Es 2.8

$$\ddot{x} + 3\dot{x} + 2 = 0$$

1. Trasformo Tramite le regole $\mathcal{L}[\dot{f}(t)] = s \cdot F(s) - f(0)$

$$\mathcal{L}[\ddot{x}] = \mathcal{L}\left[\frac{d}{ds}\cdot\left(\frac{d}{ds}x\right)\right] = s^2 X(s) - s x(0) - \dot{x}(0)$$

$$\rightarrow \ddot{x} + 3\dot{x} + 2 = 0 \rightleftharpoons s^2 X(s) - s x(0) - \dot{x}(0) + 3\big(sX(s) - x(0)\big) + 2X(s)$$

- Condizioni iniziali

$$\begin{cases} x(0) = a \\ \dot{x}(0) = b \end{cases} \rightarrow \quad s^2 X(s) - sa - b + 3sX(s) - 3a + 2X(s) = 0$$

$$X(s)(s^2 + 3s + 2) = sa + 3a + b$$

$$\Rightarrow X(s) = \frac{sa + 3a + b}{s^2 + 3s + 2}$$ Trovo Poli e zeri

(2) Anti trasformato

$$P_{1,2} = \frac{-3 \pm \sqrt{9 - 4\cdot 2}}{2} \quad \begin{matrix} \nearrow -\frac{3+1}{2} = -1 \; P_1 \\ \searrow -\frac{3-1}{2} = -2 \; P_2 \end{matrix}$$

$$\Rightarrow X(s) = \frac{r_1}{s+1} + \frac{r_2}{s+2}$$

$$\rightarrow r_1 = \lim_{s \to -1} \frac{sa + 3a + b}{s+2} \rightarrow \frac{-a + 3a + b}{1} = 2a + b \quad r_1$$

$$r_2 = \lim_{s \to -2} \frac{sa + 3a + b}{s+1} \rightarrow \frac{-2a + 3a + b}{-1} \rightarrow -a - b \quad r_2$$

$$\Rightarrow X(s) = \frac{\overset{K_1}{2a+b}}{s+1} - \frac{\overset{K_2}{a+b}}{s+2} \rightsquigarrow x(t) = (2a+b)e^{-t} - (a+b)e^{-2t}, \quad t \geq 0$$

Costanti prese come Valori iniziali

ES 2.9 Equazione completa

$$\ddot{x} + 2\dot{x} + 5x = 3 \qquad \begin{cases} x(0) = 0 \\ \dot{x}(0) = 0 \end{cases}$$

$$s^2 X(s) - s\,\cancel{x(0)}^{\,0} - \cancel{\dot{x}(0)}^{\,0} + 2sX(s) + 5X(s) = 3 \cdot \frac{1}{s}$$

$$\to s^2 X(s) + 2sX(s) + 5X(s) = \frac{3}{s} \quad \to X(s) = \frac{3}{s(s^2+2s+5)} = \frac{r_1}{s} + \quad ??$$

$$\begin{cases} p_1 = 0 \\ p_{2,3} = \dfrac{-2 \pm \sqrt{4 - 4\cdot 5}}{2} \end{cases} \begin{array}{l} \nearrow -1+2j \quad p_2 \\ \searrow -1-2j \quad p_3 \end{array}$$

$$X(s) = \frac{r_1}{s} + \frac{r_2}{s+1-2j} + \frac{r_3}{s+1+2j} = \frac{}{s^2 + \cancel{s} + \cancel{2sj} + \cancel{s} + \cancel{1} + \cancel{2j} - \cancel{2js} - \cancel{2j} - \cancel{4j^2}} =$$

$2s \qquad 5 \qquad 4$

$$= \frac{r_1}{s} + \frac{r_2 s + r_3}{s^2+2s+5}$$

$$r_1 = \lim_{s\to 0} \cancel{s} \cdot \frac{3}{\cancel{s}(\cancel{s^2} + \cancel{2s} + 5)} \to \frac{3}{5} \quad r_1$$

← Calcolo l'unico residuo calcolabile

$$\frac{3}{s(s^2+2s+5)} = \frac{r_1}{s} + \frac{r_2 s + r_3}{s^2+2s+5} = \frac{r_1 s^2 + 2r_1 s + 5r_1 + r_2 s^2 + r_3 s}{//}$$

$$\to \begin{cases} s^2(r_1 + r_2) = 0 \to r_2 = -\frac{3}{5} \quad r_2 \\ s\,(2r_1 + r_3) = 0 \quad \to \frac{6}{5} + r_3 = 0 \to r_3 = -\frac{6}{5} \quad r_3 \\ 5r_1 = 3 \to r_1 = \frac{3}{5} \text{ QED} \quad r_1 \end{cases}$$

$$\to \frac{-3s+6}{5}$$

$$X(s) = \frac{3}{5}\cdot\frac{1}{s} + \frac{-\frac{3}{5}s - \frac{6}{5}}{s^2+2s+5} = \frac{3}{5}\frac{1}{s} - \frac{3}{5}\frac{s+2}{s^2+2s+5} = \frac{3}{5}\left(\frac{1}{s} - \frac{s+2}{s^2+2s+5}\right)$$

$$= \frac{3}{5}\left(\frac{1}{s} - \frac{s+2}{(s+1)^2+2^2}\right) = \frac{3}{5}\left(\overset{1}{\frac{1}{s}} - \frac{2}{(s+1)^2+2^2} - \frac{s+1}{(s+1)^2+2^2} + \frac{1}{(s+1)^2+2^2}\right)$$

$e^{-t}\cos(2t)$

$$= \frac{3}{5}\frac{1}{s} - \frac{3}{5}\frac{2}{2}\frac{1}{(s+1)^2+2^2} + \frac{s+1}{(s+1)^2+2^2} = \frac{3}{5}\cdot\frac{1}{s} - \frac{3}{10}\frac{2}{(s+1)^2+2^2} + \frac{3}{5}\frac{s+1}{(s+1)^2+2^2}$$

$$\Rightarrow x(t) = \frac{3}{5} - \frac{3}{10}e^{-t}\sin(2t) - \frac{3}{5}e^{-t}\cos(2t) \quad , \; t \geq 0$$