METODO DEI FASORI

TEMPO → FASORE

$$e(t) = E\cos(\omega t + \alpha) \rightleftharpoons \bar{E} = E\,e^{j\alpha} \leftarrow \text{Fasore}$$

FASORE → TEMPO

(1) $A\cos(\omega t + \varphi) = \frac{A}{2}\cdot\left[e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)}\right]$ ← Eulero

(2) $A\cos(\omega t + \varphi) = \mathrm{Re}\{A\,e^{j(\omega t+\varphi)}\} = \mathrm{Re}\{A\,e^{j\varphi}\cdot e^{j\omega t}\}$

ES: $\bar{A} = A e^{j\varphi} \rightarrow a(t) = \mathrm{Re}\{\underset{\uparrow\,\text{Fasore}}{\bar{A}}\cdot \underset{\uparrow\,\text{moltiplico}}{e^{j\omega t}}\} = \mathrm{Re}\{A e^{j\varphi}\cdot e^{j\omega t}\} =$

$= \mathrm{Re}\{A\,e^{j(\omega t+\varphi)}\} = \mathrm{Re}\{A\,e^{j\alpha}\}$ con $\alpha = \omega t + \varphi$

Siccome

$e^{j\alpha} = \underset{\uparrow\,\mathrm{Re}}{\cos(\alpha)} + \cancel{j\sin(\alpha)}^{\,\mathrm{Imm}}$ ⇝ $= A\cos(\alpha) = \boxed{A\cos(\omega t+\varphi)}$ QED

PROPRIETA'

UNICITA'

Se $a(t) = A\cos(\omega t+\alpha)$; $b(t) = B\cos(\omega t + \beta)$ con $\alpha \neq \beta$, $A \neq B$

$\Rightarrow \bar{A} = A\,e^{j\alpha}$, $\bar{B} = B\,e^{j\beta} \rightarrow \bar{A} \neq \bar{B}$

LINEARITA'

$a(t) = A\cos(\omega t+\alpha)$
$b(t) = B\cos(\omega t+\beta)$ $\rightarrow c(t) = K_1\,a(t) + K_2\,b(t) \rightleftharpoons \bar{C} = K_1\bar{A} + K_2\bar{B}$

DERIVAZIONE

$$a(t) = A\cos(\omega t+\varphi) \rightleftharpoons \bar{A} = A\,e^{j\varphi}$$

$$\frac{d}{dt}[a(t)] = -A\omega\sin(\omega t+\varphi) = -A\omega\cos\left[\omega t + \left(\varphi - \tfrac{\pi}{2}\right)\right] \rightleftharpoons \dot{\bar{A}} = -A\omega e^{j(\varphi - \frac{\pi}{2})}$$

$$= -A\omega e^{j\varphi}\cdot e^{-j\frac{\pi}{2}} \quad * \; -j$$

$$= j\omega\cdot \underbrace{A e^{j\varphi}}_{\bar{A}}$$

$* \; e^{-j\frac{\pi}{2}} = \cancel{\cos(\tfrac{\pi}{2})}^{\,0} - j\cancel{\sin(\tfrac{\pi}{2})}^{\,1} = -j$

derivare nel tempo ⇄ moltiplicare per $j\omega$

IMPEDENZE

Tempo

$$\begin{cases} v = R \cdot i \\ i_c = C\,\dot{v}_c \\ v_L = L\,\dot{i}_L \end{cases} \rightleftharpoons \begin{cases} \bar{V} = \underbrace{R}_{\dot{Z}_R} \cdot \bar{I} \\ \bar{I}_c = C \cdot j\omega\,\bar{V}_c \quad \Rightarrow \quad \bar{V}_c = \underbrace{-\dfrac{j}{\omega C}}_{\dot{Z}_c}\,\bar{I}_c \\ \bar{V}_L = \underbrace{L \cdot j\omega}_{\dot{Z}_L}\,\bar{I}_L \end{cases}$$

REATTANZA

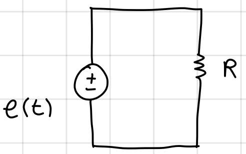
$$X_L = L j\omega \cdot \frac{1}{j} = L\omega \qquad \text{INDUTTIVA}$$

$$X_c = -\frac{j}{C\omega} \cdot \frac{1}{j} = -\frac{1}{C\omega} \qquad \text{CAPACITIVA}$$

AMMETTENZA

$$\dot{Y} = \frac{1}{\dot{Z}} \quad \Rightarrow \quad \bar{I} = \dot{Y} \cdot \bar{V}$$

VALORE EFFICACE



Sappiamo che $P = v(t) \cdot i(t)$ ma $\begin{cases} i(t) = \dfrac{e(t)}{R} \\ v(t) = e(t) \end{cases}$

$$\Rightarrow \quad P = \frac{e^2(t)}{R}$$

$\Rightarrow$ Voglio trovare la potenza media :

T. media integrale : $\text{Valore medio} = \dfrac{1}{T}\displaystyle\int_0^T f(x)\,dx$

$$\Rightarrow \quad P_{med} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\int_0^T \frac{e^2(t)}{R}\,dt$$

CASO 1: $e(t) = E_0 = \text{cost}$

$$\Rightarrow \quad P_{med} = \frac{1}{T}\int_0^T \frac{E^2}{R}\,dt = \frac{1}{T}\,\frac{E^2}{R} \cdot T = \frac{E_0^2}{R} \qquad e(t) = E_0$$

CASO 2: $e(t) = E_M \cdot \cos(\omega t) \qquad \varphi = 0$

$$\Rightarrow \quad P_{med} = \frac{1}{T}\int_0^T \frac{E_M^2 \cos^2(\omega t)}{R}\,dt = \frac{E_M^2}{RT}\int_0^T \cos^2(\omega t)\,dt$$

↳ Voglio trovare il valore di E_0 tale da dissipare la stessa potenza di e(t)

$$\to \frac{E_0^2}{\cancel{R}} = \frac{E_M^2}{\cancel{R}\,T}\int_0^T \cos^2(\omega t)\,dt \qquad \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(x)$$

$$= \frac{E_M^2}{T}\int_0^T \frac{1+\cos(\omega t)}{2}\,dt = \frac{E_M^2}{T}\,2\cdot[t]_0^T$$

$$\Rightarrow E_0^2 = \frac{E_M^2}{T}\cdot\frac{1}{2}T \to E_0 = \frac{E_M}{\sqrt{2}}$$

↖ Valore efficace sinusoide

POTENZA IN REGIME SINUSOIDALE

$$P = v(t)\cdot i(t) \to P = V_m \cos(\underset{a}{\omega t} + \underset{b}{\alpha}) \cdot I_m \cos(\underset{a'}{\omega t} + \underset{b'}{\beta})$$

Sappiamo che $2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$

$$\to P(t) = \frac{1}{2} V_m I_m \left[\cos(\omega t + \alpha + \omega t + \beta) + \cos(\omega t + \alpha - \omega t - \beta)\right]$$

ist.

$$= K\left[\cos(2\omega t + \alpha + \beta) + \cos(\alpha - \beta)\right]$$

Potenza istantanea

non è di puls ω ma 2ω!

manca ω

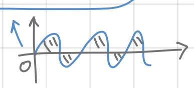
POTENZA MEDIA

Teorema media integrale $\to$

$$P_{med} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\cdot\frac{V_m I_m}{2}\int_0^T \underbrace{\cos(\alpha-\beta)}_{\text{cost}} + \cos(2\omega t + \underbrace{\alpha+\beta}_{\varphi})\,dt$$

$$= \frac{V_m I_m}{2T}\cdot\cos(\alpha-\beta)\cdot T + \frac{V_m I_m}{2T}\int_0^T \cos(2\omega t + \varphi)\,dt$$

$\emptyset$



$$\Rightarrow P_{media} = \frac{V_m I_m}{2}\cdot\cos(\alpha - \beta)$$

fasi di i e v

RESISTORE

$$v(t) = V_m \cos(\omega t + \alpha)$$
$$i(t) = I_m \cos(\omega t + \beta)$$
$$\to v = R\cdot i \rightsquigarrow V_m\cos(\omega t + \alpha) = R\cdot I_m\cos(\omega t + \beta)$$

$\to$ Abbiamo $V_m = R\cdot I_m \Leftrightarrow \alpha = \beta$ STESSA FASE TRA i e v

$$P(t) = \frac{V_m I_m}{2}\left(\cancel{\cos(\alpha-\beta)}^{\emptyset} + \cos(2\omega t + \alpha + \beta)\right) = \frac{R\cdot I_m^2}{2}\cdot\cos(2\omega t + 2\beta)$$ P. ist

$$P = \frac{R I_m^2}{2} \overset{\substack{V=R\cdot I \\ I=\frac{V}{R}}}{=} \frac{V_m^2}{2R}$$ P med

INDUTTORE

$$\begin{cases} v(t) = V_m \cos(\omega t + \alpha) \\ i(t) = I_m \cos(\omega t + \beta) \end{cases} \rightarrow \quad v_L = L \cdot \dot{i}_L \rightarrow V_m \cos(\omega t + \alpha) = -L \cdot \omega I_m \sin(\omega t + \beta)$$

$$\rightsquigarrow V_m \cos(\omega t + \alpha) = L\omega I_m \cos\left(\omega t + \beta + \frac{\pi}{2}\right) \Leftrightarrow \alpha = \beta + \frac{\pi}{2}$$

$v(t)$ in anticipo di $\frac{\pi}{2}$ rispetto a $i(t)$

$$P(t) = \frac{L\omega I_m^2}{2} \cos\left(2\omega t + \alpha + \beta + \frac{\pi}{2}\right) + \cos\left(\cancel{\beta + \beta - \frac{\pi}{2}}\right)^{\emptyset}$$

$$P = \frac{L\omega I_m^2}{2} \cos(\varphi) = \emptyset$$

$\varphi = \alpha - \beta = \beta \cdot \beta - \frac{\pi}{2} = \cdot \frac{\pi}{2}$

CONDENSATORE

$$\begin{cases} v(t) = V_m \cos(\omega t + \alpha) \\ i(t) = I_m \cos(\omega t + \beta) \end{cases} \rightarrow \quad i_C = C \dot{v}_C$$

$$\rightarrow I_m \cos(\omega t + \alpha) = -C V_m \omega \sin(\omega t + \beta) = C V_m \omega \cos\left(\omega t + \beta - \frac{\pi}{2}\right)$$

la rel $I_m = C V_m$ è rispettata $\Leftrightarrow \alpha = \beta - \frac{\pi}{2}$, se $\alpha = \beta$

$\rightarrow \alpha = \alpha - \frac{\pi}{2}$ $v(t)$ è In RITARDO di $\frac{\pi}{2}$ rispetto a $i(t)$

$$P(t) = \frac{C\omega I_m^2}{2} \cos\left(2\omega t + \alpha + \beta - \frac{\pi}{2}\right) + \cos\left(\cancel{\beta - \beta + \frac{\pi}{2}}\right)^{\emptyset}$$

$$P = \frac{C\omega I_m^2}{2} \cos(\varphi) = \emptyset$$

POTENZA COMPLESSA

$$\dot{S} = \frac{\bar{V} \cdot \bar{I}^*}{2} = \underbrace{\frac{V_m I_m}{2} \cos(\alpha - \beta)}_{P} + \underbrace{j \frac{V_m I_m}{2} \sin(\alpha - \beta)}_{Q}$$

uguale alla P media

P: POTENZA ATTIVA

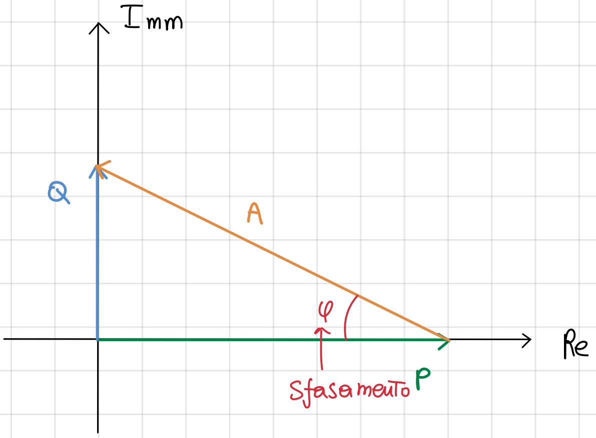
Q: POTENZA REATTIVA

* Se lo sfasamento φ è nullo, Q è ZERO

Es: circuito resistivo $\rightarrow Q = \emptyset$

* Se φ è max $\rightarrow \varphi = \pm \frac{\pi}{2} \rightarrow P = K \cdot \sin(\varphi) = \emptyset$

Siccome sono numeri complessi.



$$A = |\dot{S}| = \sqrt{P^2 + Q^2}$$

$$\frac{Q}{P} = \tan(\varphi) \rightarrow \varphi = \operatorname{atan}\left(\frac{Q}{P}\right)$$

POTENZA FASORI CON VALORE EFFICACE

$$\begin{cases} v(t) = V_m \cos(\omega t + \alpha) \rightleftharpoons \\ i(t) = I_m \sin(\omega t + \beta) \rightleftharpoons \end{cases} \begin{aligned} \bar{V} &= \frac{V_m}{\sqrt{2}} e^{j\alpha} = V_0 e^{j\alpha} \\ \bar{I} &= \frac{I_m}{\sqrt{2}} e^{j\beta} = I_0 e^{j\beta} \end{aligned}$$

TESI: $\dot{S} = V_0 I_0 e^{j(\alpha-\beta)}$

↳ proof $V_0 I_0 e^{j(\alpha-\beta)} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} e^{j(\alpha-\beta)} = \frac{V_m I_m}{2} \cdot [\cos(\alpha-\beta) + j\sin(\alpha-\beta)]$

$= P + jQ$ QED

CONSERVAZIONE DELLE POTENZE COMPLESSE (BOUCHEROT)

$$\sum_{k=1}^{e} \dot{S}_k = \emptyset \rightarrow \sum_k (P_k + jQ_k) = \emptyset \; \xrightarrow{\text{AND}} \begin{cases} \sum P_k = 0 \\ \sum jQ_k = \emptyset \end{cases}$$

DIM: $\sum_k \dot{S}_k = \sum \frac{V_k \bar{I}_k^*}{2} = \frac{1}{2} \bar{V}^T \cdot \bar{I}^* = \frac{1}{2} (A^T U)^T \cdot \bar{I}^* = \frac{1}{2} U^T \underbrace{A \bar{I}^*}_{\emptyset} =$

↓

DIM

Se $\underline{\underline{A}} \bar{I} = 0 \rightarrow \underline{\underline{A}} \, Re\{\bar{I}\} + \underline{\underline{A}} \, Imm\{\bar{I}\} = 0$

$\Rightarrow \begin{cases} \underline{\underline{A}} \, Re\{\bar{I}\} = 0 \\ \underline{\underline{A}} \, Im\{\bar{I}\} = 0 \end{cases}$ AND $\Rightarrow \begin{cases} \underline{\underline{A}} \, Re\{\bar{I}\} = 0 \\ - \underline{\underline{A}} \, Im\{\bar{I}\} = 0 \end{cases}$

$\Rightarrow \underline{\underline{A}} \bar{I} = \underline{\underline{A}} \bar{I}^* = \emptyset$ QED