

$$\begin{aligned} \vec{P}_\mu &= m \cdot g \cdot \cos \theta \\ \vec{P}_\tau &= m \cdot g \cdot \sin \theta \end{aligned}$$

$$\begin{cases} \vec{\mu} \\ \vec{\tau} \end{cases} \left\{ \begin{array}{l} mg \cos \theta - T = m \cdot \vec{a}_\mu \\ -mg \sin \theta = m \cdot \vec{a}_\tau \end{array} \right.$$

$$\begin{aligned} \vec{a}_\mu &= \text{Centripeta} = -\frac{v^2}{R} = \left(-\frac{\dot{s}^2}{\ell} \right) \hat{\vec{\mu}} \\ \vec{a}_\tau &= \text{Tangenziale} = ?? = \ddot{s} \cdot \hat{\vec{\tau}} \\ &\quad |\vec{a}| \end{aligned}$$

$$\Rightarrow \begin{cases} mg \cos \theta - T = -\frac{m \dot{s}^2}{\ell} \hat{\vec{\mu}} \\ -mg \sin \theta = m \cdot \ddot{s} \hat{\vec{\tau}} \end{cases} \quad \text{Approssimazione per piccole oscillazioni} \quad \sin \theta \sim \theta$$

$$\Rightarrow -m \vec{g} \theta = m \ddot{s} \hat{\vec{\tau}} \quad \text{Se } m_I = m_g \Rightarrow -\vec{g} \theta = \ddot{s} \hat{\vec{\tau}} \Rightarrow \ddot{s} \hat{\vec{\tau}} + \vec{g} \theta = 0$$

$$\text{ma } \theta \text{ Radianti} \Rightarrow 1 \text{ Rad} = \frac{\ell}{R} \Rightarrow \ddot{s} \hat{\vec{\tau}} + \frac{g}{R} \cdot \theta = 0 \quad \text{ponendo } R = \ell \text{ ovvero il "FILO" del pendolo}$$

$$\Rightarrow \ddot{s} \hat{\vec{\tau}} + \left(\frac{g}{\ell} \right) \cdot s = 0 \quad \Rightarrow \frac{\ddot{s} \hat{\vec{\tau}} + \kappa^2 s = 0}{\text{Eq. diff}}$$

Battezzo $\frac{g}{\ell} = \kappa^2$

$$\Rightarrow \text{Soluuzione: } S(t) = A \cdot \cos(\kappa t + \varphi)$$

\cos e' periodico
di $T = 2\pi$

$$\text{Trovare } \kappa: \cos \text{ e' periodico} \Rightarrow S(t+T_0) = A \cdot \cos(\kappa t + \varphi + 2\pi)$$

$$\Rightarrow A \cos(\kappa t + \kappa T_0 + \varphi) = A \cos(\kappa t + \varphi + 2\pi) \Rightarrow \cancel{\kappa t} + \kappa T_0 + \varphi = \cancel{\kappa t} + \varphi + 2\pi$$

$$\Rightarrow \kappa T_0 = 2\pi \Rightarrow \kappa = \frac{2\pi}{T_0}$$

$$\Rightarrow S(t) = A \cos\left(\frac{2\pi}{T_0} t + \varphi\right) = A \cos(wt + \varphi)$$

$w = \frac{d\omega}{dt} = \frac{2\pi}{T_0}$

$$\begin{aligned} \text{Periodo} \quad \kappa &= w = \frac{2\pi}{T} \Rightarrow \kappa^2 = \frac{g}{\ell} \Rightarrow \kappa = \sqrt{\frac{g}{\ell}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} \\ \Rightarrow 2\pi \sqrt{\frac{g}{\ell}} &= T \quad \text{Periodo del pendolo} \end{aligned}$$

1° eq EI.

$$\phi_E = \int \vec{E} \cdot \hat{n} dS = \frac{Q}{\epsilon_0} \quad f = \frac{dQ}{dV} \Rightarrow dQ = f dV \Rightarrow Q = \int f dV$$

$$= \int_S \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int f dV \quad \Rightarrow \int_V \nabla \cdot \vec{E} dV = \frac{1}{\epsilon_0} \int f dV = 0 \quad \boxed{\nabla \cdot \vec{E} = \frac{f}{\epsilon_0}} \quad (a)$$

2° eq

$$C = \oint \vec{E} \cdot d\vec{e} = 0 \Rightarrow \int \nabla \times \vec{E} \cdot \hat{n} dS = 0 \quad \boxed{\nabla \times \vec{E} = 0}$$

Legge di Coulomb

$$L = F \cdot S = \int \vec{F}_c \cdot d\vec{e} = U_A - U_B = q(V_A - V_B) \Rightarrow \frac{L}{q} = \frac{F}{q} d\vec{e} = -dV \Rightarrow \frac{L}{q} = \vec{E} \cdot d\vec{e} = -dV$$

$$\text{V e del tipo } V(x, y, z) \Rightarrow dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

otteniamo dV

$$\begin{cases} d\vec{e} = \hat{i} dx + \hat{j} dy + \hat{k} dz \\ \nabla V = \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \end{cases} \Rightarrow dV = \nabla V \cdot d\vec{e}$$

$$\Rightarrow \vec{E} \cdot d\vec{e} = -\nabla V \cdot d\vec{e} \quad \boxed{\vec{E} = -\nabla V} \quad (b)$$

Uniamo (a) e (b)

$$\Rightarrow \nabla(-\nabla V) = \frac{f}{\epsilon_0} \Rightarrow \nabla^2 V = -\frac{f}{\epsilon_0} \quad \text{Eq di Poisson}$$

$$V = \frac{1}{4\pi\epsilon_0} \int_V \frac{f(\vec{r}')}{4\pi r} dV$$

Potenziale Vettore

$$\vec{B} = \nabla \times \vec{A}$$

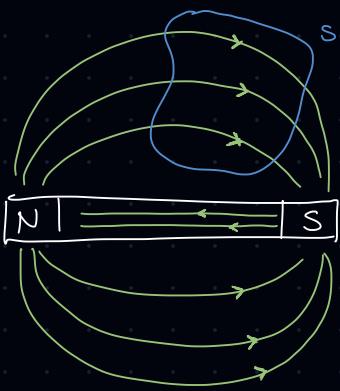
$$\Rightarrow \begin{cases} \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{B} = \mu_0 \vec{J} \end{cases} \quad \begin{matrix} \text{GAUSS} \\ \text{AMPERE} \end{matrix} \Rightarrow \begin{cases} \nabla \cdot (\nabla \times \vec{A}) = 0 \\ \nabla \times (\nabla \times \vec{A}) = \mu_0 \vec{J} \end{cases} \quad \Rightarrow \begin{matrix} \text{Sempre Vera} \\ \nabla \cdot (\nabla \times \vec{A}) = \nabla^2 \vec{A} = \mu_0 \vec{J} \end{matrix}$$

$$\text{Se } \nabla \cdot \vec{A} = 0 \Rightarrow \nabla^2 \vec{A} = -\mu_0 \vec{J} \quad \text{Simile all'eq di Poisson}$$

$$\Rightarrow A = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \cdot (\vec{r}')}{4\pi r} dV \quad \text{ma} \quad \vec{J} dV = \left(\frac{\vec{J}}{4\pi} \cdot \vec{S} \right) d\vec{e} \Rightarrow A = \frac{\mu_0}{4\pi} \oint \frac{\vec{I} \cdot \vec{e}'}{4\pi r} d\vec{e}'$$

$$\vec{A}' = \vec{A} + \vec{\nabla} f \Rightarrow \vec{B} = \nabla \times \vec{A}' = \nabla \times \vec{A}'$$

Maxwell Magnetostatica



$$\oint \vec{B} \cdot \hat{n} dS = 0 \quad \nabla \cdot \vec{B} = 0 \quad \left. \begin{array}{l} \text{1° Eq di Maxwell} \\ \text{Obiettivo} \end{array} \right\}$$

Biot - Savart

Sperimentalmente:

$$\vec{B} = K \cdot \frac{\vec{I}}{R}$$

Campo Magnetico attorno un filo percorso da corrente

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{I}}{R} \quad \text{Circuazione: } C = \oint \vec{B} \cdot d\vec{e} = K \oint \frac{\vec{I}}{R} d\vec{e}$$

$$\text{ma } d\vec{e} = \text{Arco di circonferenza} \Rightarrow I_{\text{Rad}} = \frac{R}{\ell} \Rightarrow \ell = \varphi \cdot R \Rightarrow d\ell = R d\varphi$$

$$\Rightarrow C = \oint \vec{B} d\vec{e} = K \oint \frac{\vec{I} \cdot R}{R} d\varphi = \oint \vec{B} d\vec{e} = K I \cdot \oint d\varphi = \oint \vec{B} d\vec{e} = \frac{\mu_0}{4\pi} \cdot I \cdot 4\pi$$

$$\oint \vec{B} d\vec{e} = \mu_0 I \quad \text{Legge di Ampère Integrale}$$

$$\int_S (\nabla \times \vec{B}) \cdot \hat{n} dS = \mu_0 \int_J \vec{J} \cdot \hat{n} dS \quad \nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{Differenziale}$$

Mancava un pezzo!



$$\oint \vec{J} \cdot \hat{n} dS = 0 \quad \nabla \cdot \vec{J} = 0 \quad \text{corrente stazionaria}$$



$$\oint \vec{J} \cdot \hat{n} dS \neq 0! \quad \text{Troviamo il pezzo}$$

Eq di continuità

$$- dq = \int \vec{J} \cdot \hat{n} dS \cdot dt \quad \Rightarrow \quad - \frac{dq}{dt} = \int \vec{J} \cdot \hat{n} dS \quad \text{ma } \rho = \frac{dq}{dV} \Rightarrow dq = \rho dV$$

$$\Rightarrow - \frac{d}{dt} \int_V \rho(x, y, z) dV = \int_S \vec{J} \cdot \hat{n} dS \quad \Rightarrow - \int_V \frac{\partial \rho}{\partial t} dV = \int_S \vec{J} \cdot \hat{n} dS$$

$$\Rightarrow - \int_V \frac{\partial \rho}{\partial t} dV = \int_V \nabla \rho dV \quad \Rightarrow \quad \nabla \rho = - \frac{\partial \rho}{\partial t} \quad \text{Obiettivo}$$

Perché manca un pezzo?

Ampère: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ -> Div = 0 $\cancel{\nabla \cdot (\nabla \times \mathbf{B})} = \mu_0 \nabla \cdot \mathbf{J}$ -> $\nabla \cdot \mathbf{J} = 0$ Valida solo in regime di corrente stazionaria!

-> Termine di Maxwell $\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

-> Ampère Maxwell //: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

proof -> Div = 0 $\cancel{\nabla \cdot (\nabla \times \mathbf{B})} = \mu_0 \nabla \cdot \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial (\nabla \times \mathbf{E})}{\partial t}$ -> $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$

-> $\mu_0 \left(\nabla \cdot \mathbf{J} + \epsilon_0 \frac{1}{\epsilon_0} \frac{\partial \mathcal{S}}{\partial t} \right) = 0$ $\nabla \cdot \mathbf{J} = - \frac{\partial \mathcal{S}}{\partial t}$ QED

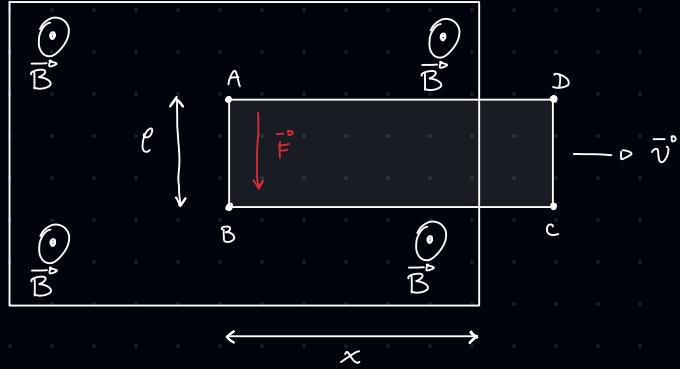
Maxwell campo elettrico

$$\left\{ \begin{array}{l} \vec{E} = \frac{\rho}{\epsilon_0} \\ \oint \vec{E} \cdot d\vec{e} = 0 \Rightarrow \nabla \times \vec{E} = 0 \end{array} \right. \rightarrow \text{Mancava un pezzo}$$

Faraday $f_{em} = - \frac{d\phi_B}{dt}$ $\phi_B = \int \vec{B} \cdot \vec{n} ds = \int B ds \cos\theta$

Flusso Tagliato Varia Superficie

$$\vec{F} = \text{Lorentz} = q \vec{E} + q(\vec{v} \times \vec{B})$$



$$\Rightarrow f_{em} = \frac{L}{q} = \frac{q \int \vec{E} \cdot d\vec{e}}{q} + \frac{q \int (\vec{v} \times \vec{B}) \cdot d\vec{e}}{q}$$

$$\text{Se } E = 0 \rightarrow f_{em} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{e}$$

$$f_{em} = \int_A^B (\vec{v} \times \vec{B}) \cdot d\vec{e} + \int_B^C (\vec{v} \times \vec{B}) \cdot d\vec{e} + \int_C^D (\vec{v} \times \vec{B}) \cdot d\vec{e} + \int_D^A (\vec{v} \times \vec{B}) \cdot d\vec{e} = \int_A^B (\vec{v} \times \vec{B}) \cdot d\vec{e} = \int v \cdot B \cdot \frac{\sin\theta}{1} d\vec{e} = \underline{v B e}$$

Forza EM

Troviamo lo stesso risultato con $f_{em} = - \frac{d\phi_B}{dt}$?

La superficie S dipende da x: $S = b \times h = l \cdot x$

$$\phi_B = \int \vec{B} \cdot \vec{n} ds = B \underbrace{\int_S ds}_{l \cdot x} \Rightarrow \phi_B = Bex$$

FLUSSO

ad un tempo $t' = t + \Delta t \rightarrow \phi_B(t + \Delta t) = Bl(x - \Delta x)$ con $\Delta x = v \cdot \Delta t$

$$\Rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\phi_B(t + \Delta t) - \phi_B(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{Blx - Blx - Blx}{\Delta t} = - Bl \underbrace{\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}}_v$$

$$\Rightarrow - \frac{d\phi_B}{dt} = Blv \quad QED$$

Flusso concatenato \rightarrow varia \vec{B}

$$\rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\int \vec{B}(t + \Delta t, \vec{r}) \cdot \hat{n} dS - \int \vec{B}(t, \vec{r}) \cdot \hat{n} dS}{\Delta t}$$

Approssimo: $\vec{B} \approx \vec{B}(t, \vec{r}) + \frac{\partial \vec{B}}{\partial t}(t + \Delta t, \vec{r}) \cdot \Delta t$

$$\rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\int \vec{B}(t, \vec{r}) \cdot \hat{n} dS + \int \frac{\partial \vec{B}}{\partial t}(t + \Delta t, \vec{r}) \cdot \Delta t \cdot \hat{n} dS - \int \vec{B}(t, \vec{r}) \cdot \hat{n} dS}{\Delta t}$$

$$\rightarrow \frac{d\phi_B}{dt} = \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

$$\Rightarrow f_{em} = \frac{q}{\epsilon_0} \quad \text{se } F = q\vec{E} + q(\vec{v} \wedge \vec{B}) \rightarrow f_{em} = \int \vec{E} \cdot d\vec{e} + \int (\vec{v} \wedge \vec{B}) \cdot d\vec{e}$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{e} + \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e} = \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS \quad \text{se } v=0 \rightarrow \int_e \vec{E} \cdot d\vec{e} = \int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

$$\rightarrow \int_S (\vec{v} \wedge \vec{E}) \cdot \hat{n} dS = \int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS \rightarrow \boxed{\vec{v} \wedge \vec{E} = \frac{\partial \vec{B}}{\partial t}} \quad \text{Eq. di maxwell campo elettromagn.}$$

Tutto insieme

Tagliato: $-\frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e}$

Concatenato: $-\frac{d\phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$

Generale $\Rightarrow -\frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e} - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$

$$\Rightarrow \text{Scrivo } f_{em} = -\frac{d\phi}{dt} \rightarrow \oint \vec{E} \cdot d\vec{e} + \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e} = \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e} - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

$$\rightarrow \oint_e \vec{E} \cdot d\vec{e} = - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS \rightarrow \int_S \vec{v} \wedge \vec{E} \cdot d\vec{S} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \rightarrow \vec{v} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Autoinduzione

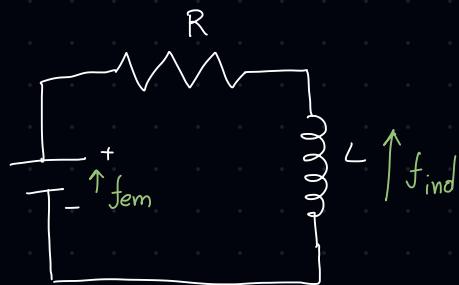
Legge di Laplace $B = \frac{\mu_0 I}{4\pi} \int \frac{d\ell \wedge \vec{r}}{4\pi^3}$

flusso -o $\phi_B = \int_S \frac{\mu_0 I}{4\pi} \int \frac{d\ell \wedge \vec{r}}{4\pi^3} \cdot \vec{n} dS$ tutto costante tranne I

$\Rightarrow \phi_B = L \cdot I$ -o Se $B \neq \text{cost}$ -o $\frac{d\phi_B}{dt} = L \cdot \frac{dI}{dt}$
 Induttanza

Siccome $f_{em} = -\frac{d\phi_B}{dt} = -L \frac{dI}{dt}$

Circuito R-L



Dalle leggi di Kirchhoff

$$f_{em} - f_{ind} = R \cdot I$$

$$\Rightarrow f_{em} - L \frac{dI}{dt} = R \cdot I \Rightarrow f_{em} = R \cdot I + L \frac{dI}{dt}$$

Eq differenziale

Soluzione del tipo $I = A e^{\alpha t} + D$

$$\begin{aligned} \Rightarrow \frac{dI}{dt} &= A \alpha e^{\alpha t} \Rightarrow f_{em} = R \cdot A e^{\alpha t} + L A \alpha e^{\alpha t} + RD \quad \text{ma } f_{em} = \text{cost} \\ \Rightarrow RAe^{\alpha t} + LA\alpha e^{\alpha t} &= 0 \Rightarrow A e^{\alpha t} (R + L\alpha) = 0 \Rightarrow R + L\alpha = 0 \Rightarrow \alpha = -\frac{R}{L} \quad \downarrow f_{em} = RD \\ \Rightarrow I &= A e^{-\frac{R}{L}t} + \frac{f}{R} \quad \text{Soluzione generale} \end{aligned}$$

$$\Rightarrow \text{Trovo } A: \quad I(0) = 0 \Rightarrow I(0) = A \left(e^{-\frac{R}{L} \cdot 0} + \frac{f}{R} \right) = 0 \quad \text{per } A + \frac{f}{R} = 0 \Rightarrow A = -\frac{f}{R}$$

$$\Rightarrow I = -\frac{f}{R} e^{-\frac{R}{L}t} + \frac{f}{R} = \frac{f}{R} \left(-e^{-\frac{R}{L}t} + 1 \right)$$

Sol Particolare

Legge di Lorentz



Sperimentalmente Lorentz Trova queste qualità nella Forza

$$1) \vec{F} \perp \vec{v}, \vec{F} \perp \vec{B}$$

$$2) \vec{F} \text{ dipende da } \sin \theta$$

$$3) \vec{F} \text{ dipende dalla carica } \rightarrow F \propto q$$

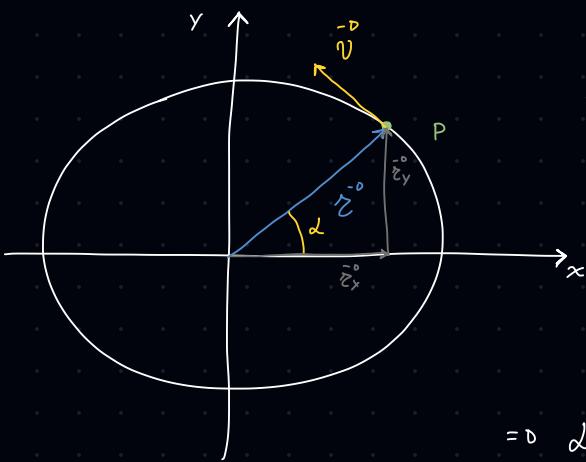
$$\Rightarrow \vec{F}_L = q \cdot (\vec{v} \wedge \vec{B}) \text{ perche' } \vec{v} \wedge \vec{B} = v \cdot B \cdot \sin \theta \quad \text{inoltre } \vec{v} \wedge \vec{B} \perp \vec{F}$$

Se esiste un campo elettrico

$$F_{\text{Coulomb}} = \frac{1}{4\pi} \frac{q_1 \cdot q_2}{R^2} \Rightarrow E = \frac{F}{q} \Rightarrow F = q \cdot E \quad \underline{\text{Forza d. Coulomb}}$$

$$\Rightarrow \vec{F}_{\text{TOT}} = \vec{F}_L + \vec{F}_C = qE + q(\vec{v} \wedge \vec{B})$$

Moto circolare



$$\bar{\omega} = \text{Velocità angolare} = \frac{d\alpha}{dt} \Rightarrow$$

$$\omega = \frac{d\alpha}{dt} \Rightarrow d\alpha = \omega dt \Rightarrow \int_{\alpha_0}^{\alpha_f} d\alpha = \int_{t_0}^{t_f} \omega dt$$

$$\Rightarrow \alpha_f - \alpha_0 = \omega(t_f - t_0)$$

pongo $\alpha_0 = 0, t_0 = 0$

$$\Rightarrow \underline{\alpha(t) = \omega t}$$

$$\bar{v} = \bar{\epsilon}_x + \bar{\epsilon}_y = \hat{i} R \cos \alpha + \hat{j} R \sin \alpha \quad \text{Siccome } V = \frac{ds}{dt} = \dot{s}$$

$$\Rightarrow R \cos \alpha + R \sin \alpha \Rightarrow R \cos(\omega t) + R \sin(\omega t) \quad \underline{\text{Dipende da } t}$$

$$\bar{v} = -\hat{i} \omega R \sin(\omega t) + \hat{j} \omega R \cos(\omega t) = \omega R \left[\hat{j} \cos(\omega t) - \hat{i} \sin(\omega t) \right]$$

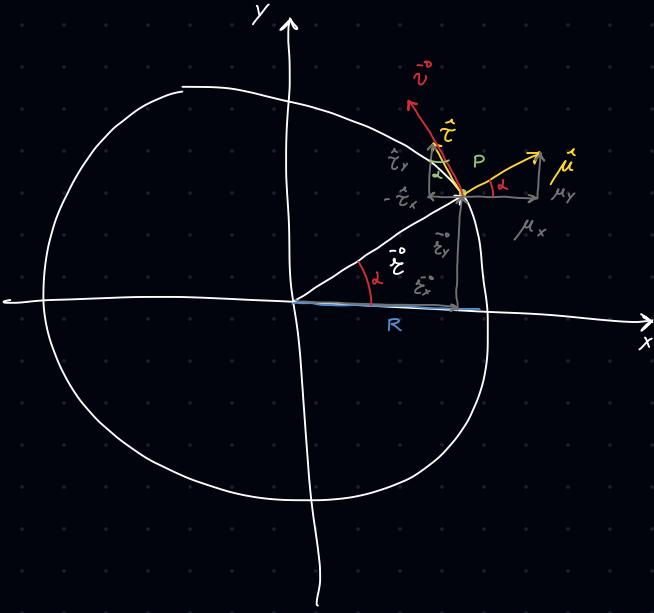
$$\bar{\alpha} = -\hat{i} \omega^2 R \cos(\omega t) - \hat{j} \omega^2 R \sin(\omega t) = -\omega^2 R \left[\hat{i} \cos(\omega t) + \hat{j} \sin(\omega t) \right] = -\frac{\omega^2 R \bar{\epsilon}}{R}$$

Direzione
opposta ad
 $\bar{\epsilon}$

Modulo

$$|\bar{v}| = \sqrt{\omega^2 R^2 \left[\cos^2(\alpha) + \sin^2(\alpha) \right]} = \sqrt{\omega^2 R^2} = \omega R \quad |\bar{v}| \Rightarrow V = \omega R \Rightarrow \omega = \frac{V}{R} \quad (1)$$

$$|\bar{\alpha}| = \sqrt{\omega^4 R^2 \left[1 \right]} = \frac{\omega^2 R}{|\bar{\alpha}|} \quad \Rightarrow |\bar{\alpha}| = \omega^2 R \Rightarrow \alpha = \frac{V^2}{R^2} \cdot R \Rightarrow \alpha = \frac{V^2}{R} \quad (2)$$



$$\begin{aligned}\hat{\mu} &= \hat{\mu}_x + \hat{\mu}_y = i\mu \cos \alpha + j\mu \sin \alpha = i \cos \alpha + j \sin \alpha \\ \hat{\tau} &= \hat{\tau}_x + \hat{\tau}_y = -i \sin \alpha + j \cos \alpha \\ \Rightarrow \begin{cases} \hat{\mu} = i \cos \alpha + j \sin \alpha \\ \hat{\tau} = -i \sin \alpha + j \cos \alpha \end{cases} & \Rightarrow \begin{cases} \hat{\mu} = i \cos(\omega t) + j \sin(\omega t) \\ \hat{\tau} = -i \sin(\omega t) + j \cos(\omega t) \end{cases} \\ \omega &= \frac{d\alpha}{dt} \quad \alpha = \omega t\end{aligned}$$

$$\begin{aligned}\Rightarrow \begin{cases} \dot{\mu} = -i \omega \sin(\omega t) + j \omega \cos(\omega t) \\ \dot{\tau} = -i \omega \cos(\omega t) - j \omega \sin(\omega t) \end{cases} &= \omega \left[\begin{matrix} -i \sin(\omega t) + j \cos(\omega t) \\ i \cos(\omega t) + j \sin(\omega t) \end{matrix} \right] = (\omega \hat{\tau}) \hat{\mu} \\ &= -\omega \left[\begin{matrix} i \cos(\omega t) + j \sin(\omega t) \\ -i \sin(\omega t) + j \cos(\omega t) \end{matrix} \right] = (-\omega \hat{\mu}) \hat{\tau}\end{aligned}$$

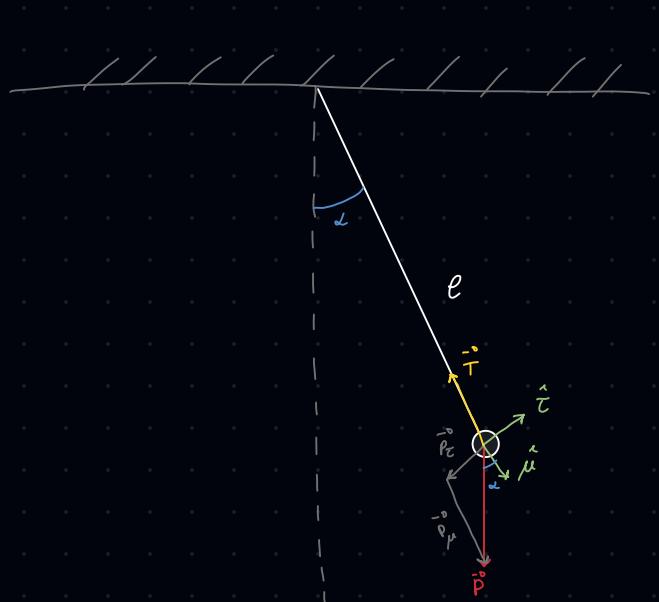
$$\Rightarrow \text{Se } v = \frac{ds}{dt} = \frac{d}{dt} [r_x + r_y] = \frac{d}{dt} [iR \cos(\omega t) + jR \sin(\omega t)] = \frac{d}{dt} \left[R \left(i \cos(\omega t) + j \sin(\omega t) \right) \right]$$

$$\Rightarrow \dot{s} = \frac{d}{dt} \left[R \hat{\mu} \right] = R [\omega \hat{\tau}] \Rightarrow \overset{\circ}{v} = R \omega \hat{\tau} \quad (1)$$

$$\text{Se } v = \omega s t \Rightarrow \ddot{a} = R \omega \left[-\omega \hat{\mu} \right] = (-R \omega^2 \hat{\mu}) \quad (2)$$

$$\text{Se } v \neq \text{cost} \Rightarrow \ddot{a} = \frac{d \ddot{v}}{dt} = R \cdot \frac{d\omega}{dt} \hat{\tau} + R \omega \frac{d \hat{\tau}}{dt} = \left(R \ddot{\omega} \hat{\tau} \right) \left(-R \omega^2 \hat{\mu} \right) \quad \begin{array}{l} \text{Acc centripeta} \\ \text{Acc Tang.} \end{array}$$

Pendolo Semplice



$$\begin{cases} \text{lungo } \hat{\mu} : & \vec{P}_{\mu} - \vec{T} = m \vec{a}_{\mu} \\ \text{lungo } \hat{\tau} : & \vec{P}_{\tau} = m \cdot \vec{a}_{\tau} \end{cases}$$

$$\begin{cases} \vec{P}_{\mu} = mg \cos \alpha \\ \vec{P}_{\tau} = -mg \sin \alpha \end{cases}$$

$$\Rightarrow \begin{cases} mg \cos \alpha - T = m \cdot \vec{a}_{\mu} \\ -mg \sin \alpha = m \cdot \vec{a}_{\tau} \end{cases}$$

Ci sono due Acc perche' $\omega \neq \text{cost}$

$$\rightarrow \text{Acc centripeta} : \vec{a}_{\mu} = - \frac{v^2}{R} = - \frac{\ddot{s}}{l} \quad (1)$$

$$\rightarrow \text{Acc tangenziale} : \vec{a}_{\tau} = \ddot{s} \cdot \hat{\tau}$$

$$\Rightarrow \begin{cases} mg \cos \alpha - T = - \frac{m \ddot{s}}{l} \\ -mg \sin \alpha = m \ddot{s} \hat{\tau} \end{cases} \quad \text{Inutile} \quad \rightarrow \text{Approssimazione per piccole oscillazioni} \rightarrow \sin \alpha \sim \alpha$$

$$\Rightarrow -mg \alpha = m \ddot{s} \hat{\tau} \quad \rightarrow \ddot{s} \hat{\tau} + g \alpha = 0 \quad \text{Siccome } 1 \text{ Rad} = \frac{l}{R} \quad \rightarrow \alpha = \frac{s}{l}$$

$$\Rightarrow \ddot{s} \hat{\tau} + s \frac{g}{l} = 0 \quad \text{battezzo} \quad \frac{g}{l} = \kappa^2 \quad \rightarrow \ddot{s} + \kappa^2 s = 0 \quad \text{Eq. differenziale}$$

$$\text{Soluzione : } s(t) = A \cos(\kappa t + \varphi)$$

Trovare κ

$$\cos e' periodico di $2\pi \Rightarrow s(t+T_0) = A \cos(\kappa t + \varphi + 2\pi)$$$

$$\rightarrow A \cos(\kappa t + \kappa T_0 + \varphi) = A \cos(\kappa t + \varphi + 2\pi) \Leftrightarrow \kappa t + \kappa T_0 + \varphi = \kappa t + \varphi + 2\pi$$

$$\rightarrow \kappa T_0 = 2\pi \Rightarrow \kappa = \frac{2\pi}{T_0} \quad \text{Battutto} \quad \omega = \frac{2\pi}{T_0} \quad \text{velocità Angolare}$$

$$\Rightarrow s(t) = A \cos(\omega t + \varphi)$$

Periodo del pendolo

$$\text{Se } \kappa = \frac{2\pi}{T_0} = \omega \rightarrow T_0 = \frac{2\pi}{\omega} \quad m a \quad \kappa^2 = \frac{a}{l} \Rightarrow T_0 = 2\pi \sqrt{\frac{l}{a}}$$

Energia Cinetica

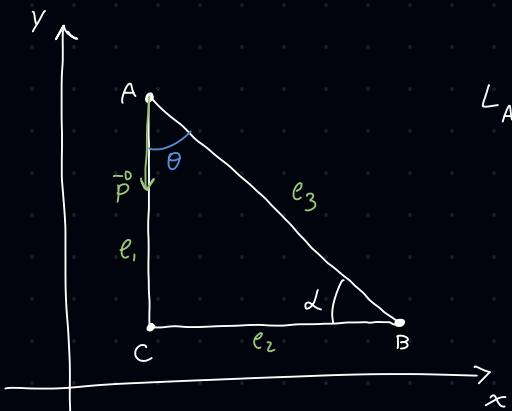
$$L = \vec{F} \cdot \vec{S} \Rightarrow L = \int_{S_0}^{\vec{S}} \vec{F} \cdot d\vec{S} \quad \text{ma} \quad \vec{F} = m \cdot \vec{a} \Rightarrow L = m \int_{S_0}^{\vec{S}} \vec{a} \cdot d\vec{S} \quad a = \frac{d\vec{v}}{dt}$$

$$\Rightarrow L = m \int_{S_f}^{\vec{v}} \frac{d\vec{v}}{dt} \cdot d\vec{S} \Rightarrow L = m \int_{v_0}^{v_f} v \, dv = m \left[\frac{v^2}{2} \right]_{v_0}^{v_f} = \underline{\frac{\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2}{Energia cinetica}}$$

G = Energia cinetica , U = Energia potenziale

$$\Rightarrow L = G_B - G_A \quad \text{ma se pongo } U = -G \Rightarrow L = -U_B - (-U_A) \Rightarrow \underline{L = U_A - U_B}$$

Campi conservativi : Peso



$$L_{AC} = \int_A^C \vec{P} \cdot d\vec{e} = mg(c-A) = \underline{mg e_1}$$

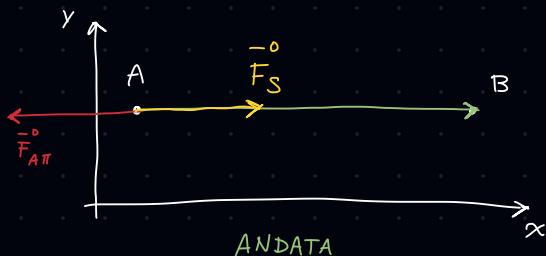
$$L_{ABC} = \int_A^B \vec{P} \cdot d\vec{e}_3 + \int_B^C \vec{P} \cdot d\vec{e}_2 = \int_A^B P \cos \theta \, de_3 + \int_B^C P \cos \alpha \, de_2$$

$$\Rightarrow \int_A^B P \cos de_3 \quad \text{ma} \quad de_3 \cos \theta = de_1 \Rightarrow \underline{L_{ABC} = mg e_1}$$

\Rightarrow Conservativo

Non Conservativo : Attrito

$$|\vec{F}_{ATT}| = \mu_0 |\vec{N}| \quad \text{con verso opposto al moto}$$



$$\Rightarrow L_1 = \int_A^B \vec{F}_s \cdot d\vec{e} - \int_A^B \vec{F}_{ATT} \cdot d\vec{e}$$

$$= F_s \cdot S - F_{ATT} \cdot S$$

$$L_2 = - \int_B^A \vec{F}_s \cdot d\vec{e} - \int_B^A \vec{F}_{ATT} \cdot d\vec{e}$$

$$= -F_s \cdot S - F_{ATT} \cdot S$$

$$\Rightarrow L_1 + L_2 \text{ dovrebbe essere zero, invece } L_1 + L_2 = S [F_s - F_A - F_s - F_A] = \underline{-2 F_{ATT}}$$

Non conservativo!

Impulso

$$\bar{I} = \int_{t_0}^{t_f} \bar{F} dt \quad \bar{P} = m \cdot \bar{v}$$

$$\bar{I} = \int_{t_0}^{t_f} \bar{F} dt = \int_{t_0}^{t_f} m \cdot \bar{a} dt = m \int_{t_0}^{t_f} \frac{d\bar{v}}{dt} dt = m (\bar{v}_f - \bar{v}_0) = \bar{P}_f - \bar{P}_0$$

Teorema Impulso

Momenti: FORZA

$$\bar{\tau} \wedge \bar{F} = \bar{M}$$

ANGOLARE

$$\bar{\tau} \wedge \bar{P} = \bar{L}$$

Faccio la derivata -o

$$\frac{d\bar{L}}{dt} = \frac{d}{dt} (\bar{\tau} \wedge \bar{P}) \Rightarrow \frac{d\bar{L}}{dt} = \frac{d\bar{\tau}}{dt} \wedge \bar{P} + \bar{\tau} \wedge \frac{d\bar{P}}{dt}$$

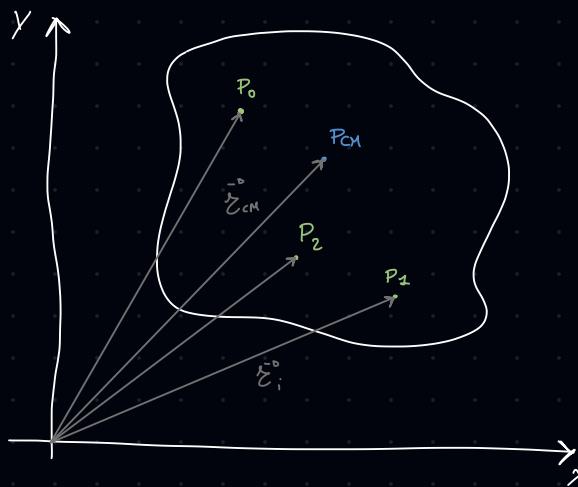
$$-\circ \quad \frac{d\bar{L}}{dt} = (\bar{v} \wedge \bar{P}) + \bar{\tau} \wedge (m \cdot \bar{a}) \Rightarrow \frac{d\bar{L}}{dt} = \bar{v} \wedge \bar{F} \Rightarrow \bar{M} = \frac{d\bar{L}}{dt}$$

\downarrow

$$\bar{v} \wedge m \cdot \bar{v} = \emptyset$$

QED

CenTro massa



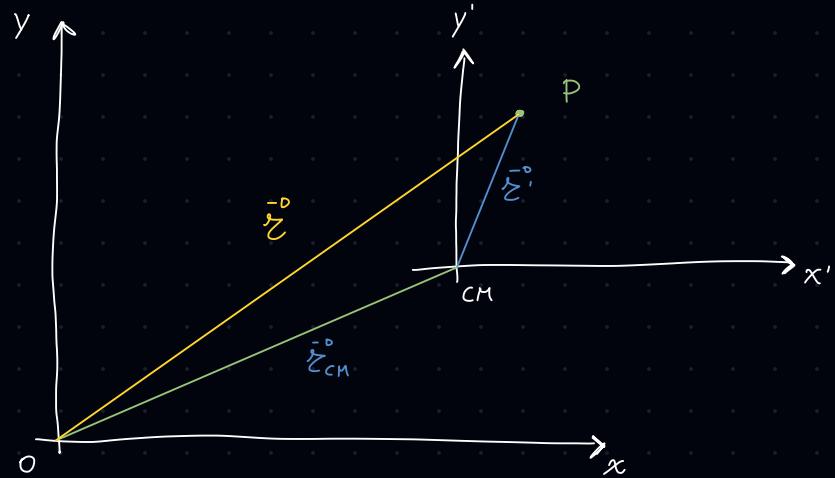
Per definizione $\vec{R}_{CM} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{M}$

$$\Rightarrow \vec{v}_{CM} = \frac{\sum_i m_i v_i}{M} \quad \text{ma } m \cdot v = \vec{p} \Rightarrow \vec{v}_{CM} = \frac{\sum_i \vec{p}_i}{M}$$
$$\Rightarrow \vec{p}_{TOT} = M \cdot \vec{v}_{CM} \quad (1)$$

$$\vec{a}_{CM} = \frac{\sum_i m_i \vec{a}_i}{M} \quad \text{ma } \vec{F} = m \cdot \vec{a} \Rightarrow \vec{a}_{CM} = \frac{\sum_i \vec{F}_i}{M}$$

$$\Rightarrow \sum_i \vec{F}_i = M \cdot \vec{a}_{CM} \quad (2)$$

Teorema di Koenig



$$\text{OSS: } \vec{\mathcal{E}} = \vec{\mathcal{E}}_{CM} + \vec{\mathcal{E}}'$$

$$\Rightarrow \vec{v} = \vec{v}_{CM} + \vec{v}'$$

↑ ↑ ↑
 Velocità Velocità Velocità
 di O di CM da CM
 Vel. DI Misurata
 CN

Momento Angolare $\vec{L} = \vec{\mathcal{E}} \wedge m \cdot v$

$$\text{ma } \vec{\mathcal{E}} = \vec{\mathcal{E}}_{CM} + \vec{\mathcal{E}}' \text{ e } \vec{v} = \vec{v}_{CM} + \vec{v}'$$

$$\Rightarrow \vec{L} = (\vec{\mathcal{E}}_{CM} + \vec{\mathcal{E}}') \wedge m \cdot (\vec{v}_{CM} + \vec{v}') = \vec{\mathcal{E}}_{CM} \wedge m \vec{v}_{CM} + \vec{\mathcal{E}}_{CM} \wedge m \vec{v}' + \vec{\mathcal{E}}' \wedge m \vec{v}_{CM} + \vec{\mathcal{E}}' \wedge m \vec{v}'$$

Si tratta di un sistema \Rightarrow si parla di $\vec{L}_{TOT} = \sum \vec{L} = \sum_i \vec{\mathcal{E}}_i \wedge m_i \vec{v}_i$

$$\Rightarrow L_{TOT} = \sum_i \vec{\mathcal{E}}_{CM} \wedge m_i v_{CM} + \sum_i \vec{\mathcal{E}}_{CM} \wedge m_i v'_i + \sum_i \vec{\mathcal{E}}' \wedge m_i v_{CM} + \sum_i \vec{\mathcal{E}}' \wedge m_i v'_i$$

$$= \vec{\mathcal{E}}_{CM} \wedge M \vec{v}_{CM} + \cancel{\vec{\mathcal{E}}_{CM} \wedge \sum_i m_i \vec{v}'_i} + \cancel{\sum_i M \vec{\mathcal{E}}' \wedge \vec{v}_{CM}} + \sum_i \vec{\mathcal{E}}' \wedge m_i \vec{v}'_i$$

$$\sum_i m_i \vec{v}'_i = M \vec{v}'_{CM}$$

$$= \vec{\mathcal{E}}_{CM} \wedge M \vec{v}_{CM} + \boxed{\sum_i \vec{\mathcal{E}}' \wedge m_i \vec{v}'_i}$$

Momento angolare
del sistema misurato
dal CM

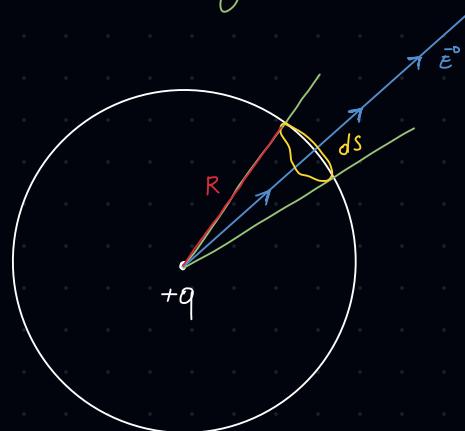
$$= \vec{L}_{CM} + \vec{L}'_{TOT}$$

Teorema di coulomb

$$\int \vec{E} \cdot \hat{n} dS = \left(\frac{Q}{\epsilon_0} \right) \xrightarrow{\text{Gauss}} \rho = \frac{dQ}{dV} \xrightarrow{\text{}} Q = \int_V \rho dV \xrightarrow{\text{}} \int \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int_V \rho dV$$

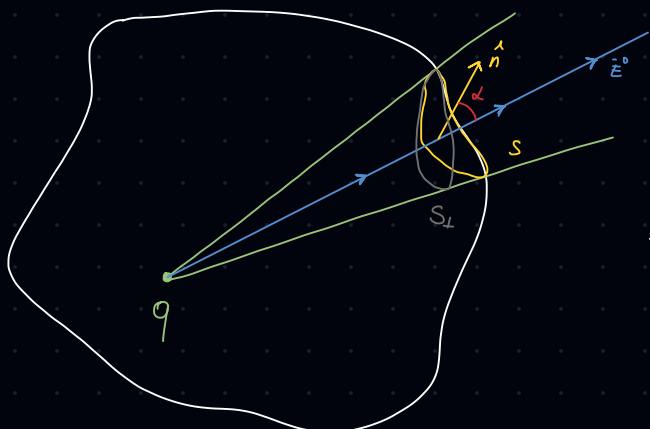
$$\xrightarrow{\text{}} \int_V (\nabla \cdot \vec{E}) dV = \frac{1}{\epsilon_0} \int_V \rho dV \xrightarrow{\text{}} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{Maxwell}$$

Dimostrare Gauss



$$\begin{aligned} \phi_E &= \int_S \vec{E} \cdot \hat{n} dS \\ &= \frac{q}{4\pi\epsilon_0 \epsilon^2} \int_S \epsilon \hat{n} dS \quad \hat{\epsilon} \perp \hat{n} \Rightarrow \int_S dS = 4\pi \epsilon^2 \\ &\Rightarrow \phi_E = \int \vec{E} \cdot \hat{n} dS = \frac{q}{\epsilon_0} \quad \text{QED} \end{aligned}$$

Superficie qualsiasi



Possiamo Trovare la sup S_\perp che e' perpend.

$$\Rightarrow S_\perp = S \cos \theta$$

$$\Rightarrow \phi_E = \oint \vec{E} \cdot \hat{n} dS = \oint \frac{q}{4\pi\epsilon_0 \epsilon^2} \cdot \epsilon \cdot \hat{n} \cdot dS$$

non costante

$$\epsilon \cdot \hat{n} = \epsilon \cdot n \cdot \cos \theta = \cos \theta$$

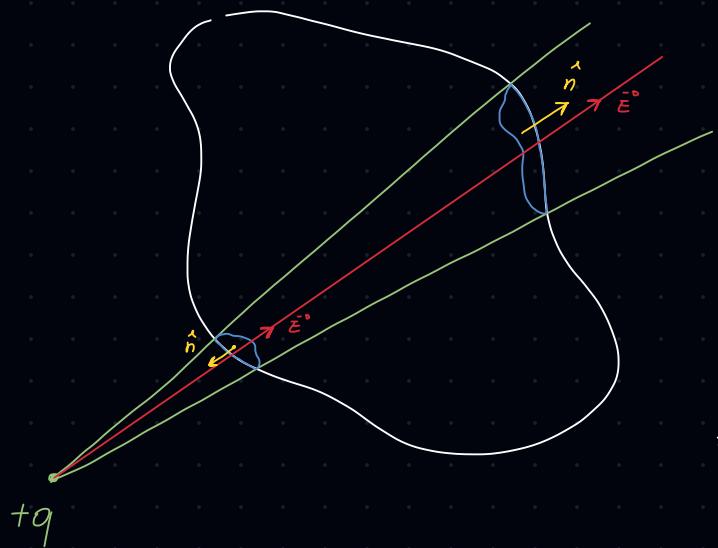
$$\begin{aligned} \Rightarrow \phi_E &= \oint \vec{E} \cdot \hat{n} dS = \frac{q}{4\pi\epsilon_0} \cdot \oint \frac{1}{\epsilon^2} \cdot \cos \theta \cdot dS = \frac{q}{4\pi\epsilon_0} \oint \frac{1}{\epsilon^2} dS' \quad \text{ma } dS' \text{ e' la sup. di una sfera} \\ &= \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{\epsilon^2} \cdot 4\pi\epsilon^2 = \frac{q}{\epsilon_0} \quad \Downarrow \\ &\quad \epsilon = \text{cost} \end{aligned}$$

Possiamo anche definire Angolo solido $d\Omega = \frac{dS_\perp}{\epsilon^2} = \frac{dS \cos \theta}{\epsilon}$

$$= \phi_E = \frac{q}{4\pi\epsilon_0} \oint \frac{\vec{E} \cdot \hat{n}}{\epsilon^2} dS = \kappa \oint \frac{\cos\theta}{\epsilon^2} dS = \kappa \oint \frac{dS_\perp}{\epsilon^2} = \kappa \oint d\Omega$$

$$\oint d\Omega = 4\pi \Rightarrow \phi_E = \frac{q}{4\pi\epsilon_0} 4\pi = \frac{q}{\epsilon_0} \quad QED$$

Carica cesterna



Per il teorema appena dimostrato

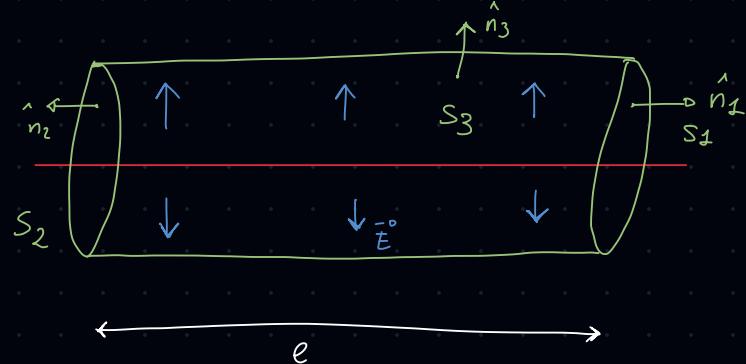
$$\phi_{S_1} = \int_S \vec{E} \cdot \hat{n}_1 dS_1 = \frac{q}{\epsilon_0} \text{ prendo } \hat{n}_1 \text{ come riferimento}$$

$$\phi_{S_2} = \int_S \vec{E} \cdot \hat{n}_2 dS_2 \text{ ma } \hat{n}_2 = -\hat{n}_1$$

$$\Rightarrow \phi_{S_2} = - \int_S \vec{E} \cdot \hat{n}_1 dS_2 = - \frac{q}{\epsilon_0}$$

$$\Rightarrow \phi_{S_1} = -\phi_{S_2} \Rightarrow \phi_{S_1} + \phi_{S_2} = 0$$

Filo carico - coulomb



$$\phi_E = \oint_{S} \vec{E} \cdot \hat{n} dS = \int_S \vec{E} \cdot \hat{n}_1 dS_1 + \int_S \vec{E} \cdot \hat{n}_2 dS_2 + \int_S \vec{E} \cdot \hat{n}_3 dS_3$$

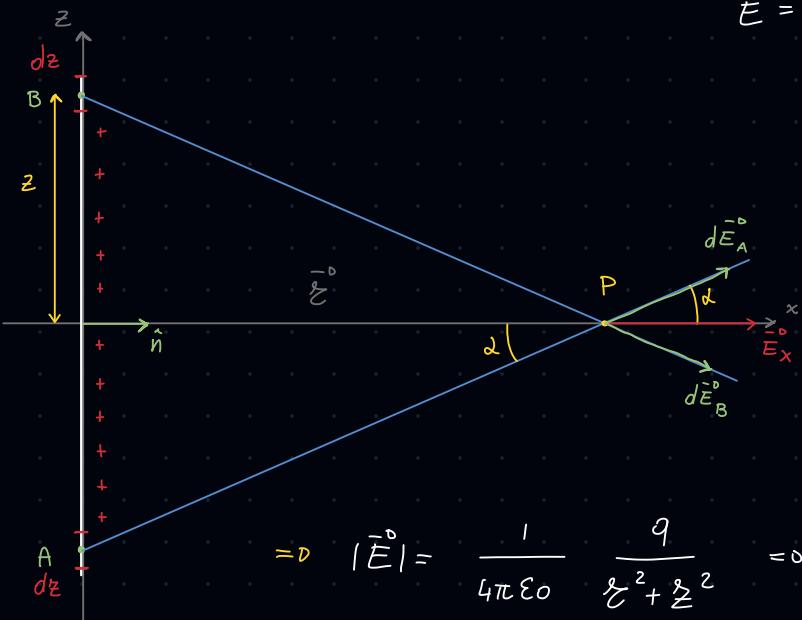
$$= \int_S \vec{E} \cdot \hat{n}_2 dS_2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \phi_E = |\vec{E}| \cdot 2\pi \epsilon l = \frac{Q}{\epsilon_0} \text{ ma } l = \infty$$

$$\Rightarrow \lambda = \text{densità lineare di carica} = \frac{Q}{l} \Rightarrow Q = \lambda l$$

$$|\vec{E}| \cdot 2\pi \epsilon l = \frac{\lambda l}{\epsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi \epsilon \epsilon_0} \hat{n}$$

Filo carico senza Coulomb



$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi \epsilon_0} \frac{\vec{q}}{\epsilon^2} \text{ Non conosco } \epsilon$$

$$\begin{cases} AP = \epsilon \cos \alpha \\ BP = \epsilon \cos \alpha \end{cases} \Rightarrow AP = BP = \epsilon \cos \alpha$$

$$AP^2 = \sum^2 + \epsilon^2$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi \epsilon_0} \frac{q}{\epsilon^2 + \sum^2} \Rightarrow dE = -\frac{1}{4\pi \epsilon_0} \frac{dq}{\epsilon^2 + \sum^2}$$

$$\text{siccome } \lambda = \frac{dQ}{dl} \Rightarrow dq = \lambda dl \quad dE = \frac{1}{4\pi \epsilon_0} \frac{\lambda dl}{\epsilon^2 + \sum^2}$$

nel nostro caso $dl = d\sum$

- le componenti \sum e \sum di $d\vec{E}_A + d\vec{E}_B$ si elidono a vicenda \Rightarrow ci serve solo \vec{E}_z

$$\Rightarrow d\vec{E}_z = d\vec{E} \cos \alpha \quad \text{ma non conosiamo } \alpha$$

$$\Rightarrow \sum = AP \cos \alpha \Rightarrow \sum = \sqrt{\sum^2 + \epsilon^2} \cos \alpha \Rightarrow \cos \alpha = \frac{\sum}{\sqrt{\sum^2 + \epsilon^2}}$$

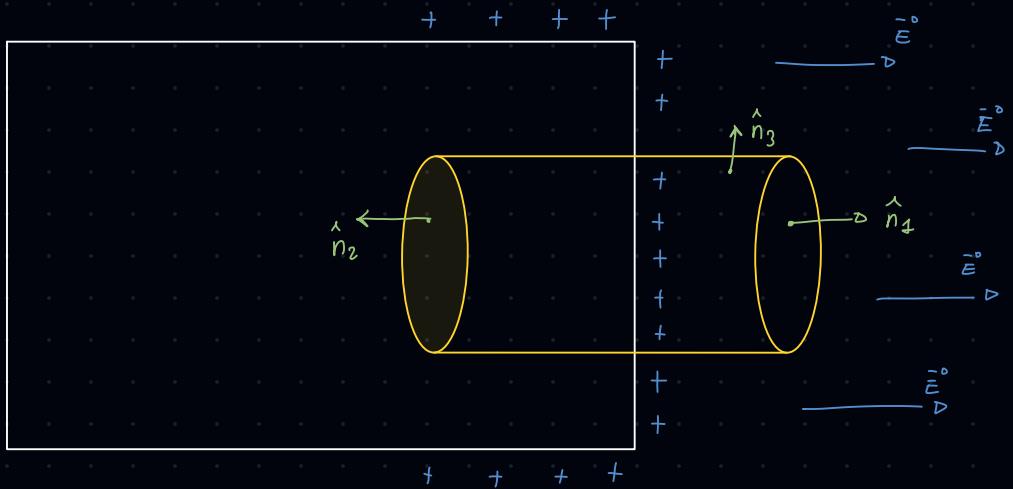
$$\Rightarrow dE_z = dE \cos \alpha = \frac{\sum}{\sqrt{\sum^2 + \epsilon^2}} dE$$

$$\Rightarrow dE_2 = \frac{1}{4\pi \epsilon_0} \cdot \frac{\lambda dz}{z^2 + z^2} \cdot \frac{z}{\sqrt{z^2 + z^2}} = \frac{1}{4\pi \epsilon_0} \frac{\lambda z dz}{(z^2 + z^2)^{\frac{3}{2}}} \quad (1)$$

Integriamo per ottenere \vec{E}_{tot}

$$E = \frac{1}{4\pi \epsilon_0} \underbrace{\int_0^e}_{\text{cost}} \frac{\lambda}{(z^2 + z^2)^{\frac{3}{2}}} dz = \frac{1}{4\pi \epsilon_0} \int_0^e \frac{z}{(z^2 + z^2)^{\frac{3}{2}}} dz = \frac{z \lambda z}{2 \cdot 4\pi \epsilon_0 z^2} \cdot \frac{e}{\sqrt{z^2 + e^2}}$$

Teorema di Coulomb



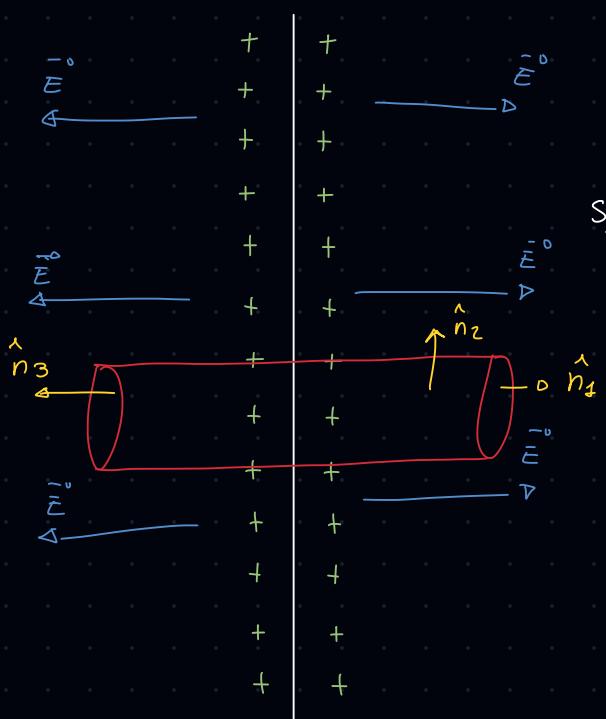
$$\phi_E = \oint \vec{E} \cdot \hat{n} dS = \int_{\partial S = \emptyset} \vec{E} \cdot \hat{n}_1 dS_1 + \int \vec{E} \cdot \hat{n}_2 dS_2 + \int \vec{E} \cdot \hat{n}_3 dS_3 = \int \vec{E} \cdot \hat{n}_1 dS_1 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow |\vec{E}| \cdot S_1 = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{S_1} \cdot \frac{1}{\epsilon_0} \text{ ma } \sigma = \frac{Q}{S} \Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$

Alternativamente

$$\sigma = \frac{dQ}{dS} \Rightarrow dQ = \sigma dS \Rightarrow Q = \int \sigma dS \Rightarrow E = \frac{\int \sigma dS}{S \epsilon_0} = \sigma \frac{\int dS}{S \epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Strato carico



$$\oint \vec{E} \cdot \hat{n} dS = \int \vec{E} \cdot \hat{n}_1 dS_1 + \int \vec{E} \cdot \hat{n}_2 dS_2 + \int \vec{E} \cdot \hat{n}_3 dS_3$$

$\parallel \quad \quad \quad \vec{E} \perp \hat{n}_2 \quad \parallel$

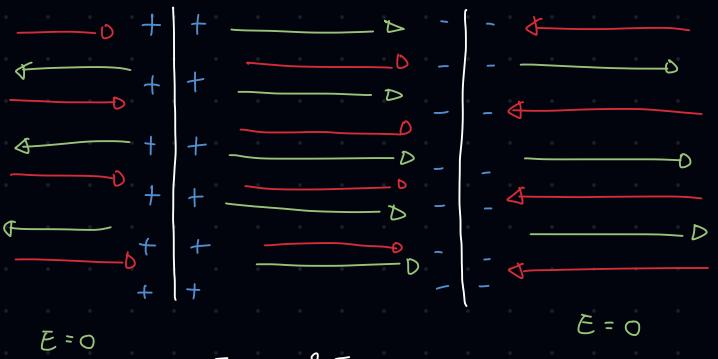
$$S_3 = S_1 \quad = 2 \int_S \vec{E} \cdot \hat{n}_1 dS_1 = 2 E \cdot S_1 = E \cdot 4\pi r$$

$$\Rightarrow E \cdot 4\pi r = \frac{Q}{\epsilon_0}$$

$$\sigma = \frac{dQ}{dS} \rightarrow Q = \int \sigma dS \rightarrow$$

$$\Rightarrow E \cdot \frac{2\pi r}{4\pi \epsilon} = \frac{\sigma \int dS}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Doppio Strato carico



$$\Rightarrow \vec{E} = \frac{\sigma}{\epsilon_0}$$

Anello Carico



$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(\zeta^2)} \rightarrow \text{nel nostro caso } \overline{AP} = ?$$

$$\Rightarrow \zeta = AP \cos \alpha \rightarrow AP^2 = \zeta^2 + R^2 \\ \Rightarrow \zeta = \sqrt{\zeta^2 + R^2} \cos \alpha$$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{\zeta^2 + R^2}$$

$\Rightarrow \vec{E}_A + \vec{E}_B$ Annullano le componenti y ed x

\Rightarrow ci serve solo $E_z = E \cos \alpha$

$$\Rightarrow dE_z = \frac{1}{4\pi\epsilon_0} \frac{dq \cos \alpha}{\zeta^2 + R^2} \rightarrow ??$$

$$\zeta = AP \cos \alpha \rightarrow \cos \alpha = \frac{\zeta}{\sqrt{\zeta^2 + R^2}}$$

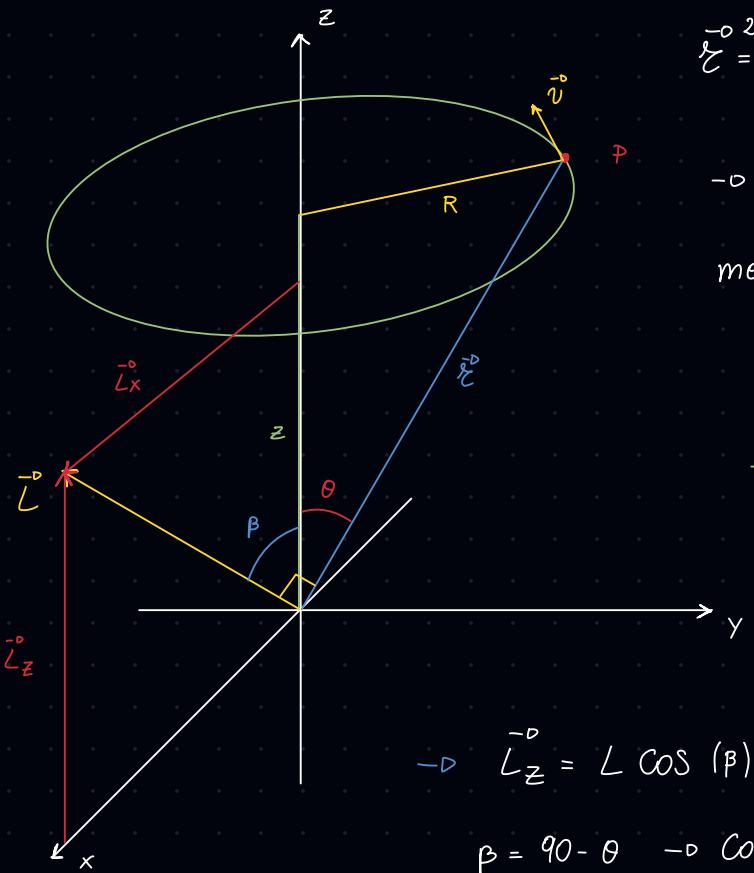
$$\Rightarrow dE_z = \frac{1}{4\pi\epsilon_0} \frac{dq}{(\zeta^2 + R^2)^{\frac{3}{2}}}$$

Siccome l'anello è una circonferenza
 \Rightarrow ha una superficie

$$\Rightarrow \lambda = \frac{dq}{ds} \Rightarrow dq = \lambda ds \rightarrow dE_z = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(\zeta^2 + R^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_z = \frac{\lambda}{4\pi\epsilon_0} \frac{1}{(\zeta^2 + R^2)^{\frac{3}{2}}} \int_{2\pi R} dS = \frac{\lambda R}{\epsilon_0 (\zeta^2 + R^2)^{\frac{3}{2}}}$$

Rotazione Asse Fisso



Momento Angolare: $\vec{L} = \vec{\varepsilon} \lambda m \vec{v} = \vec{\varepsilon} \lambda \vec{P}$

$$\vec{\varepsilon}^2 = R^2 + z^2 ; \quad R = \varepsilon \sin \theta$$

$$\Rightarrow L = \varepsilon \cdot m \cdot v \cdot \sin(90^\circ) = \varepsilon m v$$

$$\text{ma } v = \omega R$$

$$\Rightarrow L_i = \varepsilon, m, \omega R,$$

\Rightarrow Le componenti x ed y si annullano

\Rightarrow ci serve solo la componente z di L_i

$$\Rightarrow L_z = L \cos(\beta) \quad \text{ma } \beta = ?$$

$$\beta = 90^\circ - \theta \quad \Rightarrow \cos(90^\circ - \theta) = \sin(\theta)$$

$$\Rightarrow L_{z_i} = L_i \sin \theta = \varepsilon_i m_i \omega R_i \sin \theta \Rightarrow \frac{(\varepsilon_i \sin \theta) \cdot R_i m_i \omega}{R}$$

$$\Rightarrow L_z = R^2 m_i \omega \quad (1)$$

$$\Rightarrow L_{TOT} = \omega \sum_i m_i R_i^2 \quad \text{pongo } \sum_i m_i R_i^2 = \text{Momento di inerzia totale} = I_{TOT}$$

$$\Rightarrow \underline{L_{TOT} = \omega \cdot I_{TOT}}$$

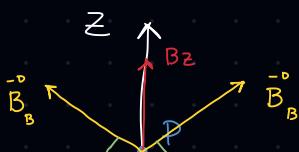
Nel caso di corpi rigidi

$$\begin{aligned} \text{---} M = \frac{d\vec{L}}{dt} &\Rightarrow M = \frac{d\vec{\omega}}{dt} I \uparrow \text{Cost} & \vec{M} = \vec{\alpha} I &\quad \text{ANALOGO A } \vec{F} = m \cdot \vec{a} \\ &\quad \text{Acc Ang.} & & \end{aligned}$$

$$\omega = \omega_0 + \int_{t_0}^{t_f} \alpha dt, \quad \theta = \theta_0 + \int_{t_0}^{t_f} \omega dt$$

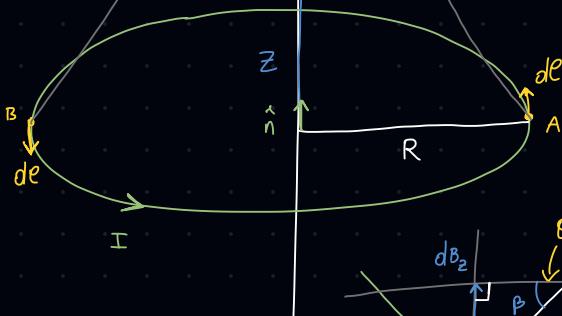
$$\Rightarrow v(t) = v_0 + \alpha \cdot t \quad \Rightarrow \omega(t) = \omega_0 + \alpha \cdot t \quad \text{con } \alpha = \frac{d\omega}{dt} = 0$$

$$S(t) = S_0 + v_0 t + \frac{1}{2} \alpha t^2 \quad \Rightarrow \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \text{con } \alpha = \frac{d\omega}{dt} \neq 0$$



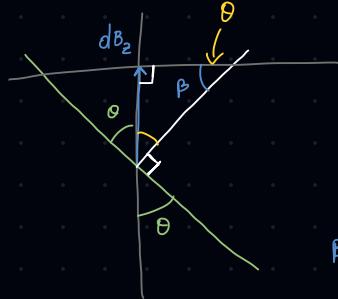
Dalla legge di Laplace

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\ell \times \vec{\epsilon}}{|\Delta\vec{\epsilon}|^3}$$



Le componenti x e dy si annullano a due a due

$$\Rightarrow \text{calcolo solo } dB_z = dB \cdot \sin \theta$$



$$\varphi = 180 - 90 - \theta \\ = 90 - \theta$$

$$\beta = 180 - 90 - \varphi \\ = 90 - 90 + \theta \\ = \theta$$

$$\begin{aligned} \Rightarrow \vec{B} &= \frac{\mu_0 I}{4\pi} \oint \frac{d\ell \times \vec{\epsilon}}{\vec{\epsilon}^3} \cdot \sin \theta \\ &= \frac{\mu_0 I}{4\pi} \oint \frac{d\ell \cdot \hat{n} \cdot \sin \theta}{\vec{\epsilon}^3} \\ &= \frac{\mu_0 I}{4\pi} \oint \left(\frac{\sin \theta}{\vec{\epsilon}^2} \right) d\ell \\ &= \frac{\mu_0 I}{4\pi} \cdot \frac{\sin \theta}{r^2} \oint d\ell \quad \cancel{2\pi r} \end{aligned}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi \vec{\epsilon}^2} \cdot \sin 2\pi R$$

$$\begin{aligned} \rightarrow \text{Non conosco } \vec{\epsilon} \rightarrow R &= \vec{\epsilon} \sin \theta \rightarrow \sin \theta = \frac{R}{\vec{\epsilon}} \\ \rightarrow \vec{\epsilon}^2 &= z^2 + R^2 \Rightarrow \vec{\epsilon} = \sqrt{z^2 + R^2} \end{aligned}$$

$$\Rightarrow \frac{\sin \theta}{\vec{\epsilon}^2} = \frac{R}{\vec{\epsilon}} \cdot \frac{1}{(z^2 + R^2)} = \frac{R}{(z^2 + R^2)^{\frac{1}{2}} \cdot (z^2 + R^2)} = \frac{R}{(z^2 + R^2)^{\frac{3}{2}}}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \frac{R^2}{(z^2 + R^2)^{\frac{3}{2}}} \cdot \cancel{2\pi} = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{\frac{3}{2}}}$$

F. Lorentz

$$\vec{F} = q \vec{E} + q \cdot (\vec{v} \wedge \vec{B})$$

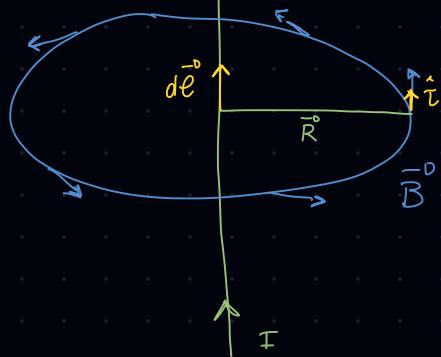
$$\vec{F} = q (\vec{v} \wedge \vec{B}) \quad \text{ma} \quad v = \frac{d\vec{r}}{dt}$$

$$= \vec{F} = \left(q \cdot \frac{d\vec{r}}{dt} \right) \wedge \vec{B} \quad \rightarrow \quad \vec{F} = I \cdot d\vec{l} \wedge \vec{B} \quad (\text{II}^{\circ} \text{ Laplace})$$

\vec{I}

sperimentale

$$\text{BiOT-Savart: } \vec{B} = \bar{k} \cdot \frac{\vec{I}}{R} \quad \bar{k} = \tau \cdot \frac{\mu_0}{4\pi}$$



$$= \vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{I}}{R} \tau \equiv \left(\frac{\mu_0}{4\pi} I \cdot \frac{d\vec{l} \wedge \vec{R}}{|R|^3} \right)$$

I° legge di Laplace

Alternativamente

$$\vec{F} \text{ con carico magnetico} = \frac{\mu_0}{4\pi} \frac{q_m \cdot q_{m'}}{\epsilon^2} \quad \Rightarrow \quad \vec{B}_{q_m} = \frac{\vec{F}_m}{q} = \frac{\mu_0}{4\pi} \frac{q_m}{\epsilon^2}$$

$$\rightarrow \text{II}^{\circ} \text{ Laplace} \rightarrow \vec{F} = I \cdot d\vec{l} \wedge \vec{B} = I \cdot d\vec{l} \wedge \left[\frac{\mu_0}{4\pi} \cdot \frac{q_m}{\epsilon^2} \cdot \left(\frac{\vec{r}}{\epsilon} \right) \right]$$

$$\vec{B}_{q_m} \quad \downarrow \quad \frac{\vec{r}}{\epsilon} = \vec{\epsilon}$$

$$\Rightarrow \vec{F} = \frac{\mu_0}{4\pi} \cdot q_m \cdot \left(I \cdot d\vec{l} \wedge \frac{\vec{\epsilon}}{\epsilon^3} \right)$$

Forza con cariche
magnetiche

$$\vec{F} = q \cdot (d\vec{l} \wedge \vec{B})$$

Forza con cariche
elettriche

$$\Rightarrow q_m = I d\vec{l} \quad \Rightarrow \quad \vec{F} = \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{l} \wedge \vec{\epsilon}}{\epsilon^3} \quad \text{I}^{\circ} \text{ Laplace}$$

Equivalenza Ampère

Campo spira percorso da corrente $\mathcal{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}$ Se $z \gg R$

$$\Rightarrow \vec{\mathcal{B}} = \frac{\mu_0 I}{2} \cdot \frac{R^2}{z^3} \hat{n} \text{ chiamo } I S \cdot \hat{n} = I \cdot \pi R^2 \cdot \hat{n} = \vec{m} \text{ spira}$$

$$\Rightarrow \vec{\mathcal{B}} = \frac{\mu_0}{2\pi} \frac{\vec{m}}{z^3} \quad (1)$$

\rightarrow Dipolo



Se P è lungo z ci interessa solo B_z

$$\Rightarrow \vec{E}_z = \frac{q_d}{4\pi\epsilon_0} \cdot \frac{3\cos\theta \cdot \vec{I}}{z^3} = \frac{q \cdot d}{4\pi\epsilon_0} \frac{\vec{I}}{z^3}$$

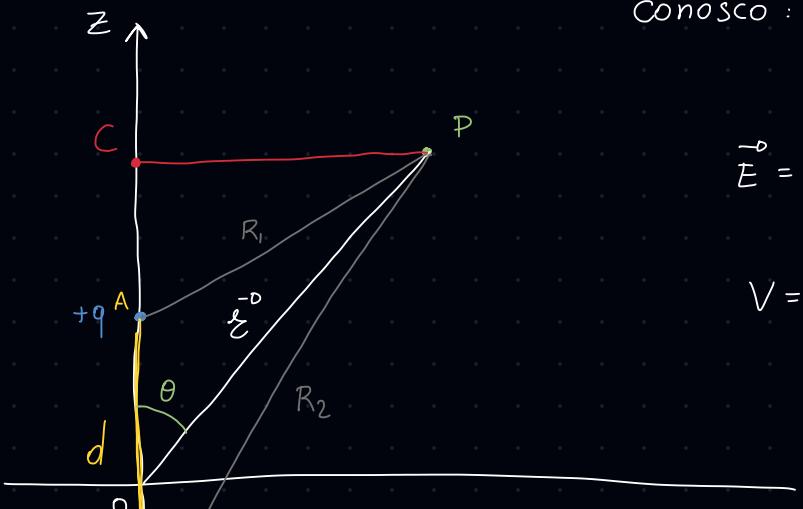
$$\text{chiamo } q \cdot d = \vec{P} \Rightarrow \vec{E} = \frac{\vec{P}}{2\pi\epsilon_0 z^3}$$

Nel caso di un dipolo magnetico

$$\rightarrow \vec{\mathcal{B}} = \frac{q \vec{d} \mu_0}{4\pi} \cdot \frac{2}{z^3} \text{ chiamo } q \cdot \vec{d} = \vec{m}$$

$$\Rightarrow \vec{\mathcal{B}} = \frac{\mu_0}{2\pi} \cdot \frac{\vec{m}}{z^3} \quad (2) \quad (1) \text{ e } (2) \text{ sono UGUALI!}$$

Dipolo elettrico



Conosco : $\begin{cases} \varepsilon & \rightarrow \text{dist di p-o} \\ \theta & \rightarrow \text{angolo tra } \varepsilon \text{ e } e \\ d & \rightarrow \text{dist tra } +q \text{ e } -q \end{cases}$

$$\vec{E} = -\vec{\nabla}V \quad \text{ma} \quad V = ?$$

$$V = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{\varepsilon_A} - \frac{1}{\varepsilon_B} \right]$$

$\uparrow \quad \uparrow$
 $R_1 \quad R_2 \quad ???$

Troviamo R_1 ed R_2

$$\begin{aligned} R_1^2 &= CP^2 + CA^2 = (\varepsilon \sin \theta)^2 + \left(\cos \frac{d}{2}\right)^2 = (\varepsilon \sin \theta)^2 + \left(\varepsilon \cos \theta - \frac{d}{2}\right)^2 \\ &= \varepsilon^2 \sin^2 \theta + \varepsilon^2 \cos^2 \theta + \left(\frac{d}{2}\right)^2 - \varepsilon \cos \theta d \\ &= \varepsilon^2 \left(\sin^2 \theta + \cos^2 \theta \right) + \left(\frac{d}{2}\right)^2 - \varepsilon \cos \theta d \\ &= \varepsilon^2 + \frac{d^2}{4} - \varepsilon \cos \theta d \end{aligned}$$

$$R_2 = \varepsilon^2 + \frac{d^2}{4} + \varepsilon \cos \theta d \quad (1)$$

$$\text{Approx di dipolo } \varepsilon \gg d \Rightarrow R_{1,2} = \sqrt{\varepsilon^2 \pm \varepsilon \cos \theta d} = \sqrt{\varepsilon(\varepsilon \pm \cos \theta d)} = \sqrt{\varepsilon} \sqrt{\varepsilon \pm \cos \theta d}$$

Approx $I(0) \rightarrow$ Taylor

$$\begin{aligned} f(d) &= \sqrt{\varepsilon + \cos \theta d} = (\varepsilon + \cos \theta d)^{\frac{1}{2}} \rightarrow f'(d) = -\frac{1}{2} \cos \theta (\varepsilon + \cos \theta d)^{-\frac{1}{2}} = \frac{\cos \theta}{2(\varepsilon + \cos \theta d)^{\frac{1}{2}}} \\ \rightarrow f'(0) &= \frac{\cos \theta}{2\sqrt{\varepsilon}} \quad f(0) = \sqrt{\varepsilon} \end{aligned}$$

$$\rightarrow f(d) \approx \sqrt{\varepsilon} + \frac{\cos \theta}{2\sqrt{\varepsilon}} \cdot d \rightarrow g(d) \approx \sqrt{\varepsilon} - \frac{\cos \theta}{2\sqrt{\varepsilon}} d$$

$$\Rightarrow \begin{cases} R_1 = \sqrt{\varepsilon} \left(\sqrt{\varepsilon} + \frac{\cos \theta d}{2\sqrt{\varepsilon}} \right) = \varepsilon - \frac{d \cos \theta}{2} \quad (2) \\ R_2 = \varepsilon + \frac{d \cos \theta}{2} \quad (3) \end{cases}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\epsilon \cdot \frac{d}{2}\cos\theta} - \frac{1}{\epsilon + \frac{d}{2}\cos\theta} \right] = K \frac{\frac{d}{2}\cos\theta - \frac{d}{2}\cos\theta}{\epsilon^2 - \left(\frac{d}{2}\right)^2\cos^2\theta}$$

$$= K \frac{\frac{d\cos\theta}{\epsilon^2 - \left(\frac{d}{2}\right)^2\cos^2\theta}}{\epsilon > d} \approx \frac{q}{4\pi\epsilon_0} \frac{d\cos\theta}{\epsilon^2}$$

-> Campo elettrico

$$\vec{E} = -\nabla V \quad \rightarrow \text{Derivate parziali} \rightarrow \text{coordinate cartesiane}$$

$$CO = \epsilon \cos\theta \quad \rightarrow \cos\theta = \frac{CO}{\epsilon} = \frac{z}{\epsilon} \quad \rightarrow z = \sqrt{x^2 + y^2 + z^2}$$

$$V = \frac{qd}{4\pi\epsilon_0} \frac{z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$\rightarrow \vec{E}_z = -\frac{\partial V}{\partial z} = -K \cdot (x^2 + y^2 + z^2)^{-\frac{3}{2}} - z \cdot \frac{3}{2} (x^2 + y^2 + z^2)^{-\frac{5}{2}} \cdot \cancel{z}$$

$$= -K \left[(x^2 + y^2 + z^2)^{-\frac{3}{2}} - \frac{3z^2}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \right] = -K \left(z^2 \right)^{-\frac{3}{2}} - \frac{3z^2}{(z^2)^{\frac{5}{2}}}$$

$$= -\frac{qd}{4\pi\epsilon_0} \cdot \left[z^{-3} - \frac{3z^2}{z^5} \right] = -\frac{qd}{4\pi\epsilon_0} \left[\frac{z^2 - 3z^2}{z^5} \right] = \frac{qd}{4\pi\epsilon_0} \left[\frac{3z^2}{z^5} - \frac{1}{z^3} \right]$$

$$= \frac{qd}{4\pi\epsilon_0 z^3} \left[\frac{3z^2}{z^2} - 1 \right] = \frac{qd}{4\pi\epsilon_0 z^3} (\cos^2\theta - 1)$$

Massa Inerziale - gravitazionale

INERZIALE

La capacità di un corpo di opporsi alla variazione di moto

GRAVITAZIONALE

Relativa alla quantità
di materia di
un corpo

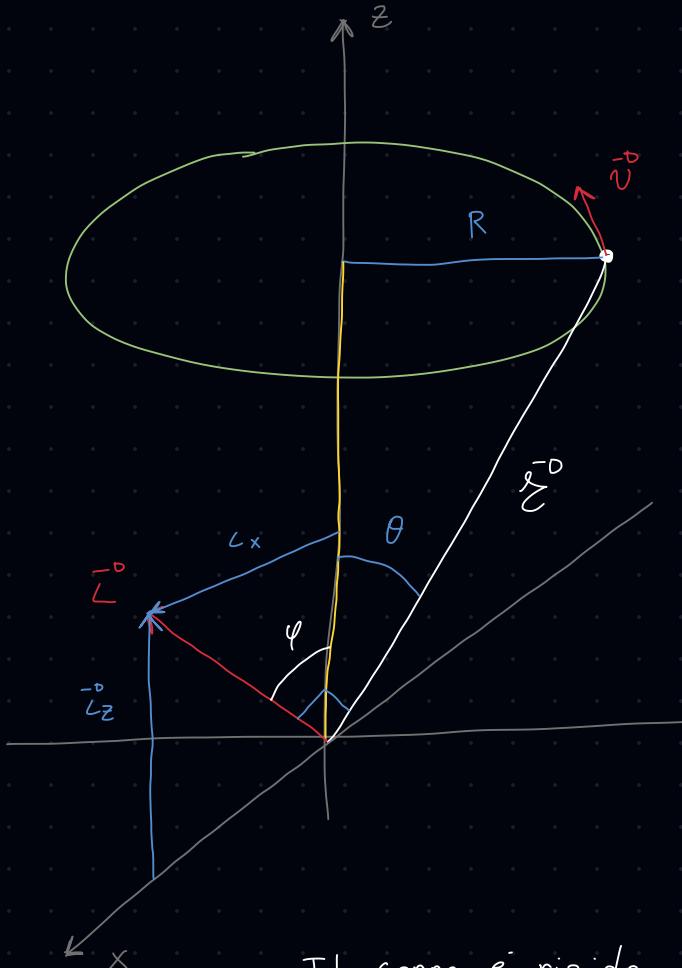
Principio di equivalenza di Einstein

$$\left\{ \begin{array}{l} \overset{\circ}{F} = G \cdot \frac{m_1 \cdot m_2}{r^2} \\ \overset{\circ}{F} = m \cdot \ddot{a} \end{array} \right. \quad \begin{array}{l} \text{Massa gravitazionale} \\ \text{Massa inerziale} \end{array}$$

$$\text{Piuma} \quad \left\{ G \cdot \frac{m_{ip} \cdot M}{\zeta^2} = m_{gp} \cdot \ddot{\alpha}^d \right.$$

$$\text{Marcello} \quad \left\{ G \cdot \frac{m_{im} \cdot M}{\zeta^2} = m_{gm} \cdot \ddot{\alpha}^d \right.$$

Rotazione Asse



$$\text{Scrivere } \vec{L} = \vec{\varepsilon} \wedge m \cdot \vec{v}$$

Le componenti x e y di \vec{L}_{TOT} si annullano

$$\Rightarrow \vec{L}_{TOT} = \vec{L}_z$$

$$\begin{aligned} \vec{L}_z &= L \cos \varphi \quad \text{ma } \varphi = 90^\circ - \theta \\ &= L \cos(90^\circ - \theta) = L \sin \theta \end{aligned}$$

$$\Rightarrow \vec{L}_{1z} = \vec{\varepsilon} \cdot m \cdot v \cdot \cancel{\sin(90^\circ)} \cdot \sin \theta$$

motocircolare $v = \omega R$

$$\Rightarrow L_i = \vec{\varepsilon} \cdot m \cdot \omega \cdot R \cdot \sin \theta$$

$$\text{ma } R = \vec{\varepsilon} \sin \theta \Rightarrow \sin \theta = \frac{R}{\vec{\varepsilon}}$$

$$\Rightarrow L_i = \cancel{\vec{\varepsilon}} \cdot m \cdot \omega \frac{R^2}{\cancel{\vec{\varepsilon}}} = m_i \omega_i R^2$$

Il corpo è rigido $\Rightarrow \omega_i = \omega$

$$\Rightarrow L_i = m_i \omega R^2$$

$$\Rightarrow L_{TOT} = (\sum_i m_i R^2) \omega = M \omega R^2$$

$$\begin{matrix} \text{momento di} \\ \text{inerzia} \\ I \end{matrix} \Rightarrow \underline{\underline{L_{TOT} = \vec{\omega} I}}$$

Dalla II eq cardinale della dinamica:

$$\left\{ \begin{array}{l} \vec{M} = \vec{\varepsilon} \wedge \vec{F} \\ \vec{L} = \vec{\varepsilon} \wedge m \cdot \vec{v} = \vec{\varepsilon} \wedge \vec{p} \end{array} \right. \Rightarrow \frac{d\vec{L}}{dt} = \vec{v} \wedge m \vec{v} + \vec{\varepsilon} \wedge m \vec{a} \\ \Rightarrow \underbrace{\frac{d\vec{L}}{dt}}_{\vec{M}} = \vec{\varepsilon} \wedge \vec{v} = \vec{M}$$

$$\Rightarrow \vec{M} = \frac{dL_{TOT}}{dt} = I \cdot \frac{d\omega}{dt} = I \alpha \quad \Rightarrow \quad \vec{M} = I \alpha \quad \equiv \quad \vec{F} = m \cdot \vec{\alpha}$$

$$\theta(t) = \theta_0 + \int_0^t \omega dt$$

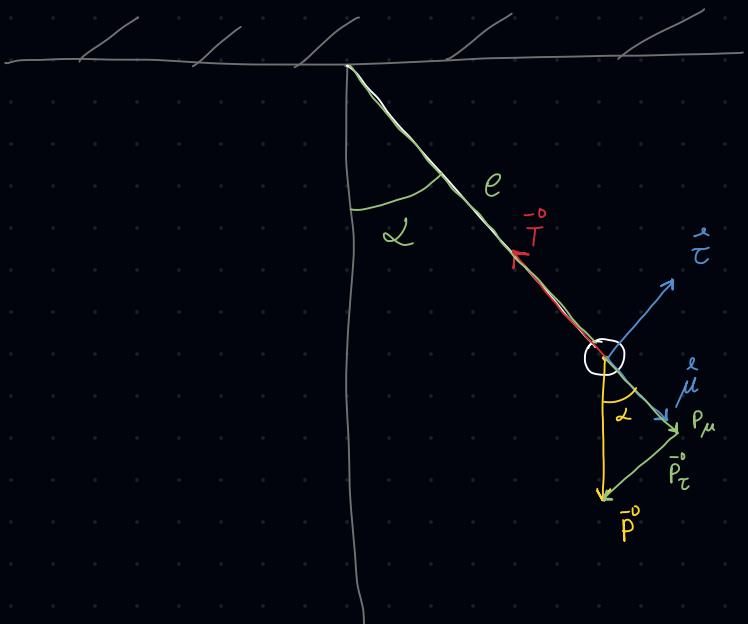
$$\omega(t) = \omega_0 + \int_0^t \alpha dt \quad \Rightarrow \quad \theta(t) = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 \quad \Rightarrow \quad S(t) = S_0 + \nu_0 t + \frac{1}{2} \alpha T^2$$

$$\omega(t) = \omega_0 + \alpha t \quad \Rightarrow \quad v(t) = v_0 + \alpha t$$

Energia cinetica Rotazioni

$$\frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 R^2 = \frac{1}{2} \omega^2 I$$

Pendolo



$$\vec{\Phi} = m \cdot \vec{g} \quad \rightarrow \quad \vec{\Phi} = \vec{P}_\mu + \vec{P}_\tau$$

$$\begin{cases} \vec{P}_\mu = m g \cos \theta \hat{\mu} \\ \vec{P}_\tau = -m g \sin \theta \hat{\tau} \end{cases}$$

Scriuw $\vec{F} = m \cdot \vec{a}$

$$\begin{cases} \vec{P}_\mu \cdot \tau = m \cdot \vec{a}_\mu \\ -\vec{P}_\tau = m \cdot \vec{a}_\tau \end{cases}$$

$$\vec{a} = \frac{d \vec{v}}{dt} = \begin{cases} \frac{v^2}{R} \text{ lungo } \hat{\mu} \\ \ddot{s} \text{ lungo } \hat{\tau} \end{cases}$$

$$\Rightarrow \begin{cases} m g \cos \theta - T = m \cdot \frac{v^2}{R} \\ -m g \sin \theta = m \cdot \ddot{s} \end{cases} \rightarrow \text{Appox piccole oscillaz. } \sin x \approx x$$

$$\Rightarrow -m g \ddot{\theta} = m \ddot{s} \quad \rightarrow -g \ddot{\theta} = \ddot{s}$$

$$1 \text{ Rad} = \frac{e \text{ ARCO}}{R \text{ Reggio}}$$

$$\rightarrow \text{Nel nostro caso} \quad \ddot{\theta} = \frac{\ddot{s}}{e}$$

$$\Rightarrow -g \ddot{\theta} = \ddot{s} \quad \Rightarrow -g \frac{\ddot{s}}{e} = \ddot{s} \quad \Rightarrow$$

$$\ddot{s} + \left(\frac{g}{e} \right) s = 0 \quad \rightarrow \text{Battazzo } \kappa^2$$

$$\rightarrow \ddot{s} + \kappa^2 s = 0 \quad \text{Eq differenziale}$$

$$\rightarrow \text{Sol} \quad s(t) = A \cos(\kappa t + \varphi)$$

$$\text{Periodo: } s(t+T_0) = A \cos(\kappa t + \varphi + 2\pi) \quad \rightarrow A \cancel{\cos}(\kappa t + \kappa T_0 + \varphi) = A \cancel{\cos}(\kappa t + \cancel{\varphi} + 2\pi)$$

$$\rightarrow \kappa T_0 = 2\pi \Rightarrow \kappa = \frac{2\pi}{T_0} = \omega \quad \rightarrow s(t) = A \cos(\omega t + \varphi)$$

$$\text{ma } \kappa = \sqrt{\frac{g}{e}} \Rightarrow T_0 = \frac{2\pi}{\kappa} = 2\pi \sqrt{\frac{e}{g}} \quad \text{Non dipende da } m!$$

Momento d'inerzia sbarro

$$I = m R^2 \quad \text{In sbarro è un continuo di punti -> corpo rigido}$$



$$\Rightarrow \rho = \frac{M}{V} \quad \text{sbarro 3D}$$

$$\lambda = \frac{M}{L} \quad \text{sbarro 2D}$$

$$\Rightarrow \int \lambda dz = \text{Massa Totale} \quad \text{inoltre} \quad \sum m_i \rightarrow \int dm = \text{massa Totale}$$

$$\Rightarrow I_{TOT} = \sum m_i R^2 \rightarrow I_{TOT} = \int R^2 dm \quad \text{sbarro 3D} \rightarrow \text{moltiplico per } dV \text{ diviso}$$

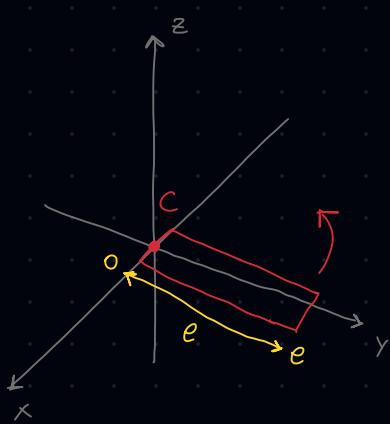
$$\Rightarrow I_{TOT} = \int R^2 \left(dm \cdot \frac{dV}{dV} \right) \rho \rightarrow I_{TOT} = \int R^2 \rho dV$$

↑ cost

Sbarro 2D:

$$I_{TOT} = \int R^2 dm \cdot \frac{de}{de} = \lambda \int_{e_0}^{e_f} R^2 de$$

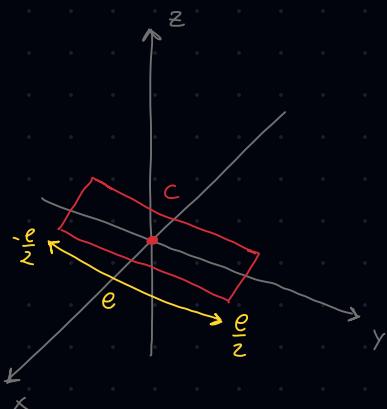
CASO 1:



$$\Rightarrow I_{TOT} = \lambda \int_0^e y^2 dy = \lambda \left[\frac{y^3}{3} \right]_0^e = \lambda \frac{e^3}{3}$$

$$= \frac{M}{R} \cdot \frac{e^3}{3} = \boxed{\frac{1}{3} M e^2} I$$

CASO 2:



$$\Rightarrow I_{TOT} = \lambda \int_{-\frac{e}{2}}^{\frac{e}{2}} y^2 dy = \lambda \left[\frac{y^3}{3} \right]_{-\frac{e}{2}}^{\frac{e}{2}} = \frac{e^3}{R} \left[\frac{e^3}{3} + \frac{e^3}{3} \right]$$

$$= \frac{M}{R} \cdot \frac{e^3}{12} = \boxed{\frac{1}{12} M e^2} I$$

II eq Card

$$\left\{ \begin{array}{l} \bar{M} = \bar{\varepsilon} \lambda \bar{F} \\ \bar{L} = \bar{\varepsilon} \lambda m \cdot \bar{v} = \bar{\varepsilon} \lambda \bar{P} \end{array} \right. \quad \Rightarrow \quad \frac{d\bar{L}}{dt} = \frac{d\bar{\varepsilon}}{dt} \lambda m \bar{v} + \bar{\varepsilon} \lambda m \frac{d\bar{v}}{dt} \quad \Rightarrow \quad \frac{d\bar{L}}{dt} = \bar{v} \lambda m \bar{u} + \bar{\varepsilon} \lambda m \bar{a}$$

$$\Rightarrow \frac{d\bar{L}}{dt} = \bar{\varepsilon} \lambda \bar{F} = \bar{M}$$

Energia cinetica

$$\bar{F} = m \cdot \bar{a} \quad \Rightarrow \quad L = \bar{F} \cdot e = \int \bar{F} \cdot d\bar{e} \quad m \bar{a} = \bar{F} = m \cdot \bar{a} \quad \Rightarrow \quad L = \int m \cdot \bar{a} \cdot d\bar{e}$$

$$m \bar{a} = \frac{d\bar{v}}{dt} \quad \Rightarrow \quad L = m \int \frac{d\bar{v}}{dt} \cdot d\bar{e} \quad \Rightarrow \quad L = m \int_{v_0}^{v_f} v \cdot dv = m \left(\frac{v_f^2}{2} - \frac{v_0^2}{2} \right)$$

$$\Rightarrow L = \int \bar{F} \cdot d\bar{e} = G$$

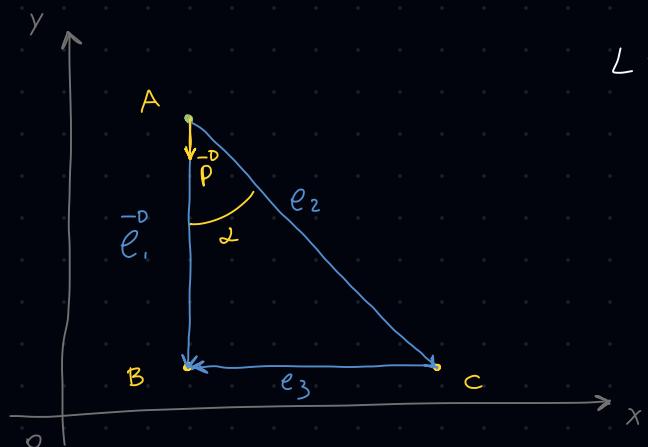
\uparrow
Energia cinetica

$$\mathcal{E}_{\text{potenziale}} : U = -G \quad \Rightarrow \quad L = \int \bar{F} \cdot d\bar{e} = -U$$

$$\Rightarrow \underline{dL = \bar{F} \cdot d\bar{e} = -U}$$

Campo conservativo:

Quando il lavoro compiuto non dipende dal cammino ma solo da stato finale ed iniziale



$$L = \oint \bar{F} \cdot d\bar{e} = \int \bar{F} \cdot d\bar{e}_1 + \left[\int \bar{F} \cdot d\bar{e}_2 + \int \bar{F} \cdot d\bar{e}_3 \right]$$

\uparrow_1

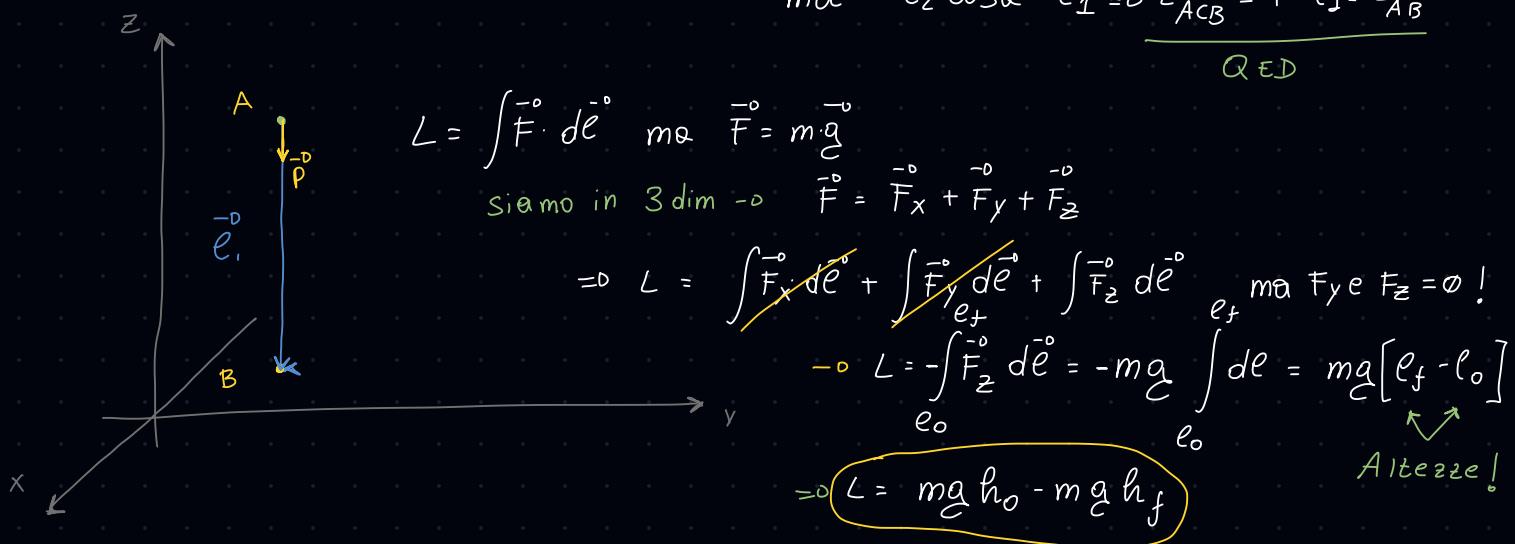
$$\bar{F} \cdot d\bar{e} = \bar{F} \cdot d\bar{e} \cdot \cos \alpha$$

$$\Rightarrow L_{AB} = \int \bar{F} \cdot d\bar{e}_1 = \bar{F} \cdot e_1$$

$$L_{ACB} = \int \bar{F} \cdot d\bar{e}_2 = \bar{F} \cdot \cos \alpha \cdot e_2$$

$$\text{ma } e_2 \cos \alpha = e_1 \Rightarrow L_{ACB} = \bar{F} \cdot e_1 = L_{AB}$$

\underline{QED}



$$L = \int \bar{F} \cdot d\bar{e} \quad \text{ma } \bar{F} = m \bar{g}$$

$$\text{Siamo in 3 dim} \Rightarrow \bar{F} = \bar{F}_x + \bar{F}_y + \bar{F}_z$$

$$\Rightarrow L = \int \bar{F}_x \cdot d\bar{e} + \int \bar{F}_y \cdot d\bar{e} + \int \bar{F}_z \cdot d\bar{e}$$

e_f ma F_y e $F_z = 0$!

$$\Rightarrow L = - \int \bar{F}_z \cdot d\bar{e} = -mg \int d\bar{e} = mg [e_f - e_0]$$

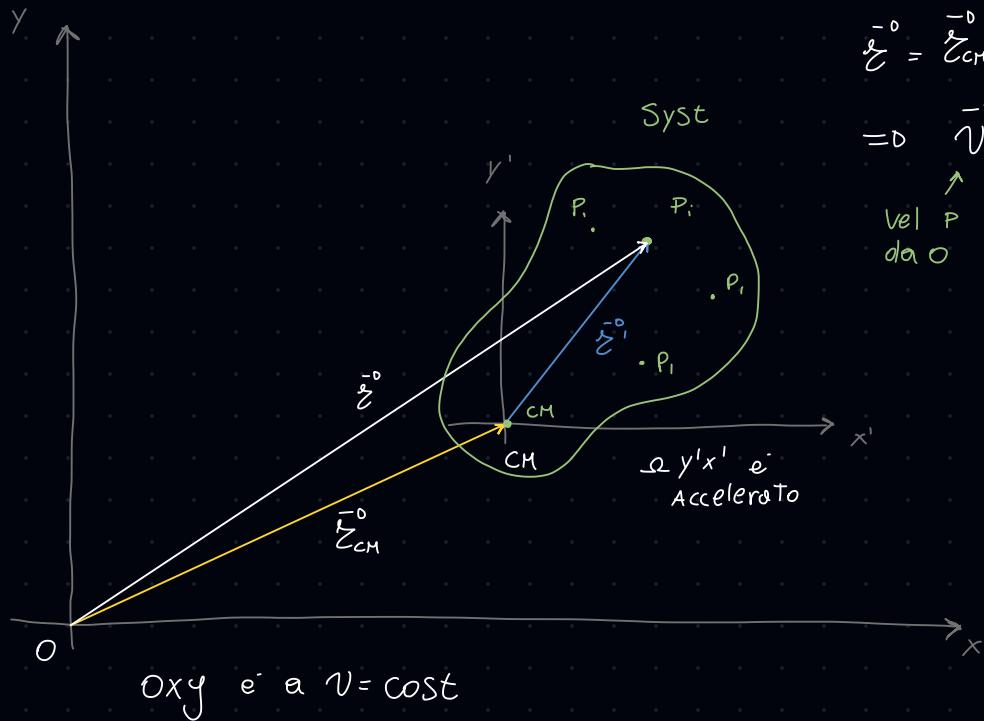
$$\Rightarrow L = mg h_0 - mg h_f$$

Altezza!

Teorema impulso

$$I = \int_{t_0}^{t_f} F dt = m \int_{t_0}^{t_f} a dt = m \int_{t_0}^{t_f} \frac{dv}{dt} dt = m \int_{v_0}^{v_f} dv = m(v_f - v_0) = \bar{P}_f - \bar{P}_0$$

Teorema di Koenig



$$\bar{\Sigma}^0 = \bar{\Sigma}_{CM}^0 + \bar{\Sigma}'^0$$

$$= \bar{V}^0 = \bar{V}_{CM}^0 + \bar{V}'^0$$

Vel P da 0 Vel P da 0 Vel P da 0

Momento Angolare: $\bar{L} = \bar{\Sigma}^0 \wedge m \bar{V}^0 = \bar{\Sigma}_i^0 \wedge m_i \bar{V}_i^0$

$$\bar{\Sigma}_i^0 = \bar{\Sigma}_{CM}^0 + \bar{\Sigma}'^0$$

$$\bar{V}_i^0 = \bar{V}_{CM}^0 + \bar{V}'^0$$

$$\bar{L}_i^0 = (\bar{\Sigma}_{CM}^0 + \bar{\Sigma}'^0) \wedge m_i (\bar{V}_{CM}^0 + \bar{V}'^0)$$

$$\bar{L}_{TOT}^0 = \sum_i \left[\bar{\Sigma}_{CM}^0 \wedge m_i \bar{V}_{CM}^0 + \bar{\Sigma}_{CM}^0 \wedge m_i \bar{V}'^0 + \bar{\Sigma}'^0 \wedge m_i \bar{V}_{CM}^0 + \bar{\Sigma}'^0 \wedge m_i \bar{V}'^0 \right]$$

$$= \bar{\Sigma}_{CM}^0 \wedge M \bar{V}_{CM}^0 + \bar{\Sigma}_{CM}^0 \wedge \sum_i m_i \bar{V}'^0 + \sum_i m_i \bar{\Sigma}'^0 \wedge \bar{V}_{CM}^0 + \sum_i \bar{\Sigma}'^0 \wedge m_i \bar{V}'^0$$

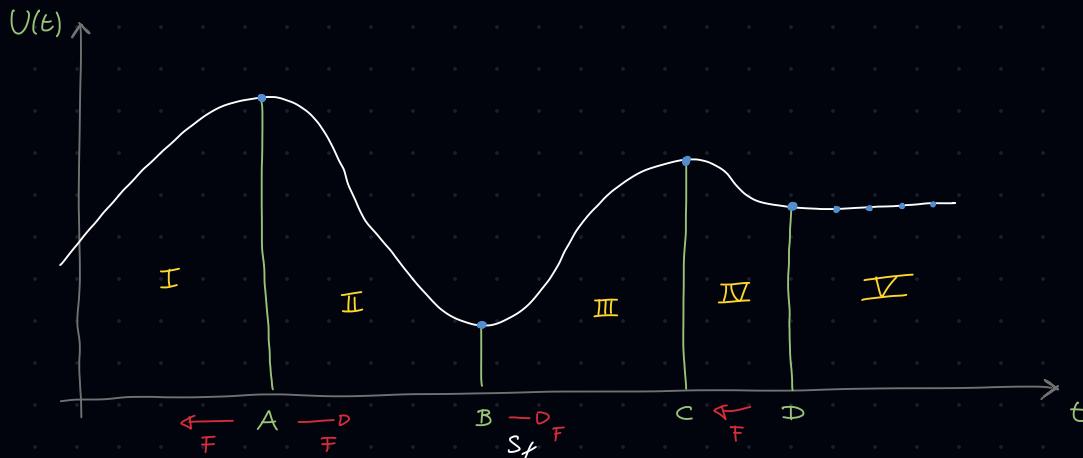
dal Centro di massa $\sum m_i \bar{V}'^0 = M \cdot \bar{V}'_{CM} = \emptyset$

$$= \bar{\Sigma}_{CM}^0 \wedge M \bar{V}_{CM}^0 + \sum_i \bar{\Sigma}'^0 \wedge m_i \bar{V}'^0$$

$\bar{L}_{TOT}^0 = \bar{L}_{CM}^0 + \bar{L}_{TOT}'$

Momento Angolare CM Momento Angolare TOTALE dei punti

Condizioni equilibrio



Teorema E. Cin: $L = \int_{S_0}^{\infty} \mathbf{F} \cdot d\mathbf{s} = \int m \cdot \mathbf{a} \cdot d\mathbf{s} = m \int \frac{d\mathbf{v}}{dt} \cdot d\mathbf{s} = m \int v \cdot dv = \frac{1}{2} m v^2 = G$

$$U = -G \Rightarrow dL = \mathbf{F} \cdot d\mathbf{s} = -dU \Rightarrow \boxed{\mathbf{F} = -\frac{dU}{ds}}$$

↑
Potenziale

Dall'analisi matematica sappiamo che se $\frac{dU}{dt} = 0 \Rightarrow$ punto min/max/cost

\Rightarrow punti dove $\frac{dU}{dt} = 0$ punti di equilibrio

Se per $I(x_0^-) \Rightarrow \frac{dU}{dt} > 0$ e $I(x_0^+) \Rightarrow \frac{dU}{dt} < 0 \Rightarrow$ **I STABILE**

Se per $I(x_0^-) \Rightarrow \frac{dU}{dt} < 0$ e $I(x_0^+) \Rightarrow \frac{dU}{dt} > 0 \Rightarrow$ **STABILE**

Legge di Ohm

$$V_A - V_B = R \cdot i$$

$$R = \rho \cdot \frac{l}{S}$$

1° legge

2 legge

$$\begin{cases} V_A - V_B = R \cdot I \\ V_A - V_B = \int \vec{E} \cdot d\vec{\ell} \end{cases}$$

$$L = F \cdot S \Rightarrow \frac{dL}{q} = E \cdot d\ell = V_A - V_B$$

$$\Rightarrow R \cdot I = \int_S \vec{E} \cdot d\vec{\ell} \quad \Rightarrow \quad R \cdot dI = \vec{E} \cdot d\vec{\ell}$$

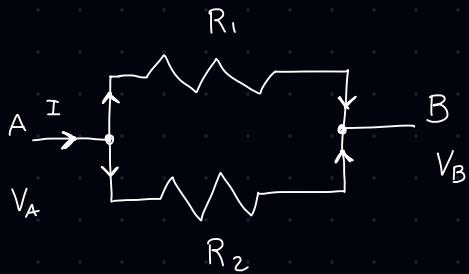
$$d\ell \cdot \hat{n} = d\vec{\ell}$$

$$R = \rho \cdot \frac{d\ell}{dS}, \quad I = \int \vec{J} \cdot \hat{n} dS \quad \Rightarrow \quad \rho \frac{d\ell}{dS} \cdot \vec{J} \cdot \hat{n} dS = \vec{E} \cdot d\vec{\ell}$$

$$\Rightarrow \vec{J} \cdot \rho \cdot d\vec{\ell} = \vec{E} \cdot d\vec{\ell} \quad \Rightarrow \quad \vec{E} = \vec{J} \cdot \rho \quad \text{Forma vettoriale}$$

Resistività Temp.

$$\rho(T) = \rho_0 (kT + 1) \quad \rho_0 = \rho(0^\circ)$$



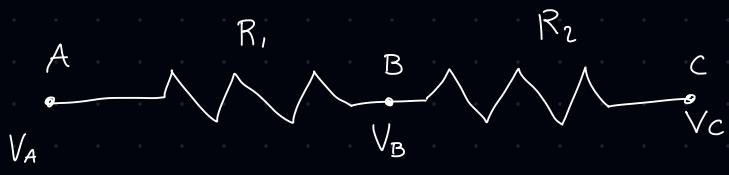
$$V_A - V_B = R_{EQ} \cdot I$$

$$\begin{cases} V_A - V_B = R_1 \cdot I_1 \\ V_A - V_B = R_2 \cdot I_2 \end{cases} \Rightarrow \begin{cases} I_1 = \frac{\Delta V}{R_1} \\ I_2 = \frac{\Delta V}{R_2} \end{cases}$$

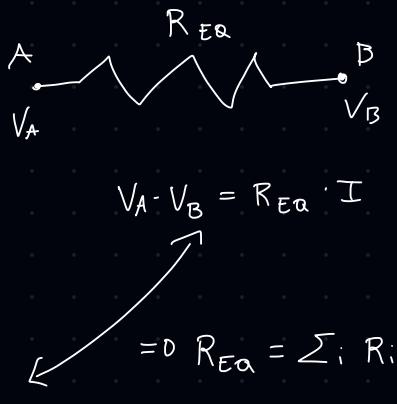
$$I = \frac{\Delta V}{R_{EQ}}$$

$$\Rightarrow I_1 + I_2 = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \Rightarrow \quad \frac{1}{R_{EQ}} = \sum_i \frac{1}{R_i}$$

I_{TOT}

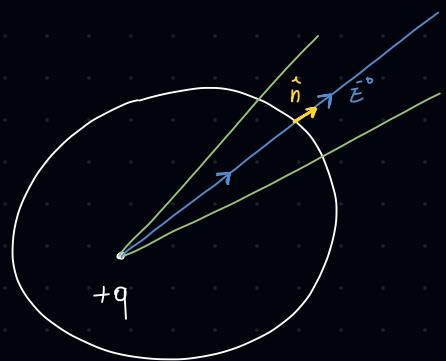


$$\begin{aligned} \rightarrow & \left\{ \begin{array}{l} V_A - V_B = R_1 I \\ V_B - V_C = R_2 I \end{array} \right. \\ \Rightarrow & V_A - V_B + V_B - V_C = I(R_1 + R_2) \\ \rightarrow & V_A - V_C = I(R_1 + R_2) \end{aligned}$$

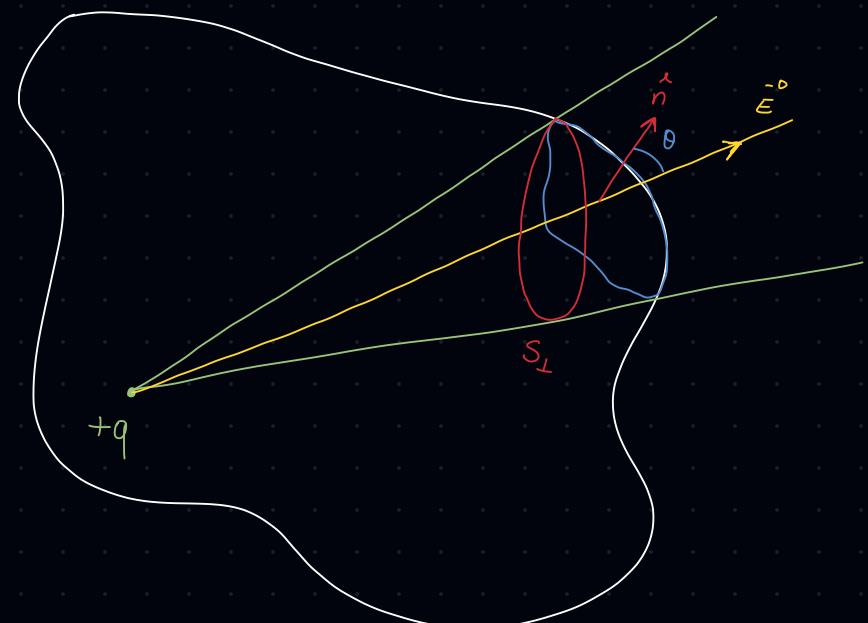


Teorema di Gauss

$$\phi_E = \int \vec{E} \cdot \hat{n} dS = \frac{Q_{int}}{\epsilon_0}$$



$$\int \vec{E} \cdot \hat{n} dS = \int \vec{E} \cdot \hat{\varepsilon} \cdot \hat{n} dS = E \int_S dS = \left| \frac{q}{4\pi\epsilon_0 R^2} \cdot 4\pi R^2 \right| = \frac{q}{\epsilon_0}$$



$$S_{\perp} = S \cos \theta$$

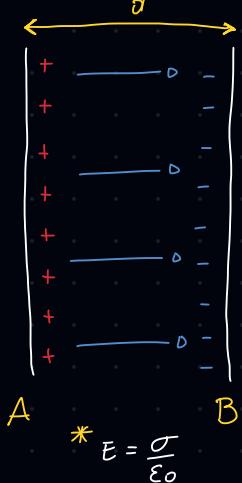
$$d\Omega = \frac{dS \cos \theta}{R^2} \quad \text{Angolo solido}$$

$$\Rightarrow \int \vec{E} \cdot \hat{n} dS = \int \frac{q}{4\pi\epsilon_0 R^2} \cdot \cos \theta \cdot dS$$

$$= \int \frac{q}{4\pi\epsilon_0} d\Omega = \frac{q}{4\pi\epsilon_0} \int d\Omega$$

$$= \frac{q}{4\pi\epsilon_0} \cdot 4\pi = \frac{q}{\epsilon_0}$$

Condensatori

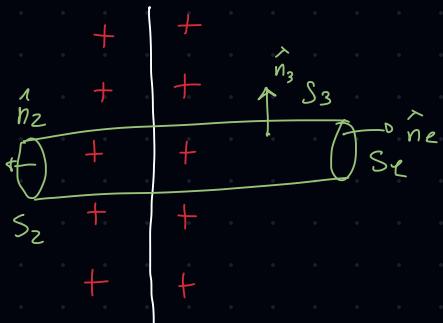


$$q_A = q_B \Rightarrow \begin{cases} V_A = k_1 Q \\ V_B = k_2 Q \end{cases} \Rightarrow V_A - V_B = Q(k_1 - k_2)$$

$$\frac{1}{C} = k_1 - k_2 \Rightarrow V_A - V_B = \frac{Q}{C}$$

$$\Rightarrow C = \frac{Q}{V_A - V_B}$$

Campo elettrico del doppio strato



$$\phi_E = \oint \vec{E} \cdot \hat{n} dS = 2 \int \vec{E} \cdot dS = 2 \epsilon S_1 = \frac{Q}{\epsilon_0}$$

$$\sigma = \frac{Q}{S} \Rightarrow Q = \sigma S$$

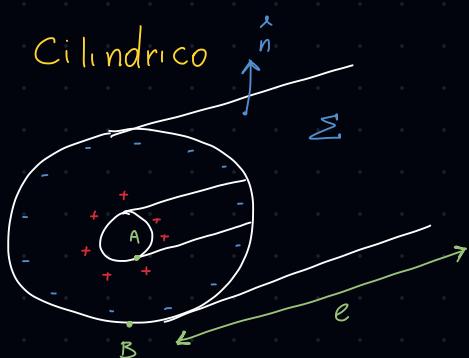
$$\Rightarrow 2 E S_1 = \frac{1}{\epsilon_0} \cdot \sigma S_1 \Rightarrow E = \frac{\sigma}{2 \epsilon_0}$$

Doppio strato $\Rightarrow E = \frac{\sigma}{\epsilon_0}$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{e} = E \cdot d \quad \text{ma} \quad E = \frac{\sigma}{\epsilon_0} \Rightarrow V_A - V_B = \frac{d \sigma}{\epsilon_0}$$

$$\Rightarrow C = \frac{Q \cdot \epsilon_0}{d \sigma} \quad \text{Siccome} \quad \sigma = \frac{Q}{S} \Rightarrow C = \frac{Q \cdot \epsilon_0}{d \cdot Q} \cdot S$$

$$C = \frac{S \cdot \epsilon_0}{d}$$



$$C = \frac{Q}{V_A - V_B} \quad \text{GAUSS} \quad \phi_E = \int \vec{E} \cdot \hat{n} dS = \frac{Q}{\epsilon_0}$$

Avvolgo il campo con una superficie gaussiana

$$\Rightarrow E \cdot \int_{\Sigma} dS = E \cdot \Sigma \quad \text{ma} \quad \Sigma = C \times l = 2\pi R \cdot l$$

$$\Rightarrow E \cdot 2\pi R \cdot l = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi R l \epsilon_0}$$

$$V_A - V_B = \int_A^B \frac{Q}{2\pi R \epsilon \epsilon_0} dR = \frac{Q}{2\pi \epsilon \epsilon_0} \int_A^B \frac{1}{R} dR$$

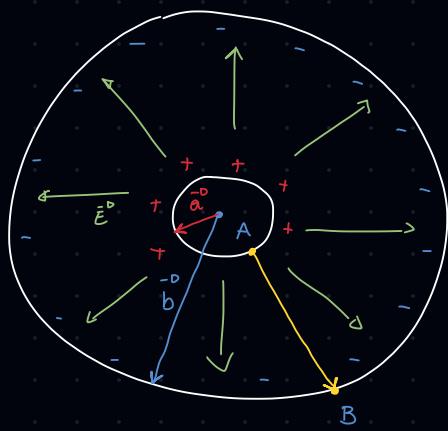
↑ Non
cost

$$\int \frac{1}{x} dx = \ln(x) + C \Rightarrow V_A - V_B = \frac{Q}{2\pi \epsilon \epsilon_0} \left[\ln(B) - \ln(A) \right] = \frac{Q \cdot \ln(\frac{B}{A})}{2\pi \epsilon \epsilon_0}$$

$$\text{ma } \frac{Q}{\epsilon} = \lambda \Rightarrow V_A - V_B = \frac{\lambda \ln(\frac{B}{A})}{2\pi \epsilon \epsilon_0}$$

$$\Rightarrow C = \frac{Q}{V_A - V_B} = \frac{Q 2\pi \epsilon \epsilon_0}{\lambda \ln(\frac{B}{A})} = \frac{Q 2\pi \epsilon \epsilon_0}{Q \ln(\frac{B}{A})} = \frac{2\pi \epsilon \epsilon_0}{\ln(\frac{B}{A})}$$

SFERICO



$$C = \frac{Q}{\Delta V} \quad \Delta V = \int_A^B E \cdot dR = \int \frac{Q \frac{1}{R}}{4\pi \epsilon_0 \epsilon^2} dR$$

$$= \int_A^B \frac{Q}{4\pi \epsilon_0 \epsilon^2} \frac{1}{R^2} dR = \frac{Q}{4\pi \epsilon_0 \epsilon} \int_A^B \frac{1}{R^2} dR$$

$$\Delta V = \frac{Q}{4\pi \epsilon_0} \left[-\frac{1}{\epsilon} \right]_A^B \quad \text{ma } A = \bar{a} \quad B = b$$

$$\Rightarrow \Delta V = \frac{Q}{4\pi \epsilon_0} \left[-\frac{1}{b} + \frac{1}{a} \right] = k \left[\frac{b-a}{ba} \right] \Rightarrow \Delta V = \frac{Q}{4\pi \epsilon_0} \left[\frac{b-a}{ba} \right]$$

$$\Rightarrow C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{4\pi \epsilon_0} \left[\frac{b-a}{ba} \right]} = 4\pi \epsilon_0 \frac{ba}{b-a}$$

CAPACITA'

il Potenziale è proporzionale alla carica , $Q_A = -Q_B \Rightarrow |Q_A| = |Q_B|$

$$\left\{ \begin{array}{l} V_A = K_1 Q \\ V_B = K_2 Q \end{array} \right. \Rightarrow V_B - V_A = Q(K_2 - K_1) \quad \frac{1}{C} = K_2 - K_1$$

$$= \frac{Q}{C} \Rightarrow C = \frac{Q}{V_A - V_B}$$

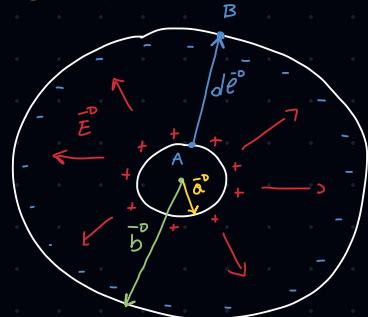
Sappiamo che E doppio strato : $E = \frac{\sigma}{\epsilon_0}$

Siccome $E = \frac{F}{q} \Rightarrow F = q \cdot E \Rightarrow \frac{F}{q} = \int_E^B dE = V_A - V_B$

$$= \Delta V = \int_A^B \frac{\sigma}{\epsilon_0} dE = \frac{Q}{S \epsilon_0} \int_A^B dE = \frac{Q \cdot d}{S \cdot \epsilon_0} \Rightarrow C = \frac{Q}{\Delta V} = \frac{S \cdot \epsilon_0}{d}$$

$$\sigma = \frac{Q}{S}$$

SFERICO



$$E = \frac{Q}{4\pi \epsilon_0 R^2} \hat{z} \Rightarrow V_A - V_B = \int_A^B \frac{Q}{4\pi \epsilon_0 R^2} d\hat{z} \Rightarrow d\hat{z} = d\theta$$

$$= \Delta V = \frac{Q}{4\pi \epsilon_0} \int_A^B \frac{1}{R^2} d\theta = \frac{Q}{4\pi \epsilon_0} \left[-\frac{1}{R} \right]_A^B$$

$$= \frac{Q}{4\pi \epsilon_0} \left[\frac{b-a}{ba} \right]$$

$$= C = 4\pi \epsilon_0 \cdot \frac{b-a}{b+a}$$

cilindrico

Campo Elettrico = ??

Teorema di Gauss

$$\phi_E = \oint \vec{E} \cdot \hat{n} dS = \int \vec{E} \cdot d\vec{S} = \vec{E} \cdot \Sigma = \frac{Q}{\epsilon_0}$$

$$\text{ma } \Sigma = C \cdot \ell = 2\pi R \ell$$

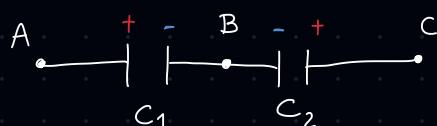
$$\Rightarrow \vec{E} \cdot 2\pi R \ell = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{2\pi R \ell \epsilon_0}$$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{r} = \int_A^B \frac{Q}{2\pi R \ell \epsilon_0} \cdot d\vec{r} = \frac{Q}{2\pi \ell \epsilon_0} \int_A^B \frac{1}{r} dr$$

$$\Rightarrow V_A - V_B = \frac{Q}{2\pi \ell \epsilon_0} \left[\ln(B) - \ln(A) \right] = \frac{Q \ln(\frac{B}{A})}{2\pi \ell \epsilon_0}$$

$$= C = \frac{2\pi \ell \epsilon_0}{\ln(\frac{B}{A})}$$

Nei circuiti



$$C_1 = \frac{Q}{V_A - V_B}$$

$$C_2 = \frac{Q}{V_B - V_C}$$

↓

$$V_A - V_B = \frac{Q}{C_1} \quad V_B - V_C = \frac{Q}{C_2}$$

$$\Rightarrow V_A - V_B - V_C = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

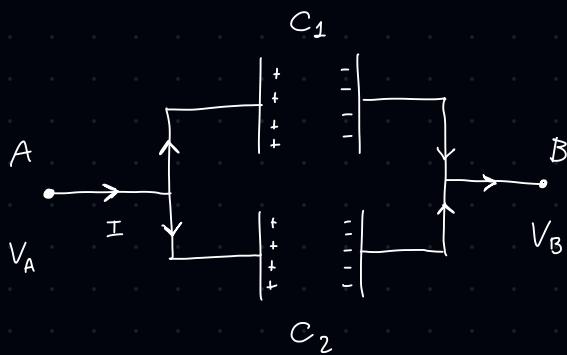


$$C_{EQ} = \frac{Q}{V_A - V_B}$$

$$\Rightarrow V_A - V_B = \frac{Q}{C_{EQ}}$$

$$\frac{1}{C_{EQ}} = \sum_i \frac{1}{C_i}$$

$$\Rightarrow V_A - V_B = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$



$$C_1 = \frac{Q_1}{V_A - V_B} \quad C_2 = \frac{Q_2}{V_A - V_B}$$

$$\Rightarrow Q_1 = C_1(V_A - V_B) \quad Q_2 = C_2(V_A - V_B)$$

$$\Rightarrow \overline{Q_{\text{tot}}} = (V_A - V_B)(C_1 + C_2) \Rightarrow Q = (V_A - V_B)(C_1 + C_2)$$

$$C_{EQ} = \frac{Q}{V_A - V_B}$$

$$\Rightarrow Q = (V_A - V_B) C_{EQ}$$

$$\Rightarrow C_{EQ} = \sum_i C_i$$

Energia Campo \vec{E}

$$V_A \left| \begin{array}{c} + \\ + \\ + \\ + \\ + \\ + \end{array} \right| V_B$$

$$A \quad E = \frac{\sigma}{\epsilon_0} \quad B$$

\Rightarrow Lavoro per spostare una carica da A o B

$$L = \int_{\text{lavoro Forza}}^B F \cdot de \quad \text{ma} \quad \vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q \cdot \vec{E} \quad \Rightarrow \frac{L}{q} = \int_A^B \vec{E} \cdot d\vec{e} = V_A - V_B$$

$$\Rightarrow dL = F \cdot de \quad \Rightarrow dL = q \cdot E \cdot de \quad \text{ma} \quad E \cdot de = V$$

$$\Rightarrow dL = V dq$$

$$\Rightarrow L = \int_A^B V dq \quad \text{dal condensatore} \quad C = \frac{Q}{V_A - V_B} \Rightarrow V_A - V_B = \frac{Q}{C}$$

$$= \int_A^B \frac{Q}{C} dq = \frac{1}{C} \int_A^B Q dQ = \frac{1}{C} \left[\frac{Q^2}{2} \right]_A^B = \frac{1}{C} \left[\frac{Q_B^2}{2} - \frac{Q_A^2}{2} \right]$$

Se pongo $\begin{cases} A = 0 \\ B = Q \end{cases} \Rightarrow L = \frac{1}{C} \frac{Q^2}{2}$ ma $\frac{Q}{C} = V_A - V_B$

$$\Rightarrow L = \frac{(V_A - V_B) Q}{2}$$

Definisco \mathcal{W} densità volumetrica di Energia $\rightarrow \mathcal{W} = \frac{\mathcal{E}}{V}$

$$V = S \cdot d \Rightarrow \mathcal{W} = \frac{(V_A - V_B) \sigma}{2(S/d)} = \frac{1}{2} (V_A - V_B) \frac{\sigma}{d}$$

ma nel cond piano $E \cdot S = \frac{Q}{\epsilon_0} \rightarrow E \cdot \epsilon_0 = \frac{Q}{S} \rightarrow E \cdot \epsilon_0 = \sigma \quad (1)$

cond piano $E \cdot d = V_A - V_B \Rightarrow E = \frac{V_A - V_B}{d} \quad (2)$

$$\rightarrow \mathcal{W} = \frac{1}{2} E \cdot E \cdot \epsilon_0 \Rightarrow \mathcal{W} = \frac{1}{2} E^2 \epsilon_0$$

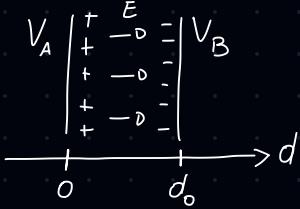
Densità di Energia
Volum.

Energia \rightarrow Necessaria a Spostare una carica

$$L = F \cdot e \rightarrow dL = F \cdot de \quad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E$$

$$\frac{dL}{q} = \text{Lavoro del C.E.} = E \cdot de \quad \text{ma} \quad \int \vec{E} \cdot d\vec{e} = -dV$$

$$= 0 \quad \boxed{L_E = q \cdot V} \quad \rightarrow \quad dL = V dq$$

$$= 0 \quad L = \int_A^B V dq$$


$$= 0 \quad \int_V^Q dq =$$

$$C = \frac{Q}{V} \quad \Rightarrow \quad V = \frac{Q}{C} \quad \rightarrow \quad L = \frac{Q}{C} \quad = 0 \quad L = \int_0^Q \frac{Q}{C} dq = \frac{1}{C} \left[\frac{Q^2}{2} \right]_0^Q$$

$$\rightarrow L = \frac{1}{C} \frac{Q^2}{2} \quad \text{ma} \quad \frac{Q}{C} = V \Rightarrow \quad \boxed{L = \mathcal{E} = \frac{Q(V_A - V_B)}{2}}$$

Energia caso generale

Caso del condensatore piano

$$W = \frac{\mathcal{E}}{V} \quad \underline{V} = S \cdot d \quad \rightarrow \quad \frac{Q(V_A - V_B)}{2 S d} = \frac{1}{2} \frac{V_A - V_B}{d} \cdot \sigma$$

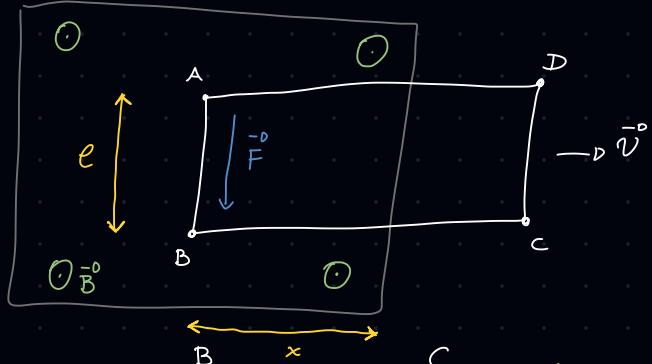
$$\begin{aligned} & \text{Sappiamo che } E \cdot d = V_A - V_B \Rightarrow E = \frac{V_A - V_B}{d} \\ & \text{inoltre } E \cdot S = \frac{Q}{\epsilon_0} \Rightarrow E \epsilon_0 = \frac{Q}{S} = \sigma \end{aligned} \quad \left. \begin{aligned} & W = \frac{1}{2} E^2 \epsilon_0 \\ & \underline{\underline{W = \frac{1}{2} E^2 \epsilon_0}} \end{aligned} \right\}$$

Faraday

$$f_{em} = - \frac{d\phi_B}{dt}$$

TAGLIATO

$$\vec{F}_{\text{Lorenz}} = q(\vec{v} \wedge \vec{B})$$



$$f_{em} = \frac{L}{q} = q \frac{\int (\vec{v} \wedge \vec{B}) d\vec{e}}{q}$$

$$= \int_A^B (\vec{v} \wedge \vec{B}) d\vec{e} + \int_B^C (\vec{v} \wedge \vec{B}) d\vec{e} + \int_C^D (\vec{v} \wedge \vec{B}) d\vec{e} + \int_D^A (\vec{v} \wedge \vec{B}) d\vec{e} = \int_A^B (\vec{v} \wedge \vec{B}) d\vec{e}_1$$

$\vec{F} \perp d\vec{e}$

$$= VB\ell$$

$$\text{ma } f_{em} = - \frac{d\phi_B}{dt} = \int B \cdot \underbrace{\cos dS}_{\text{VARIANO}}$$

Se il filo si muove, S dipende da t = 0 $S = \ell \cdot x$ ma $x(t) = x - \Delta x$
 $\Rightarrow dS = \ell \cdot dx$ con $\Delta x = v \cdot \Delta t$

$$f_{em} = - \frac{d\phi}{dt} = \int_0^x B \cdot \ell \cdot dx = \underline{B\ell x}$$

$$\bar{t} = t + \Delta t \quad \text{e} \quad \bar{x} = x - \Delta x$$

$$\phi_B(t + \Delta t) = Bx(x - \Delta x)$$

$$= \frac{d\phi_B}{dt} = \frac{\phi_B(t + \Delta t) - \phi_B(t)}{\Delta t} = \frac{Bx(x - \Delta x) - Bx}{\Delta t} = \frac{\cancel{Bx}x - \cancel{Bx}\Delta x - \cancel{Bx}}{\Delta t}$$

$$\rightarrow \frac{d\phi}{dt} = \lim_{\Delta t \rightarrow 0} - \frac{Bx \cancel{\Delta x}}{\Delta t} = -Bxv \quad \rightarrow - \frac{d\phi}{dt} = \underline{Bxv}$$

Concatenato

$$\text{Varia } \mathcal{B}, \quad S = \cos t \quad \Rightarrow \quad \mathcal{B}(t, \varepsilon)$$

$$\frac{d\phi}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\int \overset{\circ}{\mathcal{B}}(t + \Delta t, \varepsilon) \hat{n} dS - \int \overset{\circ}{\mathcal{B}}(t, \varepsilon) \hat{n} dS}{\Delta t}$$

Approssimo con Taylor $\Delta t = 0$, 1^o Termine

$$\mathcal{B}(t, \varepsilon) \approx \mathcal{B}(t, \varepsilon) + \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t}(t, \varepsilon) \cdot \Delta t$$

$$\Rightarrow \frac{d\phi}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\int \overset{\circ}{\mathcal{B}}(t, \varepsilon) \hat{n} dS + \int \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t}(t + \Delta t, \varepsilon) \Delta t - \int \overset{\circ}{\mathcal{B}}(t, \varepsilon) \hat{n} dS}{\Delta t}$$

$$= \left(\frac{d\phi}{dt} = \int_S \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t} \hat{n} dS \right)$$

$$\text{ma } f_{em} = \frac{q}{q} \quad \Rightarrow \quad F_{LOR} = q E + q (\vec{v} \wedge \vec{\mathcal{B}}) \quad \Rightarrow \quad f_{em} = \int \vec{E} \cdot d\vec{e} + \int \vec{v} \wedge \vec{\mathcal{B}} d\vec{e}$$

$$\Rightarrow \int \vec{E} d\vec{e} + \int_{V=0} \vec{v} \wedge \vec{\mathcal{B}} d\vec{e} = - \int_S \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t} \hat{n} dS \quad \text{ma } \mathcal{B} = \text{Varia}$$

$$V = \emptyset \Rightarrow \int v \wedge B d\vec{e} = 0$$

$$\Rightarrow \oint_E \vec{E} d\vec{e} = - \int \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t} \hat{n} dS \quad \Rightarrow \quad \int_S \vec{\nabla} \lambda \vec{E} \hat{n} dS = - \int \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t} \hat{n} dS$$

$$\Rightarrow \left(\vec{\nabla} \lambda \vec{E} = - \frac{\partial \overset{\circ}{\mathcal{B}}}{\partial t} \right)$$

TUTTO in sieme

$$1) -\frac{d\phi}{dt} = \oint (\vec{v} \wedge \vec{B}) d\ell \quad \left. \right\} \quad \frac{d\Phi}{dt} = \oint \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS - \oint (\vec{v} \wedge \vec{B}) d\ell$$

$$2) -\frac{d\phi}{dt} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

$$\text{ma } f_{em} = -\frac{d\phi}{dt} = \frac{L}{q} = \oint \vec{E} \cdot d\vec{e} + \int (\vec{v} \wedge \vec{B}) d\ell$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{e} + \oint (\vec{v} \wedge \vec{B}) d\ell = -\int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS + \oint (\vec{v} \wedge \vec{B}) d\ell$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{e} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS \quad \Rightarrow \quad \boxed{\nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{Maxwell}$$

AUTOinduzione

Quando circola I in un circuito \Rightarrow Campo magnetico indotto

$$\Rightarrow \text{BIOT-Savart} \Rightarrow dB = \frac{\mu_0 I}{4\pi \epsilon_0} \frac{d\ell \wedge \vec{\epsilon}}{\epsilon^3}$$

$$\Rightarrow \phi_B = \int_S \frac{\mu_0 I}{4\pi \epsilon_0} \int \frac{d\ell \wedge \vec{\epsilon}}{\epsilon^3} \cdot \hat{n} dS \quad \text{AUTO FLUSSO}$$

TUTTO COSTANTE
TRANNE I

$$\Rightarrow \phi_B = L \cdot I \quad \text{Induttanza}$$

Quando $B \neq \text{cost}$

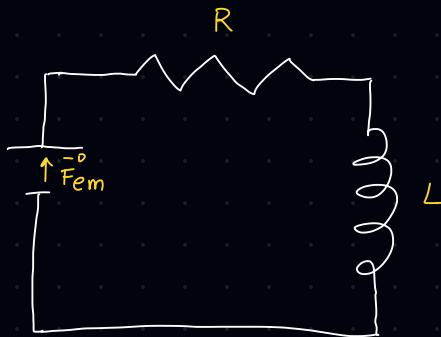
$$\phi_B \neq 0 \Rightarrow \frac{d\phi_B}{dt} \neq 0 \Rightarrow \phi_B = L \frac{dI}{dt}$$

$$\Rightarrow f_{em} = -\frac{d\phi}{dt} = \Theta L \frac{dI}{dt}$$

Segno
opposto

Esempio Induttanza

Circuito R-L



$$f_{em} - f_{ind} = R \cdot I \quad \text{ma } f_{ind} = -L \frac{dI}{dt}$$

$$- \circ \quad f_{em} = L \frac{dI}{dt} + R \cdot I \quad \text{Eq diff}$$

$$\text{Soluzione: } I(t) = A e^{\lambda t} + D$$

$$- \circ \quad \dot{I} = \lambda A e^{\lambda t} \quad - \circ \quad f_{ind} = L \lambda A e^{\lambda t} + R A e^{\lambda t} + R D \quad \text{ma } f_{ind} = \text{cost}$$

$$- \circ \quad f_{ind} = R \cdot D = 0 \quad D = \frac{f}{R} \quad - \circ \quad L \lambda A e^{\lambda t} + R A e^{\lambda t} = 0 \quad - \circ \quad A e^{\lambda t} (L \lambda + R) = 0$$

$$- \circ \quad L \lambda + R = 0 \quad - \circ \quad \lambda = -\frac{R}{L}$$

$$- \circ \quad \text{Sol gen: } I(t) = A e^{-\frac{R}{L}t} + \frac{f}{R}$$

$$\text{Pongo } I(0) = 0 \quad - \circ \quad I(0) = A e^{-\frac{R}{L}0} + \frac{f}{R} = 0 \quad - \circ \quad A + \frac{f}{R} = 0 \quad - \circ \quad A = -\frac{f}{R}$$

$$- \circ \quad \text{Sol part: } I(t) = -\frac{f}{R} e^{-\frac{R}{L}t} + \frac{f}{R}$$

OHM

$$1) \quad V_A - V_B = R \cdot I$$

Forma vettoriale

$$2) \quad R = \rho \cdot \frac{e}{S}$$

$$\begin{cases} V_A - V_B = R \cdot I \\ V_A - V_B = \int E \cdot d\ell \end{cases}$$

$$- \circ \quad R \cdot I = \int \vec{E} \cdot d\vec{\ell} \quad \text{ma} \quad I = \int \vec{J} \cdot \vec{n} dS = J \cdot S \quad R = \rho \frac{d\ell}{dS}$$

$$- \circ \quad \rho \frac{d\ell}{dS} \cdot \underbrace{\int \vec{J} \cdot \vec{n} dS}_{d\ell \cdot \vec{n} = d\vec{\ell}} = \int \vec{E} \cdot d\vec{\ell} \quad - \circ \quad \rho \int \vec{J} d\vec{\ell} = \vec{E} d\vec{\ell}$$

$$- \circ \quad \vec{E} = \int \vec{J}$$

Conservazione Carica

$$-dq = \int \bar{J} \cdot \hat{n} ds \cdot dt \Rightarrow -\frac{dq}{dt} = \int \bar{J} \cdot \hat{n} ds \Rightarrow$$

so che $f = \frac{dq}{dV}$ $\Rightarrow dq = f dV \Rightarrow -\frac{d}{dt} \int f dV = \int \bar{J} \cdot \hat{n} ds$

$$\int_V \frac{\partial f}{\partial t} dV = \int_S \bar{J} \cdot \hat{n} ds \Rightarrow -\int_V \frac{\partial f}{\partial t} dV = \int_V \bar{\nabla} J \cdot \hat{n} dV$$

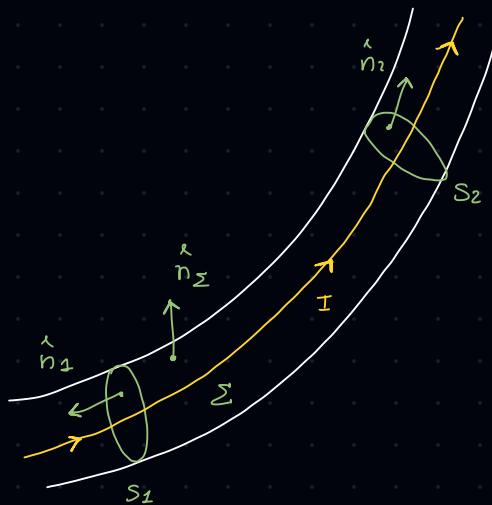
$$-\frac{\partial f}{\partial t} = \bar{\nabla} J$$

condizioni di corrente stazionaria

\Rightarrow La densità di carico è sempre lo stesso

$$\bar{\nabla} J = \emptyset$$

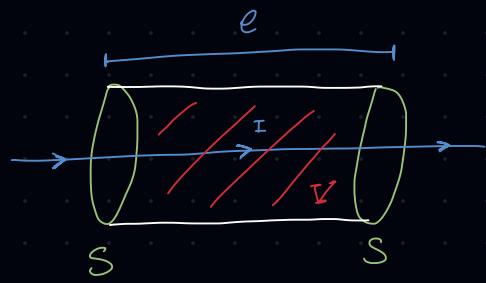
$$\frac{\partial f}{\partial t} = 0$$



$$\begin{aligned} \phi_E &= \int \bar{J} \cdot \hat{n} ds = \int_{S_1} + \int_{S_2} + \int_{S_3} \\ &= -\phi_1 + \phi_2 = \emptyset \quad \text{perché } S_1 = S_3 \\ &\Rightarrow \phi_1 = -\phi_{S_2} \end{aligned}$$

Densità di corrente: $I = \int \bar{J} \cdot \hat{n} ds$

$$f = \frac{d\alpha}{dV} \Rightarrow d\alpha = f dV \quad \text{ma } dV = S \cdot \ell \quad \text{ma } \ell = v \cdot t$$



$$\Rightarrow dV = dS \cdot v \cdot dt$$

$$\Rightarrow dq = \int f \cdot dS \cdot \bar{v} \cdot dt \Rightarrow \frac{dq}{dt} = \int f \cdot \bar{v} \cdot dS$$

$$\Rightarrow I = \int f \cdot \bar{v} \cdot dS \quad \bar{J} = f \cdot \bar{v}$$

$$\Rightarrow I = \int \bar{J} \cdot \hat{n} ds$$

Auto ind

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\hat{e} \wedge \vec{\epsilon}}{A\varepsilon^3} \rightarrow \phi_B = \int \frac{\mu_0 I}{4\pi} \int \frac{d\hat{e} \wedge \vec{\epsilon}}{A\varepsilon^3} \hat{n} dS$$

$$\rightarrow \text{Tutto cost} \rightarrow \phi_B = L \cdot I \quad \text{Se } B \neq \text{cost} \quad \frac{d\phi_B}{dt} = L \frac{dI}{dt}$$

Siccome $f_{en} = -\frac{d\phi_B}{dt}$

$$f_{ind} = -L \frac{dI}{dt}$$

Potenziale Scalare

$$\begin{cases} \vec{\nabla} \wedge \vec{E} = 0 & (1) \\ \vec{\nabla} V = -\frac{\rho}{\varepsilon_0} & (2) \end{cases} \rightarrow \text{Legge Coulomb} \quad dL = \vec{F} \cdot d\vec{e} = -dU \quad \rightarrow \frac{dL}{q} = \vec{E} \cdot d\vec{e} = -dV$$

$$\rightarrow E \cdot d\vec{e} = -dV \quad dV \text{ e del tipo} \quad \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$\begin{cases} \vec{\nabla} V = \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \\ d\vec{e} = \hat{i} dx + \hat{j} dy + \hat{k} dz \end{cases} \Rightarrow \vec{\nabla} V \cdot d\vec{e} = dV$$

$$\rightarrow \vec{E} \cdot d\vec{e} = -\vec{\nabla} V \cdot d\vec{e} \quad \rightarrow \vec{E} = -\vec{\nabla} V$$

nelle (1) e (2) \rightarrow

$$\begin{cases} \vec{\nabla} \wedge (-\vec{\nabla} V) = 0 \\ \vec{\nabla}(-\vec{\nabla} V) = -\frac{\rho}{\varepsilon_0} \end{cases} \rightarrow \text{Sempre Vero} \quad \vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0} \quad \text{Eq di Poisson}$$

Soluzione $V = \frac{1}{4\pi\varepsilon_0} \int \frac{f(z')}{|\vec{r} - \vec{r}'|} dV$

Potenziale vettore

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \end{cases} \quad \text{Ampere} \quad \oint \vec{B} \cdot d\vec{e} = \mu_0 I$$

Potenziale Vettore: $\vec{B} = \vec{\nabla} \wedge \vec{A}$

$$\left\{ \begin{array}{l} \bar{\nabla}^{\bar{B}} = 0 \rightarrow \bar{\nabla}^{\bar{B}} (\bar{\nabla}^{\bar{A}} \wedge \bar{A}) = 0 \quad \rightarrow \text{Sempre vero} \\ \bar{\nabla} \wedge \bar{B} = \mu_0 \bar{J} \rightarrow \bar{\nabla} \wedge (\bar{\nabla}^{\bar{A}} \wedge \bar{A}) = \mu_0 \bar{J} \quad \rightarrow \text{Identità} \end{array} \right.$$

$$\rightarrow \bar{\nabla}(\bar{\nabla}^{\bar{A}}) - \bar{\nabla}^2 \bar{A} = \mu_0 \bar{J} \quad \text{Se } \bar{\nabla}^2 \bar{A} = 0 \quad \rightarrow \bar{\nabla}^2 \bar{A} = -\mu_0 \bar{J}$$

$$\Rightarrow \text{Soluzione} \rightarrow \bar{A}(\bar{z}) = \frac{\mu_0}{4\pi} \int \frac{J(z')}{|\bar{z} - \bar{z}'|} dV \quad \text{ma } V = S \cdot \ell \quad \rightarrow dV = S \cdot d\ell$$

$$\Rightarrow \bar{A}(\bar{z}) = \frac{\mu_0}{4\pi} \int \frac{J \cdot S \cdot d\ell'}{|\bar{z} - \bar{z}'|} = \frac{\mu_0}{4\pi} \int \frac{S}{|\bar{z} - \bar{z}'|} d\bar{\ell}'$$

$$\text{Se } \bar{\nabla}(\bar{\nabla}^{\bar{A}}) \neq 0 \quad A' = A + \bar{\nabla}^{\bar{B}} f$$

$$\text{Invarianza di Gauge: } \bar{B} = \bar{\nabla} \wedge \bar{A} = \bar{\nabla} \wedge \bar{A}' \quad \text{proof: } \bar{\nabla} \wedge A + \cancel{\bar{\nabla} \wedge (\bar{\nabla} f)} = \bar{\nabla} \wedge \bar{A}$$

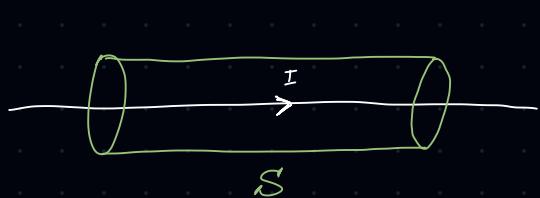
$$\text{fissiamo una } f: \quad \bar{\nabla} A' = \bar{\nabla} \bar{A} + \bar{\nabla}^2 f = 0 \quad \text{Fisso la Gauge}$$

\Rightarrow Eq di poisson

Corrente Spostamento

$$\text{Legge di Ampere: } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad \leftrightarrow \quad \int \vec{B} \cdot d\vec{s} = \mu_0 I$$

Corrente stazionaria:



$$\phi_E = \oint J \cdot \hat{n} ds = \emptyset$$

$$\rightarrow \nabla J = \emptyset \quad \text{Stazionario}$$

Non Stazionario \rightarrow Condensatore



$$\phi = \oint \vec{J} \cdot \hat{n} ds \neq \emptyset !$$

$\rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ e incompleto! Infatti Applico la divergenza

$$\vec{\nabla} (\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\nabla} \vec{J} \rightarrow \vec{\nabla} \vec{J} = 0 \quad \text{Stazionario!} \rightarrow \text{Sbagliato!}$$

$$\text{infatti} \quad -dq = \int \vec{J} \cdot \hat{n} ds \cdot dt \rightarrow -\frac{dq}{dt} = \int J \cdot \hat{n} ds \quad \text{ma} \quad f = \frac{dq}{dt} \rightarrow dq = f dt$$

$$\rightarrow -\frac{d}{dt} \int f dt = \int \vec{J} \cdot \hat{n} ds \rightarrow -\int \frac{\partial f}{\partial t} dt = \int J \cdot \hat{n} ds$$

$$\rightarrow -\int \frac{\partial f}{\partial t} dt = \int \nabla f \cdot dV \rightarrow \nabla f = -\frac{\partial f}{\partial t} \quad \text{NON ZERO!}$$

Termine Mancante: $\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ Curr Spost. m.

$$Eq \text{ Ampère-Maxwell: } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Test Div:

$$\nabla \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 \nabla \cdot \vec{J} + \mu_0 \epsilon_0 \frac{\partial \nabla \cdot \vec{E}}{\partial t} \rightarrow \mu_0 \nabla \cdot \vec{J} + \mu_0 \epsilon_0 \frac{1}{\epsilon_0} \frac{\partial f}{\partial t} = 0$$

$$\rightarrow \mu_0 \left[\nabla \cdot \vec{J} + \frac{\partial f}{\partial t} \right] = 0 \rightarrow \nabla \cdot \vec{J} = -\frac{\partial f}{\partial t} \quad \underline{\text{QED}}$$

Modello di Drude

1) Ohm vettoriale

$$\left\{ \begin{array}{l} V_A - V_B = R \cdot I \\ V_A - V_B = \int_{e^0}^{-e^0} d\vec{e} \end{array} \right. \Rightarrow R I = \int E \cdot d\vec{e} \quad \text{ma} \quad \left\{ \begin{array}{l} R = \rho \cdot \frac{l}{S} \\ dI = \int \vec{J} \cdot \vec{n} ds \end{array} \right.$$

$$d\vec{e} \cdot \vec{n} = d\vec{e}^0$$

$$\Rightarrow \rho \frac{d\vec{e}}{ds} \cdot \vec{J} \cdot \vec{n} ds = \vec{E} \cdot d\vec{e}^0 \Rightarrow \rho d\vec{e}^0 \vec{J} = \vec{E} d\vec{e}^0 \Rightarrow$$

$$\vec{E} = \int \vec{J}$$

Resistività

$$2) \text{ Corrente} \quad I = \int \vec{J} \cdot \vec{n} ds$$

$$\text{con} \quad \vec{J} = \rho \cdot \vec{v} \quad \Rightarrow \text{per non confonderci:} \quad \rho = -e n$$

Densità
Vol carica

$$\Rightarrow \vec{J} = -e n \vec{v}$$

A) Presenza di campo elettrico

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q \cdot \vec{E} \Rightarrow \vec{F} = -e \vec{E}$$

$E = 0 \quad \alpha = 0, v = \text{cost}$
 $E \neq 0 \quad \alpha \neq 0, v \neq \text{cost}$

$$\vec{v} = v_0 + \vec{a} \cdot t$$

$$F = m \cdot a \Rightarrow \vec{a} = \frac{\vec{F}}{m} \quad m a \quad F = -e \vec{E} \Rightarrow \vec{a} = -\frac{e \vec{E}}{m}$$

$$\bullet E \neq 0 \Rightarrow v = v_0 - \frac{e \vec{E}}{m} t$$

$$\bullet E = 0 \Rightarrow v = v_0$$

B) La media

Quando si lavora con le particelle quantistiche e' bene parlare di "media"

$$- E = 0$$

$$\langle v \rangle = \langle v_0 \rangle - \left\langle \frac{e \vec{E}}{m} t \right\rangle \Rightarrow \langle v \rangle = \langle v_0 \rangle$$

ma se $v_0 \in (+\infty, -\infty)$ $\Rightarrow \langle v_0 \rangle = 0$



$$- E \neq 0$$

$$\langle v \rangle = \langle v_0 \rangle - \left\langle \frac{e \vec{E}}{m} t \right\rangle = - \frac{e \vec{E}}{m} \langle t \rangle \quad \text{Tempo rilassamento medio}$$

$$\Rightarrow \langle v \rangle = - \frac{e \vec{E}}{m} \tau \quad \text{per } E \neq 0$$

c) Sostitui sco:

$$\vec{J} = -e n \vec{v} \Rightarrow J = -e n \cdot \left(-\frac{e \vec{E}}{m} \right) \tau = \underline{\underline{\frac{n e^2 E}{m} \tau}}$$

$$\vec{E} = \rho \vec{J} \Rightarrow J = \underline{\underline{\frac{E}{\rho}}}$$

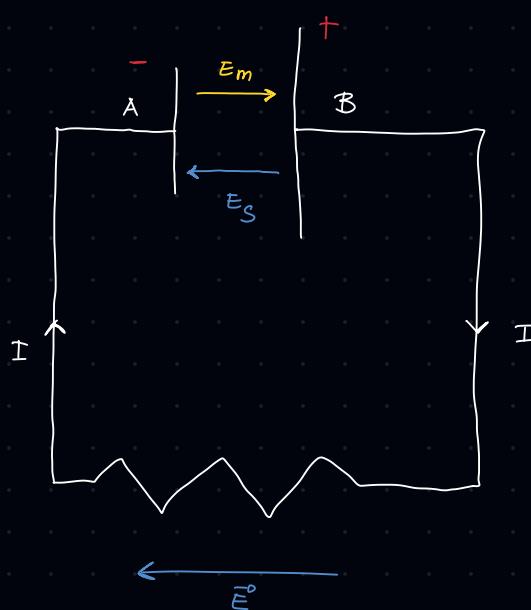
Unisco: $\frac{\vec{E}}{\rho} = \frac{n e^2 \vec{J}}{m} \tau \Rightarrow$

$$\tau = \frac{m}{n e^2 \rho}$$

Attraverso τ troviamo v e poi S (cammino medio Tra uno scontro e l'altro)

S corrisponde proprio al raggio di un Atomo -o Si Trova!

Forza Elettrica

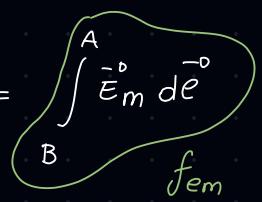


$$\mathcal{L} = \int_A^B \vec{F} \cdot d\vec{e} \quad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E$$

\rightarrow Lavoro campo E per spostare q:

$$\frac{\mathcal{L}}{q} = \int_A^B E \cdot de = f_{em} \neq 0!$$

$$f_{em} = \frac{\mathcal{L}}{q} = \int_A^B E_s de + \int_B^A (E_s + E_m) de = \int_B^A \vec{E}_m \cdot d\vec{e}$$



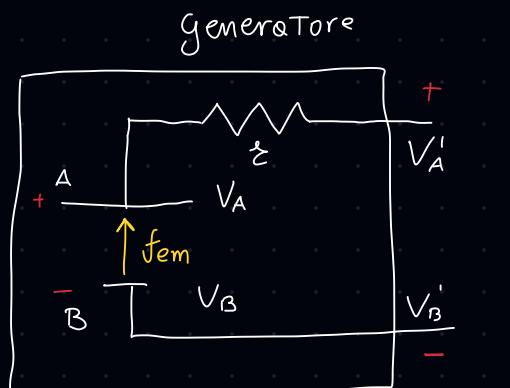
Potenziale effettivo

$$V_A' - V_B' = \xi \cdot I \rightarrow V_A - V_B - \xi I$$

$$\rightarrow V_A' - V_B' = (V_A - V_B) - \xi I$$

$$\begin{cases} V_A' - V_B' = R \cdot I \\ V_A - V_B = f_{em} \end{cases}$$

$$\rightarrow f_{em} - \xi I = RI \rightarrow f_{em} = I(R + \xi)$$



$$\Rightarrow I = \frac{f_{em}}{R + \xi} \quad \text{corrente}$$

$$\text{Sappiamo che } V_A - V_B = R \cdot I \quad \rightarrow I = \frac{V_A - V_B}{R}$$

$$\Rightarrow \frac{V_A - V_B}{R} = \frac{f_{em}}{R + \xi} \quad \rightarrow V_A - V_B = \frac{f_{em}}{R + \xi} \cdot R \quad (1)$$

$$R = \frac{V_A - V_B}{I} \quad \rightarrow I = \frac{f_{em}}{\frac{V_A - V_B}{I} - \xi} \quad \rightarrow \frac{V_A - V_B - \xi I}{I} \quad \rightarrow I = \frac{f_{em}}{V_A - V_B - \xi I} \cdot I$$

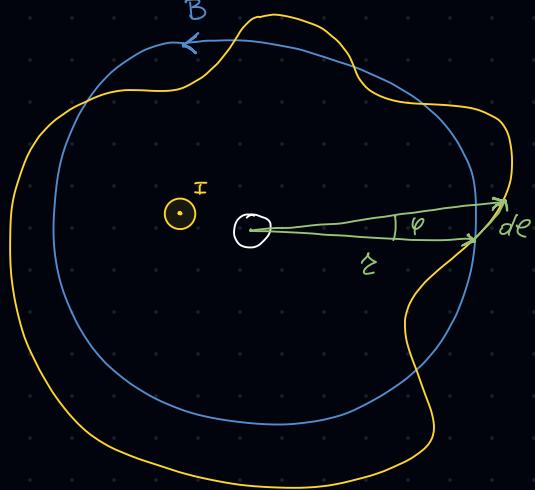
$$\rightarrow V_A - V_B = f_{em} + \xi I \quad (2)$$

(1) e (2) Equivalenti per il seguente discorso

$$\text{quando } \xi \rightarrow 0 \quad (1) \Delta V = \frac{R}{R + 0} \cdot f_{em} \quad \rightarrow \Delta V = f_{em} \quad] \quad \xi = 0 \rightarrow \text{Generatore ideale}$$

$$(2) \Delta V = f_{em} + 0 \cdot I \quad \rightarrow \Delta V = f_{em}$$

Teorema di Ampère



$$\text{Biot-Savart: } \vec{B} = \mu_0 \frac{I}{2\pi} \frac{\vec{r}}{r^2}$$

Circuazione:

$$C = \oint \vec{B} \cdot d\vec{e} \quad 1 \text{ Rad} = \frac{e}{R} \frac{\text{Arco}}{\text{Raggio}}$$

$$\Rightarrow d\varphi = \frac{d\ell}{R} \Rightarrow d\ell = R \cdot d\varphi$$

$$\Rightarrow C = \oint \frac{\mu_0}{2\pi} \frac{I}{R} \cdot R d\varphi \Rightarrow \frac{\mu_0}{2\pi} I \int d\varphi = \frac{\mu_0}{2\pi} I \cdot 2\pi$$

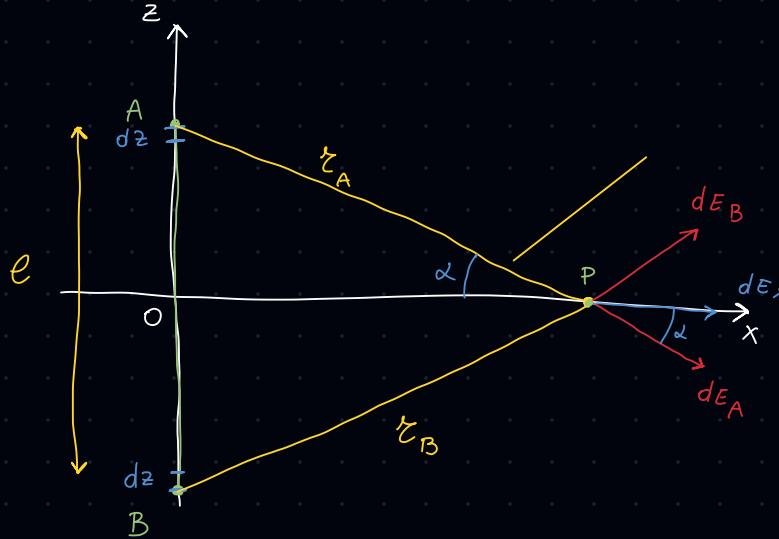
$$\Rightarrow \boxed{\int \vec{B} \cdot d\vec{e} = \mu_0 I}$$

$$\text{ma } I = \int J \cdot \hat{n} ds \Rightarrow \int \vec{B} \cdot d\vec{e} = \mu_0 \int \vec{J} \cdot \hat{n} ds \Rightarrow \int_{\Sigma} (\vec{\nabla} \times \vec{B}) \cdot \hat{n} ds = \mu_0 \int_{\Sigma} \vec{J} \cdot \hat{n} ds$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

Filo carico No Gauss

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2} \text{ ma } z = ?$$



$$\zeta_A^2 = \frac{OA^2}{z^2} + \frac{OP^2}{x^2} = z^2 + x^2$$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{(dq)}{z^2 + x^2} \hat{z}$$

Come integro dq ?? \rightarrow Non posso $\rightarrow \lambda = \frac{dq}{dz}$ $\rightarrow dq = \lambda dz$ nel caso specifico

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda(dz)}{z^2 + x^2}$$

mi serve solo $E_z = \frac{1}{4\pi\epsilon_0} \frac{\lambda \cos \alpha}{z^2 + x^2} dz$ ma $\cos \alpha = ??$

$$x = \zeta_A \cos \alpha \rightarrow \cos \alpha = \frac{x}{\zeta_A} \Rightarrow \cos \alpha = \frac{x}{\sqrt{z^2 + x^2}}$$

$$\rightarrow dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda x}{(x^2 + z^2)^{\frac{3}{2}}} dz \rightarrow E_z = \frac{\lambda x}{4\pi\epsilon_0} \int_{-e}^e \frac{1}{(z^2 + x^2)^{\frac{3}{2}}} dz$$

$$\Rightarrow E_z = \frac{2\lambda x}{4\pi\epsilon_0} \left[\frac{1}{x^2} - \frac{z}{(x^2 + z^2)^{\frac{1}{2}}} \right]_0^e = \frac{2\lambda x}{4\pi\epsilon_0} \left[\frac{1}{x^2} \cdot \frac{e}{(x^2 + e^2)^{\frac{1}{2}}} \right]$$

$$\text{Siccome } e \gg x \rightarrow \vec{E} = \frac{2\lambda x}{4\pi\epsilon_0} \frac{1}{x^2} \cancel{- \frac{e}{x^2}} = \frac{\lambda}{2\pi\epsilon_0 x} \hat{z}$$

Eq Maxwell

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = \frac{f}{\epsilon_0} \\ \nabla \times \vec{E} = \emptyset \end{array} \right. \Rightarrow \nabla \cdot \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{B} = \emptyset \\ \nabla \times \vec{B} = \mu_0 \vec{J} \end{array} \right. \Rightarrow \nabla \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$$

1)

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{dopo Rotore} \quad \nabla \times (\nabla \times \vec{E}) = - \frac{\partial (\nabla \times \vec{B})}{\partial t}$$

$$\nabla \cdot (\nabla \times \vec{E}) - \nabla^2 \vec{E} = - \mu_0 \frac{\partial \vec{J}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{ma } \nabla \cdot \vec{E} = \frac{f}{\epsilon_0}$$

$$- \nabla \cdot \frac{f}{\epsilon_0} - \nabla^2 \vec{E} = - \mu_0 \frac{\partial \vec{J}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$- \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = - \frac{f}{\epsilon_0} - \mu_0 \frac{\partial \vec{J}}{\partial t} \quad (1)$$

Campi Sorgenti

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{dopo Rotore} \quad \nabla \times (\nabla \times \vec{B}) = \mu_0 \nabla^2 \vec{J} + \mu_0 \epsilon_0 \frac{\partial (\nabla \times \vec{E})}{\partial t}$$

$$- \nabla \cdot (\nabla \times \vec{B}) - \nabla^2 \vec{B} = \mu_0 \nabla^2 \vec{J} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$- \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} - \nabla^2 \vec{B} = \mu_0 \nabla^2 \vec{J} \quad (2)$$

Campi Sorgenti

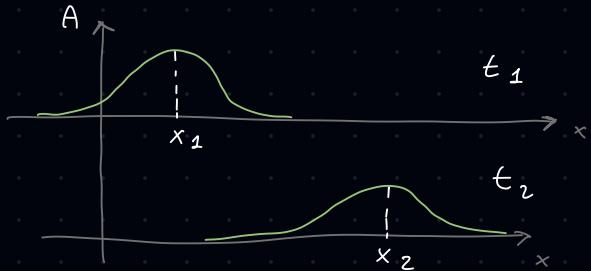
Scegliamo una eq tra (1) e (2)

$$(1) \Rightarrow \text{Siamo nel vuoto} \Rightarrow f = J = 0 \Rightarrow (1): \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = \emptyset$$

$$\Rightarrow \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{scelgo solo Asse } x$$

$$- \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{eq differenziale}$$

\Rightarrow Soluzione: $E(t) = A \sin(Kx - \omega t)$



$$\left. \begin{array}{l} E_1 = A \sin(Kx_1 - \omega t_1) \\ E_2 = A \sin(Kx_2 - \omega t_2) \end{array} \right\} E_1 = E_2 \Leftrightarrow \phi_1 = \phi_2$$

$$\Rightarrow Kx_1 - \omega t_1 = Kx_2 - \omega t_2$$

$$\Rightarrow \omega = K \nu \Rightarrow \kappa = \frac{\omega}{\nu} \Rightarrow \boxed{\frac{\omega}{\kappa} = \nu}$$

$$\Rightarrow \omega(t_2 - t_1) = K(x_2 - x_1) \Rightarrow \omega = K \left(\frac{\Delta x}{\Delta t} \right) \nu$$

$$\text{dalla dinamica} \Rightarrow x = v \cdot t \Rightarrow \underbrace{\frac{(\omega)t}{\kappa}}_{v} = x \Rightarrow x = v \cdot t$$

\uparrow
 $\frac{\omega}{\kappa}$

$$\Rightarrow E = A \sin(kx - \omega t) \stackrel{\omega = kv}{=} A \sin(kx - kvt) = A \sin[k(x - vt)]$$

$$\text{Risolviamo l'eq differenziale} \quad \frac{\partial E}{\partial x} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

$$\frac{\partial E}{\partial x} = kA \cos(kx - kvt) \Rightarrow \frac{\partial^2 E}{\partial x^2} = -k^2 A \sin(kx - vt)$$

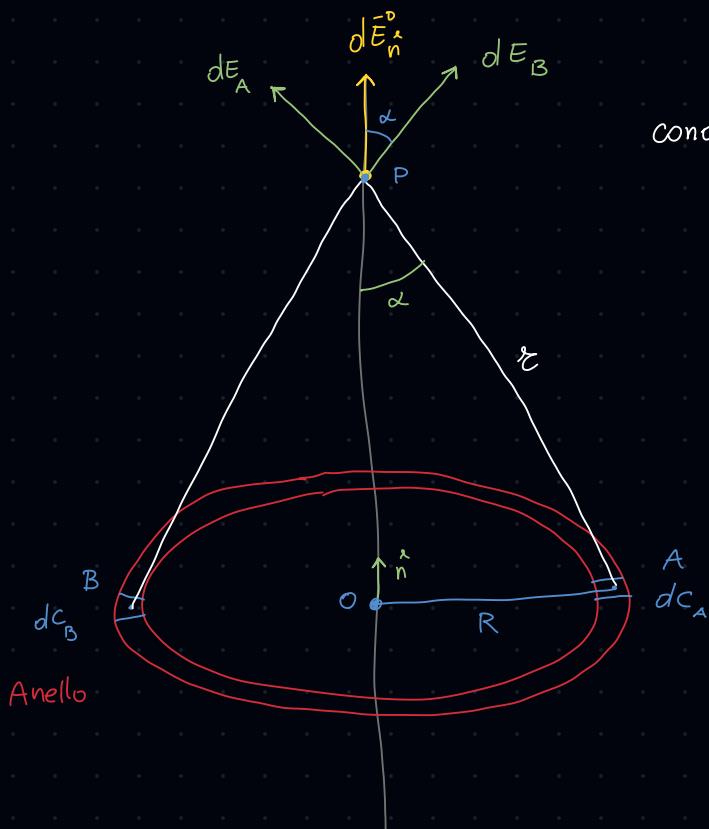
$$\frac{\partial E}{\partial t} = -v k A \cos(kx - kvt) \quad \frac{\partial^2 E}{\partial t^2} = -v^2 k^2 A \sin(kx - vt)$$

$$\Rightarrow \text{Sub} \Rightarrow -k^2 A \sin(kx - vt) - \mu_0 \epsilon_0 v^2 k^2 A \sin(kx - vt) = 0$$

$$\Rightarrow k^2 A \sin(kx - vt) [1 - \mu_0 \epsilon_0 v^2] = 0 \quad \Leftrightarrow 1 - \mu_0 \epsilon_0 v^2 = 0$$

$$\Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{c}$$

A nello carico



$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{z^2} \quad \text{ma } \epsilon = ??$$

conosco

R	Raggio
z	"Altezza"

$$z^2 = z^2 + R^2 \Rightarrow dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{R^2 + z^2}$$

$$\text{mi serve solo } dE_z = dE \cos \alpha$$

$$\Rightarrow dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{R^2 + z^2} \cdot (\cos \alpha) ??$$

$$z = \sqrt{z^2 + R^2} \Rightarrow \cos \alpha = \frac{z}{\sqrt{z^2 + R^2}} = \frac{z}{\sqrt{R^2 + z^2}}$$

$$\Rightarrow dE_n = \frac{1}{4\pi\epsilon_0} \frac{z dq}{(R^2 + z^2)^{\frac{3}{2}}} \quad \text{So che } \lambda = \frac{dq}{dc} = \frac{dq}{dc} \Rightarrow dq = \lambda dc$$

$$\Rightarrow dE_n = \frac{1}{4\pi\epsilon_0} \frac{z \cdot \lambda}{(R^2 + z^2)^{\frac{3}{2}}} dc \Rightarrow E_n = \frac{\lambda z}{4\pi\epsilon_0} \int \frac{1}{(R^2 + z^2)^{\frac{3}{2}}} dc$$

costante

$$\Rightarrow E_n = \frac{\lambda z}{4\pi\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}} \cdot 2\pi R \Rightarrow E_n = \frac{\lambda z \cdot R}{2\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}}$$

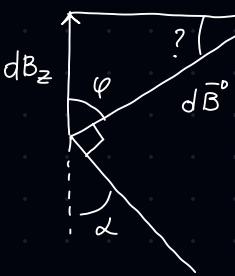
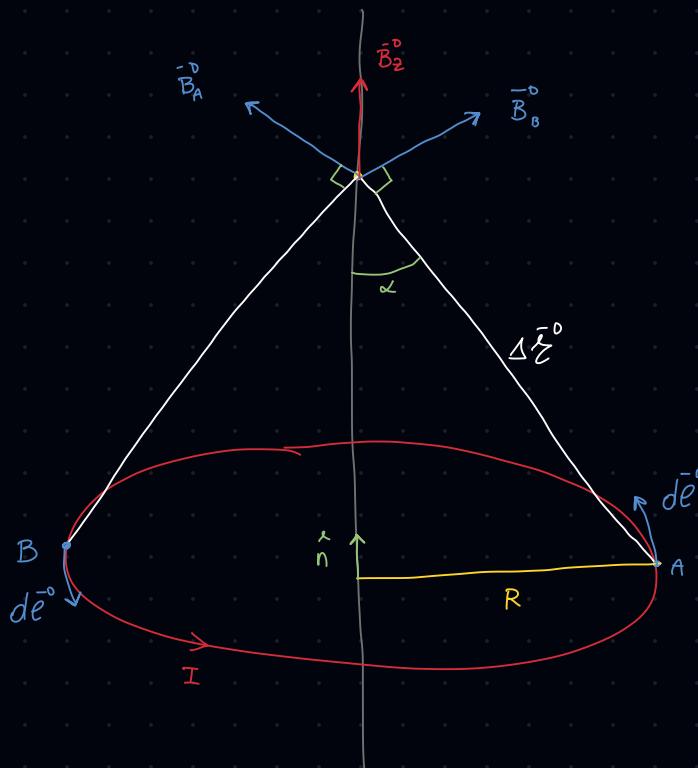
$$\text{ma } \lambda = \frac{Q}{c} \Rightarrow \frac{z \cdot 2\pi R}{4\pi\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}} \cdot \frac{Q}{c} \Rightarrow E_n = \frac{z \cdot Q}{4\pi\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}}$$

Se $z \gg R$

$$\Rightarrow E_n = \frac{z \cdot Q}{4\pi\epsilon_0 (z^2)^{\frac{3}{2}}} = \frac{Q}{4\pi\epsilon_0 z^2} \hat{n}$$

Si comporta come una carica uniforme!

Spira percorso da corrente



$$\begin{aligned}\varphi &= 180 - 90 - \alpha = 90 - \alpha \\ \Rightarrow ? &= 180 - 90 - \varphi \\ &\stackrel{!}{=} 90 - 90 + \alpha \\ &\stackrel{!}{=} \alpha\end{aligned}$$

\Rightarrow legge di Laplace

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{e} \wedge \Delta \vec{e}^o}{\Delta \vec{e}^o^3}$$

ci serve solo $dB_z = dB \sin \alpha$

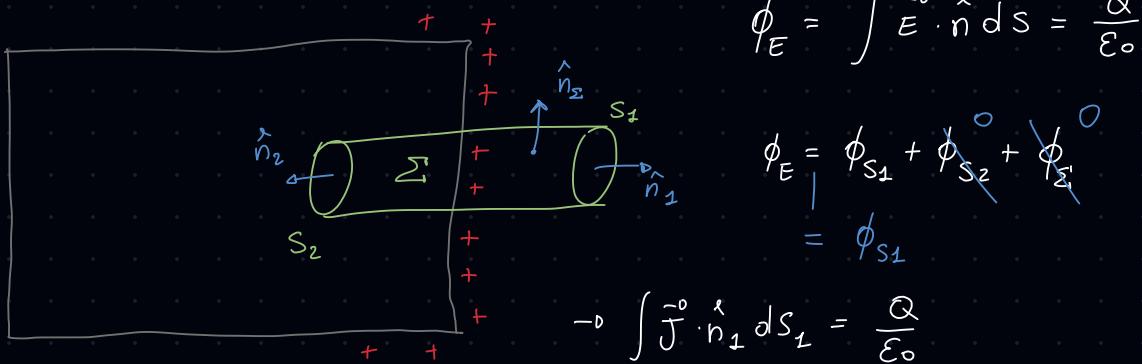
$$\text{ma } R = \Delta \vec{e}^o \sin \alpha \Rightarrow \sin \alpha = \frac{R}{\Delta \vec{e}^o}$$

$$\text{ma } \Delta \vec{e}^o^2 = R^2 + z^2 \Rightarrow \sin \alpha = \frac{R}{\sqrt{R^2 + z^2}}$$

$$\Rightarrow dB_z = \frac{\mu_0}{4\pi} I \frac{d\vec{e} \cdot \Delta \vec{e}^o \cdot \sin 90}{\Delta \vec{e}^o^3} \cdot \sin \alpha = \frac{\mu_0}{4\pi} I \frac{R}{(R^2 + z^2)^{\frac{3}{2}}} d\vec{e}$$

$$\Rightarrow B_z = \frac{\mu_0 I}{4\pi} \frac{R}{(R^2 + z^2)^{\frac{3}{2}}} \oint d\vec{e} = \frac{\mu_0}{4\pi} I \frac{2\pi R^2}{(R^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_0 I}{z} \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}$$

Teorema di Coulomb



$$\Phi_E = \int \vec{E} \cdot \hat{n} dS = \frac{Q}{\epsilon_0}$$

$$\begin{aligned}\Phi_E &= \Phi_{S1} + \Phi_{S2} + \Phi_{\Sigma} \\ &= \Phi_{S1}\end{aligned}$$

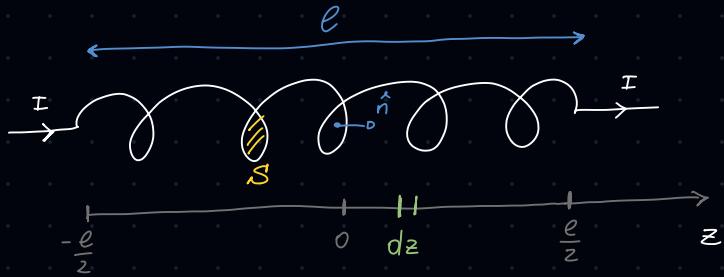
$$\Rightarrow \int \vec{J} \cdot \hat{n}_1 dS_1 = \frac{Q}{\epsilon_0}$$

$$S_1 = \pi R^2 \Rightarrow E \cdot \pi R^2 = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{\pi R^2 \epsilon_0} \equiv \frac{Q}{S \epsilon_0}$$

$$\text{ma } \frac{Q}{S} = \sigma$$

$$\Rightarrow E = \frac{\sigma}{\epsilon_0}$$

Campo B Solenoide



Definisco n = Densità di spire per unità lunghezza

$$\Rightarrow n = \frac{N}{\ell} \leftarrow \begin{array}{l} \text{numero} \\ \text{spire} \end{array}$$

$$\Rightarrow n dz = N$$

$$\Rightarrow B_{\text{spira}} = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$

$$\Rightarrow B_{\text{sol}} = \frac{\mu_0 I R^2}{2} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{1}{(R^2 + z^2)^{3/2}} n dz = \frac{N}{\ell} \cdot \frac{\mu_0 I R^2}{2} \left[\frac{1}{R^2} \cdot \frac{z}{(R^2 + z^2)^{1/2}} \right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}}$$

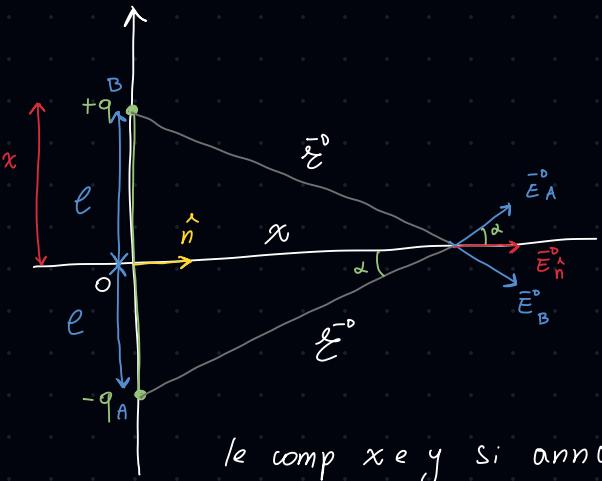
$$\Rightarrow \frac{1}{R^2} \cdot \frac{\ell}{2(R^2 + \frac{\ell^2}{4})} + \frac{1}{R^2} \cdot \frac{\ell^2}{2\sqrt{R^2 + \frac{\ell^2}{4}}} = \ell \left[\frac{1}{R^2} \frac{\ell}{2(R^2 + \frac{\ell^2}{4})} \right]$$

$$\text{ma } \ell \left(\frac{4R^2 + \ell^2}{4} \right)^{\frac{1}{2}} = \frac{(4R^2 + \ell^2)^{\frac{1}{2}}}{\sqrt{4}}$$

$$\Rightarrow B = \frac{N}{\ell} \cdot \frac{\mu_0 I R}{2} \ell \left[\frac{1}{R} \frac{\ell}{(4R^2 + \ell^2)^{\frac{1}{2}}} \right]$$

$$\Rightarrow B = \frac{N \mu_0 I R}{(4R^2 + \ell^2)^{\frac{1}{2}}} \quad \text{Se } \ell \gg R \Rightarrow B = \frac{N}{\ell} \mu_0 I R = \underline{\underline{n \mu_0 I R}}$$

CA MPO



$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{z^2} \quad z^2 = z^2 + x^2$$

$$\begin{aligned} \text{d}E &= \frac{1}{4\pi\epsilon_0} \frac{dq}{z^2 + x^2} \quad \text{ma} \quad \lambda = \frac{dq}{dx} \\ &\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dz}{z^2 + x^2} \\ &\text{L} \Rightarrow dq = \lambda dx \\ &\Rightarrow dq = \lambda dz \end{aligned}$$

le comp x e y si annullano $\Rightarrow dE_n = dE \cdot \underline{\cos \alpha}$

$$x = z \cos \alpha \Rightarrow \cos \alpha = \frac{x}{z} \Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{x \lambda dz}{(z^2 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_n = \frac{\lambda}{2\pi\epsilon_0} \int_0^e \frac{1}{(z^2 + x^2)^{\frac{3}{2}}} dz = \frac{\lambda}{2\pi\epsilon_0} \left[\frac{1}{x^2} \cdot \frac{z}{(z^2 + x^2)^{\frac{1}{2}}} \right]_0^e$$

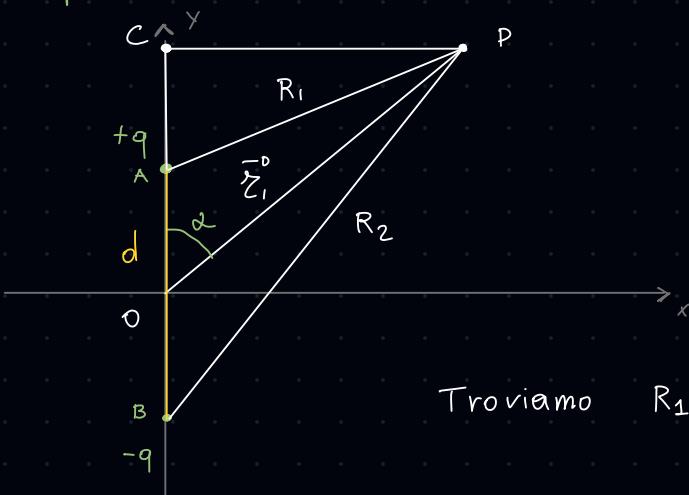
$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0} \left[\frac{1}{x^2} \cdot \frac{e}{(e^2 + x^2)^{\frac{1}{2}}} \right] = \frac{\lambda}{2\pi\epsilon_0 x} \frac{e}{(e^2 + x^2)^{\frac{1}{2}}}$$

Teatro ma d'impere

Spira \Leftrightarrow Dipolo
Lunga dist

Forza
Momento
Campo E/B

Dipolo



Sappiamo che $\vec{E} = -\vec{\nabla}V$

ma $V = ?$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

Troviamo R_1 e R_2

$$\begin{aligned} R_1^2 &= CP^2 + CA^2 = (\xi \sin \alpha)^2 + (CO - \frac{d}{2})^2 = \xi^2 \sin^2 \alpha + (\xi \cos \alpha - \frac{d}{2})^2 \\ &= \xi^2 \sin^2 \alpha + \xi^2 \cos^2 \alpha + \left(\frac{d}{2}\right)^2 - \xi d \cos \alpha = \xi^2 (\cos^2 + \sin^2) + \left(\frac{d}{2}\right)^2 - \xi d \cos \alpha \end{aligned}$$

$$\text{Hp: } \xi \gg d \Rightarrow \begin{cases} R_1^2 = \xi^2 - \xi d \cos \alpha \\ R_2^2 = \xi^2 + \xi d \cos \alpha \end{cases} \Rightarrow \begin{cases} R = \sqrt{\xi(\xi - d \cos \alpha)} \\ R = \sqrt{\xi(\xi + d \cos \alpha)} \end{cases}$$

$$\Rightarrow R_1 = \sqrt{\xi} \sqrt{\xi - d \cos \alpha} \quad \Rightarrow f(d) = (\xi - d \cos \alpha)^{\frac{1}{2}} \Rightarrow f'(d) = -\frac{1}{2} (\xi - d \cos \alpha)^{-\frac{1}{2}} \cdot \cos \alpha$$

Appross

$$f(0) = \sqrt{\xi}, \quad f'(0) = -\frac{\cos \alpha}{2\sqrt{\xi}}$$

$$\Rightarrow R_1 \approx \sqrt{\xi} \cdot \left[\sqrt{\xi} - \frac{\cos \alpha}{2\sqrt{\xi}} d \right] \approx \xi - \frac{d \cos \alpha}{2}$$

$$R_2 \approx \xi + \frac{d \cos \alpha}{2}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\xi - \frac{d}{2} \cos \alpha} - \frac{1}{\xi + \frac{d}{2} \cos \alpha} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{\xi + \frac{d}{2} \cos \alpha - \xi + \frac{d}{2} \cos \alpha}{\xi^2 - \left(\frac{d}{2}\right)^2 \cos^2 \alpha} \right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \alpha}{\xi^2 - \left(\frac{d}{2}\right)^2 \cos^2 \alpha} \quad \text{se } \xi \gg d \Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \alpha}{\xi^2}$$

$$\text{Si come } \vec{E} = -\vec{\nabla}V$$

$$\text{ci interessa } \vec{E}_z = \frac{\partial V}{\partial z} \quad \Rightarrow \text{convertiamo in coordinate cartesiane}$$

$$z = \xi \cos \alpha \Rightarrow \cos \alpha = \frac{z}{\xi}, \quad \xi = \sqrt{x^2 + y^2 + z^2}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{z \cdot d}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{dV}{dz} = \kappa \frac{d}{dz} \left[z \left(x^2 + y^2 + z^2 \right)^{\frac{3}{2}} \right] = \kappa \cdot \left(x^2 + y^2 + z^2 \right)^{\frac{3}{2}} - \frac{3}{2} z^2 \left(x^2 + y^2 + z^2 \right)^{-\frac{5}{2}}$$

$$\Rightarrow \frac{dV}{dt} = \kappa \left[\left(z^2 \right)^{-\frac{3}{2}} - \frac{3 z^2}{z \left(z^2 \right)^{-\frac{5}{2}}} \right] = \kappa \left[z^{-3} - \frac{3 z^2}{2 z^5} \right] = \kappa \left[\frac{1}{z^3} - \frac{3 z^2}{2 z^5} \right]$$

$$\Rightarrow \frac{\partial V}{\partial t} = \frac{\kappa}{z^3} \left[1 - \frac{3 z^2}{z^2} \cos^2 \alpha \right] = \frac{q_0}{4\pi \epsilon_0 z^3} \left[1 - 3 \cos^2 \alpha \right]$$

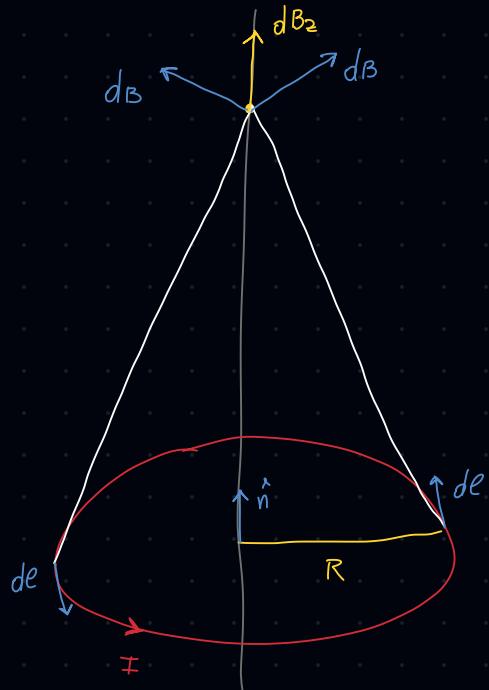
Siccome $\epsilon_z = \frac{dV}{dx} = q_0 \left(\frac{3 \cos^2 \alpha - 1}{4\pi \epsilon_0 z^3} \right)$

Se P e lungo $z \rightarrow \alpha = 0 \Rightarrow \cos \alpha = 1 \Rightarrow \epsilon_z = \frac{q_0 \bar{P}}{2 \pi \epsilon_0 z^3}$

pongo $q \cdot \bar{d} = \bar{P}$ momento Dipolo

Spira percorso da corrente

$$\epsilon_z = \frac{\bar{P}}{2\pi \epsilon_0 z^3} \quad (1)$$



$$B_z = n \frac{\mu_0 I R^2}{2(z^2 + R^2)^{\frac{3}{2}}}$$

$$\text{Se } z \gg R \Rightarrow B_z = n \frac{\mu_0 I R^2}{2 z^3}$$

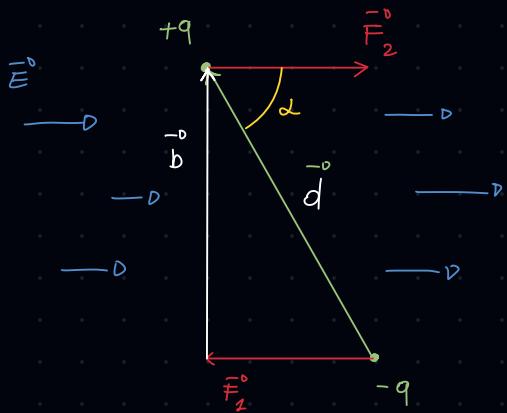
Definisco $n I S = \bar{m}$

$$\text{con } S = \pi R^2 \Rightarrow B_z = \frac{\mu_0 \bar{m}}{2\pi z^3} \quad (2)$$

(1) e (2) Equivalenti

Momento

$$F = q \cdot E ; M = \Sigma \wedge F$$



=0 Ci sono 2 Forze

$$M_{TOT} = \sum_1^{\infty} \lambda F_1 + \sum_2^{\infty} \lambda F_2$$

Sce lgo il pivot in $-q = 0$ $\zeta_1 = \emptyset$

$$\Rightarrow M_{\text{Tot}} = \vec{d} \wedge \vec{F}_2 = \vec{d} \wedge q \vec{e}$$

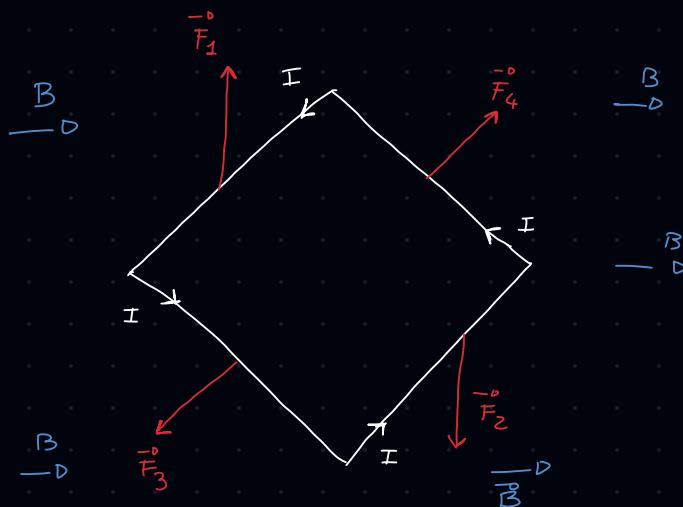
$$p \circ q = 0 \quad q \cdot \overline{d} = \overline{p}^0 = 0 \quad M_{TOT} = \overline{p}^0 \wedge \overline{E}$$

Trovo il modulo

$$|\vec{M}| = qd \cdot E \cdot \sin(\alpha) \quad \text{ma} \quad d \sin \alpha = b$$

$$=0 \quad M = q \cdot b \cdot E \quad \Leftrightarrow \quad q \cdot E = F = 0 \quad (M = F \cdot b) \quad \text{Moments}$$

momento spira



$$E = \bar{F} - q\bar{E} + q \cdot (\bar{v} \wedge \bar{\beta})$$

Immaginalo
Ribaltato

Sappiamo che $M = F \cdot b$

$$F_1 = q(\vec{v} \wedge \vec{B}) \quad m\omega \quad q = ??$$

$$\Rightarrow \vec{F} = q \frac{d\vec{e}}{dt} \wedge \vec{B} \Rightarrow d\vec{F} = I d\vec{e} \wedge \vec{B}$$

$$|\vec{F}| = I e B \Rightarrow M = I B e_1 \cdot e_3 \sin \varphi = I B S \sin \varphi$$

$$\text{battle 220} \quad \bar{m}^0 = I S_n^k - 0 \quad \bar{M}^0 = \bar{m}^0 B \sin \alpha = \bar{m}^0 \wedge \bar{B}^0$$

Effetto Joule

$$\text{Ohm: } \begin{cases} V = R \cdot I \\ R = \rho \cdot \frac{e}{S} \end{cases} \quad \text{Esprimiamo la potenza come} \quad P = \frac{dL}{dt}$$

- $L = q \cdot V \rightarrow dL = V dq$
- $P = \frac{dL}{dq} \rightarrow P = \frac{V dq}{dt} \rightarrow P = V \cdot I$
- Da Ohm (1) $\rightarrow V = R \cdot I \rightarrow P = R I^2$

Modello Drude:

$$1) \dots \quad 2) \dots \quad 3) \dots \quad 4) \dots \quad 5) \tau \rightarrow \frac{1}{\tau}$$

Scriviamo Ohm in forma vett

$$\begin{cases} V = R \cdot I \\ R = \rho \cdot \frac{e}{S} \end{cases} \rightarrow \begin{cases} V = R \cdot I \\ V = \int \vec{E} \cdot d\vec{r} \end{cases} \rightarrow R \cdot I = \int \vec{E} \cdot d\vec{r}$$

$$\rightarrow \int \frac{de}{ds} \cdot \vec{J} \cdot \hat{n} ds = \epsilon \cdot d\vec{r} \rightarrow \int \vec{J} = \vec{E}$$

$$R = \rho \cdot \frac{de}{ds}$$

$$I = \int \vec{J} \cdot \hat{n} ds$$

$$\text{inoltre } J = \bar{v} \cdot \rho \text{ ma } \text{Scrivo } f = -n \cdot e \rightarrow \bar{J} = -\bar{v} \cdot n e$$

Campo E

E accelera le cariche

$$\Rightarrow \vec{E} = 0 \Rightarrow V = V_0 \quad (1)$$

$$E = \frac{F}{q}$$

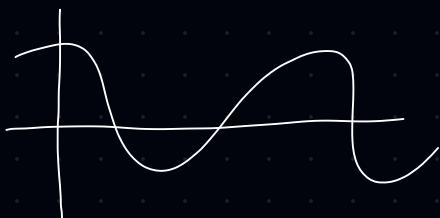
$$\rightarrow E \neq 0 \Rightarrow V = V_0 + a \cdot t \text{ ma } F = m \cdot a \rightarrow a = \frac{F}{m} \text{ con } F = q \cdot E$$

$$\Rightarrow F = -e E \rightarrow a = -\frac{e E}{m}$$

$$\Rightarrow V = V_0 - \frac{e E}{m} \cdot t \quad (2)$$

Vel media

$$\text{dalla (2)} \rightarrow \langle V \rangle = V_0 - \left\langle \frac{e E}{m} t \right\rangle \rightarrow \langle V \rangle = V_0$$



$$\text{Se } V_0 \in (-\infty, +\infty) \rightarrow \underline{\langle V_0 \rangle = 0}$$

$$\text{Se } E \neq 0 \rightarrow \langle V \rangle = V_0 - \left\langle \frac{e E}{m} t \right\rangle = \cancel{\langle V_0 \rangle} - \frac{e E}{m} \cancel{\langle t \rangle} \tau$$

$$\rightarrow \langle V \rangle = -\frac{e E}{m} \tau$$

Tornando a J ed E

$$J = -\vec{v} \cdot \vec{n} e \rightarrow J = \frac{e^2 E}{m} \tau \cdot \vec{n} \cdot e \rightarrow J = \frac{e^2 E n}{m} \tau$$

$$\vec{E} = \int \vec{J} \rightarrow \vec{E} = \int \frac{e^2 \vec{E} n \tau}{m} \rightarrow \tau = \frac{m}{\int e^2 n}$$

Impulso

$$I = \int_{t_0}^{t_f} \vec{F} dt = \int_{t_0}^{t_f} m \cdot \vec{a} dt = \int_{t_0}^{t_f} m \cdot \frac{dv}{dt} dt = m \int_{v_0}^{v_f} dv = m v_f - m v_0 = \vec{P}_f - \vec{P}_0$$

Conservazione del momento Angolare e di una forza

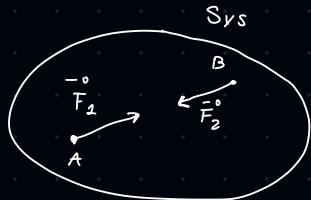
$$\left\{ \begin{array}{l} \vec{F} = m \cdot \vec{a} \\ M = \frac{d \vec{L}}{dt} \end{array} \right. \quad (a) \quad \sum_i \vec{F}_{int} + \sum_i \vec{F}_{ext} = \underbrace{\sum_i m \cdot \vec{a}_i}_{\rightarrow} \quad \sum_i m_i \frac{dv}{dt} \rightarrow \sum_i \vec{p} = \frac{d \vec{p}_{tot}}{dt}$$

$$\Rightarrow F = m \cdot a \rightarrow \sum_i F_{int} + \sum_i F_{ext} = \frac{d \vec{p}_{tot}}{dt}$$

$$(b) M = \frac{d \vec{L}}{dt} \rightarrow \sum_i M_{int} + \sum_i M_{ext} = \left(\sum_i \frac{d L}{dt} \right) \frac{d L_{tot}}{dt}$$

Momenti e Forze interni

$$\text{Se vale il III° principio: } \vec{F}_1 = -\vec{F}_2 \quad \Rightarrow \underline{\sum_i F_{int} = 0}$$



$$\vec{\epsilon}_3 = \vec{\epsilon}_2 + \vec{\epsilon}_3 \rightarrow \vec{\epsilon}_3 = \vec{\epsilon}_1 - \vec{\epsilon}_2 \quad \text{e} \quad \vec{F}_1 = -\vec{F}_2$$

$$\vec{M} = \vec{\epsilon} \wedge \vec{F} \Rightarrow M_{tot} = \vec{\epsilon}_1 \wedge \vec{F}_1 + \vec{\epsilon}_2 \wedge \vec{F}_2 = \vec{\epsilon}_1 \wedge -\vec{F}_2 + \vec{\epsilon}_2 \wedge \vec{F}_2$$

$$\rightarrow \vec{M} = (\vec{\epsilon}_1 - \vec{\epsilon}_2) \wedge \vec{F}_2 = \vec{\epsilon}_3 \wedge \vec{F}_2$$

$$\text{ma } \vec{\epsilon}_3 \parallel \vec{F}_2 \Rightarrow \underline{M_{int} = 0}$$

\Rightarrow Riscrivo le eq

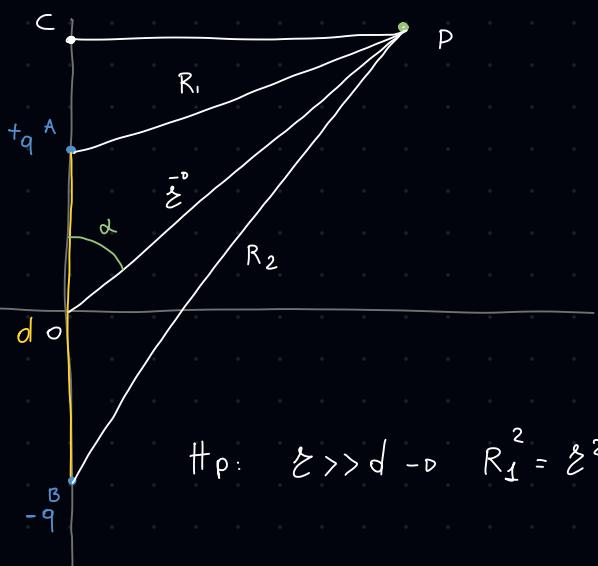
$$\left\{ \begin{array}{l} \sum_i F_{ext} = \frac{d \vec{p}_{tot}}{dt} \\ \sum_i M_{ext} = \frac{d \vec{L}}{dt} \end{array} \right. \quad \text{ma se il sys e isolato} \rightarrow \sum_i F_{ext} = \sum_i M_{ext} = 0$$

$$\Rightarrow \begin{cases} \frac{d \vec{p}}{dt} = 0 \\ \frac{d \vec{L}}{dt} = 0 \end{cases}$$

Il momento Ang e di inerzia si conservano!

Equivalezza Ampere

Bipolo:



$$E = -\nabla V \quad \text{ma} \quad V = ?$$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{ma} \quad R_1, R_2 = ?$$

$$\begin{aligned} R_1^2 &= CP^2 + CO^2 = (\xi \sin \alpha)^2 + (\xi \cos \alpha - \frac{d}{2})^2 \\ &= \xi^2 \sin^2 \alpha + \xi^2 \cos^2 \alpha + \left(\frac{d}{2}\right)^2 - \xi d \cos \alpha \\ &= \xi^2 (\sin^2 + \cos^2) + \left(\frac{d}{2}\right)^2 - \xi d \cos \alpha \\ &= \xi^2 \left(\frac{d}{2}\right)^2 - \xi d \cos \alpha \end{aligned}$$

$$\text{H.P.: } \xi \gg d \Rightarrow R_1^2 = \xi^2 - \xi d \cos \alpha = \left\{ \begin{array}{l} R_1 = \sqrt{\xi^2 - \xi d \cos \alpha} \\ R_2 = \sqrt{\xi^2 + \xi d \cos \alpha} \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} R_1 = \sqrt{\xi(\xi - d \cos \alpha)} \\ R_2 = \sqrt{\xi(\xi + d \cos \alpha)} \end{array} \right. = \left\{ \begin{array}{l} R_1 = \sqrt{\xi} \sqrt{\xi - d \cos \alpha} \\ R_2 = \sqrt{\xi} \sqrt{\xi + d \cos \alpha} \end{array} \right.$$

Approx:

$$f(d) = (\xi - d \cos \alpha)^{\frac{1}{2}} \Rightarrow f'(d) = \frac{1}{2} (\xi - d \cos \alpha)^{-\frac{1}{2}} \cdot (-\cos \alpha) = -\frac{\cos \alpha}{2\sqrt{\xi - d \cos \alpha}}$$

$$f(0) = \sqrt{\xi} \quad f'(0) = -\frac{\cos \alpha}{2\sqrt{\xi}}$$

$$\Rightarrow R_1 \approx \sqrt{\xi} \left[\sqrt{\xi} - \frac{d \cos \alpha}{2\sqrt{\xi}} \right] = \xi - \frac{1}{2} d \cos \alpha \quad \left. \begin{array}{l} \\ R_2 = \xi + \frac{1}{2} d \cos \alpha \end{array} \right\}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \left[\frac{\xi + \frac{d}{2} \cos \alpha - \xi + \frac{d}{2} \cos \alpha}{\xi^2 - \left(\frac{d}{2}\right)^2 \cos^2 \alpha} \right] = \frac{q}{4\pi\epsilon_0} \frac{d \cos \alpha}{\xi^2 - \left(\frac{d}{2}\right)^2 \cos^2 \alpha} \quad \text{H.P. } \xi \gg d \quad \boxed{V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \alpha}{\xi^2}}$$

$$E = -\nabla V \quad \text{Coord cart:} \quad \xi^2 = x^2 + y^2 + z^2 \quad CO = \xi \cos \alpha \Rightarrow \cos \alpha = \frac{z}{\xi} = \frac{z}{(x^2 + y^2 + z^2)^{\frac{1}{2}}}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{d \cdot z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

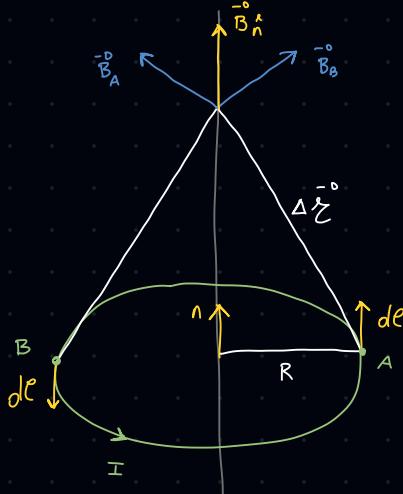
$$\Rightarrow E_z = \frac{\partial V}{\partial x} = \frac{d}{dx} \cdot K \left[(x^2 + y^2 + z^2)^{-\frac{3}{2}} \cdot z \right] = K (x^2 + y^2 + z^2)^{-\frac{3}{2}} - \frac{3}{z} \cdot z^2 (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\Rightarrow E_z = K \left[\frac{1}{(\xi^2)^{\frac{3}{2}}} - \frac{3z^2}{(\xi^2)^{\frac{5}{2}}} \right] = K \left[\frac{1}{\xi^3} - \frac{3z^2}{\xi^5} \right] = \frac{K}{\xi^3} \left[1 - 3 \left(\frac{z^2}{\xi^2} \right) \right]$$

-o

(15')

Spira



$$\vec{B} = \mu_0 \frac{I}{R} \quad \text{e} \quad \vec{B} = \frac{\mu_0}{4\pi} \int \frac{d\vec{\ell} \wedge \Delta \vec{z}}{\Delta z^3}$$

$$\Delta \vec{z} = ? \quad z^2 = R^2 + z^2$$

Ci serve solo dB_z

$$d\vec{B}_z = dB \sin \alpha$$

$$\begin{aligned} \phi &= 180 - 90 - \alpha \\ &= 180 - 90 - \psi \\ &= 90 - 90 + \alpha \\ &= \alpha \end{aligned}$$

$$\Rightarrow d\vec{B}_z = \frac{\mu_0}{4\pi} I \frac{d\ell \wedge \Delta z}{\Delta z^3} \cdot \sin \alpha = \frac{\mu_0}{4\pi} I \frac{d\ell \Delta z \cdot \frac{1}{\sin \theta}}{\Delta z^3} \sin \alpha = \frac{\mu_0}{4\pi} I \frac{\sin \alpha}{\Delta z^2} d\ell$$

$$\Rightarrow B_z = \frac{\mu_0}{4\pi} I \frac{\sin \alpha}{\Delta z^2} \int d\ell = \frac{\mu_0}{4\pi} I \frac{\sin \alpha}{\Delta z^2} 2\pi R$$

ma

$$\begin{cases} \Delta z^2 = z^2 + R^2 \\ R = \Delta z \sin \alpha \Rightarrow \sin \alpha = \frac{R}{(\Delta z)^2} \end{cases}$$

$$\Rightarrow B_z = \mu_0 I \frac{R^2}{2(R^2 + z^2)^{3/2}}$$

Bip. Magnetico

$$\frac{\mu_0 \vec{m}}{2\pi z^3}$$

Teorema Ampere

$$\vec{E}_z = \frac{q}{4\pi \epsilon_0 z^3} \left[1 - 3 \cos^2 \alpha \right]$$

Se per lungo z $\Rightarrow \frac{q d}{2\pi \epsilon_0 z^3} = \frac{\vec{P}}{2\pi \epsilon_0 z^3}$

$$E_z = \frac{\mu_0 I R^2}{2 z^3} \quad \text{pongo} \quad \vec{m} = I S n \quad \Rightarrow I \pi R^2 =$$

$$E_z = \frac{\mu_0 \vec{m}}{2\pi z^3}$$

equivalenti

Maxwell

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1) \\ \nabla \times \vec{E} = \vec{0} \quad (2) \end{array} \right.$$

$$\int \vec{E} \cdot \hat{n} dS = \frac{Q}{\epsilon_0} \quad \text{ma} \quad \oint \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int_V \rho dV$$

$$- \int_V (\nabla \times \vec{E}) dV = \frac{1}{\epsilon_0} \int_V \rho dV \quad \Rightarrow \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$C = \oint \vec{E} d\vec{e} = V_A - V_B \quad \text{ma} \quad V_A = V_B \Rightarrow \oint \vec{E} d\vec{e} = 0 \quad \Rightarrow \int (\nabla \times \vec{E}) \cdot \hat{n} dS = 0 \quad \Rightarrow \nabla \times \vec{E} = 0$$

$$dL = F \cdot de \quad \text{ma} \quad E = \frac{F}{q} \Rightarrow \frac{dL}{q} = E \cdot de = - \frac{dU}{q} \quad \underline{E \cdot de = -dV}$$

$$\text{ma} \quad V(x, y, z) \Rightarrow dV = i \frac{\partial V}{\partial x} dx + j \frac{\partial V}{\partial y} dy + k \frac{\partial V}{\partial z} dz$$

$$\left\{ \begin{array}{l} d\vec{e} = i dx + j dy + k dz \\ \nabla V = i \frac{\partial V}{\partial x} + j \frac{\partial V}{\partial y} + k \frac{\partial V}{\partial z} \end{array} \right. \Rightarrow dV = \nabla V \cdot d\vec{e} \Rightarrow \vec{E} \cdot d\vec{e} = - \nabla V \cdot d\vec{e}$$

$$\Rightarrow \boxed{\vec{E} = -\nabla V}$$

Sostituiamo (1) e (2)

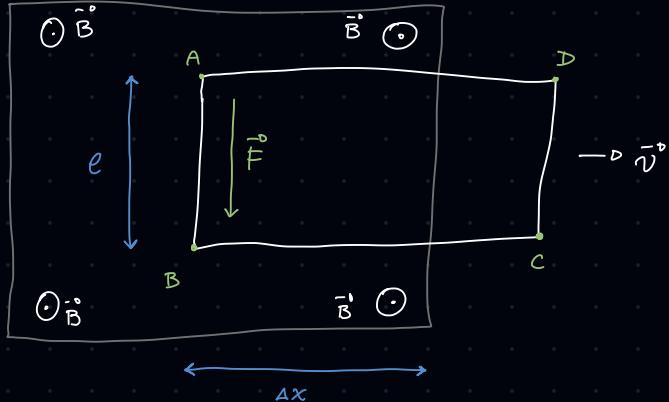
$$\left\{ \begin{array}{l} \nabla \cdot (-\nabla V) = \frac{\rho}{\epsilon_0} \\ \nabla \times (-\nabla V) = \vec{0} \end{array} \right. \Rightarrow \boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}} \quad \text{Eq di Poisson}$$

$$\text{Soluzione} \quad V = \frac{1}{4\pi\epsilon_0} \int \frac{f(\vec{r}')}{|\vec{r} - \vec{r}'|} dV$$

Compi magnetici Variabili

$$\text{Faraday: } f_{em} = - \frac{d\phi_B}{dt} \quad \Rightarrow \quad \phi_B = \int \vec{B} \cdot \hat{n} dS = \underbrace{\int_B \cos \theta dS}_{\text{compo}} \underbrace{\frac{1}{\text{Superf}}} \quad E=0$$

Flusso Tagliato: varia S



$$\text{Forza} \rightarrow \text{Lorentz} \rightarrow \vec{F} = q \vec{E} + q \cdot (\vec{v} \times \vec{B})$$

$$f_{em} = \frac{L}{q} = q \frac{\int_A^B (\vec{v} \times \vec{B}) d\vec{e}}{q} = \int_A^B (\vec{v} \times \vec{B}) d\vec{e}_1$$

$$= \underline{V_B I}$$

$$\boxed{f_{em} = \int (\vec{v} \times \vec{B}) d\vec{e}}$$

$$\text{ma Faraday dice } f_{em} = - \frac{d\phi_B}{dt}$$

\Rightarrow il flusso dipende dal tempo: $\phi_B = B \cdot S$ ma Se f del Tempo

$$\rightarrow S = b \times h = e \cdot x \text{ ma } x \text{ dipende da } t : \bar{x} = x - \Delta x$$

$$\rightarrow \phi_B = \int \vec{B} \cdot \hat{n} ds = B \cdot e \cdot x \quad \text{a } \bar{t} = t + \Delta t \rightarrow \phi_B = B \cdot e \cdot (x - \Delta x)$$

$$\Rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\phi_B(t + \Delta t) - \phi_B(t)}{\Delta t} = \frac{B e \bar{x} - B e (\bar{x} - \Delta x)}{\Delta t}$$

$\frac{d\phi_B}{dt} = B e v$ QED

Concatenato: Varia $B \rightarrow B(t, \bar{s})$

$$\frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\int B(t + \Delta t, \bar{s}) \hat{n} ds - \int B(t, \bar{s}) \hat{n} ds}{\Delta t}$$

$$\text{Approssimo } B \simeq B(t, \bar{s}) + \frac{\partial B}{\partial t}(t + \Delta t) \cdot \Delta t$$

$$\rightarrow \frac{d\phi}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\int B(t, \bar{s}) d\bar{s} + \int \frac{\partial B}{\partial t}(t + \Delta t, \bar{s}) \Delta t d\bar{s} - \int B(t, \bar{s}) d\bar{s}}{\Delta t}$$

$$\Rightarrow \frac{d\phi_B}{dt} = \int \frac{\partial B}{\partial t} \hat{n} ds \quad (2)$$

$$\text{Faraday dice: } f_{em} = \frac{d\phi}{dt} \quad \text{ma } f_{em} = \frac{L}{q} \quad F = q \vec{E} + q(\vec{v} \wedge \vec{B})$$

$$\Rightarrow f_{em} = \oint \vec{E} \cdot d\vec{e} + \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e} \quad (3) \quad \text{ma nel flusso concat. } v=0$$

$$\Rightarrow f_{em} = \oint \vec{E} \cdot d\vec{e} = - \int_S \frac{\partial B}{\partial t} \hat{n} ds \quad \rightarrow \int_S (\vec{v} \wedge \vec{E}) \cdot \hat{n} ds = - \int_S \frac{\partial B}{\partial t} \hat{n} ds$$

$$\Rightarrow \vec{v} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{II eq di Maxwell Complete!}$$

Metto tutto insieme

$$(1) \quad f_{em} = \oint (\vec{v} \wedge \vec{B}) d\vec{e}^o = - \frac{d\phi_B}{dt} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B}) d\vec{e}^o - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

Varia sia \vec{B} che S

$$(2) \quad - \frac{d\phi_B}{dt} = - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

ma $f_{em} = \frac{q}{q} = \oint \vec{E} \cdot d\vec{e}^o + \oint (\vec{v} \wedge \vec{B}) d\vec{e}^o$

$$\Rightarrow \oint \vec{E} \cdot d\vec{e}^o + \cancel{\oint (\vec{v} \wedge \vec{B}) d\vec{e}^o} = \oint (\vec{v} \wedge \vec{B}) d\vec{e}^o - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS \quad (1)$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{e}^o = - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS = 0 \quad \boxed{\vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$

Autoinduzione

$$B = \frac{\mu_0}{4\pi} I \int \frac{d\ell \lambda \Delta \vec{\Sigma}^0}{|\Delta \vec{\Sigma}|^3}$$

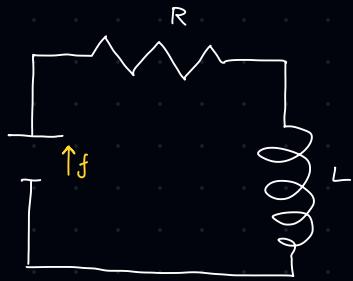
$\rightarrow \phi_B = \underbrace{\int \frac{\mu_0 I}{4\pi} \int d\ell \lambda \Delta \vec{\Sigma}^0}_{\text{Tutto costante}} \cdot \vec{n} dS$
 Tranne I

$$\rightarrow \phi_B = L \cdot I$$

$$\rightarrow \text{Se } B \text{ varia} \rightarrow \frac{d\phi}{dt} = L \cdot \frac{dI}{dt} \rightarrow f_{em} = -\frac{d\phi_B}{dt} = -L \frac{dI}{dt}$$

Verso opposto

Circuito RL



Sappiamo che

$$\sum_i f_{em} = \sum_i R_i I \quad \text{ma } L \text{ è contato come } f_{em}:$$

$$\rightarrow f_{em} - f_{ind} = R \cdot I \rightarrow f_{em} = f_{ind} + RI$$

$$\rightarrow f_{em} = L \frac{dI}{dt} + RI \quad \text{Eq diff} \quad \text{soluzione: } I(t) = Ae^{\alpha t} + D$$

$$\rightarrow I'(t) = \alpha Ae^{\alpha t} \rightarrow L \cdot \alpha Ae^{\alpha t} + RAe^{\alpha t} + RD = f_{em} \rightarrow f_{em} = RD \rightarrow D = \frac{f}{R} \quad (1)$$

$$\text{inoltre } L \alpha Ae^{\alpha t} + RAe^{\alpha t} = 0 \rightarrow Ae^{\alpha t}(L\alpha + R) = 0 \rightarrow \alpha = -\frac{R}{L} \quad (2)$$

$$\rightarrow \text{Sol gen } I(t) = A e^{-\frac{R}{L}t} + \frac{f}{R} \quad (3)$$

$$I(0) = 0 \rightarrow I(0) = A e^{0} + \frac{f}{R} = 0 \rightarrow A = -\frac{f}{R} \rightarrow I(t) = -\frac{f}{R} e^{-\frac{R}{L}t} + \frac{f}{R} = \frac{f}{R} \left(1 - e^{-\frac{R}{L}t} \right)$$

$$\text{per } t \rightarrow \infty \quad I(t) = \text{Cost} = \frac{f}{R}$$

Campo \vec{E}

$$\vec{F}_c = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_2}{\epsilon^2} \quad \text{def} \quad E = \frac{\vec{F}_c}{q} = \frac{\vec{F}_c}{q} = \frac{q_1 \cdot q_2}{4\pi\epsilon_0 \epsilon^2} \hat{\epsilon}$$

$$\begin{aligned} L &= \int_A^B \vec{F}_c \cdot d\vec{e} = \frac{1}{4\pi\epsilon_0} \int_A^B \frac{q_1 \cdot q_2}{\epsilon^2} \underbrace{\hat{\epsilon} d\vec{e}}_{\hat{\epsilon} \cdot d\vec{e} = d\vec{e}} = \frac{q_1 \cdot q_2}{4\pi\epsilon_0} \int_A^B \frac{1}{\epsilon^2} d\epsilon = \frac{q_1 \cdot q_2}{4\pi\epsilon_0} \left[-\frac{1}{\epsilon} \right]_A^B \\ &= \frac{q_1 \cdot q_2}{4\pi\epsilon_0} \left[\frac{1}{\epsilon_A} - \frac{1}{\epsilon_B} \right] = U_A - U_B \end{aligned}$$

$$= \frac{L}{q} = \int E d\epsilon = \frac{U_A - U_B}{q} = V_A - V_B \quad \text{potenziale elettrico}$$

circuitazione $C = \oint \vec{E} \cdot d\vec{e} = V_A - V_B$ ma $V_A = V_B \Rightarrow \oint \vec{E} \cdot d\vec{e} = 0 \iff \boxed{\vec{\nabla} \wedge \vec{E} = 0}$ Maxwell

ma $\vec{E} \cdot d\vec{e} = -dV \Rightarrow dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$

$$\begin{aligned} \Rightarrow \left\{ \begin{array}{l} d\vec{e} = dx + dy + dz \\ \vec{\nabla} V = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} \end{array} \right\} \Rightarrow dV = \vec{\nabla} V \cdot d\vec{e} \Rightarrow \vec{E} \cdot d\vec{e} = -\vec{\nabla} V \cdot d\vec{e} \\ \underbrace{\vec{E} = -\vec{\nabla} V}_{\text{Maxwell}} \end{aligned}$$

Sup equipot:

Presi due punti di una sup equipot: $V_A = V_B \forall A, B$

\Rightarrow per ogni punto $\Rightarrow L = q \cdot V = \emptyset \Rightarrow$ non si compie lavoro per spostare cariche

ma $L = \vec{F} \cdot d\vec{e}$ ovvero $L = F \cdot d\epsilon \underbrace{\cos \alpha}_{?} = \emptyset$
 quando $\cos \alpha = 0 \Rightarrow \alpha = 90^\circ$

\Rightarrow Linee di forza \perp alla (Spostamento)

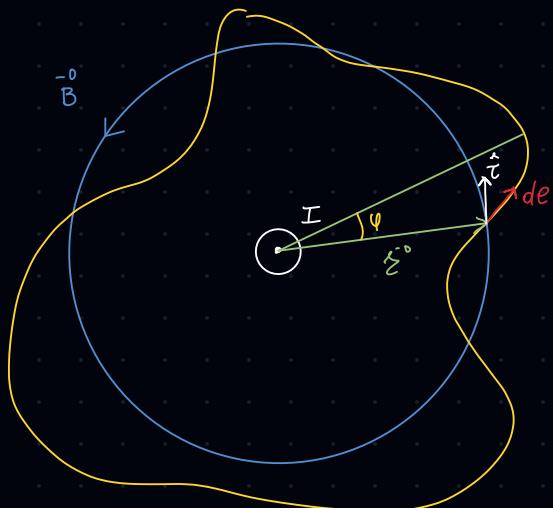
\uparrow
Superficie

Gauss: $\int \vec{E} \cdot \hat{n} dS = \frac{Q}{\epsilon_0}$ ma $\rho = \frac{Q}{V} \Rightarrow \oint \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int \rho dV$

$$\begin{aligned} \left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{Maxwell} \\ \vec{\nabla} \wedge \vec{E} = 0 \end{array} \right\} \Rightarrow \vec{E} = -\vec{\nabla} V \quad \left| \begin{array}{l} \vec{\nabla} (-\vec{\nabla} V) = \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\epsilon_0} \end{array} \right. \quad \text{Poisson} \\ \text{Sol: } E = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV \quad \boxed{\vec{\nabla} \wedge (-\vec{\nabla} V) = 0} \quad \checkmark \end{aligned}$$

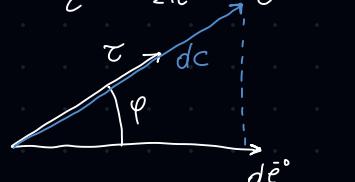
Magnetostatica

$$\left\{ \begin{array}{l} \nabla^2 B = 0 \quad \leftarrow \text{Non Esistono} \quad \text{Dipoli Magneticci} \\ \nabla \wedge B = \mu_0 J \quad \Rightarrow \quad \oint B \cdot d\vec{e} = \mu_0 I \quad \text{Ampere} \end{array} \right.$$



$$C = \oint B \cdot d\vec{e} \quad \text{ma} \quad B = K \cdot \frac{I}{\rho} = \frac{\mu_0}{2\pi} \frac{I}{\rho} \hat{\tau}$$

$$\rightarrow C = \frac{\mu_0 I}{2\pi} \int \frac{1}{\rho} \hat{\tau} \cdot d\vec{e} =$$

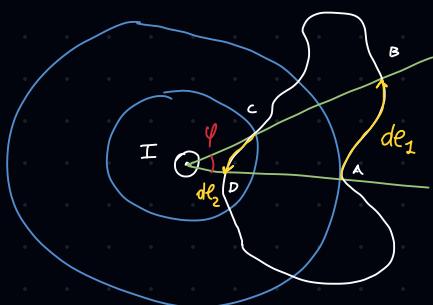


$\hat{\tau} d\tau \cos \varphi = \text{Arcodi circonf}$

$$\Rightarrow \hat{\tau} \cdot d\vec{e} = \ell \quad \Rightarrow \quad 1 \text{ Rad} = \frac{\ell}{R} \quad \Rightarrow \quad \ell = 1 \text{ Rad} \cdot R$$

$$\Rightarrow \hat{\tau} d\vec{e} = \varphi \cdot \ell \quad \Rightarrow \quad C = \frac{\mu_0}{2\pi} I \left(\int \frac{1}{\rho} \cdot \ell \cdot d\varphi \right)^{2\pi} = \frac{\mu_0}{2\pi} I \cdot 2\pi = \mu_0 I$$

$$\Rightarrow \oint B \cdot d\vec{e} = \mu_0 I \quad \Rightarrow \quad \nabla \wedge B = \mu_0 J$$



$$\rightarrow C = \oint B \cdot d\vec{e} = \int_A^B B d\ell + \int_B^C B d\ell_2$$

$$\text{Siccome } \oint B d\ell = |B| \oint d\varphi$$

$$\rightarrow C = |B| \int_A^B d\varphi + |B| \int_C^D d\varphi$$

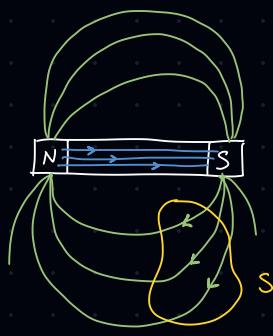
Stesso Angolo

$$\text{ma} \quad \left\{ \begin{array}{ll} A \rightarrow B & \text{ANTIORARIO} \\ C \rightarrow D & \text{ORARIO} \end{array} \right.$$

$$\Rightarrow C = \int_A^B d\varphi - \int_C^D d\varphi = \emptyset$$

Maxwell complete:

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{B}}{\partial t} \right) \\ \text{Corrente di Spostamento} \end{cases}$$



$$\phi_B = \oint_S \vec{B} \cdot \hat{n} ds = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\begin{cases} \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} & \text{Faraday} \\ \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} & \text{Gauss} \end{cases}$$

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 & \text{Monopoli magnetici} \\ \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} & \text{Ampere Maxwell} \end{cases}$$

Maxwell -o Onde

$$\begin{cases} \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{B}}{\partial t} \\ f_{em} = -\frac{d\phi_B}{dt} \quad \Rightarrow \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \end{cases}$$

I) Campo Elettrico

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Rotore} \quad \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial (\vec{\nabla} \times \vec{B})}{\partial t}$$

$$-\nabla^2 \vec{E} = -\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$-\nabla^2 \vec{E} = -\mu_0 \vec{\nabla} \vec{J} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$-\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{-\vec{\nabla} f}{\epsilon_0} + \mu_0 \vec{\nabla} \vec{J} \quad (1)$$

II) Campo B

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{Rotore} \quad \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \times \vec{J}) + \mu_0 \epsilon_0 \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$$

~~$$-\nabla^2 \vec{B} = \mu_0 (\vec{\nabla} \times \vec{J}) - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$~~

$$-\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} + \vec{\nabla}^2 \vec{B} \quad (2)$$

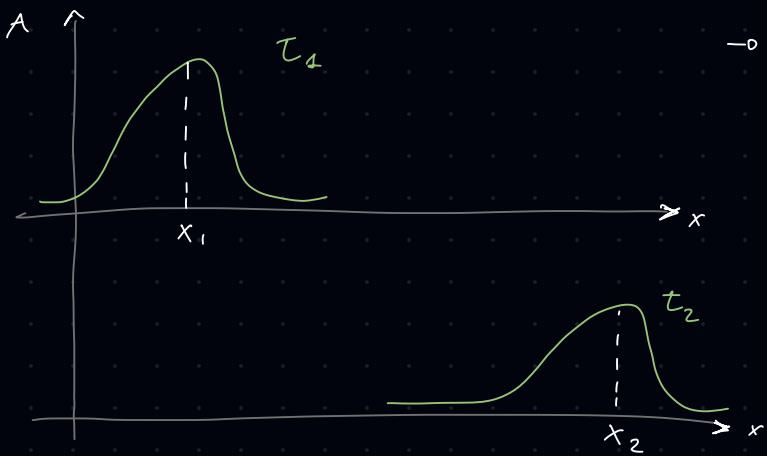
Siamo nel vuoto $\Rightarrow \mu = \epsilon = 0$

$$\Rightarrow \begin{cases} \nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 & (1) \\ \nabla^2 B - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} = 0 & (2) \end{cases}$$

dalla (1) \Rightarrow

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{Considero solo } x$$

$$\Rightarrow \frac{\partial^2 E}{\partial x^2} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{Eq diff: sol: } E(t) = A \sin(\kappa x - \omega t)$$



$$\Rightarrow E_1 = A \sin(\kappa x_1 - \omega t_1)$$

$$E_2 = A \sin(\kappa x_2 - \omega t_2)$$

$$E_1 = E_2 \Rightarrow \kappa x_1 - \omega t_1 = \kappa x_2 - \omega t_2$$

$$\Rightarrow \kappa(x_2 - x_1) = \omega(t_2 - t_1)$$

$$\Delta x = \left(\frac{\omega}{\kappa} \right) \Delta t$$

$$\Rightarrow v = \frac{\omega}{\kappa} = \omega \cdot \kappa$$

$$\Rightarrow E(t) = A \sin(\kappa x - \omega t) = A \sin[\kappa(x - vt)]$$

Derivate

$$\frac{\partial E}{\partial t} = -v \kappa A \cos(\kappa x - \kappa v t) \quad \frac{\partial^2 E}{\partial t^2} = -v^2 \kappa^2 A \sin(\kappa x - \kappa v t)$$

$$\frac{\partial E}{\partial x} = \kappa A \cos(\kappa x - \kappa v t) \quad \frac{\partial^2 E}{\partial x^2} = -\kappa^2 A \sin(\kappa x - \kappa v t)$$

$$\Rightarrow \text{Eq diff (A)} \uparrow \quad -\kappa^2 A \sin(\kappa x - \kappa v t) + \mu_0 \epsilon_0 v^2 \kappa^2 A \sin(\kappa x - \kappa v t) = 0$$

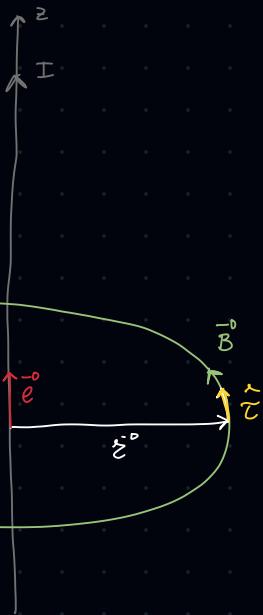
$$\Rightarrow \kappa^2 A \sin(\kappa x - \kappa v t) \left[-1 + \mu_0 \epsilon_0 v^2 \right] = 0 \quad \Leftrightarrow \mu_0 \epsilon_0 v^2 - 1 = 0$$

$$v = \sqrt{\frac{1}{\mu_0 \epsilon_0}} = C$$

Biot Savart

$$B = \kappa \cdot \frac{I}{z} = \frac{\mu_0}{2\pi} \frac{I}{z}$$

per ottenere $\hat{B} : \hat{e} \wedge \hat{z} = \frac{\hat{e} \wedge \hat{z}}{z}$ (a)



Lorenz: $\vec{F} = q \cdot (\vec{v} \wedge \vec{B}) \rightarrow F = q \cdot \frac{d\vec{e}}{dt} \wedge \vec{B}$

$\rightarrow d\vec{F} = I \frac{d\vec{e}}{dt} \wedge \vec{B}$ Cariche elettriche
II Biot Savart

Carica magnetica

$$\vec{F} = \frac{\mu_0}{4\pi} \frac{q_m \cdot q_m'}{z^2} \hat{z} \rightarrow B = \frac{\vec{F}}{q} = \vec{B}$$

$$\rightarrow d\vec{F} = \vec{B} dq$$

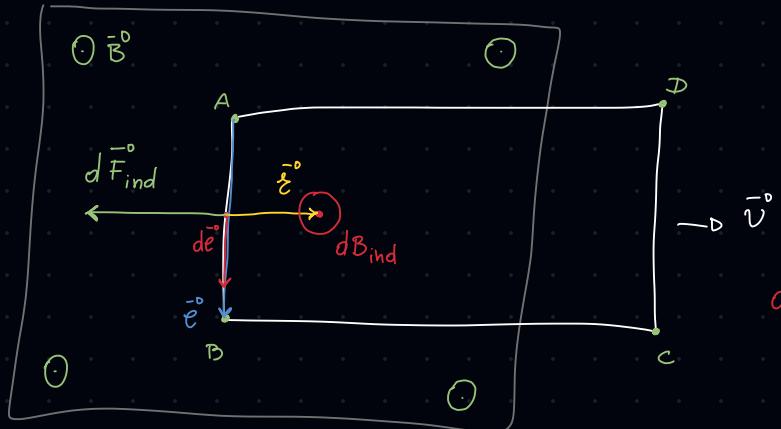
$$\Rightarrow \left\{ \begin{array}{l} d\vec{F} = \frac{I}{z} d\vec{e} \wedge \vec{B} \\ dF = \vec{B} \cdot dq \end{array} \right. \Rightarrow \underline{dq = I \cdot d\vec{e}} \quad (1)$$

Siccome $\vec{B} = \frac{\vec{F}}{q} = \frac{\mu_0}{4\pi} \frac{q_m}{z^2}$ $\rightarrow B = \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{e}}{z^2}$ vettore? (a) ↑

$$\rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{e} \wedge \vec{z}}{z^3} \quad \text{Biot-Savart I}$$

Legge di Lenz

$$f_{\text{em}} = \text{---} \frac{d\phi}{dt}$$



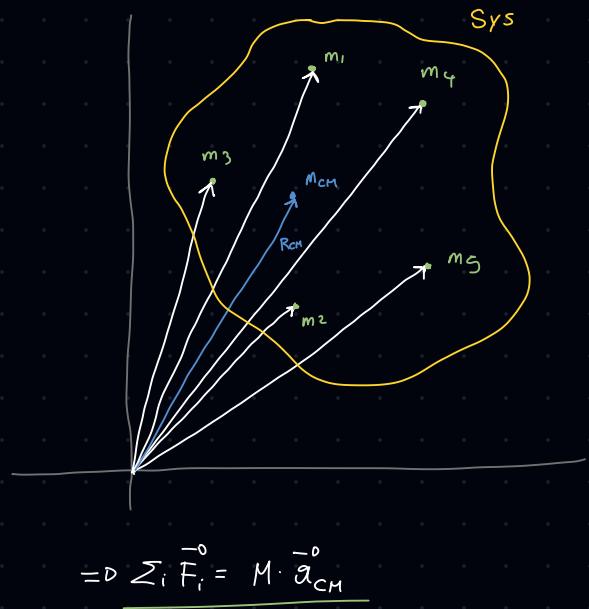
$$F = q(v \lambda B)$$

$$d\bar{B}_{\text{ind}} = \frac{\mu_0 I}{4\pi} \frac{d\bar{e} \wedge \bar{\epsilon}}{z^3} \rightarrow \text{USCENTE}$$

da Faraday

$$\bar{F}_{\text{ind}} = i d\bar{e} \wedge \bar{\epsilon}$$

Centro di Massa



$$\bar{R}_{\text{CM}} = \frac{\sum_i m_i \bar{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \bar{r}_i}{M}$$

$$\rightarrow \bar{v} = \frac{d\bar{s}}{dt} = \frac{d\bar{R}_{\text{CM}}}{dt} = \frac{\sum_i m_i \bar{v}_i}{M} \quad m \cdot \bar{v} = \bar{P}$$

$$\Rightarrow \bar{v}_{\text{CM}} = \frac{\bar{P}_{\text{TOT}}}{M} \quad \Rightarrow \bar{P}_{\text{TOT}} = \bar{v}_{\text{CM}} \cdot M$$

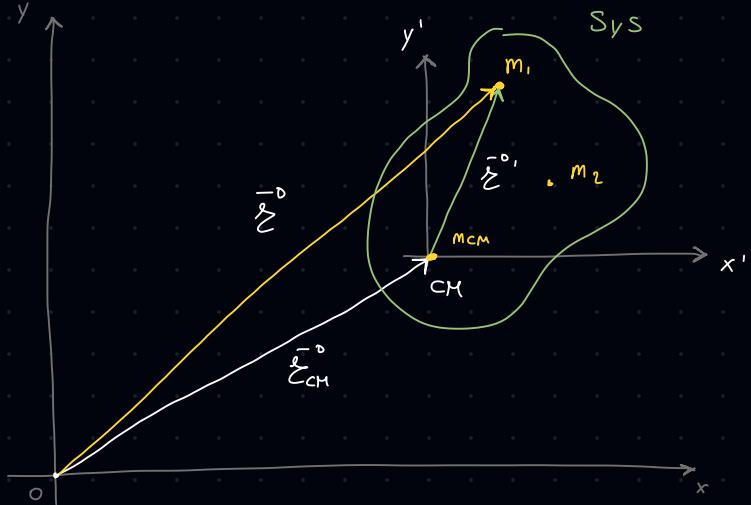
$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d\bar{v}_{\text{CM}}}{dt} = \frac{\sum_i m_i \bar{a}_i}{M} \Rightarrow \bar{a}_{\text{CM}} = \frac{\sum_i \bar{F}_i}{M}$$

$$\Rightarrow \sum_i \bar{F}_i = M \cdot \bar{a}_{\text{CM}}$$

$$\left\{ \begin{array}{l} \bar{L} = \bar{\epsilon} \wedge m_i \bar{v} = \bar{\epsilon} \wedge \bar{P} \\ \bar{M} = \bar{\epsilon} \wedge \bar{F} \end{array} \right. \Rightarrow \frac{d\bar{L}}{dt} = \frac{d\bar{\epsilon}}{dt} \wedge m \bar{v} + \bar{\epsilon} \wedge m \frac{d\bar{v}}{dt} \\ = \bar{v} \wedge m \bar{v} + \boxed{\bar{\epsilon} \wedge m \cdot \bar{a}} \bar{M}$$

$$\Rightarrow \bar{M} = \frac{d\bar{L}}{dt}$$

Teorema di Koenig



$$\vec{\epsilon} = \vec{\epsilon}_{CM} + \vec{\epsilon}', \Rightarrow \vec{v} = \vec{v}_{CM} + \vec{v}'$$

Siccome $\vec{L} = \vec{\epsilon} \wedge m \vec{v}$

$$\begin{aligned} \Rightarrow \vec{L} &= \sum_i \vec{\epsilon}_i \wedge m_i \vec{v}_i = \sum_i [\vec{\epsilon}_{CM} + \vec{\epsilon}'] \wedge [\vec{v}_{CM} + \vec{v}'] = \sum_i \vec{\epsilon}_{CM} \wedge m_i \vec{v}_{CM} + \sum_i \vec{\epsilon}_{CM} \wedge m_i \vec{v}' + \\ &\quad + \sum_i \vec{\epsilon}' \wedge m_i \vec{v}_{CM} + \sum_i \vec{\epsilon}' \wedge m_i \vec{v}' \\ &= \vec{\epsilon}_{CM} \wedge M \vec{v}_{CM} + \vec{\epsilon}_{CM} \wedge \sum_i m_i \vec{v}' + \sum_i m_i \vec{\epsilon}' \wedge \vec{v}_{CM} + \sum_i \vec{\epsilon}' \wedge m_i \vec{v}' \end{aligned}$$

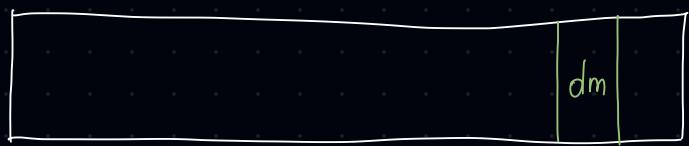
Dal CM $R_{CM} = \frac{\sum_i m_i \vec{v}_i}{M} \Rightarrow \vec{v}_{CM} = \frac{\sum_i m_i \vec{v}_i}{M} = \frac{\vec{P}_{TOT}}{M}$

- $\sum_i m_i \vec{v}' = M \cdot \vec{v}'_{CM}$ vel del CM dal CM = 0
- $\sum_i m_i \vec{\epsilon}' = M \cdot \vec{\epsilon}'_{CM}$ pos del CM dal CM = 0

$$\Rightarrow \vec{L}_{TOT} = \vec{\epsilon}_{CM} \wedge M \vec{v}_{CM} + \sum_i \vec{\epsilon}' \wedge m_i \vec{v}' \Rightarrow \vec{L}_{TOT} = \vec{L}_{CM} + \vec{L}_{TOT}$$

Momento Inerzia

$$I = m \cdot R^2$$



$$\text{e.i. } m = dm$$

$$\Rightarrow \int R^2 dm \quad \text{moltiplico per il volume} \rightarrow I_{TOT} \int R^2 \frac{dm}{dV} dV \quad \text{pongo } \int = \frac{M}{V}$$

$$\rightarrow I_{TOT} = \int_V R^2 \rho dV \quad \text{sbarro 2D} \rightarrow I_{TOT} = \int \int x^2 dx$$



$$I_{TOT} = \int \int y^2 dy \quad \text{con } \rho = \frac{M}{L}$$

$$\rightarrow I = \frac{M}{L} \int y^2 dy = \frac{M}{L} \left[\frac{y^3}{3} \right]_0^L = \frac{M}{L} \frac{L^3}{2}$$

$$\rightarrow I = \frac{1}{3} M L^2$$

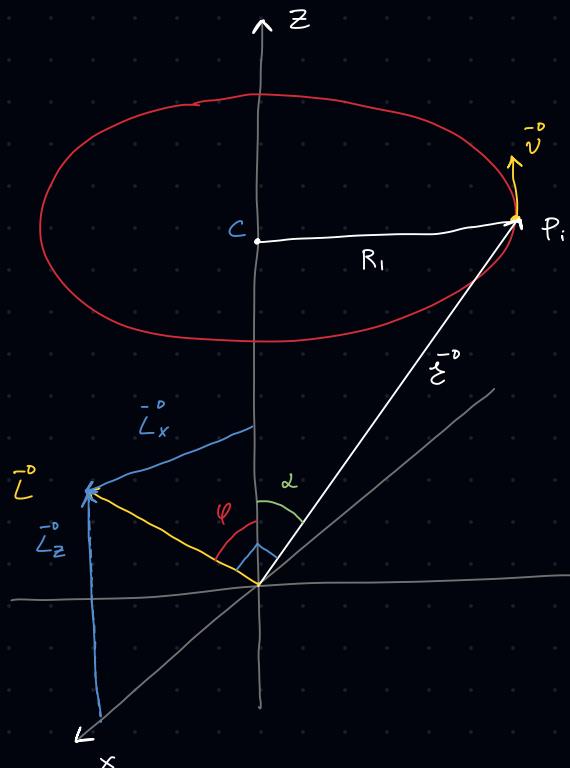


$$I_{TOT} = \frac{M}{L} \int_{-\frac{e}{2}}^{\frac{e}{2}} y^2 dy = \frac{M}{L} \left[\frac{y^3}{3} \right]_{-\frac{e}{2}}^{\frac{e}{2}}$$

$$= \frac{M}{L} \left[\frac{\frac{e^3}{8}}{3} + \frac{\frac{e^3}{8}}{3} \right] = \frac{M}{L} \cdot \frac{L^3}{12}$$

$$\rightarrow I = \frac{M L^2}{12}$$

Rotazione Asse fisso



Momento angolare

$$\vec{L} = \vec{\epsilon} \wedge m \cdot \vec{v}$$

$$! \quad \vec{\epsilon} = ?$$

conosco $\begin{cases} z & : \overline{OC} \\ R_i & : \text{Raggio} \\ \alpha & : \text{Angolo } z \hat{o} p_i \end{cases}$

$$\Rightarrow \vec{\epsilon}^2 = z^2 + R_i^2$$

$$\Rightarrow \vec{L}_i = \vec{\epsilon} \wedge m_i \vec{v}_i \quad \Rightarrow \quad L_i = \vec{\epsilon} m v \sin(\alpha)$$

Le componenti x ed y si annullano

$$\Rightarrow \vec{L}_z = \vec{L} \cos \alpha$$

$$\phi = 90 - \alpha \quad \Rightarrow \quad \cos(90 - \alpha) = \sin(\alpha) \Rightarrow \quad L_z = L \sin \alpha$$

$$\Rightarrow L_z = \vec{\epsilon} m v \sin \alpha \quad \text{ma} \quad \underbrace{\vec{\epsilon} \sin \alpha = R}_{\text{Trig}} \quad \text{e} \quad \underbrace{v = \omega R}_{\text{Moto Circolare}}$$

$$\Rightarrow L_{z_i} = m_i \omega_i R_i^2 \quad \text{ma} \quad \omega_i = \omega \quad \Rightarrow \quad L_{z_i} = \omega \left(\sum_i m_i R_i^2 \right) \quad I_{\text{inerzia}}$$

$$\Rightarrow \boxed{L_{z_i} = \vec{\omega} I}$$

Siccome

$$\vec{M} = \frac{d\vec{L}}{dt}$$

$$\Rightarrow \vec{M} = I \frac{d\vec{\omega}}{dt} \quad \Rightarrow \quad \boxed{\vec{M} = \alpha I}$$

Equividente

$$\vec{F} = m \cdot \vec{a}$$

$$\text{Se} \quad \vec{M} = \alpha I \quad \Rightarrow \quad \alpha = \frac{M}{I}$$

$$\begin{cases} \theta = \theta_0 + \int_0^t \omega dt \\ \omega = \omega_0 + \int_0^t \alpha dt \end{cases}$$

$$\text{Se} \quad M = 0 \quad \Rightarrow \quad \alpha = 0 \quad \Rightarrow \quad \begin{cases} \text{Fermo} \\ v = \text{cost} \end{cases}$$

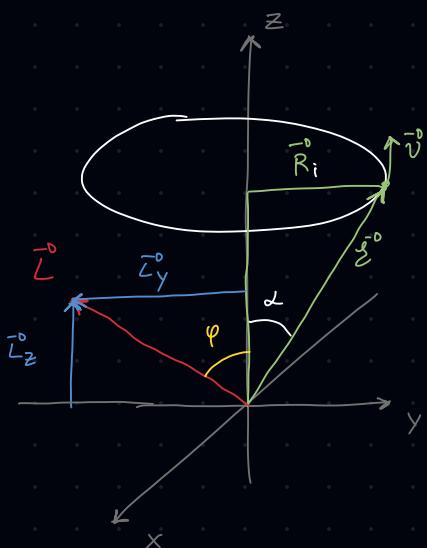
$$\Rightarrow \theta = \theta_0 + \omega t \quad \Leftrightarrow \quad S(t) = S_0 + v t$$

Se $M \neq 0$ ma cost

$$\Rightarrow \theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 \Leftrightarrow S(t) = S_0 + vt + \frac{1}{2} \alpha t^2$$

Energia

$$\frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 R^2 = \frac{1}{2} I \omega^2$$



$$\vec{L} = \epsilon_0 \lambda m \vec{v} \Rightarrow |L| = \epsilon_0 m v_i \sin \frac{1}{\sqrt{q_0}}$$

$$\text{L} \text{ ed } L_x \text{ si annullano} \Rightarrow \vec{L}_z = \vec{L}_i \cdot \cos(\varphi)$$

$$\text{ma } \varphi = 90^\circ - \alpha \Rightarrow \cos(90^\circ - \alpha) = \sin(\alpha)$$

$$\Rightarrow \vec{L}_z = L_i \sin(\alpha) = \epsilon_0 m v_i \sin \alpha$$

$$\text{ma } \epsilon_0 \sin \alpha = \frac{1}{R_i}$$

$$\Rightarrow L_{L_i} = m v_i \frac{1}{R_i} \text{ ma } \vec{v} = \vec{\omega} R$$

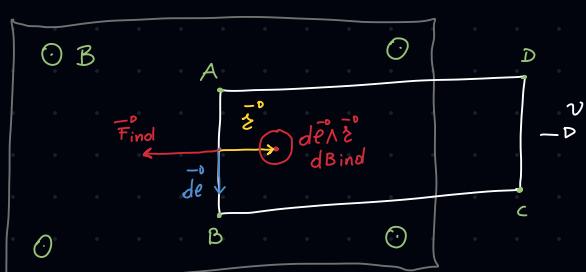
$$\Rightarrow L_{z_i} = m \omega_i R_i^2 \quad \omega_i = \omega \Rightarrow L_{z_i} = \omega m R_i^2$$

$$\sum_i m_i R_i^2 = I \Rightarrow \boxed{L_z = \omega I}$$

dalla II eq cardinale

$$\vec{M} = \frac{d\vec{L}}{dt} \Rightarrow \vec{M} = I \frac{d\vec{\omega}}{dt} = \alpha I \Rightarrow \alpha = \frac{\vec{M}}{I} \equiv \vec{F} = m \cdot \vec{a}$$

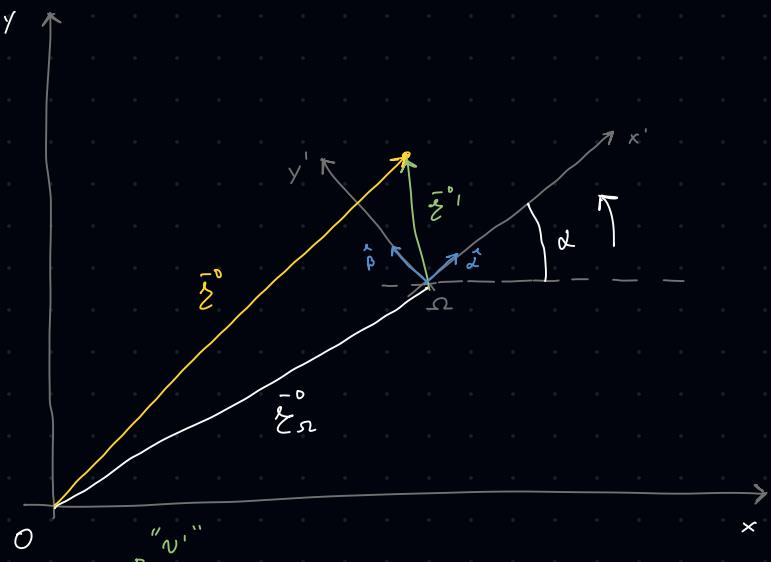
$$\begin{cases} \theta(t) = \theta_0 + \int_0^t \omega dt & M=0 \Rightarrow \theta(t) = \theta_0 + \omega t \\ \omega(t) = \omega_0 + \int_0^t \alpha dt & M \neq 0 \Rightarrow \begin{cases} \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ \omega(t) = \omega_0 + \alpha t \end{cases} \end{cases}$$



$$dB_{\text{ind}} = \frac{\mu_0}{2\pi} \cdot \frac{de \wedge \vec{z}}{z^3}$$

$$\text{Faraday} \cdot \vec{F}_{\text{ind}} = I de \wedge d\vec{B}$$

Dinamica Sys non inerziali



$$\vec{\xi} = \vec{\xi}_\alpha + \vec{\xi}' \Rightarrow \vec{v} = \frac{d\vec{\xi}_\alpha}{dt} + \frac{d\vec{\xi}'}{dt}$$

$$\Rightarrow \vec{v} = \vec{v}_\alpha + \vec{v}'$$

VERSORI

$$\begin{cases} \vec{\xi}' = \hat{\alpha} x' + \hat{\beta} y' \\ \vec{\xi} = \hat{x} x + \hat{y} y \end{cases} \Rightarrow \hat{\alpha} e \hat{\beta} \neq \cos \alpha$$

$$= \frac{d\vec{\xi}'}{dt} = \frac{d\alpha}{dt} x' + \alpha \frac{dx'}{dt} + \frac{d\beta}{dt} y' + \beta \frac{dy'}{dt} = \left(\underbrace{\left(\frac{d\hat{\alpha}}{dt} \right) x' + \frac{d\hat{\beta}}{dt} y'}_{\omega \wedge \hat{\alpha}} \right) + \underbrace{\left(\hat{\alpha} \frac{dx'}{dt} + \hat{\beta} \frac{dy'}{dt} \right)}_{\vec{v}'}$$

$$= \vec{v}' + (\omega \wedge \hat{\alpha}) x' + (\omega \wedge \hat{\beta}) y' = \vec{v}' + \omega \wedge \underbrace{(\hat{\alpha} x' + \hat{\beta} y')}_{\vec{\xi}'} = \vec{v}' + \omega \wedge \vec{\xi}' \quad (1)$$

$$= \frac{d\vec{\xi}}{dt} = \vec{v}_\alpha + \vec{v}' + \omega \wedge \vec{\xi}' \quad \text{Vel di p da o}$$

$$\vec{a} = \frac{d^2 \vec{\xi}}{dt^2} = \frac{d\vec{v}'}{dt} + \frac{d\vec{v}'}{dt} + \frac{d\vec{v}'}{dt} + \omega \wedge \underbrace{\frac{d\vec{\xi}'}{dt}}$$

$$\vec{v}' = \left(\hat{\alpha} \frac{dx'}{dt} + \hat{\beta} \frac{dy'}{dt} \right) \Rightarrow \frac{d\vec{v}'}{dt} = \underbrace{\left(\frac{d\hat{\alpha}}{dt} \frac{dx'}{dt} + \frac{d\hat{\beta}}{dt} \frac{dy'}{dt} \right)}_{(\alpha)} + \underbrace{\left(\hat{\alpha} \frac{d^2 x'}{dt^2} + \hat{\beta} \frac{d^2 y'}{dt^2} \right)}_{\vec{a}'}$$

$$(\alpha) (\omega \wedge \hat{\alpha}) \frac{dx'}{dt} + (\omega \wedge \hat{\beta}) \frac{dy'}{dt} = \omega \wedge \left(\hat{\alpha} \frac{dx'}{dt} + \hat{\beta} \frac{dy'}{dt} \right) = \omega \wedge \vec{v}'$$

$$= \frac{d\vec{v}'}{dt} = (\omega \wedge \vec{v}') + \vec{a}'$$

$$= \frac{d^2 \vec{\xi}}{dt^2} = \vec{a}_\alpha + (\vec{\omega} \wedge \vec{v}') + \vec{a}' + \frac{d\vec{w}}{dt} \wedge \vec{\xi}' + \vec{w} \wedge \left[\vec{v}' + (\vec{\omega} \wedge \vec{\xi}') \right]$$

$$= \vec{a}_\alpha + \underbrace{2(\vec{\omega} \wedge \vec{v}')}_{\text{Acc Coriolis}} + \vec{a}' + \frac{d\vec{w}}{dt} \wedge \vec{\xi}' + \underbrace{\vec{w} \wedge (\vec{\omega} \wedge \vec{\xi}')}_{(-\omega^2 \vec{\xi})}$$

$$\text{Chiamiamo } \vec{a}_T = \vec{a}_\alpha + \frac{d\vec{w}}{dt} \wedge \vec{\xi}' - \omega^2 \vec{\xi} \quad \text{Centrifuga}$$

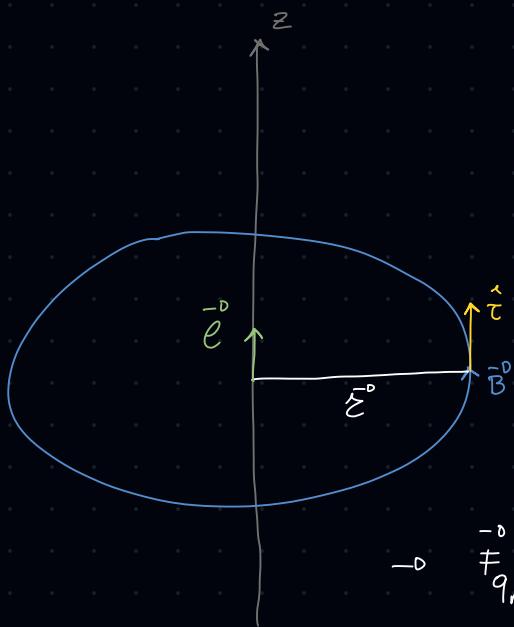
$$= \vec{a} = \vec{a}' + \vec{a}_{co} + \vec{a}_{TR}$$

$$\text{Considero} \quad \vec{F} = m \cdot \vec{\alpha} \quad \Rightarrow \quad \vec{F} = m \left(\vec{\alpha}' + \vec{\alpha}_{CO} + \vec{\alpha}_{TR} \right) \quad \Rightarrow \quad \vec{F} - \underbrace{m \vec{\alpha}_{CO}}_{\vec{F}_{CO}} - \underbrace{m \vec{\alpha}_{TR}}_{\vec{F}_{TR}} = m \cdot \vec{\alpha}'$$

$$\Rightarrow \sum_i \vec{F}_i = m \vec{\alpha}'$$

$$F_{CO} = -2m(\omega \wedge \nu) = 2m(\nu \wedge \omega)$$

Cavo percorsso da corrente



$$\text{Biot-Savart: } \vec{B} = \mu_0 \frac{\vec{I}}{2\pi R}$$

come Trovare \vec{B} ? $\vec{B} = \vec{B} \cdot \hat{\tau}$

$$\hat{\tau} = \hat{e} \wedge \vec{\epsilon} \Rightarrow \vec{B} = \frac{\mu_0}{2\pi} I \cdot \frac{d\hat{e} \wedge \vec{\epsilon}}{\vec{\epsilon}^2}$$

$$\vec{F}_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{\vec{\epsilon}} \hat{\epsilon}$$

$$\Rightarrow \vec{F}_{q_m} = \frac{\mu_0}{4\pi} \frac{q_m q_m'}{\vec{\epsilon}^2} \hat{\epsilon} \Rightarrow \vec{B} = \frac{\vec{F}}{q_m} = \frac{\mu_0}{4\pi} \frac{q_m}{\vec{\epsilon}^2} \hat{\epsilon} \quad (1)$$

$$F_{Lorentz} = q \vec{v} \wedge \vec{B} = q \frac{d\hat{e}}{dt} \wedge \vec{B} = \cancel{\frac{I}{\pi} d\hat{e} \wedge \vec{B}} \quad \text{LAPLACE}$$

$$\begin{cases} \vec{F} = \cancel{\frac{I}{\pi} d\hat{e} \wedge \vec{B}} \\ \vec{F} = \cancel{q \vec{v}} \end{cases} \Rightarrow q_m = I d\hat{e} \Rightarrow (1) \vec{B} = \frac{\mu_0}{4\pi} \frac{I d\hat{e}}{\vec{\epsilon}^2} \hat{\epsilon}$$

$$\text{ma } \hat{\epsilon} = d\hat{e} \wedge \hat{\epsilon} = \frac{d\hat{e} \wedge \vec{\epsilon}}{\epsilon} \Rightarrow \vec{B} = \frac{\mu_0}{4\pi} I \frac{d\hat{e} \wedge \vec{\epsilon}}{\epsilon} \quad \text{Laplace 1}$$

Maxwell Integrale

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = \frac{Q}{\epsilon_0} \quad \text{Gauss} \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Faraday} \end{array} \right. \Rightarrow \int \vec{E} \cdot \hat{n} dS = \frac{Q}{\epsilon_0}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{B} = 0 \quad \text{Monopol} \\ \nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \quad \text{Ampere-Maxwell} \end{array} \right. \Rightarrow \phi_B = \oint \vec{B} \cdot \hat{n} dS = 0 \quad ??$$

Bonus: $\vec{F}_L = q \vec{E} + q (\vec{v} \times \vec{B})$

Amp-Max Integrale

$$\text{Se } \frac{d\phi_B}{dt} = \int \vec{E} \cdot d\vec{e} - \oint \vec{v} \times \vec{B} de = \oint \frac{\partial \vec{B}}{\partial t} de - \oint \vec{v} \times \vec{B} de$$

$$\Rightarrow \frac{d\phi_E}{dt} = \oint \frac{\partial \vec{E}}{\partial t} de - \oint \vec{v} \times \vec{E} de$$

$$\Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\int \vec{B} \cdot \hat{n} dS = \mu_0 \int \vec{J} \cdot \hat{n} dS + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

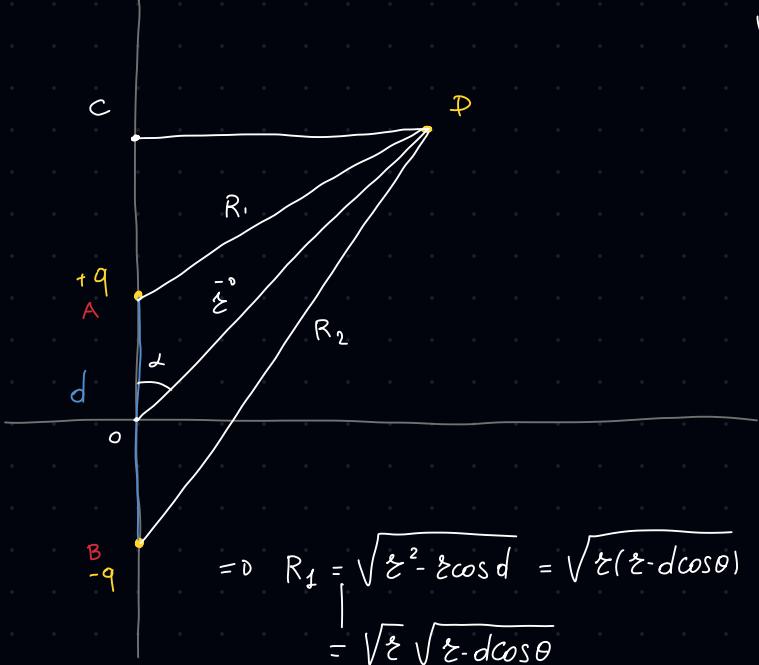
$$\Rightarrow \oint \vec{B} \cdot \hat{n} dS = \mu_0 I + \mu_0 \epsilon_0 \oint \frac{\partial \vec{E}}{\partial t} de - \mu_0 \epsilon_0 \oint \vec{v} \times \vec{E} de$$

$$\Rightarrow \oint \vec{B} \cdot \hat{n} dS - \frac{1}{c^2} \oint \vec{v} \times \vec{E} de = \mu_0 I + \mu_0 \epsilon_0 \left[\oint \frac{\partial \vec{E}}{\partial t} de - \oint \vec{v} \times \vec{E} de \right]$$

Ampere : Dipolo - Spira

$$E = -\vec{\nabla} V \quad \text{ma} \quad V = ?$$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{ma} \quad R_1, R_2 = ?$$



$$\begin{aligned} R_1^2 &= z^2 + d^2 = (z \sin \theta)^2 + (z \cos \theta - \frac{d}{2})^2 \\ &= z^2 \sin^2 \theta + (z \cos \theta - \frac{d}{2})^2 \\ &= z^2 \sin^2 \theta + z^2 \cos^2 \theta + \left(\frac{d}{2}\right)^2 - z \cos \theta d \\ &= z^2 (1 + \cos^2 \theta) + \left(\frac{d}{2}\right)^2 - z \cos \theta d \\ &\quad | \quad z \gg 0 \\ &= z^2 - z \cos \theta d \end{aligned}$$

Approx Taylor in $d \approx 0$

$$\Rightarrow f(d) = (z - d \cos \theta)^{\frac{1}{2}} \Rightarrow f'(d) = \frac{1}{2} (z - d \cos \theta)^{-\frac{1}{2}} (-\cos \theta) = -\frac{\cos \theta}{2(z - d \cos \theta)^{\frac{1}{2}}}$$

$$\Rightarrow f(0) = \sqrt{z} \quad f'(0) = -\frac{\cos \theta}{2\sqrt{z}}$$

$$\Rightarrow R_1 \approx \sqrt{z} \cdot \left[\sqrt{z} - \frac{d \cos \theta}{2\sqrt{z}} \right] = z - \frac{d \cos \theta}{2}$$

$$R_2 \approx z + \frac{d \cos \theta}{2}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{z - \frac{d}{2} \cos \theta} - \frac{1}{z + \frac{d}{2} \cos \theta} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{z + \frac{d}{2} \cos \theta - z + \frac{d}{2} \cos \theta}{z^2 - (\frac{d}{2})^2 \cos^2 \theta} \right] = \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{z^2 - (\frac{d}{2})^2 \cos^2 \theta}$$

$$\text{ma se } z \gg d \Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{z^2} \quad (1)$$

$$\Rightarrow E = -\vec{\nabla} V \Rightarrow \vec{E} = -\hat{x} \frac{\partial V}{\partial x} - \hat{y} \frac{\partial V}{\partial y} - \hat{z} \frac{\partial V}{\partial z} \quad \text{Se } \vec{e} \text{ lungo } z$$

$$\Rightarrow \vec{E}_z = -K \frac{\partial V}{\partial z} \quad \text{ma ci servono le coordinate polari}$$

$$\Rightarrow z^2 = x^2 + y^2 + z^2 ; \quad z \cos \theta = z \quad \Rightarrow \cos \theta = \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z}{\sqrt{(x^2 + y^2 + z^2)^{\frac{1}{2}}}}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{d z}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} \quad (20')$$

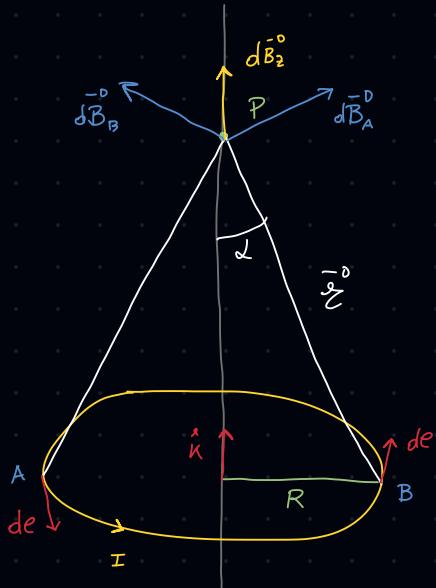
$$\begin{aligned}
 V(z) &= \left(\frac{q_d}{4\pi\epsilon_0} \right) z (x^2 + y^2 + z^2)^{-\frac{3}{2}} \Rightarrow V'(z) = -K \left[(x^2 + y^2 + z^2)^{-\frac{3}{2}} - \frac{3}{2} z (x^2 + y^2 + z^2)^{-\frac{5}{2}} \cdot 2z \right] \\
 &= -K \left[(z^2)^{\frac{3}{2}} + 2z^2 (z^2)^{-\frac{5}{2}} \right] = -K \left[\frac{1}{z^3} + \frac{3z}{z^5} \right] = + \frac{K}{z^3} \left[3 \frac{z^2}{z^2} - 1 \right] \\
 &= \frac{q}{4\pi\epsilon_0 z^3} \left[3 \cos^2 \theta - 1 \right]
 \end{aligned}$$

Se $\theta = 0$ lungo $z \rightarrow \theta = 0 \rightarrow \cos \theta = 1 \rightarrow E_z = \frac{2q_d}{4\pi\epsilon_0 z^3}$

Pongo
 $m = d q$

Spira

$$E_z = \frac{m}{2\pi\epsilon_0 z^3} \quad (1)$$

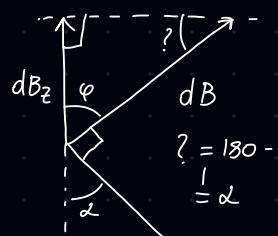


Da Biot-Savart

$$dB = \frac{\mu_0}{4\pi} I \frac{de \times \hat{z}}{z^3} \quad \text{ma } \epsilon = ??$$

$$\begin{aligned}
 z^2 &= R^2 + z^2 \rightarrow dB = \frac{\mu_0}{4\pi} I \frac{de \times \hat{z} \sin \varphi}{z^3} = \frac{\mu_0 I de}{4\pi z^2} \\
 &= \frac{\mu_0 I de}{4\pi (R^2 + z^2)}
 \end{aligned}$$

$$dB_x = dB_y \text{ si Annulloano} \rightarrow dB = dB_z = dB \sin \varphi$$



$$\Rightarrow dB_z = \frac{\mu_0 I}{4\pi} \frac{de \sin \varphi}{(R^2 + z^2)} \quad \text{ma } \epsilon \sin \varphi = R$$

$$\Rightarrow \sin \varphi = \frac{R}{z} = \frac{R}{\sqrt{R^2 + z^2}}$$

$$\Rightarrow dB_z = \frac{\mu_0 I}{4\pi} \frac{de R}{(R^2 + z^2)^{\frac{3}{2}}}$$

$$\Rightarrow B_z = \frac{\mu_0 I R}{4\pi} \cdot \frac{1}{(R^2 + z^2)^{\frac{3}{2}}} \int de = \frac{\mu_0 I R}{4\pi} \cdot \cancel{2\pi R^2} \rightarrow \overset{\cancel{2\pi R^2}}{B_z} = \frac{\mu_0 I R^2}{(R^2 + z^2)^{\frac{3}{2}}}$$

Se $z \gg R$

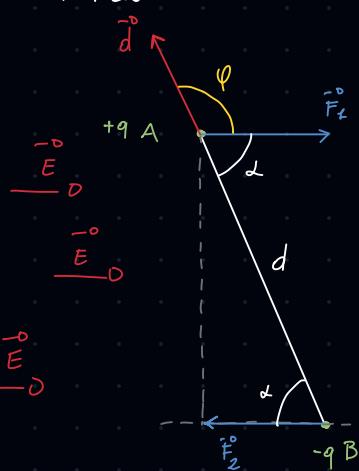
$$B_z = \frac{\mu_0 I 2\pi R^2}{4\pi z^3} \quad \text{pongo } \overset{\cancel{I}}{P} = \overset{\cancel{I}}{I} S \Rightarrow$$

$$\overset{\cancel{I}}{B_z} = \frac{\mu_0 \overset{\cancel{I}}{P}}{2\pi z^3} \quad QED$$

(15')

Momento

$$\text{Inerzia} \quad I = m R^2$$



$$\vec{F}_1 = -\vec{F}_2 \quad \Rightarrow \quad \begin{cases} \vec{F}_1 = q \cdot \vec{E} \\ \vec{F}_2 = -q \cdot \vec{E} \end{cases}$$

$$M = \sum_i \vec{r}_i \wedge \vec{F}_i \quad \text{ma} \quad M_{TOT} = \sum_i \vec{r}_i \wedge \vec{F}_i \quad \text{Polo in } B$$

$$\Rightarrow M_{TOT} = \vec{d} \wedge \vec{F}_1 + \vec{0} \wedge \vec{F}_2 = \vec{d} \wedge \vec{F}_1 = \vec{d} \wedge q \cdot \vec{E} = q \cdot \vec{d} \wedge \vec{E} = \underline{\vec{p} \wedge \vec{E}}$$

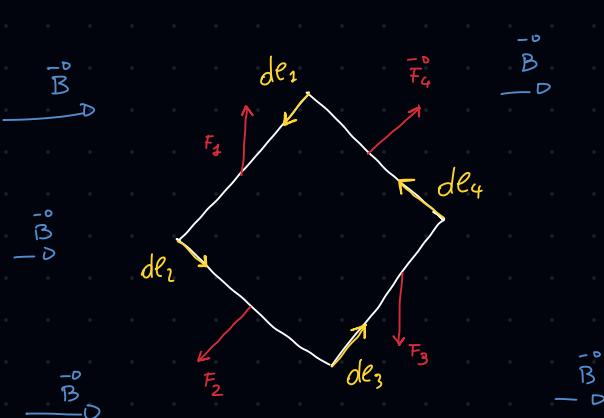
$$\text{Modulo:} \quad M_{TOT} = d \cdot F_1 \cdot \sin(\varphi)$$

$$\text{ma} \quad \varphi = 180 - \alpha = \sin(180 - \alpha) = \sin(\alpha)$$

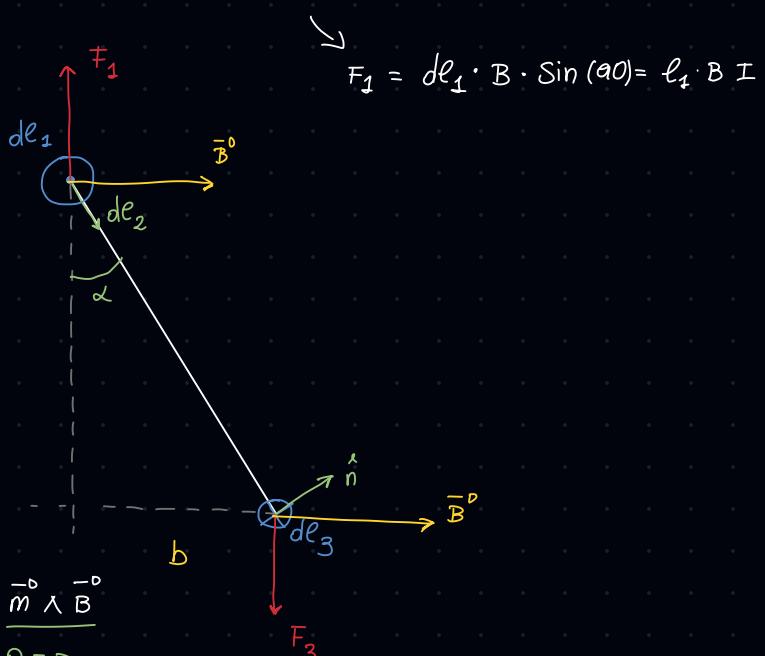
$$\Rightarrow M_{TOT} = F_1 \cdot d \cdot \sin \alpha \quad \text{ma} \quad d \cdot \sin \alpha = b \Rightarrow \boxed{M_{TOT} = F_1 \cdot b}$$

$$\rightarrow \text{Dipolo magnetico} \rightarrow \underline{\vec{M}_{TOT} = \vec{m} \wedge \vec{B}}$$

Spira



$$\vec{F} = \underbrace{q \cdot \vec{E} + q (\vec{v} \wedge \vec{B})}_{\text{Lorentz}} = \underbrace{\frac{I \cdot d\vec{l} \wedge \vec{B}}{\text{Laplace}}}_{\text{Laplace}}$$



$$M = F \cdot b = (I B l_1) \cdot (l_2 \sin \alpha) = I B S \sin \alpha$$

$$\text{definisco} \quad \vec{m} = \hat{n} \cdot I \cdot S \quad \Rightarrow \quad \vec{M} = \vec{m} \cdot B \sin \alpha = \frac{\vec{m} \wedge \vec{B}}{QED}$$

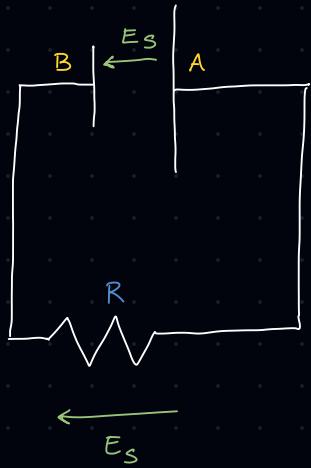
Joule

$$L = \int \vec{F} \cdot d\vec{e} \quad \text{ma } E = \frac{F}{q} \rightarrow F = q \cdot E = 0 \quad L = q \cdot V$$

$$\text{Si } L \text{ come } \Phi = \frac{dL}{dt} \rightarrow P = \frac{VdI}{dt} = P = VI \quad \text{ma } V = RI$$

$$\Rightarrow P = RI^2 \quad \text{effetto Joule}$$

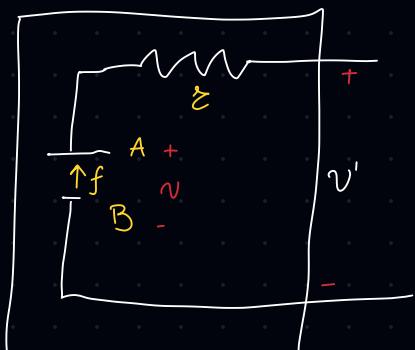
Fem



$$L = \oint E \cdot d\vec{e} = 0 \quad \text{definisco } f_{em} = \frac{L}{q} = \int \vec{E} \cdot d\vec{e} \neq 0$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{e} = \int_A^B E_S d\vec{e} + \int_B^A (E_S + E_m) d\vec{e} = \cancel{\int_A^B E_S d\vec{e}} - \cancel{\int_A^B E_S d\vec{e}} + \int_B^A E_m d\vec{e}$$

$$\Rightarrow f_{em} = \int_B^A E_m \cdot d\vec{e}$$



$$V'_A - V'_B = (V_A - V_B) - RI$$

$$\text{ma } V_A - V_B = f_{em} \Rightarrow V'_A - V'_B = f_{em} - zI$$

$$\Rightarrow V'_A - V'_B = f_{em} - zI = RI \Rightarrow f_{em} = I(R+z)$$

$$\Rightarrow I = \frac{f_{em}}{R+z} \quad \text{ma } V = RI \Rightarrow I = \frac{V}{R}$$

$$\Rightarrow \frac{V}{R} = \frac{f_{em}}{R+z} \Rightarrow V = \frac{R}{R+z} f_{em} \quad \text{per } z_{int} = 0 \Rightarrow V = f_{em}$$

Modello di Drude $\rho = \frac{1}{\tau}$

1) e-e 2) e-i 3) τ 4) $v(t)$ 5) voto elastico

$$J = \bar{n} \cdot \vec{\rho} \quad \rightarrow \quad J = -n \cdot e \cdot \vec{v}$$

↑
 dens
volum

 ↑
 numero
di elettr
on

Ohm vettoriale

$$\begin{aligned} dV &= R \cdot dI \\ \frac{dV}{dI} &= \rho \end{aligned} \quad \left. \begin{aligned} R \cdot dI &= E \cdot d\ell \\ \int \frac{dE}{d\ell} \cdot d\ell &= \int dI \end{aligned} \right\} \quad \underline{\int J \cdot \hat{n} ds = e d\ell}$$

Velocità:

$$E = \frac{F}{q} \rightarrow F = q \cdot E \quad \Rightarrow \quad q = -e \rightarrow F = -e \cdot E$$

$$\text{Se } \vec{F} = m \cdot \vec{a} \quad \rightarrow \quad -e \cdot E = m \cdot \vec{a} \quad \Rightarrow \quad a = -\frac{e \cdot E}{m}$$

$$\rightarrow v(t) = v_0 + at \quad \rightarrow \quad v(t) = v_0 - \frac{eE}{m} t$$

$$E = 0 \quad \rightarrow \quad \langle v(t) \rangle = \langle v_0 \rangle - \langle \frac{eE}{m} t \rangle = \langle v_0 \rangle = 0$$

$$E \neq 0 \quad \langle v(t) \rangle = \langle v_0 \rangle - \langle \frac{eE}{m} t \rangle = 0 - \frac{eE}{m} \langle t \rangle = -\frac{eE}{m} \tau$$



$$\Rightarrow \bar{J} = -ne \bar{v} \rightarrow \bar{J} = -ne \left(-\frac{eE}{m} \tau \right) = \frac{n e^2 E}{m} \tau \quad \text{ma} \quad \bar{J} = \frac{\bar{E}}{\rho}$$

$$\Rightarrow \frac{n e^2 E \tau}{m} = \frac{E}{\rho} \quad \rightarrow \quad \boxed{\tau = \frac{m}{\rho \cdot n e^2}}$$

$$\bar{E} = -\bar{\nabla} V \quad ??$$

$$\text{Sappiamo che} \quad \bar{E} \cdot d\bar{e} = -dV \quad \Rightarrow \quad dV = i \frac{\partial V}{\partial x} dx + j \frac{\partial V}{\partial y} dy + k \frac{\partial V}{\partial z} dz$$

$$\rightarrow \begin{cases} de = i dx + j dy + k dz \\ \nabla V = i \frac{\partial V}{\partial x} + j \frac{\partial V}{\partial y} + k \frac{\partial V}{\partial z} \end{cases} \quad \rightarrow \quad -dV = -d\bar{e} \cdot \bar{\nabla} V$$

$$\Rightarrow \bar{E} \cdot d\bar{e} = -d\bar{e} \cdot \bar{\nabla} V \quad \rightarrow \quad \bar{E} = -\bar{\nabla} V$$

Eq Continuità della corrente

$$-\alpha = \int \bar{J} \cdot \hat{n} ds \quad dt \quad \rightarrow \quad \rho = \frac{Q}{V} \quad \rightarrow \quad Q = \int \rho \cdot dV \quad \rightarrow \quad -\frac{d}{dt} \int \rho \cdot dV = \int \bar{J} \cdot \hat{n} ds$$

$$\rightarrow - \int \frac{\partial \rho}{\partial t} dV = \int (\bar{\nabla} \cdot \bar{J}) dV \quad \rightarrow \quad \bar{\nabla} \cdot \bar{J} = - \frac{\partial \rho}{\partial t}$$

Se siamo in corrente
stazionaria

$$\Rightarrow \frac{\partial \rho}{\partial t} = 0 \quad \rho = \text{cost}$$

Densità di corrente

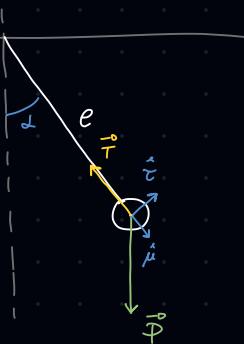
$$dq = j \cdot dV \quad \text{ma} \quad V = S \cdot e \quad \Rightarrow \quad dV = dS \cdot de$$

\downarrow
 $b \times h \quad e = v \cdot t \rightarrow de = v \cdot dt$

$$\Rightarrow dq = \int dS \cdot de = \int dS \cdot v \cdot dt \quad \Rightarrow \quad \frac{\partial q}{\partial t} = \int \vec{v} \cdot \vec{n} ds$$

$I = \int \vec{J} \cdot \vec{n} ds$

Pendolo Semplice



$$\vec{P} = \vec{P}_\mu + \vec{P}_\tau = P \cos \alpha - P \sin \alpha$$

$$\vec{F} = m \cdot \vec{a}$$

$$\begin{aligned} \dot{\mu} : \quad & \vec{P}_\mu - T = m \cdot \vec{a} \quad \Rightarrow \quad \left\{ \begin{array}{l} m g \cos \alpha - T = m \vec{a}_\mu \\ -m g \sin \alpha = m \vec{a}_\tau \end{array} \right. \\ \tau : \quad & -\vec{P}_\tau = m \vec{a}_\tau \quad \Rightarrow \quad \left\{ \begin{array}{l} m g \cos \alpha - T = m \vec{a}_\mu \\ -m g \sin \alpha = m \vec{a}_\tau \end{array} \right. \end{aligned}$$

$$\vec{a} = \frac{d \vec{v}}{dt} = \ddot{s} \quad \Rightarrow \quad \text{L'acce è centripeta} \Rightarrow \vec{a}_\mu = -\frac{v^2}{R} \vec{i}$$

$$\vec{a}_\tau = \ddot{s}$$

$$\Rightarrow \left\{ \begin{array}{l} m g \cos \alpha - T = -\frac{m v^2}{R} \\ -m g \sin \alpha = m \cdot \ddot{s} \end{array} \right. \quad \Rightarrow \quad \text{Approx per piccole oscillazioni} \quad \sin \alpha \approx \alpha$$

$$\text{ma} \quad 1 \text{ Rad} = \frac{\pi}{R} \quad \begin{matrix} \leftarrow \text{Arco} \\ \leftarrow \text{Raggio} \end{matrix} \quad \Rightarrow \quad \alpha = \frac{s}{e} \quad \Rightarrow \quad -m g \frac{s}{e} = m \cdot \ddot{s} \quad \Rightarrow \quad \ddot{s} + \frac{g}{e} s = 0$$

$$\text{chiamo} \quad K^2 = \frac{g}{e} \quad \Rightarrow \quad \ddot{s} + K^2 s = 0 \quad \text{Eq diff}$$

$$\text{Soluzione} \quad s(t) = A \sin(Kt + \phi)$$

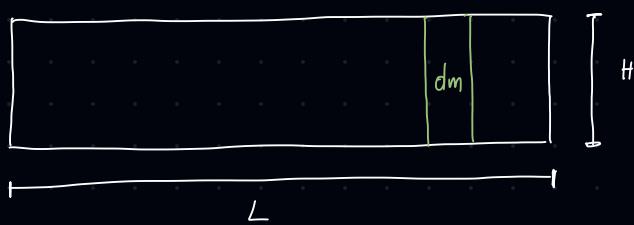
$$s(t+T) = A \sin(Kt + \phi + 2\pi) \Rightarrow A \sin(Kt + KT + \phi) = A \sin(Kt + \phi + 2\pi) \Rightarrow Kt + KT + \phi = Kt + \phi + 2\pi$$

$$\Rightarrow KT = 2\pi \quad \Rightarrow \quad K = \frac{2\pi}{T} = \omega$$

$$\Rightarrow T = \frac{2\pi}{\omega} \quad \text{ma} \quad \omega = \sqrt{\frac{g}{e}} \quad \Rightarrow \quad T = 2\pi \sqrt{\frac{e}{g}}$$

Inerzia Sbarra

$$I = m R^2$$



$\sum m_i \rightarrow \int dm$ continuo di punti

$$\text{sbarra 3D} \rightarrow I_{\text{tot}} = \int R^2 dm$$

$$\rightarrow I_{\text{tot}} = \int R^2 \frac{dm}{dv} dv \quad \rho = \frac{M}{V} \Rightarrow I_{\text{tot}} = \int V R^2 \rho dv$$

$$\text{sbarra 2D} \quad \rho = \frac{M}{V} \rightarrow \lambda = \frac{M}{L} \Rightarrow I_{\text{tot}} = \int_e R^2 \lambda de$$

Lungo un lato

$$\rightarrow I_{\text{tot}} = \frac{M}{L} \int_0^L y^2 dy = \frac{M}{L} \left[\frac{y^3}{3} \right]_0^L = \frac{ML^2}{3}$$

Lungo il centro $\frac{L}{2}$

$$I_{\text{tot}} = \frac{M}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} y^2 dy = \frac{1}{12} M L^2$$

Corrente di spostamento

$$\text{Ampere - Maxwell: } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Continuità corrente

$$-dq = \vec{J} \cdot \hat{n} ds \cdot dt \rightarrow -\frac{dq}{dt} = \vec{J} \cdot \hat{n} ds \rightarrow -\int_V \frac{\partial J}{\partial t} dv = \int \vec{J} \cdot \hat{n} ds$$

$$\rightarrow -\frac{\partial J}{\partial t} = \vec{\nabla} \cdot \vec{J} \quad \text{obbiettivo}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \vec{\nabla} \cdot \vec{J} = \mu_0 \epsilon_0 \frac{\partial (\nabla \cdot \vec{E})}{\partial t} \rightarrow \mu_0 \vec{\nabla} \cdot \vec{J} = -\mu_0 \epsilon_0 \frac{1}{\epsilon_0} \frac{\partial \rho}{\partial t} \rightarrow \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Dalle eq di Max alle Onde

$$\text{- Amp. Max: } \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\text{- Faraday: } \vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

Campo E

$$\nabla \wedge (\nabla \wedge E) = - \frac{\partial (\nabla \wedge B)}{\partial t} \quad \Rightarrow \quad \nabla (\nabla^2 E) - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \nabla \frac{f}{\epsilon_0} - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \Rightarrow \quad \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \nabla \frac{f}{\epsilon_0} \quad (1)$$

Campo B

$$\nabla \wedge (\nabla \wedge B) = \mu_0 (\nabla \wedge J) + \mu_0 \epsilon_0 \frac{\partial (\nabla \wedge E)}{\partial t} \quad \Rightarrow \quad \nabla (\nabla^2 B) - \nabla^2 B = \mu_0 (\nabla \wedge J) - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

$$\Rightarrow \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} - \nabla^2 B = \mu_0 (\nabla \wedge J) \quad (2)$$

Nel vuoto $f = J = 0$

$$\left\{ \begin{array}{l} \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} - \nabla^2 B = 0 \quad (a) \\ \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = 0 \quad (b) \end{array} \right.$$

$$(b) \quad \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = 0 \quad \Rightarrow \quad \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} - \frac{\partial^2 E_x}{\partial x^2} - \frac{\partial^2 E_y}{\partial y^2} - \frac{\partial^2 E_z}{\partial z^2} = 0$$

$$\text{Solo } x \rightarrow \frac{\partial^2 E}{\partial x^2} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

Eq Differenziale con sd: $E(t) = A \sin(\kappa x - \omega t)$

$$E_1 = A \sin(\kappa x_1 - \omega t_1) \quad E_2 = A \sin(\kappa x_2 - \omega t_2) \quad \Rightarrow \quad \kappa x_1 - \omega t_1 = \kappa x_2 - \omega t_2 \quad \Rightarrow \quad \kappa(x_2 - x_1) = \omega(t_2 - t_1)$$

$$\nu = \frac{\omega}{\kappa} \Rightarrow \omega = \nu \cdot \kappa$$

$$x_2 - x_1 = \left(\frac{\omega}{\kappa} \right) (t_2 - t_1)$$

$$\Rightarrow E(t) = A \sin(\kappa x - \nu \kappa t)$$

$$\frac{\partial E}{\partial t} = -\nu \kappa A \cos(\kappa x - \nu \kappa t)$$

$$\frac{\partial E}{\partial x} = \kappa A \cos(\kappa x - \nu \kappa t)$$

$$\frac{\partial^2 E}{\partial t^2} = -\nu^2 \kappa^2 A \sin(\kappa x - \nu \kappa t)$$

$$\frac{\partial^2 E}{\partial x^2} = -\kappa^2 A \sin(\kappa x - \nu \kappa t)$$

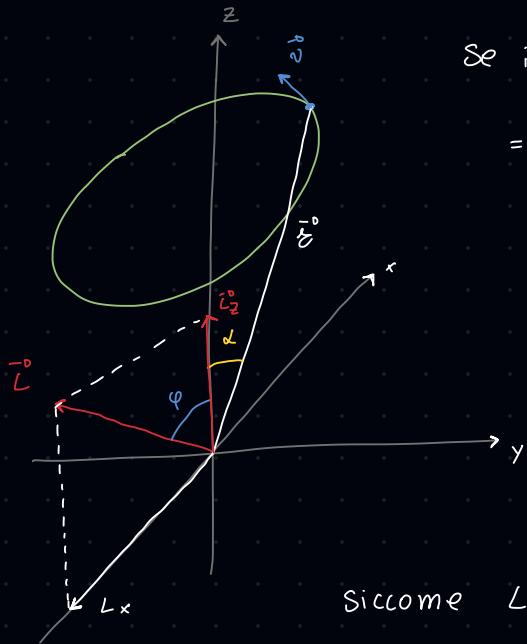
$$\Rightarrow -\kappa^2 A \sin(\kappa x - \nu \kappa t) +$$

$$+ \mu_0 \epsilon_0 \nu^2 \kappa^2 A \sin(\kappa x - \nu \kappa t) = 0$$

$$\rightarrow K^2 A \sin(\kappa x - \kappa t) \left[-1 + \mu_0 \epsilon_0 v^2 \right] = 0 \Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} = c$$

II eq cardinale della Dinamica

$$\begin{cases} \vec{L} = \vec{\epsilon} \wedge m \vec{v} = \vec{\epsilon} \wedge \vec{p} \\ \vec{M} = \vec{\epsilon} \wedge \vec{F} \end{cases} \quad \text{ma } I = m R^2$$



Se il corpo è speculare $\rightarrow L_x = L_y = 0$

$$\Rightarrow L_z = L \cos(\varphi) \quad \text{ma} \quad \varphi = 90 - \alpha \Rightarrow \cos(90 - \alpha) = \sin(\alpha)$$

$$\Rightarrow L_z = L \sin(\alpha)$$

$$L = \vec{\epsilon} \wedge m \cdot \vec{v} \Rightarrow L = \epsilon \cdot m \cdot v \cdot \sin(90)$$

$$\Rightarrow L = \epsilon m v \quad \text{ma} \quad v = \omega R$$

$$\Rightarrow L_i = \epsilon m_i \omega_i R_i$$

Siccome $L_z = L \sin \alpha \Rightarrow L_{iz} = \epsilon m_i \omega_i R_i \sin \alpha$

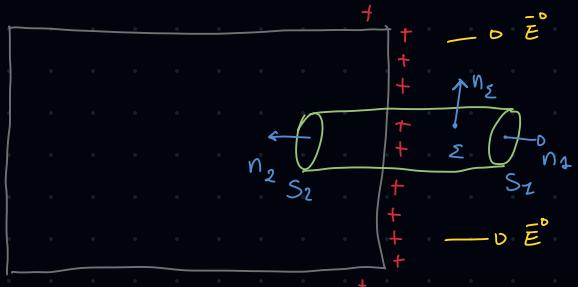
$$\text{ma } \epsilon \sin \alpha = R \Rightarrow L_{iz} = m_i \omega_i R^2 \quad \sum_i m_i R_i^2 = I_{\text{tot}}$$

$$\rightarrow \underline{L_{\text{tot}} = \omega I}$$

Siccome $\begin{cases} \vec{L} = \vec{\epsilon} \wedge m \cdot \vec{v} \\ \vec{M} = \vec{\epsilon} \wedge \vec{F} \end{cases} \Rightarrow \frac{dL}{dt} = \frac{d\vec{\epsilon}}{dt} \wedge m \cdot \vec{v} + \vec{\epsilon} \wedge m \cdot \frac{d\vec{v}}{dt} = \cancel{\vec{v} \wedge m \vec{v}} + \vec{\epsilon} \wedge m \frac{\vec{a}}{F}$

$$\rightarrow \frac{dL}{dt} = \vec{M} \Rightarrow M_{\substack{\text{corpo} \\ \text{rigido}}} = I \frac{d\omega}{dt} \quad \rightarrow \boxed{M = \alpha I} \quad \text{II eq cardinale}$$

1 Coulomb



$$E = \emptyset$$

$$\vec{E} \perp \hat{n}_z$$

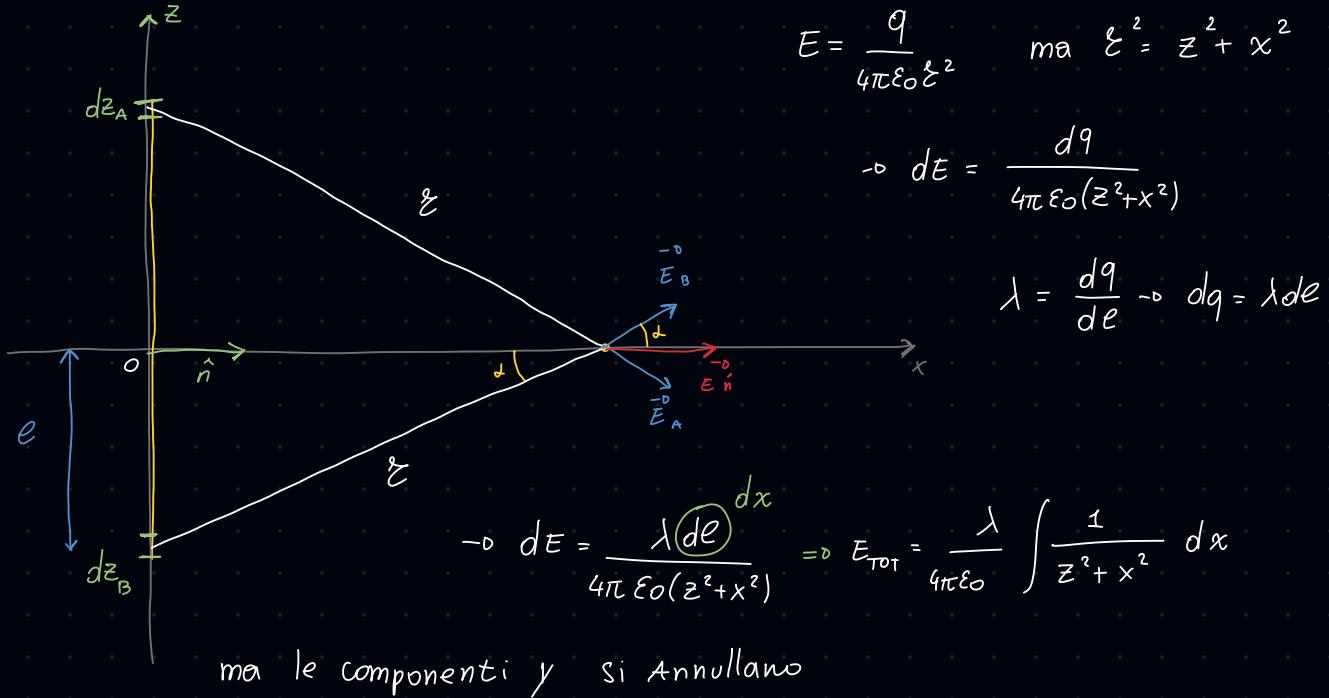
$$\phi_E = \oint_{S_1} + \oint_{S_2} + \oint_{S_2} = \int E \cdot \hat{n} dS_z = \frac{Q}{\epsilon_0}$$

$$\text{ma } \sigma = \frac{Q}{S} \rightarrow Q = \int \sigma dS$$

$$\Rightarrow \int \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int \sigma dS$$

$$\Rightarrow E = \frac{\sigma}{\epsilon_0}$$

Filo carico

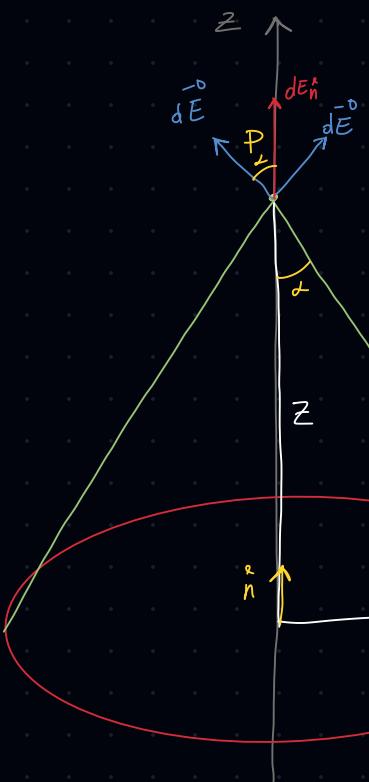


$$\rightarrow \vec{E}_n = E \cdot \cos \vartheta \cdot \hat{n} \quad \text{ma } \epsilon \cos \vartheta = x \rightarrow \cos \vartheta = \frac{x}{\epsilon} = \frac{x}{\sqrt{x^2 + z^2}}$$

$$\rightarrow E_{TOT} = \frac{\lambda}{4\pi\epsilon_0} \int_0^e \frac{x}{(x^2 + z^2)^{\frac{3}{2}}} dx = \frac{2\lambda}{4\pi\epsilon_0} \left[\frac{1}{x^2} \frac{z}{(x^2 + z^2)^{\frac{1}{2}}} \right]_0^e$$

$$= \frac{\lambda}{2\pi\epsilon_0 x^2} \cdot \frac{e}{(x^2 + e^2)^{\frac{1}{2}}} \quad \text{lunghezza finita}$$

$$\text{se } l \rightarrow \infty = 0 \quad l \gg x = 0 \quad E = \frac{\lambda}{2\pi\epsilon_0 x^2} \cdot \frac{\epsilon}{\epsilon} = \frac{\lambda}{2\pi\epsilon_0 x^2}$$



$$dE = \frac{q}{4\pi\epsilon_0 z^2} \vec{E}$$

ma $\lambda = \frac{dq}{de} \rightarrow dq = \lambda de$

$$z^2 = z^2 + R^2 \Rightarrow dE = \frac{\lambda de}{4\pi\epsilon_0 (z^2 + R^2)}$$

le comp x e y si annullano $\rightarrow dE_n = dE \cdot \cos\alpha$

$$\begin{aligned} \rightarrow dE_n &= \frac{\lambda de \cos\alpha}{4\pi\epsilon_0 (z^2 + R^2)} \quad \text{ma } \epsilon \cdot \cos\alpha = z \cdot \cos\alpha = \frac{z}{\sqrt{z^2 + R^2}} \\ &= \frac{\lambda dc \cdot z}{4\pi\epsilon_0 (z^2 + R^2)^{\frac{3}{2}}} \end{aligned}$$

$$\Rightarrow E_n = \frac{\lambda z}{4\pi\epsilon_0 (z^2 + R^2)^{\frac{3}{2}}} \left(\oint dc \right) = \frac{\lambda z R}{2\epsilon_0 (z^2 + R^2)^{\frac{3}{2}}} \cdot n \quad \text{se } z \gg R \quad E_n = \frac{\lambda z R}{2\epsilon_0 z^{\frac{3}{2}}} \cdot n$$

OPPURE $E_n = \frac{\lambda z \cdot 2\pi R}{4\pi\epsilon_0 (z^2 + R^2)^{\frac{3}{2}}}$

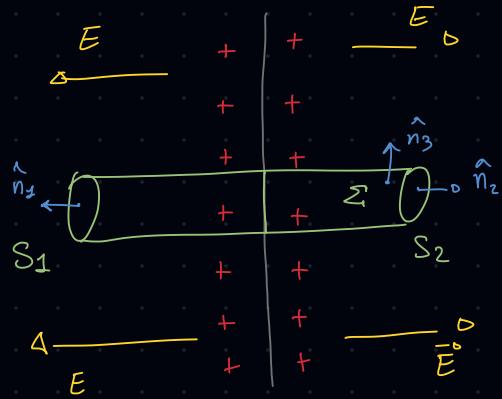
siccome $\lambda = \frac{Q}{l} \rightarrow Q = \lambda \cdot l$
 $\rightarrow \lambda \cdot 2\pi R \cdot Q$

$$\Rightarrow E_n = \frac{Q \cdot \frac{z}{z^{\frac{3}{2}}}}{4\pi\epsilon_0 z^{\frac{3}{2}}} \cdot n$$

\uparrow
 $z \gg R$

Strato Carico

$$\phi_{\text{TOT}} = \oint \vec{E} \cdot \hat{n} dS = \phi_{S_1} + \phi_{S_2} + \phi_{\Sigma} = \frac{Q}{\epsilon_0}$$



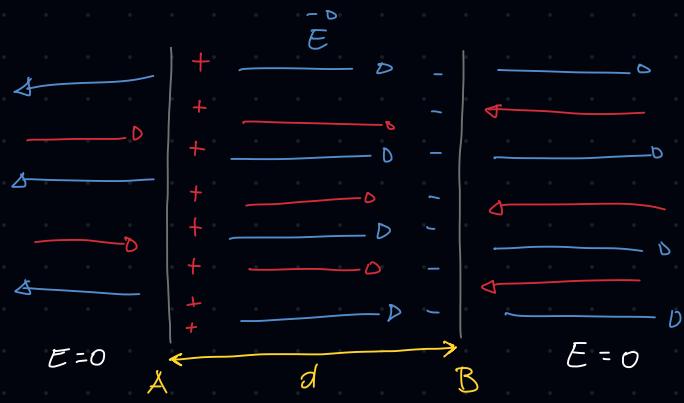
$$\Rightarrow 2\phi_S = \frac{Q}{\epsilon_0} \Rightarrow 2 \int \vec{E} \cdot \hat{n} dS_1 = \frac{Q}{\epsilon_0}$$

$$\text{ma } \sigma = \frac{Q}{S} \Rightarrow Q = \int \sigma dS$$

$$\Rightarrow 2 \int \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int \sigma dS$$

$$\Rightarrow \boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}}$$

Doppio Strato



$$\text{Tr A e B } E = 2 \cdot \frac{\sigma}{2\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$$

$$E \neq 0$$

CAPACITA'

$$\begin{cases} V_A = \kappa_1 \cdot Q \\ V_B = \kappa_2 \cdot Q \end{cases} \Rightarrow V_A + V_B = Q (\kappa_1 + \kappa_2) \Rightarrow \kappa_1 + \kappa_2 = \frac{1}{C} \Rightarrow V_A - V_B = \frac{Q}{C}$$

Cond Piano

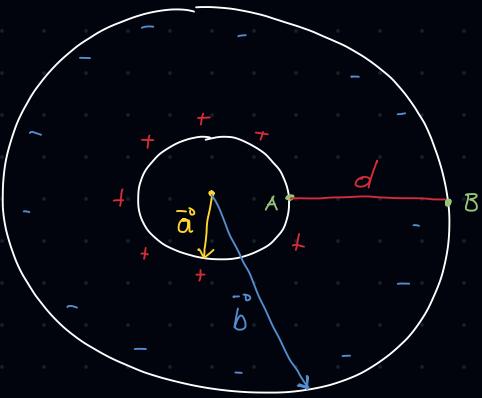
$$C = \frac{Q}{V_A - V_B}$$

$$V_A - V_B = \int \vec{E} \cdot d\vec{e} = \int \frac{\sigma}{\epsilon_0} \cdot d\vec{e} = \frac{\sigma}{\epsilon_0} \cdot d$$

$$\text{ma } \sigma = \frac{Q}{S}$$

$$\Rightarrow V_A - V_B = \frac{Q \cdot d}{S \cdot \epsilon_0}$$

$$\Rightarrow C_{\text{PIANO}} = \frac{Q \cdot S \cdot \epsilon_0}{Q \cdot d} = \boxed{\frac{S \cdot \epsilon_0}{d}}$$



$$C = \frac{Q}{V_A - V_B} \quad \text{ma } V_A - V_B = ? \rightarrow V_A - V_B = \int \vec{E} \cdot d\vec{l}$$

$$\text{ma } E = ?? \rightarrow E = \frac{q}{4\pi\epsilon_0 \epsilon^2} \vec{\epsilon}$$

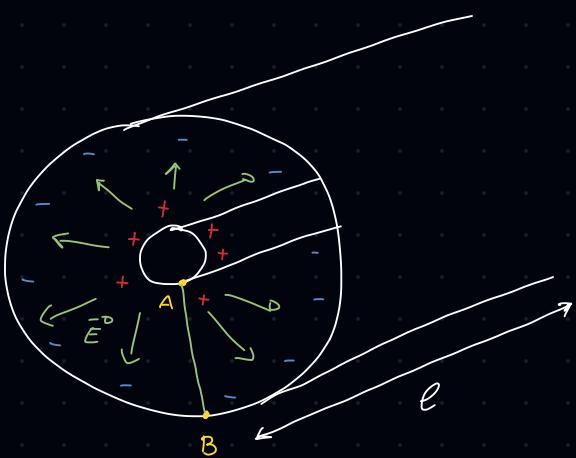
$$\Rightarrow V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\epsilon_A} - \frac{1}{\epsilon_B} \right]$$

$$\epsilon_A = \frac{-d}{|a|}, \epsilon_B = \frac{-d}{|b|} \rightarrow V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$\Rightarrow V_A - V_B = K \left[\frac{b-a}{a \cdot b} \right] \Rightarrow V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[\frac{b-a}{b \cdot a} \right]$$

$$\Rightarrow C = \frac{Q \cdot 4\pi\epsilon_0}{Q} \frac{b \cdot a}{b-a} \Rightarrow C = 4\pi\epsilon_0 \frac{b \cdot a}{b-a}$$

Cilindrico



$$C = \frac{Q}{V_A - V_B} \Rightarrow V_A - V_B = \int \vec{E} \cdot d\vec{l} \Rightarrow E = ??$$

so che $\phi_E = \oint \vec{E} \cdot d\vec{n} ds = \frac{Q}{\epsilon_0}$

$$\Rightarrow \int \vec{E} \cdot \vec{n} ds = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E \cdot (2\pi r l) \frac{Q}{\epsilon_0}$$

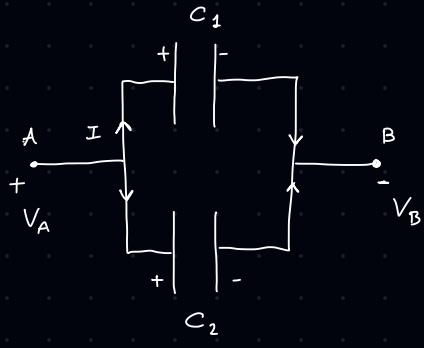
$$E = \frac{Q}{2\pi R l \epsilon_0}$$

$$\Rightarrow V_A - V_B = \int \frac{Q}{2\pi R l \epsilon_0} dl$$

$$\Rightarrow V_A - V_B = \frac{Q}{2\pi R \epsilon_0} \int_A^B \frac{1}{l} dl = \frac{Q}{2\pi R \epsilon_0} \left[\ln(l) \right]_A^B =$$

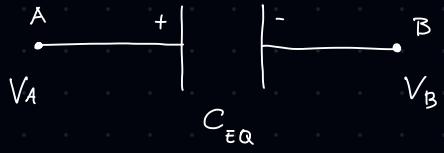
$$V_A - V_B = \frac{Q \ln \left(\frac{A}{B} \right)}{2\pi R \epsilon_0}$$

$$\Rightarrow C = \frac{2\pi R \epsilon_0}{\ln(A/B)}$$



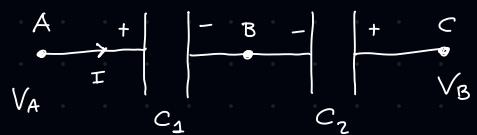
$$C_1 = \frac{Q_1}{V_A - V_B} \quad C_2 = \frac{Q_2}{V_A - V_B}$$

$$= \begin{cases} V_A - V_B = \frac{Q_1}{C_1} \\ V_A - V_B = \frac{Q_2}{C_2} \end{cases} \quad = \begin{cases} Q_1 = \frac{V_A - V_B}{C_1} \\ Q_2 = \frac{V_A - V_B}{C_2} \end{cases}$$



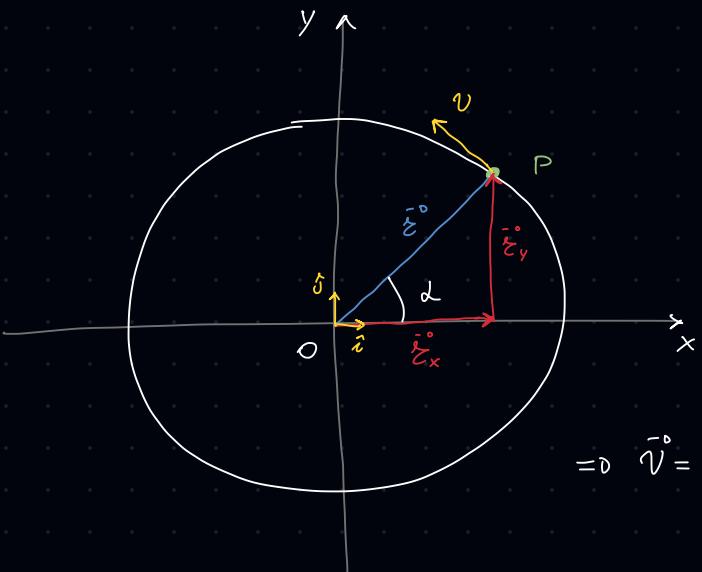
$$C_{EQ} = \frac{Q}{V_A - V_B} \rightarrow Q = (V_A - V_B) \cdot C_{EQ}$$

$$\rightarrow \frac{Q_1 + Q_2}{Q_{TOT}} = (V_A - V_B) (C_1 + C_2) \rightarrow C_{EQ} = \sum_i C_i$$



$$\begin{cases} V_A - V_B = \frac{Q}{C_1} \\ V_B - V_C = \frac{Q}{C_2} \end{cases} \Rightarrow V_A - V_C = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \rightarrow \frac{1}{C_{EQ}} = \sum_i \frac{1}{C_i}$$

Moto Circolare



Definisco

$$\omega = \frac{d\alpha}{dt} \quad \text{da} \quad d\alpha = \omega dt$$

$$\int_{\alpha_0}^{\alpha_f} d\alpha = \int_{t_0}^{t_f} \omega dt$$

$$\Rightarrow \alpha(t) = \omega \cdot t \quad \text{Se } \omega = \text{cost}$$

$$\epsilon = \dot{\epsilon}_x + \dot{\epsilon}_y = i \epsilon \cos \alpha + j \epsilon \sin \alpha$$

$$\Rightarrow v = \frac{d\epsilon}{dt} = \frac{d}{dt} [i \epsilon \cos(\omega t) + j \epsilon \sin(\omega t)]$$

$$\Rightarrow v = -i \epsilon \sin(\omega t) \cdot \omega + j \epsilon \cos(\omega t) \cdot \omega = \omega [j \epsilon \cos(\omega t) - i \epsilon \sin(\omega t)]$$

$$\ddot{\alpha} = -i \epsilon \cos(\omega t) \omega^2 - j \epsilon \sin(\omega t) \omega^2 = \underline{\omega^2 [i \epsilon \cos(\omega t) + j \epsilon \sin(\omega t)]} \quad \overline{\epsilon}$$

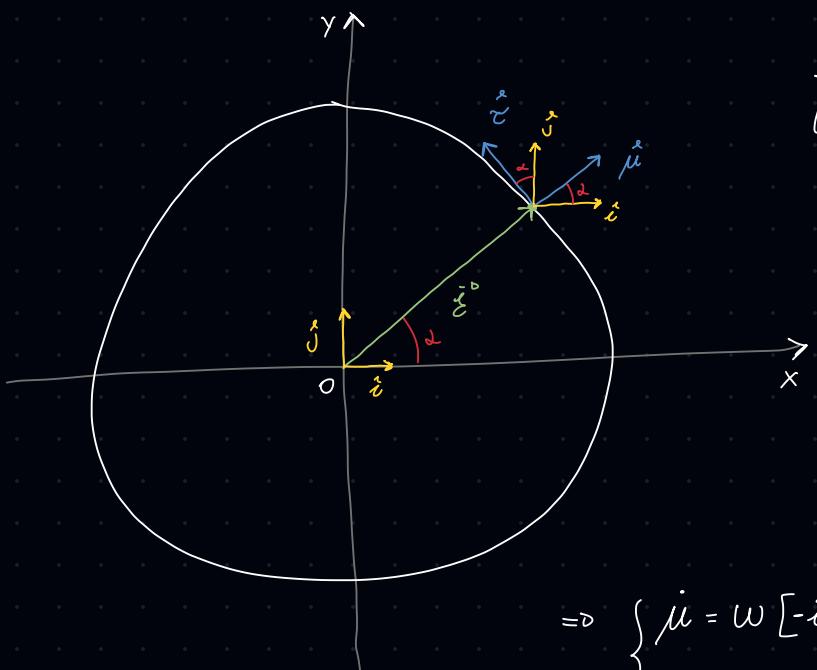
$$\Rightarrow \ddot{\alpha} = -\omega^2 \epsilon \quad \text{Acc Centripeta}$$

Modulo

$$|\vec{v}| = \sqrt{\omega^2 \epsilon^2 \sin^2(\omega t) + \omega^2 \epsilon^2 \cos^2(\omega t)} = \sqrt{\omega^2 \epsilon^2} = \omega \epsilon = \underline{\omega \epsilon} \quad (\star)$$

$$|\vec{a}| = \sqrt{\omega^4 \dots} = \sqrt{\omega^4 \epsilon^2} = \omega^2 \epsilon = \alpha = \epsilon \cdot \frac{\omega^2}{\epsilon^2} = \underline{\frac{\omega^2}{R}} \quad \text{Acc CP}$$

$$\omega = \frac{v}{r}$$



$$\begin{cases} \dot{\mu} = \hat{i} \cos \omega + \hat{j} \sin \omega \\ \dot{\tau} = -\hat{i} \sin \omega + \hat{j} \cos \omega \end{cases}$$

$$\Rightarrow \vec{\varepsilon} = \hat{i} \varepsilon \cos \omega + \hat{j} \varepsilon \sin \omega$$

$$\omega = \omega t$$

$$\begin{cases} \dot{\mu} = -\hat{i} \omega \sin(\omega t) + \hat{j} \omega \cos(\omega t) \\ \dot{\tau} = -\hat{i} \omega \cos(\omega t) - \hat{j} \omega \sin(\omega t) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{\mu} = \omega [-\hat{i} \sin(\omega t) + \hat{j} \cos(\omega t)] = \omega \hat{\tau} \\ \dot{\tau} = -\omega [\hat{i} \cos(\omega t) + \hat{j} \sin(\omega t)] = -\omega \hat{\mu} \end{cases}$$

$$\vec{\varepsilon}^o = \vec{\varepsilon}_x^o + \vec{\varepsilon}_y^o = \hat{i} \varepsilon \cos \omega + \hat{j} \varepsilon \sin \omega = \varepsilon [\hat{i} \cos(\omega t) + \hat{j} \sin(\omega t)] = \hat{\varepsilon} \hat{\mu}$$

$$\Rightarrow \frac{d\vec{\varepsilon}^o}{dt} = \vec{\varepsilon}^o \cdot \frac{d\mu}{dt} = \vec{\varepsilon}^o \cdot (\omega \hat{\tau}) = \vec{\varepsilon} \omega \hat{\tau} \vec{v}$$

Se $\omega = \text{const}$

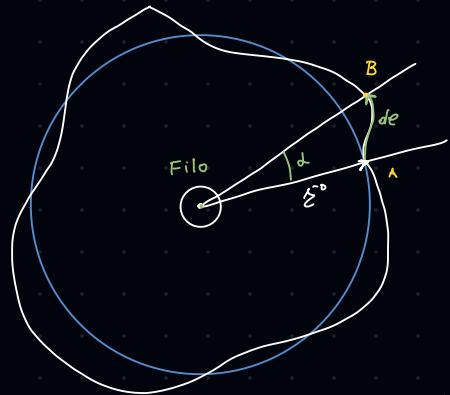
$$\frac{d\vec{\varepsilon}}{dt} = \vec{\varepsilon} \omega \frac{d\hat{\tau}}{dt} = \vec{\varepsilon} \omega (-\omega \hat{\mu}) = -\vec{\varepsilon} \omega^2 \hat{\mu}$$

Se $\omega \neq \text{const}$

$$\frac{d\vec{\varepsilon}}{dt} = \vec{\varepsilon} \omega \hat{\tau} + \vec{\varepsilon} \dot{\hat{\tau}} = -\vec{\varepsilon} \omega^2 \hat{\mu} + \vec{\varepsilon} \omega \hat{\tau}$$

Legge di Ampere

Da Biot - Savart



$$dB = K \cdot \frac{I}{r} t \cdot de$$

$$\int B \cdot de = \mu_0 I$$

$$\nabla \times B = \mu_0 J$$

$$C = \oint \vec{B} \cdot d\vec{e} = \frac{\mu_0}{2\pi} \oint \frac{1}{r} t \cdot d\vec{e}$$

ma $t \cdot d\vec{e} = \text{Arco di circonferenza}$; Siccome $1 \text{ Rad} = \frac{\ell}{R} \Rightarrow \ell = \text{Rad} \cdot R$

$$\Rightarrow t \cdot d\vec{e} = \frac{\ell}{R} d\ell \Rightarrow C = \frac{\mu_0 I}{2\pi} \oint \frac{1}{R} \ell \cdot d\ell \Rightarrow \mu_0 I \Rightarrow \oint \vec{B} \cdot d\vec{e} = \mu_0 I$$

$$\int (\nabla \times B) \cdot dS = \mu_0 \int J \cdot n \cdot dS \Rightarrow \nabla \times B = \mu_0 J$$

Joule

$$P_{\text{pow}} = \frac{dL}{dt} \quad E = \frac{F}{q} \Rightarrow F = q \cdot E \Rightarrow L = q \cdot \int E \cdot de = q \cdot V$$

$$\text{ma } V = R \cdot I \quad dL = dq \cdot R \cdot I \quad \Rightarrow P_{\text{pow}} = \frac{dL}{dt} = \frac{dq}{dt} R \cdot I = R \cdot I^2$$

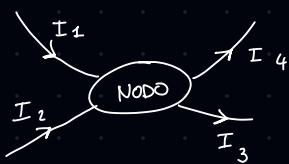
POTENZA
Dissipata

OPPURE

$$L = dq \cdot R \cdot I \quad \text{ma} \quad I = \frac{dq}{dt} \Rightarrow dq = I dt \Rightarrow dL = I^2 R dt \Rightarrow P_{\text{pow}} = I^2 R$$

Leggi Kirchhoff

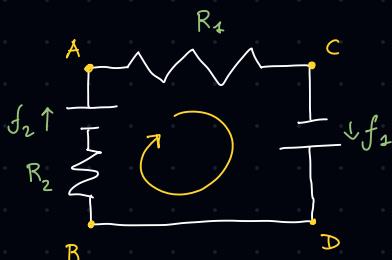
Nodi: LKC



Uscenti: Positive
Entranti: Negative

$$\Rightarrow I_4 + I_3 - I_1 - I_2 = 0$$

MAGLIE: LKT



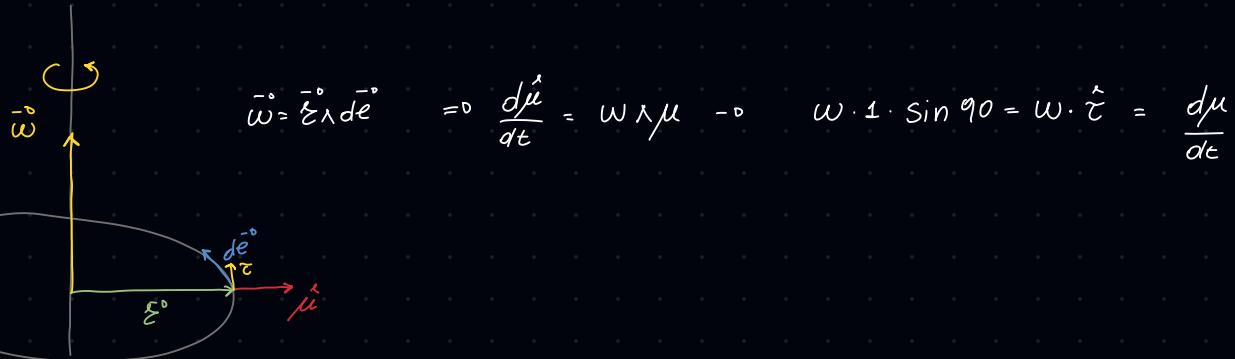
Senso ORARIO

Siccome è un loop $V_A - V_B = 0$ siccome $A \equiv B$

\Rightarrow Da Ohm $(V_A - V_B) + \sum_i f_i = \sum_i I R_i$

\Rightarrow Kirchhoff $\sum_i f_i = \sum_i I R_i$

$$\frac{d\hat{\omega}}{dt} = \omega \times \dot{\omega} \quad -\circ$$

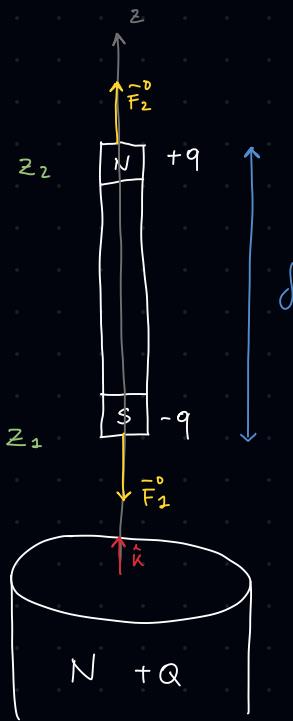


$$\left\{ \begin{array}{l} F = G \cdot \frac{m \cdot M}{r^2} \quad -\circ \quad G \cdot \frac{m \cdot M}{r^2} = m \cdot \ddot{a} \\ F = m \cdot \ddot{a} \end{array} \right.$$

Inertiale gravitazionale

Martello Piuma	$\left\{ \begin{array}{l} G \cdot \frac{m_m \cdot M_L}{r^2} = m_m \cdot \ddot{a}_m \\ G \cdot \frac{m_p \cdot M_L}{r^2} = m_p \cdot \ddot{a}_p \end{array} \right.$	$\Rightarrow \frac{m_m}{m_p} = \frac{m_m}{m_p}$ Inertiale gravitazionale
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Ampere : Force



$$F = \frac{\mu_0}{4\pi} \frac{q_m \cdot Q_m}{z^2} \quad \Rightarrow \quad F_{TOT} = F_2 + F_1$$

$$\begin{aligned} \Rightarrow F_{TOT} &= \frac{\mu_0}{4\pi} \left[\frac{q_m Q_m}{z_2^2} - \frac{q_m Q_m}{z_1^2} \right] \\ &= \frac{Q_m q_m \mu_0}{4\pi} \left[\frac{1}{(z_1 + \delta)^2} - \frac{1}{z_1^2} \right] = K \left[\frac{-z_1^2 + z_1^2 + \delta^2 - 2z_1 \delta}{(z_1 + \delta)^2 z_1^2} \right] \\ &= -K \left[\frac{Q_m q_m \mu_0 2 \delta}{4\pi z_1^2} \right] \end{aligned}$$

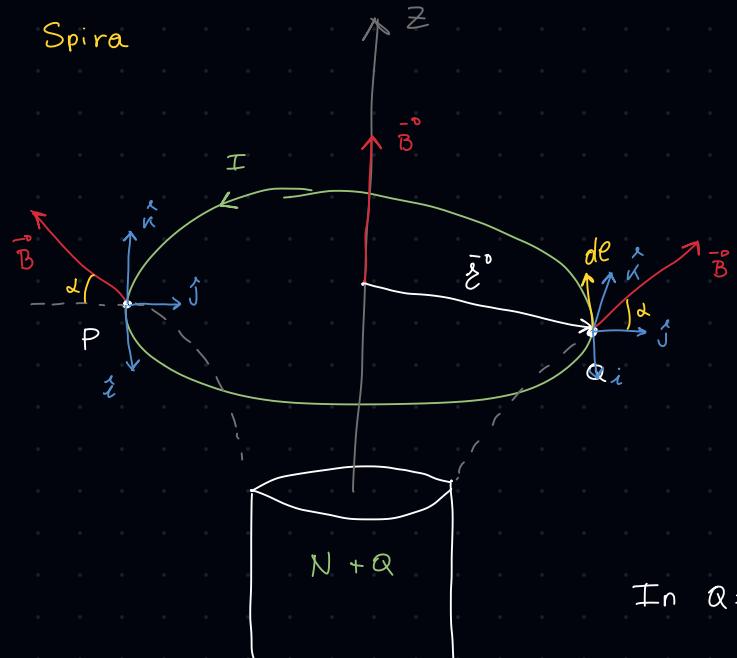
$\delta \gg z$

ma $B = \frac{F}{q} = \frac{\mu_0 Q_m}{4\pi z^2} \xrightarrow{B}$ $\Rightarrow F = -K B \cdot z \delta$

$$\vec{m} = K q_m \int$$

$$\vec{m} = K \cdot 2 \int \Rightarrow \vec{F} = -\frac{\vec{m} B}{z}$$

Spira



$$In P: d\vec{e} = \hat{i} d\theta$$

$$da \text{ Laplace } dF = I d\ell \wedge \vec{B}$$

$$\Rightarrow dF_P = I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d\theta & 0 & 0 \\ 0 & -B \cos \alpha & B \sin \alpha \end{vmatrix}$$

$$\begin{aligned} &= I \left[\hat{i}(0) - \hat{j}(d\theta B \sin \alpha) + \hat{k}(d\theta (-B \cos \alpha)) \right] \\ &= -\hat{j} I d\theta B \sin \alpha - \hat{k} I d\theta B \cos \alpha \end{aligned}$$

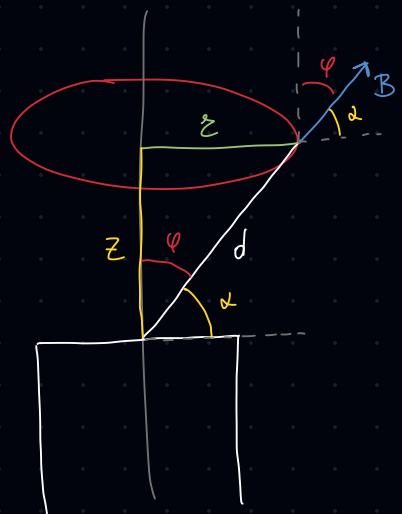
$$In Q: d\vec{e} = -\hat{i} d\theta$$

$$\Rightarrow dF_Q = I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -d\theta & 0 & 0 \\ 0 & B \cos \alpha & B \sin \alpha \end{vmatrix}$$

$$\Rightarrow dF_Q = I \left[\hat{i}(0) - \hat{j}(-d\theta B \sin \alpha) + \hat{k}(-d\theta B \cos \alpha) \right] = \hat{j} I d\theta B \sin \alpha - \hat{k} I d\theta B \cos \alpha$$

$$\Rightarrow dF_{TOT} = -2K I d\theta B \cos \alpha$$

$$\Rightarrow F_{TOT} = -2K I B \cos \alpha \int_0^\pi d\theta \Rightarrow F_{TOT} = -2\pi R I B \cos \alpha$$



$$F_{TOT} = -2\pi R I B \cos \alpha \quad \text{ma } \alpha = 90^\circ - \varphi \Rightarrow \cos(90^\circ - \varphi) = \sin(\varphi)$$

$$\Rightarrow F_{TOT} = -2\pi R I B \sin \varphi$$

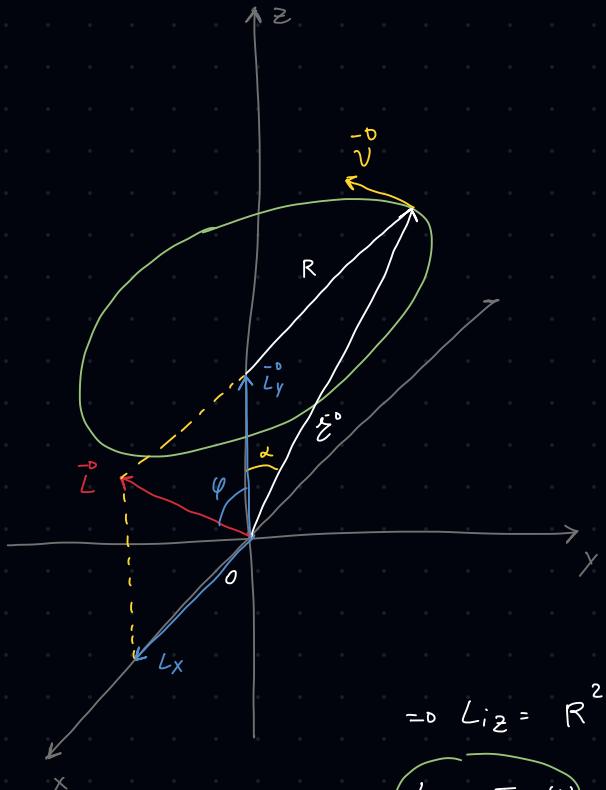
Se $z \gg r \rightarrow \sin \varphi \approx \tan \varphi$

$$\begin{cases} d \sin \varphi = z \\ d \cos \varphi = r \end{cases} \rightarrow \tan \varphi = \frac{z}{r}$$

$$\Rightarrow \overset{-\circ}{F_{TOT}} = -2\pi z I B \cdot \frac{z}{r} = -\frac{2\pi z^2 I B}{r} K$$

$$\overset{-\circ}{m} = I \cdot S \cdot K \Rightarrow$$

$$\overset{-\circ}{F_{TOT}} = -\frac{2 m \overset{-\circ}{B}}{z}$$



$$\bar{L} = \bar{\varepsilon} \wedge m \cdot \bar{v}$$

$$\bar{L}_i = \bar{\varepsilon}_i \wedge m_i \cdot \bar{v}_i \Rightarrow |L_i| = \varepsilon_i \cdot m_i \cdot v_i \cdot \sin(90^\circ)$$

$$|L_i| = \bar{\varepsilon}_i \cdot m_i \cdot v_i$$

$$\text{ma } v_i = \omega_i R \Rightarrow |L_i| = \varepsilon_i \cdot m_i \cdot \omega_i \cdot R$$

Siccome L_y e L_x si annullano

$$L_z = L_i \cdot \cos(\varphi) \quad \text{ma } \varphi = 90^\circ - \alpha$$

$$\Rightarrow \cos(90^\circ - \alpha) = \sin(\alpha)$$

$$\Rightarrow L_{i_z} = \varepsilon_i \cdot m_i \cdot \omega_i \cdot R \cdot \sin(\alpha) \quad \text{ma } \varepsilon_i \sin \alpha = R$$

$$\Rightarrow L_{i_z} = R^2 \cdot m_i \cdot \omega_i \quad \text{chiama} \quad \sum_i m_i R^2 = I_{\text{tot}}$$

$$\Rightarrow \boxed{L_{\text{tot}} = I \cdot \omega}$$

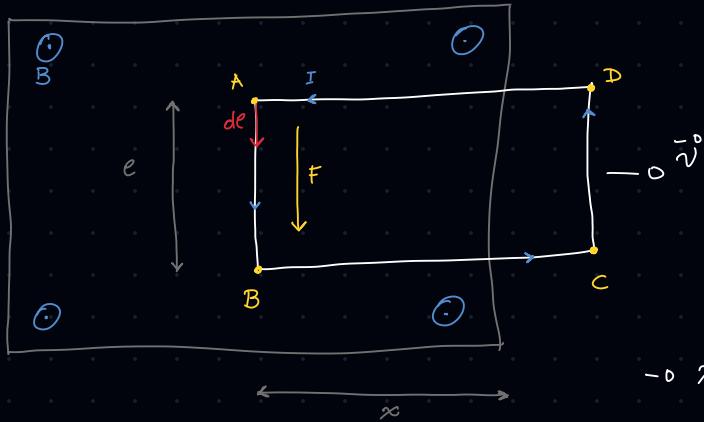
Corpo rigido

$$\Rightarrow \begin{cases} \bar{M} = \bar{\varepsilon} \wedge \bar{F} \\ \bar{L} = \bar{\varepsilon} \wedge m \bar{v} \end{cases} \Rightarrow \frac{d\bar{L}}{dt} = \frac{d\bar{\varepsilon}}{dt} \wedge m \bar{v} + \bar{\varepsilon} \wedge m \frac{d\bar{v}}{dt} \Rightarrow \frac{d\bar{L}}{dt} = \bar{M}$$

$$\Rightarrow \bar{M} = I \cdot \frac{d\omega}{dt} \Rightarrow \bar{M} = I \alpha$$

$$f_{em} = - \frac{d\phi_B}{dt}$$

Tagliato



$$\vec{F} = q \vec{E} + q (\vec{v} \wedge \vec{B})$$

$$f_{em} = \frac{L}{q} = \oint \vec{v} \wedge \vec{B} \cdot d\vec{e} = \int v \lambda_B l_1 = \underline{v B e}$$

(1)

 inoltre ϕ_B è funzione di t

$$\phi_B = \int \vec{B} \cdot \vec{n} ds = B_\perp \cdot S \quad \text{ma } S = l \cdot x$$

$\rightarrow x$ dipende da $t \rightarrow \bar{x} = t + \Delta t \rightarrow \bar{x} = x - \Delta x$
con $\Delta x = v \cdot \Delta t$

$$\rightarrow \phi_B = \int \vec{B} \cdot \vec{n} ds = B \cdot l \cdot x$$

$$\rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\phi_B(t + \Delta t) - \phi_B(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\cancel{B} \cancel{e} x - \cancel{B} \cancel{e} (\bar{x})}{\Delta t} = \cancel{B} \cancel{e} \frac{\cancel{x}}{\Delta t}$$

$$\frac{d\phi_B}{dt} = - B e v$$

QED

 Contenuto $S = \cos t !$

$$B(t, \xi)$$

$$\rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\phi_B(t + \Delta t, \xi) - \phi_B(t, \xi)}{\Delta t}$$

$$\frac{\int B(t, \xi) d\vec{s} - \int \frac{\partial B}{\partial t}(t + \Delta t, \xi) \Delta t d\vec{s} - \int B(t, \xi) d\vec{s}}{\Delta t}$$

Approx :

$$B \approx B(t, \xi) + \frac{\partial B}{\partial t}(t + \Delta t) \cdot \Delta t = \lim_{\Delta t \rightarrow 0}$$

$$\rightarrow \frac{d\phi_B}{dt} = - \oint \frac{\partial B}{\partial t}(\bar{t}, \xi) d\vec{s} \rightarrow - \frac{d\phi_B}{dt} = + \oint \frac{\partial B}{\partial t} \vec{n} d\vec{s} \quad (2)$$

$$\text{Tutto insieme} \quad - \frac{d\phi_B}{dt} = - \oint \frac{\partial B}{\partial t} d\vec{s} + \underbrace{\int (\vec{v} \wedge \vec{B}) de}_{\text{nel flusso concatenato } \vec{v} = \emptyset} \quad \text{CASO GENERALE}$$

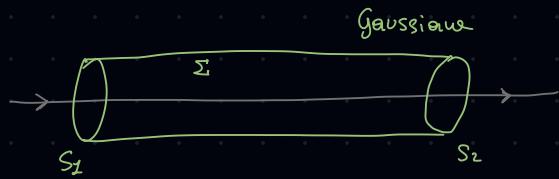
$$\rightarrow \text{ma } f_{em} = \frac{L}{q} \rightarrow \frac{- \oint \phi_E de + \int (\vec{v} \wedge \vec{B}) de}{q}$$

$$\rightarrow \oint \vec{E} d\vec{e} + \oint (\vec{v} \wedge \vec{B}) d\vec{e} = - \oint \frac{\partial B}{\partial t} \vec{n} dS + \oint (\vec{v} \wedge \vec{B}) d\vec{e}$$

$$\rightarrow \oint \vec{E}^o d\vec{s} = - \oint \frac{\partial \vec{B}^o}{\partial t} \cdot \hat{n} dS^o \quad \rightarrow \quad \vec{\nabla} \wedge \vec{E}^o = - \frac{\partial \vec{B}^o}{\partial t}$$

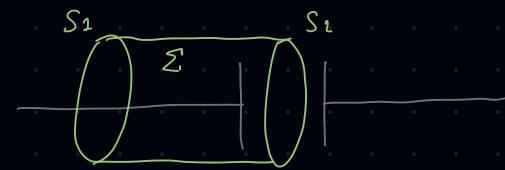
Maxwell
E.S.

Corrente di spostamento



$$I = \oint J \cdot \hat{n} dS = 0 \quad \Rightarrow \quad \nabla \cdot J = 0$$

Corrente Stazionaria



$$\oint J \cdot \hat{n} dS \neq 0 \quad \text{ma come ??}$$

$$\rightarrow \text{Legge conservazione: } -dq = \int J \cdot \hat{n} ds \cdot dt \rightarrow -\frac{dq}{dt} = \int \bar{J} \cdot \hat{n} ds \quad \oint = \frac{dq}{dt}$$

$$\Rightarrow -\frac{d}{dt} \int_{V'} J dV = \int \bar{J} \cdot \hat{n} ds \quad \rightarrow \quad -\int \frac{\partial J}{\partial t} dV = \int J \cdot \hat{n} ds$$

$$\Rightarrow \vec{\nabla} \cdot \vec{J} = -\frac{\partial J}{\partial t}$$

Non corrente
Stazionario

$$\rightarrow \text{Legge di Ampere: } \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \quad \rightarrow \text{Diverg} \rightarrow \vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{B}) = \mu_0 \vec{\nabla} \cdot \vec{J} \quad \rightarrow \quad \vec{\nabla} \cdot \vec{J} = 0$$

$$\text{Termine Maxwell} \quad \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow A.M: \vec{\nabla} \wedge \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Valido solo per
corrente staz.

$$\rightarrow \text{Div} \rightarrow \nabla \cdot (\nabla \wedge \vec{B}) = \mu_0 \left(\nabla \cdot \vec{J} + \epsilon_0 \frac{\partial (\nabla \cdot \vec{E})}{\partial t} \right)$$

$$\rightarrow \vec{\nabla} \cdot \vec{J} + \epsilon_0 \frac{1}{\mu_0} \frac{\partial J}{\partial t} \quad \rightarrow \quad \nabla \cdot \vec{J} = -\frac{\partial J}{\partial t}$$

$$\text{Ampere-Maxwell: } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\text{Faraday: } \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \Leftrightarrow \quad f_{\text{em}} = - \frac{d\phi_B}{dt}$$

CAMPO E

$$\nabla \times (\nabla \times \vec{E}) = - \frac{\partial (\nabla \times \vec{B})}{\partial t} \quad \Rightarrow \quad \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = - \mu_0 \frac{\partial \vec{J}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$- \circ \quad \frac{\nabla \cdot \vec{E}}{\epsilon_0} - \nabla^2 \vec{E} = - \mu_0 \frac{\partial \vec{J}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad - \circ \quad \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = - \mu_0 \frac{\partial \vec{J}}{\partial t} - \frac{\nabla \cdot \vec{E}}{\epsilon_0} \quad (1)$$

CAMPO B

$$\nabla \times (\nabla \times \vec{B}) = \mu_0 \nabla \cdot \vec{J} + \mu_0 \epsilon_0 \frac{\partial (\nabla \cdot \vec{E})}{\partial t} \quad - \circ \quad \cancel{\nabla (\nabla \cdot \vec{B})} - \nabla^2 \vec{B} = \mu_0 \nabla \cdot \vec{J} - \frac{\partial^2 \vec{B}}{\partial t^2} \mu_0 \epsilon_0$$

$$- \circ \quad \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} - \nabla^2 \vec{B} = \mu_0 \nabla \cdot \vec{J} \quad (2)$$

Solo x

$$(1) \cdot \int = J = 0 \quad - \circ \quad \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = 0 \quad - \circ \quad \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} - \frac{\partial^2 E}{\partial x^2} = 0$$

$$\text{Eq Diff Sol: } E(t) = A \sin(\kappa x - \omega t)$$

Trovo "ω"

$$\begin{aligned} E_1 &= A \sin(\kappa x_1 - \omega t_1) \\ E_2 &= A \sin(\kappa x_2 - \omega t_2) \end{aligned} \quad \left. \begin{array}{l} E_1 = E_2 \Leftrightarrow \\ \kappa x_1 - \omega t_1 = \kappa x_2 - \omega t_2 \end{array} \right\} \quad \underbrace{\kappa(x_2 - x_1)}_{V} = \omega(t_2 - t_1)$$

$$- \circ \quad x_2 - x_1 = \frac{\omega}{\kappa} (t_2 - t_1) = 0 \quad V = \frac{\omega}{\kappa} \quad - \circ \quad \omega = V \cdot \kappa$$

$$- \circ \quad E(t) = A \sin(\kappa x - V \kappa t)$$

$$- \circ \quad \frac{\partial E}{\partial t} = - V \kappa A \cos(\kappa x - V \kappa t) \quad - \circ \quad \frac{\partial^2 E}{\partial t^2} = - V^2 \kappa^2 A \sin(\kappa x - V \kappa t)$$

$$\frac{\partial E}{\partial x} = + \kappa A \cos(\kappa x - V \kappa t) \quad - \circ \quad \frac{\partial^2 E}{\partial x^2} = - \kappa^2 A \sin(\kappa x - V \kappa t)$$

$$- \circ \quad - \mu_0 \epsilon_0 V^2 \kappa^2 A \sin(\kappa x - V \kappa t) + \kappa^2 A \sin(\kappa x - V \kappa t) = 0$$

$$- \circ \quad \kappa^2 A \sin(\kappa x - V \kappa t) \left[- \mu_0 \epsilon_0 V^2 + 1 \right] = 0 \quad - \circ \quad V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$E \quad \left\{ \begin{array}{l} \nabla E = \frac{\rho}{\epsilon_0} \Rightarrow \int E \cdot d\vec{e} = \frac{Q}{\epsilon_0} \quad (2) \\ \nabla \wedge E = 0 \Rightarrow \int \vec{E} \cdot \hat{n} ds = 0 \Rightarrow \phi_E = 0 \end{array} \right. \quad (2.A)$$

$$\hookrightarrow \vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t} \Rightarrow \int \nabla \wedge E \cdot ds = - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} ds \Rightarrow \int \vec{E} \cdot d\vec{e} = - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} ds$$

$$\Rightarrow \int E \cdot d\vec{e} + \int (\vec{v} \wedge \vec{B}) \cdot d\vec{e} = - \int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} ds + \oint (\vec{v} \wedge \vec{B}) \cdot d\vec{e} \quad (2.B)$$

$$B \quad \left\{ \begin{array}{l} \nabla B = 0 \Rightarrow \int B \cdot \hat{n} ds = 0 \Rightarrow \phi_B = 0 \\ \nabla \wedge B = \mu_0 J \end{array} \right. \quad (3)$$

(4.A)

$$\hookrightarrow \int \nabla \wedge B \cdot \hat{n} ds = \mu_0 \int J \cdot \hat{n} ds \Rightarrow \int B \cdot d\vec{e} = \mu_0 I \quad \text{Ampère}$$

$$\hookrightarrow \vec{\nabla} \wedge \vec{B} = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\int \nabla \wedge B \cdot \hat{n} ds = \mu_0 \int J \cdot \hat{n} ds + \mu_0 \epsilon_0 \int \frac{\partial E}{\partial t} \cdot \hat{n} ds$$

$$\Rightarrow \oint E \cdot d\vec{e} - \frac{1}{c^2} \oint (\vec{v} \wedge \vec{E}) \cdot d\vec{e} = \mu_0 \underbrace{\int J \cdot \hat{n} ds}_{I} + \mu_0 \epsilon_0 \left[\int \frac{\partial E}{\partial t} \cdot \hat{n} ds - \oint (\vec{v} \wedge \vec{E}) \cdot d\vec{e} \right]$$

$$\text{Lorenz} \quad F = q \cdot E + q (\vec{v} \wedge \vec{B})$$

- 1) Prop. alla carica
- 2) legata al Sin dell angolo tra ve B
- 3) $F \perp B$ e $F \perp v$

$$\Rightarrow F = q \cdot v \cdot B \cdot \sin \alpha \Rightarrow F = q (\vec{v} \wedge \vec{B})$$

Prodotto scalare

$$\vec{A} = i x + j y + k z \quad \vec{B} = i' x' + j' y' + k' z' \Rightarrow \vec{A} \cdot \vec{B} = A \cdot B \cos \alpha$$

$$\begin{aligned} \vec{i} x \cdot \vec{i} x' + \vec{i} x \cdot \vec{j} y' + \vec{i} x \cdot \vec{k} z' + \\ \vec{j} y \cdot \vec{i} x' + \vec{j} y \cdot \vec{j} y' + \vec{j} y \cdot \vec{k} z' + \\ \vec{k} z \cdot \vec{i} x' + \vec{k} z \cdot \vec{j} y' + \vec{k} z \cdot \vec{k} z' = x x' + y y' + z z' \end{aligned}$$

$$\cos(90^\circ) = 0 \Rightarrow \text{versori perp} = 0$$

Prodotto vettoriale $\sin(\theta) = 0 \Rightarrow \text{versori paralleli} = 0$

$$\vec{A} \cdot \vec{A}' = \det \begin{vmatrix} i & j & k \\ x & y & z \\ x' & y' & z' \end{vmatrix} =$$

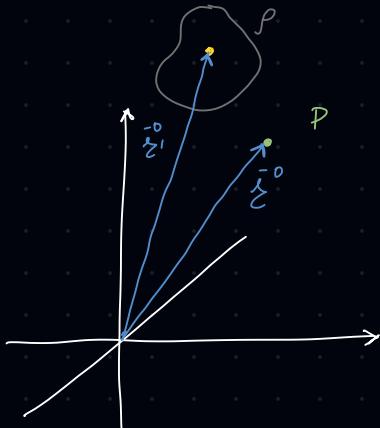
Potenziale Vettore

$$\left\{ \begin{array}{l} \nabla E = \frac{\rho}{\epsilon_0} \\ \nabla \cdot E = 0 \end{array} \right. \quad d\vec{e} = E d\vec{e} = -dV \quad \rightarrow \vec{E} d\vec{e} = -dV \quad \text{ma} \quad dV = \hat{i} \frac{\partial V}{\partial x} dx + \hat{j} \frac{\partial V}{\partial y} dy + \hat{k} \frac{\partial V}{\partial z} dz$$

$$\left\{ \begin{array}{l} d\vec{e} = \hat{i} dx + \hat{j} dy + \hat{k} dz \\ \vec{\nabla} V = \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \end{array} \right. \Rightarrow -dV = d\vec{e} \cdot \vec{\nabla} V = 0 \quad \vec{E} d\vec{e} = -d\vec{e} \cdot \vec{\nabla} V \Rightarrow \vec{\epsilon} = -\nabla V$$

$$\rightarrow \left\{ \begin{array}{l} \nabla(-\nabla V) = \frac{\rho}{\epsilon_0} \\ \nabla \cdot (\nabla V) = 0 \end{array} \right. \quad \text{Eq di Poisson}$$

$$\text{Soluzione} \quad V = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|} dV$$



POT Vettore \vec{A} / $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\rightarrow \text{MAXWELL:} \quad \left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \end{array} \right. \quad \rightarrow \quad \left\{ \begin{array}{l} \nabla \cdot (\nabla \times \vec{A}) = 0 \\ \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J} \end{array} \right. \quad \checkmark$$

$$\rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A} = \mu_0 \vec{J}$$

$$\text{SE} \quad \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) = 0 \quad \rightarrow \quad \vec{\nabla}^2 \vec{A} = \mu_0 \vec{J} \quad \text{Eq di "Poisson"}$$

$$\rightarrow \vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{J(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|} dV = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{\xi}') dS}{|\vec{\xi} - \vec{\xi}'|} d\vec{e} \quad \text{ma} \quad d\vec{V} = d\vec{e} \cdot d\vec{S} \quad \text{e} \quad \vec{J} \cdot d\vec{S} = I$$

$$\rightarrow \vec{A} = \frac{\mu_0}{4\pi} I \oint \frac{1}{|\vec{\xi} - \vec{\xi}'|} d\vec{e}$$

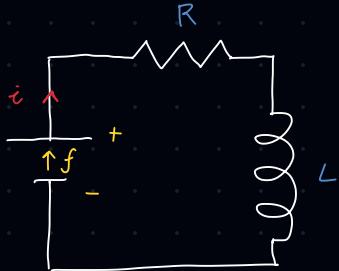
Autoinduzione

Da Biot-Savart $B = \mu_0 \frac{I}{R}$ \rightarrow Laplace $\rightarrow B = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{e} \wedge \vec{\epsilon}}{r^3} dS$

$\phi_B = \int_S \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{e} \wedge \vec{\epsilon}}{r^3} dS$ Tutto costante Tranne I

$\rightarrow \phi_B = L \cdot I$ ma $f_{em} = -\frac{d\phi}{dt} \rightarrow f_{ind} = -L \cdot \frac{dI}{dt}$

Circuito $R L$



$$f_{em} - f_{ind} = R \cdot i \quad \rightarrow \quad f_{em} - L \frac{dI}{dt} = R \cdot i$$

$$\rightarrow f_{em} = R \cdot I + L \frac{dI}{dt} \quad \text{Eq Diff}$$

1) Sol: $I(t) = Ae^{\lambda t} + D$

$$\rightarrow I'(t) = A\lambda e^{\lambda t} \quad \rightarrow \quad f_{em} = RAe^{\lambda t} + RD + LA\lambda e^{\lambda t}$$

$$f_{em} = \text{Cost} \quad \rightarrow \quad f_{em} = RD \quad \rightarrow \quad D = \frac{f}{R} \quad (1)$$

$$RAe^{\lambda t} + LA\lambda e^{\lambda t} = 0 \quad \rightarrow \quad R + L\lambda = 0 \quad \rightarrow \quad \lambda = -\frac{R}{L} \quad (2)$$

$$\rightarrow \text{Sol generale } I(t) = Ae^{-\frac{R}{L}t} + \frac{f}{R}$$

$$\text{pongo } I(0) = 0 \quad \rightarrow \quad I(0) = A + \frac{f}{R} = 0 \quad \rightarrow \quad A = -\frac{f}{R} \quad (3)$$

$$\rightarrow \text{Sol Part: } I(t) = -\frac{f}{R} e^{-\frac{R}{L}t} + \frac{f}{R} \quad \rightarrow \quad \frac{f}{R} \left(1 - e^{-\frac{R}{L}t}\right) \quad \text{per } t \rightarrow \infty$$

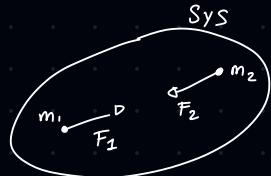
$$\Rightarrow I(t) \rightarrow \frac{f}{R}$$

Conservazione Momento Angolare

$$\left\{ \begin{array}{l} \vec{F} = m \cdot \vec{\alpha} \\ \vec{M} = \frac{d\vec{L}}{dt} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \sum_i \vec{F}_i = \sum_i m_i \vec{\alpha}_i \\ \sum_i \vec{M}_i = \sum_i \frac{d\vec{L}_i}{dt} \end{array} \right. \quad \text{ma} \quad \vec{\alpha} = \frac{d\vec{v}}{dt} \Rightarrow \sum_i \vec{F}_i = \sum_i m_i \frac{d\vec{v}}{dt}$$

$$\sum_i \vec{F}_{int} + \sum_k \vec{F}_{ext} = \frac{\vec{P}_{TOT}}{dt} \quad (2)$$

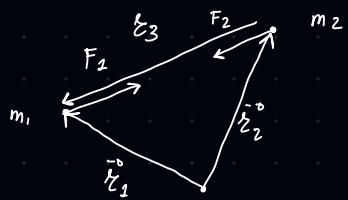
Forze interne



$$\text{III Principio: } \vec{F}_1 = -\vec{F}_2$$

$$\Rightarrow \sum_i \vec{F}_{int} = \emptyset \quad \Rightarrow \sum_i \vec{F}_{ext} = \frac{\vec{P}_{TOT}}{dt}$$

Momenti interni



$$\vec{\epsilon}_2 + \vec{\epsilon}_3 = \vec{\epsilon}_1$$

$$L = \epsilon \wedge m \cdot v$$

$$\Rightarrow M_{TOT} = \vec{\epsilon}_1 \wedge F_1 + \vec{\epsilon}_2 \wedge F_2 \quad \text{ma} \quad F_1 = -F_2$$

$$\Rightarrow M_{TOT} = \vec{\epsilon}_1 \wedge (-F_2) + \vec{\epsilon}_2 \wedge F_2 \Rightarrow M_{TOT} = (\vec{\epsilon}_1 - \vec{\epsilon}_2) \wedge \vec{F}_2 = \vec{\epsilon}_3 \wedge F_2$$

$$\Rightarrow \sum_i M_{ext} = \sum_i \frac{dL_i}{dt}$$

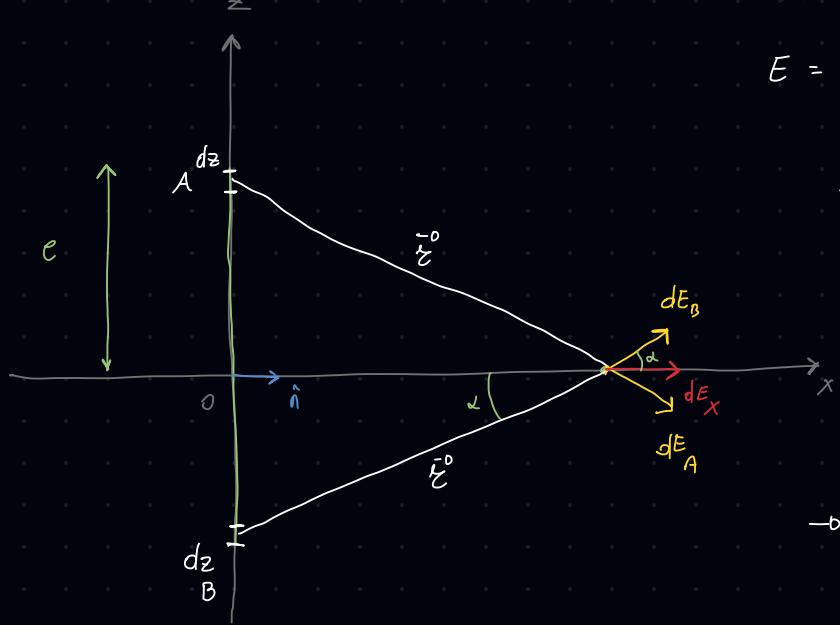
Eq Aggiornata

$$\left\{ \begin{array}{l} \sum_i \vec{F}_{ext} = \frac{d\vec{P}_{TOT}}{dt} \\ \sum_i M_{ext} = \frac{dL_{TOT}}{dt} \end{array} \right. \quad \text{Se sys e' solo TOT} \Rightarrow \sum_i \vec{F}_{ext} = \sum_i M_{ext} = 0$$

$$\left\{ \begin{array}{l} \frac{d\vec{P}_{TOT}}{dt} = 0 \\ \frac{d\vec{L}_{TOT}}{dt} = 0 \end{array} \right.$$

CAMPIONE E IN P

$$E = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{z^2} \quad \text{ma } \epsilon^2 = z^2 + x^2$$



$$\rightarrow dE_p = \frac{dq}{4\pi\epsilon_0} \cdot \frac{1}{(x^2+z^2)}$$

ma dE_y si annulla

$$\rightarrow dE_{TOT} = dE_x = dE \cos \alpha$$

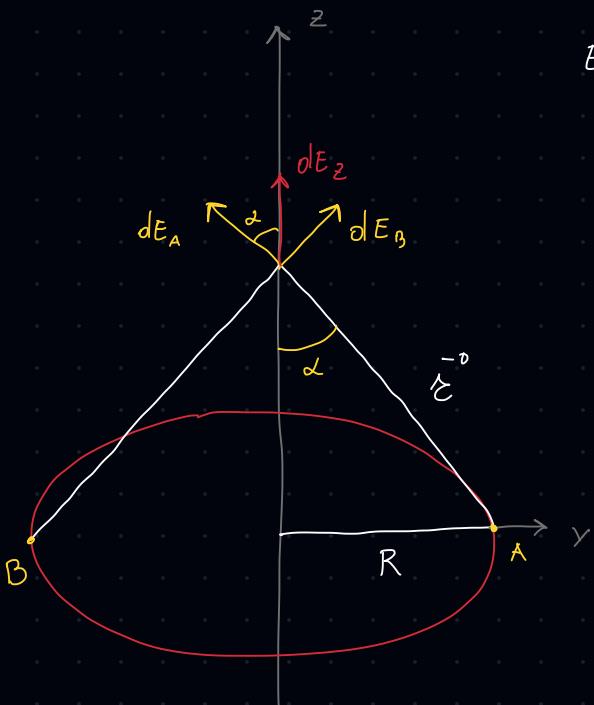
$$\rightarrow dE_p = \frac{dq}{4\pi\epsilon_0} \cdot \frac{\cos \alpha}{(x^2+z^2)}$$

$$\text{ma } \epsilon \cdot \cos \alpha = x \rightarrow \cos \alpha = \frac{x}{\epsilon} = \frac{x}{\sqrt{x^2+z^2}}$$

$$\rightarrow dE_p = \frac{dq}{4\pi\epsilon_0} \cdot \frac{x}{(x^2+z^2)^{\frac{3}{2}}} \quad \text{ma } \lambda = \frac{dq}{dz} \rightarrow dq = \lambda dz = \lambda de$$

$$\begin{aligned} \rightarrow E_x &= \left| \frac{\lambda x}{4\pi\epsilon_0} \right|^2 \int_0^e \frac{1}{(x^2+z^2)} dz = \left| \frac{\lambda x}{4\pi\epsilon_0} \right|^2 \left[\frac{1}{x^2} - \frac{z}{(x^2+z^2)^{\frac{1}{2}}} \right]_0^e = \\ &= \frac{2\lambda x}{4\pi\epsilon_0} \cdot \frac{1}{x^2} \cdot \frac{e}{(x^2+e^2)^{\frac{1}{2}}} \quad E = \frac{\lambda}{2\pi\epsilon_0 x} \cdot \frac{e}{(x^2+e^2)^{\frac{1}{2}}} \end{aligned}$$

$$\text{per } e \gg x \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 x} \cdot \frac{e}{e}$$



$$E = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{z^2} \quad z^2 = z^2 + R^2$$

$$\rightarrow dE = \frac{dq}{4\pi\epsilon_0} \cdot \frac{1}{R^2 + z^2}$$

$$dE_A \text{ e } dE_B \text{ si Annull} \rightarrow dE_{TOT} = dE \cos \alpha$$

$$\text{ma } \epsilon \cdot \cos \alpha = z \rightarrow \cos \alpha = \frac{z}{R} = \frac{z}{(R^2 + z^2)^{\frac{1}{2}}}$$

$$\rightarrow dE_{TOT} = \frac{dq}{4\pi\epsilon_0} \cdot \frac{z}{(R^2 + z^2)^{\frac{3}{2}}}$$

$$\text{ma } \lambda = \frac{dq}{de} \rightarrow dq = \lambda de = \lambda dC$$

$$\rightarrow E_{TOT} = \frac{z}{4\pi\epsilon_0(R^2 + z^2)^{\frac{3}{2}}} \oint dC = \frac{2\pi R z}{4\pi\epsilon_0(R^2 + z^2)^{\frac{3}{2}}}$$

$$\text{ma } \lambda = \frac{Q}{C} \rightarrow Q = \lambda \cdot C$$

$$C = 2\pi R$$

$$\rightarrow E_{TOT} = \frac{z \cdot Q_{TOT}}{4\pi\epsilon_0(R^2 + z^2)^{\frac{3}{2}}}$$

$$\text{se } z \gg R \rightarrow E_{TOT} = \frac{Q_{TOT}}{4\pi\epsilon_0 z^3}$$



$$dE_{Anello} = \frac{dQ \cdot z}{4\pi\epsilon_0 \cdot (z^2 + \epsilon^2)^{\frac{3}{2}}}$$

devo integrarlo da 0 ad R

$$\sigma_{disc} = \frac{Q}{S} \Rightarrow Q_{TOT} = \sum_{disc} \sigma = \sigma \cdot \pi R^2$$

chiamo $R = \epsilon$

$$\rightarrow dq = \sigma 2\pi \epsilon dz$$

$$\rightarrow dE = \frac{\sigma 2\pi \epsilon dz}{2\pi\epsilon_0 (z^2 + \epsilon^2)^{\frac{3}{2}}}$$

$$\rightarrow E_{TOT} = \frac{z\sigma}{2\epsilon_0} \int \frac{\epsilon}{(z^2 + \epsilon^2)^{\frac{3}{2}}} dz = \frac{z\sigma}{2\epsilon_0} \left[-\frac{1}{(z^2 + \epsilon^2)^{\frac{1}{2}}} \right]_0^R = -\frac{z\sigma}{2\epsilon_0} \left[\frac{1}{(z^2 + R^2)^{\frac{1}{2}}} - \frac{1}{z} \right]$$

$$\rightarrow E_{TOT} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{(z^2 + R^2)^{\frac{1}{2}}} \right] \quad R \gg z \quad E_{TOT} = \frac{\sigma}{2\epsilon_0} \left[1 - 0 \right]$$

Continuità Corrente

$$-\mathrm{d}q = \int J \cdot \hat{n} \mathrm{d}s \cdot \mathrm{d}t \quad \Rightarrow \quad -\frac{\mathrm{d}q}{\mathrm{d}t} = \int \bar{J} \cdot \hat{n} \mathrm{d}s \quad \text{ma} \quad f = \frac{Q}{V} \quad \Rightarrow \quad qQ = \int \rho \mathrm{d}V$$

$$\Rightarrow -\frac{\partial}{\partial t} \int \rho \mathrm{d}V = \int \bar{J} \cdot \hat{n} \mathrm{d}s \quad \Rightarrow \quad -\int \frac{\partial f}{\partial t} \mathrm{d}V \int \bar{J} \cdot \hat{n} \mathrm{d}s \quad \Rightarrow \quad \int_V \bar{\nabla} \bar{J} \mathrm{d}V = -\int \frac{\partial f}{\partial t} \mathrm{d}V$$

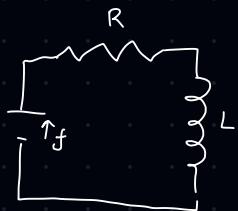
$$\Rightarrow \bar{\nabla} \bar{J} = -\frac{\partial f}{\partial t} \quad \text{Se} \quad \text{in condizioni stazionarie} \quad f = \text{cost} \quad \Rightarrow \quad \bar{\nabla} \bar{J} = 0$$

Autoinduzione

$$\bar{B} = \frac{\mu_0 I}{4\pi \epsilon_0} \oint \frac{d\bar{e} \wedge \bar{\epsilon}}{4\pi^3} \quad \Rightarrow \quad \phi = \oint \bar{B} \cdot \hat{n} \mathrm{d}s = \int_S \frac{\mu_0 I}{4\pi \epsilon_0} \oint \frac{d\bar{e} \wedge \bar{\epsilon}}{4\pi^3} \cdot \hat{n} \mathrm{d}s \quad \begin{matrix} \text{Tutto costante} \\ \text{Tremme } I \end{matrix}$$

$$\Rightarrow \phi = L \cdot I \quad \Rightarrow \quad f_{em} = V_{AB} = -\frac{d\phi}{dt} = \left(-L \frac{dI}{dt} \right) f_{em} \text{ indotta}$$

Circuito $L R$



$$f_{em} - f_{ind} = RI \quad \Rightarrow \quad f_{em} = RI + L \frac{dI}{dt}$$

$$\text{Sol: } I(t) = A e^{\frac{xt}{L}}$$

Vettore magnetico Potenziale

$$\bar{B} = \bar{\nabla} \wedge \bar{A} \quad \Rightarrow \quad \text{Eq Max} \quad \begin{cases} \bar{\nabla} \cdot \bar{B} = 0 \\ \bar{\nabla} \wedge \bar{B} = \mu_0 \bar{J} + \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t} \end{cases} \quad \Rightarrow \quad \begin{cases} \bar{\nabla} \cdot (\bar{\nabla} \wedge \bar{A}) = 0 & \checkmark \\ \bar{\nabla} \wedge (\bar{\nabla} \wedge \bar{A}) = \mu_0 \bar{J} + \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t} \end{cases}$$

$$\Rightarrow \bar{\nabla}(\bar{\nabla} \wedge \bar{A}) - \bar{\nabla}^2 \bar{A} = \mu_0 \bar{J} \quad \text{Se} \quad \bar{\nabla} \bar{A} = 0$$

$$\Rightarrow \bar{\nabla}^2 \bar{A} = -\mu_0 \bar{J} \quad \equiv \quad \bar{\nabla}^2 V = -\frac{\rho}{\epsilon_0}$$

Eq Poisson

$$E = -\bar{\nabla} V \quad \Rightarrow \quad \int E \cdot \hat{n} \mathrm{d}s = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad \int E \cdot \hat{n} \mathrm{d}s = \frac{1}{\epsilon_0} \int \rho \cdot \mathrm{d}V \quad \Rightarrow \quad \int \bar{\nabla} E \mathrm{d}V = \int \frac{\rho}{\epsilon_0} \mathrm{d}V$$

$$\nabla(-\bar{\nabla} V) = \frac{\rho}{\epsilon_0} \quad \Rightarrow \quad \bar{\nabla}^2 V = -\frac{\rho}{\epsilon_0}$$

$$\text{Sol: } V = \frac{1}{4\pi \epsilon_0} \int_V \frac{J(\vec{\xi}')}{|\vec{\xi} \cdot \vec{\xi}'|} \mathrm{d}V \quad \Rightarrow \quad A(\vec{\xi}) = \frac{\mu_0}{4\pi} \int_V \frac{J(\vec{\xi}')}{|\vec{\xi} \cdot \vec{\xi}'|} \mathrm{d}V \quad \text{ma} \quad \mathrm{d}V = \mathrm{d}\vec{s} \cdot \mathrm{d}e$$

$$= \oint A(\xi) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{\xi}') \cdot d\xi' \cdot d\ell}{|\xi - \xi'|} \quad \text{ma} \quad \vec{J} \cdot \vec{S} = \vec{I}$$

$$= \oint A(\xi) = \frac{\mu_0}{4\pi} \int \frac{I(\xi')}{|\xi - \xi'|} d\ell$$

Se $\nabla A \neq 0$

$$\text{Pongo } \vec{A}' = \vec{A} + \vec{\nabla} f \quad \rightarrow \quad B = \vec{\nabla} \wedge \vec{A}' = \vec{\nabla} \wedge \vec{A} + \vec{\nabla}^2 f = \vec{\nabla} \wedge (\vec{A} + \vec{\nabla} f) = \nabla \wedge A + \nabla \Delta(\nabla f)$$

$$\text{Trovo una } f : \quad \nabla(\nabla A') = \vec{\nabla}^2 \vec{A} + \vec{\nabla}^2 f = \emptyset$$

Joule

$$\text{Pow} = \frac{dL}{dt} \quad L = \int F \cdot d\ell \quad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E \quad \rightarrow \quad L = q \cdot \int E \cdot d\ell$$

$$\rightarrow \text{Pow} = \frac{dq \cdot E \cdot d\ell}{dt} \quad \rightarrow \quad \text{Pow} = I \cdot E \cdot d\ell \quad \text{ma} \quad E \cdot d\ell = dV$$

$$\rightarrow \text{Pow} = I \cdot V \quad \text{ma} \quad V = R \cdot I \quad \rightarrow \quad \underline{\text{Pow} = I^2 R}$$

Energia Campo \mathcal{E}

$$L = \int F \cdot d\ell \quad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E \quad \rightarrow \quad L = q \int E \cdot d\ell = q \cdot V$$

$$\text{Dai condensatori} \quad C = \frac{Q}{U_A - U_B} \quad \rightarrow \quad V = \frac{Q}{C} \quad \rightarrow \quad dL = V \cdot dq \quad \rightarrow \quad L = \int V \cdot dq$$

$$\rightarrow L = \int \frac{q}{C} \cdot dq \\ = \frac{1}{C} \int q \cdot dq = \frac{1}{C} \frac{q^2}{2} + d$$

$$\rightarrow L = \int_A^B \frac{q}{C} \cdot dq = \frac{1}{C} \left[\frac{Q_B^2}{2} - \frac{Q_A^2}{2} \right] = \frac{Q_B^2}{2C} - \frac{Q_A^2}{2C}$$

$$\text{Se cond iniziale } Q_A = 0 \rightarrow L = \frac{Q^2}{2C} \quad \text{ma} \quad \frac{Q}{C} = V$$

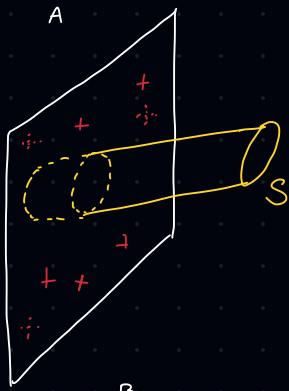
$$\rightarrow \mathcal{E} = \frac{Q \cdot V}{2}$$

Cond Piano

Condensatore piano

$$\begin{array}{c}
 \left| \begin{array}{ccccc}
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 + & - & - & & \\
 \end{array} \right| \quad \left\{ \begin{array}{l} V_A = K_1 Q \\ V_B = K_2 Q \end{array} \right. \quad \Rightarrow \quad V_B - V_A = Q(K_2 - K_1) \quad \text{dunque } K_2 - K_1 = \frac{1}{C}
 \end{array}$$

$$V_A - V_B = \int E \cdot d\ell \quad , \quad E = \text{Strato carico}$$



$$\begin{aligned}
 \phi_E &= \int E \cdot \hat{n} dS = \frac{Q_{TOT}}{\epsilon_0} \quad \text{ma} \quad \sigma = \frac{Q}{S} \quad \Rightarrow \quad Q = \sigma \cdot S \\
 &\Rightarrow \int E \cdot \hat{n} dS = \frac{1}{\epsilon_0} \int \sigma \cdot dS \\
 &\Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}
 \end{aligned}$$

$$\Rightarrow V = \int_A^B \frac{\sigma}{\epsilon_0} d\ell \quad \Rightarrow \boxed{\frac{\sigma}{\epsilon_0} \cdot d} \quad V = \sigma = \frac{Q}{S} \quad \Rightarrow \quad V = \frac{Q \cdot d}{S \cdot \epsilon_0}$$

$$C = \frac{Q}{V} = \frac{\sigma S \epsilon_0}{Q \cdot d} \quad \Rightarrow \quad C = \frac{S \epsilon_0}{d}$$

$$\Rightarrow \mathcal{E} = \frac{Q \cdot V}{2} = \frac{Q \cdot Q \cdot d}{S \cdot \epsilon_0 \cdot 2} = \boxed{\frac{Q^2 d}{2 S \epsilon_0}}$$

E caso particolare

Densità Volum. Energia

$$W = \frac{\mathcal{E}}{V} \quad \Rightarrow \quad W = \frac{\mathcal{E}}{S \cdot d} = \frac{Q \cdot V}{2 S d} = \frac{\sigma V}{2 d} \quad \text{Siccome } V = \int E \cdot d\ell = E \cdot d \quad \Rightarrow \quad \frac{V}{d} = E$$

$$\Rightarrow W = \frac{\sigma \cdot E}{2} \quad \int E \cdot n dS = \frac{Q}{\epsilon_0} \Rightarrow E \cdot \epsilon_0 = \frac{Q}{S} = \sigma$$

$$\Rightarrow W = \frac{1}{2} E^2 \epsilon_0$$

Energia

$$L = \int F \cdot de \quad \text{ma} \quad E = \frac{F}{q} \rightarrow F = q \cdot E \Rightarrow L = q \int E \cdot de = q \cdot V$$

Del cond

$$C = \frac{Q}{V_A - V_B} \Rightarrow V_A - V_B = \frac{Q}{C} \rightarrow L = \int V \cdot dq = \int_A^B \frac{q}{C} dq = \mathcal{E}$$

$$\rightarrow \mathcal{E} = \frac{1}{2} \left[\frac{q_B^2}{2} - \frac{q_A^2}{2} \right] \quad \text{se } q_A = 0 \rightarrow \mathcal{E} = \frac{Q^2}{2C} = \frac{Q \cdot (V_A - V_B)}{2} \quad \text{Energia (1)}$$

$$\text{Definisco } w = \frac{\mathcal{E}}{V} = \frac{\mathcal{E}}{S \cdot e} = \frac{Q \cdot (V_A - V_B)}{2 \cdot S \cdot e} = \frac{\sigma \cdot (V_A - V_B)}{2e} = \frac{\sigma_e}{2} \int E \cdot de = E \cdot e = V \rightarrow E = \frac{V}{e}$$

$$\text{mo do Gauß: } \int E \cdot d\vec{S} = \frac{Q}{\epsilon_0} \rightarrow \sigma = \frac{Q}{S} \rightarrow Q = \sigma \cdot S \rightarrow E \cdot S = \frac{\sigma \cdot S}{\epsilon_0} \rightarrow E \cdot \epsilon_0 = \sigma$$

$$\Rightarrow \mathcal{E} = \frac{E^2 \epsilon_0}{2} \quad \text{Energia (2)}$$

Condensatore piano

$$\begin{array}{c|c} + & - \\ + & - \\ + & - \\ + & - \end{array} \quad V_A \quad E = \frac{\sigma}{\epsilon_0} \quad V_B$$



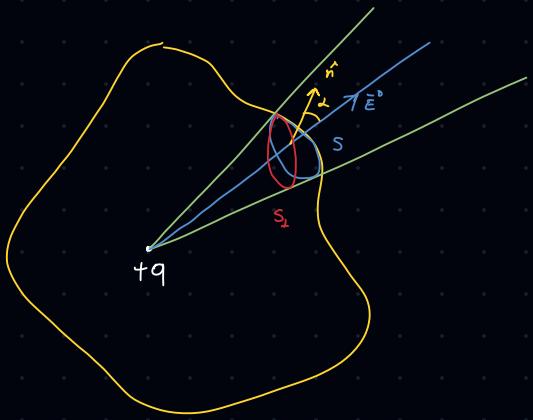
$$\int E \cdot \hat{n} ds = \frac{Q}{\epsilon_0} \rightarrow \sigma = \frac{Q}{S} \rightarrow Q = \sigma \cdot S$$

$$\rightarrow \int \tilde{E} \cdot \hat{n} ds = \frac{1}{\epsilon_0} \int \sigma \cdot ds$$

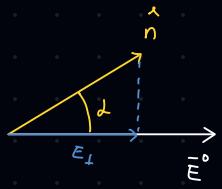
$$\rightarrow E = \frac{\sigma}{\epsilon_0}$$

$$V = \int E \cdot de = \int \frac{\sigma}{\epsilon_0} de = \frac{\sigma}{\epsilon_0} \cdot d$$

$$\rightarrow \mathcal{E} = \frac{Q \cdot \sigma \cdot d}{\epsilon_0 \cdot 2} \quad \text{siccome } \sigma = \frac{Q}{S} \rightarrow \mathcal{E} = \frac{Q^2 d}{\epsilon_0 S 2}$$



$$\text{definisco } d\Omega = \frac{dS_\perp}{r^2} = \frac{ds \cos\alpha}{r^2}$$



$$\phi_E = \int \vec{E} \cdot \hat{n} ds$$

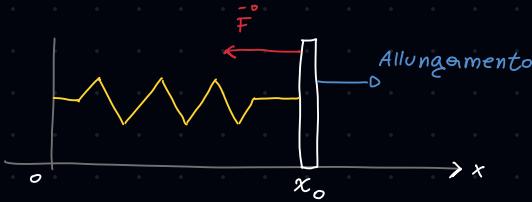
$$= \frac{q}{4\pi\epsilon_0} \int \frac{\cos\alpha}{r^2} ds = \frac{q}{4\pi\epsilon_0} \oint d\Omega = \frac{q}{4\pi\epsilon_0} \cdot 4\pi = \frac{q}{\epsilon_0}$$

Alternative

$$\phi_E = \int E \cdot \hat{n} ds = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \cdot \hat{e} \cdot \hat{n} ds = \frac{q}{4\pi\epsilon_0} \oint \frac{\cos\alpha \cdot ds}{r^2} = \frac{q}{4\pi\epsilon_0} \oint \frac{1}{r^2} ds_\perp$$

$\cancel{r \cdot n \cos\alpha}$

$$\rightarrow \frac{q}{4\pi\epsilon_0} \oint ds_\perp = \frac{q}{4\pi\epsilon_0 r^2} \cdot 4\pi r^2$$



$$\vec{F} = -Kx \hat{x}$$

$$\sum_i F_i = m \cdot a_i$$

$$\rightarrow -Kx = m \cdot a_x$$

$$\rightarrow Kx + ma = 0 \quad ma = \ddot{x} \rightarrow Kx + m\ddot{x} = 0 \quad \rightarrow \ddot{x} + \frac{K}{m}x = 0$$

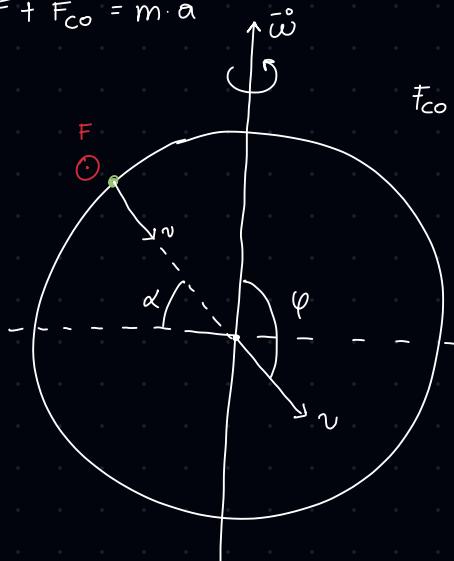
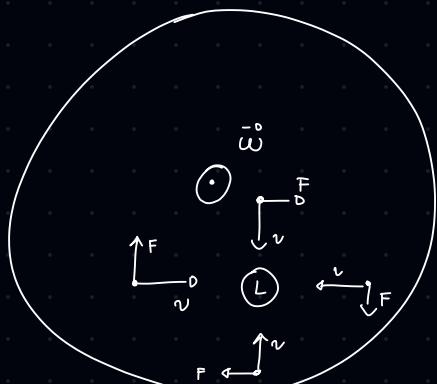
$$\text{pongo } \omega^2 = \frac{K}{m} = 0 \quad \ddot{x} + \omega^2 x = 0 \quad \text{eq diff}$$

$$\text{Sol: } x(t) = A \cos(\omega t + \varphi)$$

$$\text{Periodo: } x(t+\tau) = A \cos(\omega t + \varphi + 2\pi) \quad \Leftrightarrow \quad \omega t + \omega\tau + \varphi = \omega t + \varphi + 2\pi$$

$$\alpha_{co} = 2(\vec{\omega} \wedge \vec{v}) \quad \rightarrow \quad F = m \cdot a \quad \rightarrow \quad F = m/a + 2(\omega \wedge v) = F - 2m(\vec{\omega} \wedge \vec{v}) = m \cdot a$$

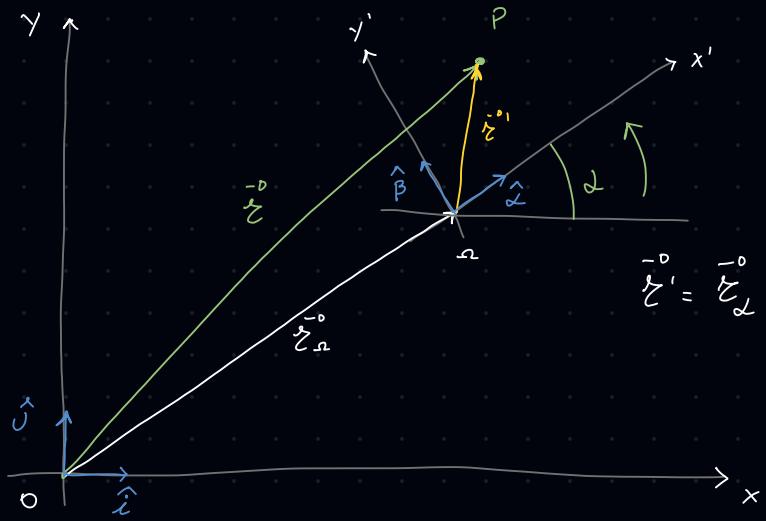
$$\rightarrow F_{co} = -2m(\omega \wedge v) = 2m(v \wedge \omega) \quad \rightarrow \quad F + F_{co} = m \cdot a$$



$$F_{co} = 2m(\vec{v} \wedge \vec{\omega}) = 2m v \omega \cdot \sin\varphi$$

$$\varphi = 180^\circ - \alpha \Rightarrow \sin\varphi = \sin\alpha$$

$$\Rightarrow \sin\alpha = \max \alpha = 90^\circ$$



$$\vec{\varepsilon} = \vec{\varepsilon}_\alpha + \vec{\varepsilon}'$$

$$\Rightarrow \vec{v} = \frac{d\vec{\varepsilon}}{dt} + \frac{d\vec{\varepsilon}'}{dt} = \vec{v}_\alpha + \frac{d\vec{\varepsilon}'}{dt}$$

$$\vec{\varepsilon}' = \vec{\varepsilon}_\alpha + \vec{\varepsilon}_\beta = \alpha x' + \beta y' \quad \text{con } \alpha \in \beta \text{ variab.}$$

$$\begin{aligned} \Rightarrow \frac{d\vec{\varepsilon}'}{dt} &= \frac{d\alpha}{dt} x' + \frac{d\beta}{dt} y' + \left(\frac{dx'}{dt} \hat{\alpha} + \frac{dy'}{dt} \hat{\beta} \right) = (\omega \lambda \hat{\alpha}) x' + (\omega \lambda \hat{\beta}) y' + \vec{v}' \\ &= \omega \lambda (\hat{\alpha} x' + \hat{\beta} y') + \vec{v}' = (\omega \lambda \vec{\varepsilon}') + \vec{v}' \quad (1) \end{aligned}$$

$$\Rightarrow \vec{v} = \vec{v}_\alpha + \vec{v}' + \vec{\omega} \lambda \vec{\varepsilon}'$$

$$\vec{v}_T = \vec{v}_\alpha + \vec{\omega} \lambda \vec{\varepsilon}'$$

$$\Rightarrow \vec{v} = \vec{v}' + \vec{v}_{TR}$$

$$\vec{\alpha} = \frac{d\vec{v}}{dt} = \left(\frac{d\vec{v}_\alpha}{dt} \right) + \frac{d\vec{v}'}{dt} + \frac{d\vec{\omega}}{dt} \lambda \vec{\varepsilon}' + \omega \lambda \frac{d\vec{\varepsilon}'}{dt}$$

$$\Rightarrow \frac{d}{dt} \left(\frac{dx'}{dt} \hat{\alpha} + \frac{dy'}{dt} \hat{\beta} \right) = \left(\frac{d^2 x'}{dt^2} \hat{\alpha} + \frac{d^2 y'}{dt^2} \hat{\beta} \right) + \left(\frac{dx'}{dt} \frac{d\hat{\alpha}}{dt} + \frac{dy'}{dt} \frac{d\hat{\beta}}{dt} \right)$$

$$\Rightarrow \frac{d}{dt} \left(\frac{dx'}{dt} \hat{\alpha} + \frac{dy'}{dt} \hat{\beta} \right) = \vec{\alpha}' + \omega \lambda \left(\frac{dx'}{dt} \hat{\alpha} + \frac{dy'}{dt} \hat{\beta} \right) = \vec{\alpha}' + \omega \lambda \vec{v}'$$

$$\vec{\omega} \lambda \frac{d\vec{\varepsilon}'}{dt} = \omega \lambda \left[(\omega \lambda \vec{\varepsilon}') + \vec{v}' \right] = \vec{\omega} \lambda (\vec{\omega} \lambda \vec{\varepsilon}) + \vec{\omega} \lambda \vec{v}' = \underline{-\omega^2 \vec{\varepsilon} + \vec{\omega} \lambda \vec{v}'}$$

$$\Rightarrow \vec{\alpha} = \vec{\alpha}_\alpha + \frac{d\vec{\omega}}{dt} \lambda \vec{\varepsilon}' + \vec{\alpha}' + \vec{\omega} \lambda \vec{v}' - \underline{\vec{\alpha}_{CP}} = \vec{\alpha}_\alpha + \frac{d\vec{\omega}}{dt} \lambda \vec{\varepsilon}' + \vec{\alpha}' + \vec{\alpha}_{CP} + \underline{\vec{\alpha}_{CO}}$$