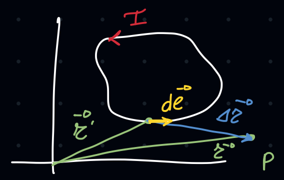
Campo B di un filo

2 Laplace: $\vec{F} = q \cdot \vec{v} \wedge \vec{B} \to \vec{F} = q \cdot \frac{d\vec{l}}{dt} \wedge \vec{B} \to \underline{\vec{F} = I \cdot d\vec{l} \wedge \vec{B}}$

1 Laplace $\vec{B} = \frac{\mu_0}{4\pi} \cdot \oint I \cdot \frac{d\vec{l} \wedge \Delta\vec{r}}{|\Delta\vec{r}|^3}$

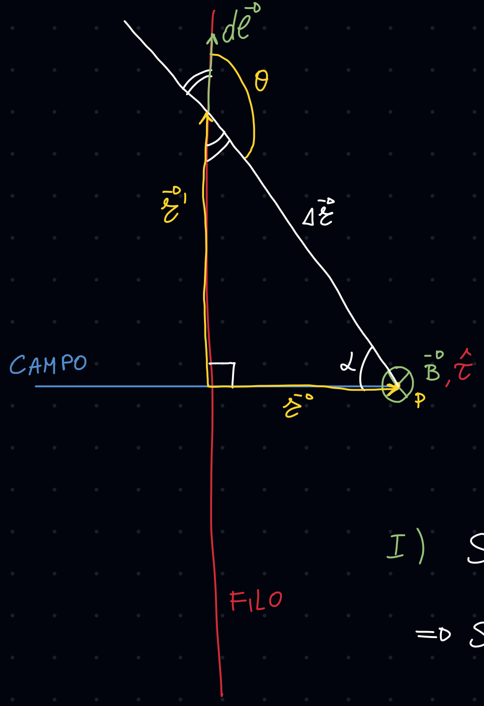
proof

$$\vec{F}_m = \frac{\mu_0}{4\pi} \cdot \frac{q_m \cdot q_m'}{r^2} \to \vec{B} = \frac{\vec{F}_m}{q_m} = \frac{\mu_0}{4\pi} \frac{q_m}{r^2} \hat{r} = \frac{\mu_0}{4\pi} \frac{q_m \cdot \vec{r}}{r^3}$$



$$\to \vec{B} = \frac{\mu_0}{4\pi} \oint I \cdot \frac{d\vec{l} \wedge \Delta\vec{r}}{|\Delta\vec{r}|^3}$$

Scopo del gioco Trovare $\vec{B}_P = \frac{\mu_0}{2\pi} \frac{I}{r}$



$$\vec{B} = \overbrace{\frac{\mu_0 I}{4\pi}}^{K'} \int \frac{d\vec{l} \wedge \Delta\vec{r}}{|\Delta\vec{r}|^3} \hat{\tau} = K' \int \frac{dl \cancel{\Delta r} \sin\theta}{\Delta r^{\cancel{3}\,2}} \hat{\tau}$$

$$= K' \oint \frac{dl \cdot \sin\theta \cdot \hat{\tau}}{\Delta r^2} \qquad \text{ma } dl \parallel dr' \Rightarrow dl = dr'$$

$$|\vec{B}| = K' \int \frac{\sin\theta}{\Delta r^2} dr'$$

Scriviamo tutto in funzione di α

I) $\sin(\theta)$ · $\Delta^\circ = 90 + \alpha + \cancel{180} - \theta = \cancel{180} \Rightarrow \alpha = \theta - 90 \Rightarrow \theta = \alpha + 90$

$$\Rightarrow \sin(\theta) = \underbrace{\sin(\alpha + 90) = \cos(\alpha)}_{\text{TRIG}} \quad (1)$$

II) $\begin{cases} \vec{r}' = \Delta\vec{r} \sin\alpha \\ \vec{r} = \Delta\vec{r} \cos\alpha \end{cases} \Rightarrow \frac{\vec{r}'}{\vec{r}} = \tan\alpha \Rightarrow \vec{r}' = \vec{r} \tan\alpha \to d\vec{r}' = d(\vec{r} \tan\alpha)$ ma $\vec{r} = \text{cost}$

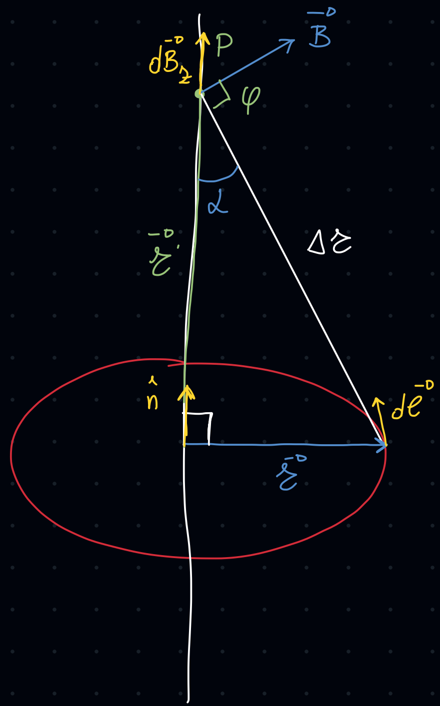
$$\Rightarrow d\vec{r}' = \frac{r}{\cos^2\alpha} d\alpha \quad (2)$$

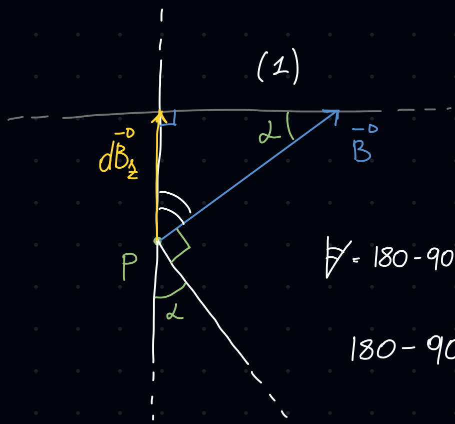
III) $\vec{r} = \Delta\vec{r} \cos\alpha \Rightarrow \Delta\vec{r} = \frac{\vec{r}}{\cos\alpha}$ (3)

$$\to B = K' \int \frac{\cos(\alpha)}{\frac{r^{\cancel{2}}}{\cancel{\cos^2}\alpha}} \cdot \frac{\cancel{r}}{\cancel{\cos^2\alpha}} d\alpha \to B = \frac{K'}{r} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\alpha)\, d\alpha = \frac{\mu_0 I}{4\pi r} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right]$$

$$\to B = \frac{\mu_0 I}{\cancel{4}_2 \pi r} \cdot [\cancel{1 - (-1)}] = \frac{\mu_0 I}{2\pi r} \qquad \underline{\text{QED}}$$

Campo B di una Spira





$\gamma = 180-90-\alpha = 90-\alpha$

$180-90-90+\alpha = \alpha$ QED

1) Laplace: $dB = \frac{\mu_0}{4\pi} I \cdot \frac{d\vec{\ell} \wedge \Delta\vec{r}}{|\Delta\vec{r}|^3}$ (2)

2) Consideriamo solo $d\vec{B}_z$ perchè le altre componenti si eliminano a due a due

$\rightarrow d\vec{B} = d\vec{B}_z = \hat{n}\, dB_z \rightarrow \vec{B} = \hat{n} \oint dB_z$

ma $dB_z = \overset{(2)}{dB} \sin\alpha$ dal disegno (1)

$$\rightarrow \vec{B} = \underbrace{\hat{n} \cdot \frac{\mu_0 I}{4\pi}}_{\vec{K}'} \oint \frac{d\vec{\ell} \wedge \Delta\vec{r}}{\Delta r^3} \cdot \sin\alpha = \vec{K}' \cdot \oint \frac{d\ell \cdot \cancel{\Delta r}\, \overset{\to 1}{\sin\varphi}}{\Delta r^{\cancel{3}}} \sin\alpha = \vec{K}' \oint \frac{d\ell}{\underbrace{\Delta r^2}_{\text{costanti}}} \sin\alpha$$

$$\rightarrow \vec{B} = \vec{K}' \cdot \frac{\sin\alpha}{\Delta r^2} \oint d\ell = \hat{n} \frac{\mu_0 I}{4\pi} \cdot \frac{\sin\alpha}{\Delta r^2}\, 2\pi r$$

ma: $\sin\alpha \rightarrow r = \Delta r \sin\alpha \Rightarrow \sin\alpha = \frac{r}{\Delta r}$ (a)

$\Delta r = \sqrt{r^2 + \vec{r}'^2} \rightarrow \Delta r^2 = r^2 + r'^2$ Battezzo $r' = z \Rightarrow \Delta r^2 = r^2 + z^2$

$$\Rightarrow \vec{B} = \hat{n} \frac{\mu_0 I}{\overset{2}{\cancel{4\pi}}} \frac{r}{\sqrt{r^2+z^2}} \cdot \frac{\cancel{2\pi} r}{r^2+z^2} \rightarrow (r^2+z^2)^{\frac{1}{2}} \cdot (r^2+z^2)^1 = (r^2+z^2)^{\frac{3}{2}} = \sqrt{(r^2+z^2)^3}$$

$$\Rightarrow \vec{B} = \hat{n} \frac{\mu_0 I R^2}{2(r^2+z^2)^{3/2}}$$ QED

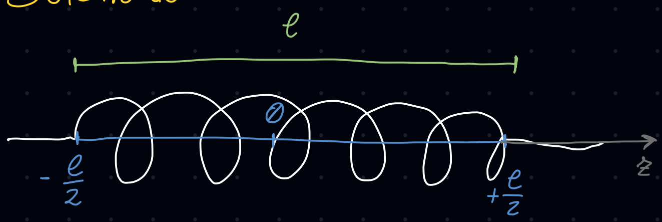
Momento magnetico $\vec{m} = \hat{n} \cdot I \cdot S$ dove S è la superficie $S_0 = \pi R^2$

$$\Rightarrow \vec{m}_{spira} = \hat{n} \cdot I \cdot \pi R^2 \qquad \Rightarrow \vec{B}_{spira} = \frac{\hat{n} \cdot I R^2}{2(r^2+z^2)^{\frac{3}{2}}} \cdot \frac{\pi}{\pi}$$

$\vec{m}_{spira}$

$$\Rightarrow \vec{B}_{spira} = \frac{\mu_0 \vec{m}}{2\pi(r^2+z^2)^{\frac{3}{2}}}$$

Solenoide



Ci sono N spire in una lunghezza ℓ

$\Rightarrow$ n = densità di spire = $\dfrac{N}{\ell}$

$\rightarrow$ dz è la porzione infinitesimale della LUNGHEZZA del filo

$\Rightarrow$ $n\,dz$ = Spire nella porzione

COSTANTI

$$\Rightarrow \vec{B}_{TOT} = \int \underbrace{\vec{B}}_{\text{CAMPO SINGOLA SPIRA}} \cdot n\,dz = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \hat{n} \cdot \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}} \cdot n\,dz \rightarrow \vec{B}_{TOT} = \hat{n} \cdot \frac{\mu_0 I R^2}{2} n \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{1}{(R^2+z^2)^{\frac{3}{2}}}\,dz$$

$$\text{Primitiva} = \frac{1}{R^2} \cdot \frac{z}{(R^2+z^2)^{\frac{1}{2}}} \rightarrow \vec{B}_{TOT} = \overbrace{\hat{n} \cdot \frac{\mu_0 I R^2}{2}}^{\vec{K}'} \cdot \frac{N}{\ell} \cdot \left[\frac{1}{R^2} \cdot \frac{z}{\sqrt{R^2+z^2}} \right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}}$$

$$\rightarrow \vec{B}_{TOT} = \vec{K}' \cdot \frac{N}{\ell} \cdot \left[\frac{1}{R^2} \cdot \frac{\ell}{2\sqrt{R^2+\frac{\ell^2}{4}}} + \frac{1}{R^2} \cdot \frac{\ell}{2\sqrt{R^2+\frac{\ell^2}{4}}} \right]$$

$$= \hat{n} \frac{\mu_0 I R^2 N}{\ell \cdot \cancel{2}} \cdot \cancel{2} \left[\frac{1}{R^2} \frac{\ell}{2\sqrt{R^2+\frac{\ell^2}{4}}} \right] \rightarrow 2\sqrt{R^2+\frac{\ell^2}{4}} = 2\sqrt{\frac{4R^2-\ell^2}{4}} = \frac{\cancel{2}\sqrt{4R^2-\ell^2}}{\sqrt{\cancel{4}}}$$

$$= \hat{n} \frac{\mu_0 I \cancel{R^2} N}{\cancel{\ell}} \cdot \frac{\cancel{\ell}}{\sqrt{4R^2+\ell^2}} \cdot \frac{1}{\cancel{R^2}} \rightarrow \vec{B} = \hat{n} \cdot \frac{\mu_0 I N}{\sqrt{4R^2+\ell^2}}$$

Sosteniamo che $\ell >> R$ ovvero lunghezza del Solenoide molto maggiore del suo raggio

$$\rightarrow \sqrt{4R^2+\ell^2} \simeq \sqrt{\ell^2} \simeq \ell \Rightarrow \vec{B} = \hat{n} \frac{\mu_0 I \overbrace{N}^{n}}{\ell} = \underline{\hat{n} \mu_0 I n}$$