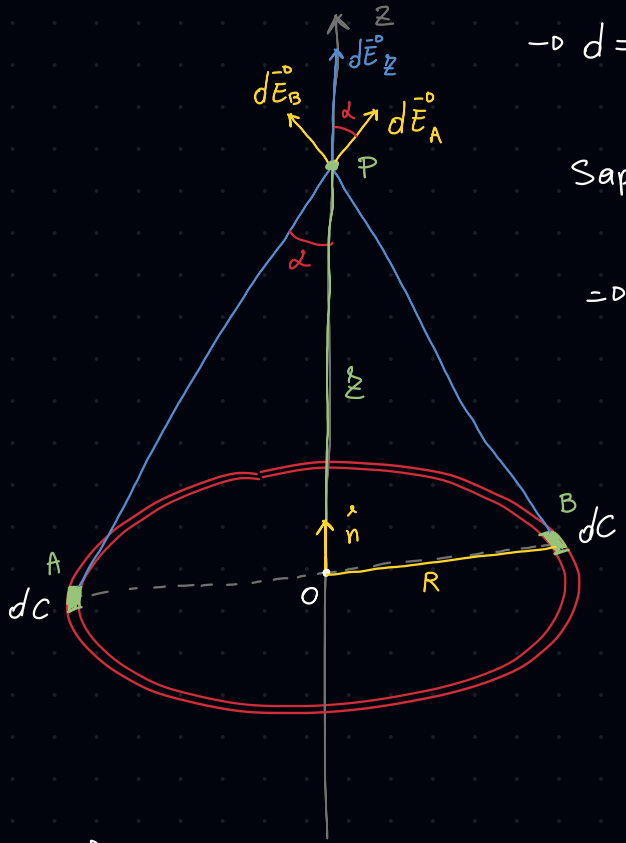
ANELLO CARICO

$\overline{AP} = \sqrt{R^2 + z^2}$ Siccome $|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{d^2}$



$\rightarrow d = AP \Rightarrow dE_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{R^2+z^2}$

Sappiamo che λ = dens. lineare $\Rightarrow \lambda = \frac{dQ}{dl}$

$\Rightarrow dQ = \lambda\, dl \Rightarrow Q_{TOT} = \int \lambda\, dl$

ma l non è altro che la circonferenza

$\Rightarrow Q_{TOT} = \int \lambda\, dc$

Dal disegno sappiamo che $d\vec{E}_{\hat{n}} = d\vec{E}_A + d\vec{E}_B$

è SOLO LUNGO z $\Rightarrow$ Sommiamo solo le comp lungo z, dato che le altre si eliminano a 2 a 2.

$\rightarrow d\vec{E}_z = d\vec{E}_A \cos\alpha = \frac{\lambda\, dc}{4\pi\varepsilon_0 (R^2+z^2)} \cdot \cos\theta$

Per definizione $\cos\alpha = \frac{Cat}{Hip} = \frac{z}{AP} = \frac{z}{\sqrt{R^2+z^2}}$

$\Rightarrow d\vec{E}_z = \frac{\lambda\, dc}{4\pi\varepsilon_0 (R^2+z^2)} \cdot \frac{z}{\sqrt{R^2+z^2}} \Rightarrow E_z = \int_C^C \frac{\lambda}{4\pi\varepsilon_0 (R^2+z^2)} \frac{z}{\sqrt{R^2+z^2}}\, dc$

$\frac{1}{R^2+z^2} \cdot \frac{1}{\sqrt{R^2+z^2}} = \frac{1}{(R^2+z^2)\cdot(R^2+z^2)^{\frac{1}{2}}} = \frac{1}{(R^2+z^2)^{1+\frac{1}{2}}}$

$\Rightarrow E_z = \frac{\lambda z}{4\pi\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}} \cdot \int_C dc = \frac{\cancel{2}\cancel{\pi} R\, \lambda z}{2\,\cancel{4}\cancel{\pi}\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}$ (1)

$$\boxed{\Rightarrow \vec{E}_z = \frac{R z \lambda}{2\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}\,\hat{n}}$$

E se $z >> R$? Siccome $Q = \int \lambda\, dc = \lambda 2\pi R$

$\Rightarrow$ dalla (1) $\frac{\overbrace{2\pi R\lambda}^{Q_{TOT}} z}{4\pi\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}} = \frac{Q_{TOT}\cdot z}{4\pi\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}$

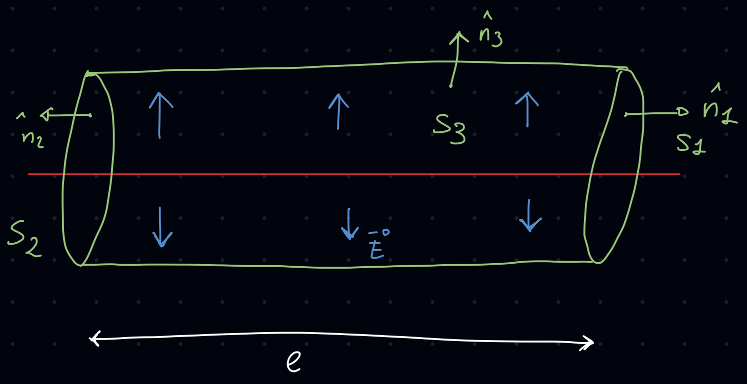
Se $z >> R$ $E_z = \frac{Q_{TOT}\cdot \cancel{z}}{4\pi\varepsilon_0 (\cancel{z}^2)^{\cancel{\frac{3}{2}}\,2}}$

$$\boxed{\Rightarrow \vec{E}_z = \frac{Q_{TOT}}{4\pi\varepsilon_0 z^2}\,\hat{n}}$$

Legge di Coulomb per la carica puntiforme

Filo carico - Coulomb

$\vec E \perp \hat n_1$ $\quad$ $\vec E \perp \hat n_2$



$$\phi_E = \oint \vec E \cdot \hat n\, dS = \cancel{\int_S E \cdot \hat n_1\, dS_1} + \cancel{\int_S E \hat n_2\, dS_2} + \int_S E \hat n_3\, dS_3$$

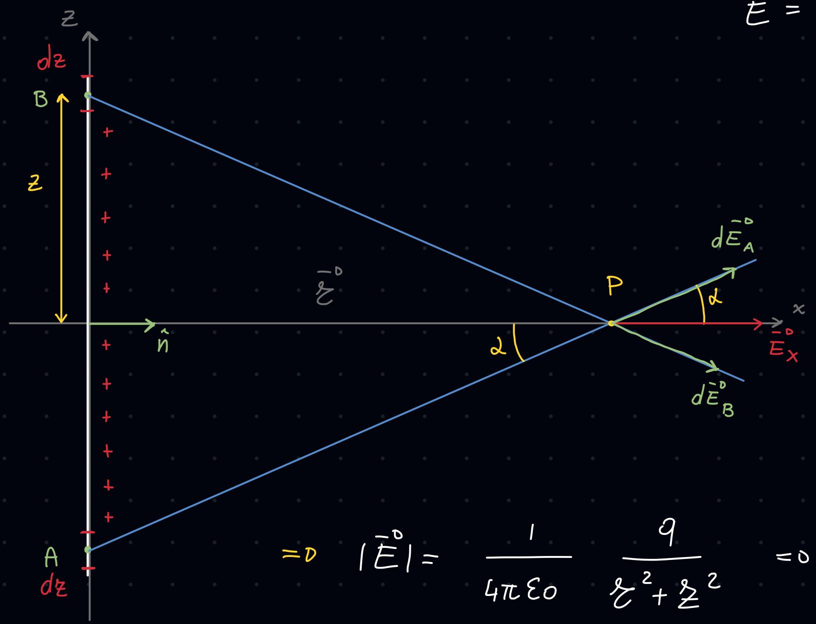
$$= \int \vec E \cdot n_2\, dS_2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \phi_E = |\vec E| \cdot 2\pi r \ell = \frac{Q}{\varepsilon_0} \quad \text{ma } \ell = \infty$$

$\Rightarrow \lambda =$ densità lineare di carica $= \dfrac{Q}{\ell} \Rightarrow Q = \lambda \ell$

$$|\vec E|\, 2\pi r \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0} \Rightarrow \vec E = \frac{\lambda}{2\pi r \varepsilon_0}\hat n$$

Filo carico senza Coulomb



$$\vec E = \frac{\vec F}{q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ Non conosco r $\rightarrow$ AP / BP

$\begin{cases} AP = r\cos\alpha \\ BP = r\cos\alpha \end{cases} \Rightarrow AP = BP = r\cos\alpha$

Non conosco α

$$AP^2 = z^2 + r^2$$

$$\Rightarrow |\vec E| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+z^2} \Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}$$

siccome $\lambda = \dfrac{dQ}{d\ell} \Rightarrow dq = \lambda\, d\ell \Rightarrow dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dz}{r^2+z^2}$

nel nostro caso $d\ell = dz$

$\rightarrow$ le componenti z ed y di $d\vec E_A + d\vec E_B$ si elidono a vicenda $\Rightarrow$ ci serve solo $\vec E_z$

$\Rightarrow d\vec E_z = d\vec E \cos\alpha$ ma non conosciamo α

$\rightarrow z = AP\cos\alpha \Rightarrow z = \sqrt{z^2+r^2}\cos\alpha \Rightarrow \cos\alpha = \dfrac{z}{\sqrt{z^2+r^2}}$

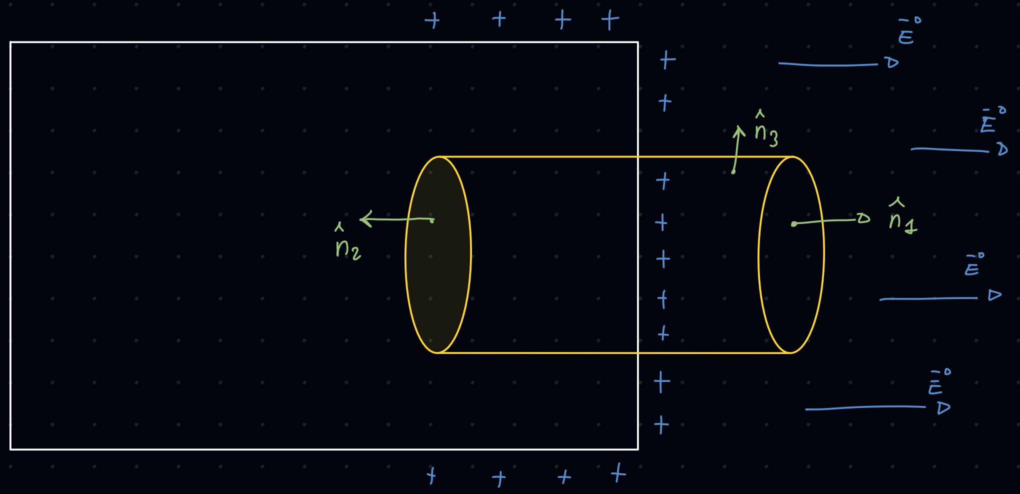
$$\Rightarrow dE_z = dE\cos\alpha = \frac{z}{\sqrt{z^2+r^2}}\, dE$$

$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dz}{r^2+z^2} \cdot \frac{z}{\sqrt{z^2+r^2}} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda z\, dz}{(r^2+z^2)^{\frac{3}{2}}} \quad (1)$$

Integriamo per ottenere $\vec{E}_{TOT}$

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{\overset{\text{cost}}{\lambda}}{(r^2+z^2)^{\frac{3}{2}}}\, dz = 2\frac{\lambda}{4\pi\varepsilon_0} \int_0^{\ell} \frac{z}{(r^2+z^2)^{\frac{3}{2}}}\, dz = \underbrace{\frac{\cancel{2}\lambda \cancel{z}}{\cancel{2}4\pi\varepsilon_0 r^{\cancel{2}}} \cdot \frac{\ell}{\sqrt{r^2+\ell^2}}}_{|\vec{E}|}$$

Teorema di Coulomb

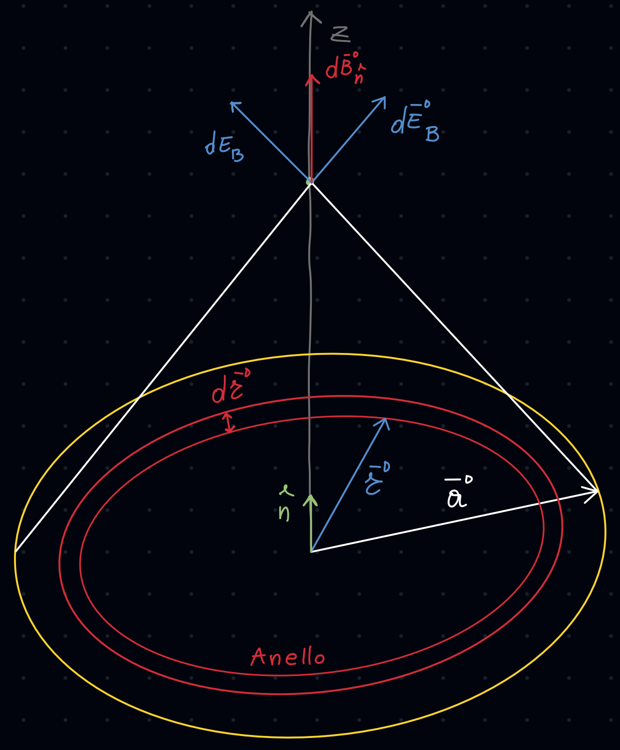


$$\phi_E = \oint \vec{E} \cdot \hat{n}\, dS = \int \vec{E} \cdot \hat{n}_1\, dS_1 + \underbrace{\int \vec{E} \cdot \hat{n}_2\, dS_2}_{\int = \emptyset} + \underbrace{\int \vec{E} \cdot \hat{n}_3\, dS_3}_{\hat{n}_3 \perp \vec{E}} = \int \vec{E} \cdot \hat{n}_1\, dS_1 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow |\vec{E}| \cdot S_1 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{S_1} \cdot \frac{1}{\varepsilon_0} \quad \text{ma} \quad \sigma = \frac{Q}{S} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Alternativamente

$$\sigma = \frac{dQ}{dS} \Rightarrow dq = \sigma\, dS \rightarrow q = \int \sigma\, dS \rightarrow E = \frac{\int \sigma\, dS}{S \varepsilon_0} = \sigma \frac{\cancel{\int dS}}{\cancel{S} \varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$



Disco

Per Trovare B_{Disco} ci basta sommare dB_{Anello} per Tutto il raggio del disco

$$\rightarrow \quad d\vec{B}_{Anello} = \frac{dQ\, z}{4\pi\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}\hat{n} \qquad (1)$$

$$Q_{TOT}\ \text{Disco} = \sigma \cdot S_{disco} = \sigma \cdot \pi a^2$$

Carica di 1 Anello = dQ_{Anello}

chiamo $a = r$ $\rightarrow$ Faccio il differenziale

$$\rightarrow \quad dQ_{Anello} = \sigma \cdot 2\pi r \cdot dr \qquad (2)$$

Sostituisco (2) in (1)

$$\rightarrow \quad d\vec{B} = \frac{z\,\sigma\, \cancel{2\pi}\, r \cdot dr}{\underset{2}{\cancel{4\pi}}\,\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}} = \frac{\sigma\, z\, r\, dr}{2\varepsilon_0 (R^2+z^2)^{\frac{3}{2}}}$$

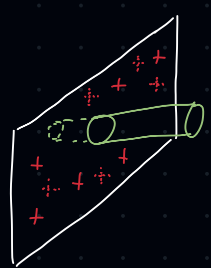
$$\Rightarrow \quad B = \frac{\sigma}{2\varepsilon_0} z \int_0^a \frac{r}{(R^2+z^2)^{\frac{3}{2}}}\, dr = \frac{\sigma}{2\varepsilon_0} z \left[-\frac{1}{(z^2+R^2)^{\frac{1}{2}}} \right]_0^a = -\frac{\sigma}{2\varepsilon_0} z \left[\frac{1}{(z^2+a^2)^{\frac{1}{2}}} - \frac{1}{z} \right]$$

$$\rightarrow \quad B = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}} \right]$$

Se $a \rightarrow \infty$ Disco $\rightarrow$ Lamina

infatti $\displaystyle \lim_{a \to \infty} \frac{\sigma}{2\varepsilon_0}\left[1 - \underbrace{\frac{z}{+\infty}}_{0} \right] \rightarrow \frac{\sigma}{2\varepsilon_0}$ lamina carica

LAMINA



$$\phi_E = \oint \vec{E} \cdot \hat{n}\, ds = \frac{Q}{\varepsilon_0} \quad \rightarrow \quad \phi_E = \phi_{S_1} + \phi_{S_2} + \cancelto{0}{\phi_L} = 2\phi_{S_1}$$

$$\phi_{S_1} = E \cdot S_1 = \frac{Q}{\varepsilon_0} \quad \rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \quad \phi_E = \frac{\sigma}{2\varepsilon_0}$$