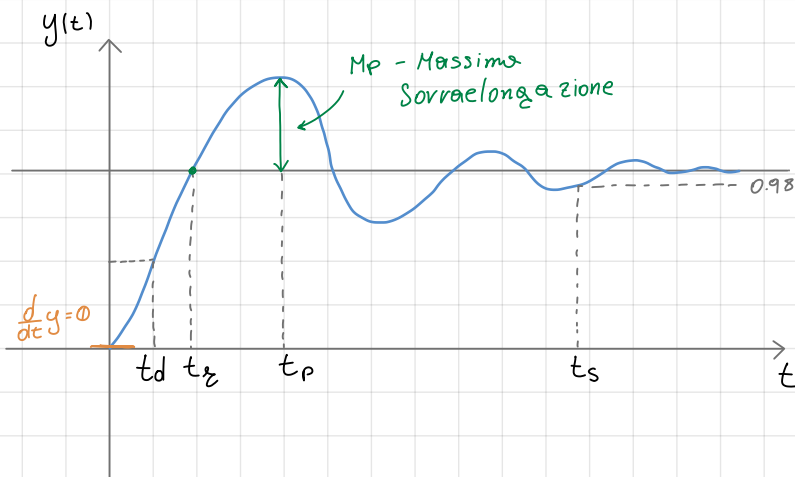


RISPOSTA AL GRADINO (DEL 2nd ORDINE)



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

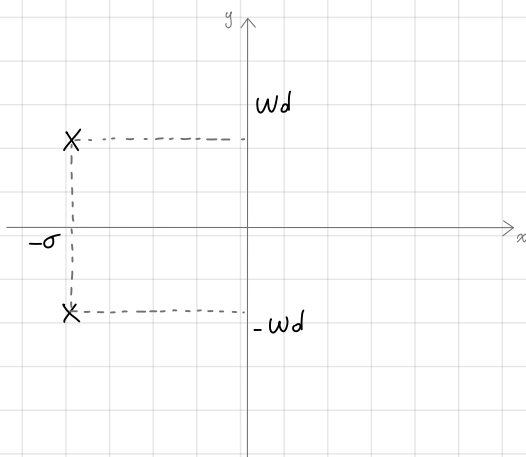
$$\text{Con } 0 < \zeta < 1$$

$$y(t) = 1 \cdot e^{-\zeta\omega_n t} \left(\cos(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t) \right)$$

$$\text{con } \omega_d = \omega_n \sqrt{1-\zeta^2}$$

Poli complessi e coniugati

$$\sigma = -\zeta\omega_n$$



CON CHE VELOCITA' RISPONDE IL SYS?

PARAMETRI

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \cdot 100$$

Tempo di picco

$$\begin{cases} t_r = \text{Rise Time} \\ t_p = \text{Peak Time} \\ t_d = \text{delay Time} \rightarrow \text{Valore per raggiungere } 50\% \text{ } t_r \\ t_s = \text{Settling Time} \end{cases}$$

CALCOLO DEI PARAMETRI

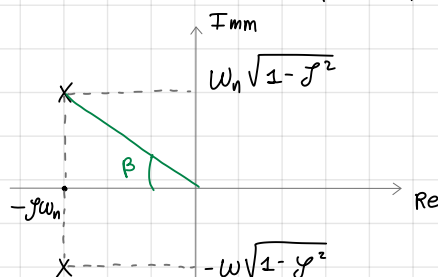
RISE TIME

$$y(t_r) = 1 \quad \leadsto \quad 1 - e^{-\zeta\omega_n t_r} \left(\cos(\omega_d t_r) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r) \right) = 1$$

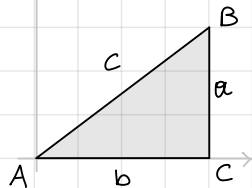
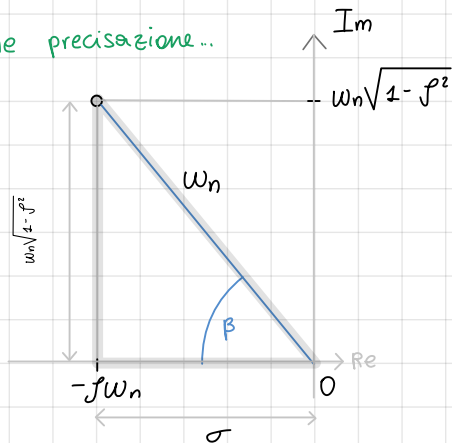
sempre $\neq 0$

$$\text{ovvero} \quad \cos(\omega_d t_r) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r) = 0 \quad \rightarrow \quad \sin(\omega_d t_r) = -\frac{\sqrt{1-\zeta^2}}{\zeta} \cos(\omega_d t_r)$$

$$\Rightarrow \tan(\omega_d t_r) = -\frac{\sqrt{1-\zeta^2}}{\zeta} \quad (a)$$



Qualche precisazione...



$$\begin{cases} \cos(A) = \frac{b}{c} \\ \sin(A) = \frac{a}{c} \end{cases} \Rightarrow \tan(A) = \frac{a}{b}$$

$$\Rightarrow \tan(\beta) = \frac{w_n \sqrt{1-f^2}}{f w_n} = \frac{\sqrt{1-f^2}}{f} \quad (b)$$

Se confrontiamo la (b) con la (a) notiamo che manca un "-", ma

$$-\tan(\alpha) = \tan(\pi - \alpha)$$

Riscriviamo la (a): $\tan(w_d \cdot t_z) = \tan(\pi - \beta) \Rightarrow w_d \cdot t_z = \pi - \beta \Rightarrow$

$$t_z = \frac{\pi - \beta}{w_d}$$

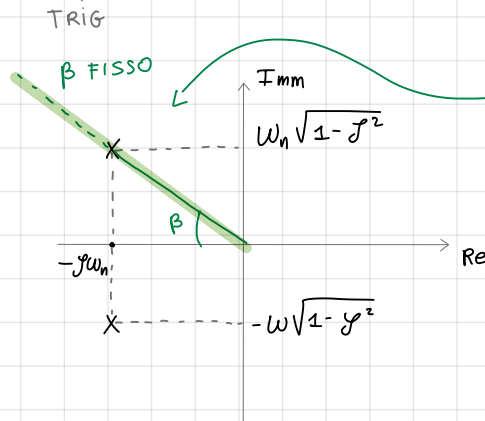
QED

Trigonometria...

$$\omega_n \sqrt{1-\gamma^2} = \gamma \omega_n \tan(\beta) \Rightarrow \tan(\beta) = \frac{\sqrt{1-\gamma^2}}{\gamma} \Rightarrow \omega_d t_z = \pi - \beta$$

proof $\tan(\omega_d t_z) = \tan(\pi - \beta) = -\tan(\beta)$

$$\Rightarrow t_z = \frac{\pi - \beta}{\omega_d}$$



Per Accorciare t_z

pongo $\beta = \cos t$

\Rightarrow Mi serve ω_d piu grande

Siccome $\omega_d = \omega_n \sqrt{1-\gamma^2}$
 \uparrow
 cambio ω_n

PEAK TIME

$y = \text{MAX}$ per la 1st volta

Bisogna notare che la derivata *tecnicamente* si azzera infinite volte, e la prima è in zero. Quindi quella che interessa a noi è la seconda!

$\dot{y}(t) = 0$ anche in $t=0$

$\lim_{s \rightarrow \infty} \underbrace{s}_{\text{TVI}} \cdot \underbrace{\frac{1}{s}}_{\text{Derivata}} \cdot \underbrace{G(s)}_{\text{gradino}} \cdot \underbrace{\frac{1}{s}}_{\text{gradino}} = \lim_{s \rightarrow \infty} \frac{\omega_n^2 s}{s^2 + \dots} = \lim_{s \rightarrow \infty} \omega_n^2 \frac{s}{s^2} \rightarrow 0$

Calcolo la derivata...

quando $\dot{y}(t) = 0$?

$$\frac{d}{dt} y(t) = \gamma \omega_n e^{-\gamma \omega_n t} \left(\cos(\omega_d t) + \frac{\gamma}{\sqrt{1-\gamma^2}} \sin(\omega_d t) \right) - e^{-\gamma \omega_n t} \left(\omega_d (-\sin(\omega_d t) + \frac{\gamma}{\sqrt{1-\gamma^2}} \cos(\omega_d t)) \right) \stackrel{?}{=} 0$$

proof
simplific.

$$\omega_d \frac{\gamma}{\sqrt{1-\gamma^2}} = \omega_n \sqrt{1-\gamma^2} \cdot \frac{\gamma}{\sqrt{1-\gamma^2}} = \omega_n \gamma$$

* \downarrow vedi qiv'

solo il sin può annullarsi

$$\Rightarrow \frac{d}{dt} y(t) = e^{-\gamma \omega_n t} \left(\underbrace{\frac{\gamma^2 \omega_n}{\sqrt{1-\gamma^2}} + \omega_d}_{\text{costante}} \right) \underbrace{\sin(\omega_d t)}_{\text{?}} \stackrel{?}{=} 0$$

ovvero per $\sin(\omega_d t) = 0 \Rightarrow$ per $\omega_d t_p = \pi \Rightarrow t_p = \frac{\pi}{\omega_d}$

$$\dot{y}(t) = f W_n e^{-f W_n t} \left(\cos(W_d t) + \frac{f}{\sqrt{1-f^2}} \sin(W_d t) \right) - e^{-f W_n t} W_d \left(-\sin(W_d t) + \frac{f}{\sqrt{1-f^2}} \cos(W_d t) \right)$$

$$= \cancel{e^{-f W_n t} f W_n \cos(W_d t)} + \frac{f^2 W_n e^{-f W_n t}}{\sqrt{1-f^2}} \sin(W_d t) + e^{-f W_n t} W_d \sin(W_d t) + e^{-f W_n t} \cdot \frac{W_d f}{\sqrt{1+f}} \cos(W_d t)$$

$$= \frac{f^2 W_n e^{-f W_n t}}{\sqrt{1-f^2}} \sin(W_d t) + e^{-f W_n t} W_d \sin(W_d t)$$

$$W_d = W_n \sqrt{1-f^2}$$

$$\hookrightarrow \frac{W_d f}{\sqrt{1+f}} \cos(W_d t) = \frac{W_n \sqrt{1-f^2}}{\sqrt{1-f^2}} \cos(W_d t)$$

$$= \cancel{e^{-f W_n t} W_n \cos(W_d t)}$$

MASSIMA SOVRAELONGAZIONE

$$y(t_p) = 1 - e^{-\gamma \omega_n \cdot \frac{\pi}{\omega_d}} \left(\cos\left(\omega_d \cdot \frac{\pi}{\omega_d}\right) + \frac{\gamma}{\sqrt{1-\gamma^2}} \sin\left(\omega_d \cdot \frac{\pi}{\omega_d}\right) \right) = 1 + e^{-\frac{\gamma \pi}{\sqrt{1-\gamma^2}}} \quad \text{Valore picco } y(t_p)$$

per $\gamma \rightarrow 0$: $1 + e^0 = 2$

$$y(\infty) = \lim_{s \rightarrow 0} s \cdot G(s) \cdot U(s) = \lim_{s \rightarrow 0} s \cdot G(s) \cdot \frac{1}{s} \Rightarrow$$

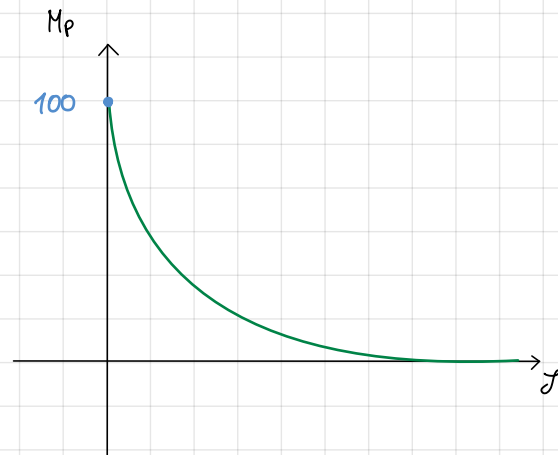
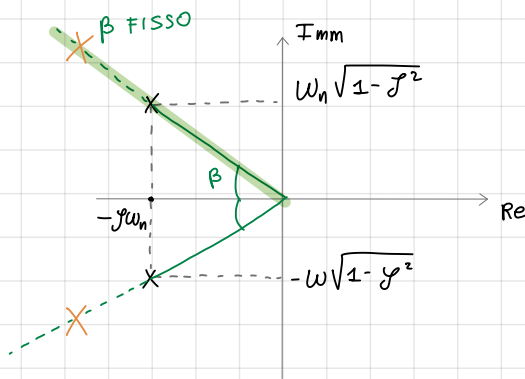
Valore a regime

$$y(\infty) = \lim_{s \rightarrow 0} G(s)$$

guadagno statico

$$M_p = \frac{y(t_p) - 1}{1} \cdot 100 = e^{-\frac{\pi \gamma}{\sqrt{1-\gamma^2}}} \cdot 100$$

funzione solo di $\gamma \rightarrow$ con $\omega_n = \text{cost}$ e uguale



TEMPO DI ASSESTAMENTO

* Scrivere γ come Angolo

* $0 < \gamma < 1$

$$y(t) = 1 - \frac{1}{\sqrt{1-\gamma^2}} e^{-\gamma \omega_n t} \left(\sqrt{1-\gamma^2} \cos(\omega_d t) + \gamma \sin(\omega_d t) \right)$$

\uparrow Sin β \uparrow Cos β

$$= 1 - \frac{1}{\sqrt{1-\gamma^2}} e^{-\gamma \omega_n t} \sin(\omega_d t + \beta)$$

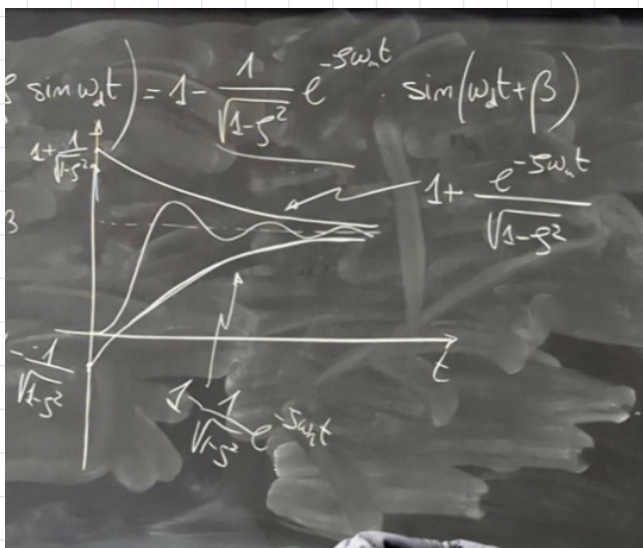
$$\sin^2 \alpha + \cos^2 \alpha = 1, \forall \alpha$$

Il sin resta sempre all'interno di una coppia di curve di inviluppo

curve: $1 \pm \frac{1}{\sqrt{1-\gamma^2}} e^{-\gamma \omega_n t}$

Si considera $4/5 \tau$

$$\Rightarrow \tau \frac{1}{\gamma \omega_n} \Rightarrow t_s = \frac{4}{\gamma \omega_n}$$

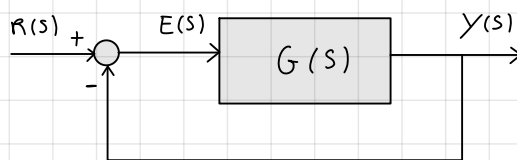


Recap

$$e^{-\frac{t}{\tau}} = e^{-2t} \Rightarrow \tau = \frac{1}{2}$$

ESEMPIO 5.1

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$



DATI

$$\zeta = 0.6$$

$$\omega_n = 5 \text{ rad/s}$$

Q: Calcolare t_z , t_p , M_p e t_s con $U(t) = 1(t)$

$$U(t) = 1(t) \Rightarrow U(s) = \frac{1}{s}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 4$$

$$\sigma = \zeta\omega_n = 3$$

$$t_z = \frac{\pi - \beta}{\omega_d} \quad \text{con} \quad \omega_d \quad \beta \quad \zeta\omega_n$$

$$= 0.55 \text{ secondi} \quad \text{Ans 1}$$

$$\tan(\beta) = \frac{\omega_d}{\sigma}$$

$$\Rightarrow \beta = \arctan\left(\frac{\omega_d}{\sigma}\right)$$

$$= 0.93 \text{ rad}$$

$$t_p = \frac{\pi}{\omega_d} = 0.78 \text{ s} \quad \text{Ans 2}$$

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)}$$

$$\text{con } y(\infty) = \lim_{s \rightarrow 0} s \cdot F(s) \quad \text{con } F(s) = Y(s) = G(s) \cdot U(s)$$

$$\leadsto Y(s) = \frac{\omega_n^2}{s^2(s + 2\zeta\omega_n)} = \frac{\xi_1}{s^2} + \frac{\xi_2}{s + 2\zeta\omega_n} \quad \text{[LUNGO]}$$

$$\text{calcolo invece } M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \cdot 100 = e^{-\left(\frac{\sigma}{\omega_d}\right)\pi} = 9.48 \times 10^{-2} \approx 9.5\%$$

$$\text{Ans 3}$$

$$t_s = \frac{4}{\omega_d \zeta} = \frac{4}{\sigma} = 1.3 \text{ s}$$

$$\text{Ans 4}$$

(RECAP)

Block diagram: $U(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$

$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad 0 < \zeta < 1$

$G(s) = \frac{Y(s)}{U(s)} \Big|_{u(t)=\delta(t)} = Y(s)$

$G(s) = \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_d^2}$ *proof*

$s^2 + (\zeta\omega_n)^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2(1 - \zeta^2) + (\zeta\omega_n)^2$
 $= s^2 + 2\zeta\omega_n s + \omega_n^2$

$G(s) = \frac{\omega_n^2}{\omega_d} \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} \rightarrow \frac{\omega_n^2}{\omega_d \sqrt{1 - \zeta^2}} \cdot e^{-\zeta\omega_n t} \cdot \sin \omega_d t$

$y(t)$ *Risposta impulsiva*

$0 < \zeta < 1$

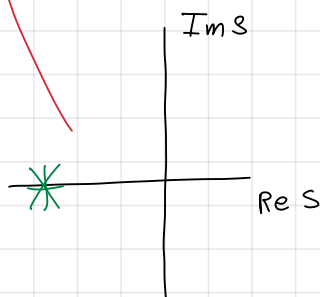
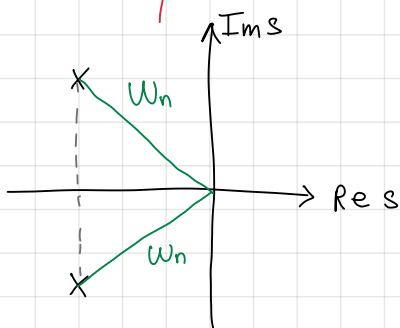
$G(s) = \frac{\omega_n^2}{\omega_d} \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} \cdot \frac{\omega_n^2}{\omega_d \sqrt{1 - \zeta^2}} \cdot e^{-\zeta\omega_n t} \cdot \sin \omega_d t$

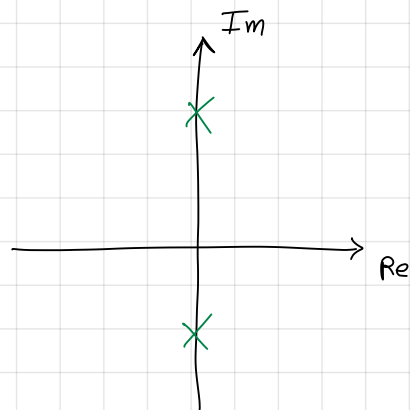
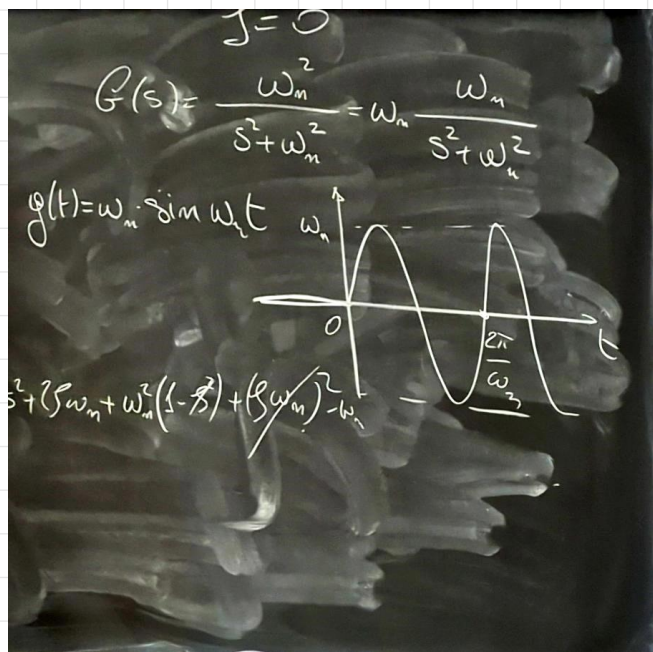
$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \zeta = 1$

$G(s) = \frac{\omega_n^2}{(s + \omega_n)^2}$

$g(t) = \omega_n^2 t \cdot e^{-\omega_n t}$

$e^{-\omega_n t} + \omega_n t e^{-\omega_n t} = 0$
 $(1 - \omega_n t) e^{-\omega_n t} = 0 \quad t^* = \frac{1}{\omega_n}$

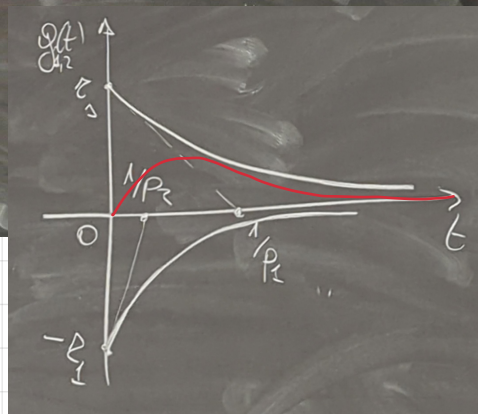
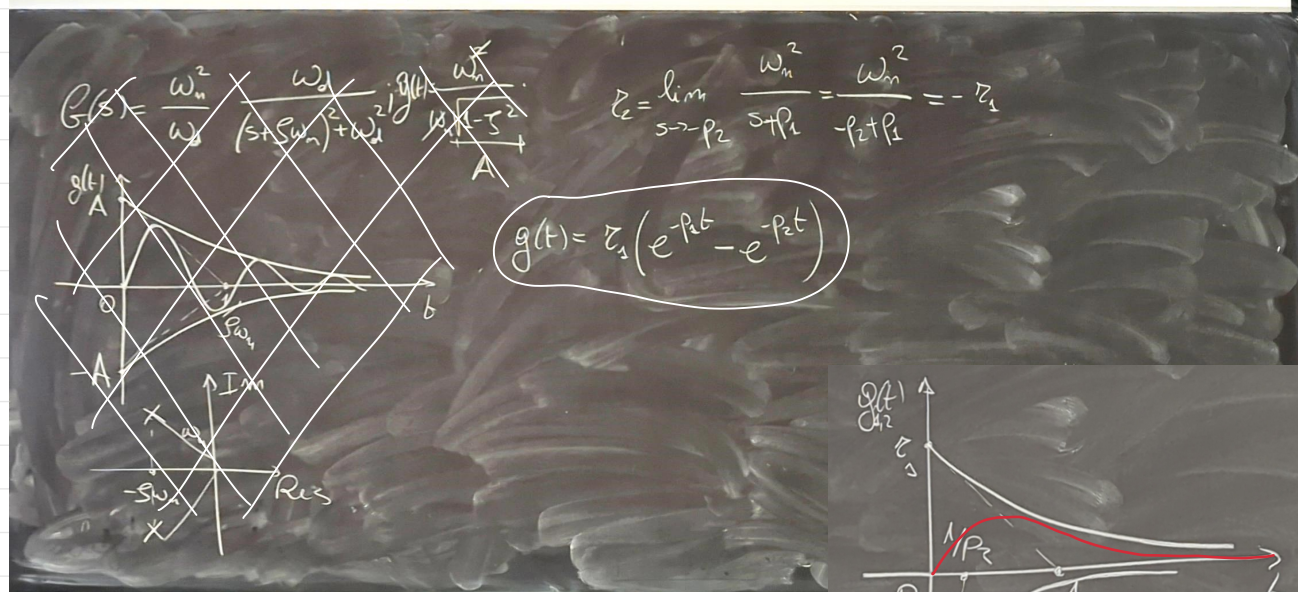
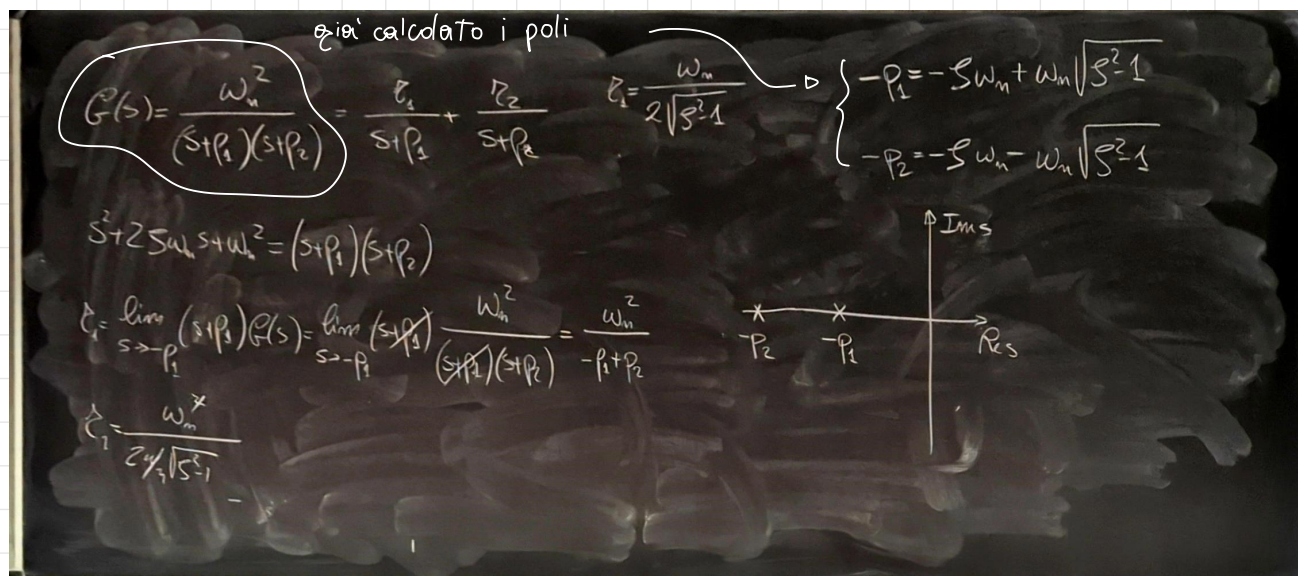




$\zeta > 0$

SOVRASMOZZATO

AUDIO * Diversi CASI

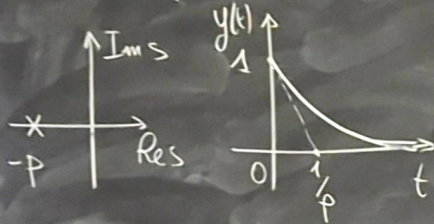


COME VALUTARE UN SYS

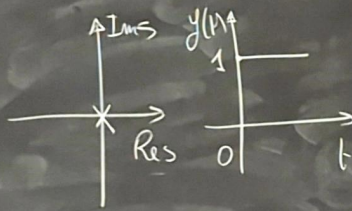
* Posizione del polo

$$\frac{1}{s+p} \xrightarrow{\mathcal{L}^{-1}} e^{-pt}$$

$$\frac{1}{s+p} \xrightarrow{\mathcal{L}^{-1}} e^{-pt}$$



$$\frac{1}{s} \xrightarrow{\mathcal{L}^{-1}} 1(t)$$



$$\frac{1}{s^2} \xrightarrow{\mathcal{L}^{-1}} t \cdot 1(t)$$

