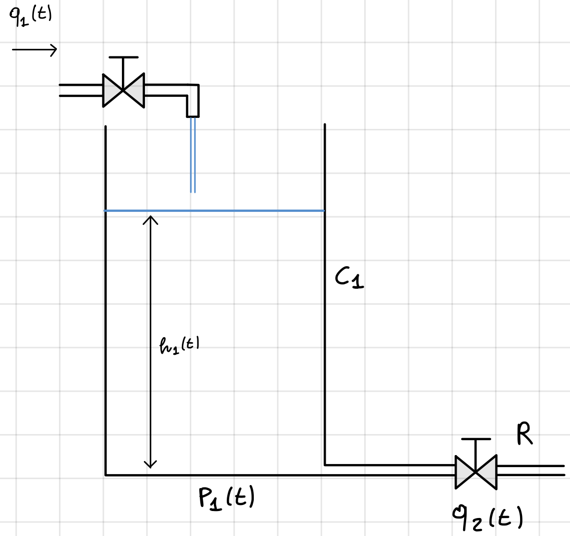
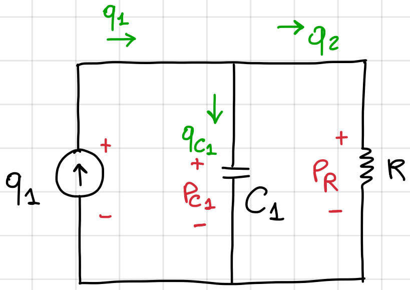
SINGOLO SERBATOIO





$$q_{C_1} = C_1 \frac{dP_{C_1}}{dt}$$

ma $q_{C_1} = q_1 - q_2$

$$\Rightarrow C_1 \frac{dP_{C_1}}{dt} = q_1 - q_2$$

$$P_R = R \cdot q_2 \rightarrow q_2 = \frac{P_R}{R}$$

ma $P_R = P_{C_1} \Rightarrow C_1 \frac{dP_{C_1}}{dt} = q_1 - \frac{P_{C_1}}{R}$

FUNZIONE DI TRASFERIMENTO

$$S C_1 P_{C_1}(S) = Q_1(S) - \frac{1}{R_1} P_{C_1}(S) \rightarrow$$

$$\rightarrow P_{C_1}\left(SC_1 + \frac{1}{R_1}\right) = Q_1(S) \rightarrow P_{C_1}(S) = \frac{R_1}{SC_1R_1 + 1} Q(S)$$

Se voglio l'altezza

$\rho \cdot g \cdot h = P - Pa$ se $P >> Pa$

$$\Rightarrow h = \frac{P_1}{\rho \cdot g}$$

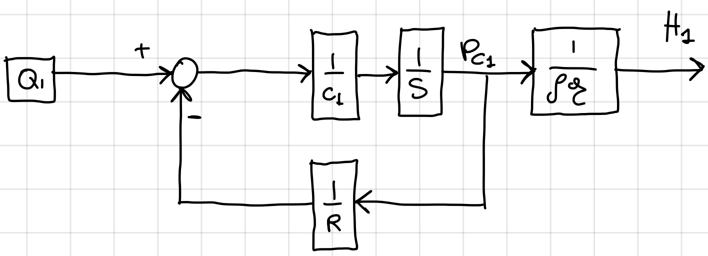
- Se IN: $Q(S)$ e OUT: $P_{C_1}(S)$

$$\Rightarrow \frac{P_{C_1}(S)}{Q(S)} = G_1(S) = \frac{R_1}{SC_1R_1 + 1}$$

- Se IN: $Q(S)$ e OUT: $H(S) = \frac{P_1(S)}{\rho g} = \frac{R_1}{\rho g(SC_1R_1 + 1)} Q(S)$

$$\Rightarrow G_2(S) = \frac{R_1}{\rho g(SC_1R_1 + 1)}$$

SCHEMA A BLOCCHI

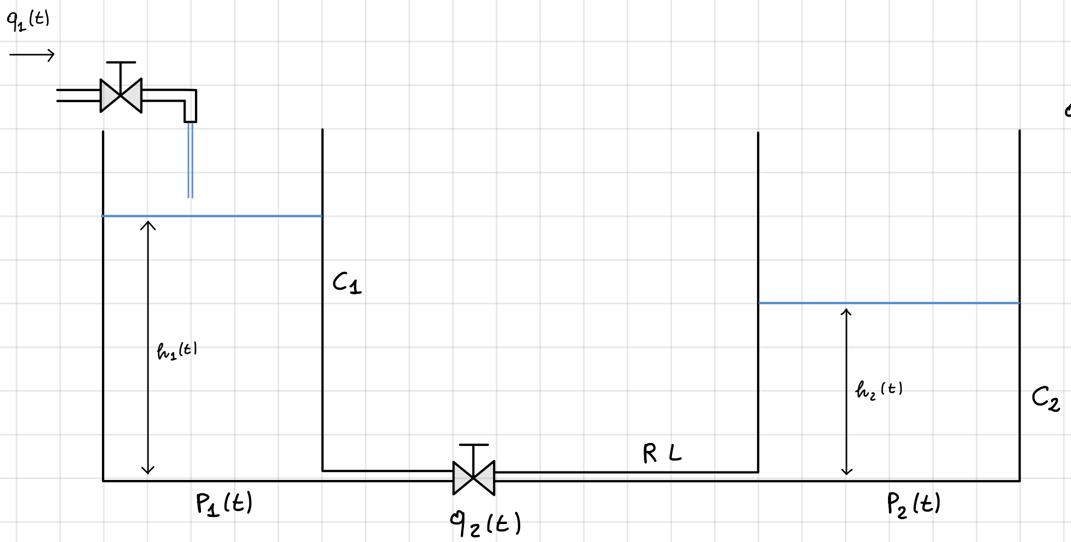


SPAZIO DI STATO

$X_1 = P_{C_1}$ $\quad U = q_1$ $\quad y = P_{C_1}$

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1} \cdot U - \frac{1}{C_1R_1} x_1 \\ y = x_1 \end{cases} \rightarrow \begin{cases} \dot{x}_1 = -\frac{1}{C_1R_1} x_1 + \frac{1}{C_1} U \\ y = 1 \quad x_1 + 0 \quad U \end{cases}$$

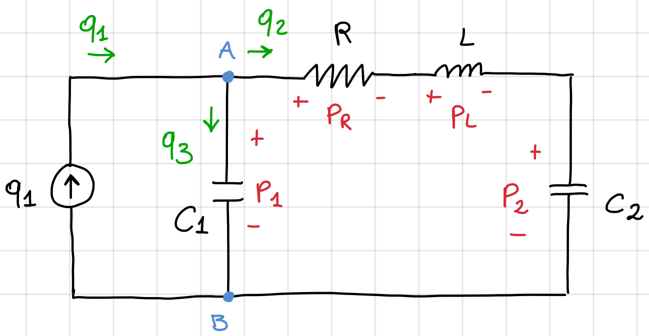
ESEMPIO 4.10 — Lezione 14



$$q_a - q_b = \frac{S}{\rho g} \cdot \frac{dP}{dt} \quad (C)$$

$$R_{id} = \frac{\Delta P}{q}$$

$$\Delta P = \frac{\rho h}{S} \cdot \frac{dq}{dt} \quad (L)$$



Rel Caratt

$$\begin{cases} v_L = L \cdot \dfrac{di_L}{dt} \\ i_C = C \cdot \dfrac{dv_C}{dt} \end{cases}$$

(P ↔ v, q ↔ i)

$$\begin{cases} q_3 = C_1 \dfrac{dP_1}{dt} \Rightarrow q_1 - q_2 = C_1 \dfrac{dP_1}{dt} \\ q_2 = C_2 \cdot \dfrac{dP_2}{dt} \\ P_L = -P_R + P_1 - P_2 = -R \cdot q_2 + P_1 - P_2 = L \dfrac{dq_2}{dt} \end{cases}$$

SPAZIO DI STATO

Scelgo $x_1 = P_1$; $x_2 = P_2$; $x_3 = q_2$; $u = q_1$; $y = h_2$

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1}(u - x_3) \\ \dot{x}_2 = \dfrac{1}{C_2}(x_3) \\ \dot{x}_3 = \dfrac{1}{L}(-R \cdot x_3 + x_1 - x_2) \\ y = \dfrac{1}{\rho g} x_2 \end{cases}$$

Se abbiamo un cilindro di lunghezza e molto Alto $\Rightarrow P >> P_a$

Pressione alla base — Pressione ATM all'apice

$\Rightarrow$ Bernoulli : $\rho \cdot g \cdot h \cong P$

$$\Rightarrow h = \frac{P_2}{\rho g}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \underbrace{\begin{pmatrix} \frac{1}{C_1} \\ 0 \\ 0 \end{pmatrix}}_{B} \cdot u$$

$$y = \underbrace{\begin{pmatrix} 0 & \frac{1}{\rho g} & 0 \end{pmatrix}}_{C'} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \underbrace{(0)}_{D} \cdot u$$

$$\begin{cases} sC_1\, P_1(s) = Q_1(s) - Q_2(s) & (1) \\ sC_2\, P_2(s) = Q_2(s) & (2) \\ sL\, Q_2(s) = -R\,Q_2(s) + P_1(s) - P_2(s) & (3) \end{cases}$$

$$\boxed{H_2(s) = \frac{1}{\rho g} P_2(s)}$$

$$P_2 = \frac{1}{sC_2} Q_2 \qquad ; \qquad Q_2(s) = \frac{1}{R}P_1 - \frac{1}{R}P_2 - \frac{sL}{R}Q_2$$

$$P_1 = \frac{Q_1}{sC_1} - \frac{Q_2}{sC_1} \quad \rightarrow \quad Q_2(s) = \frac{1}{sC_1R} Q_1 - \frac{1}{sRC_1} Q_2 - \frac{1}{R} P_2 - \frac{sL}{R} Q_2$$

$$\rightarrow \quad Q_2\left(1 + \frac{1}{sRC_1} + \frac{sL}{R}\right) = \frac{1}{sC_1R} Q_1 - \frac{1}{R} P_2 \quad \rightsquigarrow \quad \frac{sRC_1 + 1 + s^2LC_1}{sRC_1}$$

$$\rightarrow \quad Q_2 = \frac{1}{sRC_1 + 1 + s^2LC_1} Q_1 - \frac{sC_1}{sRC_1 + 1 + s^2LC_1} P_2$$

$$\Rightarrow \quad P_2 = \frac{1}{s^2RC_1C_2 + sC_2 + s^3LC_1C_2} Q_1 - \frac{C_1}{sRC_1C_2 + C_2 + s^2LC_1C_2} P_2$$

$$\Rightarrow \quad P_2\left(1 + \frac{C_1}{sRC_1C_2 + C_2 + s^2LC_1C_2}\right) = \frac{1}{s^2RC_1C_2 + sC_2 + s^3LC_1C_2} Q_1$$

$$P_2\left(\frac{sRC_1C_2 + C_2 + s^2LC_1C_2 + C_1}{sRC_1C_2 + C_2 + s^2LC_1C_2}\right) = \frac{1}{s(s^2LC_1C_2 + sRC_1C_2 + C_2)} Q_1$$

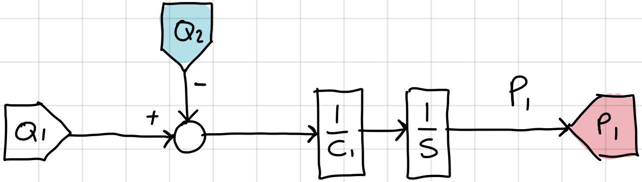
$$P_2(s) = \frac{\cancel{s^2LC_1C_2 + sRC_1C_2 + C_2}}{s^2LC_1C_2 + sRC_1C_2 + C_1 + C_2} \cdot \frac{1}{s(\cancel{s^2LC_1C_2 + sRC_1C_2 + C_2})} Q_1$$

$$\Rightarrow \quad P_2(s) = \frac{1}{s(s^2LC_1C_2 + sRC_1C_2 + C_1 + C_2)} Q_1(s) \quad \Rightarrow \quad H_2(s) = \frac{\frac{1}{\rho g} Q_1(s)}{s(s^2LC_1C_2 + sRC_1C_2 + C_1 + C_2)}$$

Voglio la FDT tra IN: $Q_1(s)$ e OUT: $H_2(s)$

$$\Rightarrow \quad \frac{H_2(s)}{Q_1(s)} = G_1(s) = \frac{\dfrac{\frac{1}{\rho g}\cancel{Q_1(s)}}{s(s^2LC_1C_2 + sRC_1C_2 + C_1 + C_2)}}{\cancel{Q_1(s)}} = \boxed{\frac{\frac{1}{\rho g}}{s(s^2LC_1C_2 + sRC_1C_2 + C_1 + C_2)}}$$

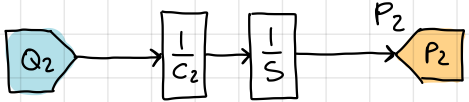
SCHEMA A BLOCCHI

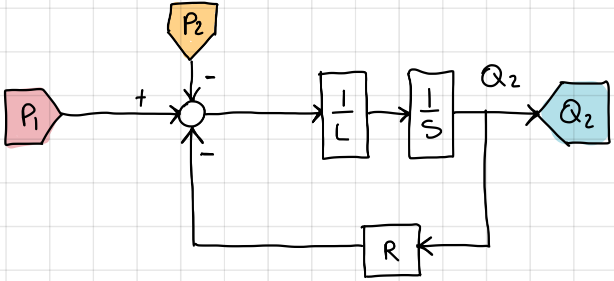


$$q_1 - q_2 = C_1 \frac{dP_1}{dt}$$

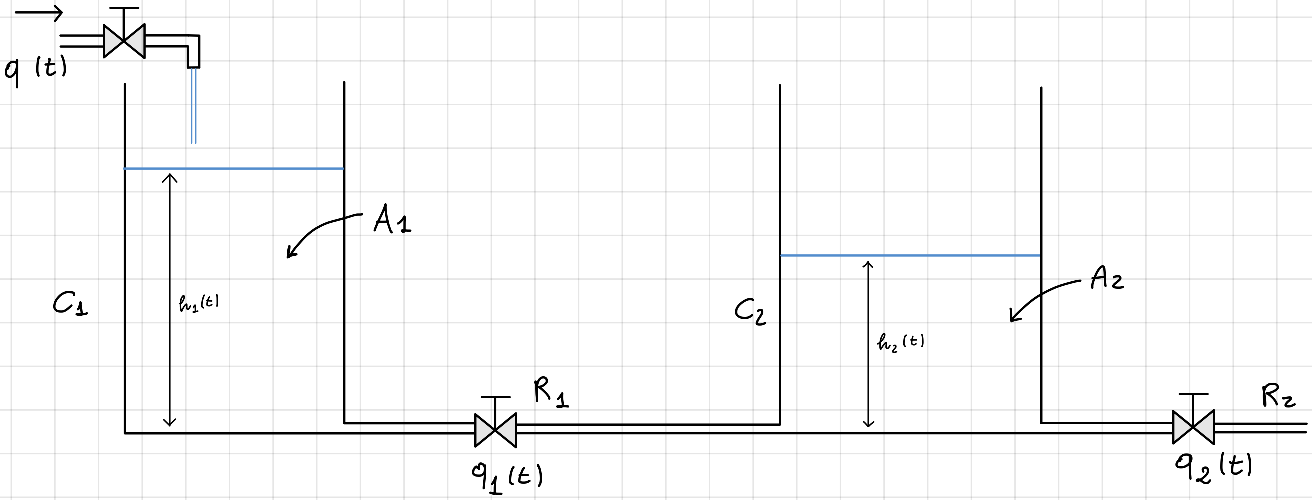
$$q_2 = C_2 \cdot \frac{dP_2}{dt}$$

$$-R \cdot q_2 + P_1 - P_2 = L \frac{d q_2}{dt}$$





SERBATOI CON UN SOLO FLUSSO DI ENTRATA



TANK 1 $\begin{cases} q - q_1 = A_1 \dfrac{dh_1}{dt} \; ; \; q_1 = \dfrac{1}{R_1}(h_1 - h_2) \\ q_1 - q_2 = A_2 \dfrac{dh_2}{dt} \; ; \; q_2 = \dfrac{h_2}{R_2} \end{cases}$ TANK 2

Voglio OUT: H_1 IN: Q

$$\begin{cases} 1) \; q - \dfrac{h_1 - h_2}{R_1} = A_1 \dfrac{dh_1}{dt} \rightleftharpoons SA_1H_1 = Q - \dfrac{H_1}{R_1} + \dfrac{H_2}{R_1} \\ 2) \; \dfrac{h_1 - h_2}{R_1} - \dfrac{h_2}{R_2} = A_2 \dfrac{dh_2}{dt} \rightleftharpoons SA_2H_2 = \dfrac{H_1}{R_1} - \dfrac{H_2}{R_1} - \dfrac{H_2}{R_2} \to H_2\left(SA_2 + \dfrac{1}{R_1} + \dfrac{1}{R_2}\right) = \dfrac{H_1}{R_1} \end{cases}$$

$$\Rightarrow H_1 = H_2\left(SR_1A_2 + 1 + \frac{R_1}{R_2}\right)$$

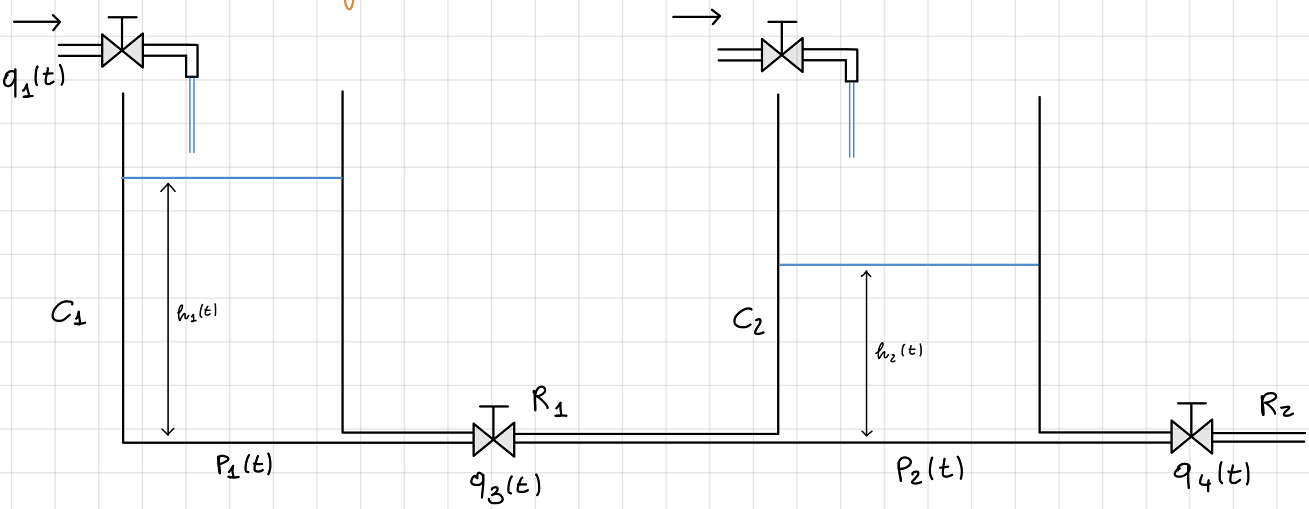
Nello 1 $\to$ $H_1 \left(\underbrace{SA_1 + \frac{1}{R_1}}_{\frac{SA_1R_1+1}{R_1}}\right) = Q + \frac{1}{R_1} H_2 \to H_1 = \frac{R_1}{SA_1R_1+1} + \frac{1}{SA_1R_1+1} H_2$

$$\to H_2\left(\frac{SR_1R_2A_2 + R_2 + R_1}{R_2}\right) = \frac{R_1}{SA_1R_1+1} Q + \frac{1}{SA_1R_1+1} H_2$$

$$\to H_2\left(\frac{SR_1R_2A_2 + R_2 + R_1}{R_2} - \frac{1}{SA_1R_1+1}\right) = \frac{R_1}{SA_1R_1+1} Q \to H_2\left(\frac{SR_1R_2A_2 + R_2 + R_1 \; (SA_1R_1+1) - R_2}{R_2(SA_1R_1+1)}\right) = \frac{R_1}{//} Q$$

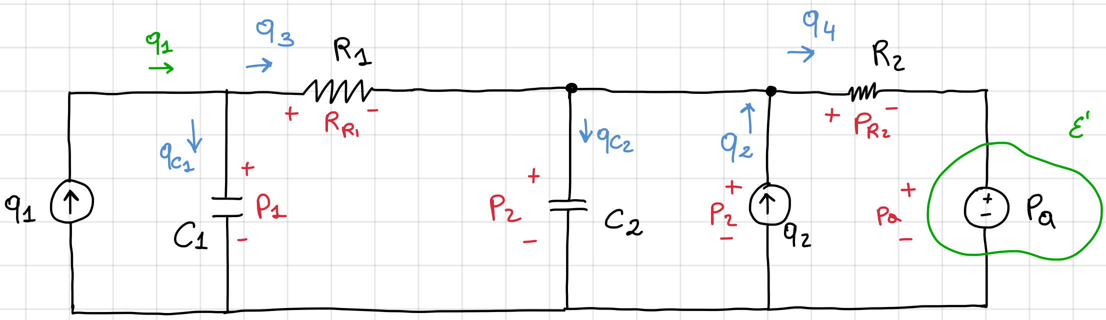
$$H_2\left(\frac{S^2R_1^2R_2A_1A_2 + SR_1R_2A_2 + SA_1R_1R_2 + R_2 + SA_1R_1^2 + R_1 - R_2}{SA_1R_1R_2 + R_2}\right) = \frac{R_1}{SA_1R_1+1} Q$$

Serbatoi con flussi separati



$* P_a \neq \emptyset$

Circuito equivalente

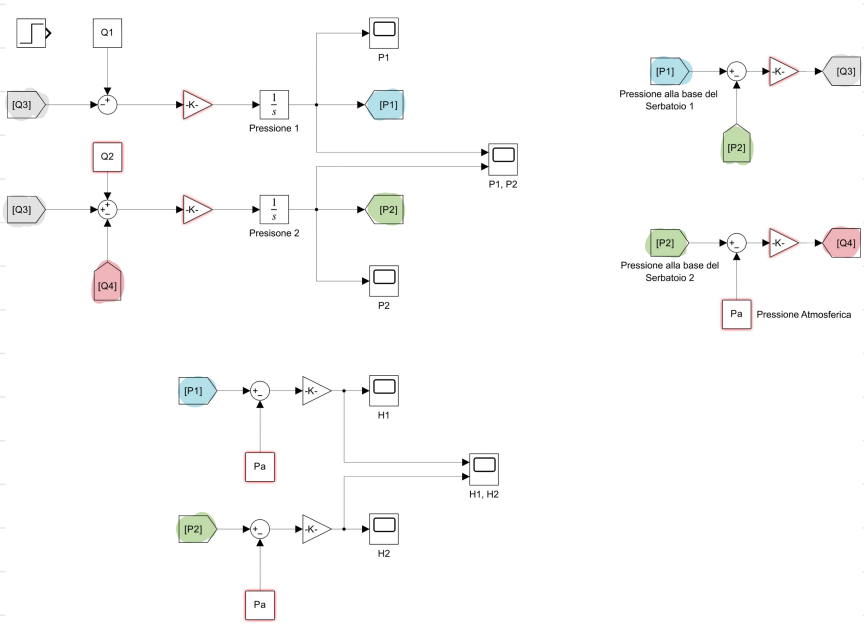


É la ΔPressione tra P_2 e P_a, se $P_2 = P_a \Rightarrow \Delta P = \emptyset$;
Se $P_2 >> P_a \rightarrow P_a \simeq \emptyset \Rightarrow$ Non c'è il gen Tens.

$$
\begin{cases}
q_{C_1} = C_1 \dfrac{dP_1}{dt} \rightarrow q_{C_1} + q_3 = q_1 \rightarrow q_{C_1} = q_1 - q_3 \Rightarrow C_1 \dfrac{dP_1}{dt} = q_1 - q_3 \\
q_{C_2} = C_2 \dfrac{dP_2}{dt} \rightarrow q_{C_2} + q_4 - q_3 - q_2 = 0 \rightarrow q_{C_2} = q_2 + q_3 - q_4 \Rightarrow C_2 \dfrac{dP_2}{dt} = q_2 + q_3 - q_4 \\
P_{R_1} = R_1 \cdot q_3 \rightarrow q_3 = \dfrac{P_{R_1}}{R_1} \quad \text{ma LKT: } P_{R_1} = P_1 - P_2 \Rightarrow q_3 = \dfrac{P_1 - P_2}{R_1} \\
P_{R_2} = R_2 \cdot q_4 \rightarrow q_4 = \dfrac{P_{R_2}}{R_2} \quad \text{ma LKT: } P_{R_2} = P_2 - P_a \Rightarrow q_4 = \dfrac{P_2 - P_a}{R_2}
\end{cases}
$$

SCHEMA A BLOCCHI

Da Bernoulli: $\rho g h = P - P_a \Rightarrow h = \dfrac{P_h - P_a}{\rho g}$



$$
\begin{cases}
\dfrac{dP_1}{dt} = \dfrac{1}{C_1}\left(q_1 - \dfrac{P_1 - P_2}{R_1}\right) \\
\dfrac{dP_2}{dt} = \dfrac{1}{C_2}\left(q_2 + \dfrac{P_1 - P_2}{R_1} - \dfrac{P_2 - P_a}{R_2}\right)
\end{cases}
$$

SPAZIO DI STATO

Scelgo $x_1 = P_1$ $x_2 = P_2$ $U_1 = q_1$ $U_2 = q_2$

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1}\left(U_1 - \frac{x_1}{R_1} + \frac{x_2}{R_1}\right) \\ \dot{x}_2 = \frac{1}{C_2}\left(U_2 + \frac{x_1}{R_1} - \frac{x_2}{R_1} - \frac{x_2}{R_2} + \cancel{\frac{P_a}{R_2}}\right) \end{cases}$$

$y = h_1 = \frac{P_1 - \cancel{P_a}}{\rho g}$ Trascuro P_a

$\Rightarrow y_1 = \frac{P_1}{\rho g}$; $y_2 = \frac{P_2}{\rho g}$

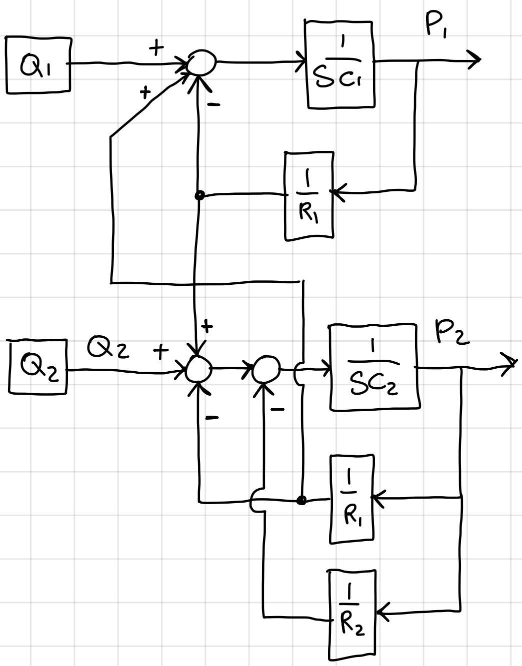
$$\begin{cases} y_1 = \frac{1}{\rho g} \cdot x_1 \\ y_2 = \frac{1}{\rho g} x_2 \\ y_3 = \frac{1}{\rho g}(x_1 - x_2) \end{cases}$$ Differenza di Altezza

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{C_1 R_1} & \frac{1}{C_1 R_1} \\ \frac{1}{C_2 R_1} & -\frac{R_2 + R_1}{C_2 R_1 R_2} \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho g} & 0 \\ 0 & \frac{1}{\rho g} \\ \frac{1}{\rho g} & -\frac{1}{\rho g} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 0 \cdot U$$

SCHEMA A BLOCCHI

A Mano



Funzione di Trasferimento

IN Q_1 OUT P_1

$$\begin{cases} C_1 \dfrac{dP_1}{dt} = q_1 - q_3 \\ C_2 \dfrac{dP_2}{dt} = q_2 + q_3 - q_4 \end{cases} \rightarrow \begin{cases} \dfrac{dP_1}{dt} = \dfrac{1}{C_1}\left(q_1 - \dfrac{P_1 - P_2}{R_1}\right) \\ \dfrac{dP_2}{dt} = \dfrac{1}{C_2}\left(q_2 + \dfrac{P_1 - P_2}{R_1} - \dfrac{P_2 - \cancel{Pa}}{R_2}\right) \end{cases}$$

Assumo $Pa \cong 0$

$$\begin{cases} q_3 = \dfrac{P_1 - P_2}{R_1} & \rightleftharpoons \quad Q_3 = \dfrac{1}{R_1} P_1(s) - \dfrac{1}{R_1} P_2(s) \\ q_4 = \dfrac{P_2 - Pa}{R_2} & \rightleftharpoons \quad Q_4 = \dfrac{1}{R_2} P_2(s) \end{cases}$$

$$\begin{cases} sC_1 P_1 = Q_1 - \dfrac{1}{R_1} P_1 + \dfrac{1}{R_1} P_2 \\ sC_2 P_2 = Q_2 + \dfrac{1}{R_1} P_1 - \dfrac{1}{R_1} P_2 - \dfrac{1}{R_2} P_2 \end{cases}$$

dalla 1 $\quad P_1\left(sC_1 + \dfrac{1}{R_1}\right) = Q_1 + \dfrac{1}{R_1} P_2 \quad \leftrightarrow \quad P_2 = P_1(sC_1 R_1 + 1) - R_1 Q_1$

dalla 2 $\quad P_2\left(sC_2 + \dfrac{1}{R_1} + \dfrac{1}{R_2}\right) = Q_2 + \dfrac{1}{R_1} P_1 \quad \rightarrow \quad P_2 = \dfrac{R_1 R_2}{sC_2 R_1 R_2 + R_1 + R_2} Q_2 + \dfrac{R_2}{sC_2 R_1 R_2 + R_1 + R_2} P_1$

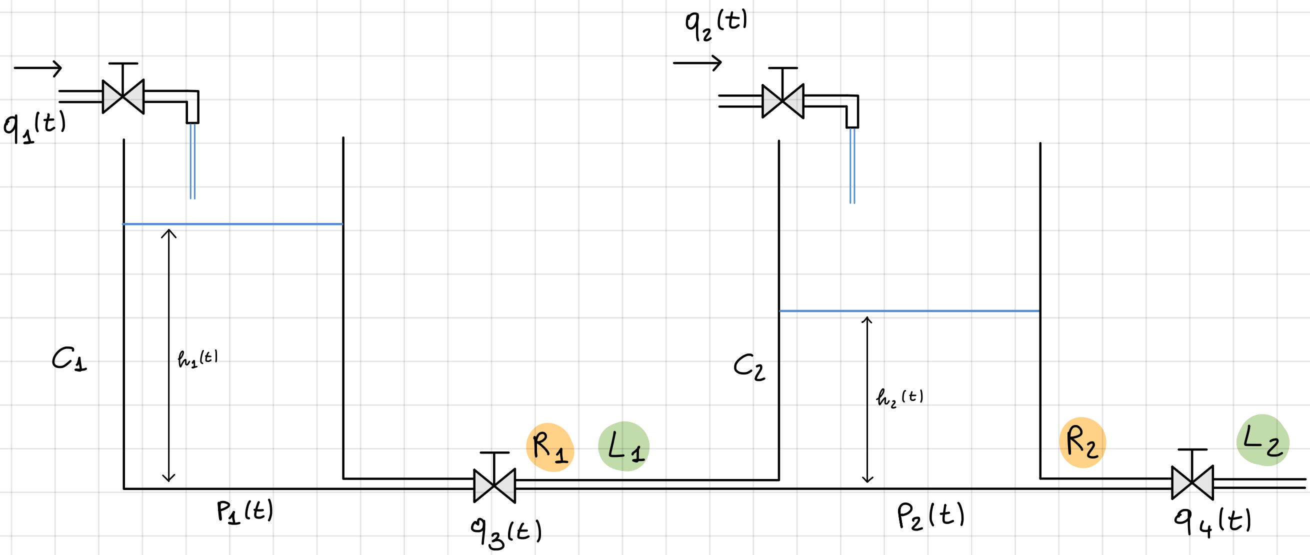
$$P_1\left(sC_1 + \frac{1}{R_1}\right) = Q_1 + \frac{R_2}{sC_2 R_1 R_2 + R_1 + R_2} Q_2 + \frac{R_2}{R_1(sC_2 R_1 R_2 + R_1 + R_2)} P_1$$

$$\Rightarrow P_1\left(sC_1 + \frac{1}{R_1} - \frac{R_2}{R_1(sC_2 R_1 R_2 + R_1 + R_2)}\right) = Q_1 + \frac{R_2}{sC_2 R_1 R_2 + R_1 + R_2} Q_2$$

$$P_1 = \frac{sC_2 R_1 R_2 + R_1 + R_2}{s^2 C_1 C_2 R_1 R_2 + s(C_1 R_1 + C_1 R_2 + C_2 R_2) + 1} \cdot \left(Q_1 + \frac{R_2}{sC_2 R_1 R_2 + R_1 + R_2} Q_2\right)$$

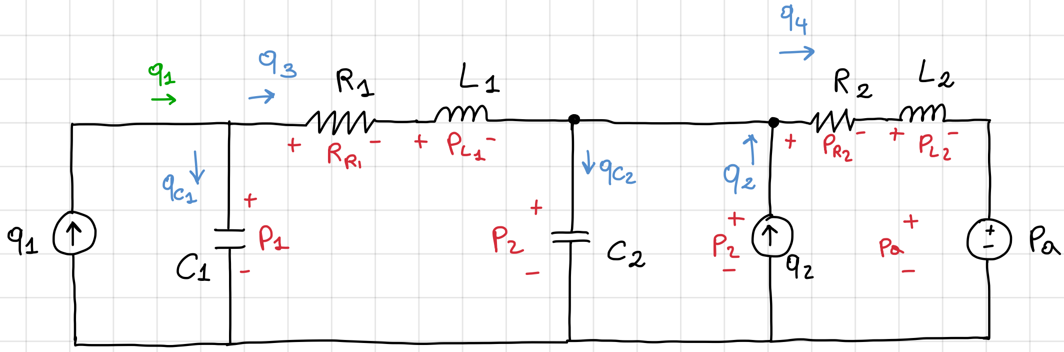
Serbatoi con flussi separati ed INDUTTANZE

MODELLO ESAME



$* P_a \neq \emptyset$

CIRCUITO ASSOCIATO



$$\begin{cases} C_1 \dfrac{dP_1}{dt} = q_{c_1} & q_{c_1} = q_1 - q_3 \rightarrow C_1 \dfrac{dP_1}{dt} = q_1 - q_3 \\ C_2 \dfrac{dP_2}{dt} = q_{c_2} & q_3 - q_{c_2} + q_2 - q_4 = \emptyset \rightarrow q_{c_2} = q_2 + q_3 - q_4 \rightarrow C_2 \dfrac{dP_2}{dt} = q_2 + q_3 - q_4 \end{cases}$$

$$\begin{cases} L_1 \dfrac{dq_3}{dt} = P_{L_1} & P_{L_1} = -P_{R_1} + P_1 - P_2 \rightarrow L_1 \dfrac{dq_3}{dt} = -R_1 q_3 + P_1 - P_2 \\ L_2 \dfrac{dq_4}{dt} = P_{L_2} & P_{L_2} = -P_{R_2} + P_2 - P_a \rightarrow L_2 \dfrac{dq_4}{dt} = -R_2 q_4 + P_2 - P_a \end{cases}$$

DISTURBO

$$\begin{cases} P_{R_1} = R_1 \cdot q_3 \rightarrow q_3 = \dfrac{P_{R_1}}{R_1} & P_{R_1} = P_1 - P_2 - P_{L_1} \Rightarrow \\ P_{R_2} = R_2 \cdot q_4 \rightarrow q_4 = \dfrac{P_{R_2}}{R_2} & P_{R_2} = P_2 - P_a - P_{L_2} \Rightarrow \end{cases}$$

$$\boxed{\begin{aligned} q_3 &= \frac{P_1 - P_2 - P_{L_1}}{R_1} \\ q_4 &= \frac{P_2 - P_a - P_{L_2}}{R_2} \end{aligned}} \quad (1)$$

$$\begin{cases} C_1 \dfrac{dP_1}{dt} = q_1 - \dfrac{P_1 - P_2 - P_{L_1}}{R_1} \\ C_2 \dfrac{dP_2}{dt} = q_2 + \dfrac{P_1 - P_2 - P_{L_1}}{R_1} - \dfrac{P_2 - P_a - P_{L_2}}{R_2} \end{cases}$$

$x_1 = P_1 \quad x_2 = P_2 \quad x_3 = q_3 \quad x_4 = q_4 \quad U_1 = q_1 \quad U_2 = q_2 \quad U_3 = P_a \quad y = P_1$

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1}\left(U_1 - x_3\right) \\ \dot{x}_2 = \frac{1}{C_2}\left(U_2 + x_3 - x_4\right) \\ \dot{x}_3 = \frac{1}{L_1}\left(-R_1 x_3 + x_1 - x_2\right) \\ \dot{x}_4 = \frac{1}{L_2}\left(-R_2 x_4 + x_2 - U_3\right) \\ y = x_1 \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = 0x_1 + 0x_2 - \frac{1}{C_1}x_3 + 0x_4 + \frac{1}{C_1}U_1 + 0U_2 + 0U_3 \\ \dot{x}_2 = 0x_1 + 0x_2 + \frac{1}{C_2}x_3 - \frac{1}{C_2}x_4 + 0U_1 + \frac{1}{C_2}U_2 + 0U_3 \\ \dot{x}_3 = \frac{1}{L_1}x_1 - \frac{1}{L_1}x_2 - \frac{R_1}{L_1}x_3 + 0x_4 + 0U_1 + 0U_2 + 0U_3 \\ \dot{x}_4 = 0x_1 + \frac{1}{L_2}x_2 + 0x_3 - \frac{R_2}{L_2}x_4 + 0U_1 + 0U_2 - \frac{1}{L_2}U_3 \\ y = 1\,x_1 \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} & 0 \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{C_2} \\ \frac{1}{L_1} & -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ 0 & \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_2} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T + \underline{\underline{\emptyset}} \cdot \underline{U}^T$$

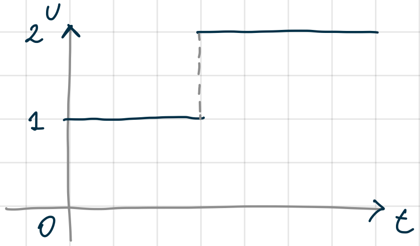
Calcolo Dell'Altezza

Dall'eq di Bernoulli: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_1^2 + \rho g h_2$

Con $v_1 = v_2$ e $h_2 = \emptyset$ $(h_{P_2} = 0)$ $\rightarrow$ $P_A + \rho g h_1 = P$

$\rightarrow$ $h = \frac{P - P_A}{\rho g}$ $\Rightarrow$ $h_1 = \frac{P_1 - P_A}{\rho g}$; $h_2 = \frac{P_2 - P_A}{\rho g}$

$10^{-2} = 0.01$



$$\begin{cases} U_1(t) = \mathbb{1}(t) \rightleftharpoons \dfrac{1}{s} \\ U_2(t) = \mathbb{1}(t-100) \rightleftharpoons \dfrac{1}{s} e^{-100s} \end{cases}$$

$$U_{TOT}(s) = \frac{1}{s} + \frac{1}{s} e^{-100s}$$