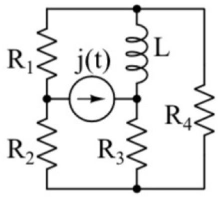
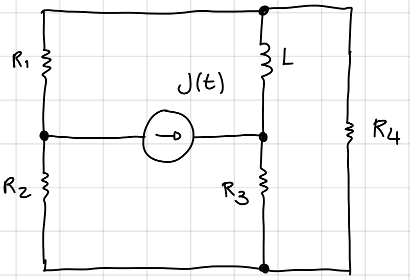
Esercizio 1 (PROVA D'ESAME 26/2/20)

Il circuito è a riposo per $t < 0$. Determinare la corrente che scorre nell'induttore in ogni istante di tempo.
Dati: $R_1 = 120\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 110\ \Omega$, $R_4 = 180\ \Omega$, $L = 0.4$ H,

$$j(t) = \begin{cases} 0\,A & t < 0 \\ 0.8\,A & 0 < t < 5\text{ms} \\ 0\,A & t > 5\text{ms} \end{cases}$$





DATI

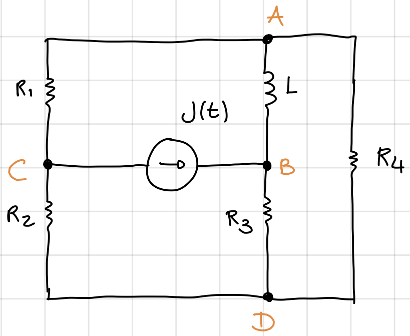
A $R_1 = 120\ \Omega$ B $R_2 = 60\ \Omega$ C $R_3 = 110\ \Omega$
D $R_4 = 180\ \Omega$ L $L = 0.4\ H$

$$j(t) = \begin{cases} 0\ A & t < 0 \\ 0.8\ A & 0 < t < 5\ ms \\ 0\ A & t > 5\ ms \end{cases} \quad J$$

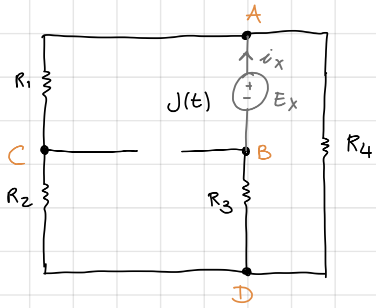
$\boxed{t < 0}$ $\rightarrow$ T_1 Stazionario

per $T_1 \rightarrow J(t) = 0\ A \Rightarrow \boxed{i_L = \emptyset}$ Ans 1

$\boxed{0 < t < 5ms}$ $\rightarrow$ T_2 DINAMICO

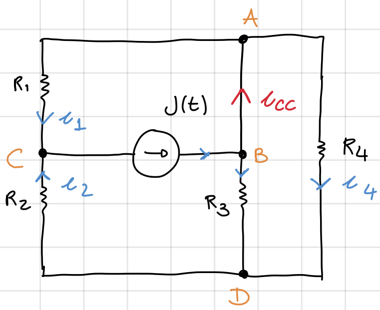


(1) NORTON$_{(AB)}$ (a) Req_{AB}



$$Req_{AB} = \left[(R_1 + R_2) \,//\, R_4\right] + R_3 = 200\ \Omega \quad R_N$$

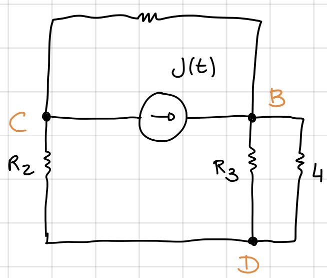
(b) Corrente di C.C. $\rightarrow i_{cc} = ?$



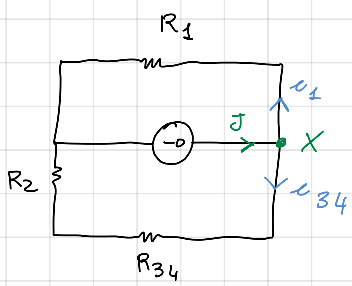
LKC_B : $i_{cc} + i_3 - J(t) = 0$
$\rightarrow i_{cc} = J(t) - i_3$?

Ci serve scrivere prima l'equazione perché tra poco non vedremo più Icc (visto che A=B e scompare il c.c.)

$\equiv$



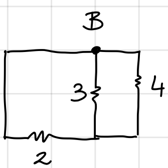
$\rightarrow$



$\rightarrow R_{234} = (R_3 \,//\, R_4) = 68.27\ \Omega$ (C D F)

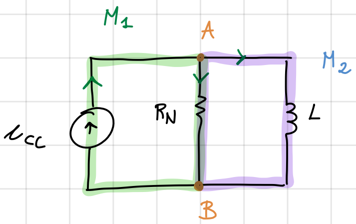
$\Rightarrow i_1 = J(T_2) \cdot \dfrac{R_{234}}{R_{234} + R_1} = 0.413\ A$

LKC_X : $i_{34} = J(t) \cdot \dfrac{R_1}{R_1 + R_{34} + R_2} = J(t) \cdot 0.48\ A$



$\rightarrow i_3 = J(t) \cdot 0.48 \cdot \dfrac{R_4}{R_4 + R_3} = J(t) \cdot 0.298\ A$

$$\Rightarrow \quad i_{cc} = J(t) - i_3 = J(t)\,(1 - 0.298) = 0.561 \text{ A} \quad i_N$$



RELAZIONI CARAT $\rightarrow$ EQ STATO

$$V = R \cdot i \rightarrow i_{R_N} = \frac{v_{AB}}{R_N} \quad \text{ma} \quad LKT_{M_1}: \; -V_{cc} + V_{AB} = 0 \rightarrow V_{AB} = V_{cc}$$

ma $LKT_{M_2}: \; -V_{AB} + V_L = 0 \rightarrow V_{AB} = V_L \Rightarrow i_R = \dfrac{V_L}{R_N}$

$$LKC_A: \; -i_{cc} + i_R + i_L = 0 \rightarrow i_{cc} = \frac{V_{cc}}{R_N} + i_L = 0 \rightarrow i_{cc} = \frac{V_L}{R_N} + i_L = 0$$

Inoltre $v_L = L \cdot \dfrac{di_L}{dt}$

$$\Rightarrow \boxed{\frac{L}{R}\dot{i}_L + i_L = i_{cc}} \qquad \frac{L}{R}\lambda + 1 = 0 \rightarrow \lambda = -\frac{R}{L}$$

Sappiamo che $i(t) \propto C_1 e^{\lambda t} \rightarrow \dot{i}(t) = C_1 \lambda e^{\lambda t}$

mentre la soluzione è del tipo $i_L(t) = i_{c0}(t) + i_{cp}(t)$

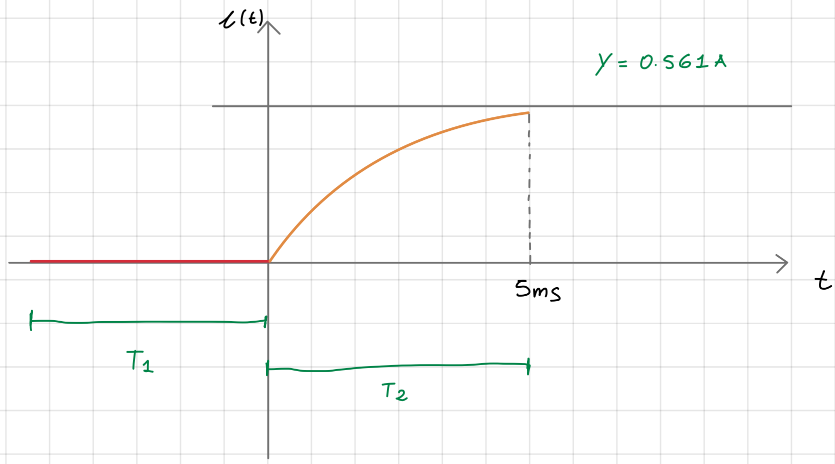
FORZAMENTO $\rightarrow J(T_1)$

Trovo $i_{c0}(t) \propto C_1 e^{\lambda t} = C_1 e^{-\frac{R}{L}t}$

$$\begin{cases} i_L(t) = C_1 e^{-\frac{R}{L}t} + i_{cc} \\ i_L(0^+) = 0 \end{cases} \rightarrow i_L(0^+) = C_1 \cancel{e^{-\frac{R}{L}0^+}}^{1} + i_{cc} = 0 \rightarrow C_1 = -i_{cc} = -0.561 \text{ A}$$

$$\Rightarrow i_L(T_2) = -0.561\, e^{-\frac{R}{L}t} + 0.561 \rightarrow i_L(t_1)\Big|_{5ms} = 0.5149$$

$\rightarrow$ Funzione Tensione per $0 < t < 5$ ms $\qquad i_L(t) = 0.561\left(1 - e^{-\frac{R}{L}t}\right)$



$\boxed{t > 5\ ms}$ $t \to \infty$

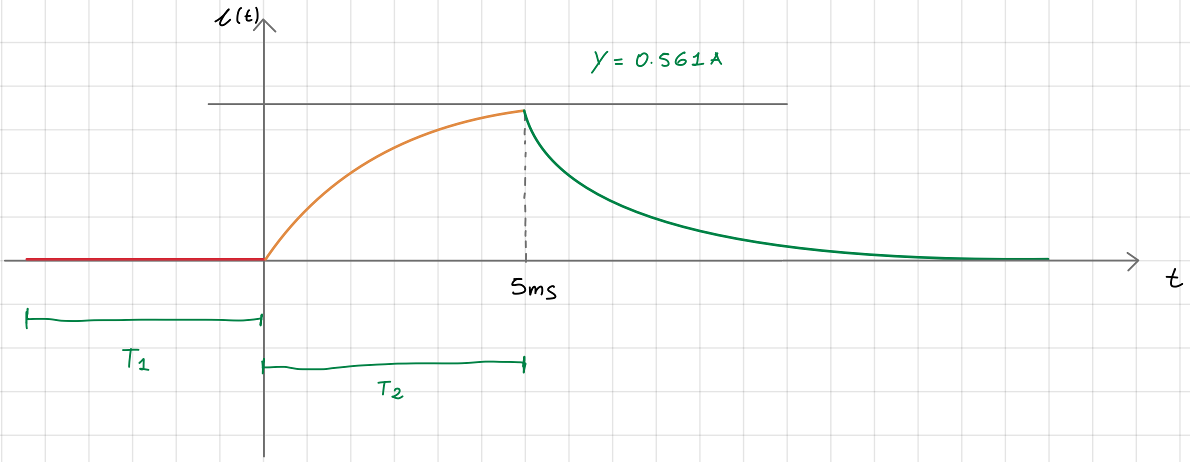
$$i_c(T_3) = i_{c0}(t) + \cancel{i_{cp}(t)}^{\,0} = i_{c0}(t) \propto C_2 e^{\lambda t}$$

ma $\lambda = -\frac{R}{L} \to i_{c0} = C_2 e^{-\frac{R}{L}t}$

Trovo $C_2 \to$ C.I. $\to i_L(t_1^+) = 0.515\ A$

$$\to \begin{cases} i_L(t) = C_2 e^{-\frac{R}{L}t} \\ i_L(5ms) = 0.515\ A \end{cases} \to i_L(5ms) = C_2 \cdot e^{-\frac{R}{L}\cdot 5ms} = 0.515 \to C_2 = 0.515 \cdot e^{\frac{R}{L}\cdot 5ms}$$

$$\Rightarrow i_{c0} = 0.515\, e^{\frac{R}{L}t_1} \cdot e^{-\frac{R}{L}t} = 0.515\, e^{-\frac{R}{L}(t-t_1)}$$

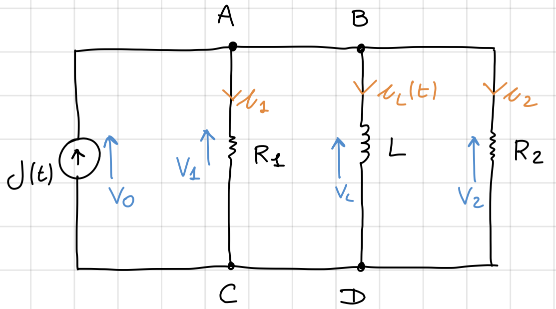


$$60 : t = 100 : x$$

$$\frac{60}{t} = \frac{100}{x} \to x = 100 \cdot \frac{t}{60}$$

ESERCITAZIONE 02/12/21

Esercizio 1



DATI

A $R_1 = 100\ \Omega$ C $L_1 = 0.2H$

B $R_2 = 47\ \Omega$

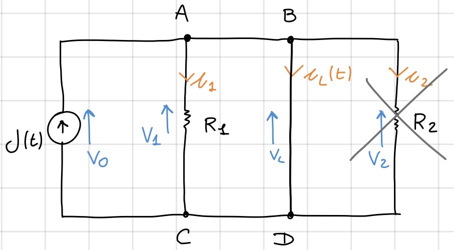
$$J(t) = \begin{cases} 0\ A & t<0 \\ 0.4\ A & t>0 \end{cases}$$

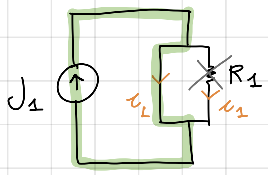
Q: $i_L(t) = \ ?\ \ \forall t$

$\boxed{t<0}$

Se $J(t) = \emptyset \Rightarrow$ $\underline{i_L(0) = 0\ A}$ $\underline{V_L(0) = \emptyset\ V}$

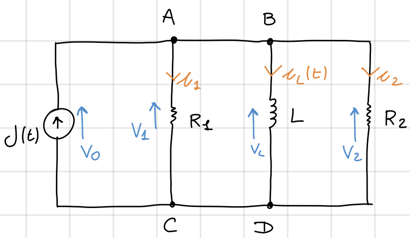
$\boxed{t \to \infty}$

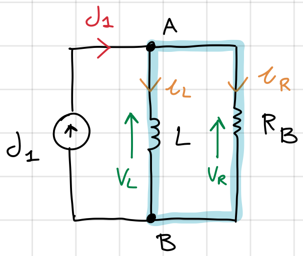




$\Rightarrow i_L(\infty) = J_1 = 0.4\ A$

$\boxed{t>0}$





Con $R_B = R_1 // R_2 = 31.97\Omega$

$-V_L + V_R = 0 \Rightarrow V_L = V_R$

$-J_1 + i_L + i_R = 0 \rightarrow i_R = J_1 - i_L$

$V_R = i_R \cdot R_B$

$V_L = L \cdot \frac{di_L}{dt}$

$\Rightarrow V_R = L\,\dot{i}_L \Rightarrow i_R R_B = L\,\dot{i}_L$

$\hookrightarrow R_B(J_1 - i_L) = L\,\dot{i}_L$

$\hookrightarrow \dot{i}_L + \frac{R_B}{L} i_L = \frac{R_B}{L} J_1$

Pongo $\tau = \frac{L}{R_B} \rightarrow \dot{i}_L + \frac{i_L}{\tau} = \frac{J_1}{\tau}$

Omogenea Associata: $\lambda + \frac{R_B}{L} = 0 \rightarrow \lambda = -\frac{R_B}{L}$

Risolvo l'eq diff $\rightarrow$ separazione Var

$\frac{di_L}{dt} = -\frac{R_B}{L} i_L \rightarrow \int \frac{di}{dt} = -\frac{R_B}{L}\int i_L \rightarrow \int di = -\frac{R_B}{L} i_L \int dt$ $t+K$

$\ln(i_L)$

$\rightarrow \int \frac{di_L}{i_L} = -\frac{R_B}{L}t + K \rightarrow \ln(i_L) = -\frac{R_B}{L}t + K \rightarrow i_L(t) = e^{\left(-\frac{R_B}{L}t + K\right)} = e^{-\frac{R_B}{L}t} \cdot e^{K}$ $\hat{K}$

→ Sol generale + particolare

$$i_L(t) = i_{L\emptyset}(t) + i_{Lp}(t) = \hat{k}\, e^{-\frac{R_B}{L}t} + 0.4$$

→ Trovo k → Cond iniziali

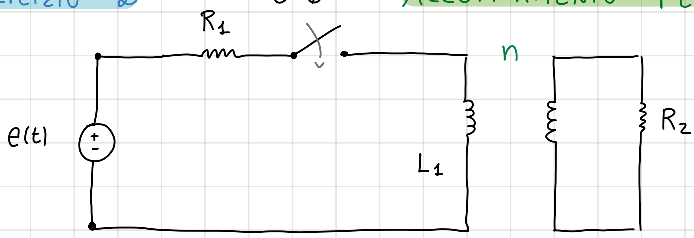
$i_L(0^-) = \emptyset$ → Continuità delle **grandezze di stato** → $i_L(0^+) = \emptyset$

$$\begin{cases} i_L(t) = \hat{k}\, e^{-\frac{R_B}{L}t} + 0.4 \\ i_L(0^+) = 0 \end{cases} \rightarrow \quad i_L(0) = \hat{k}\, e^{-\frac{R_B}{L}\emptyset} + 0.4 = 0 \rightarrow \quad \hat{k} = -0.4$$

→ SOLUZIONE

$$i_L(t) = -0.4\, e^{-\frac{R_B}{L}t} + 0.4 = 0.4\left(1 - e^{-\frac{R_B}{L}t}\right)$$

Esercizio 2 — ACCOPPIAMENTO PERFETTO



DATI

$e(t) = E = 20\,V$

A $R_1 = 10\,\Omega$ B $R_2 = 0.2\,\Omega$

C $L_1 = 50\,mH$ D $L_2 = 2\,mH$

Mutua induzione
$\downarrow$
$M = 10\,mH$
E

Q_1: v_{R_2} per $t>0$ Q_2: P_E A regime

$$K \triangleq \frac{M}{\sqrt{L_1 L_2}} \overset{?}{=} 1 \rightarrow \text{Accoppiamento Perfetto} \Leftrightarrow M = \sqrt{L_1 \cdot L_2}$$

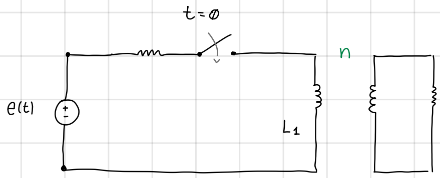
$$\rightarrow 10\,mH \overset{?}{=} \sqrt{50\,mH \cdot 2\,mH} \Rightarrow 10\,mH = \sqrt{\cancel{100}\,mH} \rightarrow A.P. \checkmark$$

Se A.P. $\rightarrow$ $n = \frac{M}{L_1} = \frac{L_2}{M} = 5$ n

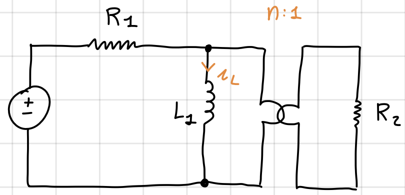
$\uparrow$
Rapporto di Conversione

$t < 0$

Circuito a riposo $\rightarrow$ $v_{R_2} = \emptyset$



$\longrightarrow$



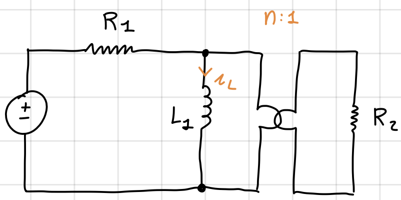
N.B. per $t<0$ il circuito e' Aperto

$\boxed{i_L(0^-) = \emptyset}$

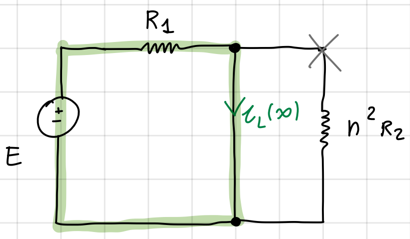
$t \rightarrow \infty$

$$\boxed{Z_1 = Z_2\, n^2}$$
$\uparrow$

Usiamo la FORMULA DEL TRASPORTO DI IMPEDENZA PRIMARIO per trasformare...



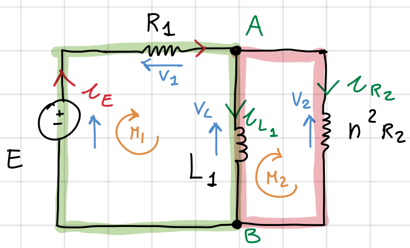
$\longrightarrow$



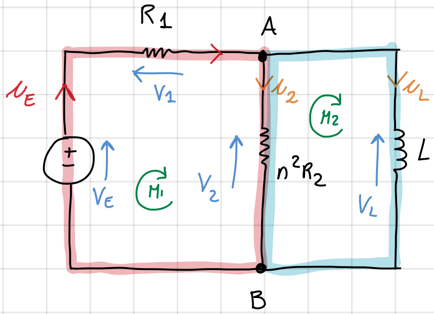
$$\rightarrow i_E(\infty) = \frac{E}{R_1} = 2\,A \Rightarrow P_E^e(\infty) = E \cdot i_E = 20 \cdot 2 = 40\,W$$

Ans 1

$t > 0$



$\equiv$



$$\begin{cases} -v_E + v_1 + v_2 = 0 \\ -v_2 + v_L = 0 \\ -i_E + i_2 + i_L = 0 \\ v_L = L\,\dot{i}_L \end{cases} \longrightarrow \begin{cases} -E + i_E R_1 + i_2 \cdot n^2 R_2 = 0 \\ -i_2 \cdot n^2 R_2 + L\,\dot{i}_L \\ i_E = i_2 + i_L \end{cases} \longrightarrow i_2 = \frac{L\,\dot{i}_L}{n^2 R_2}$$

$$\Rightarrow -E + \left(\frac{L\,\dot{i}_L}{n^2 R_2} + i_L\right) R_1 + \frac{L\,\dot{i}_L}{n^2} = \emptyset \longrightarrow -E + \frac{L R_1}{n^2 R_2}\dot{i}_L + i_L R_1 + L\,\dot{i}_L = \emptyset$$

Eq Diff

$$\longrightarrow \boxed{L_1\left(\frac{R_1}{n^2 R_2} + 1\right)\dot{i}_L + i_L R_1 = E} \longrightarrow \dot{i}_L + \frac{i_L R_1}{L_1\left(\frac{R_1}{n^2 R_2} + 1\right)} = \emptyset$$

$$\longrightarrow \lambda = \frac{i_2 R_1}{\frac{L_1 R_1}{n^2 R_2} + L_1} = \emptyset \longrightarrow \lambda = \frac{i_L R_1}{\frac{L_1 R_1 + L_1 n^2 R_2}{n^2 R_2}} \longrightarrow \lambda = \frac{(R_1)(n^2 R_2)}{(L_1 R_1) + (L_1 n^2 R_2)} i_L$$

$$\lambda = -\frac{1}{\tau}$$

trovare τ alternativamente dal circuito...

$$\tau = \frac{L_1}{R_1 /\!/ (n^2 R_2)} = \frac{L_1}{\frac{R_1 \cdot n^2 R_2}{R_1 + n^2 R_2}} = \frac{L_1 (R_1 + n^2 R_2)}{R_1 \; n^2 R_2} \Rightarrow \tau = \frac{L_1 R_1 + L_1 n^2 R_2}{R_1 \; n^2 R_2}$$

$\Rightarrow$ Posso scrivere l'eq diff (O.A.) come $\dot{i}_L + \frac{1}{\tau} i_L = \emptyset$

$$\longrightarrow \dot{i}_L + \frac{i_L}{\tau} = 0 \longrightarrow \int \frac{di_L}{dt} = -\frac{1}{\tau}\int i_L \longrightarrow \int di = -\frac{1}{\tau} i_L \int dt$$

$$\longrightarrow \int \frac{1}{i_L} di_L = -\frac{1}{\tau}\int dt \longrightarrow \ln(i_L) = -\frac{1}{\tau} t + K \longrightarrow i_L(t) = \hat{K} e^{-\frac{1}{\tau} t} \quad i_L(t)$$

Determino la Costante K

$$\begin{cases} i_L(t) = \hat{K} e^{-\frac{1}{\tau} t} + i_p(\infty) = \hat{K} e^{-\frac{t}{\tau}} + 2 \\ i_L(0^+) = i_L(0^-) = \emptyset \end{cases} \longrightarrow i_L(0) = \hat{K} e^{-\frac{\emptyset}{\tau}} + 2 = \emptyset$$

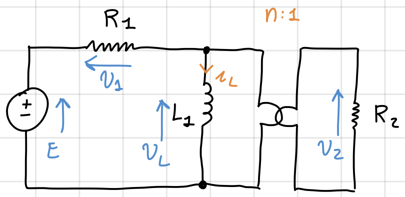
$$\Rightarrow \hat{K} = -2 \quad \hat{K}$$

$$\Rightarrow i_L(t) = -2 e^{-\frac{t}{\tau}} + 2 = \boxed{2\left(1 - e^{-\frac{t}{\tau}}\right)} \quad i_L(t)$$

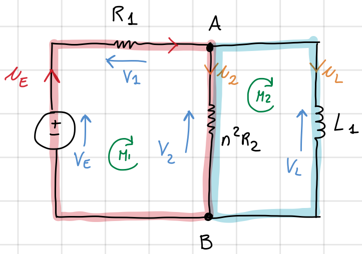
$$\longrightarrow \dot{i}_L(t) = 2\left[+\frac{1}{\tau} e^{-\frac{t}{\tau}}\right]$$

Q_1: $v_{R_2} = ?$

C



C'



dalla $M_2(c')$ $\rightarrow$ $-v_2' + v_{L_1}' = 0$ $\rightarrow$ $v_2' = v_{L_1}'$ $\rightarrow$ $v_2' = L \cdot \dot{i}_L$

$\rightarrow$ Derivo la soluzione $\rightarrow$ $\dot{i}_L(t) = \frac{2}{\tau} e^{-\frac{t}{\tau}} = \underline{133.\bar{3}\, e^{-6.67t} \quad A}$

$\Rightarrow v_2' = L \cdot \dot{i}_L = L \cdot 133.3\, e^{-\frac{t}{.133}} = \underline{6.67\, e^{-6.67t}} \quad \checkmark$

* i risultati numerici sono errati

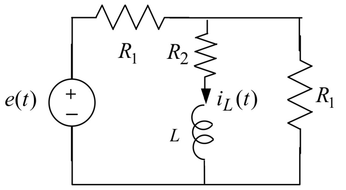
Dalla Formula del trasporto:

$$v_{R_2} = \frac{v_{R_2}'}{n} = \underline{1.\bar{3}\, e^{-6.67t}} \quad V$$

Ans 2

ESERCIZIO 10.1

Considerato il seguente circuito nel quale all'istante $t = 0$ il generatore inverte la sua polarità, calcolare la corrente nell'induttore per ogni t.

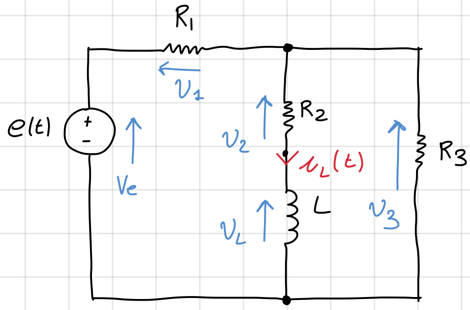


$$e(t) = \begin{cases} 10\ V & t < 0 \\ -10\ V & t > 0 \end{cases}$$

$R_1 = 10\,\Omega$

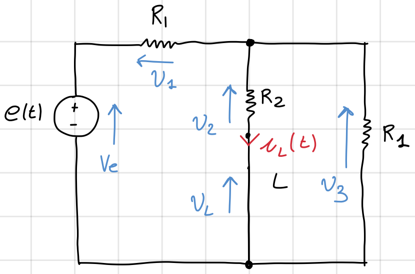
$R_2 = 20\,\Omega$

$L = 2\,mH$



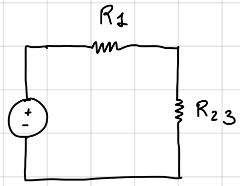
$t < 0$

STAZIONARIO

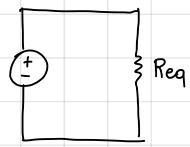


$E_0 = 10\ V$

A: $R_1 = 10\,\Omega$ B: $R_2 = 20\,\Omega$ C: $L = 2\,mH$



E



$R_{eq} = (R_2 \parallel R_1) + R_1 = 16.67\,\Omega$

D

$$\Rightarrow i_E = \frac{E_0}{R_{eq}} = \frac{3}{5} = 0.6\,A \qquad i_E$$

$$\Rightarrow i_L = i_E \cdot \frac{R_1}{R_1 + R_2} = 1/5 = 0.2\,A$$

i_L per $t<0$

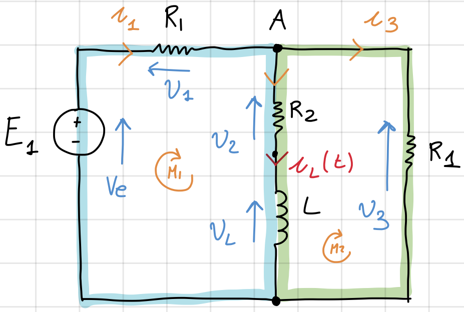
$t \to \infty$

$$i_{E_1}(\infty) = -0.6\,A \to i_L(\infty) = -0.2\,A$$

i_L per $t \to \infty$

$t > 0$

con $E_1 = -10\,V$



$$\begin{cases} -v_E + v_1 + v_2 + v_L = 0 \\ -v_L - v_2 + v_3 = 0 \\ -i_1 + i_L + i_3 = 0 \end{cases} \to \begin{cases} -E + i_1 R_1 + i_L R_2 + L\dot{i}_L = 0 \\ -L\dot{i}_L - i_L R_2 + i_3 R_1 = 0 \\ i_1 = i_L + i_3 \end{cases}$$

Dalla (2) $i_3 = \frac{L}{R_1}\dot{i}_L + \frac{R_2}{R_1} i_L$

$\to$ Nella (1) $\to$ $R_1 i_L + R_1 i_3 + i_L R_2 + L\dot{i}_L = E \to R_1 i_L + L\dot{i}_L + R_2 i_L + i_L R_2 + L\dot{i}_L = E$

$$2L\dot{i}_L + i_L(R_1 + R_2 + R_2) = E \to 2L\dot{i}_L + i_L(2R_2 + R_1) = E \to \dot{i}_L + i_L\left(\frac{2R_2 + R_1}{2L}\right) = E$$

$$\to \lambda + \frac{2R_2 + R_1}{2L} = 0 \to \lambda = -\frac{2R_2 + R_1}{2L} \qquad \lambda$$

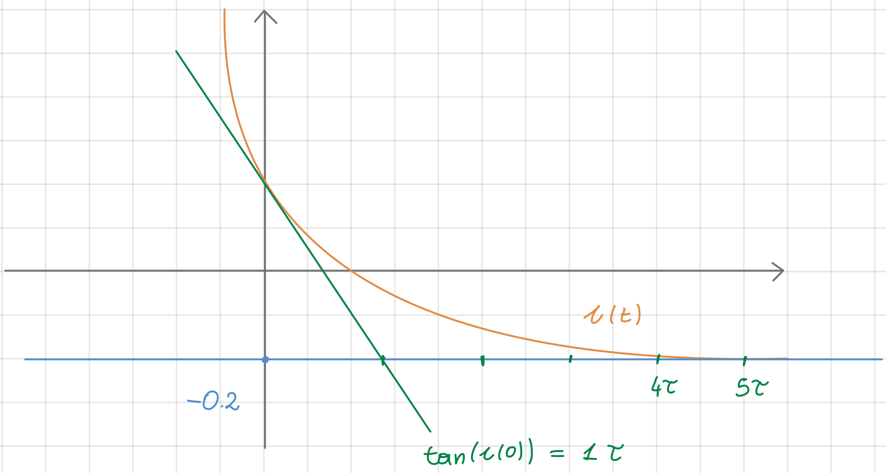
$$\Rightarrow \tau = \frac{2L}{2R_2 + R_1} \qquad \tau$$

$$\Rightarrow i_L(t) = K e^{-\lambda t} + i_P = K e^{-\frac{2R_2 + R_1}{2L}t} - 0.2$$

Trovo le C.I. : Cauchy

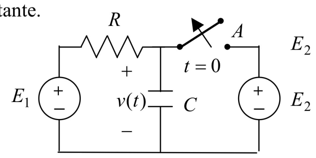
$$\begin{cases} i_L(t)\ K e^{-\frac{R_1-R_2}{2L} t} - 0.2 \\ i_L(0^+) = i_L(0^-) = 0.2\ A \end{cases} \Rightarrow i_L(0) = K e^{0} - 0.2 = 0.2 \Rightarrow K = 0.4$$

$$\Rightarrow i_L(t) = 0.4\, e^{-\frac{2R_2+R_1}{2L} t} - 0.2 \cong \boxed{0.4\, e^{-12.5\times 10^{3}\, t} - 0.2}$$ $i(t)$ per $t > \emptyset$

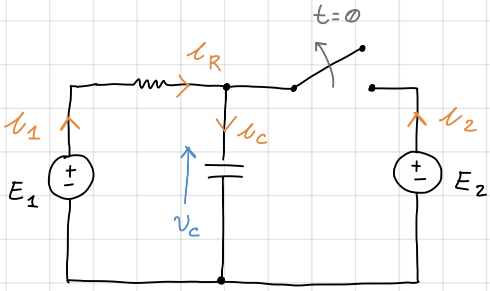


ESERCIZIO 10.2

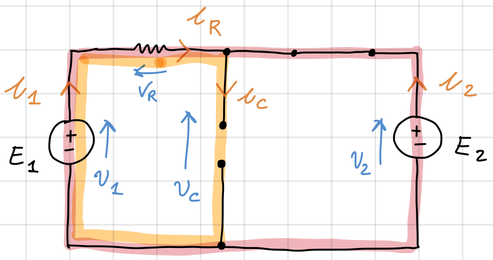
Nel seguente circuito all'istante $t = 0$ si apre l'interruttore A. Calcolare la tensione sul condensatore $v(t)$ per ogni istante.



$E_1 = 8\,V$, $E_2 = 2\,V$
$R = 10\,k\Omega$, $C = 2\,mF$



$\boxed{t<0}$ Regime



1 Sola maglia $\rightarrow$ $LKT_M: \begin{cases} -v_1 + v_R + v_2 = 0 \\ LKC_A: -i_R - i_2 = 0 \end{cases}$ $\quad i_R = i_1$

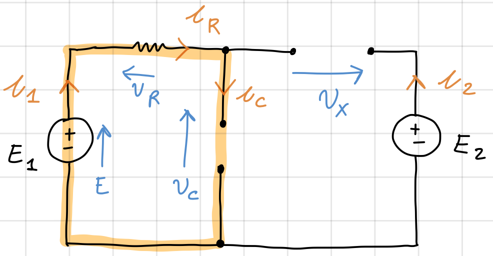
$$\begin{cases} -E_1 + i_1 R_1 + E_2 = 0 \quad \rightarrow \quad -8 + 2 + i_1 R_1 = 0 \\ i_1 = -i_2 \end{cases}$$

$$\rightarrow \quad i_1 = \frac{6}{R_1} = 0.6\ mA$$

$$LKT_{M_2}: \quad -v_1 + v_R + v_C = 0 \rightarrow v_C = v_1 - v_R = E_1 - i_1 R_1 = 2\ V$$

v_C $t<0$
Ans 1

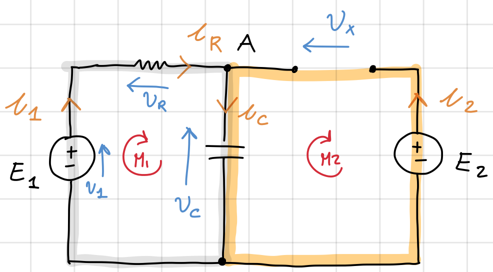
$\boxed{t \rightarrow \infty}$ Regime



v_C per $t \rightarrow \infty$

$$-v_E + \cancel{v_R} + v_C = 0 \rightarrow v_C = E$$
$i_1 = 0$ $\quad$ 8V $\quad$ Ans 2

$\boxed{t>0}$



$$i_C = C\,\dot{v}_C$$

$$\begin{cases} -E_1 + v_R + v_C = 0 \\ -v_C + \cancel{v_x} + E_2 = 0 \quad (i=0) \\ -i_R + i_C = 0 \end{cases} \rightarrow \quad \begin{matrix} i_C R + v_C = E_1 \\ v_C = E_2 \leftarrow ?? \\ i_R = i_C \end{matrix}$$

$$\rightarrow \quad \dot{v}_C RC + v_C = E_1 \quad \rightarrow \quad \dot{v}_C + \frac{1}{RC} v_C = E_1$$

$$\rightarrow \quad \lambda + \frac{1}{RC} = 0 \quad \rightarrow \quad \lambda = -\frac{1}{RC} \quad \lambda$$

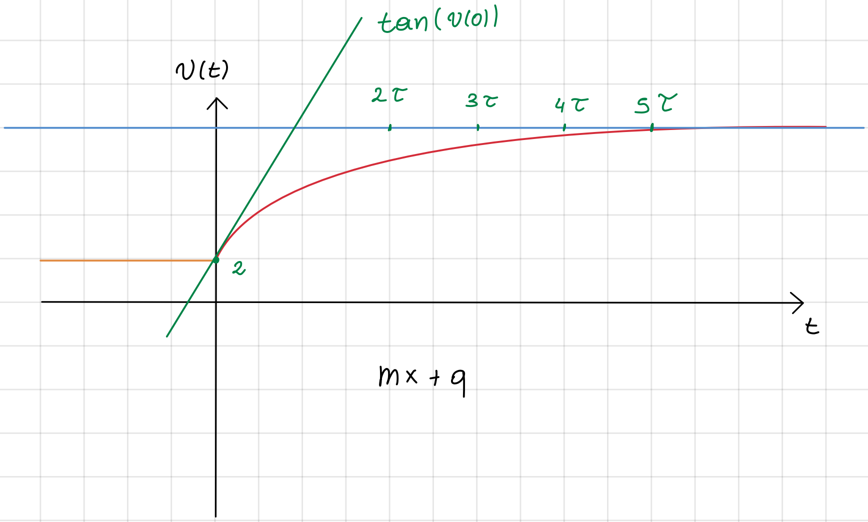
$$\rightarrow \quad \int \frac{dv_C}{dt} = -\frac{1}{RC}\int v_C \quad \rightarrow \quad \int \frac{1}{v_C}\,dv_C = -\frac{1}{RC}\int dt$$

$$\rightarrow \quad \ln(v_C) = -\frac{1}{RC}t + K \quad \rightarrow \quad v_C = e^{-\frac{1}{RC}t} \cdot \hat{K} \quad \lambda$$

Determino $\hat{K}$ ...

$$\begin{cases} v_c(t) = e^{\lambda t}\hat{K} + v_{cp} = \hat{K}e^{\lambda t} + 8 \\ v_c(0^+) = v_c(0^-) = 2 \end{cases} \Rightarrow \quad v_c(0^+) = \hat{K}e^{0} + 8 = 2 \rightarrow \hat{K} = 2-8 = -6\,V$$

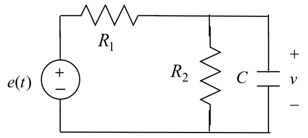
$$\Rightarrow \boxed{v_c(t) = -6\,e^{-\frac{1}{RC}t} + 8}$$ $v_c(t)$ per $t>0$



$mx + q$

ESERCIZIO 10.3

Il circuito in esame è in regime stazionario per $t<0$. Valutare la tensione $v(t)$ per $t>0$.

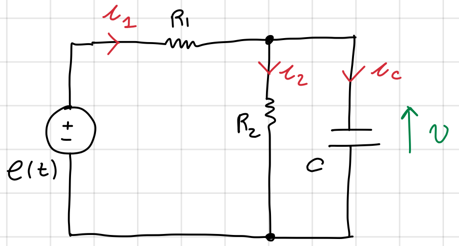


$e(t) = 50\,V \quad t>0$

$v(t=0) = 10\,V, \quad C = 1\,mF$

$R_1 = 20\,\Omega, \quad R_2 = 24\,\Omega.$

Risultato: $v(t) = 27.3 - 17.3e^{-91.7t}\ V$.



DATI

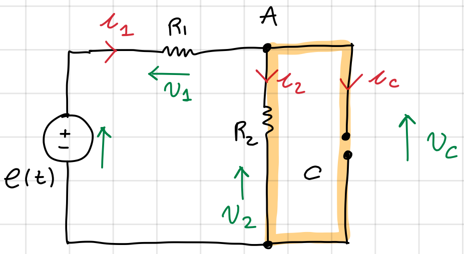
$e(t) = 50\,V$ E $\qquad t>0$

C.I. $v(t=0) = 10\,V \qquad t=0$

$C = 1\,mF \qquad R_1 = 20\,\Omega \quad R_2 = 24\,\Omega$

C $\qquad$ A $\qquad$ B

$\boxed{t \to \infty}$



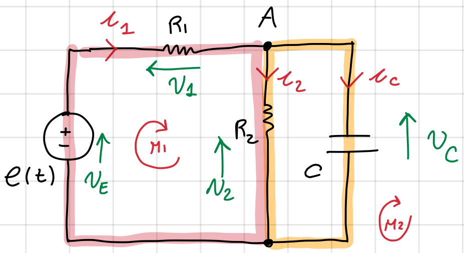
$$\begin{cases} LKC_A: & -i_1 + i_2 + i_c = 0 \rightarrow i_1 = i_2 \\ LKT_{M_1}: & -v_2 + v_c = 0 \rightarrow v_c = v_2 \\ LKT_{M_2}: & -v_E + v_1 + v_2 = 0 \rightarrow v_2 = E - v_1 = E - i_1 R_1 \end{cases}$$

↑?

$v_c(\infty)$ Regime

$$i_E = \frac{E}{Req} = \frac{E}{R_1+R_2} = 1.136\,A \rightarrow v_2 = v_c = E - i_1 R_1 = 27.27\,V$$

$\boxed{t>0}$ $\qquad$ C.I. $\quad v(0^+) = 10\,V$



$$LKT \begin{cases} -v_E + v_1 + v_2 = 0 \rightarrow i_1 R_1 + i_2 R_2 = E \\ -v_2 + v_c = 0 \rightarrow v_c = v_2 \\ -v_E + v_1 + v_c = 0 \rightarrow i_1 R_1 + v_c = E \end{cases}$$

$$\Rightarrow \cancel{i_1 R_1} + i_2 R_2 = \cancel{i_1 R_1} + v_c$$

$$\rightarrow i_2 R_2 = v_c \rightarrow i_2 = \frac{1}{R_2} v_c$$

$$\rightarrow i_1 R_1 + v_c = E$$

LKC $\quad -i_1 + i_2 + i_c = 0 \rightarrow i_1 = i_2 + i_c$

R.C $\quad i_c = C \cdot \dot{v}_c$

$$\rightarrow i_2 R_1 + i_c R_1 + v_c = E \rightarrow \frac{R_1}{R_2} v_c + C R_1 \dot{v}_c + v_c = E \rightarrow C R_1 \dot{v}_c + v_c\left(\frac{R_1}{R_2} + 1\right) = E$$

$$\rightarrow \dot{v}_c + v_c \left(\frac{\frac{R_1}{R_2} + 1}{C R_1}\right) = \frac{E}{R_1 C} \rightarrow \dot{v}_c + v_c\left(\frac{R_1 + R_2}{C R_1 R_2}\right) = \frac{E}{R_1 C}$$

$$\lambda + \frac{R_1 + R_2}{C R_1 R_2} = 0 \rightarrow \lambda = -\frac{R_1 + R_2}{C R_1 R_2}$$

$$\int \frac{dv_c}{dt} = -\frac{R_1+R_2}{CR_1R_2}\int v_c \quad \rightarrow \quad \int \frac{1}{v_c}\,dv_c = -\frac{R_1+R_2}{CR_1R_2}\int dt$$

$$\rightarrow \ln(v_c) = -\frac{R_1+R_2}{CR_1R_2}\,t + K \quad \rightarrow \quad v_c = e^{-\frac{R_1+R_2}{R_1R_2C}t}\,\hat{K}$$

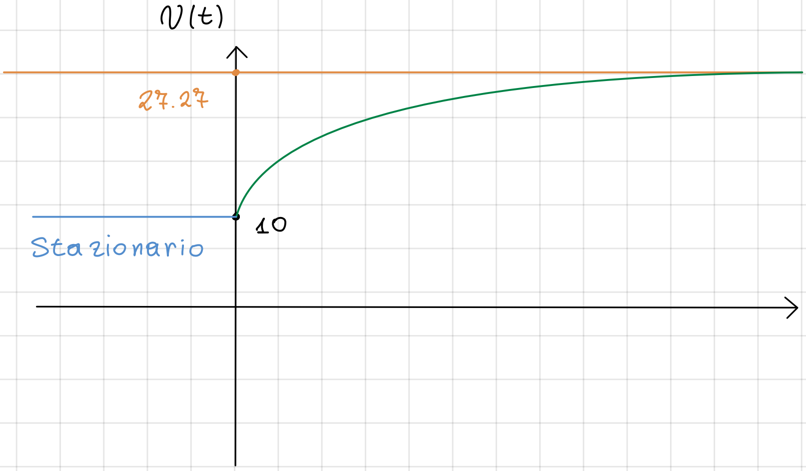
$$\Rightarrow v_c(t) = v_{c0}(t) + v_{cp}(t) = \hat{K}\,e^{-\frac{R_1+R_2}{R_1R_2C}t} + 27.27$$

Trovo $\hat{K}$ → Cauchy

$$\begin{cases} v(t) = \hat{K}\,e^{-\frac{R_1+R_2}{R_1R_2C}t} + 27.27 \\ v(0) = 10 \end{cases} \Rightarrow v(0) = \hat{K}\,e^{\lambda 0} + 27.27 = 10 \rightarrow \hat{K} = -17.27$$

$$\Rightarrow v_c(t) = 27.27 - 17.27\,e^{-\frac{R_1+R_2}{R_1R_2C}t} = 27.27 - 17.27\,e^{-91.67\,t}$$

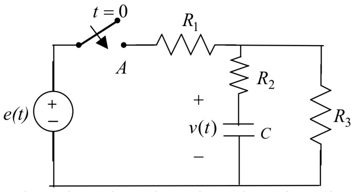
$v_c(t)$ per $t>0$



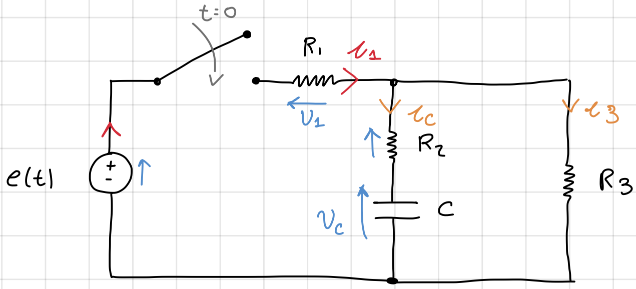
ESERCIZIO 10.4

Il seguente circuito è a riposo fino a $t = 0$, istante in cui si chiude l'interruttore A. Calcolare:

a) la costante di tempo τ del circuito;
b) la tensione ai capi del condensatore per $t > 0$.



$e(t) = 10\cos(\omega t)$
$\omega = 100 rad/s$
$R_1 = 20\,\Omega$, $R_2 = 5\,\Omega$
$R_3 = 10\,\Omega$, $C = 1\,mF$

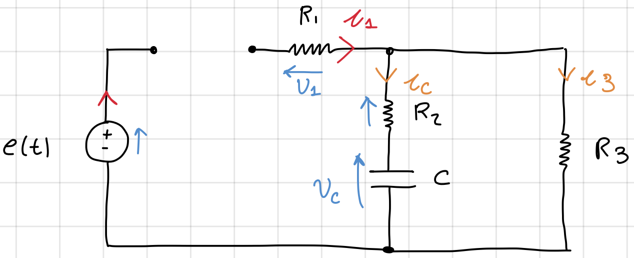


$$e(t) = 10\cos(\omega t) \underset{\omega = 100\, r/s}{=} 10\cos(100t) \rightleftharpoons 10\, e^{0t} = 10\,V \quad \bar{E}$$

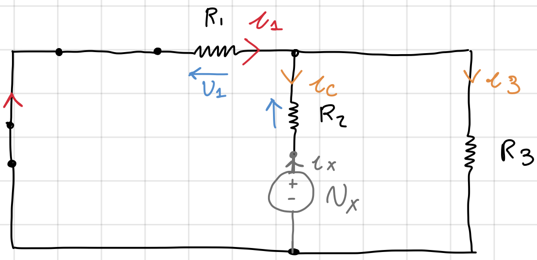
$R_1 = 20\,\Omega$	$\rightleftharpoons$	$\dot{Z}_1 = 20\,\Omega$	A
$R_2 = 5\,\Omega$	$\rightleftharpoons$	$\dot{Z}_2 = 5\,\Omega$	B
$R_3 = 10\,\Omega$	$\rightleftharpoons$	$\dot{Z}_3 = 10\,\Omega$	C
$C = 1\,mF$	$\rightleftharpoons$	$-\frac{j}{100 \cdot 1m} = -10j$	D

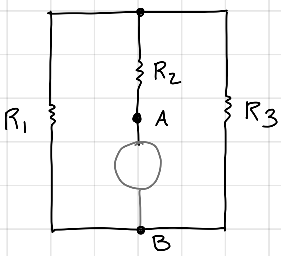
$\boxed{t<0}$ Stazionario, Riposo

$\boxed{V_c(0^-) = \emptyset}$ C.I.



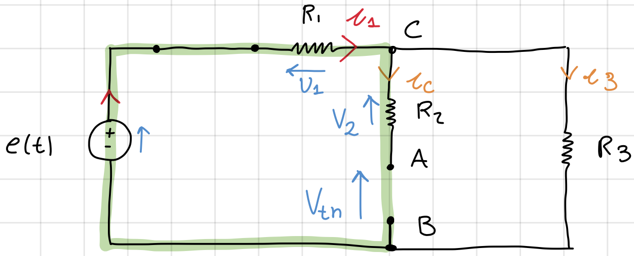
$\boxed{t>0}$ Trovo l'eq di thevenin (condensatore)





$$R_{th} = (R_1 // R_3) + R_2 = \frac{35}{3} = 11.67\,\Omega \quad R_{th}$$

$$\boxed{\tau = R_{th} \cdot C} = 0.01167\,s = 11.67\,mS \quad \tau$$

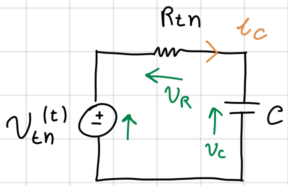


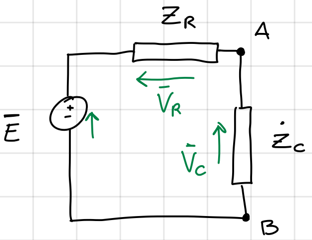
$$V_{th} = V_{CB}$$

$$-V_e + V_1 + \cancel{V_2} + V_{th} = 0 \quad \underset{i_2 = 0}{} \quad \rightarrow V_1 + V_{th} = e(t)$$

$$\rightarrow V_{th} = e(t) - R_1 \cdot i_1 \qquad \text{ma} \quad i_1 = \frac{e(t)}{R_1 + R_3} \Rightarrow V_{th} = e(t) - \frac{R_1\, e(t)}{R_1 + R_3} = e(t)\left[\frac{\cancel{R_1} + R_3 - \cancel{R_1}}{R_1 + R_3}\right]$$

$$\rightarrow V_{th} = \frac{e(t) \cdot R_3}{R_1 + R_3} \quad V_{th}(t)$$





$\dot Z_C = -10j$

$\dot Z_R = \frac{35}{3}\,\Omega$

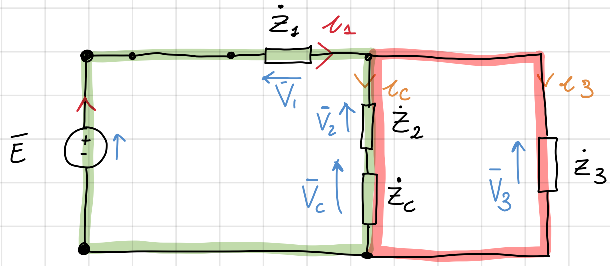
$$-v_{tn} + v_R + v_C = 0 \Rightarrow i_C \cdot R_{tn} + v_C = v_{tn} \quad \text{ma } i_C = C\,\dot v_C \Rightarrow C R_{tn}\,\dot v_C + v_C = v_{tn}$$

$$\Rightarrow \dot v_C + \frac{1}{CR_{tn}} v_C = \frac{v_{tn}}{CR_{tn}} \Rightarrow \lambda + \frac{1}{CR_{tn}} = 0 \Rightarrow \lambda = -\frac{1}{CR_{tn}} \quad \lambda$$

$$\Rightarrow v_C(t) = K\,e^{-\frac{1}{CR_{tn}}t} + v_{cp}(t)$$

circuito per $t \to \infty$

$\boxed{t \to \infty}$



$R_1 = 20\,\Omega \rightleftarrows \dot Z_1 = 20\,\Omega$ A

$R_2 = 5\,\Omega \rightleftarrows \dot Z_2 = 5\,\Omega$ B

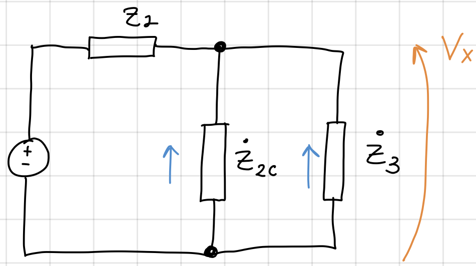
$R_3 = 10\,\Omega \rightleftarrows \dot Z_3 = 10\,\Omega$ C

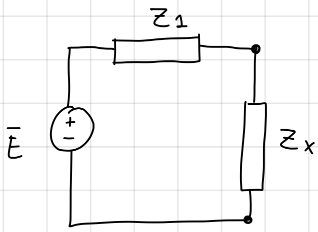
$C = 1\text{ mF} \rightleftarrows -\frac{j}{100 \cdot 1m} = -10j$ D

$\bar E = 10\text{ V}$

$$\Rightarrow \dot Z_{2C} = \dot Z_2 + \dot Z_C = 5 - 10j \quad , \quad \dot Z_x = \dot Z_{2C} // \dot Z_3 \quad L$$

$$\bar I = \frac{\bar E}{\dot Z_1 + \dot Z_x} = K$$



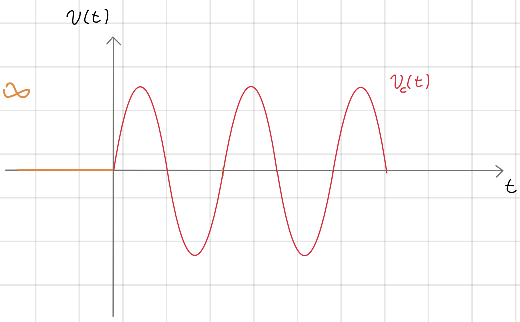


$$\Rightarrow \bar V_x = \dot Z_x \cdot \bar I = J$$

$$V_C = V_x \cdot \frac{\overset{D}{\dot Z_C}}{\underset{D}{\dot Z_C} + \underset{B}{\dot Z_2}} = 1.41 - 1.65j = 2.17 \angle -0.86 = 2.17\,e^{-0.86j}$$

$v_c(t)$ per $t \to \infty$

$$V_x \rightleftarrows \boxed{v_c(t) = 2.17\cos(100t - 0.86)}$$



TORNANDO A $\boxed{t>0}$ ...

$$v_c(t) = K\,e^{-\frac{1}{CR_{tn}}t} + v_{cp}(t) = \begin{cases} K\,e^{-\frac{1}{CR_{tn}}t} + 2.17\cos(100t - 0.86) \\ v_c(0) = 0 \end{cases}$$

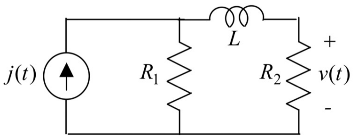
RADIANTI

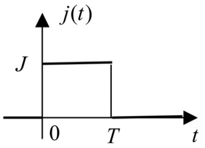
$$\Rightarrow v_C(0) = K + 2.17\cos(-0.86) = 0 \Rightarrow K = -2.17\cos(-0.86) = -1.42$$

$$\Rightarrow \boxed{v_c(t) = -1.42\,e^{-\frac{1}{CR_{tn}}t} + 2.17\cos(100t - 0.86)} = -1.42\,e^{-85.7t} + 2.17\cos(100t - 0.86)\ \text{V}$$

ESERCIZIO 10.5

In figura è riportato lo schema equivalente di un grilletto elettronico per pistola. L'uscita del sistema è il segnale di tensione $v(t)$ prelevato ai capi di R_2. Determinare tale segnale per $0 < t < 0.3\ s$.

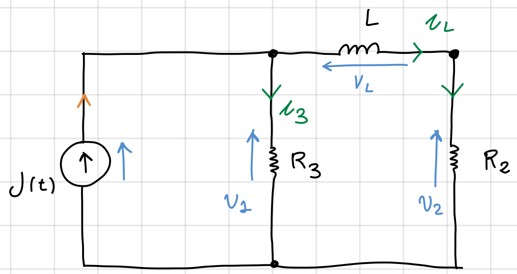


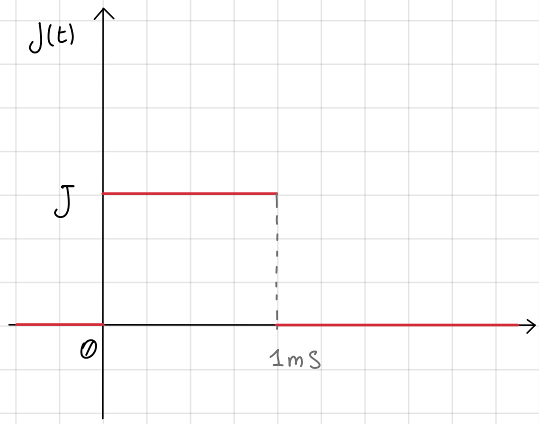


$J = 40\ A$, $T = 1\ ms$

$R_1 = 30\ \Omega$, $R_2 = 20\ \Omega$

$L = 50\ mH$





$\boxed{t<0}$

$\hookrightarrow J(t)=0 \rightarrow v_2(t<0)=\varnothing$

$\boxed{v_L(0^-) = \varnothing}$ c.i.

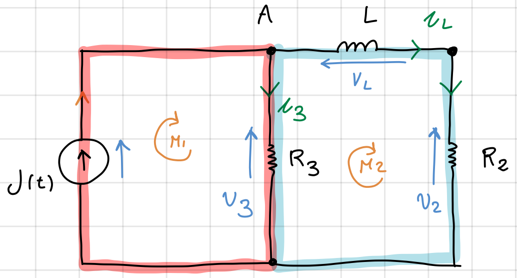
$$v_L(t) = L\frac{di}{dt}$$

A $R_3 = 30\Omega$ $\quad$ $J = 40A$

B $R_2 = 20\Omega$ $\quad$ $T = 1\ mS$

C $L = 50\ mH$

$\boxed{t>0}$ Transitorio



$$\begin{cases} -v_J + v_3 = 0 \\ -v_3 + v_L - v_2 = 0 \end{cases} \rightarrow i_3 R_3 + L\dot{i}_L - i_L R_2 = \varnothing$$

$$-J + i_3 + i_L = 0 \rightarrow i_3 = J - i_L$$

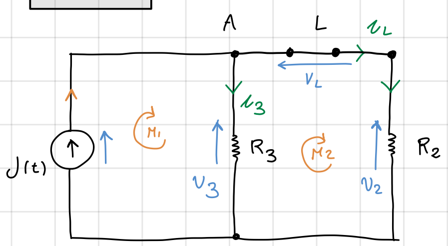
$$\Rightarrow J R_3 - i_L R_3 + L\dot{i}_L - i_L R_2 = 0 \rightarrow L\dot{i}_L + i_L(-R_2 - R_3) = -J R_3$$

$$\rightarrow \dot{i}_L + i_L\left(-\frac{R_2+R_3}{L}\right) = -\frac{R_3}{L}J \rightarrow \lambda - \frac{R_2+R_3}{L} = 0 \rightarrow \lambda = \frac{R_2+R_3}{L} = 1000\ s^{-1}$$

!! $\lambda > 0$

$$\Rightarrow i_L(t) = K e^{\lambda t} + i_{Lp}(t)$$ Regime

$\boxed{t \rightarrow \infty}$



$$\Rightarrow R_{eq} = R_3 // R_2 = 12\Omega$$

$$\Rightarrow V_J = J \cdot R_{eq} = 480\ V$$ c.i.

Torno a $\boxed{t>0}$

$$\Rightarrow i_L(t) = K e^{\lambda t} + i_{Lp}(t)$$ $\rightarrow$ A noi serve $v(t)$ $\rightarrow$ $v_L(t) = K e^{\lambda t} + v_{Lp}(t)$

$$= K e^{\lambda t} + 480$$

Trovo K

$$\begin{cases} v_L(t) = K e^{\lambda t} + 480 \\ v_L(0^+) = v_L(0^-) = 0 \end{cases} \rightarrow \quad v_L(0) = K e^{\lambda 0} + 480 = 0 \rightarrow K = -480$$

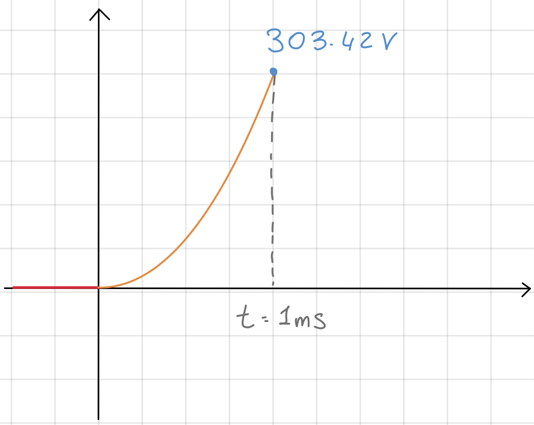
K

$$\Rightarrow \quad v_L(t) = -480\, e^{-\lambda t} + 480 = 480\left(1 - e^{-1000\, t}\right) \qquad \text{per } 0 < t < T$$

$$\Rightarrow \quad v_L(T) = 480\left(1 - e^{\overset{-1}{\cancel{1000 \cdot 1ms}}}\right) = 303.42 \ V$$

$$\Updownarrow$$

$$= 480\left(1 - e^{-1}\right)$$



$\Rightarrow$ Eq Diff per $t > T$

$$v_L(t) = v_{L0}(t) + \cancel{v_{Lp}(t)}$$

non c'e' forzamento!

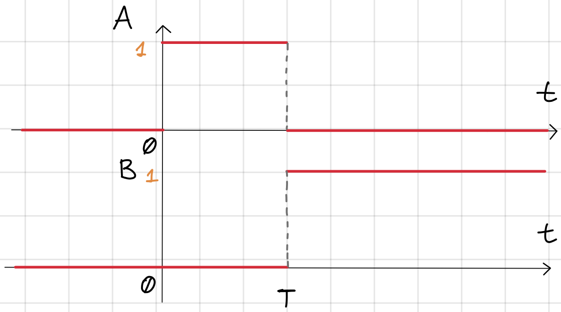
$$\rightarrow \begin{cases} v_L(t) = v_{L0}(t) = K\, e^{-1000\, t} \\ v_L(T^+) = v_L(T^-) = 480(1-e) \end{cases}$$

$$\rightarrow \quad v_L(T^+) = K\, e^{\overset{-1}{\cancel{-1000\, t}}} = 480(1-e) \rightarrow K_1 e^{-1} = 480(1 - e^{-1})$$

$$\hookrightarrow K_1 = 480\left(\frac{1}{e^{-1}} - \frac{\cancel{e^{-1}}}{\cancel{e^{-1}}}\right) = 480(e-1) \quad K_1$$

$$\Rightarrow \quad v_L(t) = 480(e-1)\, e^{-1000\, t} \ V \qquad \text{per } t > T$$

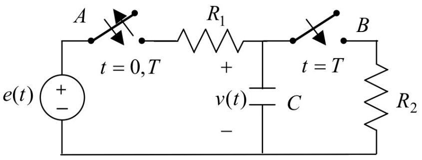
ESERCIZIO 10.6

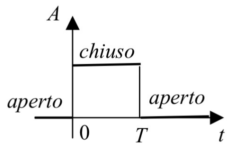


La seguente rete rappresenta lo schema elettrico equivalente del circuito di carica della stazione spaziale orbitante. La carica avviene tra l'istante $t=0$ e l'istante $t=T$, intervallo in cui l'interruttore A resta chiuso. Per $t>T$, invece, il condensatore C viene collegato al resto della rete attraverso la chiusura dell'interruttore B. Supponedo la rete a riposo per $t<0$, valutare:

a) la tensione sul condensatore $v(t)$ per $0<t<T$;

b) l'energia massima $W_{\max}$ erogabile da C per $t>T$;



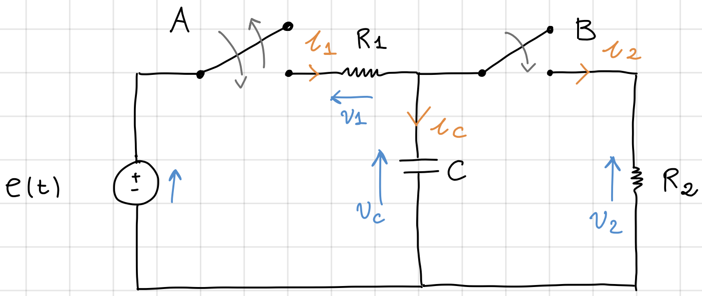


$e(t)=100\sin(20t)\ V$

$R_1=10\ \Omega$

$C=10\ mF$

$T=2\ s$



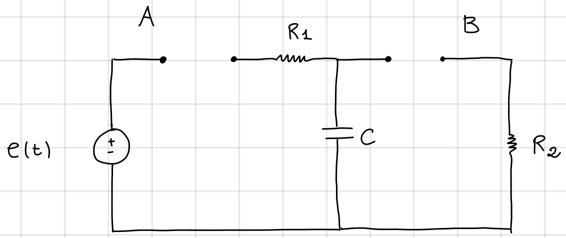
DATI

$e(t)=100\sin(20t) = 100\cos(20t-\frac{\pi}{2})\ V$

$R_1=10\ \Omega \qquad C=10\ mF \qquad T=2\ s$

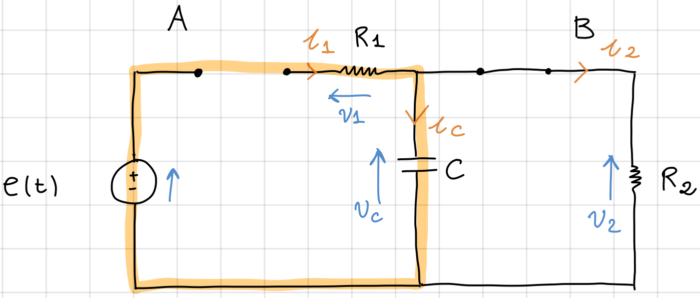
$t<0$

R.C. $\cdot\ i_c = C\,\dot{v}_c$



$$\begin{cases} v_c(t)=0 \\ i_c(t)=0 \end{cases} \quad \text{per} \quad t<0$$

$t \to \infty$



$-e(t)+\cancel{v_1}+v_c=0 \to v_c=e(t)$