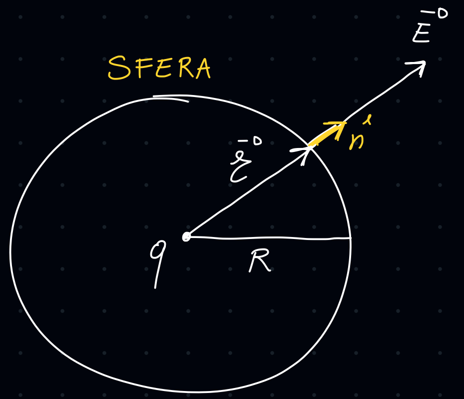
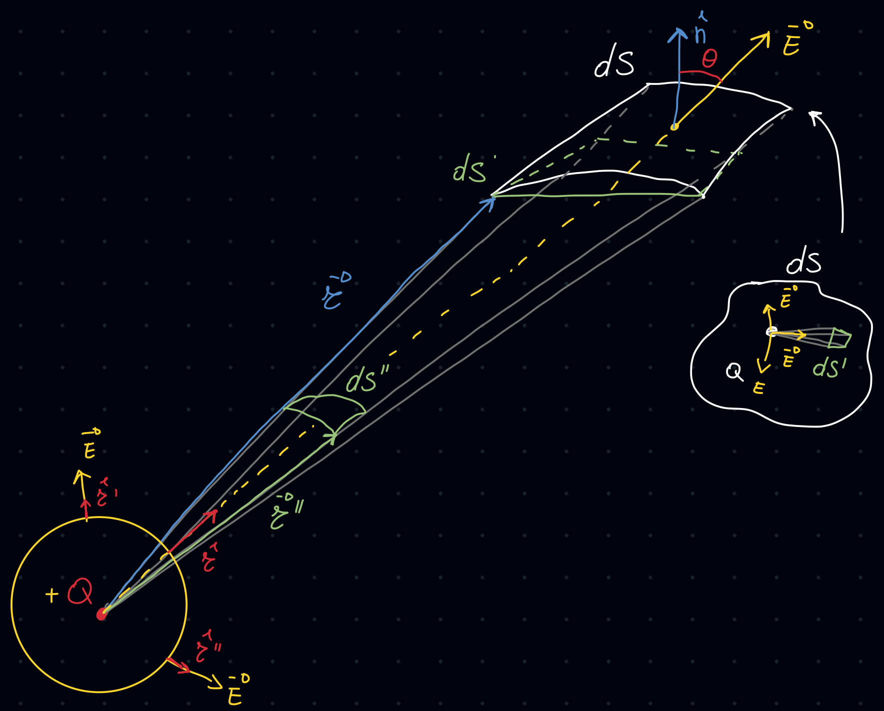



$$\Phi = \oint_S \vec{E}\cdot\hat{n}\,d\vec{s} = \oint \underbrace{\frac{1}{4\pi\varepsilon_0}}_{c}\cdot\underbrace{\overset{c}{\frac{q}{r^2}}}_{c}\cdot\hat{r}\cdot\hat{n}\,d\vec{s}$$

$$\Rightarrow \phi = \frac{q}{4\pi\varepsilon_0 r^2}\int \hat{r}\cdot\hat{n}\cdot ds \Rightarrow \hat{r}\cdot\hat{n} = r\cdot n\cdot\cos(0) = 1$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\int ds = \frac{q}{\cancel{4\pi}\varepsilon_0\cancel{r^2}}\cdot\cancel{4\pi}\cancel{r^2}$$ S

$$= \frac{q}{\varepsilon_0}$$ Teorema di Gauss




$$ds\cos\theta = ds'$$

$$\frac{ds}{r^2} = \frac{ds''}{\vec{r}''^2}$$

Non costante
sup qualsiasi!

$$\Rightarrow \phi = \oint_S(\vec{E}\cdot\hat{n})ds = \oint_S \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,\hat{r}\cdot\hat{n}\,ds = \frac{q}{4\pi\varepsilon_0}\cdot\oint_S\frac{1}{r^2}\,ds\cos\theta$$ ds'

$$\hat{r}\cdot\hat{n} = r\cdot n\cdot\cos\theta$$

Radiale Sfera NORMALE

$$\Rightarrow \phi = \frac{q}{4\pi\varepsilon_0}\cdot\oint_{S'}\frac{1}{r^2}\,ds'$$

$$\frac{1}{r^2}\,ds$$ Sup $$\Rightarrow \frac{1}{\cancel{r^2}}\cdot 4\pi\cancel{r^2}$$

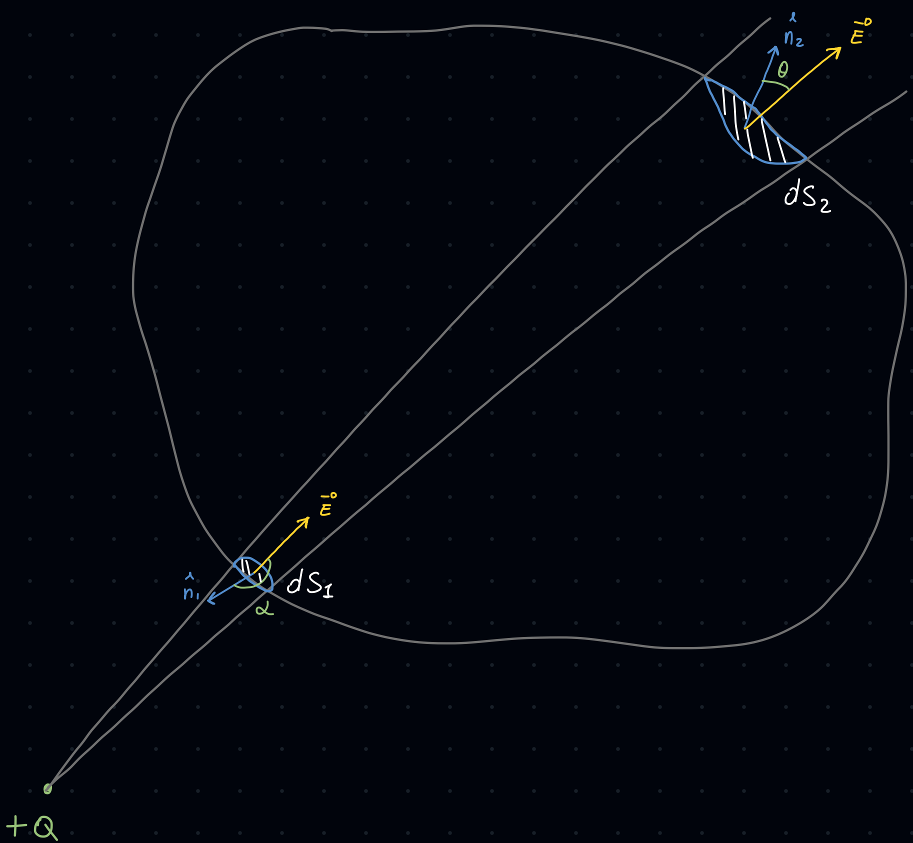
$$\phi = \frac{q}{\cancel{4\pi}\varepsilon_0}\cdot\cancel{4\pi} = \frac{q_{int}}{\varepsilon_0}$$ QED

Inoltre definiamo Angolo Solido $d\Omega = \frac{\perp dS}{R^2} = \frac{dS\cos\theta}{R^2}$

$$\Rightarrow \phi = \oint_S \vec{E}\cdot\hat{n}\,dS = \frac{Q}{4\pi\varepsilon_0}\cdot\oint \frac{1}{R^2}\cdot\underbrace{\hat{r}\cdot\hat{n}}_{\cos\theta}dS = \frac{Q}{4\pi\varepsilon_0}\oint_S \underbrace{\frac{1}{R^2}dS\cos\theta}_{d\Omega} = \frac{Q}{4\pi\varepsilon_0}\oint d\Omega =$$

$$= \frac{Q}{4\pi\varepsilon_0}\cdot\overbrace{4\pi}^{\Omega} = \frac{Q}{\varepsilon_0} \quad \text{QED}$$

Inoltre $\oint \Omega = \oint \frac{1}{R^2}dS \Rightarrow \frac{Q}{4\pi\varepsilon_0}\cdot\oint\frac{1}{R^2}dS = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{R^2}\cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$ QED




$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2}$$

$$\phi_{S_2} = \oint_{S_2}(\vec{E}\cdot\hat{n}_1)\cdot dS_2 = \frac{q}{4\pi\varepsilon_0}\int_{S_2}\frac{1}{R^2}\cdot\underbrace{\hat{r}\cdot\hat{n}_2\,dS_2}_{dS_2\cos\theta = dS_2'} = \frac{q}{4\pi\varepsilon_0}\int_{S_2'}\frac{1}{R^2}dS_2'$$

$$\phi_{S_1} = \oint_{S_1}(\vec{E}\cdot\hat{n}_1)\cdot dS_1 = \frac{q}{4\pi\varepsilon_0}\oint_{S_1}\frac{1}{R^2}\cdot\underbrace{\hat{r}\cdot\hat{n}_1\cdot dS_1}_{dS_2\cdot\cos(\alpha)\ \text{con}\ \alpha>90 \Rightarrow \cos(\alpha)<0} = -\frac{q}{4\pi\varepsilon_0}\oint\frac{1}{R^2}dS_1'$$

$$\Rightarrow \phi_{TOT} = \phi_1 + \phi_2 = \emptyset$$

Gauss : $\phi_E = \oint_S \vec{E} \cdot \hat{n} \, dS = \frac{Q_{int}}{\varepsilon_0}$

ma Tramite il Teorema della divergenza $\int_S \vec{A} \cdot \hat{n} \, dS = \int_V \vec{\nabla} \vec{A} \cdot dV$

$\Rightarrow \phi_E = \oint_S \vec{E} \cdot \hat{n} \, dS = \int_V \vec{\nabla} \cdot \vec{E} \cdot dV = \frac{Q}{\varepsilon_0}$

Definiamo $\rho = \frac{dQ}{dV} \Rightarrow \rho \, dV = dQ$

$\Rightarrow Q = \int_V \rho \, dV = \rho V$

Densità Volumetrica di carica

$\Rightarrow \phi_E = \cancel{\int_V} \vec{\nabla} \vec{E} \, \cancel{dV} = \frac{1}{\varepsilon_0} \cancel{\int_V} \rho \cdot \cancel{dV}$

$\Rightarrow \phi_E = \vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0}$ II Eq di maxwell (sarebbe la prima)