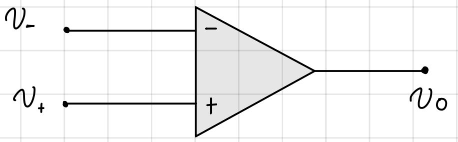
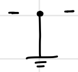
AMPLIFICATORI OPERAZIONALI

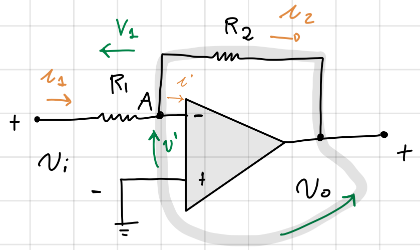


$$v_0 = K\,(v_+ - v_-)$$

guadagno



Tipica connessione INVERTENTE



$$\begin{cases} i_1 = \dfrac{v_{R_1}}{R_1} = \dfrac{v_1 - v'}{R_1} \\[2ex] i_2 = \dfrac{v' - v_0}{R_2} \end{cases}$$

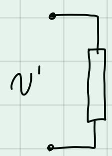
[uso la maglia v', v_0, v_2]

LKC_A : $-i_1 + i' + i_2 = 0 \rightarrow i_1 = i' + i_2$

nell'A.O. la corrente in entrata e' ZERO

Hp. $i' \simeq 0$

$\Rightarrow \boxed{i_1 \simeq i_2} \Rightarrow (1) \simeq (2)$



$v' = Z\,i'$ $\rightarrow i' = \dfrac{v'}{Z}$ con $Z \to \infty$

$\Rightarrow i' \sim 0$

$\rightarrow \dfrac{v_1 - v'}{R_1} = \dfrac{v' - v_0}{R_2} \Rightarrow$ $\boxed{\dfrac{v_1}{R_1} = -\dfrac{v_0}{R_2}}$ $\Rightarrow$ $\boxed{v_0 = -\dfrac{R_2}{R_1} v_i}$

$v' \simeq 0$

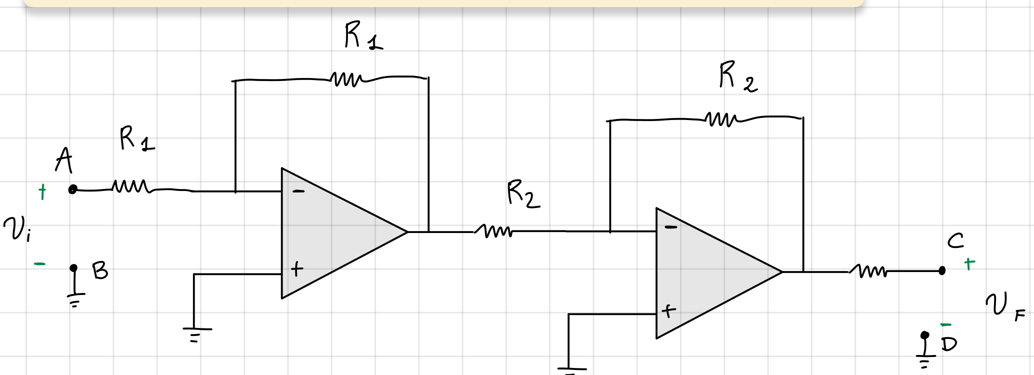
GUADAGNO a seconda delle R cambia K

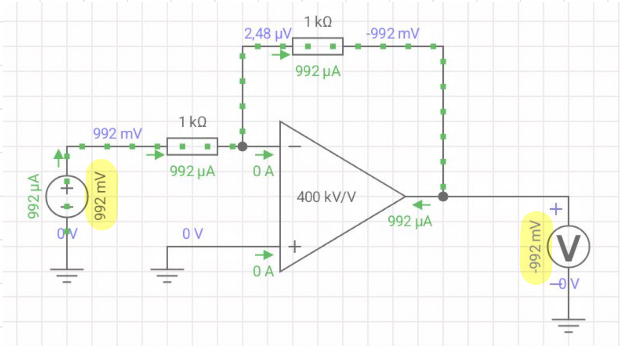
Configurazione invertente

Se metto due amplificatori operazionali (entrambi con delle resistenze uguali, anche diverse tra i due amplificatori) otteniamo esattamente la tensione iniziale (senza segno opposto)

Se $R_1 = R_2$

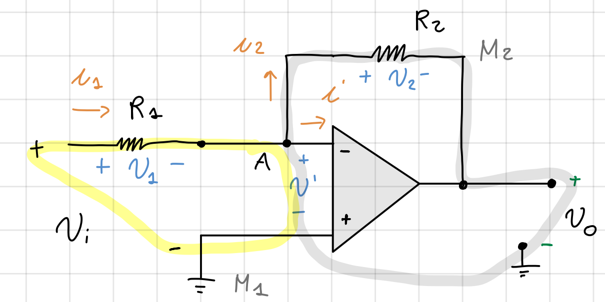
$v_0 \rightsquigarrow$ ▷ $\rightsquigarrow -v_0$





Amplificatore invertente

QUALCHE SPIEGAZIONE IN PIÙ..



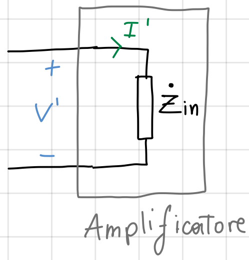
Scopo del gioco: Trovare v_o in relazione a R_1, R_2

$$i_2 = \frac{v_2}{R_2} \underset{M_2}{=} \frac{v' - v_o}{R_2}$$

$$i_1 = \frac{v_{R_1}}{R_1} \underset{M_1}{=} \frac{v_i - v'}{R_1}$$

ma LKC$_A$: $-i_1 + i_2 + i' = 0 \Rightarrow i_1 = i_2 + i'$

$\rightarrow$ Proprietà degli A.Op. $\dot{Z}_{interna} >>$



Troviamo i'

$$V' = \dot{Z} \cdot I' \Rightarrow I' = \frac{V'}{\dot{Z}}$$

ma se $\dot{Z} >> V \Rightarrow \frac{V'}{\dot{Z}} \simeq 0$

$\Rightarrow I' \simeq 0$ QED

$$\Rightarrow i_1 = i_2 + \underbrace{\cancel{i'}}_{\sim 0} = i_2$$

$\Rightarrow i_1 \simeq i_2$ ovvero $\dfrac{v_i - v'}{R_1} = \dfrac{v' - v_o}{R_2}$ (1)

Inoltre gli op-amp sono fatti per avere un guadagno molto elevato in modo da amplificare anche il minimo segnale:

$v_o = K(v_+ - v_-)$ con $K >> (v_+ - v_-)$ ma (vedi figura) v_+ è messo a Terra!

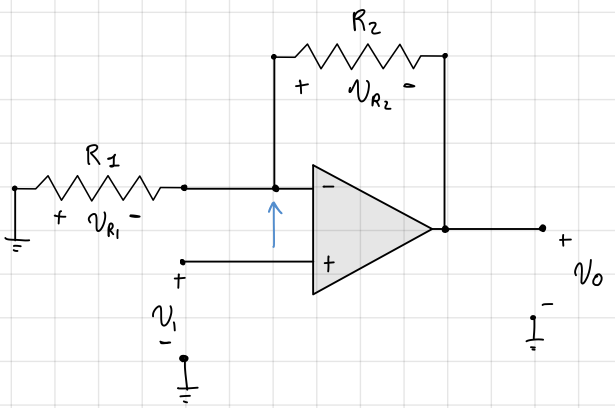
$\Rightarrow v_o = K(0 - v')$ con $K >> 1 \Rightarrow v' \simeq \emptyset$

(1) $\dfrac{v_i - \cancel{v'}}{R_1} = \dfrac{\cancel{v'} - v_o}{R_2} \rightarrow \dfrac{v_i}{R_1} = -\dfrac{v_o}{R_2} \Rightarrow v_o = - \dfrac{R_2}{R_1} v_i$

guadagno

INVERTENTE

CONFIGURAZIONE NON INVERTENTE



$i_1 \simeq i_2 \;\rightarrow\; \bullet\; i_1 = \frac{v_{R_1}}{R_1} = -\frac{v_i}{R_2}$

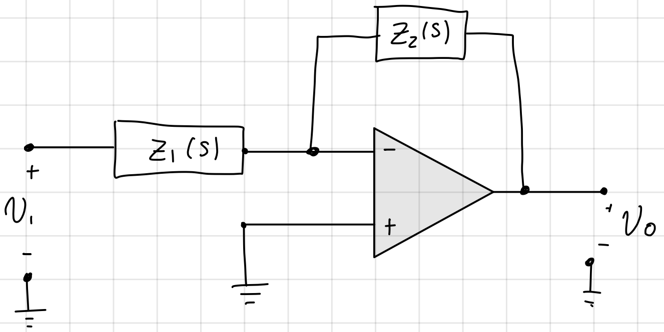
$\bullet\; i_2 = \frac{v_{R_2}}{R_2} = \frac{v_i - v_o}{R_2}$

$\rightarrow -\frac{v_i}{R_2} = \frac{v_i - v_o}{R_2} \;\rightarrow\; \frac{v_o}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_i \;\Rightarrow\; \boxed{v_o = R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_i}$

GUADAGNO

METODO DELLE IMPEDENZE

pg 125



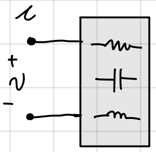
$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}} \equiv \text{F. DI TRANSF.}$$

TRASFORMATE

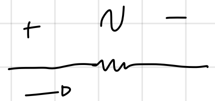
impedenze

Recap Impedenza complessa

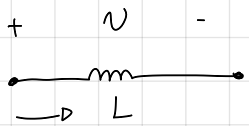
$Z(s) = \frac{V(s)}{I(s)}$ ← OUT ← IN



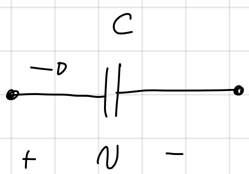
La differenza tra l'impedenza complessa ed una semplice funzione di trasferimento è che nel caso dell'impedenza blocchiamo come ingresso la corrente e come uscita la tensione.



$\frac{v}{i} = R = Z_R(s)$



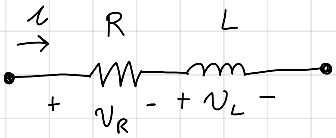
$v = L\frac{di}{dt} \Rightarrow V(s) = sL\, I(s) \Rightarrow Z_L(s) = sL$



$i = C\frac{dv}{dt} \Rightarrow I(s) = sC\, V(s) \Rightarrow Z_C(s) = \frac{1}{sC}$

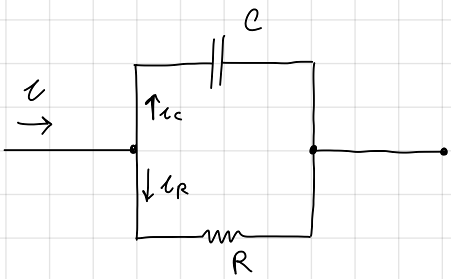
FOTO 1

COMBINAZIONI DI IMPEDENZE



$v = R\,i_L + L\,\dot{i}_L \;\rightarrow\; V(s) = R\,I(s) + sL\,I(s) = I(s)(R + sL)$

$\rightarrow$ Siccome $Z = \frac{V(s)}{I(s)} \rightarrow$ $\boxed{Z = R + sL}$ RL Serie



$i = i_C + i_R \rightarrow i = C\,\dot{v}_C + \frac{v}{R}$

$\rightleftarrows I(s) = sC\,V(s) + \frac{1}{R}V = V(s)\left(sC + \frac{1}{R}\right)$

$\Rightarrow Z(s) = \frac{1}{sC + \frac{1}{R}} = \boxed{\frac{R}{RsC + 1}}$ $Z(s)$ RC PARALL

Implementare un controllore di tipo proporzionale

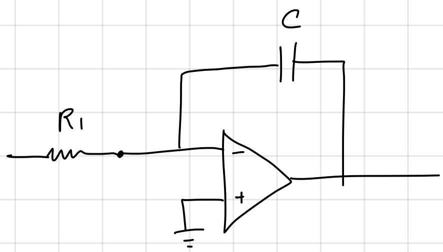
$G(s) = K_P \rightarrow$ visto prima

$G(s) = K_P + \frac{K_I}{s} \rightarrow$ vediamo dopo

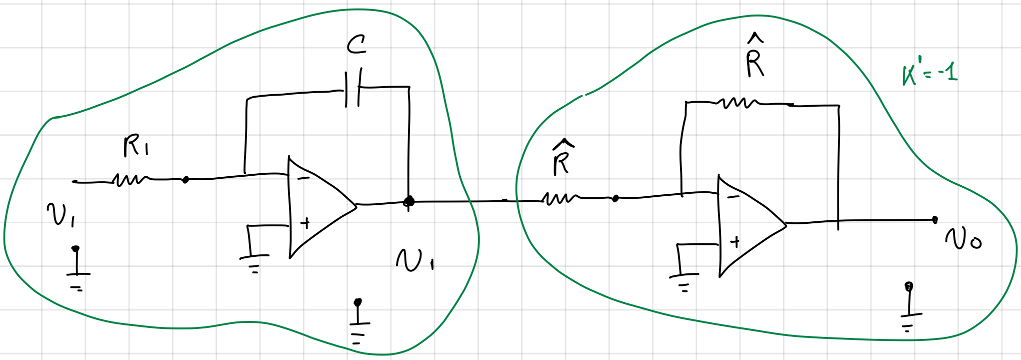
$G(s) = \frac{K_I}{s} \rightarrow$ vediamo ora

voglio impl. $G(s) = \frac{K_i}{s}$

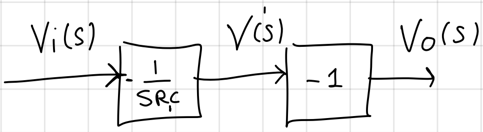
$$\begin{cases} Z_2(s) = \frac{1}{sC} \\ Z_1(s) = R_1 \end{cases}$$



$K = \frac{1}{RC}$



$K_I = -\frac{1}{R_1 C}$

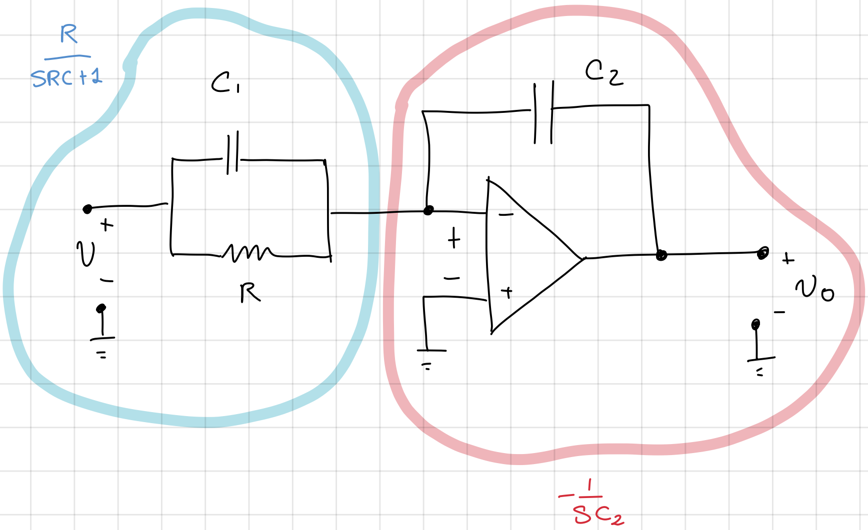


$$G(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s}$$

1 zero e 1 polo in 0

$$\frac{Z_1}{Z_2} = G(s) \quad \rightarrow \quad Z_2 = C = \frac{1}{sC_2} \leftarrow \text{POLO}$$

$$Z_1(s) = \frac{R}{sRC+1}$$



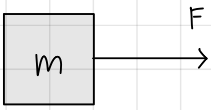
$$\frac{V_0(s)}{V_i(s)} = \frac{1}{sRC} \cdot \frac{sRC_1+1}{R} = \frac{s\underbrace{\frac{C_1}{C_2}}_{K_p} + \underbrace{\frac{1}{RC_2}}_{K_i}}{s}$$

Modellistica dei sistemi meccanici

* Primo e secondo es clip audio 8/9
e domanda orale
2° e 3° orale → elenco domande

Caratterizzati da MASSA molla e SMORZATORE

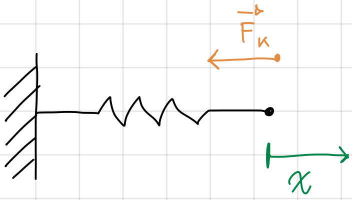
MASSA



$$F = ma \;[N], \quad a = \frac{dv}{dt} = \dot{v} \;[m/s^2], \quad v = \frac{dx}{dt} = \dot{x} \;[m]$$

$\dot{x}$ ↑ Spazio

MOLLA



Legge di Hooke : $F_k = -Kx$ (la molla viene Allungata)

$$E_m(t) = \int_0^t \underbrace{F(t)\cdot v(t)}_{\text{Potenza}}\, dt = \int_0^t m \frac{dv}{dt}\cdot v(t)\, dt = \frac{1}{2} m \int_0^t \frac{d v^2(t)}{dt}\, dt = \frac{1}{2} m \left[v^2(t) \right]_0^t$$

$$= \frac{1}{2} m v^2(t)$$

Energia di un corpo in movim.

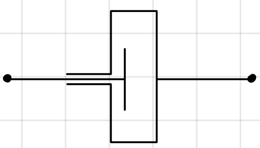
Possibile Var di Stato

$$E_k(t) = \int_0^t F_k(t)\cdot v(t)\, dt = \frac{1}{2} k\, x^2(t)$$

Energia della molla

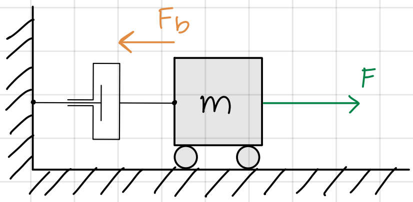
Possibile Var di Stato

SMORZATORE



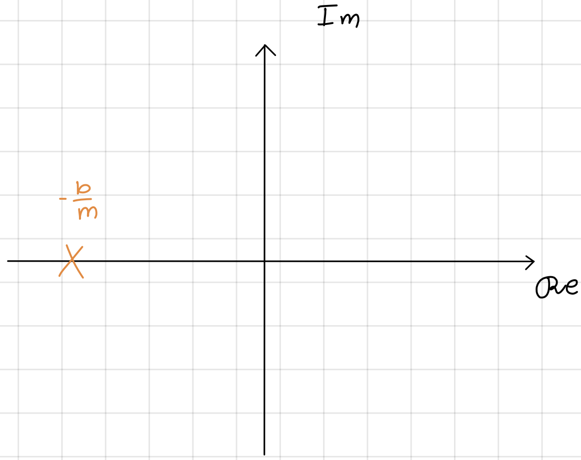
$$v = \frac{dx}{dt} \rightarrow F_b = -bv \equiv F_b = -b\cdot\dot{x}$$

SISTEMA 1

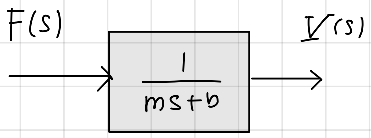


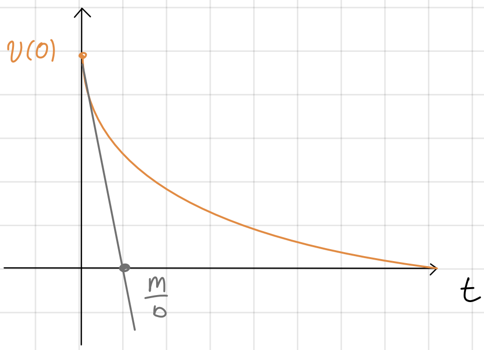
$$\begin{cases} F + F_b = m\dot{v} \\ F_b = -bv \end{cases}$$

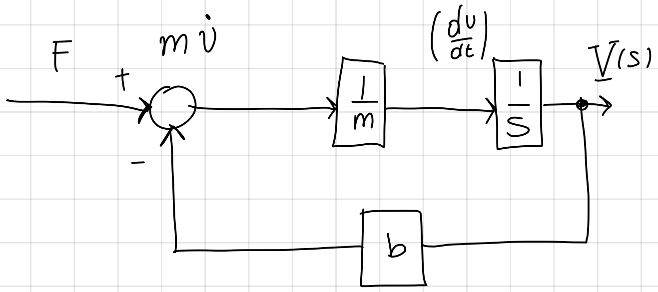
$$\mathcal{L}.T. \rightarrow \quad F(s) - b\cdot V = m\cdot s\, V(s) \quad \rightarrow \quad (ms+b)\, V(s) \quad F(s)$$



$$\Rightarrow \quad \frac{V(s)}{F(s)} = \frac{1}{ms+b}$$





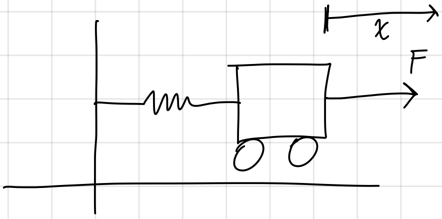


VAR. ST. $\quad v$

$$x = v \quad , \quad u = F \quad , \quad y = v$$

$$\begin{cases} \dot{x} = \frac{1}{m}(u - b\cdot x) \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \frac{b}{m}x + \frac{1}{m}u \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \overset{\frac{b}{m}}{\underline{\underline{A}}}\, x + \overset{\frac{1}{m}}{\underline{\underline{B}}}\, u \\ y = \underset{1}{\underline{\underline{C}}}\, x + \underset{0}{\underline{\underline{D}}}\, y \end{cases}$$

SENZA SMORZATORE



$$\begin{cases} F + F_K = m\dot{v} \longrightarrow \dot{v} = \frac{1}{m}(F + F_K) \\ F_K = -Kx \\ v = \dot{x} \end{cases}$$

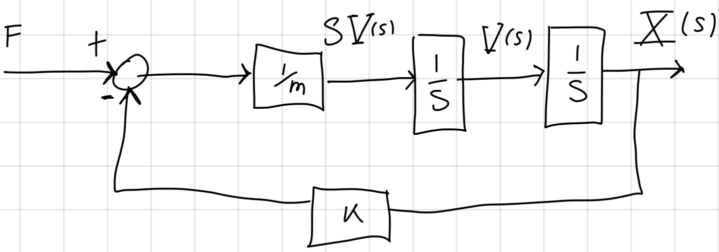
$x_1 = x \qquad x_2 = v \qquad u = F \qquad y = x_1$ ← la scelgo io

↑ pos in x

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m}(u - Kx_1) \\ y = x_1 \end{cases} \longrightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{m}x_1 + \frac{1}{m}u \\ y = x_1 \end{cases}$$

$$\longrightarrow \underset{x_1, x_2}{\underline{A}} = \begin{array}{cc} \begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix} & \begin{array}{c} x_1 \\ x_2 \end{array} \end{array} \qquad \underset{u}{\underline{B}} \begin{pmatrix} 0 \\ 1/m \end{pmatrix} \qquad \underset{y}{\underline{C}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

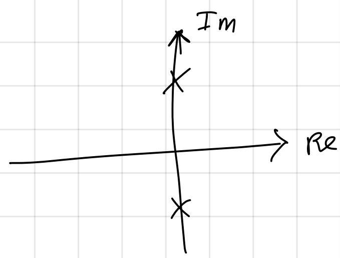
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \overset{\underline{A}}{\begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix}} \cdot \overset{\underline{B}}{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} + \overset{\underline{C}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} u$$



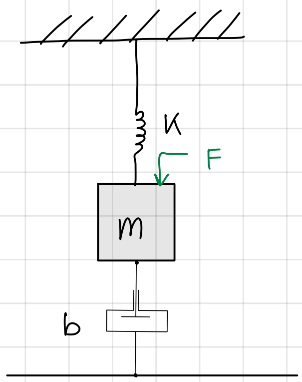
$$G(s) = \frac{X}{S} = \frac{\frac{1}{ms^2}}{1 + K\frac{1}{ms^2}} = \frac{1}{ms^2 + K}$$

Sin(t)

con $S = \pm j\sqrt{\frac{K}{m}}$



CON SMORZATORE



$$\begin{cases} F + F_K + F_b = m\dot{v} \\ F_K = -Kx \\ F_b = -bv \\ v = \dot{x} \end{cases}$$

$ms\,V(s) = -F(s) - K\,X(s) - b \cdot V(s)$ ma $V(s) = s\,X(s)$

$\to\ ms\,V(s) + bs\,X(s) + K\,X(s) = F(s)$

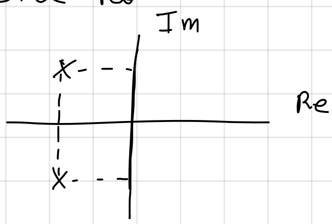
$\to\ \dfrac{X(s)}{F(s)} = \dfrac{1}{ms^2 + bs + K}$ $\to$ $\boxed{b^2 - 4m \cdot b > 0}$ ovvero se $b^2 > 4mb$

Se c'è uno smorzamento abbiamo anche la parte reale

ReP Negativo

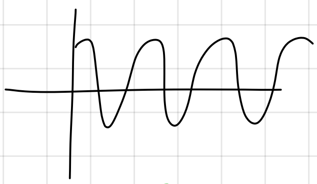
$\lambda = -a \pm jb$ $\to$

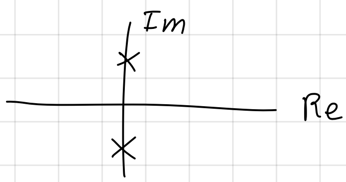
Asintoticamente stabili



STABILE

$\lambda = 0 \pm jb$ $\to$

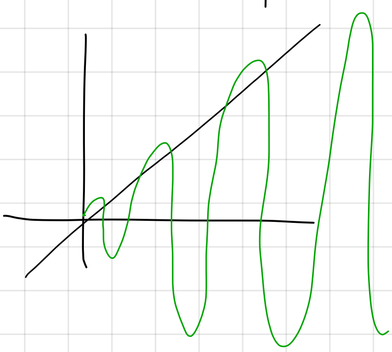


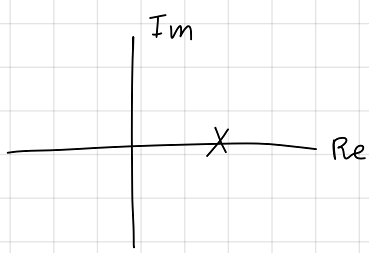


$\lambda = +a \pm jb$

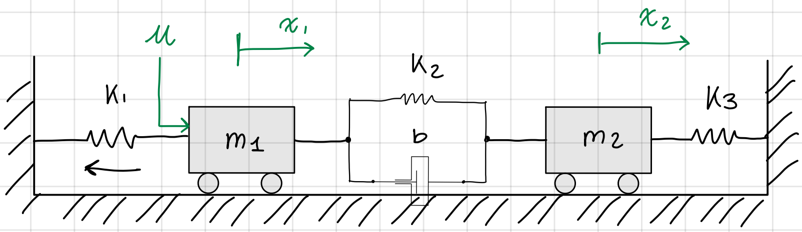
ReP Pos

INSTABILE





ACCOPPIAMENTO FERROVIARIO



Scriviamo $m \cdot a = \sum F$ Tenendo presente che $a = \dot{v} = \ddot{x}$ e le rel car

$$\begin{cases} m_1 \ddot{x}_1 = u - K_1 x_1 - K_2 (x_1 - x_2) - b(\underbrace{\dot{x}_1 - \dot{x}_2}_{v_1 - v_2}) \\ m_2 \ddot{x}_2 = K_2 (x_1 - x_2) + b(\underbrace{\dot{x}_1 - \dot{x}_2}_{v_1 - v_2}) - K_3 x_2 \end{cases}$$

mancano $\dot{x}_1$ e $\dot{x}_2$ $= 0$ Pongo

$$x_3 = \dot{x}_1 \leftarrow v_{m1}$$
$$x_4 = \dot{x}_2 \leftarrow v_{m2}$$

ottengo:

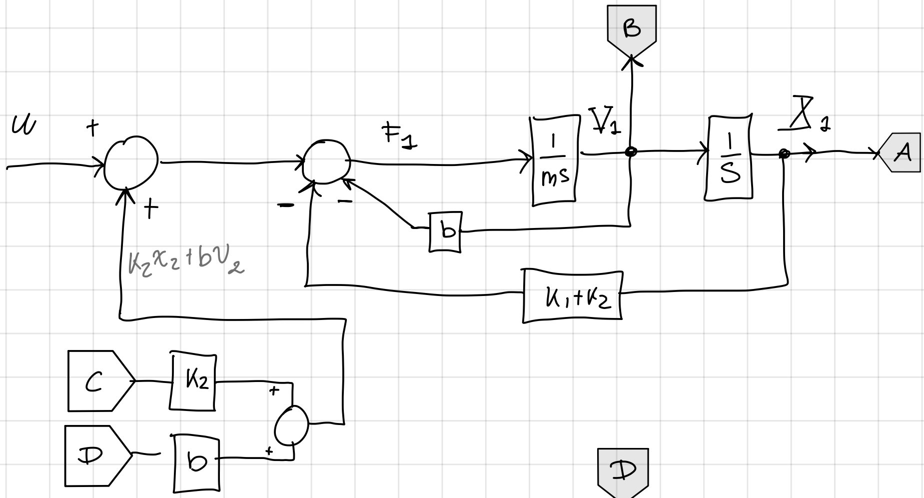
$$\begin{cases} \dot{x}_3 = \frac{1}{m_1} \left[u - K_1 x_1 - K_2 (x_1 - x_2) - b(x_3 - x_4) \right] \\ \dot{x}_4 = \frac{1}{m_2} \left[K_2 (x_1 - x_2) + b(x_3 - x_4) - K_3 x_2 \right] \end{cases}$$

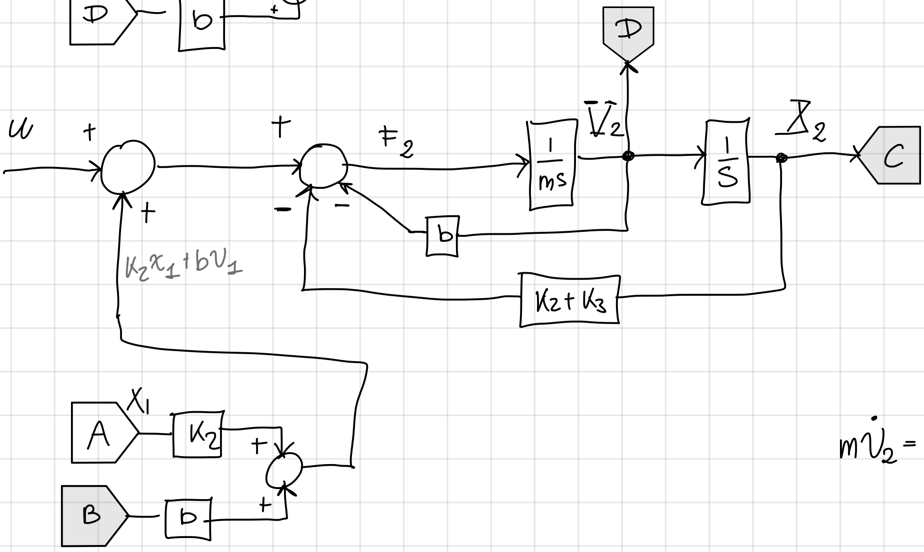
$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \end{cases}$$

$$\underline{\underline{A}} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ & \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1 + K_2}{m_1} & \frac{K_2}{m_1} & -\frac{b}{m_1} & +\frac{b}{m_1} \\ \frac{K_2}{m_2} & -\frac{K_2 + K_3}{m_2} & \frac{b}{m_2} & -\frac{b}{m_2} \end{pmatrix} \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{array} \end{array}$$

$$\underline{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix} \cdot u$$

$$y = x_2 - x_1 = (0, 0, -1, 1)$$





$$m\dot{v}_2 = -(k_2+k_3)\,x_2 - b v_2 + k_2 x_1 + b v_1$$