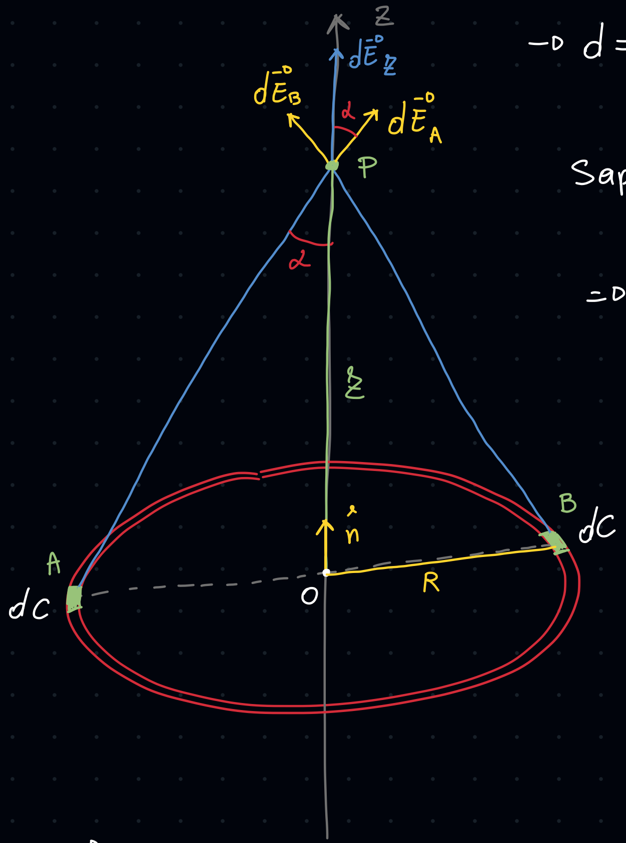
ANELLO CARICO



$\overline{AP} = \sqrt{R^2+z^2}$ Siccome $|\vec E| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{d^2}$

$\to d = AP \Rightarrow dE_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{R^2+z^2}$

Sappiamo che λ = dens. lineare $\Rightarrow \lambda = \frac{dQ}{dl}$

$\Rightarrow dQ = \lambda\, dl \Rightarrow Q_{TOT} = \int \lambda\, dl$

ma l non è altro che la circonferenza

$\Rightarrow Q_{TOT} = \int \lambda\, dc$

Dal disegno sappiamo che $d\vec E_{\hat n} = d\vec E_A + d\vec E_B$

è SOLO LUNGO z $\Rightarrow$ Sommiamo solo le comp lungo z, dato che le altre si eliminano a 2 a 2.

$\to d\vec E_z = d\vec E_A \cos\alpha = \frac{\lambda\, dc}{4\pi\varepsilon_0(R^2+z^2)}\cdot\cos\theta$

Per definizione $\cos\alpha = \frac{Cat}{Hip} = \frac{z}{AP} = \frac{z}{\sqrt{R^2+z^2}}$

$\Rightarrow d\vec E_z = \frac{\lambda\, dc}{4\pi\varepsilon_0(R^2+z^2)}\cdot\frac{z}{\sqrt{R^2+z^2}} \Rightarrow E_z = \int_C \frac{\lambda}{4\pi\varepsilon_0(R^2+z^2)}\,\frac{z}{\sqrt{R^2+z^2}}\, dc$

$\frac{1}{R^2+z^2}\cdot\frac{1}{\sqrt{R^2+z^2}} = \frac{1}{(R^2+z^2)\cdot(R^2+z^2)^{\frac12}} = \frac{1}{(R^2+z^2)^{1+\frac12}}$

$\Rightarrow E_z = \frac{\lambda z}{4\pi\varepsilon_0(R^2+z^2)^{\frac32}}\cdot\int_C dc = \frac{\cancel{2}\cancel{\pi} R\,\lambda z}{2\,\cancel{4}\cancel{\pi}\,\varepsilon_0(R^2+z^2)^{\frac32}}$ (1)

$$\boxed{\Rightarrow \vec E_z = \frac{R z \lambda}{2\varepsilon_0(R^2+z^2)^{\frac32}}\hat n}$$

E se $z >> R$?

Siccome $Q = \int \lambda\, dc = \lambda 2\pi R$

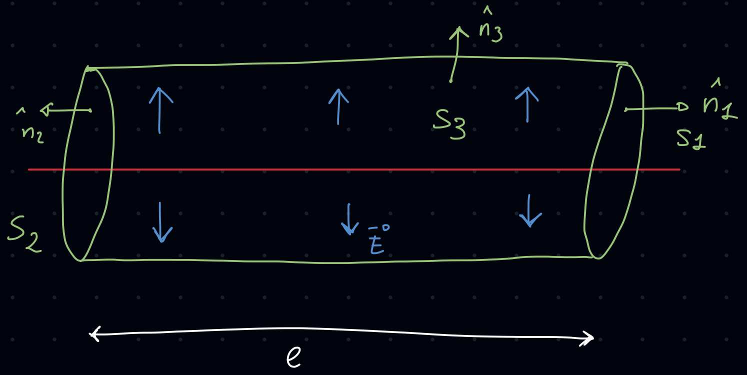
$\Rightarrow$ dalla (1) $\frac{\overbrace{2\pi R\lambda}^{Q_{TOT}}\, z}{4\pi\varepsilon_0(R^2+z^2)^{\frac32}} = \frac{Q_{TOT}\cdot z}{4\pi\varepsilon_0(R^2+z^2)^{\frac32}}$

Se $z>>R$ $E_z = \frac{Q_{TOT}\cdot \cancel{z}}{4\pi\varepsilon_0(\cancel{z^2})^{\frac32}}$

$$\boxed{\Rightarrow \vec E_z = \frac{Q_{TOT}}{4\pi\varepsilon_0 z^2}\hat n}$$

Legge di Coulomb per la carica puntiforme

Filo carico - Coulomb



$\vec{E} \perp \hat{n}_1$ $\quad$ $\vec{E} \perp \hat{n}_2$

$$\phi_E = \oint \vec{E} \cdot \hat{n}\, dS = \cancel{\int_S E \cdot \hat{n}_1\, dS_1} + \cancel{\int_S E \hat{n}_2 dS_2} + \int_S E \hat{n}_2 dS_3$$

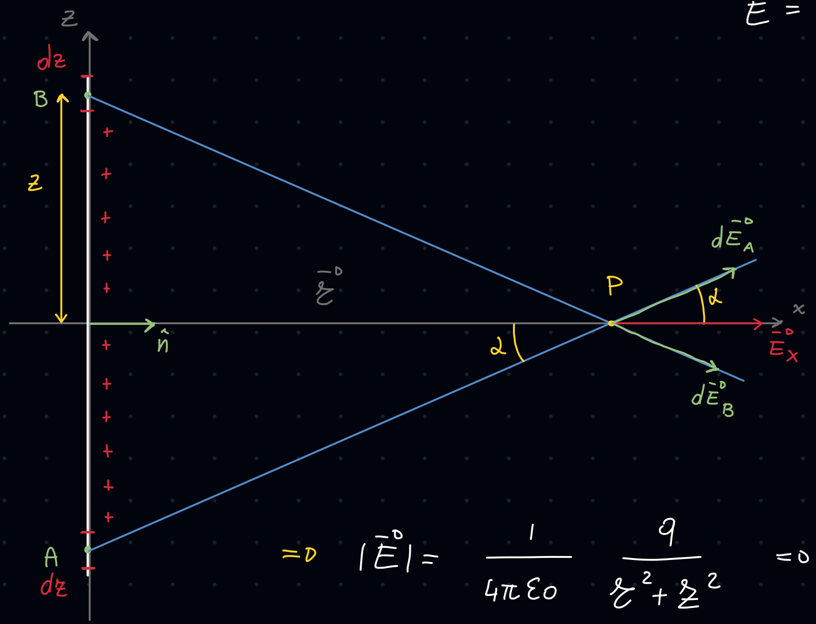
$$= \int \vec{E} \cdot n_2 dS_2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \phi_E = |\vec{E}| \cdot 2\pi r e = \frac{Q}{\varepsilon_0} \quad \text{ma } e = \infty$$

$\Rightarrow \lambda =$ densità lineare di carica $= \dfrac{Q}{e} \Rightarrow Q = \lambda e$

$$|\vec{E}|\, 2\pi r \cancel{e} = \frac{\lambda \cancel{e}}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{n}$$

Filo carico senza Coulomb



$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Non conosco r $\rightarrow$ AP / BP

$$\begin{cases} AP = r\cos\alpha \\ BP = r\cos\alpha \end{cases} \Rightarrow AP = BP = r\cos\alpha$$

Non conosco α

$$AP^2 = z^2 + r^2$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+z^2} \Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}$$

siccome $\lambda = \dfrac{dQ}{de} \Rightarrow dq = \lambda\, de \Rightarrow dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dz}{r^2+z^2}$

nel nostro caso $de = dz$

$\rightarrow$ le componenti z ed y di $d\vec{E}_A + d\vec{E}_B$ si elidono a vicenda $\Rightarrow$ ci serve solo $\vec{E}_z$

$\Rightarrow d\vec{E}_z = d\vec{E}\cos\alpha$ ma non conosciamo α

$\rightarrow z = AP\cos\alpha \Rightarrow z = \sqrt{z^2+r^2}\cos\alpha \Rightarrow \cos\alpha = \dfrac{z}{\sqrt{z^2+r^2}}$

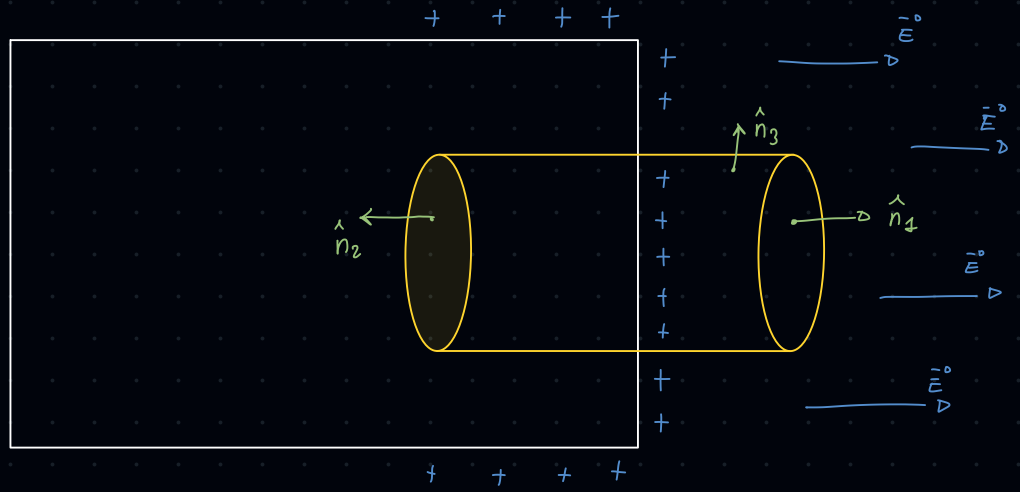
$$\Rightarrow dE_z = dE\cos\alpha = \frac{z}{\sqrt{z^2+r^2}}\, dE$$

$$\Rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\,dz}{r^2+z^2}\cdot\frac{z}{\sqrt{z^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda z\,dz}{(r^2+z^2)^{\frac{3}{2}}} \qquad (1)$$

Integriamo per ottenere $\vec{E}_{TOT}$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\overset{\text{cost}}{\lambda}}{(r^2+z^2)^{\frac{3}{2}}}\,dz = 2\,\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\ell}\frac{r}{(r^2+z^2)^{\frac{3}{2}}}\,dz = \underbrace{\frac{\cancel{2}\lambda\cancel{r}}{2\,4\pi\varepsilon_0\,r^{\cancel{2}}}\cdot\frac{\ell}{\sqrt{r^2+\ell^2}}}_{|\vec{E}|}$$

Teorema di Coulomb



$$\phi_E = \oint \vec{E} \cdot \hat{n}\, dS = \int \vec{E} \cdot \hat{n}_1 dS_1 + \underbrace{\int \vec{E} \cdot \hat{n}_2 dS_2}_{\int = \emptyset} + \underbrace{\int \vec{E} \cdot \hat{n}_3 dS_3}_{\hat{n}_3 \perp \vec{E}} = \int \vec{E} \cdot \hat{n}_1 dS_1 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow |\vec{E}| \cdot S_1 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{S_1} \cdot \frac{1}{\varepsilon_0} \quad \text{ma} \quad \sigma = \frac{Q}{S} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Alternativamente

$$\sigma = \frac{dQ}{dS} \Rightarrow dq = \sigma\, dS \rightarrow q = \int \sigma\, dS \rightarrow E = \frac{\int \sigma\, dS}{S \varepsilon_0} = \sigma \frac{\cancel{\int dS}}{\cancel{S} \varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$