

Campo B di un filo

2 Laplace: $\vec{F} = q \cdot \vec{v} \wedge \vec{B} \rightarrow \vec{F} = q \cdot \frac{d\vec{e}}{dt} \wedge \vec{B} \rightarrow \underline{\vec{F} = I \cdot d\vec{e} \wedge \vec{B}}$

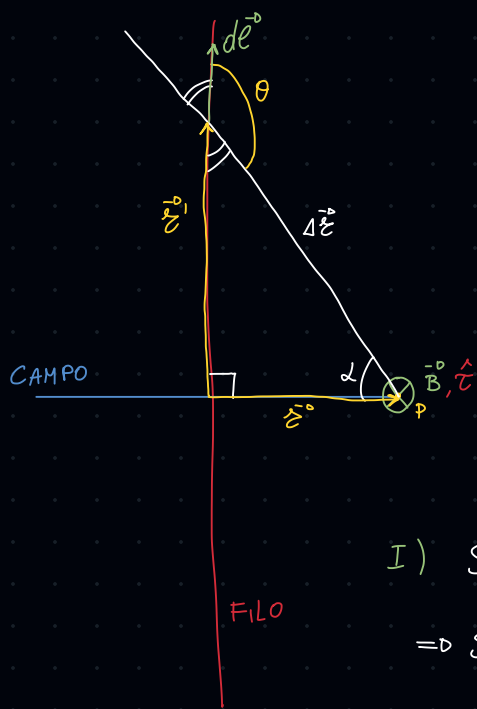
1 Laplace $\vec{B} = \frac{\mu_0}{4\pi} \oint I \cdot \frac{d\vec{e} \wedge \Delta\vec{e}}{|\Delta\vec{e}|^3}$

proof $\vec{F}_m = \frac{\mu_0}{4\pi} \cdot \frac{q_m \cdot q_m'}{r^2} \rightarrow \vec{B} = \frac{\vec{F}_m}{q_m} = \frac{\mu_0}{4\pi} \frac{q_m}{r^2} \hat{r} = \frac{\mu_0}{4\pi} \frac{q_m \cdot \vec{r}}{r^3}$



$\rightarrow \vec{B} = \frac{\mu_0}{4\pi} \oint I \cdot \frac{d\vec{e} \wedge \Delta\vec{e}}{|\Delta\vec{e}|^3}$

Scopo del gioco Trovare $\vec{B}_p = \frac{\mu_0}{2\pi} \frac{I}{r}$



$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{e} \wedge \Delta\vec{e}}{|\Delta\vec{e}|^3} \hat{r} = \kappa' \int \frac{de \cdot \Delta z \cdot \sin\theta}{\Delta z^3} \hat{r}$
 $= \kappa' \oint \frac{de \cdot \sin\theta}{\Delta z^2} \hat{r}$ ma $de \parallel dz' \Rightarrow de = dz'$

$|\vec{B}| = \kappa' \int \frac{\sin\theta}{\Delta z^2} dz'$

Scriviamo tutto in funzione di α

I) $\sin(\theta) \quad \Delta = 90 + \alpha + 180 - \theta = 180 \Rightarrow \alpha = \theta - 90 \Rightarrow \theta = \alpha + 90$

$\Rightarrow \sin(\theta) = \sin(\alpha + 90) = \cos(\alpha) \quad (1)$
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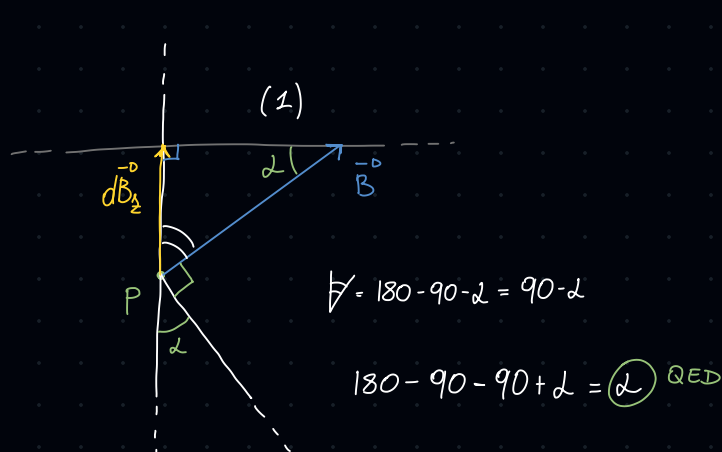
II) $\begin{cases} z' = \Delta z \sin\alpha \\ z = \Delta z \cos\alpha \end{cases} \Rightarrow \frac{z'}{z} = \tan\alpha \Rightarrow z' = z \tan\alpha \rightarrow dz' = d(z \tan\alpha) \text{ ma } z = \text{cost}$
 $\Rightarrow dz' = \frac{z}{\cos^2\alpha} d\alpha \quad (2)$

III) $z = \Delta z \cos\alpha \Rightarrow \Delta z = \frac{z}{\cos\alpha} \quad (3)$

$\rightarrow B = \kappa' \int \frac{\cos(\alpha)}{\frac{z}{\cos^2\alpha}} \cdot \frac{z}{\cos^2\alpha} d\alpha \rightarrow B = \frac{\kappa'}{z} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\alpha) d\alpha = \frac{\mu_0 I}{4\pi z} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right]$

$\rightarrow B = \frac{\mu_0 I}{4\pi z} [1 - (-1)] = \frac{\mu_0 I}{2\pi z} \quad \underline{QED}$

Campo B di una Spira



1) Laplace:
$$dB = \frac{\mu_0}{4\pi} I \cdot \frac{d\vec{e} \wedge \Delta\vec{z}}{|\Delta\vec{z}|^3} \quad (2)$$

2) Consideriamo solo \vec{B}_z perché le altre componenti si eliminano a due a due

$$\rightarrow d\vec{B} = d\vec{B}_z = \frac{\mu_0}{4\pi} n d\vec{B}_z \rightarrow \vec{B} = \frac{\mu_0}{4\pi} n \oint d\vec{B}_z$$

ma $dB_z = \overset{(2)}{dB} \sin \alpha$ dal disegno (1)

$$\rightarrow \vec{B} = \underbrace{\frac{1}{n} \cdot \mu_0 I}_{\vec{K}'} \oint \frac{d\vec{e} \wedge \Delta \vec{z}}{\Delta z^3} \cdot \sin \alpha = \vec{K}' \oint \frac{de \cdot \cancel{\Delta z} \cdot \sin \varphi}{\cancel{\Delta z}^3} \sin \alpha = \vec{K}' \oint \underbrace{\frac{de}{\Delta z^2}}_{\text{costanti}} \sin \alpha$$

$$\rightarrow \vec{B} = \vec{K} \cdot \frac{\sin \theta}{\Delta x^2} \oint d\ell = n \frac{\mu_0 I}{4\pi} \cdot \frac{\sin \theta}{\Delta x^2} 2\pi \ell$$

ma: $\sin \alpha \approx 0 \quad \epsilon = \Delta \epsilon \sin \alpha \Rightarrow \sin \alpha = \frac{\epsilon}{\Delta \epsilon} \quad (a)$

$$\Delta z = \sqrt{z^2 + z'^2} \rightarrow \Delta z^2 = z^2 + z'^2 \quad \text{Bei } \pi/2 \text{ zu } z' = z \Rightarrow \Delta z^2 = z^2 + z^2$$
$$\Rightarrow \beta = n \frac{\mu_0 I}{24\pi} \frac{z}{\sqrt{z^2 + z^2}} \cdot \frac{2\pi z}{z^2 + z^2} \rightarrow (z^2 + z^2)^{\frac{1}{2}} \cdot (z^2 + z^2)^1 = (z^2 + z^2)^{\frac{3}{2}} = \sqrt{(z^2 + z^2)^3}$$

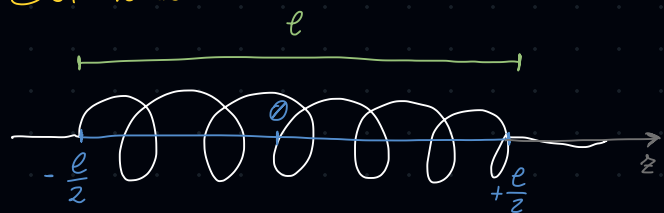
$$\Rightarrow \vec{B} = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}} \quad \text{QED}$$

Momento magnetico $\vec{m} = \vec{n} \cdot I \cdot S$ dove S è la superficie $S_0 = \pi R^2$

$$\Rightarrow \vec{m}_{\text{spira}} = \vec{n} \cdot I \cdot \pi R^2 \quad \Rightarrow \vec{B}_{\text{spira}} = \frac{\vec{n} \cdot I R^2}{2(z^2 + R^2)^{\frac{3}{2}}} \cdot \frac{\pi}{\pi} \vec{m}_{\text{spira}}$$

$$\Rightarrow \vec{B}_{\text{spira}} = \frac{\mu_0 \vec{m}}{2\pi(z^2 + R^2)^{\frac{3}{2}}}$$

Solenoide



Ci sono N spire in una lunghezza l

$$\Rightarrow n = \text{densità di spire} = \frac{N}{l}$$

$\rightarrow dz$ è la porzione infinitesimale della LUNGHEZZA del filo

$\Rightarrow n dz =$ spire nella porzione

$$\Rightarrow \vec{B}_{TOT} = \int \underbrace{\vec{B}}_{\text{CAMPO SINGOLA SPIRA}} n dz = \int_{-l/2}^{l/2} \underbrace{\left(\frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \right)}_{\text{COSTANTI}} \cdot n dz \quad \rightarrow \quad \vec{B}_{TOT} = \hat{n} \cdot \frac{\mu_0 I R^2}{2} n \int_{-l/2}^{l/2} \frac{1}{(R^2 + z^2)^{3/2}} dz$$

$$\text{Primitiva} = \frac{1}{R^2} \frac{z}{(R^2 + z^2)^{1/2}} \quad \rightarrow \quad \vec{B}_{TOT} = \hat{n} \cdot \frac{\mu_0 I R^2}{2} \cdot \frac{N}{l} \cdot \left[\frac{1}{R^2} \frac{z}{\sqrt{R^2 + z^2}} \right]_{-l/2}^{l/2}$$

$$\begin{aligned} \rightarrow \vec{B}_{TOT} &= \hat{n} \cdot \frac{N}{l} \cdot \left[\frac{1}{R^2} \frac{l}{2\sqrt{R^2 + \frac{l^2}{4}}} + \frac{1}{R^2} \frac{l}{2\sqrt{R^2 + \frac{l^2}{4}}} \right] \\ &= \hat{n} \cdot \frac{\mu_0 I R^2 N}{l \cdot 2} \cdot 2 \left[\frac{1}{R^2} \frac{l}{2\sqrt{R^2 + \frac{l^2}{4}}} \right] \rightarrow 2\sqrt{R^2 + \frac{l^2}{4}} = 2\sqrt{\frac{4R^2 + l^2}{4}} = \frac{2\sqrt{4R^2 + l^2}}{\sqrt{4}} \\ &= \hat{n} \cdot \frac{\mu_0 I R^2 N}{l} \cdot \frac{l}{\sqrt{4R^2 + l^2}} \cdot \frac{1}{R^2} \rightarrow \vec{B} = \hat{n} \cdot \frac{\mu_0 I N}{\sqrt{4R^2 + l^2}} \end{aligned}$$

Sosteniamo che $l \gg R$ ovvero lunghezza del Solenoide molto maggiore del suo raggio

$$\rightarrow \sqrt{4R^2 + l^2} \simeq \sqrt{l^2} \simeq l \quad \Rightarrow \quad \vec{B} = \hat{n} \cdot \frac{\mu_0 I N}{l} = \hat{n} \mu_0 I n$$