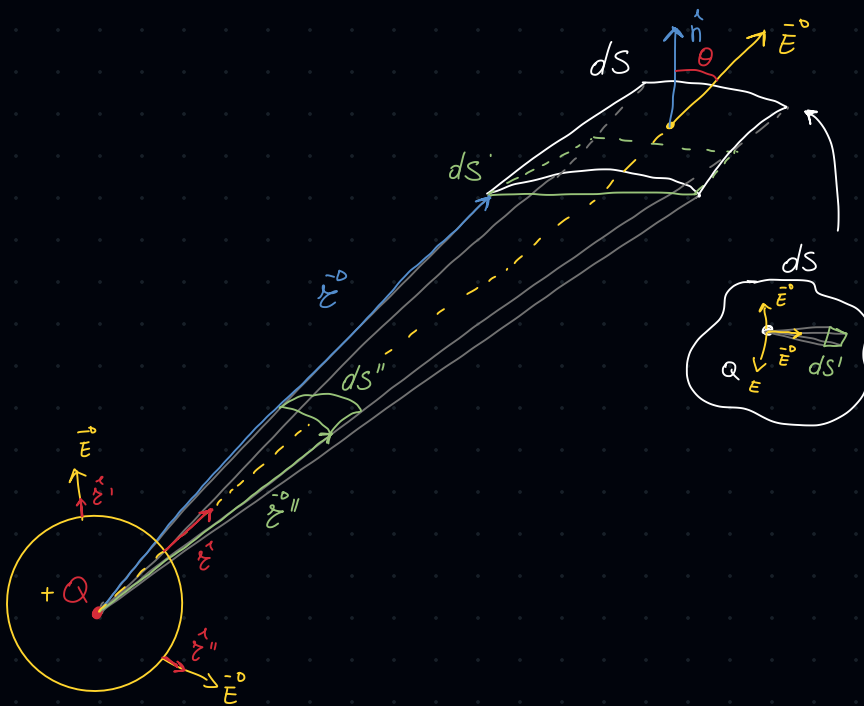


$$\Phi = \oint_S \vec{E} \cdot \hat{n} d\vec{s} = \oint_S \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cdot \hat{r} \cdot \hat{n} d\vec{s}$$

$$\Rightarrow \Phi = \frac{q}{4\pi\epsilon_0 r^2} \int \hat{r} \cdot \hat{n} d\vec{s} = 0 \quad \hat{r} \cdot \hat{n} = \hat{r} \cdot \hat{n} \cdot \cos(0) = 1$$

$$= \frac{q}{4\pi\epsilon_0 r^2} \int d\vec{s} = \frac{q}{4\pi\epsilon_0 r^2} \cdot 4\pi r^2$$

$$= \frac{q}{\epsilon_0} \quad \text{Teorema di Gauss}$$



$$ds \cos \theta = ds'$$

$$\frac{ds}{r^2} = \frac{ds''}{r''^2}$$

$$\Rightarrow \Phi = \oint_S (\vec{E} \cdot \hat{n}) d\vec{s} = \oint_S \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cdot \hat{r} \cdot \hat{n} d\vec{s} = \frac{q}{4\pi\epsilon_0} \oint_S \frac{1}{r^2} ds \cos \theta$$

$$\hat{r} \cdot \hat{n} = \hat{r} \cdot \hat{n} \cdot \cos \theta = \hat{r} \cdot \hat{n} \cdot \cos \theta$$

$$\Rightarrow \Phi = \frac{q}{4\pi\epsilon_0} \oint_S \frac{1}{r^2} ds' = \frac{1}{r^2} ds'_{\text{sup}} = \frac{1}{r^2} \cdot 4\pi r^2$$

Radiale Sfera NORMALE

$$\Phi = \frac{q}{4\pi\epsilon_0} \cdot 4\pi = \frac{q_{\text{INT}}}{\epsilon_0} \quad \text{QED}$$

Non costante
sup qualsiasi!

Inoltre definiamo Angolo Solido $d\Omega = \frac{1}{R^2} dS = \frac{dS \cos \theta}{R^2}$

$$\Rightarrow \phi = \oint_S \vec{E} \cdot \hat{n} dS = \frac{Q}{4\pi\epsilon_0} \oint_S \frac{1}{R^2} \underbrace{\hat{r} \cdot \hat{n}}_{\cos \theta} dS = \frac{Q}{4\pi\epsilon_0} \oint_S \underbrace{\left(\frac{1}{R^2} dS \cos \theta \right)}_{d\Omega} = \frac{Q}{4\pi\epsilon_0} \oint d\Omega =$$

$$= \frac{Q}{4\pi\epsilon_0} \cdot \underbrace{4\pi}_{\Omega} = \frac{Q}{\epsilon_0} \quad QED$$

Inoltre $\oint \Omega = \oint \frac{1}{R^2} dS \Rightarrow \frac{Q}{4\pi\epsilon_0} \oint \frac{1}{R^2} dS = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{R^2} \cdot 4\pi R^2 = \frac{Q}{\epsilon_0} \quad QED$



$$\phi_{TOT} = \phi_{S1} + \phi_{S2}$$

$$\phi_{S2} = \oint_{S2} (\vec{E} \cdot \hat{n}_2) dS_2 = \frac{q}{4\pi\epsilon_0} \int_{S2} \frac{1}{R^2} \underbrace{\hat{r} \cdot \hat{n}_2}_{\cos \theta} dS_2 = \frac{q}{4\pi\epsilon_0} \int_{S'_2} \frac{1}{R^2} dS'_2$$

$\hookrightarrow dS_2 \cos \theta = dS'_2$

$$\phi_{S1} = \oint_{S1} (\vec{E} \cdot \hat{n}_1) dS_1 = \frac{q}{4\pi\epsilon_0} \oint_{S1} \frac{1}{R^2} \cdot \underbrace{\hat{r} \cdot \hat{n}_1}_{\cos \alpha} dS_1 = - \frac{q}{4\pi\epsilon_0} \oint_{S'_1} \frac{1}{R^2} dS'_1$$

$\hookrightarrow dS_2 \cdot \cos(\alpha) \text{ con } \alpha > 90 \Rightarrow \cos(\alpha) < 0$

$$\Rightarrow \underline{\phi_{TOT} = \phi_1 + \phi_2 = 0}$$

Gauss: $\phi_E = \oint_S \vec{E} \cdot \vec{n} \, dS = \frac{Q_{int}}{\epsilon_0}$

$\Rightarrow \phi_E = \oint_S \vec{E} \cdot \vec{n} \, dS = \int_V \vec{\nabla} \cdot \vec{E} \, dV = \frac{Q}{\epsilon_0}$

$\Rightarrow \phi_E = \int_V \vec{\nabla} \cdot \vec{E} \, dV = \frac{1}{\epsilon_0} \int_V \rho \, dV$

$\Rightarrow \phi_E = \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{II Eq di Maxwell (sarebbe la prima)}$

ma Tramite il Teorema della divergenza $\int_S \vec{A} \cdot \vec{n} \, dS = \int_V \vec{\nabla} \cdot \vec{A} \, dV$

Definiamo $\rho = \frac{dQ}{dV} \Rightarrow \rho \, dV = dQ$
 $\Rightarrow Q = \int_V \rho \, dV = \rho V$
Densità Volumetrica di carica