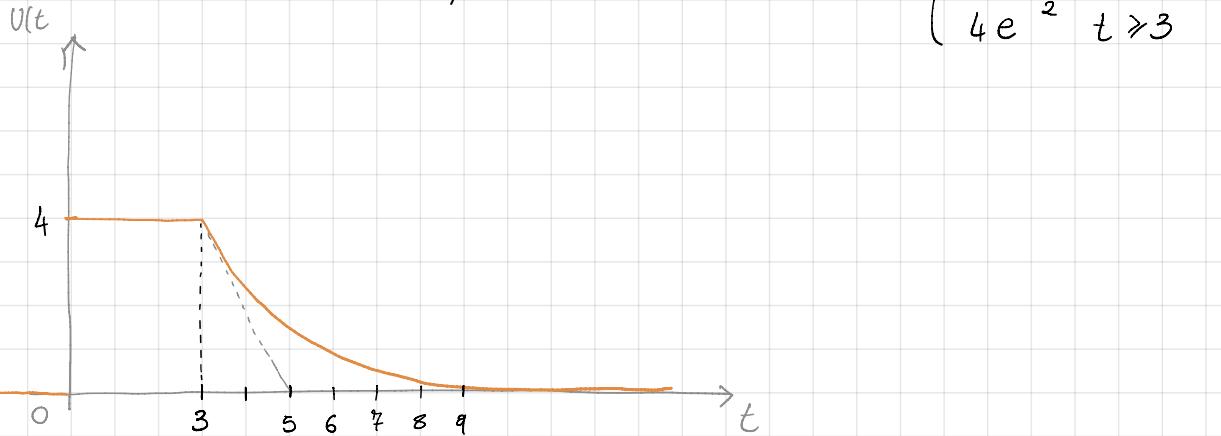


## ES: POLI COMPLESSI E CONIUGATI

$$G(s) = \frac{s + 0.1}{(s^2 + 10s + 100)(s+2)}$$



(1) Scrivo  $u(t)$  con segnali noti.

• Per  $t \in (0, 3)$   $\Rightarrow u_1(t) = 4 \cdot \mathbb{1}(t)$

• Per  $t \geq 3$   $\Rightarrow u_1(t) + u_2(t) + u_3(t)$  con  $u_2(t) = -4 \cdot \mathbb{1}(t-3)$  e  $u_3(t) = 4e^{-\frac{t-3}{2}} \cdot \mathbb{1}(t-3)$   
 Annullo  $u_1(t)$

(2) Determino le Trasformate

$$U_1(t) \Leftrightarrow U_1(s) = 4 \cdot \frac{1}{s}$$

$$U_2(t) \Leftrightarrow U_2(s) = -4 \cdot \frac{1}{s} \cdot e^{-3s}$$

$$U_3(t) \Leftrightarrow U_3(s) = \mathcal{L}\left[4 \cdot e^{-\frac{t}{2}}\right] \cdot e^{-3s} = \frac{4}{s + \frac{1}{2}} e^{-3s}$$

(3) Individuare il segnale che ci permette di ricondursi agli altri segnali

→ I segnali elementari da considerare sono:

$$\hat{U}_1(t) = \mathbb{1}(t) \quad \text{e} \quad \hat{U}_2(t) = e^{-\frac{t}{2}}$$

(5) Antitrasformata

$$\cdot \hat{U}_1(s) = \frac{1}{s} \Rightarrow \hat{Y}_1(s) = \frac{s + 0.1}{s(s^2 + 10s + 100)(s+2)} \quad (1)$$

$$\cdot \hat{U}_2(s) = \frac{1}{s + \frac{1}{2}} \Rightarrow \hat{Y}_2(s) = \frac{1}{s + \frac{1}{2}} \cdot \frac{s + 0.1}{(s^2 + 10s + 100)(s+2)} \quad (2)$$

Trovo l'uscita al primo segnale:  $\hat{y}_1(t)$  (1)

Verifico i poli:  $s^2 + 10s + 100 \Rightarrow s = -5 \pm \sqrt{25-100} = -5 \pm j\sqrt{75}$  Complx

5.1)  $y_1(t)$  con poli non complx e conj

$$\Rightarrow Y_1(s) = \frac{\zeta_1}{s} + \frac{\zeta_2}{s+2} + \frac{\zeta_3 + \zeta_4 s}{s^2 + 10s + 100}$$

$$\zeta_1 = \lim_{s \rightarrow 0} s \cdot \frac{s+0.1}{s(s^2 + 10s + 100)(s+2)} \Rightarrow \frac{0.1}{200} = \frac{1}{2} \cdot 10^{-3} \zeta_1$$

$$\zeta_2 = \lim_{s \rightarrow -2} (s+2) \frac{s+0.1}{s(s^2 + 10s + 100)(s+2)} = \frac{-1.9}{2 \cdot 84} = \frac{1}{11} \cdot 10^{-3} \zeta_2$$

$$\frac{(s+2)\zeta_1(s^2 + 10s + 100) + s\zeta_2(s^2 + 10s + 100) + (\zeta_3 + \zeta_4 s)(s+2)s}{s(s^2 + 10s + 100)(s+2)} \rightsquigarrow \begin{cases} s^3(\zeta_1 + \zeta_2 + \zeta_4) = 0 \\ s^2(2\zeta_1 + 10\zeta_1 + 10\zeta_2 + 2\zeta_4) = 0 \\ s(100\zeta_1 + 20\zeta_1 + 100\zeta_2 + 2\zeta_3) = 1 \\ 200\zeta_4 = 0.1 \end{cases}$$

$$\zeta_1 + \zeta_2 + \zeta_4 = 0 \Rightarrow \zeta_4 = -(z_1 + z_2) = 11.5 \times 10^{-3}$$

$$\zeta_3 = -(12\zeta_1 + 10\zeta_2 + 2\zeta_4) = -93 \times 10^{-3} \zeta_3$$

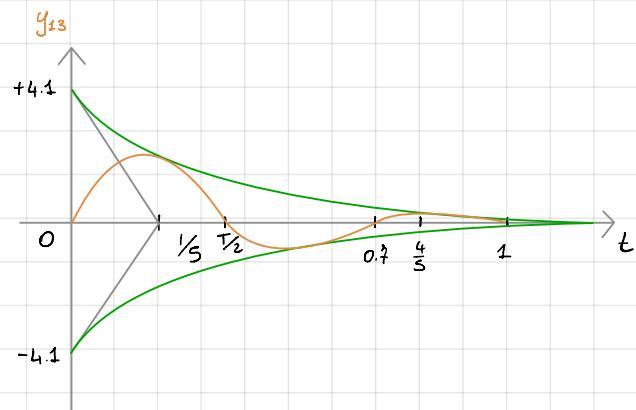
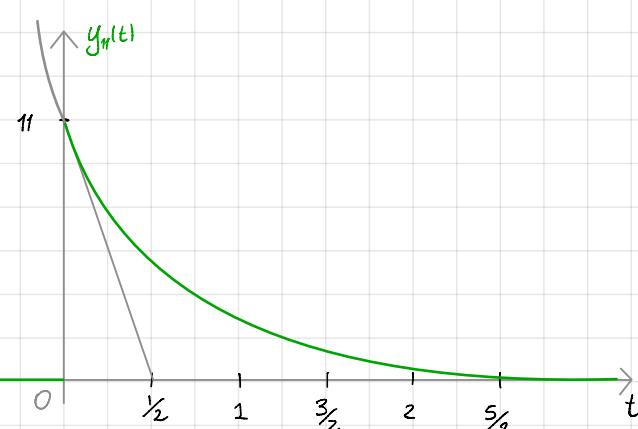
$$\Rightarrow \hat{Y}_1(s) = 10^{-3} \left[ \frac{1}{2} + \frac{11}{s+2} - \frac{93 + 11.5s}{(s+5)^2 + (5\sqrt{3})^2} \right] = 10^{-3} \left[ \frac{1}{2} + \frac{11}{s+2} - 11.5 \cdot \frac{s+5}{(s+5)^2 + (5\sqrt{3})^2} \right]$$

$$= 10^{-3} \left[ \frac{1}{2} + \frac{11}{s+2} - 11.5 \cdot \frac{s+5}{(s+5)^2 + (5\sqrt{3})^2} - \frac{11.5}{5\sqrt{3}} \left( \frac{93}{11.5} - 5 \right) \frac{5\sqrt{3}}{(s+5)^2 + (5\sqrt{3})^2} \right]$$

$$\Rightarrow \hat{y}_1(t) = 10^{-3} \left( \frac{1}{2} + 11 \cdot e^{-2t} - 11.5 \cdot e^{-5t} \cos(5\sqrt{3}t) - 4.1 \cdot e^{-5t} \sin(5\sqrt{3}t) \right) \cdot u(t)$$

$$T = \frac{2\pi}{5\sqrt{3}} = 0.75$$

DISEGNI Dei Modi



Trovo l'uscita al secondo segnale:  $\hat{y}_2(t)$  (2)

$$\hat{Y}_2(s) = \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{\varepsilon_1}{s+\frac{1}{2}} + \frac{\varepsilon_2}{s+2} + \frac{\varepsilon_3 + \varepsilon_4 s}{s^2+10s+100}$$

$$\varepsilon_1 = \lim_{s \rightarrow -\frac{1}{2}} (s+\frac{1}{2}) \cdot \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{-0.4}{\frac{3}{2} \cdot (\frac{1}{4} - 5 + 100)} \approx -2.8 \times 10^{-3} \quad \varepsilon_1$$

$$\varepsilon_2 = \lim_{s \rightarrow -2} (s+2) \cdot \frac{s+0.1}{(s+\frac{1}{2})(s+2)(s^2+10s+100)} = \frac{-1.9}{-\frac{3}{2} (4 - 20 + 100)} = 15 \times 10^{-3} \quad \varepsilon_2$$

$$\hat{Y}_2(s) = \frac{(s+2)\varepsilon_1(s^2+10s+100) + (s+\frac{1}{2})\varepsilon_2(s^2+10s+100) + (\varepsilon_3 + \varepsilon_4 s)(s+2)(s+\frac{1}{2})}{(s+\frac{1}{2})(s+2)(s^2+10s+100)}$$

$$= s^3(\varepsilon_1 + \varepsilon_2 + \varepsilon_4) = 0 \quad \text{da cui } \varepsilon_1 \text{ ed } \varepsilon_2 = 0 \quad \varepsilon_4 = -\varepsilon_1 - \varepsilon_2 = -12.5 \times 10^{-3} \quad \varepsilon_4$$

$$s^2(10\varepsilon_1 + 2\varepsilon_1 + \frac{1}{2}\varepsilon_2 + 10\varepsilon_2 + \varepsilon_3 + 2\varepsilon_4 + \frac{1}{2}\varepsilon_4) = 0$$

$$\text{da } \varepsilon_1(10+2) + \varepsilon_2(\frac{1}{2}+10) + \varepsilon_3(1) + \varepsilon_4(2+\frac{1}{2}) = 0$$

$$\Rightarrow \varepsilon_3 = -(\dots) = -0.092 = 92.65 \times 10^{-3} \quad \varepsilon_3$$

\* Forma che ci interessa

$\Rightarrow$  A questo punto dovranno sostituire i residui, ma possiamo riscrivere nella forma che ci torna più comoda per fare l'antitrasformata PRIMA di sostituire:

$$s^2+10s+100 \Rightarrow p_{1,2} = -5 \pm j5\sqrt{3} \Rightarrow s^2+10s+100 = (s+5)^2 + (5\sqrt{3})^2$$

$$\begin{aligned} \Rightarrow \hat{Y}_2(s) &= \frac{\varepsilon_1}{s+\frac{1}{2}} + \frac{\varepsilon_2}{s+2} + \varepsilon_4 \frac{\frac{\varepsilon_3}{\varepsilon_4} + s}{(s+5)^2 + (5\sqrt{3})^2} = \frac{\varepsilon_1}{s+\frac{1}{2}} + \frac{\varepsilon_2}{s+2} + \varepsilon_4 \frac{(s+5) + \frac{\varepsilon_3}{\varepsilon_4} - 5}{(s+5)^2 + (5\sqrt{3})^2} \\ &\quad \text{mi serve } (s+5) \\ &= \frac{\varepsilon_1}{s+\frac{1}{2}} + \frac{\varepsilon_2}{s+2} + \varepsilon_4 \frac{s+5}{(s+5)^2 + (5\sqrt{3})^2} + \varepsilon_4 \frac{\left(\frac{\varepsilon_3}{\varepsilon_4} - 5\right)}{(s+5)^2 + (5\sqrt{3})^2} \cdot \frac{5\sqrt{3}}{5\sqrt{3}} \\ &= \frac{\varepsilon_1}{s+\frac{1}{2}} + \frac{\varepsilon_2}{s+2} + \varepsilon_4 \frac{(s+5)}{(s+5)^2 + (5\sqrt{3})^2} + \frac{\varepsilon_4 \left(\frac{\varepsilon_3}{\varepsilon_4} - 5\right)}{5\sqrt{3}} \cdot \frac{(5\sqrt{3})}{(s+5)^2 + (5\sqrt{3})^2} \end{aligned}$$

$$\Rightarrow \hat{y}_2(t) = \varepsilon_1 e^{-\frac{t}{2}} + \varepsilon_2 e^{-2t} + \varepsilon_4 e^{-5t} \cdot \cos(5\sqrt{3}t) + \frac{\varepsilon_4 \left(\frac{\varepsilon_3}{\varepsilon_4} - 5\right)}{5\sqrt{3}} \cdot e^{-5t} \cdot \sin(5\sqrt{3}t)$$

## USCITE REALI

$$\begin{cases} U_1(t) = 4 \cdot \mathbb{1}(t) \\ U_2(t) = -4 \cdot \mathbb{1}(t-3) \\ U_3(t) = 4e^{-\frac{t-3}{2}} \cdot \mathbb{1}(t-3) \end{cases}$$

ed Abbiamo scelto

$$\begin{cases} \hat{U}_1(s) = \frac{1}{s} \\ \hat{U}_2(s) = \frac{1}{s+1} \end{cases} \Rightarrow \begin{cases} U_1(t) = \mathbb{1}(t) \\ U_2(t) = e^{-\frac{t}{2}} \cdot \mathbb{1}(t) \end{cases}$$

$$\Rightarrow \begin{cases} y_1(t) = 4 \hat{y}_1(t) \cdot \mathbb{1}(t) \\ y_2(t) = -4 \hat{y}_2(t-3) \cdot \mathbb{1}(t-3) \\ y_3(t) = 4 \hat{y}_3(t-3) \cdot \mathbb{1}(t-3) \end{cases} \Rightarrow y(t) = y_1(t) + y_2(t) + y_3(t)$$

## Esercizio Risposta in freq

$$G(s) = 100 \cdot \frac{0.2s + 1}{s^2 + 2s + 10}$$

$$* s^2 + 2j\omega_n s + \omega_n^2$$

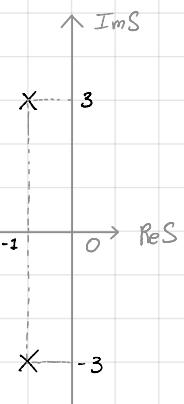
### (1) Forma Standard, Poli e Zeri

$$G(s) = 100 \cdot \frac{1 + 0.2s}{s^2 + \frac{2}{10}s + 1}$$

Forma Standard

$$Z_1 = -\frac{1}{0.2} = -5$$

$$P_{1,2} = -1 \pm 3j$$



### (2) Punti di rottura

$$s^2 + 2s + 10 = s^2 + 2j\omega_n s + \omega_n^2 = 0 \quad \begin{cases} j\omega_n = 3 \\ 10 = \omega_n^2 \end{cases} \Rightarrow \omega_n = \sqrt{10} \approx 3.16$$

$$\mathcal{T} = \frac{1}{\omega_n} = \frac{1}{\sqrt{10}} \approx 0.32$$

•  $\omega_1 = 5$

### (3) Intervallo di Visual

$$\min(\omega_i) = \omega_n = \sqrt{10} \Rightarrow t \in \left( \frac{1}{10}, 10 \right) [\text{Almeno}]$$

$$\max(\omega_i) = \omega_1 = 5$$

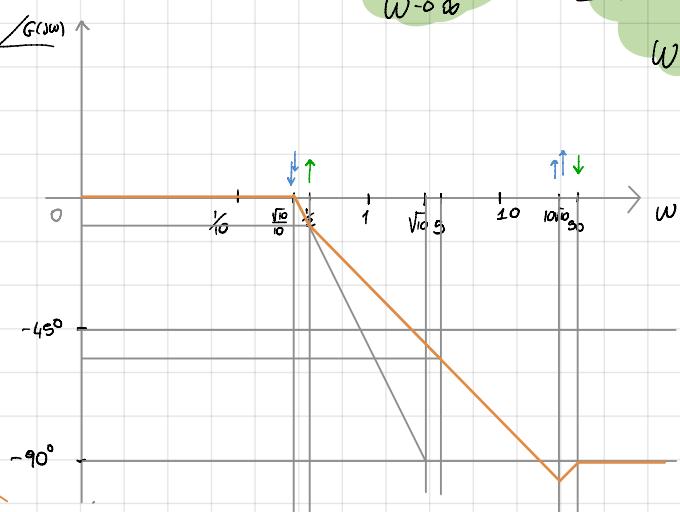
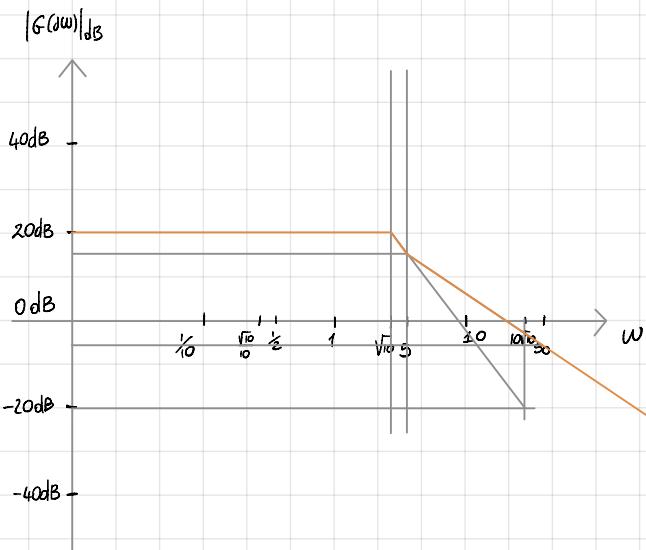
### (4) Andamenti iniziali e finali

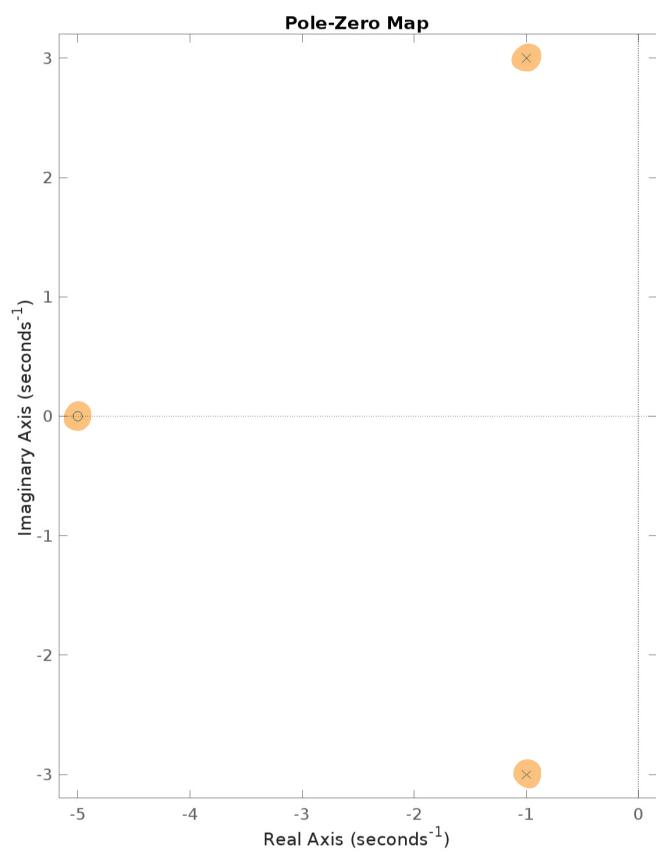
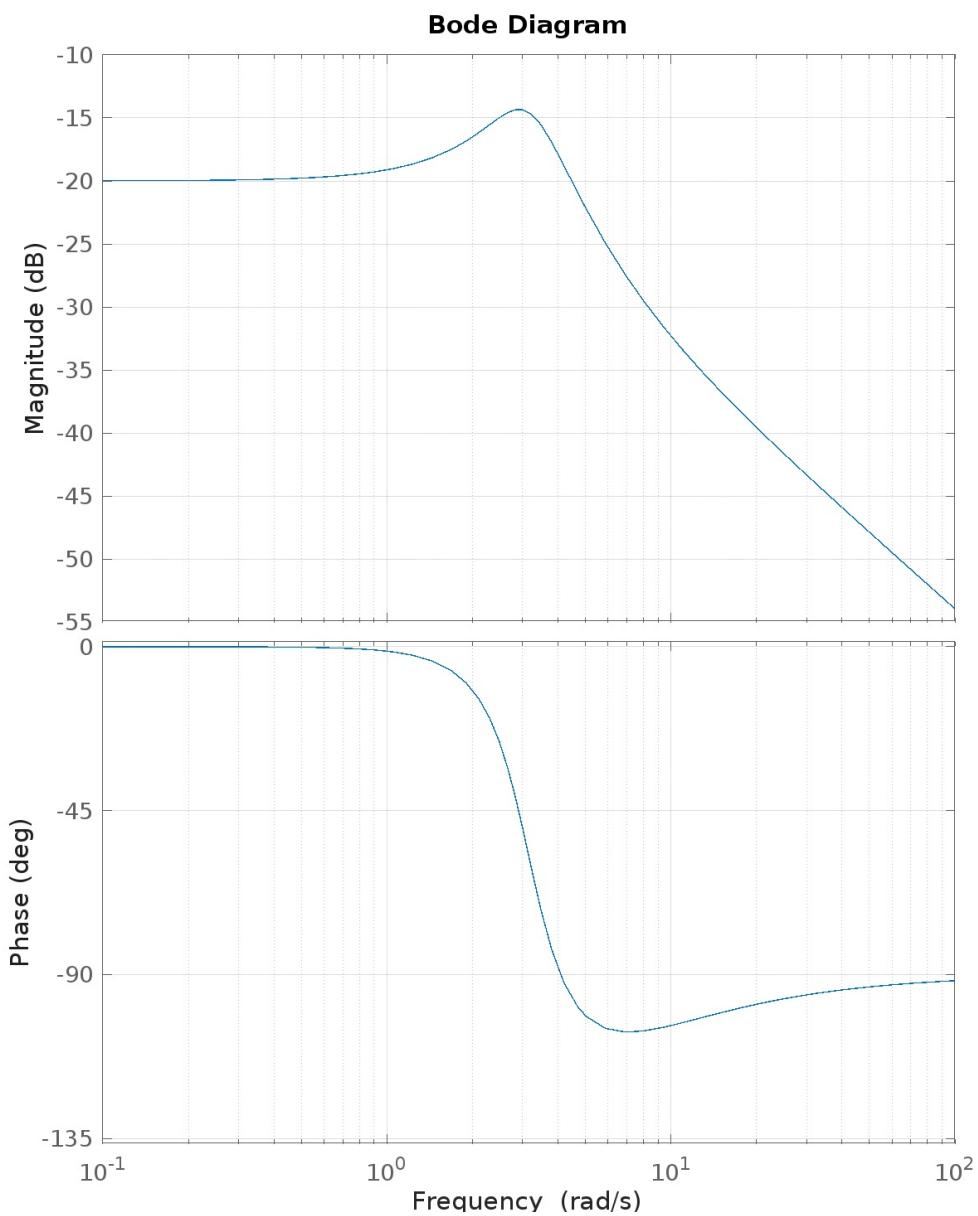
Modulo • No poli in 0, guadagno statico:  $K_p = 10 \Rightarrow |G(j\omega)|_{dB} \Big|_{w=0} = 20 \log_{10}(10) = 20dB$

per  $w=0 \rightarrow \infty$   $\begin{cases} 1 \text{ Polo} \rightarrow +20 dB/\text{dec} \\ 1 \omega_n \rightarrow -40 dB/\text{dec} \end{cases} \Rightarrow |G(j\omega)|_{dB} \Big|_{w=0 \infty} = -20 dB/\text{dec}$

Fase  $\begin{cases} 1 \text{ zero a } \text{Re } \omega < 0 \rightarrow \text{Anticipo di } 90^\circ \\ 1 \omega_n \rightarrow \text{Ritardo } 180^\circ \end{cases} \Rightarrow$

$$\begin{aligned} |G(j\omega)|_{dB} \Big|_{w=0 \infty} &= -90^\circ \\ |G(j\omega)|_{dB} \Big|_{w=0} &= 0^\circ \end{aligned}$$





# SCRIVIAMO LE USCITE REALI

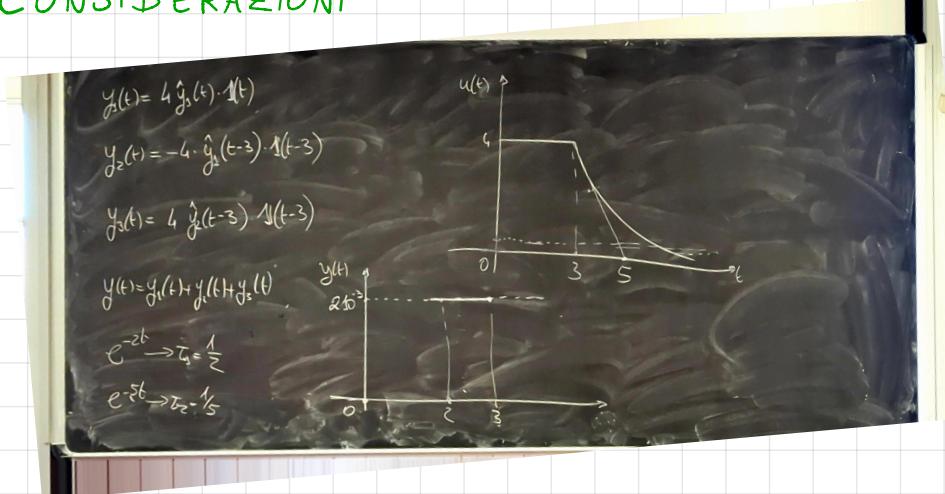
$$y_1(t) = 4 \hat{y}_1(t) \cdot 1(t)$$

$$y_2(t) = -4 \cdot \hat{y}_2(t-3) \cdot 1(t-3)$$

$$y_3(t) = 4 \hat{y}_3(t-3) \cdot 1(t-3)$$

$$y(t) = y_1(t) + y_2(t) + y_3(t)$$

## CONSIDERAZIONI

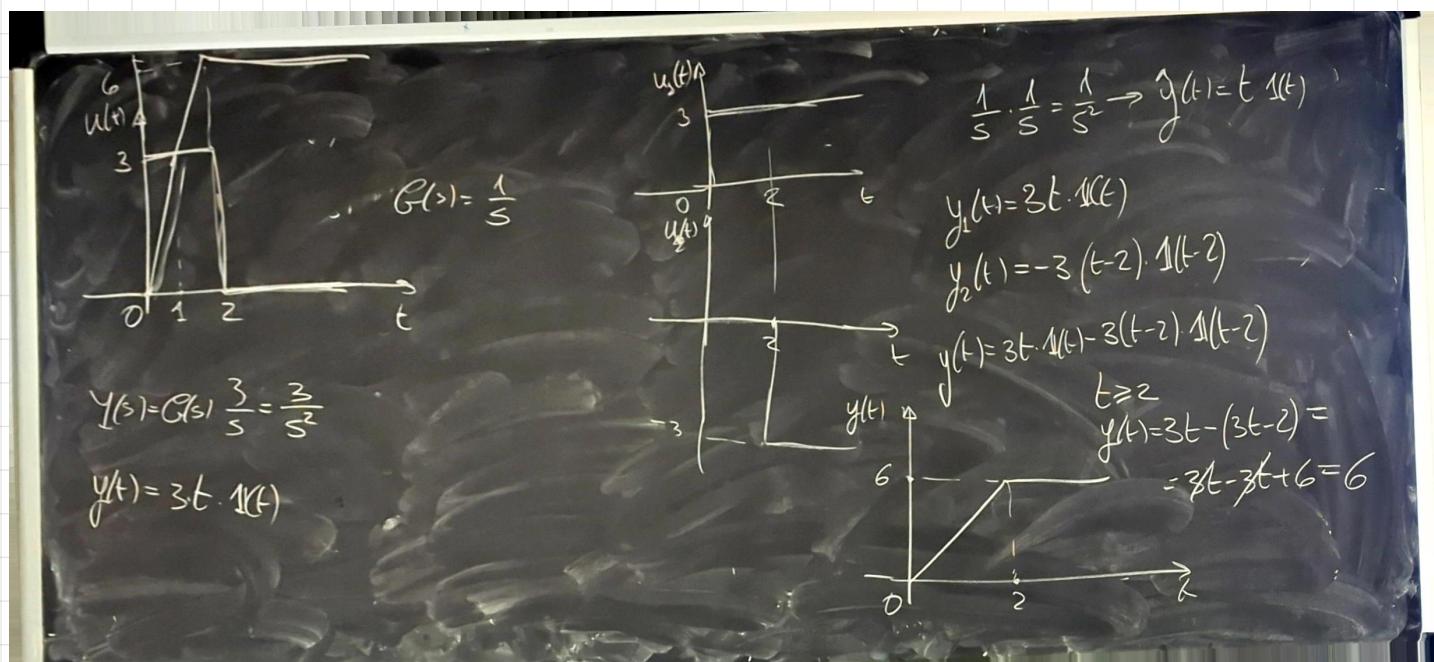


Asint. ST  $\Rightarrow$  Non rimane ad un valore stabile  
 $\Rightarrow$  Tende a zero

$$G(s) = \frac{s+0.1}{(s+2)((s+5)^2 + (5\sqrt{3})^2)}$$

$$\lim_{s \rightarrow 0^+} s G(s) \cdot \frac{4}{s} = 4 G(0)$$

## BONUS



# RISPOSTA IN FREQUENZA

$$G(s) = \frac{0.2s+1}{s^2 + 2s + 10}$$

$$(s+1)^2 + 9$$

$$\bar{\omega} = -5$$

$$\tilde{p} = -1 \pm j\sqrt{3}$$

$$\omega_n = \sqrt{10}, \omega_d = 5$$

$$R \tilde{s} = Z, s = \frac{1}{\omega_d} = \frac{1}{\sqrt{10}}$$

