ES. 1.1 **Esprimere la corrente $i(t)$ in termini di fasore nei seguenti tre casi:**

a) $i(t) = 4\sin(\omega t - 1.14)$ b) $i(t) = 10\sin(\omega t - \pi)$ c) $i(t) = 8\sin(\omega t + \pi/2)$

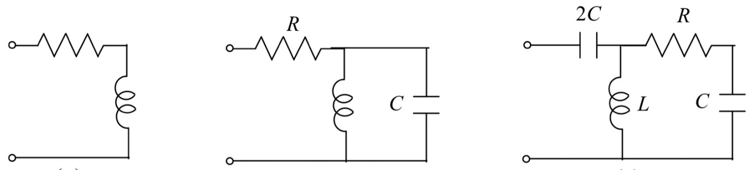
Risultato: a) $\bar{I} = 4\exp(-j1.14)$; *b)* $\bar{I} = -10$; *c)* $\bar{I} = 8j$.

(a) $\bar{I} = 4\, e^{-J1.14}$

(b) $\bar{I}_2 = 10\, e^{-J\pi} \rightarrow -1 = -10$

(c) $\bar{I} = 8\, e^{j\frac{\pi}{2}} = 8\left[\cancel{\cos\left(\tfrac{\pi}{2}\right)}^{0} + J\,\cancel{\sin\left(\tfrac{\pi}{2}\right)}^{1}\right] = 8j$

ES. 1.2 **Valutare (in coordinate cartesiane e polari) le impedenze viste ai capi dei morsetti:**



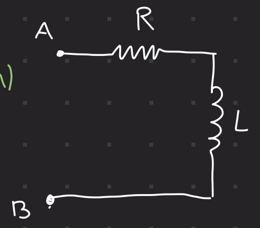
(a) $R = 10\,\Omega$ $L = 1\,mH$ $\omega = 10^4\ rad/s$

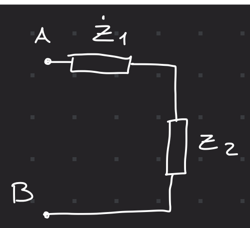
(b) $R = 8\,\Omega$, $L = 15\,mH$ $C = 0.4\,mF$, $f = 50\,Hz$

(c) $R = 200\,\Omega$, $L = 16\,mH$ $C = 10\,\mu F$, $\omega = 2.5\cdot 10^3\ rad/s$

Risultato: *a)* $\dot{Z} = 10 + 10j = 10\sqrt{2}\exp(j\pi/4)\,\Omega$; *b)* $\dot{Z} = 8 + 11.54j = 14\exp(j0.965)\,\Omega$; *c)* $\dot{Z} = 8 + 20j = 21.5\exp(j1.19)\,\Omega$;

(a)





$\dot{Z}_1 = 10\,\Omega$

$\dot{Z}_2 = j\cdot 10^4 \cdot 1\times 10^{-3} = 10j$

$\Rightarrow \dot{Z}_{eq} = \dot{Z}_1 + \dot{Z}_2 = 10 + 10j$

$\Rightarrow$ Exp Form $= |z|\, e^{J\angle z}$

$\Rightarrow |\dot{Z}_{Eq}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

$\angle z = \operatorname{atan}\left(\frac{10}{10}\right) = \frac{1}{4}\pi$

$\Rightarrow$ Exp $\dot{Z}_{Eq} = 10\sqrt{2}\; e^{\frac{1}{4}\pi}$

Sulla calcolatrice basta convertire in $r\angle\theta$

(b)

$\omega = 2\pi f = 100\pi$ rad/s

$\dot{Z}_R = 8\,\Omega$

$\dot{Z}_L = J\, 10^2\pi \cdot 15\times 10^{-3} = \frac{J\pi\, 15}{10} = 1.5\pi j$

$\dot{Z}_C = -\frac{J}{10^2\pi\cdot\frac{2}{5}10^{-3}} = -\frac{5j}{\pi\cdot 2\cdot 10^{-1}} = -\frac{25}{\pi}J$

$\Rightarrow \dot{Z}_{Eq} = \left(\dot{Z}_C \parallel \dot{Z}_L\right) + \dot{Z}_R = 8 + 11.55j$ Ans $\Rightarrow \dot{Z}_{eq} = r\, e^{J\angle\theta} = 14\, e^{J0.965}$ Ans

(c)

$$\dot{Z}_R = 200\ \Omega$$

$$\dot{Z}_L = j \cdot 2.5\times\cancel{10^{3}} \cdot 16\times\cancel{10^{-3}} = 40j$$

$$\dot{Z}_c = -\frac{j}{10\cdot\cancel{10^{-6}}_{-3}\cdot 2.5\times\cancel{10^{3}}} = -40j$$

Boh !

$$\Rightarrow \dot{Z}_{eq} = \left[(\dot{Z}_R + \dot{Z}_c) \parallel \dot{Z}_L \right] + 2\dot{Z}_c = 8 - 40j = 8\sqrt{26}\, e^{-1.37j}$$

ES. 1.3 Le seguenti coppie di fasori esprimono tensione e corrente relative ad un dato bipolo. Dire, nei tre casi, se si tratta di un resistore, un condensatore o un induttore e valutare il valore dei parametri corrispondenti *R*, *C* o *L*

a) $v(t) = 15\cos(400t + 1.2)$, $i(t) = 3\sin(400t + 1.2)$;

b) $v(t) = 8\cos(900t - \pi/3)$, $i(t) = 2\sin(900t + 2\pi/3)$;

c) $v(t) = 20\cos(250t + \pi/3)$, $i(t) = 5\sin(250t + 5\pi/6)$;

(a)

$$\bar{V} = 15\, e^{1.2j} \qquad i(t) = 3\sin(400t + 1.2) = 3\cos\left(400t + 1.2 - \frac{\pi}{2}\right)$$

$$\Rightarrow \bar{I} = 3\, e^{\left(1.2 - \frac{\pi}{2}\right)j}$$

Per capire se si tratta di un resistore, induttore o condensatore dobbiamo guardare la **fase tra tensione e corrente**:

- **Resistore:** 0 o pi/2 (in fase) $\alpha = \beta$
- **Induttore**: la tensione è in anticipo rispetto alla corrente di pi/2 $\alpha = \beta + \frac{\pi}{2}$ $\beta = \alpha - \frac{\pi}{2}$
- **Condensatore**: la corrente è in anticipo rispetto alla tensione di pi/2 (la corrente è in ritardo) $\alpha = \beta - \frac{\pi}{2}$ $\beta = \alpha + \frac{\pi}{2}$

Controllo la fas, o arg

$$\angle \dot{Z} = \underbrace{\angle \bar{V}}_{\alpha} - \underbrace{\angle \bar{I}}_{\beta} = \cancel{1.2} - \left(\cancel{1.2} + \frac{\pi}{2}\right) = \frac{\pi}{2} \Rightarrow \alpha = \beta - \frac{\pi}{2} \Rightarrow \text{INDUTTORE}$$

$$\Rightarrow \dot{Z}_L = j\omega L \quad (1) \qquad \bar{V} = \dot{Z}\bar{I} \quad (2)$$

INDUTTORE

$$\Rightarrow \bar{V} = j\omega L \bar{I} \rightarrow L = \frac{|\bar{V}|}{\omega |\bar{I}|} = \frac{15}{\omega\, 3} = 0.0125\ H = 12.5\ mH$$

(b) $\bar{V} = 8\, e^{-\frac{\pi}{3}j}$ $\qquad i(t) = 2\sin\left(900t + \frac{2\pi}{3}\right) = 2\cos\left(900t + \frac{2}{3}\pi - \frac{\pi}{2}\right)$

$= 2\cos\left(900t + \frac{1}{6}\pi\right)$

$\Rightarrow \bar{I} = 2e^{\frac{1}{6}\pi j}$

$$\Rightarrow \angle \dot{Z} = \underbrace{\angle \bar{V}}_{\alpha} - \underbrace{\angle \bar{I}}_{b} = -\frac{\pi}{3} - \frac{1}{6}\pi = -\frac{1}{2}\pi \Rightarrow \alpha = \beta - \frac{\pi}{2} \Rightarrow \text{CONDENSATORE}$$

$$\Rightarrow \dot{Z}_C = -\frac{j}{\omega C} \Rightarrow \bar{V} = \dot{Z}_C \bar{I} = -\frac{j\bar{I}}{\omega C} \Rightarrow C = \frac{|\bar{I}|}{\omega|\bar{V}|} = 0.28\, mF$$

Ans

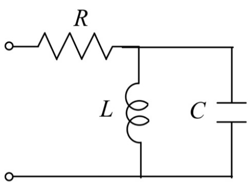
(c) $\bar{V} = 20\, e^{\frac{\pi}{3}j}$ $\qquad \bar{I} = 5\, e^{\frac{1}{3}\pi}$

$$\angle \dot{Z} = \angle \bar{V} - \angle \bar{I} = \frac{\pi}{3} - \frac{\pi}{3} = 0 \Rightarrow \alpha - \beta = 0 \Rightarrow \text{RESISTORE}$$

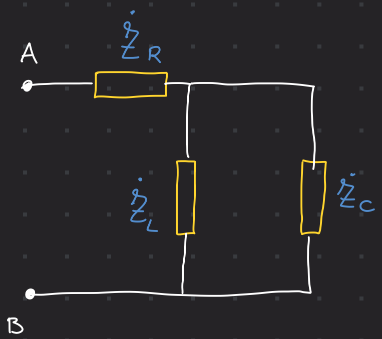
$$\dot{Z} = R \Rightarrow \bar{V} = R \cdot \bar{I} \Rightarrow R = \frac{|\bar{I}|}{|\bar{V}|} = \frac{20}{5} = 4\,\Omega$$

Ans

ES. 1.4 - **Si consideri il circuito in figura, determinando L tale che la parte immaginaria dell'impedenza vista ai capi dei morsetti risulti** $\mathrm{Im}\{\dot{Z}\} = 100\,\Omega$.



$C = 10\ \mu F$
$f = 1\ kHz$



$L \,/\, \mathrm{Im}\{\dot{Z}_{eq}\} = 100\,\Omega$ ovvero $\dot{Z}_{eq} = n + 100j$ $\qquad \Omega = 2\pi f = 2\pi \times 10^{3}$

$$\left.\begin{aligned} \dot{Z}_L &= j\omega L \\ \dot{Z}_C &= -\frac{j}{\omega C} \end{aligned}\right\} \quad \dot{Z}_{eq} = \left(\dot{Z}_L \parallel \dot{Z}_C\right) + \dot{Z}_R = \frac{j^2 \frac{\cancel{\omega} L}{\cancel{\omega} C}}{j\left(\omega L - \frac{1}{\omega C}\right)} + R$$

$$= R + j\,\frac{L/\cancel{C}}{\frac{\omega^2 LC - 1}{\omega \cancel{C}}} = R + j\,\frac{\omega L}{1 - \omega^2 LC}$$

$$\Rightarrow \mathrm{Im}\{\dot{Z}_{eq}\} = \frac{\omega L}{1 - \omega^2 LC} = 100$$?

$$\omega L = 100 - 100\,\omega^2 LC \Rightarrow \omega L + 100\,\omega^2 LC = 100 \Rightarrow L(\omega + 100\,\omega^2 C) = 100$$

$$\Rightarrow L = \frac{100}{\omega + 100\,\omega^2 C} = 2.185 \times 10^{3}\, H = 2.185\, mH$$

ES. 1.5 - **A quale di queste impedenze corrisponde la fase $\varphi = -\pi/4$?**

1: R-L serie	**2:** R-C serie	**3:** R-C parallelo	**4:** L-C serie
$R = 10\,\Omega$	$R = 10\,\Omega$	$R = 0.5\,\Omega$	$C = 1\,F$
$L = 10\,mH$	$C = 10\,mF$	$C = 0.2\,F$	$L = 1\,H$
$\omega = 100\,rad/s$	$\omega = 100\,rad/s$	$\omega = 10\,rad/s$	$\omega = 1\,rad/s$

(1)

$\dot{Z}_R = 10\,\Omega$

$\dot{Z}_L = j\omega L = j\,\cancel{10}^{2} \cdot \cancel{10} \times 10^{\cancel{-3}\,-1} = j\,\Omega$

$\Rightarrow \dot{Z}_{eq} = 10 + j = 10.05\, e^{j(0.1)}$

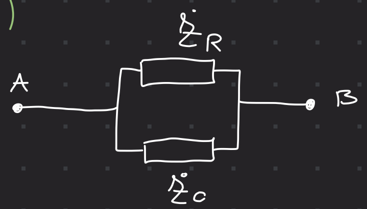
(2)

$\dot{Z}_R = 10\,\Omega$

$\dot{Z}_C = -\frac{j}{\omega C} = -\frac{j}{\cancel{10}^{2}\ \cancel{10} \times 10^{\cancel{-3}\,-1}} = -j$

$\Rightarrow \dot{Z}_{eq} = 10 - j = 10.05\, e^{-j(0.1)}$

(3)



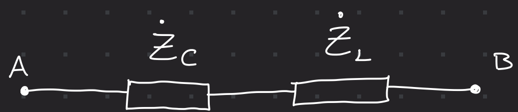
$\dot{Z}_R = 0.5$

$\dot{Z}_C = -\frac{j}{10 \cdot \frac{1}{5}} = -\frac{1}{2}j$

$\Rightarrow \dot{Z}_{eq} = \frac{\dot{Z}_R \cdot \dot{Z}_C}{\dot{Z}_R + \dot{Z}_C} = \frac{1}{4} - \frac{1}{4}j = \frac{\sqrt{2}}{4}\, e^{-j\left(\frac{1}{4}\pi\right)}$

(3) Ans

(4)



$\dot{Z}_C = -\frac{j}{1 \cdot 1} = -j$

$\dot{Z}_L = j \cdot 1 \cdot 1 = j$

$\Rightarrow \dot{Z}_{eq} = \emptyset$ WTF??

ES. 1.6 - **Dati i seguenti fasori** $\overline{V}_1 = 10\exp(j\pi/6)$, $\overline{V}_2 = 10\exp(-j\pi/6)$, $\overline{V}_3 = 5\exp(-j\pi/3)$:

a) **rappresentare nel piano complesso i fasori** $\overline{V}_1, \overline{V}_2, \overline{V}_3$;

b) **calcolare i fasori:** $\overline{V}_1 + \overline{V}_2, \overline{V}_1 - \overline{V}_2, \overline{V}_1 + \overline{V}_3, \overline{V}_1 - \overline{V}_3$;

c) **rappresentare nel piano complesso i fasori valutati al punto b)**

d) **rappresentare nel tempo le tensioni corrispondenti ai fasori dei punti a) e b), definito la trasformazione fasoriale come segue:**

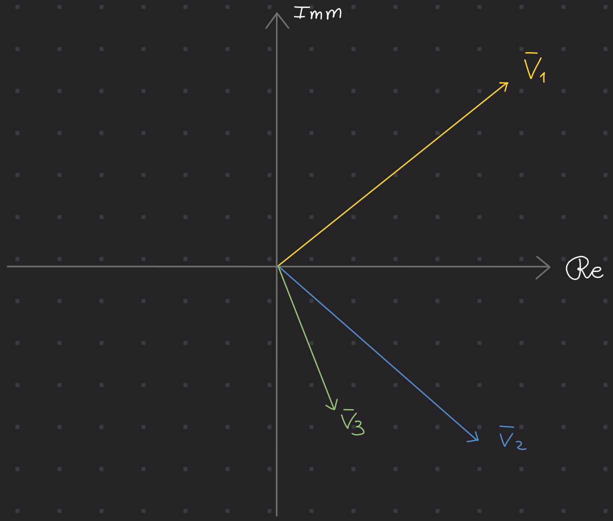
$$v(t) = V_M \sin(\omega t + \alpha) \leftrightarrow \overline{V} = V_M \exp(j\alpha)$$

(a)

$$\bar{V}_1 = 10\, e^{j\frac{\pi}{6}} = 10\left[\cos\left(\tfrac{\pi}{6}\right) + j\sin\left(\tfrac{\pi}{6}\right)\right] = 10\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right] = 5\sqrt{3} + 5j$$

$$\bar{V}_2 = 10\, e^{-j\frac{\pi}{6}} = 10\left[\cos\left(\tfrac{\pi}{6}\right) + j\sin\left(\tfrac{\pi}{6}\right)\right] = 5\sqrt{3} - 5j$$

$$\bar{V}_3 = 5\, e^{-j\frac{\pi}{3}} = 5\left[\cos\left(\tfrac{\pi}{3}\right) - j\sin\left(\tfrac{\pi}{3}\right)\right] = \frac{5}{2} - j\frac{5}{2}\sqrt{3}$$



(b)

- $V_1 + V_2 = 10\sqrt{3}$
- $V_1 - V_2 = 10j$
- $V_1 + V_3 = 11.16 + 0.67j$
- $V_1 - V_3 = 6.16 + 9.33j$

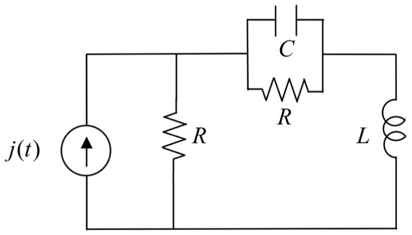
(d) dato $v(t) = V_M \sin(\omega t + \alpha) \rightleftharpoons \bar{V} = V_M e^{j\alpha}$

$$V_1 = 10\, e^{j\frac{\pi}{6}} \rightleftharpoons v_1(t) = 10\cos\left(\omega t + \tfrac{\pi}{6}\right)$$

$$\bar{V}_2 = 10\, e^{-j\frac{\pi}{6}} \rightleftharpoons v_2(t) = 10\cos\left(\omega t - \tfrac{\pi}{6}\right)$$

$$\bar{V}_3 = 5\, e^{-j\frac{\pi}{3}} \rightleftharpoons v_3(t) = 5\cos\left(\omega t - \tfrac{\pi}{3}\right)$$

ES. 2.1 - Con riferimento al seguente circuito, valutare l'impedenza $\dot{Z}_{eq}$ vista ai capi del generatore e la potenza complessa $\dot{S}$ erogata dal generatore.

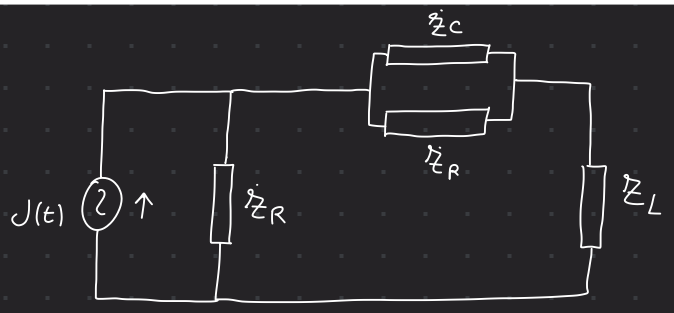


$j(t) = 10\sin(2t)\ A$

$R = 2\,\Omega$

$L = 1\,H$

$C = 0.25\,F$



$\dot{Z}_R = 2\,\Omega$

$\dot{Z}_C = -\dfrac{j}{2 \cdot 0.25} = -2j$

$\dot{Z}_L = 2j$

$$\dot{Z}_{eq} = \left[\left(\dot{Z}_C \parallel \dot{Z}_R\right) + \dot{Z}_L\right] \parallel \dot{Z}_R = \frac{4}{5} + \frac{2}{5}j$$

Ans 1

$$j(t) = 10\sin(2t) = 10\cos\left(2t - \frac{\pi}{2}\right) A \rightleftharpoons \bar{I} = 10\, e^{-j\frac{\pi}{2}} = 10\left[\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right] = -10j$$

Potenza complessa

$$\rightarrow \boxed{\dot{S} = \frac{1}{2}\bar{V}_J \cdot \bar{I}^*} = \frac{1}{2} \cdot \dot{Z}_{eq} \cdot \bar{I} \cdot \bar{I}^*$$

$$= \frac{1}{2}\dot{Z}_{eq} \cdot \left(10^2 + \frac{1}{4}\pi^2\right)$$

$$= 40 + 20.5j$$

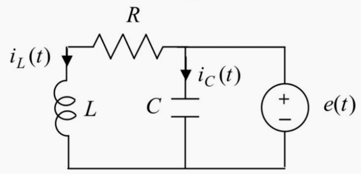
$\rightarrow |z|^2$ modulo quadro

$z \cdot z^* = (a+ib)\cdot(a-ib) =$

$= a^2 - \cancel{iab} + \cancel{iab} - i^2 b =$

$= a^2 + b^2$

ES. 2.2 - **Con riferimento al seguente circuito, valutare l'impedenza $\dot{Z}_{eq}$ vista ai capi del generatore e le correnti $i_L(t)$ e $i_C(t)$**



$e(t) = 10\cos(1000t)\ V$
$R = 10\ \Omega \quad L = 20\ mH$
$C = 0.1\ mF$

Risultato: $\dot{Z}_{eq} = 5 - j15\ \Omega$; $i_L(t) = 0.45\cos(1000t - 1.11)\ A$, $i_C(t) = -sin(1000t)\ A$.

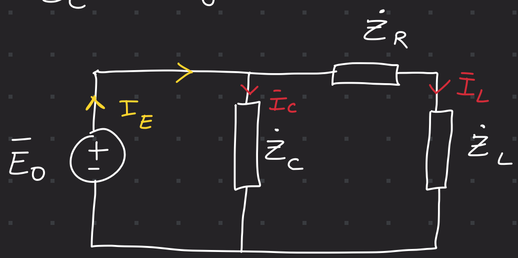
$\dot{Z}_R = 10\ \Omega$

$\dot{Z}_L = 20j\ \Omega$

$\dot{Z}_C = -10j\ \Omega$

$\dot{Z}_{eq} = 5 - 15j$

$e(t) = 10\cos(10^3 t) \rightleftharpoons \bar{E}_0 = 10V$



$$\bar{I}_E = \frac{\bar{E}_0}{\dot{Z}_{eq}} = \frac{1}{5} + \frac{3}{5}j$$

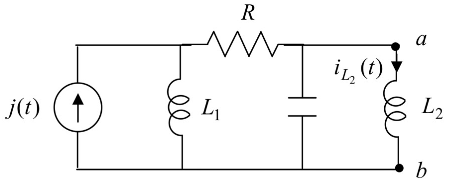
$$\bar{I}_L = \bar{I}_E \cdot \frac{\dot{Z}_C}{\dot{Z}_C + \dot{Z}_R + \dot{Z}_L} = \frac{1}{5} - \frac{2}{5}j\ A$$

$$\Rightarrow \bar{I}_C = \bar{I}_E - \bar{I}_L = \frac{1}{5}j\ A$$

$$\Rightarrow \bar{I}_C \rightleftharpoons i(t) = \frac{1}{5}\cos\left(1000t \ \ \frac{\pi}{2}\right) = -\frac{1}{5}\sin(1000\,t)$$

$$\bar{I}_L \rightleftharpoons i_L(t) = \frac{\sqrt{5}}{5}\cos(1000t - 1.107)$$

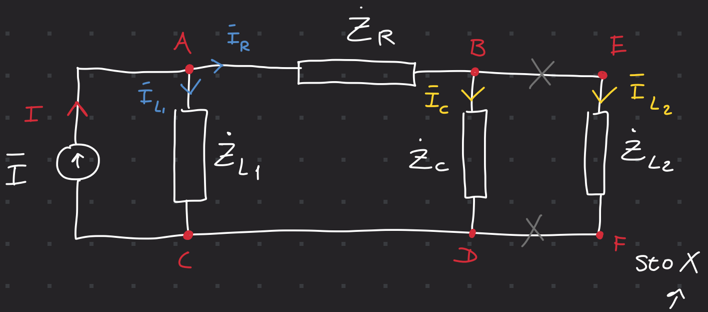
ES. 2.3 - **Applicando il teorema di Thévenin, valutare la potenza complessa e la potenza istantanea assorbita dall'induttore L_2.**



$j(t) = 10\sqrt{2}\sin(100t + 0.35)$ A

$R = 4\,\Omega$, $C = 3$ mF,

$L_1 = 2$ mH, $L_2 = 5$ mH



$j(t) = 10\sqrt{2}\,\sin(100t + 0.35)\,A = 10\sqrt{2}\cos(100t + 0.35 - \frac{\pi}{2})$

$j(t) \rightleftharpoons \bar{I} = 10\, e^{j(0.35 - \frac{\pi}{2})} = 10\left[\cos(0.35 - \frac{\pi}{2}) + j\sin(0.35 - \frac{\pi}{2})\right]$

Sto Y

$\dot{Z}_R = 4 \rightarrow A$

$\dot{Z}_C = -\frac{10}{3}j \rightarrow C$

$\dot{Z}_{L_1} = \frac{1}{5}j \rightarrow D$

$\dot{Z}_{L_2} = \frac{1}{2}j \rightarrow E$

(1) $\dot{Z}_{eqBD} = (\dot{Z}_{L_1} + \dot{Z}_R) \,||\, \dot{Z}_C = 1.72 - 1.98j$

(2) Thevenin $\rightarrow$ Tensione a vuoto

M

$\bar{V}_{EF} = \bar{V}_{BD} \Rightarrow V_{BD} = \dot{Z}_C \cdot \bar{I}_C$ ma $\bar{I}_R = \bar{I} \cdot \dfrac{Z_{L_1}}{Z_{L_1} + (Z_R + Z_C)} = 0.21 + 0.33j$

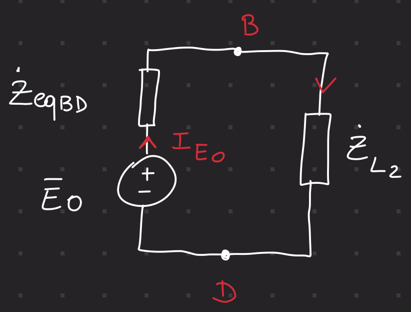
$\rightarrow \bar{I}_C = \bar{I}_R \Rightarrow V_{BD} = 1.114 - 0.69j$

Sugli es è al contrario (??)

$1.31 \angle -0.554$

E_0

$\Rightarrow$ Thevenin:



$\bar{I}_{L_2}$

$\bar{I}_{E_0} = \dfrac{\bar{E}_0}{\dot{Z}_{eq}} = 0.477 + 0.147j$

$\dot{Z}_{L_2}$ $\quad$ $\dot{A}_{L_2}$

$\Rightarrow \dot{A}_{L_2} = j \cdot X_{L_2} \cdot I^2 = j\omega L_2 \cdot I^2 = 0.07 - 0.103j$

Reattanza Induttiva

$X_{L_2} = \omega L_2$

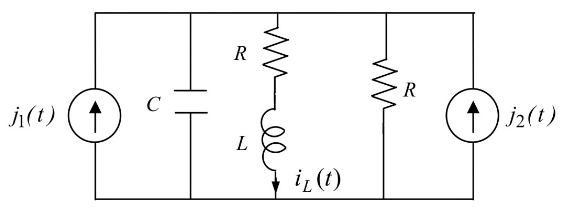
$p(t) = v(t) \cdot i(t)$

$\rightarrow i_{L_2}(t) = 0.499\,\sqrt{2}\cdot\cos(100t + 0.299)$

$= 0.499\,\sqrt{2}\,\sin(100t + 0.299 + \frac{\pi}{2})$

$v_0(t) = 1.31\cos(100t - 0.554)$

ES. 2.4 - **Con riferimento al seguente circuito valutare la corrente $i_L(t)$.**



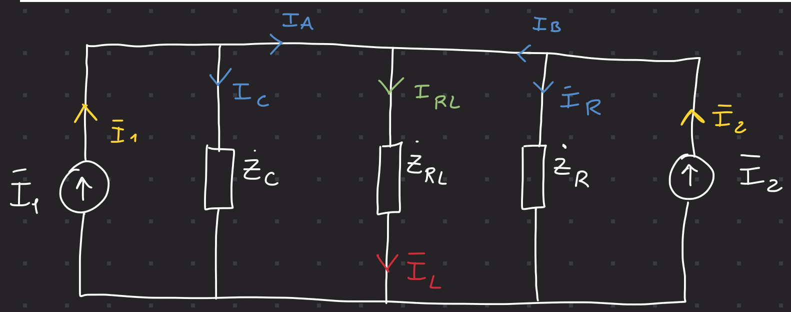
$j_1(t) = 10\cos(1000t)\ A$

$j_2(t) = 10\sin(1000t)\ A$

$R = 2\,\Omega$

$L = 2\text{ mH}$

$C = 1\text{ mF}$



C $\dot{Z}_C = -j\ \Omega$

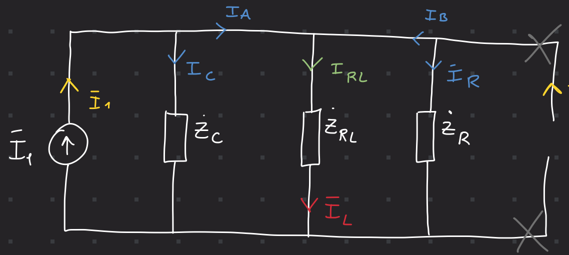
A $\dot{Z}_R = 2\ \Omega$

$\dot{Z}_L = 2j$

$\left.\begin{matrix} \dot{Z}_R = 2\ \Omega \\ \dot{Z}_L = 2j \end{matrix}\right\}$ B $\dot{Z}_{RL} = 2+2j$

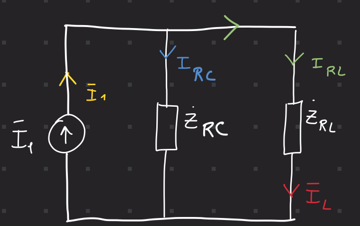
$\bar{I}_1 = 10\,A \qquad \bar{I}_2 = 10\left[\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right] = -10j$

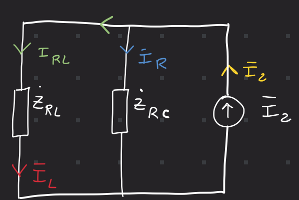
C'



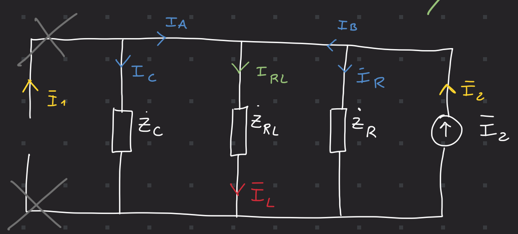
E

$\bar{I}'_{RL} = \bar{I}_1 \cdot \frac{\dot{Z}_{RC}}{\dot{Z}_{RC} + \dot{Z}_{RL}} = -\frac{10}{3}j = -3.\bar{3}j$ $\bar{I}'_{RL}$





D

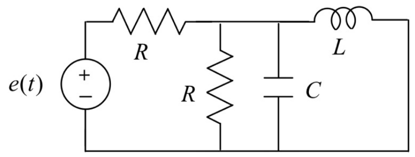


$\bar{I}''_{RL} = \bar{I}_2 \cdot \frac{\bar{Z}_{RC}}{\bar{Z}_{RC} + \bar{Z}_{RL}} = -3.\bar{3}\,A$

$\Rightarrow \bar{I} = \bar{I}' + \bar{I}'' = -3.33 - 3.33j = 4.71 \angle -\frac{3}{4}\pi$

$\Rightarrow i(t) = 4.71\cos\left(1000t - \frac{3}{4}\pi\right) = 4.71\sin\left(1000t - \frac{3}{4}\pi + \frac{\pi}{2}\right) = \underbrace{4.71\sin\left(1000t - \frac{1}{4}\pi\right)}_{\text{SI TROVA}}$

ES. 2.5 - **Applicando il teorema di Norton, valutare la potenza complessa e la potenza istantanea assorbita dal parallelo *R-C* in figura.**



$e(t) = 5\sqrt{2}\,\sin(1000t + \pi/3)\,V$

$R = 0.21\,\Omega$, $L = 1.12\,\text{mH}$

$C = 1.23\,\text{mF}$.

Risultato: $\dot{A} = 29.72\text{ W} - j7.68\text{ VAr}$; $p(t) = [\,29.72 - 30.70\cos(2000t + 2.27)]\text{ W}$.

$e(t) = 5\sqrt{2}\,\sin(1000t + \frac{\pi}{3})$

$\rightleftarrows \bar{E} = 5\sqrt{2}$