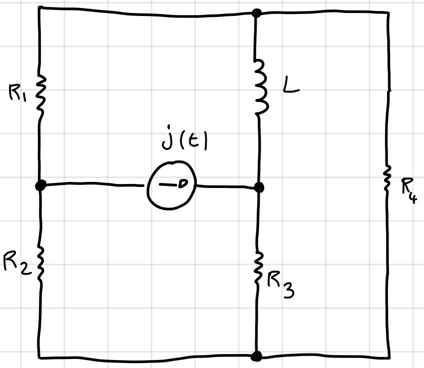
PROVA ESAME 26/02/20



Il circuito è a riposo per $t<0$, determinare la corrente che scorre in L per ogni istante di tempo.

DATI

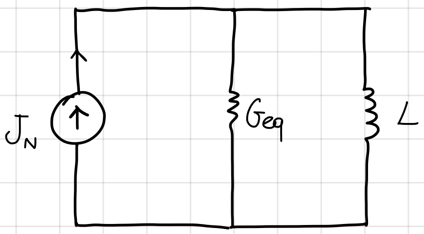
A $R_1 = 120\,\Omega$ B $R_2 = 60\,\Omega$

C $R_3 = 110\,\Omega$ D $R_4 = 180\,\Omega$

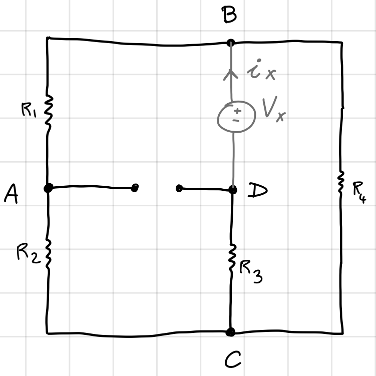
E $L = 0.4\,H$

$$j(t) = \begin{cases} 0\,A & t<0 \\ 0.8\,A & 0<t<5\,ms \\ 0\,A & t>5\,ms \end{cases}$$

(1) Analisi agli intervalli con un circuito eq di Norton ai capi dell'induttore



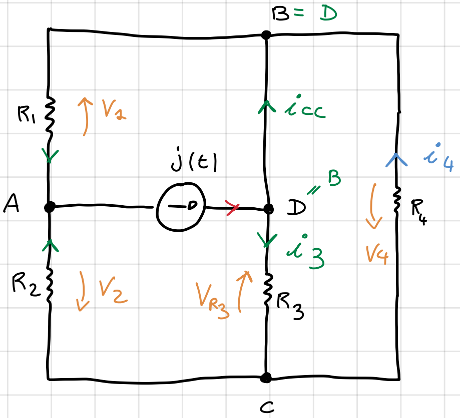
(1.a) Resistenza eq Norton $R_N = R_{BC}$



F

$$R_N = \left[(R_1 + R_2) \,//\, R_4\right] + R_3 = 200\,\Omega \quad R_N$$

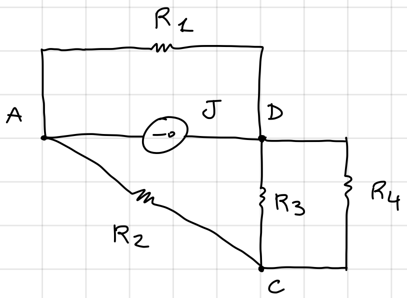
(1.b) Trovo i_{cc} → Sostituisco ad L un C.C.

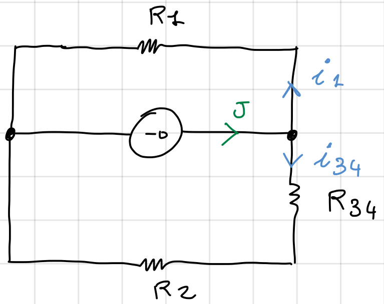


$LKC_D: \; -j + i_{cc} + i_3 = 0$

$\Rightarrow \; i_{cc} = J - i_3 \,?$

Semplifico il circuito





$R_{34} = R_3 // R_4 = 68.27\ \Omega$

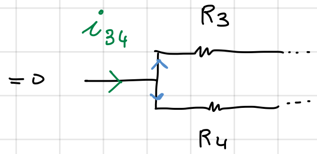
$$i_{34} = J \cdot \frac{R_1}{R_1 + (R_{34} + R_2)} =$$

esce 0.48

$$= J(t) \cdot 0.64$$

è la corrente che arriva al parallelo 3-4 che viene poi ripartita

$\Rightarrow$



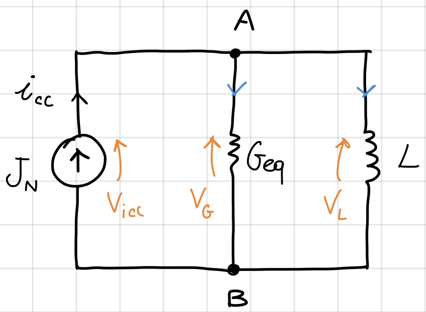
$$i_3 = i_{34} \cdot \frac{R_4}{R_4 + R_3} = J(t) \cdot 0.397\ A$$

i_3

$$\Rightarrow \quad i_{cc} = J(t) - i_3 = J(t) - J(t) \cdot 0.397 = J(t)\left(1 - 0.397\right) = 0.6 \cdot J(t)$$

$$j(t) = \begin{cases} 0\ A & t < 0 \\ 0.8\ A & 0 < t < 5\,ms \\ 0\ A & t > 5\,ms \end{cases} \Rightarrow \quad i_{cc}(t) = \begin{cases} 0\ A & t < 0 \\ 0.48\ A & 0 \leq t \leq 5\,ms \\ 0\ A & t > 5\,ms \end{cases}$$

Tornando al circuito di Norton



Trovo $i_L \rightarrow$ LKC_A: $-i_{cc} + i_G + i_L = 0$

$$\Rightarrow \quad i_L = i_{cc} - i_G$$

ma $i_G = G_{eq} \cdot V_{AB}$

$V_{AB} = V_{icc} = V_L$

Eq Diff

$$V_L = L\,\frac{di_L}{dt} \quad \Rightarrow \quad i_L + G_{eq} L \cdot \frac{di_L}{dt} = i_{cc} \quad \rightarrow \quad i_L + \frac{L}{R_{eq}}\,\frac{di_L}{dt} = i_{cc}$$

Rel Car

(2) Associo l'eq diff ad una condizione iniziale

$$\begin{cases} i_L + \dfrac{L}{R_{eq}} \cdot \dfrac{di_L}{dt} = i_{cc} \\ i_L(0^+) = i_L(0^-) = 0 \end{cases}$$

$$j(t) = \begin{cases} 0\ A & t < 0 \\ 0.8\ A & 0 < t < 5\,ms \\ 0\ A & t > 5\,ms \end{cases}$$

(3) Soluzione

$$i_L \sim K e^{-\frac{t}{\tau}} + i_{cc}$$

Soluzione particolare

(3.a) Costante K uso la condizione iniziale

$$\begin{cases} i_L(t) = K e^{-\frac{t}{\tau}} + i_{cc} \\ i_L(0) = 0 \end{cases} \Rightarrow i_L(0) = K \cancel{e^{-\frac{0}{\tau}}}^{1} + i_{cc} = 0 \rightarrow K + i_{cc} = 0$$

$$\Rightarrow K = -i_{cc} = -0.48\ A \qquad (0 < t < 5\ ms)$$

$$\Rightarrow i_L(t) = -i_{cc}\, e^{-\frac{t}{\tau}} + i_{cc} = i_{cc}\left(1 - e^{-\frac{t}{\tau}}\right)\ A$$

$i_L(t)$

INTERVALLO $t > 5\ ms$

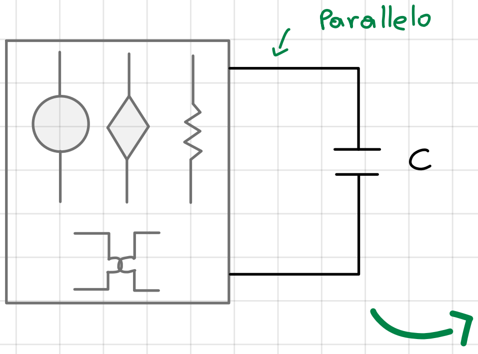
Per calcolare cosa succede da quando il generatore si spegne nuovamente (t=t1) a t->inf dobbiamo valutare la **nuova condizione iniziale**, ovvero quella che abbiamo appena calcolato:

$$i_L(t_1) = 0.48\left(1 - e^{-\frac{t}{\tau}}\right)\ A = I_1$$

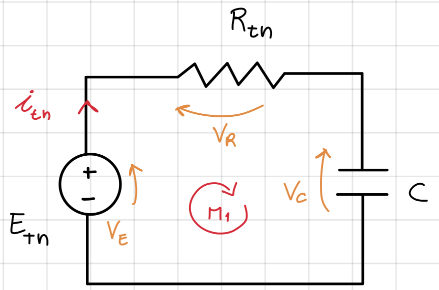
NUOVA Cond. Iniz.

Esempio Pilota

Parallelo



(1) Circuito eq con thevenin



LKT_{M_1}: $-V_E + V_R + V_C = 0$

$\rightarrow V_E = V_R + V_C$

Eq Caratt: $V_R = R_{th}\, i_{th}$

$i_{th}(t) = C \cdot \frac{dv(t)}{dt}$

$\Rightarrow \begin{cases} V_E = V_R + V_C \\ V_R = R_{th} \cdot i_{th}(t) \\ i_{th}(t) = C \cdot \frac{dv(t)}{dt} \end{cases}$ $\Rightarrow V_E = R_{th} \cdot C \cdot \frac{dv(t)}{dt} + v_c(t)$

$\rightarrow \boxed{e(t) = RC\, \dot{v}_c + v_c}$ eq diff

(2) PROBLEMA DI CAUCHY

(2) $\begin{cases} RC \cdot \dot{v}_c + v_c = e(t) \\ v_c(0^+) = V_0 \end{cases}$ $\longrightarrow$ La soluzione sara' $V_c(t) = \underbrace{V_{c0}(t)}_{\text{Transitorio}} + \underbrace{V_{cp}(t)}_{\text{Regime}}$

Omogenea, Particolare

(2.a) Omogenea Associata

Sappiamo per Hp. che $v_c \sim K e^{\lambda t}$ $\rightarrow \dot{v}_c \triangleq \lambda K e^{\lambda t}$

$\rightarrow$ Sostituisco nella (2) $\rightarrow RC\lambda \cancel{K} \cancel{e^{\lambda t}} + \cancel{K} \cancel{e^{\lambda t}} = \emptyset$ ← L'omogenea e' uguale a 0!

$\rightarrow RC\lambda + 1 = 0 \rightarrow \lambda = -\frac{1}{RC}$ FREQUENZA NATURALE

$\rightarrow v_{c0} = K e^{-\frac{t}{RC}}$

(2.b) Soluzione particolare

$\boxed{V_{cp}(t) \triangleq e(t)}$ $\rightarrow$ Hp. $e(t) = E_0 \Rightarrow V_{cp}(t) = E_0$

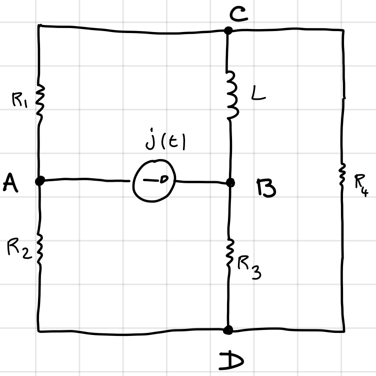
$\Rightarrow V_c(t) = V_{c0}(t) + V_{cp}(t) = K e^{-\frac{t}{RC}} + E_0$

Trovo la costante K

$\begin{cases} v_c(t) = K e^{-\frac{t}{RC}} + \underbrace{E_0}_{V_{cp}} \\ v_c(0^+) = v_0 \end{cases}$ ← Condizione iniziale

$\Rightarrow v_c(0^+) = K e^{-\frac{0}{RC}} + E_0 = v_0 \rightarrow \underbrace{v_0 = K + E_0}_{\text{Cond Iniz.}} \rightarrow \underbrace{K = v_0 - E_0}_{\text{COSTANTE}}$

ESEMPIO INIZIALE RIFATTO



DATI

A $R_1 = 120\ \Omega$

B $R_2 = 60\ \Omega$

C $R_3 = 110\ \Omega$

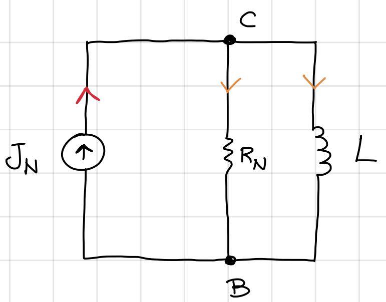
D $R_4 = 180\ \Omega$

X $L = 0.4\ H$

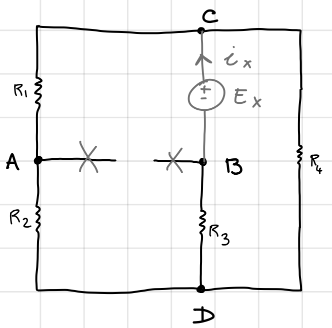
$$j(t) = \begin{cases} 0\ A & t < 0 \\ 0.8\ A & 0 < t < 5ms \\ 0\ A & t > 5ms \end{cases}$$

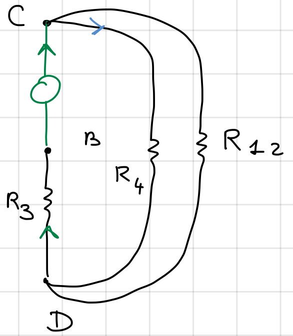
(1) Abbiamo $L \rightarrow$ NORTON N_{CB}

Obbiettivo



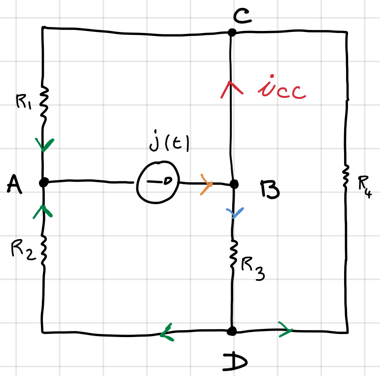
- $R_{BC} = (R_{12} \,//\, R_4) + R_3 = 200\ \Omega$ R_N

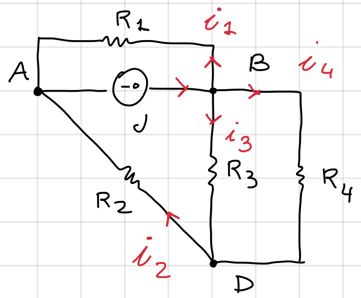




$\rightarrow$

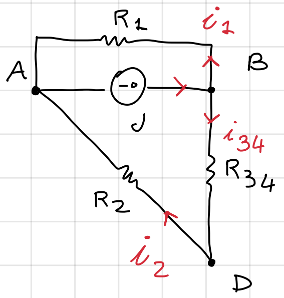
- $J_{th} = i_{cc} = ?$





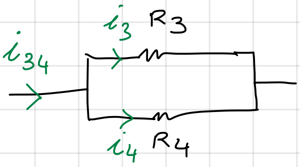
LKC$_D$: $-j(t) + i_{cc} + i_3 = 0$

$\rightarrow i_{cc} = j(t) - i_3$?



$$\rightarrow \quad i_{34} = j(t)\,\frac{R_1}{R_1 + R_{34} + R_2} = J(t)\cdot\frac{29}{60} \quad A$$

$\Rightarrow$



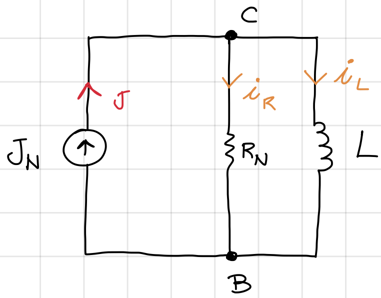
$$\rightarrow \quad i_3 = i_{34}\cdot\frac{R_4}{R_4 + R_3} = J(t)\cdot\frac{3}{10} \quad A$$

i_3

$$\Rightarrow \quad i_{cc} = J(t) - i_3 = J(t) - \frac{3}{10}J(t) = J(t)\left[1 - \frac{3}{10}\right] = \frac{7}{10}\cdot J(t)$$

i_{cc}

TORNANDO AL C.E. th



$$j(t) = \begin{cases} 0\ A & t<0 \\ 0.8\ A & 0<t<5ms \\ 0\ A & t>5ms \end{cases} \Rightarrow i_{cc}(t) = \begin{cases} 0\ A \\ 0.56\ A & 0<t<5 \\ 0\ A \end{cases}$$

$$LKC_c: \quad -J(t) + i_R + i_L = 0 \rightarrow \begin{cases} i_L + i_R - J(t) = 0 \\ v_L(t) = L\,\dfrac{d i_L}{dt} \\ i_R = \dfrac{v_L(t)}{R_N} \end{cases}$$

$$\Rightarrow \quad i_L + \frac{v_L(t)}{R_N} - \underbrace{J(t)}_{i_{cc}} = 0 \rightarrow i_L + \frac{L}{R_N}\frac{d i_L}{dt} = i_{cc}$$

$$\boxed{i_L + \frac{L}{R_N}\,\dot{i}_L = i_{cc}}$$

eq diff 1° ordine

Condizione iniziale

$$\begin{cases} i_L + \dfrac{L}{R_N}\,\dot{i}_L = i_{cc} \\ i_L(0^+) = 0 \end{cases}$$

Soluzione $\quad i_L(t) = i_{L0}(t) + i_{Lp}(t)$

Hp. $\quad i_{L0}(t) \sim K e^{\lambda t} \rightarrow \dot{i}_L = \lambda K e^{\lambda t}$

$$\Rightarrow \quad 1\,\cancel{K}\cancel{e^{\lambda t}} + \frac{L}{R_N}\lambda \cancel{K}\cancel{e^{\lambda t}} = 0 \rightarrow 1 + \frac{L}{R_N}\lambda = 0 \rightarrow \lambda = -\frac{R_N}{L}$$

$$\rightarrow \quad i_{L0}(t) = K e^{-\frac{R_N}{L}t}$$

- $i_{Lp}(t) \sim i_{cc}(t)$ SOL PARTICOLARE

$$\rightarrow \quad i_L(t) = i_{L0}(t) + i_{Lp}(t) = K e^{-\frac{R_N}{L}t} + \underset{\text{cost}}{i_{cc}}$$

Determino K

$$\begin{cases} i_L(t) = K e^{-\frac{R_N}{L}t} + i_{cc} \\ i_L(0^+) = 0 \end{cases} \quad \rightarrow \quad i_L(0) = K \cancel{e^{-\frac{R_N}{L}0}}^{\,1} + i_{cc} = 0$$

$$\hookrightarrow \quad K + i_{cc} = 0 \rightarrow K = -i_{cc}$$

$$\Rightarrow \quad i_L(t) = -i_{cc}\, e^{-\frac{R_N}{L}t} + i_{cc} = i_{cc}\left(1 - e^{-\frac{R_N}{L}t}\right) \quad i_L(t)$$

QUINDI negli intervalli

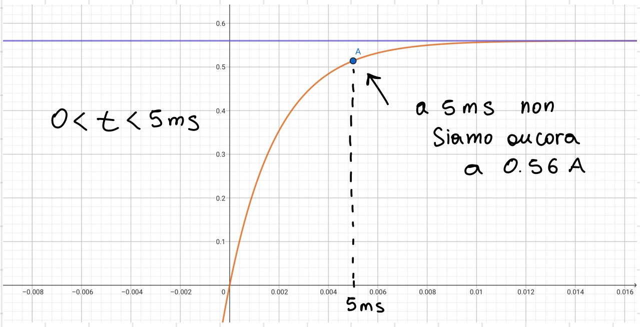
$$\frac{R_N}{L} = \frac{1}{L/R_N}$$

$\boxed{t<0}$ $\quad J(t) = 0\,A \rightarrow i_{cc} = 0 \rightarrow i_L(0^-) = 0$

$\boxed{0<t<5ms}$ $\quad J(t) = 0.8\,A \rightarrow i_{cc}(t) = 0.56\,A \rightarrow i_L(t_1) = 0.56\left(1 - e^{-\frac{1}{L/R_N}t}\right)A$

$\qquad$ " t_1

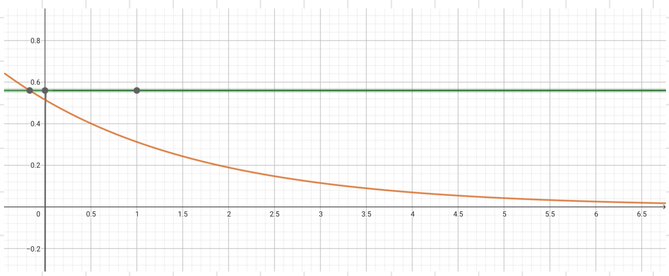
$$= 0.56\left(1 - e^{-\frac{t}{2}}\right)A$$



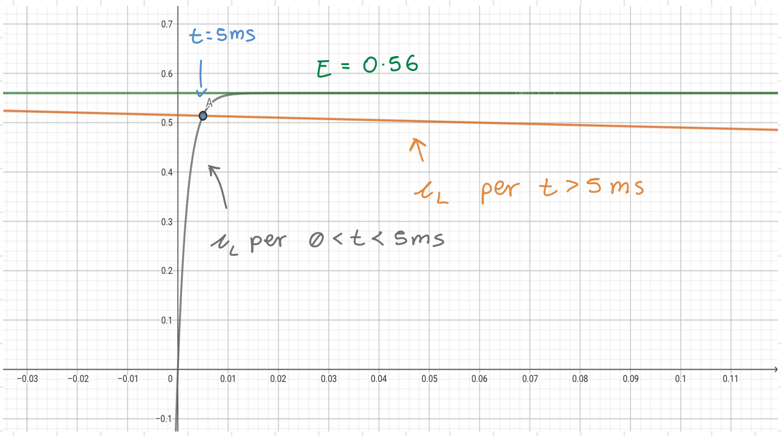
$\boxed{t>5}$ $\quad$ Nuova condizione iniziale: $\quad i_L(t_1) = 0.56\left(1 - e^{\frac{1}{2\cancel{ms}} t_1 \cancel{ms}}\right) = 0.56\left(1 - e^{\frac{5}{2}}\right)A$

$$= 0.514\,A = I_1$$

$$\Rightarrow \quad i_L(t) = K_2\, e^{-(t-t_1)/\tau} = 0.514\, e^{-(t-5ms)/2ms}$$



Espando l'asse t $\rightarrow$



DATI

$R_1 = R_2 = 1\,k\Omega$ (A, B)

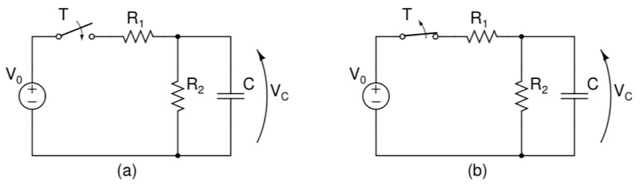
C: $C = 0.1\,\mu F$

X: $V_0 = 5V$

Teoria dei circuiti – Esercitazione di Laboratorio

Transitori e dominio dei fasori

Esercizio 1



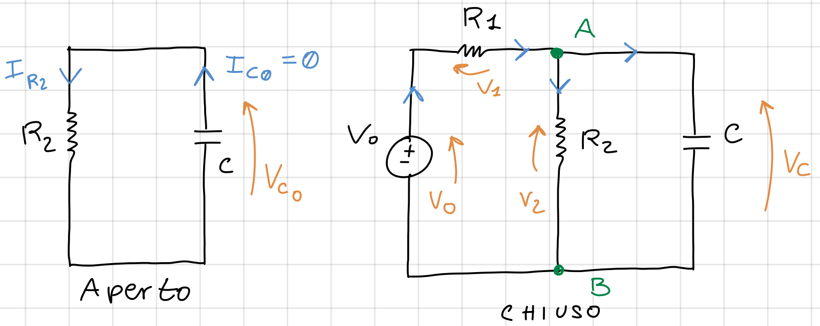
Con riferimento al circuito di figura, si assumano i seguenti valori:
$R_1 = R_2 = 1\,\text{k}\Omega$, $C = 0.1\,\mu\text{F}$, $V_0 = 5\,\text{V}$.
Determinare l'andamento della tensione $V_C(t)$ nei due casi seguenti:

(1) • per $t < t_0 = 0\,\text{s}$ l'interruttore T è aperto ed il circuito è a regime. All'istante $t = t_0$ l'interruttore T si chiude;

(2) • per $t < t_0 = 0\,\text{s}$ l'interruttore T è chiuso ed il circuito è a regime. All'istante $t = t_0$ l'interruttore T si apre.

CASO (a) T Aperto → Chiuso

$\boxed{t < t_0}$ con $t_0 = 0s$

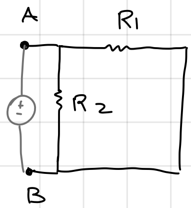


Hp $I_C = 0 \rightarrow I_{R_2} = 0$

$\Rightarrow V = R_2 \cdot I_{R_2} = 0$

$t_0 = 0 \rightarrow \boxed{t > t_0}$ (1)

• Trovo thevenin (condensatore → th)

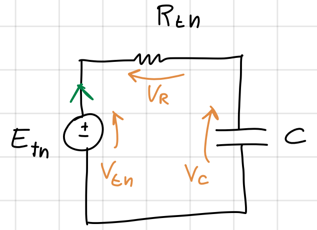


• $R_{th} = R_1 // R_2 = 500\,\Omega$ R_{th}

• $V_{AB} = V_{R_2} = V_C$

Part Tens: $V_{th} = V_0 \cdot \dfrac{R_2}{R_2 + R_1} = 2.5\,V$ E_{th}

$\Rightarrow$



• Trovo l'eq diff del circuito eq

$$\begin{cases} -e_{th}(t) + V_R + V_C = 0 & \text{LKT}_M \\ V_R = R_{th} \cdot i & \text{Res} \\ i(t) = C\,\dfrac{dv}{dt} & \text{Cond} \end{cases}$$

$\Rightarrow -e(t) + R_{th}\,i + V_c = 0 \rightarrow R_{th} \cdot C \cdot \dfrac{dV_c}{dt} + v_c = e(t)$

$\Rightarrow RC\,\dot{v} + v = e(t)$

$v_c \sim K e^{\lambda t} \to \dot{v}_c = K\lambda e^{\lambda t} \to RC\cancel{K}\lambda \cancel{e^{\lambda t}} + \cancel{K}\cancel{e^{\lambda t}} = 0$

$\to \lambda RC + 1 = 0 \to \lambda = -\frac{1}{RC}$

$\Rightarrow v_c(t) = v_{c0}(t) + v_{cp}(t)$

$\to v_{c0}(t) \sim K e^{\lambda t} \Rightarrow v_{c0}(t) = K e^{-\frac{t}{RC}}$

$v_{cp}(t) \sim e(t) \Rightarrow V_0 = 5V = \text{cost} = E_0$

$\Rightarrow v_{cp}(t) = E_0 \qquad \Rightarrow \underline{v_c(t) = K e^{-\frac{t}{RC}} + E_0}$

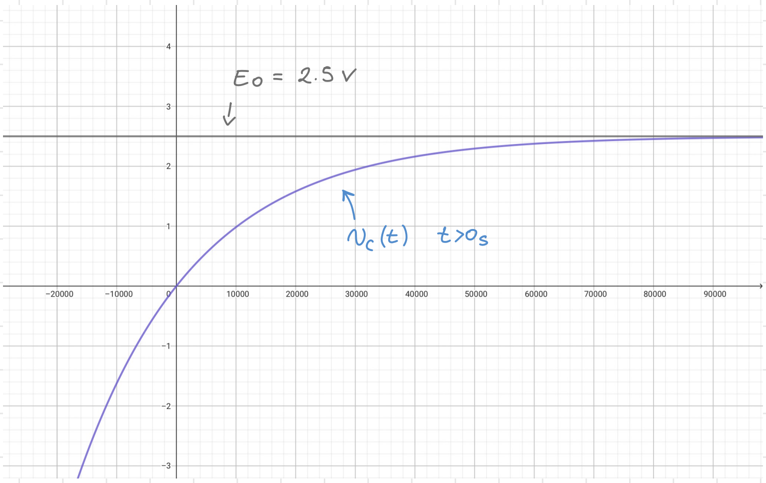
Cond Iniz

$$\begin{cases} v_c(t) = K e^{-\frac{t}{RC}} + E_0 \\ v_c(0^+) = 0 \end{cases} \Rightarrow v_c(0) = K e^{-\frac{0}{RC}} + E_0 = 0$$

$\to K = -E_0$

$\Rightarrow$ SOL PART $\to v = -E_0 e^{-\frac{t}{RC}} + E_0 \Rightarrow v_c(t) = E_0\left(1 - e^{-\frac{t}{RC}}\right) = E_0\left(1 - e^{20\times10^{3}\cdot t}\right)$

$$\Rightarrow \begin{cases} v_c(t) = 0 & t < 0\,s \\ v_c(t) = 2.5 \cdot \left(1 - e^{20\times10^{3}\cdot t}\right) & t > 0\,s \end{cases}$$

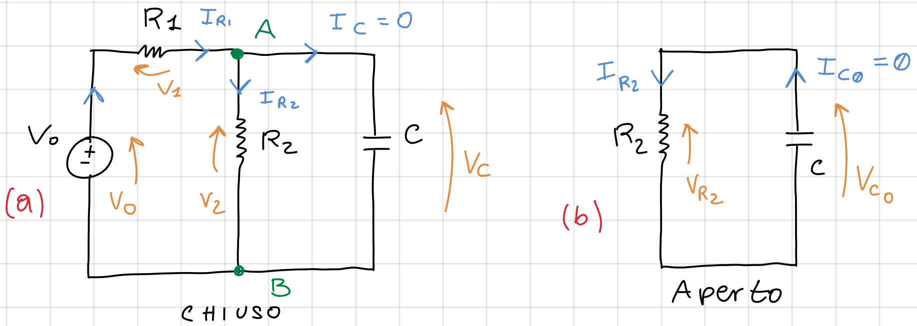


Costante di tempo

$\tau = \frac{1}{\lambda} = \frac{1}{1/RC} = RC = 5 \times 10^{-5}$

$\to \tau = 50 \times 10^{-6}\,s = 50\,\mu s$

(2) L'interruttore si apre



$\boxed{t < t_0}$ (a)

Hp $I_C = \emptyset$ $\Rightarrow$ LKC_A: $I_{R_2} + \cancel{I_C}^{\emptyset} = I_{R_1}$ $\Rightarrow$ $I_{R_1} = I_{R_2}$

$\rightarrow$ calcolo V_C A REGIME (dal circuito a)

$$V_{C\emptyset} = V_0 \cdot \frac{R_2}{R_2 + R_1} = 2.5\ V$$ $V_{C\emptyset}$

$\boxed{t > t_0}$ (b)

- Scrivo le eq del circuito

$$\begin{cases} LKT_M: & -V_{R_2} + V_{C\emptyset} = \emptyset \\ CAR: & V_{R_2} = R_2 \cdot i_2 \\ CAR: & i_C = C \dfrac{dV_{C0}}{dt} \end{cases} \Rightarrow -R \cdot C \cdot \frac{dv}{dt} + v = \emptyset$$

- Derivata

Hp. $v_c(t) \sim K e^{\lambda t}$ $\rightarrow$ $\dot{v}(t) = K\lambda e^{\lambda t}$

- Sostituisco $\rightarrow$ Frequenza

$$-RC \cdot \lambda \cancel{K e^{\lambda t}} + \cancel{K e^{\lambda t}} = \emptyset \rightarrow -RC\lambda + 1 = 0 \rightarrow \lambda = \frac{-1}{-RC} = \frac{1}{RC}$$ Freq

- Soluzione iniziale

$$v_c = v_{c\emptyset}(t) + v_e(t) \rightarrow \begin{cases} v_c(0) = K e^{-\frac{t}{RC}} + v_c(t) \\ v_c(\emptyset) = 2.5 \end{cases}$$

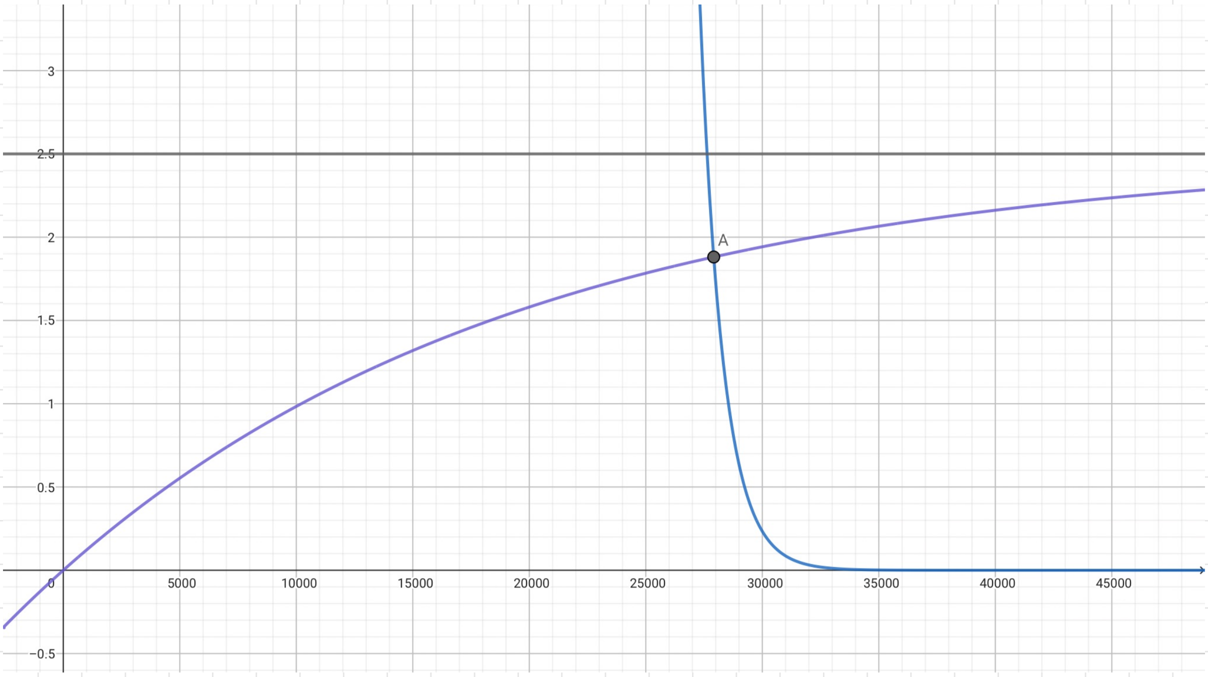
$$\Rightarrow K e^{-\frac{\emptyset}{RC}} + v_c(t) = 2.5 \rightarrow K = 2.5 - v_c(t)$$ K

- Metto insieme

$$v_c(t) = [2.5 - \cancel{v_c(t)}^{\emptyset}] \cdot e^{-\frac{t}{RC}} + \cancel{v_c(t)}^{\emptyset} = 2.5\, e^{-\frac{t}{RC}}$$

$v_c(t)$ $t > t_0$

$* \ \frac{1}{RC} = 10 \times 10^3$

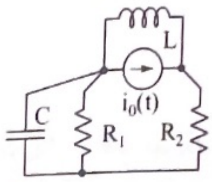


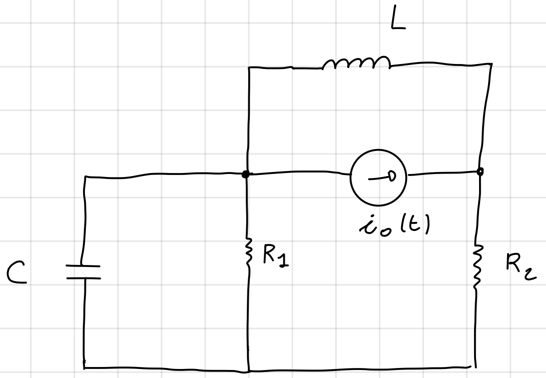
L'immagine è illustrativa, in ealtà il secondo transitorio (blu) è molto più ripido del primo (viola)

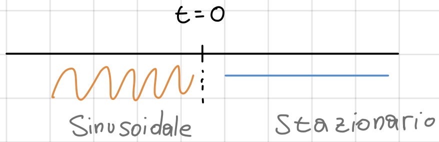
Esercizio 1 - *obbligatorio per tutti*

La rete in figura è in regime sinusoidale per $t < 0$. All'istante $t = 0$ il generatore diventa stazionario. Determinare la corrente nell'induttore L per ogni t. Dati: $R_1 = 170\Omega$, $R_2 = 340\Omega$, $L = 0.4$H, $C = 33\mu$F,

$$i_0(t) = \begin{cases} 0.8\cos(100t)\ A & t < 0 \\ 0.8\ A & t > 0 \end{cases}$$







$$i_0(t) = \begin{cases} 0.8 \ \cos(100t) A & t<0 \\ 0.8 \ A & t>0 \end{cases}$$

(A) $\boxed{t<0}$ Sinusoidale

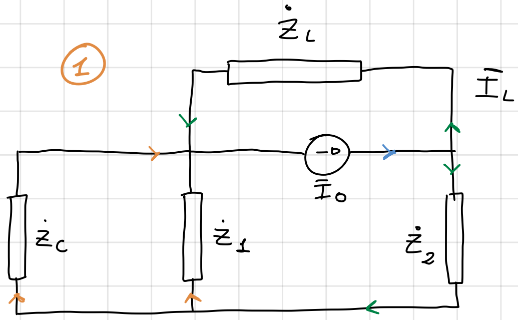
(1) Impedenze e circuito eq

$R_1 = 170\ \Omega$ → A $\dot{Z}_1 = 170\ \Omega$

$R_2 = 340\ \Omega$ → B $\dot{Z}_2 = 340\Omega$

$L = 0.4\ H$ → D $\dot{Z}_L = j\ 100 \cdot 0.4\ H = 40j\ H$

$C = 33\mu F$ → C $\dot{Z}_c = -\frac{j}{100 \cdot 33 \times 10^{-6}} F = -303.03j\ F$



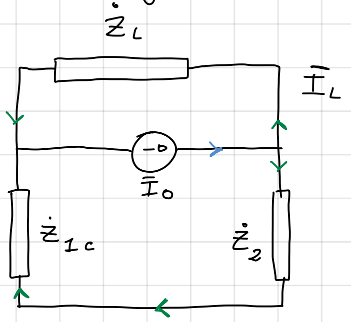
$i_0(t) = 0.8\ \cos(100t)\ A \rightleftarrows 0.8 \cdot e^{0} = \bar{I}_0 = 0.8\ A$

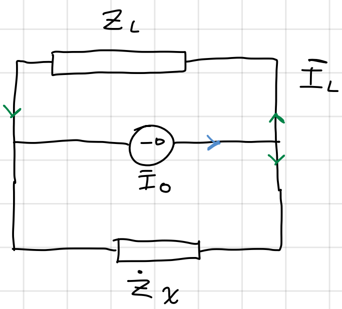
(2) Trovare $\bar{I}_L$

Semplifico il circuito

$\dot{Z}_{1C} = \dot{Z}_1 \parallel \dot{Z}_c = 128.12 - 24.89j$

$\dot{Z}_x = \dot{Z}_{1C} + \dot{Z}_2 = 468.12 - 24.89j$

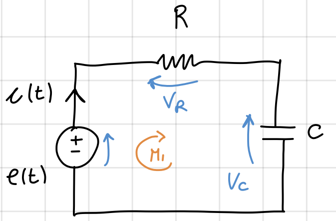




$\Rightarrow \bar{I}_L = \bar{I}_0 \cdot \frac{\dot{Z}_x}{\dot{Z}_x + \dot{Z}_L} = 0.798 - 0.068j\ A$

X D

Esercizio visto nella lez 25



$e(t)$ varia nel tempo $\rightarrow$ $e(t) = \begin{cases} 0 & t<0 \\ E_0 & 0<t<t_1 \\ 0 & t>t_1 \end{cases}$

DATI

$R = 10\ \Omega$ $\qquad t_1 = 20\ ms = 2\times10^{-3}\ s$

$C = 1\ mF$

$E_0 = 10V$

$\boxed{t<0}$

C.I. (A)

$e(t) = 0 \Rightarrow \boxed{i_c(0^-) = 0} \Rightarrow V_R = 0 \quad V_c = 0 \rightsquigarrow V_c(t) = 0 \rightarrow V_c(0^-) = 0$

$\boxed{0<t<t_1}$ $\quad e(t) = E_0 = 10V$

la C.I. dell'intervallo è quella finale dell'int. prec: $V_c(0^+) = V_c(0^-) = 0\ V$

DALLE LKM$_1$: $\underbrace{-e(t) + V_R + V_c = 0}_{(1)} \rightarrow i_c(t)R + V_c(t) = e(t)$

C.I. (B)

$\Rightarrow i_c(t) = \dfrac{e(t) - V_c(t)}{R}\ \forall t \rightarrow$ nell'intervallo corrente $\rightarrow i_c(0^+) = \dfrac{\overset{10}{E_0} - \overset{0}{V_c(0^+)}}{R} \Rightarrow \boxed{i_c(0^+) = \dfrac{E_0}{R}}$

▲ Determinare $V_c(t)$ $\rightarrow$ dalla (1)

$\rightarrow V_c = e(t) - V_R$ ma $\begin{cases} i_c(t) = C \cdot \dfrac{dV_c(t)}{dt} \\ V_R = \underset{i_c(t)}{i(t)} \cdot R \end{cases} \Rightarrow V_c = e(t) - R\cdot C\cdot \dot{V}_c$

(2)

$\Rightarrow \boxed{RC\dot{V}_c + V_c - e(t) = 0}$ eq diff lin 1° ordine

ha sol $V_c(t) = V_{c0} + V_{cp}$ con $\underset{(a)}{V_{c0}(t) \propto Ke^{\lambda t}} \Rightarrow \underset{(b)}{\dot{V}_{c0}(t) = K\lambda e^{\lambda t}}$

- Trovo $V_{c0}(t)$

dalla (2)

$\rightarrow$ Associata $\rightarrow RC\dot{V}_c + V_c = 0 \rightarrow$ Sostituisco (a) (b) $\rightarrow RC\cancel{K}\lambda \cancel{e^{\lambda t}} + \cancel{K}\cancel{e^{\lambda t}} = 0$

$\rightarrow RC\lambda + 1 = 0 \rightarrow \lambda = -\dfrac{1}{RC}$ Freq. Naturale

Intervallo considerato

- trovo $V_{cp}(t)$ Soluzione di regime $\rightarrow V_{cp}(t) \propto e(t)$

$\Rightarrow V_{cp}(t) = E_0$

- Metto insieme

$V_c(t) = V_{c0} + V_{cp} = \underbrace{Ke^{\lambda t}}_{V_{c0}} + \underbrace{E_0}_{V_{cp}} = Ke^{\overset{\lambda}{-\frac{t}{RC}}} + E_0$

- Trovo la Sol COMPLETA → trovo K → Problema di Cauchy

Siccome $v_C(t) = K e^{-\frac{t}{RC}} + E_0$

e C.I. $(0<t<t_1)$ = Sol regime $(t<0) \Rightarrow v_C(0^+) = v_C(0^-) = 0$

$$\Rightarrow v_C(0) = K e^{-\frac{0}{RC}} + E_0 = 0 \rightarrow K = -E_0$$ K

- Metto Tutto insieme

$$v_C(t) = v_{C0} + v_{Cp} = K e^{\lambda t} + E_0 = K e^{-\frac{t}{RC}} + E_0 = -E_0 e^{-\frac{t}{RC}} + E_0$$

$$\Rightarrow \boxed{v_C(t) = E_0 \left(1 - e^{-\frac{t}{RC}}\right)}$$ $v_C(t)$ per $0<t<t_1$

▲ Determinare $i_C(t)$ per $0<t<t_1$

(1) $v_C + v_R = e(t) \rightarrow v_C = e(t) - R \cdot i(t) \Rightarrow \frac{dv}{dt} = \frac{d}{dt}(E_0 - Ri)$

ma $i_C(t) = C\frac{dv}{dt}$

const

$$i_C(t) = C\frac{d}{dt}(\cancel{E_0} - Ri) \Rightarrow \boxed{i_C(t) = -RC\,\dot{i}(t)}$$

Eq Diff (2)

- Risolvo la (2)

$RC\,\dot{i} + i = 0$ $\quad i_{C0}(t) \propto K e^{\lambda t} \rightarrow \dot{i}_{C0}(t) \propto \lambda K e^{\lambda t}$ $\quad$ e $\quad i(t) = i_{C0} + i_{Cp}$

C.I. (A) ↑

ma $i_{Cp} \propto i_0(t) = 0 \quad \Rightarrow i(t) = i_{C0}$

$\Rightarrow RC\lambda \cancel{K e^{\lambda t}} + \cancel{K e^{\lambda t}} = 0 \rightarrow RC\lambda + 1 = 0 \rightarrow \lambda = -\frac{1}{RC}$ (2)

Freq per i = Freq per v

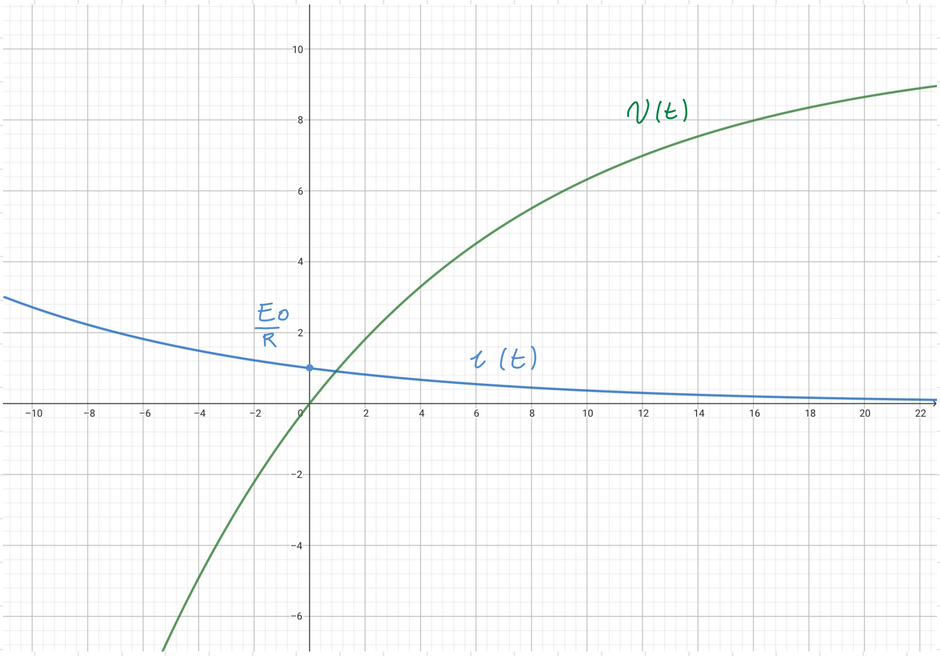
$$\rightarrow i(t) = K e^{-\frac{t}{RC}}$$

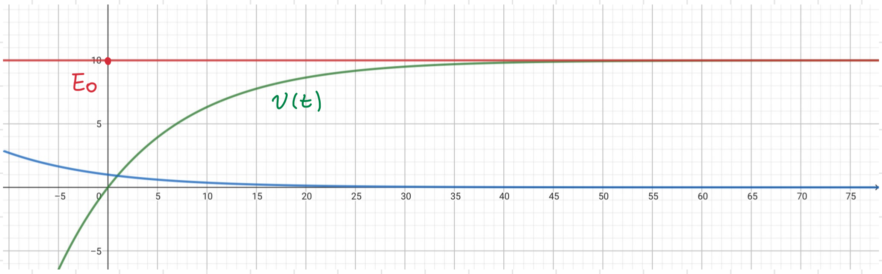
- Trovo K → Cauchy

$$\begin{cases} i(t) = K e^{-\frac{t}{RC}} \\ i(0) = \frac{E_0}{R} = 1 \end{cases} \Rightarrow i(0) = K e^{-\frac{0}{RC}} = \frac{E_0}{R} \Rightarrow K = \frac{E_0}{R}$$

K per i

$$\Rightarrow \boxed{i(t) = \frac{E_0}{R} e^{-\frac{t}{RC}}}$$ $i_C(t)$ per $0<t<t_1$





$\boxed{t > t_1}$

- C.I.

$$v_c(t_1^-) = E_0\left(1 - e^{-\frac{t_1}{RC}}\right) = 8.65\ V = V_1 \Rightarrow v_c(t_1^-) = v_c(t_1^+) = V_1$$

$t_1 = 20ms$

$$i_c(t_1^-) = \frac{E_0}{R} e^{-\frac{t}{RC}} = 0.135\ A$$

Non so perché al prof si trova 0.865 A

$$-e(t) + i(t)R + V_c = 0 \rightarrow i(t_1^+) = \frac{e(t) - V_c(t_1^+)}{R} = \frac{0 - 8.65}{10} = -0.865\ A$$

gen spento

i_1^+

$\rightsquigarrow$ Soluzione $\rightarrow v_{c0}(t) \propto K_2 e^{\lambda t} \rightarrow \dot{v}_{c0}(t) \propto \lambda K_2 e^{\lambda t} \rightsquigarrow \lambda = -\frac{1}{RC}$

dalla (2)

$$v(t) = v_{c0} + \cancel{v_{cp}} = v_{c0}$$

gen spento

$\Rightarrow$ Trovo K $\quad v_c(t) = K_2 e^{-\frac{t}{RC}}$

TENSIONE

$$\begin{cases} v_c(t) = K_2 e^{-\frac{t}{RC}} \\ v_c(t_1^+) = 8.65\ V \end{cases} \Rightarrow v_c(20ms) = K_2 e^{-\frac{20ms}{2}} = 8.65\ V \Rightarrow K_2 = V_1 e^{+\frac{t_1}{RC}} = 63.92\ V$$

-2

$$\boxed{v_C(t) = V_1 e^{+\frac{t_1}{RC}} \cdot e^{-\frac{t}{RC}}}$$ $v_C(t)$ per $t > t_1$

$\hookrightarrow \quad v_C(t) = V_1 \, e^{(t_1 - t)\frac{1}{RC}}$

$\rightsquigarrow$ Soluzione $\rightarrow i_{C0}(t) \propto K_3 e^{\lambda t} \rightarrow \dot{i}_{C0}(t) = K_3 \lambda e^{\lambda t} \rightsquigarrow \lambda = -\frac{1}{RC}$

dalla (2)

$\rightarrow i_C(t) = i_{C0} + i_{Cp} = i_{C0} \quad \rightarrow \quad i_C(t) = K_3 \, e^{-\frac{t}{RC}}$

- Trovo $K_3 \rightsquigarrow$ C.I : $i(t_1^+) = -0.865\,A$

$$\begin{cases} i_C(t) = K_3 \, e^{-\frac{t}{RC}} \\ i_C(t_1^+) = -0.865 \end{cases} \Rightarrow i_C(20ms) = K_3 \, e^{-\frac{t_1}{RC}} = -0.865 \rightarrow K_3 = -0.865 \cdot e^{\frac{t_1}{RC}} = -6.39$$

$$\Rightarrow \quad i_C(t) = -0.865 e^{\frac{t_1}{RC}} \cdot e^{\frac{t}{RC}} \Rightarrow \quad \boxed{i_C(t) = -0.865 \cdot e^{(t - t_1)\frac{1}{RC}}}$$ $i_C(t)$ per $t > t_1$