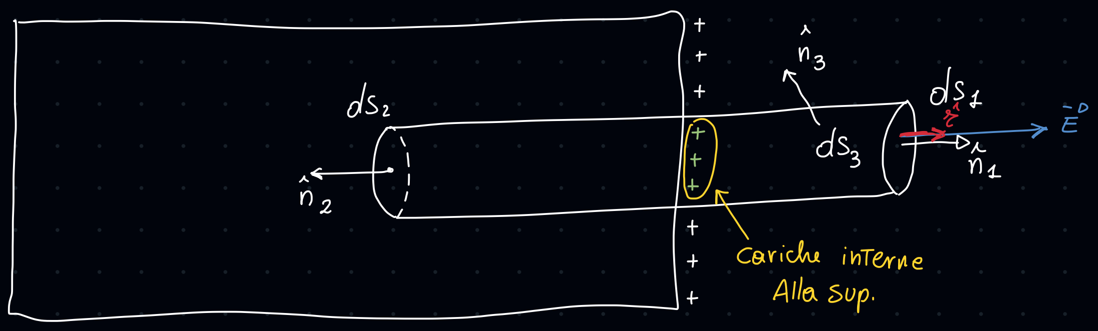
TEOREMA DI COULOMB



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} = \oint |\vec{E}| \cdot \hat{z} \cdot \hat{n}_1 dS_1 + \oint E \, \hat{z} \, \hat{n}_2 dS_2 + \oint E \, \hat{z} \, \hat{n}_3 dS_3$$

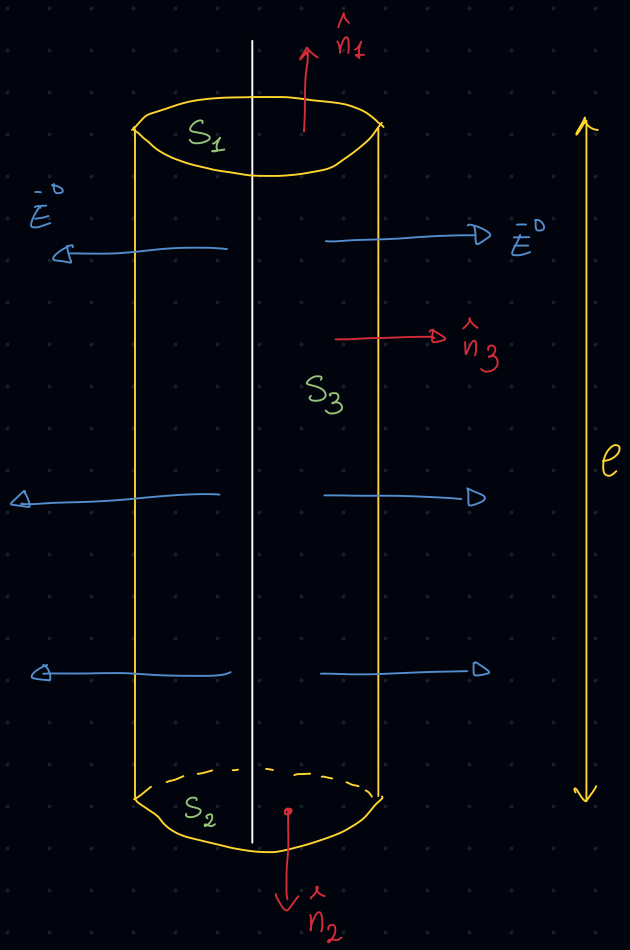
- $\hat{z} \cdot \hat{n}_1 \, dS = dS$
- $\hat{z} \cdot \hat{n}_2 \, dS = \underline{0}$ perché non esiste $\vec{E}$ in quella zona!
- $\hat{z} \cdot \hat{n}_3 \, dS = z \, n \cdot \cos(\theta) \, dS = \underline{0}$ perché $\theta = 90 \rightarrow \hat{n}_3 \perp \vec{E}$

$$\Rightarrow \phi_{TOT} = \oint \vec{E} \cdot \hat{n}_1 \, dS = \frac{Q_{int}}{\varepsilon_0} = |\vec{E}| \cdot S_1 \Rightarrow |\vec{E}| = \frac{Q_{int}}{S_1} \cdot \frac{1}{\varepsilon_0}$$

$\frac{Q_{int}}{S_1} \rightarrow \sigma$ densità di carica superficiale

$$\rightarrow |\vec{E}| = \frac{\sigma}{\varepsilon_0}$$

FILO CARICO



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} \quad \text{ma} \quad \vec{E} \perp \hat{n}_1 \, , \, \vec{E} \perp \hat{n}_3$$

$$\Rightarrow \phi_{TOT} = \phi_{S_3} = \oint \vec{E} \cdot \hat{n}_3 \, dS_3 = \frac{Q_{int}}{\varepsilon_0}$$

$$\phi = \oint \vec{E} \cdot \hat{n}_3 \, dS_3 = |\vec{E}| \cdot [S_2 \cdot e] = |\vec{E}| \cdot 2\pi R \cdot e = \frac{Q}{\varepsilon_0} \quad (1)$$

ma $e = \infty \Rightarrow$ Densità LINEARE di carica

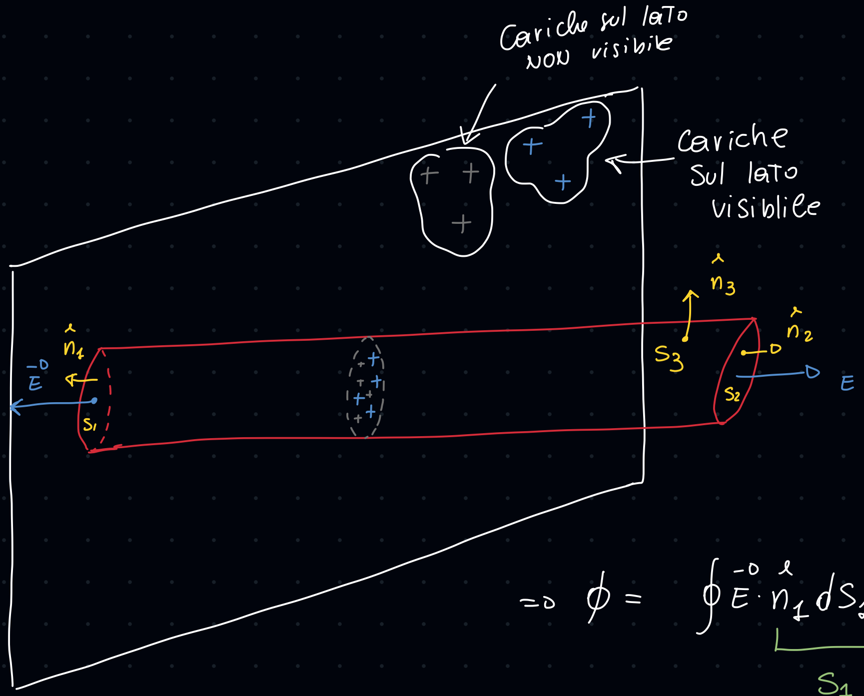
$$\lambda = \frac{Q}{e} \Rightarrow Q = \lambda \cdot e \quad (2)$$

$\Rightarrow$ dalla (1) e (2) $\rightarrow$ $|\vec{E}| \cdot 2\pi R \cdot \cancel{e} = \frac{\lambda \cancel{e}}{\varepsilon_0}$

$$\rightarrow |\vec{E}| \, 2\pi R = \frac{\lambda}{\varepsilon_0} \Rightarrow |\vec{E}| = \frac{\lambda}{2\pi R \varepsilon_0}$$

Inoltre $\vec{E} = \frac{\lambda}{2\pi R \varepsilon_0} \cdot \hat{n}_3$

STRATO CARICO



$$\phi_{TOT} = \phi_1 + \phi_2 + \phi_3$$

ma $\hat{n}_3 \perp \vec{E} \Rightarrow \phi_3 = 0$

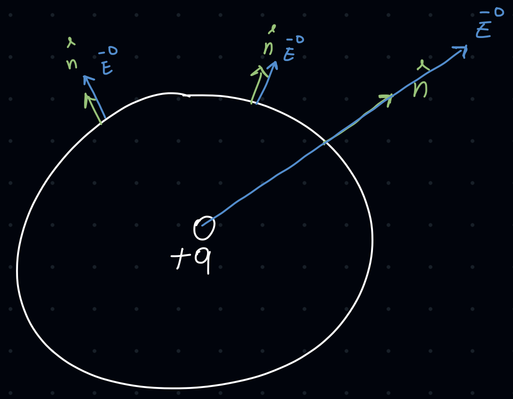
$$\Rightarrow \phi_{TOT} = \phi_1 + \phi_2$$

$$\Rightarrow \phi = \underbrace{\oint \vec{E} \cdot \hat{n}_1 dS_1 + \oint \vec{E} \cdot \hat{n}_2 dS_2}_{S_1 = S_2} = \frac{Q_{int}}{\varepsilon_0}$$

$$\rightarrow \phi = 2 \oint \vec{E} \cdot \hat{n}\, dS = 2 |\vec{E}| S = \frac{Q}{\varepsilon_0}$$

$$\sigma = \frac{Q}{S} \Rightarrow Q = \sigma \cdot S \Rightarrow 2 |\vec{E}| \cdot \cancel{S} = \frac{\sigma \cancel{S}}{\varepsilon_0} \rightarrow |\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$

Gauss $\qquad \phi = \oint \vec{E} \cdot \hat{n}\, ds = \dfrac{Q_{INT}}{\varepsilon_0}$

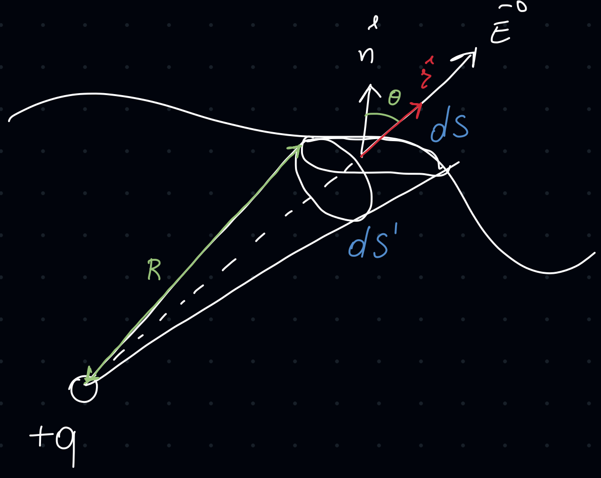


Sfera

$$\phi = \oint \vec{E} \cdot \hat{n}\, ds = |\vec{E}| \oint (\hat{r} \cdot \hat{n})\, ds \Rightarrow \phi = |\vec{E}| \oint ds = |\vec{E}| S$$

$$|\hat{r}| \cdot |\hat{n}| \cdot \cos(0) = 1$$

$$\Rightarrow \phi = |\vec{E}| \cdot 4\pi R^2 = \frac{1}{\cancel{4\pi}\varepsilon_0} \cdot \frac{Q}{\cancel{R^2}} \cdot \cancel{4\pi}\cancel{R^2} = \frac{Q}{\varepsilon_0}$$



$$ds \cdot \cos\theta = ds'$$

$$d\Omega = \frac{\perp ds}{r^2} = \frac{ds \cos\theta}{r^2}$$

$$\Rightarrow \phi = \oint \vec{E} \cdot \hat{n}\, ds = \frac{Q}{4\pi\varepsilon_0} \oint \frac{1}{R^2}\, \hat{r} \cdot \hat{n}\, ds$$

$$= \frac{Q}{4\pi\varepsilon_0} \oint \underbrace{\frac{1}{R^2} \cdot ds \cos\theta}_{d\Omega}$$

$$= \frac{Q}{4\pi\varepsilon_0} \cdot \oint d\Omega = \frac{Q}{4\pi\varepsilon_0} \oint \frac{1}{R^2}\, ds$$

$$= \frac{Q}{\cancel{4\pi}\varepsilon_0} \cdot \frac{1}{\cancel{R^2}} \cdot \cancel{R^2} \cdot \cancel{\pi 4} = \frac{Q}{\varepsilon_0}$$