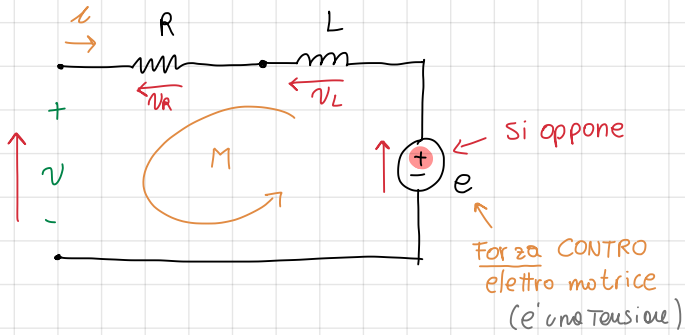
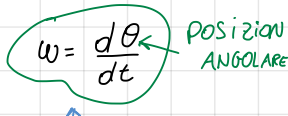
SISTEMI ELETTROMECCANICI

MOTORE ELETTRICO (a corrente continua)



→ Rappresenta l'opposizione del motore

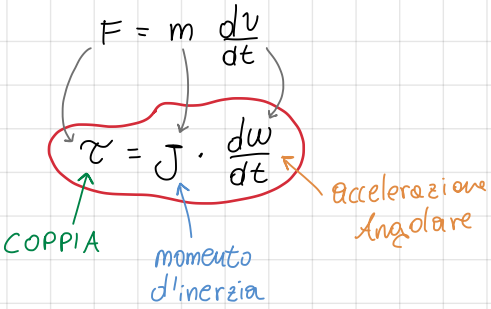


[V]

F. contro elettromotrice

velocità angolare [rad/s]

$e = K_v \omega$



Fenomeni dall'esterno che si OPPONGONO alla rotazione

$\tau_i = K_t \cdot i$ — Coppia del motore — Corrente che circola

$\tau_\omega = \ominus b \cdot \omega$ — Coppia che si OPPONE

τ_l — COPPIA DI CARICO

Può anche FAVORIRE IL MOTO → in questo caso si oppone

→ COPPIA TOTALE $\tau = \tau_i + \tau_\omega - \tau_e \rightarrow J \frac{d\omega}{dt} = \tau_i + \tau_\omega - \tau_e$

τ_e agisce come un DISTURBO → NON posso CONTROLLARLO

$\Rightarrow J \frac{d\omega}{dt} = K_t i - b\omega - \tau_e$

$LKT_M: \quad v - v_R - v_L - e = 0 \rightarrow v - K_v \omega - L \dot{i}_L - R i = 0 \rightarrow L \dot{i}_L = v - Ri - K_v \omega$

$$\begin{cases} J \frac{d\omega}{dt} = K_t i - b\omega - \tau_e & (2) \\ L \frac{di}{dt} = -Ri - K_v \omega + v & (1) \end{cases}$$

SPAZIO DI STATO

POSSO MISURARLI DIRETTAMENTE

$x_1 = i \quad x_2 = \omega \quad u_1 = v \quad u_2 = \tau_e$ (ingresso NON controllabile)

obbiettivo: $\dot{x} = \underline{A} x + \underline{B} u$

$$\begin{cases} \dot{x}_1 = -\frac{R}{L} x_1 - \frac{K_v}{L} x_2 + \frac{1}{L} u_1 \\ \dot{x}_2 = \frac{K_t}{J} x_1 - \frac{b}{J} x_2 - \frac{1}{J} u_2 \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{K_v}{L} \\ \frac{K_t}{J} & -\frac{b}{J} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} +\frac{1}{L} & 0 \\ 0 & -\frac{1}{J} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

2×1 — $\underline{A}$ 2×2 — 2×1

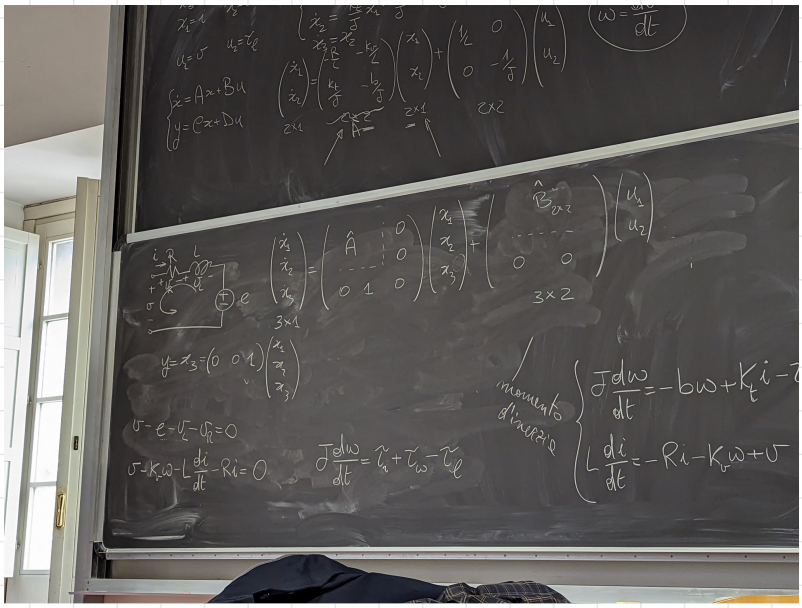
$$y = w = x_2 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \rightsquigarrow \quad y = \underline{\underline{C}} x + \underline{\underline{D}} y$$

Se voglio considerare anche la **posizione angolare** devo aggiungere una variabile di stato:

$$\dot{x}_3 = x_2 \qquad \text{Poiché } \left(x_3 = w = \frac{d\theta}{dt} \right)$$

↑ Non derivata!

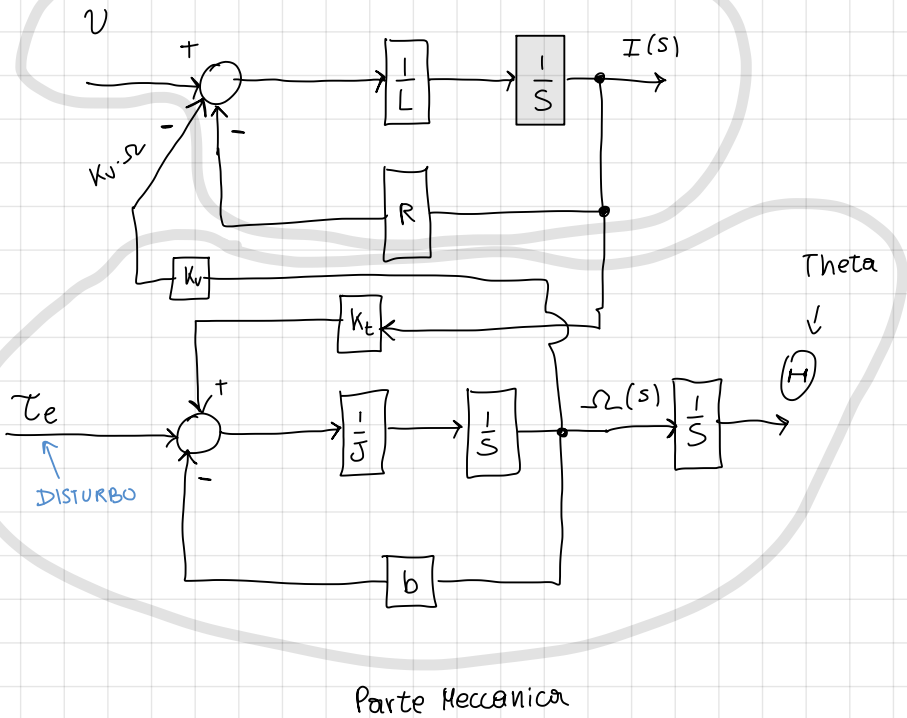
FOTO



Parte elettrica

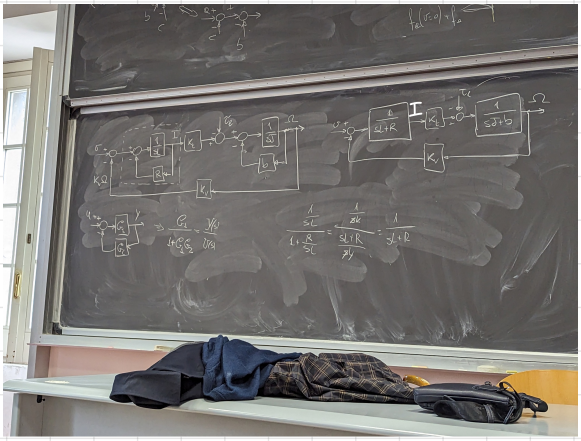
SCHEMA A BLOCCHI

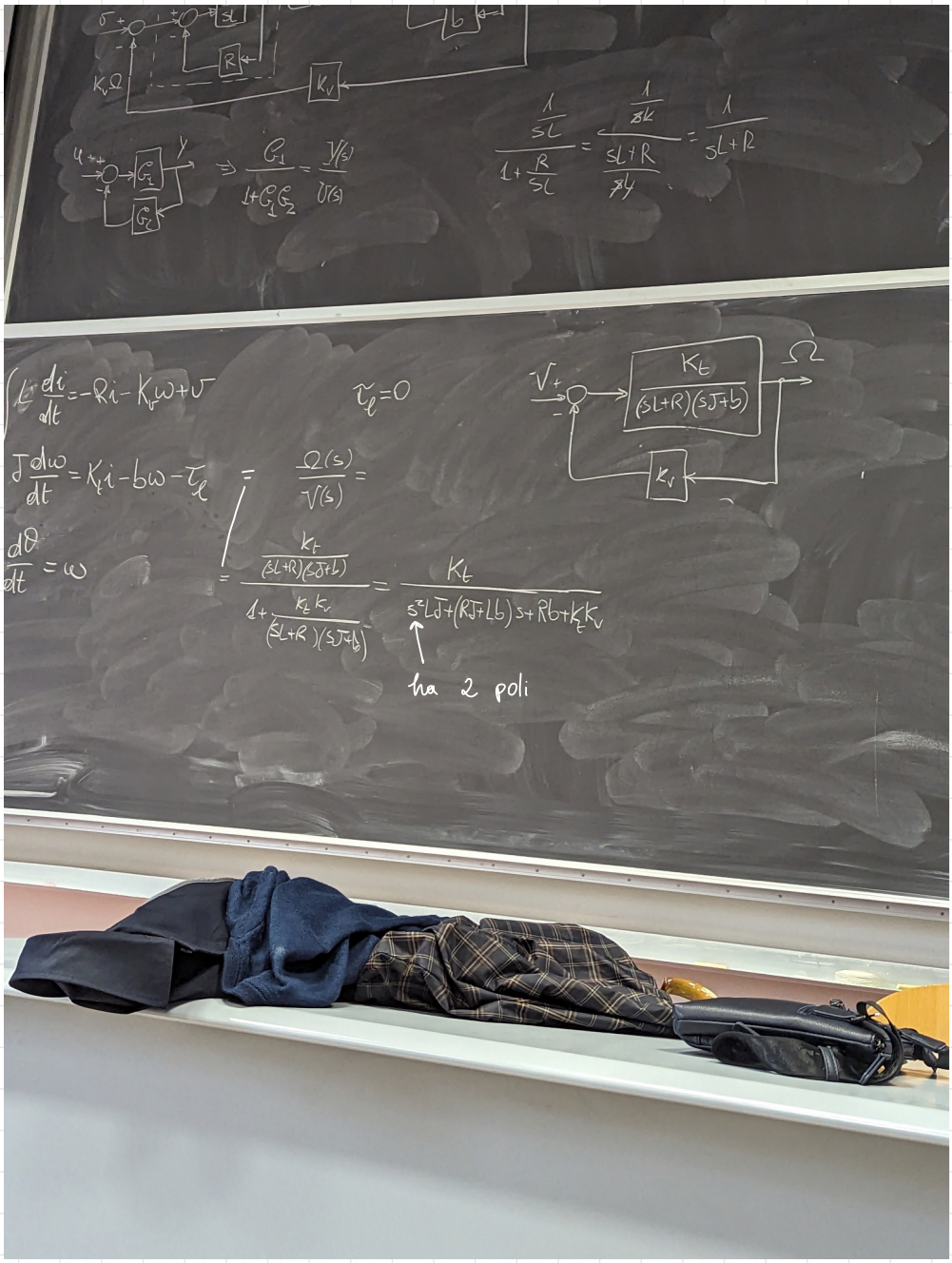
$$\begin{cases} L\dot{i} = -Ri - K_v w + v \\ J\dot{w} = K_t i - bw - \tau_e \\ \dot{\theta} = w \end{cases}$$



Parte Meccanica

FOTO





* Trasformata
54:23

Schema a blocchi lez 11

FOTO Schema a blocchi con GOTO e FROM

Dallo Spazio di Stato alla funzione di Trasferimento (MATRICI!)

(1) $\dot{x} = Ax + Bu$
(2) $y = Cx + Du$

$\rightarrow$ Suppongo C.I. $= 0 \rightarrow \mathcal{L}(1) \rightarrow sX(s) = AX(s) + BU(s)$

$Ix = x$ ← Non posso sottrarre uno scalare a una matrice!

Posso mettere in evidenza solo a Destra!

$\rightarrow$ Ricavo $X(s) \rightarrow sX(s) - AX(s) = BU(s) \rightarrow (sI - A)X(s) = BU(s)$

Sfrutto $A^{-1} \cdot A = I \rightarrow$ ~~$\frac{A}{B} = A \cdot B^{-1}$~~ SBAGLIATO! $\rightarrow$ $\frac{A}{B} = A \cdot B^{-1}$ giusto

$\rightarrow (sI-A)^{-1}(sI-A)X(s) = (sI-A)^{-1}BU(s)$

$\rightarrow X(s) = (sI-A)^{-1} \cdot BU(s)$

$\mathcal{L}(2) \rightarrow Y(s) = CX(s) + DU(s) \rightarrow Y(s) = [C(sI-A)^{-1} \cdot B + D]U(s)$

OUT IN

Funzione di Trasferimento tra Y e U

Esempio

$$A = \begin{pmatrix} -\frac{R}{L} & -\frac{K_v}{L} \\ \frac{K_t}{J} & -\frac{b}{J} \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \quad C = (0 \;\; 1) \quad D = 0$$

$x_1 = i \quad x_2 = \omega$
$y = \omega \quad u = v$

$$sI - A = s\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{R}{L} & \frac{K_v}{L} \\ -\frac{K_t}{J} & \frac{b}{J} \end{pmatrix} = \begin{pmatrix} s+\frac{R}{L} & \frac{K_v}{L} \\ -\frac{K_t}{K} & s+\frac{b}{J} \end{pmatrix}$$

$$\rightarrow (sI-A) = \begin{pmatrix} s+\frac{R}{L} & \frac{K_v}{L} \\ -\frac{K_t}{J} & s+\frac{b}{J} \end{pmatrix}^{-1} = \frac{1}{\det(sI-A)} \cdot \begin{pmatrix} \text{Posto} & \text{segno} \\ \text{segno} & \text{Posto} \end{pmatrix} =$$

ES
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow (sI-A)^{-1} = \underbrace{\frac{1}{\left(s+\frac{R}{L}\right)\left(s+\frac{b}{J}\right) - \left(K_v \cdot \left(-\frac{K_t}{JL}\right)\right)}}_{d(s)} \cdot \begin{pmatrix} s+\frac{b}{J} & -\frac{K_v}{L} \\ \frac{K_t}{J} & s+\frac{R}{L} \end{pmatrix}$$

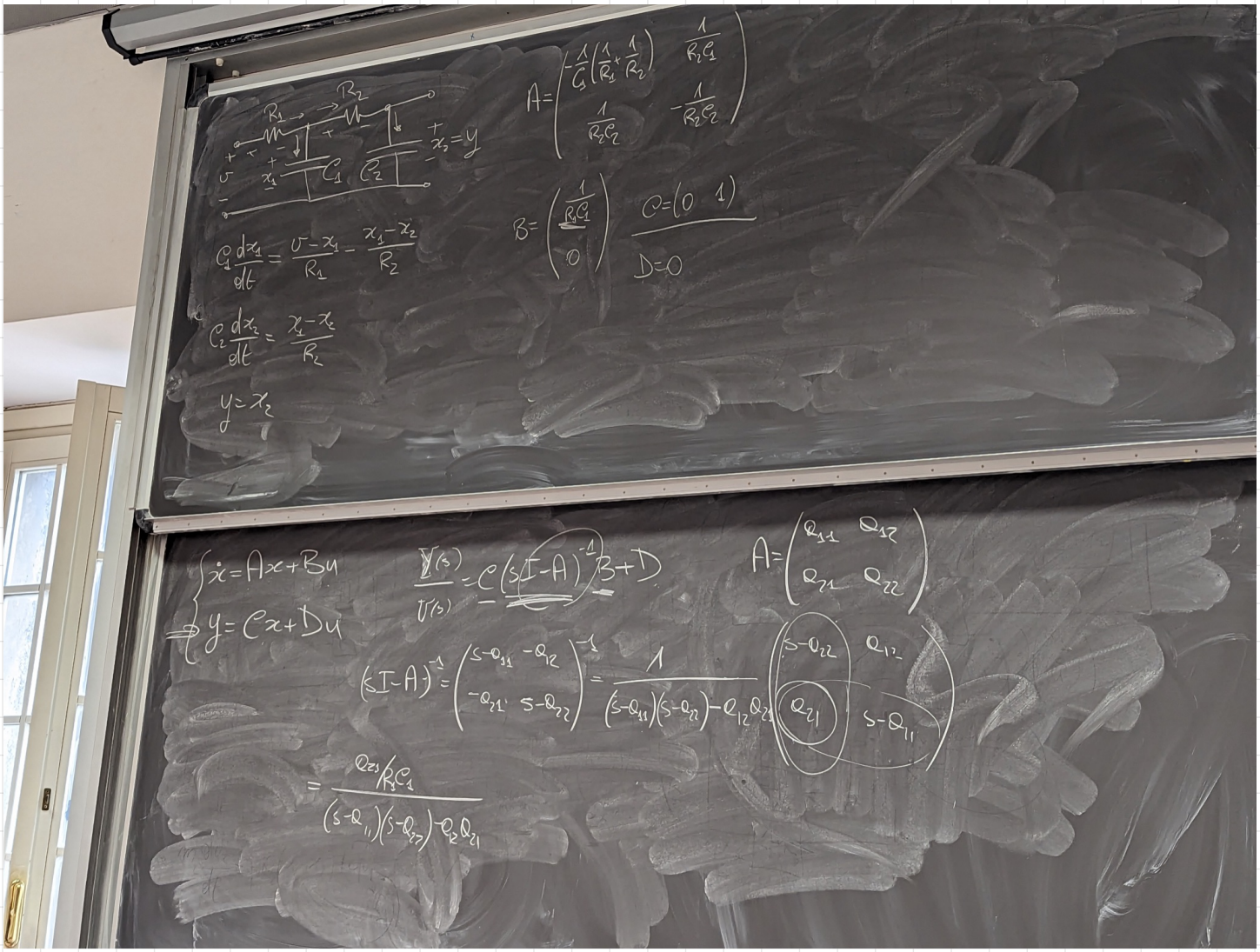
$$C \cdot (sI-A)^{-1} = (0 \;\; 1)\frac{1}{d(s)}\begin{pmatrix} s+\frac{b}{J} & -\frac{K_v}{L} \\ \frac{K_t}{J} & s+\frac{R}{L} \end{pmatrix} = \frac{1}{d(s)} \cdot \begin{pmatrix} \frac{K_t}{J} & s+\frac{R}{L} \end{pmatrix}$$

$$\left\{\left[C \cdot (sI-A)^{-1}\right] \cdot B\right\} + D = \frac{1}{d(s)} \cdot \begin{pmatrix} \frac{K_t}{J} & s+\frac{R}{L} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} + 0 = \frac{K_t/JL}{s^2 + s\left(\frac{R}{L}+\frac{b}{J}\right) + \frac{Rb}{LJ} + \frac{K_v K_t}{JL}}$$

$$\rightarrow Y(s) = \left[C \cdot (sI-A)^{-1} \cdot B\right] + D = \frac{K_t}{s^2 JL + (RJ+bL)s + Rb + K_t K_v}$$

QED

↖ Stesso risultato ottenuto nel caso del motore elettrico


R1
R2
C1
C2
x2 = y
A = (-1/C1 (1/R1 + 1/R2) 1/(R2C1)
     1/(R2C2) -1/(R2C2))
B = (1/(R1C1)
     0)
C = (0 1)
D = 0
C1 dx1/dt = (v - x1)/R1 - (x1 - x2)/R2
C2 dx2/dt = (x1 - x2)/R2
y = x2
ẋ = Ax + Bu
y = Cx + Du
Y(s)/U(s) = C(sI - A)^-1 B + D
A = (a11 a12
     a21 a22)
(sI - A)^-1 = (s - a11 -a12
               -a21 s - a22)^-1 = 1/((s - a11)(s - a22) - a12 a21) (s - a22 a12
                                                                     a21 s - a11)
= a21/(R1C1) / ((s - a11)(s - a22) - a2 a21)