/s$	$\omega = 10 rad/s$	$\omega = 1 rad/s$

(1)



$$\dot{Z}_R = 10 \Omega$$

$$\dot{Z}_L = j\omega L = j10^2 \cdot 10 \times 10^{-3} = j10 \Omega$$

$$\Rightarrow \dot{Z}_{eq} = 10 + j10 = 10.05 e^{j(0.1)}$$

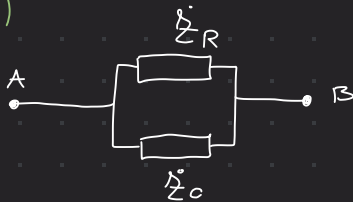
(2)



$$\dot{Z}_R = 10 \Omega$$

$$\dot{Z}_C = -\frac{j}{\omega C} = -\frac{j}{10^2 \cdot 10 \times 10^{-3}} = -j10 \Omega \quad \Rightarrow \dot{Z}_{eq} = 10 - j10 = 10.05 e^{-j(0.1)}$$

(3)



$$\dot{Z}_R = 0.5 \Omega$$

$$\dot{Z}_C = -\frac{j}{10 \cdot \frac{1}{5}} = -\frac{j}{2} \Omega \quad \Rightarrow \dot{Z}_{eq} = \frac{\dot{Z}_R \cdot \dot{Z}_C}{\dot{Z}_R + \dot{Z}_C} = \frac{\frac{1}{4} - \frac{j}{4}}{\frac{1}{4} - \frac{j}{4}} = \frac{\sqrt{2}}{4} e^{-j(\frac{1}{4}\pi)}$$

(3) Ans

(4)



$$\dot{Z}_C = -\frac{j}{1 \cdot 1} = -j$$

$$\dot{Z}_L = j \cdot 1 \cdot 1 = j \quad \Rightarrow \dot{Z}_{eq} = 0 \text{ WTF??}$$

ES. 1.6 - Dati i seguenti fasori $\bar{V}_1 = 10 \exp(j\pi/6)$, $\bar{V}_2 = 10 \exp(-j\pi/6)$, $\bar{V}_3 = 5 \exp(-j\pi/3)$:

- rappresentare nel piano complesso i fasori $\bar{V}_1, \bar{V}_2, \bar{V}_3$;
- calcolare i fasori: $\bar{V}_1 + \bar{V}_2, \bar{V}_1 - \bar{V}_2, \bar{V}_1 + \bar{V}_3, \bar{V}_1 - \bar{V}_3$;
- rappresentare nel piano complesso i fasori valutati al punto b)
- rappresentare nel tempo le tensioni corrispondenti ai fasori dei punti a) e b), definito la trasformazione fasoriale come segue:

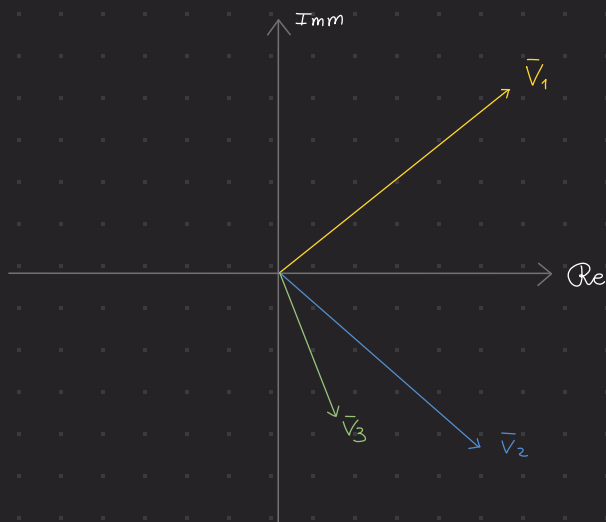
$$v(t) = V_M \sin(\omega t + \alpha) \leftrightarrow \bar{V} = V_M \exp(j\alpha)$$

(a)

$$\bar{V}_1 = 10 e^{j\frac{\pi}{6}} = 10 \left[\cos\left(\frac{\pi}{6}\right) + j \sin\left(\frac{\pi}{6}\right) \right] = 10 \left[\frac{\sqrt{3}}{2} + j \frac{1}{2} \right] = \underline{5\sqrt{3} + 5j}$$

$$\bar{V}_2 = 10 e^{-j\frac{\pi}{6}} = 10 \left[\cos\left(\frac{\pi}{6}\right) - j \sin\left(\frac{\pi}{6}\right) \right] = \underline{5\sqrt{3} - 5j}$$

$$\bar{V}_3 = 5 e^{-j\frac{\pi}{3}} = 5 \left[\cos\left(\frac{\pi}{3}\right) - j \sin\left(\frac{\pi}{3}\right) \right] = \underline{\frac{5}{2} - j \frac{5}{2}\sqrt{3}}$$



(b) • $V_1 + V_2 = 10\sqrt{3}$

• $V_1 - V_2 = 10j$

• $V_1 + V_3 = 11.16 + 0.67j$

• $V_1 - V_3 = 6.16 + 9.33j$

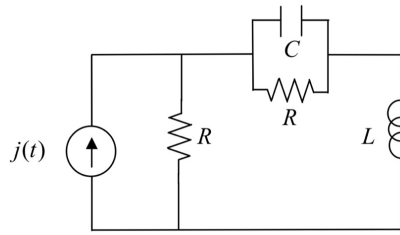
(d) dato $v(t) = V_M \sin(\omega t + \alpha) \Leftrightarrow \bar{V} = V_M e^{j\alpha}$

$$\bar{V}_1 = 10 e^{j\frac{\pi}{6}} \Leftrightarrow v_1(t) = 10 \cos\left(\omega t + \frac{\pi}{6}\right)$$

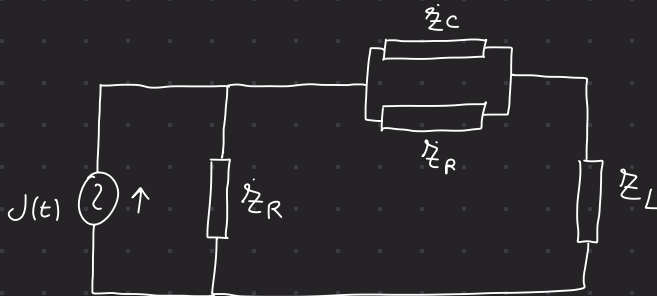
$$\bar{V}_2 = 10 e^{-j\frac{\pi}{6}} \Leftrightarrow v_2(t) = 10 \cos\left(\omega t - \frac{\pi}{6}\right)$$

$$\bar{V}_3 = 5 e^{-j\frac{\pi}{3}} \Leftrightarrow v_3(t) = 5 \cos\left(\omega t - \frac{\pi}{3}\right)$$

ES. 2.1 - Con riferimento al seguente circuito, valutare l'impedenza \dot{Z}_{eq} vista ai capi del generatore e la potenza complessa \dot{S} erogata dal generatore.



$$\begin{aligned} j(t) &= 10 \sin(2t) \text{ A} \\ R &= 2 \Omega \\ L &= 1 \text{ H} \\ C &= 0.25 \text{ F} \end{aligned}$$



$$\dot{Z}_R = 2 \Omega$$

$$\dot{Z}_C = -\frac{j}{2 \cdot 0.25} = -2j$$

$$\dot{Z}_L = 2j$$

$$\dot{Z}_{eq} = \left[(\dot{Z}_C \parallel \dot{Z}_R) + \dot{Z}_L \right] \parallel \dot{Z}_R = \left(\frac{4}{5} + \frac{2}{5}j \right) \text{ Ans 1}$$

$$j(t) = 10 \sin(2t) = 10 \cos\left(2t - \frac{\pi}{2}\right) \text{ A} \Rightarrow \bar{I} = 10 e^{-j\frac{\pi}{2}} = 10 \left[\cos\left(\frac{\pi}{2}\right) - j \sin\left(\frac{\pi}{2}\right) \right] = -10j$$

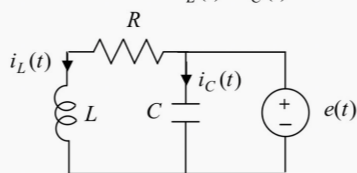
Potenza complessa

$$\begin{aligned} \dot{S} &= \frac{1}{2} \bar{V}_j \cdot \bar{I}^* = \frac{1}{2} \cdot \dot{Z}_{eq} \cdot \bar{I} \cdot \bar{I}^* \\ &= \frac{1}{2} \dot{Z}_{eq} \cdot (10^2 + \frac{1}{4}\pi^2) \\ &= 40 + 20.5j \end{aligned}$$

$|Z|^2$ modulo quadro

$$\begin{aligned} Z \cdot Z^* &= (a+ib) \cdot (a-ib) = \\ &= a^2 - iab + iab - i^2 b^2 = \\ &= a^2 + b^2 \end{aligned}$$

ES. 2.2 - Con riferimento al seguente circuito, valutare l'impedenza \dot{Z}_{eq} vista ai capi del generatore e le correnti $i_L(t)$ e $i_C(t)$



$$e(t) = 10 \cos(1000t) \text{ V}$$

$$R = 10 \, \Omega \quad L = 20 \text{ mH}$$

$$C = 0.1 \text{ mF}$$

Risultato: $\dot{Z}_{eq} = 5 - j15 \, \Omega$; $i_L(t) = 0.45 \cos(1000t - 1.11) \text{ A}$, $i_C(t) = -\sin(1000t) \text{ A}$.

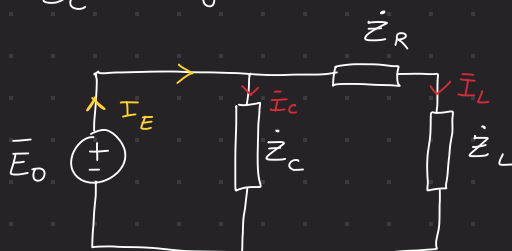
$$\dot{Z}_R = 10 \, \Omega$$

$$\dot{Z}_L = 20j \, \Omega$$

$$\dot{Z}_C = -10j \, \Omega$$

$$\dot{Z}_{eq} = 5 - 15j$$

$$e(t) = 10 \cos(10^3 t) \Rightarrow \bar{E}_0 = 10 \text{ V}$$



$$\bar{I}_E = \frac{\bar{E}_0}{\dot{Z}_{eq}} = \frac{1}{5} + \frac{3}{5}j$$

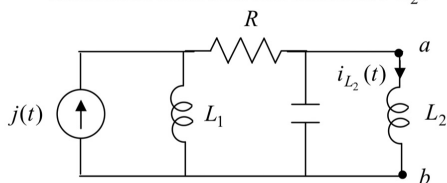
$$\bar{I}_L = \bar{I}_E \cdot \frac{\dot{Z}_C}{\dot{Z}_C + \dot{Z}_R + \dot{Z}_L} = \frac{1}{5} - \frac{2}{5}j \text{ A}$$

$$\Rightarrow \bar{I}_C = \bar{I}_E - \bar{I}_L = \frac{1}{5}j \text{ A}$$

$$\Rightarrow \bar{I}_C \Rightarrow i(t) = \frac{1}{5} \cos(1000t - \frac{\pi}{2}) = -\frac{1}{5} \sin(1000t)$$

$$\bar{I}_L \Rightarrow i_L(t) = \frac{\sqrt{5}}{5} \cos(1000t - 1.107)$$

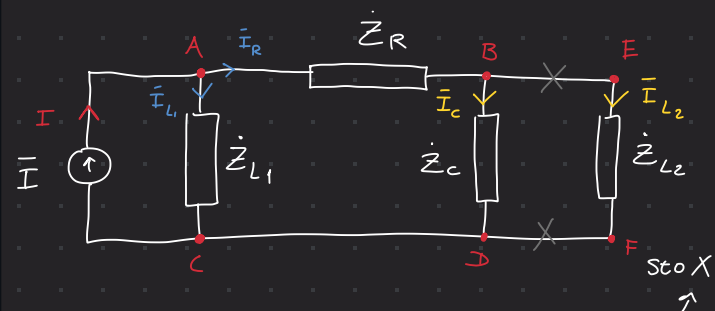
ES. 2.3 - Applicando il teorema di Thévenin, valutare la potenza complessa e la potenza istantanea assorbita dall'induttore L_2 .



$$j(t) = 10\sqrt{2} \sin(100t + 0.35) \text{ A}$$

$$R = 4 \Omega, \quad C = 3 \text{ mF},$$

$$L_1 = 2 \text{ mH}, \quad L_2 = 5 \text{ mH}$$



$$(1) \quad \dot{Z}_{eqBD} = (\dot{Z}_{L1} + \dot{Z}_R) \parallel \dot{Z}_C = 1.72 - 1.98j$$

(2) Thevenin \rightarrow Tensione a vuoto

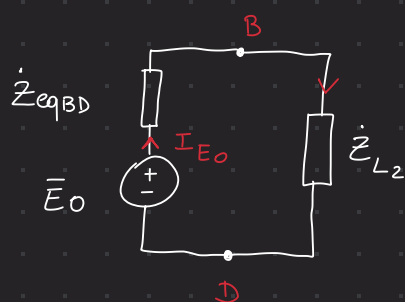
$$\bar{V}_{EF} = \bar{V}_{BD} \Rightarrow V_{BD} = \dot{Z}_C \cdot \bar{I}_C \quad \text{ma} \quad \bar{I}_R = \bar{I} \cdot \frac{\dot{Z}_{L1}}{\dot{Z}_{L1} + (\dot{Z}_R + \dot{Z}_C)} = 0.21 + 0.33j$$

$$\Rightarrow \bar{I}_C = \bar{I}_R \Rightarrow V_{BD} = 1.114 - 0.69j$$

Sugli es
e al contrario (??)

$$1.31 \angle -0.554$$

\Rightarrow Thevenin:



$$\bar{I}_{E0} = \frac{\bar{E}_0}{\dot{Z}_{eq}} = 0.477 + 0.147j$$

$$\Rightarrow \dot{A}_{L2} = j \cdot \underbrace{X_{L2}}_{\text{Reattanza induttiva}} \cdot I^2 = j\omega L_2 I^2 = 0.07 - 0.103j$$

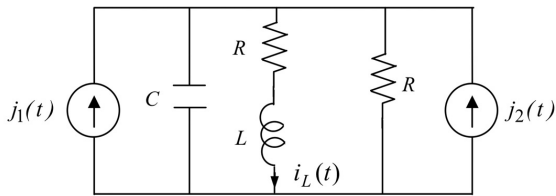
$$X_{L2} = \omega L_2$$

$$p(t) = v(t) \cdot i(t)$$

$$\Rightarrow i_{L2}(t) = 0.499 \sqrt{2} \cos(100t + 0.299) = 0.499 \sqrt{2} \sin(100t + 0.299 + \frac{\pi}{2})$$

$$v_0(t) = 1.31 \cos(100t - 0.554)$$

ES. 2.4 - Con riferimento al seguente circuito valutare la corrente $i_L(t)$.



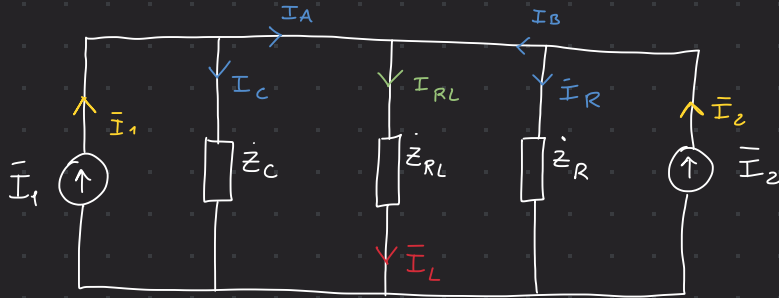
$$j_1(t) = 10 \cos(1000t) \text{ A}$$

$$j_2(t) = 10 \sin(1000t) \text{ A}$$

$$R = 2 \Omega$$

$$L = 2 \text{ mH}$$

$$C = 1 \text{ mF}$$



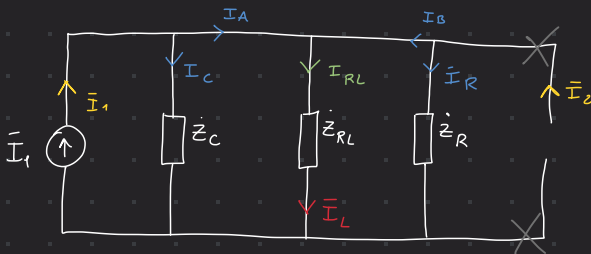
$$C \quad Z_C = -j \Omega$$

$$A \quad Z_R = 2 \Omega$$

$$Z_L = 2j$$

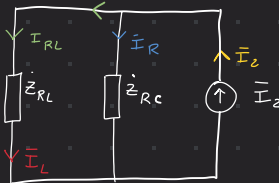
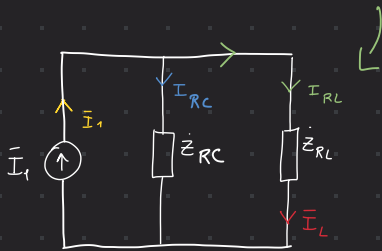
$$\left. \begin{array}{l} Z_R \\ Z_L \end{array} \right\} Z_{RL} = 2 + 2j$$

C'



$$\bar{I}_1 = 10 \text{ A} \quad \bar{I}_2 = 10 \left[\cos\left(\frac{\pi}{2}\right) - j \sin\left(\frac{\pi}{2}\right) \right] = -10j$$

$$\bar{I}'_{RL} = \bar{I}_1 \cdot \frac{Z_{RC}}{Z_{RC} + Z_{RL}} = -\frac{10}{3}j = -3.\bar{3}j$$



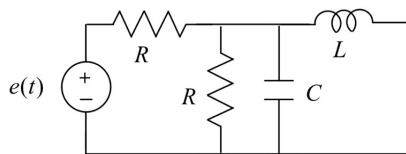
$$\bar{I}''_{RL} = \bar{I}_2 \cdot \frac{Z_{RC}}{Z_{RC} + Z_{RL}} = -3.\bar{3} \text{ A}$$

$$\Rightarrow \bar{I} = \bar{I}' + \bar{I}'' = -3.33 - 3.33j = 4.71 \angle -\frac{3}{4}\pi$$

$$\Rightarrow i(t) = 4.71 \cos\left(1000t - \frac{3}{4}\pi\right) = 4.71 \sin\left(1000t - \frac{3}{4}\pi + \frac{\pi}{2}\right) = 4.71 \sin\left(1000t - \frac{1}{4}\pi\right)$$

SI TROVA

ES. 2.5 - Applicando il teorema di Norton, valutare la potenza complessa e la potenza istantanea assorbita dal parallelo R-C in figura.



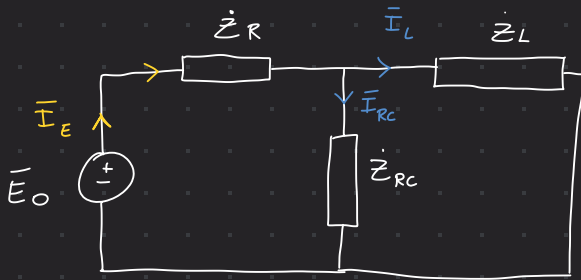
$$e(t) = 5\sqrt{2} \sin(1000t + \pi/3) \text{ V}$$

$$R = 0.21 \Omega, \quad L = 1.12 \text{ mH}$$

$$C = 1.23 \text{ mF.}$$

Risultato: $\dot{A} = 29.72 \text{ W} - j7.68 \text{ VAR}$; $p(t) = [29.72 - 30.70 \cos(2000t + 2.27)] \text{ W}$.

$$\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi}$$



$$e(t) = 5\sqrt{2} \sin(1000t + \frac{\pi}{3}) \text{ V}$$

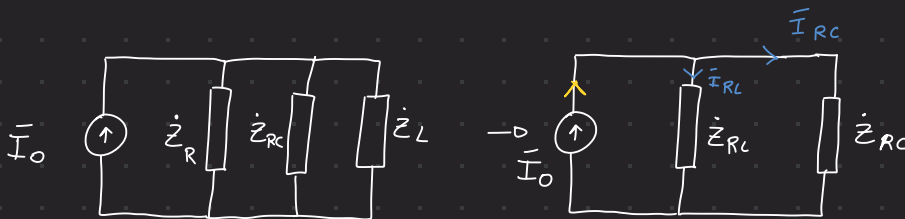
$$= 5\sqrt{2} \cos(1000t - \frac{1}{6}\pi) \text{ V}$$

$$\Leftrightarrow \bar{E}_0 = \frac{5\sqrt{2}}{\sqrt{2}} e^{-\frac{\pi}{6}j} \bar{E}_0 \text{ V}$$

$$\bar{E}_0 = 5 \left[\cos(\frac{\pi}{6}) - j \sin(\frac{\pi}{6}) \right]$$

$$= \frac{5\sqrt{3}}{2} - \frac{5}{2}j \text{ (F)}$$

$$\begin{aligned} \dot{Z}_R &= 0.21 \Omega \\ \dot{Z}_L &= 1.12j \Omega \\ \dot{Z}_C &= -0.813j \Omega \end{aligned} \quad \rightarrow \quad \dot{Z}_{RC} = 0.196 - 0.051j \Omega$$



$$\bar{V} = \bar{Z} \cdot \bar{I} \rightarrow \bar{I}_0 = \frac{\bar{E}_0}{\dot{Z}_{Eq}} = \frac{\bar{E}_0}{(\dot{Z}_R \parallel \dot{Z}_{RC}) \parallel \dot{Z}_L} = 42.032 - 22.34j$$

$$\Rightarrow \bar{I}_{RC} = \bar{I}_0 \cdot \frac{\dot{Z}_{RL}}{\dot{Z}_{RL} + \dot{Z}_{RC}} = 23.69 - 6.58j \text{ A}$$

$$\dot{S} = \dot{Z}_{RC} \cdot \bar{I}_{RC}^2 =$$

