SISTEMI IDRAULICI

$q \rightleftarrows i$

$p \rightleftarrows v$

$R_{id} \rightleftarrows R_e$

"RO"

$$\boxed{\rho = \frac{\text{PESO}}{\text{UNITA' DI VOLUME}}}$$

$[\rho] = Kg/m^3$

$[V] = m^3$

VOLUME

Metro Cubo

$$\boxed{\text{Pressione} = \text{Forza} \times \text{unità di superficie}} \rightarrow [P] = \boxed{\frac{N}{m^2}} = Pa = \frac{Kg}{m^2 s^2} = \boxed{\frac{Kg}{m s^2}}$$

(1) (1)

Pascal

$$1 \text{ atm} = 10^5 \text{ Pa} = 10^5 \frac{N}{m^2}$$

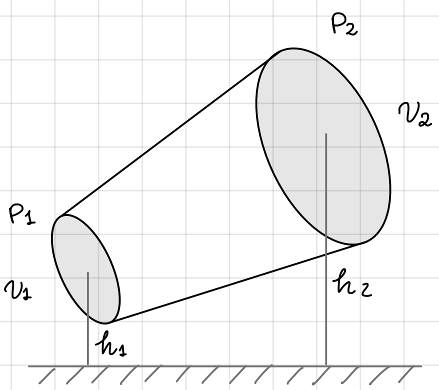
$$1 \, dm^2 = 10^{-2} m \Rightarrow \frac{10^5}{10^4} \frac{N}{m^2} \Rightarrow 1 \, dm^2 = 10^{-2} \frac{N}{m^2}$$

$$\boxed{q = \frac{dV}{dt}}$$

Portata Volumetrica

Variazione del Volume nel tempo con $\boxed{[q] = \frac{m^3}{s}}$

TEOREMA DI BERNOULLI



$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 \frac{1}{2} v_2^2 + \rho \cdot g \, h_2}$$

COSTANTE

GRANDEZZE IDRAULICHE

RESISTENZA IDRAULICA

- ATTrito - Resistore

$$R_{id} = \frac{\Delta P}{q}$$

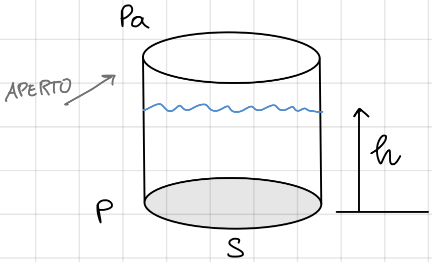
Diff di portata

Portata

Con $\Delta P = P_1 - P_2$

CAPACITA' IDRAULICA

- Serbatoio - Condensatore



$$P_a + \rho \cdot g\, h = P \;\rightarrow\; \rho g\, h = P - P_a$$

P: NON COSTANTE; P_a: COSTANTE

$\rightarrow$ DERIVO $\rightarrow$ $\rho g \dfrac{dh}{dt} = \dfrac{dP}{dt}$

moltiplico per $\dfrac{S}{S}$ $\rightarrow$ $\dfrac{\rho g}{S} \underbrace{\dfrac{d(Sh)}{dt}}_{\text{Portata}} = \dfrac{dP}{dt}$ $\rightarrow$ $\dfrac{\rho g}{S} \cdot q = \dfrac{dP}{dt}$

$\Rightarrow$ $q = \underbrace{\dfrac{S}{\rho g}}_{\text{Capacità idraulica}} \cdot \dfrac{dP}{dt}$

Capacità idraulica $\equiv$ Capacità condensatore

INDUTTANZA IDRAULICA

$v = L_{el}\, \dot{i}$

Esempio: Quando la velocità all'interno di una condotta non è UNIFORME

$$m \cdot \frac{dv}{dt} = F_1 - F_2$$

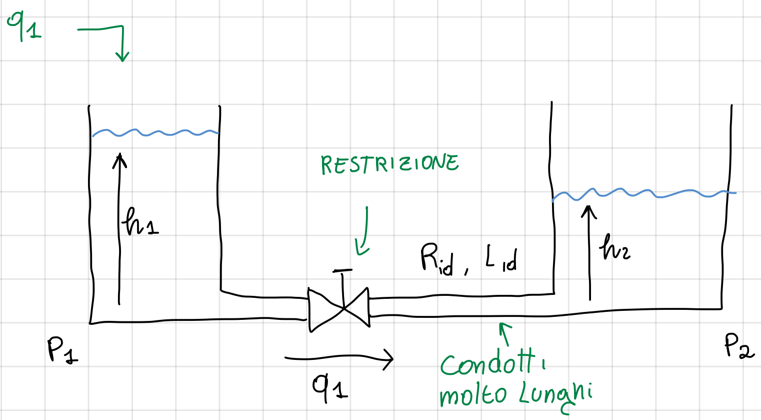
$$m^2 \cdot \frac{m^2}{s} = \frac{m^3}{s} \equiv q$$

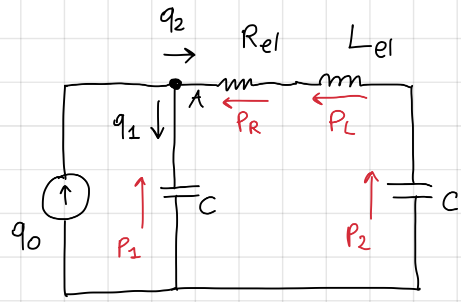
$\underbrace{\rho \cdot \underbrace{Sh}_{V}}_{m}$ $\rightarrow$ $\rho\, Sh \cdot \dfrac{dv}{dt} = F_1 - F_2$ $\rightarrow$ $\rho h \cdot \dfrac{d(\overbrace{Sv}^{q})}{dt} = F_1 - F_2$

$\rightarrow$ $\dfrac{\rho h}{S} \dfrac{dq}{dt} = \underbrace{(F_1 - F_2)\, S}_{N \cdot m^2 \equiv P}$ $\rightarrow$ $\boxed{\dfrac{\rho h}{S}\, \dot{q} = P_1 - P_2}$

ESEMPIO: CONVERSIONE E MODELLO

$q_1 = i_N =$ gen. Corr.





$-q_0 + q_1 + q_2 = 0$

$\to q_1 = q_0 - q_2$

R.C. $\begin{cases} i_C = C\,\dot{v}_C \\ v_L = L\,\dot{i}_L \end{cases}$

$$C_1 \cdot \frac{dP_1}{dt} = q_0 - q_2$$

$\leftarrow$ equivale a i_C

$$L\,\frac{dq_2}{dt} = \overbrace{-R\cdot q_2 + P_1 - P_2}^{v_L}$$

$P_L + P_2 - P_1 - P_R = 0 \xrightarrow{=0} P_L = P_R + P_1 - P_2$

$$C_2\,\frac{dP_2}{dt} = \overbrace{q_2}^{i_{C_2} = i_L}$$

$$\boxed{\begin{matrix} x_1 = P_1 & x_2 = P_2 & x_3 = q_2 \\ y = h_2 & & u = q_0 \end{matrix}}$$

$$\rho \cdot g \cdot h \cong P_2 \;\Rightarrow\; \frac{1}{\rho g} P_2 = \frac{1}{\rho g} x_2$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \to \quad A = \begin{array}{c} \begin{matrix} P_1 & P_2 & q_2 \end{matrix} \\ \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} \end{pmatrix} \end{array} \quad B = \begin{pmatrix} 1/C_1 \\ 0 \\ 0 \end{pmatrix} \quad C: \begin{pmatrix} 0 & \frac{1}{\rho g} & 0 \end{pmatrix} \quad D = \emptyset$$

Funz. di Trasferimento

$IN = q_1 \quad OUT = h_2 \quad \to \quad G(s) = \dfrac{H_2(s)}{Q_1(s)}$

$$\mathcal{L}.\mathcal{T}. \begin{cases} sC_1\,P_1(s) = Q_0(s) - Q_2(s) & (1) \\ sL\,Q_2(s) = -R\,Q_2(s) + P_1(s) - P_2(s) & (2) \\ sC_2\,P_2(s) = Q_2(s) & (3) \end{cases}$$

$$H_2(s) = \frac{1}{\rho g} P_2(s) = \frac{1}{\rho g C_2} \cdot \frac{1}{s}\,Q_2(s) \;\to (2) \to\; sL\,Q_2 + R\,Q_2 = P_1 - \frac{1}{sC_1}\,Q_2 \quad (A)$$

dalla (1) $\quad P_1 = \dfrac{Q_0 - Q_2}{sC_1}$

→ La (B) nella (A) → $(R+SL)Q_2 + \frac{1}{SC_2}Q_2 = \frac{1}{SC_1}Q_0 - \frac{1}{SC_1}Q_2$

→ $Q_2\left(R+SL+\frac{1}{SC_2}+\frac{1}{SC_1}\right) = \frac{1}{SC_1}\cdot Q_1$ m.c.m $\frac{(R+SL)SC_1C_2+C_1+C_2}{SC_1C_2}$

mcm → $\frac{(R+SL)SC_1C_2+C_1+C_2}{\cancel{SC_1}C_2}\cdot Q_2 = \frac{1}{\cancel{SC_1}}Q_1$

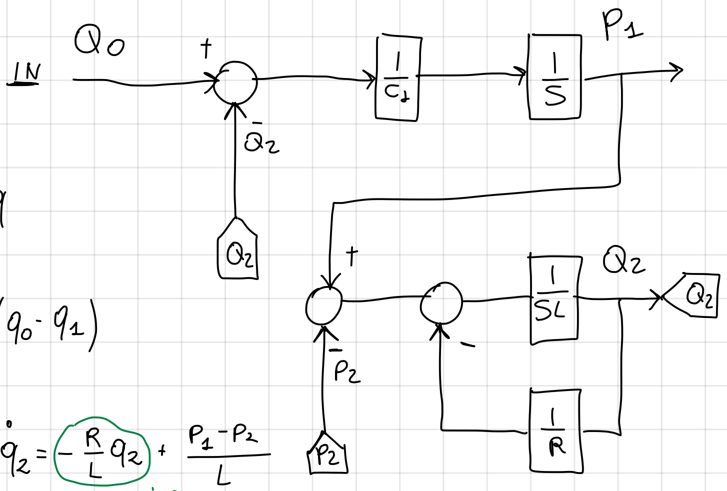
→ $Q_2(S) = \frac{C_2}{S^2LC_1C_2+SRC_1C_2+C_1C_2}Q_1(S)$

Effetto

$$\Rightarrow G(S) = \frac{H_2(S)}{Q_1(S)} = \frac{\frac{1}{\rho g \cancel{C_2} S}\cdot\frac{\cancel{C_2}}{S^2LC_1C_2+SRC_1C_2+C_1C_2}\cancel{Q_1(S)}}{\cancel{Q_1}} = \frac{\frac{1}{\rho g}}{S(S^2LC_1C_2+SRC_1C_2+C_1+C_2)}$$

CAUSA

Schema a blocchi



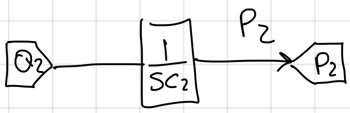
* Ricavato dalle eq

$C_1\cdot\frac{dP_1}{dt} = q_0 - q_2 \to \dot{P}_1 = \frac{1}{C}(q_0 - q_1)$

$L\frac{dq_2}{dt} = -R\cdot q_2 + P_1 - P_2 \to \dot{q}_2 = -\frac{R}{L}q_2 + \frac{P_1-P_2}{L}$

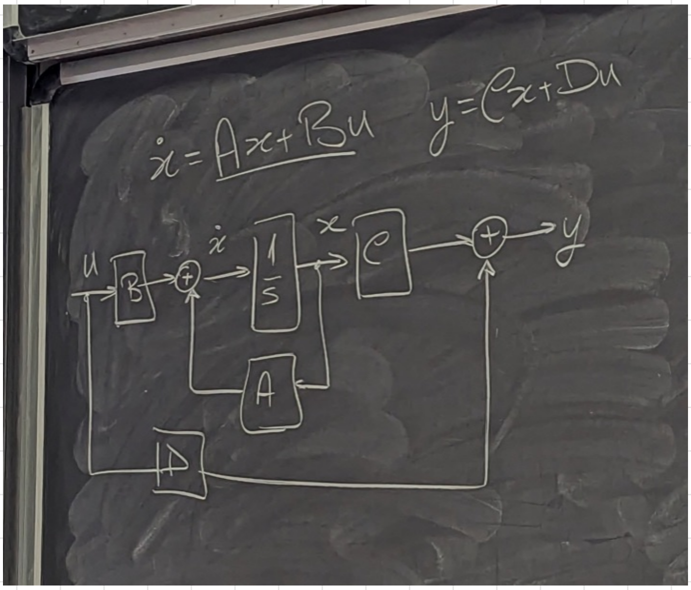
Retroazione

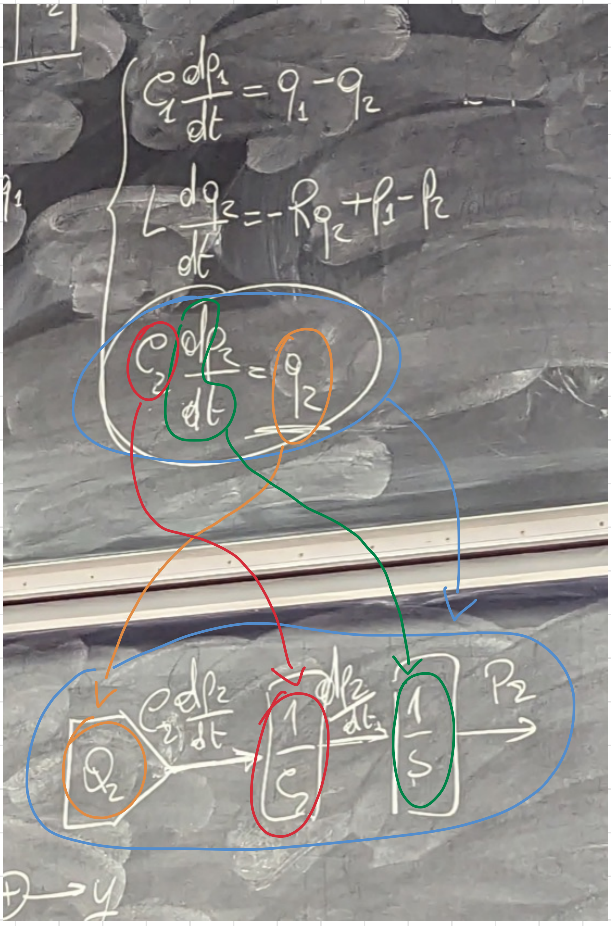
$C_2\frac{dP_2}{dt} = q_2 \to \dot{P}_2 = \frac{q_2}{C_2}$



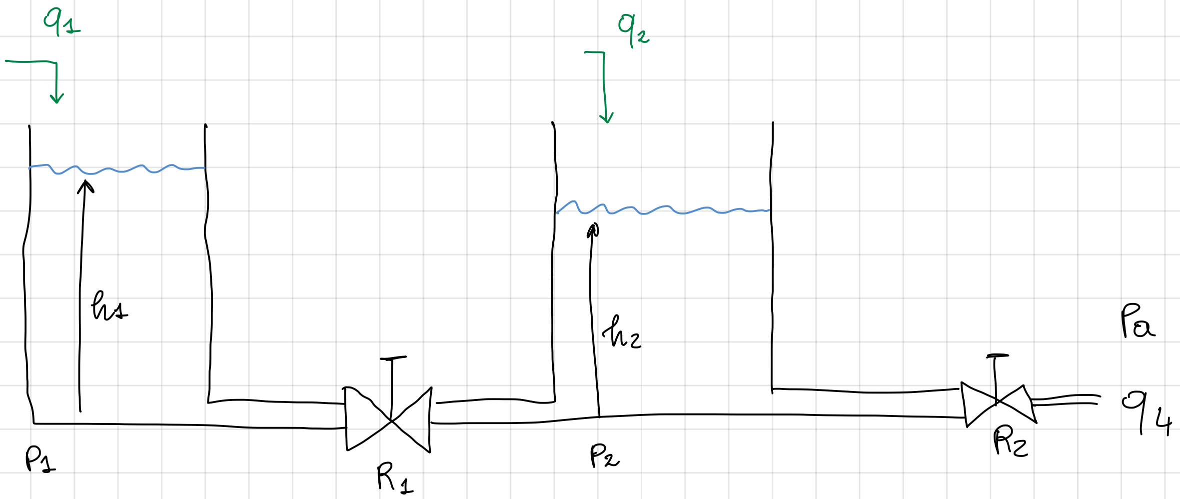
Domanda

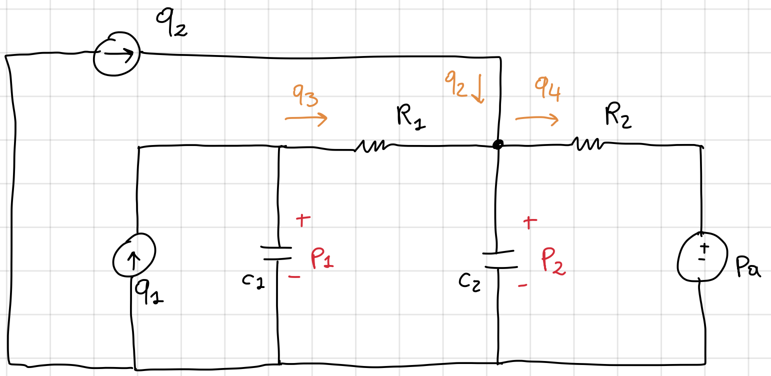
Rappr. a blocchi Con MATRICI





DUE SERBATOI...

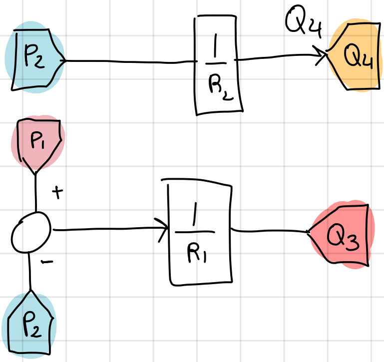


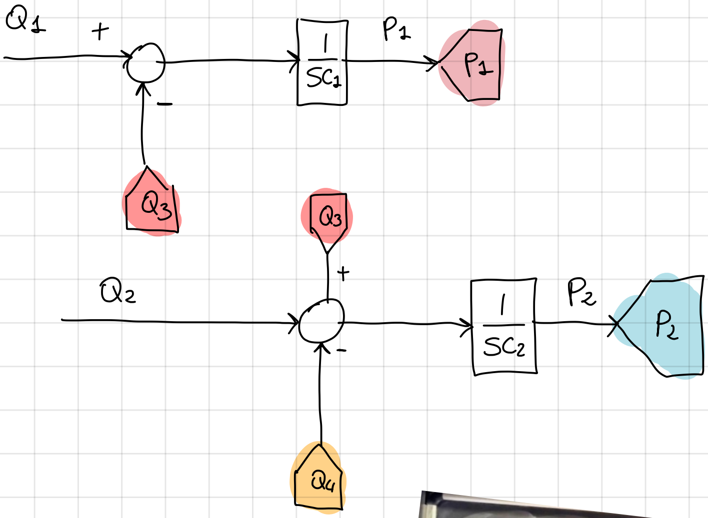


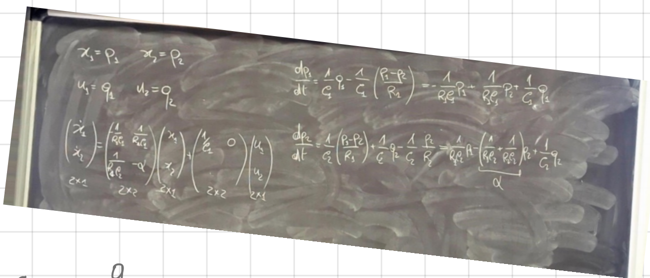
$$
\begin{cases}
C_1 \dfrac{dP_1}{dt} = q_1 - q_3 \\[2mm]
q_3 = \dfrac{P_1 - P_2}{R_1} \\[2mm]
C_2 \dfrac{dP_2}{dt} = q_3 + q_2 - q_4 \\[2mm]
q_4 = \dfrac{P_2 - P_a}{R_2} \cong \dfrac{P_2}{R_2}
\end{cases}
$$

$$\dot{P}_1 = \frac{q_1}{C_1} - \frac{1}{C_1}\left(\frac{P_1 - P_2}{R_1}\right)$$

$P_2 >> P_a$







$x_1 = P_1 \qquad x_2 = P_2$

$u_1 = q_1 \qquad u_2 = q_2$

$$
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}
=
\begin{array}{cc} P_1 & P_2 \\ \end{array}
\begin{pmatrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} \\ \frac{1}{R_1C_1} & -\frac{1}{R_1C_2} + \frac{1}{R_2C_2} \end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}
+
\begin{array}{cc} q_1 & q_2 \\ \end{array}
\begin{pmatrix} \frac{1}{C_1} & \emptyset \\ 0 & \frac{1}{C_2} \end{pmatrix}
\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}
$$

$2 \times 2 \qquad\qquad 2 \times 2$

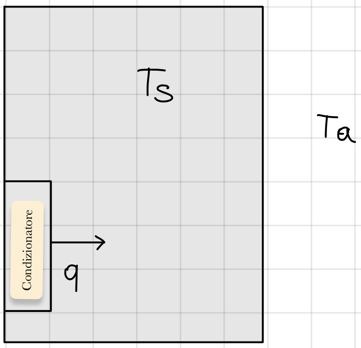
Possibile Uscita $y_1 = h_1 - h_2 = \frac{1}{\delta g} P_1 - \frac{1}{\delta g} P_2$

$\rightarrow \quad C = \left(\frac{1}{\delta g} \quad - \frac{1}{\delta g} \right)$

Funzione di Trasferimento

Fattill Tu

MODELLI TERMICI



$$\underset{\uparrow}{C}\,\frac{dT}{dt} = q$$

Capacità Termica

q ← Flusso Termico

$\hookrightarrow q = \dfrac{dQ}{dt}$

$$C = \frac{Q}{\Delta T}$$

Q ← CALORE

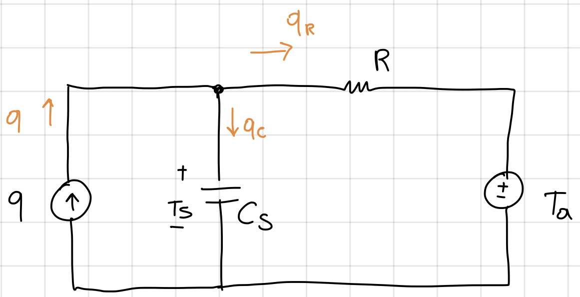
ΔT ← Variazione di Temp.

$$\begin{cases} i \rightarrow q \\ v \rightarrow T \end{cases}$$

Circuiti → Termic.

$$R = \frac{\Delta T}{q}$$

Resistenza Termica → funziona per $\begin{cases} \text{CONDUZIONE} \\ \text{CONVEZIONE} \end{cases}$



$$(1)\quad C_s\frac{dT_s}{dt} = q - q_R \quad \text{con} \quad q_R = \frac{T_s - T_a}{R}$$

$u_1 = q \qquad u_2 = T_a \qquad x = T_s$

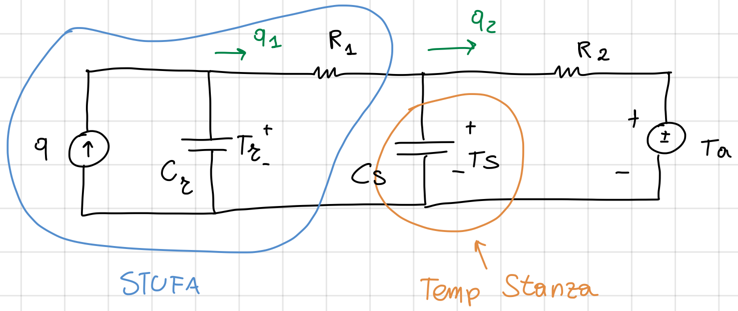
$$G(s) = \frac{T_s(s)}{Q(s)} : \rightarrow$$

$$s\,C_s\,T_s = Q - \frac{T_s - T_a}{R} \rightarrow \left(sC_s + \frac{1}{R}\right)T_s = Q + \frac{1}{R}T_a$$

$$\rightarrow \text{mcm} \rightarrow \quad T_s = \frac{R}{sRC_s + 1}Q + \frac{1}{sRC_s + 1}T_a$$

Influenza della Temp. esterna

IL RADIATORE



$$C_r \frac{dT_r}{dt} = q - q_1$$

$$q_1 = \frac{T_r - T_s}{R_1}$$

$$C_s \frac{dT_s}{dt} = q_1 - q_2$$

$$q_2 = \frac{T_s - T_a}{R_2}$$

$u_1 = T_a \qquad u_2 = q$

$x_1 = T_r \qquad x_2 = T_s$

$s C_r T_r = Q - Q_1 \to$ Sub $Q_1 \to s C_r T_r = Q - \frac{T_r}{R_1} + \frac{T_s}{R_1} \to$ Trovo T_r (1)

$s C_s T_s = Q_1 - Q_2 \to$ Sub $Q_2 \to s C_s T_s = \frac{T_r}{R_1} - \frac{T_s}{R_1} + \frac{T_s}{R_2} - \frac{T_a}{R_2}$ (2)

dalla (2) $\to \left(sC_s + \frac{1}{R_1} + \frac{1}{R_2}\right) T_s = \frac{T_r}{R_1} + \frac{T_a}{R_2}$

$\to \left(sC_s + \frac{1}{R_1} + \frac{1}{R_2}\right) T_s = \frac{1}{1 + sR_1C_r} Q + \frac{1/R_1}{1 + sR_1C_r} T_s + \frac{T_a}{R_2}$