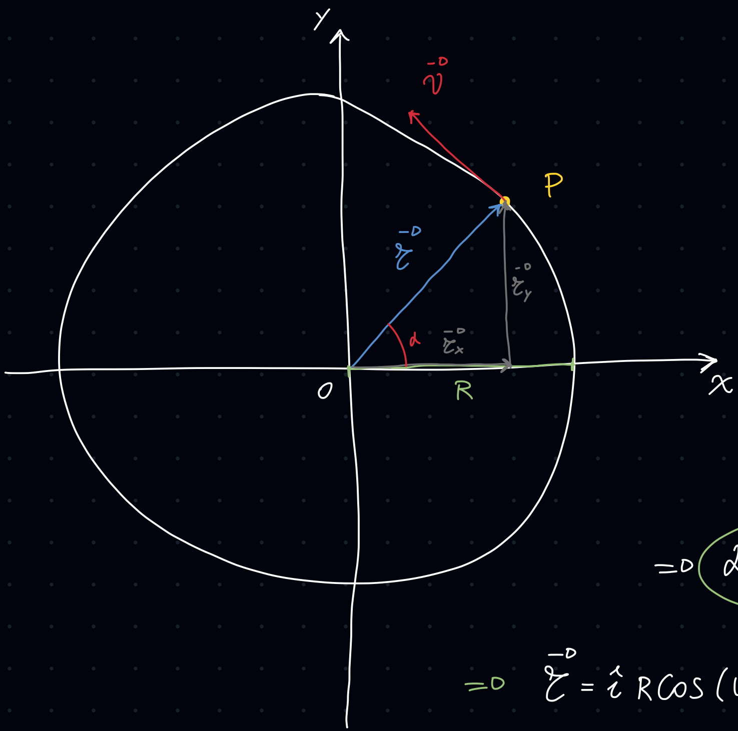


$$\vec{r} = \vec{r}_x + \vec{r}_y = R\cos\alpha + R\sin\alpha$$

Definisco $\quad \vec{\omega} = \frac{d\alpha}{dt}$

$$\Rightarrow \omega\, dt = d\alpha \rightarrow \int_{t_0}^{t_f} \omega\, dt = \int_{\alpha_0}^{\alpha_f} d\alpha$$

$$\Rightarrow \omega(t_f - t_0) = \alpha_f - \alpha_0 \qquad \text{pongo } \alpha_0 = t_0 = \emptyset$$

$$\Rightarrow \alpha(t) = \omega \cdot t$$

$$\Rightarrow \vec{r} = \hat{i} R\cos(\omega t) + \hat{j} R\sin(\omega t)$$

Siccome $\quad \vec{v} = \frac{d\vec{s}}{dt} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = -\omega R\sin(\omega t)\hat{i} + \omega R\cos(\omega t)\hat{j} = \omega R\left[\cos(\omega t)\hat{j} - \sin(\omega t)\hat{i}\right]$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 R\cos(\omega t)\hat{i} - \omega^2 R\sin(\omega t)\hat{j} = -\omega^2 R\,\underbrace{\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right]}_{\vec{r}}$$

$$= \underline{-\omega^2 R\,\vec{r}} \quad \overset{(1)}{\vec{a}}$$

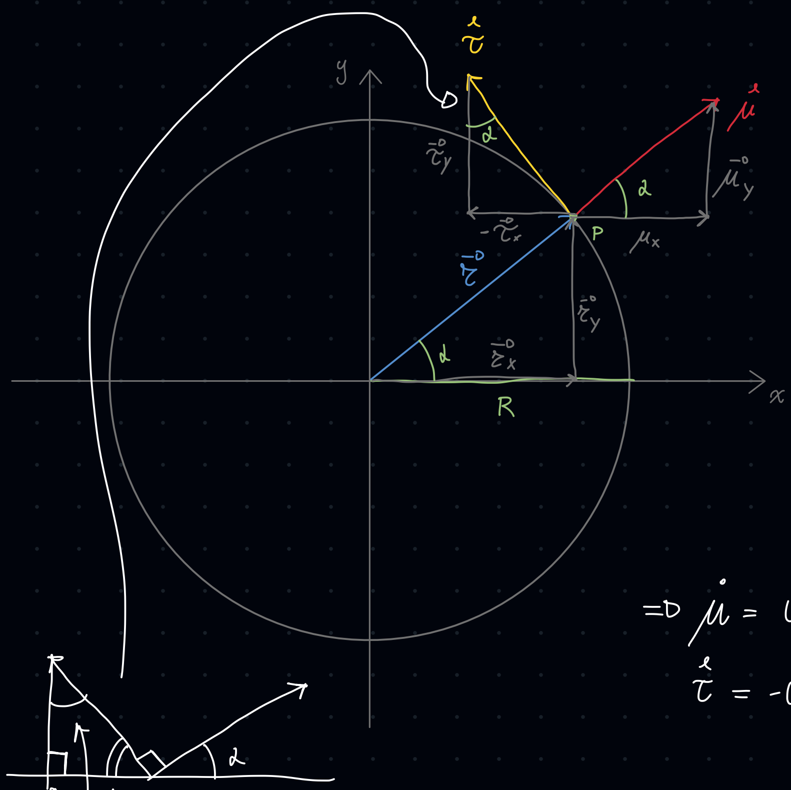
$$|\vec{v}| = \sqrt{\omega^2 R^2 [\underbrace{\cos^2(\omega t) + \sin^2(\omega t)}_{1}]} = \sqrt{\omega^2 R^2} = \omega R \; |\vec{v}| \rightarrow v = \omega R \Rightarrow \omega = \frac{v}{R}$$

$$|\vec{a}| = \sqrt{\omega^4 R^2 [\ldots]} = \omega^2 R \quad \text{ma} \qquad a = \omega^2 R = \frac{v^2}{R^2} R = \frac{v^2}{R} \; |\vec{a}|_{cp}$$

OSS.

(1) $\vec{a} = -\omega^2 R\,\vec{r} \Rightarrow$ Acc rivolta verso l'interno!

Riferimento in movimento



$$\begin{cases}\hat{\mu} = \hat{i}\cos\alpha + \hat{j}\sin\alpha \\ \hat{\tau} = -\hat{i}\sin\alpha + \hat{j}\cos\alpha\end{cases}$$

$$\omega = \frac{d\alpha}{dt} \Rightarrow \underline{\alpha(t) = \omega t}$$

$$\Rightarrow \begin{cases}\dfrac{d\mu}{dt} = -\hat{i}\omega\sin(\omega t) + \omega\hat{j}\cos(\omega t) \\ \dfrac{d\hat{\tau}}{dt} = -\omega\hat{i}\cos(\omega t) - \omega\hat{j}\sin(\omega t)\end{cases}$$

$$\Rightarrow \dot{\hat{\mu}} = \omega\Big[\overbrace{\hat{j}\cos(\alpha) - \hat{i}\sin(\alpha)}^{\hat{\tau}}\Big] = \omega\hat{\tau}$$

$$\dot{\hat{\tau}} = -\omega\Big[\underbrace{\hat{i}\cos(\alpha) + \hat{j}\sin(\alpha)}_{\hat{\mu}}\Big] = -\omega\hat{\mu}$$

$\Rightarrow$ 90

$180-90-\alpha = 90-\alpha$

$\Rightarrow 90+90-\alpha = 180-\alpha$

$\underline{180-180+\alpha = \alpha}$

$$\vec{r} = \hat{i}R\cos\alpha + R\hat{j}\sin\alpha = R\Big[\underbrace{\hat{i}\cos\alpha + \hat{j}\sin\alpha}_{\hat{\mu}}\Big] = R\hat{\mu}$$

$$\Rightarrow \frac{d\vec{r}}{dt} = R\omega\hat{\tau} \quad \vec{v}$$

$$\frac{d\vec{v}}{dt} = R\omega\left[-\omega\hat{\mu}\right] = -R\omega^2\hat{\mu}$$

Acc cp.

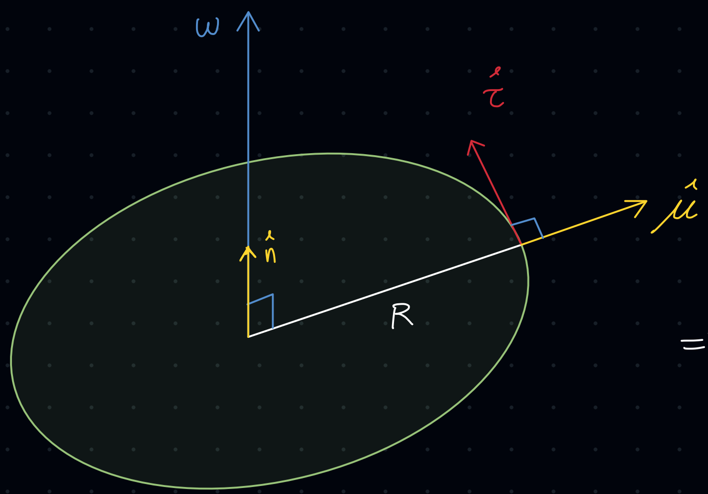
Se $\omega \neq cost \Rightarrow \dfrac{d\vec{v}}{dt} = R\,\dfrac{d\omega\hat{\tau}}{dt} = R\Big[\underbrace{\dot{\omega}\hat{\tau}}_{\vec{a}_{tg}} + \underbrace{\omega\dot{\hat{\tau}}}_{\vec{a}_{cp}}\Big]$

$\vec{a}_{cp} \to -R\omega^2\hat{\mu}$

Vettore $\vec{\omega}$

$\dot{\mu} = \omega\,\hat{\tau}$

se $\dot{\mu} \parallel \hat{\tau}$ $\Rightarrow$ l'unico modo per ottenere $\hat{\tau}$ e' con il prodotto vett tra $\hat{n}$ e μ



$$\Rightarrow \dot{\mu} = \vec{\omega} \wedge \hat{\mu} = \omega \cdot \mu \cdot \sin(90) =$$

$$= \omega\,[\text{Regola Mano Dx}] = \omega\,\hat{\tau}$$

QED

$$\Rightarrow \frac{d\hat{\mu}}{dt} = \omega \wedge \hat{\mu} \qquad \forall \hat{\mu} \text{ VERSORE}$$