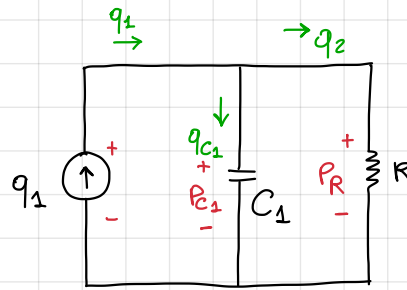
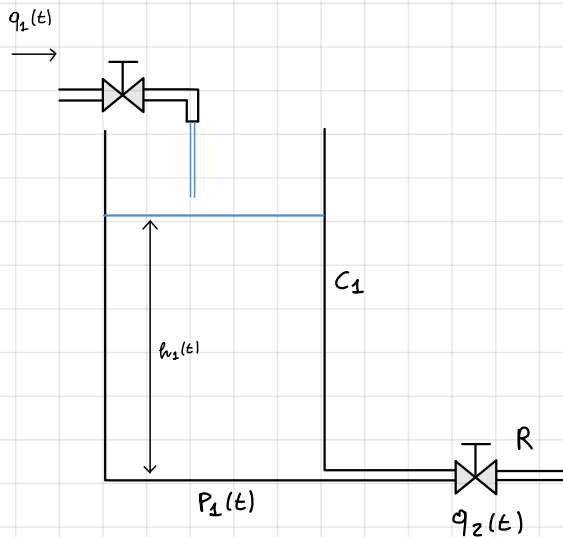


SINGOLO SERBATOIO



$$q_{C1} = C_1 \frac{dP_{C1}}{dt}$$

$$\text{ma } q_{C1} = q_1 - q_2$$

$$\Rightarrow C_1 \frac{dP_{C1}}{dt} = q_1 - q_2$$

$$P_R = R \cdot q_2 \Rightarrow q_2 = \frac{P_R}{R}$$

$$\text{ma } P_R = P_{C1} \Rightarrow C_1 \frac{dP_{C1}}{dt} = q_1 - \frac{P_{C1}}{R}$$

FUNZIONE DI TRASFERIMENTO

$$SC_1 P_{C1}(s) = Q_1(s) - \frac{1}{R_1} P_{C1}(s) \rightarrow$$

$$\Rightarrow P_{C1} \left(SC_1 + \frac{1}{R_1} \right) = Q_1(s) \Rightarrow P_{C1}(s) = \frac{R_1}{SC_1 R_1 + 1} Q(s)$$

- Se IN: $Q(s)$ e OUT: $P_{C1}(s)$

$$\Rightarrow \frac{P_{C1}(s)}{Q(s)} = G_1(s) = \frac{R_1}{SC_1 R_1 + 1}$$

- Se IN: $Q(s)$ e OUT: $H(s) = \frac{P_1(s)}{\rho g} = \frac{R_1}{\rho g (SC_1 R_1 + 1)} Q(s)$

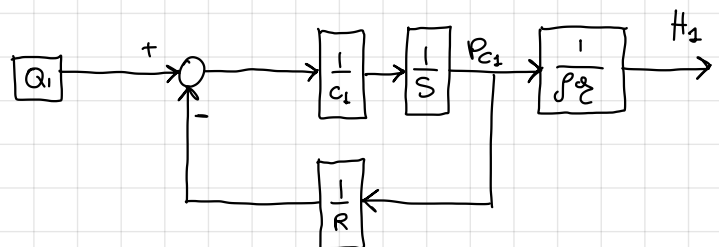
$$\Rightarrow G_2(s) = \frac{R_1}{\rho g (SC_1 R_1 + 1)}$$

Se Voglio l'altezza

$$\rho g \cdot h = P - P_a \quad \text{Se } P \gg P_a$$

$$\Rightarrow h = \frac{P_1}{\rho g}$$

SCHEMA A BLOCCHI



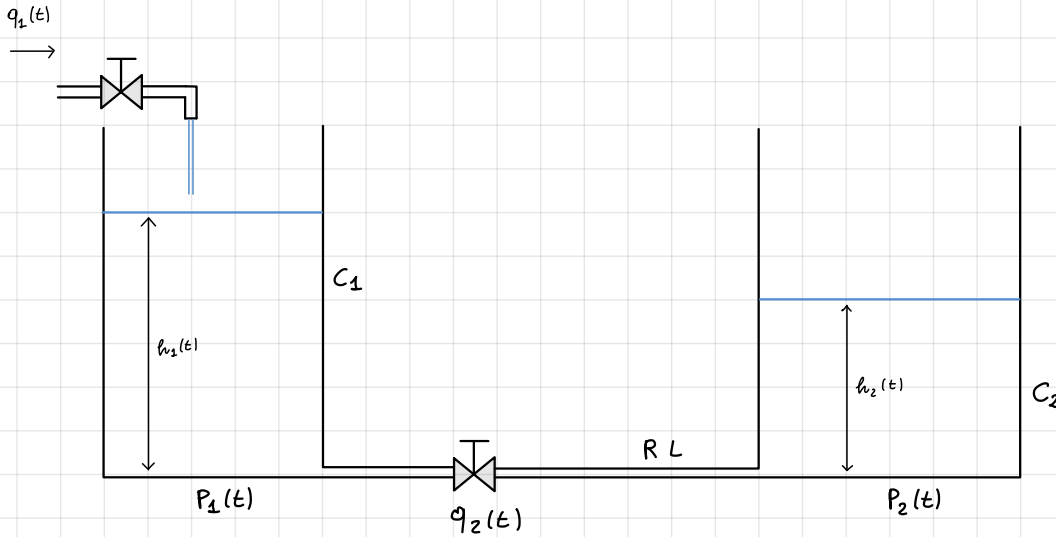
SPAZIO DI STATO

$$x_1 = P_{C1} \quad U = q_1 \quad y = P_{C1}$$

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1} \cdot U - \frac{1}{C_1 R_1} x_1 \\ y = x_1 \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = -\frac{1}{C_1 R_1} x_1 + \frac{1}{C_1} U \\ y = 1 x_1 + 0 U \end{cases}$$

ESEMPIO 4.10

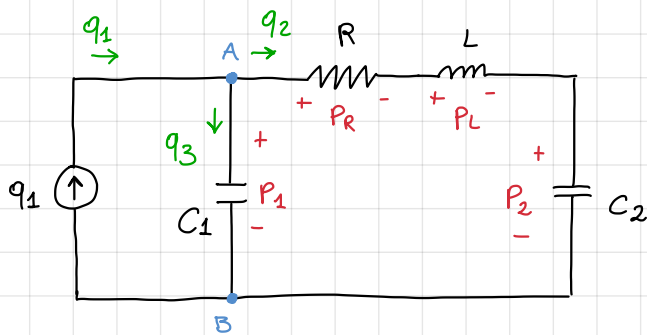
lezione 14



$$q_a - q_b = \frac{S}{\rho g} \cdot \frac{dP}{dt}$$

$$R_{id} = \frac{\Delta P}{q}$$

$$\Delta P = \frac{\rho h}{S} \cdot \frac{dq}{dt}$$



Rel Caratt

$$\begin{cases} U_L = L \cdot \frac{di_L}{dt} \\ i_C = C \cdot \frac{dv_C}{dt} \end{cases}$$

$$\begin{cases} q_3 = C_1 \frac{dP_1}{dt} \Rightarrow q_1 - q_2 = C_1 \frac{dP_1}{dt} \\ q_2 = C_2 \cdot \frac{dP_2}{dt} \\ P_L = -P_R + P_1 - P_2 = -R \cdot q_2 + P_1 - P_2 = L \frac{dq_2}{dt} \end{cases}$$

SPAZIO DI STATO

Sceglgo $x_1 = P_1$; $x_2 = P_2$; $x_3 = q_2$; $U = q_1$; $y = h_2$

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1} (U - x_3) \\ \dot{x}_2 = \frac{1}{C_2} (x_3) \\ \dot{x}_3 = \frac{1}{L} (-R \cdot x_3 + x_1 - x_2) \\ y = \frac{1}{\rho g} x_2 \end{cases}$$

Se abbiamo un cilindro di lunghezza l molto Alto $\Rightarrow P \gg P_a$

Pressione alla base \uparrow Pressione ATM all'apice \uparrow

$$\Rightarrow \text{Bernoulli: } \rho \cdot g \cdot h \approx P$$

$$\Rightarrow h = \frac{P_2}{\rho g}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1} \\ 0 \\ 0 \end{pmatrix} \cdot U$$

$$y = \begin{pmatrix} 0 & \frac{1}{\rho g} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \end{pmatrix} \cdot U$$

$$\begin{cases} SC_1 P_1(s) = Q_1(s) - Q_2(s) & (1) \\ SC_2 P_2(s) = Q_2(s) & (2) \\ SL Q_2(s) = -R Q_2(s) + P_1(s) - P_2(s) & (3) \end{cases}$$

$$H_2(s) = \frac{1}{s^2} P_2(s)$$

$$P_2 = \frac{1}{SC_2} Q_2 \quad ; \quad Q_2(s) = \frac{1}{R} P_1 - \frac{1}{R} P_2 - \frac{SL}{R} Q_2$$

$$P_1 = \frac{Q_1}{SC_1} - \frac{Q_2}{SC_1} \quad \rightarrow Q_2(s) = \frac{1}{SC_1 R} Q_1 - \frac{1}{SRC_1} Q_2 - \frac{1}{R} P_2 - \frac{SL}{R} Q_2$$

$$\rightarrow Q_2 \left(1 + \frac{1}{SRC_1} + \frac{SL}{R} \right) = \frac{1}{SC_1 R} Q_1 - \frac{1}{R} P_2 \quad \leadsto \frac{SRC_1 + 1 + S^2 LC_1}{SRC_1}$$

$$\rightarrow Q_2 = \frac{1}{SRC_1 + 1 + S^2 LC_1} Q_1 - \frac{SC_1}{SRC_1 + 1 + S^2 LC_1} P_2$$

$$\Rightarrow P_2 = \frac{1}{S^2 RC_1 C_2 + SC_2 + S^3 LC_1 C_2} Q_1 - \frac{C_1}{SRC_1 C_2 + C_2 + S^2 LC_1 C_2} P_2$$

$$\Rightarrow P_2 \left(1 + \frac{C_1}{SRC_1 C_2 + C_2 + S^2 LC_1 C_2} \right) = \frac{1}{S^2 RC_1 C_2 + SC_2 + S^3 LC_1 C_2} Q_1$$

$$P_2 \left(\frac{SRC_1 C_2 + C_2 + S^2 LC_1 C_2 + C_1}{SRC_1 C_2 + C_2 + S^2 LC_1 C_2} \right) = \frac{1}{S(S^2 LC_1 C_2 + SRC_1 C_2 + C_2)} Q_1$$

$$P_2(s) = \frac{\cancel{S^2 LC_1 C_2 + SRC_1 C_2 + C_2}}{S^2 LC_1 C_2 + SRC_1 C_2 + C_1 + C_2} \cdot \frac{1}{S(\cancel{S^2 LC_1 C_2 + SRC_1 C_2 + C_2})} Q_1$$

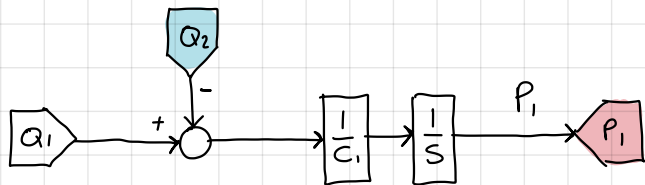
$$\Rightarrow P_2(s) = \frac{1}{S(S^2 LC_1 C_2 + SRC_1 C_2 + C_1 + C_2)} Q_1(s) \Rightarrow H_2(s) = \frac{\frac{1}{s^2} Q_1(s)}{S(S^2 LC_1 C_2 + SRC_1 C_2 + C_1 + C_2)}$$

Voglio la FDT Tra IN: $Q_1(s)$ e OUT: $H_2(s)$

$$\Rightarrow \frac{H_2(s)}{Q_1(s)} = G_1(s) = \frac{\frac{1}{s^2} Q_1(s)}{S(S^2 LC_1 C_2 + SRC_1 C_2 + C_1 + C_2)} = \frac{1}{S(S^2 LC_1 C_2 + SRC_1 C_2 + C_1 + C_2)}$$

$$\frac{1}{s^2} \frac{Q_1(s)}{S(S^2 LC_1 C_2 + SRC_1 C_2 + C_1 + C_2)}$$

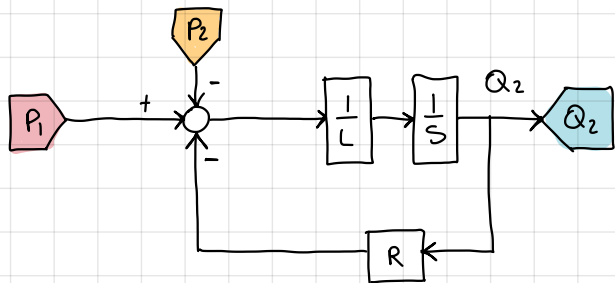
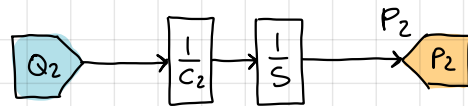
SCHEMA A BLOCCHI



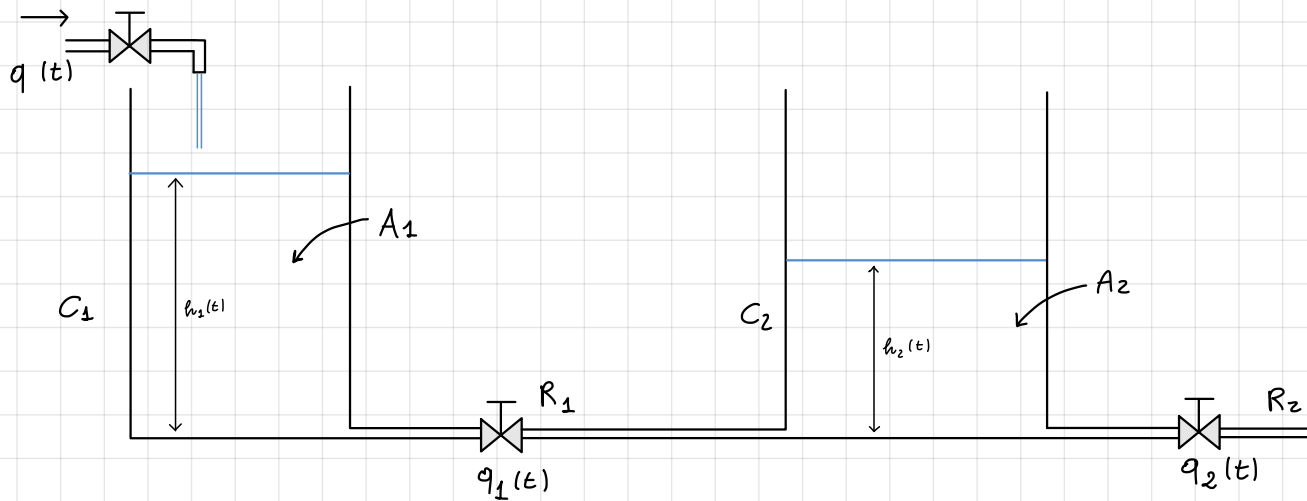
$$q_1 - q_2 = C_1 \frac{dP_1}{dt}$$

$$q_2 = C_2 \cdot \frac{dP_2}{dt}$$

$$-R \cdot q_2 + P_1 - P_2 = L \frac{dq_2}{dt}$$



SERBATOI CON UN SOLO FLUSSO DI ENTRATA



$$\text{TANK 1} \begin{cases} q - q_1 = A_1 \frac{dh_1}{dt} ; & q_1 = \frac{1}{R_1} (h_1 - h_2) \\ \text{TANK 2} \begin{cases} q_1 - q_2 = A_2 \frac{dh_2}{dt} ; & q_2 = \frac{h_2}{R_2} \end{cases} \end{cases}$$

Voglio OUT: h_1 IN: Q

$$\begin{aligned} 1) & \begin{cases} q - \frac{h_1 - h_2}{R_1} = A_1 \frac{dh_1}{dt} \\ \end{cases} \Leftrightarrow SA_1 H_1 = Q - \frac{H_1}{R_1} + \frac{H_2}{R_1} \\ 2) & \begin{cases} \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt} \\ \end{cases} \Leftrightarrow SA_2 H_2 = \frac{H_1}{R_1} - \frac{H_2}{R_1} - \frac{H_2}{R_2} \Rightarrow H_2 \left(SA_2 + \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{H_1}{R_1} \\ & \Rightarrow H_1 = H_2 \left(SA_1 A_2 + 1 + \frac{R_1}{R_2} \right) \end{aligned}$$

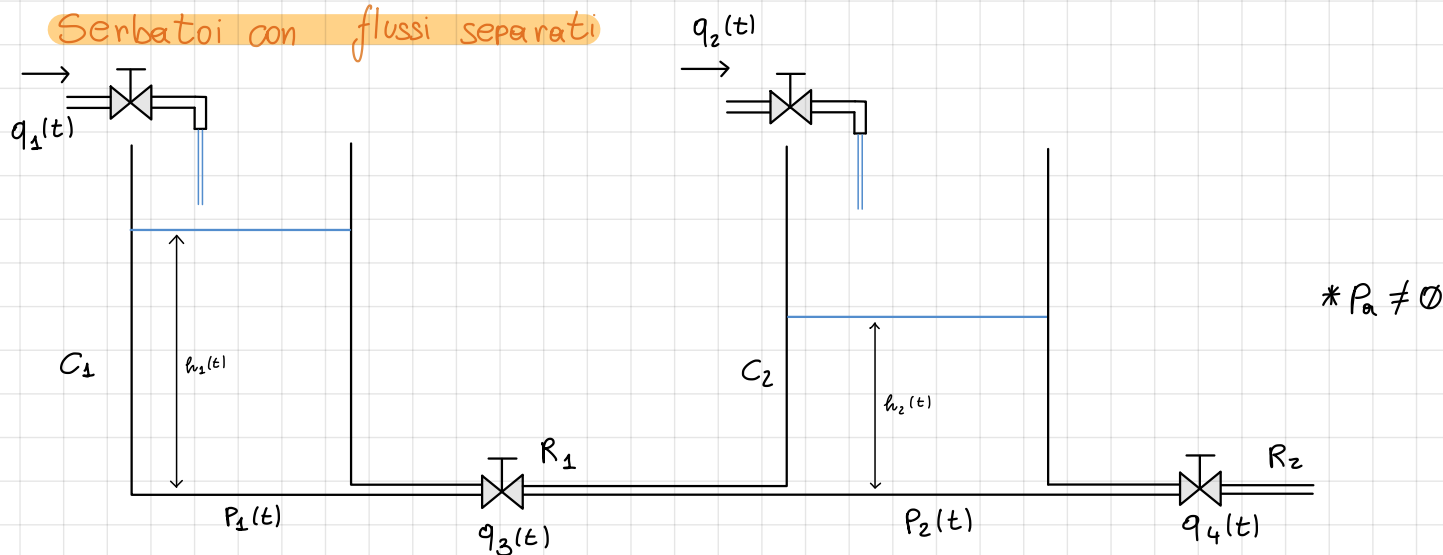
$$\text{Nella 1} \Rightarrow H_1 \left(\frac{SA_1 + \frac{1}{R_1}}{SA_1 R_1 + 1} \right) = Q + \frac{1}{R_1} H_2 \Rightarrow H_1 = \frac{R_1}{SA_1 R_1 + 1} + \frac{1}{SA_1 R_1 + 1} H_2$$

$$\Rightarrow H_2 \left(\frac{SA_1 R_2 A_2 + R_2 + R_1}{R_2} \right) = \frac{R_1}{SA_1 R_1 + 1} Q + \frac{1}{SA_1 R_1 + 1} H_2$$

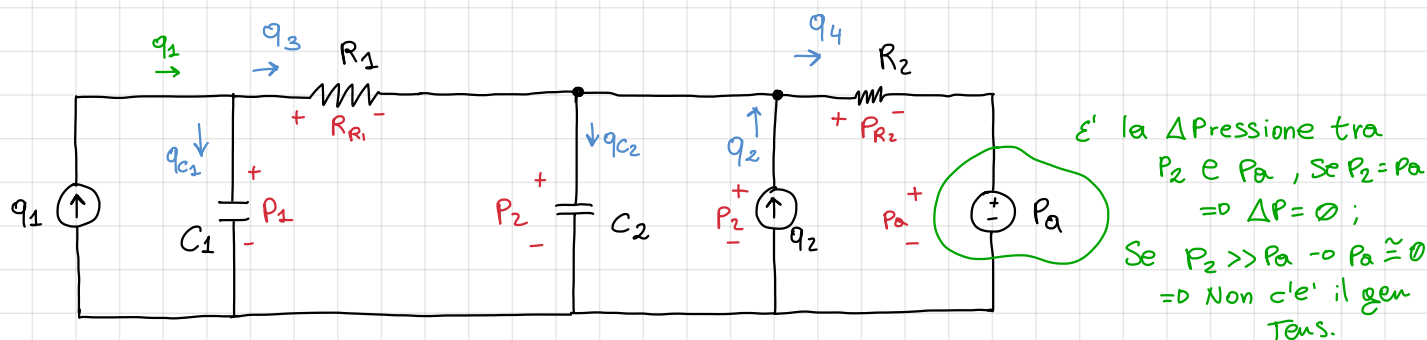
$$\Rightarrow H_2 \left(\frac{SA_1 R_2 A_2 + R_2 + R_1}{R_2} - \frac{1}{SA_1 R_1 + 1} \right) = \frac{R_1}{SA_1 R_1 + 1} Q \Rightarrow H_2 \left(\frac{SA_1 R_2 A_2 + R_2 + R_1 (SA_1 R_1 + 1) - R_2}{R_2 (SA_1 R_1 + 1)} \right) = \frac{R_1}{SA_1 R_1 + 1} Q$$

$$H_2 \left(\frac{SA_1^2 R_2 A_1 A_2 + SA_1 R_2 A_2 + SA_1 R_1 R_2 + R_2 + SA_1 R_1^2 + R_1 - R_2}{SA_1 R_1 R_2 + R_2} \right) = \frac{R_1}{SA_1 R_1 + 1} Q$$

Serbatoi con flussi separati



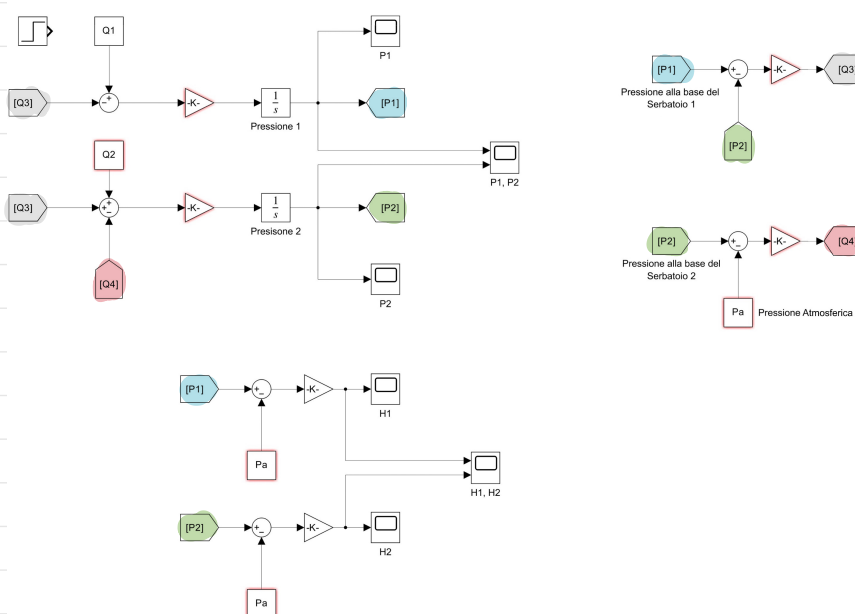
Circuito equivalente



$$\begin{cases}
 q_{c1} = C_1 \frac{dP_1}{dt} \rightarrow q_{c1} + q_3 = q_1 \rightarrow q_{c1} = q_1 - q_3 \Rightarrow C_1 \frac{dP_1}{dt} = q_1 - q_3 \\
 q_{c2} = C_2 \frac{dP_2}{dt} \rightarrow q_{c2} + q_4 - q_3 - q_2 = 0 \rightarrow q_{c2} = q_2 + q_3 - q_4 \Rightarrow C_2 \frac{dP_2}{dt} = q_2 + q_3 - q_4 \\
 P_{R1} = R_1 \cdot q_3 \rightarrow q_3 = \frac{P_{R1}}{R_1} \text{ ma LKT: } P_{R1} = P_1 - P_2 \Rightarrow q_3 = \frac{P_1 - P_2}{R_1} \\
 P_{R2} = R_2 \cdot q_4 \rightarrow q_4 = \frac{P_{R2}}{R_2} \text{ ma LKT: } P_{R2} = P_2 - P_a \Rightarrow q_4 = \frac{P_2 - P_a}{R_2}
 \end{cases}$$

SCHEMA A BLOCCHI

Da Bernoulli: $\rho g h = P - P_a \Rightarrow h = \frac{P - P_a}{\rho g}$



$$\begin{cases}
 \frac{dP_1}{dt} = \frac{1}{C_1} \left(q_1 - \frac{P_1 - P_2}{R_1} \right) \\
 \frac{dP_2}{dt} = \frac{1}{C_2} \left(q_2 + \frac{P_1 - P_2}{R_1} - \frac{P_2 - P_a}{R_2} \right)
 \end{cases}$$

SPAZIO DI STATO

Scelgo

$$x_1 = P_1$$

$$x_2 = P_2$$

$$u_1 = q_1$$

$$u_2 = q_2$$

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1} \left(U_1 - \frac{x_1}{R_1} + \frac{x_2}{R_1} \right) \\ \dot{x}_2 = \frac{1}{C_2} \left(U_2 + \frac{x_1}{R_1} - \frac{x_2}{R_1} - \frac{x_2}{R_2} + \frac{P_a}{R_2} \right) \end{cases}$$

$$y = h_1 = \frac{P_1 - P_a}{\rho g} \quad \text{Trascuro } P_a$$

$$\Rightarrow y_1 = \frac{P_1}{\rho g} ; y_2 = \frac{P_2}{\rho g}$$

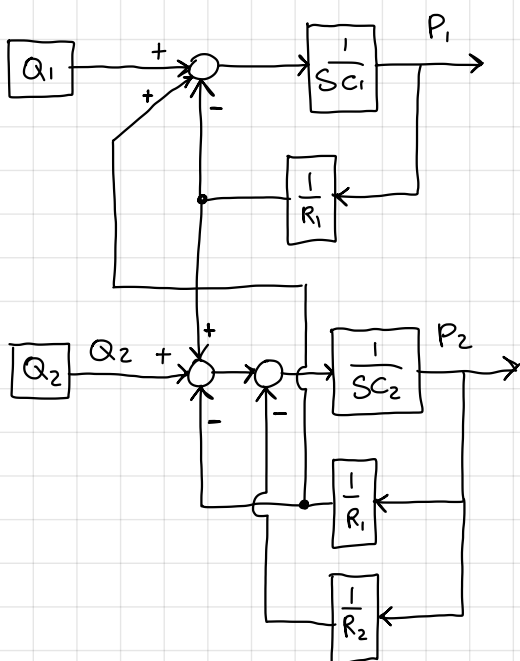
$$\begin{cases} y_1 = \frac{1}{\rho g} x_1 \\ y_2 = \frac{1}{\rho g} x_2 \\ y_3 = \frac{1}{\rho g} (x_1 - x_2) \end{cases} \quad \text{Differenza di Altezza}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{C_1 R_1} & \frac{1}{C_1 R_1} \\ \frac{1}{C_2 R_1} & -\frac{R_2 + R_1}{C_2 R_1 R_2} \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho g} & 0 \\ 0 & \frac{1}{\rho g} \\ \frac{1}{\rho g} & -\frac{1}{\rho g} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 0 \cdot U$$

SCHEMA A BLOCCHI

A Mano



Funzione di Trasferimento

IN Q_1 OUT P_1

$$\begin{cases} C_1 \frac{dP_1}{dt} = q_1 - q_3 \\ C_2 \frac{dP_2}{dt} = q_2 + q_3 - q_4 \end{cases} \rightarrow \begin{cases} \frac{dP_1}{dt} = \frac{1}{C_1} \left(q_1 - \frac{P_1 - P_2}{R_1} \right) \\ \frac{dP_2}{dt} = \frac{1}{C_2} \left(q_2 + \frac{P_1 - P_2}{R_1} - \frac{P_2 - P_a}{R_2} \right) \end{cases}$$

Assumo $P_a \approx 0$

$$\begin{cases} q_3 = \frac{P_1 - P_2}{R_1} \Rightarrow Q_3 = \frac{1}{R_1} P_1(s) - \frac{1}{R_1} P_2(s) \\ q_4 = \frac{P_2 - P_a}{R_2} \Rightarrow Q_4 = \frac{1}{R_2} P_2(s) \end{cases}$$

$$\begin{cases} SC_1 P_1 = Q_1 - \frac{1}{R_1} P_1 + \frac{1}{R_1} P_2 \\ SC_2 P_2 = Q_2 + \frac{1}{R_1} P_1 - \frac{1}{R_1} P_2 - \frac{1}{R_2} P_2 \end{cases}$$

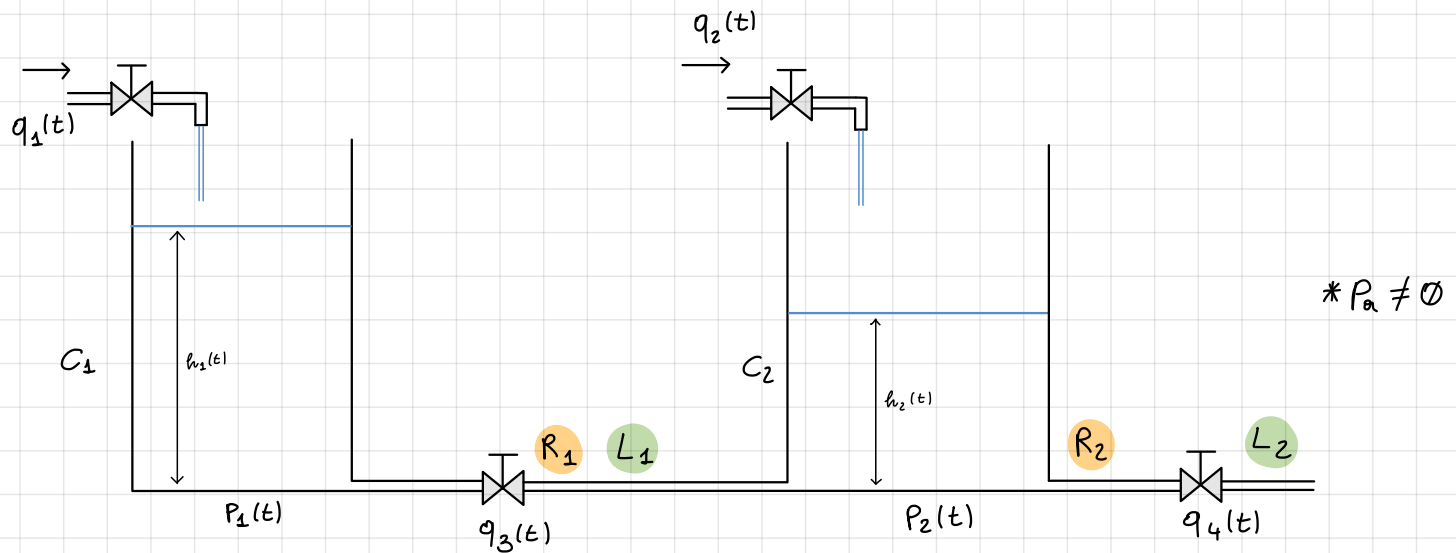
dalla 1 $P_1 \left(SC_1 + \frac{1}{R_1} \right) = Q_1 + \frac{1}{R_1} P_2 \Leftrightarrow P_2 = P_1 (SC_1 R_1 + 1) - R_1 Q_1$

dalla 2 $P_2 \left(SC_2 + \frac{1}{R_1} + \frac{1}{R_2} \right) = Q_2 + \frac{1}{R_1} P_1 \Rightarrow P_2 = \frac{R_1 R_2}{SC_2 R_1 R_2 + R_1 + R_2} Q_2 + \frac{R_2}{SC_2 R_1 R_2 + R_1 + R_2} P_1$

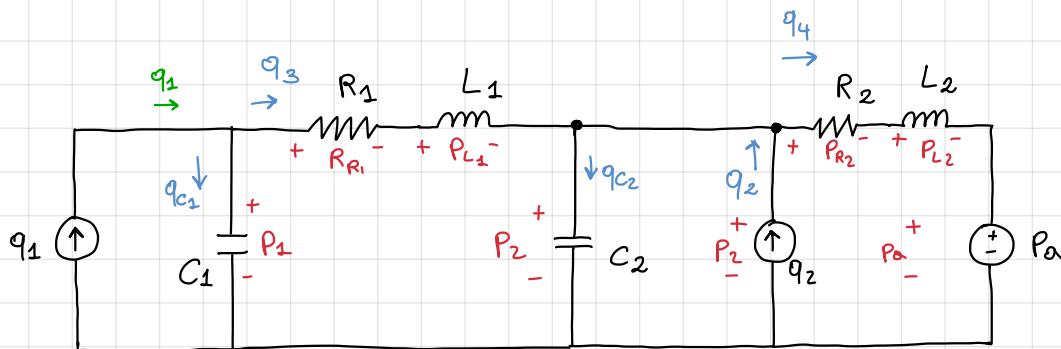
$$P_1 \left(SC_1 + \frac{1}{R_1} \right) = Q_1 + \frac{R_2}{SC_2 R_1 R_2 + R_1 + R_2} Q_2 + \frac{R_2}{R_1 (SC_2 R_1 R_2 + R_1 + R_2)} P_1$$

$$\Rightarrow P_1 \left(SC_1 + \frac{1}{R_1} - \frac{R_2}{R_1 (SC_2 R_1 R_2 + R_1 + R_2)} \right) = Q_1 + \frac{R_2}{SC_2 R_1 R_2 + R_1 + R_2} Q_2$$

$$P_1 = \frac{SC_2 R_1 R_2 + R_1 + R_2}{SC_1 C_2 R_1 R_2 + S(C_1 R_1 + C_1 R_2 + C_2 R_2) + 1} \cdot \left(Q_1 + \frac{R_2}{SC_2 R_1 R_2 + R_1 + R_2} Q_2 \right)$$



CIRCUITO ASSOCIATO



$$\begin{cases} C_1 \frac{dP_1}{dt} = q_{c1} & q_{c1} = q_1 - q_3 \quad \rightarrow \quad C_1 \frac{dP_1}{dt} = q_1 - q_3 \\ C_2 \frac{dP_2}{dt} = q_{c2} & q_3 - q_{c2} + q_2 - q_4 = 0 \quad \rightarrow \quad q_{c2} = q_2 + q_3 - q_4 \quad \rightarrow \quad C_2 \frac{dP_2}{dt} = q_2 + q_3 - q_4 \\ L_1 \frac{dq_3}{dt} = P_{L1} & P_{L1} = -P_{R1} + P_1 - P_2 \quad \rightarrow \quad L_1 \frac{dq_3}{dt} = -R_1 q_3 + P_1 - P_2 \\ L_2 \frac{dq_4}{dt} = P_{L2} & P_{L2} = -P_{R2} + P_2 - P_a \quad \rightarrow \quad L_2 \frac{dq_4}{dt} = -R_2 q_4 + P_2 - P_a \quad \text{DISTURBO} \end{cases}$$

$$\begin{cases} P_{R1} = R_1 \cdot q_3 \quad \rightarrow \quad q_3 = \frac{P_{R1}}{R_1} & P_{R1} = P_1 - P_2 - P_{L1} = 0 \\ P_{R2} = R_2 \cdot q_4 \quad \rightarrow \quad q_4 = \frac{P_{R2}}{R_2} & P_{R2} = P_2 - P_a - P_{L2} = 0 \end{cases}$$

$$\begin{cases} q_3 = \frac{P_1 - P_2 - P_{L1}}{R_1} \\ q_4 = \frac{P_2 - P_a - P_{L2}}{R_2} \end{cases} \quad (1)$$

$$\begin{cases} C_1 \frac{dP_1}{dt} = q_1 - \frac{P_1 - P_2 - P_{L1}}{R_1} \\ C_2 \frac{dP_2}{dt} = q_2 + \frac{P_1 - P_2 - P_{L1}}{R_1} - \frac{P_2 - P_a - P_{L2}}{R_2} \end{cases}$$

$$x_1 = p_1 \quad x_2 = p_2 \quad x_3 = q_3 \quad x_4 = q_4 \quad U_1 = q_1 \quad U_2 = q_2 \quad U_3 = p_a \quad y = p_1$$

$$\begin{cases} \dot{x}_1 = \frac{1}{c_1} (U_1 - x_3) \\ \dot{x}_2 = \frac{1}{c_2} (U_2 + x_3 - x_4) \\ \dot{x}_3 = \frac{1}{L_1} (-R_1 x_3 + x_1 - x_2) = 0 \\ \dot{x}_4 = \frac{1}{L_2} (-R_2 x_4 + x_2 - U_3) \\ y = x_1 \end{cases} \quad \begin{cases} \dot{x}_1 = 0x_1 + 0x_2 - \frac{1}{c_1}x_3 + 0x_4 + \frac{1}{c_1}U_1 + 0U_2 + 0U_3 \\ \dot{x}_2 = 0x_1 + 0x_2 + \frac{1}{c_2}x_3 - \frac{1}{c_2}x_4 + 0U_1 + \frac{1}{c_2}U_2 + 0U_3 \\ \dot{x}_3 = \frac{1}{L_1}x_1 - \frac{1}{L_1}x_2 - \frac{R_1}{L_1}x_3 + 0x_4 + 0U_1 + 0U_2 + 0U_3 \\ \dot{x}_4 = 0x_1 + \frac{1}{L_2}x_2 + 0x_3 - \frac{R_2}{L_2}x_4 + 0U_1 + 0U_2 - \frac{1}{L_2}U_3 \\ y = 1 \cdot x_1 \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\frac{1}{c_1} & 0 \\ 0 & 0 & \frac{1}{c_2} & -\frac{1}{c_2} \\ \frac{1}{L_1} & -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ 0 & \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} \frac{1}{c_1} & 0 & 0 \\ 0 & \frac{1}{c_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_2} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}$$

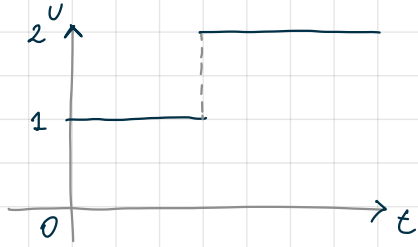
$$y = (1 \ 0 \ 0 \ 0) \cdot (x_1 \ x_2 \ x_3 \ x_4)^T + \underline{0} \cdot \underline{U}^T$$

Calcolo dell'Altezza

Dall'eq di Bernoulli: $p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

Con $v_1 = v_2$ e $h_2 = 0$ ($h_{p_2} = 0$) \rightarrow $p_A + \rho g h_1 = p$

$$\rightarrow h = \frac{p - p_A}{\rho g} \Rightarrow h_1 = \frac{p_1 - p_A}{\rho g}; \quad h_2 = \frac{p_2 - p_A}{\rho g}$$



$$\begin{cases} U_1(t) = 1(t) & \Leftrightarrow \frac{1}{s} \\ U_2(t) = 1(t-100) & \Leftrightarrow \frac{1}{s} e^{-100s} \end{cases}$$

$$U_{TOT}(s) = \frac{1}{s} + \frac{1}{s} e^{-100s}$$