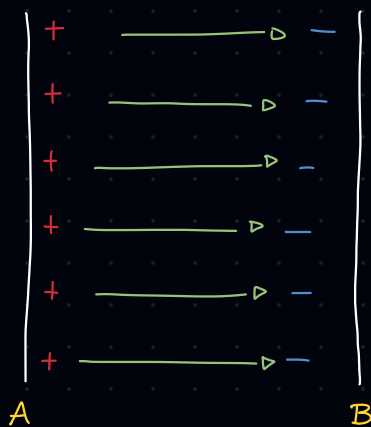


CAPACITA' DEL CONDENSATORE

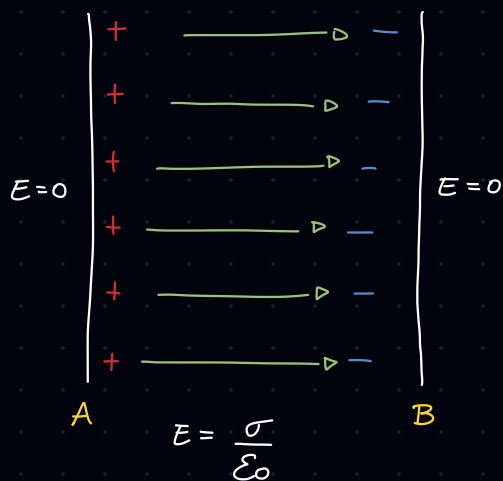


$$Q_A = Q_B \Rightarrow \begin{cases} V_A = \kappa_1 Q \\ V_B = \kappa_2 Q \end{cases} \Rightarrow V_B - V_A = Q(\kappa_2 - \kappa_1)$$

Battezzo $\kappa_2 - \kappa_1 = \frac{1}{C}$ Capacita' $\Rightarrow V_B - V_A = Q \cdot C$

$$\Rightarrow Q = C(V_B - V_A) \Rightarrow C = \frac{Q}{V_B - V_A}$$

Condensatore Piano



Dal doppio Strato $|\vec{E}| = \frac{\sigma}{\epsilon_0}$

Sappiamo che $C = \frac{Q}{V_B - V_A}$?

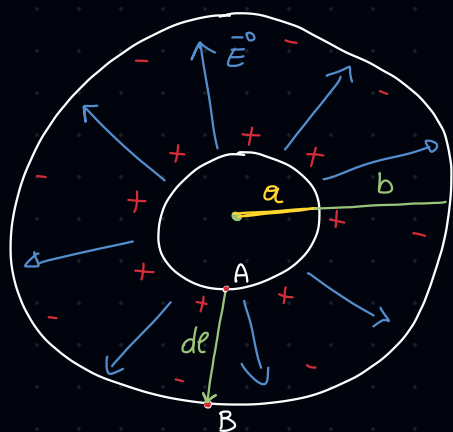
$$\Delta V = L = \int_A^B \vec{E} \cdot \vec{n} \, d\ell = |\vec{E}| \int_A^B d\ell = E \cdot d = \frac{\sigma}{\epsilon_0} \cdot d$$

Siccome $\sigma = \frac{Q}{S} \Rightarrow \Delta V = \frac{Q}{S \cdot \epsilon_0} \cdot d$

$$\Rightarrow C = \frac{Q}{V_B - V_A} = \frac{\cancel{Q}}{\cancel{Q} \cdot d} \cdot S \cdot \epsilon_0 = \frac{S \cdot \epsilon_0}{d}$$

Capacita' del cond. Piano

Condensatore sferico



$$C = \frac{Q}{\Delta V} \quad (1) \quad , \quad \vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}$$

$$\Delta V = \int_A^B \vec{E} \cdot \hat{z} \, d\ell = \int_A^B \frac{Q}{4\pi\epsilon_0 z^2} d\ell$$

$$= \frac{Q}{4\pi\epsilon_0} \int_A^B \frac{1}{z^2} \hat{z} \, d\ell$$

$$\ell = b - a$$

$$\text{ma } \hat{z} \cdot d\vec{\ell} = d\vec{z}$$

$$\Rightarrow \Delta V = \frac{Q}{4\pi\epsilon_0} \int_A^B \frac{1}{z^2} d\vec{z} = \frac{Q}{4\pi\epsilon_0} \cdot \left[-\frac{1}{z} \right]_A^B = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{z_A} - \frac{1}{z_B} \right]$$

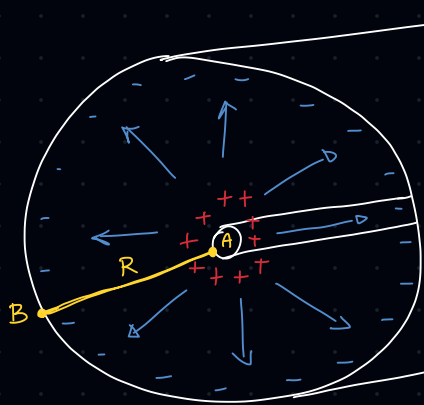
$$\text{ma } \begin{cases} z_A = a \\ z_B = b \end{cases} \Rightarrow \Delta V = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right] = \frac{Q}{4\pi\epsilon_0} \left[\frac{b-a}{ab} \right]$$

dalla (1)

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\cancel{Q} (b-a)} \cdot 4\pi\epsilon_0 \cdot ab = \frac{4\pi\epsilon_0 ab}{b-a}$$

capacità
cond. sferico

Condensatore cilindrico

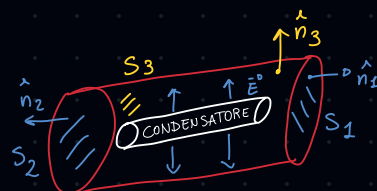


Sappiamo che $C = \frac{Q}{V_B - V_A}$

e che $\Delta V = \int_A^B \vec{E} \cdot d\vec{\ell}$ ma non abbiamo \vec{E} !

$$\phi_{TOT} = \int \vec{E} \cdot \hat{n} dS = \frac{Q_{int}}{\epsilon_0} \quad \text{Teorema di Gauss}$$

→ "Abbracciamo" il condensatore con un CILINDRO



$$\phi_{TOT} = \phi_{S1} + \phi_{S2} + \phi_{S3} \quad \text{ma} \quad \hat{n}_1 \perp \vec{E} \quad \text{e} \quad \hat{n}_2 \perp \vec{E} \quad \Rightarrow \quad \phi_{S1} = \phi_{S2} = 0$$

$$\Rightarrow \phi_{TOT} = \int \vec{E} \cdot \hat{n}_3 dS_3 = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad E \cdot 2\pi R \cdot \ell = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad E = \frac{Q}{2\pi R \ell \epsilon_0}$$

→ Integriamo E

$$\Delta V = \int_A^B \vec{E} \cdot d\vec{\ell} \quad \Rightarrow \quad \vec{E} = E \cdot \hat{z} \quad \Rightarrow \quad \Delta V = \int_A^B \frac{Q}{2\pi R \ell \epsilon_0} \cdot \hat{z} \cdot d\vec{\ell} = \frac{Q}{2\pi \ell \epsilon_0} \int_A^B \frac{1}{R} d\vec{z}$$

$$\Rightarrow \Delta V = \frac{Q}{2\pi \ell \epsilon_0} \left[\ln(R) \right]_A^B \quad \Rightarrow \quad \Delta V = \frac{Q}{2\pi \ell \epsilon_0} \ln(B) - \ln(A) = \frac{Q}{2\pi \ell \epsilon_0} \ln\left(\frac{B}{A}\right)$$

→ Troviamo C

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{2\pi \ell \epsilon_0} \ln\left(\frac{B}{A}\right)} = \frac{2\pi \ell \epsilon_0}{\ln\left(\frac{B}{A}\right)} \quad \text{Capacità condensatore cilindrico}$$

TOT

Parte

CONDENSATORI IN UN CIRCUITO

SERIE



$$\begin{cases} C_1 = \frac{Q}{V_A - V_B} \\ C_2 = \frac{Q}{V_B - V_C} \end{cases} \Rightarrow \begin{cases} V_A - V_B = \frac{Q}{C_1} \\ V_B - V_C = \frac{Q}{C_2} \end{cases}$$



$$C_{EQ} = \frac{Q}{V_A - V_B}$$

$$\Rightarrow V_A - \cancel{V_B} + \cancel{V_B} - V_C = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow \underline{V_A - V_C = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)}$$

$$\Rightarrow C_{EQ} = \sum_n \frac{1}{C_n}$$

PARALLELO



$$\begin{cases} V_A - V_B = \frac{Q_1}{C_1} \\ V_A - V_B = \frac{Q_2}{C_2} \end{cases} \Rightarrow \begin{cases} Q_1 = C_1 (\Delta V) \\ Q_2 = C_2 (\Delta V) \end{cases}$$

$$\Rightarrow Q_1 + Q_2 = \Delta V (C_1 + C_2) \quad \text{ma } Q_1 + Q_2 = Q_{TOT} \Rightarrow Q = \Delta V (C_1 + C_2)$$

$$\Delta V = \frac{Q}{C_{EQ}} \Rightarrow Q = \Delta V \cdot C_{EQ} \Rightarrow \underline{C_{EQ} = \sum_n C_n}$$