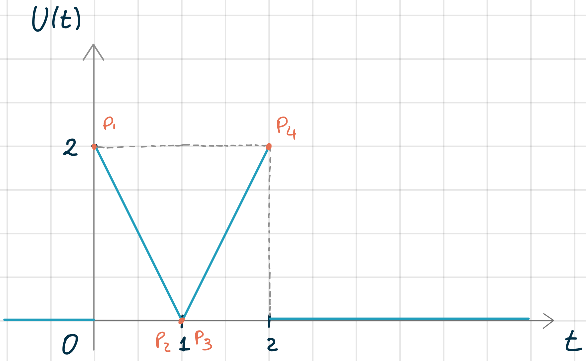
$$G(s) = \frac{1+s}{1+0.1\cdot s}$$

$$u(t) = \begin{cases} 0 & t<0 \\ 2-2t & t \in (0,1] \\ -2+2t & t\in(1,2] \\ 0 & t \geq 2 \end{cases}$$

- Segnale in input



t	A
0	2
1	∅

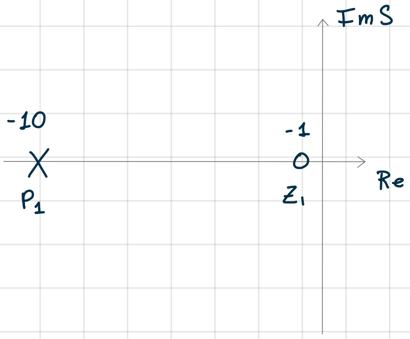
$\rightsquigarrow \begin{cases} P_1(0,2) \\ P_2(1,0) \end{cases}$

t	A
1	∅
2	2

$\rightsquigarrow \begin{cases} P_3(1,0) \\ P_4(2,2) \end{cases}$

* Non in Scala

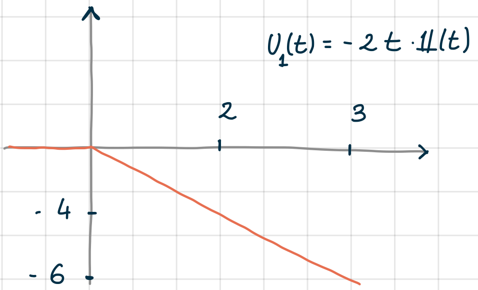
- Poli e zeri

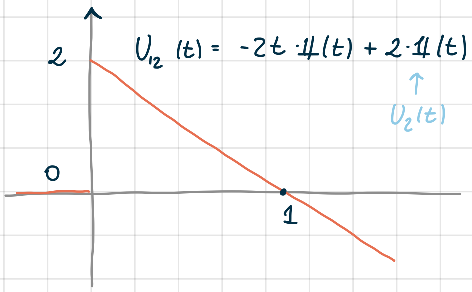


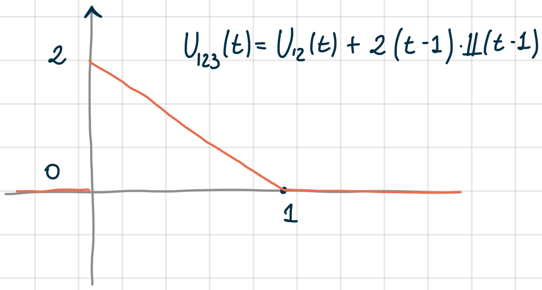
$Z: \; \bar{s} = -1$

$P: \; \bar{s} = -\frac{1}{0.1} = -10$

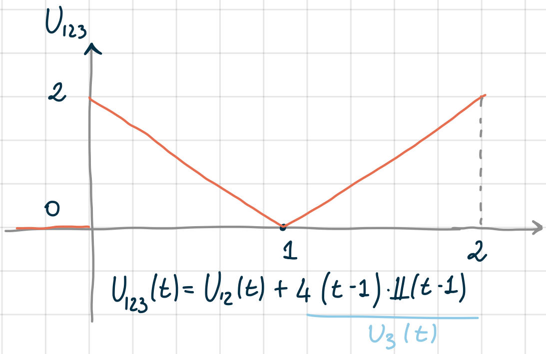
- Segnali elementari

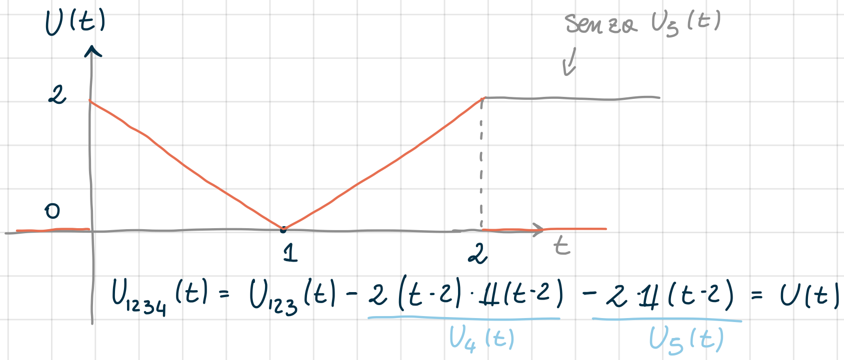






Ma siccome in 1 deve risalire, posso scrivere direttamente





$$\begin{cases} U_1(t) = -2t \cdot 1\!\!1(t) \\ U_2(t) = 2 \cdot 1\!\!1(t) \\ U_3(t) = 4(t-1)\cdot 1\!\!1(t-1) \\ U_4(t) = -2(t-2)\cdot 1\!\!1(t-2) \\ U_5(t) = -2\cdot 1\!\!1(t-2) \end{cases} \rightleftharpoons \begin{cases} U_1(s) = -2\,\frac{1}{s^2} \\ U_2(s) = 2\cdot \frac{1}{s} \\ U_3(s) = 4\cdot \frac{1}{s^2}\cdot e^{-s} \\ U_4(s) = -2\,\frac{1}{s^2}\cdot e^{-2s} \\ U_5(s) = -2\cdot\frac{1}{s}\cdot e^{-2s} \end{cases}$$

Time 30'

- Segnali fittizi

Siccome il segnale è composto di Rampe e gradini, mi basta trovare l'uscita del solo segnale rampa:

$$\hat{U}(t) = t \cdot \mathbb{1}(t) \rightleftarrows U(S) = \frac{1}{S^2}$$

$$\Rightarrow \hat{Y}(S) = G(S) \cdot \hat{U}(S) = \frac{1+S}{S^2(1+0.1S)} = \frac{r_1}{S} + \frac{r_2}{S^2} + \frac{r_3}{1+0.1S} = \frac{10+10S}{S^2(S+10)}$$

$$r_2 = \lim_{S \to 0} \cancel{S^2} \cdot \frac{10+\cancel{10S}^{\,0}}{\cancel{S^2}(\cancel{S}_{0}+10)} \to \frac{10}{10} \to 1 \quad r_2$$

$$r_3 = \lim_{S \to -10} \cancel{(S+10)} \cdot \frac{10+\cancel{10S}^{\,-100}}{\cancel{S^2}_{100}\cancel{(S+10)}} = -\frac{90}{100} = -0.9 \quad r_3$$

$$\frac{r_1 S(S+10) + r_2(S+10) + r_3 S^2}{S^2(S+10)} = \frac{10+10S}{S^2(S+10)} \rightsquigarrow \begin{cases} S^2(r_1+r_3) = 0 \\ S(10r_1 + r_2) = 10 \\ \cancel{10}\, r_2 = \cancel{10} \to r_2 = 1 \text{ QED} \end{cases}$$

$$\to r_1 = -r_3 \Rightarrow r_1 = 0.9 \quad r_1$$

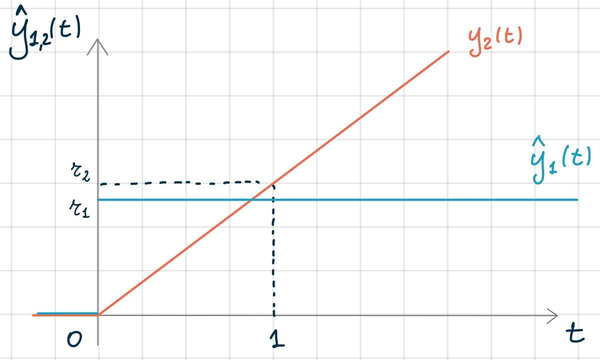
Quindi $\hat{Y}(S) = \dfrac{1+S}{S^2\, 0.1(S+10)} = \dfrac{10S+10}{S^2(S+10)} = \dfrac{r_1}{S} + \dfrac{r_2}{S^2} + \dfrac{r_3}{S+10}$

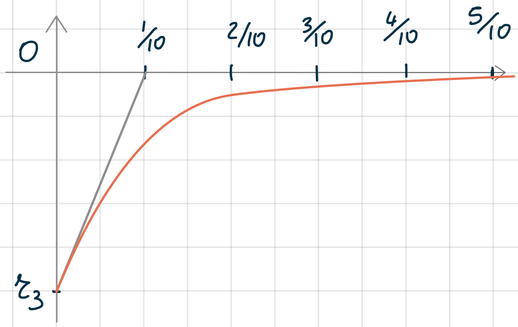
$r_1 \cdot \mathbb{1}(t)$ $\quad$ $r_2 \cdot t \cdot \mathbb{1}(t)$ $\quad$ $r_3 \cdot e^{-10t} \cdot \mathbb{1}(t)$

$$\Rightarrow \hat{y}(t) = (r_1 + r_2 t + r_3 e^{-10t}) \cdot \mathbb{1}(t)$$

$\uparrow$ $* r_3 < 0$

$$\hat{y}_3(t) = -|r_3|\, e^{-10t} \rightsquigarrow \tau = \frac{1}{10}$$





- Uscite Reali $\quad \to$ l'uscita è data da $y(t) = \sum_{i=1}^{5} y_i(t)$

$$\begin{cases} y_1(t) = -2 \cdot \hat{y}(t) \longrightarrow y_1(t) = -(2r_1 + 2r_2 t + 2r_3 e^{-10t}) \cdot \mathbb{1}(t) \\ y_2(t) = 2 \cdot \frac{d}{dt}\hat{y}(t) \longrightarrow y_2(t) = (2r_2 - 20 r_3 e^{-10t}) \cdot \mathbb{1}(t) \\ y_3(t) = 4 \cdot \hat{y}(t-1) \\ y_4(t) = -2\, \hat{y}(t-2) \\ y_5(t) = -2 \cdot \frac{d}{dt}\hat{y}(t-2) \longrightarrow y_5(t) = -(2r_2 + 20 r_3 e^{-10t-2}) \mathbb{1}(t-2) \end{cases}$$

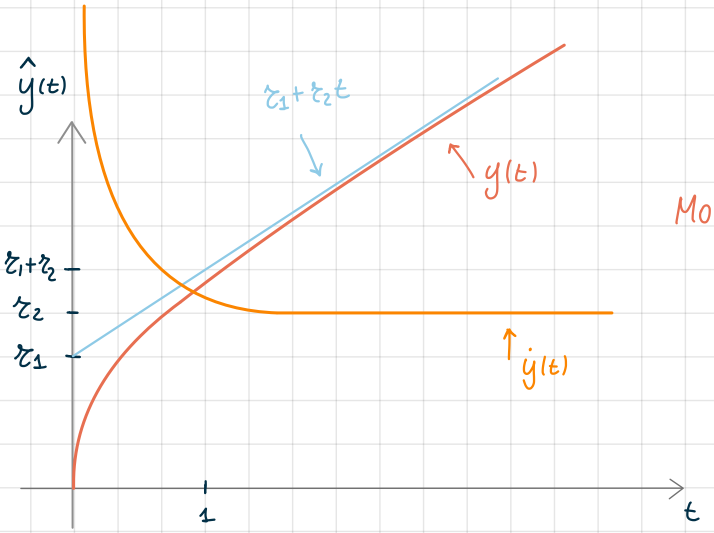
Costante

• Considerazioni

Valore iniziale

TVI: $y(t)\big|_{t=0} = \lim_{s\to\infty} s\cdot \hat{y}(s) = \cancel{s}\cdot\left(\frac{r_1}{\cancel{s}} + \frac{r_2}{s^{\cancel{2}}} + \frac{r_3}{s+1}\right) = r_1 + \frac{r_2}{s} + \frac{\cancel{s}\, r_3}{\cancel{s}(1+\frac{1}{s})} \to r_1 + r_3 = 0$

TVF: $y(t\to\infty) = \lim_{s\to 0} s\cdot \hat{y}(s) = \cancel{s}\cdot\left(\frac{r_1}{\cancel{s}} + \frac{r_2}{s^{\cancel{2}}} + \frac{r_3}{s+1}\right) = r_1 + \frac{r_2}{s} + \frac{r_3}{s+1} \to +\infty$



Morale della favola: $y(t)$ è composta da combinazioni di $\hat{y}(t)$ graficata a Sx

$$\frac{d}{dt}\hat{y}(t) = \left(r_2 - 10 r_3 e^{-10t}\right)\cdot 1\!\!1(t)$$

$$y(0) = \lim_{t\to 0} r_2 - 10 r_3 e^{-10t} = r_2 - 10 r_3 = 10$$

$$y(\infty) = r_2 - 10 e^{-10t} = r_2 = 1$$