$$G(s) = \frac{s + 0.1}{(s^2 + 10s + 100)(s+2)}$$

con $u(t) = \begin{cases} 0 & t < 0 \\ 4 & t \in (0,3) \\ 4e^{-\frac{t-3}{2}} & t \geq 3 \end{cases}$

(1) Scrivo $u(t)$ con segnali noti

- Per $t \in (0,3) \rightsquigarrow$ $U_1(t) = 4 \cdot \mathbb{1}(t)$
- Per $t \geq 3 \rightsquigarrow U_1(t) + U_2(t) + U_3(t)$ con $U_2(t) = -4\,\mathbb{1}(t-3)$ e $U_3(t) = 4e^{-\frac{t-3}{2}} \cdot \mathbb{1}(t-3)$

Annulla $U_1(t)$

(2) Determino le Trasformate

$U_1(t) \rightleftharpoons U_1(s) = 4 \cdot \frac{1}{s}$

$U_2(t) \rightleftharpoons U_2(s) = -4 \cdot \frac{1}{s} \cdot e^{-3s}$

$U_3(t) \rightleftharpoons U_3(s) = \mathcal{L}\left[4 \cdot e^{-\frac{t}{2}}\right] \cdot e^{-3s} = \frac{4}{s+\frac{1}{2}}\, e^{-3s}$

(3) Individuare il segnale che ci permette di ricondurci agli altri segnali

$\rightsquigarrow$ I segnali elementari da considerare sono:

$\hat{U}_1(t) = \mathbb{1}(t)$ e $\hat{U}_2(t) = e^{-\frac{t}{2}}$

(5) Anti trasformata

- $\hat{U}_1(s) = \frac{1}{s} \Rightarrow \hat{Y}_1(s) = \frac{s + 0.1}{s\,(s^2 + 10s + 100)(s+2)}$
- $\hat{U}_2(s) = \frac{1}{s+\frac{1}{2}} \Rightarrow \hat{Y}_2(s) = \frac{1}{s+\frac{1}{2}} \cdot \frac{s + 0.1}{(s^2 + 10s + 100)(s+2)}$

Verifico i poli: $s^2 + 10s + 100 \rightsquigarrow \bar{s} = -5 \pm \sqrt{25-100} = -5 \pm j\sqrt{75}$ Complx

5.1) $y_1(t)$ con poli Non complx e conj

$$\Rightarrow Y_1(s) = \frac{r_1}{s} + \frac{r_2}{s+2} + \frac{r_3 + r_4 s}{s^2 + 10s + 100}$$

$$\tau_1 = \lim_{s \to 0} s \cdot \frac{\cancel{s}+0.1}{s(\cancel{s^2}+\cancel{10s}+100)(s+2)} \to \frac{0.1}{200} \to \frac{1}{2}\,10^{-3} \quad \tau_1$$

$$\tau_2 = \lim_{s \to -2} \cancel{(s+2)} \frac{\cancel{s}+0.1}{s(\underset{4}{s^2}+\underset{-20}{10s}+100)\cancel{(s+2)}} = \frac{1.9}{2\cdot 84} = 11\cdot 10^{-3} \quad \tau_2$$

$$\frac{(s+2)\tau_1(s^2+10s+100)+s\tau_2(s^2+10s+100)+(\tau_3+\tau_4 s)(s+2)s}{s(s^2+10s+100)(s+2)} \leadsto \begin{cases} s^3(\tau_1+\tau_2+\tau_4) = 0 \\ s^2(2\tau_1+10\tau_1 \quad +10\tau_2+2\tau_4) = 0 \\ s(100\tau_1+20\tau_1+100\tau_2+2\tau_3) = 1 \\ 200\tau_1 = 0.1 \end{cases}$$

$$\tau_1+\tau_2+\tau_4 = 0 \Rightarrow \tau_4 = -(\tau_1+\tau_2) = 11.5\times 10^{-3} \quad \tau_4$$

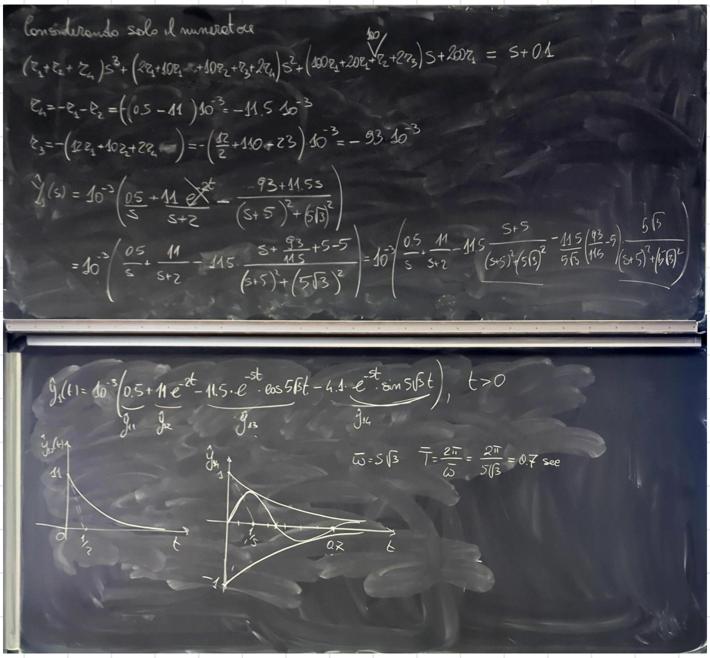
$$\tau_3 = -(12\tau_1+10\tau_2+2\tau_4) = -93\times 10^{-3} \quad \tau_3$$

$$\Rightarrow \hat{Y}_1(s) = 10^{-3}\left[\frac{1}{2} + \frac{11}{s+2} - \frac{93+11.5s}{(s+5)^2+(5\sqrt{3})^2}\right] = 10^{-3}\left[\frac{1/2}{s} + \frac{11}{s+2} - 11.5\cdot\frac{s+\frac{93}{11.5}+5-5}{(s+5)^2+(5\sqrt{3})^2}\right]$$

$$= 10^{-3}\left[\frac{1/2}{s} + \frac{11}{s+2} - 11.5\cdot\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} - \frac{11.5}{5\sqrt{3}}\left(\frac{93}{11.5}-5\right)\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}\right]$$

$$\Rightarrow \hat{y}_1(t) = 10^{-3}\left(\frac{1}{2} + \underbrace{11\cdot e^{-2t}}_{y_{11}} - \underbrace{11.5\cdot e^{-5t}\cos(5\sqrt{3}\,t)}_{y_{12}} - \underbrace{4.1\cdot e^{-5t}\cdot\sin(5\sqrt{3}\cdot t)}_{y_{13}}\right)\cdot \mathbb{1}(t)$$

INSERISCI DISEGNI



Calcolo $\hat{y}_2(t)$

$$\hat{Y}_2(s) = \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3 + r_4 s}{s^2+10s+100}$$

$$\hat{Y}_2(s) = \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{s^2+10s+100}$$

$$r_1 = \lim_{s\to -\frac{1}{2}} s\,\hat{Y}_2(s) = \frac{-\frac{1}{2}+0.1}{(-\frac{1}{2}+2)(\frac{1}{4}-\frac{10}{2}+100)} = \frac{-0.4}{1.5\cdot(95.25)} = -2.8\cdot 10^{-3}$$

$$r_2 = \lim_{s\to -2} (s+2)\,\hat{Y}_2(s) = \frac{-2+0.1}{(-2+\frac{1}{2})(4-20+100)} = \frac{1.9}{1.5\cdot 84} = 15\cdot 10^{-3}$$

$$r_1(s+2)(s^2+10s+100) + r_2(s+\tfrac{1}{2})(s^2+10s+100) + (r_3+r_4 s)(s+\tfrac{1}{2})(s+2) =$$

$$= r_1(s^3+12s^2+120s+200) + r_2(s^3+10.5s^2+105s+50) + (r_3+r_4 s)(s^2+2.5s+1)$$

$$\hat{y}_1(t) = 10^{-3}\Big(\underbrace{0.5}_{\hat{y}_{11}} + \underbrace{11 e^{-2t}}_{\hat{y}_{12}} - \underbrace{11.5\cdot e^{-5t}\cdot\cos 5\sqrt{3}t}_{\hat{y}_{13}} - \underbrace{4.1\cdot e^{-5t}\cdot\sin 5\sqrt{3}t}_{\hat{y}_{14}}\Big), \quad t>0$$

$$r_1 + r_2 + r_4 = 0 \Rightarrow r_4 = -(r_1+r_2) = -12.2\cdot 10^{-3}$$

$$12r_1 + 10.5r_2 + r_3 + 2.5r_4 = 0 \Rightarrow r_3 = -12r_1 - 10.5r_2 - 2.5r_4 = -10^{-3}(12\cdot 2.8 + 10.5\cdot 15 + 2.5\cdot 12.2) = -93.4\cdot 10^{-3}$$

$$\hat{Y}_2(s) = \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{s^2+10s+100}$$

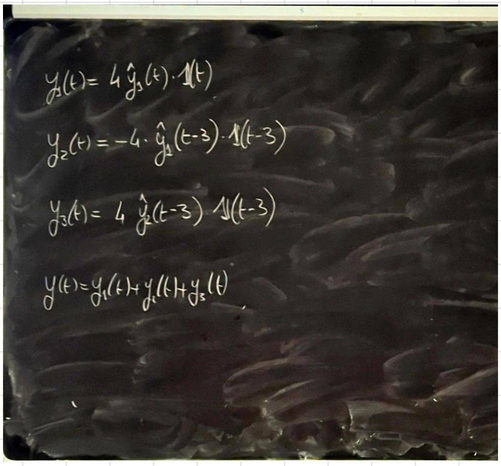
$$\hat{Y}_2(s) = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{s+5+\frac{r_3}{r_4}-5}{(s+5)^2+(5\sqrt{3})^2} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} + r_4\left(\frac{r_3}{r_4}-5\right)\frac{1}{5\sqrt{3}}\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}$$

$$\hat{Y}_2(s) = \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{s^2+10s+100}$$

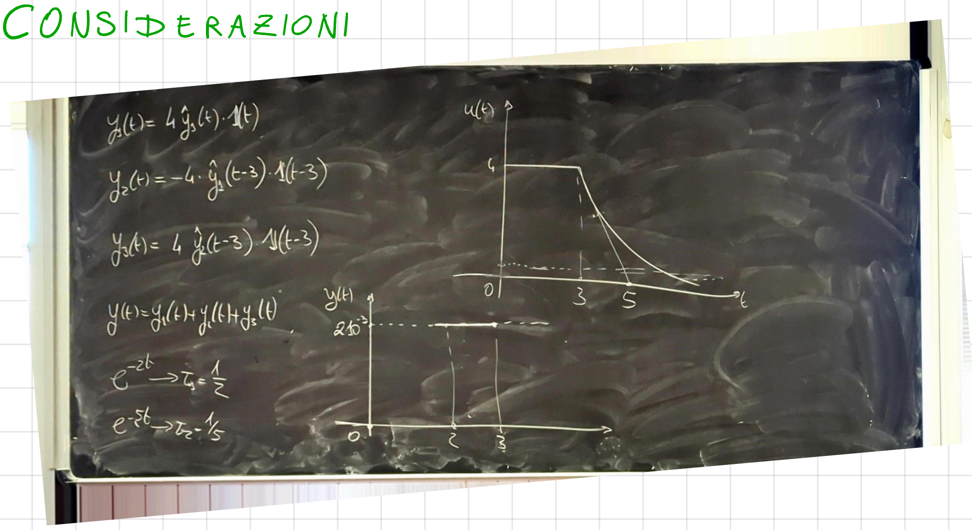
$$\hat{Y}_2(s) = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{s+5+\frac{r_3}{r_4}-5}{(s+5)^2+(5\sqrt{3})^2} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} + r_4\left(\frac{r_3}{r_4}-5\right)\frac{1}{5\sqrt{3}}\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}$$

$$\underbrace{\hat{y}_2(t) = r_1 e^{-t/2} + r_2 e^{-2t} + r_4\cdot e^{-5t}\cdot\cos 5\sqrt{3}t + \frac{r_4}{5\sqrt{3}}\left(\frac{r_3}{r_4}-5\right) e^{-5t}\sin 5\sqrt{3}t}_{}$$ uscita

SCRIVIAMO LE USCITE REALI



CONSIDERAZIONI



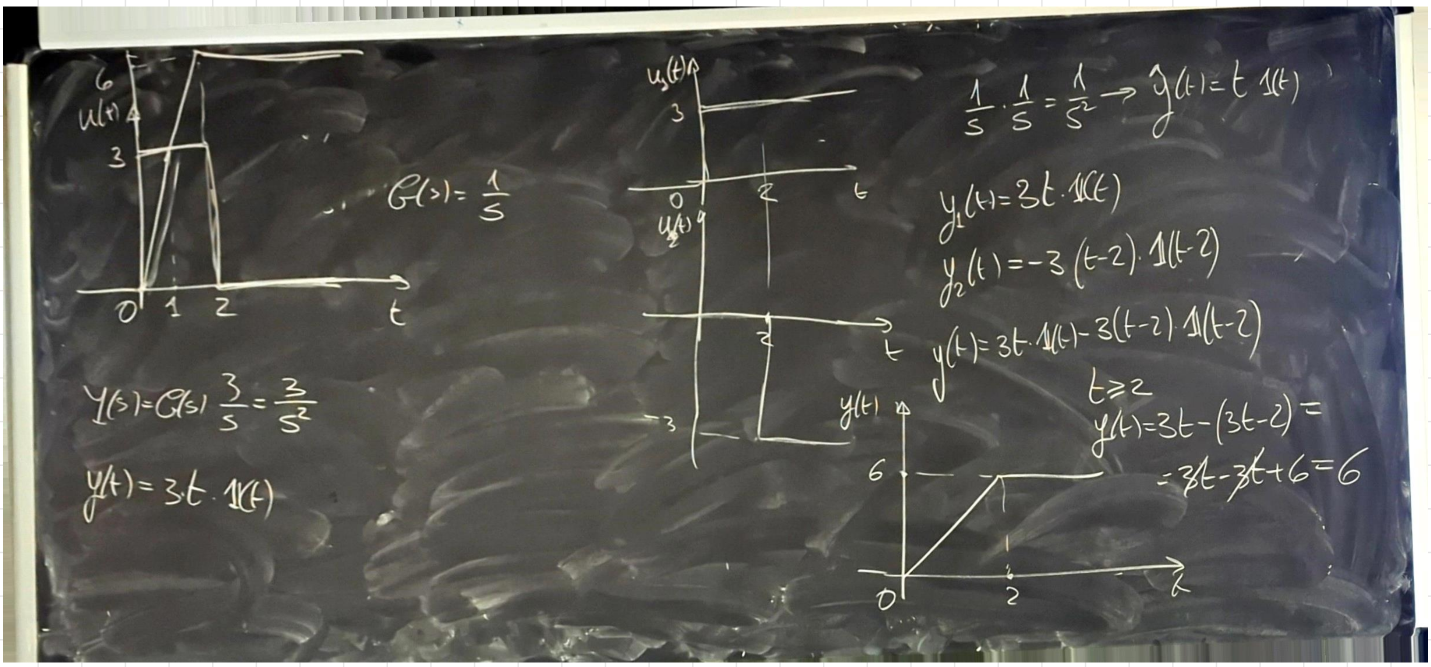
Asint. ST $\Rightarrow$ Non rimane ad un valore dato

$\Rightarrow$ Tende a zero

$$G(s) = \frac{s+0.1}{(s+2)\left((s+5)^2+(5\sqrt{3})^2\right)}$$

$$\lim_{s\to 0} \cancel{s}\, G(s)\cdot \frac{4}{\cancel{s}} = 4G(0)$$

BONUS



RISPOSTA IN FREQUENZA