Teorema di equivalenza di Ampère

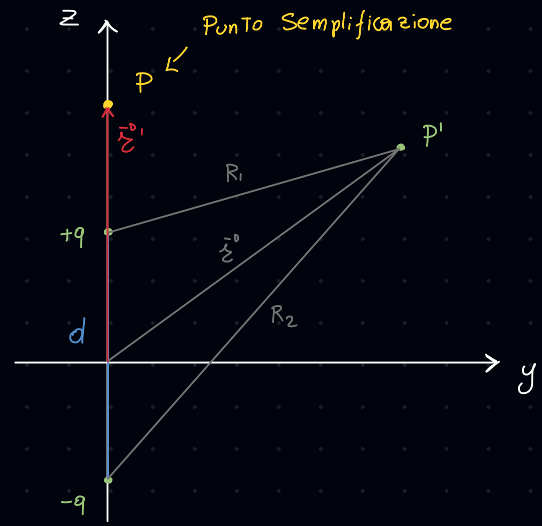
Ci dice che una spira percorsa da corrente si comporta come un dipolo magnetico, se osservata da grande distanza.

$\Rightarrow$ Questo si traduce in 3 casi:

1. Campo $\vec{E}/\vec{B}$ prodotto dalla spira con $z \gg R$ $\equiv$ Campo $\vec{E}/\vec{B}$ Bipolo
1. Forza prodotta dalla spira immersa in $\vec{B}/\vec{E}$ $\equiv$ Forza Bipolo
1. Momento agente sulla spira con $z \gg R$ $\equiv$ Momento Bipolo

Caso 1: Campo

- Bipolo Elettrico



Poniamo il caso che P sia SEMPRE lungo z

$$\Rightarrow \vec{E}_z = \frac{q \cdot d}{4\pi\varepsilon_0} \frac{3\cos^2\theta - 1}{r^3}$$

Se P è lungo $z \rightarrow \theta_{r'z} = 0 \Rightarrow \cos\theta = 1$

$$\Rightarrow \vec{E}_z = \frac{q \cdot d}{4\pi\varepsilon_0} \cdot \frac{2}{r'^3} = \frac{q \cdot d}{2\pi\varepsilon_0 \vec{r}\,'^3} \hat{k}$$

Pongo $\vec{p} = q \cdot \vec{d}$ = Momento del dipolo elettrico

$$\Rightarrow \vec{E} = \frac{\vec{p}}{2\pi\varepsilon_0 \vec{r}\,'^3}$$

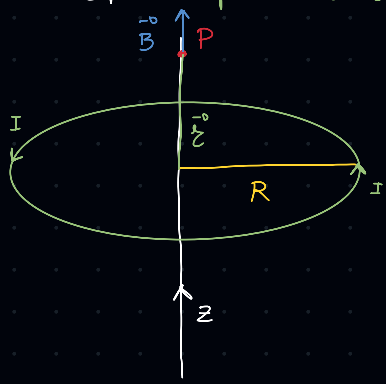
Bipolo Magnetico $\rightarrow q_m$ carica magnetica

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q_m \cdot q_m'}{r^2} \Rightarrow \vec{B}_z = \frac{\mu_0}{4\pi} \frac{2 q_m \cdot d}{\vec{r}\,'^3} \quad (1) \Rightarrow \vec{m} = q_m \cdot \vec{d}$$

Momento dipolo magnetico

$\rightarrow$ Stessa cosa del Dipolo Elettrico. $\leftarrow$ $\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{m}}{\vec{r}\,'^3}$

- Spira percorsa da corrente



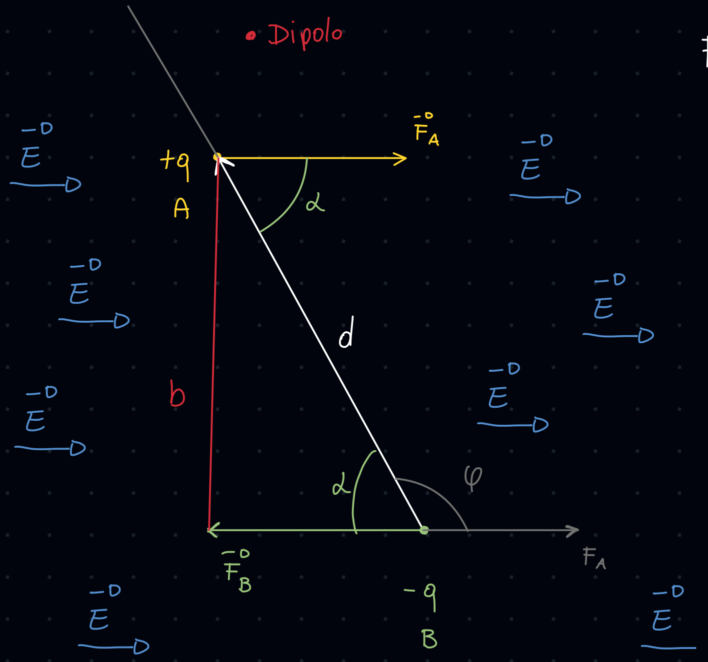
$$\vec{B}_z = \hat{n} \cdot \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}} \quad \text{se } z \gg R \rightarrow B_z = \hat{n} \frac{\mu_0 I R^2}{2 z^3}$$

Definisco $\vec{m} = I S \hat{n} \Rightarrow S = \pi R^2$

$$\Rightarrow \frac{\mu_0 \overbrace{I R^2 \hat{n} \cdot \frac{\pi}{\pi}}^{\vec{m}}}{2 z^3} = \frac{\mu_0}{2\pi} \cdot \frac{\vec{m}}{z^3} \quad \text{QED}$$

Caso 3: Momento di una Forza

• Dipolo



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \text{ma} \quad E = \frac{F}{q} \Rightarrow \vec{F} = q\cdot\vec{E}$$

↑ Forza diretta come il campo elettrico

Momento di una forza $\vec{M} = \vec{r}\wedge\vec{F}$

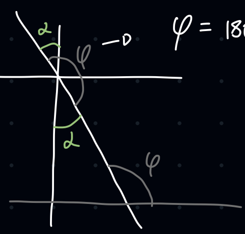
$$\Rightarrow \vec{M}_{TOT} = \vec{r}_1\wedge\vec{F}_A + \vec{r}_2\wedge\vec{F}_B$$

Fisso il polo in B

$$= \vec{r}_1\wedge\vec{F}_A + 0\wedge\vec{F}_B = \vec{r}_1\wedge\vec{F}_A$$

$$= \vec{r}_1\wedge q\cdot\vec{E} = \underbrace{d\wedge q}_{\vec{p}}\vec{E} = \vec{p}\wedge\vec{E}$$

Modulo $|\vec{M}_{TOT}| = d\cdot q\cdot E\cdot\sin\varphi = p\,E\cdot\sin\varphi = p\cdot E\cdot\sin(180-\alpha)$

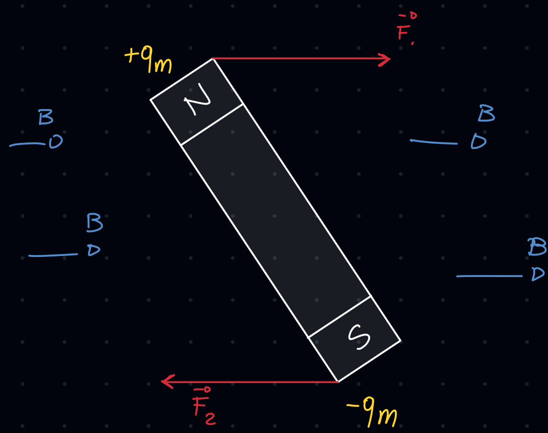


$\varphi = 180-\alpha$

$\Rightarrow M = p\cdot E\sin\alpha$ $\qquad \sin(180-\alpha) = \sin\alpha$

Inoltre: $M = dqE\sin\alpha$ ma $d\sin\alpha = b$ ← "Braccio della coppia"

$\rightarrow M = q\cdot E\cdot b$ ma $q\vec{E} = \vec{F} \Rightarrow \boxed{M = F\cdot b}$ VALIDO SEMPRE



$$\vec{F} = q_m\cdot\vec{B} \Rightarrow \vec{M}_{TOT} = \vec{d}\wedge q\cdot\vec{B}$$
$$= \vec{m}\wedge\vec{B}$$

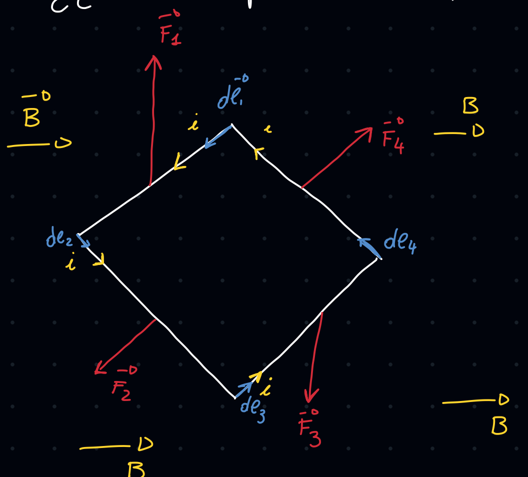
uguale a sopra ↑

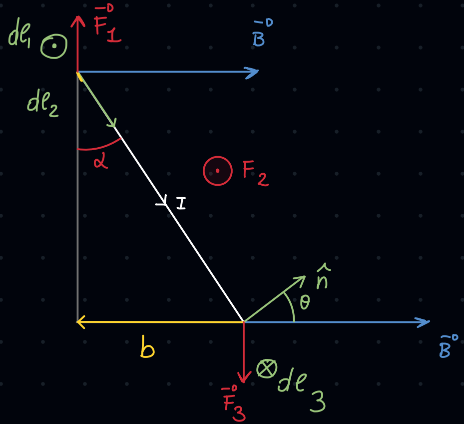
• Spira

Legge di Laplace: Forza Lorentz: $F = q(\vec{v}\wedge\vec{B}) \rightarrow F = q\frac{d\vec{l}}{dt}\wedge\hat{B} = i\,d\vec{l}\wedge\vec{B}$

LAPLACE (2)

Le forze sono date da Laplace





$$d\vec{F}_1 = I\, d\vec{e}_1 \wedge \vec{B} \Rightarrow \vec{F}_1 = I \int B \cdot de_1 = \underline{I B e_1} \qquad \underline{F_3 = I B e_3}$$

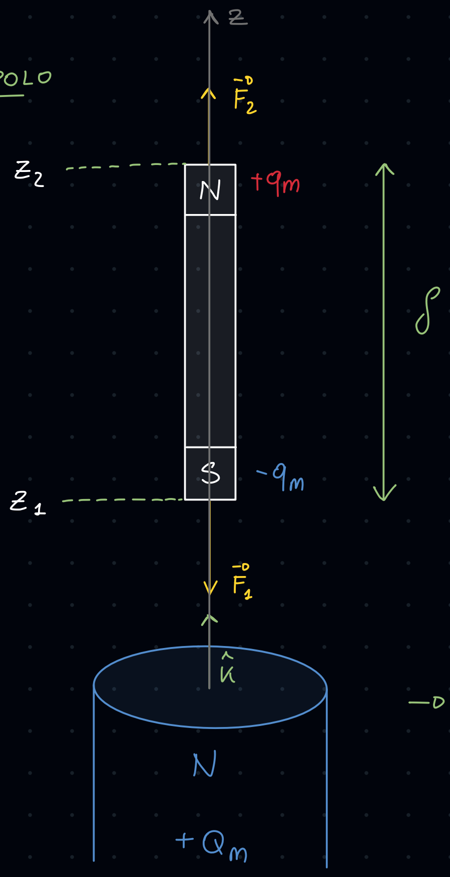
$\Rightarrow$ Momenti

$$M = F \cdot b \Rightarrow I B \underbrace{e_1 \cdot e_2}_{b \times h = S} \sin\theta \rightarrow \underline{M_{spira} = I B S \sin\alpha}$$

Definisco $\vec{m} = I \cdot S \hat{n} \Rightarrow m \cdot B \sin\alpha = \vec{m} \wedge \vec{B}$

Forza su un dipolo $\rightleftharpoons$ Spira

DIPOLO



Siccome $\vec F_c = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1\cdot q_2}{r^2} \to \frac{\vec F}{q_m} = \frac{\mu_0}{4\pi}\frac{q_m\, q_m'}{r^2}\hat r$

$$\vec F_{TOT} = \vec F_1 + \vec F_2 = \frac{\mu_0}{4\pi}\left[-\frac{q_m Q_m}{z_1^2} + \frac{q_m Q_m}{z_2^2}\right]$$

Siccome $z_2 = z_1 + \delta \to \vec F_{TOT} = \frac{\mu_0}{4\pi}\left[\frac{q_m Q_m}{(z_1+\delta)^2} - \frac{q_m Q_m}{z_1^2}\right]$

$$\to F_{TOT} = \frac{q_m Q_m \mu_0}{4\pi}\left[\frac{\cancel{z_1^2} - \cancel{z_1^2} - \delta^2 - 2z_1\delta}{(z_1+\delta)^2 z_1^2}\right]$$

$$\to \vec F_{TOT} = -\frac{q_m Q_m \mu_0}{4\pi}\left[\frac{\delta^2 + 2z\delta}{z_1(\delta+z_1)^2}\right]\hat k$$

$2z\delta \gg \delta^2$

Ipotesi $z \gg \delta \Rightarrow \vec F_{TOT} = -\frac{q_m Q_m \mu_0}{4\pi}\left[\frac{\cancel{\delta^2} + 2z\delta}{z_1^2(\cancel{\delta} + z_1)^2}\right]$

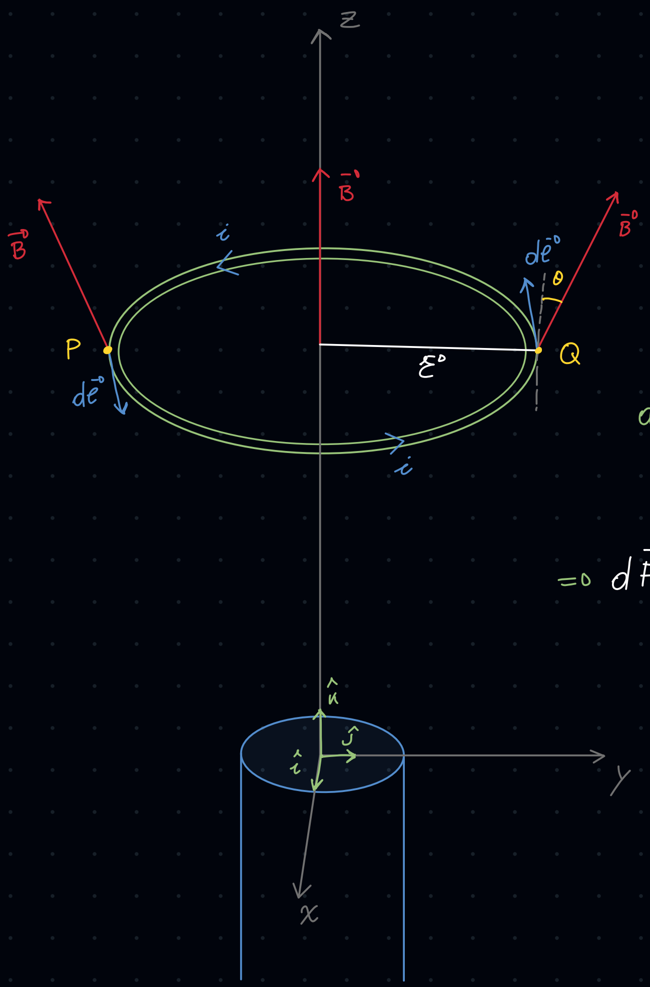
$z_1 \gg \delta$

$$\to \vec F_{TOT} = -\frac{q_m Q_m \mu_0}{4\pi}\left[\frac{2\cancel{z}\delta}{z_1^{\cancel{4}\,3}}\right]\hat k = -\hat k\,\frac{q_m Q_m \mu_0\, 2\delta}{4\pi z_1^3} \quad (1)$$

Siccome $|\vec B| = \frac{\vec F}{q} = \frac{\mu_0}{4\pi}\frac{Q_m}{z_1^2} \to \vec F_{TOT} = -\hat k\,\frac{q_m\, 2\delta}{z_1}B$ (2)

definisco $\vec m = q_m\delta\hat u \Rightarrow \vec F_{TOT} = -\frac{2\vec m B}{z_1}$ (3)

SPIRA:



Campo magnetico in P

$$d\vec{l} = dl\,\hat{i} \rightarrow \vec{B}_P = -\hat{j}\,B\sin\theta + \hat{k}\,B\cos\theta$$

$$d\vec{l} = -dl\,\hat{i} \rightarrow \vec{B}_Q = \hat{j}\,B\sin\theta + \hat{k}\,B\cos\theta$$

da Lorenz: $F = q\,\vec{v}\wedge\vec{B} \rightarrow \underbrace{I\,d\vec{l}\wedge\vec{B}}_{\text{Laplace}}$

$$\Rightarrow d\vec{F}_P = I\,d\vec{l}\wedge\vec{B} = I\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ dl & 0 & 0\\ 0 & -B\sin\theta & B\cos\theta\end{vmatrix}$$

$$= I\left[\hat{i}(0-0) - \hat{j}(dl\cdot B\cos\theta - 0) + \hat{k}(-dl\,B\sin\theta - 0)\right]$$

$$= -dl\,B\cos\theta\,I\,\hat{j} - dl\,B\sin\theta\,I\,\hat{k} \quad (1)$$

$$d\vec{F}_Q = I\,d\vec{l}\wedge\vec{B} = I\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -dl & 0 & 0\\ 0 & B\sin\theta & B\cos\theta\end{vmatrix} = I\left[\hat{i}(0) - \hat{j}(-dl\,B\cos\theta - 0) + \hat{k}(-dl\,B\sin\theta)\right]$$

$$= \hat{j}\,dl\,B\cos\theta\,I - \hat{k}\,dl\,B\sin\theta\,I \quad (2)$$

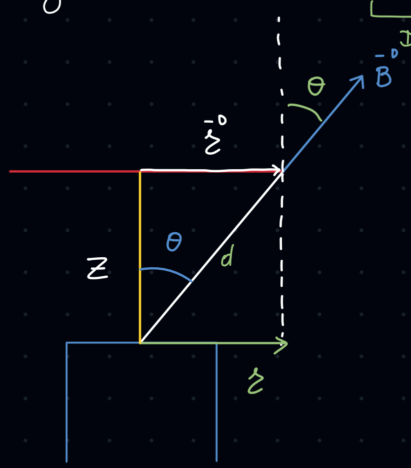
$$\Rightarrow d\vec{F}_{TOT} = d\vec{F}_P + d\vec{F}_Q = -\cancel{dl\,B\cos\theta\,I\,\hat{j}} - dl\,B\sin\theta\,I\,\hat{k} + \cancel{dl\,B\cos\theta\,I\,\hat{j}} - dl\,B\sin\theta\,I\,\hat{k}$$

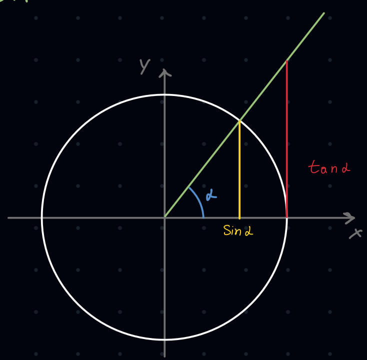
$$= -2\,dl\,B\sin\theta\,I\,\hat{k}$$

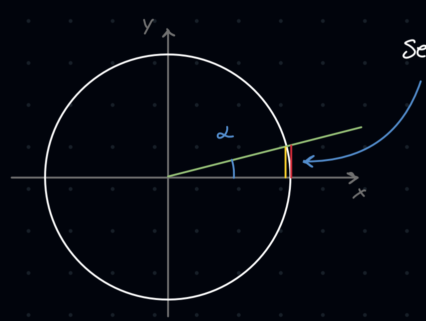
Mezza Spira

$$\Rightarrow \vec{F}_{TOT} = -2\,B\sin\theta\,I\,k\int_0^{\pi} dl = -2\pi r\,B\sin\theta\,I\,\hat{k} \quad (3)$$

Hp: Grandi distanze : $\underbrace{z_1 >> \delta}_{\text{DIPOLO}} \rightarrow \underbrace{Z >> r}_{\text{SPIRA}}$







Se $\alpha \rightarrow 0 \Rightarrow \sin\alpha \approx \tan\alpha$

$\Rightarrow$ ma $\begin{cases} \varepsilon = d \sin\alpha \\ z = d \cos\alpha \end{cases} \Rightarrow \tan\alpha = \dfrac{\varepsilon}{z}$

$\Rightarrow$ per $z >> \varepsilon \rightarrow \vec{F}_{TOT} = -2\pi\varepsilon B \tan\theta I \hat{k} = -2\pi\varepsilon B \dfrac{\varepsilon}{z} I \hat{k} = -\dfrac{2\pi\varepsilon^2 BI}{z} \hat{k}$

Chiamo $\vec{m} = I S \hat{k} = I \pi \varepsilon^2 \hat{k} \rightarrow \vec{F}_{TOT} = -\dfrac{2\vec{m}B}{z}$ QED