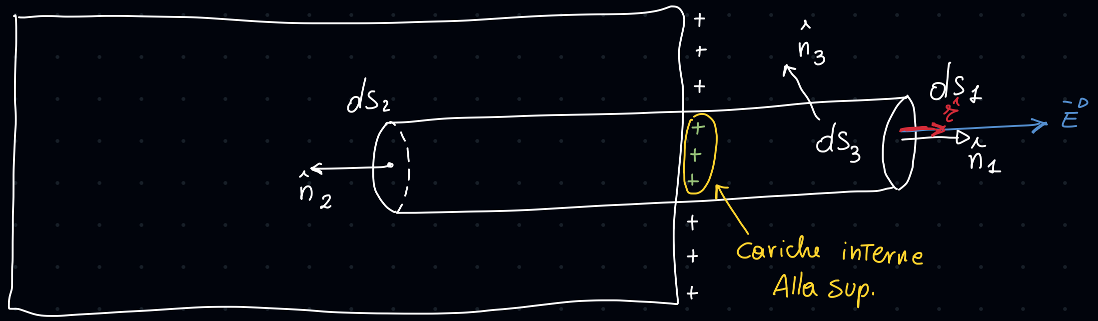
TEOREMA DI COULOMB



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} = \oint |\vec{E}| \cdot \hat{\tau} \cdot \hat{n}_1 \, dS_1 + \oint E \, \hat{\tau} \, \hat{n}_2 \, dS_2 + \oint E \, \hat{\tau} \, \hat{n}_3 \, dS_3$$

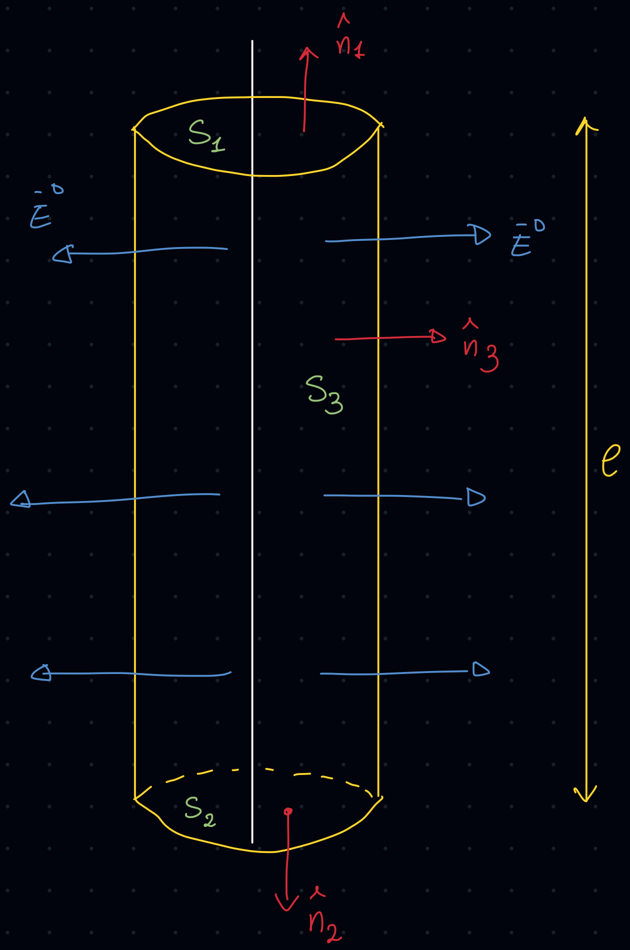
- $\hat{\tau} \cdot \hat{n}_1 \, dS = dS$
- $\hat{\tau} \cdot \hat{n}_2 \, dS = \underline{0}$ perchè non esiste $\vec{E}$ in quella zona!
- $\hat{\tau} \cdot \hat{n}_3 \, dS = \tau n \cdot \cos(\theta) \, dS = \underline{0}$ perchè $\theta = 90 \rightarrow \hat{n}_3 \perp \vec{E}$

$$\Rightarrow \phi_{TOT} = \oint \vec{E} \cdot \hat{n}_1 \, dS = \frac{Q_{int}}{\varepsilon_0} = |\vec{E}| \cdot S_1 \Rightarrow |\vec{E}| = \frac{Q_{int}}{S_1} \cdot \frac{1}{\varepsilon_0}$$

$\frac{Q_{int}}{S_1} \rightarrow \sigma$ densità di carica superficiale

$$\rightarrow |\vec{E}| = \frac{\sigma}{\varepsilon_0}$$

FILO CARICO



$$\phi_{TOT} = \phi_{S_1} + \phi_{S_2} + \phi_{S_3} \quad \text{ma} \quad \vec{E} \perp \hat{n}_1 \,,\ \vec{E} \perp \hat{n}_3$$

$$\Rightarrow \phi_{TOT} = \phi_{S_3} = \oint \vec{E} \cdot \hat{n}_3 \, dS_3 = \frac{Q_{int}}{\varepsilon_0}$$

$$\phi = \oint \vec{E} \cdot \hat{n}_3 \, dS_3 = |\vec{E}| \cdot \left[S_2 \cdot \ell \right] = |\vec{E}| \cdot 2\pi R \cdot \ell = \frac{Q}{\varepsilon_0} \quad (1)$$

ma $\ell = \infty \Rightarrow$ Densità LINEARE di carica

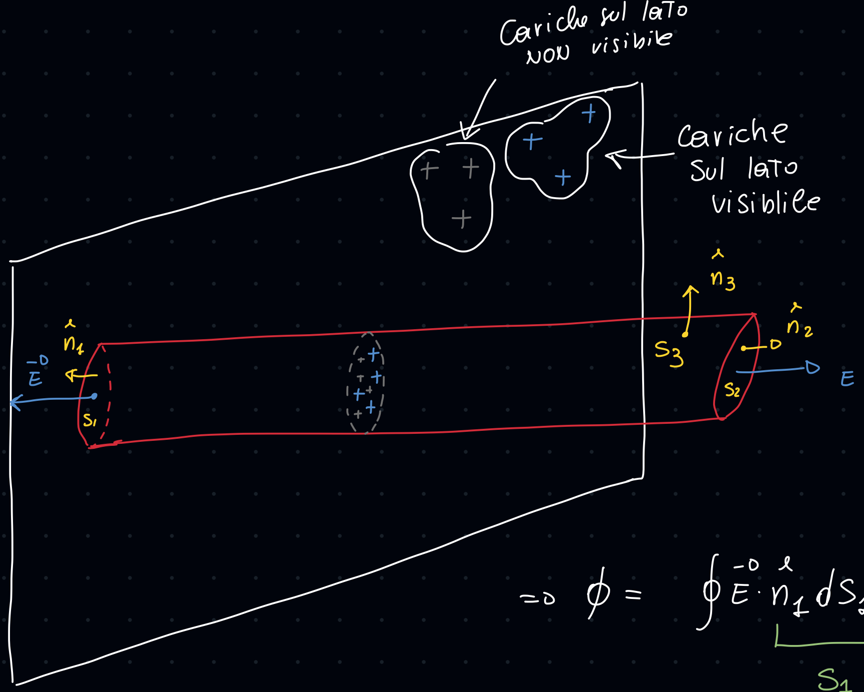
$$\lambda = \frac{Q}{\ell} \Rightarrow Q = \lambda \cdot \ell \quad (2)$$

$$\Rightarrow \text{dalla } (1) \text{ e } (2) \rightarrow |\vec{E}| \cdot 2\pi R \cdot \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0}$$

$$\rightarrow |\vec{E}| \, 2\pi R = \frac{\lambda}{\varepsilon_0} \Rightarrow |\vec{E}| = \frac{\lambda}{2\pi R \varepsilon_0}$$

Inoltre $\vec{E} = \dfrac{\lambda}{2\pi R \varepsilon_0} \cdot \hat{n}_3$

STRATO CARICO



$$\phi_{TOT} = \phi_1 + \phi_2 + \phi_3$$

ma $\hat{n}_3 \perp \vec{E} \Rightarrow \phi_3 = 0$

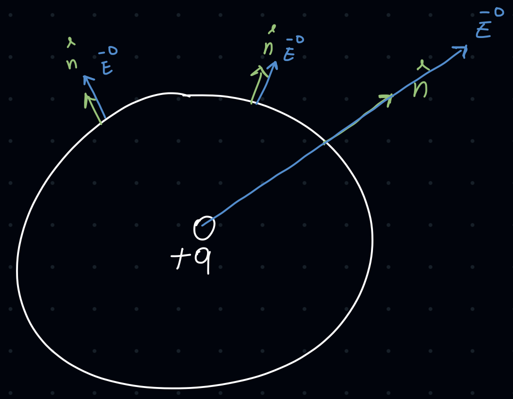
$$\Rightarrow \phi_{TOT} = \phi_1 + \phi_2$$

$$\Rightarrow \phi = \underbrace{\oint \vec{E} \cdot \hat{n}_1 \, dS_1 + \oint \vec{E} \cdot \hat{n}_2 \, dS_2}_{S_1 = S_2} = \frac{Q_{int}}{\varepsilon_0}$$

$$\rightarrow \phi = 2 \oint \vec{E} \cdot \hat{n} \, dS = 2 |\vec{E}| S = \frac{Q}{\varepsilon_0}$$

$$\sigma = \frac{Q}{S} \Rightarrow Q = \sigma \cdot S \Rightarrow 2 |\vec{E}| \cdot \cancel{S} = \frac{\sigma \cancel{S}}{\varepsilon_0} \rightarrow |\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$

Gauss $\qquad \phi = \oint \vec{E}\cdot\hat{n}\,dS = \frac{Q_{int}}{\varepsilon_0}$



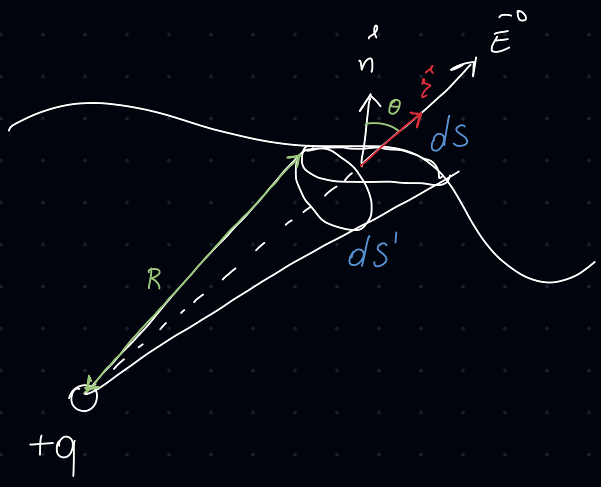
Sfera

$$\phi = \oint \vec{E}\cdot\hat{n}\,dS = |\vec{E}|\oint(\hat{r}\cdot\hat{n})\,dS \Rightarrow \phi = |\vec{E}|\oint dS = |\vec{E}|S$$

$$|\hat{r}|\cdot|\hat{n}|\cdot\cos(0) = 1$$

$$\Rightarrow \phi = |\vec{E}|\cdot 4\pi R^2 = \frac{1}{\cancel{4\pi}\varepsilon_0}\cdot\frac{Q}{\cancel{R^2}}\cdot\cancel{4\pi}\cancel{R^2} = \frac{Q}{\varepsilon_0}$$

$$dS\cdot\cos\theta = dS'$$



$$d\Omega = \frac{\perp dS}{r^2} = \frac{dS\cos\theta}{r^2}$$

$$\Rightarrow \phi = \oint \vec{E}\cdot\hat{n}\,dS = \frac{Q}{4\pi\varepsilon_0}\oint\frac{1}{R^2}\hat{r}\cdot\hat{n}\,dS$$

$$= \frac{Q}{4\pi\varepsilon_0}\oint\underbrace{\frac{1}{R^2}\cdot dS\cos\theta}_{d\Omega}$$

$$= \frac{Q}{4\pi\varepsilon_0}\cdot\oint d\Omega = \frac{Q}{4\pi\varepsilon_0}\oint\frac{1}{R^2}dS$$

$$= \frac{Q}{\cancel{4\pi}\varepsilon_0}\cdot\frac{1}{\cancel{R^2}}\cdot\cancel{R^2}\cdot\cancel{\pi 4} = \frac{Q}{\varepsilon_0}$$