

"RO"

$$[\rho] = \text{kg/m}^3$$

$[V] = m^3$

↑ VOLUME ↑ Metro Cubo

$$R_{id} \rightleftharpoons z_e$$

$$\rightarrow [P] = \frac{N}{m^2} = P_a = \frac{\frac{kg \cdot m}{s^2}}{m^2} = \frac{kg}{m \cdot s^2}$$

$$1 \text{ atm} = 10^5 \text{ Pa} = 10^5 \frac{\text{N}}{\text{m}^2}$$

$$1 \text{ dm}^2 = 10^{-2} \text{ m}^2 \Rightarrow \frac{10^5}{10^4} \frac{\text{N}}{\text{m}^2} \Rightarrow 1 \text{ dm}^2 = 10^{-2} \frac{\text{N}}{\text{m}^2}$$

$$q = \frac{dV}{dt}$$

Con

$$[q] = \frac{m^3}{s}$$

Portata Volumetrica

A diagram of a tapered pipe. The left end is a smaller circular cross-section with pressure P_1 and velocity v_1 . The right end is a larger circular cross-section with pressure P_2 and velocity v_2 . The height of the fluid column at the left is h_1 and at the right is h_2 . The pipe is shown on a horizontal surface with diagonal hatching below it.

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

COSTANTE

* Equivalenze

RESISTENZA IDRAULICA

- Attrito - Resistore

$R_{id} = \frac{\Delta P}{q}$

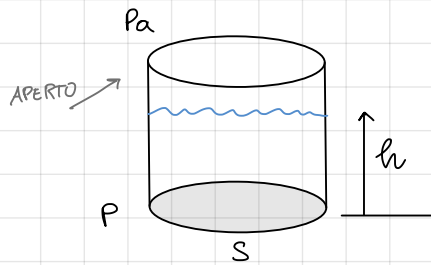
Diff di pressione

portata

Con $\Delta p = p_1 - p_2$

CAPACITA' IDRAULICA

- Serbatoio - Condensatore



$$P_a + \rho \cdot g \cdot h = P \rightarrow \rho \cdot g \cdot h = P - P_a$$

\uparrow COSTANTE
 \uparrow NON COSTANTE

$$\rightarrow \text{DERIVO} \rightarrow \rho \cdot g \cdot \frac{dh}{dt} = \frac{dP}{dt}$$

moltiplico per $\frac{S}{S} \rightarrow$

$$\frac{\rho \cdot g}{S} \cdot \frac{d(S \cdot h)}{dt} = \frac{dP}{dt} \rightarrow \frac{\rho \cdot g}{S} \cdot q = \frac{dP}{dt}$$

Portata

$$\Rightarrow q = \frac{S}{\rho \cdot g} \cdot \frac{dP}{dt}$$

Capacità idraulica \equiv Capacità condensatore

$$\frac{m^2}{\frac{kg}{m^3} \cdot \frac{m}{s^2}} = \frac{m^2}{\frac{kg}{m^2 s^2}}$$

INDUTTANZA IDRAULICA

$$V = L_e \cdot i$$

Esempio: Quando la velocità all'interno di una condotta non è UNIFORME

$$m \cdot \frac{dv}{dt} = F_1 - F_2$$

$$m^2 \cdot \frac{m}{s} = \frac{m^3}{s} \equiv q$$

$$\rho \cdot \frac{d(S \cdot v)}{dt} = F_1 - F_2 \rightarrow \rho \cdot S \cdot h \cdot \frac{dv}{dt} = F_1 - F_2 \rightarrow \rho \cdot S \cdot h \cdot \frac{d(S \cdot v)}{dt} = F_1 - F_2$$

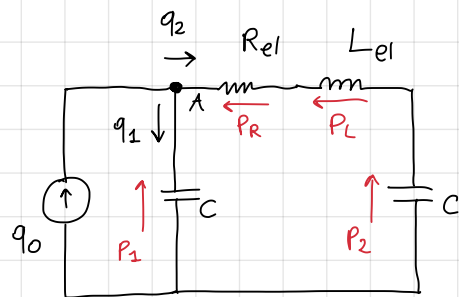
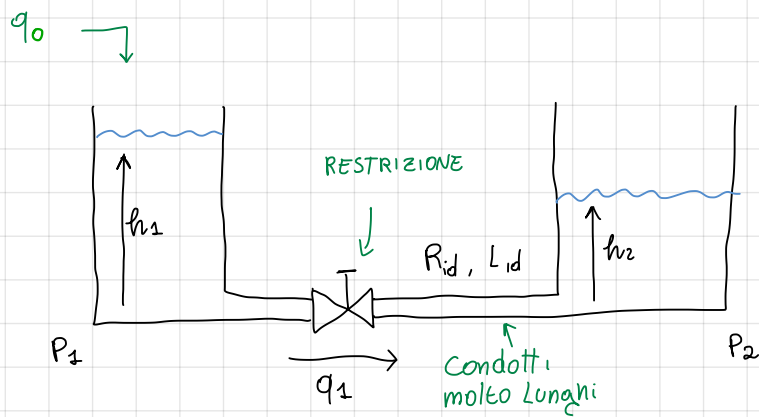
INDUTTANZA IDRAULICA

$$\rightarrow \rho \cdot S \cdot h \cdot \frac{dq}{dt} = (P_1 - P_2) \cdot S \rightarrow \left(\frac{\rho \cdot h}{S} \right) \dot{q} = P_1 - P_2$$

Perché $P = \frac{F}{S} \Rightarrow F = P \cdot S$

ESEMPIO: CONVERSIONE E MODELLO

$$q_1 = i_N = q_{gen} \cdot Corr.$$



$$\begin{aligned} -q_0 + q_1 + q_2 &= 0 \\ L \cdot q_1 &= q_0 - q_2 \end{aligned}$$

$$R.C. \begin{cases} \mathcal{L}_C = C \dot{V}_C \\ \mathcal{V}_L = L \dot{i}_L \end{cases}$$

$$C_1 \cdot \frac{dP_1}{dt} = q_0 - q_2 \quad \leftarrow \text{equivalente a } \mathcal{L}_C$$

$$L \cdot \frac{dq_2}{dt} = -R \cdot q_2 + P_1 - P_2 \quad \begin{aligned} &\text{con } \mathcal{V}_L \\ &= P_L = P_R + P_1 - P_2 \\ &P_L + P_2 - P_1 - P_R = 0 \end{aligned}$$

$$C_2 \cdot \frac{dP_2}{dt} = q_2 \quad \text{con } \mathcal{L}_{C2} = \mathcal{L}_L$$

| | | |
|-------------|-------------|-------------|
| $x_1 = P_1$ | $x_2 = P_2$ | $x_3 = q_2$ |
| $y = h_2$ | $u = q_0$ | |

$$P \cdot q \cdot h \approx P_2 \Rightarrow h = \frac{1}{\rho g} P_2 = \frac{1}{\rho g} x_2$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \rightarrow A = \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{C_1} \\ 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 & \frac{1}{\rho g} & 0 \end{pmatrix} \quad D = 0$$

Funz. di Trasferimento

$$IN = q_1 \quad OUT = h_2 \quad \rightarrow \quad G(s) = \frac{H_2(s)}{Q_1(s)}$$

* INIZIO

$$\mathcal{L.T.} \begin{cases} sC_1 P_1(s) = Q_0(s) - Q_2(s) & (1) \\ sL Q_2(s) = -R Q_2(s) + P_1(s) - P_2(s) & (2) \\ sC_2 P_2(s) = Q_2(s) & (3) \end{cases}$$

OBBIETTIVO: $\underline{H_1(Q_1)}$

(3) in $\rightarrow H_2(s) = \frac{1}{\rho g} P_2(s) = \frac{1}{\rho g C_2} \cdot \frac{1}{s} Q_2(s)$

(3) $\rightarrow (2) \rightarrow sL Q_2 + R Q_2 = P_1 - \frac{1}{sC_1} Q_2 \quad (A)$

dalla (2) $P_1 = \frac{Q_0 - Q_2}{sC_1}$

$$\rightarrow \text{La } (B) \text{ nella } (A) \rightarrow (R+SL)Q_2 + \frac{1}{SC_2} Q_2 = \frac{1}{SC_1} Q_0 - \frac{1}{SC_1} Q_2$$

$$\rightarrow Q_2 \left(R+SL + \frac{1}{SC_2} + \frac{1}{SC_1} \right) = \frac{1}{SC_1} \cdot Q_1$$

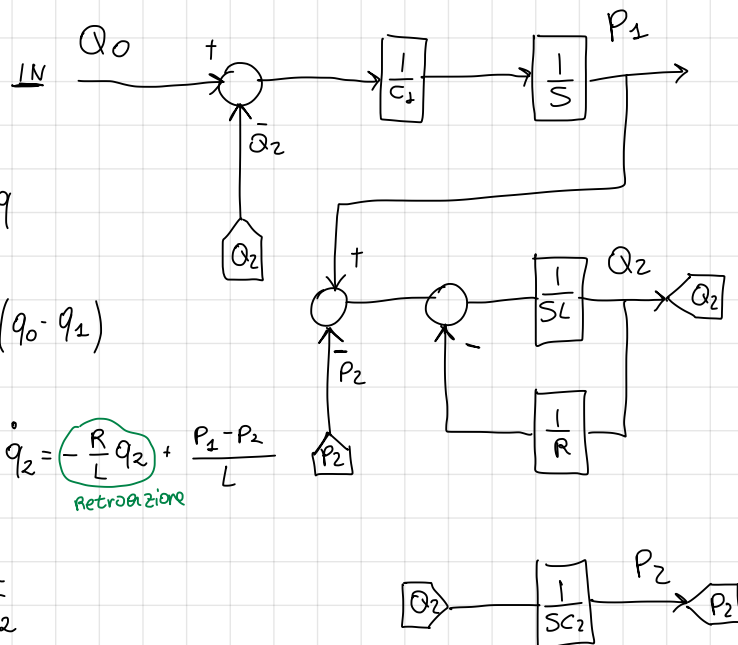
$$\text{m.c.m } \frac{(R+SL)SC_1C_2 + C_1 + C_2}{SC_1C_2}$$

$$\text{m.c.m } \rightarrow \frac{(R+SL)SC_1C_2 + C_1 + C_2}{SC_1C_2} \cdot Q_2 = \frac{1}{SC_1} Q_1$$

$$\rightarrow Q_2(s) = \frac{C_2}{S^2LC_1C_2 + SRC_1C_2 + C_1C_2} Q_1(s)$$

$$\Rightarrow G(s) = \frac{\text{Effetto } H_2(s)}{\text{CAUSA } Q_1(s)} = \frac{1}{\int \frac{1}{C_2} S} \frac{C_2}{S^2LC_1C_2 + SRC_1C_2 + C_1C_2} \frac{Q_1(s)}{Q_1} = \frac{\frac{1}{\int C_2}}{S(S^2LC_1C_2 + SRC_1C_2 + C_1 + C_2)}$$

Schema a blocchi



* Ricavato dalle eq

$$C_1 \cdot \frac{dP_1}{dt} = q_0 - q_2 \rightarrow \dot{P}_1 = \frac{1}{C_1} (q_0 - q_1)$$

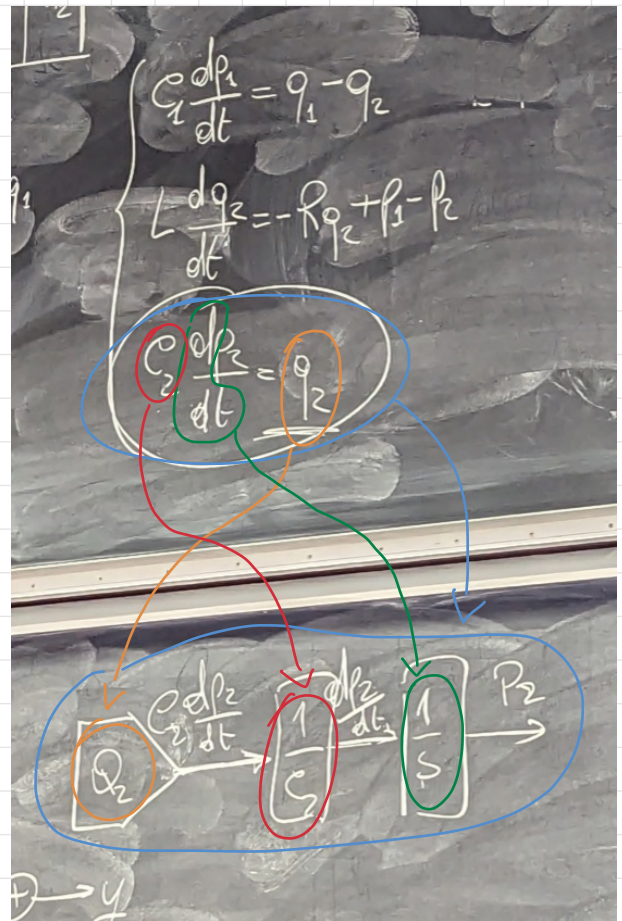
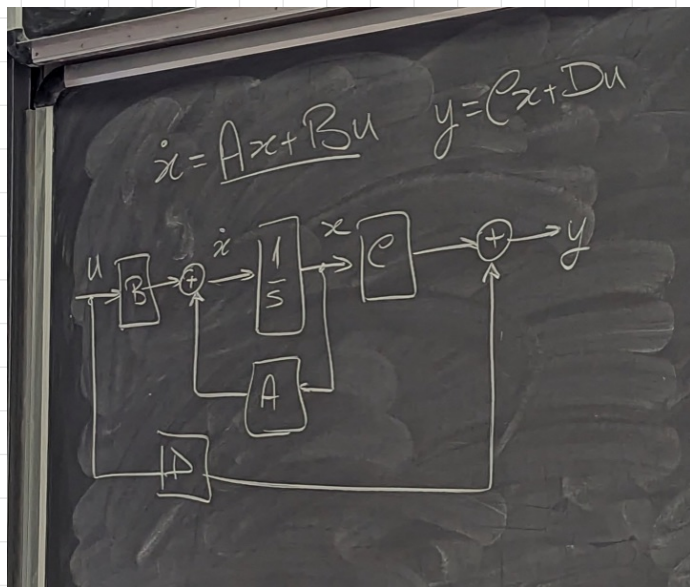
$$L \frac{dq_2}{dt} = -R \cdot q_2 + P_1 - P_2 \rightarrow \dot{q}_2 = -\frac{R}{L} q_2 + \frac{P_1 - P_2}{L}$$

Retroazione

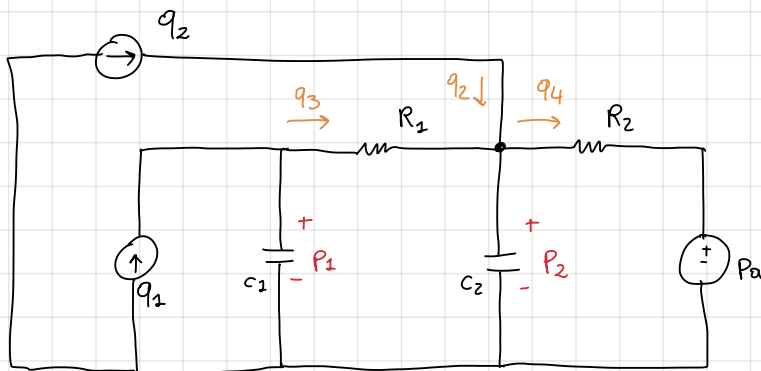
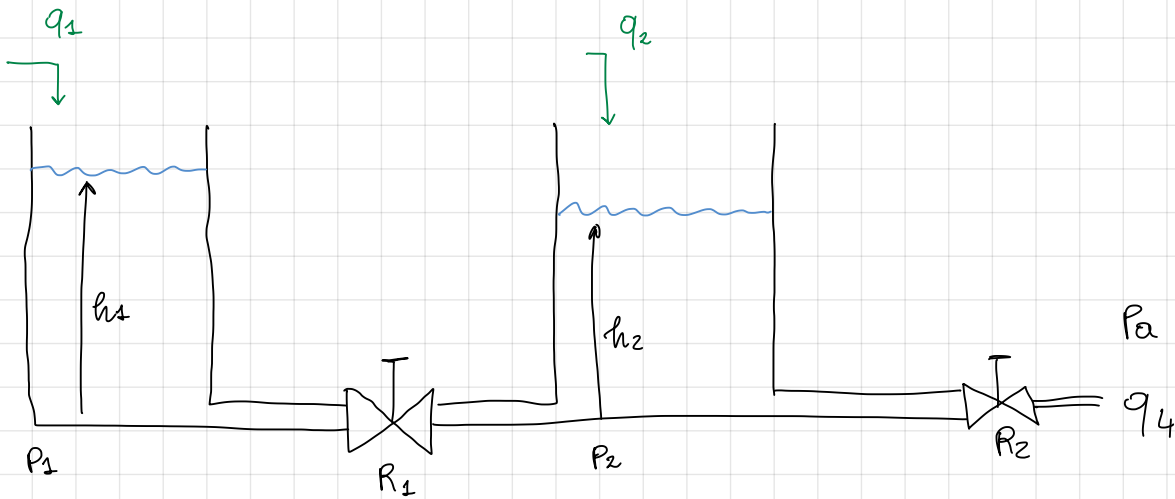
$$C_2 \frac{dP_2}{dt} = q_2 \rightarrow \dot{P}_2 = \frac{q_2}{C_2}$$

Domanda

Rappr. a blocchi Con MATICI



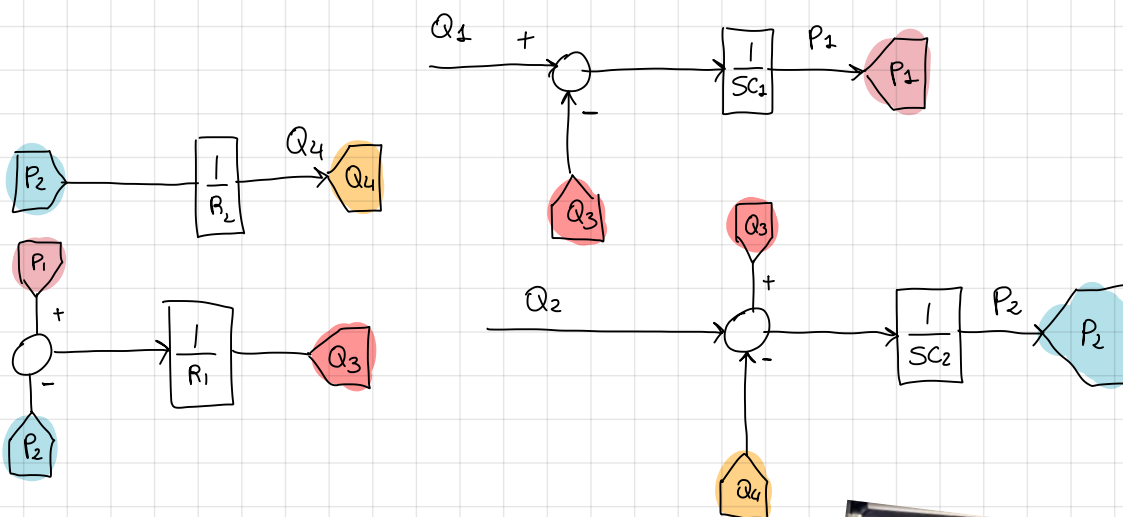
DUE SERBATOI...



$$\left\{ \begin{array}{l} C_1 \frac{dP_1}{dt} = q_1 - q_3 \\ q_3 = \frac{P_1 - P_2}{R_1} \end{array} \right\} \cdot P_1 = \frac{q_1}{C_1} - \frac{1}{C_1} \left(\frac{P_1 - P_2}{R_1} \right)$$

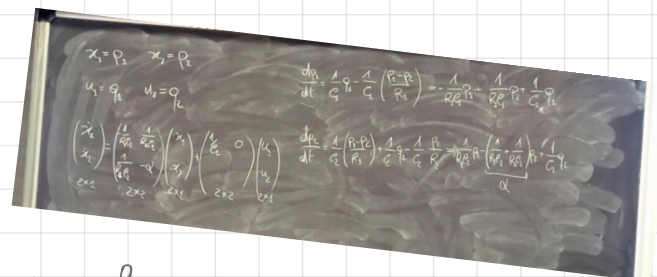
$$C_2 \frac{dP_2}{dt} = q_3 + q_2 - q_4 \quad P_2 \gg P_a$$

$$q_4 = \frac{P_2 - P_a}{R_2} \approx \frac{P_2}{R_2}$$



$$x_1 = P_1 \quad x_2 = P_2$$

$$u_1 = q_1 \quad u_2 = q_2$$



$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{R_1 C_1} & \frac{1}{R_1 C_1} \\ \frac{1}{R_1 C_1} & -\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

2×2 2×2

Possibile Uscita

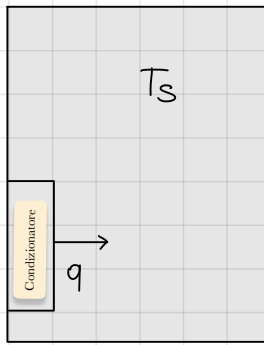
$$y_1 = h_1 - h_2 = \frac{1}{s_2} p_1 - \frac{1}{s_2} p_2$$

$$\rightarrow C = \left(\frac{1}{s_2} \quad -\frac{1}{s_2} \right)$$

Funzione di Trasferimento

Fattori T_u

MODELLI TERMICI



$$\textcircled{C} \frac{dT}{dt} = q$$

↑ Capacità Termica

← Flusso Termico

$$q = \frac{dQ}{dt}$$

$$C = \frac{Q}{\Delta T}$$

↑ Variazione di Temp

← CALORE

$$\begin{cases} 1 \rightarrow q \\ 2 \rightarrow T \end{cases}$$

Circuiti → Termic.

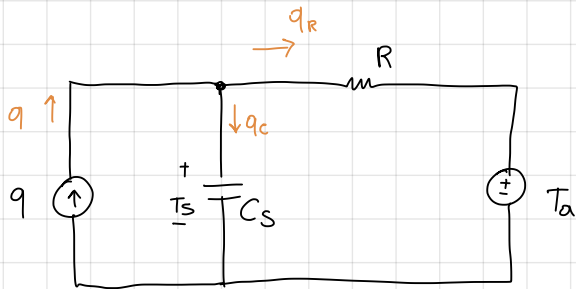
$$R = \frac{\Delta T}{q}$$

Resistenza Termica

→

funziona per

$\begin{cases} \text{CONDUZIONE} \\ \text{CONVEZIONE} \end{cases}$



$$(1) \quad C_s \frac{dT_s}{dt} = q - q_R \quad \text{con} \quad q_R = \frac{T_s - T_a}{R}$$

$$u_1 = q$$

$$u_2 = T_a$$

$$x = T_s$$

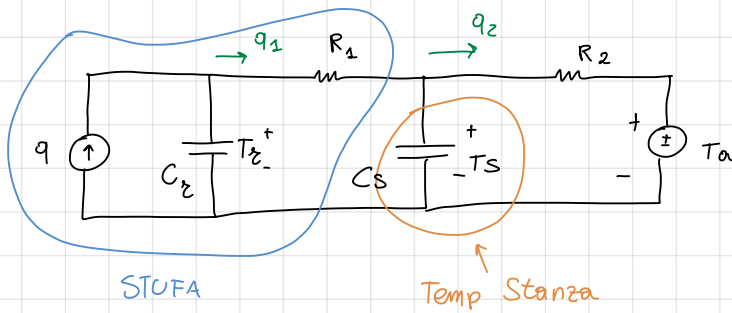
$$G(s) = \frac{T_s(s)}{Q(s)} = \dots$$

$$s C_s T_s = Q - \frac{T_s - T_a}{R} \rightarrow (s C_s + \frac{1}{R}) T_s = Q + \frac{1}{R} T_a$$

$$\rightarrow mcm \rightarrow T_s = \frac{R}{s R C_s + 1} Q + \frac{1}{s R C_s + 1} T_a$$

Influenza della Temp. esterna

IL RADIATORE



$$C_z \frac{dT_z}{dt} = q - q_1$$

$$q_1 = \frac{T_z - T_s}{R_1}$$

$$C_s \frac{dT_s}{dt} = q_1 - q_2$$

$$q_2 = \frac{T_s - T_a}{R_2}$$

$$u_1 = T_a \quad u_2 = q$$

$$x_1 = T_z \quad x_2 = T_s$$

$$S C_z T_z = Q - Q_1 \quad \rightarrow \text{Sub } Q_1 \rightarrow S C_z T_z = Q - \frac{T_z}{R_1} + \frac{T_s}{R_1} \rightarrow \text{Trovo } T_z \quad (1)$$

$$S C_s T_s = Q_1 - Q_2 \quad \rightarrow \text{Sub } Q_2 \rightarrow S C_s T_s = \frac{T_z}{R_1} - \frac{T_s}{R_2} + \frac{T_s}{R_2} - \frac{T_a}{R_2} \quad (2)$$

$$\text{dalla (2)} \rightarrow \left(S C_s + \frac{1}{R_1} + \frac{1}{R_2} \right) T_s = \frac{T_z}{R_1} + \frac{T_a}{R_2}$$

$$L \left(S C_s + \frac{1}{R_1} + \frac{1}{R_2} \right) T_s = \frac{1}{1 + S R_1 C_z} Q + \frac{\frac{1}{R_1}}{1 + S R_1 C_z} T_s + \frac{T_a}{R_2}$$