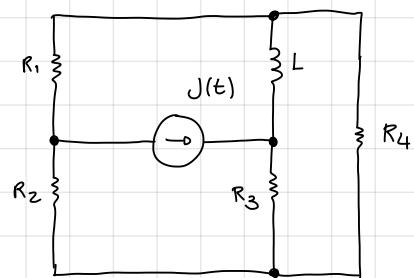
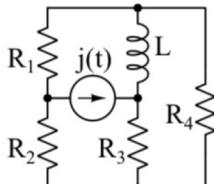


Esercizio 1 (Prova D'esame 26/2/20)

Il circuito è a riposo per $t < 0$. Determinare la corrente che scorre nell'induttore in ogni istante di tempo.
Dati: $R_1 = 120 \Omega$, $R_2 = 60 \Omega$, $R_3 = 110 \Omega$, $R_4 = 180 \Omega$, $L = 0.4 \text{ H}$,

$$j(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 0.8 \text{ A} & 0 < t < 5\text{ms} \\ 0 \text{ A} & t > 5\text{ms} \end{cases}$$



DATI

$$A \quad R_1 = 120 \Omega$$

$$B \quad R_2 = 60 \Omega$$

$$C \quad R_3 = 110 \Omega$$

$$D \quad R_4 = 180 \Omega$$

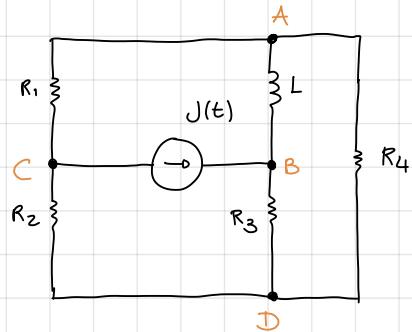
$$L \quad L = 0.4 \text{ H}$$

$$j(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 0.8 \text{ A} & 0 < t < 5 \text{ ms} \\ 0 \text{ A} & t > 5 \text{ ms} \end{cases}$$

$t < 0 \rightarrow T_1$ Stazionario

per $T_1 \rightarrow j(t) = 0 \text{ A} \Rightarrow v_L = 0$ Ans 1

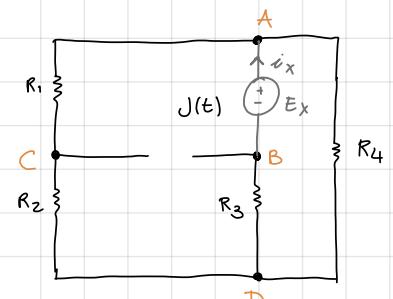
$0 < t < 5 \text{ ms} \rightarrow T_2$ Dinamico



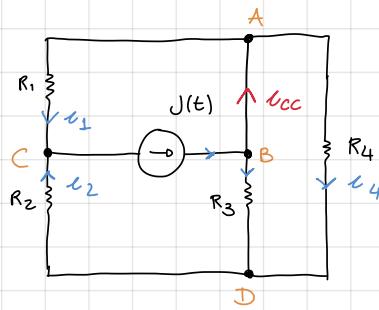
(1) NORTON_(AB)

$$\begin{aligned} R_{eqAB} &= [(R_1 + R_2) // R_4] + R_3 \\ &= 200 \Omega \end{aligned}$$

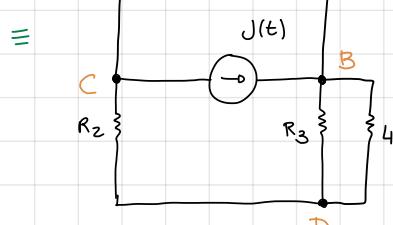
(a) R_{eqAB}



(b) Correnti di C.C. $\rightarrow I_{CC} = ?$



$$LKC_B: \quad I_{CC} + I_3 - j(t) = 0 \rightarrow I_{CC} = j(t) - I_3$$

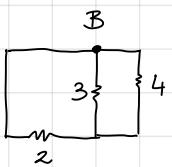


Ci serve scrivere prima l'equazione perché tra poco non vedremo più Icc (visto che A=B e scompare il c.c.)

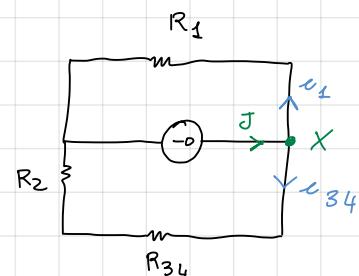
$$\rightarrow R_{234} = (R_3 // R_4) = 68.27 \Omega$$

$$\Rightarrow I_1 = j(T_e) \cdot \frac{R_{234}}{R_{234} + R_1} = 0.413 \text{ A}$$

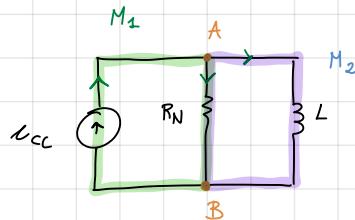
$$LKC_X: \quad I_{34} = j(t) \cdot \frac{R_1}{R_1 + R_{34} + R_2} = j(t) \cdot 0.48 \text{ A}$$



$$\rightarrow I_3 = j(t) \cdot 0.48 \cdot \frac{R_4}{R_4 + R_3} = j(t) \cdot 0.298 \text{ A}$$



$$\Rightarrow \iota_{CC} = J(t) - \iota_3 = J(t) (1 - 0.298) = 0.561 A$$



RELAZIONI CARATT -> EQ STATO

$$V = R \cdot \iota \Rightarrow \iota_{RN} = \frac{V_{AB}}{R_N} \text{ ma } LK T_{M_1} \cdot -V_{CC} + V_{AB} = 0 \Rightarrow V_{AB} = V_{CC}$$

$$\text{ma } LK T_{M_2} : -V_{AB} + V_L = 0 \Rightarrow V_{AB} = V_L \Rightarrow \iota_R = \frac{V_L}{R_N}$$

$$LK C_A : -\iota_{CC} + \iota_R + \iota_L = 0 \Rightarrow \iota_{CC} = \frac{V_{CC}}{R_N} + \iota_L = 0 \Rightarrow \iota_{CC} = \frac{V_L}{R_N} + \iota_L = 0$$

$$\text{Inoltre } V_L = L \cdot \frac{d\iota_L}{dt}$$

$$\Rightarrow \frac{L}{R} \dot{\iota}_L + \iota_L = \iota_{CC}$$

$$\frac{L}{R} \lambda + 1 = 0 \Rightarrow \lambda = -\frac{R}{L}$$

$$\text{Sappiamo che } \iota(t) \propto C_1 e^{\lambda t} \Rightarrow \dot{\iota}(t) = C_1 \lambda e^{\lambda t}$$

mentre la soluzione è del tipo $\iota_L(t) = \iota_{C_0}(t) + \iota_{ep}(t)$

$$\text{Trovo } \iota_{C_0}(t) \propto C_1 e^{-\frac{R}{L}t} = C_1 e^{-\frac{R}{L}t}$$

FORZAMENTO $\rightarrow J(T_1)$

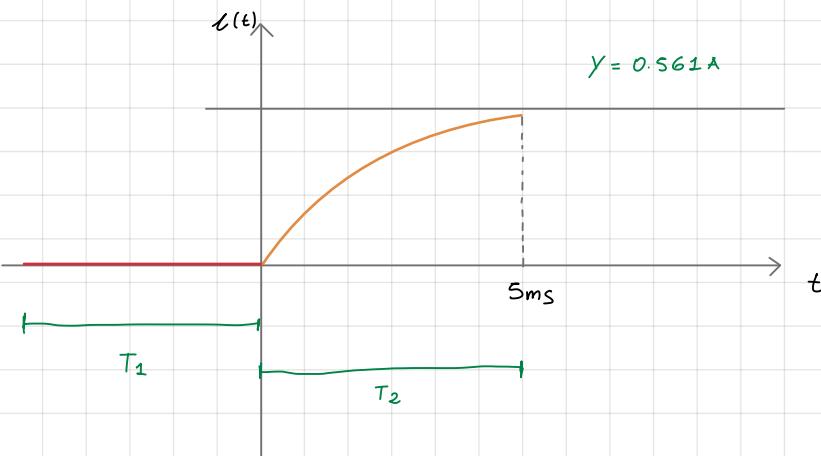
$$\begin{cases} \iota_L(t) = C_1 e^{-\frac{R}{L}t} + \iota_{CC} \\ \iota_L(0^+) = 0 \end{cases} \Rightarrow \iota_L(0^+) = C_1 e^{-\frac{R}{L}0^+} + \iota_{CC} = 0 \Rightarrow C_1 = -\iota_{CC} = -0.561 A$$

$$\Rightarrow \iota_L(T_1) = -0.561 e^{-\frac{R}{L}T_1} + 0.561 \Rightarrow \iota_L(T_1) = 0.5149$$

\rightarrow Funzione Tensione per

$0 < t < 5 \text{ ms}$

$$\iota_L(t) = 0.561 \left(1 - e^{-\frac{R}{L}t} \right)$$



$t > 5 \text{ ms}$

$t \rightarrow \infty$

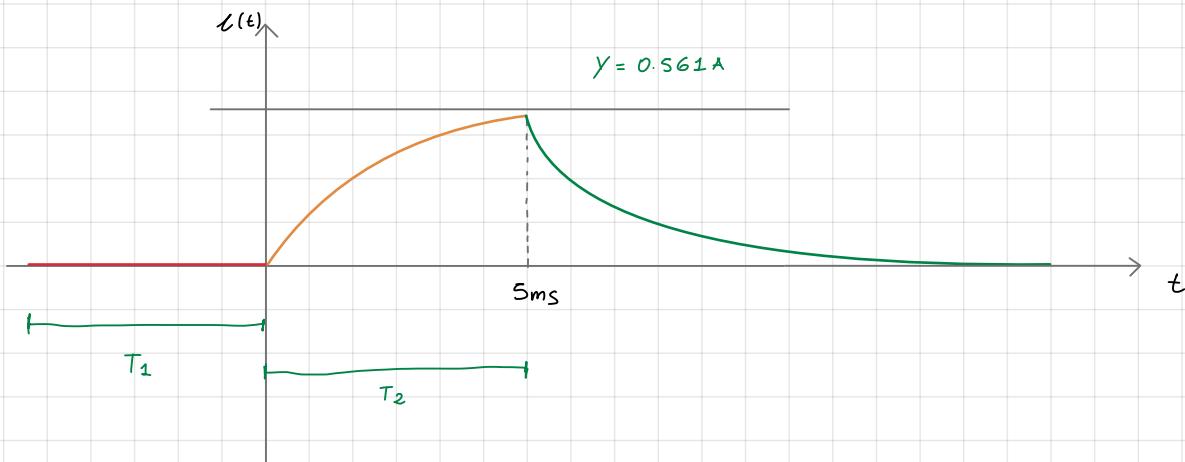
$$v_C(T_3) = v_{C0}(t) + v_{CP}(t) = v_{C0}(t) \propto C_2 e^{\lambda t}$$

$$\text{ma } \lambda = -\frac{R}{L} \rightarrow v_{C0} = C_2 e^{-\frac{R}{L} t}$$

$$\text{Troro } C_2 \rightarrow \text{C.I.} \rightarrow i_L(t_1^+) = 0.515 \text{ A}$$

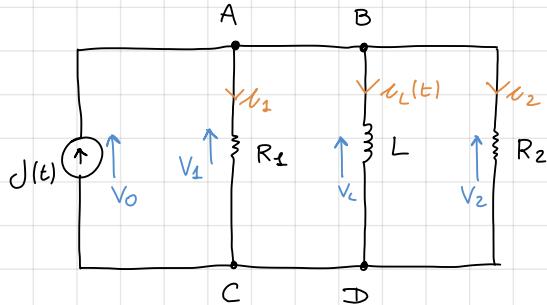
$$\begin{aligned} & \left. \begin{aligned} i_L(t) &= C_2 e^{-\frac{R}{L} t} \\ i_L(5\text{ms}) &= 0.515 \text{ A} \end{aligned} \right\} \rightarrow i_L(5\text{ms}) = C_2 \cdot e^{-\frac{R}{L} \cdot 5\text{ms}} = 0.515 \rightarrow C_2 = 0.515 \cdot e^{\frac{R}{L} \cdot 5\text{ms}} \end{aligned}$$

$$\Rightarrow i_{C0} = 0.515 e^{\frac{R}{L} t_1} \cdot e^{-\frac{R}{L} t} = 0.515 e^{-\frac{R}{L} (t-t_1)}$$



$$60 : t = 100 : x$$

$$\frac{60}{t} = \frac{100}{x} \rightarrow x = 100 \cdot \frac{t}{60}$$



DATI

A $R_1 = 100 \Omega$

B $R_2 = 4\pi \Omega$

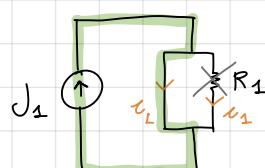
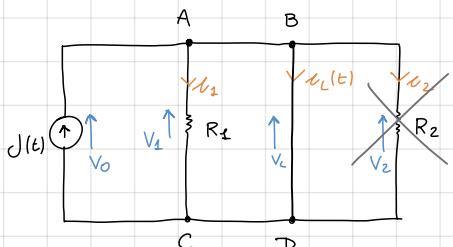
$$J(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 0.4 \text{ A} & t > 0 \end{cases}$$

Q: $\mathcal{V}_L(t) = ? \quad \forall t$ $t < 0$

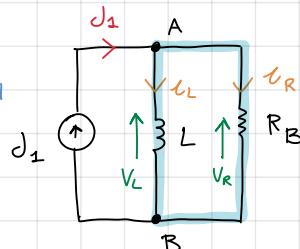
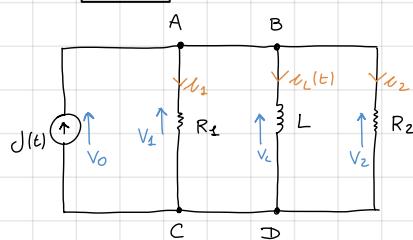
Se $J(t) = 0 \Rightarrow$

$\mathcal{V}_L(0) = 0 \text{ A}$

$\mathcal{V}_L(0) = 0 \text{ V}$

 $t \rightarrow \infty$ 

$\Rightarrow \mathcal{V}_L(\infty) = J_1 = 0.4 \text{ A}$

 $t > 0$ 

Con $R_B = R_1 \parallel R_2 = 31.97 \Omega$

$-\mathcal{V}_L + \mathcal{V}_R = 0 \Rightarrow \mathcal{V}_L = \mathcal{V}_R$

$-J_1 + \mathcal{I}_L + \mathcal{I}_R = 0 \Rightarrow \mathcal{I}_R = J_1 - \mathcal{I}_L$

$\mathcal{V}_R = \mathcal{I}_R \cdot R_B$

$\mathcal{V}_L = L \cdot \frac{d\mathcal{I}_L}{dt}$

$\Rightarrow \mathcal{V}_R = L \dot{\mathcal{I}}_L \Rightarrow \mathcal{I}_R R_B = L \dot{\mathcal{I}}_L$

$\Rightarrow R_B (J_1 - \mathcal{I}_L) = L \dot{\mathcal{I}}_L$

$\Rightarrow \dot{\mathcal{I}}_L + \left(\frac{R_B}{L}\right) \mathcal{I}_L = \left(\frac{R_B}{L}\right) J_1$

Pongo $\tau = \frac{L}{R_B} \Rightarrow \dot{\mathcal{I}}_L + \frac{\mathcal{I}_L}{\tau} = \frac{J_1}{\tau}$

Ossigeno associato: $\lambda + \frac{R_B}{L} = 0 \Rightarrow \lambda = -\frac{R_B}{L}$ Risolvo l'eq diff \Rightarrow separazione var

$\frac{d\mathcal{I}_L}{dt} = -\frac{R_B}{L} \mathcal{I}_L \Rightarrow \ln(\mathcal{I}_L)$

$\int \frac{d\mathcal{I}_L}{dt} = -\frac{R_B}{L} \int \dot{\mathcal{I}}_L \Rightarrow \int d\mathcal{I}_L = -\frac{R_B}{L} \mathcal{I}_L \int dt$

$\left(-\frac{R_B}{L} t + \kappa\right) = -\frac{R_B}{L} t + \kappa \Rightarrow \mathcal{I}_L(t) = e^{-\frac{R_B}{L} t} \cdot e^{\kappa}$

$\Rightarrow \int \frac{d\mathcal{I}_L}{\mathcal{I}_L} = -\frac{R_B}{L} t + \kappa$

$\Rightarrow \ln(\mathcal{I}_L) = -\frac{R_B}{L} t + \kappa \Rightarrow \mathcal{I}_L(t) = e^{-\frac{R_B}{L} t} \cdot e^{\kappa}$

-> Sol generale + particolare

$$\mathcal{L}_L(t) = \mathcal{L}_{L0}(t) + \mathcal{L}_{LP}(t) = \hat{K} e^{-\frac{R_B}{L} t} + 0.4$$

-> Trovo K -> Cond iniziali

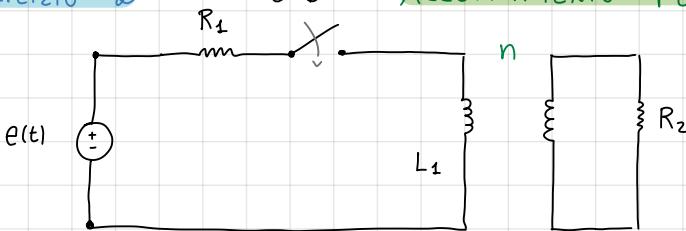
$$\mathcal{L}_L(0^-) = 0 \quad \rightarrow \quad \text{Continuità delle} \quad \text{grandezze di stato} \quad \rightarrow \quad \mathcal{L}_L(0^+) = 0$$

$$\begin{cases} \mathcal{L}_L(t) = \hat{K} e^{-\frac{R_B}{L} t} + 0.4 \\ \mathcal{L}_L(0^+) = 0 \end{cases} \rightarrow \mathcal{L}_L(0) = \hat{K} e^{-\frac{R_B}{L} 0} + 0.4 = 0 \rightarrow \hat{K} = -0.4$$

-> SOLUZIONE

$$\mathcal{L}_L(t) = -0.4 e^{-\frac{R_B}{L} t} + 0.4 = 0.4 \left(1 - e^{-\frac{R_B}{L} t} \right)$$

Esercizio 2



ACCOPIAMENTO PERFETTO

DATI

$$e(t) = E = 20 \text{ V}$$

$$A \quad R_1 = 10 \Omega \quad B \quad R_2 = 0.2 \Omega$$

$$C \quad L_1 = 50 \text{ mH} \quad D \quad L_2 = 2 \text{ mH}$$

Mutua induzione

↓

$$M = 10 \text{ mH}$$

E

$$Q_1: V_{R_2} \quad \text{Per } t > 0$$

$$Q_2: P_E \quad \text{A regime}$$

$$K \triangleq \frac{M}{\sqrt{L_1 L_2}} = 1 \quad \rightarrow \text{Accoppiamento Perfetto} \Leftrightarrow M = \sqrt{L_1 \cdot L_2}$$

$$\rightarrow 10 \text{ mH} = \sqrt{50 \text{ mH} \cdot 2 \text{ mH}} \Rightarrow 10 \text{ mH} = \sqrt{100 \text{ mH}} \rightarrow \text{A.P. } \checkmark$$

$$\text{Se A.P.} \rightarrow n = \frac{M}{L_1} = \frac{L_2}{M} = (5)^n$$

Rapporto di Conversione

$t < 0$

Circuito a riposo $\rightarrow V_{R_2} = 0$



N.B. per $t < 0$ il circuito è Aperto

$$\mathcal{L}_L(0^-) = \emptyset$$

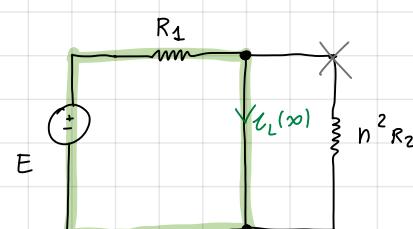
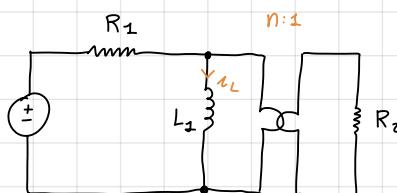
$$Z_1 = Z_2 n^2$$

$t \rightarrow \infty$

Usiamo la FORMULA

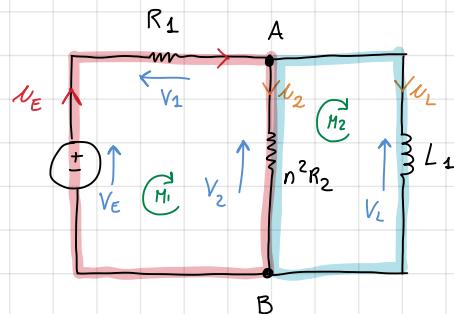
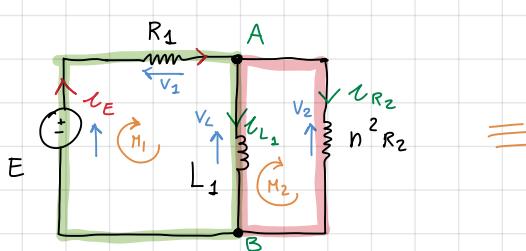
DEL TRASPORTO DI IMPEDENZA PRIMARIO

per trasformare...



$$\rightarrow I_E(\infty) = \frac{E}{R_1} = 2 \text{ A} \Rightarrow P_E(\infty) = E \cdot I_E = 20 \cdot 2 = 40 \text{ W} \quad \text{Ans 1}$$

$t > 0$



$$\left\{ \begin{array}{l} -V_E + V_1 + V_2 = 0 \\ -V_2 + V_L = 0 \\ -V_E + \dot{I}_L + V_L = 0 \\ V_L = L \dot{I}_L \end{array} \right. \Rightarrow \left\{ \begin{array}{l} -E + \nu_E R_1 + \nu_2 \cdot n^2 R_2 = 0 \\ -\nu_2 \cdot n^2 R_2 + L \dot{I}_L = 0 \\ \nu_E = \nu_2 + \nu_L \end{array} \right. \Rightarrow \nu_2 = \frac{L \dot{I}_L}{n^2 R_2}$$

$$= 0 \quad -E + \left(\frac{L \dot{I}_L}{n^2 R_2} + \nu_L \right) R_1 + \frac{L \dot{I}_L}{n^2} = 0 \quad \Rightarrow \quad -E + \frac{L R_1}{n^2 R_2} \dot{I}_L + \nu_L R_1 + L \dot{I}_L = 0$$

Eq Diff

$$\Rightarrow L_1 \left(\frac{R_1}{n^2 R_2} + 1 \right) \dot{I}_L + \nu_L R_1 = E$$

$$\Rightarrow \dot{I}_L + \frac{\nu_L R_1}{L_1 \left(\frac{R_1}{n^2 R_2} + 1 \right)} = 0$$

$$\Rightarrow \lambda = \frac{\nu_L R_1}{\frac{L_1 R_1}{n^2 R_2} + L_1} = 0 \quad \Rightarrow \quad \lambda = \frac{\nu_L R_1}{\frac{L_2 R_1 + L_1 n^2 R_2}{n^2 R_2}} \quad \Rightarrow \quad \lambda = \frac{(R_1)(n^2 R_2)}{(L_1 R_1) + (L_1 n^2 R_2)} \nu_L$$

$$\lambda = -\frac{1}{\tau}$$

trovare τ alternativamente dal circuito...

$$\tau = \frac{L_1}{R_1 \parallel (n^2 R_2)} = \frac{L_1}{R_1 \cdot \frac{n^2 R_2}{n^2 R_2 + R_1}} = \frac{L_1 (R_1 + n^2 R_2)}{R_1 n^2 R_2} \Rightarrow \tau = \frac{L_1 R_1 + L_1 n^2 R_2}{R_1 n^2 R_2}$$

\Rightarrow posso scrivere l'eq diff (0.1.) come $\dot{I}_L + \frac{1}{\tau} \nu_L = 0$

$$\Rightarrow \dot{I}_L + \frac{\nu_L}{\tau} = 0 \quad \Rightarrow \quad \int \frac{d\nu_L}{dt} = -\frac{1}{\tau} \int \nu_L \quad \Rightarrow \quad \int d\nu_L = -\frac{1}{\tau} \nu_L \int dt$$

$$\Rightarrow \int \frac{1}{\nu_L} d\nu_L = -\frac{1}{\tau} \int dt \quad \Rightarrow \quad \ln(\nu_L) = -\frac{1}{\tau} t + K \quad \Rightarrow \quad \nu_L(t) = K e^{-\frac{1}{\tau} t}$$

Determino la Costante K

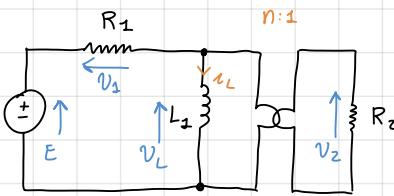
$$\left\{ \begin{array}{l} \nu_L(t) = K e^{-\frac{1}{\tau} t} + \nu_{p(\infty)} = K e^{-\frac{t}{\tau}} + 2 \\ \nu_L(0) = \nu_L(0^-) = 0 \end{array} \right. \Rightarrow \nu_L(0) = K e^{-\frac{0}{\tau}} + 2 = 0 \Rightarrow K = -2$$

$$\Rightarrow \nu_L(t) = -2 e^{-\frac{t}{\tau}} + 2 = 2 \left(1 - e^{-\frac{t}{\tau}} \right)$$

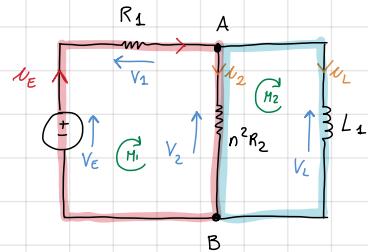
$$\Rightarrow \dot{I}_L(t) = 2 \left[+ \frac{1}{\tau} e^{-\frac{t}{\tau}} \right]$$

$$Q_1: V_{R_2} = ?$$

C



C'



$$\text{dalla } M_2(\text{c}') \rightarrow -V'_2 + V'_{L_1} = 0 \rightarrow V'_2 = V'_{L_1} \rightarrow V'_2 = L \cdot \dot{i}_L$$

$$\rightarrow \text{Derivo la soluzione} \rightarrow i_L(t) = \frac{2}{\tau} e^{-\frac{t}{\tau}} = 133.3 e^{-6.67 t} \text{ A}$$

$$\Rightarrow V'_2 = L \cdot \dot{i}_L = L \cdot 133.3 e^{-\frac{t}{133}} = 6.67 e^{-6.67 t} \quad \checkmark$$

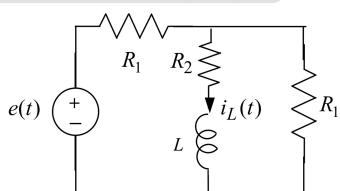
Dalla Formula del trasporto:

$$V_{R_2} = \frac{V'_2}{n} = 1.3 e^{-6.67 t} \quad V \quad \text{Ans 2}$$

* i risultati numerici
Sono errati

ESERCIZIO 10.1

Considerato il seguente circuito nel quale all'istante $t = 0$ il generatore inverte la sua polarità, calcolare la corrente nell'induttore per ogni t .

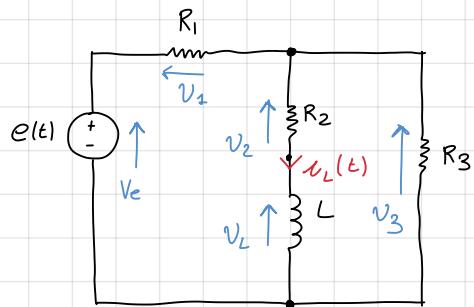


$$e(t) = \begin{cases} 10 \text{ V} & t < 0 \\ -10 \text{ V} & t > 0 \end{cases}$$

$$R_1 = 10 \Omega$$

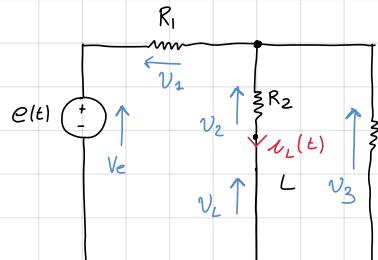
$$R_2 = 20 \Omega$$

$$L = 2 \text{ mH}$$



$t < 0$

STAZIONARIO



$$E_0 = 10 \text{ V}$$

$$R_1 = 10 \Omega$$

$$R_2 = 20 \Omega$$

$$L = 2 \text{ mH}$$

$$\begin{aligned} \text{Req} &= (R_2 \parallel R_1) + R_3 \\ &= 16.67 \Omega \end{aligned}$$

$$\Rightarrow \mathcal{V}_E = \frac{E_0}{\text{Req}} = \frac{3}{5} = 0.6 \text{ A}$$

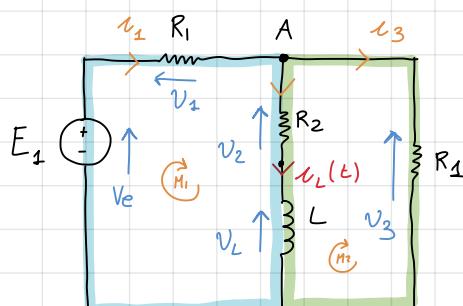
$$\Rightarrow \mathcal{V}_L = \mathcal{V}_E \cdot \frac{R_1}{R_1 + R_2} = \frac{1}{5} = 0.2 \text{ A}$$

\mathcal{V}_L per $t < 0$

$$\mathcal{V}_{E_1}(\infty) = -0.6 \text{ A} \quad \Rightarrow \quad \mathcal{V}_L(\infty) = -0.2 \text{ A}$$

$t \rightarrow \infty$

con $E_1 = -10 \text{ V}$



$$\begin{cases} -\mathcal{V}_E + \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{V}_L = 0 \\ -\mathcal{V}_L - \mathcal{V}_2 + \mathcal{V}_3 = 0 \\ -\mathcal{V}_1 + \mathcal{V}_L + \mathcal{V}_3 = 0 \end{cases} \Rightarrow \begin{cases} -E + \mathcal{V}_1 R_1 + \mathcal{V}_L R_2 + L \dot{\mathcal{V}}_L = 0 \\ -L \dot{\mathcal{V}}_L - \mathcal{V}_L R_2 + \mathcal{V}_3 R_1 = 0 \\ \mathcal{V}_1 = \mathcal{V}_L + \mathcal{V}_3 \end{cases}$$

$$\text{Dalla (2)} \quad \mathcal{V}_3 = \frac{L}{R_1} \dot{\mathcal{V}}_L + \frac{R_2}{R_1} \mathcal{V}_L$$

$$\Rightarrow \text{Nella (1)} \quad \Rightarrow R_1 \mathcal{V}_L + R_1 \mathcal{V}_3 + \mathcal{V}_L R_2 + L \dot{\mathcal{V}}_L = E \Rightarrow R_1 \mathcal{V}_L + L \dot{\mathcal{V}}_L + R_2 \mathcal{V}_L + \mathcal{V}_L R_2 + L \dot{\mathcal{V}}_L = E$$

$$2L \dot{\mathcal{V}}_L + \mathcal{V}_L (R_1 + R_2 + R_3) = E \Rightarrow 2L \dot{\mathcal{V}}_L + \mathcal{V}_L (2R_2 + R_1) = E \Rightarrow \dot{\mathcal{V}}_L + \mathcal{V}_L \left(\frac{2R_2 + R_1}{2L} \right) = E$$

$$\Rightarrow \lambda + \frac{2R_2 + R_1}{2L} = 0 \Rightarrow \lambda = -\frac{2R_2 + R_1}{2L}$$

$$\Rightarrow \tau = \frac{2L}{2R_2 + R_1}$$

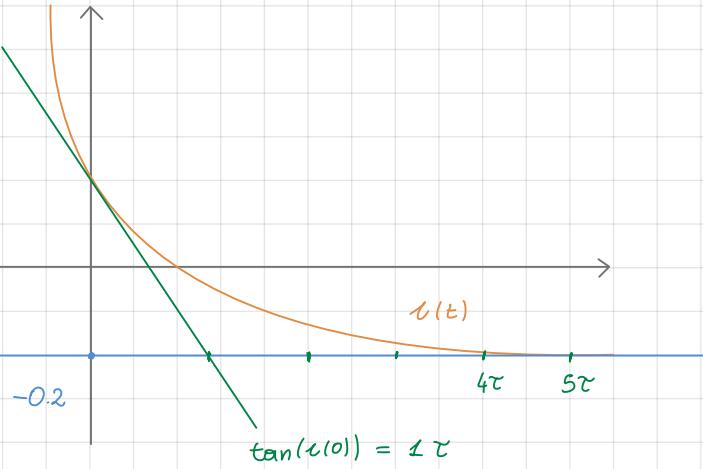
$$\Rightarrow \mathcal{V}_L(t) = K e^{-\lambda t} + \mathcal{V}_P = K e^{-\frac{2R_2 + R_1}{2L} t} - 0.2$$

$$-\frac{2R_2 + R_1}{2L} t$$

Trovo le C.I. : Couchy

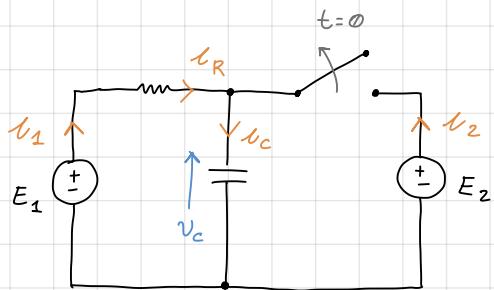
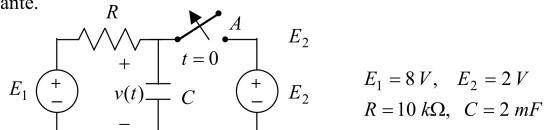
$$\begin{cases} \mathcal{L}_L(t) = K e^{-\frac{R_1+R_2}{2L}t} - 0.2 \\ \mathcal{L}_L(0^+) = \mathcal{L}_L(0^-) = 0.2 \text{ A} \end{cases} \Rightarrow \mathcal{L}_L(0) = K e^0 - 0.2 = 0.2 \Rightarrow K = 0.4$$

$$\Rightarrow \mathcal{L}_L(t) = 0.4 e^{-\frac{R_1+R_2}{2L}t} - 0.2 \stackrel{\sim}{=} 0.4 e^{-12.5 \times 10^3 t} - 0.2 \quad \mathcal{L}(t) \text{ per } t > 0$$



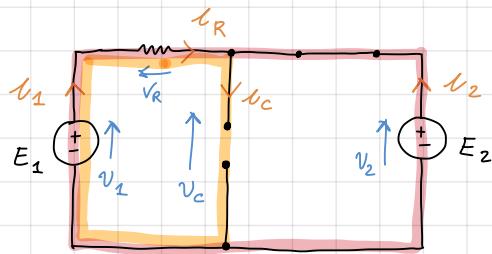
ESERCIZIO 10.2

Nel seguente circuito all'istante $t = 0$ si apre l'interruttore A . Calcolare la tensione sul condensatore $v(t)$ per ogni istante.



$t < 0$

Regime



$$1 \text{ sola maglia} \rightarrow LKT_M: \begin{cases} -v_1 + v_R + v_2 = 0 \end{cases}$$

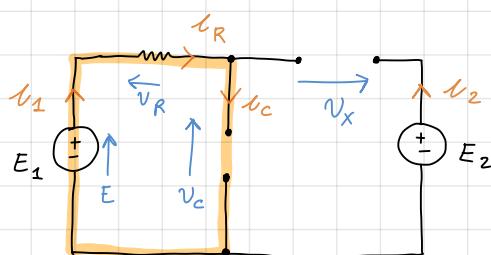
$$LKC_A: \begin{cases} -v_R - v_2 = 0 \\ v_1 = v_2 \end{cases}$$

$$\begin{cases} -E_1 + \mathcal{I}_1 R_1 + E_2 = 0 \rightarrow -8 + 2 + \mathcal{I}_1 R_1 = 0 \\ \mathcal{I}_1 = -\mathcal{I}_2 \end{cases}$$

$$\rightarrow \mathcal{I}_1 = \frac{6}{R_1} = 0.6 \text{ mA}$$

$t \rightarrow \infty$

Regime



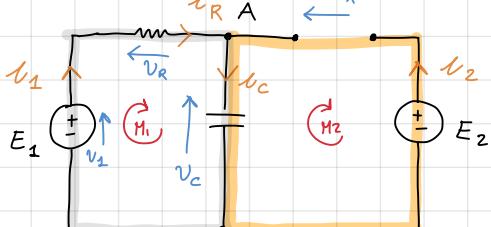
$$-v_E + v_R + v_c = 0 \rightarrow v_c = E = E_1 - \mathcal{I}_1 R_1 = 2 \text{ V}$$

Ans₁

v_c per $t \rightarrow \infty$

8V

$t > 0$



$$\begin{cases} -E_1 + v_R + v_c = 0 \\ -v_c + v_x + E_2 = 0 \\ \mathcal{I}_1 = 0 \\ -v_R + \mathcal{I}_c = 0 \end{cases} \rightarrow$$

$$v_c = C \dot{v}_c$$

$$\mathcal{I}_c R + v_c = E_1$$

$$v_c = E_2 \leftarrow ??$$

$$v_R = \mathcal{I}_c$$

$$\rightarrow \dot{v}_c RC + v_c = E_1 \rightarrow \dot{v}_c + \frac{1}{RC} v_c = E_1$$

$$\rightarrow \lambda + \frac{1}{RC} = 0 \rightarrow \lambda = -\frac{1}{RC}$$

$$\rightarrow \int \frac{dv_c}{dt} = -\frac{1}{RC} \int v_c \rightarrow \int \frac{1}{v_c} dv_c = -\frac{1}{RC} \int dt$$

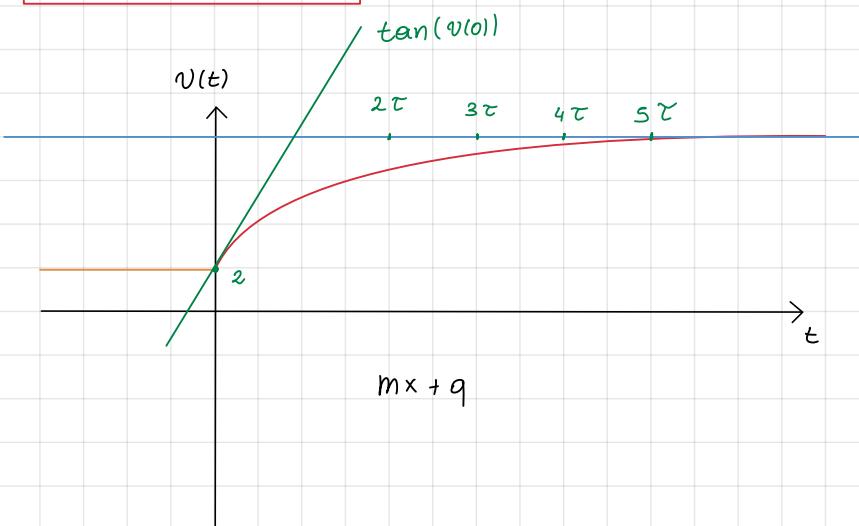
$$-\frac{1}{RC} t$$

$$\rightarrow \ln(v_c) = -\frac{1}{RC} t + K \rightarrow v_c = e^{-\frac{t}{RC}} \cdot \hat{K}$$

Determino \hat{K} ...

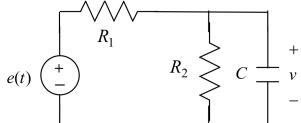
$$\begin{cases} V_c(t) = e^{\frac{\lambda t}{R}} + V_{CP} = \hat{K} e^{\frac{\lambda t}{R}} + 8 \\ V_c(0^+) = V_c(0^-) = 2 \end{cases} \Rightarrow V_c(0^+) = \hat{K} e^0 + 8 = 2 \Rightarrow \hat{K} = 2 - 8 = -6 \text{ V}$$

$$\Rightarrow V_c(t) = -6 e^{-\frac{1}{RC}t} + 8 \quad V_c(t) \text{ per } t > 0$$



ESERCIZIO 10.3

Il circuito in esame è in regime stazionario per $t < 0$. Valutare la tensione $v(t)$ per $t > 0$.

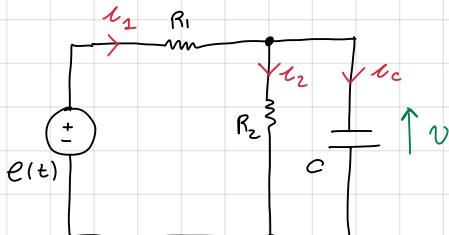


$$e(t) = 50 \text{ V} \quad t > 0$$

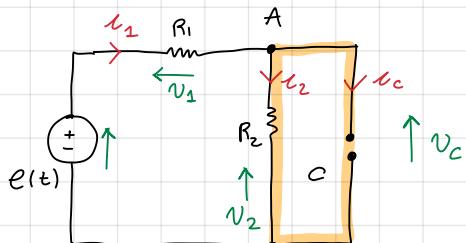
$$v(t=0) = 10 \text{ V}, \quad C = 1 \text{ mF}$$

$$R_1 = 20 \Omega, \quad R_2 = 24 \Omega$$

Risultato: $v(t) = 27.3 - 17.3e^{-91.7t} \text{ V}$



$t \rightarrow \infty$



$$LKC_A: -v_1 + v_2 + v_c = 0 \quad \Rightarrow \quad (v_1 = v_2)$$

$$LKT_{M_1}: -v_2 + v_c = 0 \quad \Rightarrow \quad v_c = v_2$$

$$LKT_{M_2}: -v_E + v_1 + v_2 = 0 \quad \Rightarrow \quad v_2 = E - v_1 = E - i_1 R_1$$

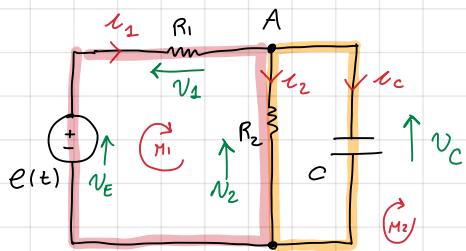
?

$v_c(\infty)$ Regime

$$i_E = \frac{E}{R_{\text{req}}} = \frac{E}{R_1 + R_2} = 1.136 \text{ A} \quad \Rightarrow \quad v_2 = v_c = E - i_1 R_1 = 27.27 \text{ V}$$

$t > 0$

C.I. $v(0^+) = 10 \text{ V}$



$$\begin{aligned} LKT: & \begin{cases} -v_E + v_1 + v_2 = 0 & \Rightarrow i_1 R_1 + i_2 R_2 = E \\ -v_2 + v_c = 0 & \Rightarrow v_c = v_2 \\ -v_E + v_1 + v_c = 0 & \Rightarrow i_1 R_1 + v_c = E \end{cases} \\ & \Rightarrow i_1 R_1 + i_2 R_2 = i_1 R_1 + v_c \end{aligned}$$

$$\Rightarrow i_2 R_2 = v_c \quad \Rightarrow i_2 = \frac{1}{R_2} v_c$$

$$LKC: -v_1 + i_2 + v_c = 0 \quad \Rightarrow \quad i_2 = i_1 + i_c$$

$$\Rightarrow i_1 R_1 + v_c = E$$

$$R.C \quad v_c = C \cdot \dot{v}_c$$

$$\Rightarrow i_2 R_2 + i_c R_1 + v_c = E \quad \Rightarrow \quad \frac{R_1}{R_2} v_c + C R_1 \dot{v}_c + v_c = E \quad \Rightarrow \quad C R_1 \dot{v}_c + v_c \left(\frac{R_1}{R_2} + 1 \right) = E$$

$$\Rightarrow \dot{v}_c + v_c \left(\frac{\frac{R_1}{R_2} + 1}{C R_1} \right) = \frac{E}{R_1 C} \quad \Rightarrow \quad \dot{v}_c + v_o \left(\frac{\frac{R_1 + R_2}{C R_1 R_2}} \right) = \frac{E}{R_1 C}$$

$$\lambda + \frac{R_1 + R_2}{C R_1 R_2} = 0 \quad \Rightarrow \quad \lambda = -\frac{R_1 + R_2}{C R_1 R_2}$$

$$\int \frac{dV_C}{dt} = -\frac{R_1 + R_2}{CR_1R_2} \int V_C \quad \rightarrow \quad \int \frac{1}{V_C} dV_C = \frac{-R_1 + R_2}{CR_1R_2} \int dt$$

$$-\ln(V_C) = -\frac{R_1 + R_2}{CR_1R_2} t + K \quad \rightarrow \quad V_C = e^{-\frac{R_1 + R_2}{CR_1R_2} t}$$

$$\Rightarrow V_C(t) = V_{C0}(t) + V_{CP}(t) = e^{-\frac{R_1 + R_2}{CR_1R_2} t} + 27.27$$

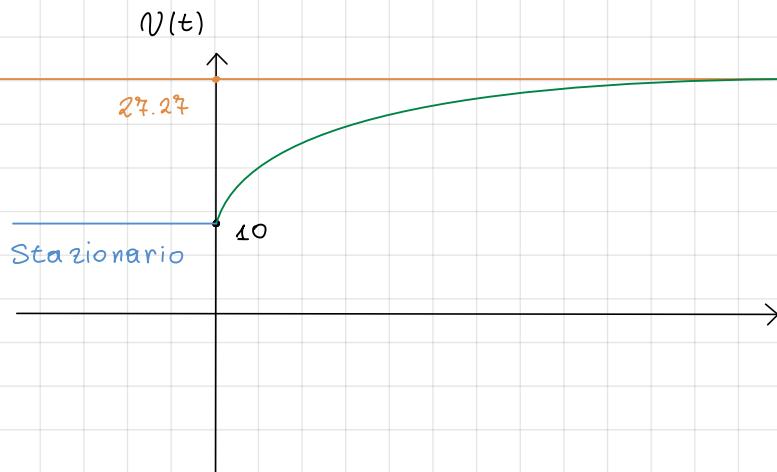
Trovo \hat{K} \rightarrow Cauchy

$$\begin{cases} V(t) = \hat{K} e^{-\frac{R_1 + R_2}{CR_1R_2} t} + 27.27 \\ V(0) = 10 \end{cases}$$

$$\lambda \emptyset$$

$$\hat{K} = -17.27$$

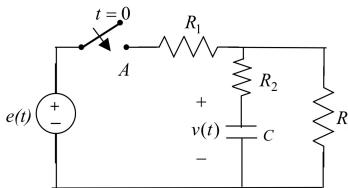
$$\Rightarrow V_C(t) = 27.27 - 17.27 e^{-\frac{R_1 + R_2}{CR_1R_2} t} = 27.27 - 17.27 e^{-91.67 t} \quad V_C(t) \text{ per } t > 0$$



ESERCIZIO 10.4

Il seguente circuito è a riposo fino a $t = 0$, istante in cui si chiude l'interruttore A. Calcolare:

- la costante di tempo τ del circuito;
- la tensione ai capi del condensatore per $t > 0$.

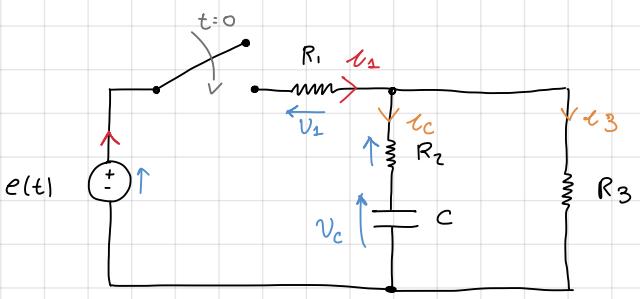


$$e(t) = 10 \cos(\omega t)$$

$$\omega = 100 \text{ rad/s}$$

$$R_1 = 20 \Omega, R_2 = 5 \Omega$$

$$R_3 = 10 \Omega, C = 1 \text{ mF}$$



$$e(t) = 10 \cos(\omega t) = 10 \cos(100t) \Rightarrow 10 e^{\frac{\omega t}{\tau}} = 10 V$$

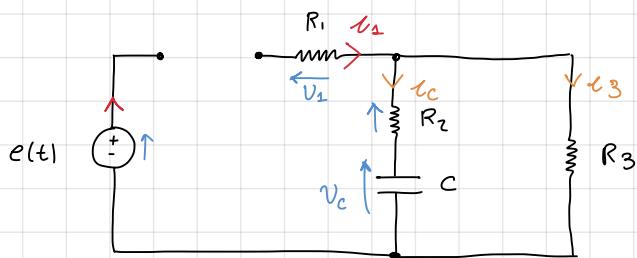
$$R_1 = 20 \Omega \Leftrightarrow \tau_1 = 20 \text{ s} \quad A$$

$$R_2 = 5 \Omega \Leftrightarrow \tau_2 = 5 \text{ s} \quad B$$

$$R_3 = 10 \Omega \Leftrightarrow \tau_3 = 10 \text{ s} \quad C$$

$$C = 1 \text{ mF} \Leftrightarrow -\frac{j}{100 \cdot 1 \text{ m}} = -10j \quad D$$

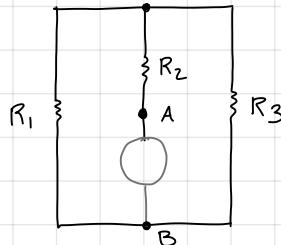
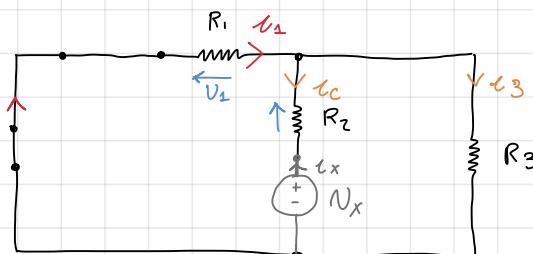
$t < 0$ Stazionario, Riposo



C.I.

$$V_C(0^+) = 0$$

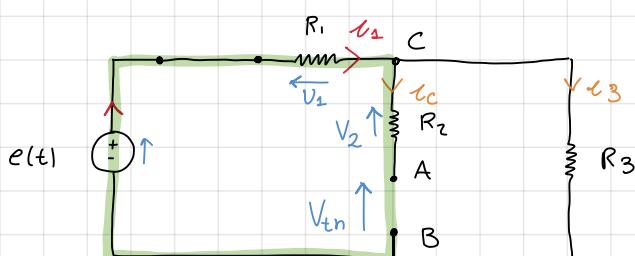
$t > 0$ Trovo l'eq di Thevenin (condensatore)



$$R_{thn} = (R_1 || R_3) + R_2$$

$$= \frac{35}{3} = 11.67 \Omega$$

$$\tau = R_{thn} \cdot C = 0.0167 \text{ s} = 11.67 \text{ ms}$$



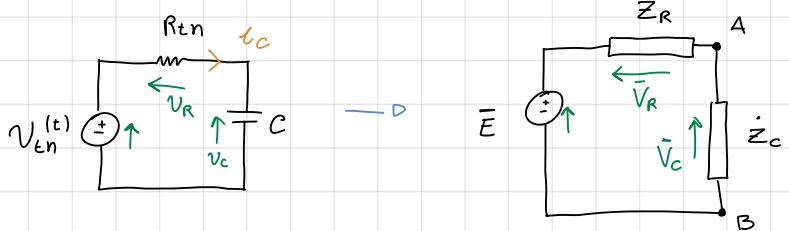
$$V_{thn} = V_{CB}$$

$$-V_e + V_1 + V_2 + V_{thn} = 0 \quad \rightarrow \quad V_1 + V_{thn} = e(t)$$

$$V_2 = 0$$

$$\therefore V_{thn} = e(t) - R_1 \cdot V_1 \quad \text{ma} \quad V_1 = \frac{e(t)}{R_1 + R_3} \quad \therefore V_{thn} = e(t) - \frac{R_1 e(t)}{R_1 + R_3} = e(t) \left[\frac{R_1 + R_3 - R_1}{R_1 + R_3} \right]$$

$$\therefore V_{thn} = \frac{e(t) \cdot R_3}{R_1 + R_3}$$



$$\dot{V}_c = -10j$$

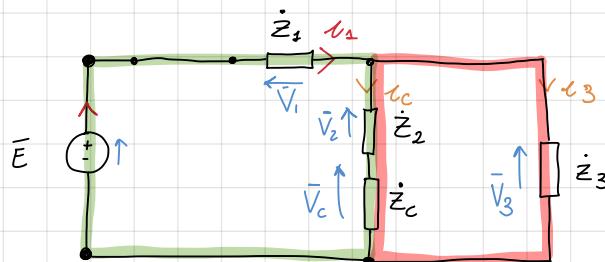
$$\dot{Z}_R = \frac{35}{3} \Omega$$

$$-V_{en} + V_R + V_C = 0 \Rightarrow V_C \cdot R_{en} + V_C = V_{en} \text{ ma } \dot{I}_C = C \dot{V}_C \Rightarrow CR_{en} \dot{V}_C + V_C = V_{en}$$

$$\Rightarrow \dot{V}_C + \frac{1}{CR_{en}} V_C = \frac{V_{en}}{CR_{en}} \Rightarrow 1 + \frac{1}{CR_{en}} = 0 \Rightarrow \lambda = -\frac{1}{CR_{en}}$$

$$\Rightarrow V_C(t) = K e^{-\frac{1}{CR_{en}} t} + V_{CP}(t) \quad \text{circuito per } t \rightarrow \infty$$

$t \rightarrow \infty$



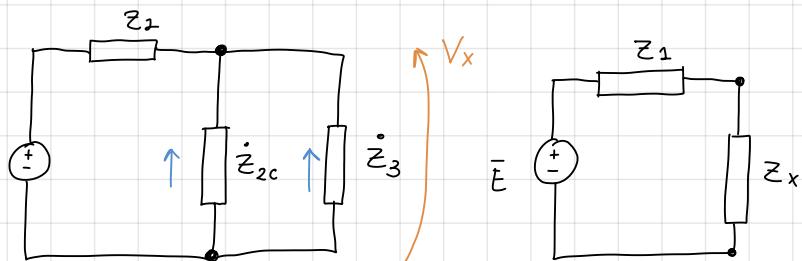
$$\begin{aligned} R_1 &= 20 \Omega \Rightarrow Z_1 = 20 \Omega \text{ A} \\ R_2 &= 5 \Omega \Rightarrow Z_2 = 5 \Omega \text{ B} \\ R_3 &= 10 \Omega \Rightarrow Z_3 = 10 \Omega \text{ C} \\ C &= 1 \text{ mF} \Rightarrow -\frac{j}{100 \cdot 1 \text{ m}} = -10j \text{ D} \\ E &= 10 \text{ V} \end{aligned}$$

$$\Rightarrow \dot{Z}_{2c} = \dot{Z}_2 + \dot{Z}_c = 5 - 10j$$

$$\dot{Z}_x = \dot{Z}_{2c} // \dot{Z}_3$$

$$\bar{I} = \frac{\bar{E}}{\dot{Z}_1 + \dot{Z}_x} = K$$

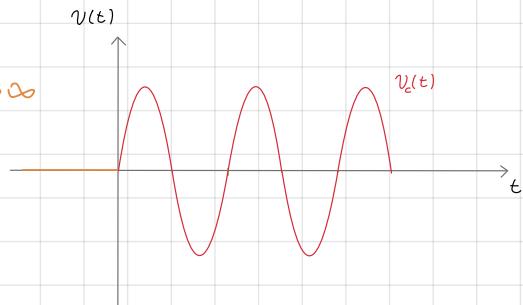
$$\Rightarrow \bar{V}_x = \dot{Z}_x \cdot \bar{I} = J$$



$$V_C = V_x \cdot \frac{\dot{Z}_c}{\dot{Z}_c + \dot{Z}_2} = 1.41 - 1.65j = 2.17 \angle -0.86 = 2.17e^{-0.86j}$$

$$V_x \Rightarrow V_C(t) = 2.17 \cos(400t - 0.86)$$

$V_C(t)$ per $t \rightarrow \infty$



TORNANDO A $t > 0$...

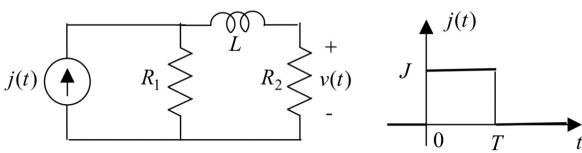
$$V_C(t) = K e^{-\frac{1}{CR_{en}} t} + V_{CP}(t) = \begin{cases} K e^{-\frac{1}{CR_{en}} t} + 2.17 \cos(400t - 0.86) \\ V_C(0) = 0 \end{cases} \text{ RADIANTI}$$

$$\Rightarrow V_C(0) = K + 2.17 \cos(-0.86) = 0 \Rightarrow K = -2.17 \cos(-0.86) = -1.42$$

$$\Rightarrow V_C(t) = -1.42 e^{-\frac{1}{CR_{en}} t} + 2.17 \cos(400t - 0.86) = -1.42 e^{-85.7t} + 2.17 \cos(400t - 0.86) \quad \checkmark$$

ESERCIZIO 10.5

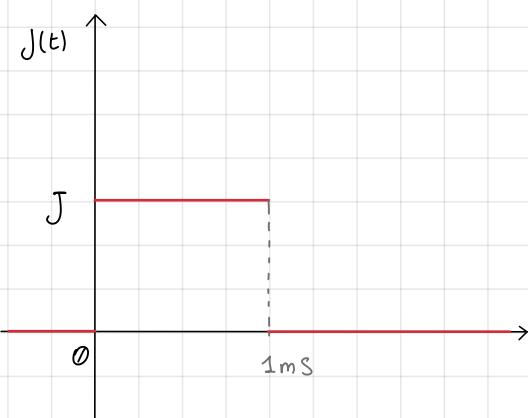
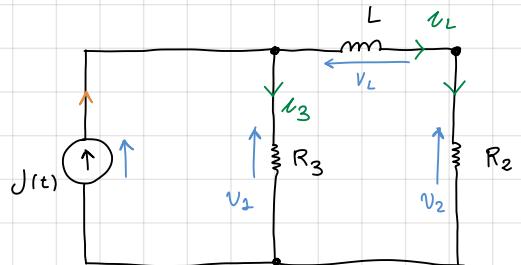
In figura è riportato lo schema equivalente di un grilletto elettronico per pistola. L'uscita del sistema è il segnale di tensione $v(t)$ prelevato ai capi di R_2 . Determinare tale segnale per $0 < t < 0.3 \text{ s}$.



$$J = 40 \text{ A}, \quad T = 1 \text{ ms}$$

$$R_1 = 30 \Omega, \quad R_2 = 20 \Omega$$

$$L = 50 \text{ mH}$$



$$t < 0$$

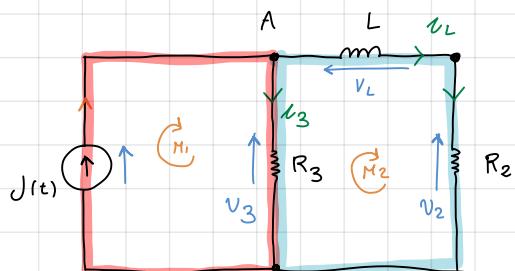
$$\text{L} \Rightarrow J(t) = 0 \rightarrow v_2(t < 0) = \emptyset$$

$$v_L(0^-) = \emptyset \quad \text{c.i.}$$

$$v_L(t) = L \frac{dv}{dt}$$

$t > 0$ Transitorio

$$\begin{array}{ll} \text{A} \quad R_3 = 30 \Omega & J = 40 \text{ A} \\ \text{B} \quad R_2 = 20 \Omega & T = 1 \text{ ms} \\ \text{C} \quad L = 50 \text{ mH} & \end{array}$$



$$\begin{cases} -v_J + v_3 = 0 \\ -v_3 + v_L - v_2 = 0 \end{cases} \rightarrow v_3 R_3 + L \dot{i}_L - v_L R_2 = 0$$

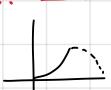
$$-J + i_3 + i_L = 0 \rightarrow v_3 = J - v_L$$

$$\Rightarrow J R_3 - i_L R_3 + L \dot{i}_L - v_L R_2 = 0 \rightarrow L \dot{i}_L + i_L (-R_2 - R_3) = -J R_3$$

$$\rightarrow i_L + \dot{i}_L \left(-\frac{R_2 + R_3}{L} \right) = -\frac{R_3}{L} J$$

$$\rightarrow \lambda - \frac{R_2 + R_3}{L} = 0 \rightarrow \lambda = \frac{R_2 + R_3}{L}$$

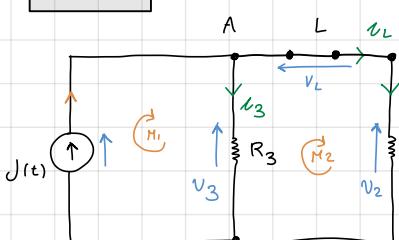
!! $\lambda > 0$



$$= 1000 \text{ s}^{-1}$$

$$\Rightarrow i_L(t) = K e^{\lambda t} + v_{LP}(t) \quad \text{req. imme}$$

$t \rightarrow \infty$



$$\Rightarrow \text{Req} = R_3 // R_2 = 12 \Omega$$

$$\Rightarrow V_J = J \cdot \text{Req} = 480 \text{ V} \quad \text{c.i.}$$

Torno α

$t > 0$

$$\Rightarrow i_L(t) = K e^{\lambda t} + v_{LP}(t) \quad \rightarrow \text{A noi serve } v(t) \rightarrow v_L(t) = K e^{\lambda t} + v_{LP}(t)$$

$$\begin{aligned} v_L(t) &= K e^{\lambda t} + v_{LP}(t) \\ &= K e^{\lambda t} + 480 \end{aligned}$$

Trovato K

$$\begin{cases} V_L(t) = K e^{\lambda t} + 480 \\ V_L(0^+) = V_L(0^-) = 0 \end{cases} \Rightarrow V_L(0) = K e^{\lambda 0} + 480 = 0 \Rightarrow K = -480$$

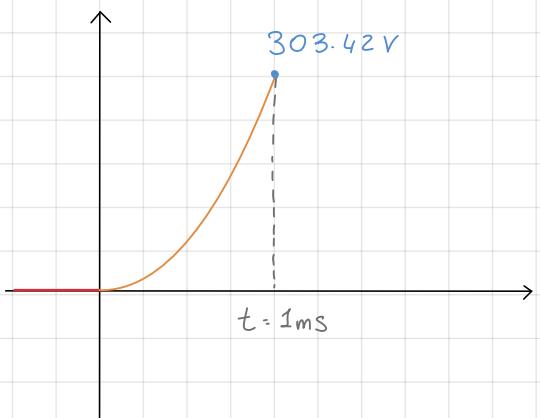
$$\Rightarrow V_L(t) = -480 e^{-\lambda t} + 480 = 480(1 - e^{-1000t}) \quad \text{per } 0 < t < T$$

$$\Rightarrow V_L(T) = 480(1 - e^{-1000 \cancel{ms}}) = 303.42 \quad \checkmark \\ = 480(1 - e^{-1})$$

\Rightarrow Eq Diff per $t > T$

$$V_L(t) = V_{L0}(t) + \cancel{V_p(t)}$$

↑
non c'è forzamento!



$$\Rightarrow \begin{cases} V_L(t) = V_{L0}(t) = K e^{-1000t} \\ V_L(T^+) = V_L(T^-) = 480(1 - e^{-1}) \end{cases} \Rightarrow V_L(T^+) = K e^{-1000T} = 480(1 - e^{-1}) \Rightarrow K_1 e^{-1} = 480(1 - e^{-1})$$
$$\therefore K_1 = 480 \left(\frac{1}{e^{-1}} - \frac{1}{e^{1-1}} \right) = 480(e - 1)$$

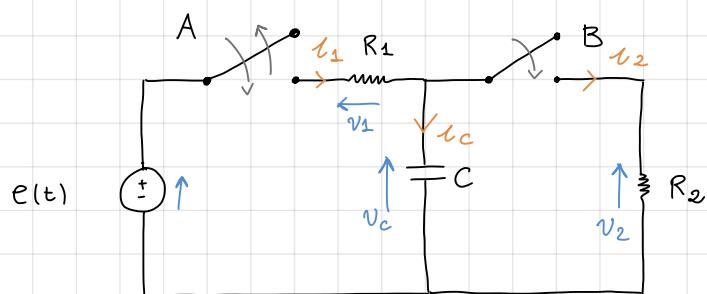
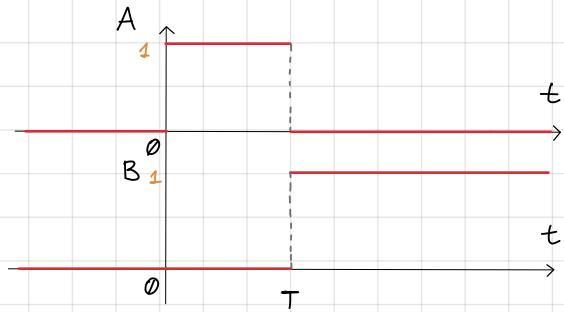
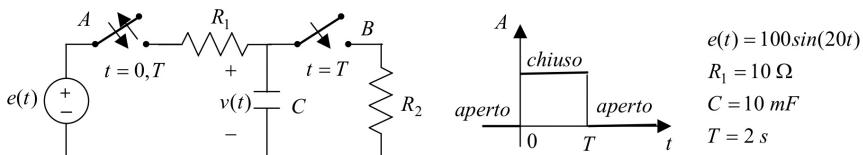
$$\Rightarrow V_L(t) = 480(e - 1) e^{-1000t} \quad \checkmark \quad \text{per } t > T$$

ESERCIZIO 10.6

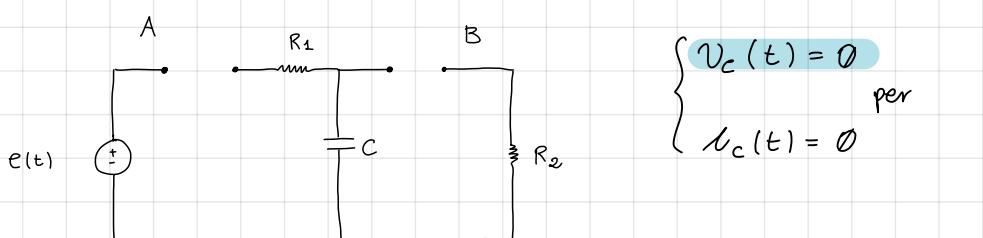
La seguente rete rappresenta lo schema elettrico equivalente del circuito di carica della stazione spaziale orbitante. La carica avviene tra l'istante $t = 0$ e l'istante $t = T$, intervallo in cui l'interruttore A resta chiuso. Per $t > T$, invece, il condensatore C viene collegato al resto della rete attraverso la chiusura dell'interruttore B. Supponendo la rete a riposo per $t < 0$, valutare:

a) la tensione sul condensatore $v(t)$ per $0 < t < T$;

b) l'energia massima W_{\max} erogabile da C per $t > T$;

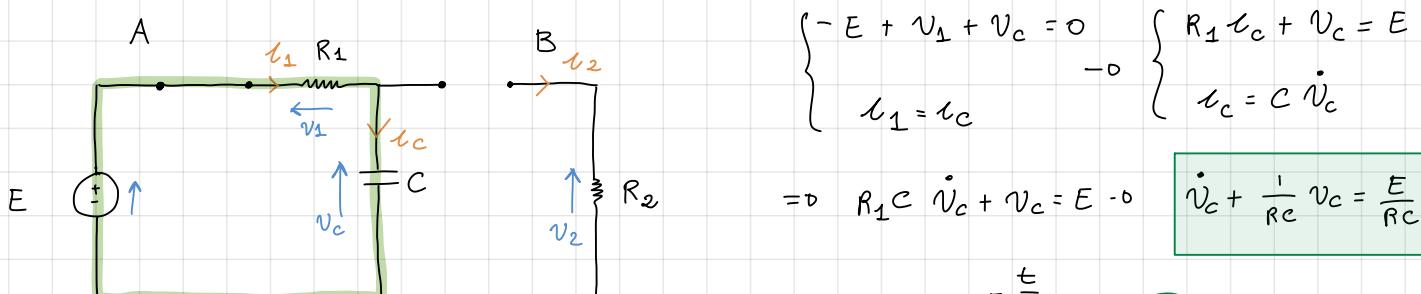


$t < 0$



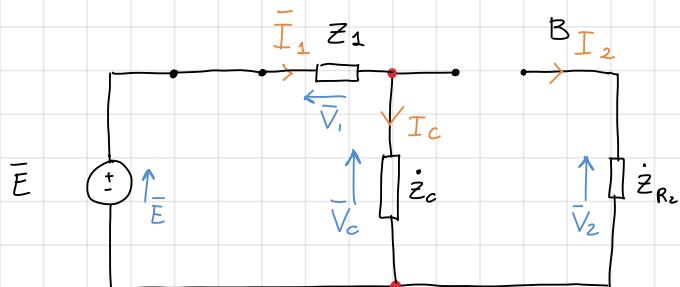
$$R.C \cdot \dot{v}_C = C \dot{i}_C$$

$0 < t < T$



$$\Rightarrow v_c(t) = K e^{-\frac{t}{RC}} + v_{cp}(t) \quad \text{Sol a regime}$$

$t \rightarrow \infty \text{ per } 0 < t < T$



$$\text{PART TEN: } V_C = \bar{E} \cdot \frac{\bar{Z}_C}{\bar{Z}_C + \bar{Z}_1} = 20 \cdot 40j = 44.72 \angle -1.12^\circ = 44.72 e^{-1.12j}$$

\Rightarrow torno al tempo $\Rightarrow \bar{V}_C \Rightarrow v_c(t) = 44.72 \sin(20t - 1.12)$

$$R_1 = 10 \Omega \Rightarrow \bar{Z}_1 = 10 \Omega$$

$$C = 10 \text{ mF} \Rightarrow \bar{Z}_C = -\frac{j}{20 \cdot 10 \text{ mF}} = -5j$$

$$e(t) = 100 \sin(20t) \Rightarrow \bar{E} = 100 \text{ V}$$

Sugli es e - 10V??

$$-1.12j$$

torno a $0 < t < T$

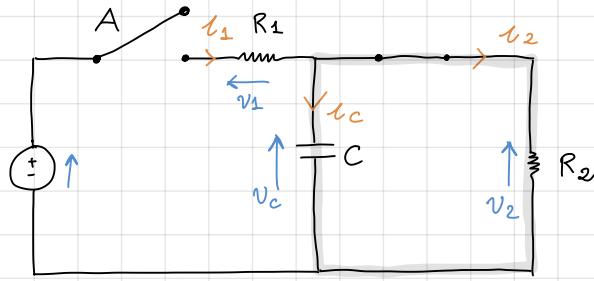
Determino K dalla C.I. $\mathcal{V}_c(0^-) = \mathcal{V}_c(0^+) = 0$ V

$$\begin{cases} \mathcal{V}_c(t) = K e^{-\frac{t}{RC}} + 44.72 \sin(20t - 1.12) \\ \mathcal{V}_c(0^+) = 0 \end{cases} \rightarrow \mathcal{V}_c(0) = K + 44.72 \sin(-1.12) = 0 \rightarrow K = +40$$

$$\Rightarrow \mathcal{V}_c(t) = 40 e^{-\frac{t}{RC}} + 44.72 \sin(20t - 1.12) \text{ V}$$

ANS 1

$t > T$



$$\begin{aligned} -\mathcal{V}_c + \mathcal{V}_2 &= 0 \rightarrow -\mathcal{V}_c + R_2 \mathcal{V}_c = 0 \\ \rightarrow R_2 C \dot{\mathcal{V}}_c - \mathcal{V}_c &= 0 \\ \Rightarrow \dot{\mathcal{V}}_c - \frac{1}{R_2 C} \mathcal{V}_c &= 0 \\ \rightarrow \mathcal{V}_c(t) &= K_2 e^{\frac{1}{R_2 C} t} + 0 \end{aligned}$$

↑ NO FORZ.

$$\rightarrow \mathcal{V}_c(T^-) = 40 e^{-\frac{10T}{RC}} + 44.72 \sin(20T - 1.12) \text{ V} = 41.36 \text{ V}$$

c.i. per T^+

\Rightarrow Energia erogabile dal condensatore \rightarrow dipende dalla tensione al momento in cui si stacca l'alimentazione

$$\rightarrow W_{\max} = \frac{1}{2} C \cdot \mathcal{V}_c(T)^2 = \frac{1}{2} \cdot 10 \mu F \cdot 41.36^2 = 8.55 \text{ J}$$

$$\Rightarrow \begin{cases} \mathcal{V}_c(t) = K_2 e^{\frac{1}{R_2 C} t} + 0 \\ \mathcal{V}_c(T^+) = 41.36 \end{cases} \rightarrow K_2 = 41.36 \Rightarrow \mathcal{V}_c(t) = 41.36 e^{\frac{1}{R_2 C} t}$$

