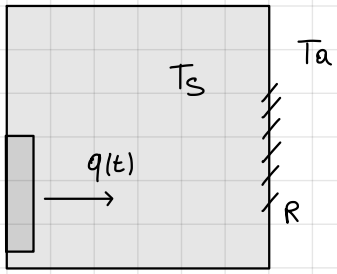
# STANZA E RADIATORE

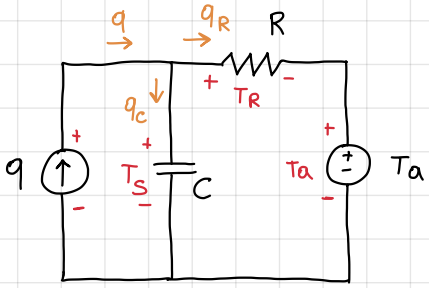





Questo è un semplice sistema dinamico di tipo termico: si compone di una stanza con delle pareti isolanti (coefficiente di isolamento dato dalla resistenza **R**) al cui interno è posto un radiatore avente un flusso **q**.

Possiamo modellare il sistema con un circuito equivalente, stando attenti a mettere la resistenza (delle pareti) *schematicamente* tra la temperatura *esterna* ed i componenti *interni* alla stanza.

## Circuito equivalente




## Scrivo le eq del sys

$$\begin{cases} C\dfrac{dT_S}{dt} = q_C \\ T_R = R \cdot q_R \end{cases} \qquad q_C = q - q_R \text{ ma } q_R = \frac{T_R}{R}$$

$$\Rightarrow C\frac{dT_S}{dt} = q - \frac{T_R}{R}$$

$$T_R = T_S - T_a \Rightarrow \boxed{C\frac{dT_S}{dt} = q - \frac{T_S - T_a}{R}}$$

## Funzione di Trasferimento

$$CS\,T_S(S) = Q(S) - \frac{1}{R}T_S(S) - \frac{1}{R}T_A(S) \rightarrow T_S(S)\left[CS + \frac{1}{R}\right] = Q(S) - \frac{1}{R}T_A(S)$$

($CS + \frac{1}{R} = \frac{SRC+1}{R}$; $-\frac{1}{R}T_A(S)$: DISTURBO)

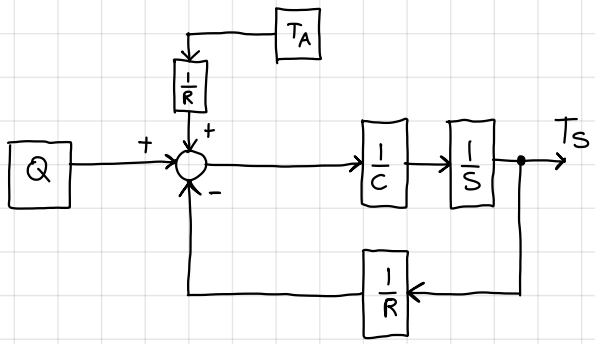
Voglio OUT: $T_S(S)$ IN: $Q(S)$

$$\Rightarrow \frac{T_S(S)}{Q(S)} = G(S) = \frac{Q(S)}{\frac{SRC+1}{R}} - \frac{\frac{1}{R}}{\frac{SRC+1}{R}}T_A(S)$$

$$\rightsquigarrow \boxed{G(S) = \frac{R}{SRC+1}Q(S) - \frac{1}{SRC+1}T_a}$$

$T_a$: Temperatura Esterna

## Schema a Blocchi



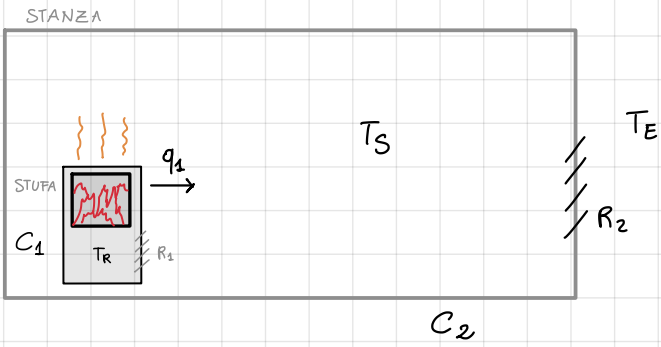

## SPAZIO DI STATO

$x_1 = T_S \quad U_1 = q \quad U_2 = T_a \quad y = T_S$

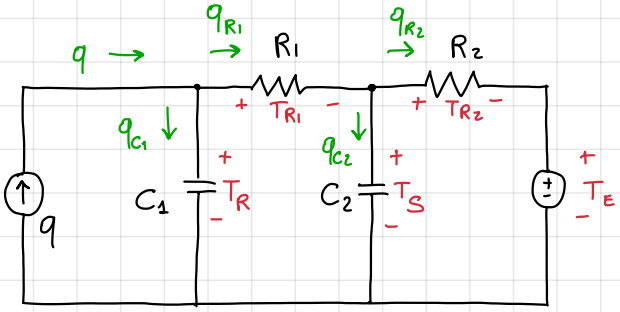
$$\begin{cases} \dot{x} = \frac{1}{C}\left[U_1 - \frac{1}{R}(x_1 - U_2)\right] \\ y = x_1 \end{cases}$$

$$\begin{cases} \dot{x} = -\frac{1}{RC}\cdot x_1 + \begin{pmatrix} \frac{1}{C} & \frac{1}{RC} \end{pmatrix}\cdot\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \\ y = 1 \cdot x_1 \end{cases}$$

# "LA STUFA" o Radiatore




CIRCUITO EQUIVALENTE




$$\begin{cases} C_1 \dfrac{dT_R}{dt} = q_{C1} \\ C_2 \dfrac{dT_S}{dt} = q_{C2} \end{cases}$$

LKC: $q_{C_1} = q - q_{R_1} \Rightarrow C_1 \dfrac{dT_R}{dt} = q - q_{R_1} = q - \dfrac{T_{R_1}}{R_1}$

LKC: $q_{C_2} = q_{R_1} - q_{R_2} \Rightarrow C_2 \dfrac{dT_S}{dt} = q_{R_1} - q_{R_2} = \dfrac{T_{R_1}}{R_1} - \dfrac{T_{R_2}}{R_2}$

$$\begin{cases} T_{R_1} = q_{R_1} \cdot R_1 \Rightarrow q_{R_1} = \dfrac{T_{R_1}}{R_1} \\ T_{R_2} = q_{R_2} \cdot R_2 \Rightarrow q_{R_2} = \dfrac{T_{R_2}}{R_2} \end{cases}$$

LKT: $T_{R_1} = T_R - T_S \quad ; \quad T_{R_2} = T_S - T_E$

$$\Rightarrow \begin{cases} C_1 \dfrac{dT_R}{dt} = q - \dfrac{1}{R_1}(T_R - T_S) = q - \dfrac{1}{R_1} T_R + \dfrac{1}{R_1} T_S \\ C_2 \dfrac{dT_S}{dt} = \dfrac{1}{R_1}(T_R - T_S) - \dfrac{1}{R_2}(T_S - T_E) = \dfrac{1}{R_1} T_R + \left(-\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) T_S + \dfrac{1}{R_2} T_E \end{cases}$$

## SPAZIO DI STATO

$x_1 = T_R \quad x_2 = T_S \quad u_1 = q \quad u_2 = T_E \quad y = T_S$

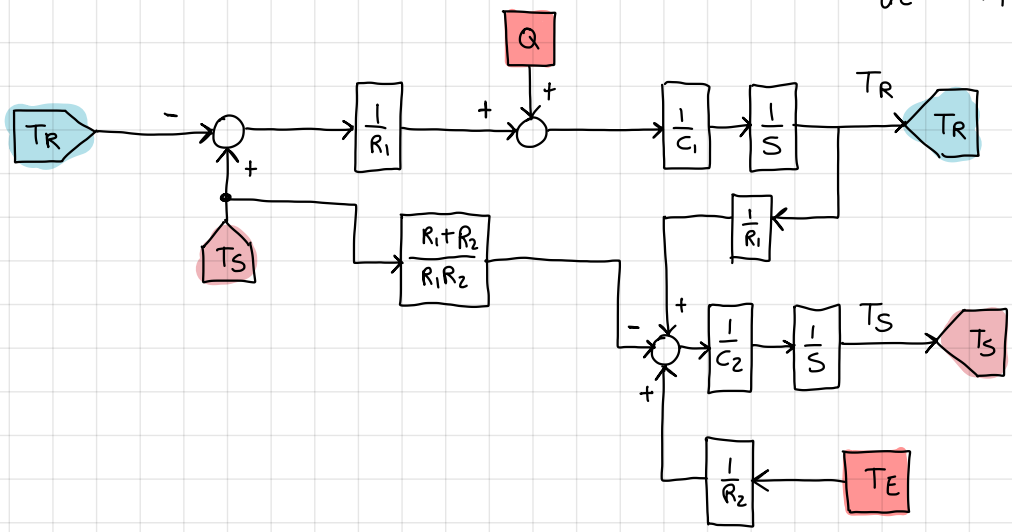
$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1}\left(u_1 - \dfrac{1}{R_1} x_1 + \dfrac{1}{R_1} x_2\right) = -\dfrac{1}{C_1 R_1} x_1 + \dfrac{1}{C_1 R_1} x_2 + \dfrac{1}{C_1} u_1 + 0\, u_2 \\ \dot{x}_2 = \dfrac{1}{C_2}\left(\dfrac{1}{R_1} x_1 - \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) x_2 + \dfrac{1}{R_2} u_2\right) = +\dfrac{1}{C_2 R_1} x_1 - \dfrac{1}{C_2}\left(\dfrac{R_2 + R_1}{R_1 R_2}\right) x_2 + 0 u_1 + \dfrac{1}{C_2 R_2} u_2 \\ y = x_2 = 0 x_1 + 1 x_2 + 0 u_1 + 0 u_2 \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{C_1 R_1} & \dfrac{1}{C_1 R_1} \\ \dfrac{1}{C_2 R_1} & -\dfrac{R_1 + R_2}{C_2 R_1 R_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \dfrac{1}{C_1} & 0 \\ 0 & \dfrac{1}{C_2 R_2} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \emptyset \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

## SCHEMA A BLOCCHI

$$\begin{cases} C_1 \dfrac{dT_R}{dt} = q - \dfrac{1}{R_1} T_R + \dfrac{1}{R_1} T_S \\ C_2 \dfrac{dT_S}{dt} = \dfrac{1}{R_1} T_R + \left( -\dfrac{1}{R_1} - \dfrac{1}{R_2} \right) T_S + \dfrac{1}{R_2} T_E \end{cases}$$




| Conducibilità termica | |
|---|---|
| **sostanza** | **k(W/m°K)** |
| argento | 430 |
| rame | 390 |
| alluminio | 240 |
| ferro | 80 |
| vetro | 0,93 |
| acqua | 0,68 |
| legno | 0,2 |
| aria secca | 0,02 |
| cemento | 1,5 |
| mattoni | 0,35 |
| muratura | 0,7 |

# FUNZIONE DI TRASFERIMENTO

$$\begin{cases} C_1 \dfrac{dT_R}{dt} = q - \dfrac{1}{R_1} T_R + \dfrac{1}{R_1} T_S \\ C_2 \dfrac{dT_S}{dt} = \dfrac{1}{R_1} T_R + \left( -\dfrac{1}{R_1} - \dfrac{1}{R_2} \right) T_S + \dfrac{1}{R_2} T_E \end{cases}$$

$$\begin{cases} sC_1 T_R = Q - \dfrac{1}{R_1} T_R + \dfrac{1}{R_1} T_S \\ sC_2 T_S = \dfrac{1}{R_1} T_R - \left( \dfrac{R_2 + R_1}{R_1 R_2} \right) T_S + \dfrac{1}{R_2} T_E \end{cases}$$

IN : $Q$ OUT : $T_S$

$$T_S \left( \frac{sC_2 R_1 R_2 + R_1 + R_2}{R_1 R_2} \right) = \underbrace{\frac{1}{R_1} T_R}_{\text{RADIATORE}} + \underbrace{\frac{1}{R_2} T_E}_{\text{DISTURBO}}$$

dalla (1): $T_R \left( \dfrac{sC_1 R_1 + 1}{R_1} \right) = Q + \dfrac{1}{R_1} T_S \Rightarrow T_R = \underbrace{\dfrac{R_1}{sC_1 R_1 + 1} Q}_{\text{IN PUT}} + \underbrace{\dfrac{1}{sC_1 R_1 + 1} T_S}_{\text{STANZA}}$

$$\Rightarrow T_S \left( \frac{sC_2 R_1 R_2 + R_1 + R_2}{R_1 R_2} \right) = \frac{1}{R_2} T_E + \frac{1}{sC_1 R_1 + 1} Q + \frac{1}{sC_1 R_1^2 + R_1} T_S$$

$$T_S \left( \frac{sC_2 R_1 R_2 + R_1 + R_2}{R_1 R_2} - \frac{1}{sC_1 R_1^2 + R_1} \right) = \frac{1}{sC_1 R_1 + 1} Q + \frac{1}{R_2} T_E$$

$$\Rightarrow T_S \left( \frac{C_1 C_2 R_1 R_2 s + C_1 R_1 s + C_1 R_2 s + C_2 R_2 + 1}{C_1 R_1 R_2 s + R_2} \right) = \frac{1}{sC_1 R_1 + 1} Q + \frac{1}{R_2} T_E$$

$$sC_1 \left( C_2 R_1 R_2 + R_1 + R_2 \right) + C_2 R_2 + 1$$

$$s^2 C_1^2 R_1 \left( C_2 R_1 R_2 + R_1 + R_2 \right) + sC_1 \left( C_2 R_1 R_2 + R_1 + R_2 \right) +$$