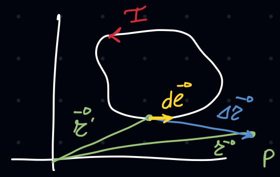
Campo B di un filo

2 Laplace: $\vec F = q\cdot \vec v \wedge \vec B \to \vec F = q\cdot \frac{d\vec l}{dt}\wedge \vec B \to \vec F = I\cdot d\vec l \wedge \vec B$

1 Laplace $\vec B = \frac{\mu_0}{4\pi}\cdot \oint I\cdot \frac{d\vec l\wedge \Delta\vec r}{|\Delta \vec r|^3}$

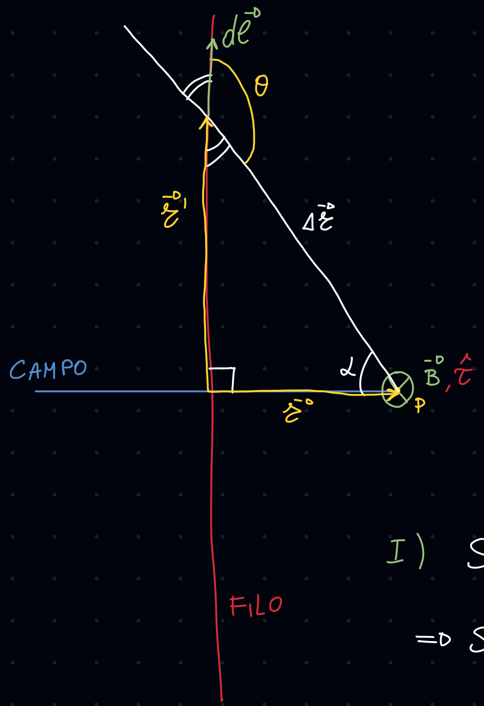
proof

$$\vec F_m = \frac{\mu_0}{4\pi}\cdot\frac{q_m\cdot q_m'}{r^2} \to \vec B = \frac{\vec F_m}{q_m} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat r = \frac{\mu_0}{4\pi}\frac{q_m\cdot \vec r}{r^3}$$



$$\to \vec B = \frac{\mu_0}{4\pi}\oint I\cdot\frac{d\vec l\wedge\Delta\vec r}{|\Delta\vec r|^3}$$

Scopo del gioco Trovare $\vec B_p = \frac{\mu_0}{2\pi}\frac{I}{r}$



$$\vec B = \overbrace{\frac{\mu_0 I}{4\pi}}^{K'} \int \frac{d\vec l\wedge\Delta\vec r}{|\Delta\vec r|^3}\hat\tau = K'\int \frac{dl\,\cancel{\Delta r}\,\sin\theta}{\Delta r^{\cancel{3}\,2}}\hat\tau$$

$$= K'\oint \frac{dl\cdot\sin\theta\cdot\hat\tau}{\Delta r^2}$$ ma $dl \parallel dr' \Rightarrow dl = dr'$

$$|\vec B| = K'\int\frac{\sin\theta}{\Delta r^2}\,dr'$$

Scriviamo tutto in funzione di α

I) $\sin(\theta)$ · $\triangle^\circ = 90+\alpha+\cancel{180}-\theta = \cancel{180} \Rightarrow \alpha=\theta-90 \Rightarrow \theta = \alpha+90$

$$\Rightarrow \sin(\theta) = \sin(\alpha+90) \underset{\text{TRIG}}{=} \cos(\alpha) \quad (1)$$

II) $\begin{cases}\vec r' = \Delta\vec r \sin\alpha \\ \vec r = \Delta\vec r\cos\alpha\end{cases} \Rightarrow \frac{\vec r'}{\vec r} = \tan\alpha \Rightarrow \vec r' = \vec r\tan\alpha \to d\vec r' = d(\vec r\tan\alpha)$ ma $\vec r = cost$

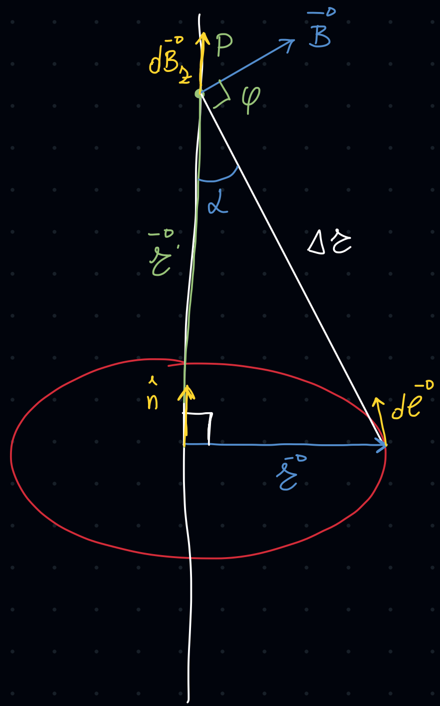
$$\Rightarrow d\vec r' = \frac{r}{\cos^2\alpha}\,d\alpha \quad (2)$$

III) $\vec r = \Delta\vec r\cos\alpha \Rightarrow \Delta\vec r = \frac{\vec r}{\cos\alpha}$ (3)

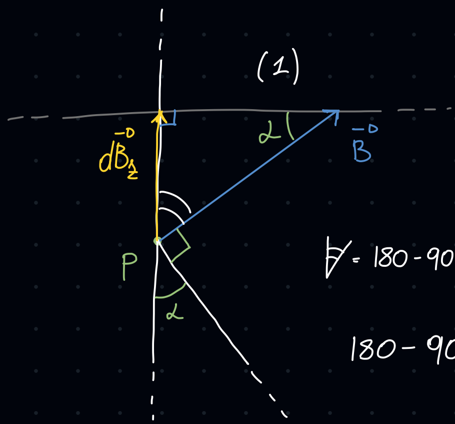
$$\to B = K'\int \frac{\cos(\alpha)}{\frac{r^{\cancel{2}}}{\cancel{\cos^2}\alpha}}\cdot\frac{\cancel{r}}{\cancel{\cos^2\alpha}}\,d\alpha \to B = \frac{K'}{r}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(\alpha)\,d\alpha = \frac{\mu_0 I}{4\pi r}\left[\sin\left(\frac{\pi}{2}\right)-\sin\left(-\frac{\pi}{2}\right)\right]$$

$$\to B = \frac{\mu_0 I}{\cancel{4}_{2}\pi r}\cdot\left[\cancel{1-(-1)}\right] = \frac{\mu_0 I}{2\pi r}$$ QED

Campo B di una Spira



(1)



$\gamma = 180 - 90 - \alpha = 90 - \alpha$

$180 - 90 - 90 + \alpha = \alpha$ QED

1) Laplace : $dB = \frac{\mu_0}{4\pi} I \cdot \frac{\vec{dl} \wedge \vec{\Delta r}}{|\vec{\Delta r}|^3}$ (2)

2) Consideriamo solo $\vec{dB_z}$ perchè le altre componenti si eliminano a due a due

$\rightarrow \vec{dB} = \vec{dB_z} = \hat{n}\, dB_z \rightarrow \vec{B} = \hat{n} \oint dB_z$

ma $dB_z = \overset{(2)}{dB} \sin\alpha$ dal disegno (1)

$\rightarrow \vec{B} = \underbrace{\hat{n} \cdot \frac{\mu_0 I}{4\pi}}_{\vec{K}'} \oint \frac{\vec{dl} \wedge \vec{\Delta r}}{\Delta r^3} \cdot \sin\alpha = \vec{K}' \cdot \oint \frac{dl \cdot \cancel{\Delta r}\, \overset{1}{\sin\varphi}}{\Delta r^{\cancel{3}}} \sin\alpha = \vec{K}' \oint \frac{dl}{\Delta r^2} \sin\alpha$

costanti

$\rightarrow \vec{B} = \vec{K}' \cdot \frac{\sin\alpha}{\Delta r^2} \oint dl = \hat{n} \frac{\mu_0 I}{4\pi} \cdot \frac{\sin\alpha}{\Delta r^2} 2\pi r$

ma: $\sin\alpha \rightarrow r = \Delta r \sin\alpha \Rightarrow \sin\alpha = \frac{r}{\Delta r}$ (a)

$\Delta r = \sqrt{r^2 + \vec{r}'^2} \rightarrow \Delta r^2 = r^2 + r'^2$ Battezzo $r' = z \Rightarrow \Delta r^2 = r^2 + z^2$

$\Rightarrow \vec{B} = \hat{n} \frac{\mu_0 I}{\cancel{2}\cancel{4\pi}} \frac{r}{\sqrt{r^2 + z^2}} \cdot \frac{\cancel{2\pi} r}{r^2 + z^2} \rightarrow (r^2+z^2)^{\frac{1}{2}} \cdot (r^2+z^2)^1 = (r^2+z^2)^{\frac{3}{2}} = \sqrt{(r^2+z^2)^3}$

$\Rightarrow \vec{B} = \hat{n} \frac{\mu_0 I R^2}{2(r^2+z^2)^{3/2}}$ QED

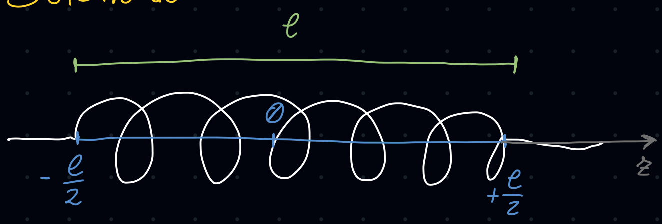
Momento magnetico $\quad \vec{m} = \hat{n} \cdot I \cdot S \quad$ dove S è la superficie $\quad S_0 = \pi R^2$

$$\Rightarrow \vec{m}_{spira} = \hat{n} \cdot I \cdot \pi R^2 \quad \Rightarrow \vec{B}_{spira} = \frac{\hat{n} \cdot I R^2}{2(r^2+z^2)^{\frac{3}{2}}} \cdot \frac{\pi}{\pi}$$

$\vec{m}_{spira}$

$$\Rightarrow \vec{B}_{spira} = \frac{\mu_0 \vec{m}}{2\pi (r^2+z^2)^{\frac{3}{2}}}$$

Solenoide



Ci sono N spire in una lunghezza ℓ

$\Rightarrow$ n = densità di spire = $\frac{N}{\ell}$

$\rightarrow$ dz è la porzione infinitesimale della <u>LUNGHEZZA</u> del filo

$\Rightarrow$ $n\,dz$ = spire nella porzione

COSTANTI

$$\Rightarrow \vec{B}_{TOT} = \int \vec{B} \cdot n\,dz = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \hat{n} \cdot \frac{\mu_0 I R^2}{2(z^2+z^2)^{\frac{3}{2}}} \cdot n\,dz \rightarrow \vec{B}_{TOT} = \hat{n} \cdot \frac{\mu_0 I R^2}{2} n \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{1}{(z^2+z^2)^{\frac{3}{2}}} dz$$

CAMPO SINGOLA SPIRA

Primitiva = $\frac{1}{R^2} \frac{z}{(R^2+z^2)^{\frac{1}{2}}}$

$$\rightarrow \vec{B}_{TOT} = \overbrace{\hat{n} \cdot \frac{\mu_0 I R^2}{2}}^{\vec{K}'} \cdot \frac{N}{\ell} \cdot \left[\frac{1}{R^2} \frac{z}{\sqrt{R^2+z^2}} \right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}}$$

$$\rightarrow \vec{B}_{TOT} = \vec{K}' \frac{N}{\ell} \cdot \left[\frac{1}{R^2} \frac{\ell}{2\sqrt{R^2+\frac{\ell^2}{4}}} + \frac{1}{R^2} \frac{\ell}{2\sqrt{R^2+\frac{\ell^2}{4}}} \right]$$

$$= \hat{n} \frac{\mu_0 I R^2 N}{\ell \cdot \cancel{2}} \cdot \cancel{2} \left[\frac{1}{R^2} \frac{\ell}{2\sqrt{R^2+\frac{\ell^2}{4}}} \right] \rightarrow 2\sqrt{R^2+\frac{\ell^2}{4}} = 2\sqrt{\frac{4R^2-\ell^2}{4}} = \frac{\cancel{2}\sqrt{4R^2-\ell^2}}{\sqrt{\cancel{4}}}$$

$$= \hat{n} \frac{\mu_0 I \cancel{R^2} N}{\cancel{\ell}} \cdot \frac{\cancel{\ell}}{\sqrt{4R^2+\ell^2}} \frac{1}{\cancel{R^2}} \rightarrow \vec{B} = \hat{n} \cdot \frac{\mu_0 I N}{\sqrt{4R^2+\ell^2}}$$

Sosteniamo che $\ell >> R$ ovvero lunghezza del solenoide molto maggiore del suo raggio

$$\rightarrow \sqrt{4R^2+\ell^2} \simeq \sqrt{\ell^2} \simeq \ell \Rightarrow \vec{B} = \hat{n} \overbrace{\frac{\mu_0 I N}{\ell}}^{n} = \underline{\hat{n} \mu_0 I n}$$