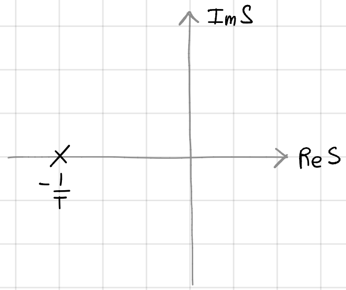
SISTEMI DEL 1° ORDINE

ES:

$$G(S) = \frac{1}{ST+1} \qquad \text{Con} \qquad U_1(t) = 1\!\!1(t)$$

Op. Preliminare : Piano complx

$$ST+1 \rightarrow \bar{S} = -\frac{1}{T}$$



Svolgimento

1) Analisi INPUT

$$U_1(t) = 1\!\!1(t) \rightleftarrows U_1(S) = \frac{1}{S}$$

2) Calcolo uscita in S

Forma Standard

$$Y_1(S) = G(S) \cdot U(S) = \frac{1}{ST+1} \cdot \frac{1}{S} = \frac{1}{T(S+\frac{1}{T}) \cdot S} = \frac{1/T}{S(S+1/T)} = \frac{\mathcal{Z}_1}{S} + \frac{\mathcal{Z}_2}{ST+1}$$

I Termini in S sono "soli"

Polo : $\bar{S} = -\frac{1}{T}$

2.1) Calcolo residui

$$\mathcal{Z}_1 = \lim_{S \to 0} \cancel{S} \cdot \frac{\cancel{1/T}}{\cancel{S}(S+\cancel{1/T})} \rightarrow 1 \ \mathcal{Z}_1$$

$$\mathcal{Z}_2 = \lim_{S \to -\frac{1}{T}} \cancel{(1+ST)} \frac{1/T}{S\,\cancel{(S+1/T)}} = \frac{1/T}{-1/T} \rightarrow -1 \ \mathcal{Z}_2$$

Fratti Semplici

$$\Rightarrow Y_1(S) = G(S) \cdot U(S) = \frac{1}{S(ST+1)} = \frac{1}{S} - \frac{1}{ST+1}$$

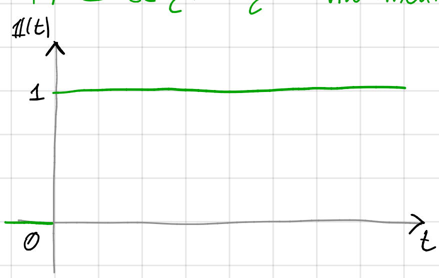
3) Antitrasformo l'uscita

- $1/S \rightleftarrows 1\!\!1(t)$
- $\frac{1}{ST+1} \rightleftarrows e^{-\frac{t}{T}}$

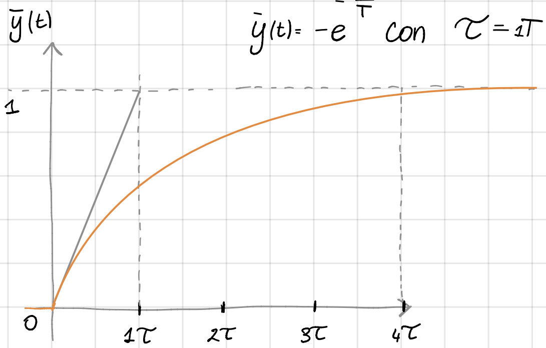
$$\Rightarrow y_1(t) = \left(1 - e^{-\frac{t}{T}}\right) 1\!\!1(t)$$

Ans 1

4) Disegno gli Andamenti



$\bar{y}(t) = -e^{-\frac{t}{T}}$ con $\tau = 1T$



ES

Stessa fdt ma input diverso $u_2(t) = t \cdot 1\!\!1(t) \rightleftarrows U(s) = \frac{1}{s^2}$

$$\Rightarrow Y_2(s) = \frac{1}{s^2} \cdot \frac{1}{sT+1} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{sT+1} \qquad \text{con } G(s) = \frac{1/T}{s+1/T} \ ; \ \bar{s} = -\frac{1}{T}$$

$$r_2 = \lim_{s \to 0} s^2 \frac{1/T}{s^2(s+1/T)} = 1 \quad r_2$$

$$r_3 = \lim_{s \to -\frac{1}{T}} \cancel{(s+1/T)} \cdot \frac{1/T}{\underbrace{s^2}_{1/T^2}\cancel{(s+1/T)}} \to \frac{1/T}{1/T^2} = T \quad r_3$$

$$= \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{s+1/T} = \frac{s r_1(s+\frac{1}{T}) + r_2(s+\frac{1}{T}) + s^2 r_3}{s^2(s+\frac{1}{T})}$$

usa il denom usato nel calcolo degli altri residui !

$$\begin{cases} s^2(r_1 + r_3) = 0 \\ s(\frac{1}{T} r_1 + r_2) = 0 \Rightarrow r_1 = -r_2 \cdot T \Rightarrow r_1 = -T \quad r_1 \\ r_2 \frac{1}{T} = \frac{1}{T} \Rightarrow r_2 = 1 \end{cases}$$

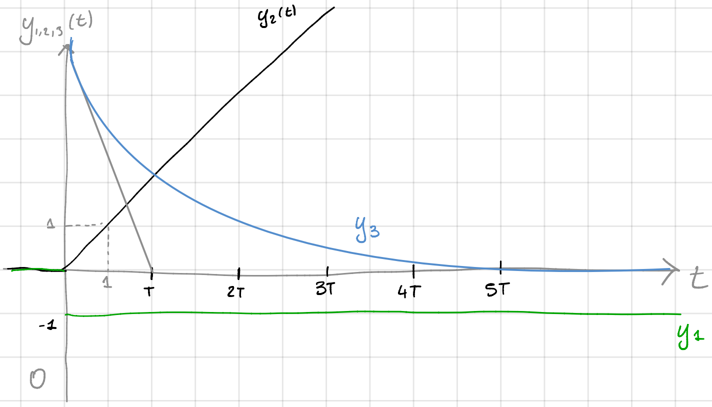
QED

$$\Rightarrow Y(s) = -\underbrace{\frac{T}{s}}_{T \cdot 1\!\!1(t)} + \underbrace{\frac{1}{s^2}}_{t \cdot 1\!\!1(t)} - \underbrace{\frac{T}{s+1/T}}_{T \cdot e^{-\frac{t}{T}}} \rightleftarrows y_1(t) = \left(-T + t - Te^{-\frac{t}{T}}\right) 1\!\!1(t)$$

• Andamento a regime e in zero

TVI: $f(0) = \lim_{t \to 0} f(t) = \lim_{s \to \infty} F(s) = \lim_{s \to \infty} -\underbrace{\frac{1}{s}}_{0} + \underbrace{\frac{1}{s^2}}_{0} + \underbrace{\frac{T}{sT+1}}_{0} \to 0 \quad f(0)$

TVF: $f(t \to \infty) = \lim_{s \to 0} -\underbrace{\frac{1}{s}}_{+\infty} + \underbrace{\frac{1}{s^2}}_{+\infty} + \underbrace{\frac{T}{sT+1}}_{+\infty} \to +\infty \quad f(t \to \infty)$



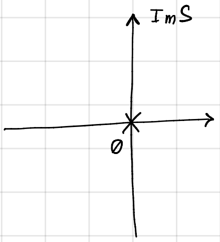
Per $t \to \infty$, $y(t) \to -1 + t + \underbrace{e^{-\frac{t}{T}}}_{0} = t - 1$ $y \simeq \bar{y}$ per $t \to \infty$

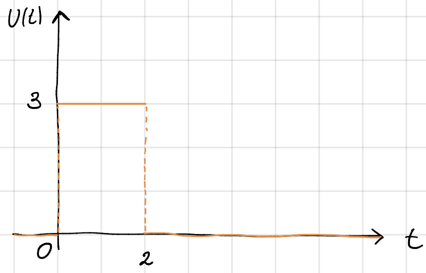
Tolgo e^{-t} perché tende a zero mentre $t \to \infty$

$\Rightarrow$ Morale: dopo 4T $y(t)$ assomiglia molto a $y(t) = t \cdot 1$

ES: INTEGRATORE CON FINESTRA

$G(s) = 1/s \Rightarrow$ No zeri, 1 Polo nell'origine $\rightarrow$ Mantiene l'uscita





$$U(t) = \begin{cases} 0 & t < 0 \\ 3 & 0 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$$

(1) Scrivo il segnale di input con segnali notevoli:

$$\begin{cases} U_1(t) = 3 \cdot 1\!\!1(t) \\ U_2(t) = -3 1\!\!1(t-2) \end{cases}$$

(2) Considero un segnale fittizio $\hat{U}(t) = 1\!\!1(t)$

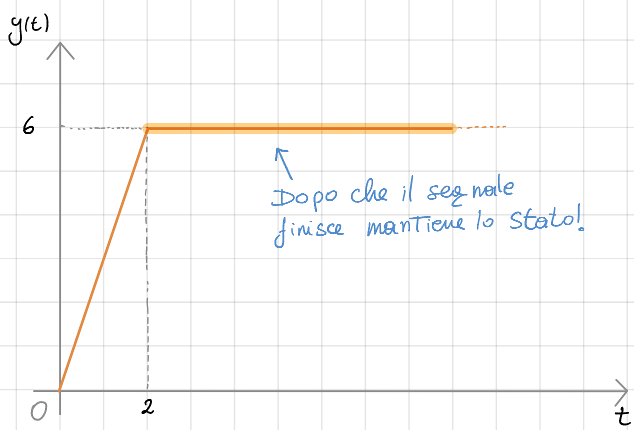
$\Rightarrow \hat{U}(s) = \frac{1}{s} \quad \Rightarrow \hat{Y}(s) = \frac{1}{s} \cdot \frac{1}{s} = 1/s^2 \rightleftarrows \hat{y}(t) = t \cdot 1\!\!1(t)$

(3) Segnali reali

$$\begin{cases} y_1(t) = 3\hat{y}(t) = 3t \cdot 1\!\!1(t) \\ y_2(t) = -3\hat{y}(t-2) = \left[-3(t-2)\right] \cdot 1\!\!1(t) \end{cases}$$

$\Rightarrow y(t) = y_1 + y_2 = 3t\, 1\!\!1(t) - 3(t-2) \cdot 1\!\!1(t-2)$

(4) Rappresento le uscite:



Per $t \in (0,2) \rightsquigarrow y(t) = 3t$

Per $t \in (2,+\infty) \rightsquigarrow y(t) = \cancel{3t} - \cancel{3t} + 6 = 6$

(5) Considerazioni

Dobbiamo tenere conto che la funzione di trasferimento è un "integratore", quindi **integra il segnale di input**. Siccome il segnale in input, per l'intervallo da 0 a 2, è un segnale costante, sappiamo che l'integrale di un segnale costante è una retta, infatti l'osserviamo in uscita.
Quando il segnale in input si azzera, l'area sottesa ad una costante di valore zero è proprio zero, quindi il contributo di quell'intervallo è zero, ma resta "l'area sottesa" calcolata fino a quel punto, quindi il segnale in uscita resta costante.

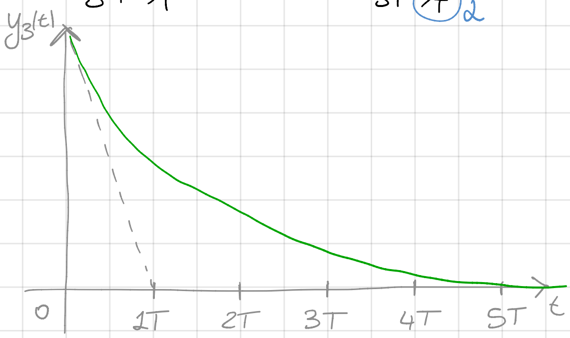
Stessa fdt ma input diverso $\qquad U_3(t) = \int \rightleftarrows U(s) = 1\!\!1(t)$

$$\Rightarrow G(s) = \frac{1}{sT+1} = \frac{1/T}{s+1/T} \rightsquigarrow Y_3(s) = \frac{1/T}{s+1/T} = \frac{1}{T}\cdot\frac{1}{s+(1/T)}$$

già in fratti semplici

α

$$\rightsquigarrow \boxed{y_3(t) = \frac{1}{T}\cdot e^{-\frac{t}{T}}}$$ Ans



CONSIDERAZIONI

INPUT	OUTPUT
$U_2(t) = t\cdot 1\!\!1(t)$	$y_2(t) = \left(-T+t-Te^{-\frac{t}{T}}\right)1\!\!1(t)$
$U_1(t) = 1\!\!1(t)$	$y_1(t) = \left(1-e^{-\frac{t}{T}}\right)1\!\!1(t)$
$U_3(t) = \int$	$y_3(t) = \frac{1}{T}\cdot e^{-\frac{t}{T}}$

Considerando $G(s) = \frac{1}{sT+1}$

$$\frac{d}{dt}\, y_2(t) = \frac{d}{dt}\left[-T+t-Te^{-\frac{t}{T}}\right] = 1-e^{-\frac{t}{T}}$$ Risposta al gradino

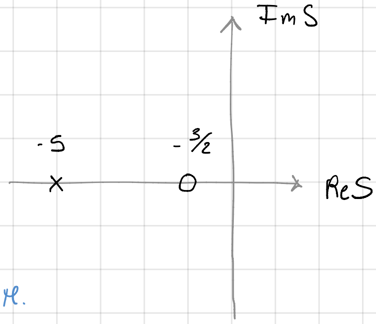
$$\frac{d}{dt}\, y_1(t) = -\frac{1}{T}\, e^{-\frac{t}{T}}$$ Risposta all'impulso

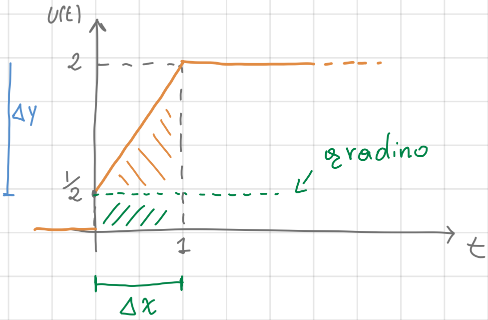
ES - Lezione 17

$$G(s) = \frac{2s+3}{s+5}$$

Zeri: $2s+3=0 \leadsto z_1 = -\frac{3}{2}$

Poli: $s+5=0 \leadsto p_1 = -5$





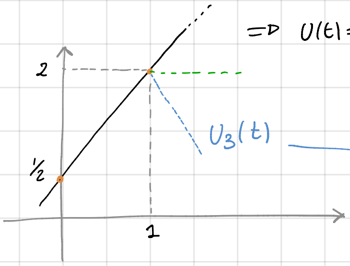
1) SCRIVO u(t) IN SEGNALI ELEM.

Trovo pendenza retta $[0;1]$: $\frac{t-0}{1-0} = \frac{u(t)-\frac{1}{2}}{2-\frac{1}{2}} \leadsto t = \frac{2}{3}u(t) - \frac{1}{3}$

What?? dovrebbe essere $\frac{1}{2}$

pendenza

$\Rightarrow u(t) = \frac{3}{2}t + \frac{1}{2}$



$\Rightarrow \begin{cases} u_2(t) = \frac{3}{2}t \cdot 1\!\!1(t) \\ u_1(t) = 1\!\!1(t) \end{cases}$

$\rightarrow \{ u_3(t) = -\frac{3}{2}(t-1)\cdot 1\!\!1(t-1)$

$\Rightarrow u(t) = u_1(t) + u_2(t) + u_3(t) = 1\!\!1(t) + \frac{3}{2}t\, 1\!\!1(t) - \frac{3}{2}(t-1)\cdot 1\!\!1(t)$

2) TRASFORMO

$$U(s) = \mathcal{L}[u(t)] = \underbrace{\frac{1}{s}}_{U_1} + \underbrace{\frac{3}{2}\frac{1}{s^2}}_{U_2} - \underbrace{\frac{3}{2}e^{-s}\cdot\frac{1}{s^2}}_{U_3}$$

3) CALCOLO USCITA IN S

Considero il segnale fittizio $\bar{u}(t) = t\,1\!\!1(t) \rightleftharpoons \bar{U}(s) = \frac{1}{s^2}$

$\Rightarrow \bar{Y}(s) = G(s)\cdot \bar{U}(s) = \frac{2s+3}{s+5}\cdot\frac{1}{s^2} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{s+5}$

$\Rightarrow r_2 = \lim_{s\to 0} \cancel{s^2}\cdot\frac{2s+3}{\cancel{s^2}(s+5)} = \frac{3}{5}$ (r_2)

$r_3 = \lim_{s\to -5}\cancel{(s+5)}\cdot\frac{2s+3}{s^2\cancel{(s+5)}} \to \frac{-7}{5} = -\frac{7}{5}$ (r_3)

$\leadsto \frac{s r_1(s+5) + r_2(s+5) + s^2 r_3}{s^2(s+5)} \Rightarrow \begin{cases} s^2(r_1+r_3) = 0 \to r_1 = -r_3 \Rightarrow r_1 = \frac{7}{5} \\ s(r_1 + r_2) = 2 \Rightarrow r_1 = (2-r_2) = \frac{10-3}{5} = \frac{7}{5} \leftarrow \text{QED} \\ 5r_2 = 3 \Rightarrow r_2 = \frac{3}{5} \quad \text{QED} \end{cases}$

$\Rightarrow \bar{Y}(s) = \frac{7}{5}\underbrace{\frac{1}{s}}_{1\!\!1(t)} + \frac{3}{5}\underbrace{\frac{1}{s^2}}_{t\,1\!\!1(t)} - \frac{7}{5}\underbrace{\frac{1}{s+5}}_{e^{-5t}} \rightleftharpoons \bar{y}(t) = \left(\frac{7}{5} + \frac{3}{5}t - \frac{7}{5}e^{-5t}\right)\cdot 1\!\!1(t)$

3.2) Calcolo segnali reali

$u_2(t) = \frac{3}{2}t\cdot 1\!\!1(t) = \frac{3}{2}\bar{u}(t) \Rightarrow y_2(t) = \frac{3}{2}\bar{y}(t) = \left(\frac{21}{10} + \frac{9}{10}t - \frac{21}{10}e^{-5t}\right)\cdot 1\!\!1(t)$ ($y_2(t)$)

$u_3(t) = -\frac{3}{2}(t-1)\cdot 1\!\!1(t-1) = -\frac{3}{2}\bar{u}(t-1) \to y_3(t) = -\frac{3}{2}\bar{y}(t-1) = \left[-\frac{21}{10} - \frac{9}{10}(t-1) + \frac{21}{10}e^{-5(t-1)}\right]\cdot 1\!\!1(t-1)$ ($y_3(t)$)

oppure $= -u_2(t-1)$

$$U_1(t) = \frac{1}{2}\,\mathbb{1}(t) = \frac{1}{2}\frac{d}{dt}\left[\bar{U}(t)\right] = \underbrace{\left(\frac{7}{10} + \frac{7}{2}e^{-5t}\right)\mathbb{1}(t)}_{U_1(t)}$$

$*$ Le Costanti potrebbero essere errate...

4) Disegno dei Singoli Termini

$$U_1(t) \simeq \frac{7}{2}e^{-5t} \Rightarrow \tau = 1/5$$

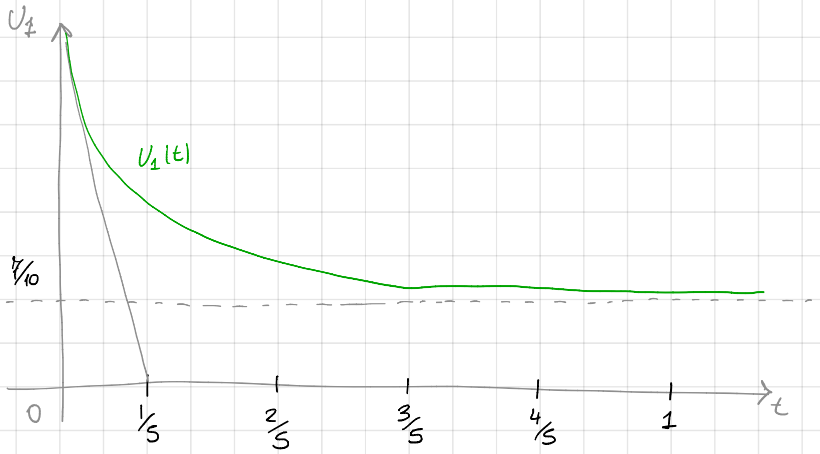
4.1) Valori iniziali e finali

Calcolo $\underset{\text{Reale}}{Y(s)} = \frac{2s+3}{s+5}\left(\frac{1}{s} + \frac{3}{2}\frac{1}{s^2} - \frac{3}{2}e^{-s}\cdot\frac{1}{s^2}\right)$

TVI : $y(0^+) = \lim_{s\to\infty} \cancel{s}\left[\frac{2s+3}{\underbrace{s+5}_{\cancel{s}(1+\frac{5}{s})}}\left(\frac{1}{\cancel{s}} + \cancel{\frac{3}{2}\frac{1}{s^2}} - \frac{3}{2}e^{-s}\cdot\frac{1}{s^2}\right)\right] \to +\infty$

TVF : $y(t\to\infty) = \lim_{s\to 0} \cancel{s}\left[\frac{2s+3}{s+5}\left(\frac{1}{\cancel{s}} + \frac{3}{2}\frac{1}{s^2} - \frac{3}{2}e^{-s}\cdot\frac{1}{s^2}\right)\right]$

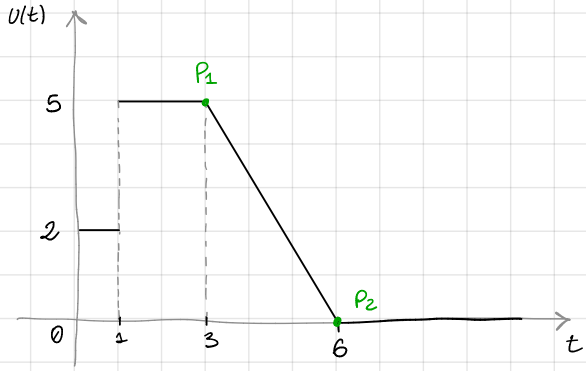
BOH



Esempio Svolgimento

$$G(S) = \frac{1}{S + \frac{1}{2}}$$

Determinare la risposta all'ingresso in figura



1) Costruiamo $u(t)$ a partire dai segnali elem.

$$\begin{cases} u_1(t) = 2 \cdot 1(t) \\ u_2(t) = 3 \cdot 1(t-1) \end{cases}$$

$u_1(t) + u_2(t)$ ottengo $5 \cdot 1(t_0)$ in $t_0 = 3$; devo trovare un segnale che ricrei la rampa in $[3;6]$

→ Il segnale sarà sicuramente una rampa, ma serve di una pendenza specifica...

$$\boxed{\frac{t-3}{6-3} = \frac{u(t)-5}{0-5}} \rightarrow u(t) = \left[\left(-1 + \frac{1}{3}t\right)\cdot(-5)\right] + 5 = 5 - \frac{5}{3}t + 5 = -\frac{5}{3}t + 10$$

Formula retta passante per 2 punti

retta che descrive il segnale in $[3;6]$

Pendenza

$$\Rightarrow U_3(t) = -\frac{5}{3}(t-3) \cdot 1(t-3)$$

Il segnale è shiftato ed inizio in $t=3$

A questo punto $\bar{U}(t) = U_1 + U_2 + U_3$ è una rampa decresc. con pendenza $-\frac{5}{3}$ (da $t=3$ in poi); dobbiamo annullarla per valori da $t=6$ in poi...

pendenza opposta

$$U_4(t) = +\frac{5}{3}(t-6) \cdot 1(t-6)$$

⇒ Segnale in figura: $U(t) = U_1(t) + U_2(t) + U_3(t) + U_4(t)$

2) Trasformate di Laplace

$$U_1(t) \rightleftarrows U_1(s) = \frac{2}{s}$$

gain, gradino

$$U_2(t) \rightleftarrows U_2(s) = \frac{3}{s} e^{-s}$$

$$U_3(t) \rightleftarrows U_3(s) = -\frac{5}{3} \frac{e^{-3s}}{s^2}$$

$$U_4(t) \rightleftarrows U_4(s) = \frac{5}{3} \frac{e^{-6s}}{s^2}$$

Ritardo, Rampa, guadagno

3) Calcolo dell'uscita nel dominio S

→ Ci conviene usare il segnale fittizio $\hat{U}(t) = t \cdot 1(t) \rightleftarrows \hat{U}(s) = \frac{1}{s^2}$

$$\Rightarrow \hat{Y}(s) = G(s) \cdot \hat{U}(s) = \frac{1}{s + \frac{1}{2}} \cdot \frac{1}{s^2} = \frac{1}{s^2\left(s + \frac{1}{2}\right)} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{s + \frac{1}{2}}$$

$$r_2 = \lim_{s \to 0} \cancel{s^2} \cdot \frac{1}{\cancel{s^2}\left(s + \frac{1}{2}\right)} \rightarrow \frac{1}{\frac{1}{2}} = 2 \quad r_2$$

$$r_3 = \lim_{s \to -\frac{1}{2}} \cancel{\left(s + \frac{1}{2}\right)} \frac{1}{s^2 \cancel{\left(s + \frac{1}{2}\right)}} \rightarrow 4 \quad r_1$$

$\frac{1}{4}$

$$\frac{s\left(s + \frac{1}{2}\right) r_1 + \left(s + \frac{1}{2}\right) r_2 + s^2 r_3}{s^2\left(s + \frac{1}{2}\right)} = \frac{s^2 r_1 + s\frac{1}{2} r_1 + s r_2 + \frac{1}{2} r_2 + s^2 r_3}{s^2\left(s + \frac{1}{2}\right)}$$

$s^2(r_1 + r_3) = 0 \Rightarrow r_1 = -r_3 = -4 \quad r_1$

$s\left(\frac{1}{2} r_1 + r_2\right) = 0 \quad r_1 = -2 r_2 = -4$

$\frac{1}{2} r_2 = 1 \Rightarrow r_2 = 2$

$$\Rightarrow \hat{Y}(s) = -\frac{4}{s} + \frac{2}{s^2} + \frac{4}{s + \frac{1}{2}}$$

Uscita in S

4.a) Antitrasformata

$$\hat{Y}(s) \rightleftharpoons \hat{y}(t) = -4\cdot \mathbb{1}(t) + 2\,t\cdot \mathbb{1}(t) + 4\cdot e^{-\frac{1}{2}t}\,\mathbb{1}(t) = \boxed{\left[-4+2t+4e^{-\frac{t}{2}}\right]\cdot \mathbb{1}(t)}$$

5) Valore iniziale e finale

Valore iniziale 0

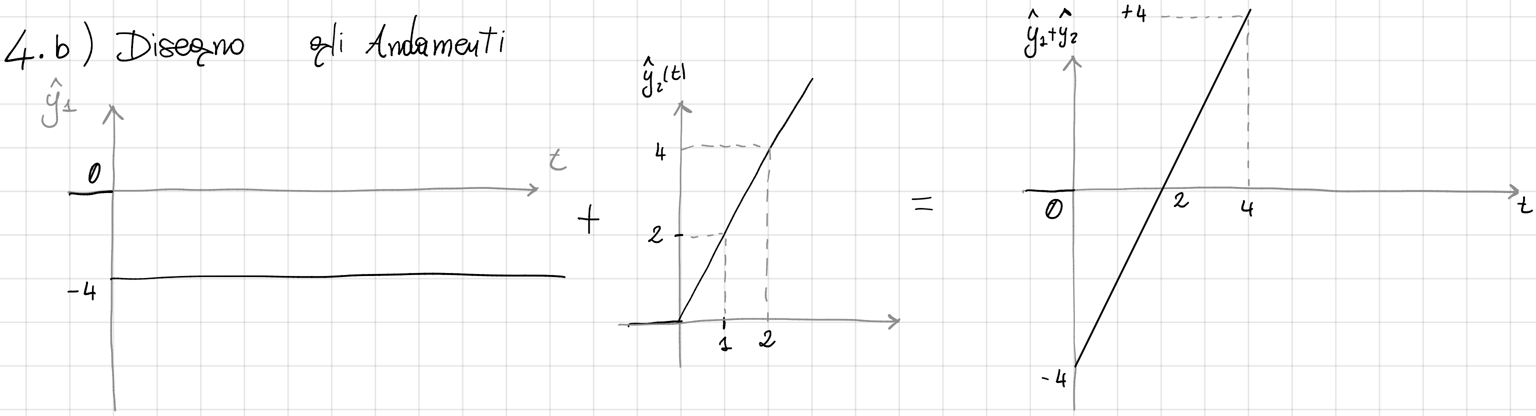
$$\hat{y}(0^+) = \lim_{t\to 0}\hat{y}(t) = \lim_{s\to\infty} s\cdot\hat{Y}(s) = \lim_{s\to\infty}\cancel{s}\cdot\left(-\frac{4}{\cancel{s}}+\frac{2}{\cancel{s}^2}+\frac{4}{s+\frac{1}{2}}\right) = \lim_{s\to\infty}\cancel{-4}+\frac{2}{s}+\frac{\cancel{s}\,4}{\cancel{s}(1+\frac{1}{2s})} \quad \cancel{4}$$

$$\hat{y}(\infty) = \lim_{t\to\infty}\hat{y}(t) = \lim_{s\to 0} -4+\frac{2}{s}+\frac{s\,4}{s+\frac{1}{2}} \to +\infty$$

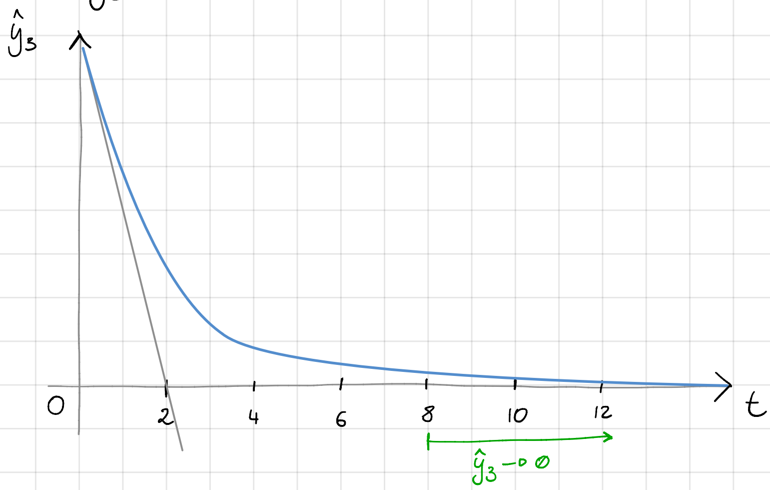
$-4 \ll \infty$, $+\infty$, $\to 0$, 0

Valore finale ← la rampa fa tendere l'uscita a $+\infty$

4.b) Disegno gli Andamenti



$\hat{y}_3(t) = 4e^{-\frac{t}{2}}$ con $\tau = 2$



All'entrata $\hat{u}(t) = t\,\mathbb{1}(t)$

↑

5. Trovo le uscite specifiche

Siccome le entrate reali sono:

$$\begin{cases} u_1(t) = 2\cdot\mathbb{1}(t) \\ u_2(t) = 3\,\mathbb{1}(t-1)\end{cases} \qquad \begin{cases} u_3(t) = -\frac{5}{3}(t-3)\cdot\mathbb{1}(t-3) \\ u_4(t) = \frac{5}{3}(t-6)\cdot\mathbb{1}(t-6)\end{cases}$$

mentre le uscite fittizie sono:

$$\begin{cases} \hat{y}_1(t) = -4\,\mathbb{1}(t) \\ \hat{y}_2(t) = 2t\,\mathbb{1}(t)\end{cases} \qquad \left\{\hat{y}_3(t) = 4e^{-\frac{1}{2}t}\,\mathbb{1}(t)\right.$$

Ricordando che $\frac{d}{dt}t = \mathbb{1}(t)$ e che $\frac{d}{dt}e^{t} = t$

$$\Rightarrow y_1(t) = 2\cdot\frac{d}{dt}(\hat{y}(t)) = 2\cdot\frac{d}{dt}\left[-4+2t+4e^{-\frac{t}{2}}\right]\cdot\mathbb{1}(t) = 2\left(2-\frac{1}{2}4e^{-\frac{t}{2}}\right)\mathbb{1}(t) = \underbrace{4\left(1-e^{-\frac{t}{2}}\right)\mathbb{1}(t)}_{y_1(t)}$$

$$y_2(t) = \frac{3}{\cancel{2}}\cdot y_1(t-1) = \frac{3}{\cancel{2}}\cdot\cancel{2}\left(2-2e^{-\frac{t-1}{2}}\right)\mathbb{1}(t) = \underbrace{6\left(1-e^{-\frac{t-1}{2}}\right)\cdot\mathbb{1}(t-1)}_{y_2(t)}$$

$$y_3(t) = -\frac{5}{3}\cdot \hat{y}(t-3)\,1\!\!1(t) = -\frac{5}{3}\cdot\left(-4+2(t-3)+4e^{-\frac{t-3}{2}}\right) = +\frac{20}{3} - \frac{10}{3}t + \frac{\cancel{30}}{\cancel{3}}_{10} - \frac{20}{3}e^{-\frac{t-3}{2}}$$

$$= \frac{50}{3} - \frac{10}{3}t - \frac{20}{3}e^{-\frac{t-3}{2}} = \underbrace{\frac{10}{3}\left(5-t-2e^{-\frac{t-3}{2}}\right)}_{y_3(t)}\cdot 1\!\!1(t-3)$$

$$y_4(t) = \frac{5}{3}\,y(t-6) = \frac{5}{3}\left(-4+2(t-6)+4e^{-\frac{t-6}{2}}\right) = -\frac{20}{3} + \frac{10}{3}t - 2\cdot\frac{30}{3} + \frac{20}{3}e^{-\frac{t}{2}}\cdot e^{6}$$

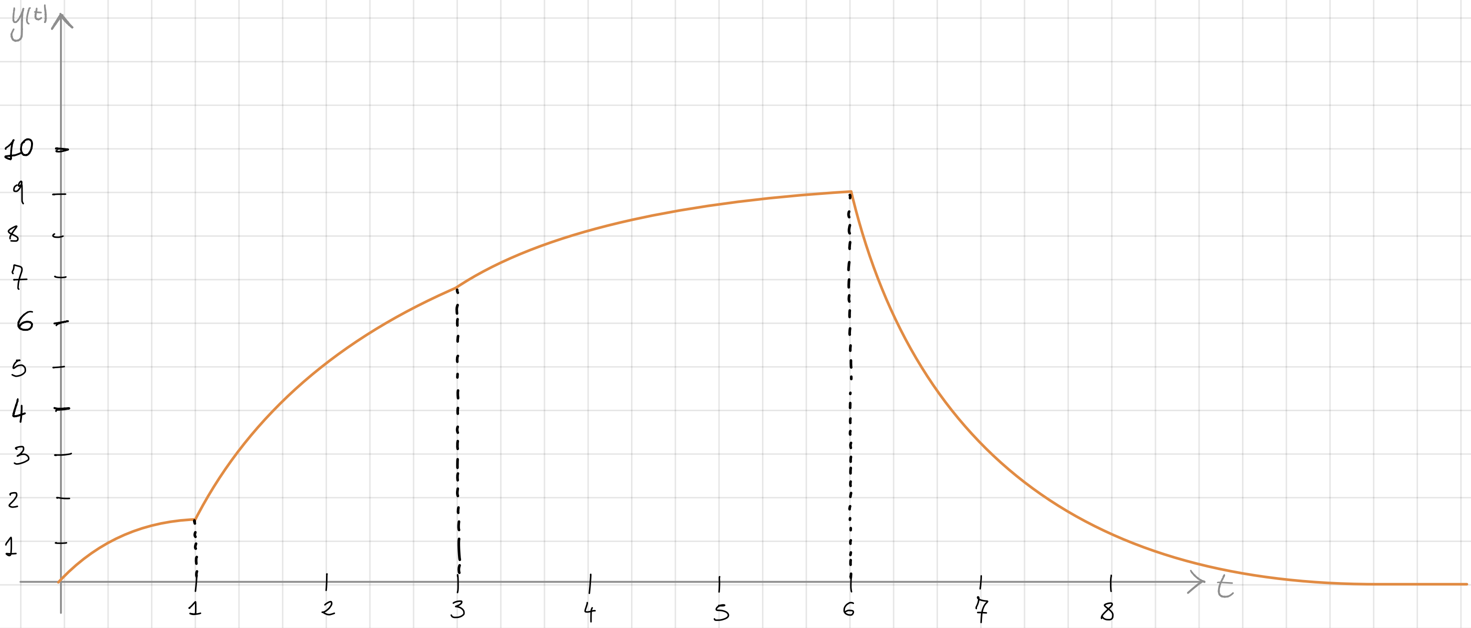
$$= -\frac{80}{3} + \frac{10}{3}t + \frac{20}{3}e^{-\frac{t-6}{2}} = \frac{10}{3}\left(-8+t+2e^{-\frac{t-6}{2}}\right)\cdot 1\!\!1(t)$$

6) Possiamo scrivere la risposta considerando diversi intervalli di tempo, a seconda dei ritardi

- T_1 : $[-\infty, 0] \rightsquigarrow y(t) = \varnothing$
- T_2 : $[0, 1] \rightsquigarrow y(t) = 4\left(1-e^{-\frac{t}{2}}\right)$
- T_3: $[1, 3] \rightsquigarrow y(t) = y_1(t) + y_2(t) = 4\left(1-e^{-\frac{t}{2}}\right) + 6\left(1-e^{-\frac{t-1}{2}}\right) = 4-4e^{-\frac{t}{2}}+6-6e^{-\frac{t}{2}}\cdot e^{+\frac{1}{2}}$
$$= 10 + e^{-\frac{t}{2}}\left(-4-6e^{\frac{1}{2}}\right) = 10 - e^{-\frac{t}{2}}\underbrace{\left(4+6\sqrt{e}\right)}_{\sim 14}$$
- T_4 : $[3, 6] \rightsquigarrow y(t) = y_1+y_2+y_3 = 10 - e^{-\frac{t}{2}}(4+6\sqrt{e}) + \frac{10}{3}\left(5-t-2e^{-\frac{t-3}{2}}\right)$
$$= \cancel{10} + e^{-\frac{t}{2}}\left(-4-6\sqrt{e}\right) + \overset{80/3}{\cancel{\frac{50}{3}}} - \frac{10}{3}t - \frac{20}{3}e^{-\frac{t}{2}}\cdot e^{\frac{3}{2}}$$
$$= \frac{80}{3} + e^{-\frac{t}{2}}\left(-4-6\sqrt{e}-\frac{20}{3}e\sqrt{e}\right) - \frac{10}{3}t \overset{\text{OPPURE}}{=} \frac{80}{3} - e^{-\frac{t}{2}}\left(4+6\sqrt{e}-\frac{20}{3}\sqrt{e^3}\right) - \frac{10}{3}t$$
- T_5 : $[6, +\infty] = y_1+y_2+y_3+y_4 = \cancel{\frac{80}{3}} - e^{-\frac{t}{2}}\left(4+6\sqrt{e}-\frac{20}{3}\sqrt{e^3}\right) - \cancel{\frac{10}{3}t} - \cancel{\frac{80}{3}} + \cancel{\frac{10}{3}t} + \frac{20}{3}e^{-\frac{t-6}{2}}$
$$\to y(t) = +e^{-\frac{t}{2}}\left(-4-6\sqrt{e}-\frac{20}{3}\sqrt{e^3}+\frac{20}{3}e^3\right) \cong 90e^{-\frac{t}{2}}$$

7) Considerazioni

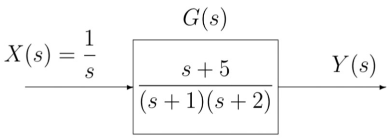
Per T_1, $y \to 4$
Per T_2, $y \to 10$



y(t)
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
t

SISTEMI DEL SECONDO ORDINE

Esercizio. Calcolare la risposta al gradino $y(t)$ del seguente sistema:



$$G(s) = \frac{s+5}{(s+1)(s+2)}$$

$$z_1 = -5 \; ; \quad p_1 = -1 \quad ; \quad p_2 = -2$$

$$u(t) = 1\!\!1(t) \rightleftarrows U(s) = 1/s$$

$$\Rightarrow Y(s) = \frac{s+5}{s(s+1)(s+2)} = \frac{r_1}{s} + \frac{r_2}{s+1} + \frac{r_3}{s+2}$$

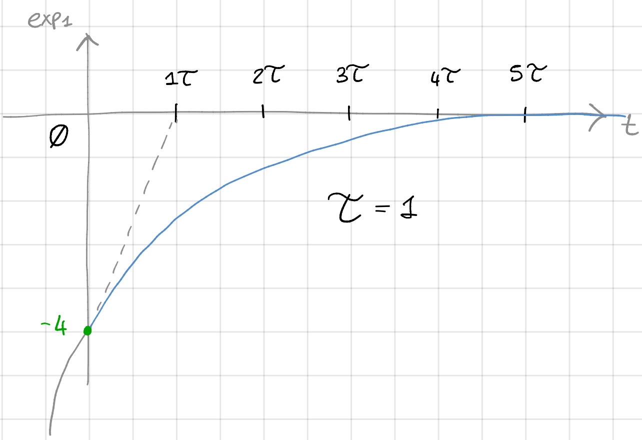
$$r_1 = \lim_{s \to 0} \cancel{s} \cdot \frac{s+5}{\cancel{s}(s+1)(s+2)} = \frac{5}{1+2} = \frac{5}{2} \quad r_1$$

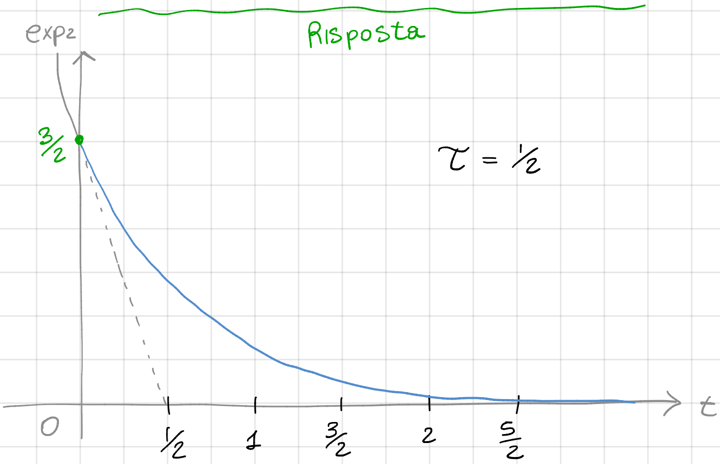
$$r_2 = \lim_{s \to -1} \cancel{(s+1)} \frac{s+5}{s\cancel{(s+1)}(s+2)} = \frac{+4}{-1} = -4 \quad r_2$$

$$r_3 = \lim_{s \to -2} \cancel{(s+2)} \frac{s+5}{s(s+1)\cancel{(s+2)}} = \frac{3}{2} \quad r_3$$

exp$_1$ exp$_2$

$$\Rightarrow Y(s) = \frac{5}{2}\,\frac{1}{s} - 4\,\frac{1}{s+1} + \frac{3}{2}\,\frac{1}{s+2} \rightleftarrows y(t) = \underbrace{\left(\frac{5}{2} - 4e^{-t} + \frac{3}{2}e^{-2t}\right) \cdot 1\!\!1(t)}_{\text{Risposta}}$$



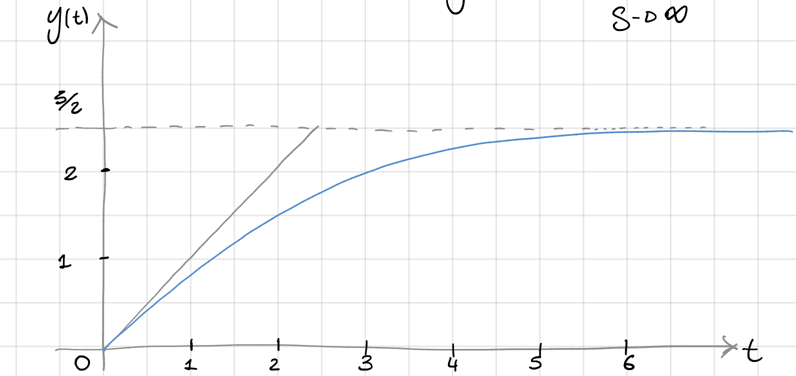


$$\text{TVI}: \; y(t=0) = \lim_{s \to \infty} \cancel{s} \left(\frac{5}{2\cancel{s}} - \frac{4}{s+1} + \frac{3}{2s+4} \right) \to \frac{5}{2} - 4 + \frac{3}{2} = \emptyset \quad \text{Valore iniz.}$$

$$s\left(1+\frac{1}{s}\right) \quad s\left(2+\frac{4}{s}\right)$$

$$\text{TVF}: \; y(t \to \infty) = \lim_{s \to 0} \cancel{s} \left(\frac{5}{2\cancel{s}} - \frac{4}{s+1} + \frac{3}{2s+4} \right) = \frac{5}{2} - \frac{\cancel{4s}}{\cancel{s+1}} + \frac{\cancel{3s}}{\cancel{2s+4}} = \frac{5}{2} \quad \text{Valore fin}$$

$$\text{Pendenza in } t=0 \leadsto \dot{y}(t=0) = \lim_{s \to \infty} s \cdot G(s) = s \cdot \frac{s+5}{(s+1)(s+2)} = \frac{\cancel{s} \cdot s\left(1+\frac{5}{s}\right)}{\cancel{s}\left(1+\frac{1}{s}\right) s\left(2+\frac{2}{s}\right)} \to 1 \quad \text{Pendenza}$$



POLI COMPLESSI E CONIUGATI

ESERCIZIO LEZIONE 24

$$G(s) = \frac{s+0.1}{(s^2+10s+100)(s+2)}$$

Con $u(t) = \begin{cases} 0 & t<0 \\ 4 & t\in(0,3) \\ 4e^{-\frac{t-3}{2}} & t \geq 3 \end{cases}$

$\hookrightarrow u_3'(t) = 4e^{-\frac{t}{2}} \cdot 1\!\!1(t)$

(1) INDIVIDUARE I SEGNALI ELEMENTARI

$$\begin{cases} U_1(t) = 4 \cdot 1\!\!1(t) & t\in[0,3] \\ U_2(t) = -4 \cdot 1\!\!1(t-3) & t\in[3,\infty) \\ U_3(t) = 4e^{-\frac{t-3}{2}} \cdot 1\!\!1(t-3) & t\in[3,\infty) \end{cases}$$

(2) TRASFORMATA Dei segnali $U_k(t)$

$$U_1(t) \rightleftharpoons U_1(s) = \frac{4}{s}$$

$$U_2(t) \rightleftharpoons U_2(s) = -\frac{4}{s} \cdot e^{-3s}$$

$$U_3: \quad U_3'(t) = 4e^{-\frac{t}{2}} \cdot 1\!\!1(t) \rightleftharpoons U_3'(s) = 4\frac{1}{s+\frac{1}{2}} \Rightarrow U_3'(s) = \mathcal{L}[U_3(t-3)]$$

Ritardo

$$\Rightarrow U_3(s) = \frac{4}{s+\frac{1}{2}} \cdot e^{-3s}$$

(3) Segnali fittizi

Scelgo $\hat{U}_1(t) = 1\!\!1(t) \rightleftharpoons \hat{U}_1(s) = \frac{1}{s}$

$\hat{U}_2(t) = e^{-\frac{t}{2}} \rightleftharpoons \hat{U}_2(s) = \frac{1}{s+\frac{1}{2}}$

(3.a) Uscita a $\hat{U}_1(s)$

$$\hat{Y}_1(s) = \frac{s+0.1}{s(s^2+10s+100)(s+2)}$$

! Controllo che i poli non siano ulteriormente scomponibili...

$s^2+10s+100 \to P_{1,2} = -5 \pm 8.7j \Rightarrow$ Non scomponibili...

$$\Rightarrow \hat{Y}_1(s) = \frac{z_1}{s} + \frac{z_2}{s+2} + \frac{z_3+z_4 s}{s^2+10s+100} = \frac{(s^2+10s+100)(s+2)z_1 + s(s+2)z_2 + s(z_3+z_4 s)(s+2)}{s(s^2+10s+100)(s+2)}$$

* A questo punto calcolo i residui e considero solo il num

$$\tau_1 = \lim_{s \to 0} \cancel{s} \cdot \frac{\overset{0}{\cancel{s}} + 0.1}{\underset{0}{\cancel{s}}(\underset{0}{\cancel{s^2}}+\underset{0}{\cancel{10s}}+100)(\underset{0}{\cancel{s}}+2)} \to \frac{0.1}{200} = 5 \times 10^{-4} \quad \tau_1$$

0.0005

$$\tau_2 = \lim_{s \to -2} \cancel{(s+2)} \frac{\overset{-2}{s} + 0.1}{\underset{-2}{s}(\underset{4}{s^2}+\underset{-20}{10s}+100)\cancel{(s+2)}} \to \frac{-1.9}{-168} = 113 \times 10^{-4} \quad \tau_2$$

$$(s^2+10s+100)(s+2)\tau_1 + s\,\tau_2(s^2+10s+100) + s(\tau_3+\tau_4 s)(s+2) = s^3\tau_1 + 2s^2\tau_1 + 10s^2\tau_1 + 20s\tau_1 + 100s\tau_1 + 200\tau_1$$
$$+ s^3\tau_2 + 10s^2\tau_2 + 100s\tau_2 +$$
$$+ s^2\tau_3 + 2s\tau_3 + s^3\tau_4 + 2s^2\tau_4$$

$$\begin{cases} s^3(\tau_1+\tau_2+\tau_4) = 0 \\ s^2(2\tau_1+10\tau_1+10\tau_2+\tau_3+2\tau_4) = 0 \\ s(20\tau_1+100\tau_1+100\tau_2+2\tau_3) = 1 \\ 200\tau_1 = 0.1 \quad \text{QED} \end{cases}$$

$\tau_3 = -954 \times 10^{-4}$ τ_3

$\tau_4 = -118 \times 10^{-4}$ τ_4

(a)

$$\hat{Y}_1(s) = \frac{\tau_1}{s} + \frac{\tau_2}{s+2} + \frac{\tau_3+\tau_4 s}{s^2+10s+100}$$

termine doppio prodotto — Termine

! Non siamo ancora pronti!

→ Dobbiamo esprimere (a) in modo da ricavare sin e cos

Forma Standard: $\frac{\omega_d}{(s+\zeta\omega_n)^2+(\omega_d)^2}$ → quindi dobbiamo riscrivere nom e det

- QUADRATO DI BINOMIO − $(s+\zeta\omega_n)^2$

→ Per trovare $\zeta\omega_n$ basta guardare il termine doppio prodotto di (a)

$$(s+\zeta\omega_n)^2 = s^2 + 2s\zeta\omega_n + (\zeta\omega_n)^2 = s^2+10s+100$$

$$\Rightarrow 2\zeta\omega_n = 10 \Rightarrow \zeta\omega_n = \frac{10}{5} \Rightarrow \zeta\omega_n = 5 \quad (1)$$

- TERMINE AL QUADRATO $(\omega_d)^2$

Con questo termine dobbiamo "Apparare" quello che resta dal quadrato di binomio:

$$(s+5)^2 = \cancel{s^2} + \cancel{10s} + 25 + (\omega_d)^2 = \cancel{s^2} + \cancel{10s} + 100$$

$$\Rightarrow \omega_d^2 = 100-25 \Rightarrow \omega_d^2 = 75 \Rightarrow \omega_d = 5\sqrt{3} \quad (2)$$

$$\Rightarrow \frac{z_3+z_4 s}{s^2+10s+100} = \frac{z_3+z_4 s}{(s+5)^2+(5\sqrt{3})^2} = z_4\,\frac{\frac{z_3}{z_4}+s+5-5}{(s+5)^2\;(5\sqrt{3})^2} = z_4\,\frac{s+5}{(s+5)^2(5\sqrt{3})^2} + z_4\,\frac{\left(\frac{z_3}{z_4}-5\right)\frac{5\sqrt{3}}{5\sqrt{3}}}{(s+5)^2(5\sqrt{3})^2}$$

OK!

Metto in evidenza per lasciare s da solo

!

$$= z_4\,\frac{s+5}{(s+5)^2(5\sqrt{3})^2} + \frac{z_3-5z_4}{z_4\cdot 5\sqrt{3}}\cdot z_4\,\frac{5\sqrt{3}}{(s+5)^2\,(5\sqrt{3})^2}$$

OK!

⤳ Sostituisco i residui

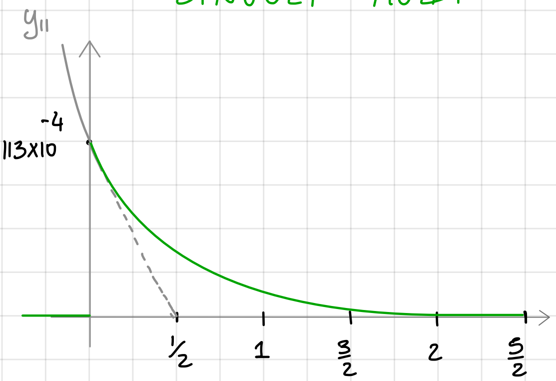
$$\hat{Y}_1(s) = \frac{z_1}{s} + \frac{z_2}{s+2} + z_4\,\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} + \frac{z_3-5z_4}{z_4\cdot 5\sqrt{3}}\;\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}$$

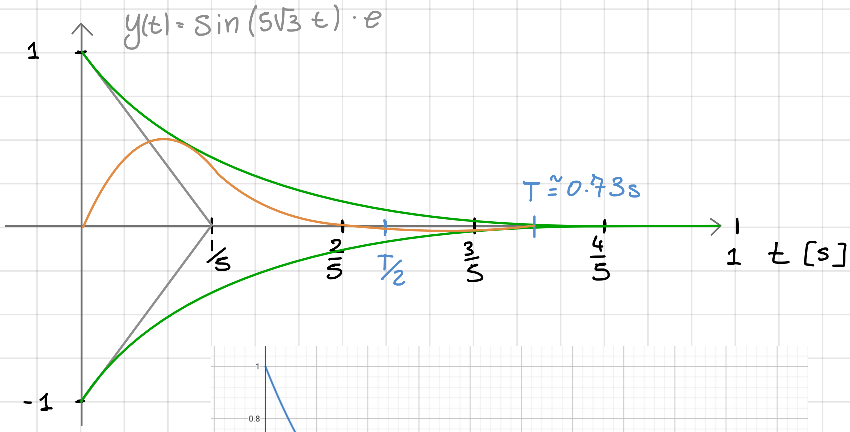
$$= \frac{5\times10^{-4}}{s} + \frac{113\times10^{-4}}{s+2} - 0.01\cdot\frac{s}{(s+5)^2+(5\sqrt{3})^2} - 0.36\cdot\frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}$$

(4.a) ANTITRASFORMO

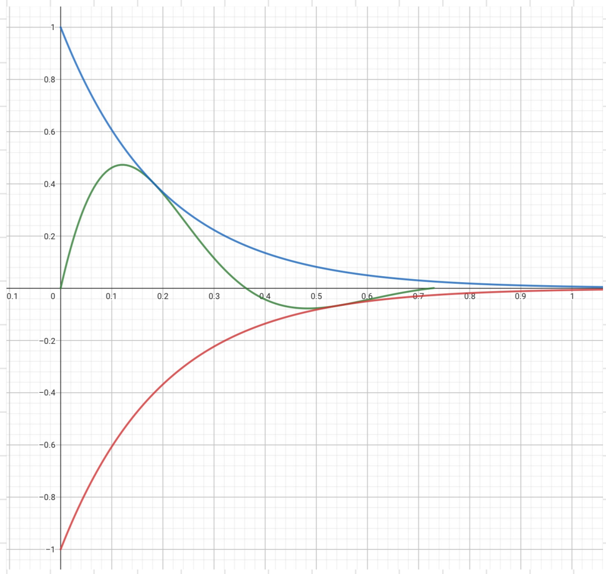
$$\hat{y}_1(t) = \Big[5\times10^{-4} + \underbrace{113\times10^{-4}\cdot e^{-2t}}_{y_{11}} \underbrace{-118\times10^{-4}\cdot\cos(5\sqrt{3}\cdot t)\cdot e^{-5t}}_{y_{12}} \underbrace{-0.36\cdot\sin(5\sqrt{3}\cdot t)\cdot e^{-5t}}_{y_{13}}\Big]\cdot 1\!\!1(t)$$

⤳ SINGOLI MODI





$\sin(5\sqrt{3}\,t) \rightarrow \quad T = \frac{2\pi}{5\sqrt{3}} \simeq 0.23\pi \simeq 0.73$



• Valore iniziale

$$TVI: \lim_{s\to\infty} s\,\hat{Y}_1(s) \rightarrow \emptyset$$

(3.b) Uscita a $\hat{U}_2(s)$

$$\hat{U}_2(s) = \frac{1}{s+\frac{1}{2}}$$

$$\Rightarrow \quad \hat{y}_2(s) = \frac{1}{(s+\frac{1}{2})} \cdot \frac{s+0.1}{(s^2+10s+100)(s+2)} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{s^2+10s+100}$$

Uso Direttamente il sistema

$$\mathrm{Num}\left[\hat{y}_2(s)\right] = r_1(s^2+10s+100)(s+2) + r_2(s+\tfrac{1}{2})(s^2+10s+100) + (r_3+r_4 s)(s+2)(s+\tfrac{1}{2})$$

$$\Rightarrow \begin{cases} s^3(r_1+r_2) = 0 \\ s^2\left(2r_1+10r_1+10r_2+\frac{1}{2}r_2+r_3+r_3+r_4+2r_4\right) = 0 \\ s\left(20r_1+100r_1+100r_2+5r_2+2r_3+\frac{1}{2}r_3+\frac{1}{2}r_3+r_4\right) = 1 \\ 200r_1+50r_2+r_3+r_3) = 0.1 \end{cases}$$

$$\Rightarrow \begin{cases} r_1 = -5.5\times 10^{-3} \\ r_2 = 5.5\times 10^{-3} \\ r_3 = 462.8\times 10^{-3} \\ r_4 = -305.8\times 10^{-3} \end{cases}$$

$$\Rightarrow \hat{y}_2(s) = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{s^2+10s+100} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + \frac{r_3+r_4 s}{(s+5)^2+(5\sqrt{3})^2}$$

$$= \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{\frac{r_3}{r_4}+s+5-5}{(s+5)^2+(5\sqrt{3})^2} = \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} + r_4\frac{\left(\frac{r_3}{r_4}-5\right)\cdot\frac{5\sqrt{3}}{5\sqrt{3}}}{(s+5)^2+(5\sqrt{3})^2}$$

OK!

Non lo Dimenticare!

$$= \frac{r_1}{s+\frac{1}{2}} + \frac{r_2}{s+2} + r_4\frac{s+5}{(s+5)^2+(5\sqrt{3})^2} + \frac{r_3-5r_4}{r_4 5\sqrt{3}}\cdot r_4 \frac{5\sqrt{3}}{(s+5)^2+(5\sqrt{3})^2}$$

OK!

(4.b) ANTITRASFORMO

$$\Rightarrow \hat{y}_2(t) = \left[r_1 e^{-\frac{t}{2}} + r_2 e^{-2t} + r_4 \cos(5\sqrt{3}\,t)\,e^{-5t} + r_4\left(\frac{r_3-5r_4}{r_4\cdot 5\sqrt{3}}\right)\sin(5\sqrt{3}\,t)\cdot e^{-5t}\right]\cdot \mathbb{1}(t)$$

(5) Segnali reali

Ricordiamo che
$$\begin{cases} u_1(t) = 4 \cdot \mathbb{1}(t) \\ u_2(t) = -4 \cdot \mathbb{1}(t-3) \\ u_3(t) = 4 \exp[(-t-3)/2] \end{cases}$$

Abbiamo scelto i segnali fittizi:
$$\begin{cases} \hat{u}_1(t) = \mathbb{1}(t) \\ \hat{u}_2(t) = e^{-\frac{t}{2}} \end{cases}$$

$$\Rightarrow \begin{cases} y_1(t) = 4\, \hat{y}_1(t) \cdot \mathbb{1}(t) \\ y_2(t) = -4\, \hat{y}_1(t-3) \cdot \mathbb{1}(t-3) \\ y_3(t) = 4\, \hat{y}_2(t-3) \cdot \mathbb{1}(t-3) \end{cases} \Rightarrow y(t) = y_1(t) + y_2(t) + y_3(t)$$

$\Rightarrow$ Molto complx da disegnare