STABILITA' DI SISTEMI LTI

(1) LINEARITA'

(2) TEMPO INVARIANZA

Se $y(t)$ è uscita con input $u(t)$, allora $y(t-\tau)$ è uscita di IN $= u(t-\tau)$

CONCETTO DI STABILITA'

- Un sys LTI si dice Asintoticamente Stabile $\Leftrightarrow$ Tutti i poli della sua T.F. sono a parte reale $< \emptyset$.

  $\uparrow$ Strettamente

- E' STABILE se la T.F. ha poli $Re\{\} < 0$
  $Re\{\} = 0$ con molteplicità semplice

ES

$\llcorner$ 1 polo in origine $\rightarrow$ $G(s) = \frac{1}{s}$ con $u(t) = \delta(t) \rightleftarrows U(s) = 1$

$\Rightarrow$ $Y(s) = G(s)\,U(s) = \frac{1}{s}$ $\Rightarrow$ $y(t) = \mathbb{1}(t)$

$\nwarrow$ gradino

- E' INSTABILE quando non è nessuno dei prec

RAMPA $\rightarrow \infty$

$G(s) = \frac{1}{s^2}$ $\Rightarrow$ $u(t) = \delta(t) \rightleftarrows U(s) = 1$ $\Rightarrow$ $Y = G \cdot U = \frac{1}{s^2}$ $\rightleftarrows$ $y(t) = t$ $\leftarrow$ INSTABILE

RISPOSTA DI UN SYS LTI

$y(t)$: USCITA $\rightsquigarrow$ $y(t) = y_{tr}(t) + y_{ss}(t)$

"Steady State"

$\uparrow$ TRANSITORIA

REGIME

Siccome i sys che ci interessano sono Asint. Stabili, $y_{tr}(t) \rightarrow \infty$

$\Rightarrow$ $y(t) \cong y_{ss}(t)$ ovvero $\lim_{t \to \infty} y(t) = \lim_{t \to \infty} y_{tr}(t) = \emptyset$

RISPOSTA DI SYS DEL 1° ORDINE

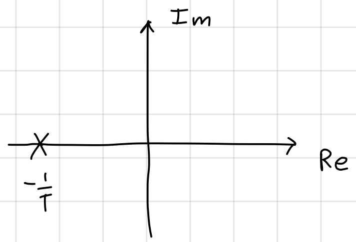
PRIMO ES:

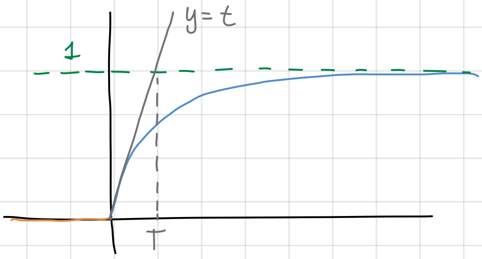
$G(s) = \frac{1}{sT+1}$ con T = costante di Tempo , $u(t) = 1(t) \rightleftharpoons U(s) = \frac{1}{s}$

$Y(s) = G(s) \cdot U(s) = \frac{1}{sT+1} \cdot \frac{1}{s} = \frac{1/T}{(s+\frac{1}{T})\, s} = \frac{\varepsilon_1}{s} + \frac{\varepsilon_2}{s+\frac{1}{T}}$

$$\begin{cases} \varepsilon_1 = \lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{1/T}{(s+1/T)\cancel{s}} = 1 \\ \varepsilon_2 = \lim_{s\to -\frac{1}{T}} (s+1/T) \cdot Y(s) = \lim_{s\to -\frac{1}{T}} \cancel{(s+1/T)} \cdot \frac{1/T}{\cancel{(s+1/T)}\, s} \to -1 \end{cases}$$

$\Rightarrow \quad Y(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}} \rightleftharpoons y(t) = 1 - e^{-t/T} \quad t \geq 0$





(1) $y(t) = 1 - \left(1 - \frac{1}{T} e^{-\frac{t}{T}}\Big|_{t=0} \cdot t + \dots\right) = 1 - 1 + \frac{t}{T} + \dots \underset{t \ll T}{\simeq} t$

(2) Teorema V.F. L.T. :

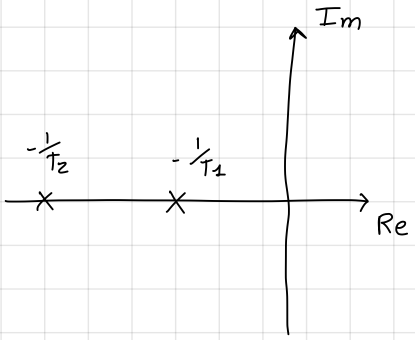
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\, Y(s) = \lim_{s\to 0} s \frac{1}{s(sT+1)} = 1$$

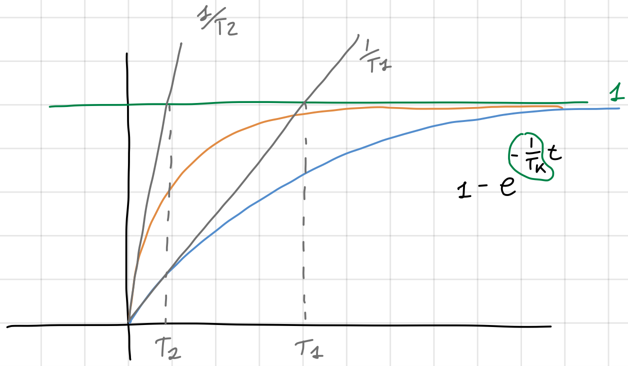
Valore a regime

(3) $\mathcal{L}[\dot{y}] = s\, Y(s) = s \cdot \frac{1}{s(sT+1)} = \frac{1}{sT+1}$

T.V.I : $\lim_{t\to 0} \frac{dy}{dt} = \lim_{s\to\infty} s \cdot \frac{1}{sT+1} \to 1/T$

Derivata in origine





IMPO: Quanto più il polo è vicino allo zero, tanto più impiegherà il sys ad andare a regime.

TEMPO PER ARRIVARE A REGIME

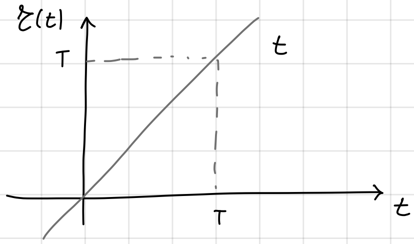
Ovviamente dipende dalla costante di tempo → Prendiamo $\boxed{4T}$

$$e(t) = 1 - y(t) = 1 - 1 - e^{-\frac{t}{T}} = e^{-\frac{t}{T}} \qquad \text{se} \quad e(4T) = e^{-\frac{4T}{T}} \cong \boxed{0.018 < 2\%\ y_{ss}}$$

errore ⇄ Distanza da 1

✳ Come scegliere l'intervallo visualiz. MATLAB.

Risposta alla Rampa



$$R(s) = \mathcal{L}[t \cdot 1\!\!1(t)] \rightleftharpoons R(s) = \frac{1}{s^2}$$
$$G(s) = 1/(sT+1)$$
$$Y(s) = ?$$

Il Sys e' A Stabile

e' instabile a seguito dell'input

$$Y = R \cdot G = \frac{1}{s^2} \cdot \frac{1}{sT+1} = \frac{1/T}{s^2(s+\frac{1}{T})} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{s+1/T}$$

Pow Max

POTENZA MAX

$$r_2 = \lim_{s \to 0} s \cdot Y = \lim_{s \to 0} \cancel{s} \cdot \frac{1/T}{s^{\cancel{2}}(s+\frac{1}{T})} \to 1 \quad r_2$$

$$r_3 = \lim_{s \to -\frac{1}{T}} \cancel{(s+\tfrac{1}{T})} \cdot \frac{1/T}{s^2 \cancel{(s+\frac{1}{T})}} = \frac{1/T}{(-\frac{1}{T})^2} = T \quad r_3$$

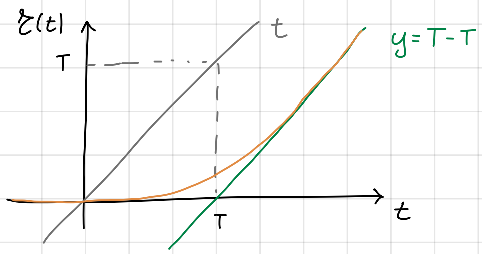
$$r_1 = \text{SOSTITUZIONE} \to \frac{r_1}{s} + \frac{1}{s} + \frac{T}{s+1/T} = \frac{1/T}{s^2(s+\frac{1}{T})} = \frac{r_1 s(s+\frac{1}{T}) + s + \frac{1}{T} + s^2 T}{s^2(s+\frac{1}{T})}$$

$$\rightsquigarrow \frac{\cancel{(r_1+T)}^{\,0} s^2 + \cancel{(\frac{r_1}{T}+1)}^{\,0} s + \frac{1}{T}}{s^2(s+\frac{1}{T})} = \frac{1/T}{s^2(s+\frac{1}{T})} \Rightarrow r_1 + T = 0 \Rightarrow r_1 = -T \quad r_1$$

$$\Rightarrow Y = -\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s+1/T} \rightleftharpoons y(t) = -T + t + Te^{-\frac{t}{T}} \quad t \geq 0$$

Trovare l'andamento a regime e Transitorio

$$y_{ss}(t) = \lim_{t \to \infty} y(t) = \lim_{t \to \infty} -T + t + \underbrace{Te^{-\frac{t}{T}}}_{\to 0} = \lim_{t \to \infty} (t - T)$$



Valore iniziale se valutato in $t=0$

$$\frac{dy}{dt} = 1 - \frac{T}{T} e^{-\frac{t}{T}} = 1 - e^{-\frac{t}{T}}$$

la derivata della risposta alla RAMPA e' la risposta al gradino!

$\Rightarrow$ Per la linearità:

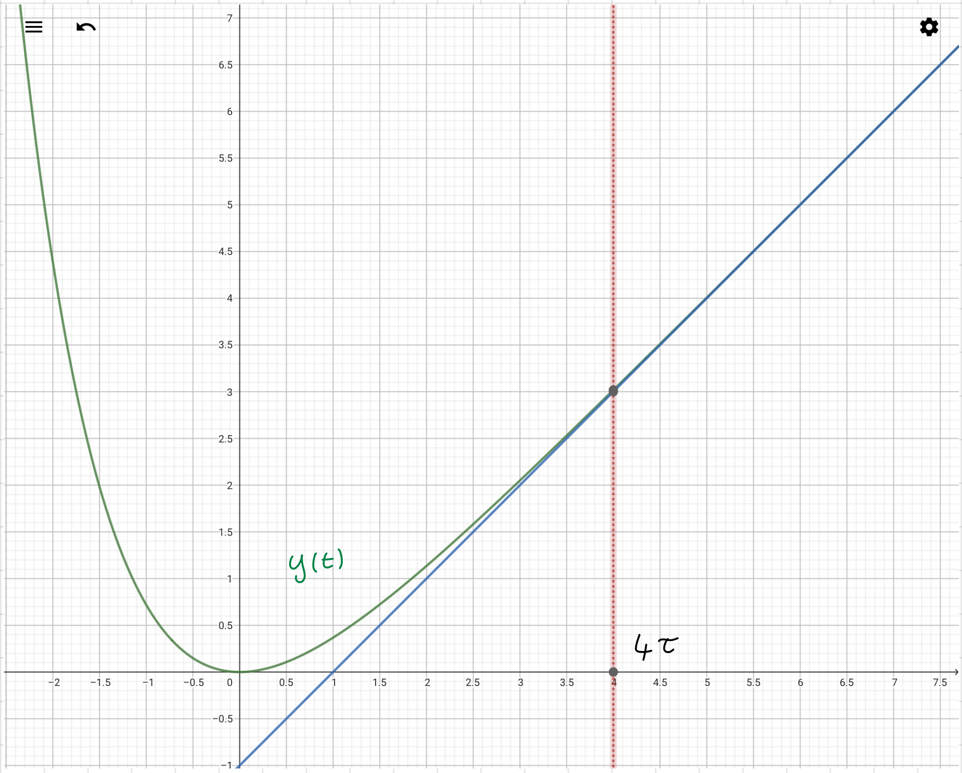
$$\boxed{\frac{d\,U(s)}{dt} \cdot G(s) = \frac{d\,Y(s)}{dt}}$$

Valore iniziale (con Teorema)

$$\lim_{s \to \infty} s \cdot \mathcal{L}\left[\frac{dy}{dt}\right] = \lim_{s \to \infty} s \cdot s \cdot Y(s) = \lim_{s \to \infty} \cancel{s^2} \cdot \frac{1/T}{\cancel{s^2}(s+\frac{1}{T})} = 0$$

Errore

$$e(t) = t - y(t) = t - (-T + t + Te^{-\frac{t}{T}}) = T(1 - e^{-\frac{t}{T}})$$



RISPOSTA IMPULSIVA

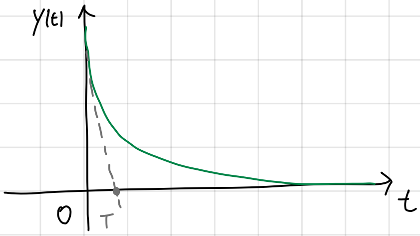
$$G(s) = \frac{1}{sT+1} = \frac{1/T}{s+\frac{1}{T}}$$

c'è 1 ma ha una dimensione!!

$$u(t) = \delta(t) \rightleftarrows R(s) = 1 \Rightarrow Y(s) = R(s)\cdot G(s) = G(s) \Rightarrow y(t) = \frac{1}{T}e^{-\frac{t}{T}} \quad t>0$$

V.I = ...
V.F. = ...

Taylor



DISCONTINUA

T.V.I.

$$y(t) = \frac{1}{T}e^{-\frac{1}{T}t}, \quad t \geq 0$$

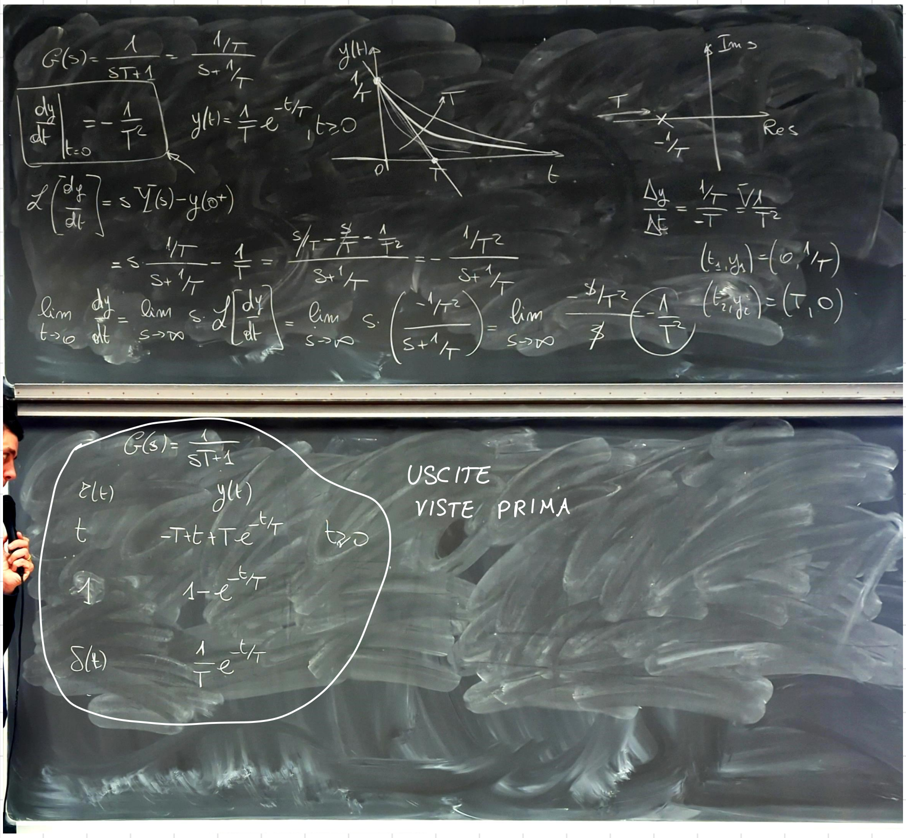
$$\left.\frac{dy}{dt}\right|_{t=0} = -\frac{1}{T^2}$$

$$\rightsquigarrow \mathcal{L}\left[\frac{dy}{dt}\right] = sY(s) - y(0^+) = s\cdot\frac{1/T}{s+1/T} - \frac{1}{T} = \frac{s/T - \frac{s}{T} - \frac{1}{T^2}}{s+1/T} = -\frac{1/T^2}{s+\frac{1}{T}}$$

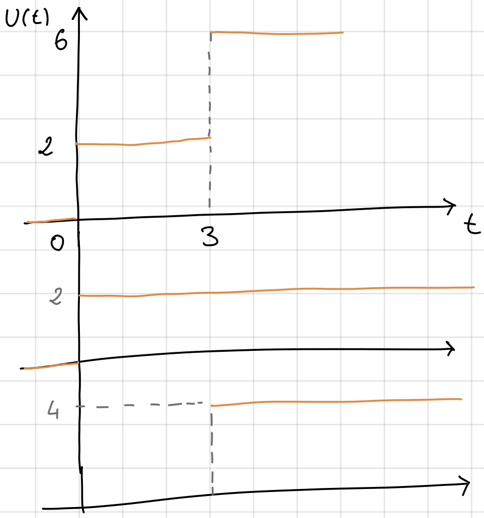
$$\rightsquigarrow \text{Teorema} \quad \lim_{t\to 0}\frac{dy}{dt} = \lim_{s\to\infty} s\cdot Y(s) = \lim_{s\to\infty} -s\cdot\frac{1/T^2}{s+\frac{1}{T^2}} \to -\frac{1}{T^2}$$

QED

FOTO



RISPOSTA DI UN SYS (1th) ad un segnale composto



$$G(s) = \frac{1}{s+10}$$

$$u(t) = u_1(t) + u_2(t) \quad \text{con} \quad \begin{cases} u_1(t) = 2 \cdot 1\!\!1(t) \\ u_2(t) = 4 \cdot 1\!\!1(t-3) \end{cases}$$

$$Y_1(s) = G \cdot U_1 = \frac{1}{s+10} \cdot \frac{2}{s} = \frac{r_1}{s} + \frac{r_2}{s+10}$$

$$r_1 = \lim_{s \to 0} s \cdot Y_1(s) = \frac{1}{5} \quad \rightsquigarrow \quad r_2 = \lim_{s \to -10} s \cdot Y_1(s) = -\frac{1}{5}$$

$y_1(t)$

$$\Rightarrow Y_1(s) = \frac{1}{5}\left(\frac{1}{s} - \frac{1}{s+10}\right) \rightleftharpoons y_1(t) = \frac{1}{5} \cdot \left(1 - e^{-10t}\right), \; t \geq 0$$

$$\hat{u}_2(t) = 4 \cdot 1\!\!1(t) = 2 \cdot \left(2 \cdot 1\!\!1(t)\right) = 2\, u_1(t) \Rightarrow \hat{y}_2(t) = 2\, y_1(t) = \frac{2}{5}\left(1 - e^{-10t}\right), \; t \geq 0$$

senza ritardo

!!!

$$\Rightarrow y_2(t) = \hat{y}_2(t-3) = \frac{2}{5}\left(1 - e^{-10(t-3)}\right), \; t \geq 3$$

$y_2(t)$

$$\Rightarrow y(t) = \frac{1}{5}\left(1 - e^{-10t}\right) \cdot 1\!\!1(t) + \frac{2}{5}\left(1 - e^{-10(t-3)}\right) \cdot 1\!\!1(t-3)$$

Ans