Autoinduzione

I in un circuito genera un campo magnetico

Da Biot-Savart $\to$ $\vec{B}_{FILO} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{\ell} \wedge \Delta\vec{r}}{|\Delta \vec{r}|^3}$

$\Rightarrow$ $\phi_B = \int_S \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{\ell} \wedge \Delta\vec{r}}{\Delta r^3} \hat{n}\, dS$ flusso di $\vec{B}$ generato da I

AUTOFLUSSO

$\to$ Tutto dipende dalla geometria del circuito, Tranne I

$\Rightarrow$ $\phi_B = L \cdot I$

INDUTTANZA

Se $B \neq cost$

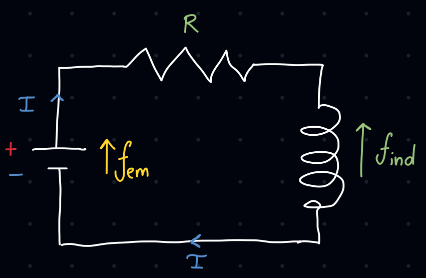
Se $B \neq cost$ $\to$ $\phi_B \neq Cost$ $\Rightarrow$ $\frac{d\phi_B}{dt} \neq 0$

$\to$ $\frac{d\phi_B}{dt} = L \frac{dI}{dt}$ ma $f_{em} = -\frac{d\phi_B}{dt}$ $\Rightarrow$ $f_{em} = -L \frac{dI}{dt}$

Forza elettro motrice Autoindotta

Circuito RI

Secondo la legge di Ohm generalizzata



$$\sum_i f_{em} = I \cdot \sum_i R_i \Rightarrow f_{em} - f_{ind} = R \cdot I$$

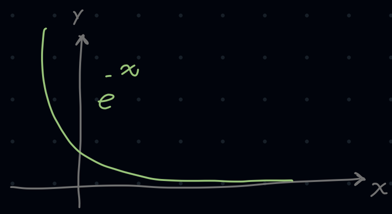
$$\Rightarrow f_{em} = f_{em} + R \cdot I \quad \text{ma} \quad f_{ind} = -L \frac{dI}{dt}$$

$\Rightarrow f_{em} = RI + L\dfrac{dI}{dt}$ Equazione differenziale con soluzione del tipo: $I = Ae^{\alpha t} + D$

$$\Rightarrow \frac{dI}{dt} = A\alpha e^{\alpha t} \to f_{em} = L\alpha A e^{\alpha t} + RAe^{\alpha t} + RD \qquad f_{em} = 0 \Rightarrow f_{em} = R \cdot D \to D = \frac{f}{R}$$

per cui $LAe^{\alpha t} + RAe^{\alpha t} = 0 \to Ae^{\alpha t}(\alpha L + R) = 0 \to \alpha L + R = 0 \to \alpha = -\dfrac{R}{L}$

$\Rightarrow I(t) = Ae^{-\frac{R}{L}t} + \dfrac{f}{R}$ Troviamo A



$$\to f(0) = 0 \to A\underbrace{\cancel{e^{-\frac{R}{L}\cdot 0}}}_{1} + \frac{f}{R} = 0 \to A + \frac{f}{R} = 0 \to A = -\frac{f}{R}$$

$$\Rightarrow I(t) = -\frac{f}{R}e^{-\frac{R}{L}t} + \frac{f}{R}$$

Se $t \to \infty$ $\quad \lim_{t\to\infty} -\dfrac{f}{R}e^{-\frac{R}{L}t} + \dfrac{f}{R} = \lim_{t\to\infty} \dfrac{f}{R}\Big(1 - \underbrace{e^{-\frac{R}{L}t}}_{0}\Big)$

$\hookrightarrow I\big|_{t\to\infty} = \dfrac{f}{R}$ CORRENTE DI REGIME