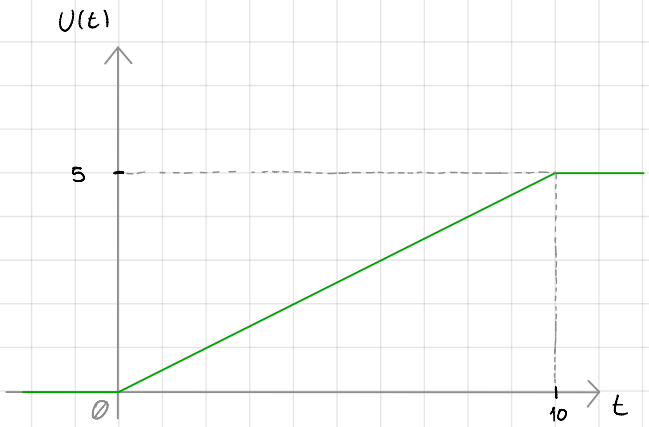
Funzione Con Polo Doppio

$$G(S) = 14 \cdot \frac{10S + 1}{(S+2)^2} \qquad \text{con} \qquad U(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2}t & 0 \leq t < 10 \\ 5 & t \geq 10 \end{cases}$$

(1) grafico $U(t)$

$$\begin{cases} U_1(t) = \frac{1}{2}\, t \cdot 1\!\!1(t) \\ U_2(t) = -\frac{1}{2}(t-10) \cdot 1\!\!1(t-10) \end{cases}$$



* Forma Standard

(2) scelgo il segnale fittizio: $\hat{U}(t) = t \cdot 1\!\!1(t) \rightleftharpoons \hat{U}(S) = \frac{1}{S^2}$

$$\Rightarrow \hat{Y}(S) = 14 \cdot \frac{10S+1}{S^2(S+2)^2} = \underbrace{140}_{\text{moltiplico alla fine}} \cdot \underbrace{\frac{S+0.1}{S^2(S+2)^2}}_{\text{Forma Standard}} = \frac{r_1}{S} + \frac{r_2}{S^2} + \frac{r_3}{S+2} + \frac{r_4}{(S+2)^2}$$

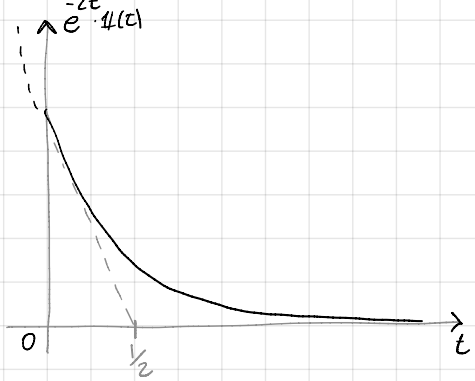
$$= r_1 S(S+2)^2 + r_2(S+2)^2 + S^2 r_3(S+2) + r_3 S^2 = r_1 S(S^2+4S+4) + r_2(S^2+4S+S^2) + S^2 r_3(S+2) + r_3 S^2$$

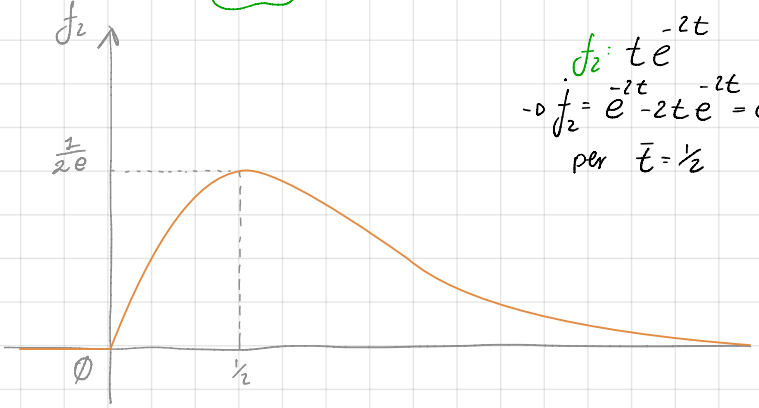
$$= S^3(r_1 + r_3) + S^2(4r_1 + r_2 + 2r_3 + r_4) S^2 + (4r_1 + 4r_2) + \frac{4r_2}{0.1}$$

- $r_2 = \lim_{S \to 00} S^2 \cdot Y(S) = \frac{0.1}{4} = 0.025$
- $r_4 = \lim_{S \to -2} (S+2)^2 \cdot Y(S) = -\frac{1.9}{4} = 0.475$

$r_1 + r_3 = 0 \Rightarrow r_3 = -r_1 = 0.475$, $r_1 + r_2 = 0.25 \Rightarrow r_1 = 0.225$

$$\hat{y}(t) = 140 \cdot \left(0.225 + 0.025\, t - 0.225\, e^{-2t} - 0.475 \underbrace{t\, e^{-2t}}_{f_2}\right) t \geq 0 \qquad \text{con } \hat{U}(t) = t \cdot 1\!\!1(t)$$





$f_2 : t e^{-2t}$

$\Rightarrow f_2' = e^{-2t} - 2te^{-2t} = 0$

per $t = 1/2$

(3) Scriro $y(t)$ Reale

* $\hat{U}_2(t) = \frac{1}{2}(t-10)\cdot \mathbb{1}(t-10) \rightleftharpoons \hat{U}_2(s) = \frac{1}{s^2} e^{-10s}$

$$y(t) = y_1(t) + y_2(t)$$

$$\begin{cases} y_1(t) = \frac{1}{2}\hat{y}(t) \\ y_2(t) = -\frac{1}{2}\hat{y}(t-10)\cdot \mathbb{1}(t-10) \end{cases}$$

(4) Considerazioni

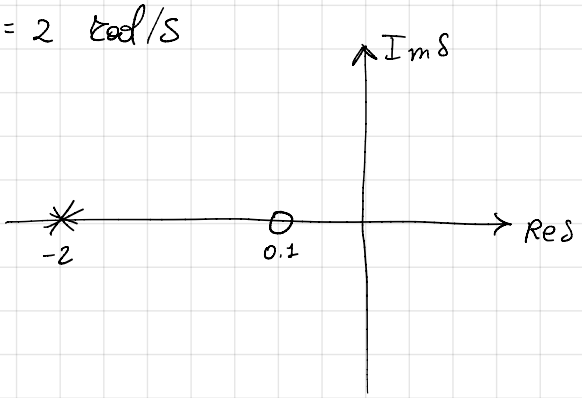
* Audio

$$G(s) = 7\,\frac{10s+1}{\left(\frac{1}{2}s+1\right)^2}$$

$z_1 = -\frac{1}{10} \rightarrow \omega_1 = 0.1$ rad/s

$p: \quad \omega_2 = 2$ rad/s

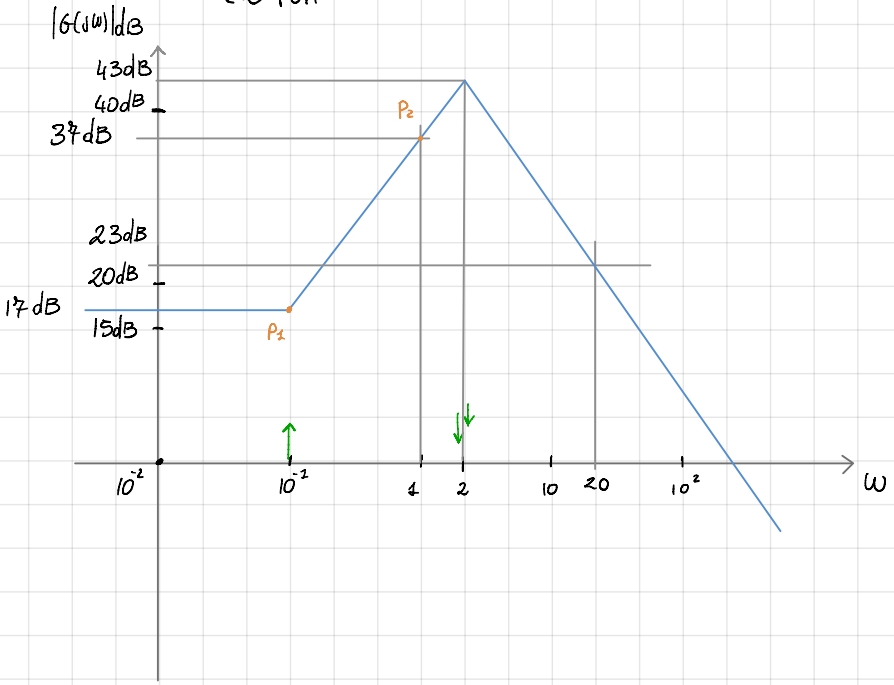
$\Rightarrow \quad \omega \in [10^{-2}; 10^{2}]$ rad/s

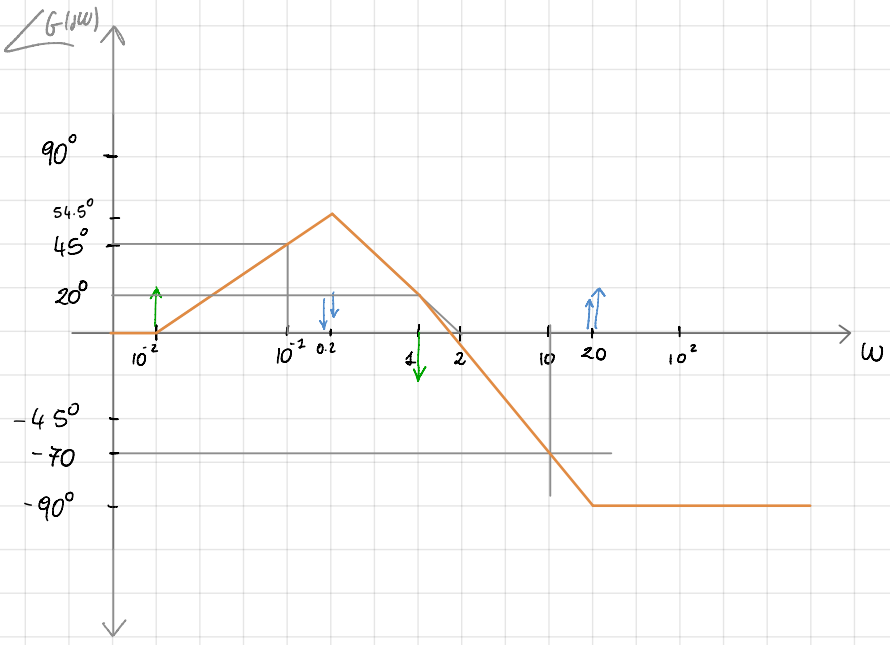


$G(0) = 7 \cdot \frac{1}{1^2} = 7 \Rightarrow |G(0)|_{dB} \cong 17\,dB$

$G(\omega \to \infty) = \begin{cases} 1 \text{ zero} \\ 2 \text{ poli} \end{cases} \Rightarrow -20\,dB/dec$

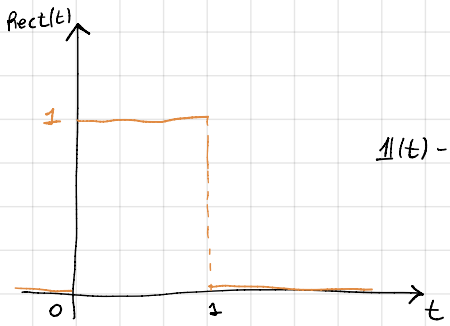
* Calcola retta tra P_1 e P_2





* Impo

* Risposta in freq



$1(t) - 1(t-1) \Rightarrow \frac{1}{s} - \frac{e^{-s}}{s} = \frac{-e^{-s}+1}{s}$

Come lo scrivo in Matlab ??