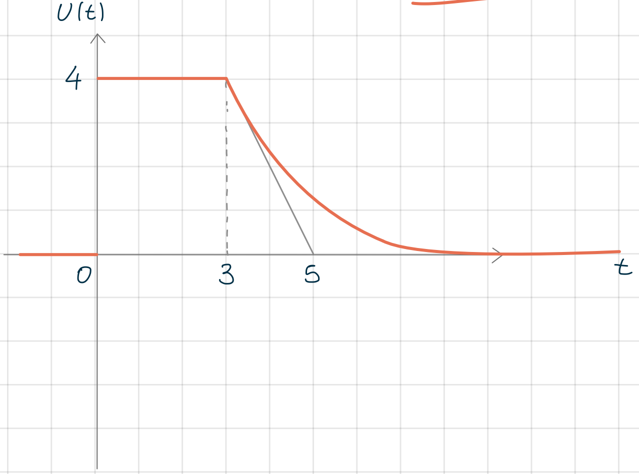
Data la FDT:

$$G(s) = \frac{s+0.1}{(s^2+10s+100)(s+2)}$$

TROVARE L'USCITA nel TEMPO ad $u(t)$



$$u(t) = \begin{cases} 0 & t<0 \\ 4 & t\in[0,3) \\ 4\,e^{-\frac{t-3}{2}} & t \geq 3 \end{cases}$$

$$e^{3/2}\cdot 4 \cdot e^{-\frac{t}{2}} \rightsquigarrow \tau = 2$$

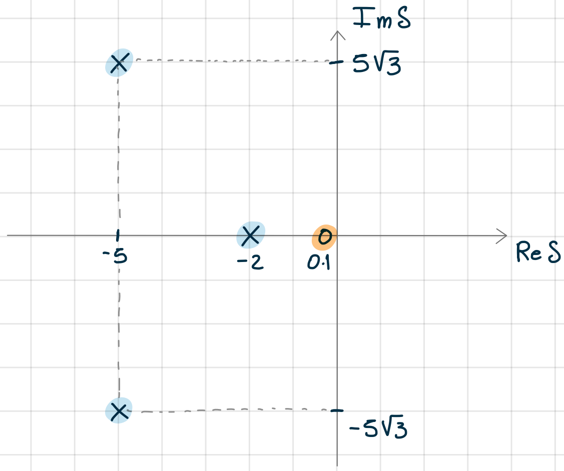
- Poli, zeri:

$$s^2+10s+100 \rightsquigarrow \Delta = 100-4\cdot 1\cdot 100 = -300 < 0 \Rightarrow \text{Poli complx e conj}$$

$$\rightsquigarrow P_{1,2} = \frac{-10 \pm j\sqrt{300}}{2} \begin{array}{l} \nearrow -5+j5\sqrt{3} \quad P_1 \\ \searrow -5-j5\sqrt{3} \quad P_2 \end{array}$$

$s+2 \rightsquigarrow P_3 = -2$ P_3

$s+0.1 \rightsquigarrow z_1 = -0.1$ z_1



1) Segnali elementari 2) Trasformo

$$\begin{cases} u_1(t) = 4\cdot 1\!\!1(t) & \rightleftharpoons U_1(s) = 4\cdot\frac{1}{s} \\ u_2(t) = -4\cdot 1\!\!1(t-3) & \rightleftharpoons U_2(s) = -4\cdot\frac{1}{s}\cdot e^{-3s} \end{cases}$$

$$\{\, u_3(t) = 4\,e^{-\frac{t-3}{2}}\cdot 1\!\!1(t-3) = \mathcal{L}\left[4\,e^{-\frac{t}{2}}\right]\cdot e^{-3s} \rightleftharpoons U_3(s) = 4\cdot\frac{1}{s+\frac{1}{2}}\,e^{-3s}$$

Tolgo il ritardo da qui

e lo Trasformo

3) Segnali fittizi

Scelgo $\hat{u}_1(t) = 1\!\!1(t)$ e $\hat{u}_2(t) = e^{-\frac{t}{2}} \rightsquigarrow \hat{U}_1(s) = \frac{1}{s}$ $\hat{U}_2(s) = \frac{1}{s+\frac{1}{2}}$

3.1) Uscita $\hat{Y}_1$

$$\hat{Y}_1(s) = \hat{U}_1(s)\cdot G(s) = \frac{s+0.1}{s\cdot(s^2+10s+100)(s+2)} = \frac{z_1}{s} + \frac{z_2}{s+2} + \frac{z_3+z_4 s}{(s^2+10s+100)}$$

$$r_1 = \lim_{s\to 0} s\cdot \hat{Y}_1(s) = \lim_{s\to 0} \cancel{s}\cdot \frac{s+0.1}{\cancel{s}(s^2+10s+100)(s+2)} \to \frac{0.1}{100\cdot 2} = 5\times 10^{-4}$$ R1

$$r_2 = \lim_{s\to -2} \cancel{(s+2)} \frac{s+0.1}{s(s^2+10s+100)\cancel{(s+2)}} \to \frac{-2+0.1}{-2(4-20+100)} = 113\times 10^{-4}$$ R2

Determino r_3 ed r_4 con un sistema

Considero solo il Num

$$\frac{(s^2+10s+100)(s+2)r_1 + r_2 s(s^2+10s+100) + (r_3+r_4 s)(s^2+2s)}{s(s^2+10s+100)(s+2)}$$

$$\begin{cases} s^3(r_1+r_2+r_4) = 0 \to r_4 = -(r_1+r_2) = -118\times 10^{-4} \\ s^2(2r_1+10r_1+10r_2+r_3+2r_4) = 0 \\ s\,(20r_1+100r_1+100r_2+2r_3) = 1 \to 120r_1+100r_2 = 2r_3 \\ 200r_1 = 0.1 \Rightarrow r_1 = \frac{1}{100\cdot 200} = 5\times 10^{-4} \ \checkmark \text{QED} \end{cases}$$

r_4

$$\rightsquigarrow r_3 = -(120r_1+100r_2)\cdot \frac{1}{2} = -5950\times 10^{-4}$$ r_3

- Rimaneggiare $\hat{y}(s)$ per l'anti trasformata

$$\hat{y}(s) = \frac{r_1}{s} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{(s^2+10s+100)}$$

Mi occupo di questo Termine

Forma Std

$$\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

per Termini complx e conj

- Termine Quadratico

$$(s+\xi\omega_n)^2 = \cancel{s^2}+2\xi\omega_n s+(\xi\omega_n)^2 = \cancel{s^2}+10s+100$$

$$\rightsquigarrow \begin{cases} 2\xi\omega_n = 10 \Rightarrow \xi\omega_n = 5 \\ (\xi\omega_n)^2 = 100 \end{cases} \to (s+\xi\omega_n)^2 = (s+5)^2$$

- Termine Noto

$$(s+5)^2 = (\cancel{s^2}+\cancel{10s}+25) \overset{?}{=} \cancel{s^2}+\cancel{10s}+100$$

Manca $75 = (\omega_n)^2 \Rightarrow \omega_n = 5\sqrt{3}$

$\to$ Riscrivo

$$\frac{r_3+r_4 s}{(s^2+10s+100)} = \frac{r_3+r_4 s}{(s+5)^2+(5\sqrt{3})^2}$$

$\leftarrow$ non è ancora pronto !

Devo ottenere una somma di sin e cos

$$\sin(\omega t) \rightleftharpoons \frac{\omega}{s^2+\omega^2} \quad , \quad \cos(\omega t) \rightleftharpoons \frac{s}{s^2+\omega^2}$$

- Metto in evidenza r_4 per lasciare da solo s

manca $5\sqrt{3}$

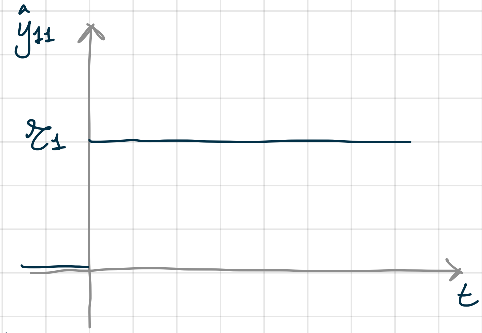
$$\Rightarrow r_4 \frac{\frac{r_3}{r_4} + s+5-5}{(s+5)^2+(5\sqrt{3})^2} = r_4 \cdot \frac{\frac{r_3}{r_4} - 5}{(s+5)^2+(5\sqrt{3})^2} + r_4 \cdot \frac{s+5}{(s+5)^2+(5\sqrt{3})^2} \quad \cos(\omega t)\cdot e^{-5t}$$

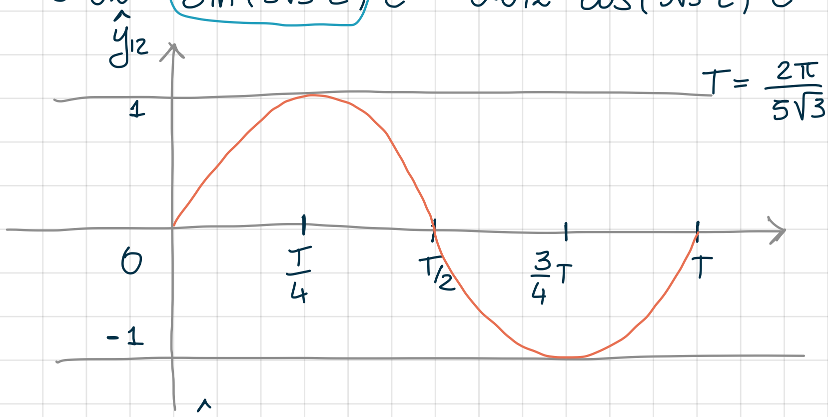
mi serve il ritardo!

$$\Rightarrow \frac{r_3 + r_4 s}{(s^2+10s+100)} = \frac{r_4\left(\frac{r_3}{r_4}-5\right)}{5\sqrt{3}} \cdot \frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2} \text{ OK!} + r_4 \cdot \frac{s+5}{(s+5)^2+(5\sqrt{3})^2} \text{ OK!}$$

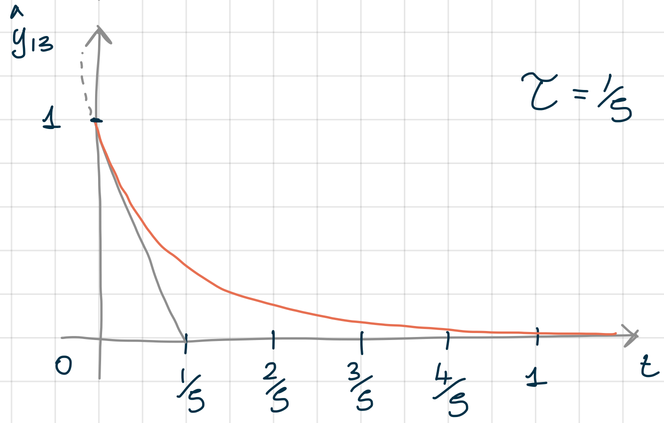
$$\hat{y}(s) = \frac{r_1}{s} + \frac{r_2}{s+2} + \frac{r_4\left(\frac{r_3}{r_4}-5\right)}{5\sqrt{3}} \cdot \underbrace{\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}}_{\hat{y}_{12}} \text{ OK!} + r_4 \cdot \frac{s+5}{(s+5)^2+(5\sqrt{3})^2} \text{ OK}$$

$$\Rightarrow \hat{y}(s) \rightleftharpoons \hat{y}(t) = \left[\overset{\hat{y}_{11}}{r_1} + r_2 t - 0.62 \cdot \sin(5\sqrt{3}\,t)\cdot e^{-5t} - 0.012 \cdot \cos(5\sqrt{3}\,t)\cdot e^{-5t}\right]\cdot 1\!\!1(t)$$



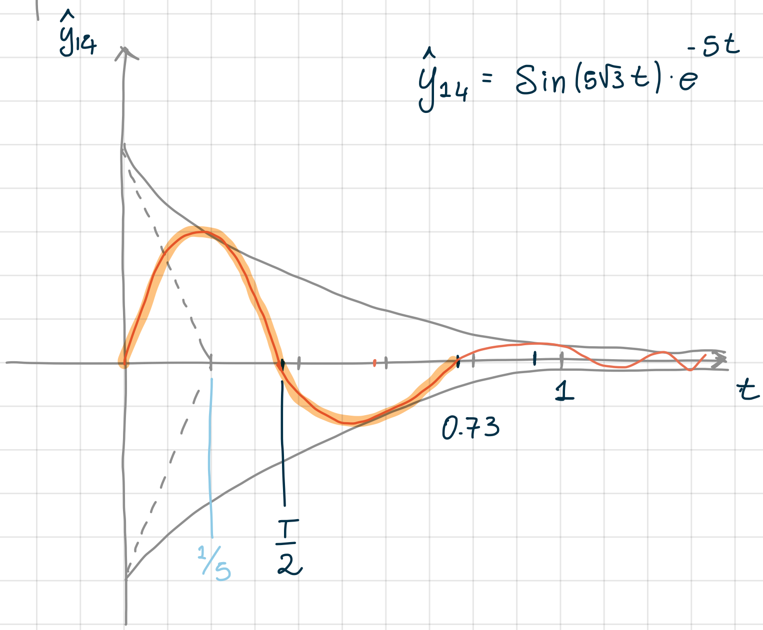


$$T = \frac{2\pi}{5\sqrt{3}} = 0.73\,s$$



$$\tau = 1/5$$

$$\hat{y}_{14} = \sin(5\sqrt{3}\,t)\cdot e^{-5t}$$



3.2) Uscita $\hat{y}_2$

$$\hat{U}_2(s) = \frac{1}{s+\frac{1}{2}}$$

$$\Rightarrow \hat{y}_3(s) = \frac{s+0.1}{(s+\frac{1}{2})(s^2+10s+100)(s+2)} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{s^2+10s+100}$$

Calcolo direttamente col sistema

$$(r_1 s + 2 r_1)(s^2+10s+100) + (r_2 s + \tfrac{1}{2} r_1)(s^2+10s+100) + (r_3 + r_4 s)(s+\tfrac{1}{2})(s+2)$$

$$\begin{cases} s^3(r_1 + r_2 + r_4) = 0 \\ s^2(2r_1 + 10 r_1 + 100 r_1 + 10 r_2 + \frac{1}{2} r_1 + r_3 + 2 r_4 \frac{1}{2} r_4 = 0 \\ s(20 r_1 + 100 r_2 + 5 r_1 + 2 r_3 + \frac{1}{2} r_3 + r_4 = 1 \\ 200 r_1 + 50 r_1 + r_3 = 0.1 \end{cases}$$

$$\begin{cases} r_1 + r_2 + 0 + r_4 = 0 \\ 112.5 r_1 + 10 r_2 + 1 r_3 + 2.5 r_4 = 0 \\ 25 r_1 + 100 r_2 + 2.5 r_3 + 1 r_4 = 1 \\ 250 r_1 + r_3 = 0.1 \end{cases}$$

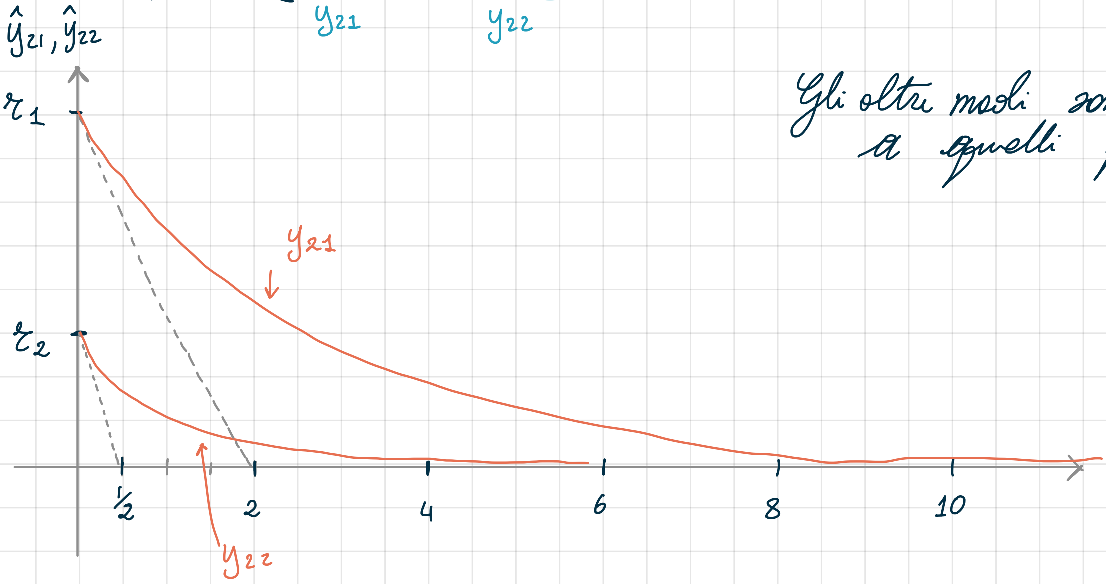
Residui

$$r_1 = 16 \times 10^{-4}$$
$$r_2 = 176 \times 10^{-4}$$
$$r_3 = -.32$$
$$r_4 = -.19$$

3^0 membro:
$$\frac{r_3 + r_4 s}{s^2+10s+100} = \frac{r_3 + r_4 s}{(s+5)\,(5\sqrt{3})^2} = r_4 \frac{\frac{r_3}{r_4} + s + 5 - 5}{(s+5)^2 (5\sqrt{3})^2}$$

$$= \frac{r_4\left(\frac{r_3}{r_4} - 5\right)}{5\sqrt{3}} \cdot \frac{5\sqrt{3}}{(s+5)^2(5\sqrt{3})^2} + r_4 \cdot \frac{s+5}{(s+5)^2(5\sqrt{3})^2}$$

$$\longrightarrow \hat{y}_2(s) = \Big[\underbrace{r_1 \cdot e^{-\frac{t}{2}}}_{y_{21}} + \underbrace{r_2\, e^{-2t}}_{y_{22}} + 0.36 \cdot \sin(5\sqrt{3}\, t) \cdot e^{-5t} - 0.7 \cos(5\sqrt{3}\, t)\, e^{-5t}\Big] \cdot \mathbb{1}(t)$$



Gli altri modi sono molto simili a quelli per $\hat{y}_1$

- Segnali Reali

Ricordiamo i segnali reali e fittizi:

$$\begin{cases} U_1(t) = 4 \cdot 1\!\!1(t) \\ U_2(t) = -4 \cdot 1\!\!1(t-3) \end{cases} \qquad \begin{cases} \hat{U}_1(t) = 1\!\!1(t) \\ \hat{U}_2(t) = e^{-\frac{t}{2}} \end{cases}$$

$$\left\{ U_3(t) = 4\, e^{-\frac{t-3}{2}} \cdot 1\!\!1(t-3) \right.$$

$$\Rightarrow \begin{cases} y_1(t) = 4 \cdot y_1(t) \cdot 1\!\!1(t) \\ y_2(t) = -4 \cdot y_1(t-3) \cdot 1\!\!1(t) \\ y_3(t) = 4 \cdot y_2(t-3) \cdot 1\!\!1(t) \end{cases}$$

Stando bene attenti a sostituire $(t-\tau)$

RISPOSTA IN FREQUENZA

$$G(S) = \frac{S+0.1}{(S^2+10S+100)(S+2)}$$

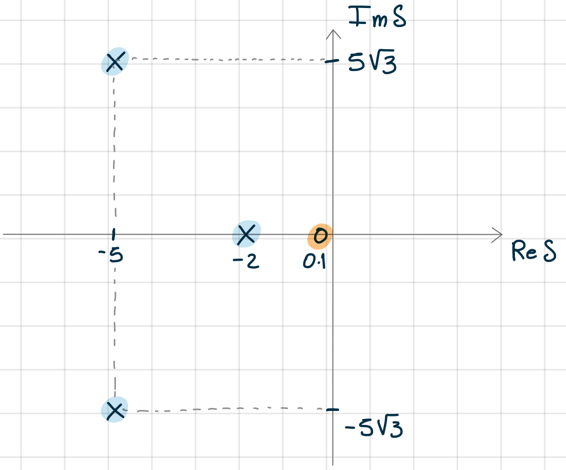
- Poli, zeri:

$S^2+10S+100 \rightsquigarrow \Delta = 100-4\cdot 1\cdot 100 = -300 < 0 \Rightarrow$ Poli complx e conj

$\rightsquigarrow P_{1,2} = \dfrac{-10 \pm j\sqrt{300}}{2}$ $\nearrow$ $\underline{-5+j5\sqrt{3}}$ P_1 ; $\searrow$ $\underline{-5-j5\sqrt{3}}$ P_2

$S+2 \rightsquigarrow \underline{P_3 = -2}$ P_3

$S+0.1 \rightsquigarrow \underline{z_1 = -0.1}$ z_1



- Frequenze di rottura

$\omega_1 = 0.1$ rad/s , $\omega_2 = 2$ rad/s per ω_3 dobbiamo trovare ω_n!

forma std: $\dfrac{\omega_d}{(S+\omega_n)^2+(\omega_d)^2} \rightsquigarrow S^2+2\zeta\omega_n S+\omega_n^2 = S^2+10S+100$

$$\Rightarrow \begin{cases} 2\zeta\omega_n = 10 & \Rightarrow 2\zeta\cdot\cancel{10} = \cancel{10} \Rightarrow \underline{\zeta = \tfrac{1}{2}} = 0.5 < 0.707 \\ \omega_n^2 = 100 & \Rightarrow \omega_n = \sqrt{100} = \underline{10}\ \omega_n \end{cases}$$

↳ Poca risonanza!

$\Rightarrow \omega_3 = 10$ rad/s

- Forma Standard

$$G(S) = \frac{0.1\left(1+\frac{S}{0.1}\right)}{100\left(\frac{S^2}{100}+\frac{S}{10}+1\right)\cdot 2\left(1+\frac{1}{2}S\right)} = 5\times 10^{4} \cdot \frac{1+10S}{\left(\frac{S^2}{100}+\frac{S}{10}+1\right)\left(1+\frac{S}{2}\right)}$$

- Valori inziali e finali

<u>MODULI</u>

<u>INIZIALE</u> No zeri/poli in 0 $\rightarrow |G(j\bar{\omega})|_{\bar{\omega}<<} = K_p = \underline{5\times 10^{-4}}$

in dB: $|G(j\bar{\omega})|_{dB} = 20\log(5\times 10^{-4}) = \underline{-66\ dB}$

<u>FINALE</u> 1 zero, 1 polo semplice ed 1 polo "complx"

+20 dB/dec, -20dB/dec, -40 dB/dec

dopo $\omega_1 = 0.1$; dopo $\omega_2 = 2$; dopo $\omega_3 = \omega_n = 10$

$\Rightarrow$ TOT mod finale : $(+20-20-40)\,dB/dec = -40\,dB/dec$

FASI

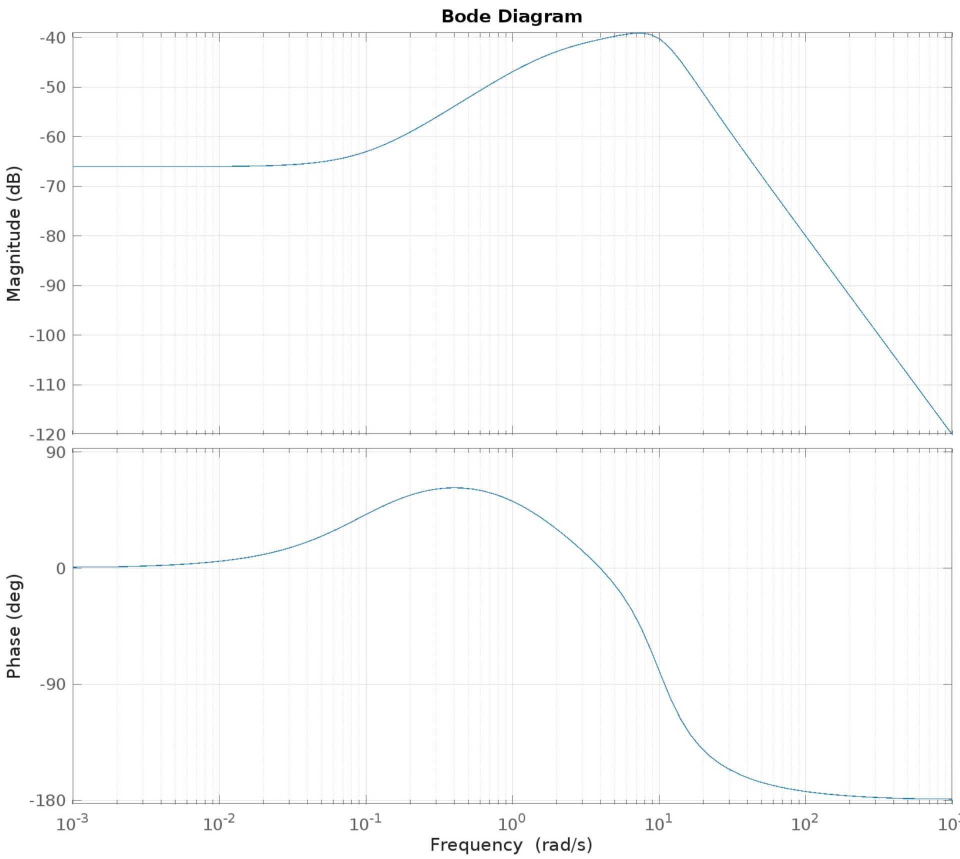
INIZIALE — NO zeri / Poli in origine $\Rightarrow \angle G(j\bar{\omega}) = 0°$ ($\bar{\omega} <<$)

FINALE
1 zero $Re P < 0 \rightarrow +90°$ Tra $0.1 \cdot 10^{-1}$ e $0.1 \cdot 10^{1}$
1 Polo semplice $\rightarrow -90°$ Tra $2 \cdot 10^{-1}$ e $2 \cdot 10^{1}$
1 Polo Complx $\rightarrow -180°$ Tra $10 \cdot 10^{-1}$ e $10 \cdot 10^{1}$

$\Rightarrow$ Totale : $-180° = -\pi$

- Scelta della Banda

$\omega \in [10^{-2} ; 10^{2}]$



Matricola: ____________________ Nome: ____________________