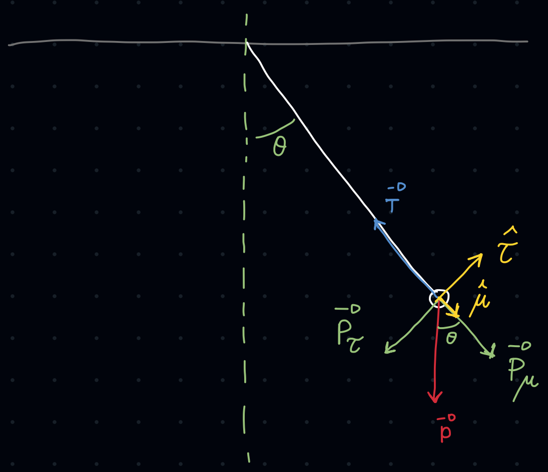


$$\vec{P}_\mu = m \cdot g \cdot \cos\theta$$
$$\vec{P}_\tau = m\, g\, \sin\theta$$

$$\begin{cases} \hat{\mu} & mg\cos\theta - T = m \cdot \vec{a}_\mu \\ \hat{\tau} & -mg\sin\theta = m\, \vec{a}_\tau \end{cases}$$

$$\vec{a}_{\hat{\mu}} = \text{Centripeta} = -\frac{v^2}{R} = \left(-\frac{\dot{S}^2}{\ell}\right) \vec{a}_{\hat{\mu}}$$
$$\vec{a}_{\hat{\tau}} = \text{Tangenziale} = ?? = \underset{|\vec{a}|}{\ddot{S}} \cdot \hat{\tau}$$

$$\Rightarrow \begin{cases} mg\cos\theta - T = -\frac{m\dot{S}^2}{\ell}\hat{\mu} \\ -mg\sin\theta = m \cdot \ddot{S}\hat{\tau} \end{cases}$$

Approssimazione per piccole oscillazioni $\sin\theta \sim \theta$

$\Rightarrow -m\vec{g}\,\theta = m\,\ddot{S}\hat{\tau}$ se $m_I = m_g \rightarrow -\vec{g}\,\theta = \ddot{S}\hat{\tau} \rightarrow \ddot{S}\hat{\tau} + \vec{g}\,\theta = 0$

ma θ Radianti $\rightarrow 1\,\text{Rad} = \frac{\ell}{R} \Rightarrow \ddot{S}\hat{\tau} + \frac{g}{R} \cdot \ell = 0$ pongo $R = \ell$ ovvero il "FILO" del pendolo

$\ell = S$ ovvero l'Arco

$$\Rightarrow \ddot{S}\hat{\tau} + \left(\frac{g}{\ell}\right) \cdot S = 0 \rightarrow \underbrace{\ddot{S}\hat{\tau} + K^2 \cdot S = 0}_{\text{Eq. diff}}$$

Battezzo $\frac{g}{\ell} = K^2$

$\Rightarrow$ Soluzione: $S(t) = A \cdot \cos(Kt + \varphi)$

Cos e' periodico di $T = 2\pi$

Trovare K: Cos e' periodico $\rightarrow S(t+T_0) = A \cdot \cos(Kt + \varphi + 2\pi)$

$$\rightarrow \cancel{A}\cos(Kt + KT_0 + \varphi) = \cancel{A}\cos(Kt + \varphi + 2\pi) \rightarrow \cancel{Kt} + KT_0 + \cancel{\varphi} = \cancel{Kt} + \cancel{\varphi} + 2\pi$$

$$\rightarrow KT_0 = 2\pi \Rightarrow K = \frac{2\pi}{T_0}$$

$$\Rightarrow S(t) = A\cos\left(\frac{2\pi}{T_0}t + \varphi\right) = A\cos(\omega t + \varphi)$$

$$\omega = \frac{d\alpha}{dt} = \frac{2\pi}{T_0}$$

Periodo $K = \omega = \frac{2\pi}{T} \rightarrow K^2 = \frac{g}{\ell} \Rightarrow K = \sqrt{\frac{g}{\ell}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}}$

$\Rightarrow 2\pi\sqrt{\frac{\ell}{g}} = T$ Periodo del pendolo

1° eq El.

$$\phi_E = \int \vec{E}\cdot\hat{n}\,dS = \frac{Q}{\varepsilon_0} \qquad \rho = \frac{dQ}{dV} \rightarrow dQ = \rho\, dV \rightarrow Q = \int \rho\, dV$$

$$\Rightarrow \int_S \vec{E}\cdot\hat{n}\,dS = \frac{1}{\varepsilon_0}\int \rho\, dV \rightarrow \int_V \vec{\nabla}\cdot\vec{E}\, dV = \frac{1}{\varepsilon_0}\int_V \rho\, dV \Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \quad (a)$$

2° eq

$$C = \oint \vec{E}\cdot d\vec{\ell} = \emptyset \Rightarrow \int \vec{\nabla}\wedge\vec{E}\;\hat{n}\,dS = 0 \Rightarrow \vec{\nabla}\wedge\vec{E} = 0$$

Legge di Coulomb

$$L = F\cdot S = \int \vec{F}_c\cdot d\vec{\ell} = U_A - U_B = q(V_A - V_B) \Rightarrow \frac{L}{q} = \frac{F}{q}d\ell = -dV \rightarrow \frac{L}{q} = \vec{E}\,d\vec{\ell} = -dV$$

V è del tipo $V(x,y,z) \Rightarrow dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$

otteniamo dV

$$\begin{cases} d\ell = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz \\ \nabla V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \end{cases} \Rightarrow dV = \vec{\nabla}V\cdot d\vec{\ell}$$

$$\Rightarrow \vec{E}\cdot\cancel{d\vec{\ell}} = -\vec{\nabla}V\cancel{d\vec{\ell}} \rightarrow \vec{E} = -\vec{\nabla}V \quad (b)$$

Uniamo (a) e (b)

$$\rightarrow \nabla(-\nabla V) = \frac{\rho}{\varepsilon_0} \rightarrow \vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0} \quad \text{Eq di Poisson}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}\,')}{\Delta r}\,dV$$

Potenziale Vettore $\quad \vec{B} = \vec{\nabla}\wedge\vec{A}$

$$\Rightarrow \begin{cases} \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} \end{cases} \begin{matrix} \text{GAUSS} \\ \text{AMPÈRE} \end{matrix} \Rightarrow \begin{cases} \vec{\nabla}(\vec{\nabla}\wedge\vec{A}) = 0 & \rightarrow \text{Sempre Vera} \\ \vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{A}) = \mu_0\vec{J} & \rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A} = \mu_0 J \end{cases}$$

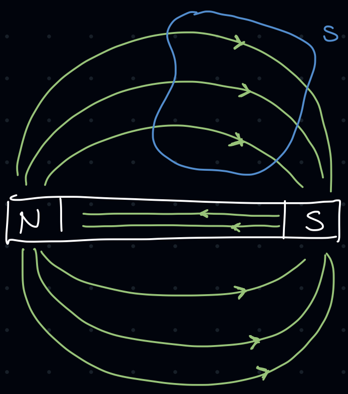
Se $\vec{\nabla}\cdot\vec{A} = 0 \rightarrow \vec{\nabla}^2 A = -\mu_0\vec{J}$ Simile all'eq di Poisson

Soluz.

$$\rightarrow A = \frac{\mu_0}{4\pi}\int \frac{J\cdot(\vec{r}\,')}{\Delta r}\,dV \quad \text{ma} \quad \vec{J}\,dV = (\overbrace{\vec{J}\cdot S}^{I})\,d\vec{\ell} \Rightarrow A = \frac{\mu_0}{4\pi}\oint \frac{I\cdot\vec{r}\,'}{\Delta r}\,d\vec{\ell}$$

$$A' = \vec{A} + \vec{\nabla}f \rightarrow \vec{B} = \vec{\nabla}\wedge\vec{A} = \vec{\nabla}\wedge\vec{A}'$$

Maxwell Magnetostatica



$$\phi_B = \int \vec{B} \cdot \hat{n}\, dS = 0 \\ \rightarrow \vec{\nabla} \vec{B} = \emptyset$$ } 1° Eq di Maxwell

Biot - Savart

Campo Magnetico attorno un filo percorso da corrente

Sperimentalmente: $B = K \cdot \frac{I}{R}$

$\rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{I}{R} \hat{\tau} \rightarrow$ Circuitazione: $C = \oint \vec{B} \cdot d\vec{\ell} = K \oint \frac{I}{R} d\vec{\ell}$

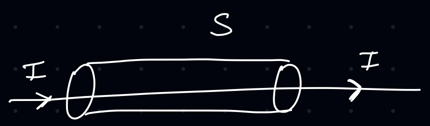
ma $d\vec{\ell}$ = Arco di circonferenza $\Rightarrow 1\,\text{Rad} = \frac{R}{\ell} \rightarrow \ell = \varphi \cdot R \rightarrow d\ell = R\, d\varphi$

$\Rightarrow C = \oint \vec{B}\, d\vec{\ell} = K \oint \frac{I}{\cancel{R}} \cdot \cancel{R}\, d\varphi \Rightarrow \oint \vec{B} d\vec{\ell} = K I \cdot \oint d\varphi \Rightarrow \oint \vec{B} d\vec{\ell} = \frac{\mu_0}{\cancel{4\pi}} \cdot I \cdot \cancel{4\pi}$

$\rightarrow \oint \vec{B}\, d\vec{\ell} = \mu_0 I$ Legge di Ampère Integrale

$\int_S (\vec{\nabla} \wedge \vec{B}) \cdot \hat{n}\, dS = \mu_0 \int_S \vec{J} \cdot \hat{n}\, dS \rightarrow \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$ Differenziale

Manca un pezzo!



$\phi = \int \vec{J} \cdot \hat{n}\, dS = \emptyset \rightarrow \underset{\text{Div}}{\vec{\nabla} \vec{J}} = \emptyset$ Corrente stazionaria

$\phi = \int \vec{J} \cdot \hat{n}\, dS \neq 0$! ?? Troviamo il pezzo

Eq di continuità

$- dq = \int \vec{J} \cdot \hat{n}\, dS \cdot dt \rightarrow - \frac{dq}{dt} = \int \vec{J} \cdot \hat{n}\, dS$ ma $\rho = \frac{dq}{dV} \rightarrow dq = \rho\, dV$

$\rightarrow - \frac{d}{dt} \int_V \rho(x,y,z)\, dV = \int_S \vec{J} \cdot \hat{n}\, dS \rightarrow - \int_V \frac{\partial \rho}{\partial t} dV = \int_S \vec{J} \cdot \hat{n}\, dS$

$\rightarrow - \int_V \frac{\partial \rho}{\partial t} dV = \int_V \vec{\nabla} \vec{J}\, dV \rightarrow \vec{\nabla} \vec{J} = - \frac{\partial \rho}{\partial t}$ Obbiettivo

Perchè manca un pezzo?

Ampère: $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$ $\to$ Div $\to$ $\vec{\nabla} \cdot \underbrace{(\cancel{\vec{\nabla} \wedge \vec{B}})}_{0} = \mu_0 \vec{\nabla} \cdot \vec{J}$ $\to$ $\vec{\nabla} \cdot \vec{J} = 0$ Valida solo in regime di correnti stazionario!

$\to$ Termine di Maxwell $\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\to$ Ampère Maxwell: $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Proof $\to$ Div $\to$ $\vec{\nabla} \cdot \underbrace{(\cancel{\vec{\nabla} \wedge \vec{B}})}_{0} = \mu_0 \vec{\nabla} \cdot \vec{J} + \mu_0 \varepsilon_0 \frac{\partial (\vec{\nabla} \cdot \vec{E})}{\partial t}$ $\to$ $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

$\to$ $\cancel{\mu_0} \left(\vec{\nabla} \cdot \vec{J} + \cancel{\varepsilon_0} \frac{1}{\cancel{\varepsilon_0}} \frac{\partial \rho}{\partial t} \right) = 0$ $\qquad \vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho}{\partial t}$ QED

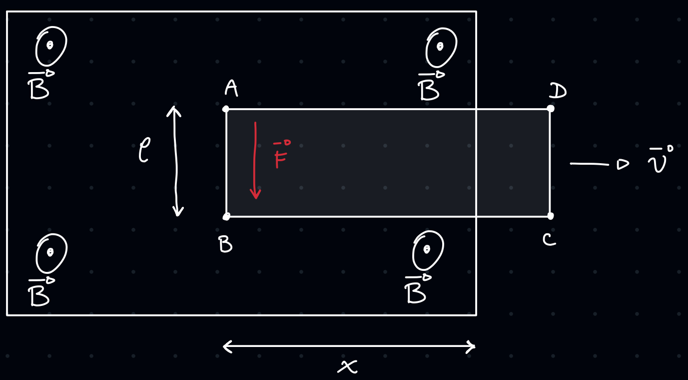
Maxwell campo elettr.

$$\begin{cases} \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \oint \vec{E}\, d\vec{l} = 0 \to \vec{\nabla} \wedge \vec{E} = 0 \end{cases} \to \text{Manca un pezzo}$$

Faraday $\quad f_{em} = -\dfrac{d\phi_B}{dt} \to \phi_B = \int \vec{B} \cdot \hat{n}\, ds = \int B\, ds \cos\theta$

Flusso Tagliato   Varia Superficie

$\vec{F} = \text{Lorentz} = q\vec{E} + q(\vec{v} \wedge \vec{B})$



$$\Rightarrow f_{em} = \frac{L}{q} = \frac{\cancel{q}\int \vec{E} \cdot d\vec{l}}{\cancel{q}} + \frac{\cancel{q}\int (\vec{v} \wedge \vec{B})\, d\vec{l}}{\cancel{q}}$$

Se $E = 0 \to f_{em} = \oint (\vec{v} \wedge \vec{B})\, d\vec{l}$

$$f_{em} = \int_A^B (\vec{v} \wedge \vec{B})\, dl + \cancel{\int_B^C (\vec{v} \wedge \vec{B})\, dl}_{\vec{F} \perp \vec{B}} + \cancel{\int_C^D (\vec{v} \wedge \vec{B})\, dl}_{\vec{B} = 0} + \cancel{\int_D^A (\vec{v} \wedge \vec{B})\, dl}_{\vec{F} \perp \vec{B}} = \int_A^B (\vec{v} \wedge \vec{B})\, dl = \int v \cdot B \cdot \underbrace{\sin\theta}_{1}\, dl = \underbrace{vBl}_{\text{Forza EM}}$$

Troviamo lo stesso risultato con $f_{em} = -\dfrac{d\phi_B}{dt}$?

la superficie S dipende da x : $\quad S = b \times h = l \cdot x$

$$\phi_B = \int \vec{B} \cdot \hat{n}\, ds = B \underbrace{\int_S ds}_{l \cdot x} \Rightarrow \underbrace{\phi_B = Blx}_{\text{FLUSSO}}$$

ad un tempo $t' = t + \Delta t \to \quad \phi_B(t+\Delta t) = Bl(x - \Delta x)$ con $\Delta x = v \cdot \Delta t$

$$\Rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \frac{\phi_B(t+\Delta t) - \phi(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\cancel{Blx} - Bl\Delta x - \cancel{Blx}}{\Delta t} = -Bl \overbrace{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}^{v}$$

$$\Rightarrow -\frac{d\phi_B}{dt} = Blv \quad \text{QED}$$

Flusso concatenato $\to$ varia $\vec{B}$

$$\to \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \frac{\int \vec{B}(t+\Delta t, \vec{r})\, \hat{n}\, dS - \int B(t, \vec{r})\, \hat{n}\, dS}{\Delta t}$$

Approssimo: $\vec{B} \simeq \vec{B}(t, \vec{r}) + \frac{\partial \vec{B}(t+\Delta t, \vec{r})}{\partial t} \cdot \Delta t$

$$\to \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \cancel{\int \vec{B}(t, \vec{r})\, \hat{n}\, dS} + \frac{\int \frac{\partial \vec{B}}{\partial t}(t+\cancel{\Delta t}, \vec{r}) \cdot \cancel{\Delta t} \cdot \hat{n}\, dS}{\cancel{\Delta t}} - \cancel{\int \vec{B}(t, \vec{r})\, \hat{n}\, dS}$$

$$\to \frac{d\phi_B}{dt} = \int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS$$

$$\Rightarrow f_{em} = \frac{L}{q} \qquad \text{se } F = q\vec{E} + q(\vec{v} \wedge \vec{B}) \to f_{em} = \int \vec{E} \cdot d\vec{l} + \int (\vec{v} \wedge \vec{B})\, d\vec{l}$$

$$\Rightarrow \oint \vec{E}\, d\vec{l} + \oint (\vec{v} \wedge \vec{B})\, d\vec{l} = \int \frac{\partial B}{\partial t}\, \hat{n}\, dS \qquad \text{se } v = 0 \to \int_l \vec{E}\, d\vec{l} = \int_S \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS$$

$$\to \int_S (\vec{\nabla} \wedge \vec{E})\, \hat{n}\, dS = \int_S \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS \to \vec{\nabla} \wedge \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

Eq. di maxwell
campo elettromagn.

Tutto insieme

Tagliato: $-\frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B})\, d\vec{l}$

Concatenato: $-\frac{d\phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS$

generale $\Rightarrow$ $-\frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B})\, d\vec{l} - \int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS$

$\Rightarrow$ Scrivo $f_{em} = -\frac{d\phi}{dt}$ $\to$ $\oint \vec{E} \cdot d\vec{l} + \cancel{\oint (\vec{v} \wedge \vec{B})\, d\vec{l}} = \cancel{\oint (\vec{v} \wedge \vec{B})\, d\vec{l}} - \int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS$

$$\to \oint_l \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t}\, \hat{n}\, dS \to \int_S \vec{\nabla} \wedge \vec{E}\, d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t}\, d\vec{S} \to \underline{\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Auto induzione

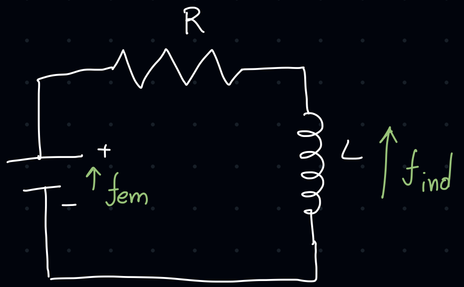
Legge di Laplace $\quad \vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\ell} \wedge \vec{r}}{\Delta r^3}$

flusso $\rightarrow \quad \phi_B = \int_S \frac{\mu_0 I}{4\pi} \int_\ell \frac{d\vec{\ell} \wedge \vec{r}}{\Delta r^3} \hat{n}\, dS \qquad$ Tutto costante tranne I

$\Rightarrow \quad \underset{\text{Induttanza}}{\phi_B = L \cdot I} \quad \rightarrow \quad$ Se $B \neq cost \rightarrow \quad \frac{d\phi_B}{dt} = L \cdot \frac{dI}{dt}$

Siccome $\quad f_{em} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$

Circuito R-L



Dalle leggi di Kirchhoff

$$f_{em} - f_{ind} = R \cdot I$$

$\rightarrow \quad f_{em} - L\frac{dI}{dt} = R \cdot I \quad \Rightarrow \quad \underline{f_{em} = RI + L\frac{dI}{dt}}$

Eq differenziale

Soluzione del tipo $\quad I = Ae^{\alpha t} + D$

$\rightarrow \quad \frac{dI}{dt} = A\alpha e^{\alpha t} \quad \Rightarrow \quad f_{em} = R \cdot Ae^{\alpha t} + LA\alpha e^{\alpha t} + RD \qquad$ ma $f_{em} = cost$

$f_{em} = R \cdot D$

$\Rightarrow \quad RAe^{\alpha t} + LA\alpha e^{\alpha t} = \emptyset \quad \rightarrow \quad Ae^{\alpha t}(R + L\alpha) = 0 \quad \rightarrow \quad R + L\alpha = 0 \quad \rightarrow \quad \alpha = -\frac{R}{L}$

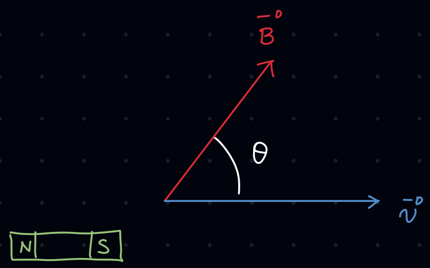
$\rightarrow D = \frac{f}{R}$

$\Rightarrow \quad I = Ae^{-\frac{R}{L}t} + \frac{f}{R} \qquad$ Soluzione generale

$\rightarrow$ Trovo A: $\quad I(0) = 0 \quad \rightarrow \quad I(0) = A\overset{1}{e^{-\frac{R}{L}t}} + \frac{f}{R} = 0 \quad$ per $\quad A + \frac{f}{R} = 0 \quad \rightarrow \quad A = -\frac{f}{R}$

$\Rightarrow \quad I = -\frac{f}{R}e^{-\frac{R}{L}t} + \frac{f}{R} = \underline{\frac{f}{R}\left(-e^{-\frac{R}{L}t} + 1\right)} \qquad$ Sol particolare

Legge di Lorentz



Sperimentalmente Lorentz trova queste qualità nella Forza

1) $\vec{F} \perp \vec{v}$, $\vec{F} \perp \vec{B}$

2) $\vec{F}$ dipende da $\sin\theta$

3) $\vec{F}$ dipende dalla carica $\Rightarrow F \propto q$

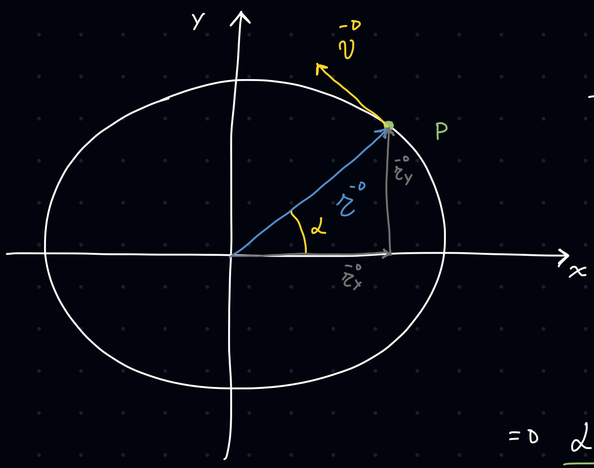
$\Rightarrow \vec{F}_L = q \cdot (\vec{v} \wedge \vec{B})$ perchè $\vec{v} \wedge \vec{B} = v \cdot B \cdot \sin\theta$ inoltre $\hat{v} \wedge \hat{B} \perp \hat{F}$

Se esiste un campo elettrico

$F_{coulomb} = \frac{1}{4\pi} \frac{q_1 \cdot q_2}{R^2} \Rightarrow E = \frac{F}{q} \Rightarrow \underline{F = q \cdot E}$ Forza di Coulomb

$\Rightarrow \underline{\vec{F}_{TOT} = \vec{F}_L + \vec{F}_C = qE + q(\vec{v} \wedge \vec{B})}$

MoTo circolare



$$\vec{\omega} = \text{Velocità Angolare} = \frac{d\alpha}{dt} \Rightarrow$$

$$\rightarrow \quad \omega = \frac{d\alpha}{dt} \rightarrow d\alpha = \omega dt \rightarrow \int_{\alpha_0}^{\alpha_f} d\alpha = \int_{t_0}^{t_f} \omega \, dt$$

$$\Rightarrow \quad \alpha_f - \alpha_i = \omega(t_f - t_0)$$

pongo $\alpha_0 = 0$, $t_0 = 0$

$$\Rightarrow \quad \underline{\alpha(t) = \omega t}$$

$\vec{r} = \vec{r}_x + \vec{r}_y = \hat{i} R \cos\alpha + \hat{j} R \sin\alpha$ Siccome $v = \frac{ds}{dt} = \dot{s}$

$\Rightarrow \quad R\cos\alpha + R\sin\alpha \rightarrow R\cos(\omega t) + R\sin(\omega t)$ <u>Dipende da t</u>

$$\rightarrow \quad \vec{v} = -\hat{i}\,\omega R \sin(\omega t) + \hat{j}\,\omega R \cos(\omega t) = \omega R\left[\hat{j}\cos(\omega t) - \hat{i}\sin(\omega t)\right]$$

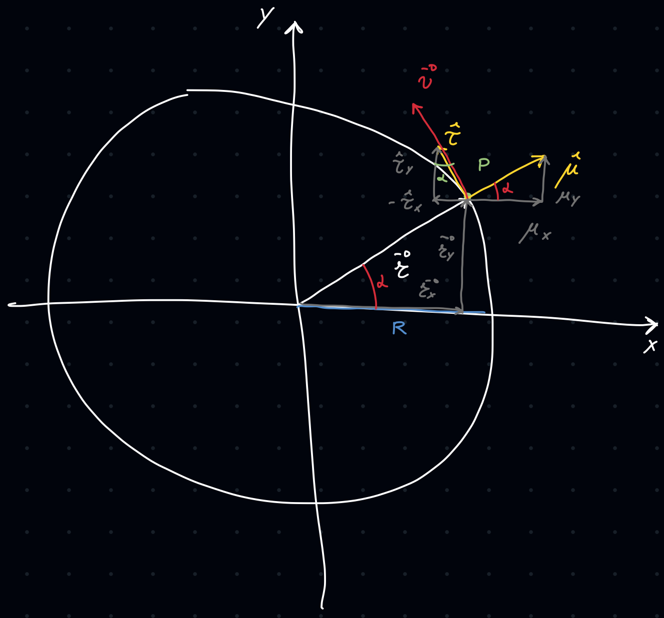
$$\vec{a} = -\hat{i}\,\omega^2 R \cos(\omega t) - \hat{j}\,\omega^2 R \sin(\omega t) = -\omega^2 R\left[\underbrace{\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)}_{R}\right] = \underline{-\omega^2 R \vec{r}}$$

Direzione opposta ad $\vec{r}$

Modulo

$$|\vec{v}| = \sqrt{\omega^2 R^2 \left[\underbrace{\cos^2(\alpha) + \sin^2(\alpha)}_{1}\right]} = \sqrt{\omega^2 R^2} = \underbrace{\omega R}_{|\vec{v}|} \Rightarrow v = \omega R \rightarrow \omega = \frac{v}{R} \quad (1)$$

$$|\vec{a}| = \sqrt{\omega^4 R^2 [1]} = \underbrace{\omega^2 R}_{|\vec{a}|} \rightarrow |\vec{a}| = \omega^2 R \Rightarrow a = \frac{v^2}{R^2} \cdot R \rightarrow a = \frac{v^2}{R} \quad (2)$$



$$\hat{\mu} = \hat{\mu}_x + \hat{\mu}_y = \hat{i}\mu\cos\alpha + \hat{j}\mu\sin\alpha = \hat{i}\cos\alpha + \hat{j}\sin\alpha$$

$$\hat{\tau} = \hat{\tau}_x + \hat{\tau}_y = -\hat{i}\sin\alpha + \hat{j}\cos\alpha$$

$$\Rightarrow \begin{cases} \hat{\mu} = \hat{i}\cos\alpha + \hat{j}\sin\alpha \\ \hat{\tau} = -\hat{i}\sin\alpha + \hat{j}\cos\alpha \end{cases} \Rightarrow \begin{cases} \hat{\mu} = \hat{i}\cos(\omega t) + \hat{j}\sin(\omega t) \\ \hat{\tau} = -\hat{i}\sin(\omega t) + \hat{j}\cos(\omega t) \end{cases}$$

$$\omega = \frac{d\alpha}{dt} \Rightarrow \alpha = \omega t$$

$\Rightarrow$

$$\begin{cases} \dot{\mu} = -\hat{i}\,\omega\sin(\omega t) + \hat{j}\,\omega\cos(\omega t) = \omega\big[\overbrace{-\hat{i}\sin(\omega t) + \hat{j}\cos(\omega t)}^{\hat{\tau}}\big] = \omega\hat{\tau} & \dot{\mu} \\ \dot{\tau} = -\hat{i}\,\omega\cos(\omega t) - \hat{j}\,\omega\sin(\omega t) = -\omega\big[\underbrace{\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)}_{\hat{\mu}}\big] = -\omega\hat{\mu} & \dot{\tau} \end{cases}$$

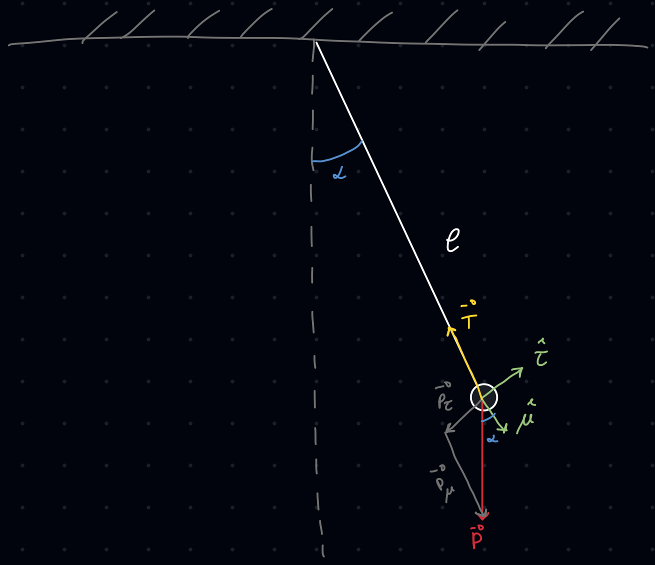
$$\Rightarrow \text{Se} \quad v = \frac{dS}{dt} = \frac{d}{dt}\left[\tau_x + \tau_y\right] = \frac{d}{dt}\left[\hat{i}R\cos(\omega t) + \hat{j}R\sin(\omega t)\right] = \frac{d}{dt}\Big[R\big(\underbrace{\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)}_{\hat{\mu}}\big)\Big]$$

$$\Rightarrow \dot{S} = \frac{d}{dt}\left[R\hat{\mu}\right] = R\left[\omega\hat{\tau}\right] \Rightarrow \vec{v} = R\omega\hat{\tau} \quad (1)$$

$$\text{Se } v = \text{cost} \rightarrow \quad \vec{a} = R\omega\left[-\omega\hat{\mu}\right] = -R\omega^2\hat{\mu} \quad (2)$$

$$\text{Se } v \neq \text{cost} \rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = R \cdot \frac{d\omega}{dt}\hat{\tau} + R\omega\frac{d\hat{\tau}}{dt} = \underbrace{R\dot{\omega}\hat{\tau}}_{\text{Acc Tang.}}\ \underbrace{-R\omega^2\hat{\mu}}_{\text{Acc centripeta}}$$

Pendolo Semplice



$$\begin{cases} \text{Lungo } \hat{\mu}: & \vec{P}_{\hat{\mu}} - \vec{T} = m\, a_\mu \\ \text{Lungo } \hat{\tau}: & \vec{P}_\tau = m \cdot \vec{a}_{\hat{\tau}} \end{cases}$$

$$\begin{cases} \vec{P}_\mu = mg \cdot \cos\alpha \\ \vec{P}_\tau = -mg \sin\alpha \end{cases}$$

$$\Rightarrow \begin{cases} mg\cos\alpha - \vec{T} = m \cdot \vec{a}_\mu \\ -mg\sin\alpha = m\vec{a}_\tau \end{cases}$$

Ci sono due Acc perchè $\omega \neq cost$

$\rightarrow$ Acc centripeta. $\vec{a}_\mu = -\dfrac{v^2}{R} = -\dfrac{\ddot{S}}{\ell}$ (1)

$\rightarrow$ Acc Tangenziale: $\vec{a}_\tau = \ddot{S} \cdot \hat{\tau}$

$$\Rightarrow \begin{cases} mg\cos\alpha - \vec{T} = -\dfrac{m\ddot{S}}{\ell} \\ -mg\sin\alpha = m\ddot{S}\hat{\tau} \end{cases}$$

Inutile

$\rightarrow$ Approssimazione per piccole oscillazioni $\rightarrow$ $\sin\alpha \sim \alpha$

$\Rightarrow -\cancel{m}g\alpha = \cancel{m}\ddot{S}\hat{\tau} \rightarrow \ddot{S}\hat{\tau} + g\alpha = 0$ Siccome $1\,\text{Rad} = \dfrac{\ell}{R} \rightarrow \alpha = \dfrac{S}{\ell}$

$\Rightarrow \ddot{S}\hat{\tau} + S\dfrac{g}{\ell} = 0$ battezzo $\dfrac{g}{\ell} = k^2 \rightarrow \ddot{S} + k^2 S = 0$ Eq. differenziale

Soluzione: $S(t) = A\cos(kt + \varphi)$

Trovare K

$\cos$ è periodico di 2π $\Rightarrow$ $S(t+T_0) = A\cos(kt + \varphi + 2\pi)$

$\rightarrow A\cos(kt + kT_0 + \varphi) = A\cos(kt + \varphi + 2\pi) \Leftrightarrow \cancel{kt} + kT_0 + \cancel{\varphi} = \cancel{kt} + \cancel{\varphi} + 2\pi$

$\rightarrow kT_0 = 2\pi \Rightarrow k = \dfrac{2\pi}{T_0}$ Battezzo $\omega = \dfrac{2\pi}{T_0}$ velocità Angolare

$$\Rightarrow S(t) = A\cos(\omega t + \varphi)$$

Periodo del pendolo

Se $k = \dfrac{2\pi}{T_0} = \omega \rightarrow T_0 = \dfrac{2\pi}{k}$ ma $k^2 = \dfrac{g}{\ell} \Rightarrow T_0 = 2\pi\sqrt{\dfrac{\ell}{g}}$

Energia Cinetica

$$L = F \cdot S \Rightarrow L = \int_{S_0}^{S_f} \vec{F}\, dS \quad \text{ma} \quad \vec{F} = m \cdot \vec{a} \Rightarrow L = m \int_{S_0}^{S_f} \vec{a} \cdot dS \qquad a = \frac{dv}{dt}$$

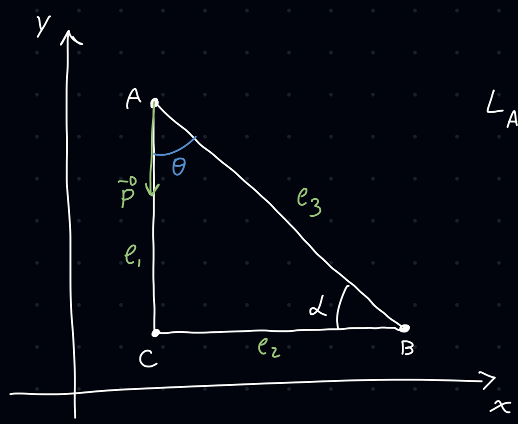
$$\Rightarrow L = m \int_{S_f} \frac{dv}{dt} \cdot dS \rightarrow L = m \int_{v_0}^{v_f} v\, dv = m \left[\frac{v^2}{2}\right]_{v_0}^{v_f} \Rightarrow \underline{\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2}$$

Energia cinetica

G = Energia cinetica , U = Energia potenziale

$$\Rightarrow L = G_B - G_A \quad \text{ma se pongo} \quad U = -G \Rightarrow L = -U_B - (-U_A) \Rightarrow \underline{L = U_A - U_B}$$

Campi conservativi : Peso



$$L_{AC} = \int_A^C \vec{P} \cdot dl = mg\,(C-A) = \underline{mg\, l_1}$$

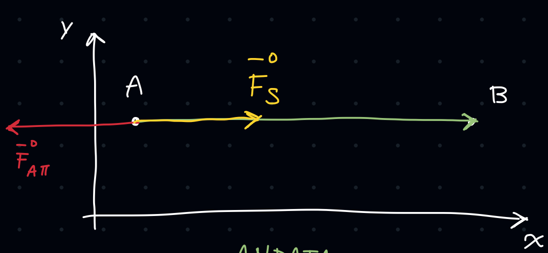
$$L_{ABC} = \int_A^B \vec{P} \cdot d\vec{l}_3 + \int_B^C \vec{P} \cdot d\vec{l}_2 = \int_A^B P \cdot \cos\theta\, dl_3 + \cancel{\int_B^C P \underbrace{\cos\alpha}_{0}\, dl_2}^{0}$$

$$\Rightarrow \int_A^B P \cdot \cos\, dl_3 \quad \text{ma} \quad dl_3 \cos\theta = dl_1 \Rightarrow \underline{L_{ABC} = mg\, l_1}$$

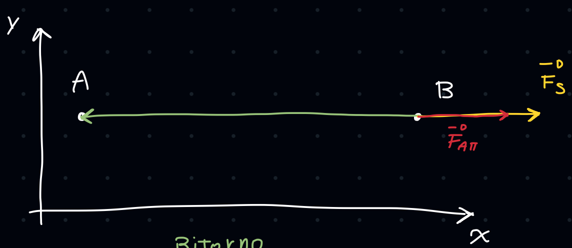
$\Rightarrow$ Conservativo

Non Conservativo : Attrito

$|\vec{F}_{Att}| = \mu_0 |\vec{N}|$ con verso opposto al moto



ANDATA



Ritorno

$$\Rightarrow L_1 = \int_A^B \vec{F}_S\, dl - \int_A^B \vec{F}_{Att}\, dl = F_S \cdot S - F_{Att} \cdot S$$

$$L_2 = -\int_B^A \vec{F}_S\, dl - \int_B^A \vec{F}_{Att}\, dl = -F_S S - F_{Att} \cdot S$$

$\Rightarrow L_1 + L_2$ dovrebbe essere zero, invece $L_1 + L_2 = S\left[F_S - F_A - F_S - F_A\right] = -2 F_{Att}$

Non conservativo!

Impulso

$$\vec{I} = \int_{t_0}^{t_f} \vec{F}\, dt \qquad \vec{p} = m \cdot \vec{v}$$

$$\vec{I} = \int_{t_0}^{t_f} \vec{F}\cdot dt = \int_{t_0}^{t_f} m\cdot\vec{a}\cdot dt = m\int_{v_0}^{v_f} \frac{dv}{\cancel{dt}}\, \cancel{dt} = m(v_f - v_0) = \vec{p}_f - \vec{p}_0$$

Teorema Impulso

Momenti:

FORZA

$$\vec{r}\wedge\vec{F} = \vec{M}$$

ANGOLARE

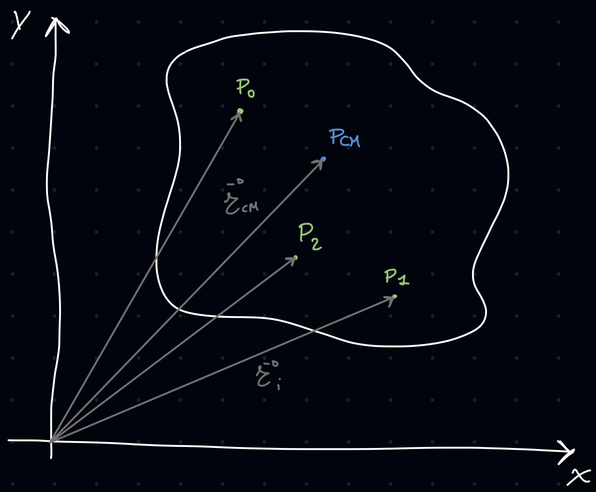
$$\vec{r}\wedge\vec{p} = \vec{L}$$

Faccio la derivata $\rightarrow$ $\dfrac{d\vec{L}}{dt} = \dfrac{d}{dt}(\vec{r}\wedge\vec{p}) \Rightarrow \dfrac{d\vec{L}}{dt} = \dfrac{d\vec{r}}{dt}\wedge\vec{p} + \vec{r}\wedge\dfrac{d\vec{p}}{dt}$

$$\rightarrow \frac{d\vec{L}}{dt} = \underbrace{\vec{v}\wedge\vec{p}}_{\vec{v}\wedge m\cdot\vec{v} = 0} + \vec{r}\wedge\overbrace{m\cdot\vec{a}}^{\vec{F}} \Rightarrow \frac{d\vec{L}}{dt} = \vec{r}\wedge\vec{F} \Rightarrow \vec{M} = \frac{d\vec{L}}{dt}$$

QED

Centro massa



Per definizione $\vec{R}_{CM} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \dfrac{\sum_i m_i \vec{r}_i}{M}$

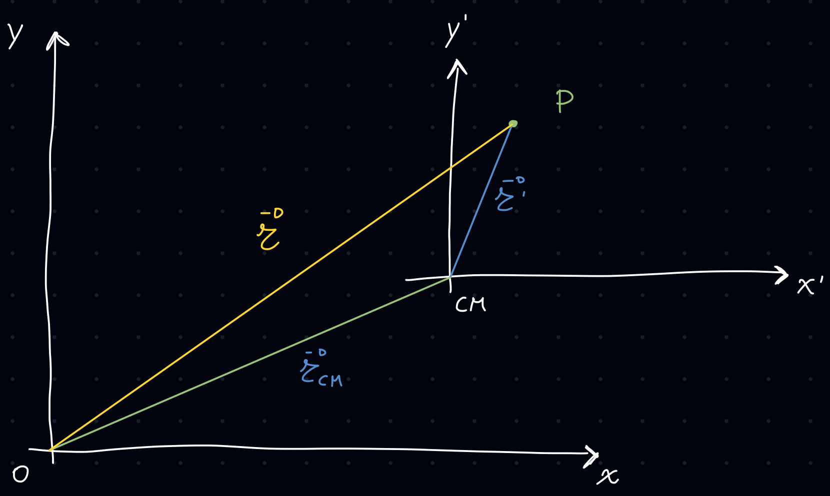
$\Rightarrow v_{CM} = \dfrac{\sum m_i v_i}{M}$ ma $m \cdot v = \vec{p} \Rightarrow \vec{v}_{CM} = \dfrac{\sum_i \vec{p}_i}{M}$

$\Rightarrow \vec{P}_{TOT} = M \cdot \vec{v}_{CM}$ (1)

$\vec{a}_{CM} = \dfrac{\sum_i m_i \vec{a}_i}{M}$ ma $\vec{F} = m \cdot \vec{a} \Rightarrow \vec{a}_{CM} = \dfrac{\sum_i \vec{F}_i}{M}$

$\Rightarrow \sum_i \vec{F}_i = M \cdot \vec{a}_{CM}$ (2)

Teorema di Konig



OSS: $\vec{r} = \vec{r}_{CM} + \vec{r}\,'$

$\Rightarrow \vec{v} = \vec{v}_{CM} + \vec{v}\,'$

Velocita' di o — Vel DI CM — Vel MISURATA da CM

Momento Angolare $\vec{L} = \vec{r} \wedge m \cdot v$

ma $\vec{r} = \vec{r}_{CM} + \vec{r}\,'$ e $\vec{v} = \vec{v}_{CM} + \vec{v}\,'$

$$\Rightarrow \vec{L} = (\vec{r}_{CM} + \vec{r}\,') \wedge m \cdot (\vec{v}_{CM} + \vec{v}\,') = \vec{r}_{CM} \wedge m \vec{v}_{CM} + \vec{r}_{CM} \wedge m \vec{v}\,' + \vec{r}\,' \wedge m \vec{v}_{CM} + \vec{r}\,' \wedge m \vec{v}\,'$$

Si tratta di un sistema $\Rightarrow$ si parla di $\vec{L}_{TOT} = \Sigma L = \Sigma_i \vec{r}_i \wedge m_i \vec{v}_i$

$$\Rightarrow L_{TOT} = \Sigma_i r_{CM} \wedge m_i v_{CM} + \Sigma_i r_{CM} \wedge m_i v_i' + \Sigma_i \vec{r}_i\,' \wedge m_i \vec{v}_{CM} + \Sigma \vec{r}_i\,' \wedge m_i \vec{v}_i\,'$$

$$= \vec{r}_{CM} \wedge M \vec{v}_{CM} + \underline{\cancel{r_{CM} \wedge \Sigma_i m_i \vec{v}_i\,'}} + \underline{\cancel{\Sigma_i M \vec{r}_i\,'} \wedge \vec{v}_{CM}} + \Sigma_i \vec{r}_i \wedge m_i \vec{v}_i$$

$\Sigma_i m_i \vec{v}_i = M \vec{v}_{CM}\,'$ $\quad$ $\Sigma_i m_i \vec{r}_i\,' = M \vec{r}_{CM}\,'$

$$= \vec{r}_{CM} \wedge M \vec{v}_{CM} + \Sigma_i \vec{r}_i \wedge m_i \vec{v}_i$$

Momento angolare del sistema misurato dal CM

$$= \vec{L}_{CM} + L'_{TOT}$$

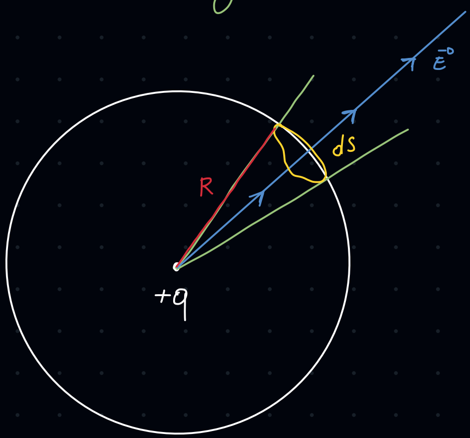
Teorema di coulomb

Gauss

$$\int \vec{E} \cdot \hat{n}\, ds = \frac{Q}{\varepsilon_0} \rightarrow \rho = \frac{dQ}{dV} \rightarrow Q = \int_V \rho\, dV \Rightarrow \int \vec{E} \cdot \hat{n}\, ds = \frac{1}{\varepsilon_0} \int_V \rho\, dV$$

$$\rightarrow \int_V (\vec{\nabla} \cdot \vec{E})\, dV = \frac{1}{\varepsilon_0} \int_V \rho\, dV \Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ Maxwell

Dimostrare Gauss



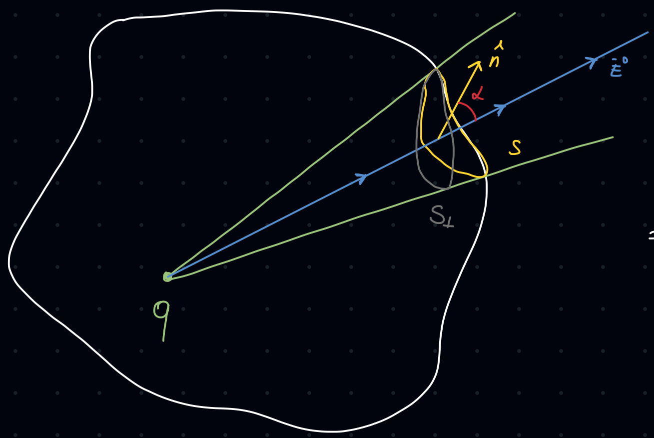
$$\phi_E = \int_S \vec{E} \cdot \hat{n}\, ds \qquad \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Sfera → Costante

$$= \frac{q}{4\pi\varepsilon_0 r^2} \int_S \hat{r} \cdot \hat{n}\, ds \qquad \hat{r} \perp \hat{n} \Rightarrow \int_S ds = 4\pi r^2$$

$$\Rightarrow \phi_E = \int \vec{E} \cdot \hat{n}\, ds = \frac{q}{\varepsilon_0}$$ QED

Superficie qualsiasi



Possiamo Trovare la sup $S_\perp$ che è perpend. al campo $\vec{E}$

$$\Rightarrow S_\perp = S \cos\theta$$

$$\Rightarrow \phi_E = \oint \vec{E} \cdot \hat{n}\, ds = \oint \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} \cdot \hat{n} \cdot dS$$

non costante

$$\hat{r} \cdot \hat{n} = r \cdot n \cdot \cos\theta = \cos\theta$$

$$\Rightarrow \phi_E = \oint \vec{E} \cdot \hat{n}\, ds = \frac{q}{4\pi\varepsilon_0} \cdot \oint \frac{1}{r^2} \cdot \cos\theta \cdot ds = \frac{q}{4\pi\varepsilon_0} \oint \frac{1}{r^2} ds'$$

ma ds' è la sup. di una sfera ⇓ $r = \text{cost}$

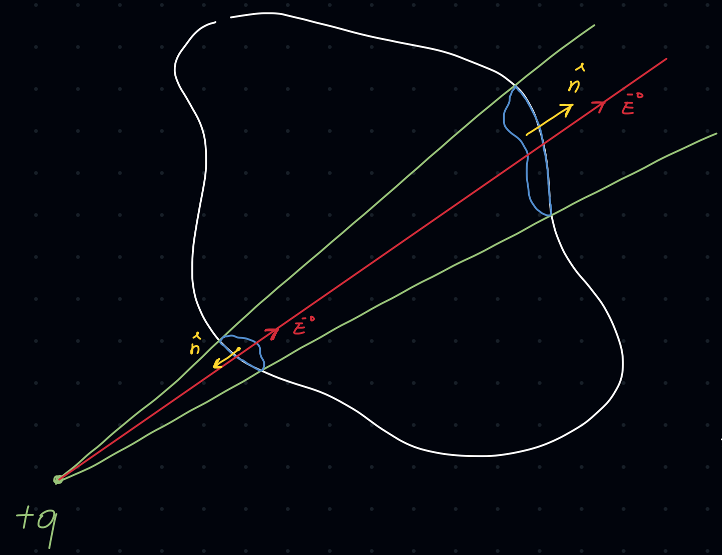
$$= \frac{q}{\cancel{4\pi}\varepsilon_0} \frac{1}{\cancel{r^2}} \cdot \cancel{4\pi r^2} = \frac{q}{\varepsilon_0}$$

Possiamo anche definire Angolo solido. $d\Omega = \dfrac{dS_\perp}{r^2} = \dfrac{dS\cos\theta}{r}$

$$\Rightarrow \phi_E = \frac{q}{4\pi\varepsilon_0}\oint \frac{\hat{r}\cdot\hat{n}}{r^2}\,dS = k\oint \frac{\cos\theta}{r^2}\,dS = k\oint \frac{dS_\perp}{r^2} = k\oint d\Omega$$

$$\oint d\Omega = 4\pi \quad \Rightarrow \quad \phi_E = \frac{q}{4\pi\varepsilon_0}\,4\pi = \frac{q}{\varepsilon_0} \quad \text{QED}$$

Carica esterna



Per il teorema appena dimostrato

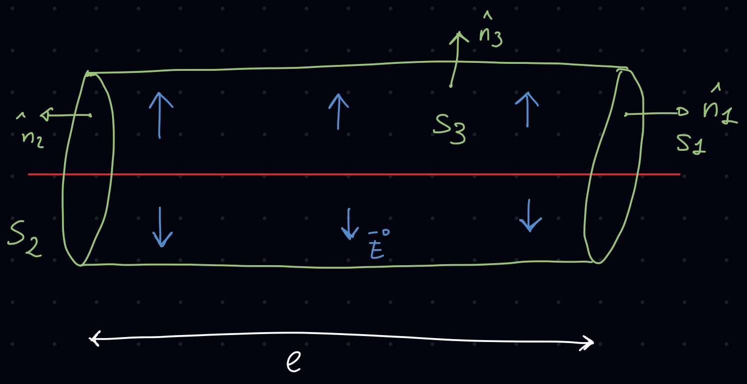
$$\phi_{S_1} = \int_S \vec{E}\cdot\hat{n}_1\,dS_1 = \frac{q}{\varepsilon_0}$$ prendo $\hat{n}_1$ come riferimento

$$\phi_{S_2} = \int_S \vec{E}\cdot\hat{n}_2\,dS_2 \quad \text{ma} \quad \hat{n}_2 = -\hat{n}_1$$

$$\Rightarrow \phi_{S_2} = -\int_S \vec{E}\cdot\hat{n}\,dS_2 = -\frac{q}{\varepsilon_0}$$

$$\Rightarrow \phi_{S_1} = -\phi_{S_2} \quad \Rightarrow \phi_{S_1} + \phi_{S_2} = \emptyset$$

Filo carico - Coulomb



$\vec{E} \perp \hat{n}_1$ $\quad$ $\vec{E} \perp \hat{n}_2$

$$\phi_E = \oint \vec{E}\cdot\hat{n}\, dS = \cancel{\int_S E\cdot \hat{n}_1\, dS_1} + \cancel{\int_S E \hat{n}_2 dS_2} + \int_S E \hat{n}_2 dS_3$$

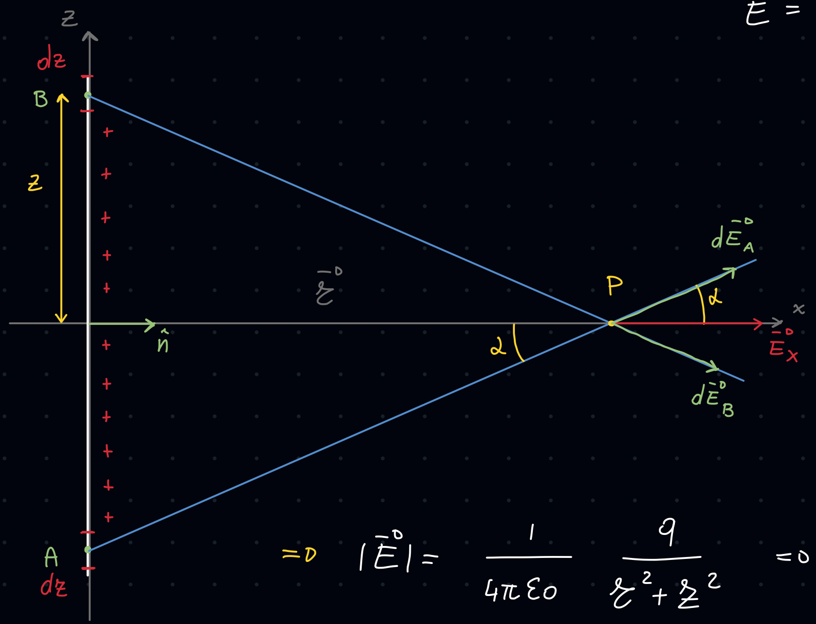
$$= \int \vec{E}\cdot n_2\, dS_2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \phi_E = |\vec{E}|\cdot 2\pi r \ell = \frac{Q}{\varepsilon_0} \quad \text{ma } \ell = \infty$$

$\Rightarrow \lambda =$ densità lineare di carica $= \dfrac{Q}{\ell} \Rightarrow Q = \lambda \ell$

$$|\vec{E}|\, 2\pi r \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{n}$$

Filo carico senza Coulomb



$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Non conosco r → AP / BP

$$\begin{cases} AP = r\cos\alpha \\ BP = r\cos\alpha \end{cases} \Rightarrow AP = BP = r\cos\alpha$$

Non conosco α

$$AP^2 = z^2 + r^2$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+z^2} \Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}$$

siccome $\lambda = \dfrac{dQ}{d\ell} \Rightarrow dq = \lambda\, d\ell \Rightarrow dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dz}{r^2+z^2}$

nel nostro caso $d\ell = dz$

→ le componenti z ed y di $d\vec{E}_A + d\vec{E}_B$ si elidono a vicenda ⇒ ci serve solo $\vec{E}_z$

$\Rightarrow d\vec{E}_z = d\vec{E}\cos\alpha$ ma non conosciamo α

$$\rightarrow z = AP\cos\alpha \Rightarrow z = \sqrt{z^2+r^2}\cos\alpha \Rightarrow \cos\alpha = \frac{z}{\sqrt{z^2+r^2}}$$

$$\Rightarrow dE_z = dE\cos\alpha = \frac{z}{\sqrt{z^2+r^2}}\, dE$$

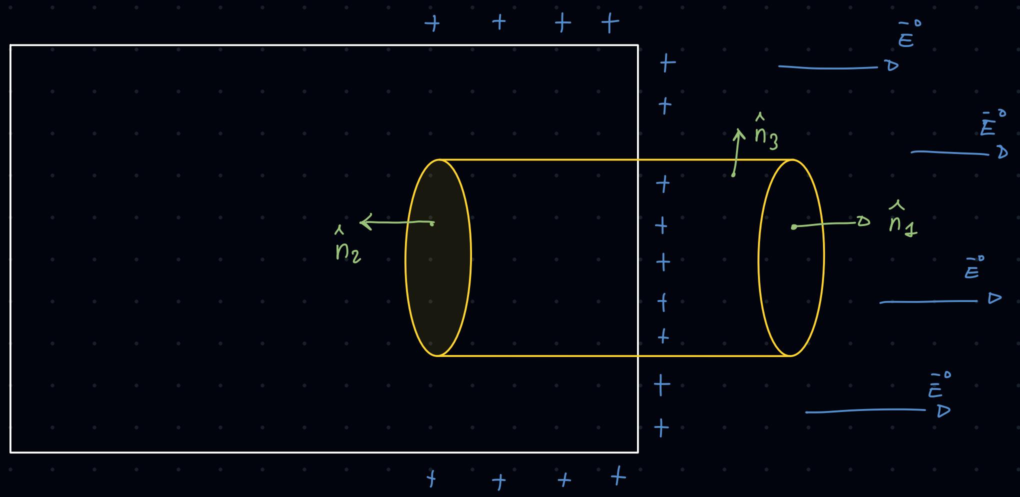
$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dz}{r^2+z^2} \cdot \frac{z}{\sqrt{z^2+r^2}} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda z\, dz}{(r^2+z^2)^{\frac{3}{2}}} \quad (1)$$

Integriamo per ottenere $\vec{E}_{TOT}$

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{\overset{\text{cost}}{\lambda}}{(r^2+z^2)^{\frac{3}{2}}}\, dz = 2\frac{\lambda}{4\pi\varepsilon_0} \int_0^{\ell} \frac{z}{(r^2+z^2)^{\frac{3}{2}}}\, dz = \frac{\cancel{2}\lambda \cancel{z}}{\cancel{2}\, 4\pi\varepsilon_0\, r^{\cancel{2}}} \cdot \frac{\ell}{\sqrt{r^2+\ell^2}}$$

$$\underbrace{\qquad\qquad}_{|\vec{E}|}$$

Teorema di Coulomb



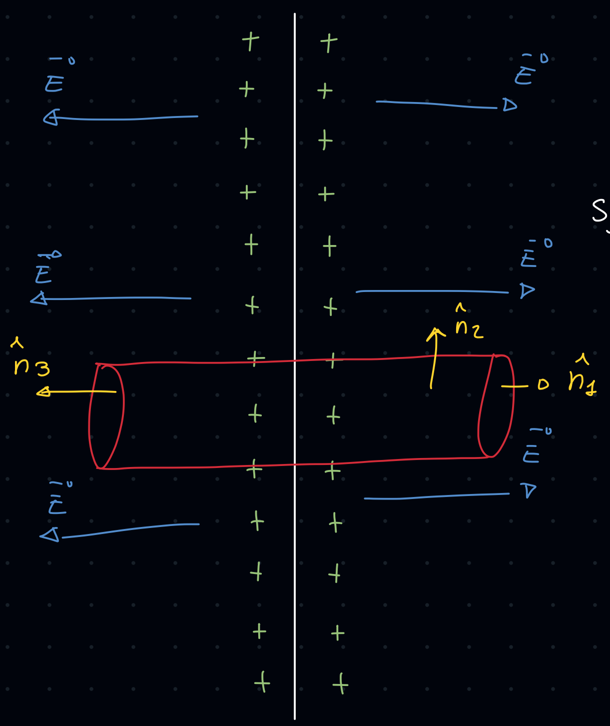
$$\phi_E = \oint \vec{E}\cdot\hat{n}\,dS = \int \vec{E}\cdot\hat{n}_1\,dS_1 + \underbrace{\int \vec{E}\cdot\hat{n}_2\,dS_2}_{\int = \emptyset} + \underbrace{\int \vec{E}\cdot\hat{n}_3\,dS_3}_{\hat{n}_3 \perp \vec{E}} = \int \vec{E}\cdot\hat{n}_1\,dS_1 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow |\vec{E}|\cdot S_1 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{S_1}\cdot\frac{1}{\varepsilon_0} \quad \text{ma} \quad \sigma = \frac{Q}{S} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Alternativamente

$$\sigma = \frac{dQ}{dS} \Rightarrow dq = \sigma\,dS \rightarrow \quad q = \int \sigma\,dS \rightarrow E = \frac{\int \sigma\,dS}{S\varepsilon_0} = \sigma\frac{\cancel{\int dS}}{\cancel{S}\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Strato carico



$$\phi_E = \oint \vec{E}\cdot\hat{n}\,dS = \int \vec{E}\cdot\hat{n}_1\,dS_1 + \overset{0}{\cancel{\int \vec{E}\cdot\hat{n}_2\,dS_2}} + \int \vec{E}\cdot\hat{n}_3\,dS_3$$

$//$ $\quad \vec{E}\perp\hat{n}_2 \quad$ $//$

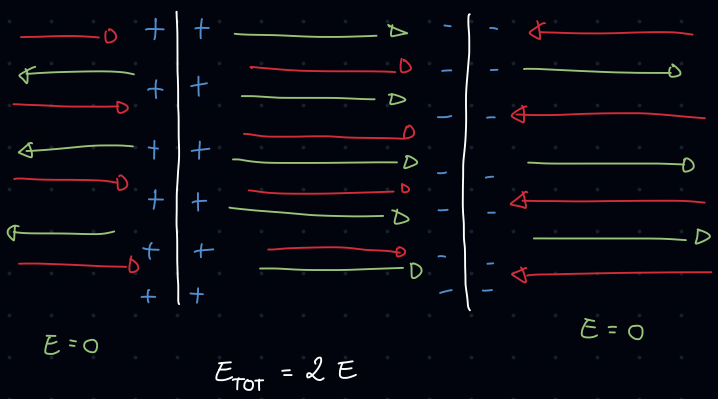
$$S_3 = S_1 \qquad = 2\int_S \vec{E}\,n_1\,dS_1 = 2\,E\cdot S_1 = E\cdot 4\pi r^2$$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\sigma = \frac{dq}{dS} \Rightarrow q = \int \sigma\,dS \Rightarrow$$

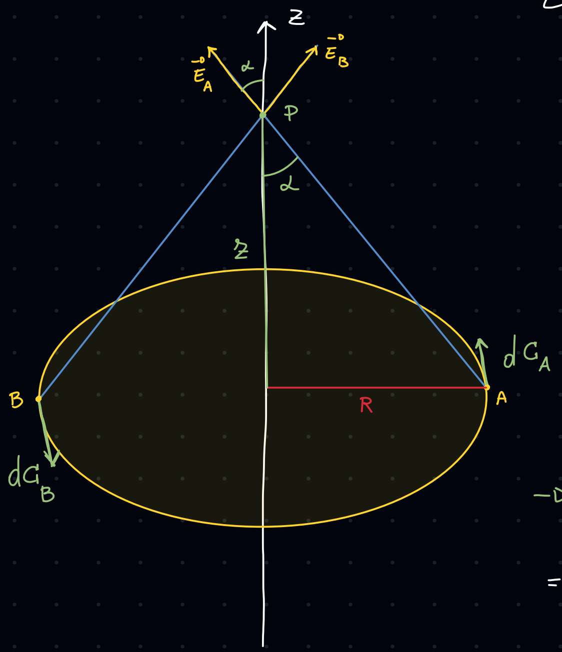
$$\Rightarrow E\cdot \overset{2}{\cancel{4\pi r^2}} = \frac{\sigma \cancel{\int dS}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Doppio Strato carico



$$\Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0}$$

Anello Carico



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \to \text{Nel nostro caso } \overline{AP} = ?$$

$$\Rightarrow z = AP\cos\alpha \to AP^2 = z^2 + R^2$$

$$\Rightarrow z = \sqrt{z^2+R^2}\cos\alpha$$

$$\Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{z^2+R^2}$$

$\to \vec{E}_A + \vec{E}_B$ Annullano le componenti y ed x

$\Rightarrow$ ci serve solo $E_z = E\cos\alpha$

$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dq\cos\alpha}{z^2+R^2} \to ??$$

$$z = AP\cos\alpha \to \cos\alpha = \frac{z}{\sqrt{z^2+z^2}}$$

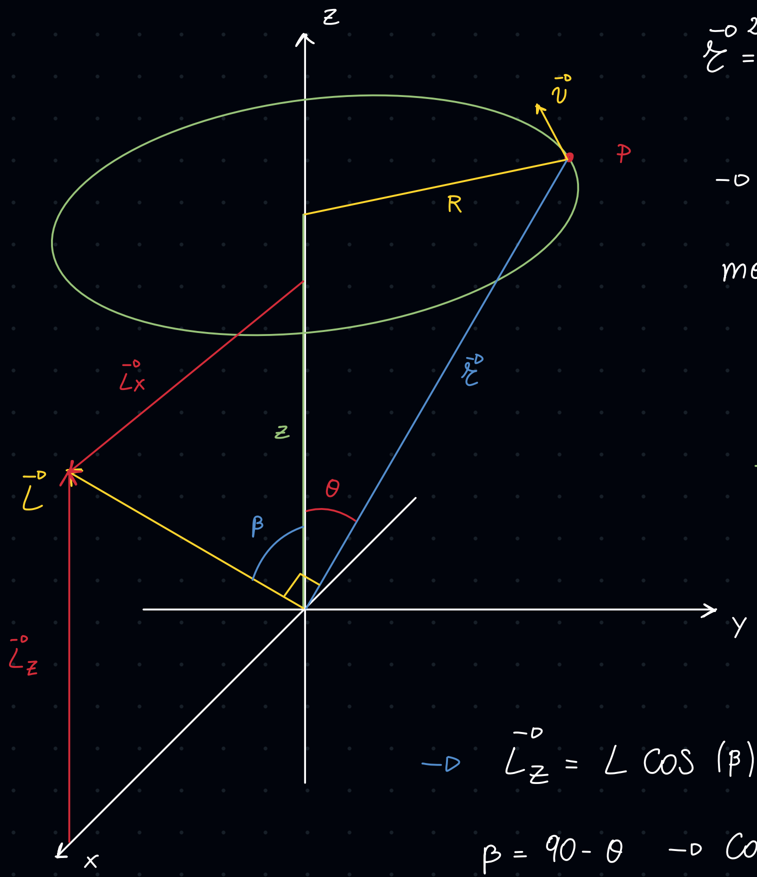
$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(z^2+R^2)^{\frac{3}{2}}}$$

Siccome l'anello è una circonferenza $\Rightarrow$ ho una superficie

$$\Rightarrow \lambda = \frac{dq}{ds} \Rightarrow dq = \lambda ds \to dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(z^2+R^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_z = \frac{\lambda}{\cancel{4\pi}_{2}\varepsilon_0}\frac{1}{(z^2+R^2)^{\frac{3}{2}}}\underbrace{\int ds}_{\cancel{2\pi}R} = \frac{\lambda R}{\varepsilon_0(z^2+z^2)^{\frac{3}{2}}}$$

Rotazione Asse Fisso



Momento Angolare: $\vec L = \vec r \wedge m\vec v = \vec r \wedge \vec p$

$\vec r^{\,2} = R^2 + z^2$; $R = r \sin\theta$

→ $L = r \cdot m \cdot v \cdot \sin(90) = r m v$

ma $v = \omega R$

⇒ $L_i = r_i m_i \omega R_i$

⇒ Le componenti x ed y si annullano

→ ci serve solo la componente z di L_i

→ $\vec L_z = L \cos(\beta)$ ma $\beta = ?$

$\beta = 90 - \theta$ → $\cos(90-\theta) = \sin(\theta)$

⇒ $L_{z_i} = L_i \sin\theta = r_i m_i \omega R_i \sin\theta$ ⇒ $\underbrace{(r_i \cdot \sin\theta)}_{R} \cdot R_i\, m_i\, \omega$

→ $L_z = R_i^2 m_i \omega$ (1)

⇒ $L_{TOT} = \omega \sum_i m_i R_i^2$ pongo $\sum_i m_i R_i^2$ = Momento di inerzia Totale = I_{TOT}

⇒ $\underline{\vec L_{TOT} = \omega \cdot I_{TOT}}$

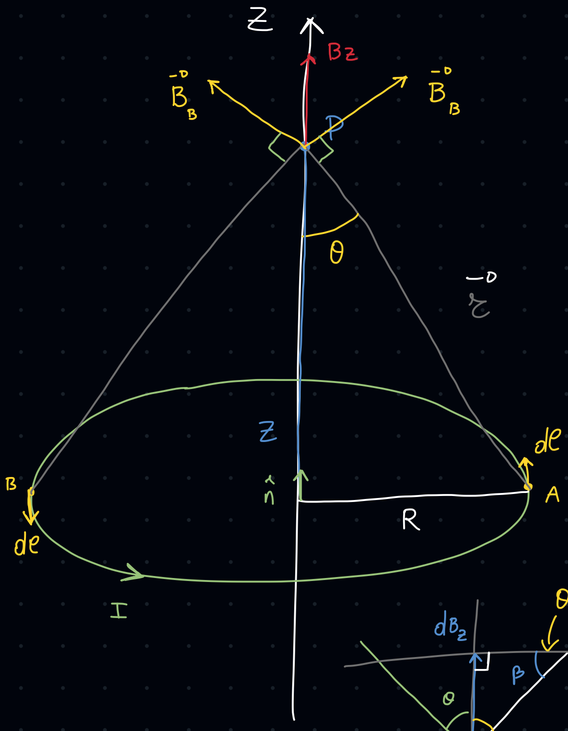
Nel caso di corpi rigidi

$\vec M = \frac{d\vec L}{dt}$ ⇒ $M = \underbrace{\frac{d\vec\omega}{dt}}_{\alpha \text{ Acc Ang.}} \underbrace{I}_{\text{cost}}$ ⇒ $\vec M = \vec\alpha I$ ANALOGO A $\vec F = m \cdot \vec a$

$\omega = \omega_0 + \int_{t_0}^{t_f} \alpha \, dt$, $\theta = \theta_0 + \int_{t_0}^{t_f} \omega \, dt$

→ $v(t) = v_0 + a \cdot t$ → $\omega(t) = \omega_0 + \alpha \cdot t$ con $\alpha = \frac{d\omega}{dt} = 0$

$S(t) = S_0 + v_0 t + \frac{1}{2} a t^2$ → $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} a t^2$ con $\alpha = \frac{d\omega}{dt} \neq 0$



Dalla legge di Laplace

$$d\vec B = \frac{\mu_0 I}{4\pi} \frac{d\ell \times \vec r}{|\Delta \vec r^{\,3}|}$$

Le componenti x ed y si annullano a due a due

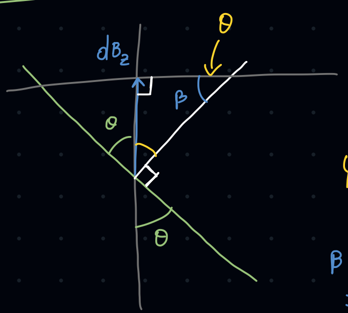
$\Rightarrow$ calcolo solo $dB_z = dB \cdot \sin\theta$

$$\Rightarrow \vec B = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec\ell \times \vec r}{r^3} \cdot \sin\theta$$

$$= \frac{\mu_0 I}{4\pi} \oint \frac{d\ell \cdot \cancel{r} \cdot \overset{1}{\cancel{\sin\gamma}}}{r^{\cancel{3}}} \cdot \sin\theta$$

$$= \frac{\mu_0 I}{4\pi} \oint \underbrace{\frac{\sin\theta}{r^2}}_{cost} d\ell$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{\sin\theta}{r^2} \cancel{\oint d\ell}\ 2\pi R$$



$\varphi = 180 - 90 - \theta = 90 - \theta$

$\beta = 180 - 90 - \varphi = 90 - 90 + \theta = \theta$

$$\Rightarrow \vec B = \frac{\mu_0 I}{4\pi r^2} \cdot \sin \cdot 2\pi R$$

$\rightarrow$ Non conosco $\rightarrow r \rightarrow R = r \sin\theta \rightarrow \sin\theta = \frac{R}{r}$

$r^2 = z^2 + R^2 \Rightarrow r = \sqrt{z^2+R^2}$

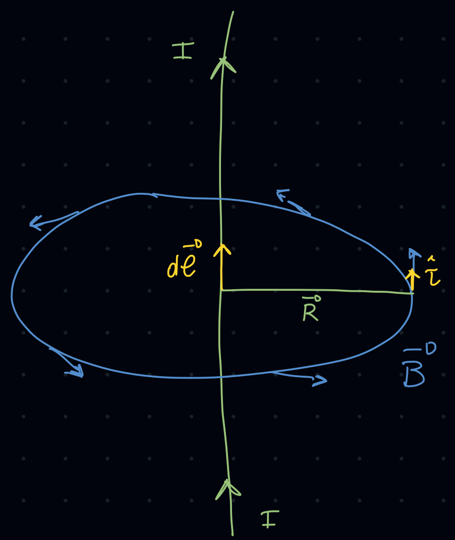
$$\Rightarrow \frac{\sin\theta}{r^2} = \frac{R}{r} \cdot \frac{1}{(z^2+R^2)} = \frac{R}{(z^2+R^2)^{\frac{1}{2}} \cdot (z^2+R^2)} = \frac{R}{(z^2+R^2)^{\frac{3}{2}}}$$

$$\Rightarrow \vec B = \frac{\mu_0 I}{\underset{2}{\cancel{4\pi}}} \frac{R^2}{(z^2+R^2)^{\frac{3}{2}}} \cdot \cancel{2\pi} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{\frac{3}{2}}}$$

F. Lorentz

$$\vec{F} = q\vec{E} + \underbrace{q \cdot (\vec{v} \wedge \vec{B})} \quad \rightarrow \quad \vec{F} = q(\vec{v} \wedge \vec{B}) \quad \text{ma} \quad v = \frac{d\vec{\ell}}{dt}$$

$$\Rightarrow \quad F = \underbrace{q \cdot \frac{d\vec{\ell}}{dt}}_{I} \wedge \vec{B} \quad \rightarrow \quad \vec{F} = I \, d\vec{\ell} \wedge \vec{B} \quad (\text{II Laplace})$$



sperimentale

Biot-Savart: $\vec{B} = \vec{k} \cdot \frac{I}{R} \qquad \vec{k} = \hat{\tau} \cdot \frac{\mu_0}{4\pi}$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi} \frac{I}{R} \hat{\tau} \equiv \frac{\mu_0}{4\pi} I \cdot \frac{d\vec{\ell} \wedge \vec{R}}{|\vec{R}^3|} \quad \vec{B}$$

I° legge di Laplace

Alternativamente

$$\vec{F} \text{ con carica magnetica} = \frac{\mu_0}{4\pi} \frac{q_m \cdot q_m'}{r^2} \quad \Rightarrow \quad \vec{B}_{q_m} = \frac{\vec{F}_m}{q} = \frac{\mu_0}{4\pi} \frac{q_m}{r^2}$$

$$\rightarrow \text{II° Laplace} \rightarrow \vec{F} = I \cdot d\vec{\ell} \wedge \underbrace{\vec{B}}_{\vec{B}_{q_m}} = I \cdot d\vec{\ell} \wedge \left[\frac{\mu_0}{4\pi} \cdot \frac{q_m}{r^2} \cdot \hat{r} \right]$$

$$\frac{\vec{r}}{r} = \hat{r}$$

$$\Rightarrow \quad \underbrace{\vec{F} = \frac{\mu_0}{4\pi} \cdot q_m \cdot (I \cdot d\vec{\ell}) \wedge \frac{\vec{r}}{r^3}}_{\text{Forza con cariche magnetiche}} \qquad \underbrace{F = q \cdot d\vec{\ell} \wedge \vec{B}}_{\text{Forza con cariche elettriche}}$$

$$\Rightarrow \quad q_m = I d\vec{\ell} \quad \Rightarrow \quad \vec{F} = \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{\ell} \wedge \vec{r}}{r^3} \quad \text{I° Laplace}$$

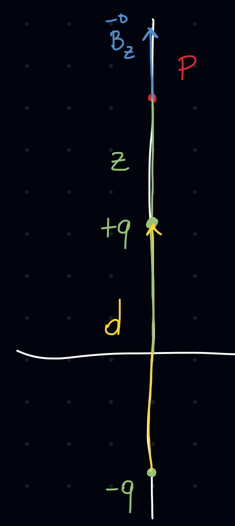
Equivalenza Ampère

Campo spira percorsa da corrente $B = \dfrac{\mu_0 I}{2\pi} \cdot \dfrac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$ Se $z >> R$

$\Rightarrow \vec{B} = \dfrac{\mu_0 I}{2} \dfrac{R^2}{z^3} \hat{n}$ chiamo $I S \cdot \hat{n} = I \cdot \pi R^2 \cdot \hat{n} = \vec{m}$ spira

$\Rightarrow \vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{\vec{m}}{z^3}$ (1)

→ Dipolo



Se P è lungo z ci interessa solo B_z

$\Rightarrow \vec{E}_z = \dfrac{q d}{4\pi \varepsilon_0} \cdot \dfrac{3 \overbrace{\cos\theta}^{1} - 1}{z^3} = \dfrac{q \vec{d}}{4\pi\varepsilon_0} \dfrac{2}{z^3}$

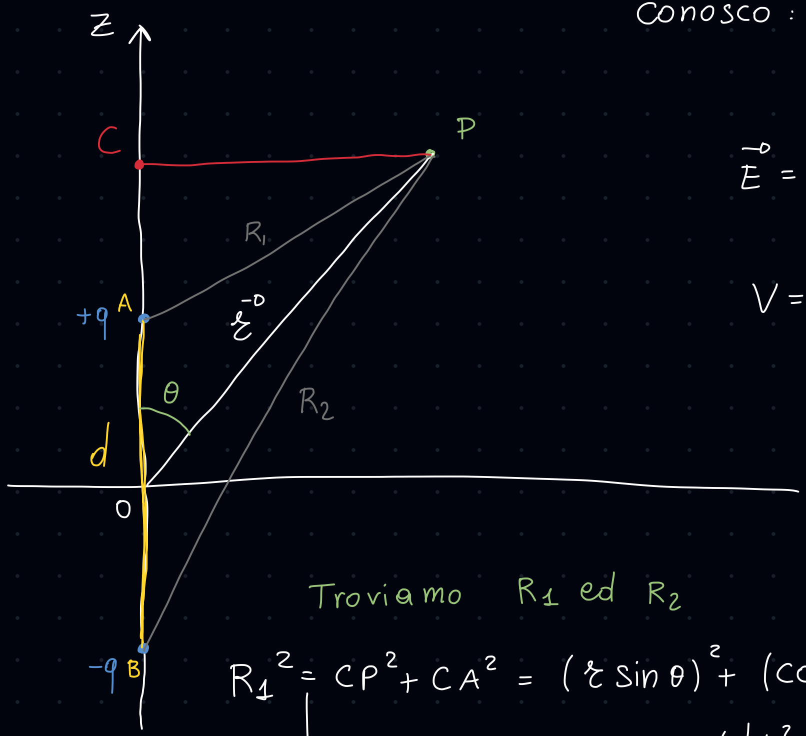
chiamo $q \cdot \vec{d} = \vec{p} \Rightarrow \vec{E} \; \dfrac{\vec{p}}{2\pi \varepsilon_0 z^3}$

Nel caso di un dipolo magnetico

→ $\vec{B} = \dfrac{q \vec{d} \mu_0}{4\pi} \cdot \dfrac{2}{z^3}$ chiamo $q \cdot \vec{d} = \vec{m}$

$\Rightarrow \vec{B} = \dfrac{\mu_0}{2\pi} \cdot \dfrac{\vec{m}}{z^3}$ (2) → (1) e (2) sono UGUALI!

Dipolo elettrico



Conosco : $\begin{cases} r \to \text{Dist di } P\text{-}O \\ \theta \to \text{Angolo tra } r \text{ e } y \\ d \to \text{dist tra } +q \text{ e } -q \end{cases}$

$\vec{E} = -\vec{\nabla} V$ ma $V = ?$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\underset{\uparrow R_1}{r_A}} - \frac{1}{\underset{\uparrow R_2}{r_B}}\right] \quad ???$$

Troviamo R_1 ed R_2

$$R_1^2 = CP^2 + CA^2 = (r\sin\theta)^2 + \left(CO - \frac{d}{2}\right) = (r\sin\theta)^2 + \left(r\cos\theta - \frac{d}{2}\right)^2$$

$$= r^2\sin^2\theta + r^2\cos^2\theta + \left(\frac{d}{2}\right)^2 - r\cos\theta\, d$$

$$= r^2(\underbrace{\sin^2\theta + \cos^2\theta}_{1}) + \left(\frac{d}{2}\right)^2 - r\cos\theta\, d$$

$$= r^2 + \frac{d^2}{4} - r\cos\theta\, d$$

$$R_2 = r^2 + \frac{d^2}{4} + r\cos\theta\, d$$

Approx di dipolo $r >> d \Rightarrow R_{1,2} = \sqrt{r^2 \pm r\cos\theta\, d} = \sqrt{r(r \pm \cos\theta d)} = \sqrt{r}\sqrt{r \pm \cos\theta d}$ (1)

Approx I(0) → Taylor

$$f(d) = \sqrt{r + \cos\theta d} = (r + \cos\theta d)^{\frac{1}{2}} \to f'(d) = -\frac{1}{2}\cos\theta\,(r + \cos\theta d)^{-\frac{1}{2}} = \frac{\cos\theta}{2(r + \cos\theta d)^{\frac{1}{2}}}$$

$$\to f'(0) = \frac{\cos\theta}{2\sqrt{r}} \qquad f(0) = \sqrt{r}$$

$$\to f(d) \simeq \sqrt{r} + \frac{\cos\theta}{2\sqrt{r}} \cdot d \qquad \to \qquad g(d) \simeq \sqrt{r} - \frac{\cos\theta}{2\sqrt{r}}\, d$$

$$\Rightarrow \begin{cases} R_1 = \cancel{\sqrt{r}} \cdot \left(\overset{r}{\cancel{\sqrt{r}}} + \dfrac{\cos\theta d}{2\cancel{\sqrt{r}}}\right) = r - \dfrac{d\cos\theta}{2} \quad (2) \\[2ex] R_2 = r + \dfrac{d\cos\theta}{2} \quad (3) \end{cases}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right] = K\,\frac{\cancel{r} + \frac{d}{2}\cos\theta - \cancel{r} + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\theta}$$

$$= K\,\frac{d\cos\theta}{r^2 - \cancel{\left(\frac{d}{2}\right)^2\cos^2\theta}} \simeq \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2}$$

$r \gg d$

→ Campo elettrico

$\vec{E} = -\vec{\nabla} V$ → Derivate parziali → coordinate cartesiane

$$CO = r\cos\theta \;\rightarrow\; \cos\theta = \frac{\overline{CO}}{r} = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$r^2 = x^2 + y^2 + z^2 \;\rightarrow\; r = \sqrt{x^2+y^2+z^2}$$

$$\Rightarrow V = \frac{qd}{4\pi\varepsilon_0}\,\frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\rightarrow \vec{E}_z = -\frac{\partial V}{\partial z} = -K \cdot (x^2+y^2+z^2)^{-\frac{3}{2}} - z \cdot \frac{3}{\cancel{2}}\,(x^2+y^2+z^2)^{-\frac{5}{2}} \cdot \cancel{2}z$$

$$= -K\left[(x^2+y^2+z^2)^{-\frac{3}{2}} - \frac{3z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}\right] = -K\,(r^{\cancel{2}})^{-\frac{3}{\cancel{2}}} - \frac{3z^2}{(r^{\cancel{2}})^{\frac{5}{\cancel{2}}}}$$

$$= -\frac{qd}{4\pi\varepsilon_0}\cdot\left[r^{-3} - \frac{3z^2}{r^5}\right] = -\frac{qd}{4\pi\varepsilon_0}\left[\frac{r^2 - 3z^2}{r^5}\right] = \frac{qd}{4\pi\varepsilon_0}\left[\frac{3z^2}{r^{5}} - \frac{1}{r^3}\right]$$

$$= \frac{qd}{4\pi\varepsilon_0 r^3}\left[\underbrace{\frac{3z^2}{r^2}}_{\cos^2\theta} - 1\right] = \frac{qd}{4\pi\varepsilon_0 r^3}\left(\cos^2\theta - 1\right)$$

Massa Inerziale - gravitazionale

INERZIALE

La capacità di un corpo di opporsi alla variazione di moto

GRAVITAZIONALE

Relativa alla quantità di materia di un corpo

Principio di equivalenza di Einstein

$$\begin{cases} \vec{F} = G \cdot \dfrac{m_1 \cdot M_2}{r^2} \\ \vec{F} = m \cdot \vec{a} \end{cases}$$

Massa gravitazionale

Massa inerziale

Piuma

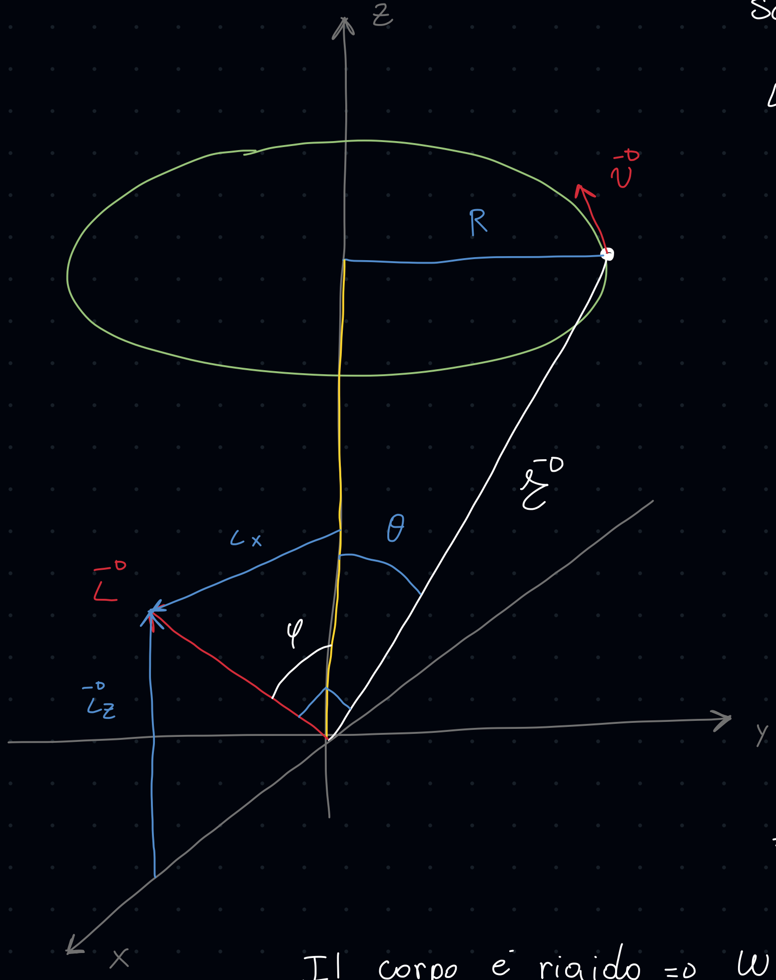
Martello

$$\begin{cases} G \cdot \dfrac{m_{ip} \cdot M}{r^2} = m_{gp} \cdot \vec{a} \\ G \cdot \dfrac{m_{im} \cdot M}{r^2} = m_{gm} \cdot \vec{a} \end{cases} \Rightarrow \frac{m_{ip}}{m_{im}} = \frac{m_{gp}}{m_{gm}}$$

INERZIALE

GRAVITAZIONALE

Rotazione Asse



Scrivere $\vec{L} = \vec{\varepsilon} \wedge m \cdot \vec{v}$

Le componenti x ed y di $\vec{L}_{TOT}$ si annullano

$\to \vec{L}_{TOT} = \vec{L}_z$

$\vec{L}_z = L\cos\varphi$ ma $\varphi = 90 - \theta$

$= L\cos(90-\theta) = L\sin\theta$

$\Rightarrow \vec{L}_{iz} = \varepsilon \cdot m \cdot v \cdot \cancel{\sin(90)}^{\,1} \cdot \sin\theta$

moto circolare $v = \omega R$

$\Rightarrow L_i = \varepsilon m \omega R \sin\theta$

ma $R = \varepsilon \sin\theta \Rightarrow \sin\theta = \frac{R}{\varepsilon}$

$\Rightarrow L_i = \cancel{\varepsilon}\, m \cdot \omega \frac{R^2}{\cancel{\varepsilon}} = m_i \omega_i R^2$

Il corpo è rigido $\Rightarrow \omega_i = \omega$

$\to L_i = m_i \omega R^2$

$\Rightarrow L_{TOT} = \sum_i m_i R^2 \omega = M\omega R^2$

momento di inerzia I $\to$ $\underline{\vec{L}_{TOT} = \vec{\omega} I}$

Dalla II eq cardinale della Dinamica:

$$\begin{cases} \vec{M} = \vec{\varepsilon} \wedge \vec{F} \\ \vec{L} = \vec{\varepsilon} \wedge m \cdot \vec{v} = \vec{\varepsilon} \wedge \vec{p} \end{cases} \Rightarrow \frac{d\vec{L}}{dt} = \underbrace{\cancel{\vec{v} \wedge m\vec{v}}}_{\emptyset} + \vec{\varepsilon} \wedge m \cdot \vec{a}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{\varepsilon} \wedge \vec{v} = \vec{M}$$

$$\Rightarrow \vec{M} = \frac{dL_{TOT}}{dt} = I \cdot \frac{d\omega}{dt} = I\alpha \to \vec{M} = I\alpha \equiv \vec{F} = m \cdot \vec{a}$$

$$\theta(t) = \theta_0 + \int_0^t \omega\, dt \qquad \omega(t) = \omega_0 + \alpha t \rightleftharpoons v(t) = v_0 + a t$$

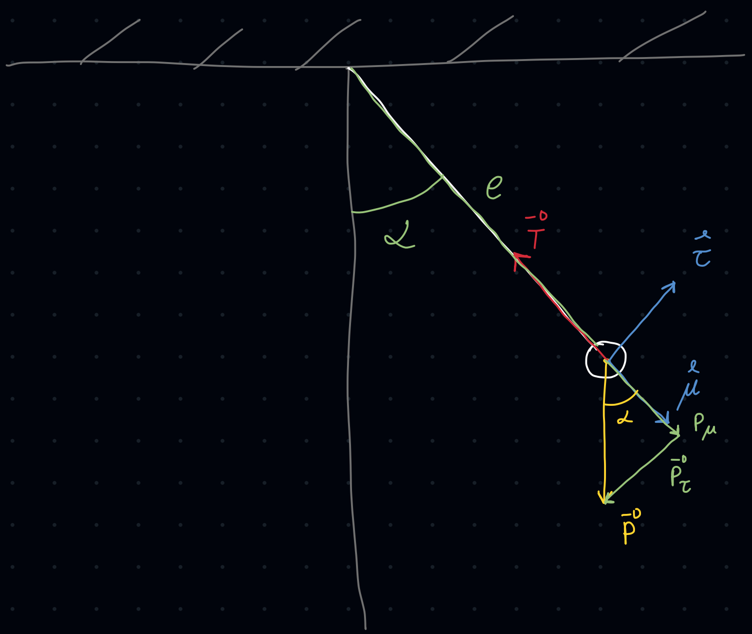
$$\Rightarrow$$

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt \qquad \theta(t) = \theta_0 + \omega t + \frac{1}{2}\alpha t^2 \rightleftharpoons S(t) = S_0 + v_0 t + \frac{1}{2} a T^2$$

Eenergia cinetica Rotazioni

$$\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2R^2 = \frac{1}{2}\omega^2 I$$

Pendolo



$$\vec{P} = m\cdot\vec{g} \rightarrow \vec{P} = \vec{P}_\mu + \vec{P}_\tau$$

$$\begin{cases} \vec{P}_\mu = mg\cos\alpha\ \hat{\mu} \\ \vec{P}_\tau = -mg\sin\alpha\ \hat{\tau} \end{cases}$$

Scrivo $\vec{F} = m\cdot\vec{a}$

$$\begin{cases} \vec{P}_\mu - T = m\cdot\vec{a}_\mu \\ -\vec{P}_\tau = m\cdot\vec{a}_\tau \end{cases} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \begin{cases} \frac{v^2}{R} \text{ lungo } \hat{\mu} \\ \ddot{s} \text{ lungo } \hat{\tau} \end{cases}$$

$$\Rightarrow \begin{cases} mg\cos\alpha - T = m\cdot\frac{v^2}{R} \\ -mg\sin\alpha = m\cdot\ddot{s} \end{cases} \rightarrow \text{Approx piccole oscillaz. } \sin x \approx x$$

$$\Rightarrow -\cancel{m}g\alpha = \cancel{m}\ddot{s} \rightarrow -g\alpha = \ddot{s} \qquad 1\text{ Rad} = \frac{l \leftarrow \text{ARCO}}{R \leftarrow \text{Raggio}}$$

$\rightarrow$ Nel nostro caso $\alpha = \frac{s}{l}$

$$\Rightarrow -g\alpha = \ddot{s} \rightarrow -g\frac{s}{l} = \ddot{s} \rightarrow \ddot{s} + \underbrace{\left(\frac{g}{l}\right)}_{\rightarrow \text{ Battezzo } k^2} s = 0$$

$\rightarrow$ $\ddot{s} + k^2 s = 0$ Eq differenziale

$\rightarrow$ Sol $s(t) = A\cos(kt+\varphi)$

Periodo: $s(t+T_0) = A\cos(kt+\varphi+2\pi) \rightarrow A\cancel{\cos}(\cancel{kt}+kT_0+\cancel{\varphi}) = A\cancel{\cos}(\cancel{kt}+\cancel{\varphi}+2\pi)$

$$\rightarrow kT_0 = 2\pi \Rightarrow k = \frac{2\pi}{T_0} = \omega \rightarrow s(t) = A\cos(\omega t+\varphi)$$

ma $k = \sqrt{\frac{g}{l}} \Rightarrow T_0 = \frac{2\pi}{k} = 2\pi\sqrt{\frac{l}{g}}$ ← Non dipende da M!

Momento inerzia Sbarra

$I = mR^2$ la sbarra è un continuo di punti → corpo rigido

⇒ $\rho = \frac{M}{V}$ Sbarra 3D

$\lambda = \frac{M}{L}$ Sbarra 2D

⇒ $\int \lambda \, dz$ = Massa Totale inoltre $\sum m_i \to \int dm$ = massa Totale

⇒ $I_{TOT} = \sum_i m_i R^2 \to I_{TOT} = \int R^2 \, dm$ sbarra 3D → moltiplico per dV
divido

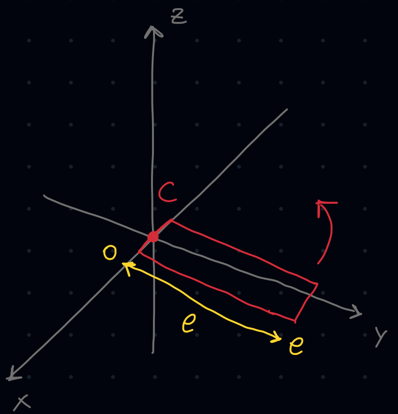
⇒ $I_{TOT} = \int R^2 \underbrace{dm \cdot \frac{dV}{dV}}_{\rho}$ → $I_{TOT} = \int R^2 \rho \, dV$

cost

Sbarra 2D:

$$I_{TOT} = \int R^2 \cdot dm \cdot \frac{d\ell}{d\ell} = \lambda \int_{\ell_0}^{\ell_f} R^2 \, d\ell$$

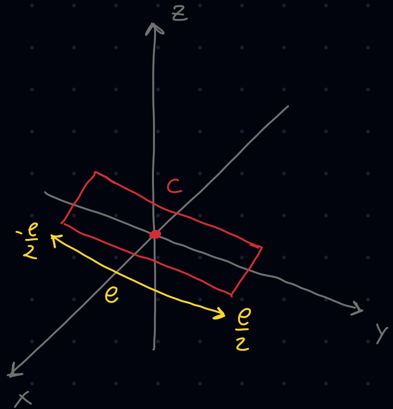
CASO 1:



$$\to I_{TOT} = \lambda \int_0^\ell y^2 \, dy = \lambda \left[\frac{y^3}{3} \right]_0^\ell = \lambda \frac{\ell^3}{3}$$

$$= \frac{M}{\cancel{\ell}} \cdot \frac{\ell^{\cancel{3}\,2}}{3} = \frac{1}{3} M \ell^2 \quad I$$

CASO 2:



$$\to I_{TOT} = \lambda \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} y^2 \, dy = \lambda \left[\frac{y^3}{3} \right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}} = \frac{\cancel{\ell}}{M} \left[\frac{\ell^3}{\cancel{8}_4 \cdot 3} + \frac{\ell^3}{\cancel{8} \cdot 3} \right]$$

$$= \frac{M}{\cancel{\ell}} \frac{\ell^{\cancel{3}\,2}}{12} = \frac{1}{12} M \ell^2 \quad I$$

II eq Card

$$\begin{cases} \vec{M} = \vec{r} \wedge \vec{F} \\ \vec{L} = \vec{r} \wedge m \cdot \vec{v} = \vec{r} \wedge \vec{p} \end{cases} \rightarrow \frac{d\vec{L}}{dt} = \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}} \wedge m\vec{v} + \vec{r} \wedge m \underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}} \rightarrow \frac{d\vec{L}}{dt} = \underbrace{\vec{v} \wedge m\vec{v}}_{0} + \vec{r} \wedge \underbrace{m\vec{a}}_{\vec{F}}$$

$$\rightarrow \frac{d\vec{L}}{dt} = \vec{r} \wedge \vec{F} \Rightarrow \frac{d\vec{L}}{dt} = \vec{M}$$

Energia cinetica

$$\vec{F} = m \cdot \vec{a} \rightarrow L = \vec{F} \cdot e \Rightarrow L = \int \vec{F}\, d\vec{e} \quad \text{ma } F = m \cdot \vec{a} \rightarrow L = \int m \cdot \vec{a}\, d\vec{e}$$

$$\text{ma } \vec{a} = \frac{d\vec{v}}{dt} \rightarrow L = m \int \underbrace{\frac{d\vec{v}}{dt} de}_{\vec{v}} \rightarrow L = m \int_{v_0}^{v_f} v\, dv = m\left(\frac{v_f^2}{2} - \frac{v_0^2}{2}\right)$$

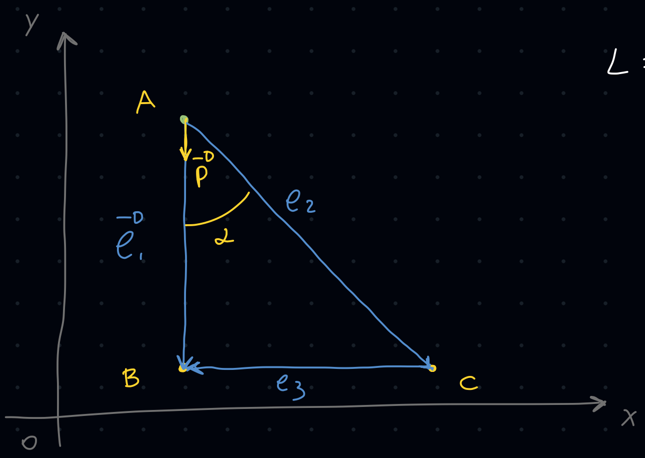
$$\Rightarrow L = \int F \cdot de = \underbrace{G}_{E\ \text{cinetica}}$$

E potenziale : $U = -G \Rightarrow L = \int F \cdot de = -U$

$$\Rightarrow dL = F \cdot de = -U$$

Campo conservativo:

Quando il lavoro compiuto non dipende dal cammino ma solo da stato finale ed iniziale



$$L = \oint F \cdot de = \underbrace{\int \vec{F}\, d\vec{e}_1}_{1} + \left[\underbrace{\int \vec{F} \cdot d\vec{e}_2}_{\neq 0 \neq 1} + \underbrace{\cancel{\int \vec{F}\, d\vec{e}_3}}_{\emptyset}\right]$$

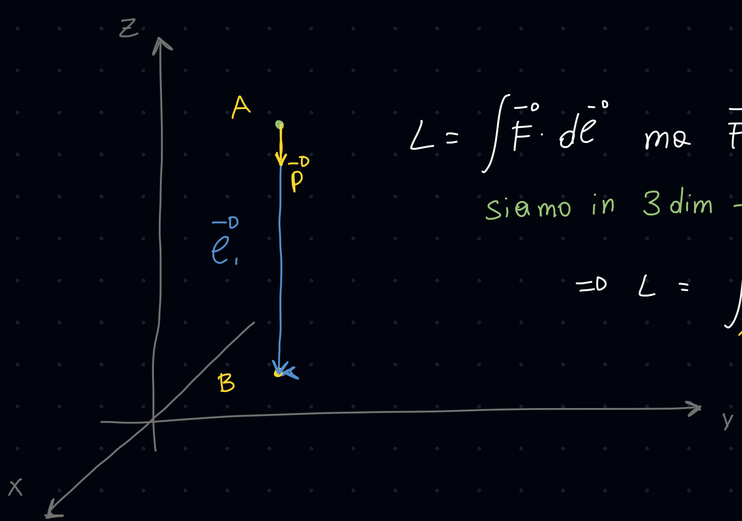
$$F \cdot de = F\, de \cdot \cos\alpha$$

$$\rightarrow L_{AB} = \int F \cdot de_1 = F \cdot e_1$$

$$L_{ACB} = \int F \cdot de_2 = F \cdot \cos\alpha\, e_2$$

$$\text{ma } e_2 \cos\alpha = e_1 \Rightarrow L_{ACB} = F\, e_1 = L_{AB}$$

QED



$$L = \int \vec{F} \cdot d\vec{e} \quad \text{ma } \vec{F} = m\vec{g}$$

siamo in 3 dim $\rightarrow \vec{F} = \vec{F}_x + \vec{F}_y + \vec{F}_z$

$$\Rightarrow L = \cancel{\int \vec{F}_x\, d\vec{e}} + \cancel{\int \vec{F}_y\, d\vec{e}} + \int \vec{F}_z\, d\vec{e} \quad \text{ma } F_y \text{ e } F_z = \emptyset\ !$$

$$\rightarrow L = -\int_{e_0}^{e_f} \vec{F}_z\, d\vec{e} = -mg \int_{e_0}^{e_f} de = mg\underbrace{[e_f - e_0]}_{\text{Altezze!}}$$

$$\Rightarrow L = mg\, h_0 - mg\, h_f$$

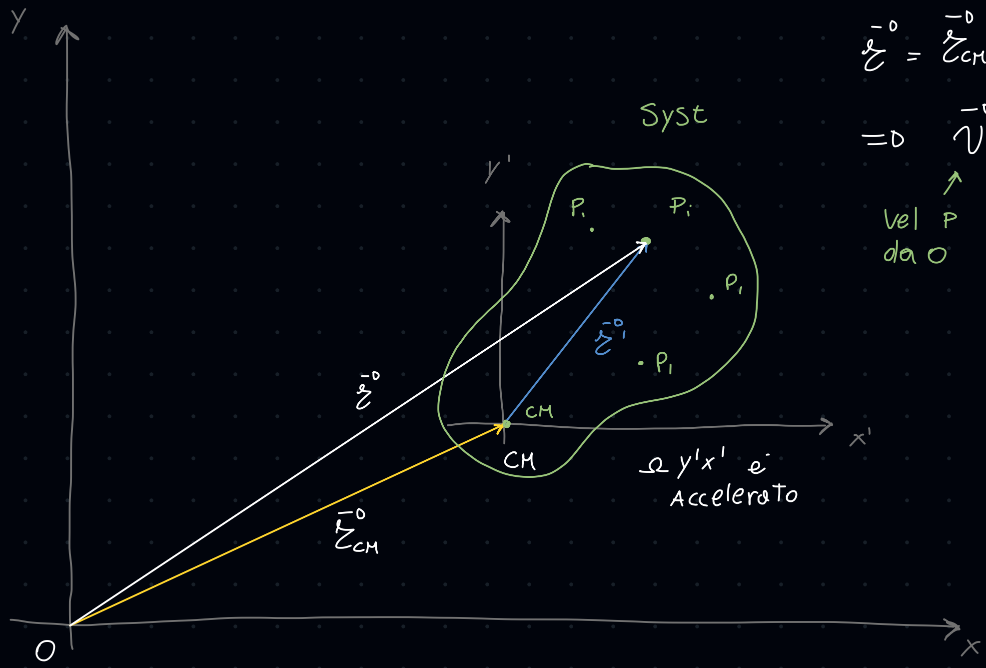
Teorema impulso

$$I = \int_{t_0}^{t_f} F\,dt = m\int_{t_0}^{t_f} a\,dt = m\int_{t_0}^{t_f} \frac{dv}{dt}\,dt = m\int_{v_0}^{v_f} dv = m(\vec v_f - \vec v_0) = \vec p_f - \vec p_0$$

Teorema di Konig

$$\vec r = \vec r_{CM} + \vec r\,'$$

$$\Rightarrow \underset{\text{vel P da O}}{\vec v} = \underset{\text{vel }\Omega\text{ da O}}{\vec v_{CM}} + \underset{\text{vel P da }\Omega}{\vec v\,'}$$



Oxy è a $v = \text{cost}$

Momento Angolare: $\vec L = \vec r \wedge m\vec v \to \vec L_i = \vec r_i \wedge m_i \vec v_i$

$$\vec r_i = \vec r_{CM} + \vec r_i\,' \qquad \vec v_i = \vec v_{CM} + \vec v_i\,'$$

$$\Rightarrow \vec L_i = (\vec r_{CM} + \vec r_i\,') \wedge m_i (\vec v_{CM} + \vec v_i\,')$$

$$\Rightarrow \vec L_{TOT} = \sum_i \left[\vec r_{CM} \wedge m_i \vec v_{CM} + \vec r_{CM} \wedge m_i \vec v_i\,' + \vec r_i\,' \wedge m_i \vec v_{CM} + \vec r_i\,' \wedge m_i \vec v_i\,' \right]$$

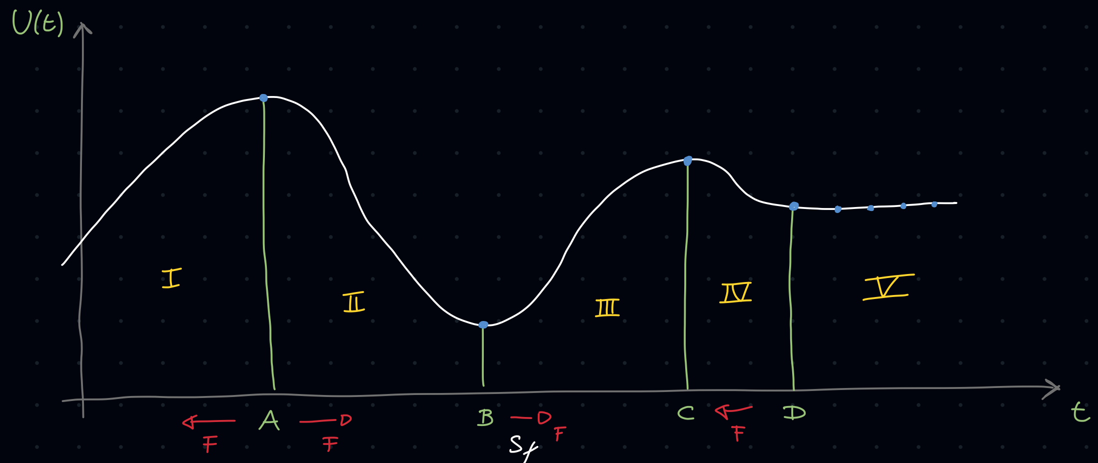
$$= \vec r_{CM} \wedge M \vec v_{CM} + \vec r_{CM} \wedge \cancel{\sum m_i \vec v_i\,'} + \cancel{\sum_i m_i \vec r_i\,'} \wedge \vec v_{CM} + \sum_i \vec r_i\,' \wedge m_i \vec v_i\,'$$

dal Centro di massa $\sum m_i \vec v_i\,' = M \cdot \vec v\,'_{CM} = \emptyset$

Dal CM $\to \sum_i m_i \vec r_i\,' = \vec r\,'_{CM} = \emptyset$

$$= \underbrace{\vec r_{CM} \wedge M \vec v_{CM}}_{\text{Momento Angolare CM}} + \underbrace{\sum_i \vec r_i\,' \wedge m_i \vec v_i\,'}_{\text{Momento Angolare TOTALE dei Punti}} \Rightarrow \vec L_{TOT} = \vec L_{CM} + \vec L\,'_{TOT}$$

Condizioni equilibrio



Teorema E. Cin: $L = \int_{S_0}^{S_f} \vec{F} \cdot ds = \int m \cdot a \, ds = m \int \frac{dv}{dt} ds = m \int_{v_0}^{v_f} v \, dv = \frac{1}{2} m v^2 = G$

cinetico

$U = -G \Rightarrow dL = \vec{F} \cdot ds = -dU \Rightarrow \vec{F} = -\frac{dU}{ds}$

Potenziale

Dall'analisi matematica sappiamo che se $\frac{dU}{dt} = 0 \Rightarrow$ punto min/max/cost

$\Rightarrow$ punti dove $\frac{dU}{dt} = 0$ Punti Di equilibrio

Se per $I(x_0^-) \to \frac{dU}{dt} > 0$ e $I(x_0^+) \to \frac{dU}{dt} < 0 \Rightarrow$ ISTABILE

Se per $I(x_0^-) \to \frac{dU}{dt} < 0$ e $I(x_0^+) \to \frac{dU}{dt} > 0 \Rightarrow$ STABILE

Leggi di Ohm

$V_A - V_B = R \cdot i$ $\qquad$ $R = \rho \cdot \frac{\ell}{S}$

1° legge $\qquad$ 2 legge

$$\begin{cases} V_A - V_B = R \cdot I \\ V_A - V_B = \int \vec{E} \cdot d\vec{\ell} \end{cases}$$

$L = F \cdot S \Rightarrow \frac{dL}{q} = E \cdot d\ell = V_A - V_B$

$$\Rightarrow R \cdot I = \int_S \vec{E} \cdot d\vec{\ell} \rightarrow R\, dI = \vec{E} \cdot d\vec{\ell}$$

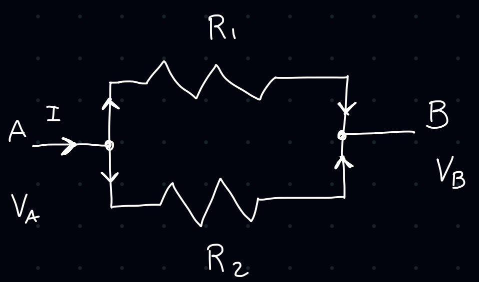
$d\ell \cdot \hat{n} = d\vec{\ell}$

$$R = \rho \cdot \frac{d\ell}{dS}, \quad I = \int \vec{J} \cdot \hat{n}\, dS \rightarrow \rho \frac{d\ell}{\cancel{dS}} \cdot J \cdot \hat{n}\, \cancel{dS} = \vec{E} \cdot d\vec{\ell}$$

$$\rightarrow \vec{J} \cdot \rho \cdot \cancel{d\vec{\ell}} = \vec{E} \cdot \cancel{d\vec{\ell}} \rightarrow \vec{E} = \vec{J}\rho$$ Forma vettoriale

Resistività Temp.

$\rho(T) = \rho_0 (KT + 1)$ $\qquad$ $\rho_0 = \rho(0°)$

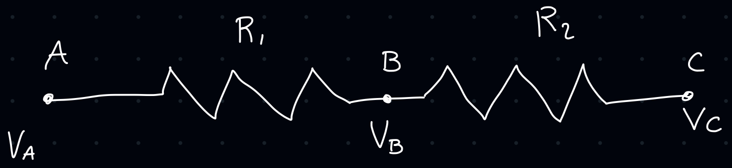


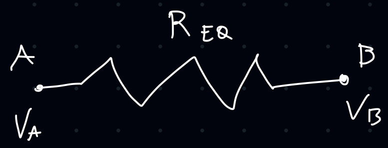
$V_A - V_B = R_{EQ} \cdot I$

$$\begin{cases} V_A - V_B = R_1 I_1 \\ V_A - V_B = R_2 I_2 \end{cases} \Rightarrow \begin{cases} I_1 = \frac{\Delta V}{R_1} \\ I_2 = \frac{\Delta V}{R_2} \end{cases}$$

$$I = \frac{\Delta V}{R_{EQ}}$$

$$\rightarrow \underbrace{I_1 + I_2}_{I_{TOT}} = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Rightarrow \frac{1}{R_{EQ}} = \sum_i \frac{1}{R_i}$$





$$V_A - V_B = R_{EQ}\cdot I$$

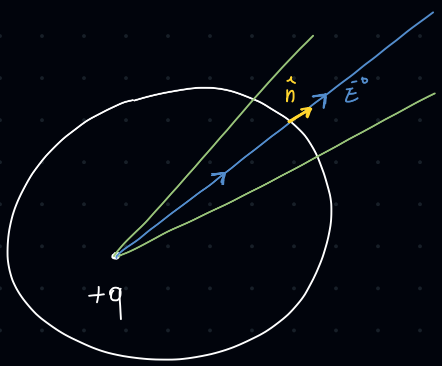
$$\to \begin{cases} V_A - V_B = R_1 I \\ V_B - V_C = R_2 I \end{cases} \Rightarrow V_A - V_B + V_B - V_C = I(R_1 + R_2)$$

$$\to V_A - V_C = I(R_1 + R_2)$$

$$\Rightarrow R_{EQ} = \sum_i R_i$$

Teorema di Gauss

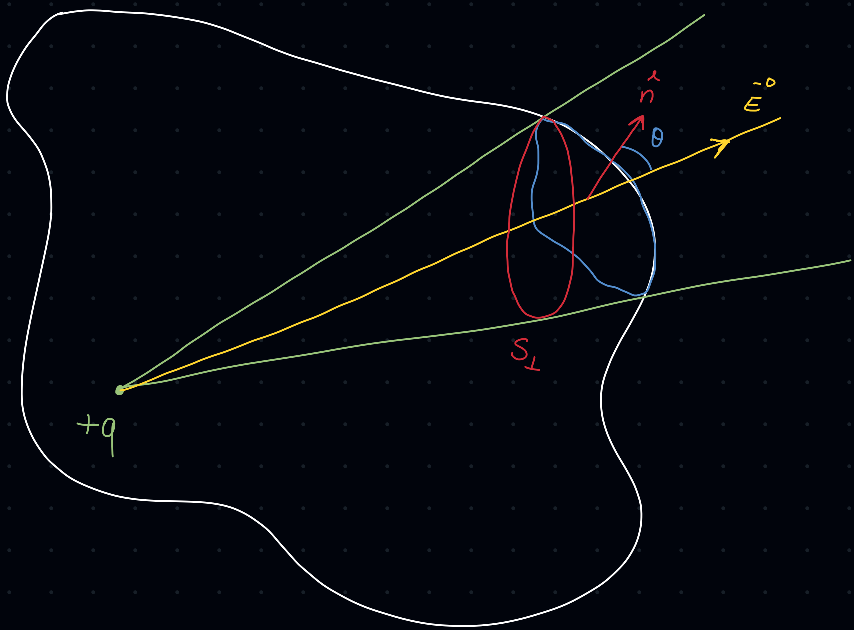
$$\phi_E = \int \vec{E}\cdot\hat{n}\,dS = \frac{Q_{int}}{\varepsilon_0}$$



$$\int \vec{E}\cdot\hat{n}\,dS = \int E\cdot\hat{r}\cdot\hat{n}\,dS = E\int_S dS = \frac{q}{4\pi\varepsilon_0 \cancel{r^2}}\cdot 4\pi\cancel{R^2}$$

$$\hat{r} \parallel \hat{n}$$

$$= \frac{q}{\varepsilon_0}$$



$$S_\perp = S\cos\theta$$

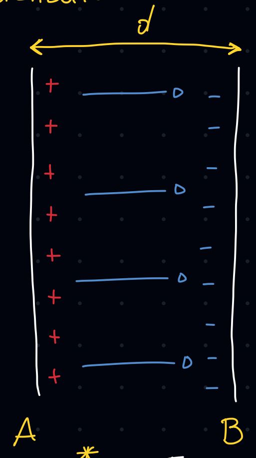
$$d\Omega = \frac{dS\cos\theta}{R^2} \quad \text{Angolo solido}$$

$$\Rightarrow \int \vec{E}\cdot\hat{n}\,dS = \int \frac{q}{4\pi\varepsilon_0}\overbrace{\frac{\cos\theta\cdot dS}{R^2}}^{d\Omega}$$

$$= \int \frac{q}{4\pi\varepsilon_0}\,d\Omega = \frac{q}{4\pi\varepsilon_0}\int d\Omega$$

$$= \frac{q}{\cancel{4\pi}\varepsilon_0}\,\cancel{4\pi} = \frac{q}{\varepsilon_0}$$

Condensatori



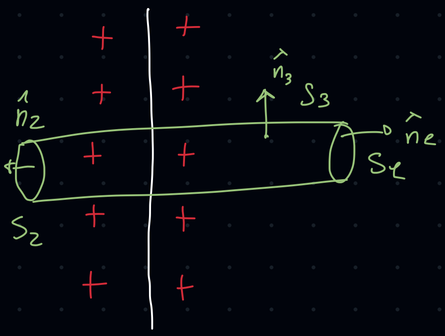
$$q_A = q_B \Rightarrow \begin{cases} V_A = k_1 Q \\ V_B = k_2 Q \end{cases} \Rightarrow V_A - V_B = Q(k_1 - k_2)$$

$$\frac{1}{C} = k_1 - k_2 \Rightarrow V_A - V_B = \frac{Q}{C}$$

$$\Rightarrow C = \frac{Q}{V_A - V_B}$$

$* \; E = \frac{\sigma}{\varepsilon_0}$

Campo elettrico del doppio strato



$$\phi_E = \oint \vec{E}\,\hat{n}\,ds = 2\int E \cdot ds = 2 E S_1 = \frac{Q}{\varepsilon_0}$$

$$\sigma = \frac{Q}{S} \Rightarrow Q = \sigma S$$

$$\Rightarrow 2 E \cancel{S_1} = \frac{1}{\varepsilon_0} \cdot \sigma \cancel{S_1} \rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

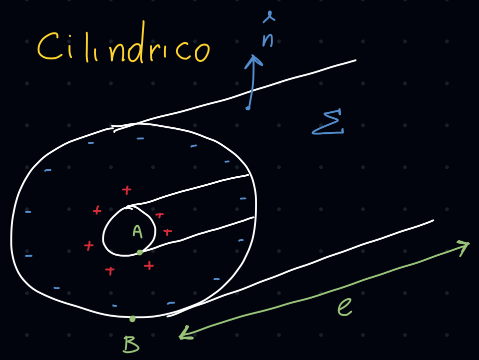
Doppio strato $\rightarrow \; E = \frac{\sigma}{\varepsilon_0} \; *$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{\ell} = E \cdot d \quad \text{ma} \quad E = \frac{\sigma}{\varepsilon_0} \Rightarrow V_A - V_B = \frac{d\sigma}{\varepsilon_0}$$

$$\Rightarrow C = \frac{Q \cdot \varepsilon_0}{d\sigma} \quad \text{Siccome} \quad \sigma = \frac{Q}{S} \Rightarrow C = \frac{\cancel{Q} \cdot \varepsilon_0}{d \cdot \cancel{Q}} \cdot S$$

$$\rightarrow C = \frac{S \cdot \varepsilon_0}{d}$$

Cilindrico



$$C = \frac{Q}{V_A - V_B}$$

GAUSS

$$\phi_E = \int \vec{E}\,\hat{n}\,ds = \frac{Q}{\varepsilon_0}$$

Avvolgo il cavo con una superficie gaussiana

$$\Rightarrow E \cdot \int_\Sigma ds = E \cdot \Sigma \quad \text{ma} \quad \Sigma = C \times \ell = 2\pi R \cdot \ell$$

$$\Rightarrow E \, 2\pi R \cdot \ell = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{2\pi R \ell \varepsilon_0}$$

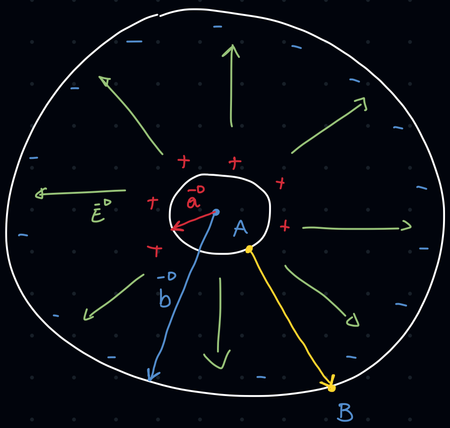
$$\to V_A - V_B = \int \vec{E}\cdot d\vec{e} = \int_A^B \frac{Q}{2\pi \underset{\text{Non cost}}{R}\, e\, \varepsilon_0}\, de = \frac{Q}{2\pi e \varepsilon_0}\int_A^B \frac{1}{R}\, dR$$

$$\int \frac{1}{x}\, dx = \ln(x) + C \Rightarrow V_A - V_B = \frac{Q}{2\pi e \varepsilon_0}\left[\ln(B) - \ln(A)\right] = \frac{Q \cdot \ln\left(\frac{B}{A}\right)}{2\pi e \varepsilon_0}$$

ma $\frac{Q}{e} = \lambda \Rightarrow V_A - V_B = \frac{\lambda \ln\left(\frac{B}{A}\right)}{2\pi \varepsilon_0}$

$$\Rightarrow C = \frac{Q}{V_A - V_B} = \frac{Q\, 2\pi \varepsilon_0}{\lambda \ln\left(\frac{B}{A}\right)} = \frac{\cancel{Q}\, 2\pi e \varepsilon_0}{\cancel{Q}\, \ln\left(\frac{B}{A}\right)} = \frac{2\pi e \varepsilon_0}{\ln\left(\frac{B}{A}\right)}$$

SFERICO



$C = \frac{Q}{\Delta V}$ $\quad \Delta V = \int_A^B E \cdot de = \int_A^B \frac{Q\, \hat{r}}{4\pi \varepsilon_0 r^2}\, d\vec{e}$ $\quad d\vec{r}$

$$\Rightarrow \int_A^B \frac{Q}{4\pi \varepsilon_0 r^2}\, d\vec{r} = \frac{Q}{4\pi \varepsilon_0}\int_A^B \frac{1}{r^2}\, d\vec{r}$$

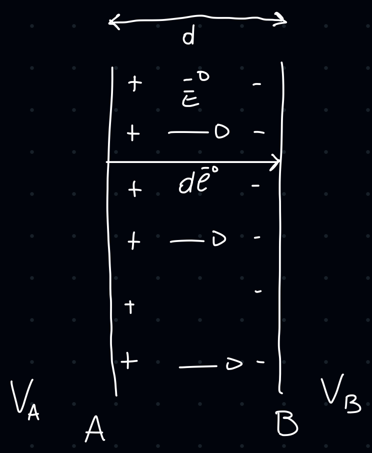
$\Delta V = \frac{Q}{4\pi \varepsilon_0}\left[-\frac{1}{r}\right]_A^B$ $\quad$ ma $A = \vec{a}$ $\quad B = b$

$$\Rightarrow \Delta V = \frac{Q}{4\pi \varepsilon_0}\left[-\frac{1}{b} + \frac{1}{a}\right] = k\left[\frac{b-a}{ba}\right] \to \Delta V = \frac{Q}{4\pi \varepsilon_0}\left[\frac{b-a}{ba}\right]$$

$$\Rightarrow C = \frac{Q}{\Delta V} = \frac{\cancel{Q}\, 4\pi \varepsilon_0}{\cancel{Q}}\left[\frac{ba}{b-a}\right] = 4\pi \varepsilon_0 \frac{ba}{b-a}$$

CAPACITÀ

il Potenziale è proporzionale alla carica, $Q_A = -Q_B \Rightarrow |Q_A| = |Q_B|$



$$\begin{cases} V_A = K_1 Q \\ V_B = K_2 Q \end{cases} \Rightarrow V_B - V_A = Q(K_2 - K_1) \qquad \frac{1}{C} = K_2 - K_1$$

$$\Rightarrow V_A - V_B = \frac{Q}{C} \Rightarrow C = \frac{Q}{V_A - V_B}$$

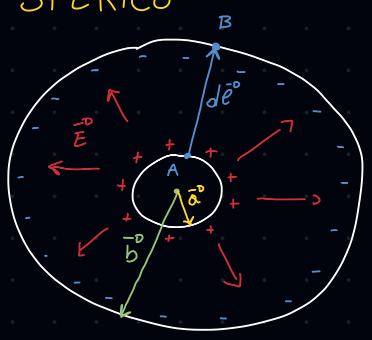
Sappiamo che $\vec{E}$ doppio strato: $E = \frac{\sigma}{\varepsilon_0}$

siccome $E = \frac{F}{q} \Rightarrow F = q \cdot E \Rightarrow \frac{L}{q} = \int_A^B \vec{E}\, d\vec{e} = V_A - V_B$

$$\Rightarrow \Delta V = \int_A^B \frac{\sigma}{\varepsilon_0}\, de = \frac{Q}{S\varepsilon_0} \int_A^B de = \frac{Q \cdot d}{S \cdot \varepsilon_0} \Rightarrow \boxed{C = \frac{Q}{\Delta V} = \frac{S \cdot \varepsilon_0}{d}}$$

$\sigma = \frac{Q}{S}$

SFERICO



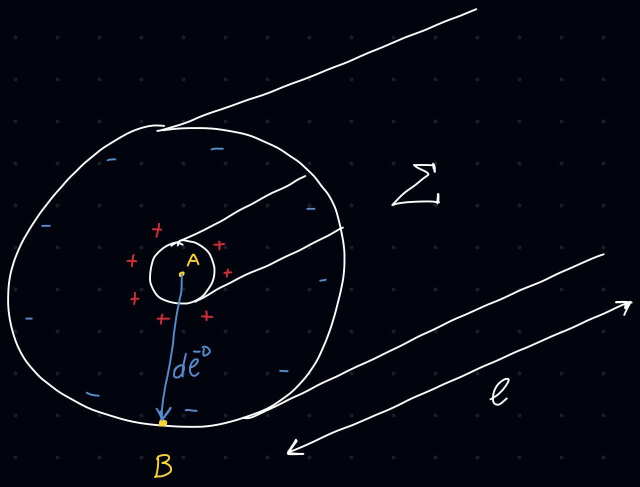
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^2} \hat{r} \Rightarrow V_A - V_B = \int_A^B \frac{Q\, \hat{r}}{4\pi\varepsilon_0 R^2}\, d\vec{e} \qquad \hat{r}\, d\vec{e} = d\vec{r}$$

$$\Rightarrow V_A - V_B = \frac{Q}{4\pi\varepsilon_0} \int_A^B \frac{1}{R^2}\, d\vec{r} = \frac{Q}{4\pi\varepsilon_0} \left[-\frac{1}{R} \right]_A^B$$

$$= \frac{Q}{4\pi\varepsilon_0} \left[\frac{b-a}{b a} \right]$$

$$\Rightarrow \boxed{C = 4\pi\varepsilon_0 \cdot \frac{b \cdot a}{b - a}}$$

cilindrico



Campo Elettrico = ??

Teorema di Gauss

$$\phi_E = \oint \vec{E}\cdot\hat{n}\,dS = \int_\Sigma E\cdot dS = E\cdot\Sigma = \frac{Q}{\varepsilon_0}$$

ma $\Sigma = C\cdot\ell = 2\pi R\cdot\ell$

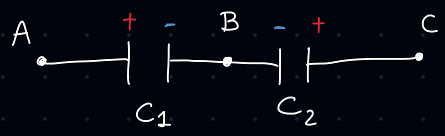
$$\Rightarrow E\cdot 2\pi R\ell = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi R\ell\,\varepsilon_0}$$

$$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{\ell} = \int_A^B \frac{Q\,\hat{r}}{2\pi R\ell\,\varepsilon_0}\cdot d\vec{\ell} = \frac{Q}{2\pi\ell\,\varepsilon_0}\int_A^B \frac{1}{r}\,d\vec{r}$$

$$\Rightarrow V_A - V_B = \frac{Q}{2\pi\ell\,\varepsilon_0}\left[\ln(B) - \ln(A)\right] = \frac{Q\ln\left(\frac{B}{A}\right)}{2\pi\ell\,\varepsilon_0}$$

$$\Rightarrow C = \frac{2\pi\ell\,\varepsilon_0}{\ln\left(\frac{B}{A}\right)}$$

Nei circuiti



$$C_1 = \frac{Q}{V_A - V_B} \qquad C_2 = \frac{Q}{V_B - V_C}$$

↓

$$V_A - V_B = \frac{Q}{C_1} \qquad V_B - V_C = \frac{Q}{C_2}$$

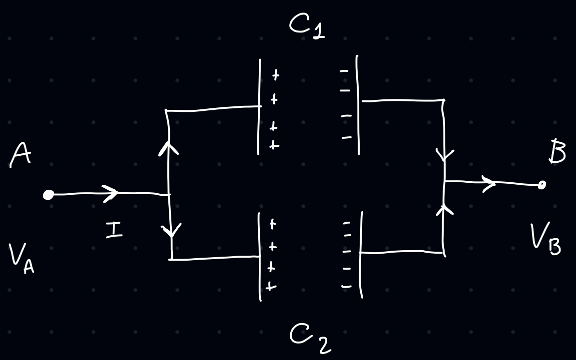
$$\Rightarrow V_A - \cancel{V_B} + \cancel{V_B} - V_C = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow V_A - V_B = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$C_{EQ} = \frac{Q}{V_A - V_B}$$

$$\Rightarrow V_A - V_B = \frac{Q}{C_{EQ}}$$

$$\frac{1}{C_{EQ}} = \sum_i \frac{1}{C_i}$$



$$C_1 = \frac{Q_1}{V_A - V_B} \qquad C_2 = \frac{Q_2}{V_A - V_B}$$

$$C_{EQ} = \frac{Q}{V_A - V_B}$$

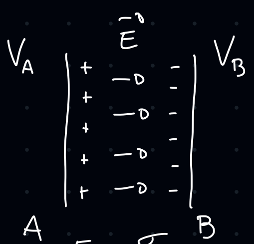
$$\to Q = (V_A - V_B)\, C_{EQ}$$

$$\to Q_1 = C_1 (V_A - V_B) \qquad Q_2 = C_2 (V_A - V_B)$$

$$\Rightarrow C_{EQ} = \Sigma_i C_i$$

$$\Rightarrow \underbrace{Q_1 + Q_2}_{Q_{TOT}} = (V_A - V_B)(C_1 + C_2) \to Q = (V_A - V_B)(C_1 + C_2)$$

Energia Campo $\vec{E}$



$$E = \frac{\sigma}{\varepsilon_0}$$

$\Rightarrow$ Lavoro per spostare una carica da A a B

$$\underbrace{L = \int_A^B F \cdot de}_{\text{lavoro Forza}} \quad \text{ma} \quad \vec{E} = \frac{\vec{F}}{q} \to \vec{F} = q \cdot \vec{E} \quad \to \quad \underbrace{\frac{L}{q} = \int_A^B \vec{E} \cdot d\vec{e} = V_A - V_B}_{\text{Lavoro del Campo } \vec{E}}$$

$$\Rightarrow dL = F \cdot de \to dL = q \cdot E\, de \quad \text{ma} \quad E de = V$$

$$\Rightarrow dL = V dq$$

$$\Rightarrow L = \int_A^B V dq \qquad \text{dal condensatore} \qquad C = \frac{Q}{V_A - V_B} \to V_A - V_B = \frac{Q}{C}$$

$$= \int_A^B \frac{Q}{C} dq = \frac{1}{C} \int_A^B Q\, dQ = \frac{1}{C} \left[\frac{Q^2}{2} \right]_A^B = \frac{1}{C} \left[\frac{Q_B^2}{2} - \frac{Q_A^2}{2} \right]$$

Se pongo $\begin{cases} A = 0 \\ B = Q \end{cases} \to L = \frac{1}{C} \frac{Q^2}{2}$ ma $\frac{Q}{C} = V_A - V_B$

$$\Rightarrow L = \frac{(V_A - V_B)\, Q}{2}$$

Definisco w densità volumetrica di Energia $\to$ $w = \frac{\mathcal{E}}{V}$

$V = S \cdot d \Rightarrow w = \frac{(V_A - V_B) Q}{2 S d} = \frac{1}{2}(V_A - V_B)\frac{\sigma}{d}$

ma nel cond piano $E \cdot S = \frac{Q}{\varepsilon_0} \to E \cdot \varepsilon_0 = \frac{Q}{S} \to E \cdot \varepsilon_0 = \sigma$ (1)

cond piano $E \cdot d = V_A - V_B \Rightarrow E = \frac{V_A - V_B}{d}$ (2)

$\to w = \frac{1}{2} E \cdot E \cdot \varepsilon_0 \Rightarrow w = \frac{1}{2} E^2 \varepsilon_0$ Dens di Energia Volum.

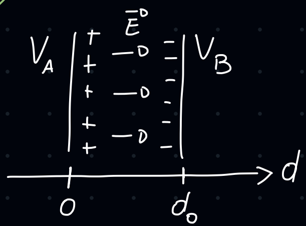
Energia $\to$ Necessaria a spostare una carica

$L = F \cdot \ell \to dL = F\, d\ell$ ma $E = \frac{F}{q} \to F = q \cdot E$

$\frac{dL}{q}$ = Lavoro del C.E = $E \cdot d\ell$ ma $\int \vec{E}\, d\vec{\ell} = -dV$

$\Rightarrow L_E = q \cdot V \to dL = V\, dq$

$\Rightarrow L = \int_A^B V\, dq$



$\Rightarrow L = \int_0^Q V\, dq =$

$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} \to L = \frac{Q}{C} \Rightarrow L = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$

$\to L = \frac{1}{C}\frac{Q^2}{2}$ ma $\frac{Q}{C} = V \Rightarrow L = \mathcal{E} = \frac{Q(V_A - V_B)}{2}$ Energia caso generale

Caso del cond Piano

$w = \frac{\mathcal{E}}{V}$ $V = S \cdot d \to \frac{Q(V_A - V_B)}{2Sd} = \frac{1}{2}\frac{V_A - V_B}{d} \cdot \sigma$

Sappiamo che $E \cdot d = V_A - V_B \Rightarrow E = \frac{V_A - V_B}{d}$

inoltre $E \cdot S = \frac{Q}{\varepsilon_0} \Rightarrow E\varepsilon_0 = \frac{Q}{S} = \sigma$

$\Bigg\}\ w = \frac{1}{2}E^2\varepsilon_0$

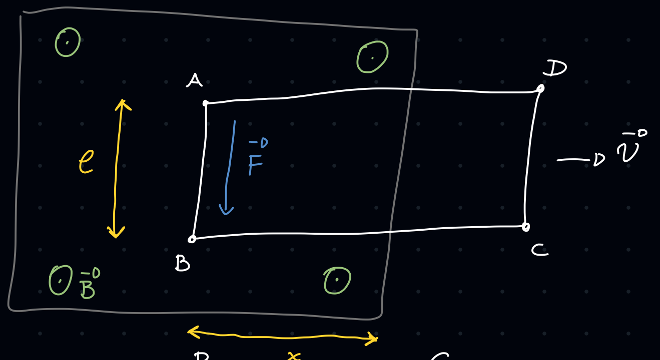
Faraday

$$f_{em} = -\frac{d\phi_B}{dt}$$

TAGLIATO

$$\vec{F}_{Lorentz} = q(\vec{v} \wedge \vec{B})$$

$$f_{em} = \frac{L}{q} = \frac{q \oint (\vec{v} \wedge \vec{B})\, d\vec{l}}{q}$$



$$\Rightarrow f_{em} = \int_A^B (\vec{v} \wedge \vec{B})\, d\vec{l} + \cancel{\int_B^C (\vec{v} \wedge \vec{B})\, d\vec{l}} + \cancel{\int_C^D (\vec{v} \wedge \vec{B})\, d\vec{l}} + \cancel{\int_D^A (\vec{v} \wedge \vec{B})\, d\vec{l}} = \int_A^B (\vec{v} \wedge \vec{B})\, dl_1$$

$\vec{F} \perp d\vec{l}$ $\quad B = 0$ $\quad F \perp dl$

$$= vBl$$

ma $$f_{em} = -\frac{d\phi_B}{dt} = \int \vec{B} \cdot \hat{n}\, dS = \int B \cdot \underbrace{\cos dS}_{\text{VARIANO}}$$

se il filo si muove, S dipende da t $\Rightarrow$ $S = l \cdot x$ ma $x(t) = x - \Delta x$ con $\Delta x = v \cdot \Delta t$

$\Rightarrow$ $dS = l \cdot dx$

$$f_{em} = -\frac{d\phi}{dt} = \int_0^x B \cdot l \cdot dx = \underline{Blx}$$

$\bar{t} = t + \Delta t$ e $\bar{x} = x - \Delta x$

$$\phi_B(t + \Delta t) = Bx(x - \Delta x)$$

$$\Rightarrow \frac{d\phi_B}{dt} = \frac{\phi_B(t+\Delta t) - \phi_B(t)}{\Delta t} = \frac{Bl(x - \Delta x) - Blx}{\Delta t} = \frac{\cancel{Blx} - Bl\Delta x - \cancel{Blx}}{\Delta t}$$

$$\rightarrow \frac{d\phi}{dt} = \lim_{\Delta t \to 0} -\frac{Bl\Delta x}{\Delta t} = -Blv \quad \rightarrow \quad \underline{-\frac{d\phi}{dt} = Blv}$$

Concatenato

Varia B, $S = \text{cost}$ $\Rightarrow B(t, r)$

$$\frac{d\phi}{dt} = \lim_{\Delta t \to 0} \frac{\int \vec{B}(t+\Delta t, r)\,\hat{n}\,ds - \int \vec{B}(t, r)\,\hat{n}\,ds}{\Delta t}$$

Approssimo con Taylor $\Delta t = 0$, 1° Termine

$$B(t,r) \approx B(t, r) + \frac{\partial \vec{B}}{\partial t}(t+\Delta t, r) \cdot \Delta t$$

$$\Rightarrow \frac{d\phi}{dt} = \lim_{\Delta t \to 0} \frac{\cancel{\int \vec{B}(t, r)\,\hat{n}\,ds} + \int \frac{\partial \vec{B}}{\partial t}(t + \cancel{\Delta t}, r)\,\cancel{\Delta t} - \cancel{\int \vec{B}(t, r)\,\hat{n}\,ds}}{\cancel{\Delta t}}$$

$$\Rightarrow \frac{d\phi}{dt} = \int_S \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,ds$$

ma $f_{em} = \frac{L}{q}$ $\Rightarrow F_{LOR} = qE + q(\vec{v} \wedge \vec{B})$ $\Rightarrow f_{em} = \int \vec{E} \cdot d\vec{l} + \int \vec{v} \wedge \vec{B}\, dl$

$$\Rightarrow \int \vec{E}\, dl + \underbrace{\cancel{\int \vec{v} \wedge \vec{B}\, dl}}_{v=0} = -\int_S \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,ds$$

ma $B = \text{varia}$, $v = \emptyset$ $\Rightarrow \int v \wedge B\, dl = 0$

$$\Rightarrow \oint \vec{E}\, dl = -\int \frac{\partial B}{\partial t}\,\hat{n}\,ds \Rightarrow \int_S \vec{\nabla} \wedge \vec{E}\,\hat{n}\,ds = -\int \frac{\partial B}{\partial t}\,\hat{n}\,ds$$

$$\Rightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Tutto insieme

1) $-\frac{d\phi}{dt} = \oint (\vec{v}\wedge\vec{B})\, dl$

2) $-\frac{d\phi}{dt} = -\int \frac{\partial \vec{B}}{\partial t}\,\hat{n}\, ds$

$$\frac{d\phi}{dt} = \oint \frac{\partial \vec{B}}{\partial t}\,\hat{n}\, ds - \oint (\vec{v}\wedge\vec{B})\, d\vec{l}$$

ma $f_{em} = -\frac{d\phi}{dt} = \frac{L}{q} = \oint \vec{E}\, d\vec{l} + \int (\vec{v}\wedge\vec{B})\, dl$

$$\Rightarrow \oint \vec{E}\, d\vec{l} + \cancel{\oint (\vec{v}\wedge\vec{B})\, d\vec{l}} = -\int \frac{\partial \vec{B}}{\partial t}\,\hat{n}\, ds + \cancel{\oint (\vec{v}\wedge\vec{B})\, dl}$$

$$\rightarrow \oint \vec{E}\cdot dl = -\int \frac{\partial B}{\partial t}\cdot\hat{n}\, ds \Rightarrow \vec{\nabla}\wedge\vec{E} = -\frac{\partial B}{\partial t}$$ Maxwell

Autoinduzione

Quando circola I in un circuito $\rightarrow$ Campo magnetico indotto

$\rightarrow$ Biot-Savart $\rightarrow$ $dB = \frac{\mu_0 I}{4\pi\varepsilon_0}\,\frac{d\vec{l}\wedge\vec{r}}{r^3}$

$$\Rightarrow \phi_B = \int_S \underbrace{\frac{\mu_0 I}{4\pi\varepsilon_0}\int \frac{d\vec{l}\wedge\vec{r}}{r^3}\,\hat{n}\, dS}_{\text{Tutto costante tranne } I}$$ AUTO FLUSSO

$\Rightarrow \phi_B = L\cdot I$

Induttanza

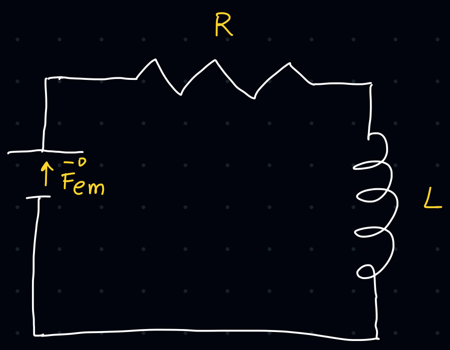
Quando $B \neq cost$

$\phi_B \neq 0 \Rightarrow \frac{d\phi_B}{dt} \neq 0 \rightarrow \phi_B = L\frac{dI}{dt}$

$\Rightarrow f_{em} = -\frac{d\phi}{dt} = - L\frac{dI}{dt}$

Segno opposto

Esempio Induttanza   Circuito R·L



$$f_{em} - f_{ind} = R\cdot I \qquad \text{ma} \quad f_{ind} = -L\frac{dI}{dt}$$

Eq Diff

$$\rightarrow \quad f_{em} = L\frac{dI}{dt} + R\cdot I$$

Soluzione: $I(t) = Ae^{\alpha t} + D$

$$\rightarrow \dot{I} = \alpha A e^{\alpha t} \rightarrow f_{ind} = L\alpha A e^{\alpha t} + RAe^{\alpha t} + RD \qquad \text{ma } f_{ind} = cost$$

$$\Rightarrow f_{ind} = R\cdot D \Rightarrow D = \frac{f}{R} \Rightarrow L\alpha A e^{\alpha t} + RAe^{\alpha t} = 0 \rightarrow Ae^{\alpha t}(L\alpha + R) = 0$$

$$\rightarrow L\alpha + R = 0 \rightarrow \alpha = -\frac{R}{L}$$

$$\Rightarrow \text{Sol gen: } I(t) = Ae^{-\frac{R}{L}t} + \frac{f}{R}$$

pongo $I(0) = 0 \rightarrow I(0) = Ae^{-\frac{R}{L}0} + \frac{f}{R} = 0 \rightarrow A + \frac{f}{R} = 0 \rightarrow A = -\frac{f}{R}$

$$\Rightarrow \text{Sol part: } I(t) = -\frac{f}{R}e^{-\frac{R}{L}t} + \frac{f}{R}$$

OHM

1) $V_A - V_B = R\cdot I$

2) $R = \rho\cdot\frac{\ell}{S}$

Forma vettoriale $\begin{cases} V_A - V_B = R\cdot I \\ V_A - V_B = \int E\cdot d\ell \end{cases}$

$$\Rightarrow R\cdot I = \int_\ell \vec{E}\cdot d\vec{\ell} \qquad \text{ma} \quad I = \int \vec{J}\cdot\hat{n}\,dS = J\cdot S \qquad R = \rho\frac{d\ell}{dS}$$

$d\ell\cdot\hat{n} = d\vec{\ell}$

$$\Rightarrow \rho\frac{d\ell}{\cancel{dS}}\cdot\int\vec{J}\cdot\hat{n}\,\cancel{dS} = \int\vec{E}\cdot d\vec{\ell} \rightarrow \rho\vec{J}\,\cancel{d\vec{\ell}} = \vec{E}\,\cancel{d\vec{\ell}}$$

$$\rightarrow \vec{E} = \rho\vec{J}$$

Conservazione Carica

$$-dq = \int \vec{J}\cdot\hat{n}\,ds \cdot dt \rightarrow -\frac{dq}{dt} = \int \vec{J}\cdot\hat{n}\,ds \Rightarrow$$

So che $\rho = \frac{dq}{dV} \rightarrow dq = \rho\, dV \rightarrow -\frac{d}{dt}\int_V \rho\, dV = \int \vec{J}\cdot\hat{n}\,ds$

$$\rightarrow -\int_V \frac{\partial \rho}{\partial t}\,dV = \int_S \vec{J}\cdot\hat{n}\,ds \Rightarrow -\int \frac{\partial \rho}{\partial t}\,dV = \int_V \vec{\nabla}\vec{J}\,dV$$

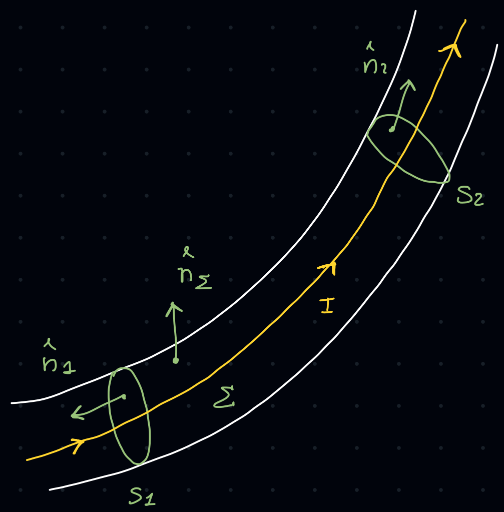
$$\rightarrow -\frac{\partial \rho}{\partial t} = \vec{\nabla}\vec{J}$$

Condizioni di corrente stazionaria

$\rightarrow$ La densità di carica è sempre la stessa

$$\rightarrow \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \vec{\nabla}\vec{J} = \varnothing$$



$$\phi_E = \oint \vec{J}\cdot\hat{n}\,ds = \int_{S_1} + \int_{S_2} + \int_{S_3}$$

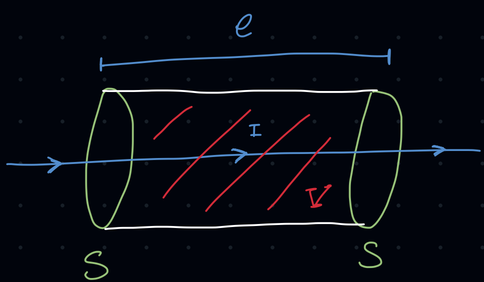
$$= -\phi_1 + \phi_2 = \varnothing \qquad \text{perché } S_1 = S_3$$

$$\Rightarrow \phi_1 = -\phi_{S_2}$$

Densità di corrente: $I = \int \vec{J}\cdot\hat{n}\,ds$

$\rho = \frac{dQ}{dV} \Rightarrow dQ = \rho\, dV$ ma $dV = S\cdot \ell$ ma $\ell = v\cdot t$

$$\Rightarrow dV = dS\cdot v\cdot dt$$



$$\Rightarrow dq = \int \rho\cdot ds\cdot \vec{v}\cdot dt \Rightarrow \frac{dq}{dt} = \int \rho\cdot\vec{v}\cdot dS$$

$$\Rightarrow I = \int \rho\cdot\vec{v}\,ds \qquad \vec{J} = \rho\cdot\vec{v}$$

$$\Rightarrow I = \int \vec{J}\cdot\hat{n}\,ds$$

Auto ind

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \wedge \Delta\vec{r}}{\Delta r^3} \rightarrow \phi_B = \int \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \wedge \Delta\vec{r}}{\Delta r^3} \hat{n}\, dS$$

$\rightarrow$ Tutto cost $\rightarrow$ $\phi_B = L \cdot I$ se $B \neq$ cost $\frac{d\phi_B}{dt} = L \frac{dI}{dt}$

Siccome $f_{em} = -\frac{d\phi_B}{dt}$

$$f_{ind} = -L \frac{dI}{dt}$$

Potenziale Scalare

$$\begin{cases} \vec{\nabla} \wedge \vec{E} = 0 & (1) \\ \nabla E = -\frac{\rho}{\varepsilon_0} & (2) \end{cases} \rightarrow \text{Legge Coulomb} \quad dL = \vec{F} \cdot d\vec{l} = -dU \rightarrow \frac{dL}{q} = \vec{E} \cdot d\vec{l} = -dV$$

$\Rightarrow$ $E \cdot d\vec{l} = -dV$ $\quad$ dV è del tipo $\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$

$$\begin{cases} \nabla V = \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} \hat{k} \\ d\vec{l} = \hat{i}\, dx + \hat{j}\, dy + \hat{k}\, dz \end{cases} \Rightarrow \vec{\nabla} V \cdot d\vec{l} = dV$$

$$\Rightarrow \vec{E} \cdot \cancel{d\vec{l}} = -\vec{\nabla} V \cancel{d\vec{l}} \rightarrow \vec{E} = -\vec{\nabla} V$$

nelle (1) e (2) $\rightarrow$

$$\begin{cases} \vec{\nabla} \wedge (-\vec{\nabla} V) = 0 \rightarrow \text{Sempre Vero} \\ \vec{\nabla}(-\nabla V) = -\frac{\rho}{\varepsilon_0} \rightarrow \vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0} \end{cases}$$

Eq di Poisson

Soluzione $V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{|\vec{r} - \vec{r}'|} dV$

Potenziale vettore

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \end{cases} \quad \text{Ampere} \quad \oint \vec{B}\, d\vec{l} = \mu_0 I$$

Potenziale Vettore: $\vec{B} = \vec{\nabla} \wedge \vec{A}$

$$\begin{cases} \vec{\nabla}\vec{B} = 0 \to \vec{\nabla}(\vec{\nabla}\wedge\vec{A}) = 0 & \to \text{Sempre vera} \\ \vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} \to \vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{A}) = \mu_0\vec{J} & \to \text{Identità} \end{cases}$$

$\to \vec{\nabla}(\vec{\nabla}\vec{A}) - \vec{\nabla}^2 A = \mu_0\vec{J}$ Se $\vec{\nabla}\vec{A} = 0 \to \vec{\nabla}^2\vec{A} = -\mu_0\vec{J}$

$\Rightarrow$ Soluzione $\to \vec{A}(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{J(r)}{|\vec{r}-\vec{r}'|}\,dV$ ma $V = S\cdot l$

$\to dV = S\cdot dl$

$\Rightarrow \vec{A}(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{J\cdot S\cdot}{|r-r'|}\,d\vec{l} = \dfrac{\mu_0}{4\pi}\int \frac{I}{|r-r'|}\,d\vec{l}$

Se $\vec{\nabla}(\vec{\nabla}\vec{A}) \neq 0$ $A' = A + \vec{\nabla}f$

Invarianza di Gauge: $\vec{B} = \vec{\nabla}\wedge\vec{A} = \vec{\nabla}\wedge\vec{A}'$ Proof: $\nabla\wedge A + \overset{0}{\cancel{\vec{\nabla}\wedge(\vec{\nabla}f)}} = \bar{\nabla}\wedge\bar{A}$

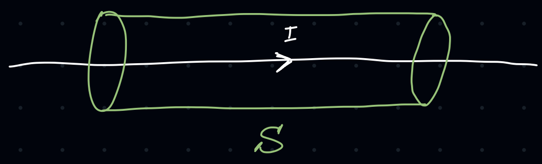
fissiamo una f: $\vec{\nabla}A' = \vec{\nabla}\vec{A} + \nabla^2 f = 0$ Fisso la gauge

$\Rightarrow$ Eq di poisson

Corrente Spostamento

Legge Ampere: $\vec{\nabla} \wedge \vec{B} = \mu_0 J \quad \longleftrightarrow \quad \int \vec{B} \cdot d\vec{e} = \mu_0 I$

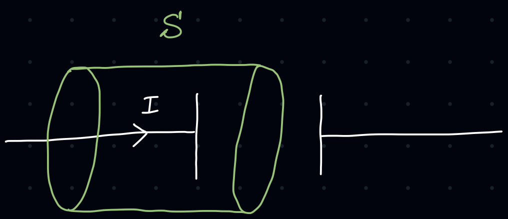
Corrente stazionaria:



$$\phi_E = \oint J \cdot \hat{n} \, ds = \emptyset$$

$$\rightarrow \nabla J = \emptyset$$ Stazionario

Non Stazionario $\rightarrow$ Condensatore



$$\phi = \oint \vec{J} \cdot \hat{n} \, ds \neq \emptyset \, !$$

$\rightarrow \quad \vec{\nabla} \wedge \vec{B} = \mu_0 I$ è incompleto! Infatti Applico la divergenza

$$\vec{\nabla} (\vec{\nabla} \wedge \vec{B}) = \mu_0 \vec{\nabla} J \rightarrow \vec{\nabla} \vec{J} = 0$$ Stazionario! $\rightarrow$ Sbagliato!

infatti $-dq = \int \vec{J} \cdot \hat{n} \, ds \, dt \rightarrow -\dfrac{dq}{dt} = \int J \cdot \hat{n} \, ds$ ma $\rho = \dfrac{dQ}{dV} \rightarrow dq = \rho \, dV$

$$\rightarrow -\frac{d}{dt} \int \rho \, dV = \int \vec{J} \cdot \hat{n} \, ds \rightarrow -\int_V \frac{\partial \rho}{\partial t} dV = \int_S J \cdot \hat{n} \, ds$$

$$\rightarrow -\int \frac{\partial \rho}{\partial t} dV = \int \nabla J \cdot dV \rightarrow \nabla J = -\frac{\partial \rho}{\partial t}$$ NON ZERO!

Termine Mancante: $\mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ Curr Spostam.

Eq Ampére-Maxwell: $\vec{\nabla} \wedge \vec{B} = \mu_0 J + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

Test Div:

$$\nabla (\nabla \wedge B) = \mu_0 \nabla J + \mu_0 \varepsilon_0 \frac{\partial \vec{\nabla} \vec{E}}{\partial t} \rightarrow \mu_0 \vec{\nabla} \vec{J} + \mu_0 \varepsilon_0 \frac{1}{\varepsilon_0} \frac{\partial \rho}{\partial t} = 0$$

$$\rightarrow \mu_0 \left[\vec{\nabla} \vec{J} + \frac{\partial \rho}{\partial t} \right] = 0 \rightarrow \vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t}$$ QED

Modello di Drude

1) Ohm vettoriale

$$\begin{cases} V_A - V_B = R \cdot I \\ V_A - V_B = \int_e \vec{E} \cdot d\vec{e} \end{cases} \Rightarrow R I = \int E \cdot de \qquad \text{ma} \begin{cases} R = \rho \cdot \frac{e}{S} \\ dI = \vec{J} \cdot \hat{n} \, dS \end{cases}$$

$de \cdot \hat{n} = d\vec{e}$

$$\Rightarrow \rho \frac{de}{\cancel{dS}} \cdot \vec{J} \cdot \hat{n} \, \cancel{dS} = \vec{E} \cdot d\vec{e} \to \rho \, d\vec{e} \, \vec{J} = \vec{E} \, d\vec{e} \to \vec{E} = \rho \vec{J}$$

ρ: Resistività

2) Corrente

$$I = \int \vec{J} \cdot \hat{n} \, dS$$

con $\vec{J} = \rho \cdot \vec{v}$ → per non confonderci: $\rho = -e n$

ρ: Densità Vol carica

$$\Rightarrow \underline{J = -e n \vec{v}}$$

A) Presenza di campo elettrico

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q \cdot \vec{E} \to \vec{F} = -e \vec{E}$$

- $E = 0$: $a = 0$, $v = \text{cost}$
- $E \neq 0$: $a \neq 0$, $v \neq \text{cost}$

$$\vec{v} = v_0 + \vec{a} \cdot t$$

$$F = m \cdot a \to \vec{a} = \frac{\vec{F}}{m} \quad \text{ma} \quad F = -e \vec{E} \to \vec{a} = -\frac{e \vec{E}}{m}$$

- $E \neq 0 \to v = v_0 - \frac{e \vec{E}}{m} t$
- $E = 0 \to v = v_0$

B) La media

Quando si lavora con le particelle quantistiche è bene parlare di "media"

- $E = \emptyset$

$$\langle v \rangle = \langle v_0 \rangle - \cancel{\left\langle \frac{e \vec{E}}{m} t \right\rangle} \to \langle v \rangle = \langle v_0 \rangle \qquad E = \emptyset$$

ma se $v_0 \in (+\infty, -\infty) \to$

$\to \underline{\langle v_0 \rangle = \emptyset}$

- $E \neq 0$

$$\langle v \rangle = \underbrace{\langle v_0 \rangle}_{\emptyset} - \langle \underbrace{\frac{e \vec{E}}{m}}_{\text{Cost}} t \rangle = -\frac{e \vec{E}}{m} \langle t \rangle$$

$\langle t \rangle$: Tempo rilassamento medio $\to \tau$

$$\Rightarrow \boxed{\langle v \rangle = -\frac{e \vec{E}}{m} \tau} \quad \text{per } E \neq \emptyset$$

c) Sostituisco:

$$\vec{J} = -env \rightarrow J = -en \cdot \left(-\frac{e\vec{E}}{m}\right)\tau = \frac{ne^2 E}{m}\tau$$

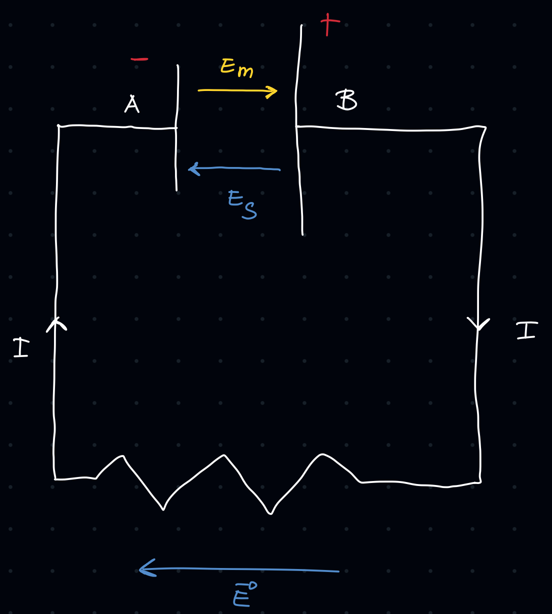
$$\vec{E} = \rho \vec{J} \Rightarrow J = \frac{\vec{E}}{\rho}$$

Unisco: $\frac{\cancel{\vec{E}}}{\rho} = \frac{ne^2 \cancel{\vec{E}}}{m}\tau \rightarrow \tau = \frac{m}{ne^2 \rho}$

Attraverso τ troviamo v e poi S (cammino medio tra uno scontro e l'altro)

S corrisponde proprio al raggio di un atomo $\rightarrow$ Si trova!

Forza E_m



$$L = \int_A^B \vec{F} \cdot d\vec{l} \quad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E$$

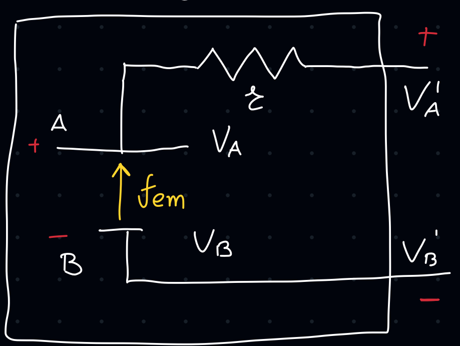
→ Lavoro campo $\mathcal{E}$ per spostare q:

$$\frac{L}{q} = \int_A^B E \cdot dl = f_{em} \neq 0 \,!$$

$$f_{em} = \frac{L}{q} = \int_A^B E_s \, dl + \int_B^A (E_s + E_m) \, dl = \boxed{\oint_B^A \vec{E}_m \, d\vec{l}}$$

fem

Generatore



Potenziale effettivo

$$V'_A - V'_B = r \cdot I \rightarrow V_A - V_B - rI$$

$$\rightarrow V'_A - V'_B = (V_A - V_B) - r \cdot I \qquad \text{ma} \begin{cases} V'_A - V'_B = R \cdot I \\ V_A - V_B = f_{em} \end{cases} \text{e}$$

$$\rightarrow f_{em} - rI = RI \rightarrow f_{em} = I(R + r)$$

$$\Rightarrow I = \frac{f_{em}}{R + r}$$ corrente

Sappiamo che $V_A - V_B = R \cdot I \rightarrow I = \frac{V_A - V_B}{R}$

$$\Rightarrow \frac{V_A - V_B}{R} = \frac{f_{em}}{R + r} \rightarrow \underline{V_A - V_B = \frac{f_{em}}{R + r} \cdot R} \quad (1)$$

$$R = \frac{V_A - V_B}{I} \rightarrow I = \frac{f_{em}}{\frac{V_A - V_B}{I} - r} \rightarrow \frac{V_A - V_B - rI}{I} \rightarrow \cancel{I} = \frac{f_{em}}{V_A - V_B - rI} \cdot \cancel{I}$$

$$\rightarrow \underline{V_A - V_B = f_{em} + rI} \quad (2)$$

(1) e (2) Equivalenti per il seguente discorso

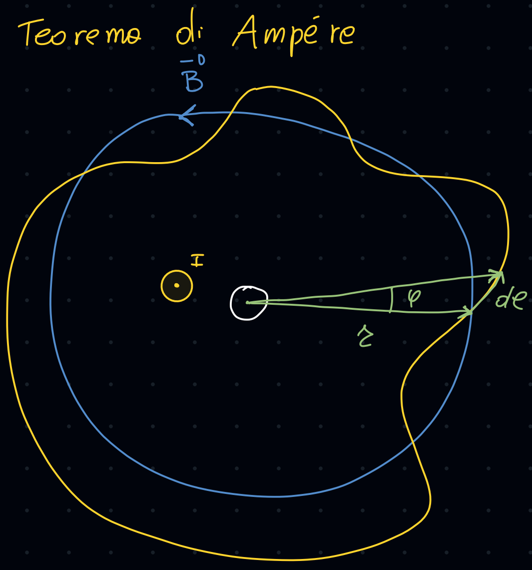
quando $r \rightarrow 0$

$$(1) \quad \Delta V = \frac{\cancel{R}^{\,1}}{\cancel{R + 0}} \cdot f_{em} \rightarrow \Delta V = f_{em}$$

$$(2) \quad \Delta V = f_{em} + 0 \cdot I \rightarrow \Delta V = f_{em}$$

$r = 0 \rightarrow$ Generatore ideale

Teorema di Ampére



Biot-Savart: $\vec{B} = K \cdot \frac{I}{r} = \frac{\mu_0}{2\pi} \frac{I}{r}$

Circuitazione:

$$C = \oint \vec{B} \cdot d\vec{\ell} \qquad 1 \text{ Rad} = \frac{\ell}{R}$$

Arco
Raggio

$$\rightarrow d\varphi = \frac{d\ell}{r} \Rightarrow d\ell = r \cdot d\varphi$$

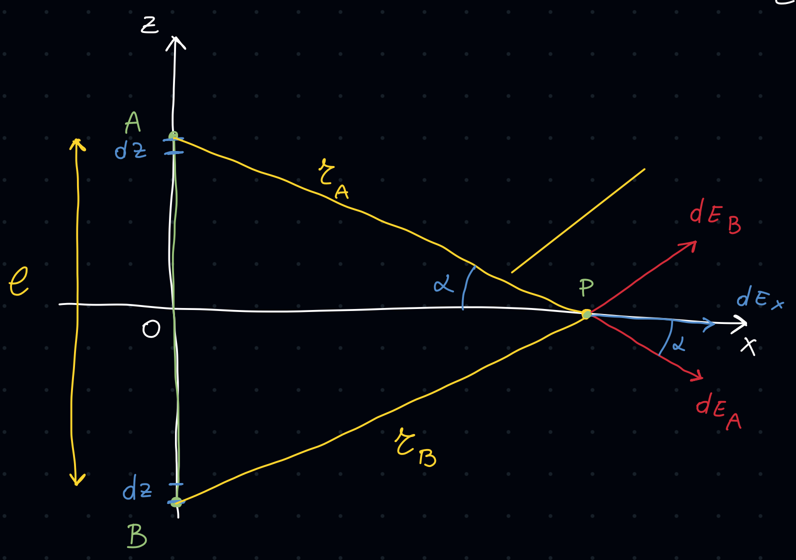
$$\Rightarrow C = \oint \frac{\mu_0}{2\pi} \frac{I}{\cancel{r}} \cdot \cancel{r} \, d\varphi \rightarrow \frac{\mu_0}{2\pi} I \int d\varphi \Rightarrow \frac{\mu_0}{\cancel{2\pi}} I \cdot \cancel{2\pi}$$

$$\rightarrow \int \vec{B} \, d\vec{\ell} = \mu_0 I$$

ma $I = \int J \cdot \hat{n} \, ds \rightarrow \int \vec{B} \, d\vec{\ell} = \mu_0 \int \vec{J} \cdot \hat{n} \, ds \rightarrow \cancel{\int_S} (\vec{\nabla} \wedge \vec{B}) \, \hat{n} \, \cancel{ds} = \mu_0 \cancel{\int} \vec{J} \cdot \hat{n} \, \cancel{ds}$

$$\rightarrow \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$$

Filo carico No Gauss



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{ma} \quad r = ?$$

$$r_A^2 = \underbrace{OA^2}_{z^2} + \underbrace{OP^2}_{x^2} = z^2 + x^2$$

$$\Rightarrow d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{z^2+x^2}\hat{r}$$

Come integro dq?? $\to$ Non posso $\to$ $\lambda = \frac{dq}{dl}$ $\to$ $dq = \lambda\, dl$ nel caso specifico $\downarrow$

$$dq = \lambda\, dz$$

$$\Rightarrow d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,\overbrace{dl \cdot \hat{r}}^{d\vec{z}}}{z^2+x^2}$$

mi serve solo $\vec{E}_z \Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\alpha}{4\pi\varepsilon_0}dz$ ma $\cos\alpha = ??$

$$x = r_A\cos\alpha \to \cos\alpha = \frac{x}{r_A} \Rightarrow \cos\alpha = \frac{x}{\sqrt{z^2+x^2}}$$

$$\to dE_z = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda x}{(x^2+z^2)^{\frac{3}{2}}}dz \to E_z = \frac{\lambda x}{4\pi\varepsilon_0}\overbrace{\int_{-l}^{l}}^{\equiv 2\int_0^l}\frac{1}{(z^2+x^2)^{\frac{3}{2}}}dz$$

$$\Rightarrow E_z = \frac{2\lambda x}{4\pi\varepsilon_0}\left[\frac{1}{x^2}\frac{z}{(x^2+z^2)}\right]_0^l = \frac{2\lambda x}{4\pi\varepsilon_0}\left[\frac{1}{x^2}\cdot\frac{l}{(x^2+l^2)^{\frac{1}{2}}}\right]$$

Siccome $l >> r$ $\to$ $\vec{E} = \frac{2\lambda\cancel{x}}{4\pi\varepsilon_0}\frac{1}{x^{\cancel{2}}}\frac{\cancel{l}}{\cancel{l}} = \frac{\lambda}{2\pi\varepsilon_0 x}\hat{r}$

Eq Maxwell

$$\begin{cases}\vec\nabla\vec E = \frac{\rho}{\varepsilon_0}\\ \vec\nabla\wedge\vec E = \emptyset\end{cases} \to \vec\nabla\vec E = -\frac{\partial\vec B}{\partial t}$$

$$\begin{cases}\vec\nabla\vec B = \emptyset\\ \vec\nabla\wedge\vec B = \mu_0\vec J\end{cases} \to \vec\nabla\wedge\vec B = \mu_0\left[\vec J + \varepsilon_0\frac{\partial\vec E}{\partial t}\right]$$

1)

$$\vec\nabla\wedge\vec E = -\frac{\partial\vec B}{\partial t} \to \text{Rotore} \to \vec\nabla\wedge(\vec\nabla\wedge\vec E) = -\frac{\partial(\vec\nabla\wedge\vec B)}{\partial t}$$

$$\vec\nabla(\vec\nabla E) - \vec\nabla^2 E = -\mu_0\frac{\partial J}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec E}{\partial^2 t}$$ ma $\vec\nabla\vec E = \frac{\rho}{\varepsilon_0}$

$$\to -\vec\nabla\frac{\rho}{\varepsilon_0} - \vec\nabla^2 E = -\mu_0\frac{\partial J}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec E}{\partial t}$$

$$\to \underbrace{\mu_0\varepsilon_0\frac{\partial^2\vec E}{\partial t} - \vec\nabla^2\vec E}_{\text{Campi}} = \underbrace{-\vec\nabla\frac{\rho}{\varepsilon_0} - \mu_0\frac{\partial\vec J}{\partial t}}_{\text{Sorgenti}} \quad (1)$$

2)

$$\vec\nabla\wedge\vec B = \mu_0\vec J + \mu_0\varepsilon_0\frac{\partial\vec E}{\partial t} \to \text{RoTore} \to \vec\nabla\wedge(\vec\nabla\wedge\vec B) = \mu_0\vec\nabla\wedge\vec J + \mu_0\varepsilon_0\frac{\partial(\vec\nabla\wedge\vec E)}{\partial t}$$

$$\to \vec\nabla(\underbrace{\cancel{\vec\nabla\vec B}}_{\emptyset}) - \vec\nabla^2 B = \mu_0\vec\nabla\wedge\vec J - \mu_0\varepsilon_0\frac{\partial^2\vec B}{\partial t}$$

$$\Rightarrow \underbrace{\mu_0\varepsilon_0\frac{\partial^2\vec B}{\partial t^2} - \vec\nabla^2\vec B}_{\text{Campi}} = \underbrace{\mu_0\vec\nabla\wedge\vec J}_{\text{Sorgenti}} \quad (2)$$

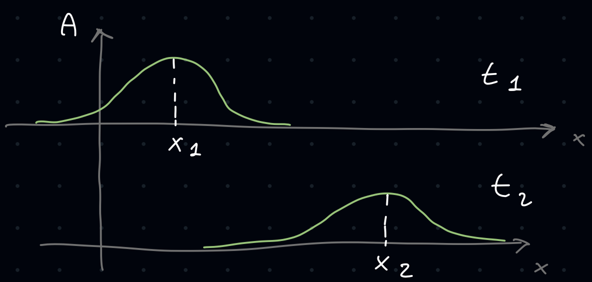
Scegliamo una eq tra (1) e (2)

(1) → Siamo nel vuoto → $\rho = J = 0 \to (1): \mu_0\varepsilon_0\frac{\partial^2\vec E}{\partial t} - \vec\nabla^2\vec E = \emptyset$

$$\to \frac{\partial^2\vec E}{\partial x^2} + \frac{\partial^2\vec E}{\partial y^2} + \frac{\partial^2\vec E}{\partial z^2} - \mu_0\varepsilon_0\frac{\partial^2\vec E}{\partial t} = 0$$ scelgo solo Asse x

$$\to \frac{\partial^2\vec E}{\partial^2 t} - \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t} = 0$$ Eq Differenziale

→ Soluzione : $E(t) = A\,\mathrm{Sin}(Kx - \omega t)$



$$\left.\begin{array}{l}E_1 = A\,\mathrm{Sin}(Kx_1 - \omega t_1)\\ E_2 = A\,\mathrm{Sin}(Kx_2 - \omega t_2)\end{array}\right\} E_1 = E_2 \Leftrightarrow \phi_1 = \phi_2$$

$$\Rightarrow Kx_1 - \omega t_1 = Kx_2 - \omega t_2$$

$$\to \omega(t_2 - t_1) = K(x_2 - x_1) \Rightarrow \omega = K\underbrace{\frac{\Delta x}{\Delta t}}_{v}$$

$$\Rightarrow \omega = Kv \Rightarrow K = \frac{\omega}{v} \Rightarrow \frac{\omega}{K} = v$$

dalla dinamica $\rightarrow$ $x = v \cdot t$ $\Rightarrow$ $\underbrace{\frac{\omega}{k}}_{v} t = x$ $\rightarrow$ $x = v \cdot t$, $v = \frac{\omega}{k}$

$\Rightarrow$ $E = A \sin(kx - \omega t) = A \sin(kx - kvt) = A \sin[k(x - v \cdot t)]$ con $\omega = kv$

Risolviamo l'eq differenziale $\frac{\partial^2 E}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$

$$\frac{\partial E}{\partial x} = kA \cos(kx - kvt) \rightarrow \frac{\partial^2 E}{\partial x^2} = -k^2 A \sin(kx - vkt)$$

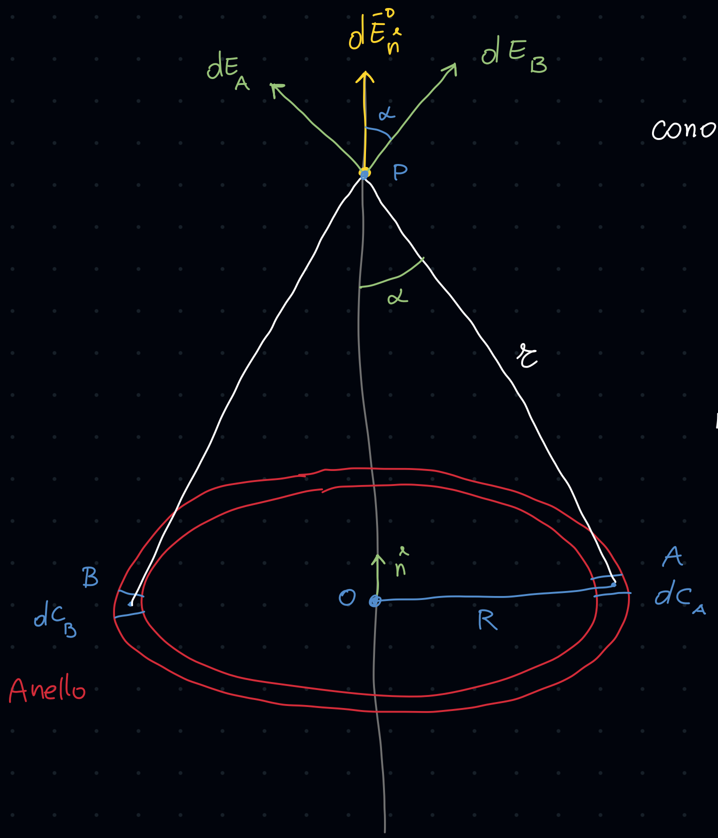
$$\frac{\partial E}{\partial t} = -vkA \cos(kx - kvt) \qquad \frac{\partial^2 E}{\partial t^2} = -v^2 k^2 A \sin(kx - kvt)$$

$\rightarrow$ Sub $\rightarrow$ $-k^2 A \sin(kx - kvt) - \mu_0 \varepsilon_0 v^2 k^2 A \sin(kx - kvt) = \varnothing$

$\rightarrow$ $k^2 A \sin(kx - kvt)\left[1 - \mu_0 \varepsilon_0 v^2\right] = 0$ $\Rightarrow$ $1 - \mu_0 \varepsilon_0 v^2 = 0$

$$\Rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{c}$$

Anello carico



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad \text{ma} \quad r = ??$$

$$\text{conosco} \begin{cases} R & \text{Raggio} \\ z & \text{"Altezza"} \end{cases}$$

$$r^2 = z^2 + R^2 \Rightarrow dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{R^2+z^2}$$

mi serve solo $dE_z = dE \cos\alpha$

$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0} \frac{dq}{R^2+z^2} \cdot \cos\alpha \quad ??$$

$$z = r\cos\alpha \rightarrow \cos\alpha = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

$$\Rightarrow d\vec{E}_{\hat{n}} = \frac{1}{4\pi\varepsilon_0} \frac{z\,dq}{(R^2+z^2)^{\frac{3}{2}}}$$

So che $\lambda = \frac{dq}{dl} = \frac{dq}{dc} \Rightarrow dq = \lambda\, dc$

$$\Rightarrow d\vec{E}_{\hat{n}} = \frac{1}{4\pi\varepsilon_0} \frac{z\cdot\lambda}{(R^2+z^2)^{\frac{3}{2}}}\, dc \rightarrow \vec{E}_{\hat{n}} = \frac{\lambda z}{4\pi\varepsilon_0} \int \underbrace{\frac{1}{(R^2+z^2)^{\frac{3}{2}}}}_{\text{costante}} dc$$

$$\rightarrow \vec{E}_{\hat{n}} = \frac{\lambda z}{\cancel{4\pi}_{2}\,\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}} \cdot \cancel{2\pi} R \rightarrow \vec{E}_{\hat{n}} = \frac{\lambda z \cdot R}{2\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}$$

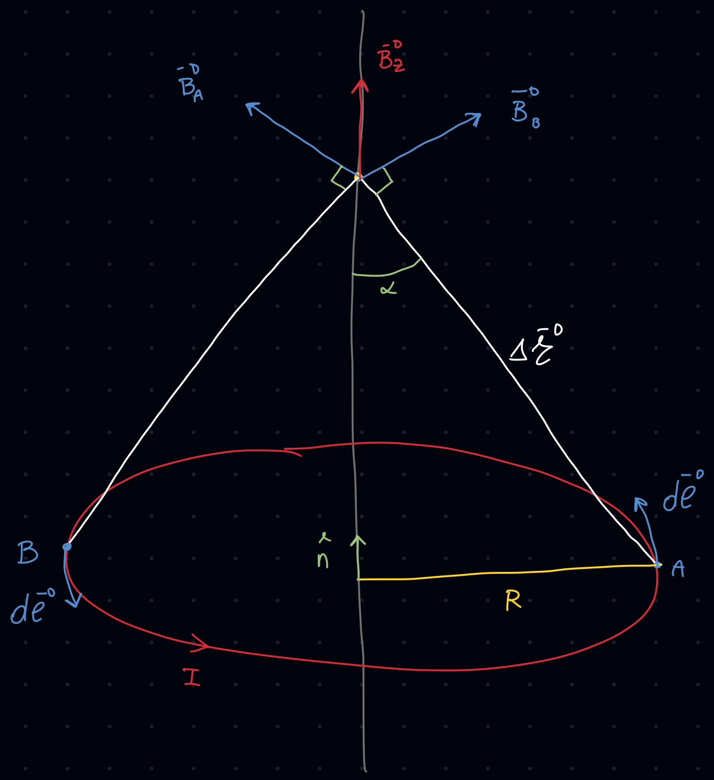
$$\text{ma } \lambda = \frac{Q}{C} \rightarrow \frac{z \cdot \cancel{2\pi} R}{4\pi\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}} \cdot \frac{Q}{\cancel{C}} \rightarrow \vec{E}_{\hat{n}} = \frac{z\cdot Q}{4\pi\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}$$

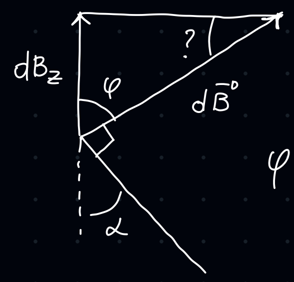
Se $z >> R$

$$\Rightarrow \vec{E}_{\hat{n}} = \frac{\cancel{z}\cdot Q}{4\pi\varepsilon_0 (z^{\cancel{2}})^{\cancel{\frac{3}{2}}\,2}} = \frac{Q}{4\pi\varepsilon_0 z^2}\,\hat{n}$$

Si comporta come una carica puntiforme!

Spira percorsa da corrente





$\varphi = 180 - 90 - \alpha = 90 - \alpha$

$\Rightarrow \; ? = 180 - 90 - \varphi \overset{!}{=} 90 - 90 + \alpha \overset{!}{=} \alpha$

$\Rightarrow$ Legge di Laplace

$$dB = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \wedge \Delta\vec{r}}{\Delta r^3}$$

ci serve solo $dB_z = dB \sin\alpha$

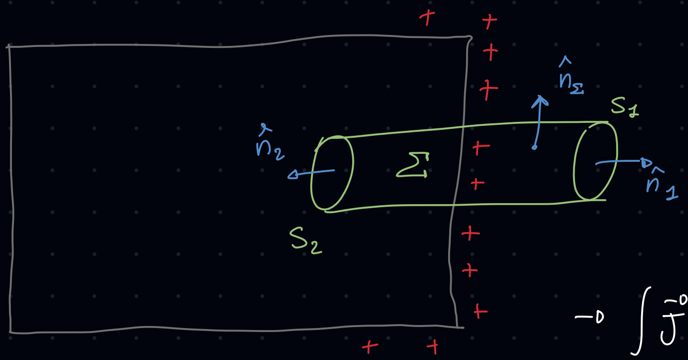
ma $R = \Delta r \sin\alpha \rightarrow \sin\alpha = \dfrac{R}{\Delta r}$

ma $\Delta r^2 = R^2 + z^2 \Rightarrow \sin\alpha = \dfrac{R}{\sqrt{R^2+z^2}}$

$$\Rightarrow dB_z = \frac{\mu_0}{4\pi} I \frac{dl \cdot \cancel{\Delta r} \cdot \sin 90}{\Delta r^{\cancel{3}\,2}} \cdot \sin\alpha = \frac{\mu_0}{4\pi} I \frac{R}{(R^2+z^2)^{\frac{3}{2}}} dl$$

$$\Rightarrow B_z = \frac{\mu_0 I}{4\pi} \frac{R}{(R^2+z^2)^{\frac{3}{2}}} \oint dl = \frac{\mu_0}{\cancel{4\pi}_{\,2}} I \cdot \frac{\cancel{2\pi} R^2}{(R^2+z^2)^{\frac{3}{2}}} = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$$

Teorema di Coulomb



$$\phi_E = \int \vec{E}\cdot\hat{n}\,dS = \frac{Q}{\varepsilon_0}$$

$$\phi_E = \phi_{S_1} + \cancel{\phi_{S_2}}^{\,0} + \cancel{\phi_{\Sigma}}^{\,0}$$

$$= \phi_{S_1}$$

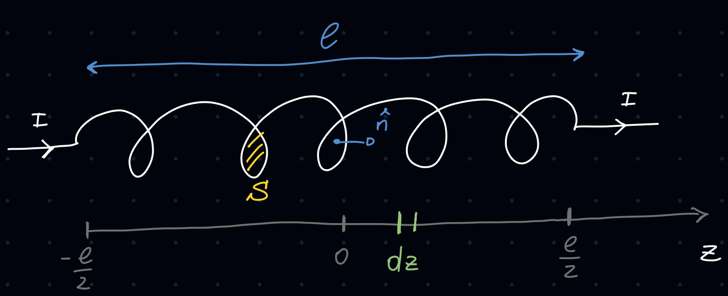
$$\to \int \vec{J}\cdot\hat{n}_1\,dS_1 = \frac{Q}{\varepsilon_0}$$

$$S_1 = \pi R^2 \Rightarrow E\cdot \pi R^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{\pi R^2 \varepsilon_0} \equiv \frac{Q}{S\,\varepsilon_0}$$

ma $\frac{Q}{S} = \sigma$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Campo B Solenoide



Definisco n = Densità di spire per unità len

$$\to n = \frac{N}{\ell}$$ ← numero spire; ← lunghezza

$$\Rightarrow n\,dz = N$$

$$\Rightarrow B_{spira} = \frac{\mu_0 I}{2}\,\frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow B_{Sol} = \frac{\mu_0 I R^2}{2}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{1}{(R^2+z^2)^{\frac{3}{2}}}\,n\,dz = \frac{N}{\ell}\cdot\frac{\mu_0 I R^2}{2}\left[\frac{1}{R^2}\cdot\frac{z}{(R^2+z^2)^{\frac{1}{2}}}\right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}}$$

$$\to \frac{1}{R^2}\cdot\frac{\ell}{2(R^2+\frac{\ell^2}{4})} + \frac{1}{R^2}\cdot\frac{\ell^2}{2\sqrt{R^2+\frac{\ell^2}{4}}} = 2\left[\frac{1}{R^2}\,\frac{\ell}{2(R^2+\frac{\ell^2}{4})}\right]$$

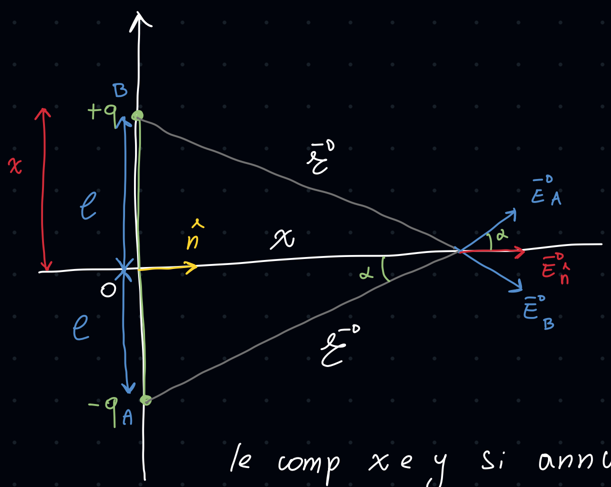
ma $2\left(\frac{4R^2+\ell^2}{4}\right)^{\frac{1}{2}} = \frac{\cancel{2}\,(4R^2+\ell^2)^{\frac{1}{2}}}{\cancel{\sqrt{4}}}$

$$\to B = \frac{N}{\cancel{\ell}}\cdot\frac{\mu_0 I R^{\cancel{2}}}{\cancel{2}}\,\cancel{2}\left[\frac{1}{\cancel{R}}\,\frac{\cancel{\ell}}{(4R^2+\ell^2)^{\frac{1}{2}}}\right]$$

$$= \frac{N\mu_0 I R}{(4R^2+\ell^2)^{\frac{1}{2}}}$$

se $\ell \gg R \to \vec{B} = \frac{N}{\ell}\cdot\mu_0 I R = n\mu_0 I R\,\hat{n}$

CAMPO



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\mathscr{r}^2} \qquad \mathscr{r}^2 = z^2 + x^2$$

$$\rightarrow \; dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{z^2+x^2} \qquad \text{ma} \quad \lambda = \frac{dq}{dl}$$

$$\hookrightarrow dq = \lambda\, dl$$

$$\equiv dq = \lambda\, dz$$

$$\Rightarrow \; dE = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dz}{z^2+x^2}$$

le comp x e y si annullano $\rightarrow \; d\vec{E}_{\hat{n}} = d\vec{E} \cdot \underline{\cos\alpha}$

$$x = \mathscr{r}\cos\alpha \rightarrow \cos\alpha = \frac{x}{\mathscr{r}} \;\Rightarrow\; dE = \frac{1}{4\pi\varepsilon_0} \frac{x\lambda\, dz}{(z^2+x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \vec{E}_n = \frac{x\lambda}{\cancel{4}_{2}\pi\varepsilon_0} \cancel{2}\int_0^l \frac{1}{(z^2+x^2)^{\frac{3}{2}}}\, dz = \frac{x\lambda}{2\pi\varepsilon_0}\left[\frac{1}{x^2} \cdot \frac{z}{(z^2+x^2)^{\frac{1}{2}}}\right]_0^l$$

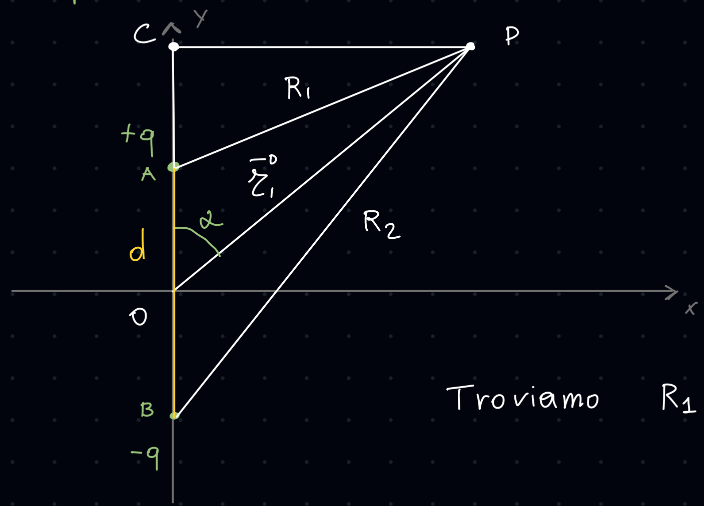
$$\rightarrow E = \frac{\cancel{x}\lambda}{2\pi\varepsilon_0}\left[\frac{1}{x^{\cancel{2}}} \frac{l}{(l^2+x^2)^{\frac{1}{2}}}\right] = \frac{\lambda}{2\pi\varepsilon_0 x} \frac{l}{(l^2+x^2)^{\frac{1}{2}}}$$

Teorema Ampere

Spira $\rightleftharpoons$ Dipolo per Forza, Momento, Campo E/B

Lunga dist

Dipolo



Sappiamo che $\vec{E} = -\vec{\nabla} V$

ma $V = ?$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Troviamo R_1 e R_2

$$R_1^2 = CP^2 + CA^2 = (\vec{r}\cdot\sin\alpha)^2 + \left(CO - \frac{d}{2}\right)^2 = r^2\sin^2\alpha + \left(r\cos\alpha - \frac{d}{2}\right)^2$$

$$= r^2\sin^2\alpha + r^2\cos^2\alpha + \left(\frac{d}{2}\right)^2 - rd\cos\alpha = r^2(\cos^2+\sin^2) + \left(\frac{d}{2}\right)^2 - rd\cos\alpha$$

Hp: $r >> d \Rightarrow \begin{cases} R_1^2 = r^2 - rd\cos\alpha \\ R_2^2 = r^2 + rd\cos\alpha \end{cases} \Rightarrow \begin{cases} R = \sqrt{r(r-d\cos\alpha)} \\ R = \sqrt{r(r+d\cos\alpha)} \end{cases}$

$$\Rightarrow R_1 = \sqrt{r}\underbrace{\sqrt{r-d\cos\alpha}}_{\text{Approx}} \quad \Rightarrow f(d) = (r-d\cos\alpha)^{\frac{1}{2}} \Rightarrow f'(d) = -\frac{1}{2}(r-d\cos\alpha)^{-\frac{1}{2}}\cdot\cos\alpha$$

$$f(0) = \sqrt{r} \quad , \quad f'(0) = -\frac{\cos\alpha}{2\sqrt{r}}$$

$$\Rightarrow R_1 \simeq \cancel{\sqrt{r}}\cdot\left[\overset{r}{\cancel{\sqrt{r}}} - \frac{\cos\alpha}{2\cancel{\sqrt{r}}}d\right] \simeq r - \frac{d\cos\alpha}{2}$$

$$R_2 \simeq r + \frac{d\cos\alpha}{2}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-\frac{d}{2}\cos\alpha} - \frac{1}{r+\frac{d}{2}\cos\alpha}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cancel{r}+\frac{d}{2}\cos\alpha - \cancel{r} + \frac{d}{2}\cos\alpha}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\alpha}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\alpha}{r^2-\left(\frac{d}{2}\right)^2\cos^2\alpha} \quad \text{se } r>>d \Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\alpha}{r^2}$$

Siccome $\vec{E} = -\vec{\nabla} V$

ci interessa $\vec{E}_z = \frac{\partial V}{\partial z}$ $\Rightarrow$ convertiamo in coordinate cartesiane

$$z = r\cos\alpha \Rightarrow \cos\alpha = \frac{z}{r} \quad , \quad r = \sqrt{x+y+z}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{z\cdot d}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{dV}{dz} = k\frac{d}{dz}\left[z\left(x^2+y^2+z^2\right)^{-\frac{3}{2}}\right] = k\cdot\left(x^2+y^2+z^2\right)^{-\frac{3}{2}} - \frac{3}{2}z^2\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$$

$$\to \frac{dV}{dt} = k\left[\left(z^2\right)^{-\frac{3}{2}} - \frac{3z^2}{2\left(z^2\right)^{-\frac{5}{2}}}\right] = k\left[z^{-3} - \frac{3z^2}{2z^5}\right] = k\left[\frac{1}{z^3} - \frac{3z^2}{2z^5}\right]$$

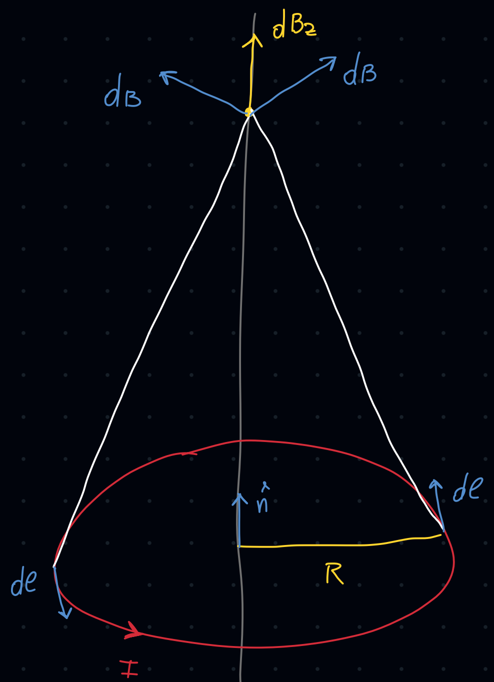
$$\to \frac{dV}{dt} = \frac{k}{z^3}\left[1 - \overset{\cos^2\alpha}{\frac{3z^2}{z^2}}\right] = \frac{qd}{4\pi\varepsilon_0 z^3}\left[1 - 3\cos^2\alpha\right]$$

Siccome $E_z = -\frac{dV}{dx} = qd\frac{(3\cos^2\alpha - 1)}{4\pi\varepsilon_0 z^3}$

Se P è lungo $z \to \alpha = 0 \Rightarrow \cos\alpha = 1 \Rightarrow \vec{E}_z = \frac{q\vec{d}\,\cancel{2}}{\cancel{4}_{2}\pi\varepsilon_0 z^3}$

pongo $q\cdot\vec{d} = \vec{p}$ momento Dipolo $\to$ $\vec{E}_z = \frac{\vec{p}}{2\pi\varepsilon_0 z^3}$ (1)

Spira percorsa da corrente



$$B_z = \hat{n}\,\frac{\mu_0 I R^2}{2\left(z^2+R^2\right)^{\frac{3}{2}}}$$

Se $z \gg R \to B_z = \hat{n}\,\frac{\mu_0 I R^2}{2z^3}$

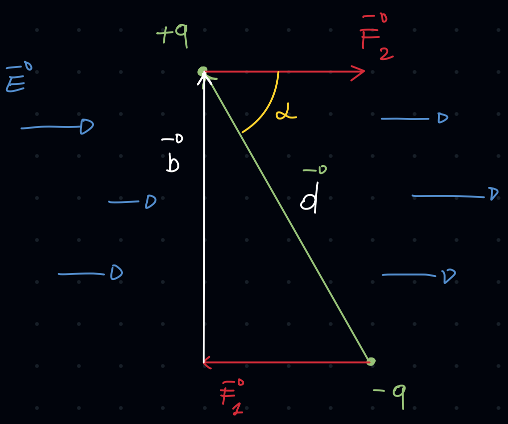
Definisco $\hat{n} I S = \vec{m}$

con $S = \pi R^2 \Rightarrow B_z = \frac{\mu_0 \vec{m}}{2\pi z^3}$ (2)

(1) e (2) Equivalenti

Momento

$\vec F = q\cdot\vec E$; $\vec M = \vec r \wedge \vec F$



$\Rightarrow$ Ci sono 2 Forze

$$\vec M_{TOT} = \vec r_1 \wedge \vec F_1 + \vec r_2 \wedge \vec F_2$$

Scelgo il pivot in $-q$ $\Rightarrow$ $r_1 = \emptyset$

$$\Rightarrow \vec M_{TOT} = \vec d \wedge \vec F_2 = \vec d \wedge q\cdot\vec E$$

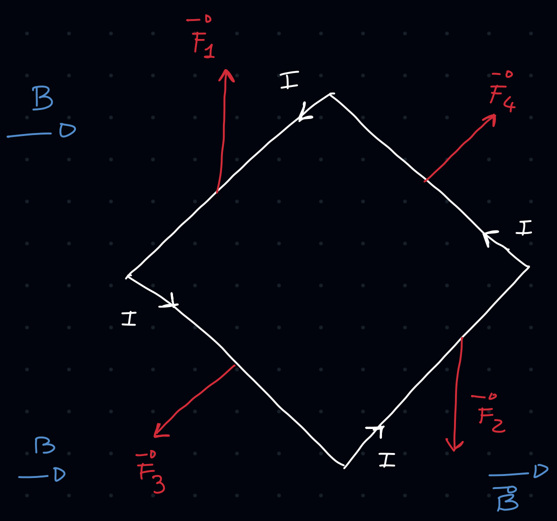
pongo $q\cdot\vec d = \vec p$ $\Rightarrow$ $M_{TOT} = \vec p \wedge \vec E$

Trovo il modulo

$|\vec M| = qd\cdot E\cdot \sin(\alpha)$ ma $d\sin\alpha = b$

$\Rightarrow M = q\cdot b\cdot E \longleftrightarrow q\cdot E = F \Rightarrow \boxed{M = F\cdot b}$ Momento

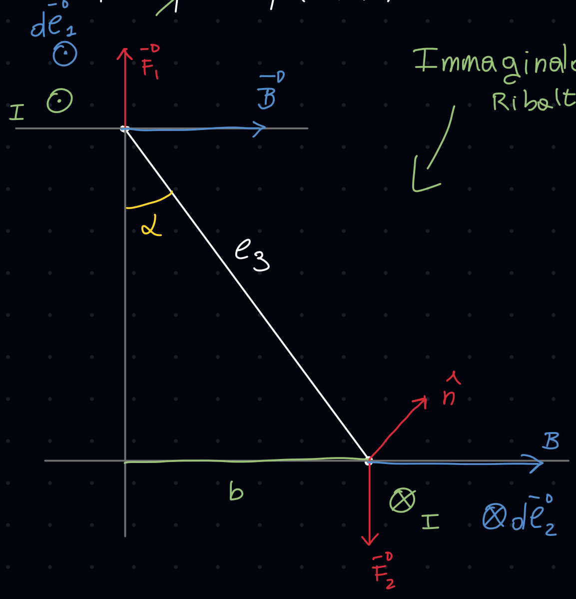
momento spira



$E = 0$

$\vec F = q\cancel{\vec E} + q\cdot(\vec v \wedge \vec B)$

Immaginalo Ribaltato



Sappiamo che $M = F\cdot b$

$F_1 = q(\vec v \wedge \vec B)$ ma $q = ??$

$\Rightarrow \vec F = q\,\frac{d\vec e}{dt}\wedge \vec B \Rightarrow d\vec F = I\, d\vec e \wedge \vec B$

$|\vec F| = I\,eB \Rightarrow M = IBe_1\cdot e_3\cdot \sin\alpha = IBS\sin\alpha$

battezzo $\vec m = IS\hat n \rightarrow \vec M = \vec m\, B\sin\alpha = \vec m \wedge \vec B$

Effetto Joule

Ohm: $\begin{cases} V = R \cdot I \\ R = \rho \cdot \frac{e}{S} \end{cases}$ Esprimo la potenza come $P = \frac{dL}{dt}$

- $L = q \cdot V \rightarrow dL = V dq$
- $P = \frac{dL}{dq} \rightarrow P = \frac{V dq}{dt} \rightarrow P = V \cdot I$
- Da Ohm (1) $\rightarrow V = R \cdot I \rightarrow P = R I^2$

Modello Drude:

1) ...   2) ...   3) ...   4) ...   5) $\tau \rightarrow \frac{1}{\tau}$

Scriviamo Ohm in forma Vett

$$\begin{cases} V = R \cdot I \\ R = \rho \cdot \frac{e}{S} \end{cases} \rightarrow \begin{cases} V = R I \\ V = \int \vec{E} \cdot de \end{cases} \rightarrow R \cdot I = \int \vec{E} d\vec{e}$$

$$R = \rho \cdot \frac{de}{dS} \qquad I = \vec{J} \cdot \hat{n} dS$$

$de \cdot \hat{n} = \cancel{d\vec{e}}$

$$\rightarrow \rho \frac{de}{\cancel{dS}} \cdot J \cdot \hat{n} \cancel{dS} = E \cdot \cancel{d\vec{e}} \rightarrow \rho \vec{J} = \vec{E}$$

inoltre $J = \vec{v} \cdot \rho$ ma scrivo $\rho = -n \cdot e \Rightarrow \vec{J} = -\vec{v} \cdot n e$

Campo E

E Accelera le cariche

$\Rightarrow \vec{E} = 0 \Rightarrow v = v_0$ (1)

$E = \frac{F}{q}$

$\vec{E} \neq 0 \Rightarrow v = v_0 + a \cdot t$ ma $F = m \cdot a \rightarrow a = \frac{F}{m}$ con $F = q \cdot E$

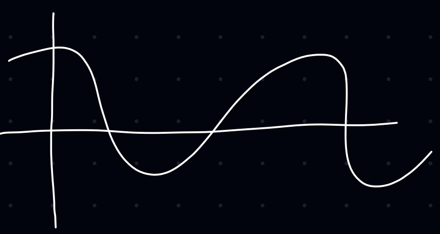
$\Rightarrow F = -eE \rightarrow a = -\frac{e\vec{E}}{m}$

$\Rightarrow v = v_0 - \frac{e\vec{E}}{m} \cdot t$ (2)

Vel media

$E = 0$

dalla (2) $\rightarrow \langle v \rangle = \langle v_0 \rangle - \langle \cancel{\frac{e\vec{E}}{m}} t \rangle \rightarrow \langle v \rangle = \langle v_0 \rangle$



Se $v_0 \in (-\infty, +\infty) \rightarrow \langle v_0 \rangle = 0$

Se $E \neq 0$

$\langle v \rangle = \langle v_0 \rangle - \langle \frac{e\vec{E}}{m} t \rangle = \cancel{\langle v_0 \rangle}^{0} - \frac{e\vec{E}}{m} \langle t \rangle$ τ

$\rightarrow \langle v \rangle = -\frac{eE}{m} \tau$

Tornando a J ed E

$$J = -\vec{v} \cdot n e \rightarrow J = \frac{e \vec{E}}{m} \tau \cdot n \cdot e \rightarrow J = \frac{e^2 E n}{m} \tau$$

$$\vec{E} = \rho \vec{J} \rightarrow \vec{E} = \rho \frac{e^2 \cancel{E} n \tau}{m} \rightarrow \tau = \frac{m}{\rho e^2 n}$$

Impulso

$$I = \int_{t_0}^{t_f} \vec{F} \, dt = \int_{t_0}^{t_f} m \cdot a \, dt = \int_{t_0}^{t_f} m \cdot \frac{dv}{\cancel{dt}} \cancel{dt} = m \int_{v_0}^{v_f} dv = m \vec{v}_f - m \vec{v}_0 = \vec{P}_f - \vec{P}_0$$

Conservazione del momento Angolare e di una forza

$$\begin{cases} \vec{F} = m \cdot \vec{a} \\ M = \frac{d\vec{L}}{dt} \end{cases}$$

(a) $\sum_i \vec{F}_{int} + \sum_i \vec{F}_{ext} = \underline{\sum_i m_i \vec{a}_i}$

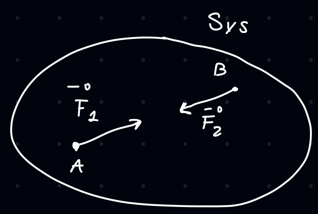
$\hookrightarrow \sum_i m_i \frac{dv}{dt} \rightarrow \sum_i \frac{\vec{P}}{dt} = \frac{d\vec{P}_{TOT}}{dt}$

$$\Rightarrow F = m \cdot a \rightarrow \sum_i F_{int} + \sum_i F_{ext} = \frac{d\vec{P}_{TOT}}{dt}$$

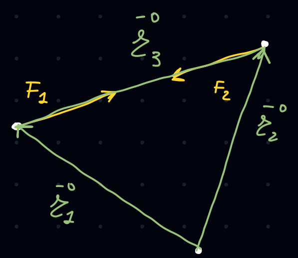
(b) $M = \frac{d\vec{L}}{dt} \rightarrow \sum_i M_{int} + \sum_i M_{ext} = \sum_i \frac{dL}{dt} \quad \frac{dL_{TOT}}{dt}$

Momenti e Forze interni

Se vale il III° principio: $\vec{F}_1 = -\vec{F}_2 \quad \Rightarrow \underline{\sum_i \vec{F}_{int} = 0}$



$$\vec{r}_1 = \vec{r}_2 + \vec{r}_3 \rightarrow \vec{r}_3 = \vec{r}_1 - \vec{r}_2 \quad e \quad \vec{F}_1 = -\vec{F}_2$$



$$\vec{M} = \vec{r} \wedge \vec{F} \Rightarrow M_{TOT} = \vec{r}_1 \wedge \vec{F}_1 + \vec{r}_2 \wedge \vec{F}_2 = \vec{r}_1 \wedge -\vec{F}_2 + \vec{r}_2 \wedge \vec{F}_2$$

$$\rightarrow \vec{M} = (\vec{r}_1 - \vec{r}_2) \wedge \vec{F}_2 = \vec{r}_3 \wedge \vec{F}_2$$

ma $r_3 \parallel F_2 \Rightarrow \underline{\vec{M}_{int} = 0}$

$\Rightarrow$ Riscrivo le eq

$$\begin{cases} \sum_i F_{ext} = \frac{d\vec{P}_{TOT}}{dt} \\ \sum_i M_{ext} = \frac{d\vec{L}}{dt} \end{cases}$$

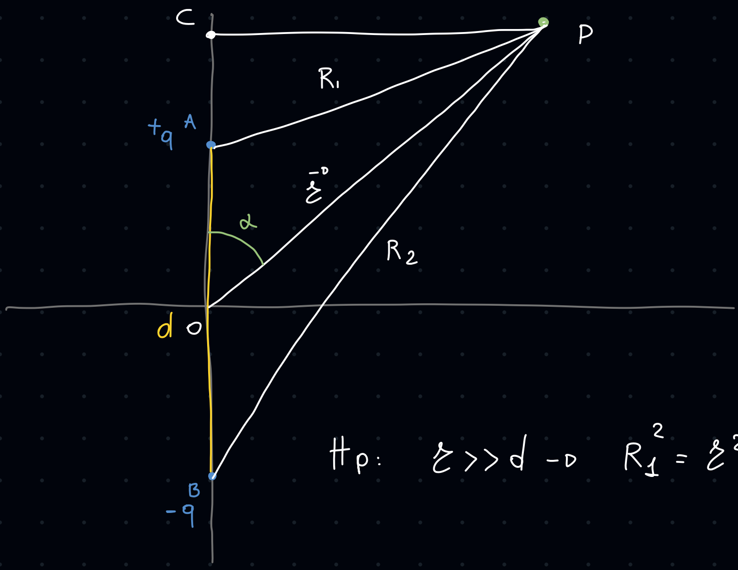
ma se il sys è isolato $\rightarrow \sum_i F_{ext} = \sum_i M_{ext} = 0$

$$\Rightarrow \begin{cases} \frac{d\vec{P}}{dt} = 0 \\ \frac{d\vec{L}}{dt} = 0 \end{cases}$$

Il momento Ang e di inerzia si conservano!

Equivalenza Ampere

Bipolo:



$E = -\vec{\nabla} V$ ma $V = ?$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{ma } R_1, R_2 = ?$$

$$R_1^2 = CP^2 + CO^2 = (r \sin\alpha)^2 + \left(r\cos\alpha - \frac{d}{2}\right)^2$$
$$= r^2\sin^2\alpha + r^2\cos^2\alpha + \left(\frac{d}{2}\right)^2 - r d\cos\alpha$$
$$= r^2(\sin^2 + \cos^2) + \left(\frac{d}{2}\right)^2 - r d \cos\alpha$$
$$= r^2 \left(\frac{d}{2}\right)^2 - r d\cos\alpha$$

Hp: $r >> d \rightarrow R_1^2 = r^2 - r d\cos\alpha \Rightarrow \begin{cases} R_1 = \sqrt{r^2 - r d\cos\alpha} \\ R_2 = \sqrt{r^2 + r d\cos\alpha} \end{cases}$

$$\Rightarrow \begin{cases} R_1 = \sqrt{r(r - d\cos\alpha)} \\ R_2 = \sqrt{r(r + d\cos\alpha)} \end{cases} = \begin{cases} R_1 = \sqrt{r}\,\underline{\sqrt{r - d\cos\alpha}} \\ R_2 = \sqrt{r}\,\underline{\sqrt{r + d\cos\alpha}} \end{cases}$$

Approx:

$$f(d) = (r - d\cos\alpha)^{\frac{1}{2}} \Rightarrow f'(d) = \frac{1}{2}(r - d\cos\alpha)^{-\frac{1}{2}} \cdot (-\cos\alpha) = -\frac{\cos\alpha}{2\sqrt{r - d\cos\alpha}}$$

$$f(0) = \sqrt{r} \qquad f'(0) = -\frac{\cos\alpha}{2\sqrt{r}}$$

$$\Rightarrow R_1 \simeq \sqrt{r}\cdot\left[\sqrt{r} - \frac{d\cos\alpha}{2\sqrt{r}}\right] = r - \frac{1}{2}d\cos\alpha$$
$$R_2 = r + \frac{1}{2}d\cos\alpha$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{r + \frac{d}{2}\cos\alpha - r + \frac{d}{2}\cos\alpha}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\alpha}\right] = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\alpha}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\alpha}$$

Hp $r >> d$ $\quad V = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\alpha}{r^2}$

$E = -\nabla V$ Coord cart: $r^2 = x^2 + y^2 + z^2$ $\quad CO = r\cos\alpha \rightarrow \cos\alpha = \frac{z}{r} = \frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}$

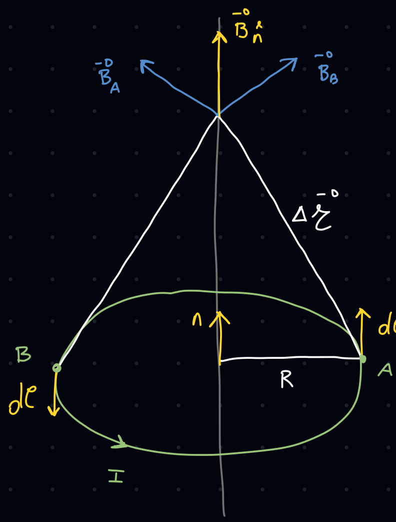
$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cdot z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_z = \frac{\partial V}{\partial x} = \frac{d}{dx}\cdot K\left[(x^2+y^2+z^2)^{-\frac{3}{2}}\cdot z\right] = K\,(x^2+y^2+z^2)^{-\frac{3}{2}} - \frac{3}{\cancel{2}}\cdot\cancel{2}z^2(x^2+y^2+z^2)^{-\frac{5}{2}}$$

$$\rightarrow E_z = K\left[\frac{1}{(r^2)^{\frac{3}{2}}} - \frac{3z^2}{(r^2)^{\frac{5}{2}}}\right] = K\left[\frac{1}{r^3} - \frac{3z^2}{r^5}\right] = \frac{K}{r^3}\left[1 - 3\overbrace{\frac{z^2}{r^2}}^{\cos^2\alpha}\right]$$

$\rightarrow$

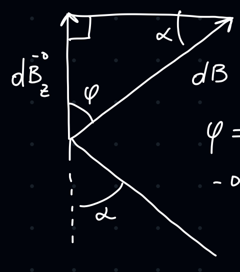
Spira



$$B = K\frac{I}{R} \rightarrow B = \frac{\mu_0}{2\pi}\int \frac{d\vec{l} \wedge \Delta\vec{r}}{\Delta r^3}$$

$\Delta\vec{r} = ?$ $\quad r^2 = R^2 + z^2$

ci serve solo dB_z



$$\left.\begin{aligned} &\varphi = 180 - 90 - \alpha \\ &\rightarrow ? = 180 - 90 - \varphi \\ &\quad = 90 - 90 + \alpha \\ &\quad = \alpha \end{aligned}\right\} \; dB_z = dB \sin\alpha$$

$$\Rightarrow dB_z = \frac{\mu_0}{2\pi} I \frac{dl \wedge \Delta r}{\Delta r^3} \cdot \sin\alpha = \frac{\mu_0}{4\pi} I \frac{dl\,\cancel{\Delta r} \cdot \cancelto{1}{\sin\theta}}{\Delta r^{\cancel{3}\,2}} \sin\alpha = \frac{\mu_0}{4\pi} I \frac{\sin\alpha}{\Delta r^2} dl$$

$$\Rightarrow B_z = \frac{\mu_0}{4\pi} I \frac{\sin\alpha}{\Delta r^2} \oint dl = \frac{\mu_0}{\cancel{4\pi}\;2} I \frac{\sin\alpha}{\Delta r^2} \cancel{2\pi} R$$

ma $\begin{cases} \Delta r^2 = z^2 + R^2 \\ R = \Delta r \sin\alpha \rightarrow \sin\alpha = \dfrac{R}{(R^2+z^2)^{\frac{1}{2}}} \end{cases}$ $\Rightarrow B_z = \mu_0 I \dfrac{R^2}{2\,(R^2+z^2)^{\frac{3}{2}}}$

$\sim 15'$

Teorema Ampere

$$\vec{E}_z = \frac{q}{4\pi\varepsilon_0 r^3}\left[1 - 3\cos^2\alpha\right]$$

se p lungo z $\rightarrow \dfrac{\cancel{2}\, q\vec{d}}{2\,\cancel{4}\pi\varepsilon_0 r^3} = \dfrac{\vec{p}}{2\pi\varepsilon_0 r^3}$

Elettrico

Bip. Magnetico

$$\frac{\mu_0 \vec{m}}{2\pi r^3}$$

equivalenti

$E_z \underset{z \gg R}{=} \dfrac{\mu_0\, I\, R^2}{2\, z^3}$ pongo $\vec{m} = I S \hat{n} \rightarrow I \pi R^2 \Rightarrow E_z = \dfrac{\mu_0\, \vec{m}}{2\pi z^3}$

Maxwell

$$\begin{cases}\vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0} & (1)\\ \vec{\nabla}\wedge\vec{E} = 0 & (2)\end{cases}$$

$$\int E\cdot\hat{n}\,ds = \frac{Q}{\varepsilon_0} \quad \text{ma} \quad \rho = \frac{Q}{V} \Rightarrow \int E\cdot\hat{n}\,ds = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

$$\to \int_V \vec{\nabla}\vec{E}\,dV = \frac{1}{\varepsilon_0}\int \rho\,dV \to \nabla E = \frac{\rho}{\varepsilon_0}$$

$$C = \oint \vec{E}\,d\vec{l} = V_A - V_B \quad \text{ma} \quad V_A = V_B \Rightarrow \oint \vec{E}\,d\vec{l} = 0 \to \int(\vec{\nabla}\wedge\vec{E})\cdot\hat{n}\,ds = 0 \to \vec{\nabla}\wedge\vec{E} = 0$$

$$dL = F\cdot dl \quad \text{ma} \quad E = \frac{F}{q} \to \frac{dL}{q} = E\cdot dl = -\frac{dU}{q} \to \underline{E\,dl = -dV}$$

ma $V(x,y,z) \to dV = \hat{i}\dfrac{\partial V}{\partial x}dx + \hat{j}\dfrac{\partial V}{\partial y}dy + \hat{k}\dfrac{\partial V}{\partial z}dz$

$$\begin{cases} d\vec{l} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz \\ \vec{\nabla}V = \hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y} + \hat{k}\dfrac{\partial V}{\partial z}\end{cases} \to dV = \vec{\nabla}V\cdot d\vec{l} \Rightarrow \vec{E}\cdot\cancel{d\vec{l}} = -\vec{\nabla}V\cdot\cancel{d\vec{l}}$$

$$\to \vec{E} = -\vec{\nabla}V$$

Sostituisco (1) e (2)

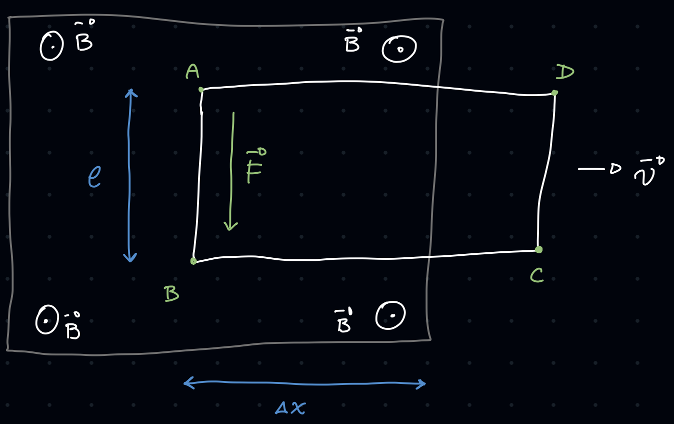
$$\begin{cases}\vec{\nabla}(-\vec{\nabla}V) = \dfrac{\rho}{\varepsilon_0} \to \vec{\nabla}^2 V = -\dfrac{\rho}{\varepsilon_0} \quad \text{Eq di Poisson}\\ \vec{\nabla}\wedge(-\vec{\nabla}V) = 0 \to \text{Sempre vera}\end{cases}$$

Soluzione $\quad V = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV$

Campi magnetici Variabili

Faraday: $f_{em} = -\dfrac{d\phi_B}{dt} \to \phi_B = \displaystyle\int \vec{B}\cdot\hat{n}\,ds = \int \underbrace{B}_{\text{campo}}\cdot\underbrace{\cos\alpha\,ds}_{\text{Superf.}}$

Flusso Tagliato: Varia S



$$\text{Forza} \to \text{Lorentz} \to \vec{F} = \cancel{q\vec{E}} + q\cdot(\vec{v}\wedge\vec{B}) \quad (E=0)$$

$$\underbrace{f_{em}}_{= vBl} = \frac{L}{q} = \frac{q\oint(\vec{v}\wedge\vec{B})\,dl}{q} = \int_A^B (\vec{v}\wedge\vec{B})\,dl_1$$

$$\underline{f_{em} = \int(\vec{v}\wedge\vec{B})\,d\vec{l}} \quad (1)$$

ma Faraday dice $f_{em} = -\dfrac{d\phi_B}{dt}$

$\to$ il flusso dipende dal tempo: $\phi_B = B\cdot S$ ma S è f del Tempo

$\Rightarrow S = b \times h = \ell \cdot x$ ma x dipende da t: $\bar{x} = x - \Delta x$

$\Rightarrow \phi_B = \int \vec{B} \cdot \hat{n}\, ds = \underline{B \cdot \ell \cdot x}$ a $\bar{t} = t + \Delta t \Rightarrow \phi_B = B \cdot \ell \cdot (x - \Delta x)$

$$\Rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \frac{\phi_B(t+\Delta t) - \phi_B(t)}{\Delta t} = \frac{\cancel{B\ell x} - B\ell \overbrace{\Delta x}^{v} - \cancel{B\ell x}}{\Delta t}$$

$$\Rightarrow -\frac{d\phi_B}{dt} = B\ell v$$ QED

Concatenato: Varia $B \Rightarrow B(t, \vec{z})$

$$\frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \frac{\int B(t+\Delta t, \vec{z})\, \hat{n}\, ds - \int B(t, \vec{z})\, \hat{n}\, ds}{\Delta t}$$

Approssimo $B \simeq B(t, \vec{z}) + \frac{\partial B}{\partial t}(t+\Delta t) \cdot \Delta t$

$$\Rightarrow \frac{d\phi}{dt} = \lim_{\Delta t \to 0} = \frac{\cancel{\int B(t, \vec{z})\, d\vec{s}} + \int \frac{\partial B}{\partial t}(t+\Delta t, \vec{z}) \cdot \cancel{\Delta t}\, d\vec{s} - \cancel{\int B(t, \vec{z})\, d\vec{s}}}{\cancel{\Delta t}}$$

$$\Rightarrow \frac{d\phi_B}{dt} = \int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, ds \quad (2)$$

Faraday dice: $f_{em} = -\frac{d\phi}{dt}$ ma $f_{em} = \frac{L}{q}$ $F = q\vec{E} + q(\vec{v} \wedge \vec{B})$

$$\Rightarrow f_{em} = \oint \vec{E}\, d\vec{\ell} + \oint (\vec{v} \wedge \vec{B})\, d\vec{\ell} \quad (3)$$ ma nel flusso concat. $v = 0$

$$\Rightarrow f_{em} = \oint_{\ell} \vec{E} \cdot d\vec{\ell} = -\int_S \frac{\partial B}{\partial t}\, \hat{n}\, ds \Rightarrow \int_S (\vec{\nabla} \wedge \vec{E})\, \hat{n}\, ds = -\int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, ds$$

$$\Rightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

II eq di Maxwell

Completa!

Metto tutto insieme

$$(1)\quad f_{em} = \oint (\vec{v} \wedge \vec{B})\, d\vec{l} = -\frac{d\phi_B}{dt}$$

$$(2)\quad -\frac{d\phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS$$

$$\left.\begin{array}{l}(1)\\(2)\end{array}\right\} \frac{d\phi_B}{dt} = \underline{\oint (\vec{v} \wedge \vec{B})\, d\vec{l} - \int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS}$$

Varia sia B che S

ma $f_{em} = \frac{L}{q} = \oint \vec{E} \cdot d\vec{l} + \oint (\vec{v} \wedge \vec{B})\, d\vec{l}$

$$\Rightarrow \oint \vec{E} \cdot d\vec{l} + \cancel{\oint (\vec{v} \wedge \vec{B})\, d\vec{l}} = \cancel{\oint (\vec{v} \wedge \vec{B})\, d\vec{l}} - \int \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, dS \quad (1)$$

$$\Rightarrow \oint \vec{E}\, d\vec{l} = -\int \frac{\partial B}{\partial t}\, \hat{n}\, dS \Rightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial B}{\partial t}$$

Autoinduzione

$$B = \frac{\mu_0}{4\pi} I \int \frac{d\vec{e} \wedge \Delta \vec{z}}{|\Delta z|^3} \rightarrow \phi_B = \int \underbrace{\frac{\mu_0}{4\pi} I \int \frac{d\vec{e} \wedge \Delta\vec{z}}{|\Delta z|^3}}_{\text{Tutto costante tranne } I} \hat{n}\, ds$$

$\rightarrow \phi_B = L \cdot I$

$\rightarrow$ Se B varia $\rightarrow \frac{d\phi}{dt} = L \cdot \frac{dI}{dt} \rightarrow \underline{f_{em} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}}$

Verso opposto

Circuito RL

Sappiamo che

$\sum_i f_{em} = \sum_i R_i I$ ma L è contato come f_{em}:

$\rightarrow f_{em} - f_{ind} = R \cdot I \rightarrow f_{em} = f_{ind} + RI$

$\rightarrow f_{em} = L\frac{dI}{dt} + R \cdot I$ Eq diff Soluzione: $I(t) = Ae^{\alpha t} + D$

$\rightarrow I'(t) = \alpha A e^{\alpha t} \rightarrow L \cdot \alpha A e^{\alpha t} + RAe^{\alpha t} + RD = f_{em} \rightarrow f_{em} = RD \rightarrow D = \frac{f}{R}$ (1)

inoltre $L\alpha A e^{\alpha t} + RAe^{\alpha t} = 0 \rightarrow Ae^{\alpha t}(L\alpha + R) = 0 \rightarrow \alpha = -\frac{R}{L}$ (2)

$\rightarrow$ Sol gen $I(t) = Ae^{-\frac{R}{L}t} + \frac{f}{R}$ (3)

$I(0) = 0 \rightarrow I(0) = Ae^{-\frac{R}{L}0} + \frac{f}{R} = 0 \rightarrow A = -\frac{f}{R} \rightarrow I(t) = -\frac{f}{R}e^{-\frac{R}{L}t} + \frac{f}{R} = \frac{f}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

per $t \rightarrow \infty$ $\underline{I(t) = \text{cost} = \frac{f}{R}}$

Campo $\vec{E}$

$$\vec{F}_c = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 \cdot q_2}{r^2} \qquad \text{def} \quad E = \frac{\vec{F}_c}{q} \Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$L = \int_A^B \vec{F}_c \cdot d\vec{e} = \frac{1}{4\pi\varepsilon_0}\int_A^B \overbrace{\frac{q_1 \cdot q_2}{r^2}}^{\text{cost}} \underbrace{\hat{r}\, d\vec{e}}_{\hat{r}\cdot d\vec{e} = d\vec{r}} = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0}\int_A^B \frac{1}{r^2}\,dr = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_A^B$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = U_A - U_B$$

$$\Rightarrow \frac{L}{q} = \int E\, de = \frac{U_A - U_B}{q} = V_A - V_B \qquad \text{potenziale elettrico}$$

circuitazione $\quad C = \oint \vec{E}\cdot d\vec{e} = V_A - V_B \quad$ ma $\quad V_A = V_B \Rightarrow \oint_e \vec{E}\cdot d\vec{e} = 0 \rightleftharpoons \underbrace{\vec{\nabla}\wedge\vec{E} = 0}_{\text{Maxwell}}$

ma $\quad \vec{E}\cdot d\vec{e} = -dV \rightarrow \quad dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$

$$\rightarrow \left\{\begin{array}{l} d\vec{e} = dx + dy + dz \\ \vec{\nabla}v = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} \end{array}\right\} \rightarrow dV = \vec{\nabla}v \cdot d\vec{e} \rightarrow \vec{E}\cancel{d\vec{e}} = -\vec{\nabla}v\,\cancel{d\vec{e}}$$

$$\rightarrow \underbrace{\vec{E} = -\vec{\nabla}V}_{} \quad \text{Maxwell}$$

Sup equipot:

Presi due punti di una sup equipot: $\quad V_A = V_B \quad \forall A, B$

$\Rightarrow$ per ogni punto $\rightarrow \quad L = q \cdot V = \emptyset \quad \rightarrow$ non si compie lavoro per spostare cariche

ma $\quad L = \vec{F}\cdot d\vec{e} \quad$ ovvero $\quad L = F \cdot de \cdot \underline{\cos\alpha} \overset{?}{=} \emptyset$

quando $\cos\alpha = 0 \rightarrow \underline{\alpha = 90°}$

$\rightarrow$ Linee di forza $\perp$ allo (spostamento) superficie

Gauss: $\quad \int \vec{E}\cdot\hat{n}\,ds = \frac{Q}{\varepsilon_0} \qquad$ ma $\quad \rho = \frac{Q}{V} \rightarrow \oint \vec{E}\,\hat{n}\,ds = \frac{1}{\varepsilon_0}\int_V \rho\, dV$

$$\rightarrow \left\{\begin{array}{ll} \vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0} & \text{Maxwell} \\ \vec{\nabla}\wedge\vec{E} = 0 \rightleftharpoons \vec{E} = -\vec{\nabla}V \end{array}\right.$$

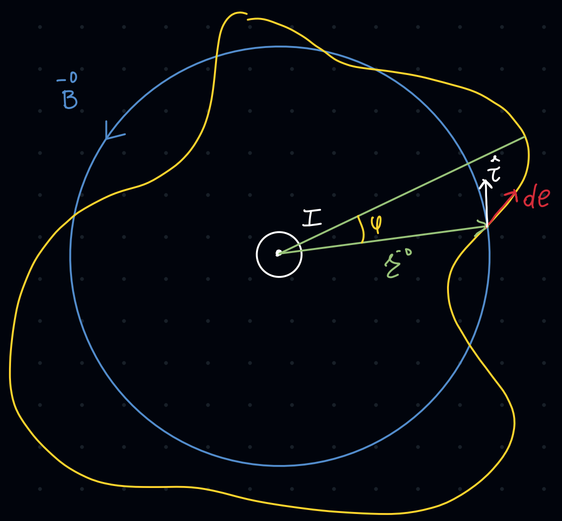
$\rightarrow \vec{\nabla}(-\vec{\nabla}V) = \frac{\rho}{\varepsilon_0} \rightarrow \vec{\nabla}^2\vec{E} = -\frac{\rho}{\varepsilon_0} \qquad$ Poisson

Sol: $\quad E = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r})}{|\vec{r}-\vec{r}'|}\,dV$

$\rightarrow \vec{\nabla}\wedge(-\vec{\nabla}V) = \emptyset \quad$ ✓

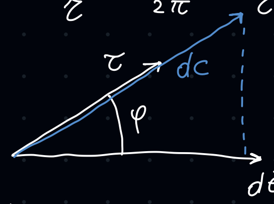
Magnetostatica

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 & \leftarrow \text{Non Esistono Dipoli Magnetici} \\ \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \end{cases} \quad \rightleftarrows \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{Ampère}$$



$$C = \oint \vec{B} \cdot d\vec{l} \qquad \text{ma} \quad B = K \cdot \frac{I}{r} = \frac{\mu_0}{2\pi} \frac{I}{r} \hat{\tau}$$

$$\Rightarrow C = \frac{\mu_0}{2\pi} I \int \frac{1}{r} \hat{\tau} \cdot d\vec{l} =$$

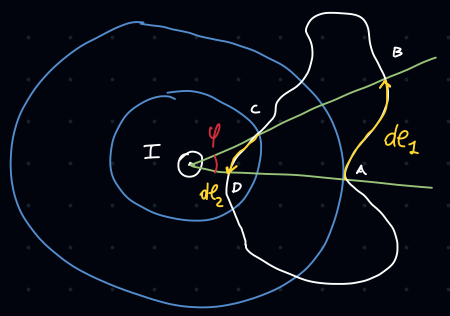


$\tau \, dl \cos\varphi$ = Arco di circonf

$$\Rightarrow \hat{\tau} \cdot d\vec{l} = \ell \quad \Rightarrow \quad 1 \text{ Rad} = \frac{\ell}{R} \quad \Rightarrow \quad \ell = 1 \text{ Rad} \cdot R$$

$$\Rightarrow \hat{\tau} \, d\vec{l} = \varphi \cdot r \quad \Rightarrow \quad C = \frac{\mu_0}{2\pi} I \left(\int \frac{1}{\cancel{r}} \cdot \cancel{r} \cdot d\varphi \right)^{2\pi} = \frac{\mu_0}{2\pi} I \cdot 2\pi = \mu_0 I$$

$$\Rightarrow \oint \vec{B} \, d\vec{l} = \mu_0 I \quad \rightleftarrows \quad \vec{\nabla} \wedge B = \mu_0 \vec{J}$$



$$\Rightarrow C = \oint \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \, dl + \int_B^C B \, dl_2$$

$$\text{Siccome } \oint \vec{B} \, d\vec{l} = |B| \oint d\varphi$$

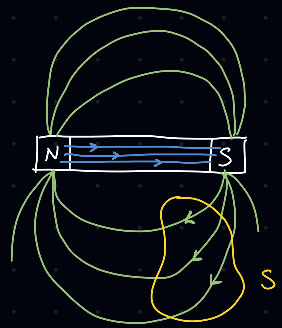
$$\Rightarrow C = |B| \int_A^B d\varphi + \underbrace{|B| \int_C^D d\varphi}_{\text{Stesso Angolo}}$$

ma $\begin{cases} A \to B & \text{ANTIORARIO} \\ C \to D & \text{ORARIO} \end{cases}$

$$\Rightarrow C = \int_A^B d\varphi - \int_C^D d\varphi = \emptyset$$

Maxwell complete:

$$\begin{cases} \vec{\nabla}\vec{B} = \emptyset \\ \vec{\nabla}\wedge\vec{B} = \mu_0\Big(J + \underbrace{\varepsilon_0 \dfrac{\partial \vec{B}}{\partial t}}_{\text{corrente di spostamento}}\Big) \end{cases}$$



$$\phi_B = \oint_S \vec{B}\cdot\hat{n}\,ds = \emptyset$$

$$\rightarrow \quad \underline{\vec{\nabla}\vec{B} = 0}$$

$$\begin{cases} \vec{\nabla}\wedge\vec{E} = -\dfrac{\partial B}{\partial t} & \text{Faraday} \\ \vec{\nabla}\vec{E} = \dfrac{\rho}{\varepsilon_0} & \text{Gauss} \end{cases}$$

$$\begin{cases} \vec{\nabla}\vec{B} = 0 & \text{Monopdi magnetici} \\ \vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0 \dfrac{\partial\vec{E}}{\partial t} & \text{Ampere Maxwell} \end{cases}$$

Maxwell → Onde

$$\begin{cases} \vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0 \dfrac{\partial\vec{B}}{\partial t} \\ f_{em} = -\dfrac{d\phi_B}{dt} \quad \rightarrow \quad \vec{\nabla}\wedge\vec{E} = -\dfrac{\partial B}{\partial t} \end{cases}$$

I) Campo Elettrico

$$\vec{\nabla}\wedge\vec{E} = -\frac{\partial B}{\partial t} \qquad \text{Rotore} \rightarrow \qquad \vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{E}) = -\frac{\partial(\vec{\nabla}\wedge\vec{B})}{\partial t}$$

$$\rightarrow \quad \vec{\nabla}(\vec{\nabla}\vec{E}) - \vec{\nabla}^2\vec{E} = -\frac{\partial(\vec{\nabla}\wedge\vec{B})}{\partial t}$$

$$\rightarrow \quad \vec{\nabla}\left(\frac{\rho}{\varepsilon_0}\right) - \vec{\nabla}^2\vec{E} = -\mu_0\vec{\nabla}\vec{J} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\rightarrow \quad \vec{\nabla}^2\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = \frac{\vec{\nabla}\rho}{\varepsilon_0} + \mu_0\vec{\nabla}\vec{J} \qquad (1)$$

II) Campo B

$$\vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad \rightarrow \text{Rotore} \qquad \vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{B}) = \mu_0(\vec{\nabla}\wedge\vec{J}) + \mu_0\varepsilon_0\frac{\partial(\vec{\nabla}\wedge\vec{E})}{\partial t}$$

$$\rightarrow \quad \underbrace{\cancel{\vec{\nabla}(\vec{\nabla}\vec{B})}}_{0} - \vec{\nabla}^2\vec{B} = \mu_0(\vec{\nabla}\wedge\vec{J}) - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\rightarrow \quad \mu_0(\vec{\nabla}\wedge\vec{J}) = -\mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} + \vec{\nabla}^2\vec{B} \qquad (2)$$

Siamo nel vuoto $\rightarrow$ $\rho = J = 0$

$$\Rightarrow \begin{cases} \vec{\nabla}^2 \vec{E} - \mu_0 \varepsilon_0 \dfrac{\partial^2 \vec{E}}{\partial t^2} = 0 & (1) \\ \vec{\nabla}^2 \vec{B} - \mu_0 \varepsilon_0 \dfrac{\partial^2 E}{\partial t^2} = 0 & (2) \end{cases}$$

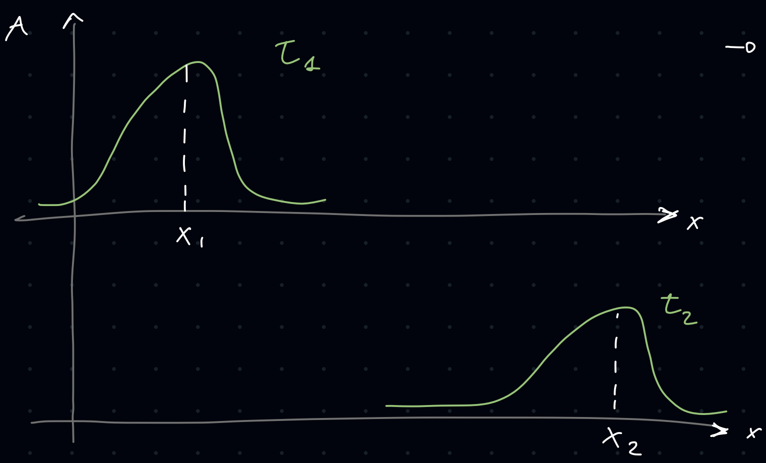
dalla (1) $\rightarrow$

$$\frac{\partial^2 \vec{E}}{\partial x} + \frac{\partial^2 \vec{E}}{\partial y} + \frac{\partial^2 \vec{E}}{\partial z} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

considero solo x

$$\rightarrow \quad \frac{\partial^2 \vec{E}}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (A)$$

Eq diff: sol: $E(t) = A \sin(kx - \omega t)$



$$\rightarrow \quad E_1 = A \sin(kx_1 - \omega t_1)$$
$$E_2 = A \sin(kx_2 - \omega t_2)$$

$$E_1 = E_2 \Leftrightarrow kx_1 - \omega t_1 = kx_2 - \omega t_2$$

$$\rightarrow k(x_2 - x_1) = \omega(t_2 - t_1)$$

$$\Delta x = \underbrace{\frac{\omega}{k}}_{v} \Delta t$$

$$\Rightarrow v = \frac{\omega}{k} \Rightarrow \omega = v \cdot k$$

$$\Rightarrow E(t) = A \sin(kx - \omega t) \rightarrow A \sin[k(x - vt)]$$

Derivate

$$\frac{\partial E}{\partial t} = -vkA\cos(kx - kvt) \qquad \frac{\partial^2 \vec{E}}{\partial t^2} = -v^2 k^2 A \sin(kx - kvt)$$

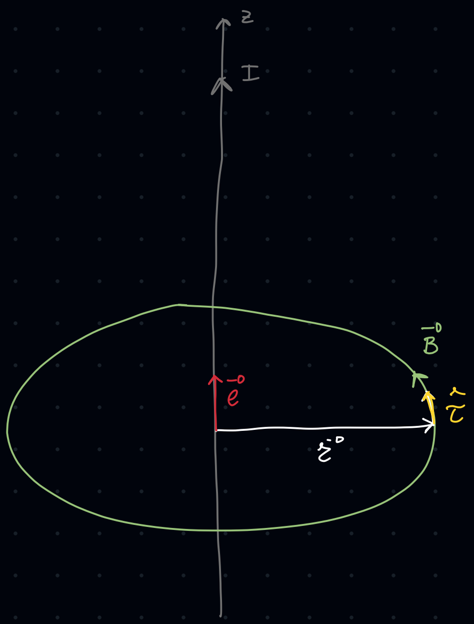
$$\frac{\partial E}{\partial x} = kA\cos(kx - kvt) \qquad \frac{\partial^2 \vec{E}}{\partial x^2} = -k^2 A \sin(kx - kvt)$$

$\Rightarrow$ Eq diff (A) $\uparrow$ $\quad -k^2 A \sin(kx - kvt) + \mu_0 \varepsilon_0 v^2 k^2 A \sin(kx - kvt) = 0$

$$\rightarrow k^2 A \sin(kx - kvt)\left[-1 + \mu_0 \varepsilon_0 v^2\right] = 0 \quad \Leftarrow \quad \mu_0 \varepsilon_0 v^2 - 1 = 0 \quad \text{per} \quad v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

Biot Savart

$$B = K \cdot \frac{I}{r} = \frac{\mu_0}{2\pi}\frac{I}{r}$$



per ottenere $\hat{B}: \hat{e} \wedge \hat{r} = \frac{\vec{e} \wedge \vec{r}}{r}$ (a)

Lorenz: $\vec{F} = q \cdot (\vec{v} \wedge \vec{B}) \rightarrow F = q \cdot \frac{d\vec{e}}{dt} \wedge \vec{B}$

$\rightarrow \underline{d\vec{F} = I \, d\vec{e} \wedge \vec{B}}$ Cariche elettriche

II Biot Savart

Carica magnetica

$$\vec{F} = \frac{\mu_0}{4\pi}\frac{q_m \cdot q_m'}{r^2}\hat{r} \rightarrow B = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q \cdot \vec{B}$$

$$\rightarrow d\vec{F} = \vec{B} \, dq$$

$$\Rightarrow \begin{cases} d\vec{F} = I \, d\vec{e} \wedge \vec{B} \\ d\vec{F} = \vec{B} \cdot dq \end{cases} \Rightarrow \underline{dq = I \cdot d\vec{e}} \quad (1)$$

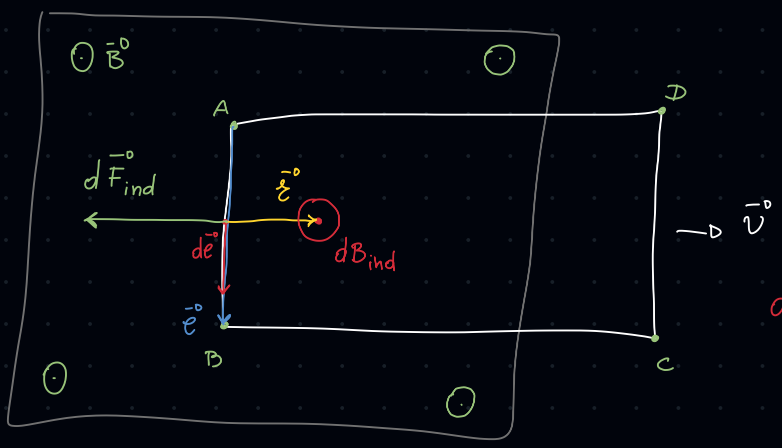
Siccome $\vec{B} = \frac{\vec{F}}{q} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2} \rightarrow B = \frac{\mu_0}{4\pi}\frac{I \cdot de}{r^2}$ vettore? (a) ↑

$$\rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{I \, d\vec{e} \wedge \vec{r}}{r^3}$$

Biot-Savart I

Legge di Lenz

$$f_{em} = \ominus \frac{d\phi}{dt}$$

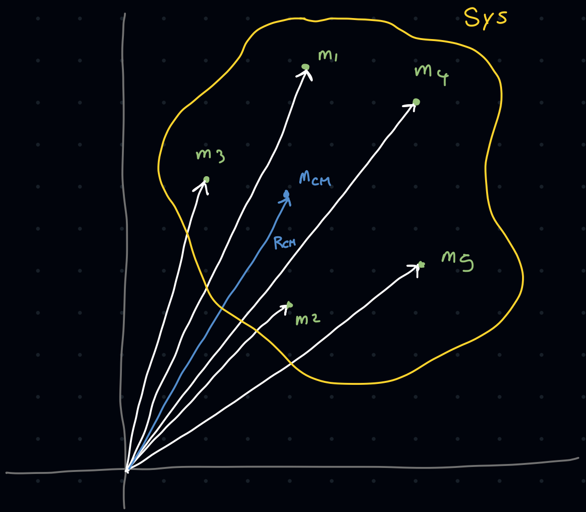


$$F = q\,(\vec v \wedge \vec B)$$

$$d\vec B_{ind} = \frac{\mu_0 I}{4\pi} \frac{d\vec e \wedge \vec r}{r^3} \rightarrow \text{USCENTE}$$

da Faraday $\vec F_{ind} = i\, d\vec e \wedge \vec r$

Centro di Massa



$$\vec R_{CM} = \frac{\sum_i m_i \vec r_i}{\sum_i m_i} = \frac{\sum_i m_i r_i}{M}$$

$$\Rightarrow v = \frac{d\vec s}{dt} = \frac{d\vec R_{CM}}{dt} = \frac{\sum_i m_i v_i}{M} \qquad m \cdot v = \vec p$$

$$\Rightarrow \vec v_{CM} = \frac{\vec p_{TOT}}{M} \Rightarrow \underline{\vec p_{TOT} = \vec v_{CM} \cdot M}$$

$$\vec a = \frac{d\vec v}{dt} = \frac{dv_{CM}}{dt} = \frac{\sum_i m_i \vec a_i}{M} \Rightarrow \vec a_{CM} = \frac{\sum_i \vec F_i}{M}$$

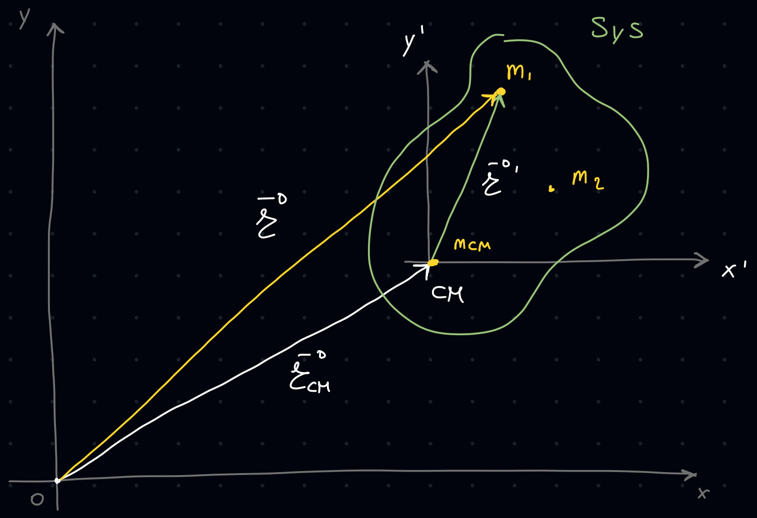
$$\Rightarrow \underline{\sum_i \vec F_i = M \cdot \vec a_{CM}}$$

$$\begin{cases} \vec L = \vec r \wedge m_i \vec v = \vec r \wedge \vec p \\ \vec M = \vec r \wedge \vec F \end{cases}$$

$$\Rightarrow \frac{d\vec L}{dt} = \frac{d\vec r}{dt} \wedge m \vec v + \vec r \wedge m \frac{d\vec v}{dt} = \cancel{\vec v \wedge m \vec v} + \overbrace{\vec r \wedge m \cdot \vec a}^{\vec M}$$

$$\Rightarrow \vec M = \frac{d\vec L}{dt}$$

Teorema di Koenig



$\vec{r} = \vec{r}_{CM} + \vec{r}' \Rightarrow \vec{v} = \vec{v}_{CM} + \vec{v}'$

Siccome $\vec{L} = \vec{r} \wedge m\vec{v}$

$$\Rightarrow \vec{L} = \sum_i \vec{r}_i \wedge m\vec{v}_i = \sum_i \left[\vec{r}_{CM} + \vec{r}'\right] \wedge \left[\vec{v}_{CM} + \vec{v}'\right] = \sum_i \vec{r}_{CM} \wedge m_i \vec{v}_{CM} + \sum_i \vec{r}_{CM} \wedge m_i \vec{v}_i' + \sum_i \vec{r}_i' \wedge m_i \vec{v}_{CM} + \sum_i \vec{r}_i' \wedge m_i \vec{v}_i'$$

$$= \vec{r}_{CM} \wedge M\vec{v}_{CM} + \vec{r}_{CM} \wedge \sum_i m_i \vec{v}_i' + \sum_i m_i \vec{r}_i' \wedge \vec{v}_{CM} + \sum_i \vec{r}_i' \wedge m_i \vec{v}_i'$$

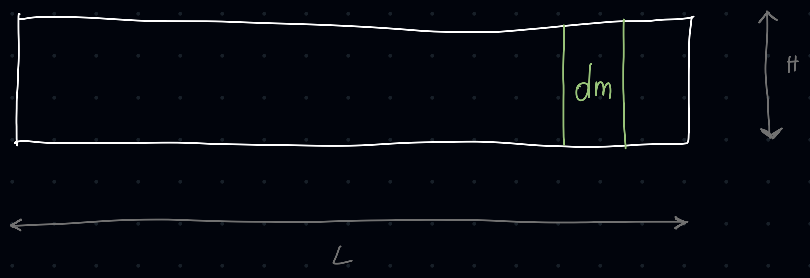
Dal CM $\quad R_{CM} = \frac{\sum_i m_i r_i}{M} \Rightarrow v_{CM} = \frac{\sum_i m_i v_i}{M} = \frac{\vec{P}_{TOT}}{M}$

- $\sum_i m_i \vec{v}_i' = M \cdot \vec{v}_{CM}'$ vel del CM dal CM $= 0$
- $\sum_i m_i \vec{r}_i' = M \cdot \vec{r}_{CM}'$ pos del CM dal CM $= 0$

$$\Rightarrow \vec{L}_{TOT} = \vec{r}_{CM} \wedge M\vec{v}_{CM} + \sum_i \vec{r}_i \wedge m_i \vec{v}_i \Rightarrow \vec{L}_{TOT} = \vec{L}_{CM} + \vec{L}'_{TOT}$$

Momento Inerzia

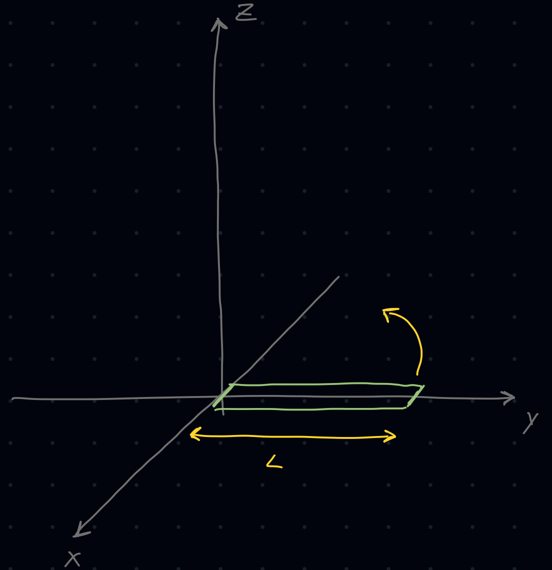
$I = m \cdot R^2$



$\Sigma_i\, m = dm$

$\Rightarrow \int R^2 dm$ moltiplico per il volume $\Rightarrow I_{TOT} \int R^2 \frac{dm}{dV} dV$ pongo $\rho = \frac{M}{V}$

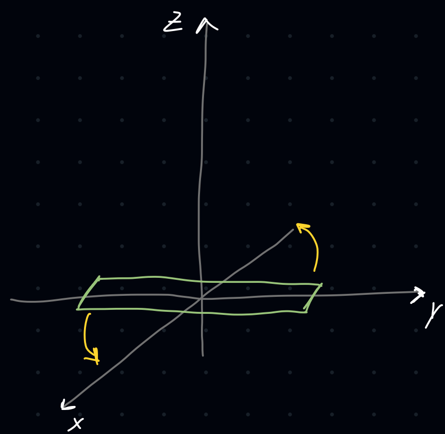
$\Rightarrow I_{TOT} = \int_V R^2 \rho \, dV$ sbarra 2D $\Rightarrow I_{TOT} = \rho \int x^2 dx$



$I_{TOT} = \rho \int y^2 dy$ con $\rho = \frac{M}{L}$

$\Rightarrow I = \frac{M}{L} \int y^2 dy = \frac{M}{L}\left[\frac{y^3}{3}\right]_0^L = \frac{M}{\cancel{L}} \frac{L^{\cancel{3}\,2}}{2}$

$\Rightarrow I = \frac{1}{3} M L^2$

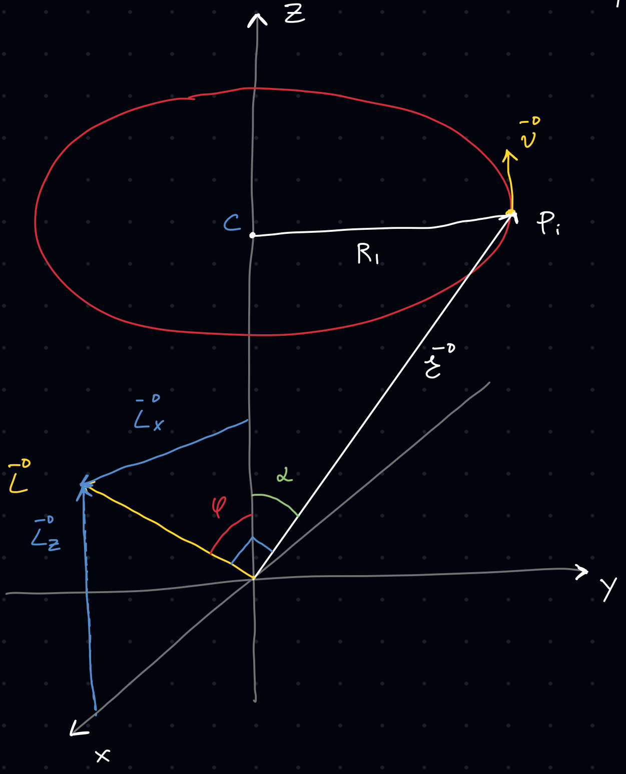


$I_{TOT} = \frac{M}{L} \int_{-\frac{e}{2}}^{\frac{e}{2}} y^2 dy = \frac{M}{L}\left[\frac{y^3}{3}\right]_{-\frac{e}{2}}^{\frac{e}{2}}$

$= \frac{M}{L}\left[\frac{e^3}{\cancel{8}_4 \cdot 3} + \frac{e^3}{\cancel{8}_4 \cdot 3}\right] = \frac{M}{\cancel{L}} \cdot \frac{L^{\cancel{3}}}{12}$

$\Rightarrow I = \frac{M L^2}{12}$

Rotazione Asse fisso



Momento Angolare

$$\vec{L} = \vec{r} \wedge m \cdot \vec{v}$$

$\nabla \quad \vec{r} = ?$

conosco $\begin{cases} z : \overline{OC} \\ R_i : \text{Raggio} \\ \alpha : \text{Angolo } Z\hat{O}P_i \end{cases}$

$$\Rightarrow \vec{r}^{\,2} = z^2 + R_i^2$$

$$\to \vec{L}_i = \vec{r}_i \wedge m_i \vec{v}_i \to L_i = r m v \overset{1}{\cancel{\sin(90)}}$$

Le componenti x ed y si Annullano

$$\to \vec{L}_z = \vec{L} \cos\varphi$$

$$\varphi = 90 - \alpha \quad \Rightarrow \quad \cos(90-\alpha) = \sin(\alpha) \to \quad \vec{L}_z = \vec{L} \sin\alpha$$

$$\Rightarrow L_z = r m v \sin\alpha \quad \text{ma} \quad \underbrace{r \sin\alpha = R}_{\text{Trig}} \quad e \quad \underbrace{v = \omega R}_{\text{Moto Circolare}}$$

$$\to L_{z_i} = m_i \omega_i R_i^2 \quad \text{ma} \quad \omega_i = \omega \quad \Rightarrow L_{z_i} = \omega \left(\Sigma_i \, m_i R_i^2 \right) \quad I\text{nerzia}$$

$$\to L_{z_i} = \vec{\omega} I \qquad \text{Siccome} \qquad \vec{M} = \frac{dL}{dt}$$

$$\Rightarrow \vec{M} = I \frac{d\vec{\omega}}{dt} \to \vec{M} = \alpha I \quad \text{Equivalente a} \quad \vec{F} = m \cdot \vec{a}\ !$$

Se $\vec{M} = \alpha I \to \alpha = \dfrac{M}{I} \Rightarrow$

$$\begin{cases} \theta = \theta_0 + \int_0^t \omega \, dt \\ \omega = \omega_0 + \int_0^t \alpha \, dt \end{cases}$$

Se $M = 0 \to \alpha = 0 \Rightarrow \begin{cases} \text{Fermo} \\ v = \text{cost} \end{cases}$

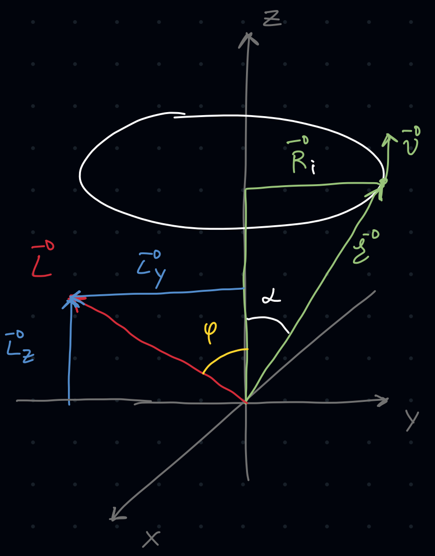
$$\to \theta = \theta_0 + \omega t \rightleftharpoons S(t) = S_0 + v \cdot t$$

Se $M \neq 0$ ma cost

$$\to \theta = \theta_0 + \omega t + \tfrac{1}{2} \alpha t^2 \rightleftharpoons S(t) = S_0 + vt + \tfrac{1}{2} a t^2$$

Energia

$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} m \omega^2 R^2 = \tfrac{1}{2} I \omega^2$$



$\vec{L} = \vec{r} \wedge m\vec{v} \Rightarrow |L_i| = r_i m_i v_i \sin(90)$ ($\sin(90) = 1$)

L_y ed L_x si annullano $\rightarrow$ $\vec{L}_{z_i} = \vec{L}_i \cdot \cos(\varphi)$

ma $\varphi = 90 - \alpha \rightarrow \cos(90-\alpha) = \sin(\alpha)$

$\Rightarrow \vec{L}_{z_i} = L_i \sin(\alpha) = r_i m_i v_i \sin\alpha$

ma $r_i \sin\alpha = \vec{R}_i$

$\Rightarrow L_{L_i} = m_i v_i \vec{R}_i$ ma $\vec{v} = \vec{\omega} R$

$\Rightarrow L_{z_i} = m_i \omega_i R_i^2 \quad \omega_i = \omega \rightarrow \quad L_{z_i} = \omega\, m_i R_i^2$

$\Sigma_i m_i R_i^2 = I \Rightarrow L_z = \omega I$

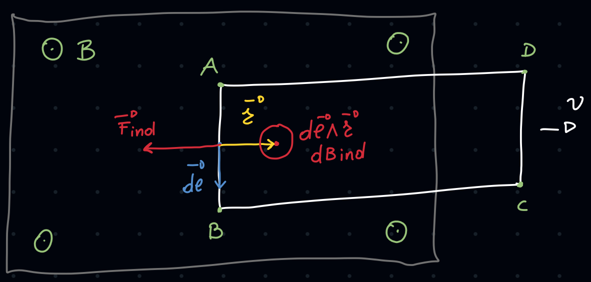
dalla II eq cardinale

$\vec{M} = \frac{d\vec{L}}{dt} \rightarrow \vec{M} = I \frac{d\omega}{dt} = \alpha I \Rightarrow \alpha = \frac{\vec{M}}{I} \equiv \vec{F} = m \cdot a$

$$\begin{cases} \theta(t) = \theta_0 + \int_0^t \omega\, dt \\ \omega(t) = \omega_0 + \int_0^t \alpha\, dt \end{cases}$$

$M = 0 \rightarrow \theta(t) = \theta_0 + \omega t$

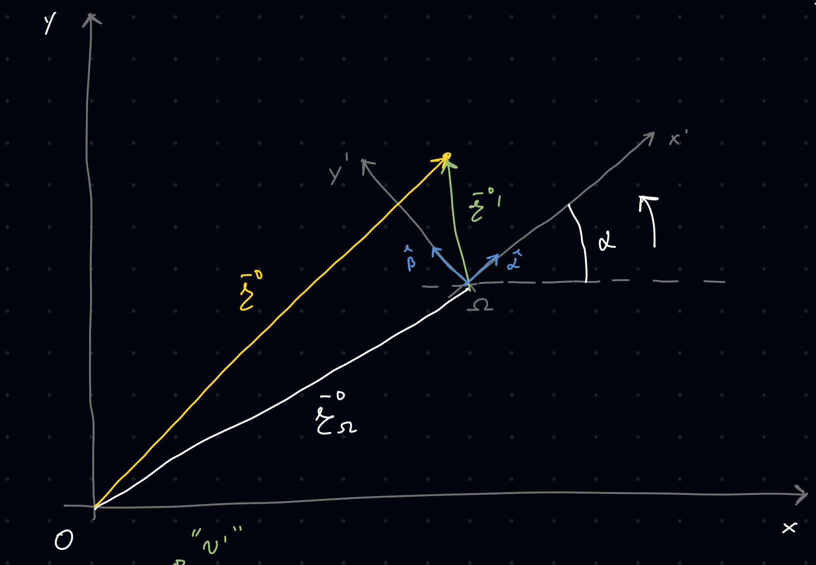
$M \neq 0 \rightarrow \begin{cases} \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega(t) = \omega_0 + \alpha t \end{cases}$



$dB_{ind} = \frac{\mu_0}{2\pi} \cdot \frac{de \wedge \vec{r}}{r^3}$

Faraday: $\vec{F}_{ind} = I\, d\vec{e} \wedge d\vec{B}$

Dinamica Sys non inerziali



$$\vec{r} = \vec{r}_\Omega + \vec{r}' \Rightarrow v = \frac{d\vec{r}_\Omega}{dt} + \frac{d\vec{r}'}{dt}$$

$$\Rightarrow \vec{v} = \vec{v}_\Omega + \vec{v}'$$

VERSORI

$$\begin{cases} \vec{r}' = \hat\alpha x' + \hat\beta y' \\ \vec{r} = \hat i x + \hat j y \end{cases} \Rightarrow \hat\alpha \text{ e } \hat\beta \neq \text{cost}$$

"v'" ~>

$$\Rightarrow \frac{d\vec{r}'}{dt} = \frac{d\alpha}{dt}x' + \alpha\frac{dx'}{dt} + \frac{d\hat\beta}{dt}y' + \hat\beta\frac{dy'}{dt} = \left(\underbrace{\frac{d\hat\alpha}{dt}}_{\omega\wedge\hat\alpha}x' + \frac{d\hat\beta}{dt}y'\right) + \underbrace{\left(\hat\alpha\frac{dx'}{dt} + \hat\beta\frac{dy'}{dt}\right)}_{\vec{v}'}$$

$$\Rightarrow \vec{v}' + (\omega\wedge\hat\alpha)x' + (\omega\wedge\hat\beta)y' = \vec{v}' + \omega\wedge\underbrace{(\hat\alpha x' + \hat\beta y)}_{\vec{r}'} = \vec{v}' + \omega\wedge\vec{r}' \quad (1)$$

$$\Rightarrow \frac{d\vec{r}}{dt} = \vec{v}_\Omega + \vec{v}' + \omega\wedge\vec{r}'$$ Vel di P da O

$$\vec{a} = \frac{d^2\vec{r}}{dt} = \frac{d\vec{v}_\Omega}{dt} + \frac{d\vec{v}'}{dt} + \frac{d\vec{\omega}}{dt}\wedge\vec{r}' + \omega\wedge\frac{d\vec{r}'}{dt}$$

$$\vec{v}' = \left(\hat\alpha\frac{dx'}{dt} + \hat\beta\frac{dy'}{dt}\right) \Rightarrow \frac{d\vec{v}'}{dt} = \underbrace{\left(\frac{d\hat\alpha}{dt}\frac{dx'}{dt} + \frac{d\hat\beta}{dt}\frac{dy'}{dt}\right)}_{(a)} + \underbrace{\left(\hat\alpha\frac{d^2x'}{dt} + \hat\beta\frac{d^2y'}{dt}\right)}_{\vec{a}'}$$

(a) $(\omega\wedge\hat\alpha)\frac{dx'}{dt} + (\omega\wedge\hat\beta)\frac{dy'}{dt} = \omega\wedge\left(\hat\alpha\frac{dx'}{dt} + \hat\beta\frac{dy'}{dt}\right) = \omega\wedge v'$

$$\Rightarrow \frac{d\vec{v}'}{dt} = (\omega\wedge v') + \vec{a}'$$

$$\Rightarrow \frac{d^2\vec{r}}{dt^2} = \vec{a}_\Omega + (\vec{\omega}\wedge\vec{v}') + \vec{a}' + \frac{d\vec{\omega}}{dt}\wedge\vec{r}' + \vec{\omega}\wedge\left[\vec{v}' + (\vec{\omega}\wedge\vec{r}')\right]$$

$$= \vec{a}_\Omega + \underbrace{2(\omega\wedge\vec{v}')}_{\text{Acc Coriolis}} + \vec{a}' + \frac{d\vec{\omega}}{dt}\wedge\vec{r}' + \underbrace{\vec{\omega}\wedge(\vec{\omega}\wedge\vec{r})}_{\to\, -\omega^2\vec{r}}$$

Chiamo $\vec{a}_T = \vec{a}_\Omega + \frac{d\vec{\omega}}{dt}\wedge\vec{r} - \omega^2\vec{r}$ ← Centrifuga

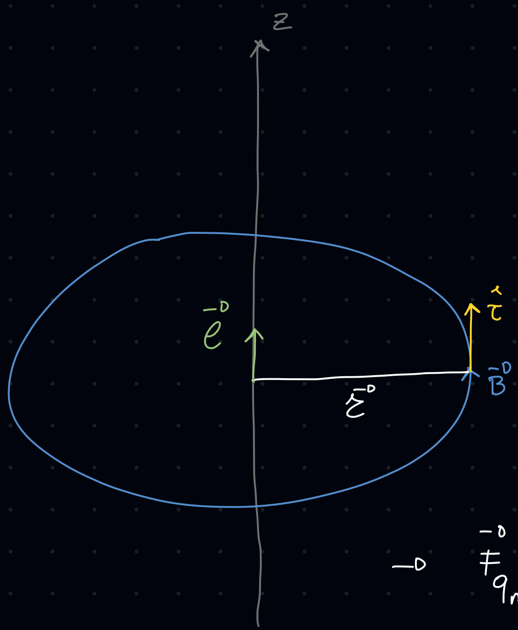
$$\Rightarrow \vec{a} = \vec{a}' + \vec{a}_{co} + \vec{a}_{TR}$$

Considero $\to \vec{F} = m \cdot \vec{a} \to \vec{F} = m\left(\vec{a}' + \vec{a}_{co} + \vec{a}_{TR}\right) \to \vec{F} \underbrace{- m\vec{a}_{co}}_{\vec{F}_{co}} \underbrace{- m\vec{a}_{TR}}_{\vec{F}_{TR}} = m \cdot \vec{a}'$

$F_{co} = -2m(\omega \wedge v) = 2m(v \wedge \omega)$

$\Rightarrow \sum_i \vec{F} = ma'$

Cavo percorso da corrente



Biot-Savart: $B = k\frac{I}{R} = \frac{\mu_0}{2\pi}\frac{I}{R}$

come Trovare $\vec{B}$? $\vec{B} = B \cdot \hat{\tau}$

$\hat{\tau} = \hat{e} \wedge \hat{\varepsilon} \Rightarrow \vec{B} = \frac{\mu_0}{2\pi} I \cdot \frac{d\vec{e} \wedge \vec{\varepsilon}}{\varepsilon^2}$

$\vec{F}_c = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 \cdot q_2}{\varepsilon}\hat{\varepsilon}$

$\to \vec{F}_{q_m} = \frac{\mu_0}{2\pi}\frac{q_m \cdot q_m'}{\varepsilon^2}\hat{\varepsilon} \Rightarrow \vec{B} = \frac{\vec{F}}{q_m} = \frac{\mu_0}{4\pi}\frac{q_m}{\varepsilon^2}\hat{\varepsilon} \quad (1)$

$\hookrightarrow F = q_m \cdot B$

$F_{Lorentz} = q\,\vec{v} \wedge \vec{B} = q\frac{d\vec{e}}{dt} \wedge \vec{B} = \underline{I\, d\vec{e} \wedge \vec{B}}$ II LAPLACE

$\to \begin{cases} F = I\,de \wedge B \\ F = q \cdot B \end{cases} \Rightarrow q_m = I\,d\vec{e} \Rightarrow (1)\ \vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{e}}{\varepsilon^2}\hat{\varepsilon}$

ma $\hat{\varepsilon} = d\hat{e} \wedge \hat{\varepsilon} = \frac{d\vec{e} \wedge \vec{\varepsilon}}{\varepsilon} \to B = \frac{\mu_0}{4\pi} I \frac{d\vec{e} \wedge \vec{\varepsilon}}{\varepsilon}$ Laplace I

Maxwell Integrale

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} & \text{Gauss} \\ \vec{\nabla} \wedge \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \text{Faraday} \end{cases}$$

Gauss $\rightarrow$ $\int \vec{E}\, \hat{n}\, ds = \dfrac{Q}{\varepsilon_0}$

Faraday $\rightarrow$ $\phi_E = \int \vec{E}\, \hat{n}\, ds = 0$ $\rightarrow$ $\oint E \cdot \hat{n}\, ds = \oint E\, dl + \int \vec{v} \wedge \vec{B}\, dl = \int v \wedge B\, dl + \int \dfrac{\partial B}{\partial t} dl$

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 & \text{Monopoli} \\ \vec{\nabla} \wedge \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \right) & \text{Ampere - Maxwell} \end{cases}$$

Monopoli $\rightarrow$ $\phi_B = \oint B \cdot \hat{n}\, ds = 0$

Ampere - Maxwell $\rightarrow$??

Bonus: $\vec{F}_L = q\vec{E} + q\,(\vec{v} \wedge \vec{B})$

Amp - Max Integrale

Se $\dfrac{d\phi_B}{dt} = \int \vec{E} \cdot d\vec{l} - \oint \vec{v} \wedge \vec{B}\, dl = \oint \dfrac{\partial \vec{B}}{\partial t} dl - \oint \vec{v} \wedge \vec{B}\, d\vec{l}$

$$\Rightarrow \frac{d\phi_E}{dt} = \oint \frac{\partial \vec{E}}{\partial t} dl - \oint \vec{v} \wedge \vec{E}\, d\vec{l}$$

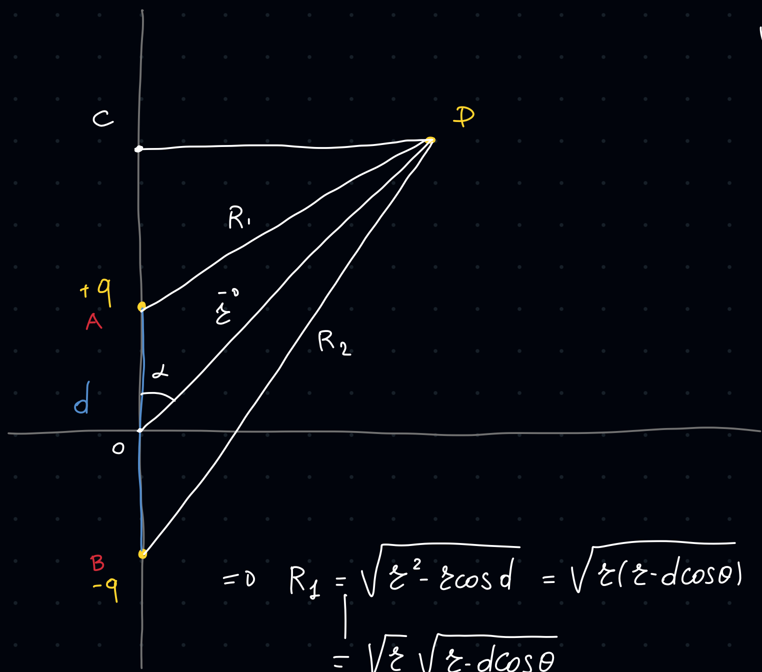
$$\Rightarrow \qquad \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\int \vec{B} \cdot \hat{n}\, ds = \mu_0 \int J \cdot \hat{n}\, ds + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\Rightarrow \oint \vec{B}\, \hat{n}\, ds = \mu_0 I + \mu_0 \varepsilon_0 \oint \frac{\partial \vec{E}}{\partial t} dl - \mu_0 \varepsilon_0 \oint \vec{v} \wedge \vec{E}\, d\vec{l}$$

$$\Rightarrow \oint \vec{B} \cdot \hat{n}\, ds - \frac{1}{c^2} \oint \vec{v} \wedge \vec{E}\, d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \left[\oint \frac{\partial \vec{E}}{\partial t} d\vec{l} - \oint \vec{v} \wedge \vec{E}\, d\vec{l} \right]$$

Ampere : Dipolo - Spira



$\vec{E} = -\vec{\nabla} V$ ma $V = ?$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} - \frac{q}{R_2}\right) \quad \text{ma } R_1, R_2 = ?$$

$$R_1^2 = CP^2 + CA^2 = (r\sin\theta)^2 + \left(CO - \frac{d}{2}\right)^2$$
$$= r^2\sin^2\theta + \left(r\cos\theta - \frac{d}{2}\right)^2 =$$
$$= r^2\sin^2\theta + r^2\cos^2\theta + \left(\frac{d}{2}\right)^2 - r\cos d$$
$$= r^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{1} + \underbrace{\cancel{\left(\frac{d}{2}\right)^2}}_{r>>0} - r\cos d$$
$$= r^2 - r\cos d$$

$$\Rightarrow R_1 = \sqrt{r^2 - r\cos d} = \sqrt{r(r - d\cos\theta)} = \sqrt{r}\,\sqrt{r - d\cos\theta}$$

Approx Taylor in $d_0 = 0$

$$\rightarrow f(d) = (r - d\cos\theta)^{\frac{1}{2}} \rightarrow f'(d) = \frac{1}{2}(r - d\cos\theta)^{-\frac{1}{2}}(-\cos\theta) = -\frac{\cos\theta}{2(r - d\cos\theta)^{\frac{1}{2}}}$$

$$\Rightarrow f(0) = \sqrt{r} \qquad f'(0) = -\frac{\cos\theta}{2\sqrt{r}}$$

$$\Rightarrow R_1 \approx \sqrt{r}\cdot\left[\sqrt{r} - \frac{d\cos\theta}{2\sqrt{r}}\right] = r - \frac{d\cos\theta}{2}$$

$$R_2 \approx r + \frac{d\cos\theta}{2}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{r + \frac{d}{2}\cos\theta - r + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\theta}\right] = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\theta}$$

ma se $r >> d \rightarrow V = \dfrac{q}{4\pi\varepsilon_0}\,\dfrac{d\cos\theta}{r^2}$ (1)

$$\rightarrow \vec{E} = -\vec{\nabla} V \Rightarrow \vec{E} = -\hat{\imath}\frac{\partial V}{\partial x} - \hat{\jmath}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$$ Se P è lungo z

$\rightarrow \vec{E}_z = -\hat{k}\dfrac{\partial V}{\partial z}$ ma ci servono le coordinate Polari

$$\rightarrow r^2 = x^2 + y^2 + z^2 \; ; \quad r\cos\theta = z \rightarrow \cos\theta = \frac{z}{r} = \frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\,\frac{dz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

(20')

$$\to V(z) = \overbrace{\frac{qd}{4\pi\varepsilon_0}}^{K} \cdot z(x^2+y^2+z^2)^{-\frac{3}{2}} \to \underbrace{V'(z)}_{\vec{E}_z} = -K\left[(x^2+y^2+z^2)^{-\frac{3}{2}} - \frac{3}{2}z(x^2+y^2+z^2)^{-\frac{5}{2}}\cdot 2z\right]$$

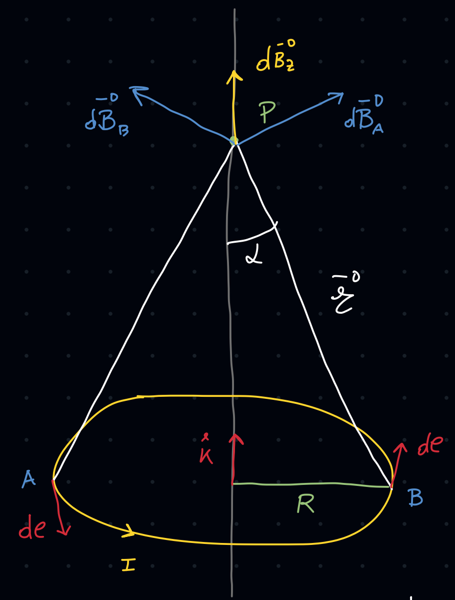
$$= -K\left[(r^2)^{-\frac{3}{2}} + 2z^2(r^2)^{-\frac{5}{2}}\right] = -K\left[\frac{1}{r^3} + \frac{3z}{r^5}\right] = +\frac{K}{r^3}\left[\underbrace{\frac{3z^2}{r^2}}_{\cos^2\theta} - 1\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r^3}\left[3\cos^2\theta - 1\right]$$

Se P è lungo $z \to \theta = 0 \to \cos\theta = 1 \to \vec{E}_z = \frac{2q\vec{d}}{4\pi\varepsilon_0 r^3}$ pongo $\vec{m} = \vec{d}q$

$$\Rightarrow \vec{E}_z = \frac{\vec{m}}{2\pi\varepsilon_0 r^3} \quad (1)$$

Spira



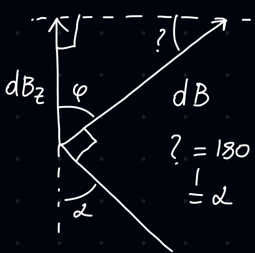
Da Biot-Savart

$$dB = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \wedge \vec{r}}{r^3} \quad \text{ma } r = ??$$

$$r^2 = R^2 + z^2 \to dB = \frac{\mu_0}{4\pi} I \frac{dl\, r \cancel{\sin 90}^{\,1}}{r^{\cancel{3}\,2}} = \frac{\mu_0 I\, dl}{4\pi r^2}$$

$$= \frac{\mu_0 I\, dl}{4\pi(R^2+z^2)}$$

dB_x e dB_y si Annullano $\to dB = dB_z = dB \sin\alpha$



$? = 180 - 90 - (180 - \alpha - 90) = \alpha$

$$\Rightarrow dB_z = \frac{\mu_0 I}{4\pi} \frac{dl \sin\alpha}{(R^2+z^2)} \quad \text{ma } r\sin\alpha = R$$

$$\Rightarrow \sin\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2+z^2}}$$

$$\Rightarrow dB_z = \frac{\mu_0 I}{4\pi} \frac{dl\, R}{(R^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow \vec{B}_z = \frac{\mu_0 I R}{4\pi} \cdot \frac{1}{(R^2+z^2)^{\frac{3}{2}}} \hat{k} \oint dl = \hat{k} \frac{\mu_0 I \cancel{R}}{\cancel{4\pi}_{\,2} (R^2+z^2)^{\frac{3}{2}}} \cdot 2\cancel{\pi} R^2 \to \vec{B}_z = \frac{\mu_0 I R^2}{(R^2+z^2)^{\frac{3}{2}}} \hat{k}$$

Se $z >> R$

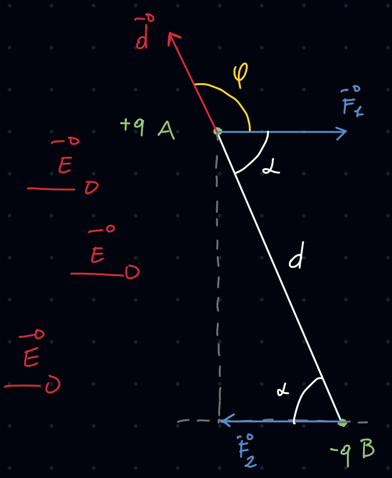
$$B_z = \hat{k} \frac{\mu_0 I\, 2\pi R^2}{4\pi\, z^3} \quad \text{pongo } \vec{p} = \hat{k} I S \Rightarrow \vec{B}_z = \frac{\mu_0 \vec{p}}{2\pi z^3}$$

QED

(15')

Momento

Inerzia · $I = m R^2$



$$\vec F_1 = -\vec F_2 \Rightarrow \begin{cases} \vec F_1 = q \cdot \vec E \\ \vec F_2 = -q \cdot \vec E \end{cases}$$

$\vec M = \vec r \wedge \vec F$ ma $\vec M_{TOT} = \sum_i \vec r_i \wedge \vec F_i$ Polo in B

$$\Rightarrow \vec M_{TOT} = \vec d \wedge \vec F_1 + 0 \wedge \vec F_2 = \vec d \wedge \vec F_1 = \vec d \wedge q \cdot \vec E = q \cdot \vec d \wedge E = \underline{\vec p \wedge \vec E}$$

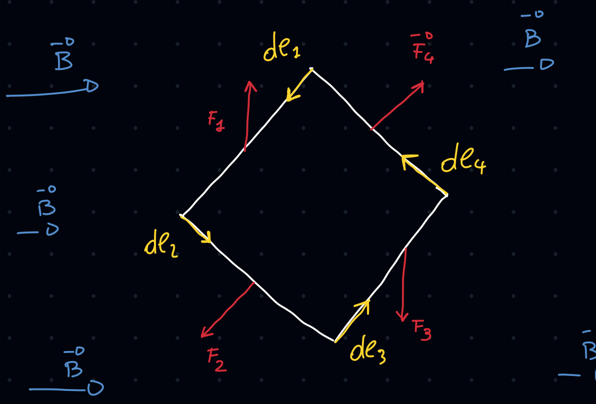
Modulo · $M_{TOT} = d \cdot F_1 \cdot \sin(\varphi)$

ma $\varphi = 180 - \alpha \Rightarrow \sin(180-\alpha) = \sin(\alpha)$

$\Rightarrow M_{TOT} = F_1 \cdot d \cdot \sin\alpha$ ma $d \sin\alpha = b \Rightarrow \boxed{M_{TOT} = F_1 \cdot b}$

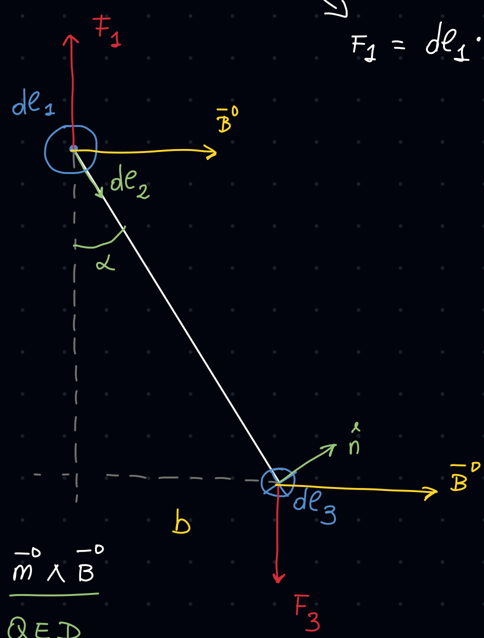
$\to$ Dipolo magnetico $\to$ $\underline{\vec M_{TOT} = \vec m \wedge \vec B}$

Spira



$$F = \underbrace{q \cdot E + q\,(\vec v \wedge \vec B)}_{\text{Lorentz}} = \underbrace{I\, d\vec\ell \wedge \vec B}_{\text{Laplace}}$$

$$F_1 = d\ell_1 \cdot B \cdot \sin(90) = \ell_1 \cdot B \cdot I$$



$$M = F \cdot b = (I B \ell_1) \cdot (\ell_2 \sin\alpha) = I B S \sin\alpha$$

definisco $\vec m = \hat n \cdot I \, S \Rightarrow \vec M = \vec m B \sin\alpha = \underline{\vec m \wedge \vec B}$

QED

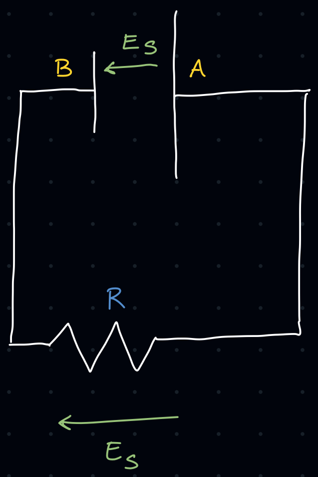
Joule

$$L = \int \vec{F} \cdot d\vec{e} \qquad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E \Rightarrow \qquad L = q \cdot V$$

Siccome $P = \frac{dL}{dt} \rightarrow P = \frac{V\,dq}{dt} \Rightarrow P = V I$ ma $V = R \cdot I$

$\Rightarrow$ $P = R I^2$ effetto Joule

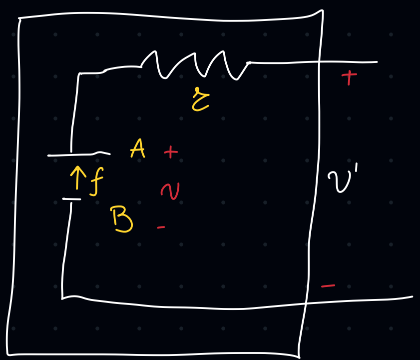
Fem



$$L = \oint E \, de = \emptyset \qquad \text{definisco} \quad f_{em} = \frac{L}{q} = \int \vec{E} \cdot d\vec{e} \neq 0$$

$$\rightarrow \oint \vec{E} \cdot d\vec{e} = \int_A^B E_s \, de + \int_B^A (E_s + E_m) \, de = \cancel{\int_A^B E_s de} - \cancel{\int_A^B E_s de} + \int_B^A E_m de$$

$$\rightarrow f_{em} = \int_B^A \vec{E}_m \cdot d\vec{e}$$



$$V_A' - V_B' = (V_A - V_B) - R \cdot I$$

ma $V_A - V_B = f_{em} \Rightarrow V_A' - V_B' = f_{em} - \mathcal{E} \cdot I$

$$\Rightarrow V_A' - V_B' = f_{em} - \mathcal{E} I = R I \Rightarrow f_{em} = I(R + \mathcal{E})$$

$\Rightarrow I = \frac{f_{em}}{R + \mathcal{E}}$ ma $V = R \cdot I \Rightarrow I = \frac{V}{R}$

$\rightarrow \frac{V}{R} = \frac{f_{em}}{R + \mathcal{E}} \rightarrow V = \frac{R}{R + \mathcal{E}} f_{em}$ per $\mathcal{E}_{int} = 0 \rightarrow V = f_{em}$

Modello di Drude

$P = \frac{1}{\tau}$

1) e-e 2) e-i 3) τ 4) $v(T)$ 5) urto elastico

$J = \vec{v} \cdot \rho$ (dens Volum) $\rightarrow$ $J = -n \cdot e \cdot \vec{v}$ (numero di Elett; elett)

Ohm Vettoriale

$$\left.\begin{array}{l} dV = R \cdot dI \\ dV = E \cdot dl \end{array}\right\} R\, dI = E\, dl \rightarrow \rho \frac{dl}{dS} \cdot J \cdot \hat{n}\, dS = E\, dl$$

$$\rightarrow \underline{\rho \vec{J} = \vec{E}}$$

Velocità

$$E = \frac{F}{q} \rightarrow F = q E \Rightarrow q = -e \rightarrow F = -e E$$

Se $\vec{F} = m \cdot \vec{a} \rightarrow -eE = m \cdot \vec{a} \Rightarrow a = -\frac{eE}{m}$

$$\rightarrow v(t) = v_0 + at \rightarrow v(t) = v_0 - \frac{eE}{m} t$$

$E = 0$ $\rightarrow$ $\langle v(t) \rangle = \langle v_0 \rangle - \langle \cancel{\frac{eE}{m}}^{0} t \rangle = \langle v_0 \rangle = 0$

$\leftarrow$ media $= 0$

$E \neq 0$ $\langle v(t) = \langle v_0 \rangle - \langle \frac{e\vec{E}}{m} t \rangle = 0 - \frac{e\vec{E}}{m} \langle t \rangle = -\frac{eE}{m} \tau$

$$\Rightarrow \vec{J} = -ne\, \vec{v} \longrightarrow \vec{J} = -ne \cdot \left(-\frac{eE}{m}\tau\right) = \frac{ne^2 E}{m}\tau \quad \text{ma} \quad \vec{J} = \frac{\vec{E}}{\rho}$$

$$\Rightarrow \frac{ne^2 E \tau}{m} = \frac{E}{\rho} \rightarrow \tau = \frac{m}{\rho \cdot n e^2}$$

$\vec{E} = -\vec{\nabla} V$??

Sappiamo che $\vec{E} \cdot \vec{dl} = -dV \Rightarrow dV = \hat{i} \frac{\partial V}{\partial x} dx + \hat{j} \frac{\partial V}{\partial y} dy + \hat{k} \frac{\partial V}{\partial z} dz$

$$\rightarrow \left\{\begin{array}{l} dl = \hat{i}\, dx + \hat{j}\, dy + \hat{k}\, dz \\ \nabla V = \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \end{array}\right. \rightarrow -dV = -\vec{dl}\, \vec{\nabla} V$$

$$\Rightarrow \vec{E} \cdot \vec{dl} = -\vec{dl}\, \vec{\nabla} V \rightarrow \vec{E} = -\vec{\nabla} V$$

Eq Continuità della corrente

$$-Q = \int \vec{J} \cdot \hat{n}\, dS \quad dt \rightarrow \quad \rho = \frac{Q}{V} \rightarrow Q = \int \rho \cdot dV \rightarrow -\frac{d}{dt} \int \rho\, dV = \int J \cdot \hat{n}\, dS$$

$$\rightarrow -\int \frac{\partial \rho}{\partial t} dV = \int (\vec{\nabla} \vec{J})\, dV \rightarrow \vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t}$$

Se siamo in corrente stazionaria

$$-\frac{\partial \rho}{\partial t} = 0 \quad \rho = \text{cost}$$

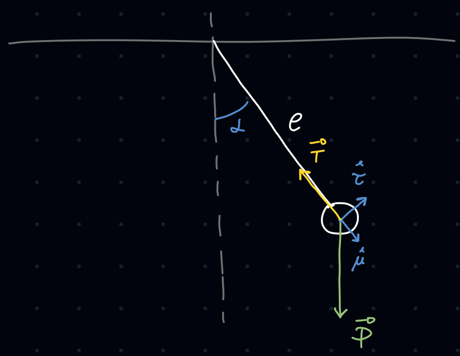
$$\Rightarrow \vec{\nabla} \vec{J} = 0$$

Densità di corrente

$dq = \rho \cdot dV$ ma $V = S \cdot \ell$ $\Rightarrow$ $dV = \underbrace{dS}_{b \times h} \cdot \underbrace{d\ell}_{\ell = v \cdot t \;\to\; d\vec{\ell} = \vec{v} \cdot dt}$

$\to$ $dq = \rho\, d\vec{S} \cdot d\ell = \rho\, d\vec{S} \cdot v \cdot dt$ $\to$ $\underbrace{\frac{dq}{dt}}_{dI} = \underbrace{\rho\, \vec{v}}_{J}\, ds$ $\to$ $I = \int \vec{J} \cdot \hat{n}\, ds$

Pendolo Semplice



$\vec{P} = \vec{P}_{\hat{\mu}} + \vec{P}_{\tau} = P \cos\alpha - P \sin\alpha$

$\vec{F} = m \cdot \vec{a}$

$\hat{\mu}$: $\vec{P}_{\mu} - T = m \cdot \vec{a}$ $\to$ $\begin{cases} mg \cos\alpha - \vec{T} = m\vec{a}_{\mu} \\ -mg \sin\alpha = m\vec{a}_{\tau} \end{cases}$

τ : $-\vec{P}_{\tau} = m\vec{a}$ $\to$

$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{S}$ $\to$ L'acc è centripeta $\to$ $\vec{a}_{\mu} = -\frac{v^2}{R}$

$\vec{a}_{\tau} = \ddot{S}$

$\Rightarrow \begin{cases} mg \cos\alpha - \vec{T} = -\frac{mv^2}{R} \\ -mg \sin\alpha = m \cdot \ddot{S} \end{cases}$ $\to$ Approx per piccole oscillazioni $\sin\alpha \approx \alpha$

ma $1 \text{ Rad} = \frac{\ell}{R}$ ← Arco / ← Raggio $\to$ $\alpha = \frac{S}{\ell}$ $\to$ $-\cancel{m} g \frac{S}{\ell} = \cancel{m} \cdot \ddot{S}$ $\to$ $\ddot{S} + \frac{g}{\ell} S = 0$

chiamo $K^2 = \frac{g}{\ell}$ $\to$ $\ddot{S} + K^2 S = 0$ Eq diff

Soluzione $S(t) = A \sin(Kt + \varphi)$

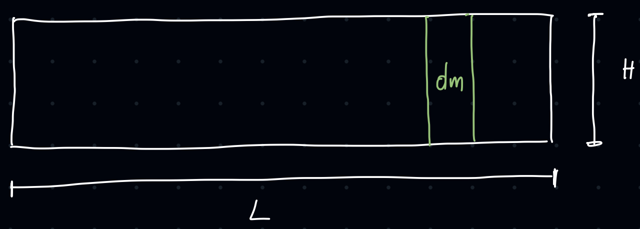
$S(t+T) = A \sin(Kt + \varphi + 2\pi)$ $\to$ $A \sin(Kt + KT + \varphi) = A \sin(Kt + \varphi + 2\pi)$ $\to$ $\cancel{Kt} + KT + \cancel{\varphi} = \cancel{Kt} + \cancel{\varphi} + 2\pi$

$\to$ $KT = 2\pi$ $\to$ $K = \frac{2\pi}{T} = \omega$

$\Rightarrow T = \frac{2\pi}{K}$ ma $K = \sqrt{\frac{g}{\ell}}$ $\to$ $T = 2\pi \sqrt{\frac{\ell}{g}}$

Inerzia Sbarra

$I = m R^2$



$\sum_i m_i \rightarrow \int dm$ Continuo di punti

Sbarra 3D $\rightarrow$ $I_{TOT} = \int R^2 dm$

$\rightarrow I_{TOT} = \int R^2 \frac{dm}{dV} dV \qquad \rho = \frac{M}{V} \Rightarrow I_{TOT} = \int_V R^2 \rho \, dV$

Sbarra 2D $\qquad \rho = \frac{M}{V} \rightarrow \lambda = \frac{M}{L} \Rightarrow I_{TOT} = \int_e R^2 \lambda \, de$

Lungo un lato

$$\rightarrow I_{TOT} = \frac{M}{L} \int_0^L y^2 dy = \frac{M}{L} \left[\frac{y^3}{3} \right]_0^L = \frac{M L^2}{3}$$

Lungo il centro

$$I_{TOT} = \frac{M}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} y^2 dy = \frac{1}{12} M L^2$$

Corrente di spostamento

Ampère - Maxwell: $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Continuità correnti

$$-dq = \vec{J} \cdot \hat{n} \, ds \cdot dt \rightarrow -\frac{dq}{dt} = \vec{J} \cdot \hat{n} \, ds \rightarrow -\int_V \frac{\partial \rho}{\partial t} dV = \int \vec{J} \cdot \hat{n} \, ds$$

$$\rightarrow -\frac{\partial \rho}{\partial t} = \vec{\nabla} \vec{J}$$ obbiettivo

$$\vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla (\nabla \wedge B) = \cancel{\mu_0} \vec{\nabla} \vec{J} = \cancel{\mu_0} \varepsilon_0 \frac{\partial (\nabla E)}{\partial t} \rightarrow \cancel{\mu_0} \vec{\nabla} \vec{J} = -\cancel{\mu_0} \varepsilon_0 \frac{1}{\varepsilon_0} \frac{\partial \rho}{\partial t} \rightarrow \vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t}$$

Dalle eq di Max alle Onde

- Amp-Max : $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

- Faraday : $\vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$

Campo E

$$\nabla \wedge (\nabla \wedge E) = - \frac{\partial(\nabla \wedge B)}{\partial t} \rightarrow \nabla(\overset{\nabla E = \frac{\rho}{\varepsilon_0}}{\nabla E}) - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t}$$

$$\rightarrow \nabla \frac{\rho}{\varepsilon_0} - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t} \rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \frac{\nabla \rho}{\varepsilon_0} \quad (1)$$

Campo B

$$\nabla \wedge (\nabla \wedge B) = \mu_0 (\nabla \wedge J) + \mu_0 \varepsilon_0 \frac{\partial(\nabla \wedge E)}{\partial t} \rightarrow \cancel{\nabla(\overset{\nabla B = 0}{\nabla B})} - \nabla^2 B = \mu_0 (\nabla \wedge J) - \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial^2 t}$$

$$\rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t} - \nabla^2 B = \mu_0 (\nabla \wedge J) \quad (2)$$

Nel vuoto $\rho = J = 0$

$$\begin{cases} \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} - \vec{\nabla}^2 \vec{B} = 0 & (a) \\ \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \vec{\nabla}^2 \vec{E} = 0 & (b) \end{cases}$$

(b) $\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t} - \vec{\nabla}^2 \vec{E} = 0 \rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t} - \frac{\partial^2 \vec{E}}{\partial x}\hat{i} - \frac{\partial^2 \vec{E}}{\partial y}\hat{j} - \frac{\partial^2 E}{\partial z}\hat{k} = \emptyset$

Solo $x \rightarrow \frac{\partial^2 \vec{E}}{\partial x} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t} = \emptyset$

Eq Differenziale con sd : $E(t) = A \sin(kx - \omega t)$

$$\left.\begin{aligned} E_1 &= A \sin(k x_1 - \omega t_1) \\ E_2 &= A \sin(k x_2 - \omega t_2) \end{aligned}\right\} E_1 = E_2 \Leftrightarrow \quad k x_1 - \omega t_1 = k x_2 - \omega t_2 \rightarrow k(x_2 - x_1) = \omega(t_2 - t_1)$$

$$\rightarrow x_2 - x_1 = \underbrace{\frac{\omega}{k}}_{v} (t_2 - t_1)$$

$v = \frac{\omega}{k} \Rightarrow \omega = v \cdot k$

$$\Rightarrow E(t) = A \sin(kx - vkt)$$

$$\frac{\partial E}{\partial t} = -vkA\cos(kx - vkt) \qquad \frac{\partial^2 E}{\partial t} = -v^2 k^2 A \sin(kx - vkt)$$

$$\frac{\partial E}{\partial x} = kA\cos(kx - vkt) \qquad \frac{\partial^2 E}{\partial x^2} = -k^2 A \sin(kx - vkt)$$

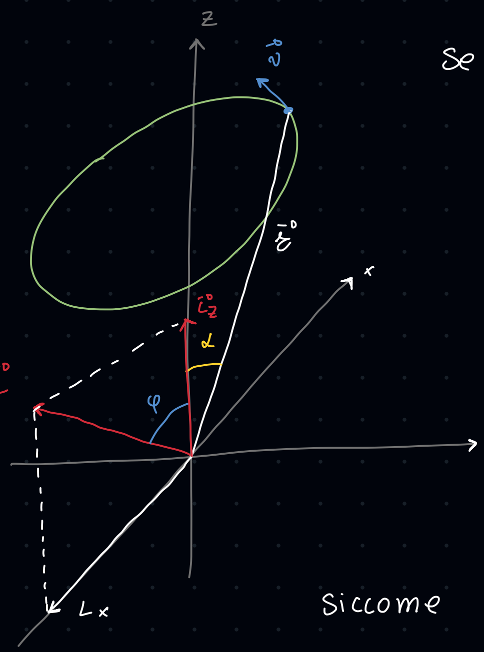
$$\Rightarrow -k^2 A \sin(kx - vkt) + \mu_0 \varepsilon_0 v^2 k^2 A \sin(kx - vkt) = 0$$

$\rightarrow \quad K^2 A \sin(Kx - vKt)\left[-1 + \mu_0 \varepsilon_0 v^2\right] = 0 \quad \Rightarrow \quad v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$

II eq cardinale della Dinamica

$$\begin{cases} \vec{L} = \vec{r} \wedge m\vec{v} = \vec{r} \wedge \vec{p} \\ \vec{M} = \vec{r} \wedge \vec{F} \end{cases}$$

ma $I = m R^2$



Se il corpo é speculare $\rightarrow L_x = L_y = \emptyset$

$\Rightarrow L_z = L\cos(\varphi)$ ma $\varphi = 90 - \alpha \rightarrow \cos(90-\alpha) = \sin(\alpha)$

$\Rightarrow \vec{L}_z = \vec{L}\sin(\alpha)$

$\vec{L} = \vec{r} \wedge m \cdot \vec{v} \Rightarrow L = r \cdot m \cdot v \cdot \sin(90)$

$\Rightarrow L = r m v$ ma $v = \omega R$

$\Rightarrow L_i = r m_i \omega_i R_i$

Siccome $L_z = L\sin\alpha \rightarrow L_{iz} = r m_i \omega_i R_i \sin\alpha$

ma $r\sin\alpha = R \Rightarrow L_{iz} = m_i \omega_i R^2$ $\quad \Sigma_i m_i R_i^2 = I_{TOT}$

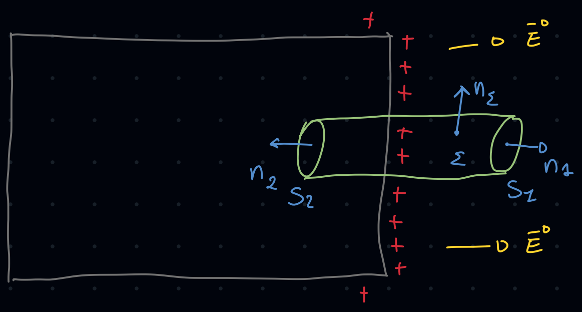
$\rightarrow L_{TOT} = \omega I$

Siccome $\begin{cases} \vec{L} = \vec{r} \wedge m \cdot \vec{v} \\ \vec{M} = \vec{r} \wedge \vec{F} \end{cases} \rightarrow \dfrac{dL}{dt} = \dfrac{d\vec{r}}{dt} \wedge m \cdot \vec{v} + \vec{r} \wedge m \dfrac{d\vec{v}}{dt} = \underbrace{\cancel{\vec{v} \wedge m\vec{v}}}_{v /\!/ v} + r \wedge \underbrace{m \cdot \vec{a}}_{\vec{F}}$

$\rightarrow \dfrac{dL}{dt} = \vec{M} \Rightarrow M_{\text{corpo Rigido}} = I\dfrac{d\omega}{dt} \rightarrow M = \alpha I$ II eq cardinale

Campi Elettrici

1 Coulomb



$E=\emptyset$ $\quad$ $\vec{E}\perp\hat{n}_\Sigma$

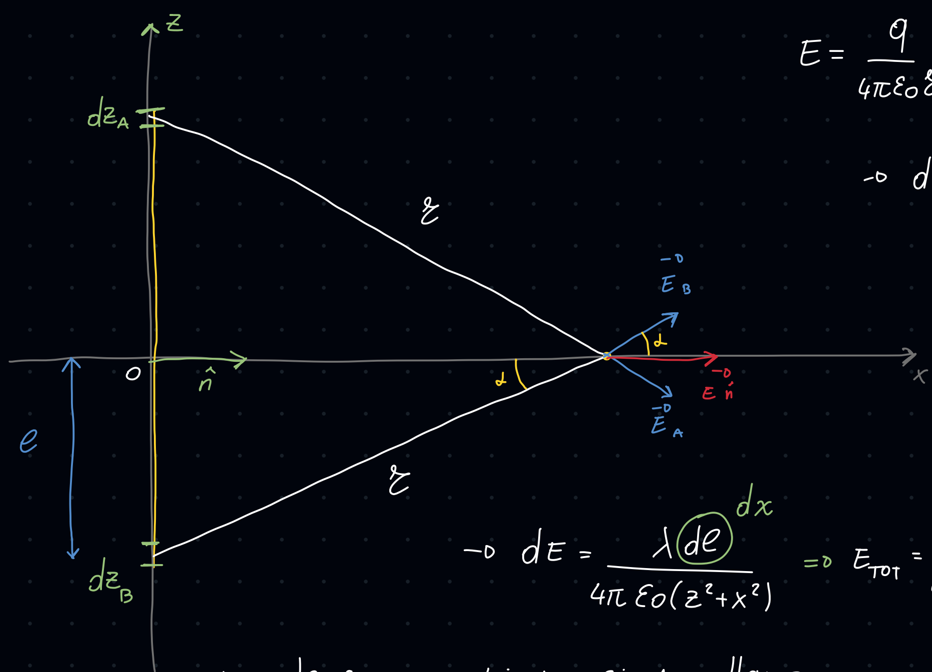
$$\phi_E = \cancel{\phi_{S_1}} + \phi_{S_2} + \cancel{\phi_{S_\Sigma}} = \int E\cdot\hat{n}\,dS_2 = \frac{Q}{\varepsilon_0}$$

ma $\sigma = \frac{Q}{S} \rightarrow Q = \int \sigma\, ds$

$$\Rightarrow \int \vec{E}\,\hat{n}\,ds = \frac{1}{\varepsilon_0}\int \sigma\, ds$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Filo carico



$E = \frac{q}{4\pi\varepsilon_0 \ell^2}$ $\quad$ ma $\ell^2 = z^2 + x^2$

$$\rightarrow dE = \frac{dq}{4\pi\varepsilon_0(z^2+x^2)}$$

$$\lambda = \frac{dq}{d\ell} \rightarrow dq = \lambda d\ell$$

$$\rightarrow dE = \frac{\lambda\, \overset{dx}{d\ell}}{4\pi\varepsilon_0(z^2+x^2)} \Rightarrow E_{TOT} = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{1}{z^2+x^2}\,dx$$

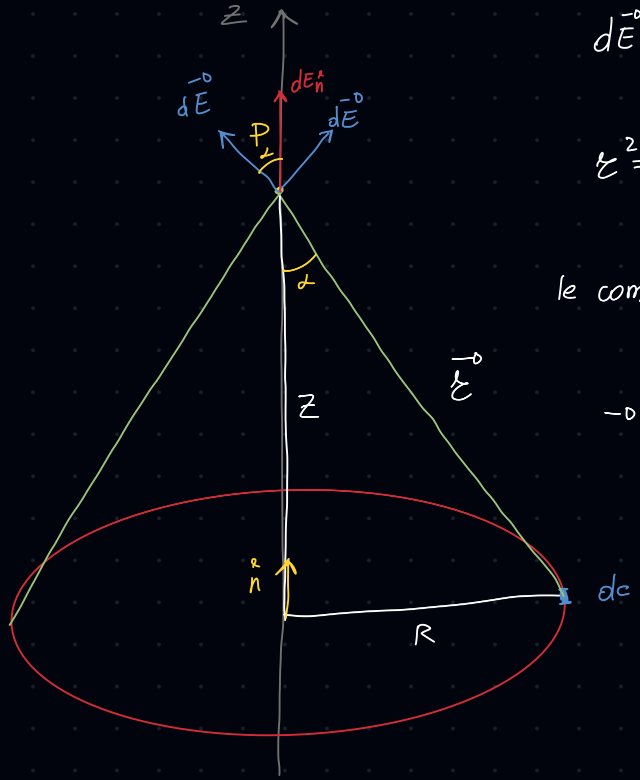
ma le componenti y si annullano

$\rightarrow \vec{E}_{\hat{n}} = E\cdot\cos\alpha\cdot\hat{n}$ $\quad$ ma $\ell\cos\alpha = x \rightarrow \cos\alpha = \frac{x}{\ell} = \frac{x}{\sqrt{x^2+z^2}}$

$$\Rightarrow E_{TOT} = \frac{\lambda}{4\pi\varepsilon_0}\,2\int_0^{\ell}\frac{x}{(x^2+z^2)^{\frac{3}{2}}}\,dx = \frac{2\lambda}{4\pi\varepsilon_0}\left[\frac{1}{x^2}\,\frac{z}{(x^2+z^2)^{\frac{1}{2}}}\right]_0^{\ell}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 x^2}\cdot\frac{\ell}{(x^2+\ell^2)^{\frac{1}{2}}}$$ Lunghezza finita

se $\ell \rightarrow \infty \Rightarrow \ell >> x \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 x^2}\,\frac{\cancel{\ell}}{\cancel{\ell}} = \frac{\lambda}{2\pi\varepsilon_0 x^2}$



$$d\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$ ma $$\lambda = \frac{dq}{dl} \to dq = \lambda\, dl$$

$$r^2 = z^2 + R^2 \Rightarrow dE = \frac{\lambda\, dl}{4\pi\varepsilon_0 (z^2+R^2)}$$

le comp x e y si annullano $\to$ $dE_{\hat n} = dE \cdot \cos\alpha$

$$\to dE_{\hat n} = \frac{\lambda\, dl \cos\alpha}{4\pi\varepsilon_0 (z^2+R^2)}$$ ma $r \cdot \cos\alpha = z \to \cos\alpha = \frac{z}{r}$

$$= \frac{\lambda\, dc \cdot z}{4\pi\varepsilon_0 (z^2+R^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_{\hat n} = \frac{\lambda z}{\cancel{4}_{2}\pi\varepsilon_0 (z^2+R^2)^{\frac{3}{2}}} \overbrace{\oint dc}^{\cancel{2\pi} R} = \frac{\lambda z R}{2\varepsilon_0 (z^2+R^2)^{\frac{3}{2}}} \cdot \hat n$$

se $z >> R$ $$E_{\hat n} = \frac{\lambda \cancel{z} R}{2\varepsilon_0 z^{\cancel{3}\,2}}\hat n$$

OPPURE $$E_{\hat n} = \frac{\lambda z \cdot 2\pi R}{4\pi\varepsilon_0 (z^2+R^2)^{\frac{3}{2}}}$$

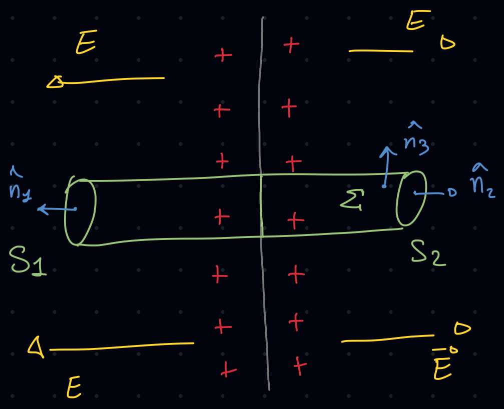
siccome $\lambda = \frac{Q}{l} \to Q = \lambda \cdot l$

$\to$ $\lambda \cdot 2\pi R$ Q

$$\Rightarrow \vec{E}_{\hat n} = \frac{Q \cdot \cancel{z}}{4\pi\varepsilon_0 \underset{\uparrow}{z^{\cancel{3}\,2}}}\hat n$$

$z >> R$

Strato Carico



$$\phi_{TOT} = \oint \vec{E} \cdot \hat{n}\, dS = \phi_{S_1} + \phi_{S_2} + \cancel{\phi_{\Sigma}}^{0} = \frac{Q}{\varepsilon_0}$$

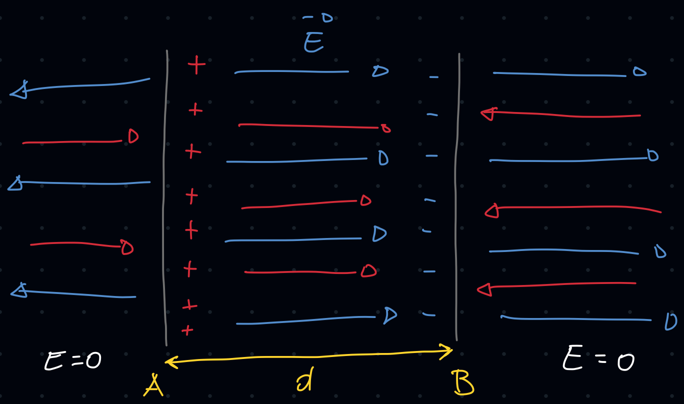
$$\rightarrow 2\phi_S = \frac{Q}{\varepsilon_0} \rightarrow 2\int \vec{E} \cdot \hat{n}\, dS_1 = \frac{Q}{\varepsilon_0}$$

ma $\sigma = \frac{Q}{S} \rightarrow Q = \int \sigma\, dS$

$$\rightarrow 2\int E \cdot \hat{n}\, dS = \frac{1}{\varepsilon_0}\int \sigma\, dS$$

$$\Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0}$$

Doppio strato



$E \neq 0$

Tra A e B $\quad E = 2 \cdot \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

CAPACITA'

$$\begin{cases} V_A = K_1 \cdot Q \\ V_B = K_2 \cdot Q \end{cases} \rightarrow V_A + V_B = Q\,(K_1 + K_2) \rightarrow K_1 + K_2 = \frac{1}{C} \Rightarrow V_A - V_B = \frac{Q}{C}$$

Cond Piano

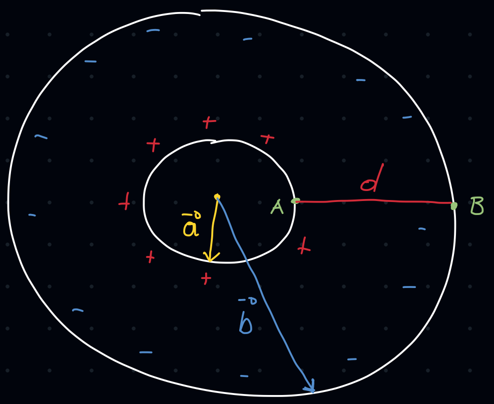
$$C = \frac{Q}{V_A - V_B}$$

$$V_A - V_B = \int \vec{E} \cdot d\vec{e} = \int \frac{\sigma}{\varepsilon_0} \cdot d\vec{e} = \frac{\sigma}{\varepsilon_0} \cdot d \qquad \text{ma} \quad \sigma = \frac{Q}{S}$$

$$\Rightarrow V_A - V_B = \frac{Q \cdot d}{S \cdot \varepsilon_0}$$

$$\Rightarrow C_{PIANO} = \frac{Q \cdot S \cdot \varepsilon_0}{Q \cdot d} = \frac{S \cdot \varepsilon_0}{Q \cdot d}$$

SFERICO



$$C = \frac{Q}{V_A - V_B} \quad \text{ma } V_A - V_B = ? \rightarrow V_A - V_B = \int \vec{E} \cdot d\vec{e}$$

$$\text{ma } E = ?? \rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

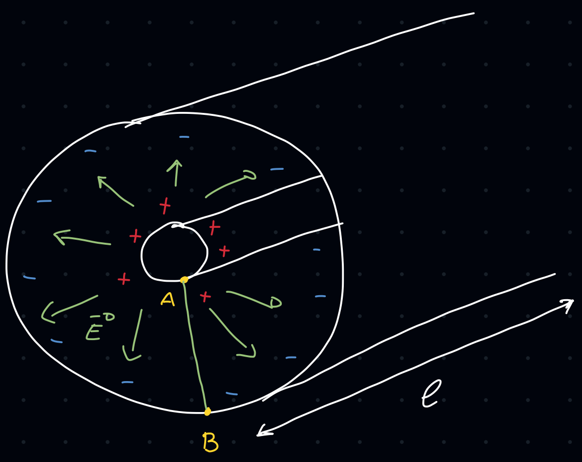
$$\Rightarrow V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$r_A = |\vec{a}|,\ r_B = |\vec{b}| \rightarrow V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\rightarrow V_A - V_B = k\left[\frac{b-a}{a\ b}\right] \rightarrow V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{b-a}{b \cdot a}\right]$$

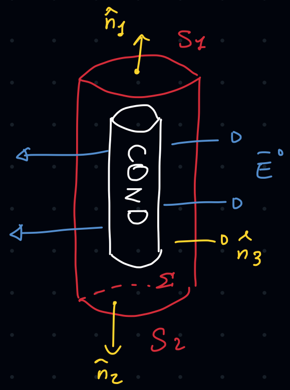
$$\Rightarrow C = \frac{Q\, 4\pi\varepsilon_0}{Q}\ \frac{b \cdot a}{b-a} \rightarrow C = 4\pi\varepsilon_0 \frac{b \cdot a}{b-a}$$

Cilindrico



$$C = \frac{Q}{V_A - V_B} \rightarrow V_A - V_B = \int E\, de \rightarrow E = ??$$

$$\text{So che } \phi_E = \oint E \cdot \hat{n}\, ds = \frac{Q}{\varepsilon_0}$$



$$\Rightarrow \int_\Sigma \vec{E} \cdot \hat{n}\, ds = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E \cdot S_\Sigma = \frac{Q}{\varepsilon_0}$$

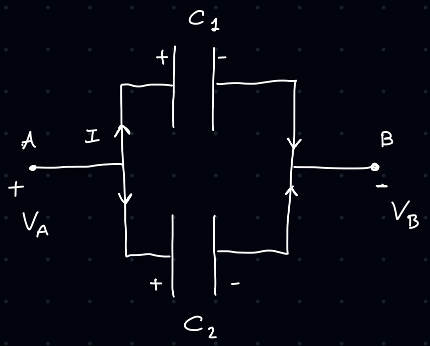
$$E = \frac{Q}{2\pi R e \varepsilon_0}$$

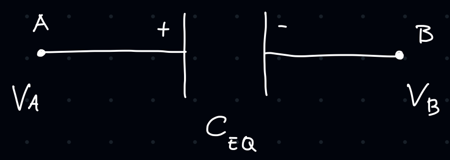
$$\Rightarrow V_A - V_B = \int \frac{Q}{2\pi R e \varepsilon_0}\, de$$

$$\rightarrow V_A - V_B = \frac{Q}{2\pi R \varepsilon_0}\int_A^B \frac{1}{e}\, de = \frac{Q}{2\pi R \varepsilon_0}\left[\ln(e)\right]_A^B = \frac{Q \ln\left(\frac{A}{B}\right)}{2\pi R \varepsilon_0}$$

$V_A - V_B$

$$\Rightarrow C = \frac{2\pi R \varepsilon_0}{\ln\left(\frac{A}{B}\right)}$$

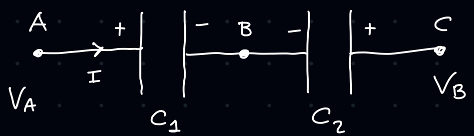




$$C_{EQ} = \frac{Q}{V_A - V_B} \rightarrow Q = (V_A - V_B) \cdot C_{EQ}$$

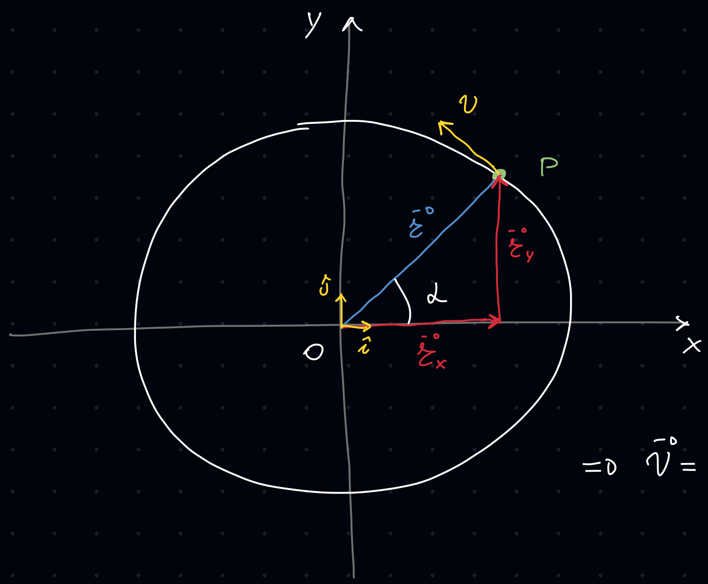
$$\begin{aligned} C_1 &= \frac{Q_1}{V_A - V_B} \\ C_2 &= \frac{Q_2}{V_A - V_B} \end{aligned} \Rightarrow \begin{cases} V_A - V_B = \dfrac{Q_1}{C_1} \\ V_A - V_B = \dfrac{Q_2}{C_2} \end{cases} \Rightarrow \begin{cases} Q_1 = \dfrac{V_A - V_B}{C_1} \\ Q_2 = \dfrac{V_A - V_B}{C_2} \end{cases}$$

$$\rightarrow \underbrace{Q_1 + Q_2}_{Q_{TOT}} = (V_A - V_B)(C_1 + C_2) \rightarrow C_{EQ} = \sum_i C_i$$



$$\begin{cases} V_A - V_B = \dfrac{Q}{C_1} \\ V_B - V_C = \dfrac{Q}{C_2} \end{cases} \Rightarrow V_A - V_C = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \rightarrow \frac{1}{C_{EQ}} = \sum_i \frac{1}{C_i}$$

MoTo Circolare



Definisco $\omega = \frac{d\alpha}{dt} \to d\alpha = \omega \, dt$

$$\to \int_{\alpha_0}^{\alpha_f} d\alpha = \int_{t_0}^{t_f} \omega \, dt$$

$$\to \alpha(t) = \omega \cdot t$$ Se $\omega = cost$

$$\vec r = \vec r_x + \vec r_y = \hat i r \cos\alpha + \hat j r \sin\alpha$$

$$\Rightarrow \vec v = \frac{d\vec r}{dt} = \frac{d}{dt}\left[\hat i r \cos(\omega t) + \hat j r \sin(\omega t)\right]$$

$$\to v = -\hat i r \sin(\omega t)\cdot \omega + \hat j r \cos(\omega t)\omega = \omega\left[\hat j r \cos(\omega t) - \hat i r \sin(\omega t)\right]$$

$$\vec a = -\hat i r \cos(\omega t)\omega^2 - \hat j r \sin(\omega t)\omega^2 = -\omega^2 [\underbrace{\hat i r \cos(\omega t) + \hat j r \sin(\omega t)}_{\vec r}]$$

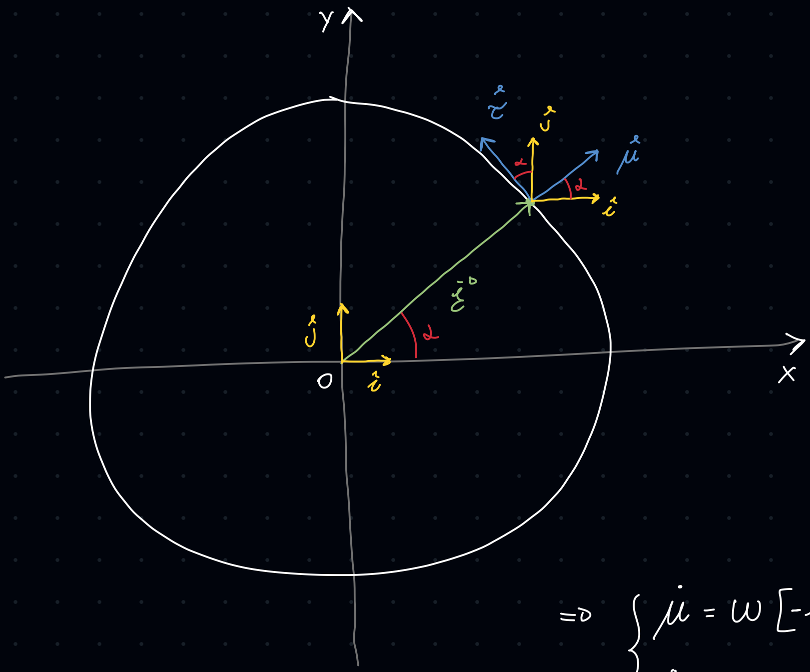
$$\to \vec a = -\omega^2 \vec r$$ Acc centripeta

Modulo

$$\omega = \frac{v}{r}$$

$$|\vec v| = \sqrt{\omega^2 r^2 \sin^2(\omega t) + \omega^2 r^2 \cos^2(\omega t)} = \sqrt{\omega^2 r^2} = \omega r \Rightarrow v = \omega r \quad (1)$$

$$|\vec a| = \sqrt{\omega^4 \dots} = \sqrt{\omega^4 r^2} = \omega^2 r \Rightarrow a = r \cdot \frac{v^2}{r^2} = \frac{v^2}{R}$$ Acc cp



$$\begin{cases} \hat{\mu} = \hat{i}\cos\alpha + \hat{j}\sin\alpha \\ \hat{\tau} = -\hat{i}\sin\alpha + \hat{j}\cos\alpha \end{cases}$$

$$\Rightarrow \; r = \hat{i}\, r\cos\alpha + \hat{j}\, r\sin\alpha$$

$$\alpha = \omega t$$

$$\begin{cases} \dot{\mu} = -\hat{i}\,\omega\sin(\omega t) + \hat{j}\,\omega\cos(\omega t) \\ \dot{\tau} = -\hat{i}\,\omega\cos(\omega t) - \hat{j}\,\omega\sin(\omega t) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{\mu} = \omega\left[-\hat{i}\sin(\omega t) + \hat{j}\cos(\omega t)\right] = \omega\hat{\tau} \\ \dot{\tau} = -\omega\left[\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)\right] = -\omega\hat{\mu} \end{cases}$$

$$\vec{r} = \vec{r}_x + \vec{r}_y = \hat{i}\, r\cos\alpha + \hat{j}\, r\sin\alpha = r\left[\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)\right] = \vec{r}\,\hat{\mu}$$

$$\Rightarrow \; \frac{d\vec{r}}{dt} = \vec{r}\cdot\frac{d\mu}{dt} = \vec{r}\cdot(\omega\hat{\tau}) = r\omega\hat{\tau} \quad \vec{v}$$

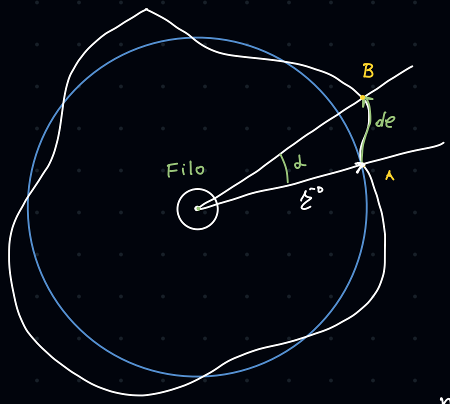
Se $\omega = \text{cost}$

$$\frac{dv}{dt} = r\omega\frac{d\hat{\tau}}{dt} = r\omega(-\omega\hat{\mu}) = -r\omega^2\hat{\mu} \quad \vec{a}$$

Se $\omega \neq \text{cost}$

$$\frac{dv}{dt} = r\omega\dot{\tau} + r\dot{\omega}\hat{\tau} = -r\omega^2\hat{\mu} + r\alpha\hat{\tau}$$

Legge Ampere



Da Biot - Savart

$$\oint B\, dl = \mu_0 I$$
$$\nabla \wedge B = \mu_0 J$$

$$dB = K \cdot \frac{I}{r}\,\hat{r}$$

$$C = \oint \vec{B} \cdot d\vec{l} = \frac{\mu_0}{2\pi} \oint \frac{1}{r}\,\hat{r}\, d\vec{l}$$

ma $\hat{r}\, d\vec{l}$ = Arco di circonferenza ; Siccome $1\,\text{Rad} = \frac{l}{R} \rightarrow l = \text{Rad} \cdot R$

$$\Rightarrow \hat{r}\, d\vec{l} = r\, d\alpha \Rightarrow C = \frac{\mu_0 I}{2\pi} \oint \frac{1}{r}\, r \cdot d\alpha \rightarrow \mu_0 I \rightarrow \boxed{\oint \vec{B}\, d\vec{l} = \mu_0 I}$$

$$\int (\nabla \wedge B)\,\hat{n}\, dS = \mu_0 \int J \cdot \hat{n}\, dS \rightarrow \boxed{\nabla \wedge B = \mu_0 J}$$

Joule

$$Pow = \frac{dL}{dt} \qquad E = \frac{F}{q} \rightarrow F = q \cdot E \Rightarrow L = q \cdot \int E\, dl = q \cdot V$$

ma $V = R \cdot I \qquad dL = dq \cdot R \cdot I \rightarrow Pow = \frac{dL}{dt} = \frac{dq}{dt} R \cdot I = R I^2$

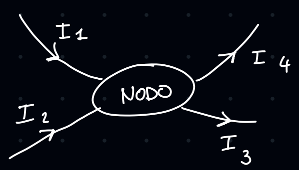
Potenza Dissipata

OPPURE

$L = dq \cdot R \cdot I$ ma $I = \frac{dq}{dt} \rightarrow dq = I\, dt \rightarrow dL = I^2 R\, dt \Rightarrow \boxed{Pow = I^2 R}$

Leggi Kirchhoff

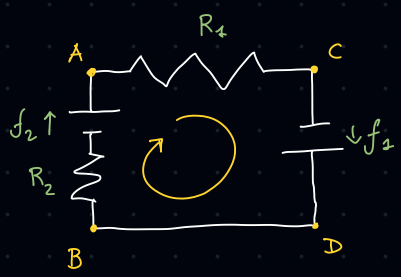
Nodi : LKC



Uscenti : Positive
Entranti : Negative

$$\Rightarrow I_4 + I_3 - I_1 - I_2 = \emptyset$$

MAGLIE : LKT



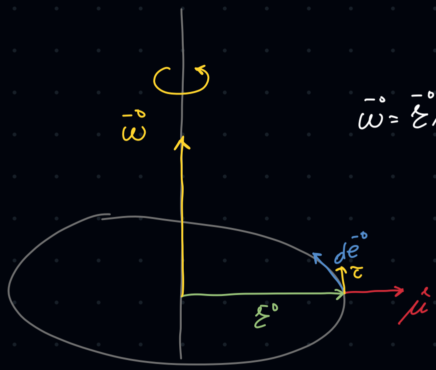
Senso ORARIO

Siccome è un loop $V_A - V_B = 0$ siccome $A \equiv B$

$\rightarrow$ Da Ohm $\cancel{(V_A - V_B)}^{0} + \Sigma_i f_i = \Sigma_i I \cdot R_i$

$\rightarrow$ Kirchhoff $\Sigma_i f_i = \Sigma_i I R_i$

$$\frac{d\hat{\alpha}}{dt} = \omega \wedge \alpha \quad \rightarrow$$



$$\vec{\omega} = \vec{\varepsilon} \wedge \vec{de} \quad \Rightarrow \frac{d\hat{\mu}}{dt} = \omega \wedge \mu \quad \rightarrow \quad \omega \cdot 1 \cdot \sin 90 = \omega \cdot \hat{\tau} = \frac{d\mu}{dt}$$

Inerziale gravitazionale

$$\begin{cases} F = G \cdot \dfrac{m \cdot M}{r^2} \\ F = m \cdot a \end{cases} \rightarrow \quad G \cdot \frac{m \cdot M}{r^2} = m \cdot \vec{a}$$

Martello

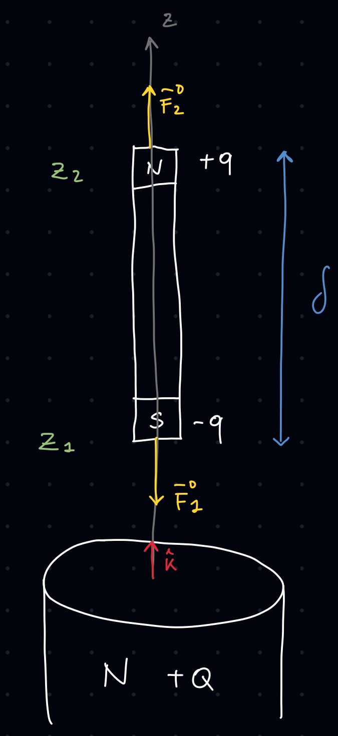
Piuma

$$\begin{cases} G \cdot \dfrac{m_m \cdot M_L}{r^2} = m_m \cdot \vec{a} \\ G \cdot \dfrac{m_p \cdot M_L}{r^2} = m_p \cdot \vec{a} \end{cases} \Rightarrow \quad \frac{m_m}{m_p} = \frac{m_m}{m_p}$$

Inerziale

gravitazionale

Ampere : Forze



$$F = \frac{\mu_0}{4\pi}\,\frac{q_m \cdot Q_m}{z^2} \quad \rightarrow \quad F_{TOT} = F_1 + F_2$$

$$\rightarrow \quad F_{TOT} = \frac{\mu_0}{4\pi}\left[\frac{q_m Q_m}{z_2^2} - \frac{q_m Q_m}{z_1^2}\right] \qquad \text{ma} \quad z_2 = (z_1+\delta)$$

$$= \frac{Q_m q_m \mu_0}{4\pi}\left[\frac{1}{(z_1+\delta)^2} - \frac{1}{z_1^2}\right] = K\left[\frac{\cancel{-z_1^2} + \cancel{z_1^2} + \cancel{\delta^2} - 2z_1\delta}{(z_1+\cancel{\delta})^2 z_1^2}\right]$$

$Z >> \delta$

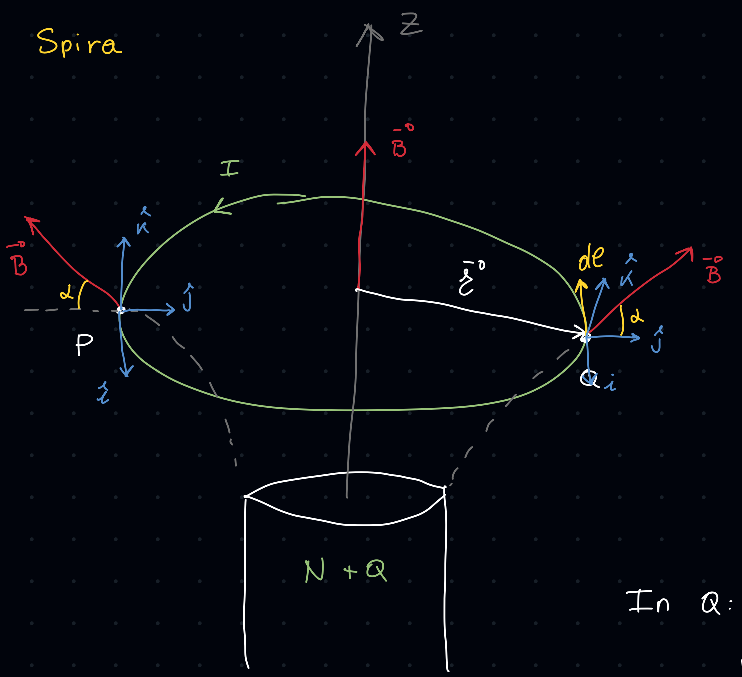
$$= -\hat{k}\,\underbrace{\frac{Q_m q_m \mu_0 2\delta \cancel{z}}{4\pi z^{\cancel{4}\,3}}}_{B}$$

$$\text{ma} \quad B = \frac{F}{q} = \frac{\mu_0 Q_m}{4\pi z^2} \quad \rightarrow \quad F = \frac{-\hat{k}\,B \cdot 2\delta}{z}$$

$$\vec{m} = \hat{k}\, q_m \delta$$

$$\vec{m} = \hat{k} \cdot 2\delta \quad \rightarrow \quad \vec{F} = -\frac{\vec{m}\, B\, 2}{z}$$

Spira



In P: $d\vec{l} = \hat{i}\, dl$

da Laplace $dF = I\, dl \wedge \vec{B}$

$$\rightarrow dF_P = I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dl & 0 & 0 \\ 0 & -B\cos\alpha & B\sin\alpha \end{vmatrix}$$

$$\Rightarrow dF_P = I\left[\hat{i}(0) - \hat{j}(dl\,B\sin\alpha) + \hat{k}(dl(-B\cos\alpha))\right]$$

$$= -\hat{j}\, I\, dl\, B\sin\alpha - \hat{k}\, I\, dl\, B\cos\alpha$$

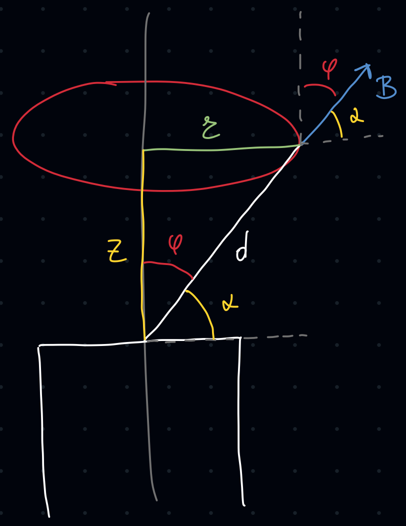
In Q: $d\vec{l} = -\hat{i}\, dl$

$$\Rightarrow dF_Q = I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -dl & 0 & 0 \\ 0 & B\cos\alpha & B\sin\alpha \end{vmatrix}$$

$$\rightarrow dF_Q = I\left[\hat{i}(0) - \hat{j}(-dl\,B\sin\alpha) + \hat{k}(-dl\,B\cos\alpha)\right] = \hat{j}\, I\, dl\, B\sin\alpha - \hat{k}\, I\, dl\, B\cos\alpha$$

$$\Rightarrow \underline{dF_{TOT} = -2\,\hat{k}\, I\, dl\, B\cos\alpha}$$

$$\Rightarrow F_{TOT} = -2\,\hat{k}\, I\, B\cos\alpha \int_0^{\pi} dl \quad \Rightarrow \quad F_{TOT} = -2\pi R\, I\, B\cos\alpha$$



$$F_{TOT} = -2\pi R I B \cos\alpha \qquad \text{ma } \alpha = 90 - \varphi \Rightarrow \cos(90-\varphi) = \sin(\varphi)$$

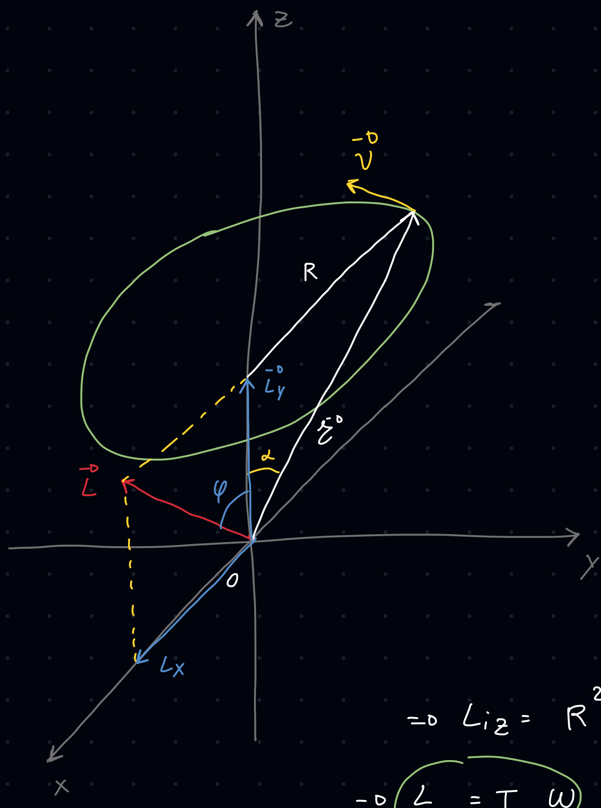
$$\Rightarrow F_{TOT} = -2\pi R I B \sin\varphi$$

se $z \gg \varrho \rightarrow \sin\varphi \approx \tan\varphi$

$$\begin{cases} d\sin\varphi = \varrho \\ d\cos\varphi = z \end{cases} \rightarrow \tan\varphi = \frac{\varrho}{z}$$

$$\rightarrow \vec{F}_{TOT} = -2\pi \varrho I B \cdot \frac{\varrho}{z} = -\frac{2\pi \varrho^2 I B}{z} \hat{k}$$

$$\vec{m} = I \cdot S \cdot \hat{k} \Rightarrow \vec{F}_{TOT} = -\frac{2\vec{m}B}{z}$$



$$\vec{L} = \vec{r} \wedge m\cdot\vec{v}$$

$$\vec{L}_i = \vec{r}_i \wedge m_i \vec{v}_i \Rightarrow |L_i| = r_i \cdot m_i v_i \overset{1}{\cancel{\sin(90)}}$$

$$\rightarrow |\vec{L}_i| = r_i\, m_i v_i$$

ma $v_i = \omega_i R \rightarrow |\vec{L}_i| = r_i\, m_i\, \omega_i R_i$

Siccome L_y e L_x si annullano

$L_z = L_i \cdot \cos(\varphi)$ ma $\varphi = 90 - \alpha$

$$\rightarrow \cos(90-\alpha) = \sin(\alpha)$$

$\rightarrow L_{iz} = r_i\, m_i\, \omega_i R_i \sin(\alpha)$ ma $r \sin\alpha = R$

$\Rightarrow L_{iz} = R^2 m_i \omega_i$ chiamo $\sum_i m_i R^2 = I_{TOT}$

$$\rightarrow L_{TOT} = I\,\omega$$

Corpo rigido

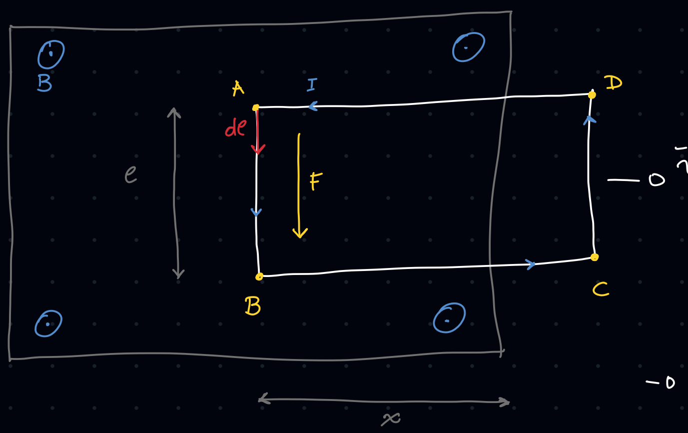
$$\Rightarrow \begin{cases} \vec{M} = \vec{r} \wedge \vec{F} \\ \vec{L} = \vec{r} \wedge m\vec{v} \end{cases} \rightarrow \frac{dL}{dt} = \underbrace{\cancel{\frac{dr}{dt} \wedge m v}}_{//} + \vec{r} \wedge m \cancel{\frac{dv}{dt}} F \rightarrow \frac{d\vec{L}}{dt} = \vec{M}$$

$$\Rightarrow \vec{M} = I \cdot \frac{d\omega}{dt} \rightarrow \vec{M} = I\alpha$$

Faraday - Neumann - Lenz

$$f_{em} = -\frac{d\phi_B}{dt}$$

Tagliato



$$\vec{F} = q\cancel{\vec{E}}^{\,0} + q(\vec{v}\wedge\vec{B})$$

$$f_{em} = \frac{L}{q} = \oint \vec{v}\wedge\vec{B}\; d\vec{l} = \int v \wedge B\, e_1 = \underline{vBe} \quad (1)$$

inoltre ϕ_B è funzione di t

$$\phi_B = \int B\cdot\hat{n}\,ds = B_\perp \cdot S \qquad \text{ma} \quad S = e\cdot x$$

$\rightarrow$ x dipende da t $\rightarrow$ a $\bar{t} = t+\Delta t$ $\rightarrow$ $\bar{x} = x - \Delta x$

con $\Delta x = v\cdot\Delta t$

$$\rightarrow \quad \phi_B = \int B\cdot\hat{n}\,ds = B\cdot e\cdot x$$

$$\rightarrow \quad \frac{d\phi_B}{dt} = \lim_{\Delta t\to 0}\frac{\phi_B(t+\Delta t)-\phi_B(t)}{\Delta t} = \lim_{\Delta t\to 0}\frac{\cancel{Bex} - Be\Delta x - \cancel{Bex}}{\Delta t} \rightarrow \frac{d\phi_B}{dt} = -Bev$$

$$\rightarrow \quad f_{em} = -\frac{d\phi}{dt} = \underline{Bev}$$

QED

Concatenato

$S = \text{cost}\,!$

$$B(t,\xi)$$

$$\Rightarrow \quad \frac{d\phi_B}{dt} = \lim_{\Delta t\to 0}\frac{\phi_B(t+\Delta t,\xi) - \phi_B(t,\xi)}{\Delta t}$$

Approx:

$$B \approx B(t,\xi) + \frac{\partial B(t+\Delta t)}{\partial t}\cdot\Delta t \quad \Rightarrow \quad \lim_{\Delta t\to 0}\frac{\cancel{\oint B(t,\xi)\,d\vec{s}} - \int\frac{\partial B}{\partial t}(t+\cancel{\Delta t}^{\,0},\xi)\cancel{\Delta t}\,d\vec{s} - \cancel{\int B(t,\xi)\,d\vec{s}}}{\cancel{\Delta t}}$$

$$\rightarrow \quad \frac{d\phi_B}{dt} = -\oint\frac{\partial B}{\partial t}(t,\xi)\,d\vec{s} \quad \rightarrow \quad -\frac{d\phi_B}{dt} = +\oint\frac{\partial B}{\partial t}\,\hat{n}\,ds \quad (2)$$

Tutto insieme

$$-\frac{d\phi_B}{dt} = -\oint\frac{\partial B}{\partial t}\,d\vec{s} + \underbrace{\int(\vec{v}\wedge\vec{B})\,de}$$

CASO GENERALE

$\leftarrow$ nel flusso concat $\vec{v} = 0$

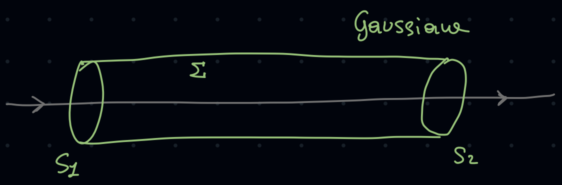
$$\rightarrow \text{ma} \quad f_{em} = \frac{L}{q} \rightarrow \frac{\cancel{q}\oint E\,de + q\int(\vec{v}\wedge\vec{B})\,de}{\cancel{q}}$$

$$\Rightarrow \quad \oint\vec{E}\,d\vec{e} + \cancel{\oint(\vec{v}\wedge\vec{B})\,d\vec{e}} = -\oint\frac{\partial\vec{B}}{\partial t}\hat{n}\,ds + \cancel{\oint(\vec{v}\wedge\vec{B})\,d\vec{e}}$$

$$\rightarrow \oint \vec{E}\, d\vec{\ell} = -\oint \frac{\partial \vec{B}}{\partial t}\, \hat{n}\, d\vec{S} \rightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

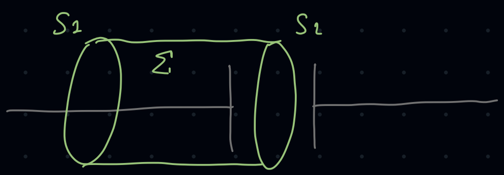
Maxwell
E.S.

Corrente di spostamento



$$I = \oint J \cdot \hat{n}\, ds = \emptyset \Rightarrow \nabla J = \emptyset$$

Corrente Stazion.



$\oint J \cdot \hat{n}\, ds \neq 0$ ma come ??

$\rightarrow$ Legge conservazione: $-dq = \int J \cdot \hat{n}\, ds \cdot dt \rightarrow -\frac{dq}{dt} = \int \vec{J} \cdot \hat{n}\, ds \qquad \rho = \frac{dq}{dV}$

$$\Rightarrow -\frac{d}{dt} \int_V \rho\, dV = \int \vec{J} \cdot \hat{n}\, ds \rightarrow -\int_V \frac{\partial \rho}{\partial t}\, dV = \int J \cdot \hat{n}\, ds$$

$$\Rightarrow \vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t}$$

Non corrente Stazionaria

$\rightarrow$ Legge Ampere: $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \rightarrow$ Diverg $\rightarrow \vec{\nabla} (\vec{\nabla} \wedge \vec{B}) = \mu_0 \vec{\nabla} \vec{J} \rightarrow \vec{\nabla} \vec{J} = 0$

Valido solo per correnti staz.

Termine Maxwell $\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow$ A-M: $\vec{\nabla} \wedge \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$

$$\rightarrow \text{DIV} \rightarrow \nabla (\nabla \wedge B) = \mu_0 \left(\nabla J + \varepsilon_0 \frac{\partial (\nabla E)}{\partial t} \right)$$

$$\rightarrow \vec{\nabla} \vec{J} + \cancel{\varepsilon_0} \frac{1}{\cancel{\varepsilon_0}} \frac{\partial \rho}{\partial t} \rightarrow \nabla J = -\frac{\partial \rho}{\partial t}$$

Amp-Max : $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Faraday : $\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightleftarrows f_{em} = -\frac{d\phi_B}{dt}$

CAMPO E

$$\nabla \wedge (\nabla \wedge E) = -\frac{\partial(\nabla \wedge B)}{\partial t} \rightarrow \nabla(\nabla E) - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t}$$

$$\rightarrow \frac{\nabla \rho}{\varepsilon_0} - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t} \rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \frac{\nabla \rho}{\varepsilon_0} \quad (1)$$

CAMPO B

$$\nabla \wedge (\nabla \wedge B) = \mu_0 \nabla \wedge J + \mu_0 \varepsilon_0 \frac{\partial(\nabla \wedge E)}{\partial t} \rightarrow \cancel{\nabla(\nabla B)}^{\,0} - \nabla^2 B = \mu_0 \nabla \wedge J - \frac{\partial^2 B}{\partial t^2} \mu_0 \varepsilon_0$$

$$\rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} - \nabla^2 B = \mu_0 \nabla \wedge J \quad (2)$$

Solo x

$$(1) \cdot \rho = J = 0 \rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = 0 \rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t} - \frac{\partial^2 E}{\partial x} = 0$$

Eq Diff Sol : $E(t) = A \sin(kx - \omega t)$

Trovo "ω"

$$\left.\begin{aligned} E_1 &= A \sin(kx_1 - \omega t_1) \\ E_2 &= A \sin(kx_2 - \omega t_2) \end{aligned}\right\} E_1 = E_2 \Leftrightarrow kx_1 - \omega t_1 = kx_2 - \omega t_2 \rightarrow k(x_2 - x_1) = \omega(t_2 - t_1)$$

$$\rightarrow x_2 - x_1 = \frac{\omega}{k}(t_2 - t_1) \Rightarrow v = \frac{\omega}{k} \rightarrow \omega = v \cdot k$$

$$\Rightarrow E(t) = A \sin(kx - vkt)$$

$$\rightarrow \frac{\partial E}{\partial t} = -vkA\cos(kx - vkt) \rightarrow \frac{\partial^2 E}{\partial t} = -v^2k^2 A \sin(kx - vkt)$$

$$\frac{\partial E}{\partial x} = +kA\cos(kx - vkt) \rightarrow \frac{\partial^2 E}{\partial x^2} = -k^2 A \sin(kx - vkt)$$

$$\Rightarrow -\mu_0 \varepsilon_0 v^2 k^2 A \sin(kx - vkt) + k^2 A \sin(kx - vkt) = 0$$

$$\rightarrow k^2 A \sin(kx - vkt)\left[-\mu_0 \varepsilon_0 v^2 + 1\right] = 0 \rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

E
$$\begin{cases} \nabla E = \dfrac{\rho}{\varepsilon_0} & \to \displaystyle\int E\, dl = \dfrac{Q}{\varepsilon_0} \quad (1) \\ \nabla \wedge E = 0 & \to \displaystyle\int \vec{E} \cdot \hat{n}\, ds = 0 \ (2.A) \to \phi_E = \emptyset \end{cases}$$

$$\hookrightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial B}{\partial t} \to \int \nabla \wedge E\, ds = -\int \frac{\partial B}{\partial t} \hat{n}\, ds \to \int \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t} \hat{n}\, ds$$

$$\Rightarrow \int E \cdot dl + \int (\vec{v} \wedge \vec{B})\, d\vec{l} = -\int \frac{\partial B}{\partial t} \cdot \hat{n}\, ds + \oint (\vec{v} \wedge \vec{B})\, d\vec{l} \quad (2.B)$$

B
$$\begin{cases} \nabla B = 0 & \to \displaystyle\int B \cdot \hat{n}\, ds = \emptyset \ (3) \to \phi_B = \emptyset \\ \nabla \wedge B = \mu_0 J \end{cases}$$

$$\hookrightarrow \int \nabla \wedge B\, \hat{n}\, ds = \mu_0 \int J \cdot \hat{n}\, ds \to \int B \cdot dl = \mu_0 I \quad (4.A)$$ Ampere

$$\hookrightarrow \vec{\nabla} \wedge \vec{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\int \nabla \wedge B\, \hat{n}\, ds = \mu_0 \int J \cdot \hat{n}\, ds + \mu_0 \varepsilon_0 \int \frac{\partial E}{\partial t} \cdot \hat{n}\, ds$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{l} \quad \frac{1}{c^2} \oint (\vec{v} \wedge \vec{E})\, dl = \mu_0 \underbrace{\int J \cdot \hat{n}\, ds}_{I} + \mu_0 \varepsilon_0 \left[\int \frac{\partial E}{\partial t} \hat{n}\, ds - \oint (\vec{v} \wedge \vec{E})\, d\vec{l} \right]$$

Lorenz $\quad F = q \cdot E + q(\vec{v} \wedge \vec{B})$

1) Prop. alla carica
2) legata al sin dell angolo tra v e B
3 $F \perp B$ e $F \perp v$

$$\Rightarrow F = q \cdot v \cdot B \cdot \sin\alpha \to F = q(\vec{v} \wedge \vec{B})$$

Prodotto scalare

$$\vec{A} = ix + \hat{j}y + \hat{k}z \qquad \vec{B} = \hat{i}x' + \hat{j}y' + \hat{k}z' \to \vec{A} \cdot \vec{B} = A \cdot B \cos\alpha$$

$\cos(90) = 0 \to$ Versori perp $= \emptyset$

$$\begin{aligned} &\hat{i}x \cdot \hat{i}x' + \cancel{ix\,\hat{j}y'} + \cancel{\hat{i}\hat{k}z'} + \\ &\cancel{\hat{j}y\,\hat{i}x'} + \hat{j}y\,\hat{j}y' + \cancel{\hat{j}y\,\hat{k}z'} + \\ &\cancel{\hat{k}z\,\hat{i}x'} + \cancel{\hat{k}z\,\hat{j}y'} + \hat{k}z\,\hat{k}z' = xx' + yy' + zz' \end{aligned}$$

Prodotto vettoriale $\quad \sin(\emptyset) = \emptyset \to$ Versori paralleli $= \emptyset$

$$\vec{A} \cdot \vec{A}' = \det \begin{vmatrix} i & \hat{j} & k \\ x & y & z \\ x' & y' & z' \end{vmatrix} =$$