ES: POLI COMPLESSI E CONIUGATI

$$G(S) = \frac{S + 0.1}{(S^2 + 10S + 100)(S+2)}$$

con $u(t) = \begin{cases} 0 & t < 0 \\ 4 & t \in (0,3) \\ 4e^{-\frac{t-3}{2}} & t \geq 3 \end{cases}$

(1) Scrivo $u(t)$ con segnali noti

- Per $t \in (0,3) \rightsquigarrow$ $U_1(t) = 4 \cdot 1\!\!1(t)$

- Per $t \geq 3 \rightsquigarrow U_1(t) + U_2(t) + U_3(t)$ con $U_2(t) = -4 \cdot 1\!\!1(t-3)$ e $U_3(t) = 4e^{-\frac{t-3}{2}} \cdot 1\!\!1(t-3)$

Annulla $U_1(t)$

(2) Determino le Trasformate

$$U_1(t) \rightleftharpoons U_1(S) = 4 \cdot \frac{1}{S}$$

$$U_2(t) \rightleftharpoons U_2(S) = -4 \cdot \frac{1}{S} \cdot e^{-3S}$$

$$U_3(t) \rightleftharpoons U_3(S) = \mathcal{L}\left[4 \cdot e^{-\frac{t}{2}}\right] \cdot e^{-3S} = \frac{4}{S+\frac{1}{2}} e^{-3S}$$

(3) Individuare il segnale che ci permette di ricondurci agli altri segnali

$\rightsquigarrow$ I segnali elementari da considerare sono:

$$\hat{U}_1(t) = 1\!\!1(t) \quad \text{e} \quad \hat{U}_2(t) = e^{-\frac{t}{2}}$$

(5) Anti trasformata

- $\hat{U}_1(S) = \frac{1}{S} \Rightarrow \hat{Y}_1(S) = \dfrac{S + 0.1}{S(S^2 + 10S + 100)(S+2)}$ (1)

- $\hat{U}_2(S) = \frac{1}{S+\frac{1}{2}} \Rightarrow \hat{Y}_2(S) = \dfrac{1}{S+\frac{1}{2}} \cdot \dfrac{S + 0.1}{(S^2 + 10S + 100)(S+2)}$ (2)

Trovo l'uscita al primo segnale: $\hat{y}_1(t)$ (1)

Verifico i poli: $s^2+10s+100 \rightsquigarrow \bar{s} = -5 \pm \sqrt{25-100} = -5 \pm j\sqrt{75}$ Complx

5.1) $y_1(t)$ con poli Non complx e conj

$$\Rightarrow Y_1(s) = \frac{r_1}{s} + \frac{r_2}{s+2} + \frac{r_3 + r_4 s}{s^2+10s+100}$$

$$r_1 = \lim_{s\to 0} \cancel{s} \cdot \frac{\cancel{s}+0.1}{\cancel{s}(s^2+10s+100)(s+2)} \to \frac{0.1}{200} \to \frac{1}{2}\, 10^{-3} \quad r_1$$

$$r_2 = \lim_{s\to -2} \cancel{(s+2)} \frac{s+0.1}{s(s^2+10s+100)\cancel{(s+2)}} = \frac{1.9}{2\cdot 84} = 11\cdot 10^{-3} \quad r_2$$

$$\frac{(s+2)\,r_1(s^2+10s+100) + s\,r_2(s^2+10s+100) + (r_3+r_4 s)(s+2)s}{s(s^2+10s+100)(s+2)} \rightsquigarrow \begin{cases} s^3(r_1+r_2+r_4) = 0 \\ s^2(2r_1+10r_1+10r_2+2r_4) = 0 \\ s(100r_1+20r_1+100r_2+2r_3) = 1 \\ 200 r_1 = 0.1 \end{cases}$$

$$r_1+r_2+r_4 = 0 \Rightarrow r_4 = -(r_1+r_2) = 11.5 \times 10^{-3} \quad r_4$$

$$r_3 = -(12 r_1 + 10 r_2 + 2 r_4) = -93 \times 10^{-3} \quad r_3$$

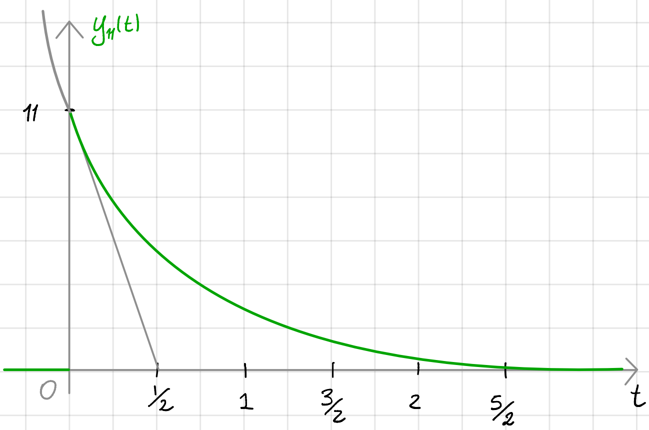
$$\Rightarrow \hat{Y}_1(s) = 10^{-3}\left[\frac{1}{2} + \frac{11}{s+2} - \frac{93+11.5s}{(s+5)^2+(5\sqrt{3})^2}\right] = 10^{-3}\left[\frac{1/2}{s} + \frac{11}{s+2} - 11.5\cdot\frac{s+\frac{93}{11.5}+5-5}{(s+5)^2+(5\sqrt{3})^2}\right]$$

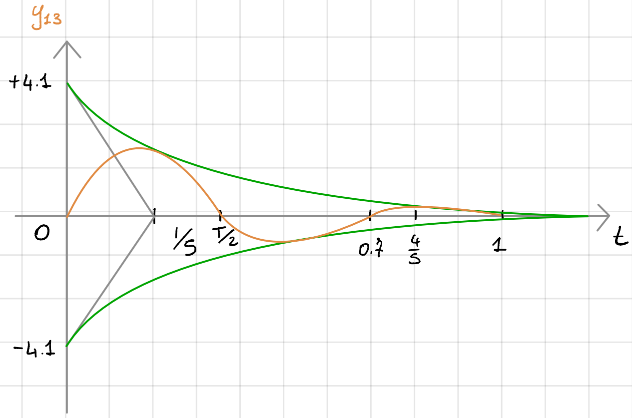
$$= 10^{-3}\left[\frac{1/2}{s} + \frac{11}{s+2} - 11.5\cdot\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} - \frac{11.5}{5\sqrt{3}}\left(\frac{93}{11.5}-5\right)\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}\right]$$

$$\Rightarrow \hat{y}_1(t) = 10^{-3}\Big(\frac{1}{2} + \underbrace{11\cdot e^{-2t}}_{y_{11}} - \underbrace{11.5\cdot e^{-5t}\cos(5\sqrt{3}\,t)}_{y_{12}} - \underbrace{4.1\cdot e^{-5t}\cdot\sin(5\sqrt{3}\cdot t)}_{y_{13}}\Big)\cdot 1(t)$$

$$T = \frac{2\pi}{5\sqrt{3}} = 0.73$$

DISEGNI Dei Modi





Trovo l'uscita al secondo segnale: $\hat{y}_2(t)$ (2)

$$\hat{Y}_2(S) = \frac{S+0.1}{(S+\frac{1}{2})(S+2)(S^2+10S+100)} = \frac{r_1}{S+\frac{1}{2}} + \frac{r_2}{S+2} + \frac{r_3+r_4 S}{S^2+10S+100}$$

$$r_1 = \lim_{S \to -\frac{1}{2}} \cancel{(S+\tfrac{1}{2})} \cdot \frac{\overset{-\frac{1}{2}}{S}+0.1}{\cancel{(S+\frac{1}{2})}\underset{-\frac{1}{2}}{(S+2)}(\underset{\frac{1}{4}}{S^2}+\underset{-5}{10S}+100)} = \frac{-0.4}{\frac{3}{2}\cdot(\frac{1}{4}-5+100)} \cong -2.8 \times 10^{-3} \quad r_1$$

$$r_2 = \lim_{S \to -2} \cancel{(S+2)} \frac{\overset{2.01}{S+0.1}}{\underset{-\frac{3}{2}}{(S+\frac{1}{2})}\cancel{(S+2)}(\underset{4}{S^2}+\underset{-20}{10S}+100)} = \frac{-1.9}{-\frac{3}{2}(4-20+100)} = 15 \times 10^{-3} \quad r_2$$

$$\hat{Y}_2(S) = \frac{(S+2)\,r_1(S^2+10S+100) + (S+\frac{1}{2})\,r_2(S^2+10S+100) + (r_3+r_4S)(S+2)(S+\frac{1}{2})}{(S+\frac{1}{2})(S+2)(S^2+10S+100)}$$

$= S^3(r_1+r_2+r_4) = 0 \rightsquigarrow$ ho già r_1 ed $r_2 \Rightarrow r_4 = -r_1 - r_2 = -12.5 \times 10^{-3}$ r_4

$$\cancel{S^2}\left(10r_1 + 2r_1 + \tfrac{1}{2}r_2 + 10r_2 + r_3 + 2r_4 + \tfrac{1}{2}r_4\right) = 0$$

$$\hookrightarrow r_1(10+2) + r_2(\tfrac{1}{2}+10) + r_3(1) + r_4(2+\tfrac{1}{2}) = 0$$

$$\Rightarrow r_3 = -(\dots) = -0.092 = 92.65 \times 10^{-3} \quad r_3$$

* Forma che ci interessa

$\Rightarrow$ A questo punto dovremmo sostituire i residui, ma possiamo riscriverci $\hat{Y}_2(S)$ nella forma che ci torna più comoda per fare l'antitrasformata PRIMA di sostituire:

$$S^2+10S+100 \rightsquigarrow P_{1,2} = -5 \pm j5\sqrt{3} \Rightarrow S^2+10S+100 = (S+5)^2 + (5\sqrt{3})^2$$

$$\Rightarrow \hat{Y}_2(S) = \frac{r_1}{S+\frac{1}{2}} + \frac{r_2}{S+2} + r_4\frac{\frac{r_3}{r_4}+S}{(S+5)^2+(5\sqrt{3})^2} = \frac{r_1}{S+\frac{1}{2}} + \frac{r_2}{S+2} + r_4\frac{(S+5)+\frac{r_3}{r_4}-5}{(S+5)^2+(5\sqrt{3})^2}$$

mi serve $(S+5)$

$$= \frac{r_1}{S+\frac{1}{2}} + \frac{r_2}{S+2} + r_4\frac{S+5}{(S+5)^2+(5\sqrt{3})^2} + r_4\frac{\left(\frac{r_3}{r_4}-5\right)}{(S+5)^2+(5\sqrt{3})^2}\cdot\frac{5\sqrt{3}}{5\sqrt{3}}$$

$$= \frac{r_1}{S+\frac{1}{2}} + \frac{r_2}{S+2} + r_4\frac{(S+5)}{(S+5)^2+(5\sqrt{3})^2} + \frac{r_4}{5\sqrt{3}}\left(\frac{r_3}{r_4}-5\right)\frac{(5\sqrt{3})}{(S+5)^2+(5\sqrt{3})^2}$$

$$\Rightarrow \hat{y}_2(t) = r_1 e^{-\frac{t}{2}} + r_2 e^{-2t} + r_4 e^{-5t}\cdot\cos(5\sqrt{3}\,t) + \frac{r_4}{5\sqrt{3}}\left(\frac{r_3}{r_4}-5\right)\cdot e^{-5t}\cdot\sin(5\sqrt{3}\cdot t)$$

⇝ USCITE REALI

$$\begin{cases} u_1(t) = 4 \cdot 1\!\!1(t) \\ u_2(t) = -4 \cdot 1\!\!1(t-3) \\ u_3(t) = 4e^{-\frac{t-3}{2}} \cdot 1\!\!1(t-3) \end{cases}$$

ed Abbiamo scelto

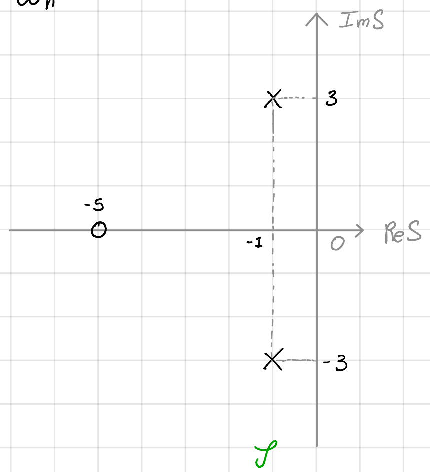
$$\begin{cases} \hat{U}_1(s) = 1/s \\ \hat{U}_2(s) = \frac{1}{s+1/2} \end{cases} \rightleftharpoons \begin{cases} u_1(t) = 1\!\!1(t) \\ u_2(t) = e^{-\frac{t}{2}} \cdot 1\!\!1(t) \end{cases}$$

$$\Rightarrow \begin{cases} y_1(t) = 4\,\hat{y}_1(t) \cdot 1\!\!1(t) \\ y_2(t) = -4\,\hat{y}_2(t-3) \cdot 1\!\!1(t-3) \\ y_3(t) = 4\,\hat{y}_3(t-3) \cdot 1\!\!1(t-3) \end{cases} \Rightarrow y(t) = y_1(t) + y_2(t) + y_3(t)$$

Esercizio Risposta in freq

$$G(S) = 100 \cdot \frac{0.2S + 1}{S^2 + 2S + 10}$$

$* \quad S^2 + 2\xi\omega_n S + \omega_n^2$



(1) Forma Standard, Poli e Zeri

$$G(S) = \cancel{100} \cdot \frac{1 + 0.2S}{\cancel{10}\left(\frac{S^2}{10} + \frac{2}{10}S + 1\right)}$$

Forma standard

$z_1 = -\frac{1}{0.2} = -5$ z

$P_{1,2} = -1 \pm 3j$ $P_{1,2}$

(2) Punti di rottura

$$S^2 + 2S + 10 = S^2 + 2\xi\omega_n S + \omega_n^2 \Rightarrow \begin{cases} \cancel{2}\xi\omega_n = \cancel{2} \\ 10 = \omega_n^2 \Rightarrow \omega_n = \sqrt{10} \simeq 3.16 \end{cases} \Rightarrow \xi = \frac{1}{\omega_n} = \frac{1}{\sqrt{10}} \simeq 0.32$$

ω_n ξ

- $\omega_1 = 5$

(3) Intervallo di Visual

$\leadsto \begin{array}{l} \min(\omega_i) = \omega_n = \sqrt{10} \\ \max(\omega_i) = \omega_1 = 5 \end{array} \Rightarrow t \in \left(\frac{1}{10} ; 10\right)$ [Almeno]

4) Andamenti iniziali e finali

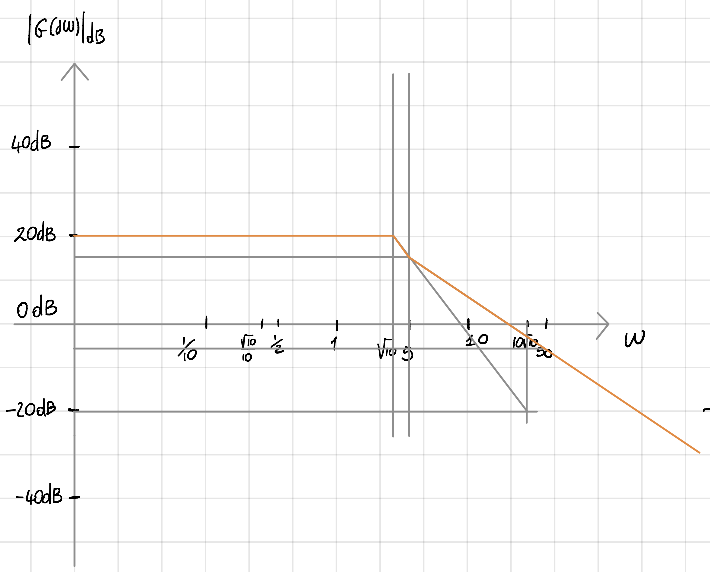
Modulo

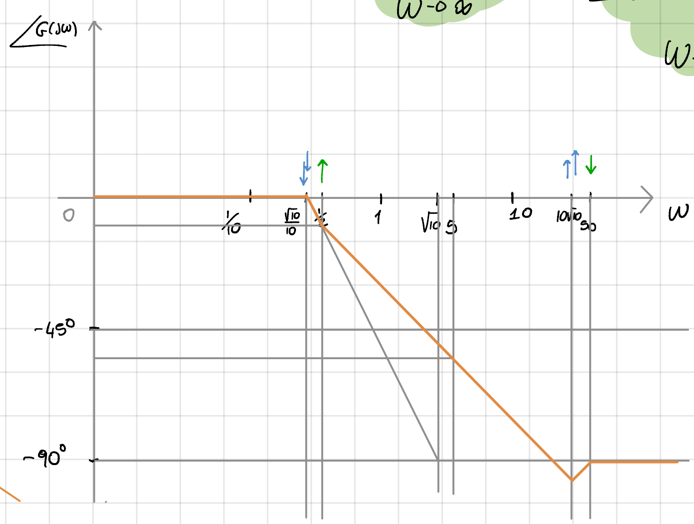
- No poli in 0, guadagno Statico: $K_p = 10 \Rightarrow |G(j\omega)|_{dB}\Big|_{\omega=0} = 20\log(10) = 20dB$

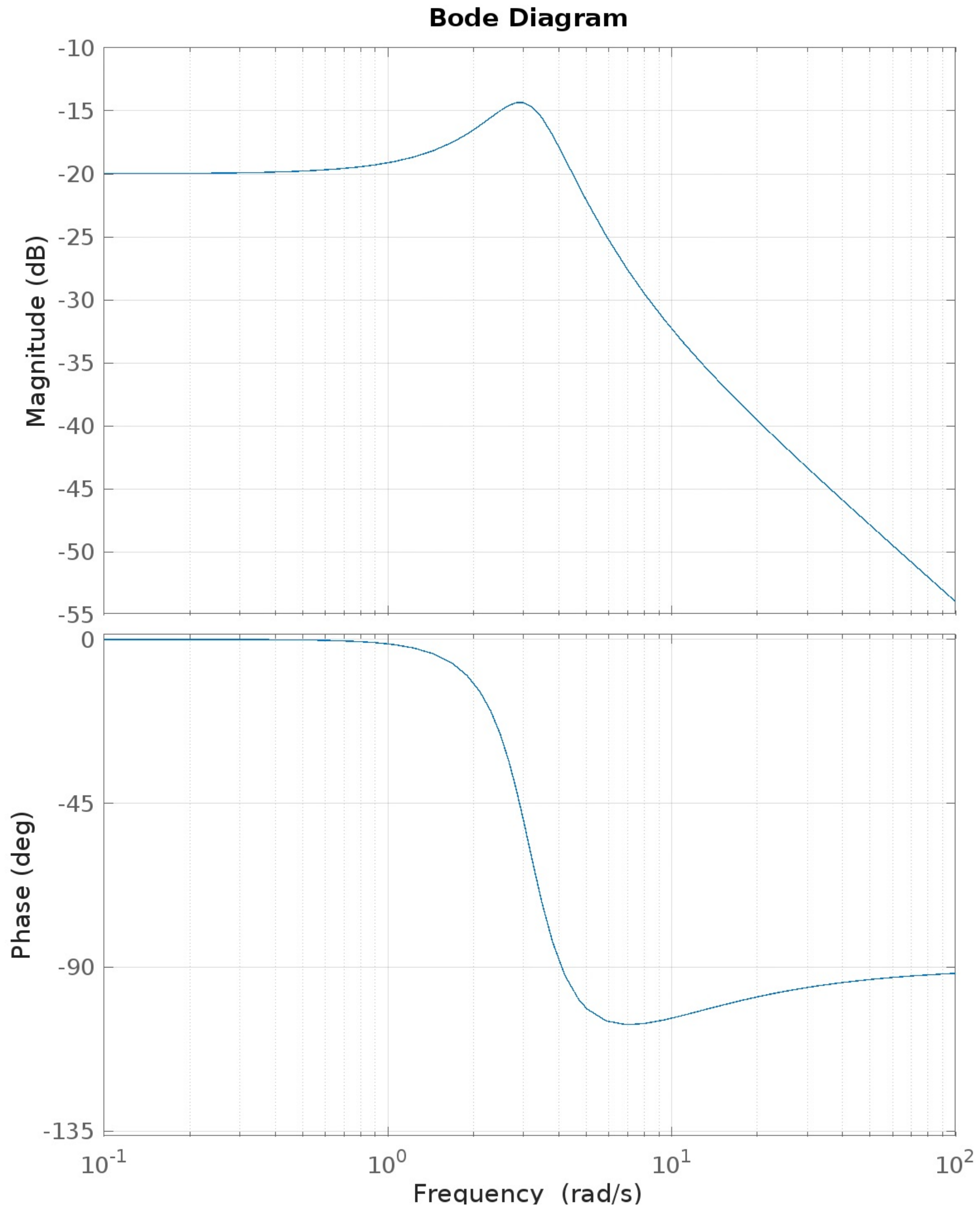
per $\omega \to \infty$ $\begin{cases} 1 \text{ Polo} \to +20\ dB/dec \\ 1\ \omega_n \to -40\ dB/dec \end{cases} \Rightarrow |G(j\omega)|_{dB}\Big|_{\omega\to\infty} = -20\ dB/dec$

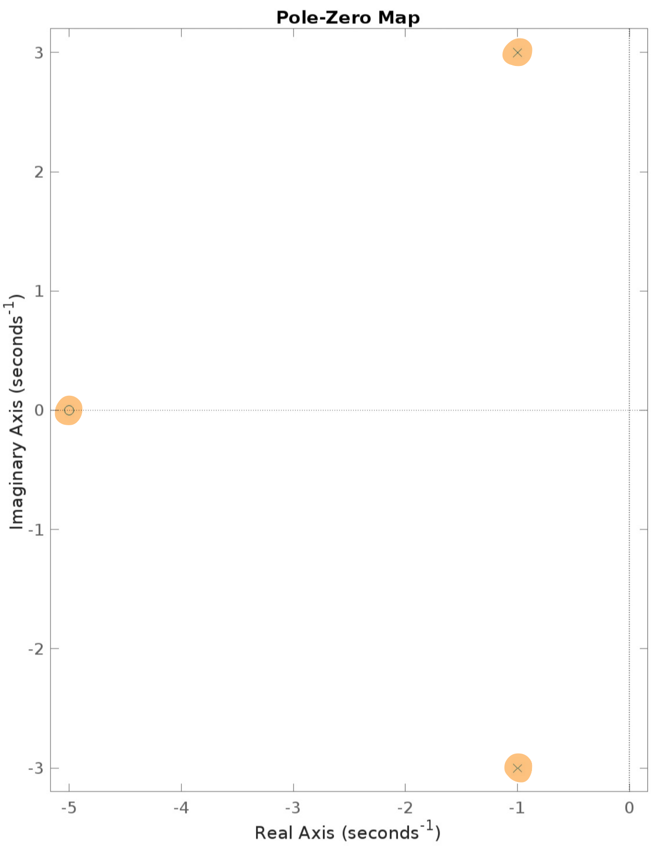
Fase

$\begin{cases} 1 \text{ zero a } ReP < 0 \to \text{Anticipo di } 90° \\ 1\ \omega_n \to \text{Ritardo } 180° \end{cases} \to \angle G(j\omega)\Big|_{\omega\to\infty} = -90° \qquad \angle G(j\omega)\Big|_{\omega=0} = 0°$

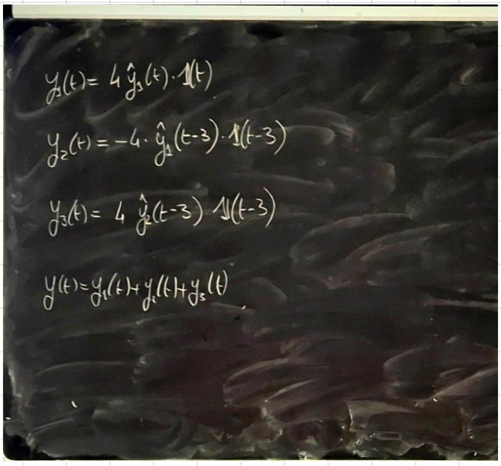




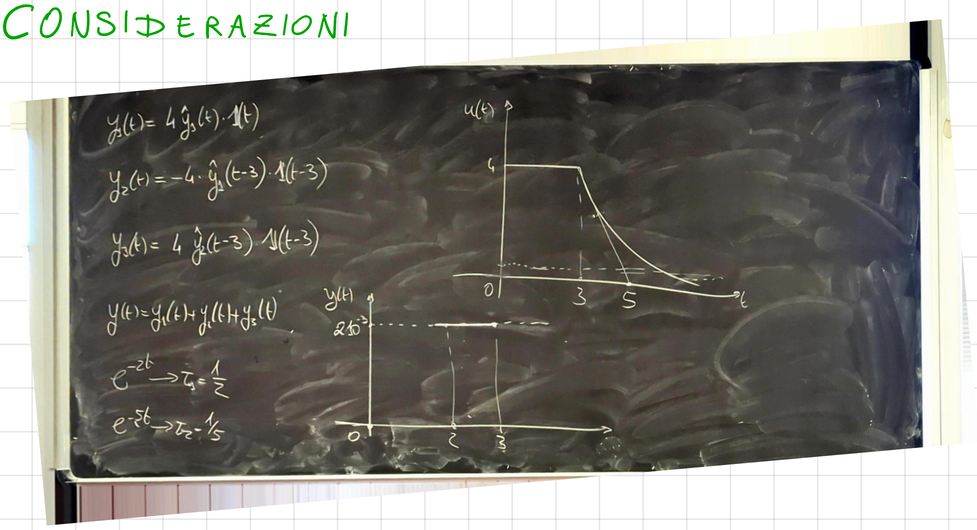

Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
0
-45
-90
-135
10^{-1}
10^{0}
10^{1}
10^{2}


Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
3
2
1
0
-1
-2
-3
-5
-4
-3
-2
-1
0

SCRIVIAMO LE USCITE REALI



CONSIDERAZIONI



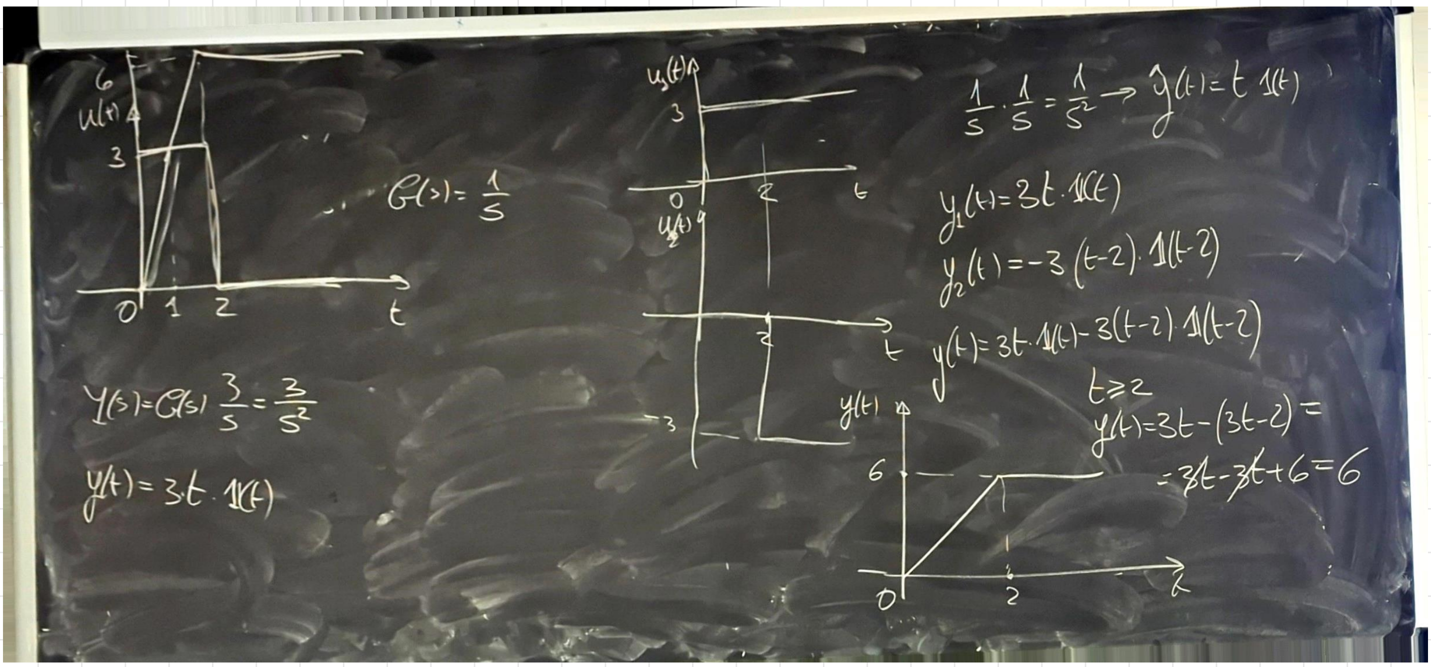
Asint. ST $\Rightarrow$ Non rimane ad un valore dato

$\Rightarrow$ Tende a zero

$$G(s) = \frac{s+0.1}{(s+2)\left((s+5)^2+(5\sqrt{3})^2\right)}$$

$$\lim_{s \to 0} \cancel{s}\, G(s) \cdot \frac{4}{\cancel{s}} = 4G(0)$$

BONUS



RISPOSTA IN FREQUENZA