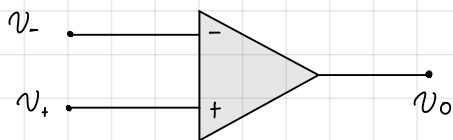


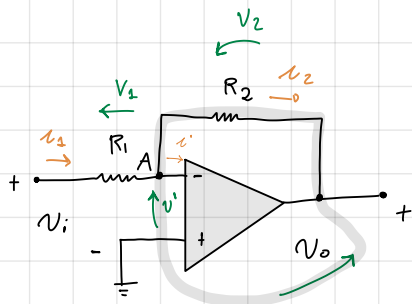
AMPLIFICATORI OPERAZIONALI



$$V_0 = K (V_+ - V_-)$$

↑
guadagno

Tipica connessione INVERTENTE



$$\int v_1 = \frac{v_{R1}}{R_1} = \frac{v_1 - v'}{R_1}$$

$$u_2 = \frac{v' - v_0}{R_2}$$

[uso la maglia v', v_0, v_2]

LKE_A: $-u_1 + u'_1 + u_2 = 0 \rightarrow u_1 = u'_1 + u_2$

Hp. $\chi' \approx 0$

nell'A.O. la corrente in entrata è ZERO

$$\Rightarrow \boxed{l_1 \approx l_2} \quad \Rightarrow (1) \approx (2)$$

$$-D \frac{v_1 - v'}{R_1} = \frac{v' - v_0}{R_2}$$

$$\frac{V_1}{R_1} = - \frac{V_o}{R_2}$$

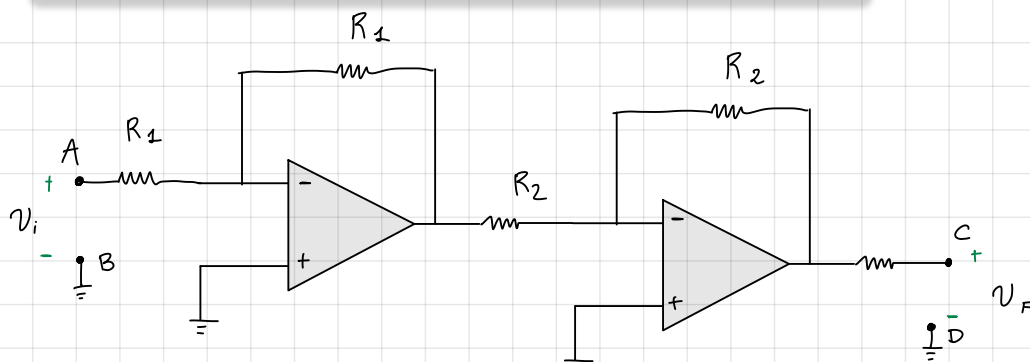
$$= 0$$

$$V_o = - \left(\frac{R_2}{R_1} \right)$$

- GUADAGNO
a seconda delle R
Combinazioni K

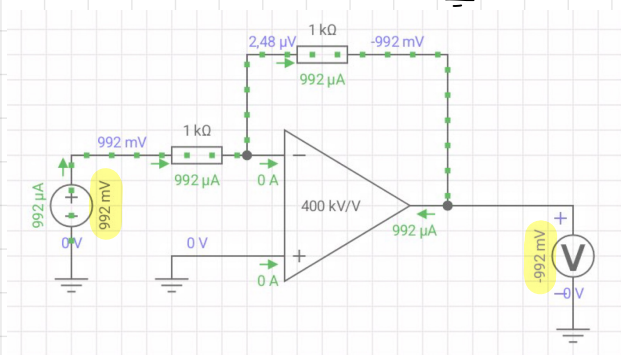
Configurazione invertute

Se metto due amplificatori operazionali (entrambi con delle resistenze uguali, anche diverse tra i due amplificatori) otteniamo esattamente la tensione iniziale (senza segno opposto)



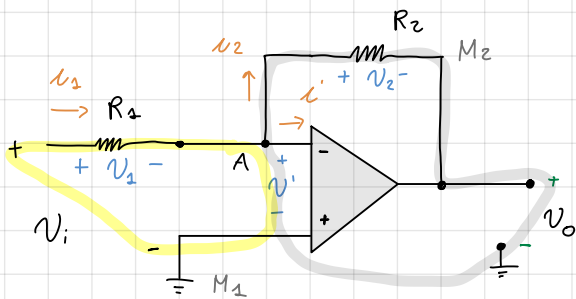
Se $R_1 = R_2$

$$V_0 \rightsquigarrow \Rightarrow \rightsquigarrow -V_0$$



Amplificatore invertente

QUALCHE SPIEGAZIONE IN PIÙ...



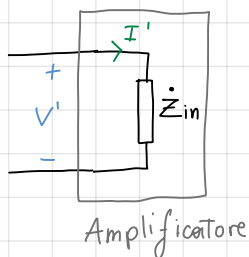
Scopo del gioco: Trovare V_o in relazione a R_1, R_2

$$I_2 = \frac{V_2}{R_2} = \frac{V_1' - V_o}{R_2}$$

$$I_1 = \frac{V_{R_1}}{R_1} = \frac{V_i - V_1'}{R_1}$$

ma LKC_A: $-I_1 + I_2 + I' = 0 \Rightarrow I_1 = I_2 + I'$

\Rightarrow Proprietà degli A.Op. $Z_{interna} \gg$



Troviamo I'

$$V_1' = Z \cdot I' \Rightarrow I' = \frac{V_1'}{Z}$$

ma se $Z \gg V \Rightarrow \frac{V_1'}{Z} \approx 0$

$\Rightarrow I' \approx 0$ QED

$\Rightarrow I_1 = I_2 + \cancel{I'} = I_2$

$\Rightarrow I_1 \approx I_2$ ovvero

$$\frac{V_i - V_1'}{R_1} = \frac{V_1' - V_o}{R_2} \quad (1)$$

Inoltre gli op-amp sono fatti per avere un guadagno molto elevato in modo da amplificare anche il minimo segnale:

$V_o = K (V_+ - V_-)$ con $K \gg (V_+ - V_-)$ ma (vedi figura) V_+ è messo a Terra!

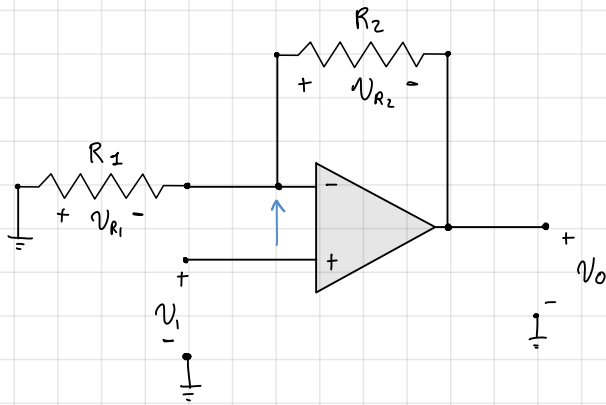
$\Rightarrow V_o = K (0 - V_1')$ con $K \gg 1 \Rightarrow V_1' \approx 0$

$$(1) \quad \frac{V_i - V_1'}{R_1} = \frac{V_1' - V_o}{R_2} \Rightarrow \frac{V_i}{R_1} = - \frac{V_o}{R_2} \Rightarrow V_o = - \left(\frac{R_2}{R_1} \right) V_i$$

INVERTENTE

guadagno

CONFIGURAZIONE NON INVERTENTE



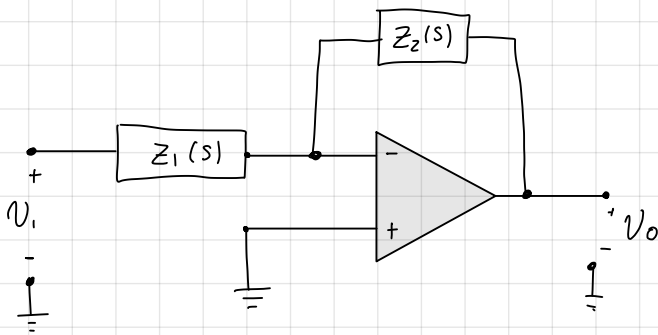
$$V_1 \approx V_2 \Rightarrow V_1 = \frac{V_{R1}}{R_1} = -\frac{V_1}{R_2}$$

$$V_2 = \frac{V_{R2}}{R_2} = \frac{V_1 - V_o}{R_2}$$

$$\Rightarrow -\frac{V_1}{R_2} = \frac{V_1 - V_o}{R_2} \Rightarrow \frac{V_o}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) V_1 \Rightarrow V_o = R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) V_i$$

GUADAGNO

METODO DELLE IMPEDENZE pg 125



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

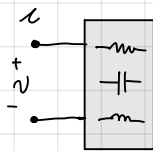
≡ F. DI TRASF.

TRASFORMATE

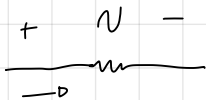
impedenze

Recap Impedenza complessa

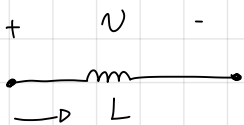
$$Z(s) = \frac{V(s)}{I(s)} \quad \leftarrow \begin{matrix} \text{OUT} \\ \text{IN} \end{matrix}$$



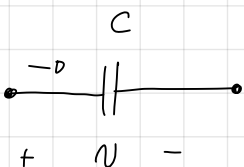
La differenza tra l'impedenza complessa ed una semplice funzione di trasferimento è che nel caso dell'impedenza blocchiamo come ingresso la corrente e come uscita la tensione.



$$\frac{V}{I} = R = Z_R(s)$$



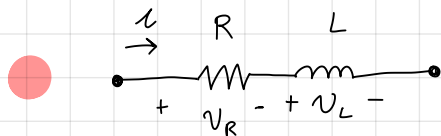
$$V = L \frac{dI}{dt} \Rightarrow V(s) = sL I(s) \Rightarrow Z_L(s) = sL$$



$$I = C \frac{dV}{dt} \Rightarrow I(s) = sC V(s) \Rightarrow Z_C(s) = \frac{1}{sC}$$

FOTO 1

COMBINAZIONI DI IMPEDENZE



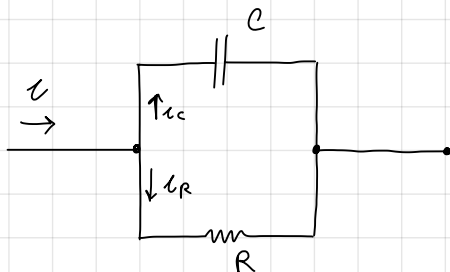
$$V = R i_L + L \dot{i}_L \rightarrow V(s) = R I(s) + sL I(s)$$

$$I = I(s)(R + sL)$$

→ Siccome $Z = \frac{V(s)}{I(s)} \rightarrow$

$$Z = R + sL$$

RL Serie



$$i = i_C + i_R \rightarrow i = C \dot{v}_C + \frac{v}{R}$$

$$\Rightarrow I(s) = sC V(s) + \frac{1}{R} V = V(s) \left(sC + \frac{1}{R} \right)$$

$$\Rightarrow Z(s) = \frac{1}{sC + \frac{1}{R}} = \frac{R}{R sC + 1} \quad \begin{matrix} Z(s) \\ RC \text{ PARALL} \end{matrix}$$

Implementare un controllore di tipo proporzionale

$$G(s) = K_P \rightarrow \text{Visto prima}$$

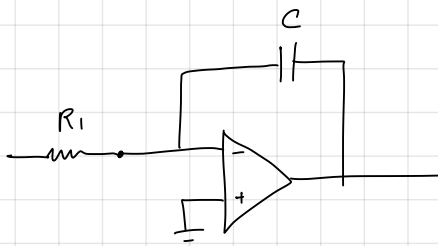
$$G(s) = K_P + \frac{K_I}{s} \rightarrow \text{vediamo dopo}$$

$$G(s) = \frac{K_I}{s} \rightarrow \text{vediamo ora}$$

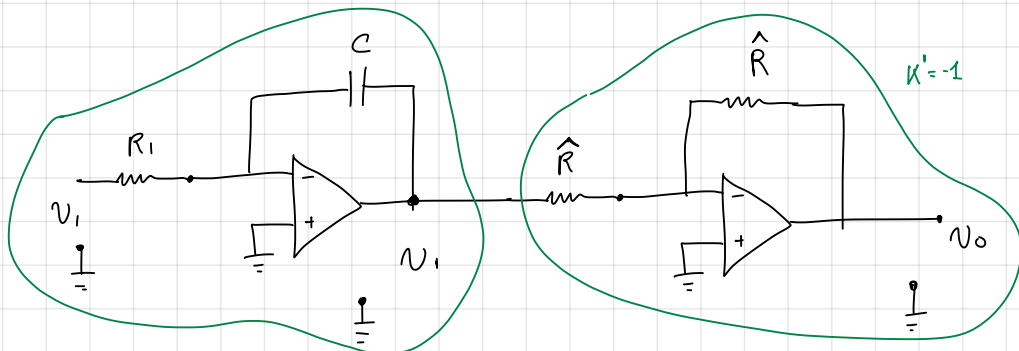
valgo impl. $G(s) = \frac{K_I}{s}$

$$\begin{cases} Z_2(s) = \frac{1}{sC} \end{cases} \quad \text{---||---}$$

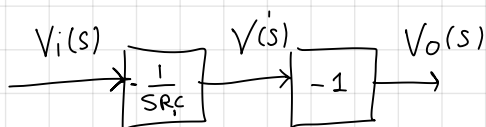
$$\begin{cases} Z_1(s) = R_1 \end{cases} \quad \text{---||---}$$



$$K = \frac{1}{RC}$$



$$K_I = -\frac{1}{R_1 C}$$

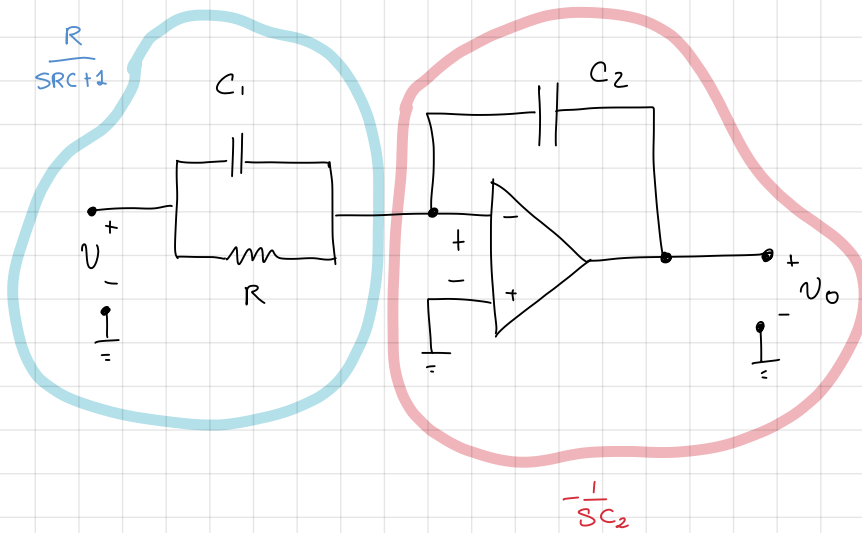


$$G(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s}$$

1 zero e 1 polo in 0

$$\frac{Z_1}{Z_2} = G(s) \quad \rightarrow \quad Z_2 = C = \frac{1}{sC_2} \quad \leftarrow \text{POLO}$$

$$Z_1(s) = \frac{R}{sRC + 1}$$



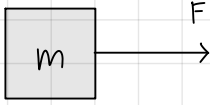
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sRC} \cdot \frac{sRC + 1}{s \left(\frac{C_1}{C_2} \right) + \left(\frac{1}{RC_2} \right)}$$

Modellistica dei sistemi meccanici

* Primo e secondo es
clip audio 8/9
e domanda orale
2 e 3 orale → cleuco domande

Caratterizzati da MASSA molla e SMORZATORE

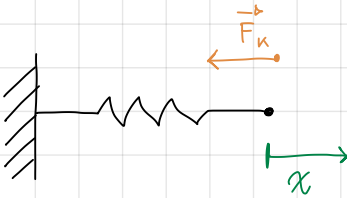
MASSA



$$F = ma \quad [N], \quad a = \frac{dv}{dt} = \dot{v} \quad [m/s^2], \quad v = \frac{dx}{dt} = \dot{x} \quad [m]$$

\uparrow
Spazio

MOLLA



Legge di Hooke: $F_k = -Kx$ (la molla viene Allungata)

$$E_m(t) = \int_0^t \underbrace{F(t) \cdot v(t)}_{\text{Potenza}} dt = \int_0^t m \frac{dv}{dt} \cdot v(t) dt = \frac{1}{2} m \int_0^t \frac{d v^2(t)}{dt} dt = \frac{1}{2} m \left[v^2(t) \right]_0^t$$

Energia di un corpo in movim.

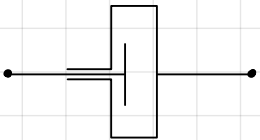
$\frac{1}{2} m v^2(t)$ ← possibile var di stato

$$E_k(t) = \int_0^t F_k(t) \cdot v(t) dt = \frac{1}{2} K x^2(t)$$

Energia della molla

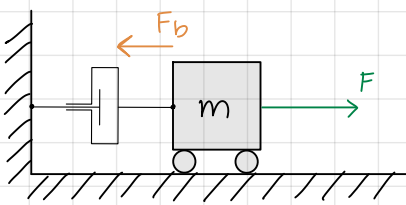
$\frac{1}{2} K x^2(t)$ ← possibile var di stato

SMORZATORE



$$v = \frac{dx}{dt} \rightarrow F_b = -bv \equiv F_b = -b \cdot \dot{x}$$

SISTEMA 1

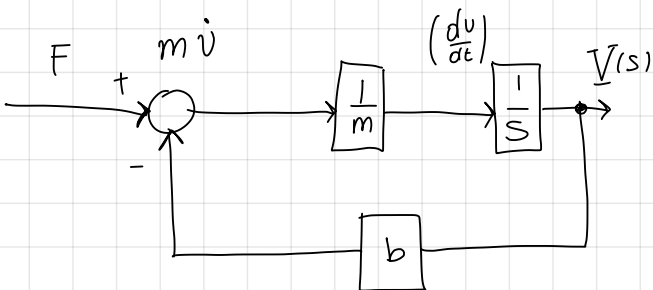
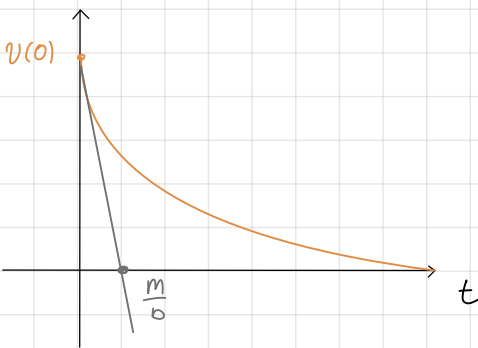
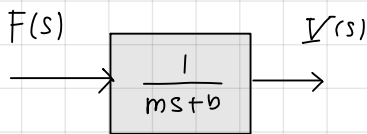
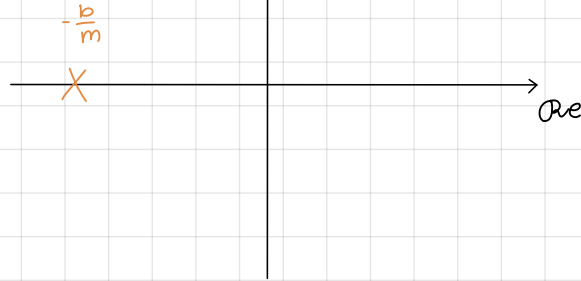


$$\begin{cases} F + F_b = m \dot{v} \\ F_b = -b v \end{cases}$$

L.T. $\rightarrow F(s) - b \cdot V = m \cdot s V(s) \rightarrow (ms + b) V(s) = F(s)$

\Rightarrow

$$\frac{V(s)}{F(s)} = \frac{1}{ms + b}$$



VAR. ST. v

$x = v$, $u = F$, $y = v$

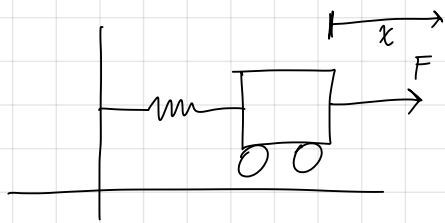
$$\begin{cases} \dot{x} = \frac{1}{m} (u - b \cdot x) \\ y = x \end{cases}$$

$$\begin{cases} \dot{x} = \frac{b}{m} x + \frac{1}{m} u \\ y = x \end{cases}$$

$$\begin{cases} \dot{x} = \underline{A} x + \underline{B} u \\ y = \underline{C} x + \underline{D} u \end{cases}$$

Annotations: $\underline{A} = \frac{b}{m}$, $\underline{B} = \frac{1}{m}$, $\underline{C} = 1$, $\underline{D} = 0$

SENZA SMORZATORE



$$x_1 = x \quad x_2 = v \quad u = F$$

$$\begin{cases} F + F_k = m \ddot{x} \rightarrow \ddot{x} = \frac{1}{m} (F + F_k) \\ F_k = -k x \\ v = \dot{x} \end{cases}$$

$$y = x_1 \leftarrow \text{la soluzione}$$

pos in x

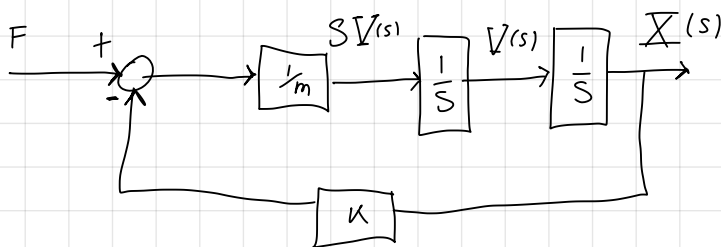
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m} (u - k x_1) \\ y = x_1 \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m} x_1 + \frac{1}{m} u \\ y = x_1 \end{cases}$$

$$\rightarrow \underline{A} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$$

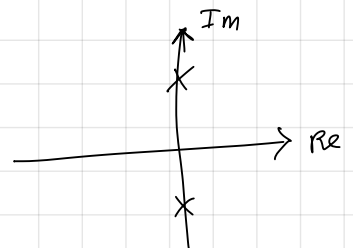
$$\underline{B} = \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \quad \underline{C} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

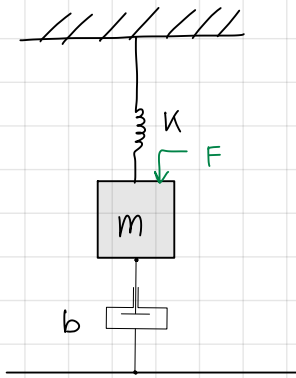


$$G(s) = \frac{X}{F} = \frac{\frac{1}{ms^2}}{1 + k \frac{1}{ms^2}} \stackrel{\text{Sin}(t)}{\parallel} = \frac{1}{ms^2 + k}$$

$$\text{con } s = \pm j \sqrt{\frac{k}{m}}$$



CON SMORZATORE



$$\begin{cases} F + F_K + F_b = m \ddot{v} \\ F_K = -Kx \\ F_b = -b\dot{v} \\ v = \dot{x} \end{cases}$$

$$ms \dot{V}(s) = -F(s) - KX(s) - b \cdot V(s)$$

$$\text{ma } V(s) = sX(s)$$

$$\rightarrow ms \dot{V}(s) + bs X(s) + KX(s) = F(s)$$

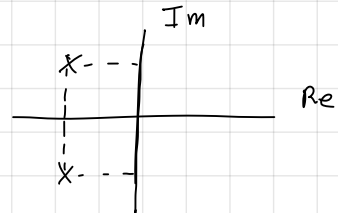
$$\rightarrow \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + K} \rightarrow \boxed{b^2 - 4m \cdot b > 0} \text{ ovvero se } b^2 > 4mb$$

Se c'è uno smorzamento abbiamo anche la parte reale

ReP Negativa
Asintoticamente stabili

$$\lambda = -a \pm jb$$

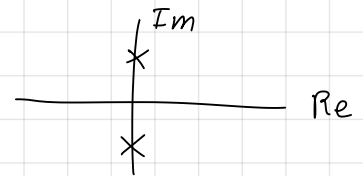
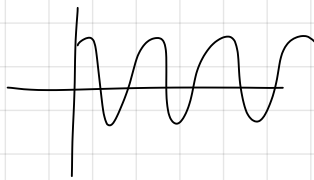
\rightarrow



STABILE

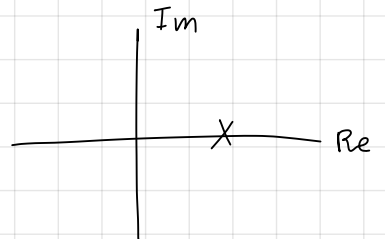
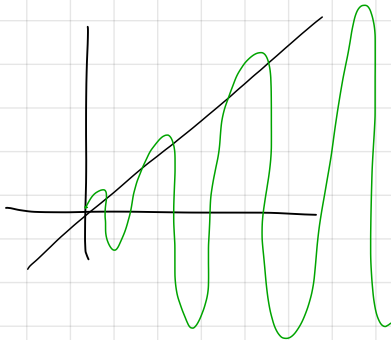
$$\lambda = 0 \pm jb$$

\rightarrow

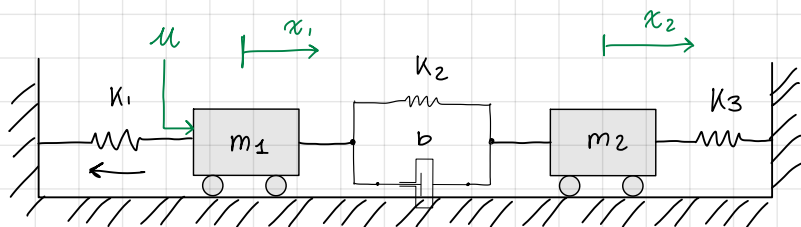


ReP Pos
INSTABILE

$$\lambda = +a \pm jb$$



ACCOPPIAMENTO FERROVIARIO



Scriviamo $m \cdot a = \sum F$ Tenendo presente che $a = \dot{v} = \ddot{x}$ e le rel car

$$\begin{cases} m_1 \ddot{x}_1 = u - k_1 x_1 - k_2 (x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2) \\ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2) - k_3 x_2 \end{cases}$$

$v_1 - v_2$ $v_1 - v_2$

manca \dot{x}_1 e $\dot{x}_2 = 0$ Pongo

$$\begin{aligned} x_3 &= \dot{x}_1 \leftarrow v_{m1} \\ x_4 &= \dot{x}_2 \leftarrow v_{m2} \end{aligned}$$

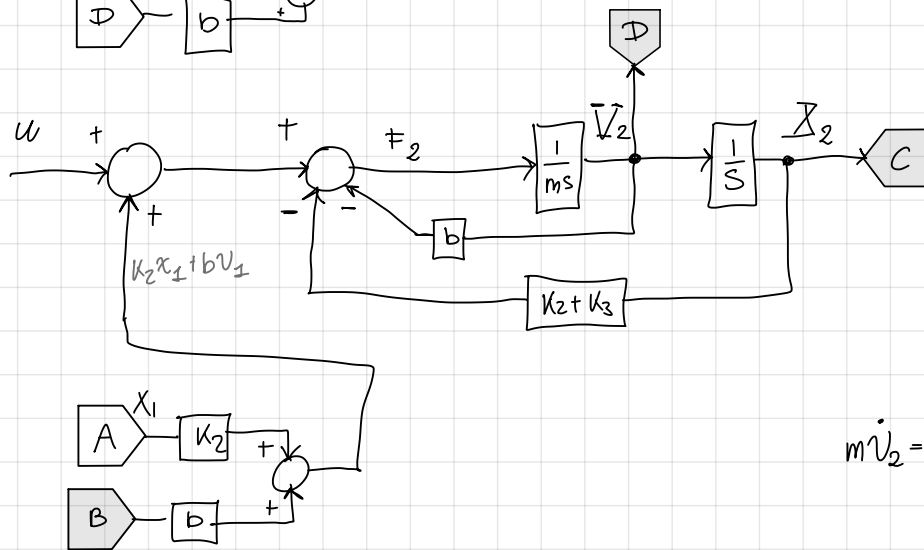
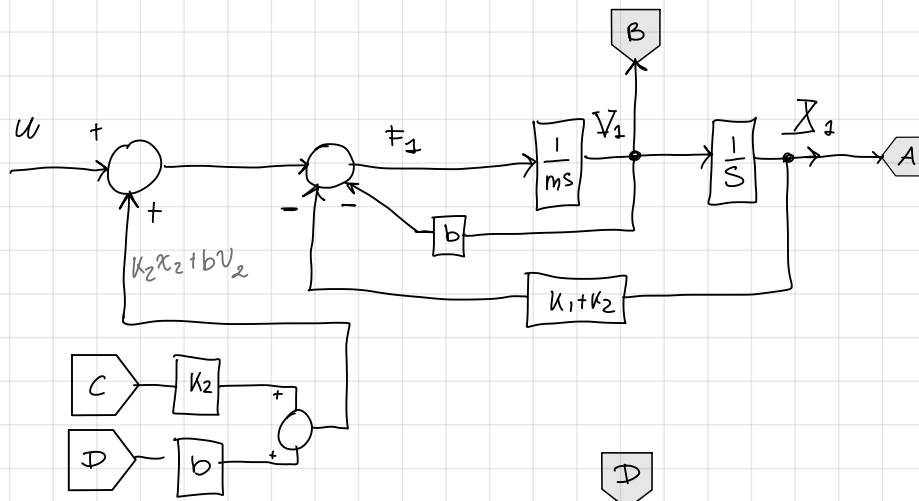
ottengo:

$$\begin{cases} \dot{x}_3 = \frac{1}{m_1} [u - k_1 x_1 - k_2 (x_1 - x_2) - b(x_3 - x_4)] \\ \dot{x}_4 = \frac{1}{m_2} [k_2 (x_1 - x_2) + b(x_3 - x_4) - k_3 x_2] \\ \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \end{cases}$$

$$\underline{A} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{b}{m_1} & \frac{b}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & \frac{b}{m_2} & -\frac{b}{m_2} \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix}$$

$$\underline{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix} \cdot u$$

$$y = x_2 - x_1 = (0, 0, -1, 1)$$



$$m\ddot{x}_2 = -(k_2 + k_3) x_2 - b\dot{x}_2 + k_2 x_1 + b\dot{x}_1$$