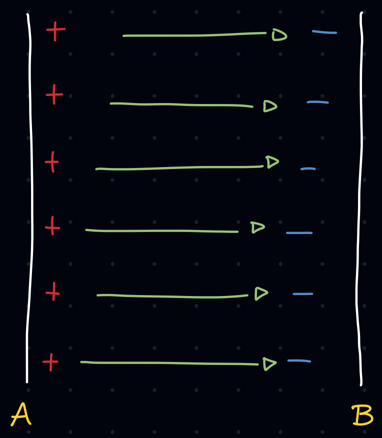
CAPACITA' DEL CONDENSATORE

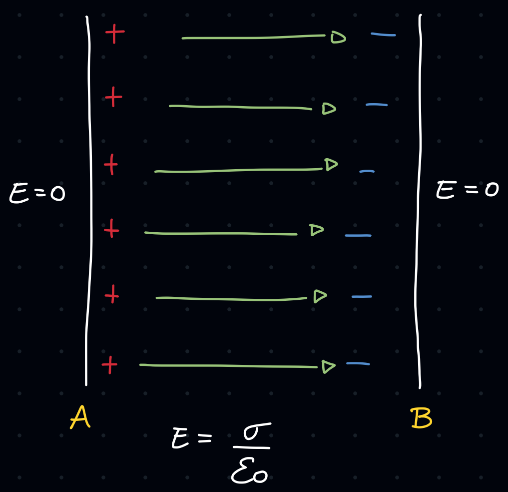


$$Q_A = Q_B \Rightarrow \begin{cases} V_A = K_1 Q \\ V_B = K_2 Q \end{cases} \Rightarrow V_B - V_A = Q(K_2 - K_1)$$

Battezzo $K_2 - K_1 = \frac{1}{C}$ Capacita' $\Rightarrow V_B - V_A = Q \cdot C$

$$\Rightarrow Q = C(V_B - V_A) \Rightarrow C = \frac{Q}{V_B - V_A}$$

Condensatore Piano



Dal doppio Strato $|\vec{E}| = \frac{\sigma}{\varepsilon_0}$

Sappiamo che $C = \frac{Q}{V_B - V_A}$?

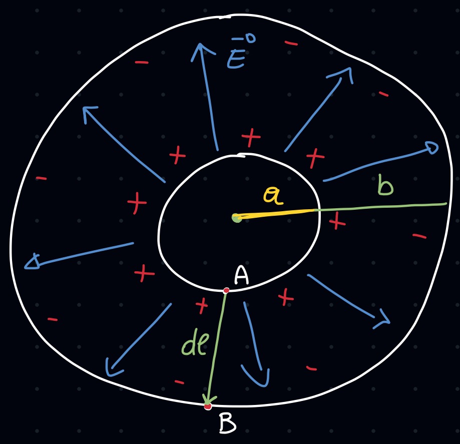
$$\Delta V = L = \int_A^B \vec{E} \cdot \hat{n}\, dl = |\vec{E}| \int_A^B dl = E \cdot d = \frac{\sigma}{\varepsilon_0} \cdot d$$

Siccome $\sigma = \frac{Q}{S} \Rightarrow \Delta V = \frac{Q}{S \cdot \varepsilon_0} \cdot d$

$$\Rightarrow C = \frac{Q}{V_B - V_A} = \frac{\cancel{Q}}{\cancel{Q} \cdot d} \cdot S \cdot \varepsilon_0 = \frac{S \cdot \varepsilon_0}{d}$$

Capacita' del cond. Piano

Condensatore sferico



(1)

$$C = \frac{Q}{\Delta V} \quad , \quad \vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}$$

$$\Delta V = \int_A^B \vec{E} \cdot \hat{r}\, dl = \int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}\, d\vec{l} \qquad l = b-a$$

$$= \frac{Q}{4\pi\varepsilon_0} \int_A^B \frac{1}{r^2}\, \hat{r}\, d\vec{l} \qquad \text{ma} \quad \hat{r} \cdot d\vec{l} = d\vec{r}$$

$\hat{r}$ $\longrightarrow$ dl

$$\Rightarrow \Delta V = \frac{Q}{4\pi\varepsilon_0} \int_A^B \frac{1}{r^2}\, d\vec{r} = \frac{Q}{4\pi\varepsilon_0} \cdot \left[-\frac{1}{r}\right]_A^B = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

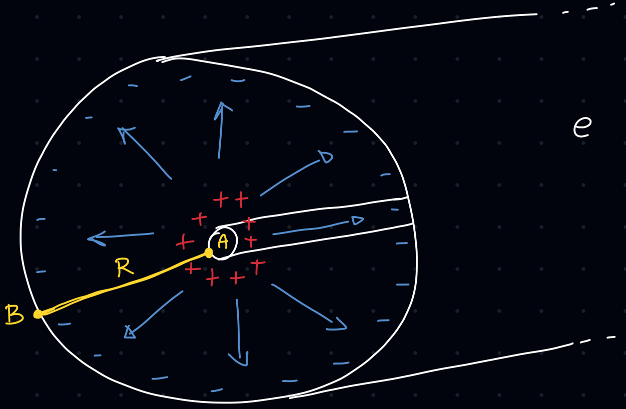
$$\text{ma} \begin{cases} r_A = a \\ r_B = b \end{cases} \Rightarrow \Delta V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{b-a}{ab}\right]$$

dalla (1)

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\cancel{Q}\,(b-a)} \cdot 4\pi\varepsilon_0 \cdot ab = \frac{4\pi\varepsilon_0\, ab}{b-a}$$

capacità cond. sferico

Condensatore Cilindrico

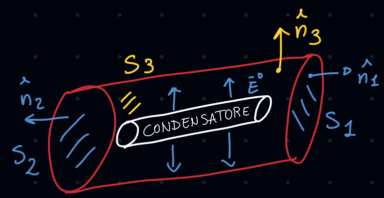


Sappiamo che $C = \dfrac{Q}{V_B - V_A}$

e che $\Delta V = L_E = \displaystyle\int_A^B \vec{E} \cdot d\vec{\ell}$ ma non abbiamo $\vec{E}$!

$$\phi_{TOT} = \int \vec{E} \cdot \hat{n}\, dS = \frac{Q_{int}}{\varepsilon_0}$$ Teorema di Gauss

→ "Abbracciamo" il condensatore con un CILINDRO



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} \quad \text{ma} \quad \hat{n}_1 \perp \vec{E} \;\; e \;\; \hat{n}_2 \perp \vec{E} \;\Rightarrow\; \phi_{S_1} = \phi_{S_2} = \emptyset$$

$$\Rightarrow \phi_{TOT} = \int \vec{E} \cdot \hat{n}_3\, dS_3 = \frac{Q}{\varepsilon_0} \Rightarrow E \cdot 2\pi R \cdot \ell = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi R \ell \varepsilon_0}$$

E

→ Integriamo E

$$\Delta V = \int_A^B \vec{E} \cdot d\vec{\ell} \;\to\; \vec{E} = E \cdot \hat{r} \Rightarrow \Delta V = \int_A^B \frac{Q}{2\pi R \ell \varepsilon_0} \cdot \hat{r} \cdot d\vec{\ell} = \frac{Q}{2\pi \ell \varepsilon_0} \int_A^B \frac{1}{R}\, d\vec{r}$$

V

$$\Rightarrow \Delta V = \frac{Q}{2\pi \ell \varepsilon_0} \Big[\ln(R) \Big]_A^B \Rightarrow \Delta V = \frac{Q}{2\pi \ell \varepsilon_0} \ln(B) - \ln(A) = \frac{Q}{2\pi \ell \varepsilon_0} \ln\left(\frac{B}{A}\right)$$

→ Troviamo C

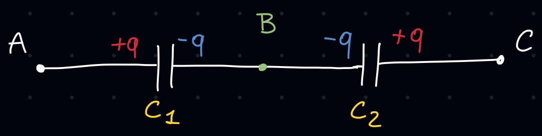
Capacità condensatore cilindrico

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\cancel{Q} \ln\left(\frac{B}{A}\right)} \cdot 2\pi \ell \varepsilon_0 = \frac{2\pi \ell \varepsilon_0}{\ln\left(\frac{B}{A}\right)}$$

TOT Parte

CONDENSATORI IN UN CIRCUITO

SERIE



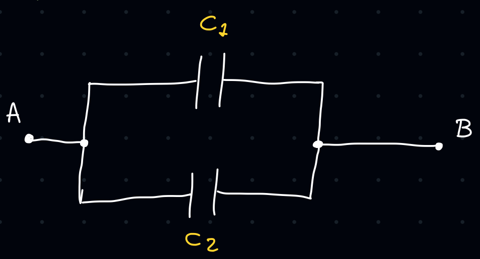
$$\begin{cases} C_1 = \dfrac{Q}{V_A - V_B} \\ C_2 = \dfrac{Q}{V_B - V_C} \end{cases} \Rightarrow \begin{cases} V_A - V_B = \dfrac{Q}{C_1} \\ V_B - V_C = \dfrac{Q}{C_2} \end{cases}$$

$$C_{EQ} = \frac{Q}{V_A - V_B}$$

$$\Rightarrow V_A - \cancel{V_B} + \cancel{V_B} - V_C = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow \underline{V_A - V_C = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

$$\Rightarrow C_{EQ} = \sum_n \frac{1}{C_n}$$

PARALLELO



$$\begin{cases} V_A - V_B = \dfrac{Q_1}{C_1} \\ V_A - V_B = \dfrac{Q_2}{C_2} \end{cases} \Rightarrow \begin{cases} Q_1 = C_1 (\Delta V) \\ Q_2 = C_2 (\Delta V) \end{cases}$$

$$\Rightarrow Q_1 + Q_2 = \Delta V (C_1 + C_2) \quad \text{ma} \quad Q_1 + Q_2 = Q_{TOT} \Rightarrow Q = \Delta V (C_1 + C_2)$$

$$\Delta V = \frac{Q}{C_{EQ}} \Rightarrow Q = \Delta V \cdot C_{EQ} \Rightarrow \underline{C_{EQ} = \sum_n C_n}$$