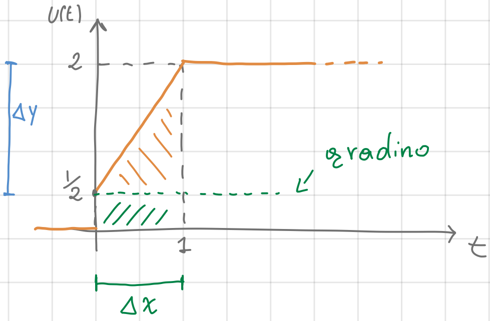
ESERCIZI

$G(s) = \frac{2s+3}{s+5}$

$G(s)$ ha
- zeri : $\bar{z} = -\frac{3}{2}$
- poli : $\bar{p} = -5$



Posso scrivere $u(t) = \underbrace{\frac{1}{2}\mathbb{1}(t)}_{\substack{u_1\\ \text{gradino}}} + \underbrace{\overset{\Delta y/\Delta x}{\frac{3}{2}}\, t\cdot\mathbb{1}(t)}_{\substack{u_2\\ \text{rampa}}} \underbrace{- \frac{3}{2}(t-1)\cdot\mathbb{1}(t-1)}_{\substack{u_3\\ \text{Rampa}}}$

Applico la linearità e la Tempo invarianza:

$y(t) = y_1(t) + y_2(t) + y_3(t)$

Considero $u_2(t)$

$U_2(s) = \frac{3}{2s^2} \to Y_2(s) = \frac{3}{\cancel{2}}\cdot\frac{\cancel{2}s+3}{s^2(s+5)} = 3\cdot\frac{s+\frac{3}{2}}{s^2(s+5)} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{s+5}$

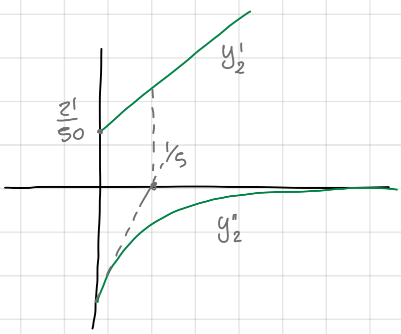
$r_2 = \lim_{s\to 0} s^2\, Y_2(s) = \lim_{s\to 0} \cancel{s^2}\cdot 3\frac{s+3/2}{\cancel{s^2}(s+5)} \to \frac{9}{10}$ r_2

$r_1 = \lim_{s\to -5} \cancel{(s+5)}\cdot 3\frac{s+3/2}{s^2\cancel{(s+5)}} \to -\frac{21}{50}$ r_1

Trovo r_1 : $3\,\frac{s+\frac{3}{2}}{s^2(s+5)} = \frac{s r_1(s+5) + r_2(s+5) + r_3 s^2}{s^2(s+5)} = \frac{(r_1+r_3)s^2 + \overset{1}{(5r_1+r_2)}s + \overset{\text{deve essere } 9/2}{5r_2}}{s^2(s+5)}$

$\Rightarrow r_3 = -r_1 = \frac{21}{50}$ r_3

$\Rightarrow Y_2(s) = \frac{21}{50}\,\frac{1}{s} + \frac{9}{10}\cdot\frac{1}{s^2} - \frac{21}{50}\,\frac{1}{(s+5)} \Leftarrow \overbrace{y_2(t) = \left(\underbrace{\frac{21}{50} + \frac{9}{10}t}_{y_2'} - \underbrace{\frac{21}{50}e^{-5t}}_{y_2''}\right)\cdot\mathbb{1}(t)}^{y_2(t)}$



Trovo $y_1(t)$

$u_1(t) = \frac{1}{2}\mathbb{1}(t) = \frac{1}{3}u_2(t) = \frac{1}{3}\left(\frac{3}{2}\mathbb{1}(t)\right) = \frac{1}{3}\frac{du_2}{dt} \Rightarrow y_1(t) = \frac{1}{3}\frac{d}{dt}y_2(t)$

$\Rightarrow y_1(t) = \frac{1}{3}\cdot\left(\frac{9}{10} + \cancel{5}\cdot\frac{21}{\cancel{50}_{10}}e^{-5t}\right) = \frac{3}{10} + \frac{21}{30}e^{-5t}, \quad t \geq 0$

Trovo $y_3(t)$

$U_3(t) = -U_2(t-1) \quad = 0 \qquad y_3(t) = -y_2(t-1) = -\frac{21}{50} - \frac{9}{10}(t-1) + \frac{21}{50} e^{-5(t-1)} \quad, t \geq 1$

$\mathbb{1}(t-1)$

Valore di regime = ?

$$Y(s) = G(s) \cdot U(s) = \frac{2s+3}{s+5} \cdot \left(\frac{1}{2s} + \frac{3}{2s^2} - \frac{3}{2s^2} e^{-s} \right)$$

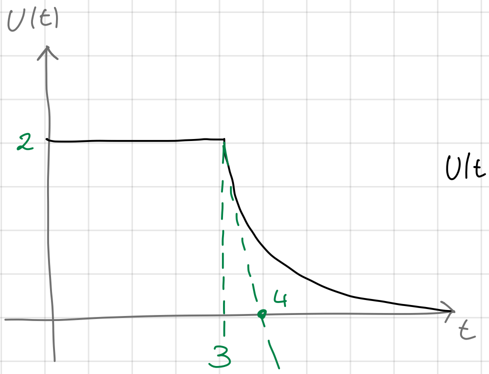
$$y_{ss}(t) = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} \frac{2s+3}{s+5} \left(\frac{1}{2} + \frac{3}{2s} - \frac{1-e^{-s}}{s} \right)$$

Taylor

$$\rightsquigarrow \lim_{s \to 0} \frac{3}{5} \left(\frac{1}{2} + \frac{3}{2} \frac{1-(1-s)}{s} \right) = \frac{6}{5}$$

Valore di regime

Esempio segnale composto



$\tau = 1$

$$U(t) = 2 \cdot \mathbb{1}(t) - 2 \cdot \mathbb{1}(t-3) + 2 \cdot e^{-(t-3)\cdot \tau} \cdot \mathbb{1}(t-3)$$

RISPOSTA TRANSITORIA PER sys del 2^{nd} Ordine

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ω_n^2: Pulsazione naturale

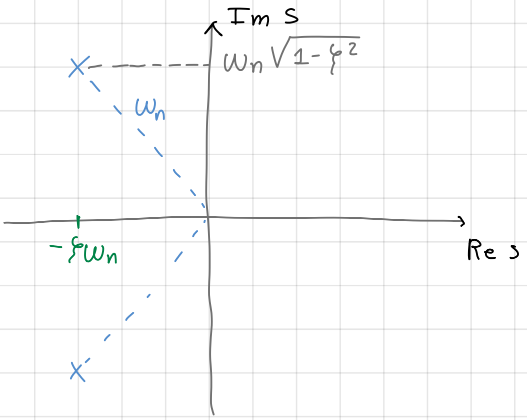
ξ: Coefficiente di smorzamento

$\rightsquigarrow$ $\bar{z}$:

$\bar{p}$: $s^2 + 2\xi\omega_n s + \omega_n^2 \to s = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$

$\to \bar{s} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

- Se $\xi = 1$ $\rightsquigarrow$ CRITICAMENTE SMORZATO
- Se $\xi > 1$ $\rightsquigarrow$ SOVRASMORZATO PARTE REALE NEGATIVA ma 2 sol.
- Se $\xi < 1$ $\rightsquigarrow$ SOTTOSMORZATO Poli complessi e coniugati $\to -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$
- $\xi = 0$ NON SMORZATO

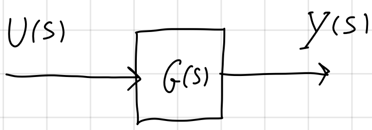


ω_n è il modulo dei poli complessi e coniugati, ovvero se $\xi < 1$

$*$ molla

Hp. $0 < \xi < 1$

$U(s) = \text{gradino} = \frac{1}{s}$



$$\Rightarrow Y(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{r_1}{s} + \frac{r_2 s + r_3}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$r_1 = \lim_{s\to 0} s \cdot Y(s) \to 1 \quad r_1$$

$$\frac{r_1(s^2 + 2\xi\omega_n s + \omega_n^2) + r_2 s^2 + r_3 s}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{(r_1 + r_2)s^2 + (2\xi\omega_n r_1 + r_3)s + r_1\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$r_1 + r_2 = 0 \Rightarrow r_2 = -r_1 \Rightarrow r_2 = -1$ $\quad r_2$

$2\xi\omega_n + r_3 = 0 \to r_3 = -2\xi\omega_n$ $\quad r_3$

$$\Rightarrow \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{1}{s} - \frac{1 + 2\xi\omega_n}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$\rightarrow$ Anti Trasformata

$$G(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = -\frac{}{(s+\xi\omega_n)^2+\omega_n^2-\underbrace{(\xi\omega_n)^2}} + \frac{1}{s} = -\frac{}{(s+\xi\omega_n)^2+\underbrace{\omega_n^2\cdot(1-\xi^2)}} + \frac{1}{s}$$

Tolgo il termine in più

$$\boxed{\begin{aligned} \omega_d &= \omega_n\sqrt{1-\xi^2} \\ \Rightarrow \omega_d^2 &= \omega_n^2(1-\xi^2) \end{aligned}}$$

$$= -\frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{1}{s}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

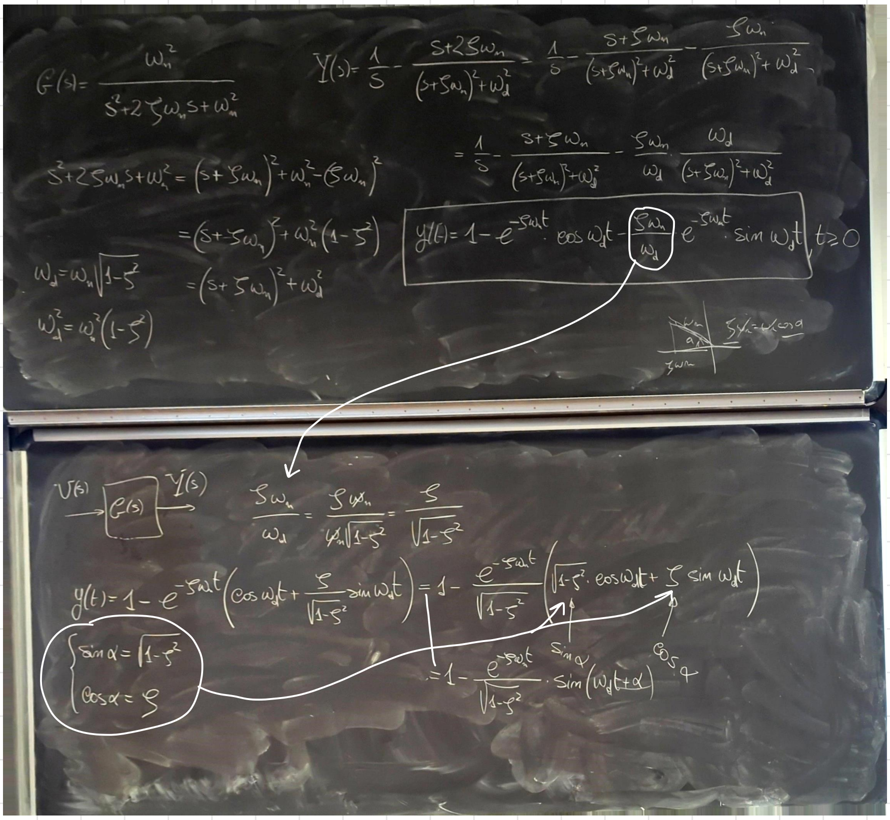
$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d} - \frac{\xi\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\rightarrow y(t) = 1 - e^{-\xi\omega_n t}\cos(\omega_d t) - \frac{\xi\omega_n}{\omega_d}\cdot e^{-\xi\omega_n t} \quad , \ t \geq 0$$

Risposta di un qualsiasi sistema del 2nd ordine

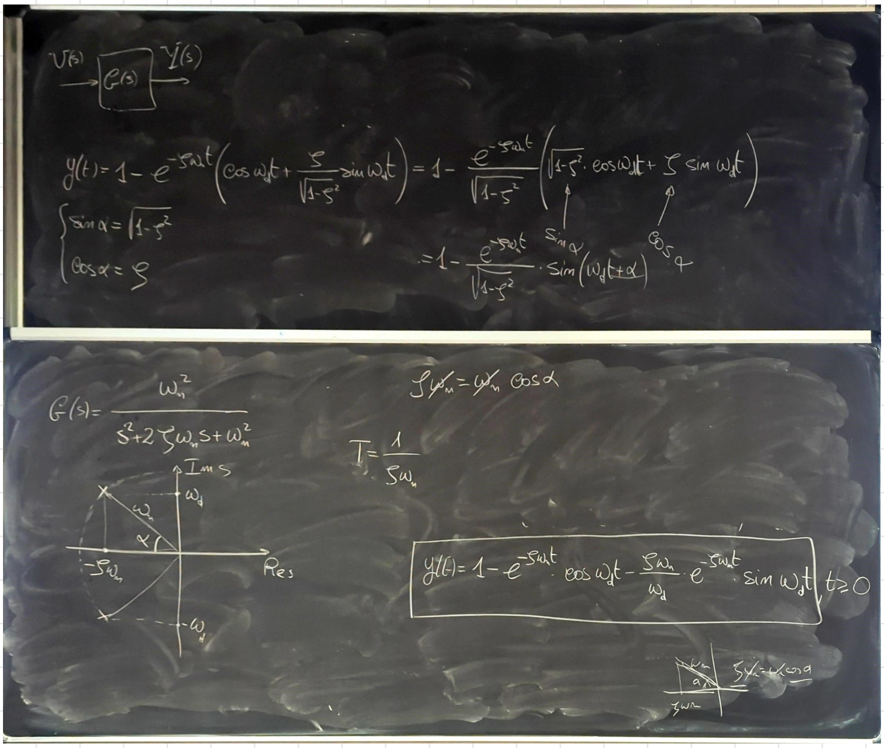
FOTO

$$\frac{\xi\omega_n}{\omega_d} = \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} = \frac{\xi}{\sqrt{1-\xi^2}} \quad \Rightarrow \quad y(t) = 1 - e$$



* Quanto più i poli si avvicinano a zero, tanto più la risposta tenderà ad oscillare; quando zetha è uguale a zero, il sistema non è smorzato e quindi i poli sono sull'asse immaginario, quindi il sistema può essere stabile ma non **asintoticamente**, ovvero abbiamo una sinusoide.

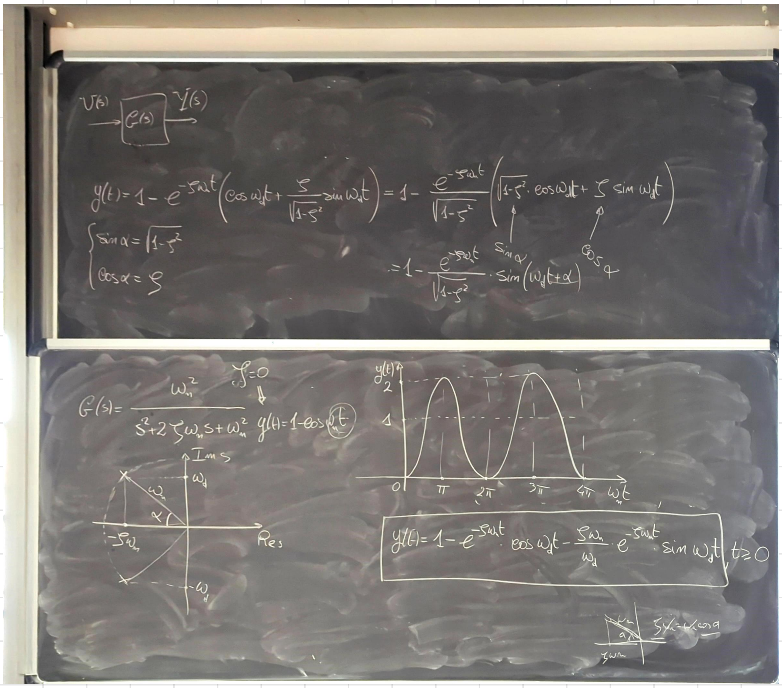
Se zetha è molto piccola, il sistema è si smorzato, ma davvero poco.
Quando zetha si avvicina ad 1, il transitorio si rimpicciolisce sempre di più fino a scomparire

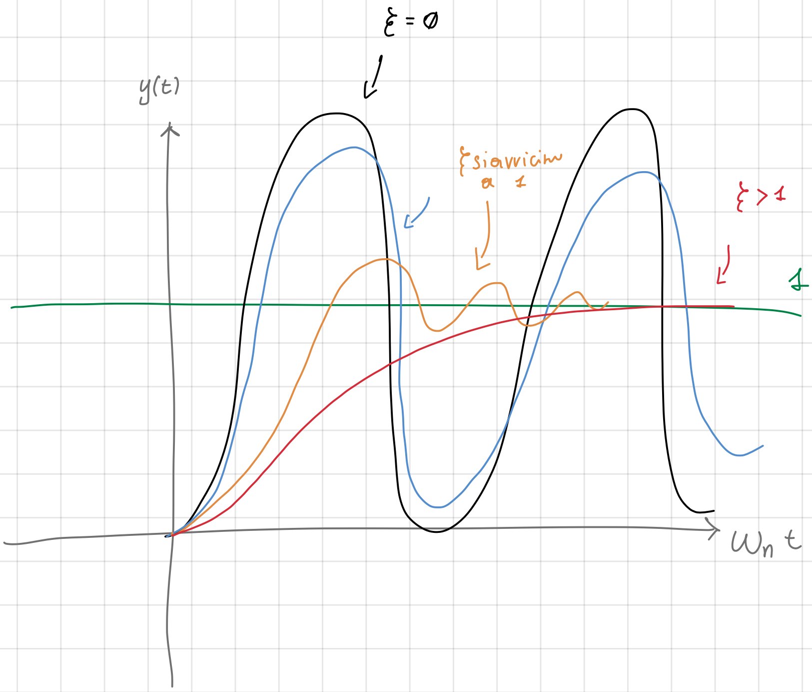


$$\omega_d = \omega_n\sqrt{1-\xi} < \omega_n$$

smorzata

Se $\xi = 0$


U(s)
G(s)
Y(s)
y(t)= 1 - e^{-ξω_n t}(cos ω_d t + ξ/√(1-ξ²) sin ω_d t) = 1 - e^{-ξω_n t}/√(1-ξ²) (√(1-ξ²) · cos ω_d t + ξ sin ω_d t)
sin α = √(1-ξ²)
cos α = ξ
sin α
cos α
= 1 - e^{-ξω_n t}/√(1-ξ²) · sin(ω_d t + α)
G(s) = ω_n² / (s² + 2ξω_n s + ω_n²)
ξ=0
y(t)=1-cos ω_n t
Im s
Re s
y(t)
y(t)= 1 - e^{-ξω_n t} cos ω_d t - ξω_n/ω_d e^{-ξω_n t} sin ω_d t   t≥0


ξ = 0
y(t)
ξ si avvicina a 1
ξ > 1
1
ω_n t

Risposta per $\xi = 1$ CRITICAMENTE SMORZATO

$$G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$Y(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{r_1}{s} + \frac{r_2}{s+\omega_n} + \frac{r_3}{(s+\omega_n)^2}$$

$$r_1 = \lim_{s \to 0} s\,Y(s) = 1$$

$$r_3 = \lim_{s \to -\omega_n} (s+\omega_n)^2\,Y(s) = -\omega_n$$

$$\frac{1}{s} + \frac{r_2}{s+\omega_n} - \frac{\omega_n}{\underset{+2\omega_n}{(s+\omega_n)^2}} = \frac{(s+\omega_n)^2 + r_2 s(s+\omega_n) - s\omega_n}{s(s+\omega_n)^2}$$

$$= \frac{(1+r_2)s^2 + (r_2\omega_n - \omega_n)s + \omega_n^2}{s(s+\omega_n)^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$Y(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{r_1}{s} + \frac{r_2}{s+\omega_n} + \frac{r_3}{(s+\omega_n)^2}$$

$$r_1 = \lim_{s \to 0} s\,Y(s) = 1$$

$$r_2 = -1$$

$$y(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}, \quad t \geq 0$$

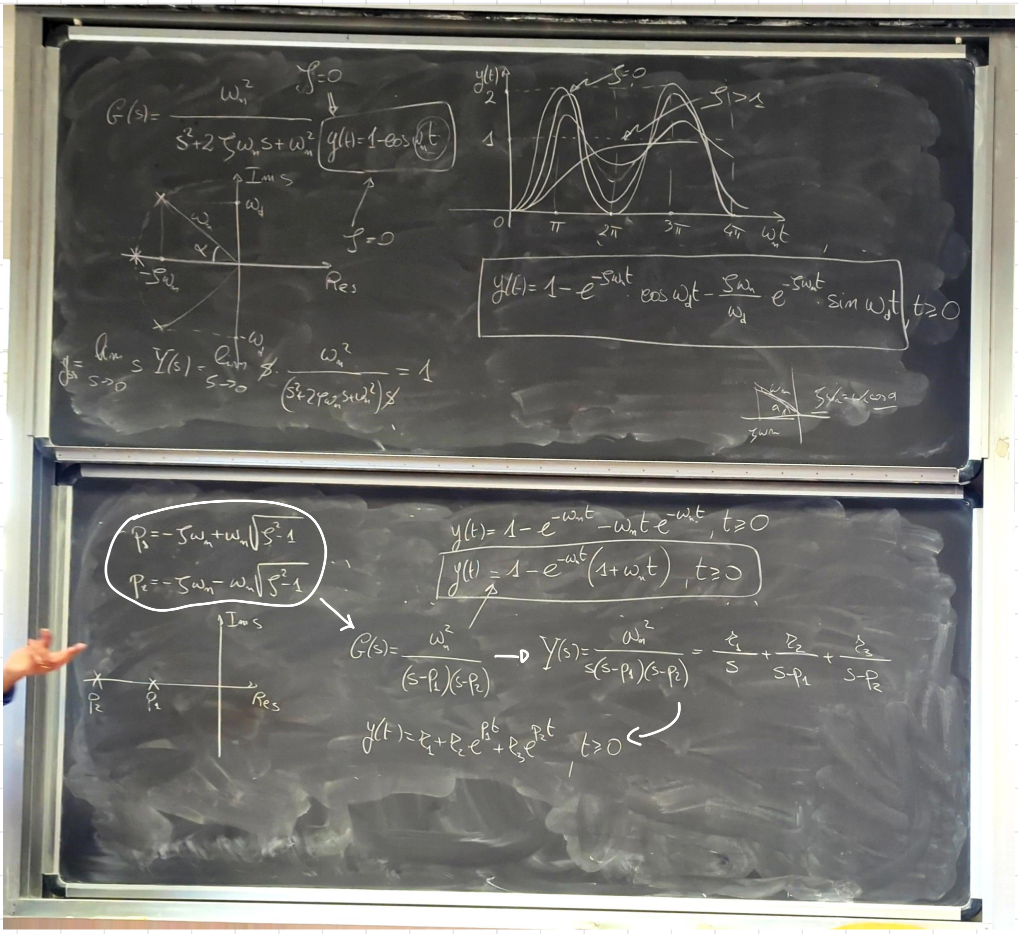
$$y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t), \quad t \geq 0$$

$$\xi = 1$$

$$\frac{1}{s} + \frac{r_2}{s+\omega_n} - \frac{\omega_n}{\underset{+2\omega_n}{(s+\omega_n)^2}} = \frac{(s+\omega_n)^2 + r_2 s(s+\omega_n) - s\omega_n}{s(s+\omega_n)^2}$$

$$= \frac{(1+r_2)s^2 + (r_2\omega_n - \omega_n)s + \omega_n^2}{s(s+\omega_n)^2}$$

Risposta per $\xi > 1$



$-\xi\omega_n + \omega_n\sqrt{\xi^2-1}$ ma $\sqrt{\xi^2-1} < \xi$