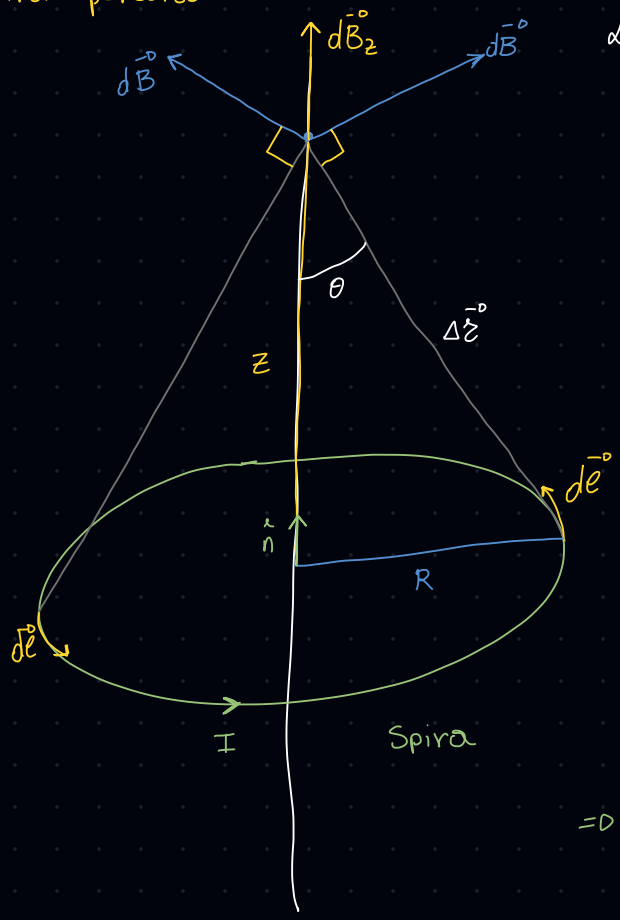
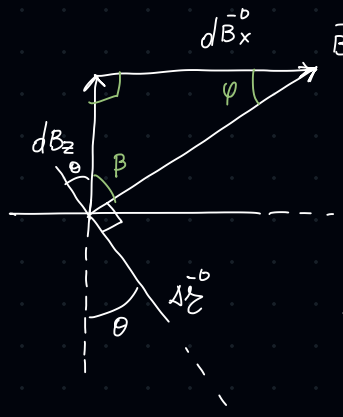
Spira percorsa da corrente



Le componenti x e y di $\vec{B}$ si annullano a due a due $\Rightarrow \vec{B} = \vec{B}_z$

Da Laplace $d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{\ell} \wedge \Delta\vec{\ell}}{\Delta\ell^3}$



$180 - \varphi - \beta$

$\beta = 180 - 90 - \theta$

$\Rightarrow \varphi = 180 - 90 - 180 + 90 + \theta = \theta$

$\Rightarrow B_z = B \sin\theta$

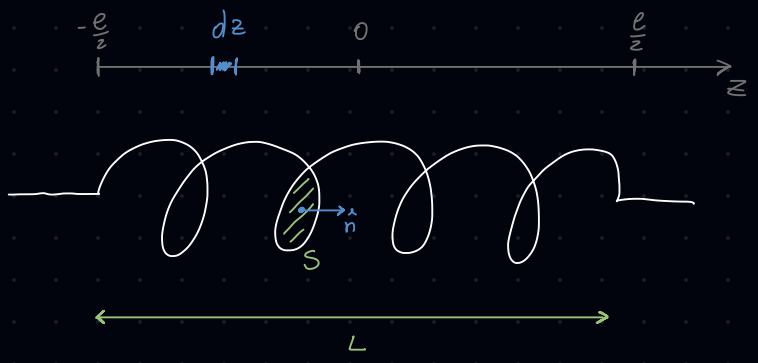
$$\Rightarrow d\vec{B}_z = \frac{\mu_0 I}{4\pi} \oint \frac{d\ell \wedge \Delta\vec{\ell}}{\Delta\ell^3} \cdot \sin\theta = K \oint \frac{d\ell \cdot \Delta\ell \cdot \sin(90)}{\Delta\ell^3} \cdot \sin(\theta) = K \cdot \frac{\sin\theta}{\Delta\ell^2} \oint d\ell$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{\sin\theta}{\Delta\ell^2} \cdot 2\pi R = \frac{\mu_0 I}{2} \cdot \frac{R \sin\theta}{\Delta\ell^2}$$

$$\Delta\ell \cdot \sin\theta = R \rightarrow \sin\theta = \frac{R}{\Delta\ell}, \quad \Delta\ell^2 = R^2 + z^2$$

$$\Rightarrow \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$$

Campo $\vec{B}$ di un Solenoide



N = Numero di spire , n = spire per unità di lunghezza

$\Rightarrow n = \frac{N}{L}$

$\Rightarrow n\,dz =$ Spire nella porzione dz

$$\Rightarrow B_{TOT} = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} B_{Spira} \cdot n\,dz = \frac{\mu_0 I}{2} \int \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}} \cdot n\,dz$$

R^2: Costante; $(R^2+z^2)^{\frac{3}{2}}$: Non costante

$$\Rightarrow \int \frac{1}{(R^2+z^2)^{\frac{3}{2}}}\,dz = \frac{1}{R^2}\frac{z}{\sqrt{R^2+z^2}} + C$$

$$\Rightarrow \vec{B}_{TOT} = \hat{n}\frac{\mu_0 I R^2 n}{2}\left[\frac{1}{R^2}\cdot\frac{z}{\sqrt{R^2+z^2}}\right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}} = \hat{n}\frac{\mu_0 I}{2}n\left[\frac{1}{R^2}\cdot\frac{\ell}{2\sqrt{R^2+(\frac{\ell}{2})^2}} + \frac{1}{R^2}\cdot\frac{\ell}{2\sqrt{R^2+(\frac{\ell}{2})^2}}\right]$$

$$= \hat{n}\,\mu_0 I R^2 \cdot \frac{N}{2\ell} \cdot 2\left[\frac{\ell}{R^2\,2\sqrt{R^2+\frac{\ell^2}{4}}}\right]$$

$$\sqrt{\frac{4R^2+\ell^2}{4}} = \frac{\sqrt{4R^2+\ell^2}}{\sqrt{4}} \quad (\sqrt{4} = 2)$$

$$= \hat{n}\,\mu_0 I R^2 \cdot \frac{N}{\ell} \cdot \frac{\ell}{\sqrt{4R^2+\ell^2}}$$

Approx $\ell >> R \Rightarrow \sqrt{4R^2+\ell^2} = \ell$

$$\Rightarrow \vec{B} = \hat{n}\mu_0 I\left(N\cdot\frac{1}{\ell}\right) \xrightarrow{n} \vec{B} = \hat{n}\mu_0 I n$$