

$$E_1 = 10 \text{ V}$$

$$R_1 = 1 \, \Omega$$

$$R_2 = 1 \, \Omega$$

$$R_3 = 2 \, \Omega$$

$$J = 1 \text{ A}$$

CORRENTI (LKC)

$$\begin{aligned} \mu_1 &\rightarrow \begin{cases} i_1 + i_4 = 0 & (i.1) \end{cases} \\ \mu_2 &\rightarrow \begin{cases} i_2 + i_3 - i_5 - i_1 = 0 & (i.2) \end{cases} \end{aligned}$$

TENSIONI (LKT)

$$\begin{aligned} v_1 &= \mu_1 - \mu_2 \\ v_2 &= \mu_2 - \mu_3 = \mu_2 \\ v_3 &= \mu_2 - \mu_3 = \mu_2 \\ v_4 &= \mu_1 - \mu_3 = \mu_1 \\ v_5 &= \mu_2 - \mu_3 = \mu_2 \end{aligned}$$

Siccome

$$v_{R_k} = R_k \cdot i_k$$

$$\rightarrow \begin{cases} v_1 = R_1 \cdot i_1 \\ v_2 = R_2 \cdot i_2 \\ v_3 = R_3 \cdot i_3 \\ v_4 = E \\ v_5 = J \end{cases} \rightarrow \begin{cases} v_1 - R_1 i_1 = 0 \\ v_2 - R_2 i_2 = 0 \\ v_3 - R_3 i_3 = 0 \\ v_4 = E \\ v_5 = J \end{cases}$$

QUINDI

$$\left\{ \begin{aligned} i_1 + i_4 &= 0 \\ i_2 + i_3 - i_5 - i_1 &= 0 \\ v_1 &= \mu_1 - \mu_2 \\ v_2 &= \mu_2 \\ v_3 &= \mu_2 \\ v_4 &= \mu_1 \\ v_5 &= \mu_2 \\ v_1 - R_1 i_1 &= 0 \\ v_2 - R_2 i_2 &= 0 \\ v_3 - R_3 i_3 &= 0 \\ v_4 &= E \\ v_5 &= J \end{aligned} \right.$$

Sistema di 12 equazioni
in 12 incognite

→ Metodo di risoluzione OTTIMALE

(1) μ_1 è nota, infatti $\mu_1 = E$

(2) Esprimiamo l'intensità di corrente in funzione della Tensione

$$\text{Siccome } v = R \cdot i \rightarrow i = \frac{v}{R}$$

$$\text{ma siccome } v_1 = \mu_1 - \mu_2 \text{ e } \mu_1 = E$$

(2.c)

$$\Rightarrow i_1 = \frac{E - \mu_2}{R_1} \quad (2.a), \quad i_2 = \frac{\mu_2}{R_2} \quad (2.b), \quad i_3 = \frac{\mu_2}{R_3}$$

(3) Imponiamo la LKC al nodo (2)

$$\rightarrow i_2 + i_3 - i_1 - J = 0 \quad \rightarrow \text{Sostituisco dalle 2.a, 2.b, 2.c}$$

↓

Isoliamo
 μ_2 (incognita)

↓

$$\frac{\mu_2}{R_2} + \frac{\mu_2}{R_3} - \frac{E - \mu_2}{R_1} - J = 0$$

$$\frac{\mu_2}{R_2} + \frac{\mu_2}{R_3} + \frac{\mu_2}{R_1} = J + \frac{E}{R_1} \quad \rightarrow \quad \mu_2 \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_1} \right) = J + \frac{E}{R_1}$$

(3.a)

$$\rightarrow \mu_2 = \frac{J + \frac{E}{R_1}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_1}}$$

Ci basta sostituire i valori numerici nella (3.a) per ottenere:

$$\mu_2 = \frac{1 \text{ A} + \frac{10 \text{ V}}{1 \Omega}}{\frac{1}{1 \Omega} + \frac{1}{2 \Omega} + \frac{1}{1 \Omega}} = \frac{1 \text{ A} + \frac{10 \text{ V}}{1 \Omega}}{1 + \frac{1}{2} + 1} = 4.4 \text{ V} \quad \text{Ans 1}$$

μ_2

4) Sostituisco Ans 1 nelle 2.a, 2.b, 2.c Per Trovare le Correnti

$$\bullet i_1 = \frac{E - \mu_2}{R_1} = \frac{10 \text{ V} - 4.4 \text{ V}}{1 \Omega} = 5.6 \text{ A}$$

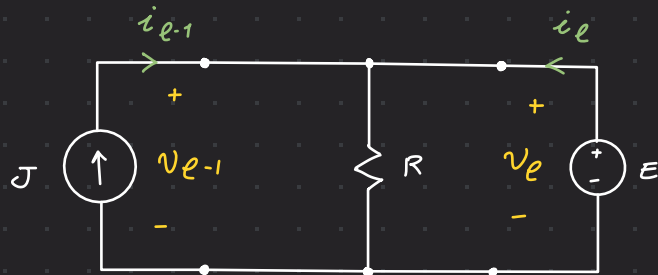
$$\bullet i_2 = \frac{\mu_2}{R_2} = \frac{4.4 \text{ V}}{1 \Omega} = 4.4 \text{ A} \quad \text{Ans 2}$$

$$\bullet i_3 = \frac{\mu_2}{R_3} = \frac{4.4 \text{ V}}{2 \Omega} = 2.2 \text{ A}$$

Inoltre dalla i.1: $i_1 + i_4 = 0 \rightarrow i_4 = -i_1 \Rightarrow i_4 = -5.6 \text{ A}$

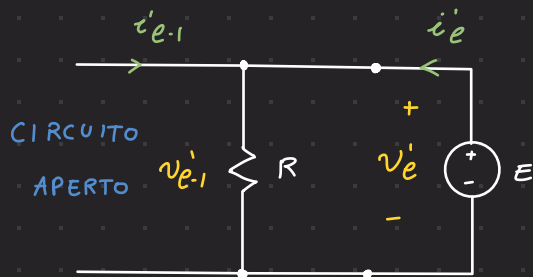
Enunciato: L'intensità di *corrente* e la *tensione* di un qualsiasi dipolo di un circuito resistivo lineare (con più generatori ideali) sono, rispettivamente, uguali alla somma delle intensità di corrente e delle tensioni che ciascuno dei generatori ideali produrrebbe se agisse da solo con tutti gli altri generatori ideali spenti.

Dimostrazione



$$\begin{aligned} Ai &= 0 \\ Bv &= 0 \\ v_k - R_k i_k &= 0 \\ v_{e-1} &= E \\ i_e &= J \end{aligned}$$

Circuito di partenza
ci riferiamo ad esso
senza apici



$$\begin{aligned} Ai' &= 0 \\ Bv' &= 0 \\ v'_k - R_k i'_k &= 0 \\ v'_{e-1} &= E \\ i'_e &= 0 \end{aligned}$$

ottenuto spegnendo
il gen di corrente
→ circuito aperto
→ singolo Apice



$$\begin{aligned} Ai'' &= 0 \\ Bv'' &= 0 \\ v''_k - R_k i''_k &= 0 \\ v''_{e-1} &= 0 \\ i''_e &= J \end{aligned}$$

ottenuto spegnendo
il gen di Tensione
→ cortocircuito
→ Doppio Apice

Tesi

L'enunciato sostiene che:

$$\begin{cases} i = i' + i'' \\ v = v' + v'' \end{cases} \quad \begin{matrix} (a) \\ (b) \end{matrix}$$

• Verifico (a) $Ai = 0 \rightarrow$ se $i = i' + i'' \rightarrow i = \underset{\downarrow 0}{Ai'} + \underset{\downarrow 0}{Ai''} = 0 \checkmark$

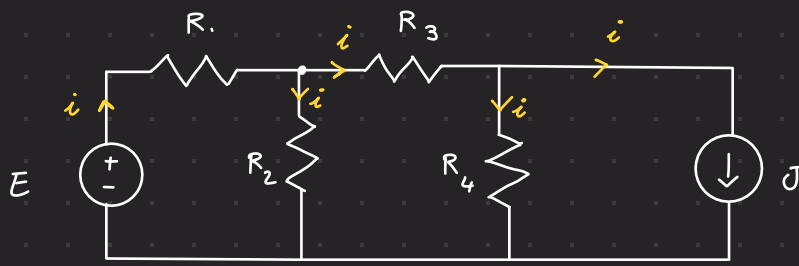
• Verifico (b) $Bv = Bv' + Bv'' = 0 \checkmark$

• Verifico i resistori $v_k - R_k i_k = 0 \rightarrow$ data la Tesi

$$\begin{aligned} \rightarrow v_k - R_k i_k &= (v'_k + v''_k) - (R_k i'_k + R_k i''_k) = v'_k + v''_k - R_k i'_k - R_k i''_k \\ &= \underset{\downarrow 0}{(v'_k - R_k i'_k)} + \underset{\downarrow 0}{(v''_k - R_k i''_k)} = 0 \checkmark \end{aligned}$$

• Verifico i generatori $v_{e-1} = E \rightarrow v'_{e-1} + v''_{e-1} = E \checkmark$
 $i_e = 0 \rightarrow \underset{\leftarrow 0}{i'_e} + \underset{\rightarrow J}{i''_e} = J \checkmark$

QED



$$\begin{aligned} R_1 &= 1 \, \Omega \\ R_2 &= 1 \, \Omega \\ R_3 &= .5 \, \Omega \\ R_4 &= .5 \, \Omega \end{aligned}$$

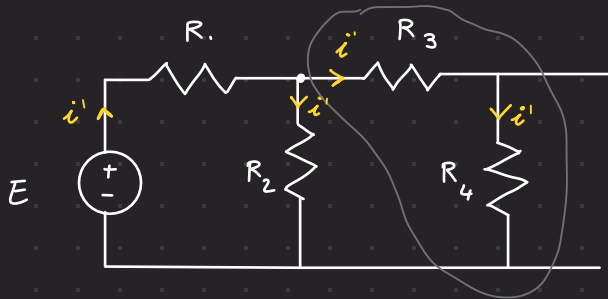
$$\begin{aligned} J &= 50 \, A \\ E &= 15 \, V \end{aligned}$$

Q1 Potenza elettrica Assorbita da $R_1 \rightarrow P_1 = R_1 i_1^2 = i_1^2$ (a)

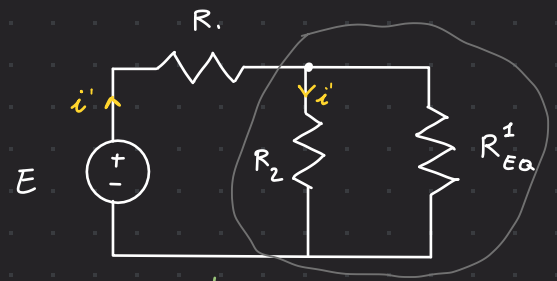
TROVIAMO i_1

-> Dalla Sovrapposizione: $i_1 = i_1' + i_1''$

(1) Rimuovo J -> C'



R_3 ed R_4
Sono in serie
 $R_{Eq}^1 = R_3 + R_4$

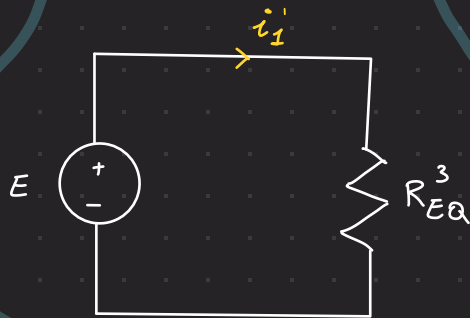


$$= \frac{R_3 + R_4 + R_2}{R_2(R_3 + R_4)} - 1$$

$$\frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

R_2 ed R_{Eq}^1
Sono in parallelo
 $\frac{1}{R_{Eq}^2} = \frac{1}{R_2} + \frac{1}{R_{Eq}^1}$
 $= \frac{1}{R_2} + \frac{1}{R_3 + R_4}$

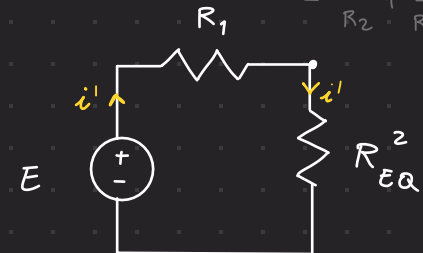
Circuito Ridotto



R_1 ed R_{Eq}^2 Sono
in Serie

$$R_{Eq}^3 = R_1 + R_{Eq}^{(2)}$$

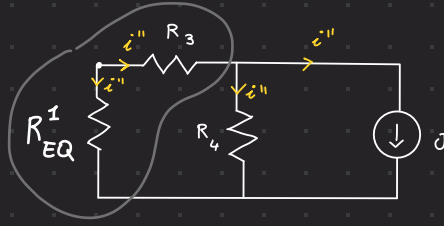
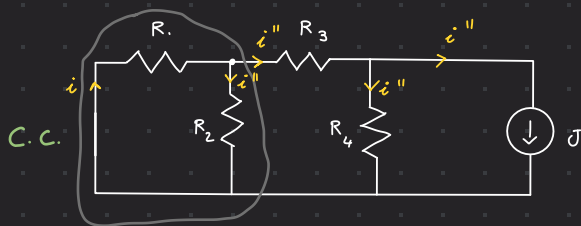
$$= R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$



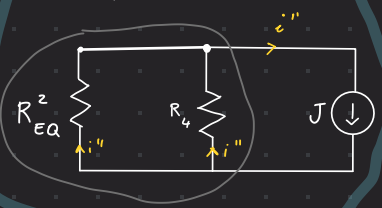
$R_{Eq}^{(3)}$

$$= 1 + \frac{1(5+5)}{1+5+5} = \frac{3}{2}$$

(2) Rimuovo E -> C''



$$\frac{1}{R_{Eq}^2} = \frac{1}{R_{Eq}^1} + \frac{1}{R_3}$$



$$R_{Eq}^{(1)} = R_1 + R_2, \quad R_{Eq}^{(2)} = \frac{1}{\frac{1}{R_{Eq}^{(1)}}} + \frac{1}{R_3} = \frac{1}{\frac{1}{R_1 + R_2}} + \frac{1}{R_3} = \frac{R_3 + R_1 + R_2}{(R_1 + R_2)R_3} \Rightarrow R_{Eq}^{(3)} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

Dalla (a) $p_1 = R_1 i_1^2 = R_1 (i_1' + i_1'')^2 = R_1 i_1'^2 + R_1 i_1''^2 + 2 R_1 i_1' i_1''$
 $i_1 = i_1' + i_1''$

(3) Determiniamo le i

Siccome $v = R \cdot i \rightarrow i = \frac{v}{R}$

$\Rightarrow i_1' = \frac{v}{R_{EA}} \quad \text{ma} \quad v = E \rightarrow i_1' = \frac{E}{R_{EA}} = \frac{15 \text{ V}}{3 \Omega} \cdot 2 = 10 \text{ A}$

i_1'' si calcola con la regola del partitore di corrente

Sappiamo che: $\begin{cases} E = R_2 \cdot i_2 \\ E = R_{EA}^3 \cdot i \end{cases} \quad \text{ma} \quad R_{EA} = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3}$

$\Rightarrow R_2 \cdot i_2 = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} \cdot i \rightarrow i_2 = \frac{R_3 (R_1 + R_2)}{(R_1 + R_2 + R_3) R_2} = ???$

BOH!

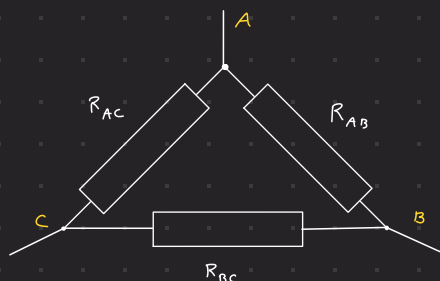
- TRIANGOLO

$$\frac{1}{R_{AB}^{(II)}} = \frac{1}{R_{AB}} + \left(\frac{1}{R_{CA} + R_{BC}} \right) = \frac{R_{AB} + R_{BC} + R_{CA}}{R_{AB} (R_{CA} + R_{BC})} \rightarrow$$

$$R_{AB}^{(I)} = \frac{R_{AB} (R_{CA} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}}$$

$$\Rightarrow R_{BC}^{(I)} = \frac{R_{BC} (R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}}$$

$$\Rightarrow R_{CA}^{(I)} = \frac{R_{CA} (R_{BC} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}}$$

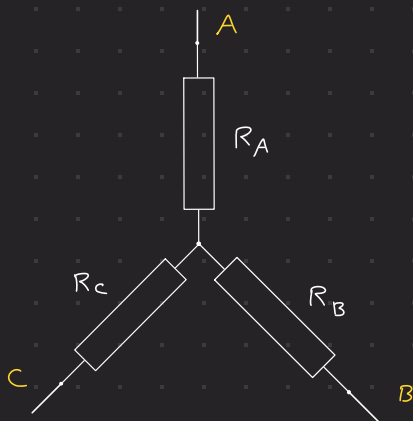


- STELLA

$$R_{AB}^{(II)} = R_A + R_B$$

$$R_{AC}^{(II)} = R_A + R_C$$

$$R_{BC}^{(II)} = R_B + R_C$$



→ Per avere l'equivalenza, tutte le resistenze DEVONO essere uguali!

$$R_{AB}^{(I)} = R_{AB}^{(II)}, \quad R_{BC}^{(I)} = R_{BC}^{(II)}, \quad R_{AC}^{(I)} = R_{AC}^{(II)}$$

OVVERO

$$\begin{cases} \frac{R_{AB} (R_{CA} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} = R_A + R_B \\ \frac{R_{BC} (R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} = R_A + R_C \\ \frac{R_{CA} (R_{BC} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} = R_B + R_C \end{cases}$$

Stella → Triangolo

$$R_{AB} = \frac{R_A \cdot R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_{BC} = \frac{R_A \cdot R_B + R_B R_C + R_C R_A}{R_A}$$

$$\bullet R_{CA} = \frac{R_A \cdot R_B + R_B R_C + R_C R_A}{R_B}$$

Triangolo → Stella

$$\bullet R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{CA}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{CA}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{CA}}$$

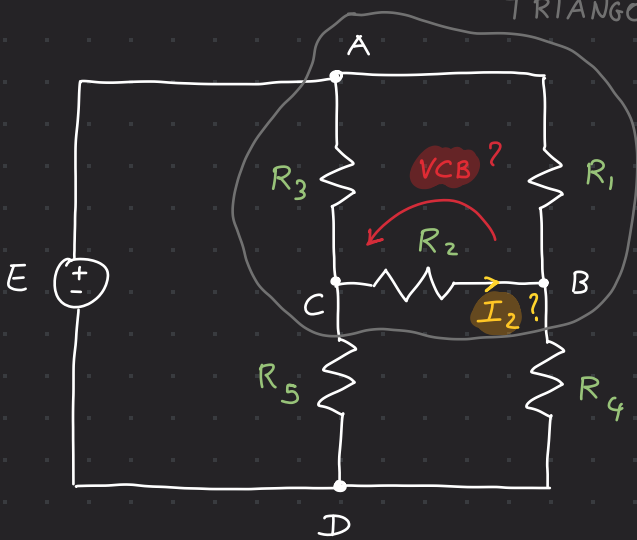
ESERCIZIO

△ → Δ

Link

Scopo del gioco: TROVARE I_2 , V_{CB}

TRIANGOLO



DATI

$$E = 24 \text{ V}$$

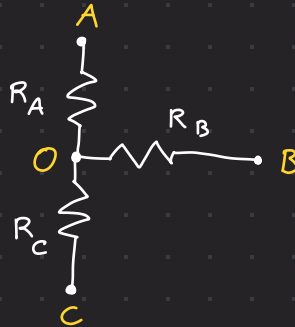
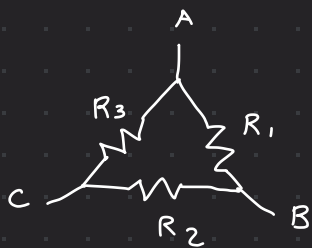
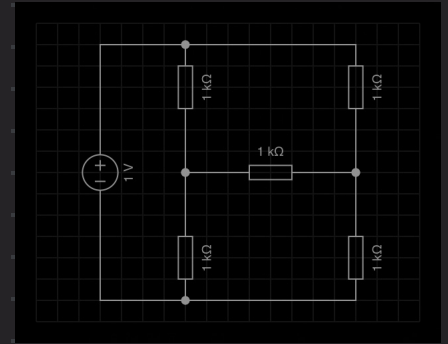
$$R_1 = 20 \Omega$$

$$R_2 = 30 \Omega$$

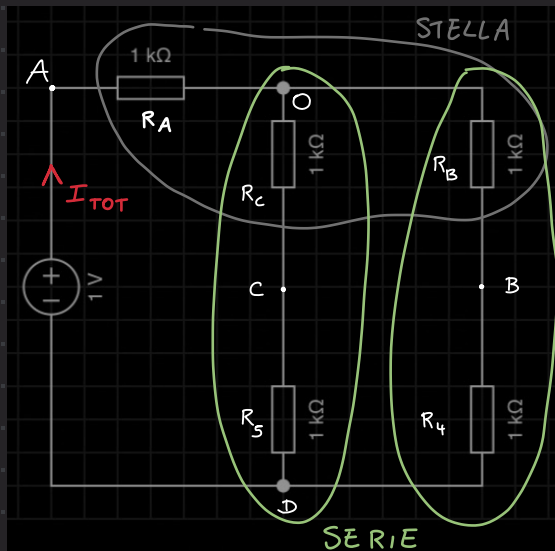
$$R_3 = 10 \Omega$$

$$R_4 = 40 \Omega$$

$$R_5 = 50 \Omega$$



→ Riscrivo il circuito



(1) Ricavo le resistenze

$$R_A = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3} = 3.3 \Omega$$

Res con il nodo A (Nel Triangolo)
in comune

$$R_B = \frac{R_1 \cdot R_2}{R_1 + R_2 + R_3} = 10 \Omega$$

$$R_C = \frac{R_2 \cdot R_3}{R_1 + R_2 + R_3} = 5 \Omega$$

$$R_{CS} = R_C + R_5 = 50 \Omega + 5 \Omega = 55 \Omega$$

$$R_{B4} = R_B + R_4 = 10 \Omega + 40 = 50 \Omega$$

SERIE

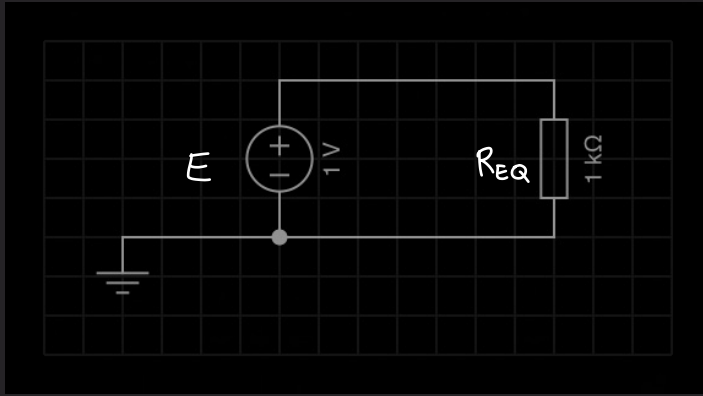
$$R_{CSB4} = R_{CS} \parallel R_{B4} = 55 \parallel 50 \Rightarrow \frac{1}{R_{CSB4}} = \frac{1}{R_{CS}} + \frac{1}{R_{B4}} = \frac{1}{55} + \frac{1}{50} = \frac{21}{550}$$

$$\text{oppure } R_{CSB4} = \frac{R_{CS} \cdot R_{B4}}{R_{CS} + R_{B4}} = 26.2 \Omega$$

$$R_{CSB4} = \frac{550}{21} = 26.2 \Omega$$

PARALLELO

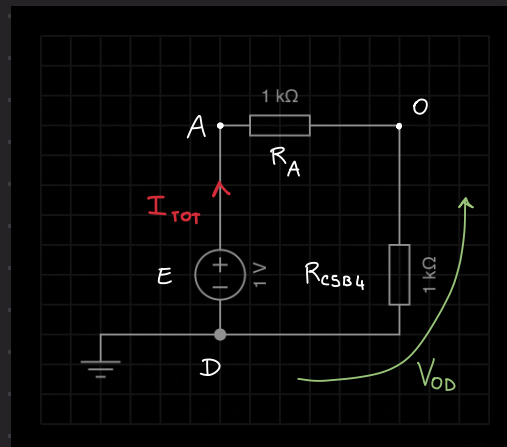
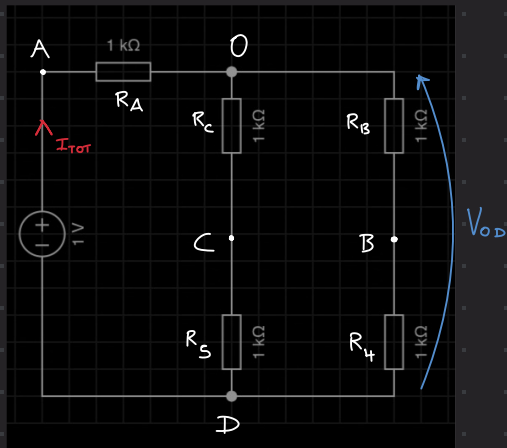
$$\Rightarrow R_{EQ} = R_A + R_{CSB4} = 29.5 \Omega$$



(2) Trovo la corrente

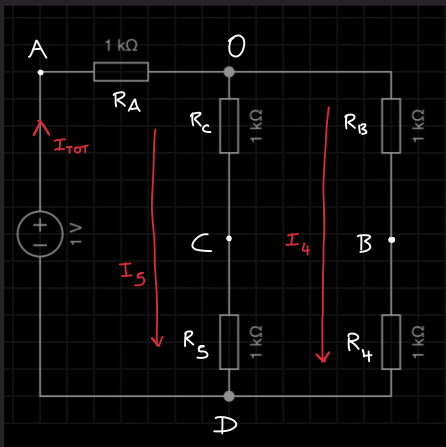
Siccome $V = R \cdot i \rightarrow I_{TOT} = \frac{V}{R_{EQ}} = \frac{E}{R_{EQ}} = \frac{24V}{29.5 \Omega} = 0.8 A$

(3) Trovo V_{OD}



$$\begin{aligned} \Rightarrow V_{OD} &= R_{CSB4} \cdot I_{TOT} \\ &= 26.2 \cdot 0.8 = \\ &= 20.96 V \end{aligned}$$

(4) Trovo le correnti

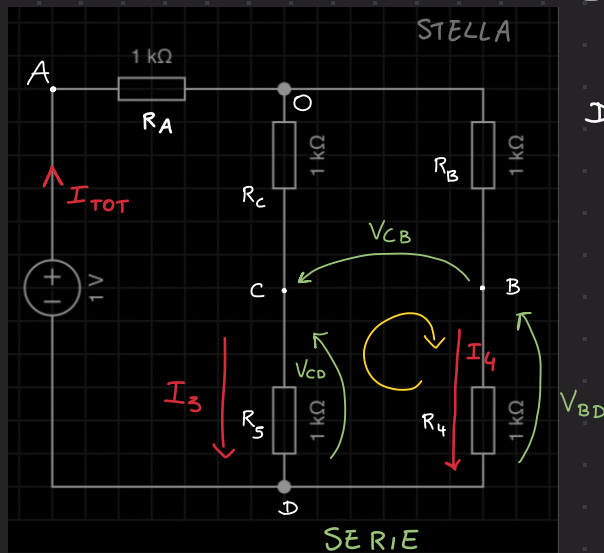


$$I_5 = \frac{V}{R_{CS}} = \frac{V_{OD}}{(R_C + R_S)} = \frac{20.96 V}{5 \Omega + 50 \Omega} = 0.38 A$$

$$I_4 = \frac{V_{OD}}{R_B + R_4} = 0.42 A$$

(5) TROVO V_{CB}

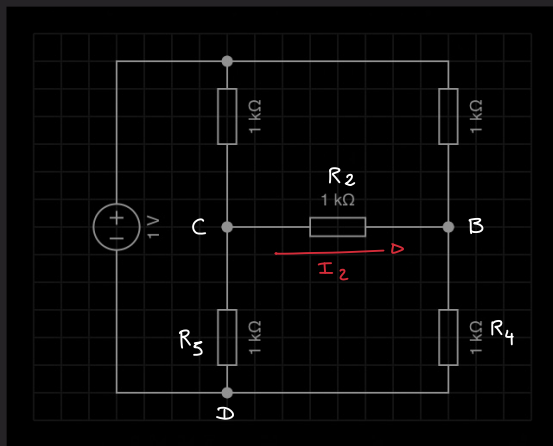
Per trovare I_2 ci serve la Tensione Tra C e B quindi



Da Kirchhoff: $V_{CD} - V_{CB} - V_{BD} = 0$

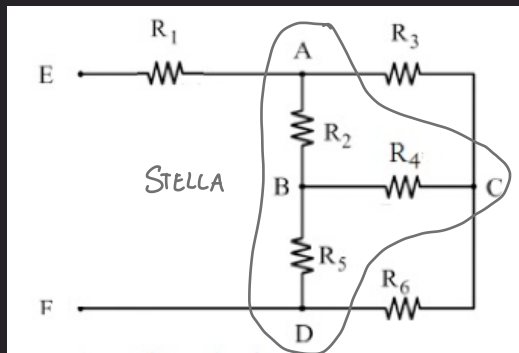
$$\Rightarrow V_{CB} = V_{CD} - V_{BD} = (R_5 \cdot I_3) - (R_4 \cdot I_4)$$

$$= (50 \cdot 0.38) - (40 \cdot 0.42) = 2.2 \text{ V } V_{CB}$$



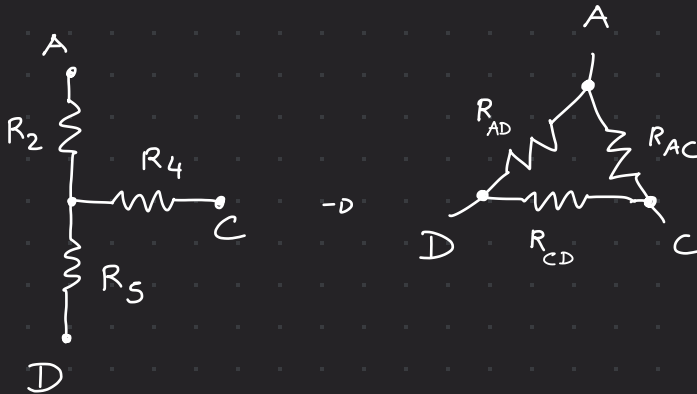
$$\Rightarrow V = R \cdot I \Rightarrow I_2 = \frac{V_{CB}}{R_2} \approx 0.073 \text{ A} = 73 \text{ mA}$$

Ans

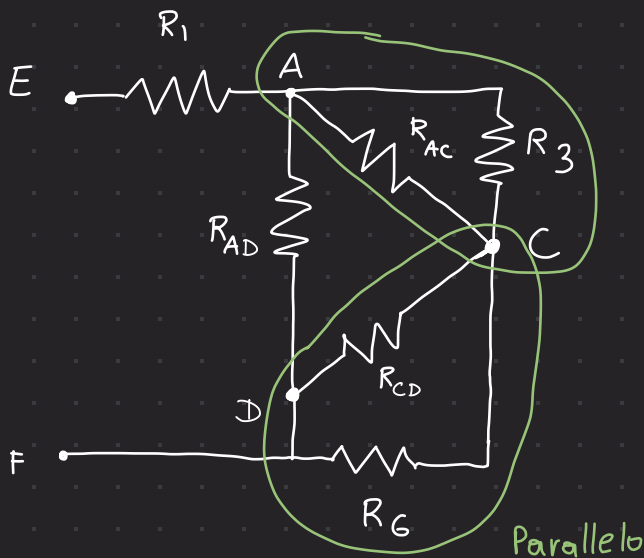


DATI

$$\begin{aligned} R_1 &= 5 \Omega \\ R_2 &= 20 \Omega \\ R_3 &= 12 \Omega \\ R_4 &= 16 \Omega \\ R_5 &= 25 \Omega \\ R_6 &= 30 \Omega \end{aligned}$$



(1) Resistenze del Triangolo



$$R_{AC} = \frac{(R_2 \cdot R_4) + (R_5 \cdot R_4) + (R_2 \cdot R_5)}{R_5} = 48.8 \Omega$$

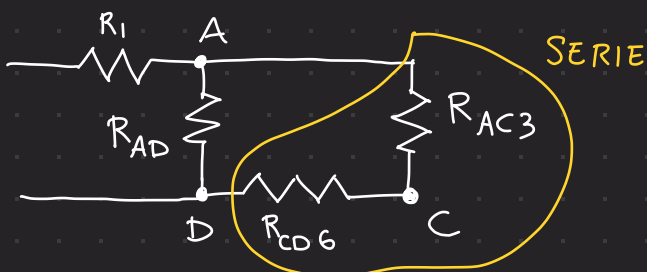
$$R_{AD} = \frac{(R_2 \cdot R_4) + (R_5 \cdot R_4) + (R_2 \cdot R_5)}{R_4} = 76.25 \Omega$$

$$R_{CD} = \frac{(R_2 \cdot R_4) + (R_5 \cdot R_4) + (R_2 \cdot R_5)}{R_2} = 61 \Omega$$

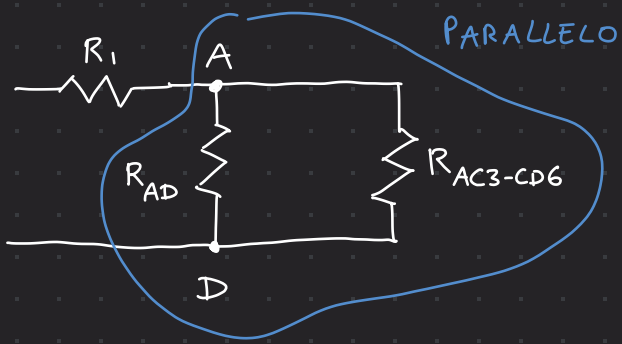
(2) Res in Parallelo

$$R_{CD6} = R_{CD} \parallel R_6 \rightarrow \frac{1}{61 \Omega} + \frac{1}{30 \Omega} \rightarrow R_{CD6} = 20.1 \Omega$$

$$R_{AC3} = R_{AC} \parallel R_3 \rightarrow \frac{1}{48.8 \Omega} + \frac{1}{12 \Omega} \rightarrow R_{AC3} = 9.6 \Omega$$



$$R_{AC3-CD6} = R_{AC3} + R_{CD6} = 29.73 \Omega$$



$$\frac{1}{R_{PAR}} = \frac{1}{R_{AC3-...}} + \frac{1}{R_{AD}} \rightarrow R_{PAR} = 21.4 \Omega$$

$$\Rightarrow R_{EQ} = R_1 + R_{PAR} = \underline{26.4 \Omega}$$

R_{EQ}



CIRCUITO RISOLTO ✓

ESERCIZIO

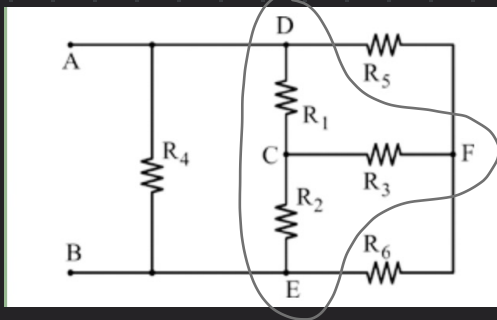
Link

DATI

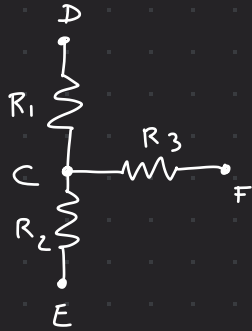
$$R_1 = R_2 = R_3 = 30 \, \Omega$$

$$R_4 = R_5 = R_6 = 150 \, \Omega$$

Q: R_{AB}

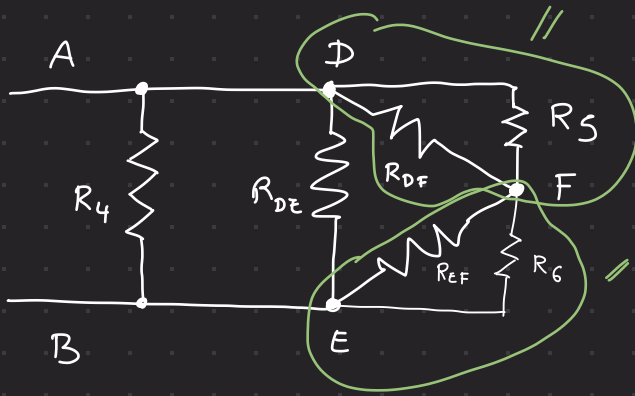


TRIANGOLO



$$R_{DE} = \frac{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}{R_3}$$

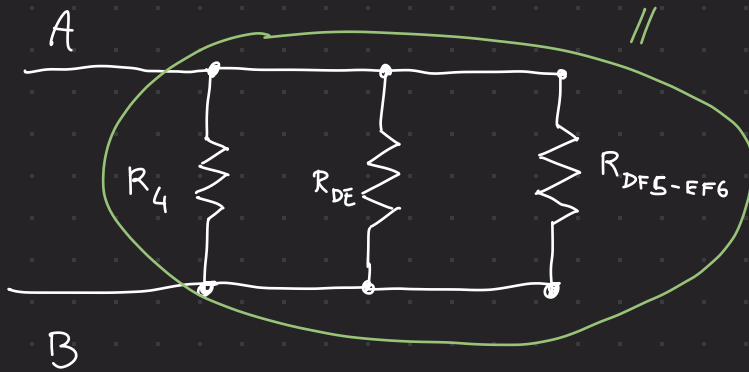
$$= R_{DF} = R_{EF} = 90 \, \Omega$$



$$\frac{1}{R_{DFS}} = \frac{1}{R_{DF}} + \frac{1}{R_5} \Rightarrow R_{DFS} = 56.25 \, \Omega$$

$$\frac{1}{R_{EF6}} = \frac{1}{R_{EF}} + \frac{1}{R_6} \Rightarrow R_{EF6} = 56.25 \, \Omega$$

$$R_{DFS-EF6} = R_{DFS} + R_{EF6} = 112.5 \, \Omega$$



$$\Rightarrow \frac{1}{R_{AB}} = \frac{1}{R_4} + \frac{1}{R_{DE}} + \frac{1}{R_{DFS-...}}$$

$$\Rightarrow R_{AB} = 37.5 \, \Omega \quad \text{Ans}$$

Esercizio 13

Nel circuito seguente con

$$R_1 = 1\text{k}\Omega$$

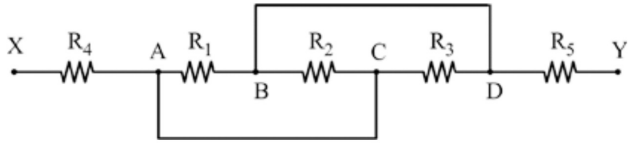
$$R_2 = 2\text{k}\Omega$$

$$R_3 = 3\text{k}\Omega$$

$$R_4 = 400\Omega$$

$$R_5 = 500\Omega$$

Calcola la resistenza R_{XY} fra i morsetti X e Y.



Considerando che $R_1 = R_2 = R_3 = 30\Omega$ e poi $R_4 = R_5 = R_6 = 150\Omega$.

[Risp.: $R_{AB} = 1,445\text{k}\Omega$]

I cortocircuiti ci fanno dire

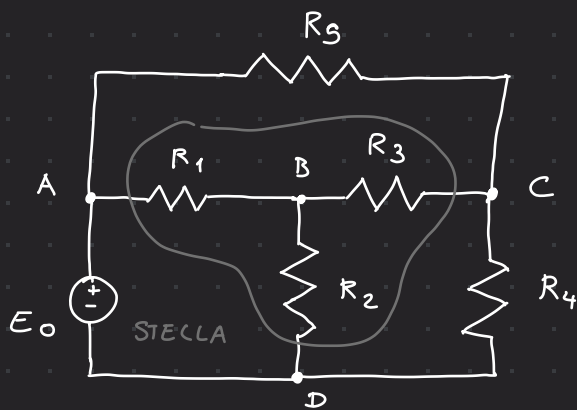
$$A = C \quad \text{e} \quad B = D$$

-> colleghiamo i dipoli di A anche a C
e viceversa

Boh

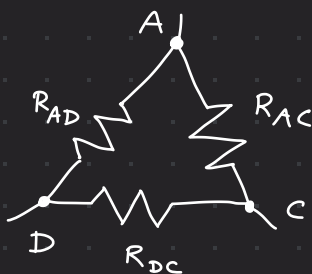
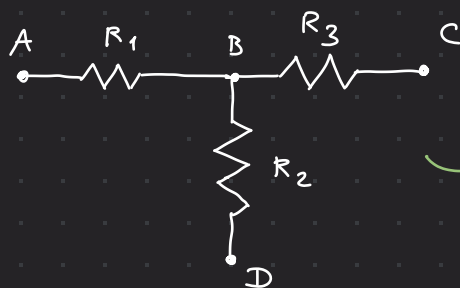
ESERCIZIO

DA COMPLETARE !!



DATI

$$\begin{aligned} E_0 &= 100 \text{ V} \\ R_1 &= 20 \, \Omega \\ R_2 &= 30 \, \Omega \\ R_3 &= 15 \, \Omega \\ R_4 &= 24 \, \Omega \\ R_5 &= 60 \, \Omega \end{aligned}$$



$$R_{AD} = \frac{R_A \cdot R_D + R_A R_C + R_D \cdot R_C}{R_C}$$

$$\text{OVVERO} \quad \frac{R_1 R_2 + R_1 R_3 + R_2 \cdot R_3}{R_3}$$

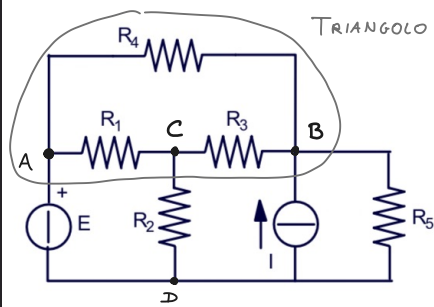
$$\Rightarrow R_{AD} = \frac{1350}{15} = 90 \, \Omega$$

$$R_{AC} = \frac{1350}{30} = 45 \, \Omega$$

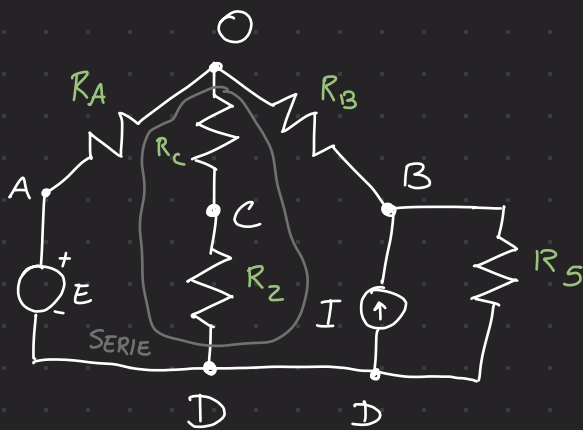
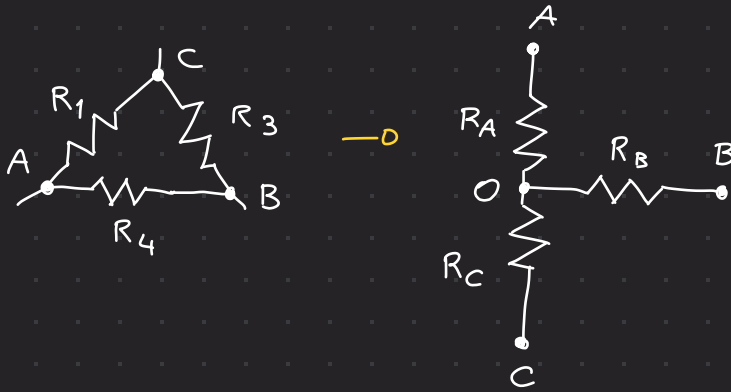
$$R_{DC} = \frac{1350}{20} = 67.5 \, \Omega$$

Calcolare la potenza assorbita da ogni resistore presente nel circuito, tensioni e correnti in ogni ramo.

[Link](#)



$E = 30 \text{ V}$
 $I = 2 \text{ A}$
 $R_1 = 40 \Omega$
 $R_2 = 25 \Omega$
 $R_3 = 20 \Omega$
 $R_4 = 400 \Omega$
 $R_5 = 100 \Omega$



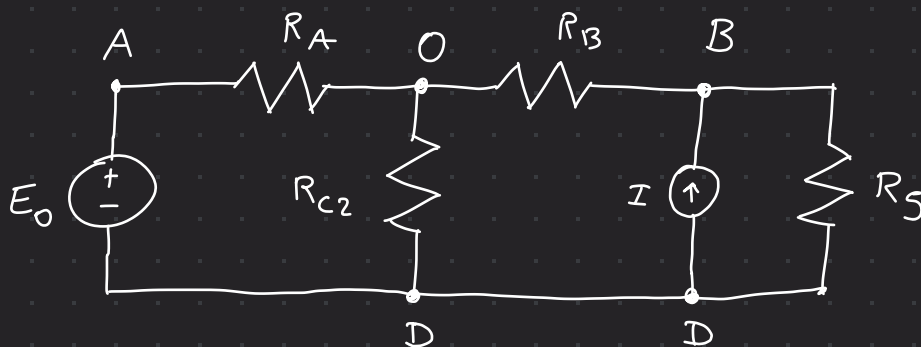
$$R_A = \frac{R_1 \cdot R_4}{R_1 + R_3 + R_4} = 34.78 \Omega$$

$$R_B = \frac{R_3 \cdot R_4}{R_1 + R_3 + R_4} = 17.39 \Omega$$

$$R_C = \frac{R_1 \cdot R_3}{R_1 + R_3 + R_4} = 1.74 \Omega$$

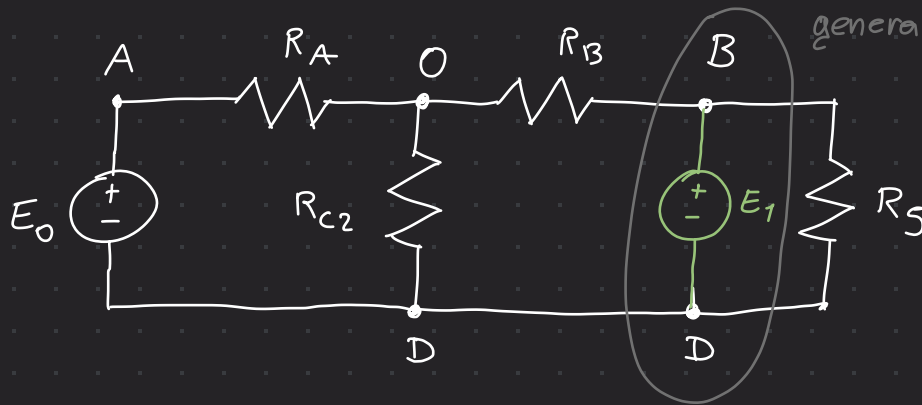
$$R_{C2} = R_C + R_2 = 1.74 + 25 = \underline{26.74 \Omega}$$

OTTENIAMO

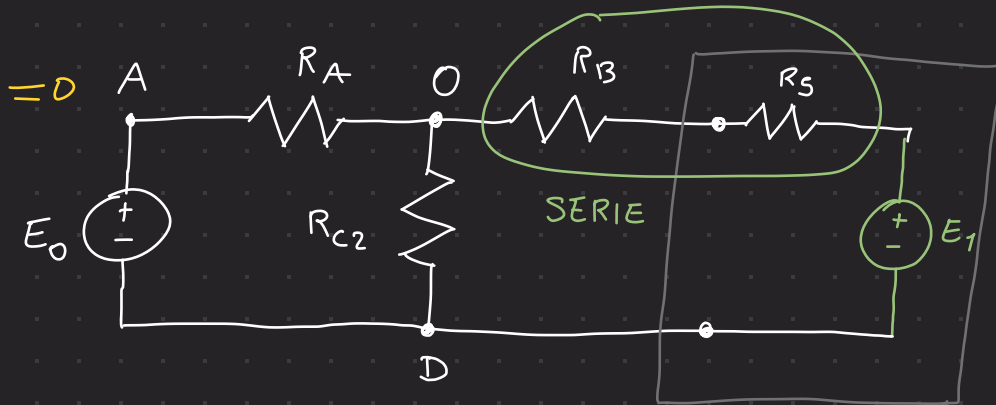
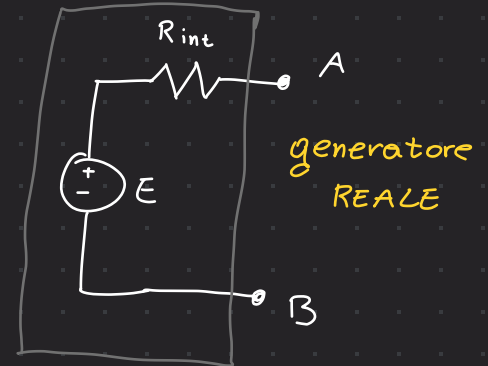


Possiamo Trasformare un generatore di corrente in uno di Tensione grazie alla legge di Ohm: Link generatore di corrente \rightarrow tensione

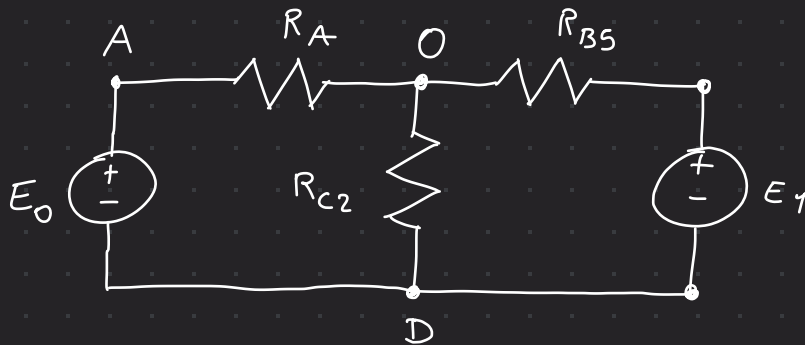
$$V = R \cdot I \Rightarrow E_1 = I \cdot R_S = 2 \cdot 100 = \underline{200V}$$



generatore Ideale



Gen Reale



$$R_{BS} = R_B + R_S = 17.39 + 100 = \underline{117.39\Omega}$$

