

$$G(j\omega) = \frac{e^{-j\omega L}}{1+j\omega T} = \frac{\cos(\omega L) - j\sin(\omega L)}{1+j\omega T}$$

$$|G(j\omega)| = \sqrt{\cos^2(\omega L) + \sin^2(\omega L)} = \frac{1}{\sqrt{1+(\omega T)^2}}$$

$$\Rightarrow |G(j\omega)|_{dB} = 20 \log(+) - 20 \log(\sqrt{1+(\omega T)^2}) = -20 \log(\sqrt{1+(\omega T)^2}) \quad \text{Modulo esatto}$$

$$\angle G(j\omega) = \angle e^{-j\omega L} - \angle 1+j\omega T = -\omega L - 0 - \angle \sqrt{1+(\omega T)^2} \cdot e^{j\omega T} = -\omega L - \tan^{-1}(\omega T) \quad \text{FASE esatta}$$

Poniamo per esempio  $L = \frac{1}{2}$ ;  $T = 1$

$$\Rightarrow |G(j\omega)|_{dB} = -20 \log(\sqrt{1+\omega^2}) ; \angle G(j\omega) = -\frac{1}{2}\omega - \tan^{-1}\left(\frac{1}{2}\omega\right)$$

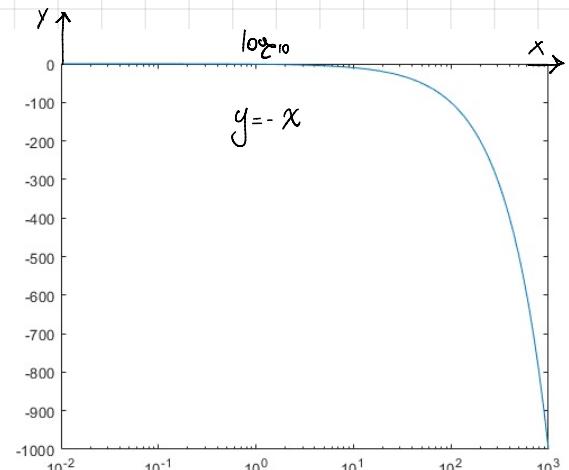
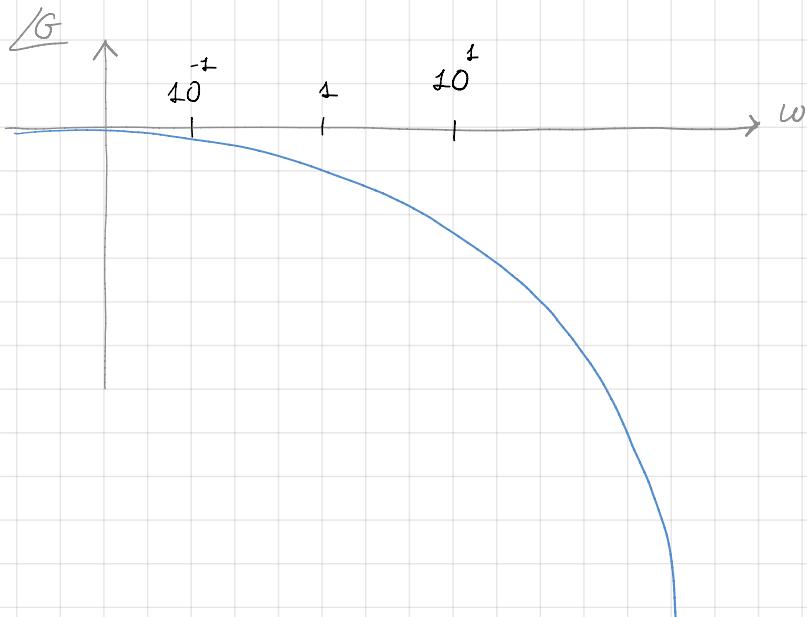
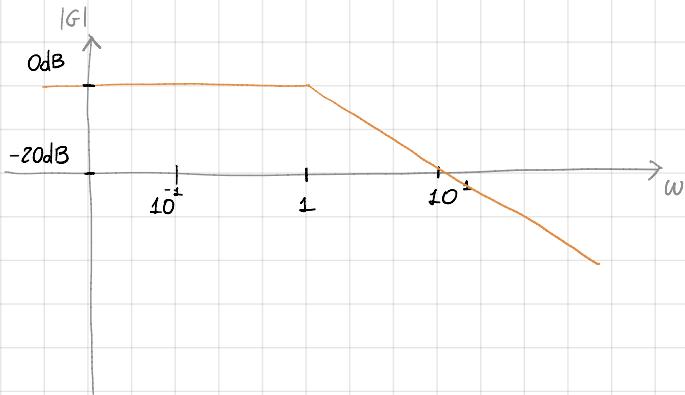
$$\text{Per } \omega \rightarrow 0 \quad |G(j\omega)|_{dB} \approx -20 \log(\sqrt{1}) = 0 \text{ dB} \quad \angle G(j\omega) \approx -\tan^{-1}(0) = 0$$

$$\text{Per } \omega \rightarrow \infty \quad |G(j\omega)|_{dB} \approx -20 \log(\omega) \quad \angle G(j\omega) \approx -\omega \quad \text{ovvero tende a } -\infty$$

-20dB/dec

$$\text{Siccome } G(j\omega) = \frac{e^{-j\omega L}}{1+j\omega T} \Rightarrow G(s) = \frac{e^{-sL}}{1+sT} \Rightarrow \bar{s}_p = -\frac{1}{T} \Rightarrow \boxed{W_1 = -1} \quad T = 1$$

Prendo come intervallo:  $\bar{\omega} \in [10^{-1}; 10^1]$



# Problema 7.1 - Pg 503

$$G(s) = \frac{10(s+4)}{(s+2)(s+5)}$$

Guadagno (forma St.)

$$G(s) = 10 \frac{1+s}{\frac{1}{2}(1+2s)\frac{1}{5}(1+5s)}$$

$$= 10^2 \cdot \frac{1+s}{(1+2s)(1+5s)}$$

guadagno Statico

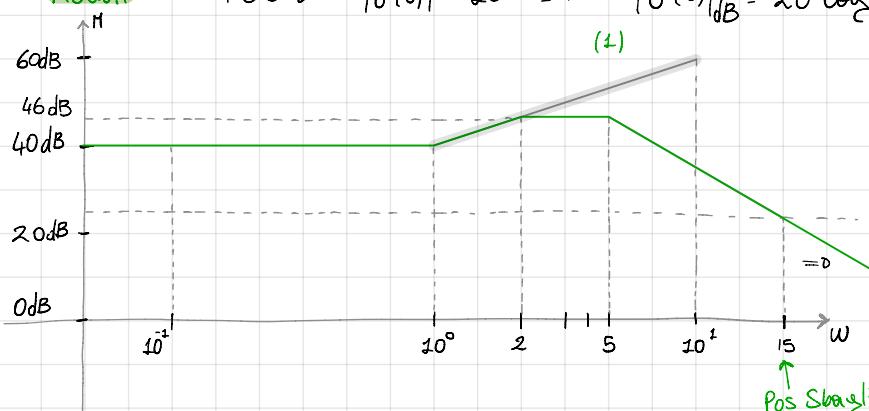
Zeri:  $s+2 = 0 \Rightarrow z_1 = -2$  Poli  $p_1 = -2 ; p_2 = -5$

**ZERO**  $\downarrow$  **POLI**  $\downarrow$   
 $w_1 = 1 , w_2 = 2 , w_3 = 5$

Scelta Banda

$$\bar{w} = [40^{-1}; 40^1]$$

Moduli



$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\frac{\log(w) - \log(1)}{\log(10) - \log(1)} = \frac{|G(jw)|_{dB} - 40}{60 - 40}$$

$$\log(w) = \frac{|G(jw)|_{dB} - 40}{20} = \boxed{|G(jw)|_{dB} = 20 \log(w) + 40}$$

eq Retta nel tratto (1)

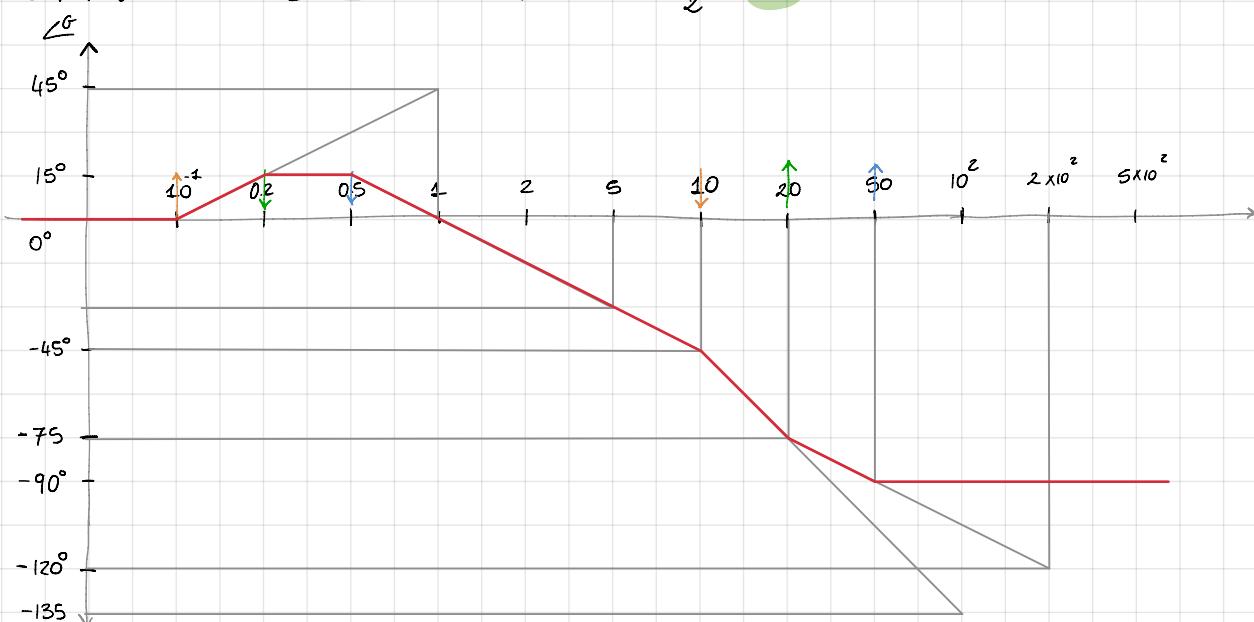
$$\text{Pos Sbagliata!} \Rightarrow |G(jw)|_{dB} = 46 dB \Rightarrow 46 dB - 20 dB = 26 dB$$

$w = 2$

Fasi

$$\angle G(jw) = \angle 4+jw - \angle 2+jw - \angle 5+jw = \tan^{-1}(w) - \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}\left(\frac{w}{5}\right)$$

$$\begin{cases} \text{Per } w=0 \Rightarrow \angle G(jw) = \tan^{-1}(0) - \tan^{-1}(0) - \tan^{-1}(0) = 0 \\ \text{Per } w=\infty \Rightarrow \angle G(jw) = -\tan^{-1}(w=\infty) = -\frac{\pi}{2} = -90^\circ \end{cases}$$



## ES

$$G(s) = \frac{(s-1)(s+10)}{s(s^2+s+16)} = \frac{-(1-s)10(1+\frac{1}{10}s)}{s(s^2+s+16)}$$

$s^2 + s + 16$  puo' essere scritta nella forma  $(s+a)(s+b)$ ?

$\Delta = 1 - 4 \cdot 1 \cdot 16 = -63 < 0 \Rightarrow$  Radici complesse e conjugate  $\Rightarrow \underline{\text{NO}}$

$$s^2 + s + 16 = 16 \left( 1 + \frac{1}{16}s + \frac{1}{16}s^2 \right) \Rightarrow G(s) = \frac{-10}{16} \frac{(1-s)(1+\frac{1}{10}s)}{s(1+\frac{1}{16}s+\frac{1}{16}s^2)} = \frac{k_b}{s} \cdot \frac{(1-\tau_1 s)(1+\tau_2 s)}{(1+\frac{2\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2)}$$

2. DOVE...

$$k_b = -\frac{5}{8}$$

$$\omega_n^2 = 16 \Rightarrow \omega_n = 4 \quad \text{e} \quad \frac{2\zeta}{4} = \frac{1}{16} \Rightarrow \zeta = \frac{4}{16} = \frac{1}{4}$$

$$\tau_1 = 4$$

$$\tau_2 = \frac{1}{10}$$

3. Punti di rottura

- $\omega_n = 4$
- $1 - s = 0 \Rightarrow \zeta_1 = 1$
- $1 + \frac{1}{10}s = 0 \Rightarrow \zeta_2 = -10$

$$\Rightarrow \begin{cases} \omega_1 = 1 \in \text{ReP} > 0 \\ \omega_2 = 4 \in \text{P} \\ \omega_3 = 10 \in \text{ReP} < 0 \end{cases}$$

4. Diagramma dei Moduli

E' presente un polo nell'origine  $\Rightarrow$  il modulo inizia con una pendenza di  $-20 \text{ dB/dec}$

Perche'? Per  $\omega$  molto piccole, avremo  $\omega \ll \omega_1 = 1$  l'unico contributo e' quello di  $\bar{G}(s) = \frac{k_b}{s}$

Termine integrante

$$\bar{G}(j\omega) = \frac{k_b}{j\omega} = \underbrace{1}_{\text{ovvero } 0 \text{ dB}} \Rightarrow \frac{|k_b|}{\omega} = 1 \Rightarrow \omega_0 = |k_b| = \underbrace{\frac{5}{8}}_{\text{ovvero } 0 \text{ dB}} \rightarrow P_1 \left( \frac{5}{8}, 0 \text{ dB} \right)$$

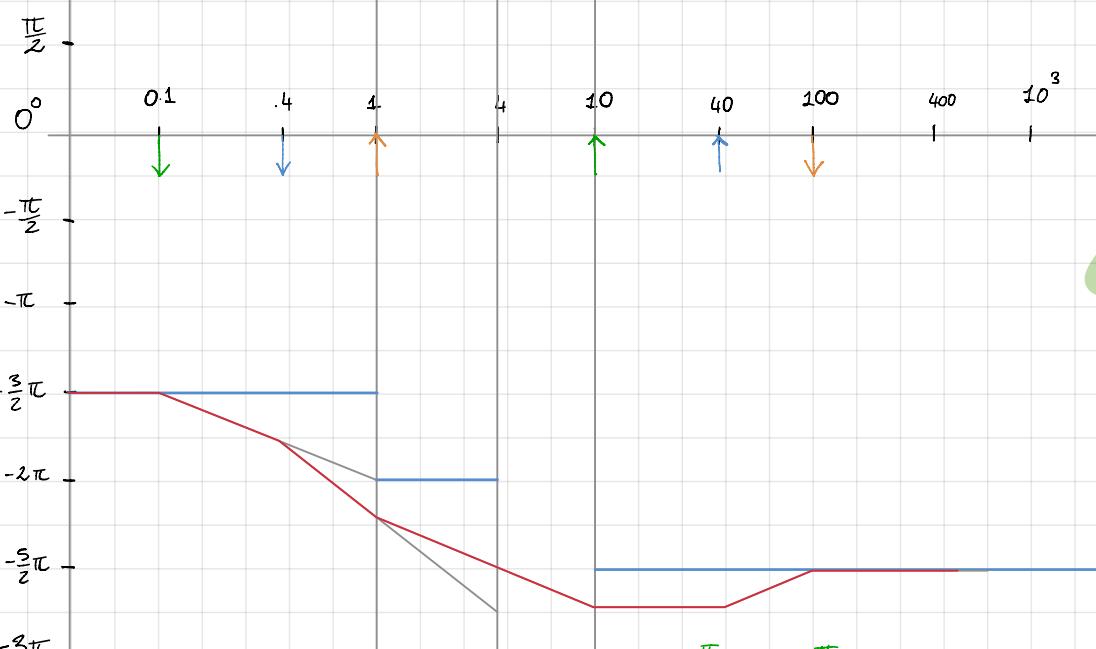
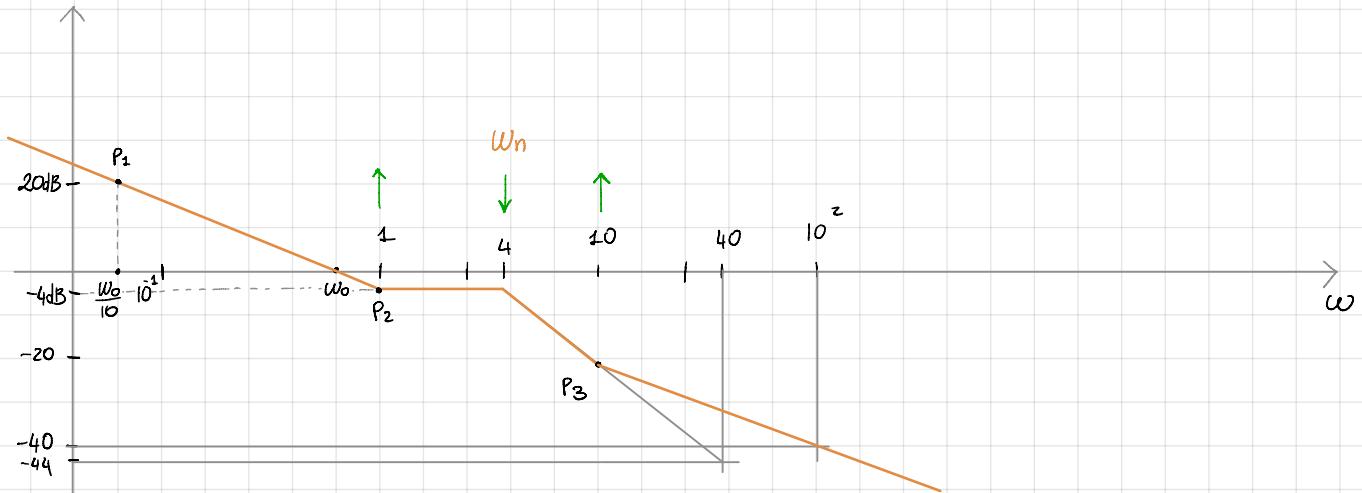
$$\frac{\log(\omega) - \log(\frac{5}{8})}{\log(\frac{5}{8}) - \log(\frac{1}{10})} = \frac{|G(j\omega)|_{\text{dB}} - 20}{-20} \rightarrow |G(j\omega)|_{\text{dB}} = \left[ - \left( \frac{\log(\omega) + 1.2}{-0.2 + 1.2} \right) \cdot 20 \right] + 20$$

$$\Rightarrow |G(j\omega)|_{\text{dB}} \Big|_{\omega=1} = -4 \text{ dB} \quad \text{e} \quad P_2 (1, -4 \text{ dB})$$

$$\frac{\log(\omega) - \log(4)}{\log(40) - \log(4)} = \frac{|G(j\omega)|_{\text{dB}} + 4}{-44 + 4} \quad \text{e} \quad |G(j\omega)|_{\text{dB}} \Big|_{\omega=10} = -(\log(\omega) - 0.6) \cdot 40 - 4$$

$$\frac{1.6}{1} = -20 \text{ dB} \quad \text{e} \quad P_3 = (10, -20 \text{ dB})$$

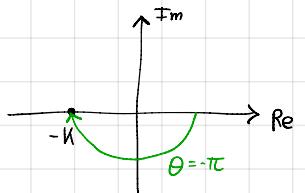
|G|



$$\left\{ \begin{array}{l} \omega_1 = 1 \text{ } \xrightarrow{\text{ReP} > 0} \\ \omega_2 = 4 \text{ } \xrightarrow{\text{ReP} < 0} \\ \omega_3 = 10 \text{ } \xrightarrow{\text{ReP} < 0} \end{array} \right.$$

Per  $\omega \ll \omega_0 \Rightarrow G_1(j\omega) \approx \frac{K_0}{j\omega} \Rightarrow \angle G = \angle K_0 - \angle j\omega$  Siccome  $K_0 < 0$

$$\Rightarrow \angle G = -\pi - \frac{\pi}{2} = -\frac{3}{2}\pi \text{ Fase iniziale}$$



Per  $1 < \omega < 4 \Rightarrow G_2(s) \stackrel{N}{=} G_1(s) (1 - \tau_1 s)$   $\Rightarrow \text{Zero ReP} = -\frac{\pi}{2}$   $\Rightarrow \text{TOT} = -\frac{3}{2}\pi - \frac{\pi}{2} = -2\pi$

Per  $4 < \omega < 10 \Rightarrow G_3(s) \stackrel{N}{=} G_2(s) (1 + \frac{2\zeta_1 s}{\omega_n} + \frac{1}{\omega_p^2} s^2)$   $\Rightarrow \text{Zero del II}^\circ = -\pi$   $\Rightarrow \text{TOT} = -2\pi - \pi = -3\pi$

Per  $\omega \geq 10 \Rightarrow G_4(s) = G_3(s) (1 + \tau_2 s)$   $\Rightarrow \text{Zero ReP} < 0 \Rightarrow +\frac{\pi}{2}$   $\Rightarrow -3\pi + \frac{\pi}{2} = -\frac{5}{2}\pi$

## Esercizio

$$G(s) = \frac{100(s-0.2)}{(s+0.8)(s^2-2s+100)}$$

Forma St:  $G(s) = \frac{1}{(1 + \frac{s}{\omega_n} + \frac{1}{\omega_n^2}s^2)}$

$$G(s) = \frac{\frac{n}{m}(1+s)}{\frac{m}{n}(s+s)}$$

(1) il termine  $(s^2-2s+100)$  ha radici reali?

$$\sim \Delta = 4 - 4 \cdot 100 = -396 < 0 \Rightarrow \text{Cmplx}$$

$$\Rightarrow G(s) = 100 \cdot \frac{-0.2(1 - \frac{s}{0.2})}{0.8(1 + \frac{s}{0.8}) 100(1 - \frac{1}{50}s + \frac{1}{100}s^2)}$$

$$\zeta_1: 1 - \frac{s}{0.2} = 0 \rightarrow \zeta_1 = +0.2 \quad \text{Zero a Rep Pos}$$

$$P_1: 1 + \frac{s}{0.8} = 0 \rightarrow P_1 = -0.8 \quad \text{Polo}$$

$$\left[ \begin{array}{l} W_1 = 0.2 \\ \zeta \text{ ReP<0} \end{array}, \quad \left[ \begin{array}{l} W_2 = 0.8 \\ P \end{array} \right], \quad \left[ \begin{array}{l} W_3 = \omega_n = 10 \\ \text{Freq} \end{array} \right] \right]$$

$$\Rightarrow K_b = 100 \cdot \frac{-0.2}{0.8 \cdot 100} = -\frac{1}{4} \quad \text{guadagno}$$

$$\sim \left\{ \begin{array}{l} -\frac{1}{50} = \frac{2\pi}{\omega_n} \Rightarrow \omega_n = \frac{1}{10} \\ \frac{1}{\omega_n^2} = \frac{1}{100} \Rightarrow \omega_n^2 = 100 \Rightarrow \omega_n = \sqrt{100} = 10 \end{array} \right.$$

\* Scelta della Banda

$$\min(P, \zeta) = 0.2 \Rightarrow \omega_{\min} = \frac{1}{10}$$

$$\max(P, \zeta) = 10 \Rightarrow \omega_{\max} = 10$$

(2) Possiamo dividere il diagramma in 4 Sezioni:

1° settore:  $\underline{W < W_1 = 0.2}$

Modulo:  $G_1(s) = K_b \Rightarrow |G_1(W)|_{dB} = 20 \log(|G_1(s)|) = 20 \log(\frac{1}{4}) \approx -12$

Fase:  $\angle K_b = -180^\circ = -\pi$  Per conv.  
con  $K_b < 0$

2° sett  $W_1 < W < W_2 \sim 0.2 < W < 0.8$

$$G_2(s) = K_b \cdot (s - 0.2)$$

gain Zero ReP<0

Modulo:  $+20 \text{ dB/dec}$

Fase:  $-\pi - \frac{\pi}{2} = -\frac{3}{2}\pi = -270^\circ$

3° sett  $W_2 < W < \omega_n = 10 \Rightarrow G_3(s) = K_b \frac{(s-0.2)}{(s+0.8)}$

Modulo:  $+20 \text{ dB/dec} - 20 \text{ dB/dec} = \text{COST} \in [?] \text{ dB}$

Fase:  $-\frac{3}{2}\pi - \frac{\pi}{2} = -2\pi = -360^\circ$

4° sett  $W > \omega_n = 10 \Rightarrow G_4(s) = G(s)$

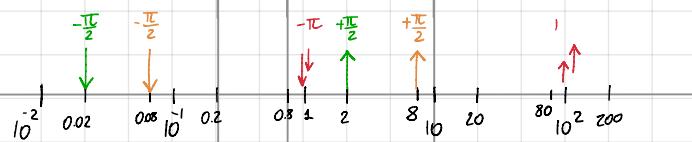
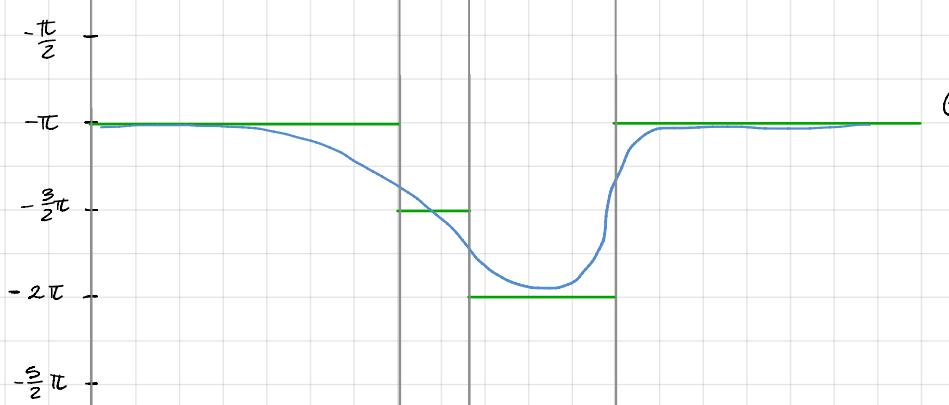
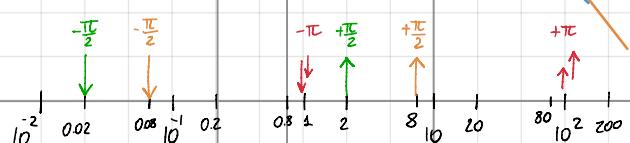
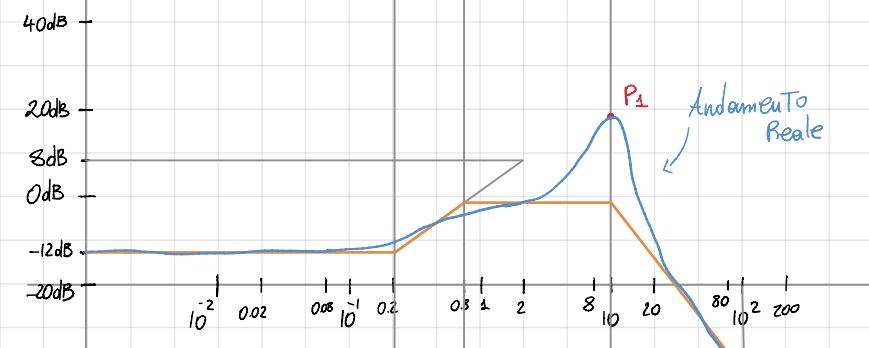
CASO NON Visto a lezione,  
lo do per buono ...

Modulo:  $0 \text{ dB/dec} - 40 \text{ dB/dec}$

Siccome  $J = -\frac{1}{10} \Rightarrow -1 < J < 0 \Rightarrow \text{Contributo} = +\pi$

Fase:  $-2\pi + \pi = -\pi = -180^\circ$

(3) Disegno dei diagrammi



$$\frac{\log(w) - 0.2}{2 - 0.2} = \frac{|G| + 12}{8 + 12}$$

$$w=0.8$$

$$= |G| \approx \left[ \left( \frac{-0.09 - 0.2}{1.8} \right) \cdot 20 \right] - 12$$

$$=$$

$$w=0.8$$

Considerazioni

$$\mathcal{J} = -\frac{1}{10} = -0.1 < 0.407 \Rightarrow \text{c'e' Risonanza!}$$

Infatti  $|G(j\omega)| \Big|_{dB} = \dots$   
 $w=w_n$

$$G(j\omega) = -\frac{1}{4} \cdot \frac{(1 - \frac{1}{0.2}j\omega)}{(1 + \frac{1}{0.8}j\omega)(1 - \frac{1}{50}j\omega + \frac{1}{100}(j\omega)^2)}$$

$$\Rightarrow \text{Mod}(G(j\omega)) \approx 5 \approx 14 \text{ dB}$$

$$w=10 \quad \text{dB}$$

$$\Rightarrow P_1 = (10, 14 \text{ dB}) \in \text{Bod}$$

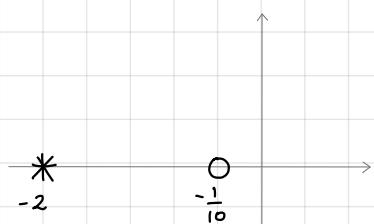
$w_1 = 0.2$   
 $\geq \text{Re} P < 0$

,  $w_2 = 0.8$ ,  $w_3 = w_n = 10$   
 $P$  Freq

# POLO DOPPIO

Es Dec 25

$$G(s) = \frac{10s+1}{\left(\frac{1}{2}s+1\right)^2} = \frac{40s+4}{4s^2+s+1}$$



1) Forma St.  $G(s)$ , Poli e Zeri

$$G_{st}(s) = K \cdot \frac{\frac{m}{n}(1 + \frac{1}{z_1}s)}{(1 + T_1 s)^2} \Rightarrow G(s) = \frac{1 + 10s}{(1 + \frac{1}{2}s)^2} \text{ già in st. F.}$$

$$z_1 = -\frac{1}{10} \quad p_{1,2} = -2$$

2) Punti di rottura

$$\text{ZERO : } w_1 = \frac{1}{10} \uparrow \quad \text{Polli : } w_{1,2} = 2 \equiv \text{POLO DOPPIO : } w_2 = 2 \downarrow$$

$$\text{A ReP pos} \Rightarrow \angle = +90^\circ$$

3) Banda di frequenze

$$w \in [10^{-2}; 10^2]$$

4) Andamenti iniziali e finali

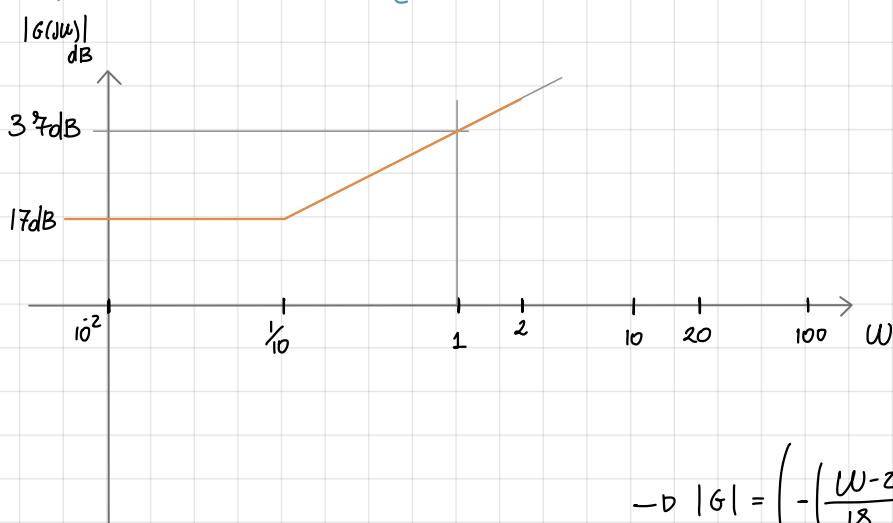
$$\text{MODULI : No Poli in origine} \Rightarrow |G(jw)|_{dB} = 20 \log(\frac{1}{2}) \approx 17dB$$

$$2 \text{ poli, 1 zero} \Rightarrow (-20 - 20 + 20) dB/\text{dec} = -20 dB/\text{dec}$$

$$\text{FASI} \quad \angle G(jw) = 0^\circ$$

$$2 \text{ poli, 1 zero} \Rightarrow -90 - 90 + 90 = -90^\circ$$

5) Traccio i diagrammi



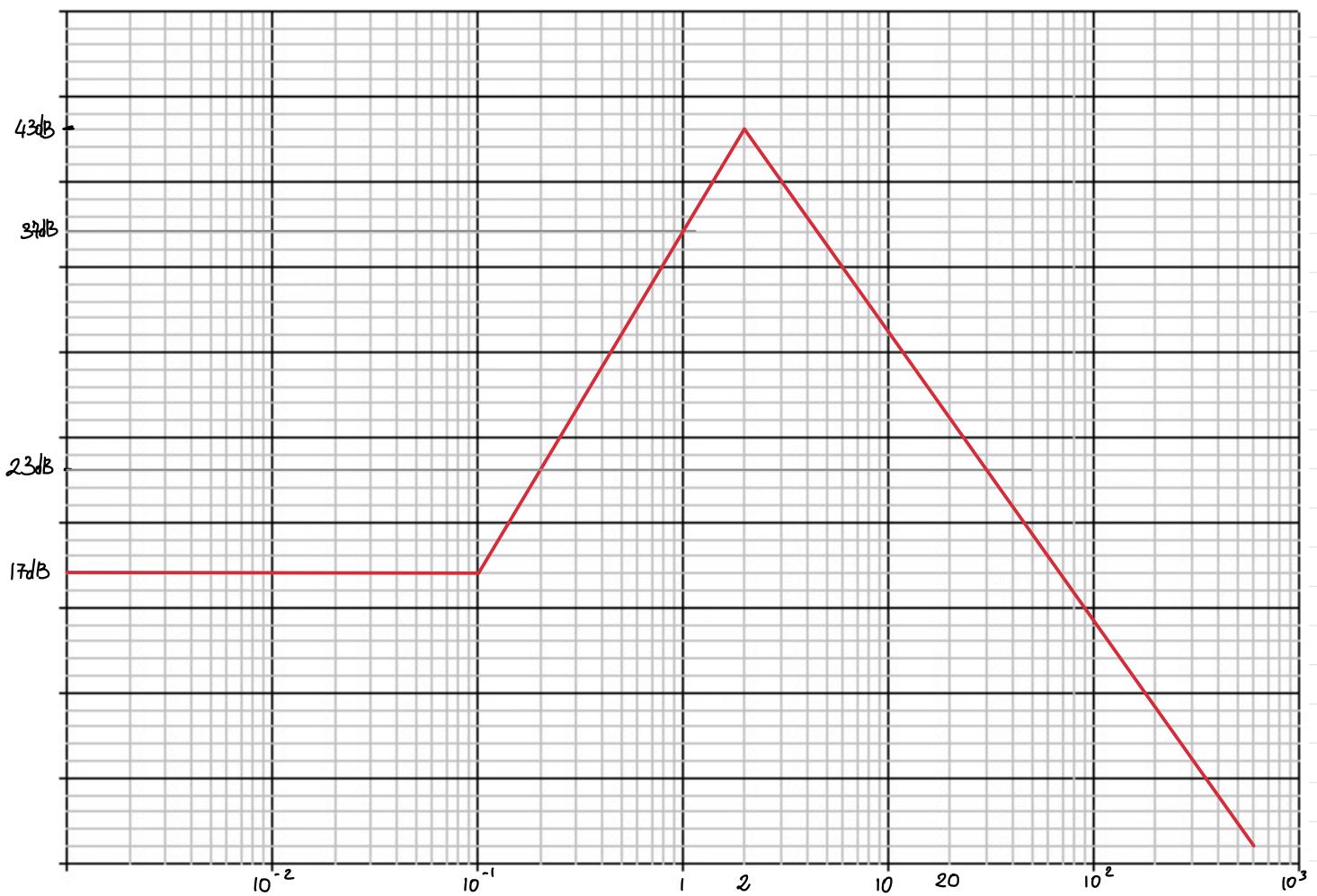
$$\frac{w-0.1}{1-0.1} = \frac{|G|-17}{37-17}$$

$$|G| = \left[ \left( \frac{w-0.1}{0.1} \right) \cdot 20 \right] + 17 \\ = 59 ??$$

$$\frac{w-2}{20-2} = \frac{|G|-59}{39-59}$$

$$\Rightarrow |G| = \left( -\left( \frac{w-2}{18} \right) \cdot 20 \right) + 59 =$$

# DIAGRAMMA DEI MODULI



### Bode Diagram

