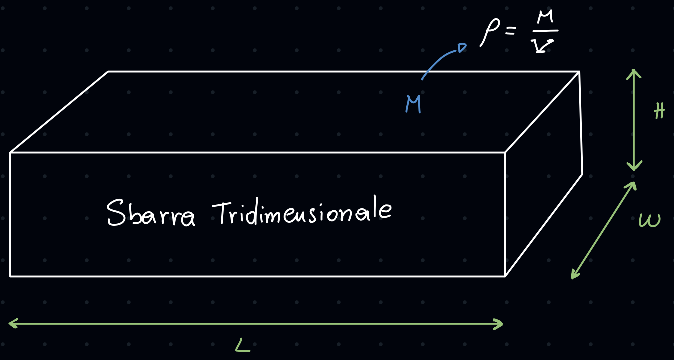
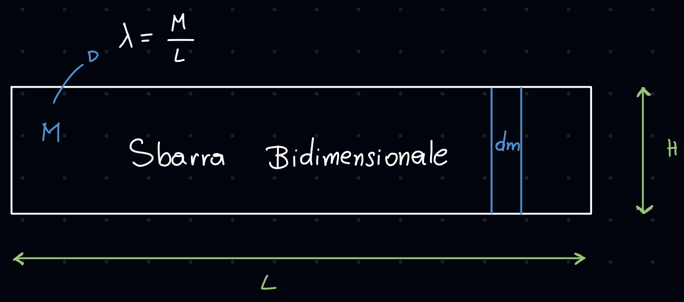
Momento di Inerzia di una Sbarra

$$I = mR^2 \to I_i = m_i R_i^2 \Rightarrow I_{TOT} = \sum_i m_i R_i^2$$





$\Rightarrow$ La sbarra e' un susseguirsi di punti CONTINUO $\Rightarrow \sum m_i \to \int dm$

$$\Rightarrow I_{TOT} = \sum_i m_i R_i^2 \to I_{TOT} = \int R^2 \, dm$$

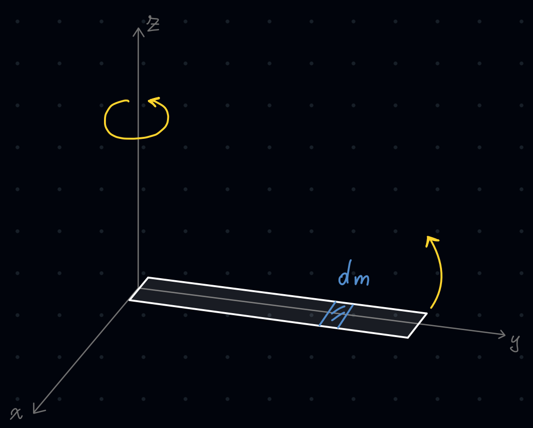
• Moltiplico e divido per dV : $I = \int_V R^2 \frac{dm}{dV} dV$ pongo $\rho = \frac{M}{V}$

$$\to I = \int_V R^2 \rho \, dV$$

• Se la sbarra e' a 2 dimensioni $\rho \to \lambda = \frac{M}{L}$ $R \to x$

$$\to I = \int_A^B \lambda x^2 dx$$

Rotazione lungo un estremo

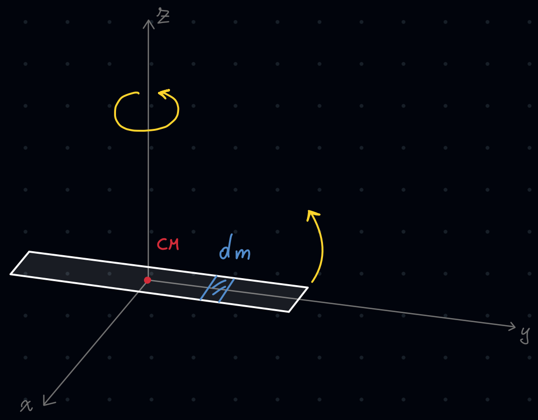


$$I = \int_A^B \lambda R^2 dV \longrightarrow I = \int_0^L \lambda y^2 dy$$

$$\to I = \lambda \int_0^L y^2 dy = \lambda \left[\frac{y^3}{3} \right]_0^L = \lambda \left[\frac{L^3}{3} \right] = \frac{1}{3} \lambda L^3$$

ma $\lambda = \frac{M}{L} \Rightarrow I = \frac{1}{3} \frac{M}{\cancel{L}} L^{\cancel{3}\,2} \to I = \frac{1}{3} M L^2$

Rotazione lungo il centro di massa

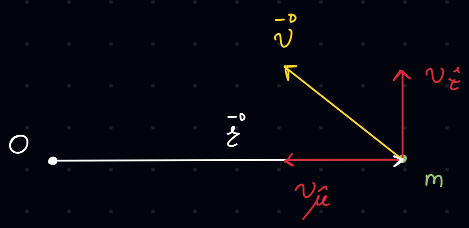


$$I = \lambda \int_{-\frac{L}{2}}^{\frac{L}{2}} y^2 \, dy = \frac{M}{L}\left[\frac{y^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{1}{3}\frac{M}{L}\left[\frac{L^3}{8} + \frac{L^3}{8}\right]$$

$$= \frac{1}{3}\frac{M}{\cancel{L}} \cdot \frac{L^{\cancel{3}\,2}}{4} = \frac{1}{12} M L^2$$

Conclusioni: E' più semplice mettere in rotazione una sbarra attorno al suo centro di massa che rispetto ad un suo estremo

Bonus: Possiamo trovare $I = m R^2$ con:



$$\vec{L} = \vec{r} \wedge \vec{p} = \vec{r} \wedge m \cdot \vec{v} = \vec{r} \wedge m(\vec{v}_{\hat{\mu}} + \vec{v}_{\hat{\tau}})$$

$$= \cancel{\vec{r} \wedge m \vec{v}_{\mu}} + \vec{r} \wedge m \vec{v}_{\tau}$$

$\vec{r} /\!/ \hat{\mu}$

$\rightarrow |\vec{L}| = r\, m\, v_{\tau} \cdot \sin\alpha = r\, m\, v$ ma $v = \omega r \rightarrow |\vec{L}| = \overbrace{(r^2 m)}^{\text{Inerzia}} \omega$