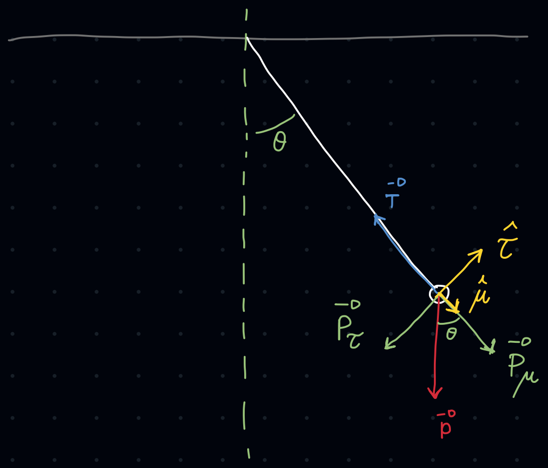


$$\vec{P}_\mu = m \cdot g \cdot \cos\theta$$
$$\vec{P}_\tau = m\, g\, \sin\theta$$

$$\begin{cases} \hat{\mu} & mg\cos\theta - T = m \cdot \vec{a}_\mu \\ \hat{\tau} & -mg\sin\theta = m\, \vec{a}_\tau \end{cases}$$

$$\vec{a}_{\hat{\mu}} = \text{Centripeta} = -\frac{v^2}{R} = \left(-\frac{\dot{S}^2}{\ell}\right) \vec{a}_{\hat{\mu}}$$

$$\vec{a}_{\hat{\tau}} = \text{Tangenziale} = ?? = \underset{\uparrow\, |\vec{a}|}{\ddot{S}} \cdot \hat{\tau}$$

$$\Rightarrow \begin{cases} mg\cos\theta - T = -\frac{m\dot{S}^2}{\ell}\hat{\mu} \\ -mg\sin\theta = m \cdot \ddot{S}\hat{\tau} \end{cases}$$

Approssimazione per piccole oscillazioni $\sin\theta \sim \theta$

$\Rightarrow -m\vec{g}\,\theta = m\,\ddot{S}\hat{\tau}$ se $m_I = m_g$ $\rightarrow$ $-\vec{g}\,\theta = \ddot{S}\hat{\tau}$ $\rightarrow$ $\ddot{S}\hat{\tau} + \vec{g}\,\theta = \emptyset$

ma θ Radianti $\rightarrow$ $1\,\text{Rad} = \frac{\ell}{R}$ $\Rightarrow$ $\ddot{S}\hat{\tau} + \frac{g}{R}\cdot \ell = 0$ pongo $R = \ell$ ovvero il "FILO" del pendolo

$\ell = S$ ovvero l'Arco

$\Rightarrow \ddot{S}\hat{\tau} + \left(\frac{g}{\ell}\right) \cdot S = 0$ $\rightarrow$ $\underline{\ddot{S}\hat{\tau} + k^2 \cdot S = 0}$

Battezzo $\frac{g}{\ell} = k^2$ Eq. diff

$\Rightarrow$ Soluzione: $S(t) = A \cdot \cos(kt + \varphi)$

Cos è periodico di $T = 2\pi$

Trovare k: Cos è periodico $\rightarrow$ $S(t + T_0) = A \cdot \cos(kt + \varphi + 2\pi)$

$\rightarrow \cancel{A}\cos(kt + kT_0 + \varphi) = \cancel{A}\cos(kt + \varphi + 2\pi)$ $\rightarrow$ $\cancel{kt} + kT_0 + \cancel{\varphi} = \cancel{kt} + \cancel{\varphi} + 2\pi$

$\rightarrow kT_0 = 2\pi$ $\Rightarrow$ $k = \frac{2\pi}{T_0}$

$\Rightarrow S(t) = A\cos\left(\frac{2\pi}{T_0}t + \varphi\right) = A\cos(\omega t + \varphi)$

$\omega = \frac{d\alpha}{dt} = \frac{2\pi}{T_0}$

Periodo $k = \omega = \frac{2\pi}{T}$ $\rightarrow$ $k^2 = \frac{g}{\ell}$ $\Rightarrow$ $k = \sqrt{\frac{g}{\ell}}$ $\Rightarrow$ $\frac{2\pi}{T} = \sqrt{\frac{g}{\ell}}$

$\Rightarrow 2\pi\sqrt{\frac{\ell}{g}} = T$ Periodo del pendolo

1° eq El.

$$\phi_E = \int \vec{E}\cdot\hat{n}\,dS = \frac{Q}{\varepsilon_0} \qquad \rho = \frac{dQ}{dV} \to dQ = \rho\, dV \to Q = \int \rho\, dV$$

$$\Rightarrow \int_S \vec{E}\cdot\hat{n}\,dS = \frac{1}{\varepsilon_0}\int \rho\, dV \to \int_V \vec{\nabla}\cdot\vec{E}\, dV = \frac{1}{\varepsilon_0}\int_V \rho\, dV \Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \quad (a)$$

2° eq

$$C = \oint \vec{E}\cdot d\vec{l} = \emptyset \Rightarrow \int \vec{\nabla}\wedge\vec{E}\ \hat{n}\,dS = 0 \Rightarrow \vec{\nabla}\wedge\vec{E} = 0$$

Legge di Coulomb

$$L = F\cdot S = \int \vec{F}_C\cdot d\vec{l} = U_A - U_B = q(V_A - V_B) \Rightarrow \frac{L}{q} = \frac{F}{q}\,dl = -dV \to \frac{L}{q} = \vec{E}\,d\vec{l} = -dV$$

V è del tipo $V(x,y,z) \Rightarrow dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$

otteniamo dV

$$\begin{cases} dl = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz \\ \nabla V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \end{cases} \Rightarrow dV = \vec{\nabla}V\cdot d\vec{l}$$

$$\Rightarrow \vec{E}\cdot \cancel{d\vec{l}} = -\vec{\nabla}V\,\cancel{d\vec{l}} \to \vec{E} = -\vec{\nabla}V \quad (b)$$

Uniamo (a) e (b)

$$\to \nabla(-\nabla V) = \frac{\rho}{\varepsilon_0} \to \vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0}$$ Eq di Poisson

$$V = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}\,')}{\Delta r}\,dV$$

Potenziale Vettore $\quad \vec{B} = \vec{\nabla}\wedge\vec{A}$

$$\Rightarrow \begin{cases} \vec{\nabla}\cdot\vec{B} = 0 & \text{GAUSS} \\ \vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} & \text{AMPÈRE} \end{cases} \Rightarrow \begin{cases} \vec{\nabla}\cdot(\vec{\nabla}\wedge\vec{A}) = 0 \to \text{Sempre Vera} \\ \vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{A}) = \mu_0\vec{J} \to \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A} = \mu_0 J \end{cases}$$

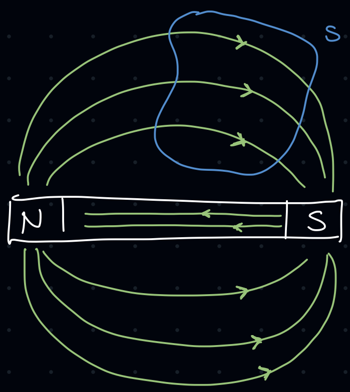
Se $\vec{\nabla}\cdot\vec{A} = 0 \to \vec{\nabla}^2 A = -\mu_0\vec{J}$ Simile all'eq di Poisson

Soluz.

$$\to A = \frac{\mu_0}{4\pi}\int \frac{J\cdot(\vec{r}\,')}{\Delta r}\,dV \quad \text{ma} \quad \vec{J}\,dV = (\underbrace{\vec{J}\cdot S}_{I})\,d\vec{l} \Rightarrow A = \frac{\mu_0}{4\pi}\oint \frac{I\cdot\vec{r}\,'}{\Delta r}\,d\vec{l}$$

$$A' = \vec{A} + \vec{\nabla}f \to \vec{B} = \vec{\nabla}\wedge\vec{A} = \vec{\nabla}\wedge\vec{A}'$$

Maxwell   Magnetostatica



$$\phi_B = \int \vec{B}\cdot\hat{n}\, dS = 0 \quad \rightarrow \quad \vec{\nabla}\cdot\vec{B} = \emptyset \qquad \Big\}\ 1° \text{ Eq di Maxwell}$$

Biot - Savart

Campo Magnetico attorno un filo percorso da corrente

Sperimentalmente: $B = K\cdot\dfrac{I}{R}$

$\rightarrow \vec{B} = \dfrac{\mu_0}{4\pi}\ \dfrac{I}{R}\ \hat{\tau}$ $\rightarrow$ Circuitazione: $C = \oint \vec{B}\cdot d\vec{\ell} = K \oint \dfrac{I}{R}\, d\vec{\ell}$

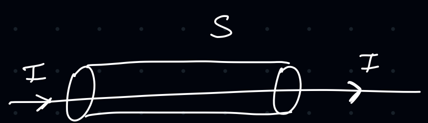
ma $d\vec{\ell}$ = Arco di circonferenza $\Rightarrow$ $1\,\text{Rad} = \dfrac{R}{\ell}$ $\rightarrow$ $\ell = \varphi\cdot R \rightarrow d\ell = R\, d\varphi$

$\Rightarrow C = \oint \vec{B}\, d\vec{\ell} = K \oint \dfrac{I}{\cancel{R}}\cdot \cancel{R}\, d\varphi \Rightarrow \oint \vec{B}\, d\vec{\ell} = K I \cdot \oint d\varphi \Rightarrow \oint \vec{B}\, d\vec{\ell} = \dfrac{\mu_0}{\cancel{4\pi}}\cdot I \cdot \cancel{4\pi}$

$\rightarrow \oint \vec{B}\, d\vec{\ell} = \mu_0 I$ Legge di Ampére Integrale

$\displaystyle\int_S (\vec{\nabla}\wedge\vec{B})\cdot\hat{n}\, dS = \mu_0 \int_S \vec{J}\cdot\hat{n}\, dS \rightarrow \vec{\nabla}\wedge\vec{B} = \mu_0 \vec{J}$ Differenziale

Manca un pezzo!



$\phi = \int \vec{J}\cdot\hat{n}\, ds = \emptyset \rightarrow \underbrace{\vec{\nabla}\cdot\vec{J}}_{\text{Div}} = \emptyset$ corrente stazionaria

$\phi = \int \vec{J}\cdot\hat{n}\, ds \neq 0$!   ??   Troviamo il pezzo

Eq di continuita'

$-dq = \int \vec{J}\cdot\hat{n}\, ds\cdot dt \rightarrow -\dfrac{dq}{dt} = \int \vec{J}\cdot\hat{n}\, ds$ ma $\rho = \dfrac{dq}{dV} \rightarrow dq = \rho\, dV$

$\rightarrow -\dfrac{d}{dt}\displaystyle\int_V \rho(x,y,z)\, dV = \int_S \vec{J}\cdot\hat{n}\, ds \rightarrow -\int_V \dfrac{\partial\rho}{\partial t}\, dV = \int_S \vec{J}\cdot\hat{n}\, ds$

$\rightarrow -\displaystyle\int_V \dfrac{\partial\rho}{\partial t}\, dV = \int_V \vec{\nabla}\cdot\vec{J}\, dV \rightarrow \vec{\nabla}\cdot\vec{J} = -\dfrac{\partial\rho}{\partial t}$ Obbiettivo

Perchè manca un pezzo?

Ampére: $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$ $\rightarrow$ Div $\rightarrow$ $\vec{\nabla}(\cancel{\vec{\nabla} \wedge \vec{B}}) = \mu_0 \vec{\nabla}\vec{J}$ $\rightarrow$ $\vec{\nabla}\vec{J} = 0$ Valida solo in regime di correnti stazionario!

$\rightarrow$ Termine di Maxwell $\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\rightarrow$ Amperé Maxwell: $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

proof $\rightarrow$ Div $\rightarrow$ $\vec{\nabla}(\cancel{\vec{\nabla} \wedge \vec{B}}) = \mu_0 \vec{\nabla}\vec{J} + \mu_0 \varepsilon_0 \frac{\partial(\vec{\nabla}\vec{E})}{\partial t}$ $\rightarrow$ $\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$

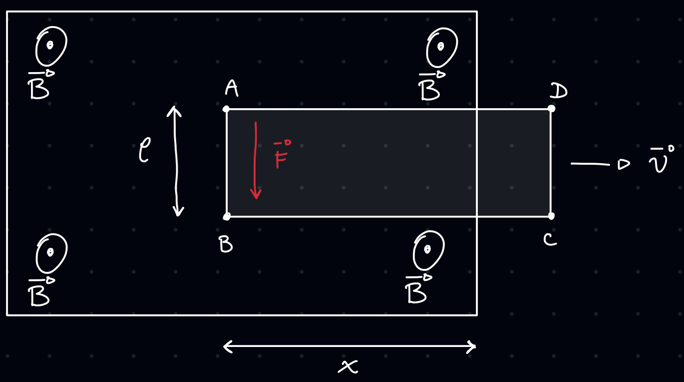
$\rightarrow$ $\cancel{\mu_0}\left(\vec{\nabla}\vec{J} + \cancel{\varepsilon_0}\frac{1}{\cancel{\varepsilon_0}}\frac{\partial \rho}{\partial t}\right) = 0$ $\vec{\nabla}\vec{J} = -\frac{\partial \rho}{\partial t}$ QED

Maxwell campo elettr.

$$\begin{cases} \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \oint \vec{E}\, d\vec{l} = 0 \to \vec{\nabla} \wedge \vec{E} = 0 \end{cases} \to \text{Manca un pezzo}$$

Faraday $\quad f_{em} = -\dfrac{d\phi_B}{dt} \quad \to \phi_B = \int \vec{B} \cdot \hat{n}\, ds = \int B\, ds \cos\theta$

Flusso Tagliato Varia Superficie



$\vec{F} = \text{Lorentz} = q\vec{E} + q(\vec{v} \wedge \vec{B})$

$$\Rightarrow f_{em} = \frac{L}{q} = \frac{\cancel{q}\int \vec{E} \cdot d\vec{l}}{\cancel{q}} + \frac{\cancel{q}\int (\vec{v} \wedge \vec{B})\, d\vec{l}}{\cancel{q}}$$

Se $E = 0 \to f_{em} = \oint (\vec{v} \wedge \vec{B})\, d\vec{l}$

$$f_{em} = \int_A^B (\vec{v} \wedge \vec{B})\, dl + \cancel{\int_B^C (\vec{v} \wedge \vec{B})\, dl} + \cancel{\int_C^D (\vec{v} \wedge \vec{B})\, dl} + \cancel{\int_D^A (\vec{v} \wedge \vec{B})\, dl} = \int_A^B (\vec{v} \wedge \vec{B})\, dl = \int v \cdot B \cdot \underbrace{\sin\theta}_{1}\, dl = \underline{vBl}$$

$\vec{F} \perp \vec{B} \qquad \vec{B} = 0 \qquad \vec{F} \perp \vec{B} \qquad$ Forza EM

Troviamo lo stesso risultato con $f_{em} = -\dfrac{d\phi_B}{dt}$?

la superficie S dipende da x: $\quad S = b \times h = l \cdot x$

$$\phi_B = \int \vec{B} \cdot \hat{n}\, ds = B \underbrace{\int_S ds}_{l \cdot x} \Rightarrow \underline{\phi_B = Blx}$$

FLUSSO

ad un tempo $t' = t + \Delta t \to \quad \phi_B(t+\Delta t) = Bl(x - \Delta x)$ con $\Delta x = v \cdot \Delta t$

$$\Rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \frac{\phi_B(t+\Delta t) - \phi(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\cancel{Blx} - Bl\Delta x - \cancel{Blx}}{\Delta t} = -Bl \overbrace{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}^{v}$$

$$\Rightarrow -\frac{d\phi_B}{dt} = Blv \quad \text{QED}$$

Flusso concatenato $\rightarrow$ varia $\vec{B}$

$$\rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \frac{\int \vec{B}(t+\Delta t, \vec{r})\,\hat{n}\,dS - \int B(t,\vec{r})\,\hat{n}\,dS}{\Delta t}$$

Approssimo: $\vec{B} \simeq \vec{B}(t,\vec{r}) + \frac{\partial \vec{B}(t+\Delta t, \vec{r})}{\partial t} \cdot \Delta t$

$$\rightarrow \frac{d\phi_B}{dt} = \lim_{\Delta t \to 0} \cancel{\int \vec{B}(t,\vec{r})\,\hat{n}\,dS} + \frac{\int \frac{\partial \vec{B}}{\partial t}(t+\cancel{\Delta t},\vec{r}) \cdot \cancel{\Delta t} \cdot \hat{n}\,dS}{\cancel{\Delta t}} - \cancel{\int \vec{B}(t,\vec{r})\,\hat{n}\,dS}$$

$$\rightarrow \frac{d\phi_B}{dt} = \int \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,dS$$

$$\Rightarrow f_{em} = \frac{L}{q} \qquad \text{se } F = q\vec{E} + q(\vec{v} \wedge \vec{B}) \rightarrow f_{em} = \int \vec{E} \cdot d\vec{l} + \int (\vec{v} \wedge \vec{B})\,d\vec{l}$$

$$\Rightarrow \oint \vec{E}\,d\vec{l} + \oint (\vec{v} \wedge \vec{B})\,d\vec{l} = \int \frac{\partial B}{\partial t}\,\hat{n}\,dS \qquad \text{se } v = 0 \rightarrow \int_l \vec{E}\,d\vec{l} = \int_S \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,dS$$

$$\rightarrow \int_S (\vec{\nabla} \wedge \vec{E})\,\hat{n}\,dS = \int_S \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,dS \rightarrow \vec{\nabla} \wedge \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

Eq. di maxwell
campo elettromagn.

Tutto insieme

Tagliato: $-\frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B})\,d\vec{l}$

Concatenato: $-\frac{d\phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,dS$

Generale $\Rightarrow$ $-\frac{d\phi_B}{dt} = \oint (\vec{v} \wedge \vec{B})\,d\vec{l} - \int \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,dS$

$\Rightarrow$ Scrivo $f_{em} = -\frac{d\phi}{dt}$ $\rightarrow$ $\oint \vec{E} \cdot d\vec{l} + \cancel{\oint (\vec{v} \wedge \vec{B})\,d\vec{l}} = \cancel{\oint (\vec{v} \wedge \vec{B})\,d\vec{l}} - \int \frac{\partial \vec{B}}{\partial t}\,\hat{n}\,dS$

$$\rightarrow \oint_l \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t}\,\hat{n}\,dS \rightarrow \int_S \vec{\nabla} \wedge \vec{E}\,d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t}\,d\vec{S} \rightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Auto induzione

Legge di Laplace $\quad B = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{e} \wedge \vec{r}}{\Delta r^3}$

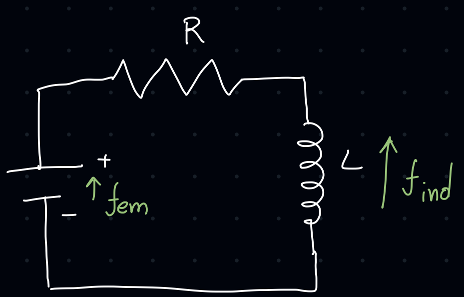
flusso $\rightarrow \quad \phi_B = \int_S \frac{\mu_0 I}{4\pi}\int_e \frac{d\vec{e}\wedge\vec{r}}{\Delta r^3}\,\hat{n}\,dS \qquad$ Tutto costante tranne I

$\Rightarrow \quad \phi_B = L \cdot I \quad \rightarrow \quad$ Se $B \neq cost \rightarrow \quad \frac{d\phi_B}{dt} = L \cdot \frac{dI}{dt}$

(L: Induttanza)

Siccome $\quad f_{em} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$

Circuito R-L



Dalle leggi di Kirchhoff

$f_{em} - f_{ind} = R \cdot I$

$\rightarrow \quad f_{em} - \frac{L\,dI}{dt} = R \cdot I \quad \Rightarrow \quad \underline{f_{em} = RI + L\frac{dI}{dt}}$

Eq differenziale

Soluzione del tipo $\quad I = Ae^{\alpha t} + D$

$\rightarrow \frac{dI}{dt} = A\alpha e^{\alpha t} \quad \Rightarrow \quad f_{em} = R \cdot Ae^{\alpha t} + LA\alpha e^{\alpha t} + RD \qquad$ ma $f_{em} = cost$

$\searrow f_{em} = R \cdot D$

$\Rightarrow RAe^{\alpha t} + LA\alpha e^{\alpha t} = \emptyset \rightarrow Ae^{\alpha t}(R + L\alpha) = 0 \rightarrow R + L\alpha = 0 \rightarrow \alpha = -\frac{R}{L} \qquad \rightarrow D = \frac{f}{R}$

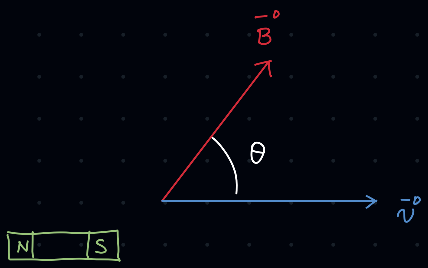
$\Rightarrow I = Ae^{-\frac{R}{L}t} + \frac{f}{R} \qquad$ Soluzione generale

$\rightarrow$ Trovo A: $\quad I(0) = 0 \rightarrow I(0) = Ae^{-\frac{R}{L}t} + \frac{f}{R} = 0 \quad$ per $\quad A + \frac{f}{R} = 0 \rightarrow A = -\frac{f}{R}$

(with $e^{-\frac{R}{L}t} = 1$)

$\Rightarrow I = -\frac{f}{R}e^{-\frac{R}{L}t} + \frac{f}{R} = \underline{\frac{f}{R}\left(-e^{-\frac{R}{L}t} + 1\right)} \qquad$ Sol Particolare

Legge di Lorentz



Sperimentalmente Lorentz trova queste qualità nella Forza

1) $\vec{F} \perp \vec{v}$, $\vec{F} \perp \vec{B}$

2) $\vec{F}$ dipende da $\sin\theta$

3) $\vec{F}$ dipende dalla carica $\Rightarrow F \propto q$

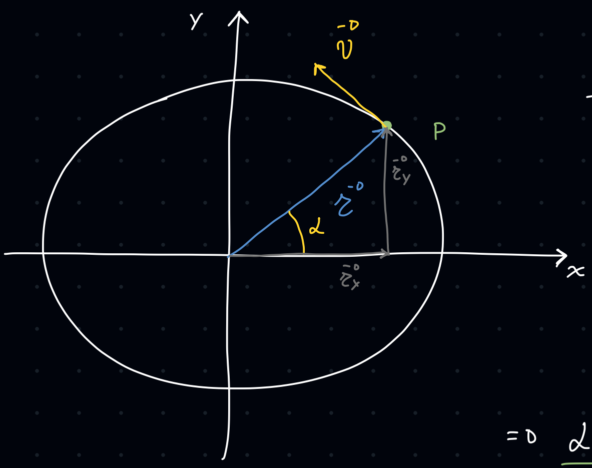
$\Rightarrow \vec{F}_L = q \cdot (\vec{v} \wedge \vec{B})$ perchè $\vec{v} \wedge \vec{B} = v \cdot B \cdot \sin\theta$ inoltre $\hat{v} \wedge \hat{B} \perp \hat{F}$

Se esiste un campo elettrico

$F_{coulomb} = \frac{1}{4\pi} \frac{q_1 \cdot q_2}{R^2} \Rightarrow E = \frac{F}{q} \Rightarrow \underline{F = q \cdot E}$ Forza di Coulomb

$\Rightarrow \underline{\vec{F}_{TOT} = \vec{F}_L + \vec{F}_C = qE + q(\vec{v} \wedge \vec{B})}$

Moto circolare



$\vec{\omega}$ = Velocità Angolare $= \frac{d\alpha}{dt} \Rightarrow$

$\rightarrow \quad \omega = \frac{d\alpha}{dt} \rightarrow d\alpha = \omega dt \rightarrow \int_{\alpha_0}^{\alpha_f} d\alpha = \int_{t_0}^{t_f} \omega \, dt$

$\Rightarrow \quad \alpha_f - \alpha_i = \omega(t_f - t_0)$

pongo $\alpha_0 = 0, \; t_0 = 0$

$\Rightarrow \quad \underline{\alpha(t) = \omega t}$

$\vec{r} = \vec{r}_x + \vec{r}_y = \hat{i} R \cos\alpha + \hat{j} R \sin\alpha$ Siccome $v = \frac{ds}{dt} = \dot{s}$

$\Rightarrow \quad R\cos\alpha + R\sin\alpha \rightarrow R\cos(\omega t) + R\sin(\omega t)$ Dipende da t

$\rightarrow \quad \vec{v} = -\hat{i}\,\omega R \sin(\omega t) + \hat{j}\,\omega R\cos(\omega t) = \omega R\left[\hat{j}\cos(\omega t) - \hat{i}\sin(\omega t)\right]$

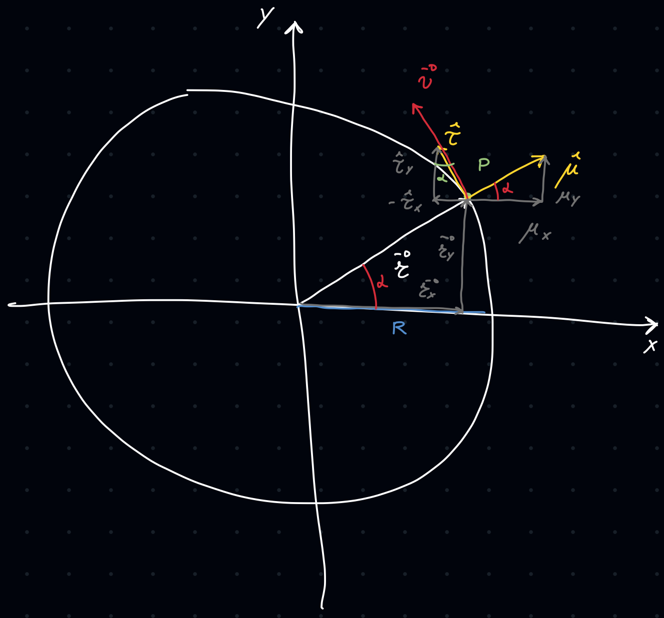
$\vec{a} = -\hat{i}\,\omega^2 R\cos(\omega t) - \hat{j}\,\omega^2 R\sin(\omega t) = -\omega^2 R\left[\underbrace{\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)}_{R}\right] = \underline{-\omega^2 R\,\vec{r}}$

Direzione opposta ad $\vec{r}$

Modulo

$|\vec{v}| = \sqrt{\omega^2 R^2\left[\underbrace{\cos^2(\alpha) + \sin^2(\alpha)}_{1}\right]} = \sqrt{\omega^2 R^2} = \underbrace{\omega R}_{|\vec{v}|} \Rightarrow v = \omega R \rightarrow \omega = \frac{v}{R}$ (1)

$|\vec{a}| = \sqrt{\omega^4 R^2 [1]} = \underbrace{\omega^2 R}_{|\vec{a}|} \rightarrow |\vec{a}| = \omega^2 R \Rightarrow a = \frac{v^2}{R^2} \cdot R \rightarrow a = \frac{v^2}{R}$ (2)



$$\hat{\mu} = \hat{\mu}_x + \hat{\mu}_y = \hat{i}\mu\cos\alpha + \hat{j}\mu\sin\alpha = \hat{i}\cos\alpha + \hat{j}\sin\alpha$$

$$\hat{\tau} = \hat{\tau}_x + \hat{\tau}_y = -\hat{i}\sin\alpha + \hat{j}\cos\alpha$$

$$\Rightarrow \begin{cases} \hat{\mu} = \hat{i}\cos\alpha + \hat{j}\sin\alpha \\ \hat{\tau} = -\hat{i}\sin\alpha + \hat{j}\cos\alpha \end{cases} \Rightarrow \begin{cases} \hat{\mu} = \hat{i}\cos(wt) + \hat{j}\sin(wt) \\ \hat{\tau} = -\hat{i}\sin(wt) + \hat{j}\cos(wt) \end{cases}$$

$$w = \frac{d\alpha}{dt} \Rightarrow \alpha = wt$$

$$\Rightarrow \begin{cases} \dot{\mu} = -\hat{i}\,w\sin(wt) + \hat{j}\,w\cos(wt) = w\underbrace{\left[-\hat{i}\sin(wt) + \hat{j}\cos(wt)\right]}_{\hat{\tau}} = w\hat{\tau} \quad \dot{\mu} \\ \dot{\tau} = -\hat{i}\,w\cos(wt) - \hat{j}\,w\sin(wt) = -w\underbrace{\left[\hat{i}\cos(wt) + \hat{j}\sin(wt)\right]}_{\hat{\mu}} = -w\hat{\mu} \quad \dot{\tau} \end{cases}$$

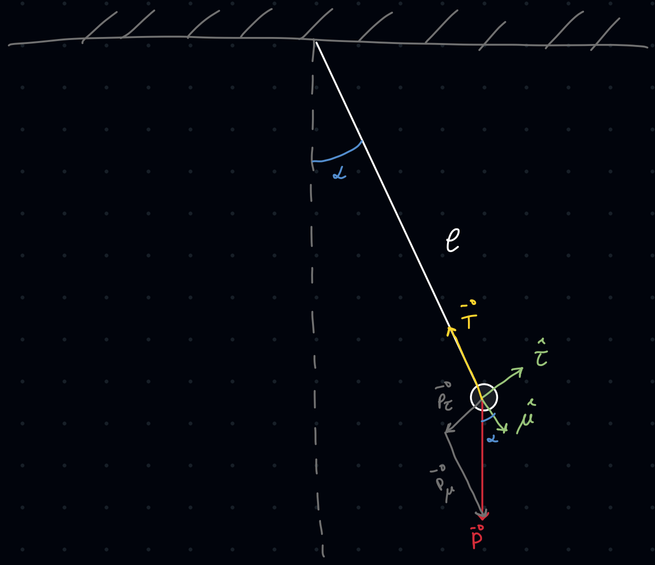
$$\Rightarrow \text{Se} \quad v = \frac{dS}{dt} = \frac{d}{dt}\left[\tau_x + \tau_y\right] = \frac{d}{dt}\left[\hat{i}R\cos(wt) + \hat{j}R\sin(wt)\right] = \frac{d}{dt}\left[R\underbrace{\left(\hat{i}\cos(wt) + \hat{j}\sin(wt)\right)}_{\hat{\mu}}\right]$$

$$\Rightarrow \dot{S} = \frac{d}{dt}\left[R\hat{\mu}\right] = R\left[w\hat{\tau}\right] \Rightarrow \vec{v} = Rw\hat{\tau} \quad (1)$$

$$\text{Se} \quad v = \text{cost} \rightarrow \quad \vec{a} = Rw\left[-w\hat{\mu}\right] = -Rw^2\hat{\mu} \quad (2)$$

$$\text{Se } v \neq \text{cost} \rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = R\cdot\frac{dw}{dt}\hat{\tau} + Rw\frac{d\hat{\tau}}{dt} = \underbrace{R\dot{w}\hat{\tau}}_{\text{Acc Tang.}}\ \underbrace{-Rw^2\hat{\mu}}_{\text{Acc centripeta}}$$

Pendolo Semplice



$$\begin{cases} \text{Lungo } \hat{\mu}: & \vec{P}_{\hat{\mu}} - \vec{T} = m\, a_\mu \\ \text{Lungo } \hat{\tau}: & \vec{P}_\tau = m \cdot \vec{a}_{\hat{\tau}} \end{cases}$$

$$\begin{cases} \vec{P}_\mu = mg \cdot \cos\alpha \\ \vec{P}_\tau = -mg \sin\alpha \end{cases}$$

$$\Rightarrow \begin{cases} mg\cos\alpha - \vec{T} = m \cdot \vec{a}_\mu \\ -mg\sin\alpha = m\vec{a}_\tau \end{cases}$$

Ci sono due Acc perchè $\omega \neq cost$

$\rightarrow$ Acc centripeta. $\vec{a}_\mu = -\dfrac{v^2}{R} = -\dfrac{\dot{S}}{l}$ (1)

$\rightarrow$ Acc Tangenziale: $\vec{a}_\tau = \ddot{S} \cdot \hat{\tau}$

$$\Rightarrow \begin{cases} mg\cos\alpha - \vec{T} = -\dfrac{m\dot{S}}{l} & \text{Inutile} \\ -mg\sin\alpha = m\ddot{S}\hat{\tau} \end{cases}$$

$\rightarrow$ Approssimazione per piccole oscillazioni $\rightarrow$ $\sin\alpha \sim \alpha$

$\Rightarrow -\cancel{m}g\alpha = \cancel{m}\ddot{S}\hat{\tau} \rightarrow \ddot{S}\hat{\tau} + g\alpha = \emptyset$ Siccome $1 \text{ Rad} = \dfrac{l}{R} \rightarrow \alpha = \dfrac{S}{l}$

$\Rightarrow \ddot{S}\hat{\tau} + S\dfrac{g}{l} = 0$ battezzo $\dfrac{g}{l} = k^2 \rightarrow \ddot{S} + k^2 S = 0$ Eq. differenziale

Soluzione: $S(t) = A\cos(kt + \varphi)$

Trovare K

Cos è periodico di 2π $\Rightarrow S(t+T_0) = A\cos(kt + \varphi + 2\pi)$

$\rightarrow A\cos(kt + kT_0 + \varphi) = A\cos(kt + \varphi + 2\pi) \Leftrightarrow \cancel{kt} + kT_0 + \cancel{\varphi} = \cancel{kt} + \cancel{\varphi} + 2\pi$

$\rightarrow kT_0 = 2\pi \Rightarrow k = \dfrac{2\pi}{T_0}$ Battezzo $\omega = \dfrac{2\pi}{T_0}$ velocità Angolare

$\Rightarrow S(t) = A\cos(\omega t + \varphi)$

Periodo del pendolo

Se $k = \dfrac{2\pi}{T_0} = \omega \rightarrow T_0 = \dfrac{2\pi}{k}$ ma $k^2 = \dfrac{g}{l} \Rightarrow T_0 = 2\pi\sqrt{\dfrac{l}{g}}$

Energia Cinetica

$$L = F \cdot S \Rightarrow L = \int_{S_0}^{S_f} \vec{F}\, ds \quad \text{ma} \quad \vec{F} = m \cdot \vec{a} \Rightarrow L = m \int_{S_0}^{S_f} \vec{a} \cdot ds \qquad a = \frac{dv}{dt}$$

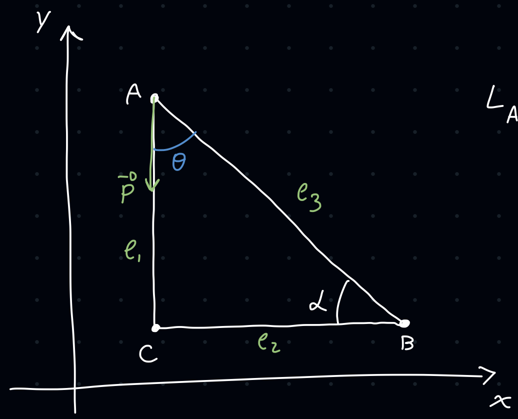
$$\Rightarrow L = m \int_{S_0}^{S_f} \frac{dv}{dt} \cdot ds \rightarrow L = m \int_{v_0}^{v_f} v\, dv = m\left[\frac{v^2}{2}\right]_{v_0}^{v_f} \Rightarrow \underline{\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2}$$

Energia cinetica

G = Energia cinetica , U = Energia potenziale

$$\Rightarrow L = G_B - G_A \quad \text{ma se pongo} \quad U = -G \Rightarrow L = -U_B - (-U_A) \Rightarrow \underline{L = U_A - U_B}$$

Campi conservativi : Peso



$$L_{AC} = \int_A^C \vec{P} \cdot de = mg\,(C-A) = \underline{mg\, e_1}$$

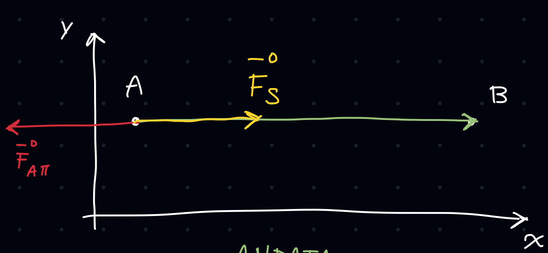
$$L_{ABC} = \int_A^B \vec{P} \cdot d\vec{e}_3 + \int_B^C \vec{P} \cdot d\vec{e}_2 = \int_A^B P \cdot \cos\theta\, de_3 + \int_B^C \cancel{P \underbrace{\cos\alpha}_{0}\, de_2}$$

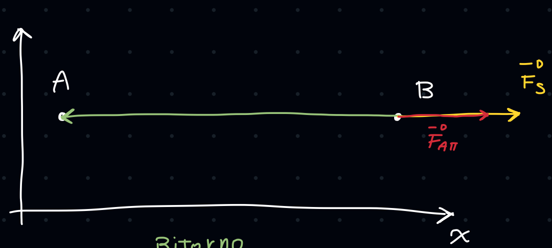
$$\Rightarrow \int_A^B P \cdot \cos de_3 \quad \text{ma} \quad de_3 \cos\theta = de_1 \Rightarrow \underline{L_{ABC} = mg\, e_1}$$

$\Rightarrow$ Conservativo

Non Conservativo : Attrito

$|\vec{F}_{AT}| = \mu_0 |\vec{N}|$ con verso opposto al moto





$$\Rightarrow L_1 = \int_A^B \vec{F}_S\, de - \int_A^B \vec{F}_{AT}\, de = F_S \cdot S - F_{AT} \cdot S$$

$$L_2 = -\int_B^A \vec{F}_S\, de - \int_B^A \vec{F}_{AT}\, de = -F_S S - F_{AT} \cdot S$$

$\Rightarrow L_1 + L_2$ dovrebbe essere zero, invece $L_1 + L_2 = S\left[F_S - F_A - F_S - F_A\right] = -2F_{AT}$

Non conservativo!

Impulso

$$\vec{I} = \int_{t_0}^{t_f} \vec{F}\, dt \qquad \vec{p} = m \cdot \vec{v}$$

$$\vec{I} = \int_{t_0}^{t_f} \vec{F} \cdot dt = \int_{t_0}^{t_f} m \cdot \vec{a} \cdot dt = m \int_{v_0}^{v_f} \frac{dv}{\cancel{dt}} \cancel{dt} = m\,(v_f - v_0) = \vec{p}_f - \vec{p}_0$$

Teorema Impulso

Momenti :

FORZA

$$\vec{r} \wedge \vec{F} = \vec{M}$$

ANGOLARE

$$\vec{r} \wedge \vec{p} = \vec{L}$$

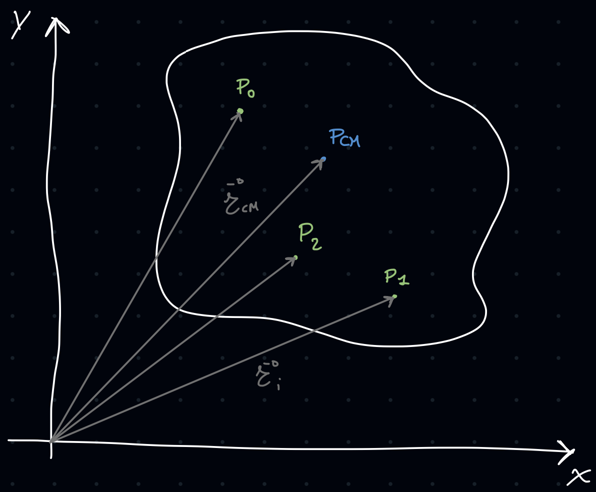
Faccio la derivata $\rightarrow$ $\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \wedge \vec{p}) \Rightarrow \frac{d\vec{L}}{dt} = \frac{dr}{dt} \wedge \vec{p} + \vec{r} \wedge \frac{d\vec{P}}{dt}$

$$\rightarrow \frac{d\vec{L}}{dt} = \vec{v} \wedge \vec{p} + \vec{r} \wedge \underbrace{m \cdot \vec{a}}_{\vec{F}} \Rightarrow \frac{d\vec{L}}{dt} = \vec{r} \wedge \vec{F} \Rightarrow \vec{M} = \frac{d\vec{L}}{dt}$$

QED

$$\vec{v} \wedge m \cdot \vec{v} = \emptyset$$

Centro massa



Per definizione $\vec{R}_{CM} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \dfrac{\sum_i m_i \vec{r}_i}{M}$

$\Rightarrow v_{CM} = \dfrac{\sum m_i v_i}{M}$ ma $m \cdot v = \vec{p} \Rightarrow \vec{v}_{CM} = \dfrac{\sum_i \vec{p}_i}{M}$

$\Rightarrow \vec{P}_{TOT} = M \cdot \vec{v}_{CM}$ (1)

$\vec{a}_{CM} = \dfrac{\sum_i m_i \vec{a}_i}{M}$ ma $\vec{F} = m \cdot \vec{a} \Rightarrow \vec{a}_{CM} = \dfrac{\sum_i \vec{F}_i}{M}$

$\Rightarrow \underline{\sum_i \vec{F}_i = M \cdot \vec{a}_{CM}}$ (2)

Teorema di König

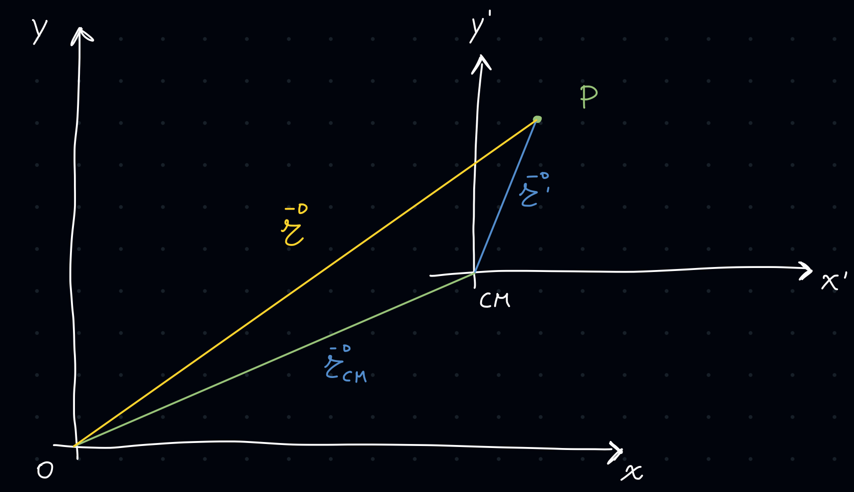
OSS: $\vec{r} = \vec{r}_{CM} + \vec{r}\,'$

$\Rightarrow \vec{v} = \vec{v}_{CM} + \vec{v}\,'$

Velocità di O

Vel DI CM

Vel MISURATA da CM



Momento Angolare $\vec{L} = \vec{r} \wedge m \cdot v$

ma $\vec{r} = \vec{r}_{CM} + \vec{r}\,'$ e $\vec{v} = \vec{v}_{CM} + \vec{v}\,'$

$$\Rightarrow \vec{L} = (\vec{r}_{CM} + \vec{r}\,') \wedge m \cdot (\vec{v}_{CM} + \vec{v}\,') = \vec{r}_{CM} \wedge m \vec{v}_{CM} + \vec{r}_{CM} \wedge m \vec{v}\,' + \vec{r}\,' \wedge m \vec{v}_{CM} + \vec{r}\,' \wedge m \vec{v}\,'$$

Si tratta di un sistema $\Rightarrow$ si parla di $\vec{L}_{TOT} = \sum L = \sum_i \vec{r}_i \wedge m_i \vec{v}_i$

$$\Rightarrow L_{TOT} = \sum_i r_{CM} \wedge m_i v_{CM} + \sum_i r_{CM} \wedge m_i v_i' + \sum_i \vec{r}_i\,' \wedge m_i \vec{v}_{CM} + \sum \vec{r}_i\,' \wedge m_i \vec{v}_i\,'$$

$$= \vec{r}_{CM} \wedge M \vec{v}_{CM} + \underbrace{\cancel{r_{CM} \wedge \sum_i m_i \vec{v}_i\,'}}_{\sum_i m_i \vec{v}_i\,' = M \vec{v}\,'_{CM}} + \underbrace{\cancel{\sum_i M \vec{r}_i\,' \wedge \vec{v}_{CM}}}_{\sum_i m_i \vec{r}_i\,' = M \vec{r}\,'_{CM}} + \sum_i \vec{r}_i \wedge m_i \vec{v}_i$$

$$= \vec{r}_{CM} \wedge M \vec{v}_{CM} + \sum_i \vec{r}_i \wedge m_i \vec{v}_i$$

Momento angolare del sistema misurato dal CM

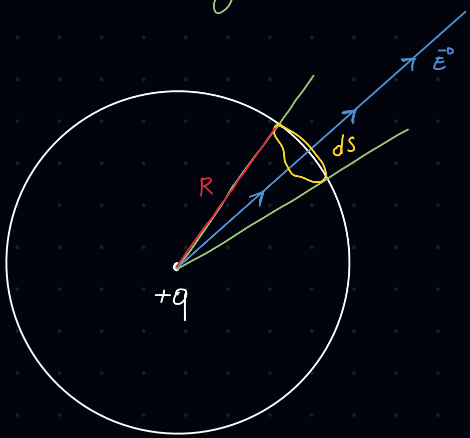
$$= \vec{L}_{CM} + L'_{TOT}$$

Teorema di coulomb

$$\int \vec{E} \cdot \hat{n}\, ds = \frac{Q}{\varepsilon_0} \quad \text{Gauss} \rightarrow \quad \rho = \frac{dQ}{dV} \rightarrow \quad Q = \int_V \rho\, dV \quad \Rightarrow \quad \int \vec{E} \cdot \hat{n}\, ds = \frac{1}{\varepsilon_0} \int_V \rho\, dV$$

$$\rightarrow \int_V (\vec{\nabla} \vec{E})\, dV = \frac{1}{\varepsilon_0} \int_V \rho\, dV \quad \Rightarrow \quad \vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{Maxwell}$$

Dimostrare Gauss



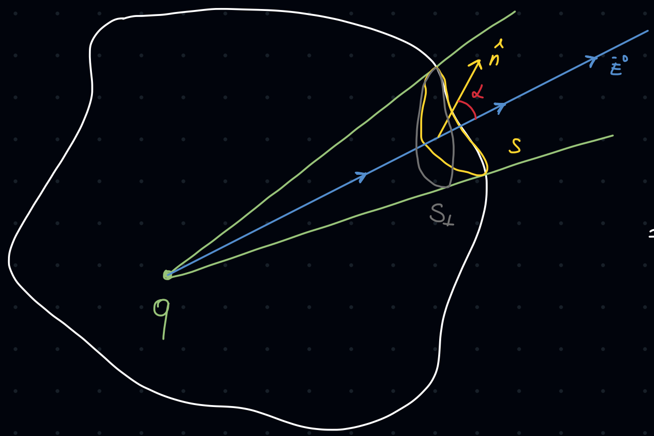
$$\phi_E = \int_S \vec{E} \cdot \hat{n}\, ds \qquad \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r}$$

Sfera → costante

$$= \frac{q}{4\pi\varepsilon_0 r^2} \int_S \hat{r} \cdot \hat{n}\, ds \qquad \hat{r} \perp \hat{n} \Rightarrow \int_S ds = 4\pi r^2$$

$$\Rightarrow \phi_E = \int \vec{E} \cdot \hat{n}\, ds = \frac{q}{\varepsilon_0} \quad \text{QED}$$

Superficie qualsiasi



Possiamo trovare la sup $S_\perp$ che è perpend. al campo $\vec{E}$

$$\Rightarrow S_\perp = S \cos\theta$$

$$\Rightarrow \phi_E = \oint \vec{E} \cdot \hat{n}\, ds = \oint \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} \cdot \hat{n} \cdot dS$$

non costante

$$\hat{r} \cdot \hat{n} = r \cdot n \cdot \cos\theta = \cos\theta$$

$$\Rightarrow \phi_E = \oint \vec{E} \cdot \hat{n}\, ds = \frac{q}{4\pi\varepsilon_0} \cdot \oint \frac{1}{r^2} \cdot \cos\theta \cdot dS = \frac{q}{4\pi\varepsilon_0} \oint \frac{1}{r^2} dS'$$

ma dS' è la sup. di una sfera ⇓ $r = \text{cost}$

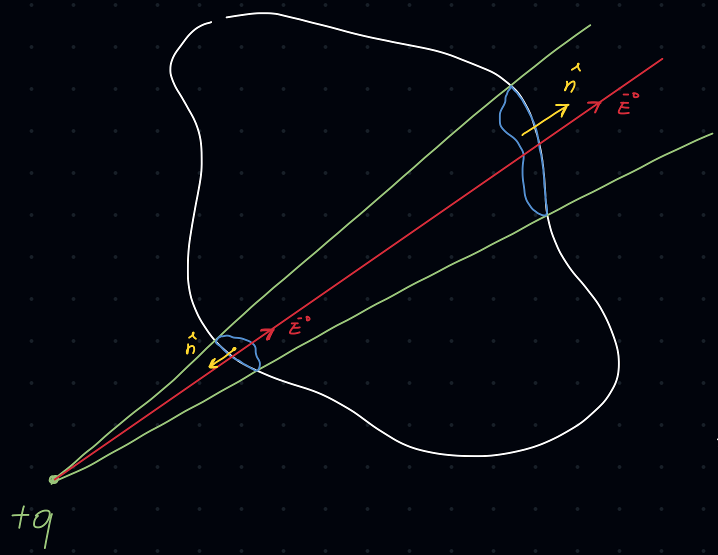
$$= \frac{q}{\cancel{4\pi}\varepsilon_0} \frac{1}{\cancel{r^2}} \cdot \cancel{4\pi r^2} = \frac{q}{\varepsilon_0}$$

Possiamo anche definire Angolo solido. $d\Omega = \frac{dS_\perp}{r^2} = \frac{dS \cos\theta}{r}$

$$\Rightarrow \phi_E = \frac{q}{4\pi\varepsilon_0} \oint \frac{\hat{r} \cdot \hat{n}}{r^2} ds = k \oint \frac{\cos\theta}{r^2} ds = k \oint \frac{dS_\perp}{r^2} = k \oint d\Omega$$

$$\oint d\Omega = 4\pi \qquad \Rightarrow \phi_E = \frac{q}{4\pi\varepsilon_0} 4\pi = \frac{q}{\varepsilon_0} \quad \text{QED}$$

Carica esterna



Per il teorema appena dimostrato

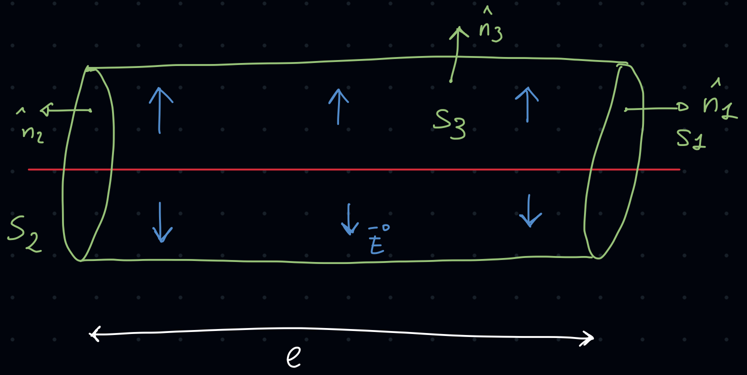
$\phi_{S_1} = \int_S \vec{E} \cdot \hat{n}_1 \, dS_1 = \frac{q}{\varepsilon_0}$ prendo $\hat{n}_1$ come riferimento

$\phi_{S_2} = \int_S \vec{E} \cdot \hat{n}_2 \, dS_2$ ma $\hat{n}_2 = -\hat{n}_1$

$$\Rightarrow \phi_{S_2} = -\int_S \vec{E} \cdot \hat{n} \, dS_2 = -\frac{q}{\varepsilon_0}$$

$$\Rightarrow \phi_{S_1} = -\phi_{S_2} \Rightarrow \phi_{S_1} + \phi_{S_2} = \emptyset$$

Filo carico - Coulomb



$\vec{E}\perp\hat{n}_1$ $\qquad$ $\vec{E}\perp\hat{n}_2$

$$\phi_E = \oint \vec{E}\cdot\hat{n}\, dS = \cancel{\int_S E\cdot\hat{n}_1\, dS_1} + \cancel{\int_S E\hat{n}_2\, dS_2} + \int_S E\hat{n}_3\, dS_3$$

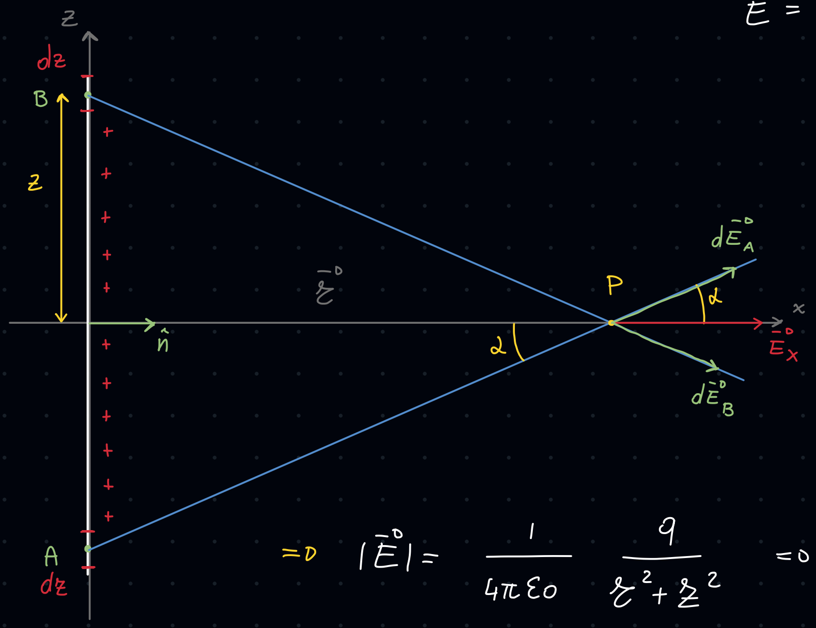
$$= \int \vec{E}\cdot n_2\, dS_2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \phi_E = |\vec{E}|\cdot 2\pi r\ell = \frac{Q}{\varepsilon_0} \quad \text{ma } \ell=\infty$$

$\Rightarrow \lambda =$ densità lineare di carica $= \dfrac{Q}{\ell} \Rightarrow Q = \lambda\ell$

$$|\vec{E}|\, 2\pi r \cancel{\ell} = \frac{\lambda\cancel{\ell}}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi r\varepsilon_0}\hat{n}$$

Filo carico senza Coulomb



$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ Non conosco r → AP / BP

$$\begin{cases} AP = r\cos\alpha \\ BP = r\cos\alpha \end{cases} \Rightarrow AP = BP = r\cos\alpha$$

Non conosco α

$$AP^2 = z^2 + r^2$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+z^2} \Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}$$

siccome $\lambda = \dfrac{dQ}{d\ell} \Rightarrow dq = \lambda\, d\ell \Rightarrow dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dz}{r^2+z^2}$

nel nostro caso $d\ell = dz$

→ le componenti z ed y di $d\vec{E}_A + d\vec{E}_B$ si elidono a vicenda $\Rightarrow$ ci serve solo $\vec{E}_z$

$\Rightarrow d\vec{E}_z = d\vec{E}\cos\alpha$ ma non conosciamo α

$$\rightarrow z = AP\cos\alpha \Rightarrow z = \sqrt{z^2+r^2}\cos\alpha \Rightarrow \cos\alpha = \frac{z}{\sqrt{z^2+r^2}}$$

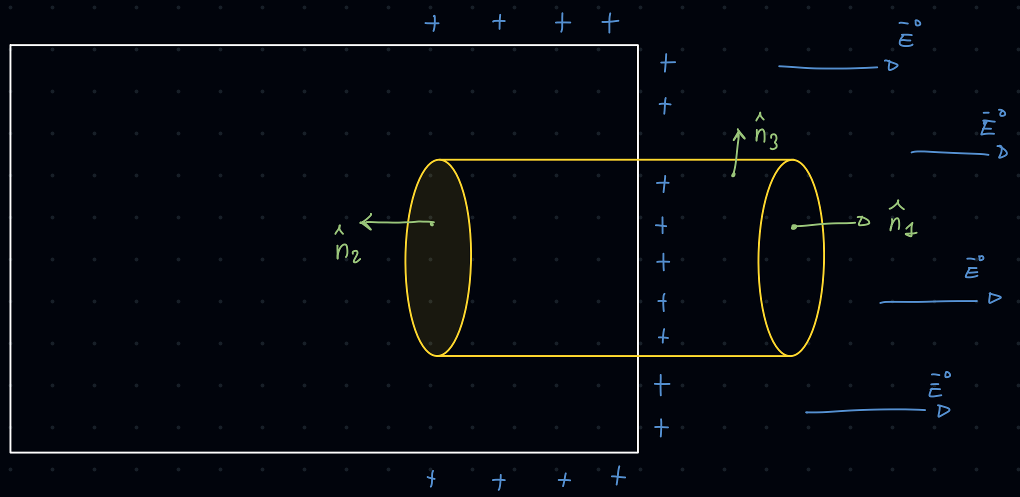
$$\Rightarrow dE_z = dE\cos\alpha = \frac{z}{\sqrt{z^2+r^2}}\, dE$$

$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\, dz}{r^2+z^2}\cdot\frac{z}{\sqrt{z^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda z\, dz}{(r^2+z^2)^{\frac{3}{2}}} \quad (1)$$

Integriamo per ottenere $\vec{E}_{TOT}$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\overset{\text{cost}}{\lambda}}{(r^2+z^2)^{\frac{3}{2}}}\,dz = 2\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\ell}\frac{z}{(r^2+z^2)^{\frac{3}{2}}}\,dz = \underbrace{\frac{\cancel{2}\lambda\cancel{z}}{2\cancel{4}\pi\varepsilon_0\, r^{\cancel{2}}}\cdot\frac{\ell}{\sqrt{r^2+\ell^2}}}_{|\vec{E}|}$$

Teorema di Coulomb



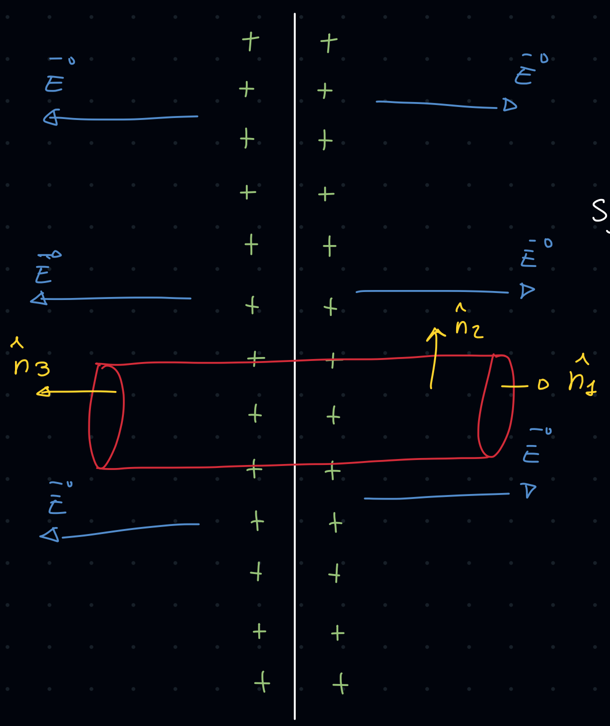
$$\phi_E = \oint \vec{E} \cdot \hat{n}\, dS = \int \vec{E} \cdot \hat{n}_1 dS_1 + \underbrace{\int \vec{E} \cdot \hat{n}_2\, dS_2}_{\rho = \emptyset} + \underbrace{\int \vec{E} \cdot \hat{n}_3 dS_3}_{\hat{n}_3 \perp \vec{E}} = \int \vec{E} \cdot \hat{n}_1\, dS_1 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow |\vec{E}| \cdot S_1 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{S_1} \cdot \frac{1}{\varepsilon_0} \quad \text{ma} \quad \sigma = \frac{Q}{S} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Alternativamente

$$\sigma = \frac{dQ}{dS} \Rightarrow dq = \sigma\, dS \to \quad q = \int \sigma\, dS \to E = \frac{\int \sigma\, dS}{S \varepsilon_0} = \sigma \frac{\cancel{\int dS}}{\cancel{S} \varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Strato carico



$$\phi_E = \oint \vec{E}\cdot\hat{n}\,dS = \int \vec{E}\cdot\hat{n}_1\,dS_1 + \cancel{\int \vec{E}\cdot\hat{n}_2\,dS_2} + \int \vec{E}\cdot\hat{n}_3\,dS_3$$

$\vec{E} \perp \hat{n}_2$

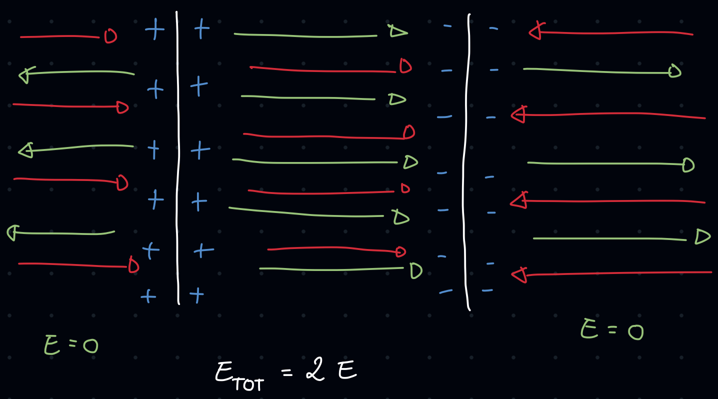
$$S_3 = S_1 \quad = 2\int_S \vec{E}\,n_1\,dS_1 = 2\,E\cdot S_1 = E\cdot 4\pi r$$

$$\Rightarrow E\cdot 4\pi r = \frac{Q}{\varepsilon_0}$$

$$\sigma = \frac{dq}{dS} \Rightarrow q = \int \sigma\,dS \Rightarrow$$

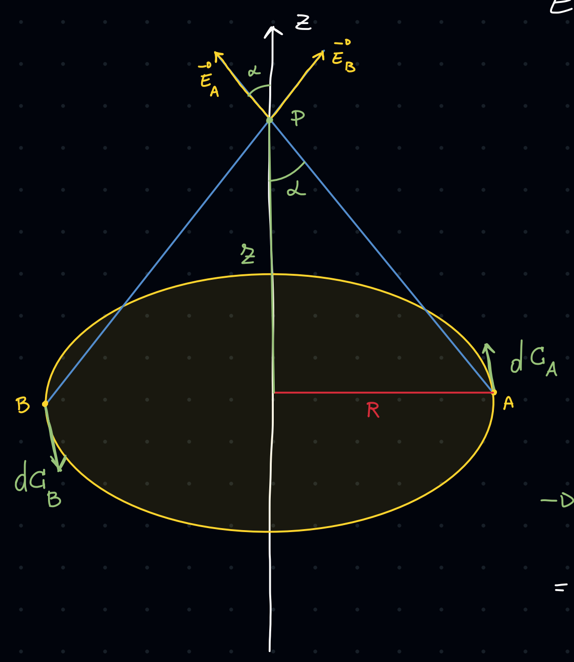
$$\Rightarrow E\cdot \cancel{4\pi r}^{\,2} = \frac{\sigma \cancel{\int dS}}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\,\varepsilon_0}$$

Doppio strato carico



$$\Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0}$$

Anello Carico



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \rightarrow \text{Nel nostro caso } \overline{AP} = ?$$

$$\Rightarrow z = AP\cos\alpha \rightarrow AP^2 = z^2 + R^2$$

$$\Rightarrow z = \sqrt{z^2+R^2}\,\cos\alpha$$

$$\Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{z^2+R^2}$$

$\rightarrow$ $\vec{E}_A + \vec{E}_B$ Annullano le componenti y ed x

$\Rightarrow$ ci serve solo $E_z = E\cos\alpha$

$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dq\cos\alpha}{z^2+R^2} \rightarrow ??$$

$$z = AP\cos\alpha \rightarrow \cos\alpha = \frac{z}{\sqrt{z^2+z^2}}$$

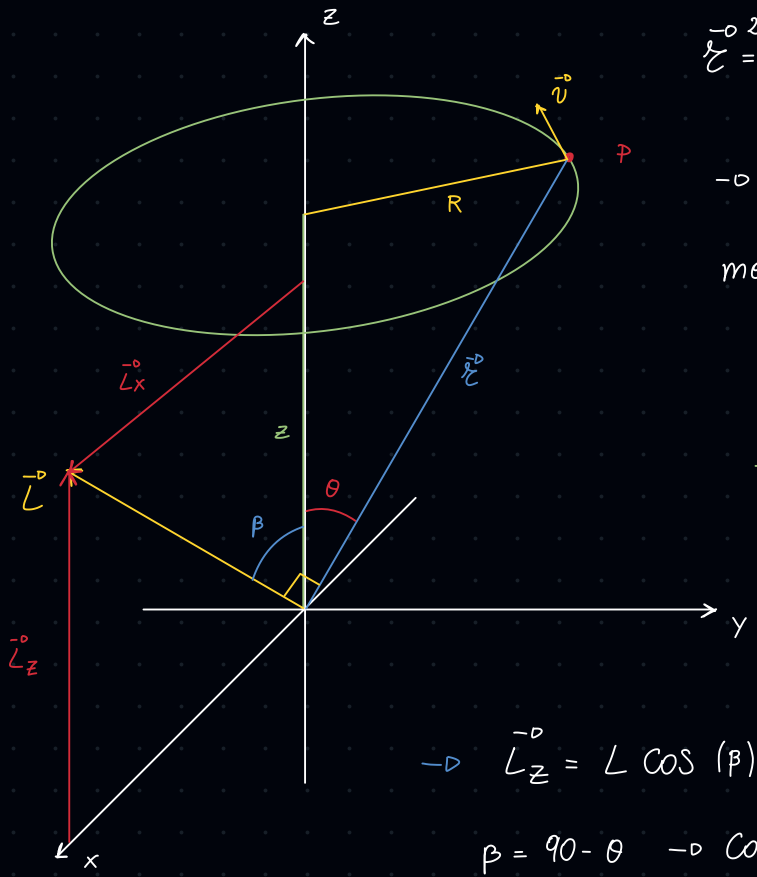
$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(z^2+R^2)^{\frac{3}{2}}}$$

Siccome l'anello è una circonferenza $\Rightarrow$ ho una superficie

$$\Rightarrow \lambda = \frac{dq}{ds} \Rightarrow dq = \lambda\, ds \rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2+R^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_z = \frac{\lambda}{\cancel{4\pi}_{2}\varepsilon_0}\frac{1}{(z^2+R^2)^{\frac{3}{2}}}\underbrace{\int ds}_{\cancel{2\pi} R} = \frac{\lambda R}{\varepsilon_0(z^2+z^2)^{\frac{3}{2}}}$$

Rotazione Asse Fisso



Momento Angolare: $\vec{L} = \vec{r} \wedge m\vec{v} = \vec{r} \wedge \vec{p}$

$\vec{r}^{\,2} = R^2 + z^2$; $R = r \sin\theta$

$\rightarrow L = r \cdot m \cdot v \cdot \sin(90) = r m v$

ma $v = \omega R$

$\Rightarrow L_i = r_i m_i \omega R_i$

$\Rightarrow$ Le componenti x ed y si annullano

$\rightarrow$ ci serve solo la componente z di L_i

$\rightarrow \vec{L}_z = L \cos(\beta)$ ma $\beta = ?$

$\beta = 90 - \theta \rightarrow \cos(90-\theta) = \sin(\theta)$

$\Rightarrow L_{z_i} = L_i \sin\theta = r_i m_i \omega R_i \sin\theta \Rightarrow \underbrace{(r_i \cdot \sin\theta)}_{R} \cdot R_i \, m_i \, \omega$

$\rightarrow L_z = R_i^2 m_i \omega$ (1)

$\Rightarrow L_{TOT} = \omega \sum_i m_i R_i^2$ pongo $\sum_i m_i R_i^2$ = Momento di inerzia Totale $= I_{TOT}$

$\Rightarrow \underline{\vec{L}_{TOT} = \omega \cdot I_{TOT}}$

Nel caso di corpi rigidi

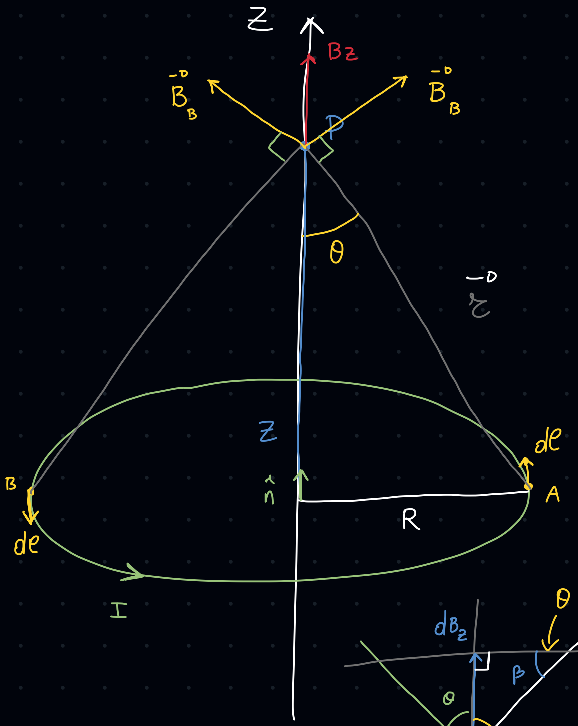
$\vec{M} = \dfrac{d\vec{L}}{dt} \Rightarrow M = \dfrac{d\vec{\omega}}{dt} I \Rightarrow \vec{M} = \vec{\alpha} I$ ANALOGO A $\vec{F} = m \cdot \vec{a}$

$\dfrac{d\vec{\omega}}{dt}$: α Acc Ang. ; I: cost

$\omega = \omega_0 + \int_{t_0}^{t_f} \alpha \, dt$, $\theta = \theta_0 + \int_{t_0}^{t_f} \omega \, dt$

$\rightarrow v(t) = v_0 + a \cdot t \rightarrow \omega(t) = \omega_0 + \alpha \cdot t$ con $\alpha = \dfrac{d\omega}{dt} = 0$

$S(t) = S_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} a t^2$ con $\alpha = \dfrac{d\omega}{dt} \neq 0$

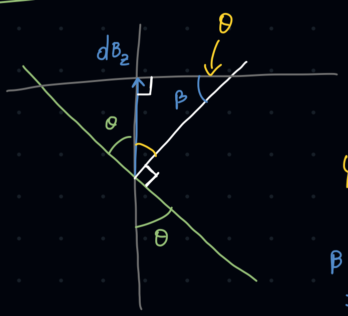


Dalla legge di Laplace

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\ell \times \vec{r}}{|\Delta \vec{r}^{\,3}|}$$

Le componenti x ed y si annullano a due a due

$\Rightarrow$ calcolo solo $dB_z = dB \cdot \sin\theta$



$\varphi = 180 - 90 - \theta = 90 - \theta$

$\beta = 180 - 90 - \varphi = 90 - 90 + \theta = \theta$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi}\oint \frac{d\vec{\ell} \times \vec{r}}{r^3}\cdot \sin\theta$$

$$= \frac{\mu_0 I}{4\pi}\oint \frac{d\ell \cdot \cancel{r}\cdot \overset{1}{\cancel{\sin\gamma}}}{r^{\cancel{3}}}\cdot \sin\theta$$

$$= \frac{\mu_0 I}{4\pi}\oint \underbrace{\frac{\sin\theta}{r^2}}_{\text{cost}}\, d\ell$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{\sin\theta}{r^2}\,\cancel{\oint d\ell}\; 2\pi R$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi r^2}\cdot \sin\cdot 2\pi R$$

$\rightarrow$ Non conosco $\rightarrow r \rightarrow R = r\sin\theta \rightarrow \sin\theta = \frac{R}{r}$

$r^2 = z^2 + R^2 \Rightarrow r = \sqrt{z^2+R^2}$

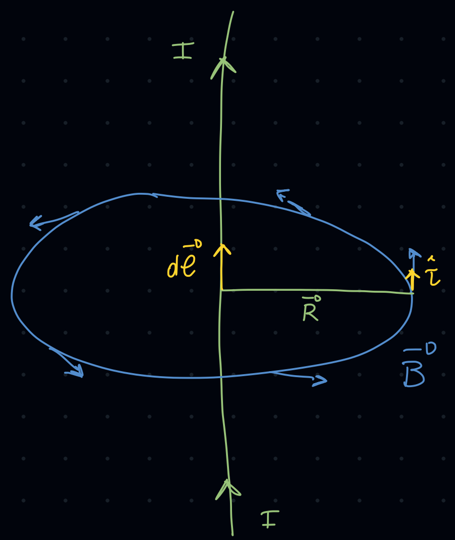
$$\Rightarrow \frac{\sin\theta}{r^2} = \frac{R}{r}\cdot\frac{1}{(z^2+R^2)} = \frac{R}{(z^2+R^2)^{\frac{1}{2}}\cdot(z^2+R^2)} = \frac{R}{(z^2+R^2)^{\frac{3}{2}}}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{\underset{2}{\cancel{4\pi}}}\,\frac{R^2}{(z^2+R^2)^{\frac{3}{2}}}\cdot\cancel{2\pi} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{\frac{3}{2}}}$$

F. Lorentz

$$\vec{F} = q\vec{E} + \underbrace{q \cdot (\vec{v} \wedge \vec{B})} \quad \rightarrow \quad \vec{F} = q(\vec{v} \wedge \vec{B}) \quad \text{ma} \quad v = \frac{d\vec{\ell}}{dt}$$

$$\Rightarrow \quad F = \underbrace{q \cdot \frac{d\vec{\ell}}{dt}}_{I} \wedge \vec{B} \quad \rightarrow \quad \vec{F} = I \, d\vec{\ell} \wedge \vec{B} \quad \text{(II Laplace)}$$



sperimentale

Biot - Savart : $\vec{B} = \vec{k} \cdot \frac{I}{R} \qquad \vec{k} = \hat{\tau} \cdot \frac{\mu_0}{4\pi}$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi} \frac{I}{R} \hat{\tau} \equiv \frac{\mu_0}{4\pi} I \cdot \frac{d\vec{\ell} \wedge \vec{R}}{|\vec{R}^3|} \qquad \vec{B}$$

I° legge di Laplace

Alternativamente

$$\vec{F} \text{ con carica magnetica} = \frac{\mu_0}{4\pi} \frac{q_m \cdot q_m'}{r^2} \quad \Rightarrow \quad \vec{B}_{q_m} = \frac{\vec{F}_m}{q} = \frac{\mu_0}{4\pi} \frac{q_m}{r^2}$$

$$\rightarrow \text{II° Laplace} \rightarrow \vec{F} = I \cdot d\vec{\ell} \wedge \underbrace{\vec{B}}_{\vec{B}_{q_m}} = I \cdot d\vec{\ell} \wedge \left[\frac{\mu_0}{4\pi} \cdot \frac{q_m}{r^2} \cdot \hat{r}\right]$$

$$\hat{r} \rightarrow \frac{\vec{r}}{r} = \hat{r}$$

$$\Rightarrow \underbrace{\vec{F} = \frac{\mu_0}{4\pi} \cdot q_m \cdot I \cdot d\vec{\ell} \wedge \frac{\vec{r}}{r^3}}_{\text{Forza con cariche magnetiche}} \qquad \underbrace{F = q \cdot d\vec{\ell} \wedge \vec{B}}_{\text{Forza con cariche elettriche}}$$

$$\Rightarrow q_m = I d\vec{\ell} \quad \Rightarrow \quad \vec{F} = \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{\ell} \wedge \vec{r}}{r^3} \quad \text{I° Laplace}$$

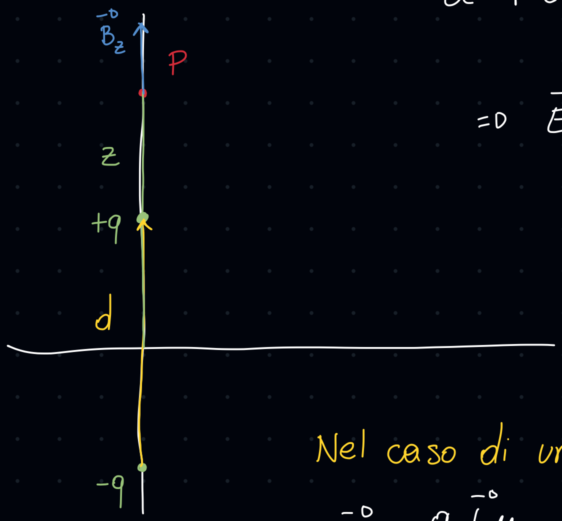
Equivalenza Ampére

Campo spira percorsa da corrente $B = \frac{\mu_0 I}{2\pi} \cdot \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$ Se $z >> R$

$\Rightarrow \vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{z^3} \hat{n}$ chiamo $I S \cdot \hat{n} = I \cdot \pi R^2 \cdot \hat{n} = \vec{m}$ spira

$\Rightarrow \vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{m}}{z^3}$ (1)

$\rightarrow$ Dipolo



Se P è lungo z ci interessa solo B_z

$\Rightarrow \vec{E}_z = \frac{q d}{4\pi \varepsilon_0} \cdot \frac{3 \overset{\rightarrow 1}{\cos\theta} - 1}{z^3} = \frac{q \cdot \vec{d}}{4\pi\varepsilon_0} \frac{2}{z^3}$

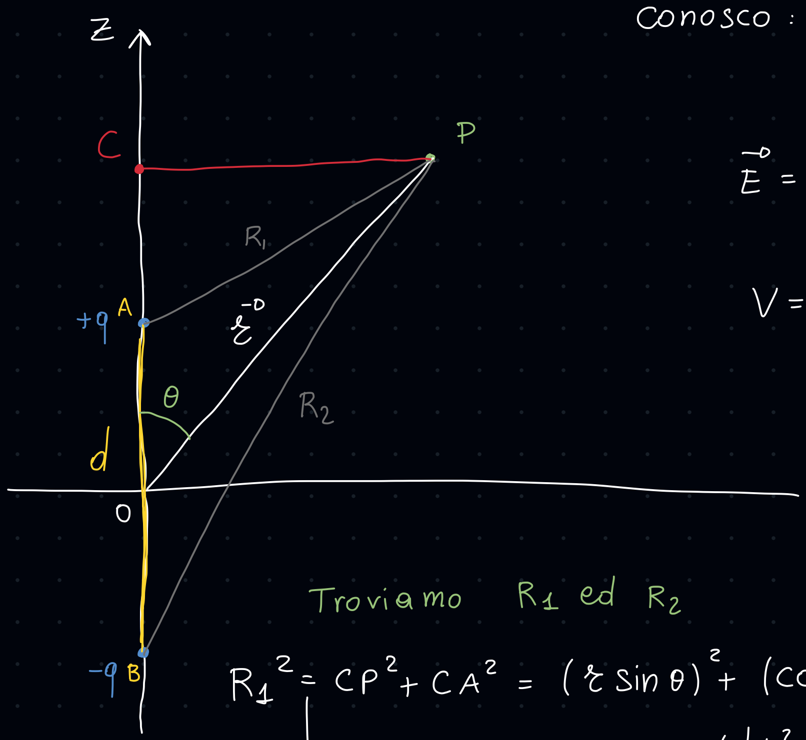
chiamo $q \cdot \vec{d} = \vec{p} \Rightarrow \vec{E} \, \frac{\vec{p}}{2\pi\varepsilon_0 z^3}$

Nel caso di un dipolo magnetico

$\rightarrow \vec{B} = \frac{q \vec{d} \mu_0}{4\pi} \cdot \frac{2}{z^3}$ chiamo $q \cdot \vec{d} = \vec{m}$

$\Rightarrow \vec{B} = \frac{\mu_0}{2\pi} \cdot \frac{\vec{m}}{z^3}$ (2) $\rightarrow$ (1) e (2) Sono UGUALI!

Dipolo elettrico



Conosco: $\begin{cases} r \to \text{Dist di P-O} \\ \theta \to \text{Angolo tra } r \text{ e } y \\ d \to \text{dist tra } +q \text{ e } -q \end{cases}$

$\vec{E} = -\vec{\nabla} V$ ma $V = ?$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$r_A \uparrow R_1$, $r_B \uparrow R_2$???

Troviamo R_1 ed R_2

$$R_1^2 = CP^2 + CA^2 = (r\sin\theta)^2 + \left(CO - \frac{d}{2}\right) = (r\sin\theta)^2 + \left(r\cos\theta - \frac{d}{2}\right)^2$$

$$= r^2\sin^2\theta + r^2\cos^2\theta + \left(\frac{d}{2}\right)^2 - r\cos\theta\, d$$

$$= r^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{1} + \left(\frac{d}{2}\right)^2 - r\cos\theta\, d$$

$$= r^2 + \frac{d^2}{4} - r\cos\theta\, d$$

$$R_2 = r^2 + \frac{d^2}{4} + r\cos\theta\, d$$

(1)

Approx di dipolo $r >> d \Rightarrow R_{1,2} = \sqrt{r^2 \pm r\cos\theta\, d} = \sqrt{r(r \pm \cos\theta d)} = \sqrt{r}\sqrt{r \pm \cos\theta d}$

Approx I(0) → Taylor

$$f(d) = \sqrt{r + \cos\theta d} = (r + \cos\theta d)^{\frac{1}{2}} \to f'(d) = -\frac{1}{2}\cos\theta(r + \cos\theta d)^{-\frac{1}{2}} = \frac{\cos\theta}{2(r+\cos\theta d)^{\frac{1}{2}}}$$

$$\to f'(0) = \frac{\cos\theta}{2\sqrt{r}} \qquad f(0) = \sqrt{r}$$

$$\to f(d) \simeq \sqrt{r} + \frac{\cos\theta}{2\sqrt{r}} \cdot d \quad \to \quad g(d) \simeq \sqrt{r} - \frac{\cos\theta}{2\sqrt{r}} d$$

$$\Rightarrow \begin{cases} R_1 = \cancel{\sqrt{r}} \left(\cancel{\sqrt{r}}^{\,r} + \dfrac{\cos\theta d}{2\cancel{\sqrt{r}}}\right) = r - \dfrac{d\cos\theta}{2} & (2) \\[2ex] R_2 = r + \dfrac{d\cos\theta}{2} & (3) \end{cases}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right] = K\,\frac{\cancel{r} + \frac{d}{2}\cos\theta - \cancel{r} + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\right)^2\cos^2\theta}$$

$$= K\,\frac{d\cos\theta}{r^2 - \cancel{\left(\frac{d}{2}\right)^2\cos^2\theta}} \simeq \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2}$$

$r \gg d$

$\rightarrow$ Campo elettrico

$\vec{E} = -\nabla\vec{V}$ $\rightarrow$ Derivate parziali $\rightarrow$ coordinate cartesiane

$$CO = r\cos\theta \rightarrow \cos\theta = \frac{\overline{CO}}{r} = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$r^2 = x^2+y^2+z^2 \rightarrow r = \sqrt{x^2+y^2+z^2}$$

$$\Rightarrow V = \frac{qd}{4\pi\varepsilon_0}\,\frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\rightarrow \vec{E}_z = -\frac{\partial V}{\partial z} = -K\cdot(x^2+y^2+z^2)^{-\frac{3}{2}} - z\cdot\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{5}{2}}\cdot\cancel{2}z$$

$$= -K\left[(x^2+y^2+z^2)^{-\frac{3}{2}} - \frac{3z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}\right] = -K\,(r^{\cancel{2}})^{-\frac{3}{\cancel{2}}} - \frac{3z^2}{(r^{\cancel{2}})^{\frac{5}{\cancel{2}}}}$$

$$= -\frac{qd}{4\pi\varepsilon_0}\cdot\left[r^{-3} - \frac{3z^2}{r^5}\right] = -\frac{qd}{4\pi\varepsilon_0}\left[\frac{r^2 - 3z^2}{r^5}\right] = \frac{qd}{4\pi\varepsilon_0}\left[\frac{3z^2}{r^5}^{\,2} - \frac{1}{r^3}\right]$$

$$= \frac{qd}{4\pi\varepsilon_0 r^3}\left[\underbrace{\frac{3z^2}{r^2}}_{\cos^2\theta} - 1\right] = \frac{qd}{4\pi\varepsilon_0 r^3}\left(\cos^2\theta - 1\right)$$