

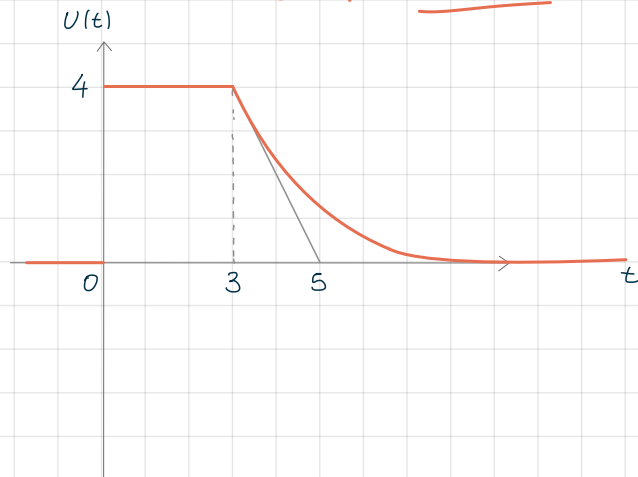
Data la FDT:

$$G(s) = \frac{s + 0.1}{(s^2 + 10s + 100)(s + 2)}$$

$$U(t) = \begin{cases} 0 & t < 0 \\ 4 & t \in [0, 3) \\ 4e^{-\frac{t-3}{2}} & t \geq 3 \end{cases}$$

$$e^{\frac{3}{2}} \cdot 4 \cdot e^{-\frac{t}{2}} \leadsto \tau = 2$$

TROVARE L'USCITA nel TEMPO ad $U(t)$



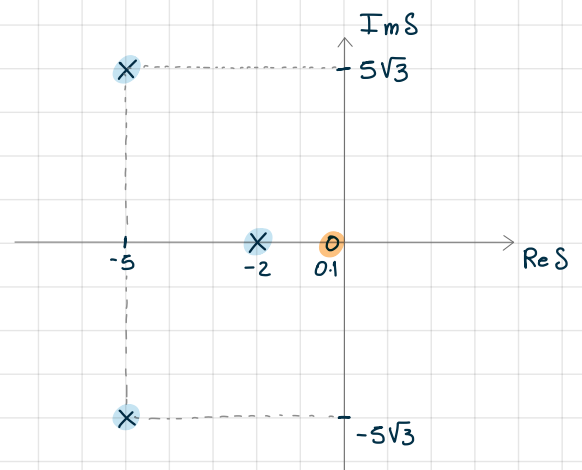
• Poli, zeri:

$$s^2 + 10s + 100 \leadsto \Delta = 100 - 4 \cdot 1 \cdot 100 = -300 < 0 \Rightarrow \text{Poli complx e conj}$$

$$\leadsto P_{1,2} = \frac{-10 \pm j\sqrt{300}}{2} \rightarrow \begin{cases} -5 + j5\sqrt{3} & P_1 \\ -5 - j5\sqrt{3} & P_2 \end{cases}$$

$$s + 2 \leadsto P_3 = -2 \quad P_3$$

$$s + 0.1 \leadsto Z_1 = -0.1 \quad Z_1$$



1) Segnali elementari

2) Trasformo

$$\begin{cases} U_1(t) = 4 \cdot \mathbb{1}(t) & \Rightarrow U_1(s) = 4 \cdot \frac{1}{s} \\ U_2(t) = -4 \cdot \mathbb{1}(t-3) & \Rightarrow U_2(s) = -4 \cdot \frac{1}{s} \cdot e^{-3s} \end{cases}$$

$$\begin{cases} U_3(t) = 4e^{-\frac{t-3}{2}} \cdot \mathbb{1}(t-3) = \mathcal{L}\left[4e^{-\frac{t}{2}}\right] \cdot e^{-3s} & \Rightarrow U_3(s) = 4 \cdot \frac{1}{s + \frac{1}{2}} e^{-3s} \end{cases}$$

↑ Tolgo il ritardo da qui
↑ e lo Trasformo

3) Segnali fittizi

$$\text{Scego } \hat{U}_1(t) = \mathbb{1}(t) \quad \text{e} \quad \hat{U}_2(t) = e^{-\frac{t}{2}} \leadsto \hat{U}_1(s) = \frac{1}{s} \quad \hat{U}_2(s) = \frac{1}{s + \frac{1}{2}}$$

3.1) Uscita \hat{Y}_1

$$\hat{Y}_1(s) = \hat{U}_1(s) \cdot G(s) = \frac{s + 0.1}{s(s^2 + 10s + 100)(s + 2)} = \frac{Z_1}{s} + \frac{Z_2}{s + 2} + \frac{Z_3 + Z_4 s}{(s^2 + 10s + 100)}$$

$$z_1 = \lim_{s \rightarrow 0} s \cdot \hat{y}_1(s) = \lim_{s \rightarrow 0} s \cdot \frac{s + 0.1}{s(s^2 + 10s + 100)(s+2)} \rightarrow \frac{0.1}{100 \cdot 2} = 5 \times 10^{-4} \quad R_1$$

$$z_2 = \lim_{s \rightarrow -2} (s+2) \frac{s + 0.1}{s(s^2 + 10s + 100)(s+2)} \rightarrow \frac{-2 + 0.1}{-2(4 \cdot 20 + 100)} = 113 \times 10^{-4} \quad R_2$$

Determino z_3 ed z_4 con un sistema

Considero solo il Num

$$\frac{(s^2 + 10s + 100)(s+2)z_1 + z_2 s(s^2 + 10s + 100) + (z_3 + z_4 s)(s^2 + 2s)}{s(s^2 + 10s + 100)(s+2)}$$

$$\begin{cases} s^3(z_1 + z_2 + z_4) = 0 \rightarrow z_4 = -(z_1 + z_2) = -118 \times 10^{-4} \quad z_4 \\ s^2(2z_1 + 10z_2 + 10z_3 + 2z_4) = 0 \\ s(20z_1 + 100z_2 + 100z_3 + 2z_4) = 1 \rightarrow 120z_1 + 100z_2 = 2z_3 \\ 200z_1 = 0.1 \Rightarrow z_1 = \frac{1}{100 \cdot 200} = 5 \times 10^{-4} \quad \text{QED} \end{cases}$$

$$\rightarrow z_3 = -(120z_1 + 100z_2) \cdot \frac{1}{2} = -5950 \times 10^{-4} \quad z_3$$

• Rimane gestire $\hat{y}(s)$ per l'anti trasformata

$$\hat{y}(s) = \frac{z_1}{s} + \frac{z_2}{s+2} + \frac{z_3 + z_4 s}{(s^2 + 10s + 100)}$$

Mi occupo di questo termine

$$\frac{\omega_d}{(s + j\omega_n)^2 + \omega_d^2}$$

Forma Std
Per Termini comp. conj

• Termine Quadratico

$$(s + j\omega_n)^2 = s^2 + 2j\omega_n s + (j\omega_n)^2 = s^2 + 10s + 100$$

$$\rightarrow \begin{cases} 2j\omega_n = 10 \Rightarrow j\omega_n = 5 \\ (j\omega_n)^2 = 100 \end{cases} \rightarrow (s + j\omega_n)^2 = (s + 5)^2$$

• Termine NoTo

$$(s + 5)^2 = (s^2 + 10s + 25) \stackrel{?}{=} s^2 + 10s + 100$$

Manca $75 = (\omega_n)^2 \Rightarrow \omega_n = 5\sqrt{3}$

\rightarrow Riscrivo

$$\frac{z_3 + z_4 s}{(s^2 + 10s + 100)} = \frac{z_3 + z_4 s}{(s + 5)^2 + (5\sqrt{3})^2}$$

\leftarrow non e' ancora pronto !

Devo ottenere una somma di sin e cos

$$\sin(\omega t) \Rightarrow \frac{\omega}{s^2 + \omega^2}, \quad \cos(\omega t) \Rightarrow \frac{s}{s^2 + \omega^2}$$

• Metto in evidenza τ_4 per lasciare da solo s

$$\Rightarrow \tau_4 \frac{\frac{\tau_3}{\tau_4} + s + 5 - 5}{(s+5)^2 + (5\sqrt{3})^2} = \tau_4 \cdot \frac{\frac{\tau_3}{\tau_4} - 5}{(s+5)^2 + (5\sqrt{3})^2} + \tau_4 \cdot \frac{s+5}{(s+5)^2 + (5\sqrt{3})^2} \quad \text{cos}(\omega t) \cdot e^{-5t}$$

\swarrow manca $5\sqrt{3}$

mi serve
il ritardo!

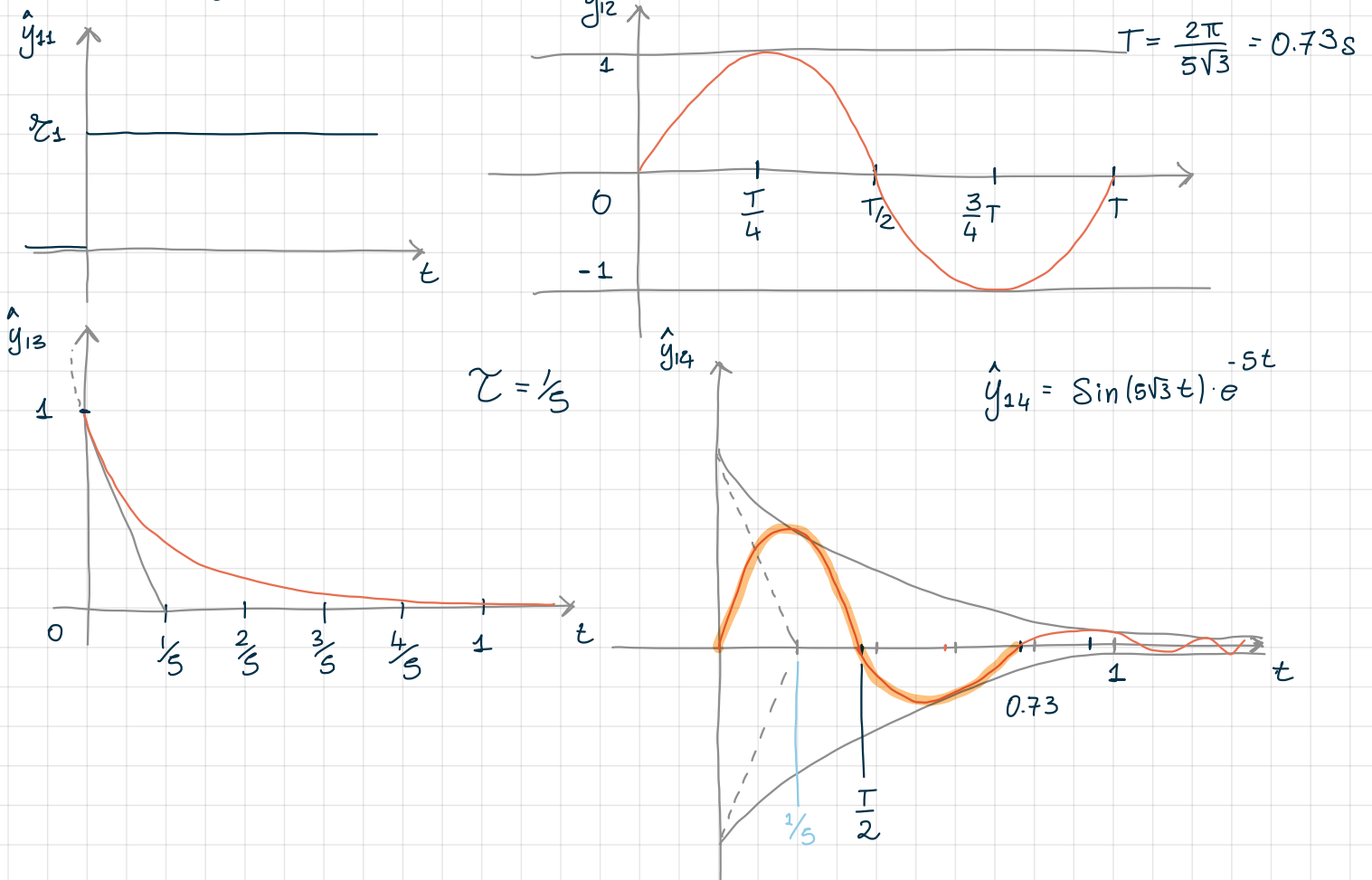
$$\Rightarrow \frac{\tau_3 + \tau_4 s}{(s^2 + 10s + 100)} = \frac{\tau_4 \left(\frac{\tau_3}{\tau_4} - 5 \right)}{5\sqrt{3}} \cdot \frac{5\sqrt{3}}{(s+5)^2 + (5\sqrt{3})^2} + \tau_4 \cdot \frac{s+5}{(s+5)^2 + (5\sqrt{3})^2}$$

$\text{OK} \nabla$ $\text{OK} \nabla$

$$\hat{y}(s) = \frac{\tau_1}{s} + \frac{\tau_2}{s+2} + \frac{\tau_4 \left(\frac{\tau_3}{\tau_4} - 5 \right)}{5\sqrt{3}} \cdot \frac{5\sqrt{3}}{(s+5)^2 + (5\sqrt{3})^2} + \tau_4 \cdot \frac{s+5}{(s+5)^2 + (5\sqrt{3})^2}$$

$\text{OK} \nabla$ OK

$$\Rightarrow \hat{y}(s) \Rightarrow \hat{y}(t) = \left[\tau_1 + \tau_2 t - 0.62 \cdot \hat{y}_{12} \cdot \sin(5\sqrt{3} \cdot t) \cdot e^{-5t} - 0.012 \cdot \cos(5\sqrt{3} \cdot t) \cdot e^{-5t} \right] \cdot 1(t)$$



3.2) Uscita \hat{y}_2 $\hat{U}_2(s) = \frac{1}{s + \frac{1}{2}}$

$$=0 \quad \hat{y}_3(s) = \frac{s + 0.1}{(s + \frac{1}{2})(s^2 + 10s + 100)(s + 2)} = \frac{r_1}{s + \frac{1}{2}} + \frac{r_2}{s + 2} + \frac{r_3 + r_4 s}{s^2 + 10s + 100}$$

Calcolo direttamente col sistema

$$(r_1 s + 2 r_1)(s^2 + 10s + 100) + (r_2 s + \frac{1}{2} r_1)(s^2 + 10s + 100) + (r_3 + r_4 s)(s + \frac{1}{2})(s + 2)$$

$$\begin{cases} s^3 (r_1 + r_2 + r_4) = 0 \\ s^2 (2 r_1 + 10 r_2 + 100 r_1 + 10 r_2 + \frac{1}{2} r_1 + r_3 + 2 r_4 \frac{1}{2} r_4) = 0 \\ s (20 r_1 + 100 r_2 + 5 r_1 + 2 r_3 + \frac{1}{2} r_3 + r_4) = 1 \\ 200 r_1 + 50 r_2 + r_3 = 0.1 \end{cases}$$

$$\begin{cases} r_1 + r_2 + 0 + r_4 = 0 \\ 112.5 r_1 + 10 r_2 + 1 r_3 + 2.5 r_4 = 0 \\ 25 r_1 + 100 r_2 + 2.5 r_3 + 1 r_4 = 1 \\ 250 r_1 + r_3 = 0.1 \end{cases}$$

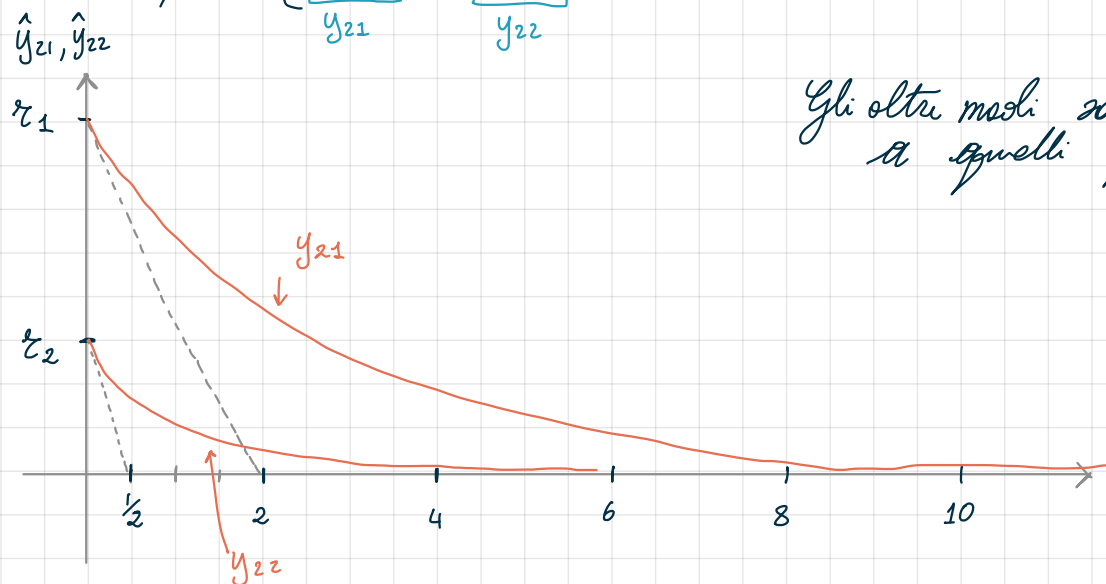
Residui

$$\begin{aligned} r_1 &= 16 \times 10^{-4} \\ r_2 &= 17.6 \times 10^{-4} \\ r_3 &= -0.32 \\ r_4 &= -0.19 \end{aligned}$$

3° membro: $\frac{r_3 + r_4 s}{s^2 + 10s + 100} = \frac{r_3 + r_4 s}{(s + 5)(5\sqrt{3})^2} = r_4 \frac{\frac{r_3}{r_4} + s + 5 - 5}{(s + 5)^2 (5\sqrt{3})^2}$

$$= \frac{r_4 \left(\frac{r_3}{r_4} - 5 \right)}{5\sqrt{3}} \cdot \frac{s\sqrt{3}}{(s + 5)^2 (5\sqrt{3})^2} + r_4 \cdot \frac{s + 5}{(s + 5)^2 (5\sqrt{3})^2}$$

$$\hat{y}_2(s) = \left[\underbrace{r_1 \cdot e^{-\frac{t}{2}}}_{y_{21}} + \underbrace{r_2 e^{-2t}}_{y_{22}} + 0.36 \cdot \sin(5\sqrt{3}t) \cdot e^{-5t} - 0.7 \cos(5\sqrt{3}t) e^{-5t} \right] \cdot 4(t)$$



Gli altri modi sono molto simili a quelli per \hat{y}_1

• Segnali Reali

Ricordiamo i segnali reali e fittizi:

$$\begin{cases} U_1(t) = 4 \cdot \underline{1}(t) \\ U_2(t) = -4 \cdot \underline{1}(t-3) \end{cases}$$

$$\begin{cases} \hat{U}_1(t) = \underline{1}(t) \\ \hat{U}_2(t) = e^{-\frac{t}{2}} \end{cases}$$

$$\begin{cases} U_3(t) = 4 e^{-\frac{t-3}{2}} \cdot \underline{1}(t-3) \end{cases}$$

$$\Rightarrow \begin{cases} y_1(t) = 4 \cdot y_1(t) \cdot \underline{1}(t) \\ y_2(t) = -4 \cdot y_1(t-3) \cdot \underline{1}(t) \\ y_3(t) = 4 \cdot y_2(t-3) \cdot \underline{1}(t) \end{cases}$$

Stando bene attenti a sostituire $(t-3)$

RISPOSTA IN FREQUENZA

$$G(s) = \frac{s + 0.1}{(s^2 + 10s + 100)(s + 2)}$$

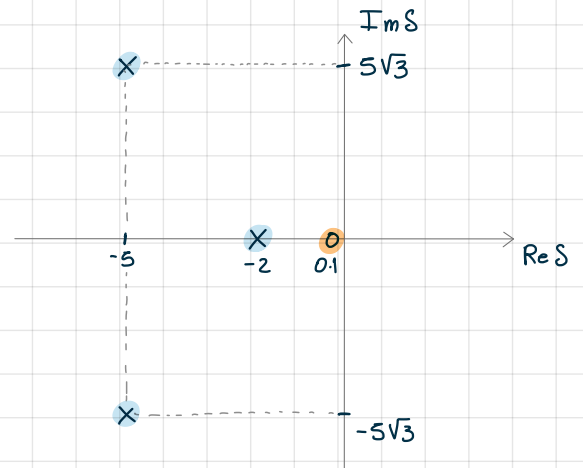
• Poli, zeri:

$$s^2 + 10s + 100 \leadsto \Delta = 100 - 4 \cdot 1 \cdot 100 = -300 < 0 \Rightarrow \text{Poli complessi e conj}$$

$$\leadsto P_{1,2} = \frac{-10 \pm j\sqrt{300}}{2} \begin{cases} \nearrow \frac{-5 + j5\sqrt{3}}{1} P_1 \\ \searrow \frac{-5 - j5\sqrt{3}}{1} P_2 \end{cases}$$

$$s + 2 \leadsto P_3 = -2 \quad P_3$$

$$s + 0.1 \leadsto \underline{z_1 = -0.1} \quad z_1$$



• Frequenze di risonanza

$\omega_1 = 0.1$ rad/s, $\omega_2 = 2$ rad/s per ω_3 dobbiamo trovare ω_n !

$$\text{forma std: } \frac{\omega_d}{(s + \omega_n)^2 + (\omega_d)^2} \leadsto s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 10s + 100$$

$$\Rightarrow \begin{cases} 2\zeta\omega_n = 10 \\ \omega_n^2 = 100 \end{cases} \Rightarrow \begin{cases} 2\zeta \cdot 10 = 10 \\ \omega_n = \sqrt{100} = 10 \end{cases} \Rightarrow \zeta = \frac{1}{2} = 0.5 < 0.707$$

↳ poca risonanza!

$$\Rightarrow \omega_3 = 10 \text{ rad/s}$$

• Forma Standard

$$G(s) = \frac{0.1(1 + \frac{s}{0.1})}{100(\frac{s^2}{100} + \frac{s}{10} + 1) \cdot 2(1 + \frac{s}{2})} = 5 \times 10^{-4} \cdot \frac{1 + 10s}{(\frac{s^2}{100} + \frac{s}{10} + 1)(1 + \frac{s}{2})}$$

• Valori iniziali e finali

MODULI INIZIALE No Zeri/Poli in 0 $\Rightarrow |G(j\omega)|_{\omega \ll 1} = K_P = 5 \times 10^{-4}$

in dB: $|G(j\omega)|_{dB} = 20 \log(5 \times 10^{-4}) = -66 \text{ dB}$

FINALE 1 Zero, 1 Polo semplice ed 1 polo "complex"
 +20 dB/dec -20 dB/dec -40 dB/dec
 dopo $\omega_1 = 0.1$ dopo $\omega_2 = 2$ dopo $\omega_3 = \omega_n = 10$

\Rightarrow Tot mod finale : $(+20 - 20 - 40) \text{ dB/dec} = -40 \text{ dB/dec}$

FASI

INIZIALE

NO Zeri / Poli in origine $\Rightarrow \angle G(j\omega) = 0^\circ$

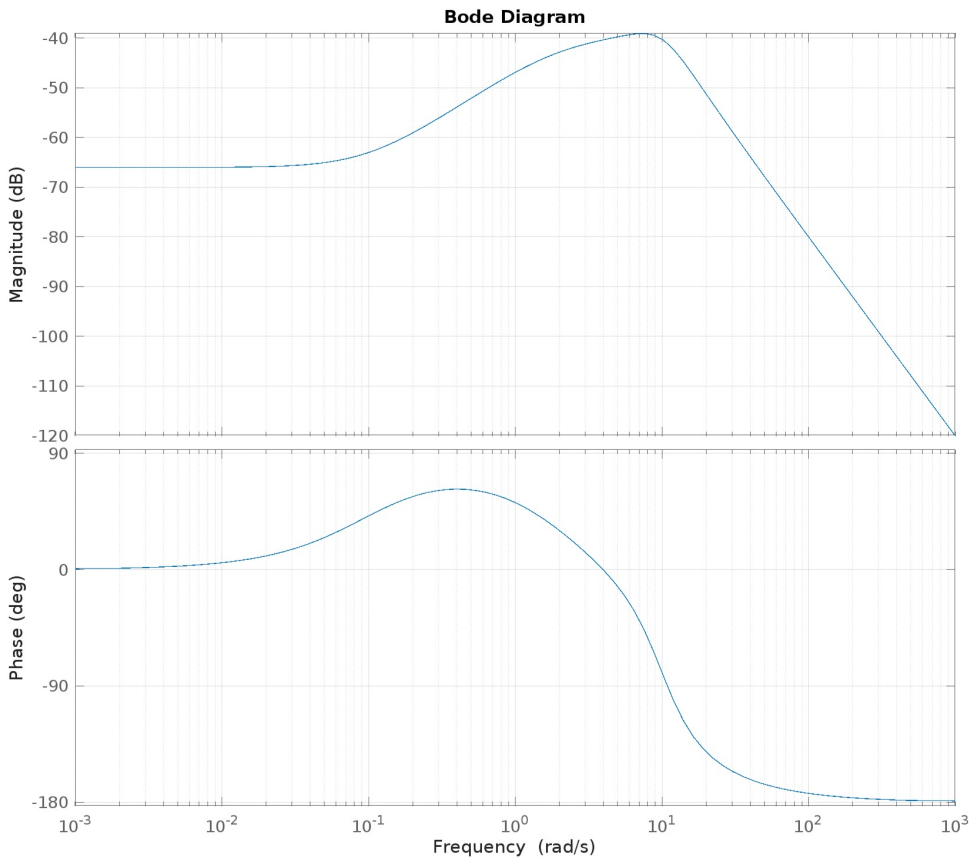
FINALE

1 Zero $\text{Re} p < 0 \rightarrow +90^\circ$	Tra $0.1 \cdot 10^{-1}$ e $0.1 \cdot 10^1$
1 Polo semplice $\rightarrow -90^\circ$	Tra $2 \cdot 10^{-1}$ e $2 \cdot 10^1$
1 Polo Complesso $\rightarrow -180^\circ$	Tra $40 \cdot 10^{-1}$ e $10 \cdot 10^1$

\Rightarrow Totale : $-180^\circ = -\pi$

- Scelta della Banda

$\omega \in [10^2; 10^3]$



Matricola: _____

Nome: _____

