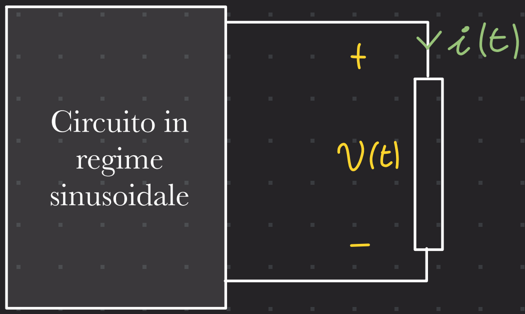
Potenza in regime sinusoidale



Cosa accade nel dominio del tempo?

$$v(t) = V_m \cos(\omega t + \alpha)$$
$$i(t) = I_m \cos(\omega t + \beta)$$

$$\rightarrow \ p(t) = v(t) \cdot i(t) = V_m I_m \cos(\omega t + \alpha)\cos(\omega t + \beta) = \frac{V_m I_m}{2}\Big[\cos(\omega t + \alpha + \omega t + \beta) + \cos(\omega t + \alpha - \omega t - \beta)\Big]$$

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$
$$\Rightarrow \cos x \cos y = \frac{1}{2}\left[\cos x+y + \cos x-y\right]$$

Potenza **istantanea**

manca ω

$$\boxed{p(t) = \frac{V_m I_m}{2}\left[\cos(\alpha - \beta) + \cos(2\omega t + \alpha + \beta)\right]}$$

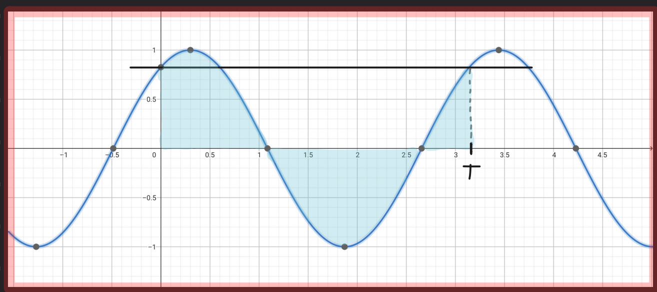
La potenza non si può rappr. come FASORE!

$\Leftarrow$ NON è una sinusoide di pulsazione ω!

Potenza media

$$P = \frac{1}{2}\int_0^T p(t)\, dt = \frac{V_m I_m}{2}\cos(\alpha + \beta) \cdot \frac{1}{T}\int_0^T dt + \frac{V_m I_m}{2}\,\frac{1}{T}\int_0^T \cos(2\omega t + \alpha + \beta)\, dt$$

$\nearrow \varnothing$



$$\Rightarrow P = \frac{V_m I_m}{2}\cos(\alpha - \beta)$$

Fattore di Potenza

$=$ pongo $\boxed{\alpha - \beta = \varphi}$

Potenza **media**

$$\Rightarrow \boxed{P = \frac{V_m I_m}{2}\cos(\varphi)}$$

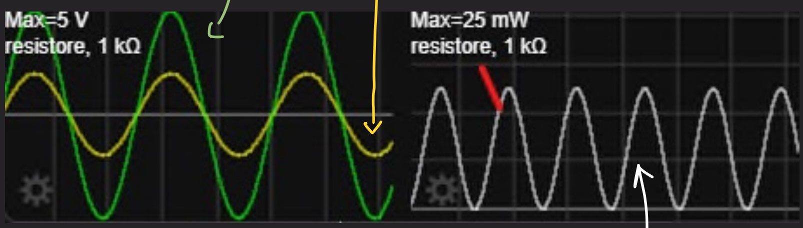
RESISTORE

$$v(t) = V_M \cos(\omega t + \alpha)$$
$$i(t) = I_M \cos(\omega t + \beta)$$

siccome $\underbrace{V = R\, i}$ $\longrightarrow$ $\underbrace{V_M \cos(\omega t + \alpha) = R \cdot I_M \cos(\omega t + \beta)}$

Vera se $\boxed{\alpha = \beta}$ $\Rightarrow$ $\boxed{V_M = R \cdot I_M}$

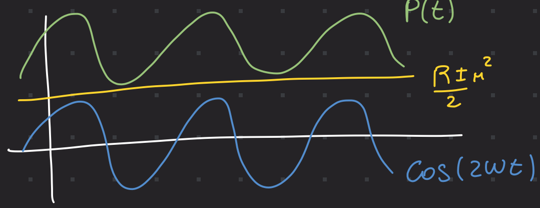
V I



P

Potenza istantanea di un resistore

$$\boxed{P(t) = \frac{R\, I_M^2}{2}\left[\cancel{\cos(0)}^{\,1} + \cos(2\omega t)\right]}$$



INDUTTORE

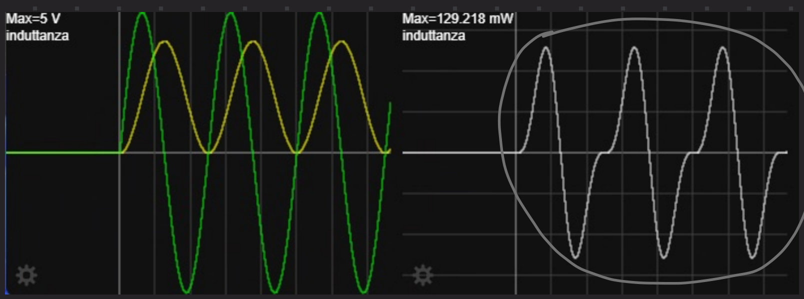
$$v(t) = V_M \cos(\omega t + \alpha)$$
$$i(t) = I_M \cos(\omega t + \beta)$$

$$v(t) = L \cdot \frac{di}{dt} \longrightarrow \frac{di}{dt} = -\omega I_M \sin(\omega t + \beta) = \omega I_M \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

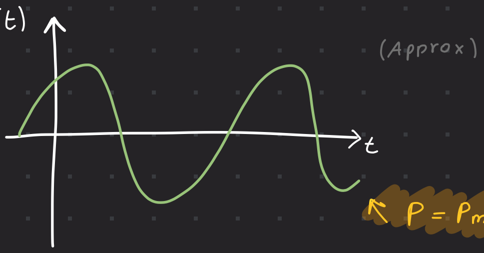
$$\Rightarrow V_M \cos(\omega t + \alpha) = L \omega I_M \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

$\Rightarrow$ se $\boxed{\alpha = \beta + \frac{\pi}{2}}$ $\Rightarrow$ $\boxed{V_M = L \omega I_M}$

La tensione è in anticipo di pi/2 rispetto alla corrente



$\longrightarrow$



$P = P_{media} = \underline{\underline{0}}$

$\left[\left(\beta + \frac{\pi}{2}\right) + \beta\right]$

proof: $P = \frac{L \omega I_M^2}{2} \cdot \cancel{\cos(\varphi)}\ 0$

$\beta - \frac{\pi}{2} - \beta = -\frac{\pi}{2}$

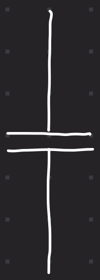
$\cos\left(\pm\frac{\pi}{2}\right) = 0$

Potenza istantanea di un induttore

$$\Rightarrow \boxed{P(t) = \frac{L \omega I_M^2}{2} \cos\left(2\omega t + 2\beta + \frac{\pi}{2}\right)}$$

CONDENSATORE

C



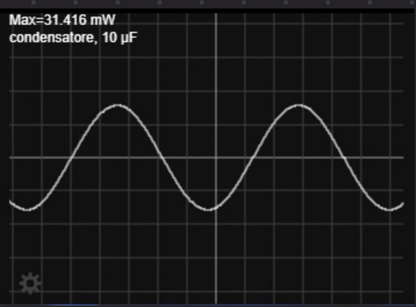
$$v(t) = V_M \cos(\omega t + \alpha)$$
$$i(t) = I_M \cos(\omega t + \beta)$$

$$i(t) = C \frac{dv}{dt} \to \frac{dv}{dt} = \omega V_M \cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$$

$$\Rightarrow I_M \cos(\cancel{\omega t} + \underbrace{\beta) = C \omega V_M \cos(\cancel{\omega t} + \alpha}_{\text{Se } \beta = \alpha + \frac{\pi}{2}} + \frac{\pi}{2})$$

Potenza istantanea condensatore

$$\Rightarrow \boxed{\begin{cases} I_M = C \omega V_M \\ \beta = \alpha + \frac{\pi}{2} \end{cases}} \to \boxed{p(t) = \frac{C \omega V_M^2}{2} \cdot \left[\cos(0) \cdot \cos\left(2\omega t + 2\alpha + \frac{\pi}{2}\right)\right]}$$



Concetto di **Potenza complessa**

Dominio dei fasori

$$\bar{V} = V_m \cdot e^{j\alpha}$$

$$\bar{I} = I_m \cdot e^{j\beta}$$

Primo Approccio (errato!)

come nel dom. del t : $p = v \cdot i$

$$\rightarrow P = \bar{V} \cdot \bar{I} = V_m e^{j\alpha} \cdot I_m e^{j\beta} = V_m I_m e^{j(\alpha+\beta)}$$

$$= V_m I_m \left[\cos(\alpha+\beta) + j \sin(\alpha+\beta) \right]$$

manca $\frac{1}{2}$

Quasi... ma manca $\alpha - \beta$

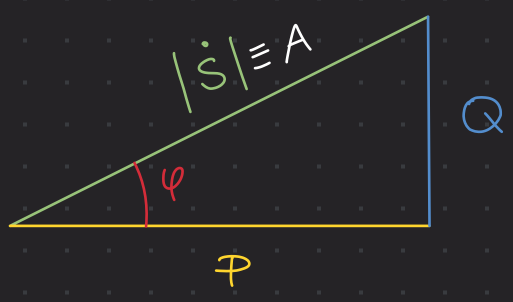
Secondo Approccio : Complesso Coniugato (giusto!)

Potenza complessa

(1)

$$\text{Potenza Complessa} = \dot{S} = \frac{\bar{V} \cdot \bar{I}^*}{2} = \underbrace{\frac{V_m I_m}{2} \cos(\alpha - \beta)}_{(P) \text{ Potenza Media}} + j \underbrace{\frac{V_m I_m}{2} \sin(\alpha - \beta)}_{Q \text{ Potenza Reattiva}}$$

Qualche grandezza utile e unità di misura…



Potenza apparente

$$A = |\dot{S}| = \sqrt{P^2 + Q^2}$$

- $P = |\dot{S}| \cos\varphi$
- $Q = |\dot{S}| \sin\varphi$
- $\frac{Q}{P} = tg\,\varphi$

- $[P] = W$
- $[Q] = V \cdot A_r$ "Volt-ampere reattivi"
- $[A] = VA$ "Volt-ampere"

Tabella riassuntiva

	φ	P	Q	A
R	$\emptyset$	$\frac{R I_m^2}{2}$	$\emptyset$	$\frac{R I_m^2}{2}$
L	$\frac{\pi}{2}$	$\emptyset$	$\frac{\omega L I_m^2}{2}$	$\frac{\omega L I_m^2}{2}$
C	$-\frac{\pi}{2}$	$\emptyset$	$-\frac{\omega C I_m^2}{2}$	$\frac{\omega C I_m^2}{2}$

Metodo dei fasori con la **convenzione dei valori efficaci**

$$v(t) = V_m \cos(\omega t + \alpha) \rightsquigarrow \bar{V} = \frac{V_m}{\sqrt{2}} e^{j\alpha} = V_0 e^{j\alpha}$$

$$i(t) = I_m \cos(\omega t + \beta) \rightsquigarrow \bar{I} = \frac{I_m}{\sqrt{2}} e^{j\beta} = I_0 e^{j\beta}$$

Valore efficace sinusoide

$$\Rightarrow \dot{S} = \bar{V} \cdot \bar{I}^* = V_0 I_0 e^{j(\alpha-\beta)}$$

$$E_0 = \frac{E_M}{\sqrt{2}}$$ (lez 14)

proof $\rightarrow$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} e^{j(\alpha-\beta)} = \frac{V_m I_m}{2} e^{j(\alpha-\beta)} =$$

$$= \frac{V_m I_m}{2} \cos(\alpha-\beta) + j \frac{V_m I_m}{2} \sin(\alpha-\beta) = P + jQ \qquad (1)$$

Conservazione della Potenza complessa

Tesi

$$\boxed{\sum_{K=1}^{\ell} \dot{S}_K = 0}$$

$$\rightarrow \sum_{K=1}^{\ell} (P_K + jQ_K) = 0 \quad \begin{cases} \sum_{K=1}^{\ell} P_K = 0 \\ \sum_{K=1}^{\ell} Q_K = 0 \end{cases}$$

Affinché la somma delle potenze complesse sia zero, sia la Potenza media che la Potenza reattiva sono zero **separatamente**; questo è ciò che dobbiamo riuscire a dimostrare:

$$\sum_{K=1}^{\ell} \frac{1}{2} \bar{V}_K \bar{I}_K^* = \frac{1}{2} \bar{\underline{V}}^T \bar{\underline{I}}^* = \frac{1}{2} \overset{\bar{V}^T}{\bar{\underline{U}}^T \underline{\underline{A}}} \, \bar{\underline{I}}^* = 0$$

FASORE-VETTORE

Fasore

Vettore

$$\bar{\underline{V}} = \begin{pmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_K \end{pmatrix} \qquad \bar{\underline{I}} = \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_K \end{pmatrix}$$

Valgono le LKI $\Rightarrow \underline{\underline{A}} \, \bar{\underline{I}} = 0$

non Trasposta! $\rightarrow$ dobbiamo dimostrare che anche $\underline{\underline{A}} \, \bar{\underline{I}}^* = 0$

Se $\underline{\underline{A}}\bar{\underline{I}} = 0 \Rightarrow \underline{\underline{A}} \, \mathrm{Re}(\bar{\underline{I}}) + j \, \mathrm{Im}(\bar{\underline{I}}) = 0 \Rightarrow \begin{cases} \underline{\underline{A}} \, \mathrm{Re}(\bar{\underline{I}}) = 0 \\ \underline{\underline{A}} \, \mathrm{Im}(\bar{\underline{I}}) = 0 \end{cases}$

Separatamente!

$\Rightarrow \underline{\underline{A}} \, \mathrm{Re}(\bar{\underline{I}}) - j \, \mathrm{Im}(\bar{\underline{I}}) = 0 \quad \Rightarrow \quad \underline{\underline{A}} \, \bar{\underline{I}}^* = 0$ QED

Energia assorbita da un dipolo in regime sinusoidale

$$\boxed{\mathcal{E} = \int_0^{mT} p(t)\, dt = mT \cdot P}$$

$\longrightarrow [\mathcal{E}] = Wh = 1W \cdot 1h$

$\longrightarrow 1Wh = 1 \cdot 60 \cdot 60\, J = 3.6\, KJ$