Dipolo elettrico

Partiamo dalla eq di Maxwell per il campo elettrico:

$$L = \int \vec F \cdot d\vec e = U_A - U_B \quad \text{ma} \quad E = \frac{F}{q} \to \frac{U}{q} = V \Rightarrow \int \vec E\, de = V_A - V_B$$

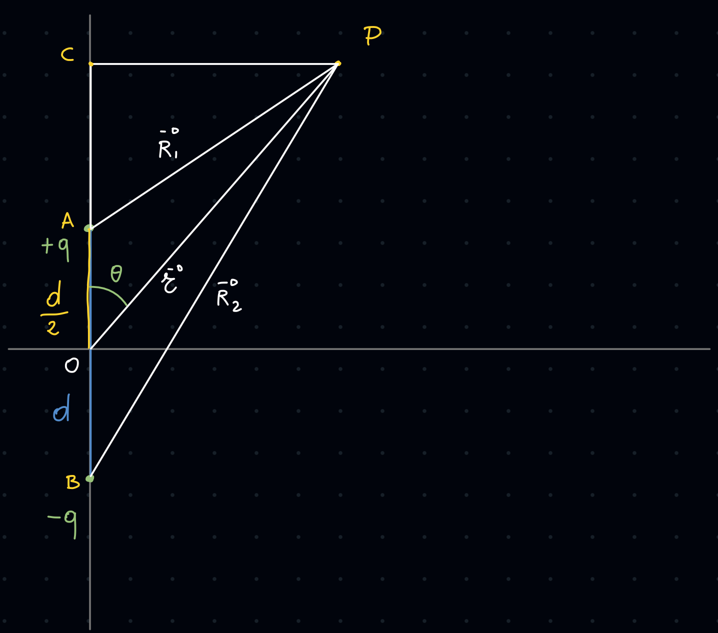
$$\text{Se} \quad L = \int \vec F\, d\vec e = -U \Rightarrow \frac{L}{q} = \int \vec E\, d\vec e = -V \Rightarrow \vec E\, d\vec e = -dV$$

ma dV è del tipo $\frac{\partial V}{\partial y}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$

$$\begin{cases} d\vec e = \hat i\, dx + \hat j\, dy + \hat k\, dz \\ \vec\nabla V = \hat i \frac{\partial V}{\partial x} + \hat j \frac{\partial V}{\partial y} + \hat k \frac{\partial V}{\partial z} \end{cases} \Rightarrow dV = \vec\nabla V \cdot de \Rightarrow \vec E\, \cancel{d\vec e} = -\vec\nabla V \cdot \cancel{d\vec e} \to \underline{\vec E = -\vec\nabla V}$$

Possiamo trovare il campo elettrico tramite il potenziale

Potenziale elettrico $V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$



Approx di dipolo: $r >> d$

$$\text{Conosco} \begin{cases} \vec r & \text{Posizione di P} \\ \theta & \text{Angolo tra } \vec r \text{ e } d \\ d & \text{distanza tra i poli} \end{cases}$$

$$R_1^2 = \overline{CP}^2 + \overline{CA}^2 = (r\sin\theta)^2 + \left(\overline{CO} - \frac{d}{2}\right)^2$$
$$= r^2\sin^2\theta + \left(r\cos\theta - \frac{d}{2}\right)$$
$$= r^2\sin^2\theta + r^2\cos^2\theta + \frac{d^2}{4} - dr\cos\theta$$
$$= r^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{1} + \frac{d^2}{4} - d\cdot r\cdot\cos\theta$$
$$= r^2 + \cancel{\frac{d^2}{4}}^{\,r>>d} - d\cdot r\cdot\cos\theta$$
$$= r^2 - dr\cos\theta$$

$$R_2^2 = \overline{CP}^2 + \overline{CB}^2 = (r\sin\theta)^2 + \left(\overline{CO} + \frac{d}{2}\right)^2 = r^2 + dr\cos\theta$$

$$\Rightarrow \begin{cases} R_1 = \sqrt{r^2 - rd\cos\theta} \\ R_2 = \sqrt{r^2 + rd\cos\theta} \end{cases} \Rightarrow \begin{cases} R_1 = \sqrt{r}\cdot\sqrt{r - d\cos\theta} \\ R_2 = \sqrt{r}\cdot\sqrt{r + d\cos\theta} \end{cases}$$

Approssimo $\sqrt{r - d\cos\theta}$ con Taylor in $x_0 = 0$

$$g_1(d) = \sqrt{r - d\cos\theta} = (r - d\cos\theta)^{\frac{1}{2}} \to g_1'(d) = \frac{1}{2}(r - d\cos\theta)^{-\frac{1}{2}}\cdot(+\sin\theta) = -\frac{\cos\theta}{2(r - d\cos\theta)}$$

$$g_1(0) = (r - 0\cos\theta)^{\frac{1}{2}} = \sqrt{r} \qquad g_1'(0) = -\frac{\cos\theta}{2(r - 0)} = -\frac{1}{2\sqrt{r}}\cos\theta$$

$$\Rightarrow \text{Taylor} \to \sqrt{r - d\cos\theta} \simeq \sqrt{r} - \frac{\cos}{2\sqrt{r}} \cdot d = \sqrt{r} - \frac{d\cos\theta}{2\sqrt{r}} = \bar{f}(d)$$

$$\Rightarrow R_1 \simeq \sqrt{r}\cdot \bar{f}(d) = \sqrt{r}\left(\sqrt{r} - \frac{d\cos\theta}{2\sqrt{r}}\right) = r - \frac{d\cos\theta}{2} \quad (1)$$

$$\Rightarrow R_2 \simeq r + \frac{d\cos\theta}{2} \quad (2)$$

Trovo il potenziale

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right) = k\left(\frac{\cancel{r} + \frac{d}{2}\cos\theta - \cancel{r} + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2} \quad \text{ma } r >> d \Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \quad (3)$$

Trovo il campo elettrico

$\vec{E} = -\vec{\nabla} V \to$ Derivate parziali $\to$ Servono le coordinate cartesiane!

$CO = r\cos\theta \Rightarrow$ Battezzo $\overline{CO} = z \to \cos\theta = \frac{z}{r}$

inoltre eq sfera: $r^2 = x^2 + y^2 + z^2$

$$\Rightarrow \frac{\cos\theta}{r^2} = \frac{z}{r}\cdot\frac{1}{r^2} = \frac{z}{(x^2+y^2+z^2)(x^2+y^2+z^2)^{\frac{1}{2}}} = \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cdot z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$ ora siamo pronti

$$-\frac{3}{2}(x^2+y^2+z^2)^{-\frac{5}{2}}\cdot 2x$$

$$\vec{E}_x = -\hat{i}\frac{\partial V}{\partial x} = \frac{q}{4\pi\varepsilon_0} d\cdot z\cdot\frac{d}{dx}(x^2+y^2+z^2)^{-\frac{3}{2}} = +\frac{q}{4\pi\varepsilon_0}\cdot\frac{3\cdot d\cdot z\, x}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

$$x^2+y^2+z^2 = r^2 \Rightarrow \vec{E}_x = +\hat{i}\frac{3qd}{4\pi\varepsilon_0}\cdot\frac{z\cdot x}{\sqrt[2]{(r^2)^5}} = \hat{i}\frac{3qd}{4\pi\varepsilon_0}\frac{z\cdot x}{r^5} \quad (A)$$

$$\vec{E}_y = \hat{j}\frac{3qd}{4\pi\varepsilon_0}\cdot\frac{zy}{r^5} \quad (B)$$

$$\vec{E}_z = -\hat{k}\frac{qd}{4\pi\varepsilon_0}\cdot\left(\frac{3z^2}{r^5} - \frac{1}{r^3}\right) = \frac{1}{4\pi\varepsilon_0 r^3}\left(\frac{3z^2}{r^2} - 1\right) = \frac{1}{4\pi\varepsilon_0 r^3}(3\cos\theta - 1) \quad (C)$$

$\cos\theta$