

$$\vec{r}^o = \vec{r}_x^o + \vec{r}_y^o = R \cos \alpha + R \sin \alpha$$

Definisco  $\bar{\omega} = \frac{d\alpha}{dt}$

$$\Rightarrow \omega dt = d\alpha \Rightarrow \int_{t_0}^{t_f} \omega dt = \int_{\alpha_0}^{\alpha_f} d\alpha$$

$$\Rightarrow \omega(t_f - t_0) = \alpha_f - \alpha_0 \quad \text{pongo } \alpha_0 = t_0 = 0$$

$$\Rightarrow \alpha(t) = \omega \cdot t$$

$$\Rightarrow \vec{r}^o = \hat{i} R \cos(\omega t) + \hat{j} R \sin(\omega t)$$

Siccome  $\vec{v}^o = \frac{d\vec{r}^o}{dt} \Rightarrow \vec{v}^o = \frac{d\vec{r}^o}{dt} = -\omega R \sin(\omega t) \hat{i} + \omega R \cos(\omega t) \hat{j} = \omega R [\cos(\omega t) \hat{j} - \sin(\omega t) \hat{i}]$

$$\vec{a}^o = \frac{d\vec{v}^o}{dt} = -\omega^2 R \cos(\omega t) \hat{i} - \omega^2 R \sin(\omega t) \hat{j} = -\omega^2 R [\cos(\omega t) \hat{i} + \sin(\omega t) \hat{j}]$$

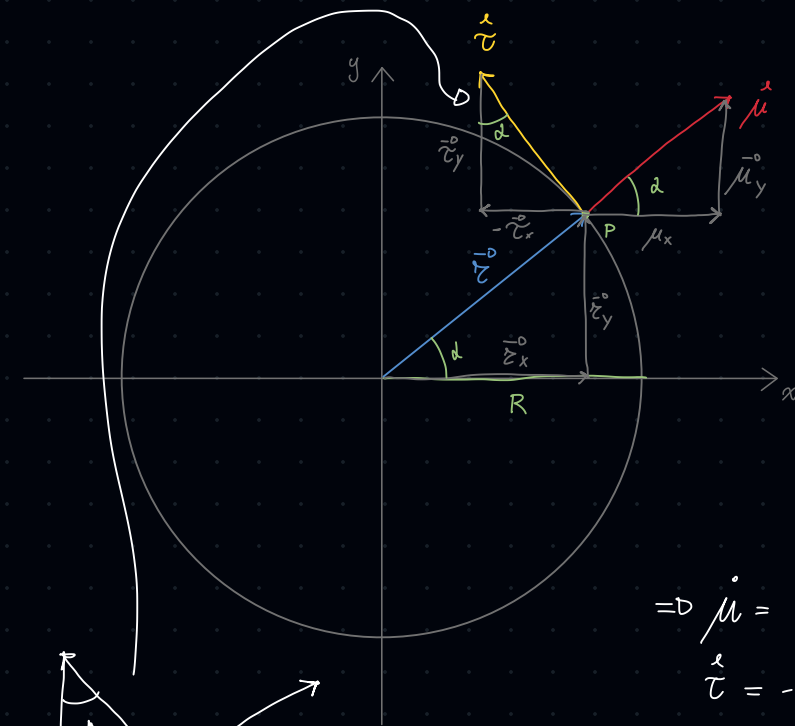
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 $= -\omega^2 R \vec{r}^o \quad (1) \quad \vec{a}^o$

$$|\vec{v}^o| = \sqrt{\omega^2 R^2 [\cos^2(\omega t) + \sin^2(\omega t)]} = \sqrt{\omega^2 R^2} = \omega R \quad |\vec{v}^o| \Rightarrow v = \omega R \Rightarrow \omega = \frac{v}{R}$$

$$|\vec{a}^o| = \sqrt{\omega^4 R^2 [\dots]} = \omega^2 R \quad \text{ma } \nearrow \quad a = \omega^2 R = \frac{v^2}{R^2} R = \left(\frac{v^2}{R}\right) \frac{|\vec{a}^o|}{c_P}$$

OSS. (1)  $\vec{a}^o = -\omega^2 R \vec{r}^o \Rightarrow$  Acc rivolta verso l'interno!

# Riferimento in movimento



$$\begin{cases} \hat{\mu} = \hat{i} \cos \alpha + \hat{j} \sin \alpha \\ \hat{\tau} = -\hat{i} \sin \alpha + \hat{j} \cos \alpha \end{cases}$$

$$\omega = \frac{d\alpha}{dt} \Rightarrow \alpha(t) = \omega t$$

$$\Rightarrow \begin{cases} \frac{d\hat{\mu}}{dt} = -\hat{i} \omega \sin(\omega t) + \omega \hat{j} \cos(\omega t) \\ \frac{d\hat{\tau}}{dt} = -\omega \hat{i} \cos(\omega t) - \omega \hat{j} \sin(\omega t) \end{cases}$$

$$\Rightarrow \dot{\hat{\mu}} = \omega [\hat{j} \cos(\alpha) - \hat{i} \sin(\alpha)] = \omega \hat{\tau}$$

$$\dot{\hat{\tau}} = -\omega [\hat{i} \cos(\alpha) + \hat{j} \sin(\alpha)] = -\omega \hat{\mu}$$

$$\begin{aligned} \vec{r} &= \hat{i} R \cos \alpha + R \hat{j} \sin \alpha = R [\hat{i} \cos \alpha + \hat{j} \sin \alpha] \\ &= R \hat{\mu} \end{aligned}$$

$$\Rightarrow \frac{d\vec{r}}{dt} = R \omega \hat{\tau} \quad \vec{v}$$

$$\frac{d\vec{v}}{dt} = R \omega [-\omega \hat{\mu}] = -R \omega^2 \hat{\mu} \quad \text{Acc. cp.}$$

$$\text{Se } \omega \neq \text{cost} \Rightarrow \frac{d\vec{v}}{dt} = R \frac{d\omega \hat{\tau}}{dt} = R \left[ \underbrace{\dot{\omega} \hat{\tau}}_{\vec{a}_{t\phi}} + \underbrace{\omega \dot{\hat{\tau}}}_{\vec{a}_{cp}} \right]$$

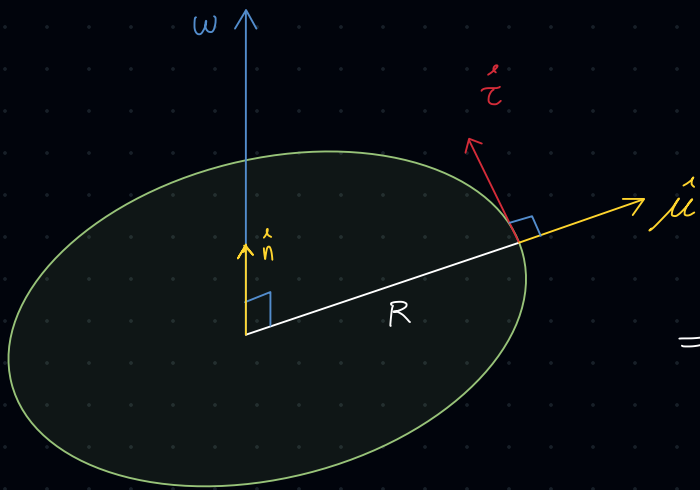
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 $-R \omega^2 \hat{\mu}$

$$\begin{aligned} 180 - 90 - \alpha &= 90 - \alpha \\ 90 &= 90 + 90 - \alpha = 180 - \alpha \\ 180 - 180 + \alpha &= \alpha \end{aligned}$$

Vettore  $\vec{\omega}$

$$\dot{\vec{\mu}} = \omega \hat{\tau}$$

se  $\vec{\mu} \parallel \hat{\tau} \Rightarrow$  l'unico modo per ottenere  $\hat{\tau}$  è  
con il prodotto vett Tra  $\hat{n}$  e  $\vec{\mu}$



$$\begin{aligned} \Rightarrow \dot{\vec{\mu}} &= \vec{\omega} \wedge \vec{\mu} = \omega \cdot \mu \cdot \sin(90) = \\ &= \omega [\text{Regola Mano D} \times] = \omega \hat{\tau} \end{aligned}$$

QED

$\Rightarrow$

$$\frac{d\vec{\mu}}{dt} = \omega \wedge \vec{\mu} \quad \forall \vec{\mu} \text{ VERSORE}$$