METODO DEI FASORI

TEMPO → FASORE

$e(t) = E\cos(\omega t + \alpha) \rightleftharpoons \bar{E} = E\, e^{j\alpha}$ ← Fasore

FASORE → TEMPO

Eulero

(1) $A\cos(\omega t + \varphi) = \frac{A}{2}\cdot\left[e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)}\right]$

(2) $A\cos(\omega t + \varphi) = \mathrm{Re}\{A\, e^{j(\omega t+\varphi)}\} = \mathrm{Re}\{A\, e^{j\varphi}\cdot e^{j\omega t}\}$

ES: $\bar{A} = A e^{j\varphi} \rightarrow a(t) = \mathrm{Re}\{\underset{\text{Fasore}}{\bar{A}} \cdot \underset{\text{moltiplico}}{e^{j\omega t}}\} = \mathrm{Re}\{A e^{j\varphi}\cdot e^{j\omega t}\} =$

$= \mathrm{Re}\{A\, e^{j(\omega t+\varphi)}\} = \mathrm{Re}\{A\, e^{j\alpha}\}$ con $\alpha = \omega t + \varphi$

Siccome

$e^{j\alpha} = \underset{\mathrm{Re}}{\cos(\alpha)} + \cancel{j\sin(\alpha)}^{\,Imm}$ ⇝ $= A\cos(\alpha) = \boxed{A\cos(\omega t + \varphi)}$ QED

PROPRIETA'

UNICITA'

Se $a(t) = A\cos(\omega t + \alpha)$; $b(t) = B\cos(\omega t + \beta)$ con $\alpha \neq \beta$, $A \neq B$

$\Rightarrow \bar{A} = A\, e^{j\alpha}$, $\bar{B} = B\, e^{j\beta} \rightarrow \bar{A} \neq \bar{B}$

LINEARITA'

$a(t) = A\cos(\omega t + \alpha)$
$b(t) = B\cos(\omega t + \beta)$ $\rightarrow c(t) = k_1 a(t) + k_2 b(t) \rightleftharpoons \bar{C} = k_1\bar{A} + k_2\bar{B}$

DERIVAZIONE

$a(t) = A\cos(\omega t + \varphi) \rightleftharpoons \bar{A} = A\, e^{j\varphi}$

$\frac{d}{dt}[a(t)] = -A\omega\sin(\omega t + \varphi) = -A\omega\cos\left[\omega t + \left(\varphi - \frac{\pi}{2}\right)\right] \rightleftharpoons \dot{\bar{A}} = -A\omega\, e^{j(\varphi - \frac{\pi}{2})}$ *

$= -A\omega\, e^{j\varphi}\cdot \underbrace{e^{-j\frac{\pi}{2}}}_{-j}$

$= \underset{\text{derivare nel tempo}}{j\omega} \cdot \underbrace{A\, e^{j\varphi}}_{\bar{A}}$

derivare nel tempo ⇄ moltiplicare per $j\omega$

* $e^{-j\frac{\pi}{2}} = \cancel{\cos\left(\frac{\pi}{2}\right)}^{\,0} - j\,\cancel{\sin\left(\frac{\pi}{2}\right)}^{\,1} = -j$

IMPEDENZE

Tempo

$$\begin{cases} v = R \cdot i \\ i_C = C \, \dot{v}_C \\ v_L = L \, \dot{i}_L \end{cases} \rightleftharpoons \begin{cases} \bar{V} = \underbrace{R}_{\dot{Z}_R} \cdot \bar{I} \\ \bar{I}_C = C \cdot j\omega \, \bar{V}_C \\ \bar{V}_L = \underbrace{L \cdot j\omega}_{\dot{Z}_L} \, \bar{I}_L \end{cases} \Rightarrow \bar{V}_C = \underbrace{-\frac{j}{\omega C}}_{\dot{Z}_C} \, \bar{I}_C$$

REATTANZA

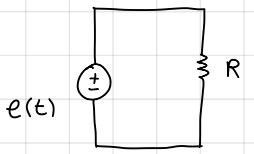
$$X_L = L j\omega \cdot \frac{1}{j} = L\omega \qquad \text{INDUTTIVA}$$

$$X_C = -\frac{j}{C\omega} \cdot \frac{1}{j} = -\frac{1}{C\omega} \qquad \text{CAPACITIVA}$$

AMMETTENZA

$$\dot{Y} = \frac{1}{\dot{Z}} \Rightarrow \bar{I} = \dot{Y} \cdot \bar{V}$$

VALORE EFFICACE



Sappiamo che $P = v(t) \cdot i(t)$ ma $\begin{cases} i(t) = \dfrac{e(t)}{R} \\ v(t) = e(t) \end{cases}$

$$\Rightarrow P = \frac{e^2(t)}{R}$$

$\Rightarrow$ Voglio trovare la potenza media :

T. media integrale : Valore medio $= \frac{1}{T} \int_0^T f(x) \, dx$

$$\Rightarrow P_{med} = \frac{1}{T} \int_0^T P(t) \, dt = \frac{1}{T} \int_0^T \frac{e^2(t)}{R} \, dt$$

CASO 1: $e(t) = E_0 = \text{COST}$

$$\rightarrow P_{med} = \frac{1}{T} \int_0^T \frac{E^2}{R} dt = \frac{1}{T} \frac{E^2}{R} \cdot T = \frac{E_0^2}{R} \qquad e(t) = E_0$$

CASO 2: $e(t) = E_M \cdot \cos(\omega t)$ $\qquad \varphi = 0$

$$\rightarrow P_{med} = \frac{1}{T} \int_0^T \frac{E_M^2 \cos^2(\omega t)}{R} \, dt = \frac{E_M^2}{RT} \int_0^T \cos^2(\omega t) \, dt$$

↳ Voglio trovare il valore di E_0 tale da dissipare la stessa potenza di e(t)

$$\rightarrow \frac{E_0^2}{\cancel{R}} = \frac{E_M^2}{\cancel{R}\,T}\int_0^T \cos^2(\omega t)\,dt \qquad \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(x)$$

$$= \frac{E_M^2}{T}\int_0^T \frac{1+\cos(\omega t)}{2}\,dt = \frac{E_M^2}{T}\,2\cdot[t]_0^T$$

$$\Rightarrow E_0^2 = \frac{E_M^2}{T}\cdot 2T \rightarrow E_0 = \frac{E_M}{\sqrt{2}}$$

Valore efficace sinusoide

POTENZA IN REGIME SINUSOIDALE

$$P = v(t)\cdot i(t) \rightarrow P = V_m \cos(\overset{a}{\omega t} + \overset{b}{\alpha}) \cdot I_m \cos(\overset{a'}{\omega t} + \overset{b'}{\beta})$$

Sappiamo che $2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$

$$\rightarrow P(t) = \frac{1}{2} V_m I_m \left[\cos(\omega t + \alpha + \omega t + \beta) + \cos(\omega t + \alpha - \omega t - \beta)\right]$$

ist.

$$= K\left[\cos(2\omega t + \alpha + \beta) + \cos(\alpha - \beta)\right]$$

Potenza istantanea

non è di puls ω ma 2ω!

manca ω

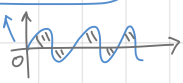
POTENZA MEDIA

Teorema media integrale $\rightarrow$ $P_{med} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\cdot\frac{V_m I_m}{2}\int_0^T \cos(\alpha-\beta) + \cos(2\omega t + \underbrace{\alpha+\beta}_{\varphi})\,dt$

cost

$$= \frac{V_m I_m}{2T}\cdot\cos(\alpha-\beta)\cdot T + \frac{V_m I_m}{2T}\int_0^T \cos(2\omega t + \varphi)\,dt$$

$\emptyset$



$$\Rightarrow P_{media} = \frac{V_m I_m}{2}\cdot\cos(\alpha - \beta)$$

fasi di i e v

RESISTORE

$$v(t) = V_m \cos(\omega t + \alpha)$$
$$i(t) = I_m \cos(\omega t + \beta)$$
$$\rightarrow v = R\cdot i \rightsquigarrow V_m\cos(\omega t + \alpha) = R\cdot I_m\cos(\omega t + \beta)$$

$\rightarrow$ Abbiamo $V_m = R\cdot I_m$ $\measuredangle = 0$ $\alpha = \beta$ STESSA FASE tra i e v

$$P(t) = \frac{V_m I_m}{2}\left(\cancel{\cos(\alpha-\beta)}^{\,\emptyset} + \cos(2\omega t + \alpha + \beta)\right) = \frac{R\cdot I_m^2}{2}\cdot\cos(2\omega t + 2\beta)$$ P. ist

$$P = \frac{R I_m^2}{2} \overset{\substack{V=R\cdot I \\ I=V/R}}{=} \frac{V_m^2}{2R}$$ P med

INDUTTORE

$$\begin{cases} v(t) = V_m\cos(\omega t+\alpha) \\ i(t) = I_m\cos(\omega t+\beta) \end{cases} \to \quad v_L = L\cdot \dot{i}_L \quad \to \quad V_m\cos(\omega t+\alpha) = -L\cdot \omega I_m \sin(\omega t+\beta)$$

$$\rightsquigarrow \quad V_m\cos(\omega t+\alpha) = L\omega I_m \cos\left(\omega t+\beta+\frac{\pi}{2}\right) \iff \alpha = \beta+\frac{\pi}{2}$$

$v(t)$ in anticipo di $\frac{\pi}{2}$ rispetto a $i(t)$

$$P(t) = \frac{L\omega I_m^2}{2}\left[\cos\left(2\omega t+\alpha+\beta+\frac{\pi}{2}\right) + \underbrace{\cancel{\cos\left(\beta-\beta-\frac{\pi}{2}\right)}}_{0}\right]$$

$$P = \frac{L\omega I_m^2}{2}\cos(\varphi) = 0$$

$\varphi = \alpha-\beta = \beta-\beta-\frac{\pi}{2} = -\frac{\pi}{2}$

CONDENSATORE

$$\begin{cases} v(t) = V_m\cos(\omega t+\alpha) \\ i(t) = I_m\cos(\omega t+\beta) \end{cases} \to \quad i_C = C\,\dot{v}_C$$

$$\to I_m\cos(\omega t+\alpha) = -CV_m\omega\sin(\omega t+\beta) = CV_m\omega\cos\left(\omega t+\beta-\frac{\pi}{2}\right)$$

la rel $I_m = CV_m$ è rispettata $\iff \alpha = \beta-\frac{\pi}{2}$, se $\alpha = \beta$

$\hookrightarrow \alpha = \alpha - \frac{\pi}{2}$

$v(t)$ è in RITARDO di $\frac{\pi}{2}$ rispetto a $i(t)$

$$P(t) = \frac{C\omega I_m^2}{2}\left[\cos\left(2\omega t+\alpha+\beta-\frac{\pi}{2}\right) + \underbrace{\cancel{\cos\left(\beta-\beta+\frac{\pi}{2}\right)}}_{0}\right]$$

$$P = \frac{C\omega I_m^2}{2}\cos(\varphi) = 0$$

POTENZA COMPLESSA

Uguale alla P media

$$\dot{S} = \frac{\dot{V}\cdot \dot{I}^*}{2} = \frac{V_m I_m}{2}\cos(\alpha-\beta) + j\,\frac{V_m I_m}{2}\sin(\alpha-\beta)$$

P: POTENZA ATTIVA

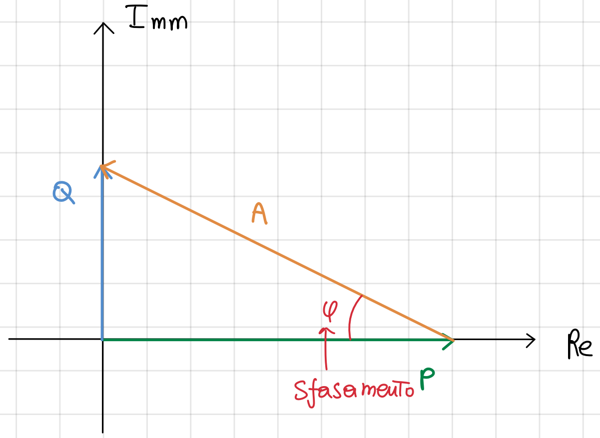
Q: POTENZA REATTIVA

* Se lo sfasamento φ è nullo, Q è ZERO

Es: circuito resistivo $\to Q = 0$

* se φ è max $\to \varphi = \pm\frac{\pi}{2} \to P = K\cdot\sin(\varphi) = 0$

Siccome sono numeri complessi.



$$A = |\dot{S}| = \sqrt{P^2+Q^2}$$

$$\frac{Q}{P} = \tan(\varphi) \to \varphi = \operatorname{atan}\left(\frac{Q}{P}\right)$$

POTENZA FASORI CON VALORE EFFICACE

$$\begin{cases} v(t) = V_m \cos(\omega t + \alpha) \rightleftharpoons \bar{V} = \frac{V_m}{\sqrt{2}} e^{j\alpha} = V_0 e^{j\alpha} \\ i(t) = I_m \sin(\omega t + \beta) \rightleftharpoons \bar{I} = \frac{I_m}{\sqrt{2}} e^{j\beta} = I_0 e^{j\beta} \end{cases}$$

TESI: $\dot{S} = V_0 I_0 e^{j(\alpha-\beta)}$

$\hookrightarrow$ proof $V_0 I_0 e^{j(\alpha-\beta)} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} e^{j(\alpha-\beta)} = \frac{V_m I_m}{2} \cdot \left[\cos(\alpha-\beta) + j \sin(\alpha-\beta)\right]$

$= P + jQ$ QED

CONSERVAZIONE DELLE POTENZE COMLESSE (BOUCHEROT)

$$\sum_{k=1}^{e} \dot{S}_k = \emptyset \rightarrow \sum_k (P_k + jQ_k) = \emptyset \;\text{AND}\; \begin{cases} \sum P_k = 0 \\ \sum jQ_k = \emptyset \end{cases}$$

DIM: $\sum_k \dot{S}_k = \sum \frac{V_k \bar{I}_k^*}{2} = \frac{1}{2} \underline{\bar{V}}^T \cdot \underline{\bar{I}}^* = \frac{1}{2}(A^T U)^T \cdot \bar{I}^* = \frac{1}{2} U^T \underbrace{A \bar{I}^*}_{\emptyset} =$

$\downarrow$

DIM

Se $\underline{\underline{A}}\,\underline{\bar{I}} = 0 \rightarrow \underline{\underline{A}}\,\mathrm{Re}\{\underline{\bar{I}}\} + \underline{\underline{A}}\,\mathrm{Imm}\{\underline{\bar{I}}\} = 0$

$\Rightarrow \begin{cases} \underline{\underline{A}}\,\mathrm{Re}\{\underline{\bar{I}}\} = 0 \\ \underline{\underline{A}}\,\mathrm{Im}\{\underline{\bar{I}}\} = 0 \end{cases}$ AND $\Rightarrow \begin{cases} \underline{\underline{A}}\,\mathrm{Re}\{\underline{\bar{I}}\} = 0 \\ -\underline{\underline{A}}\,\mathrm{Im}\{\underline{\bar{I}}\} = 0 \end{cases}$

$\Rightarrow \underline{\underline{A}}\,\underline{\bar{I}} = \underline{\underline{A}}\,\underline{\bar{I}}^* = \emptyset$ QED