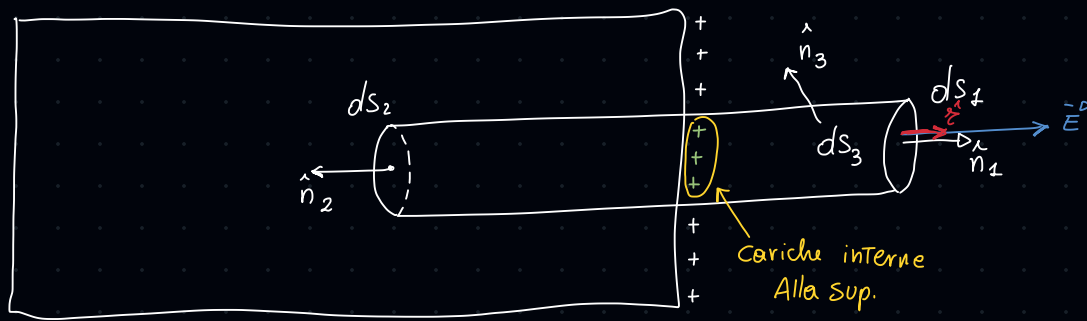


TEOREMA DI COULOMB



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} = \oint |\vec{E}| \cdot \hat{z} \cdot \hat{n}_1 ds_1 + \oint E \hat{z} \cdot \hat{n}_2 ds_2 + \oint E \hat{z} \cdot \hat{n}_3 ds_3$$

$$\bullet \hat{z} \cdot \hat{n}_1 ds = ds$$

$$\bullet \hat{z} \cdot \hat{n}_2 ds = \underline{0} \text{ perche' non esiste } \vec{E} \text{ in quella zona!}$$

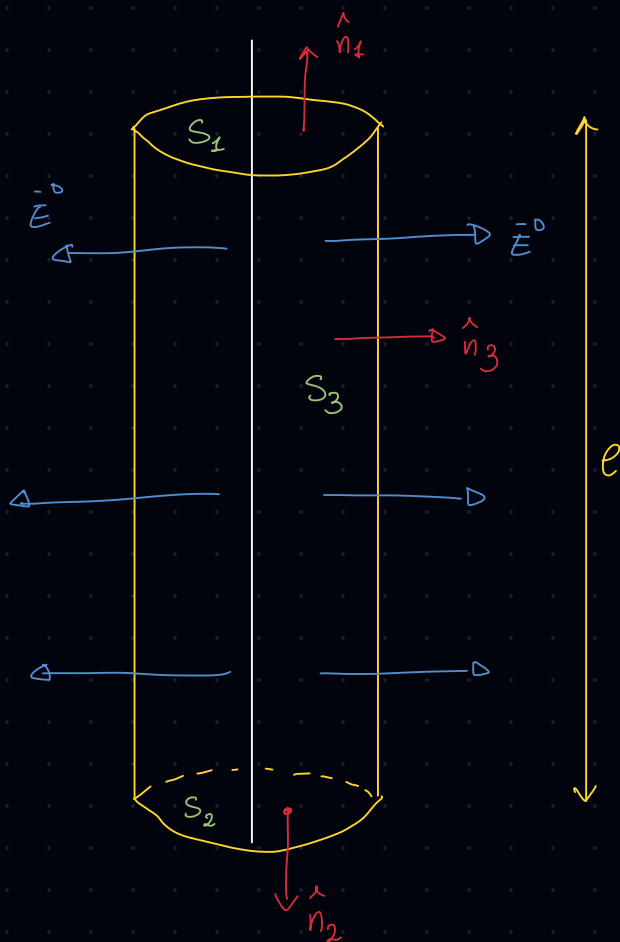
$$\bullet \hat{z} \cdot \hat{n}_3 ds = \hat{z} \cdot \hat{n} \cdot \cos(\theta) ds = \underline{0} \text{ perche' } \theta = 90 \rightarrow \hat{n}_3 \perp \vec{E}$$

$$\Rightarrow \phi_{TOT} = \oint \vec{E} \cdot \hat{n}_1 ds = \frac{Q_{int}}{\epsilon_0} = |\vec{E}| \cdot S_1 \Rightarrow |\vec{E}| = \left(\frac{Q_{int}}{S_1} \right) \cdot \frac{1}{\epsilon_0}$$

densità di
carica superficiale

$$\Rightarrow |\vec{E}| = \frac{\sigma}{\epsilon_0}$$

FILLO CARICO



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} \quad \text{ma} \quad \vec{E} \perp \hat{n}_1, \vec{E} \perp \hat{n}_2$$

$$\Rightarrow \phi_{TOT} = \phi_{S_3} = \oint \vec{E} \cdot \hat{n}_3 dS_3 = \frac{Q_{int}}{\epsilon_0}$$

$$\phi = \oint \vec{E} \cdot \hat{n}_3 dS_3 = |\vec{E}| \cdot [S_3 \cdot e] = |\vec{E}| \cdot 2\pi R \cdot e = \frac{Q}{\epsilon_0} \quad (1)$$

ma $e = \infty \Rightarrow$ Densità LINEARE di carica

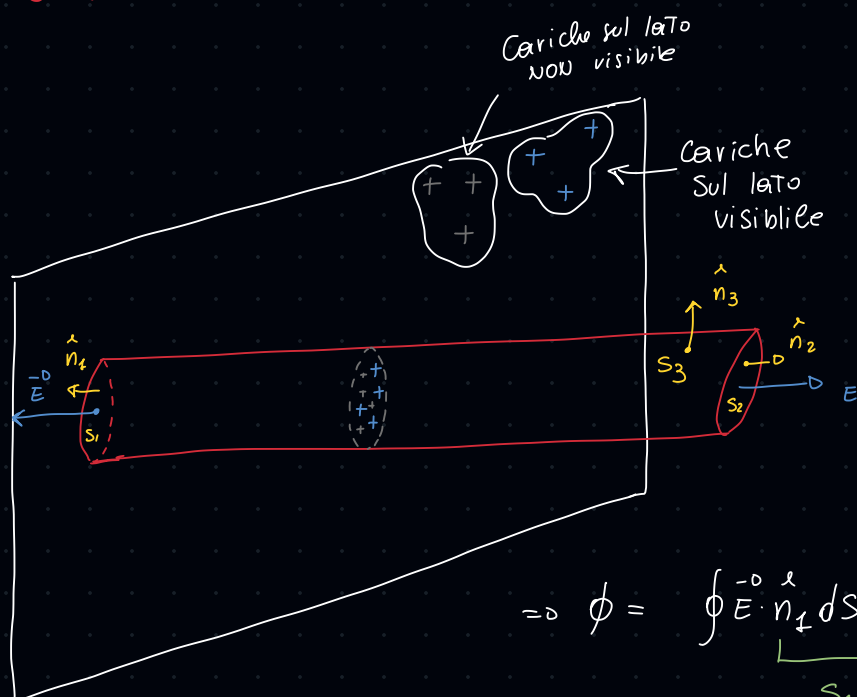
$$\lambda = \frac{Q}{e} \Rightarrow Q = \lambda \cdot e \quad (2)$$

$$\Rightarrow \text{dalla (1) e (2)} \Rightarrow |\vec{E}| \cdot 2\pi R \cdot e = \frac{\lambda e}{\epsilon_0}$$

$$\Rightarrow |\vec{E}| 2\pi R = \frac{\lambda}{\epsilon_0} \Rightarrow |\vec{E}| = \frac{\lambda}{2\pi R \epsilon_0}$$

Inoltre $\vec{E} = \frac{\lambda}{2\pi R \epsilon_0} \cdot \hat{n}_3$

STRATO CARICO



$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\text{ma } \vec{n}_3 \perp \vec{E} \Rightarrow \phi_3 = 0$$

$$\Rightarrow \phi_{\text{TOT}} = \phi_1 + \phi_2$$

$$\Rightarrow \phi = \underbrace{\oint \vec{E} \cdot \vec{n}_1 dS_1 + \oint \vec{E} \cdot \vec{n}_2 dS_2}_{S_1 = S_2} = \frac{Q_{\text{int}}}{\epsilon_0}$$

$$\Rightarrow \phi = 2 \oint \vec{E} \cdot \vec{n} dS = 2 |\vec{E}| S = \frac{Q}{\epsilon_0}$$

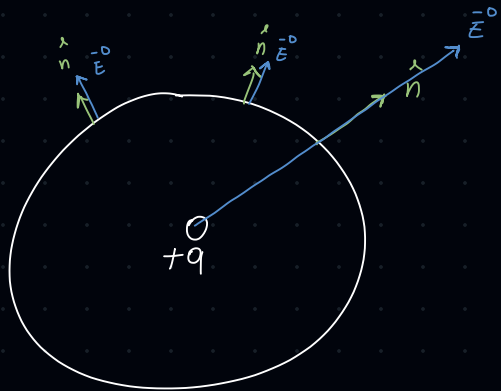
$$\sigma = \frac{Q}{S} \Rightarrow Q = \sigma \cdot S$$

$$\Rightarrow 2 |\vec{E}| \cancel{S} = \frac{\sigma \cancel{S}}{\epsilon_0}$$

$$\Rightarrow |\vec{E}| = \frac{\sigma}{2\epsilon_0}$$

Gauss

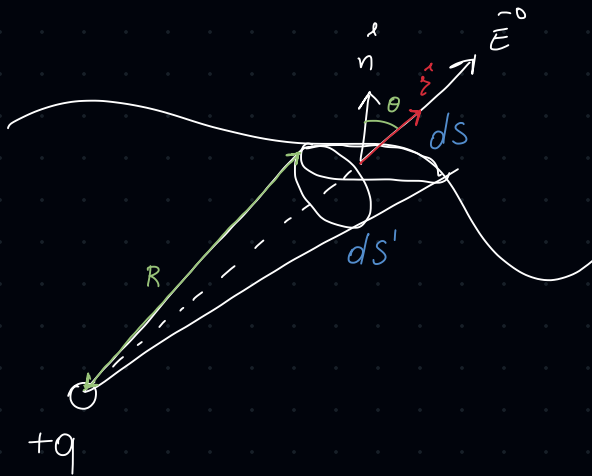
$$\phi = \oint \vec{E}^0 \cdot \vec{n} \, dS = \frac{Q_{\text{int}}}{\epsilon_0}$$



Sfera

$$\phi = \oint \vec{E}^0 \cdot \vec{n} \, dS = |\vec{E}^0| \oint \underbrace{\vec{e} \cdot \vec{n}}_{|\vec{e}| \cdot |\vec{n}| \cdot \cos(0) = 1} \, dS \Rightarrow \phi = |\vec{E}^0| \oint dS = |\vec{E}^0| S$$

$$\Rightarrow \phi = |\vec{E}^0| \cdot 4\pi R^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2} \cdot 4\pi R^2 = \frac{Q}{\epsilon_0}$$



$$dS \cdot \cos \theta = dS'$$

$$d\Omega = \frac{dS}{r^2} = \frac{dS \cos \theta}{r^2}$$

$$\Rightarrow \phi = \oint \vec{E}^0 \cdot \vec{n} \, dS = \frac{Q}{4\pi\epsilon_0} \oint \frac{1}{R^2} \vec{e} \cdot \vec{n} \, dS$$

$$= \frac{Q}{4\pi\epsilon_0} \oint \left(\frac{1}{R^2} \cdot dS \cos \theta \right) d\Omega$$

$$= \frac{Q}{4\pi\epsilon_0} \oint d\Omega = \frac{Q}{4\pi\epsilon_0} \oint \frac{1}{R^2} dS$$

$$= \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{R^2} \cdot R^2 \cdot \pi 4 = \frac{Q}{\epsilon_0}$$