Dipolo elettrico

Partiamo dalla eq di Maxwell per il campo elettrico:

$$L = \int \vec{F} \cdot d\vec{e} = U_A - U_B \quad \text{ma} \quad E = \frac{F}{q} \to \frac{U}{q} = V \Rightarrow \int \vec{E}\, de = V_A - V_B$$

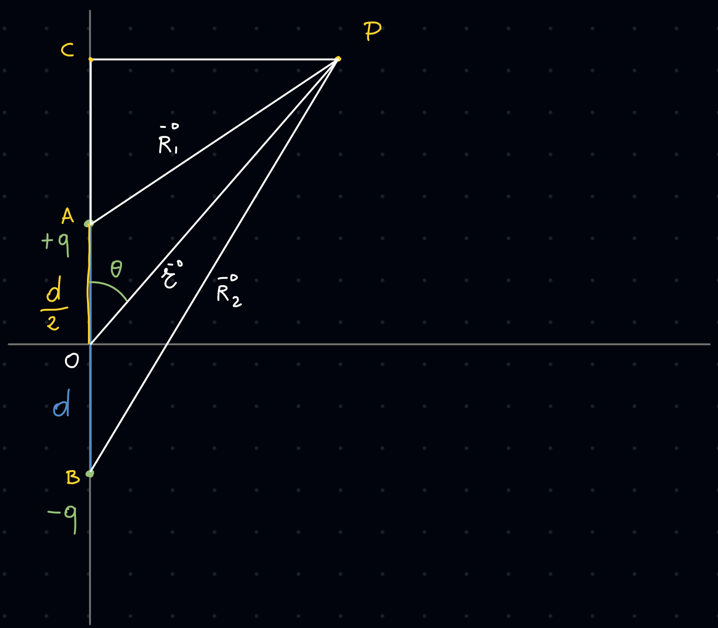
$$\text{Se} \quad L = \int \vec{F}\, d\vec{e} = -U \Rightarrow \frac{L}{q} = \int \vec{E}\, d\vec{e} = -V \Rightarrow \vec{E}\, d\vec{e} = -dV$$

ma dV è del tipo $\frac{\partial V}{\partial y} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$

$$\begin{cases} d\vec{e} = \hat{i}\, dx + \hat{j}\, dy + \hat{k}\, dz \\ \vec{\nabla} V = \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \end{cases} \Rightarrow dV = \vec{\nabla} V \cdot de \Rightarrow \vec{E}\, \cancel{d\vec{e}} = -\vec{\nabla} V \cdot \cancel{d\vec{e}} \to \underline{\vec{E} = -\vec{\nabla} V}$$

Possiamo trovare il campo elettrico tramite il potenziale

Potenziale elettrico $V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$



Approx di dipolo: $r >> d$

Conosco $\begin{cases} \vec{r} & \text{Posizione di P} \\ \theta & \text{Angolo tra } \vec{r} \text{ e } d \\ d & \text{distanza tra i poli} \end{cases}$

$$R_1^2 = \overline{CP}^2 + \overline{CA}^2 = (r\sin\theta)^2 + \left(\overline{CO} - \frac{d}{2}\right)^2$$

$$= r^2 \sin^2\theta + \left(r\cos\theta - \frac{d}{2}\right)$$

$$= r^2 \sin^2\theta + r^2\cos^2\theta + \frac{d^2}{4} - dr\cos\theta$$

$$= r^2 \underbrace{(\sin^2\theta + \cos^2\theta)}_{1} + \frac{d^2}{4} - d \cdot r \cdot \cos\theta$$

$$= r^2 + \overset{r>>d}{\cancel{\frac{d^2}{4}}} - d \cdot r \cdot \cos\theta$$

$$= r^2 - dr\cos\theta$$

$$R_2^2 = \overline{CP}^2 + \overline{CB}^2 = (r\sin\theta)^2 + \left(CO + \frac{d}{2}\right)^2 = r^2 + dr\cos\theta$$

$$\Rightarrow \begin{cases} R_1 = \sqrt{r^2 - rd\cos\theta} \\ R_2 = \sqrt{r^2 + rd\cos\theta} \end{cases} \Rightarrow \begin{cases} R_1 = \sqrt{r} \cdot \sqrt{r - d\cos\theta} \\ R_2 = \sqrt{r} \cdot \sqrt{r + d\cos\theta} \end{cases}$$

Approssimo $\sqrt{r - d\cos\theta}$ con Taylor in $x_0 = 0$

$$g_1(d) = \sqrt{r - d\cos\theta} = (r - d\cos\theta)^{\frac{1}{2}} \to g_1'(d) = \frac{1}{2}(r - d\cos\theta)^{-\frac{1}{2}} \cdot (+\sin\theta) = -\frac{\cos\theta}{2(r - d\cos\theta)}$$

$$g_1(0) = (r - 0\cos\theta)^{\frac{1}{2}} = \sqrt{r} \qquad g_1'(0) = -\frac{\cos\theta}{2(r - 0)} = -\frac{1}{2\sqrt{r}}\cos\theta$$

$\Rightarrow$ Taylor $\rightarrow \sqrt{r - d\cos\theta} \simeq \sqrt{r} - \frac{\cos}{2\sqrt{r}} \cdot d = \sqrt{r} - \frac{d\cos\theta}{2\sqrt{r}} = \bar{f}(d)$

$$\Rightarrow R_1 \simeq \sqrt{r} \cdot \bar{f}(d) = \sqrt{r} \cdot \left(\sqrt{r} - \frac{d\cos\theta}{2\sqrt{r}}\right) = r - \frac{d\cos\theta}{2} \quad (1)$$

$$\Rightarrow R_2 \simeq r + \frac{d\cos\theta}{2} \quad (2)$$

Trovo il potenziale

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right) = k\left(\frac{\cancel{r} + \frac{d}{2}\cos\theta - \cancel{r} + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{d\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}$$ ma $r >> d \Rightarrow V = \frac{q}{4\pi\varepsilon_0} \frac{d\cos\theta}{r^2} \quad (3)$

Trovo il campo elettrico

$\vec{E} = -\vec{\nabla} V \rightarrow$ Derivate parziali $\rightarrow$ Servono le coordinate cartesiane!

$CO = r\cos\theta \Rightarrow$ Battezzo $\overline{CO} = z \rightarrow \cos\theta = \frac{z}{r}$

inoltre eq sfera: $r^2 = x^2 + y^2 + z^2$

$$\Rightarrow \frac{\cos\theta}{r^2} = \frac{z}{r} \cdot \frac{1}{r^2} = \frac{z}{(x^2+y^2+z^2)(x^2+y^2+z^2)^{\frac{1}{2}}} = \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0} \frac{d \cdot z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$ ora siamo pronti

$-\frac{3}{2}(x^2+y^2+z^2)^{-\frac{5}{2}} \cdot 2x$

$$\vec{E}_x = -\hat{i}\frac{\partial V}{\partial x} = \frac{q}{4\pi\varepsilon_0} d \cdot z \cdot \frac{d}{dx}(x^2+y^2+z^2)^{-\frac{3}{2}} = +\frac{q}{4\pi\varepsilon_0} \cdot \frac{3 \cdot d \cdot z\, x}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

$$x^2+y^2+z^2 = r^2 \Rightarrow \vec{E}_x = +\hat{i}\frac{3qd}{4\pi\varepsilon_0} \cdot \frac{z \cdot x}{\sqrt[2]{(r^2)^5}} = \hat{i}\frac{3qd}{4\pi\varepsilon_0} \frac{z \cdot x}{r^5} \quad (A)$$

$$\vec{E}_y = \hat{j}\frac{3qd}{4\pi\varepsilon_0} \cdot \frac{zy}{r^5} \quad (B)$$

$$\vec{E}_z = -\hat{k}\frac{qd}{4\pi\varepsilon_0} \cdot \left(\frac{3z^2}{r^5} - \frac{1}{r^3}\right) = \frac{1}{4\pi\varepsilon_0 r^3}\left(\frac{3z^2}{r^2} - 1\right) = \frac{1}{4\pi\varepsilon_0 r^3}(3\cos\theta - 1)$$

$\cos\theta$

(C)