

ANELLO CARICO

$$\overline{AP} = \sqrt{R^2 + z^2}$$

Siccome $|\vec{E}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{d^2}$

$$\Rightarrow d = AP \Rightarrow dE_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{R^2 + z^2}$$

Sappiamo che $\lambda = \text{dens. lineare} \Rightarrow \lambda = \frac{dq}{de}$

$$\Rightarrow dq = \lambda de \Rightarrow Q_{\text{TOT}} = \int \lambda de$$

ma e non è altro che la circonferenza

$$\Rightarrow Q_{\text{TOT}} = \int \lambda dc$$

Dal disegno sappiamo che $d\vec{E}_n = d\vec{E}_A + d\vec{E}_B$

è SOLO LUNGO $z \Rightarrow$ Sommiamo solo le comp lungo z , dato che le altre si eliminano a 2 a 2.

$$\Rightarrow d\vec{E}_z = d\vec{E}_A \cos \alpha = \frac{\lambda dc}{4\pi\epsilon_0 (R^2 + z^2)} \cdot \cos \theta$$

Per definizione $\cos \alpha = \frac{\text{Cat}}{\text{Hip}} = \frac{z}{AP} = \frac{z}{\sqrt{R^2 + z^2}}$

$$\Rightarrow d\vec{E}_z = \frac{\lambda dc}{4\pi\epsilon_0 (R^2 + z^2)} \cdot \frac{z}{\sqrt{R^2 + z^2}} \Rightarrow E_z = \int \frac{\lambda}{4\pi\epsilon_0 (R^2 + z^2)} \cdot \frac{z}{\sqrt{R^2 + z^2}} dc$$

$$= \frac{1}{(R^2 + z^2)^{3/2}} = \frac{1}{(R^2 + z^2)^{1 + \frac{1}{2}}}$$

$$\Rightarrow E_z = \frac{\lambda z}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \int dc = \frac{2\pi R \lambda z}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \quad (1)$$

$$\Rightarrow \vec{E}_z = \frac{R z \lambda}{2\epsilon_0 (R^2 + z^2)^{3/2}} \hat{n}_z$$

E se $z \gg R$?

Siccome $Q = \int \lambda dc = \lambda 2\pi R$

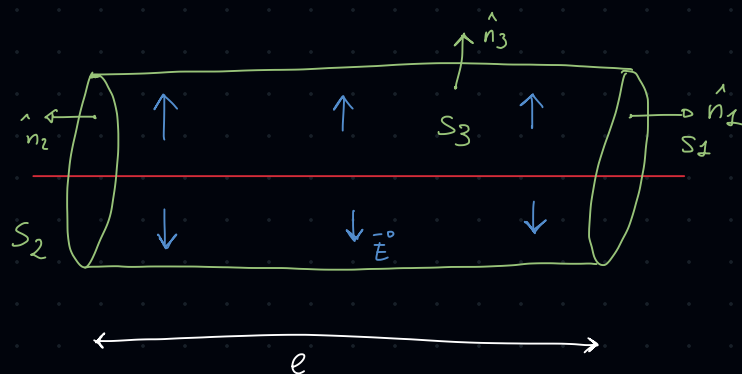
$$\Rightarrow \text{dalla (1)} \quad \frac{2\pi R \lambda z}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} = \frac{Q_{\text{TOT}} \cdot z}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}}$$

Se $z \gg R \quad E_z = \frac{Q_{\text{TOT}} \cdot z}{4\pi\epsilon_0 \left(\frac{1}{z^2}\right)^{3/2}}$

$$\Rightarrow \vec{E}_z = \frac{Q_{\text{TOT}}}{4\pi\epsilon_0 z^2} \hat{n}$$

Legge di Coulomb per la carica puntiforme

Filo carico - Coulomb



$$\phi_E = \oint \vec{E} \cdot \hat{n} ds = \int_S \vec{E} \cdot \hat{n}_1 ds_1 + \int_S \vec{E} \cdot \hat{n}_2 ds_2 + \int_S \vec{E} \cdot \hat{n}_3 ds_3$$

$\vec{E} \perp \hat{n}_1$ $\vec{E} \perp \hat{n}_2$

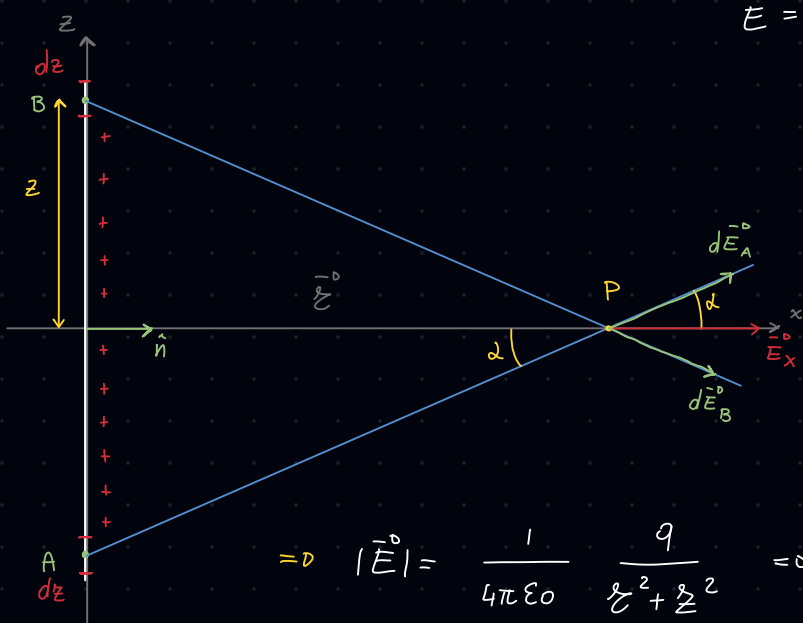
$$= \int_S \vec{E} \cdot \hat{n}_2 ds_2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \phi_E = |\vec{E}| \cdot 2\pi r l = \frac{Q}{\epsilon_0} \quad \text{ma } l = \infty$$

$$\Rightarrow \lambda = \text{densità lineare di carica} = \frac{Q}{l} \Rightarrow Q = \lambda l$$

$$|\vec{E}| 2\pi r l = \frac{\lambda l}{\epsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi r \epsilon_0} \hat{n}$$

Filo carico senza Coulomb



$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Non conosco r
AP / BP

$$\begin{cases} AP = r \cos \alpha \\ BP = r \cos \alpha \end{cases} \Rightarrow AP = BP = r \cos \alpha$$

Non conosco α

$$AP^2 = r^2 + r^2$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + r^2} \Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2 + r^2}$$

siccome $\lambda = \frac{dq}{dl} \Rightarrow dq = \lambda dl$

nel nostro caso $dl = dz$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dz}{r^2 + r^2}$$

\Rightarrow le componenti z ed y di $d\vec{E}_A + d\vec{E}_B$ si elidono a vicenda \Rightarrow ci serve solo \vec{E}_z

$$\Rightarrow d\vec{E}_z = dE \cos \alpha \quad \text{ma non conosciamo } \alpha$$

$$\Rightarrow z = AP \cos \alpha \Rightarrow z = \sqrt{r^2 + r^2} \cos \alpha \Rightarrow \cos \alpha = \frac{z}{\sqrt{r^2 + r^2}}$$

$$\Rightarrow dE_z = dE \cos \alpha = \frac{z}{\sqrt{r^2 + r^2}} dE$$

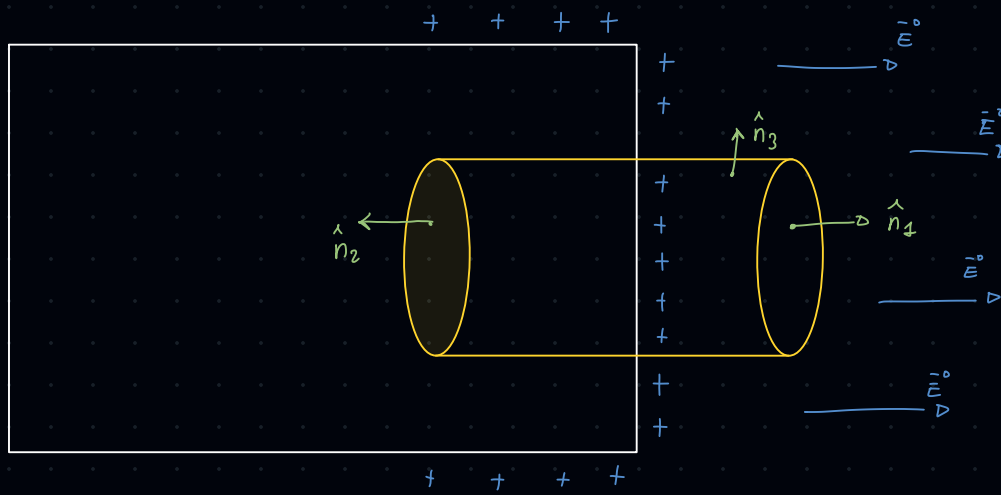
$$\Rightarrow dE_z = \frac{1}{4\pi\epsilon_0} \frac{\lambda dz}{z^2 + z^2} \cdot \frac{z}{\sqrt{z^2 + z^2}} = \frac{1}{4\pi\epsilon_0} \frac{\lambda z dz}{(z^2 + z^2)^{\frac{3}{2}}} \quad (1)$$

Integriamo per ottenere \vec{E}_{tot}

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \cos\tau}{(z^2 + z^2)^{\frac{3}{2}}} dz = 2 \frac{\lambda}{4\pi\epsilon_0} \int_0^e \frac{z}{(z^2 + z^2)^{\frac{3}{2}}} dz = \frac{2\lambda z}{2\pi\epsilon_0 z^2} \cdot \frac{e}{\sqrt{z^2 + e^2}}$$

$|\vec{E}|$

Teorema di Coulomb



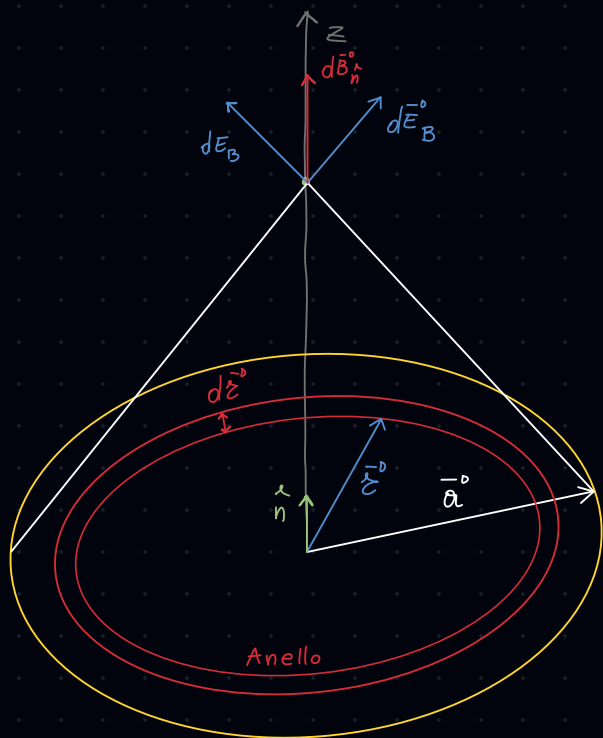
$$\phi_E = \oint \vec{E} \cdot \hat{n} ds = \int \vec{E} \cdot \hat{n}_1 ds_1 + \int \vec{E} \cdot \hat{n}_2 ds_2 + \int \vec{E} \cdot \hat{n}_3 ds_3 = \int \vec{E} \cdot \hat{n}_1 ds_1 = \frac{Q}{\epsilon_0}$$

$\int = 0$ $\hat{n}_3 \perp \vec{E}$

$$\Rightarrow |\vec{E}| \cdot S_1 = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{S_1} \cdot \frac{1}{\epsilon_0} \quad \text{ma} \quad \sigma = \frac{Q}{S} \Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$

Alternativamente

$$\sigma = \frac{dq}{ds} \Rightarrow dq = \sigma ds \Rightarrow q = \int \sigma ds \Rightarrow E = \frac{\int \sigma ds}{S \epsilon_0} = \sigma \frac{\int ds}{S \epsilon_0} = \frac{\sigma}{\epsilon_0}$$



Disco

Per trovare B_{Disco} ci basta sommare dB_{Anello} per tutto il raggio del disco

$$\rightarrow d\vec{B}_{\text{Anello}} = \frac{dQ \vec{z}}{4\pi \epsilon_0 (R^2 + z^2)^{\frac{3}{2}}} \quad (1)$$

$$Q_{\text{TOT Disco}} = \sigma \cdot S_{\text{disco}} = \sigma \cdot \pi a^2$$

$$\text{Carica di 1 Anello} = dQ_{\text{Anello}}$$

chiamo $a = z$ \rightarrow faccio il differenziale

$$\rightarrow dQ_{\text{Anello}} = \sigma \cdot 2\pi z \cdot dz \quad (2)$$

Sostituisco (2) in (1)

$$\rightarrow d\vec{B} = \frac{z \sigma 2\pi z \cdot dz}{4\pi \epsilon_0 (R^2 + z^2)^{\frac{3}{2}}} = \frac{\sigma z^2 dz}{2\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}}$$

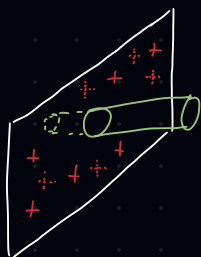
$$\rightarrow B = \frac{\sigma}{2\epsilon_0} z \int_0^a \frac{z}{(R^2 + z^2)^{\frac{3}{2}}} dz = \frac{\sigma}{2\epsilon_0} z \left[-\frac{1}{(z^2 + R^2)^{\frac{1}{2}}} \right]_0^a = -\frac{\sigma}{2\epsilon_0} z \left[\frac{1}{(z^2 + a^2)^{\frac{1}{2}}} - \frac{1}{z} \right]$$

$$\rightarrow B = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + a^2}} \right]$$

Se $a \rightarrow \infty$ Disco \rightarrow Lamina

infatti $\lim_{a \rightarrow \infty} \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + a^2}} \right] \rightarrow \frac{\sigma}{2\epsilon_0}$ lamina carica

LAMINA



$$\phi_E = \oint \vec{E} \cdot \hat{n} ds = \frac{Q}{\epsilon_0} \rightarrow \phi_E = \phi_{S_1} + \phi_{S_2} + \cancel{\phi_{\text{lat}}} = 2\phi_{S_1}$$

$$\phi_{S_1} = E \cdot S_1 = \frac{Q}{\epsilon_0} \rightarrow E = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow \phi_E = \frac{\sigma}{2\epsilon_0}$$

