

ESERCIZI ANTITRASFORMATA e rappresentazione grafica

ES 1

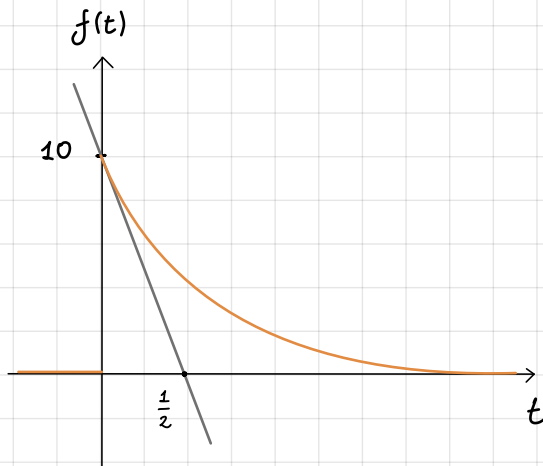
$$F(s) = \frac{10}{s+2}$$

Q. Antitrasformata ed Andamento nel tempo

$$\rightarrow f(t) = 10 \cdot e^{-2t} \cdot \mathbb{1}(t) \quad \text{oppure} \quad \begin{cases} 10 e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

T. Val inizi $\rightarrow f(0^+) = \lim_{s \rightarrow \infty} s \cdot F(s) = \lim_{s \rightarrow \infty} \frac{10s}{s+2} = 10$

T. Val fin $\rightarrow \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s) = \lim_{s \rightarrow 0} \frac{10s}{s+2} = 0$



• Costante di Tempo

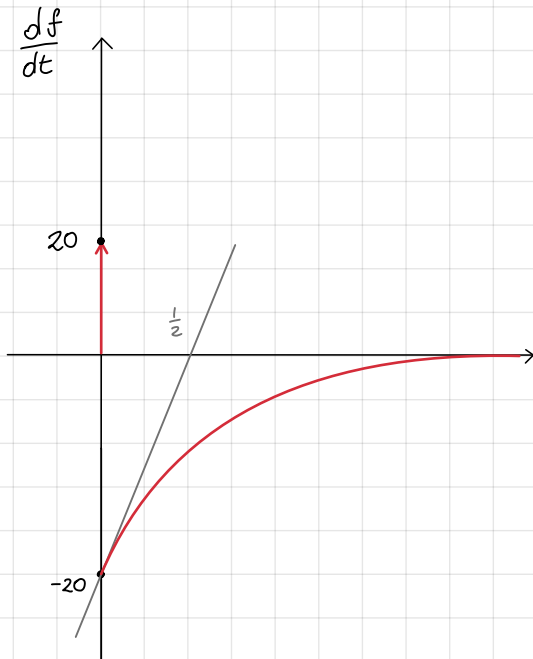
$$e^{-\frac{t}{\tau}} \rightarrow \frac{1}{\tau}$$

$$f'(t) = -20 e^{-2t} \rightarrow f'(0^+) = -20$$

$$\mathcal{L}_+ [f'(t)] = s F(s) - f(0^+) \quad \text{Regola della derivata}$$

$$= \frac{10s}{s+2} - 10 = \frac{10s - 10s - 20}{s+2} = -\frac{20}{s+2}$$

\rightarrow



$$\mathcal{L}_- [f'(t)] = s F(s) - f(0^-) = \frac{10s}{s+2}$$

\rightarrow FRATTI SEMPLICI

$$= 10 \frac{s+2-2}{s+2} = 10 \frac{s+2-2}{s+2} = 10 \left(\frac{s+2}{s+2} - \frac{2}{s+2} \right)$$

$$= 10 \left(1 - \frac{2}{s+2} \right) \xrightarrow{\text{ANTI}} \mathcal{L}^{-1} [f'(t)] = 10 \left(\delta(t) - 2 e^{-2t} \cdot \mathbb{1}(t) \right)$$

La differenza sta nell'impulso, perchè la derivata di una discontinuità è l'impulso

Perchè $\frac{1}{2}$ è la pendenza?

La derivata di una funzione e consideriamo la sua derivata in t^* , questa è la pendenza della retta tangente

$$p_1 = (0, 10) \quad \rightarrow \quad \frac{10}{\frac{1}{2}} = 20$$

$$p_2 = \left(\frac{1}{2}, 0\right)$$

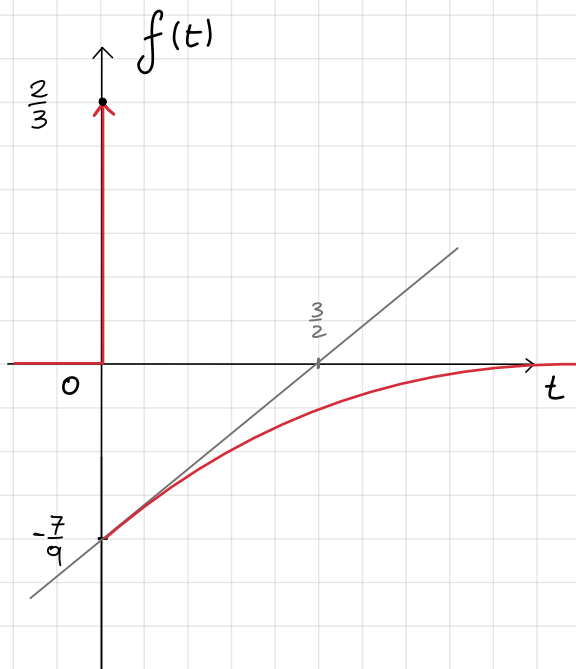
Es 2:

$$F(s) = \frac{2s-1}{3s+2} = \frac{1}{3} \left(\frac{2s-1}{s+\frac{2}{3}} \right) = \frac{2}{3} \left(\frac{s-\frac{1}{2}}{s+\frac{2}{3}} \right) = \frac{2}{3} \frac{s+\frac{2}{3}-\frac{2}{3}-\frac{1}{2}}{s+\frac{2}{3}}$$

$$= \frac{2}{3} \left(1 - \frac{\frac{7}{6}}{s+\frac{2}{3}} \right)$$

$$\Rightarrow f(t) = \frac{2}{3} \left(\delta(t) - \frac{7}{6} e^{-\frac{2}{3}t} \mathbb{1}(t) \right)$$

$$\Rightarrow \tau_0 = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$



* bisognava calcolare anche il valore iniziale e finale: bisogna sempre **applicare le regole della trasformata di Laplace per ogni esercizio svolto.**

ES 3

$$F(s) = \frac{2s}{s^2 + 3s + 2}$$

Poli e zeri

1 zero $\rightarrow 0$ zero

2 poli $\rightarrow s = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot 2}}{2}$

-2 Poli

-1 Poli

$$\Rightarrow F(s) = \frac{2s}{s^2 + 3s + 2} = \frac{z_1}{s+1} + \frac{z_2}{s+2} \Rightarrow \frac{2s}{(s+1)(s+2)} = \frac{z_1}{s+1} + \frac{z_2}{s+2}$$

+ STABILE

- INSTABILE!

$$z_1 = \lim_{s \rightarrow -1} (s+1) F(s) = \lim_{s \rightarrow -1} \frac{2s}{s+2} \rightarrow -2$$

$$z_2 = \lim_{s \rightarrow -2} (s+2) F(s) = \lim_{s \rightarrow -2} \frac{2s}{s+1} \rightarrow +4$$

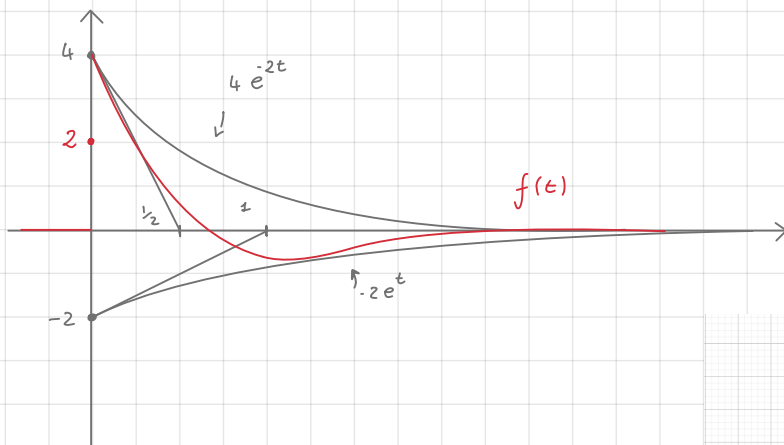
$$\Rightarrow F(s) = -\frac{2}{s+1} + \frac{4}{s+2} = -2 \left(\frac{1}{s+1} \right) + 4 \left(\frac{1}{s+2} \right)$$

$\mathcal{L}[e^{-\alpha t} u(t)]$

$\alpha = 1$

$\alpha = 2$

$$\Rightarrow f(t) = (-2 e^{-t} + 4 e^{-2t}) \cdot u(t) \leftarrow \text{SOL}$$



ES 4:

$$F(s) = \frac{s+8}{s^2+2s+2}$$

Zero di $F(s) \rightarrow s+2=0 \rightarrow s=-2$

Poli di $F(s) \rightarrow s_{1,2} = \frac{-2 \pm \sqrt{4-4 \cdot 1 \cdot 2}}{2} = -1 \pm j$

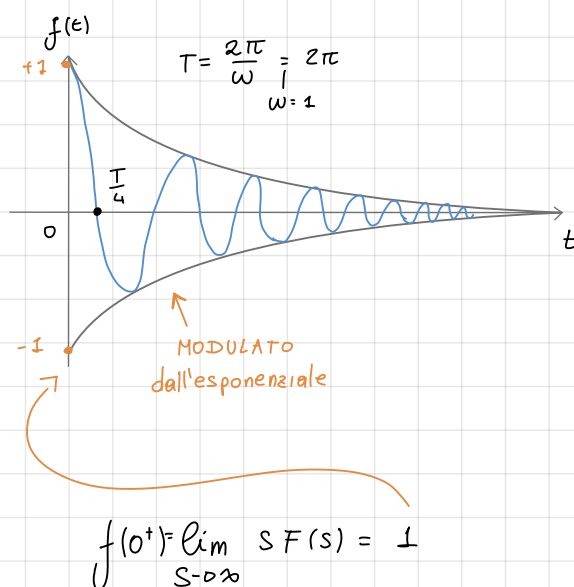
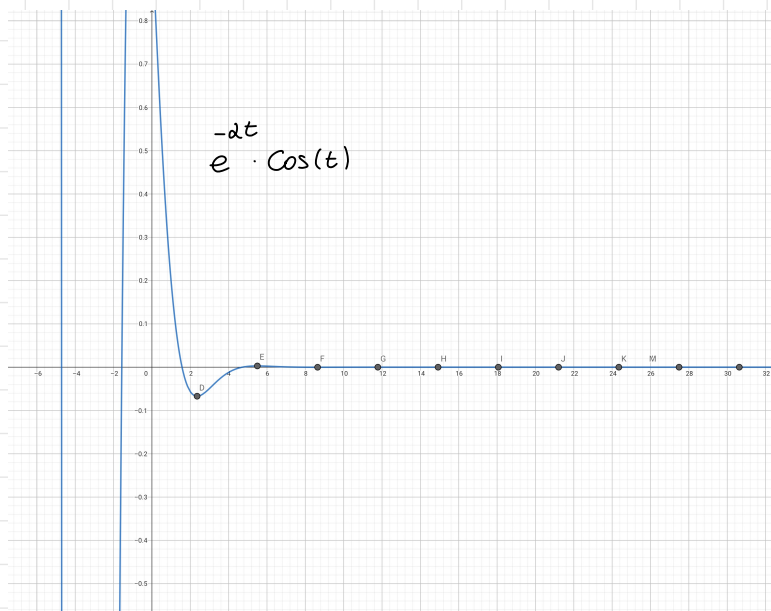
Riscrivo $F(s) = \frac{s+8}{(s+1)^2 + 2 - 1} \quad \omega=1$
 $\uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow$
 $s^2+2s+1+2-1 = s^2+2s+2 \checkmark$

Siccome $\mathcal{L}[e^{-\alpha t} \cos(\omega t)] = \frac{s+\alpha}{(s+\alpha)^2 + \omega^2} = \frac{s+2}{s^2+2s+2+1}$

$F(s) = \frac{s+1}{(s+1)^2+1} + \frac{7}{(s+1)^2+1}$
 $\uparrow \quad \quad \quad \uparrow$
 $e^{-t} \cos(\omega t) \quad \sin(\omega t)$

$$f(t) = \left[e^{-t} \cos(t) + 7 e^{-t} \sin(t) \right] 1(t) \leftarrow \text{Ans}$$

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s) = \lim_{s \rightarrow 0} \frac{s^2+8s}{s^2+2s+2} \rightarrow 0$$



Reali \rightarrow Scomposizione

$$\frac{a_1 s + a_2}{s^2 + bs + c}$$

imm \rightarrow

$$\frac{a_1 s + a_2}{\left(s + \frac{b}{2}\right)^2 + \underbrace{c - \left(\frac{b}{2}\right)^2}_{w^2}}$$

$$= a_1 \frac{s + \frac{b}{2}}{\left(s + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2} + a_1 \frac{\frac{a_2}{a_1} - \frac{b}{2}}{\underbrace{\hspace{1cm}}_{\parallel}}$$

\uparrow cos \uparrow sin

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MATLAB Drive > Sistemi Dinamici > SD_Lezione5.m

```
1 % andiamo a definire dei polinomi
2
3 B = [3 -1] % polinomio al nominatore
4
5 A = [1 3 2] % polinomio al denominatore
6
7 length(A) % stampa la lunghezza del vettore
8
9 [r, p, k] = residue(B, A) % ha come output [r, p, k]
10
11 p' %serve per fare il trasposto
12
13 % facciamo un disegno dell'anti trasformata
14 % (1) Definire un vettore tempo
15 t = 0:0.05:5 % il passo di campionamento è un decimo della costante di
    tempo del segnale più veloce (che ha una costante di 1/2)
16 % (2) definiamo la funzione
17 f = 7 * exp(-2 * t) - 4 * exp(-t) % essendo t un vettore ci viene restituito
    un altro vettore fatto da tanti elementi quanti sono quelli di t
18
19 % (3) disegno la funzione
20 plot(t, f)
```

Columns 22 through 28

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.5426 | -0.5559 | -0.5647 | -0.5698 | -0.5714 | -0.5702 | -0.5665 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 29 through 35

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.5607 | -0.5531 | -0.5440 | -0.5336 | -0.5223 | -0.5100 | -0.4971 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 36 through 42

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.4837 | -0.4699 | -0.4559 | -0.4417 | -0.4274 | -0.4131 | -0.3989 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 43 through 49

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.3849 | -0.3710 | -0.3573 | -0.3438 | -0.3307 | -0.3178 | -0.3053 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 50 through 56

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.2930 | -0.2812 | -0.2696 | -0.2585 | -0.2477 | -0.2372 | -0.2271 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 57 through 63

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.2174 | -0.2080 | -0.1989 | -0.1902 | -0.1818 | -0.1737 | -0.1660 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 64 through 70

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.1586 | -0.1514 | -0.1446 | -0.1380 | -0.1317 | -0.1257 | -0.1199 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 71 through 77

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.1144 | -0.1091 | -0.1041 | -0.0992 | -0.0946 | -0.0902 | -0.0860 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 78 through 84

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.0819 | -0.0781 | -0.0744 | -0.0709 | -0.0676 | -0.0644 | -0.0613 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 85 through 91

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.0584 | -0.0556 | -0.0530 | -0.0505 | -0.0481 | -0.0458 | -0.0436 |
|---------|---------|---------|---------|---------|---------|---------|

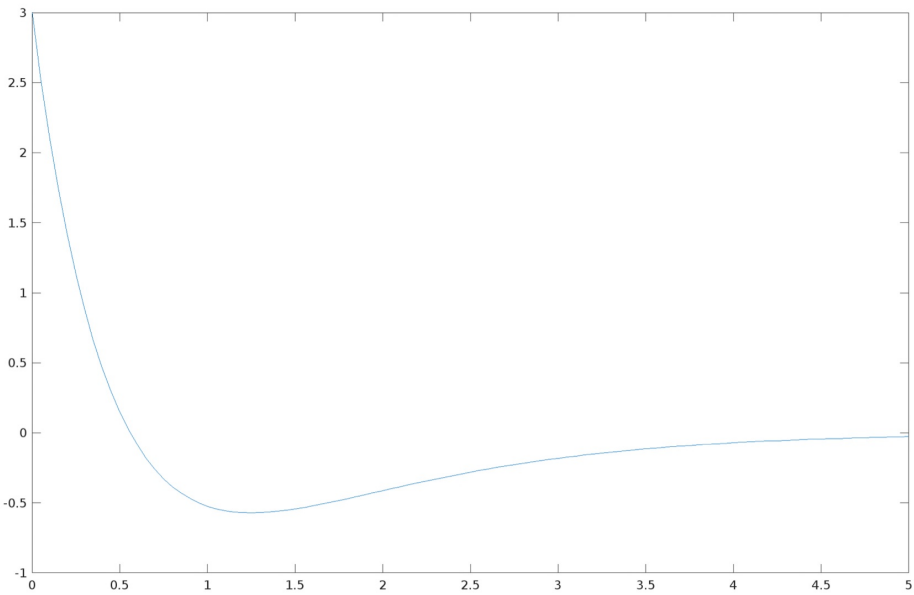
Columns 92 through 98

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| -0.0415 | -0.0395 | -0.0376 | -0.0358 | -0.0341 | -0.0324 | -0.0309 |
|---------|---------|---------|---------|---------|---------|---------|

Columns 99 through 101

| | | |
|---------|---------|---------|
| -0.0294 | -0.0280 | -0.0266 |
|---------|---------|---------|

>> clear



Output