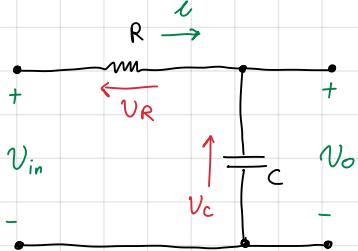




CIRCUITO RC



$$\{ V_R + V_C = V_{in}$$

$$\begin{cases} V_R = R \cdot I \\ I_C = C \dot{V}_C \Rightarrow \dot{V}_C = \frac{1}{C} I_C \Rightarrow V_C = \frac{1}{C} \int I_C dt \end{cases}$$

FUNZIONE DI TRASF.

C.I. nulle

$$\Rightarrow V_R + V_C = V_{in} \Rightarrow R I(s) + \frac{1}{C} \cdot S I(s) - \cancel{I(0^+)} = V_{in}(s) \Rightarrow V_{in}(s) = R I(s) + \frac{1}{CS} I(s) \\ = I(s) \left(R + \frac{1}{CS} \right)$$

$$\text{Siccome } V_o(s) = \frac{1}{CS} I(s) \Rightarrow G(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{\cancel{CS} \ I(s)}{I(s) \left(\frac{RCS + 1}{CS} \right)} = \boxed{\frac{1}{RCS + 1}} \\ G(s)$$

SPAZIO DI STATO

(1) STABILIAMO

$$in = u = V_{in}$$

$$out = y = V_o$$

$$\text{Variabile di stato} = x = V_C = V_o$$

(2) SCRIVO NELLA FORMA

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Dalle R.C.

$$\dot{x} \quad \left[\begin{array}{l} c \frac{d}{dt} V_o = I \Rightarrow \dot{V}_o = \frac{1}{C} I \Rightarrow \dot{V}_o = \frac{1}{RC} (V_{in} - V_o) \Rightarrow \dot{V}_o = -\frac{1}{RC} V_o + \frac{1}{RC} V_{in} \\ V_R = R \cdot I \Rightarrow I = \frac{V_R}{R} \text{ ma } V_R = V_{in} - V_o \Rightarrow I = \frac{V_{in} - V_o}{R} \end{array} \right]$$

$$y \quad \left[\text{Siccome } V_o = x, y = V_o \Rightarrow y = x \right]$$

$$\Rightarrow \begin{cases} \dot{x} = -\frac{1}{RC} x + \frac{1}{RC} u \\ y = x \end{cases}$$

\rightsquigarrow

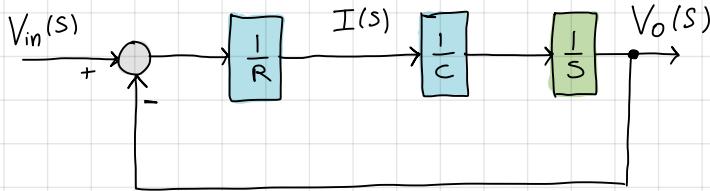
FAI MATRICI

SCHEMA A BLOCCHI

dalle eq $C \frac{dV_C}{dt} = I \Rightarrow C \frac{dV_O}{dt} = I \Leftrightarrow CS V_O(s) = I(s)$

$$V_R = R \cdot I \Rightarrow I = \frac{V_R}{R} \quad \text{ma} \quad V_R = V_{in} - V_O \Rightarrow I = \frac{V_{in} - V_O}{R}$$

$$\Leftrightarrow I(s) = \frac{V_{in}(s) - V_O(s)}{R}$$



RISPOSTA NEL TEMPO

Considero un segnale $V(t) = \mathbb{1}(t) \Rightarrow U(s) = \frac{1}{s}$

$$G(s) = \frac{1}{RCs+1} \Rightarrow Y_1(s) = \frac{1}{s(RCs+1)} = \frac{\varepsilon_1}{s} + \frac{\varepsilon_2}{RCs+1}$$

ε_1 del tipo $\frac{1}{s+1}$

$$\bar{\rho}_1 = -\frac{1}{RC}$$

$$\varepsilon_1 = \lim_{s \rightarrow 0} s \cdot \frac{1}{s(RCs+1)} = 1$$

$$\varepsilon_2 = \lim_{s \rightarrow 0 \cdot \frac{1}{RC}} (RCs+1) \cdot \frac{1}{s(RCs+1)} = -RC$$

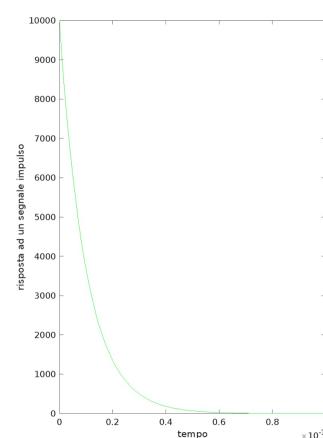
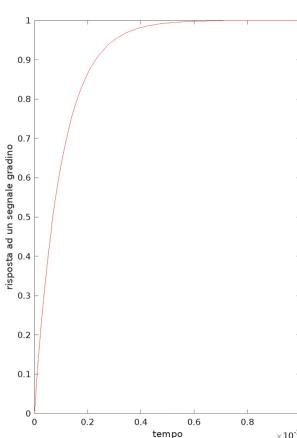
$$\therefore Y_1(s) = \frac{1}{s} - \frac{RC}{RCs+1}$$

$$\therefore y_1(t) = \left(1 - e^{-\frac{t}{RC}}\right) \cdot \mathbb{1}(t)$$

Risposta ad un segnale Step(1) per R.C generici.

Per $V(t) = \mathbb{f} \Rightarrow U(s) = 1$

$$\therefore y_2(t) = \frac{d}{dt} y_1(t) = \left(\frac{1}{RC} e^{-\frac{t}{RC}}\right) \cdot \mathbb{1}(t)$$



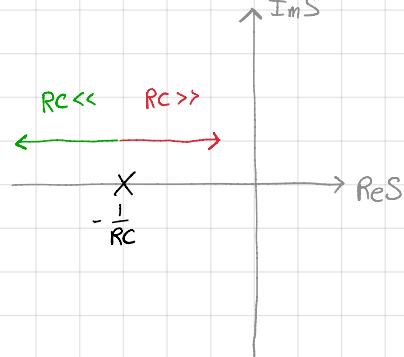
RISPOSTA IN FREQUENZA

$$G(S) = \frac{1}{RCs+1}$$

$$\Rightarrow P_1 = -\frac{1}{RC} = 0$$

$$\omega_1 = \frac{1}{RC}$$

(2)



(1) FORMA STANDARD

$$G(S) = \frac{1}{RCs+1} \quad G(S) \text{ è già nella S.F.}$$

(3) Intervallo di freq.

$\omega_1 = \frac{1}{RC} \Rightarrow$ Dipende da R, C ; Supponiamo che $\begin{cases} R = 470 \Omega \\ C = 10 \text{ mF} = 10 \times 10^{-3} \text{ F} \end{cases}$

$$\Rightarrow \omega_1 = \frac{1}{RC} = 2.13 \text{ rad/s} \Rightarrow \omega \in \left[\frac{1}{10} ; 10 \right] \text{ Almeno } \pm 1 \text{ decade}$$

(4) Andamenti iniziali e finali

MODULO: • Per $\omega < \omega_1$: $|G(j\omega)| = K_p = 1 \Rightarrow |G(j\omega)|_{dB} = 0$

• Per $\omega \gg \omega_1$: 1 Polo $\Rightarrow |G(j\omega)|_{dB} = -20 \text{ dB/dec}$

FASE: $\angle G(j\omega) = -\angle RCj\omega + 1 = -\tan^{-1}(RC \cdot \omega)$

• Per $\omega \ll \omega_1 \Rightarrow \angle G(j\omega) = 0^\circ \leftarrow \omega \ll \omega_1$

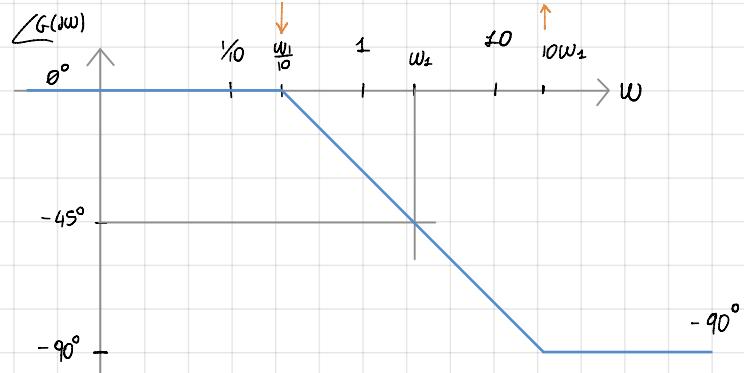
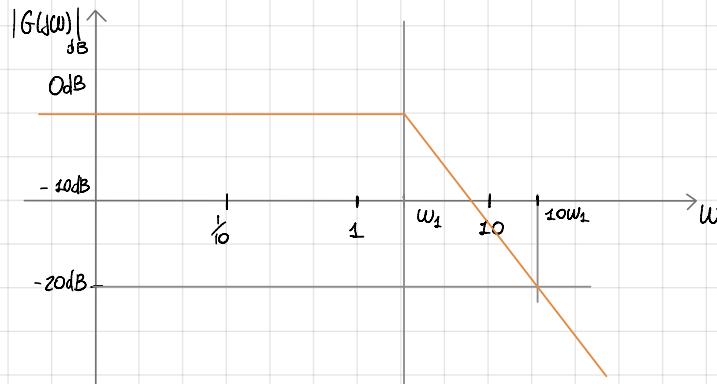
• Per $\omega \gg \omega_1 \Rightarrow \angle G(j\omega) = -90^\circ = -\frac{\pi}{2} \leftarrow \omega \gg \omega_1$

$\omega \ll \omega_1$

$$\frac{1}{dB}$$

$\omega \gg \omega_1$

(5) Diagrammi EDi Bode



(6) Uscita in frequenza

- Segnale (a): $U_1(t) = 5 \sin(0.1t) \Rightarrow Y_1(t) \triangleq 5 \cdot |G(j\omega)| \cdot \sin(t + \angle G(j\omega))$

$$|G(j0.1)|_{dB} = 0 \text{ dB} \Rightarrow 20 \log(|G(j0.1)|) = 0 = 0 \text{ dB} \quad |G(j0.1)| = 10^0 = 1 \text{ stessa modulo}$$

$$\angle G(j0.1) = 0^\circ \Rightarrow Y_{ss1}(t) = 5 \cdot 1 \cdot \sin(t)$$

Questo è il motivo per cui è chiamato filtro RC

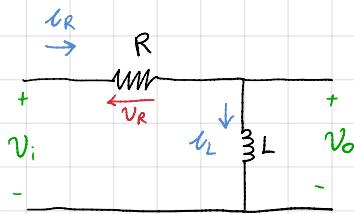
- Segnale (b): $U_2(t) = 5 \sin(21.3t) \Rightarrow |G(j\omega_2)|_{dB} = -20 \text{ dB} \Rightarrow 20 \log(|G|) = -20 \Rightarrow |G| = 10^{-4} = 0.1$

$$\angle G(j\omega_2) = -90^\circ = -\frac{\pi}{2} \Rightarrow Y_{ss2}(t) = \left(\frac{5}{10}\right) \sin(21.3t - \frac{\pi}{2})$$

Modulo attenuato *Fase ritardata*

CIRCUITO RL

CIRCUITO RL



$$(1) \begin{cases} V_R = R \cdot i_R \\ V_L = L \cdot i_L \end{cases} \text{ ma } V_o = V_L \text{ e } V_i = V_R + V_L$$

$$\Rightarrow V_i = R \cdot i_R + L \cdot i_L \quad \text{con } i_R = i_L$$

$$\Rightarrow V_i(s) = R \cdot I_R + S L I_L = I(R + S L)$$

Se prendo V_i come input e i_L come out

$$\Rightarrow G(s) = \frac{I(s)}{V_i(s)} = \frac{1}{S L + R}$$

Funzione di Trasferimento

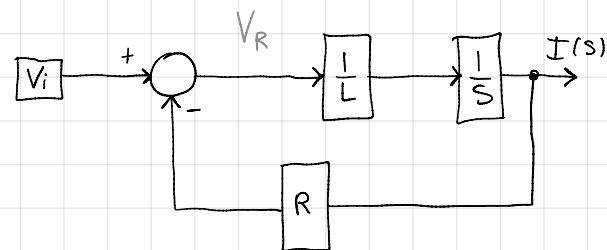
SCHEMA A BLOCHI

dalle eq (1) $\Rightarrow i_L = \frac{1}{L} \cdot V_L$ ma $V_L = V_i - V_R = V_i - R \cdot i$

$$\Rightarrow i_L = \frac{1}{L} \cdot (V_i - R \cdot i)$$

input

Retroazione



* Volendo posso trovare la Z.T. anche dallo schema a blocchi:

Siccome abbiamo una retroazione: $y(s) = \frac{G(s)}{1 + K \cdot G(s)}$ $\Rightarrow I(s) = \frac{\frac{1}{S L}}{1 + R \cdot \frac{1}{S L}}$

CATENA DIRETTA

gain sulla retroazione

$$\Rightarrow I(s) = \frac{1}{S L (1 + \frac{R}{S L})} = \frac{1}{S L + R}$$

stessa di prima!

SPAZIO DI STATO

Definisco $U = V_i$; $y = i$; $x = \dot{x}$

IN

OUT

State Var

$$\dot{i} = \frac{1}{L} \cdot V_L = \frac{1}{L} (V_i - V_R) = \frac{1}{L} (V_i - R \cdot i) \Rightarrow \dot{x} = \frac{1}{L} (U - R \cdot x)$$

$$i = i \Rightarrow y = x$$

y

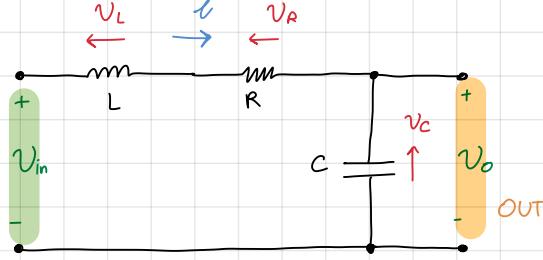
x

$$\Rightarrow \begin{cases} \dot{x} = -\frac{R}{L} x + \frac{1}{L} U \\ y = x \end{cases}$$

Spazio Di Stato

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad \begin{cases} \dot{x} = (-\frac{R}{L})(x) + (\frac{1}{L})U \\ y = (1)x + (0)U \end{cases}$$

CIRCUITO RLC



SERIE

$$L.K. \quad V_L + V_R + V_C = V_{in}$$

L.T.

$$V_L + V_R + V_C = V_{in}$$

$$\begin{aligned} L.T. \quad & V_L = V_i - V_o - V_R \\ & = V_i - V_o - R \cdot i \end{aligned}$$

$$\begin{cases} V_L = L i \\ i = C \dot{V}_c \rightarrow \dot{V}_c = \frac{1}{C} i \rightarrow V_c = \frac{1}{C} \int i dt \\ V_R = R i \end{cases} \quad \Downarrow$$

$$L i + R i + \frac{1}{C} \int i dt = V_{in}$$

$$\begin{aligned} \Rightarrow L s I(s) - i(0^+) + R I(s) + \frac{1}{C} \frac{I(s)}{s} + \frac{i(0^+)}{s} &= V_{in}(s) \\ \Downarrow \quad L s I(s) + R I(s) + \frac{1}{C s} I(s) &= V_{in}(s) \quad V_c \equiv V_o \end{aligned}$$

$$\text{Supponiamo di volere } G(s) = \frac{V_o(s)}{V_{in}(s)} \quad \text{e} \quad I(s) \left[L s + R + \frac{1}{C s} \right] = V_{in}(s)$$

$$\text{Siccome } V_o = V_c = \frac{1}{C} \int i dt \Rightarrow V_o = \frac{1}{C s} I(s)$$

$$\Rightarrow G(s) = \frac{\frac{1}{C s} I(s)}{I(s) \left[L s + R + \frac{1}{C s} \right]} = \frac{\frac{1}{C s}}{\frac{L C s^2 + R C s + 1}{C s}} = \frac{1}{L C s^2 + R C s + 1} \quad G(s)$$

SPAZIO DI STATO (1)

Scelgo

$$\begin{array}{l} \text{Uscita} \\ U = V_i \\ \text{Stato} \\ x_1 = i \quad x_2 = V_c = V_o \end{array}$$

$$\begin{array}{l} \text{Uscita} \\ y = V_o \end{array}$$

$$\dot{i} = \frac{1}{L} V_L \quad \text{ma} \quad V_L = (V_i - V_c - V_R) = (V_i - V_o - V_R) \Rightarrow \dot{x}_1 = \frac{1}{L} (U - x_2 - R x_1)$$

$$V_o = V_c \Rightarrow \dot{V}_c = \frac{1}{C} \dot{V}_o \quad \dot{x}_2 = \frac{1}{C} \dot{V}_o$$

$$\begin{array}{l} U \\ V_o = V_o \\ \dot{x}_2 = \frac{1}{C} x_1 \end{array}$$

$$y = V_o \rightarrow \text{e' e' una Var di Stato } (x_2) \quad \text{e' una Var di Stato } (x_2) \quad y = x_2$$

Riscriro il Tutto

$$\begin{cases} \dot{x}_1 = -\frac{R}{L} x_1 - \frac{1}{L} x_2 + \frac{1}{L} U \\ \dot{x}_2 = \frac{1}{C} x_1 + 0 x_2 + 0 U \\ y = 0 x_1 + 1 x_2 + 0 U \end{cases}$$

=>

$$\begin{cases} \dot{x} = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \cdot U \\ y = (0 \ 1) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + (0) \cdot U \end{cases}$$

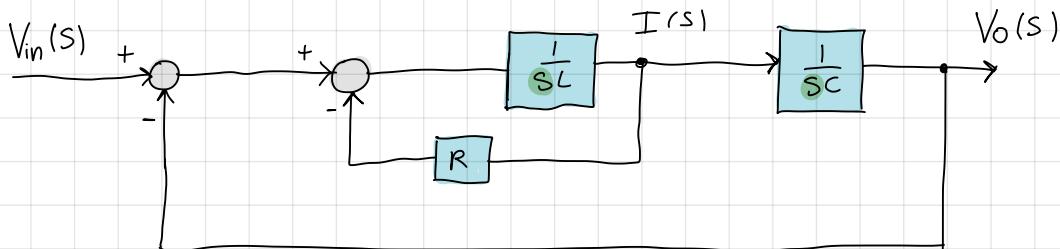
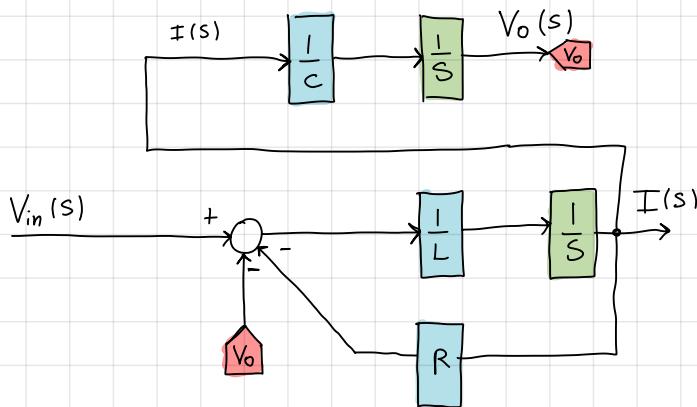
SCHEMA A BLOCCHI

Dalle eq:

$$\begin{cases} L \dot{i} = V_L \\ C \dot{V}_o = i \\ V_R = R \cdot i \end{cases}$$

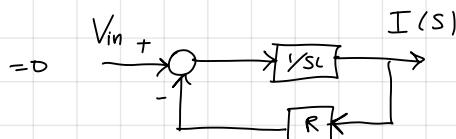
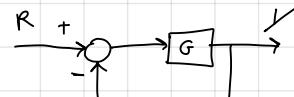
$$V_L = V_{in} - V_o - V_R$$

$\nwarrow V_R = R \cdot i$



Ricordiamo il sys in feedback:

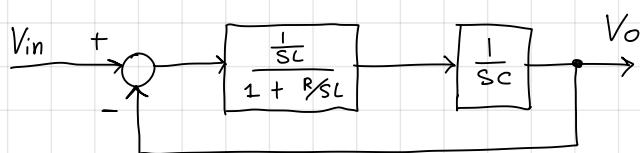
$$Y = \frac{G}{1+G} \cdot R(s)$$



\leadsto

$$\frac{I(s)}{V_{in}(s)} = \frac{\frac{1}{SL}}{1 + \frac{R}{SL}}$$

\Rightarrow Posso scrivere lo schema anche così:



SPAZIO DI STATO (2)

(1) Equazione diff dalla funzione di Trasferimento:

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{LCS^2 + RCS + 1}$$

$$\Rightarrow V_{in}(s) = LCS^2 V_o(s) + RCS V_o(s) + V_o(s)$$

$$\Rightarrow S^2 V_o(s) + \frac{RS}{L} V_o(s) + \frac{1}{LC} V_o(s) = \frac{1}{LC} V_{in}(s)$$

$$\text{Siccome } \mathcal{L} \left[\dot{f}(t) \right] = \frac{1}{s} f(s) \underset{c.f. = 0}{=} 0$$

$$\ddot{V}_o(t) + \frac{R}{L} \dot{V}_o(t) + \frac{1}{LC} V_o(t) = \frac{1}{LC} U(t)$$

(2) Scelgo le entrate ed uscite e le Variabili di stato

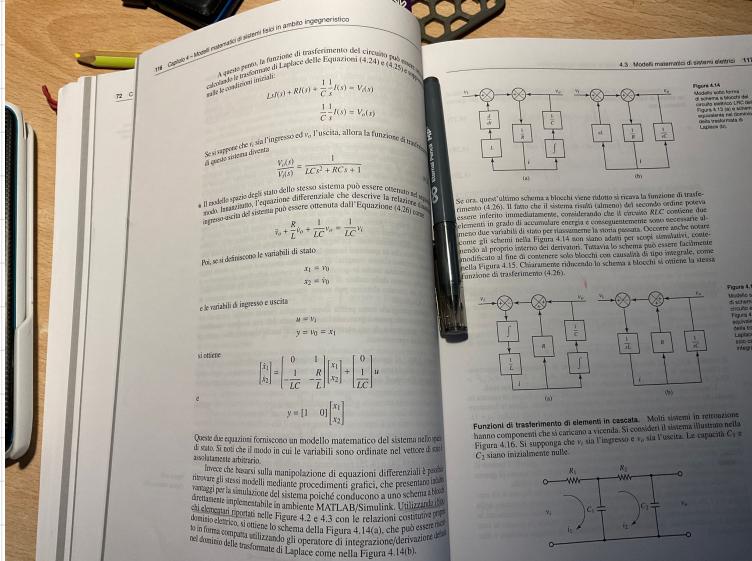
$$\text{Scelgo} \quad \begin{cases} x_1 = V_o \\ x_2 = \dot{V}_o \end{cases}$$

$$\begin{cases} u = V_{in} \\ y = V_o = x_1 \end{cases}$$

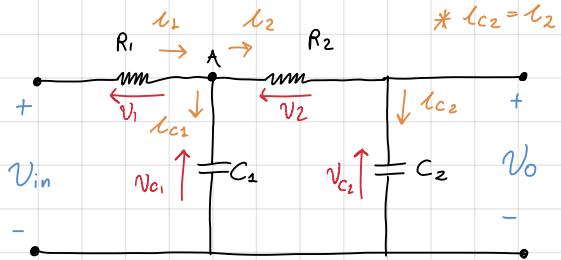
$$\begin{cases} \dot{x}_n = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{LC} & \frac{R}{L} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{matrix} V_o \\ \dot{V}_o \end{matrix}$$

BOH



Elementi in Cascata



$$\begin{cases} \dot{V}_{C_1} = C_1 \dot{V}_{C_1} \\ \dot{V}_{C_2} = C_2 \dot{V}_{C_2} \end{cases} \quad \text{so} \quad V_{C_1} = \frac{1}{C_1} \int \dot{V}_{C_1} dt$$

$$\text{so} \quad V_{C_2} = \frac{1}{C_2} \int \dot{V}_{C_2} dt$$

$$\begin{cases} V_1 = R_1 I_1 \\ V_2 = R_2 I_2 \end{cases}$$

SPAZIO DI STATO

Pongo $V_{C_1} = x_1$ $V_{C_2} = x_2$
 $U = V_{in}$ $y = V_0$

$$C \dot{V}_{C_1} = I_{C_1} \quad \text{ma} \quad LK_C_A \quad I_{C_1} = I_1 - I_{C_2}$$

$$\text{L} \quad C \dot{V}_{C_1} = I_1 - I_2$$

$$\text{ma} \cdot I_1 = \frac{V_1}{R_1}, \quad V_1 = V_{in} - V_{C_1}$$

$$= \frac{V_{in} - V_{C_1}}{R_1}$$

$$\cdot I_2 = \frac{V_2}{R_2}, \quad V_2 = V_{C_1} - V_{C_2}$$

$$= \frac{V_{C_1} - V_{C_2}}{R_2}$$

$$\Rightarrow C \dot{V}_{C_1} = \left(\frac{V_{in} - V_{C_1}}{R_1} \right) - \left(\frac{V_{C_1} - V_{C_2}}{R_2} \right)$$

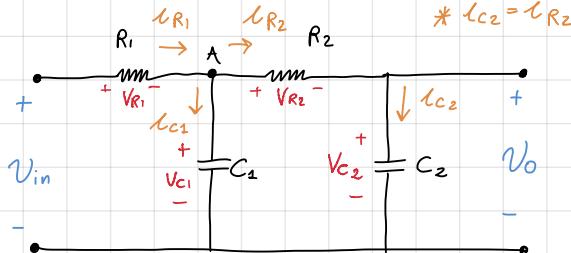
$$\dot{V}_{C_2} = \frac{x_1 - x_2}{C_2 R_2}$$

$$\Rightarrow C \ddot{x}_1 = \frac{U - x_1}{R_1} - \frac{x_2 - x_1}{R_2} = x_1 \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + x_2 \left(\frac{1}{R_2} \right) + \frac{1}{R_1} U$$

$$\begin{cases} \ddot{x}_1 = \left[\frac{1}{C_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \right] x_1 + \frac{1}{R_2} x_2 + \frac{1}{R_1} U \\ \dot{x}_2 = \frac{1}{C_2 R_2} x_1 - \frac{1}{C_2 R_2} x_2 \\ y = x_2 \end{cases}$$

$$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{C_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) & \frac{1}{R_2} \\ \frac{1}{C_2 R_2} & -\frac{1}{C_2 R_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{R_1} \\ 0 \end{pmatrix} \cdot U$$

Elementi in Cascata



$$\begin{cases} I_{C1} = C_1 \frac{dV_{C1}}{dt} \\ I_{C2} = C_2 \frac{dV_{C2}}{dt} \end{cases}$$

$$\begin{cases} V_{R1} = R_1 \cdot I_{R1} \\ V_{R2} = R_2 \cdot I_{R2} \end{cases} \Rightarrow \begin{cases} I_{R1} = \frac{V_{R1}}{R_1} \\ I_{R2} = \frac{V_{R2}}{R_2} \end{cases}$$

SPAZIO DI STATO

OBIETTIVO: Scrivere le eq nelle forme: $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

$$Scelgo \quad x_1 = V_{C1}; \quad x_2 = V_{C2};$$

$$U = V_i; \quad y = V_o$$

$$\dot{x}_1 = \frac{1}{C_1} \cdot I_{C1} \quad \text{ma} \quad I_{C1} = I_{R1} - I_{R2} = \frac{V_{R1}}{R_1} - \frac{V_{R2}}{R_2} \quad \text{con} \quad V_{R1} = V_i - V_{C1} = U - x_1 \\ V_{R2} = V_{C1} - V_{C2} = x_1 - x_2$$

$$\Rightarrow \dot{x}_1 = \frac{1}{C_1} \left(\frac{U - x_1}{R_1} - \frac{x_1 - x_2}{R_2} \right) = x_1 \left(-\frac{1}{C_1 R_1} - \frac{1}{C_1 R_2} \right) + x_2 \left(-\frac{1}{C_1 R_2} \right) + U \cdot \frac{1}{C_1 R_1}$$

$$\dot{x}_2 = \frac{1}{C_2} \cdot \frac{V_{R2}}{R_2} = \frac{1}{C_2 R_2} (V_{C1} - V_{C2}) = \frac{1}{C_2 R_2} x_1 - \frac{1}{C_2 R_2} x_2$$

$$y = x_2$$

$$\Rightarrow \begin{cases} \dot{x}_1 = x_1 \left(-\frac{1}{C_1 R_1} - \frac{1}{C_1 R_2} \right) + x_2 \left(-\frac{1}{C_2 R_2} \right) + U \cdot \frac{1}{C_1 R_1} \\ \dot{x}_2 = \frac{1}{C_2 R_2} x_1 - \frac{1}{C_2 R_2} x_2 \\ y = x_2 \end{cases} \Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{C_1 R_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) & -\frac{1}{C_1 R_2} \\ \frac{1}{C_2 R_2} & -\frac{1}{C_2 R_2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1 R_1} \\ 0 \end{pmatrix} \cdot U$$

FUNZ. DI TRASFERIMENTO

Tra out: $V_o(s)$ e In: $V_i(s)$

$$V_i = V_{R_1} + V_{C_1} = R_1 I_{R_1} + \frac{1}{C_1} \int (I_{R_1} - I_{R_2}) dt \quad (1) \quad I_{C_1} = C_1 \cdot \dot{V}_{C_1} = 0 \quad V_{C_1} = \frac{1}{C_1} \int I_{C_1} dt$$

$$+ V_{C_2} + V_{R_2} - V_{C_1} = 0$$

$$I_{C_2} = C_2 \cdot \dot{V}_{C_2} = 0 \quad V_{C_2} = \frac{1}{C_2} \int I_{C_2} dt$$

$$-\frac{1}{C_1} \int (I_{R_1} - I_{R_2}) dt + I_{R_2} R_2 + \frac{1}{C_1} \int I_{R_2} dt = 0$$

$$\frac{1}{C_1} \int (I_{R_2} - I_{R_1}) dt + I_{R_2} R_2 + \frac{1}{C_1} \int I_{R_2} dt = 0 \quad (2)$$

Calcolo le trasformate

$$I_{R_1} - I_{R_2} - I_{C_1} = 0$$

$$(1) \quad R_1 I_{R_1} + \frac{1}{C_1 s} I_{R_1} - \frac{1}{C_1 s} I_{R_2} = V_i$$

$$(2) \quad \frac{1}{C_1 s} I_{R_2} - \frac{1}{C_1 s} I_{R_1} + I_{R_2} R_2 + \frac{1}{C_1 s} I_{R_2} = 0$$

$$\Rightarrow \begin{cases} V_o(s) = \frac{1}{C_1 s} (I_{R_1} - I_{R_2}) + I_{R_2} R_2 \\ V_i(s) = \frac{1}{C_1 s} (I_{R_1} - I_{R_2}) + I_{R_1} R_1 \end{cases}$$

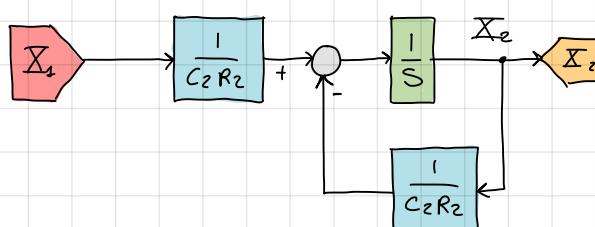
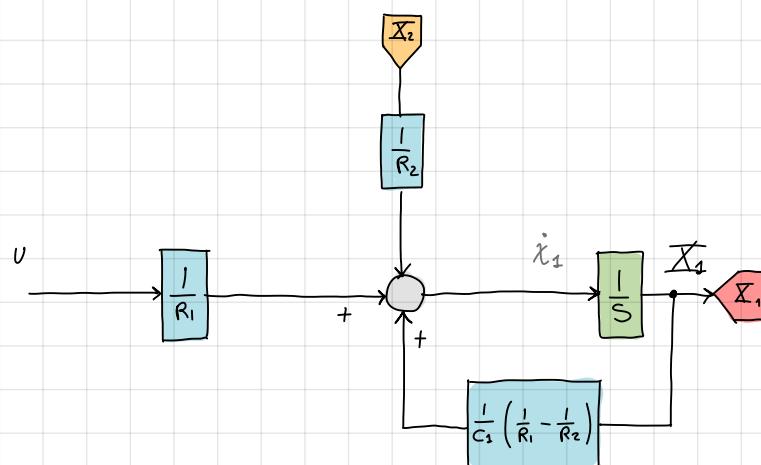
$$\Rightarrow \frac{V_o(s)}{V_i(s)} =$$

Ma che ne
so io...

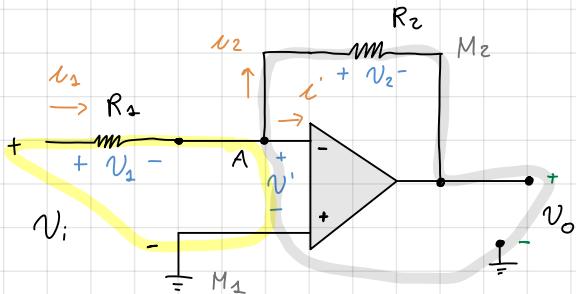
SCHEMA A BLOCCHI

(dallo spazio di stato)

$$\begin{cases} \dot{x}_1 = \left[\frac{1}{C_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \right] x_2 + \frac{1}{R_2} x_2 + \frac{1}{R_1} u \\ \dot{x}_2 = \frac{1}{C_2 R_2} x_1 - \frac{1}{C_2 R_2} x_2 \end{cases}$$



Amplificatore invertente



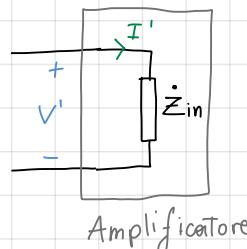
Scopo del gioco: Trovare V_o in relazione a R_1, R_2

$$\ell_2 = \frac{V_2}{R_2} = \frac{V' - V_o}{R_2}$$

$$\ell_1 = \frac{V_{R_1}}{R_1} = \frac{V_i - V'}{R_1}$$

$$\text{ma } LUC_A: -\ell_1 + \ell_2 + \ell' = 0 \Rightarrow \ell_1 = \ell_2 + \ell'$$

\rightarrow Proprietà degli A.Op. Z_{interna} >>



Troviamo ℓ'

$$V' = Z \cdot I' \Rightarrow I' = \frac{V'}{Z}$$

$$\text{ma se } Z \gg V = \frac{V'}{Z} \approx 0$$

$\Rightarrow I' \approx 0$ QED

$$\Rightarrow \ell_1 = \ell_2 + \ell' = \ell_2$$

$$\Rightarrow \ell_1 \approx \ell_2 \text{ orrero}$$

$$\frac{V_i - V'}{R_1} = \frac{V' - V_o}{R_2} \quad (1)$$

Inoltre gli op-amp sono fatti per avere un guadagno molto elevato in modo da amplificare anche il minimo segnale:

$$V_o = K (V_+ - V_-) \text{ con } K \gg (V_+ - V_-) \text{ ma (vedi figura) } V_+ \text{ è messo a Terra!}$$

$$\Rightarrow V_o = K (0 - V') \text{ con } K \gg 1 \Rightarrow V' \approx 0$$

guadagno

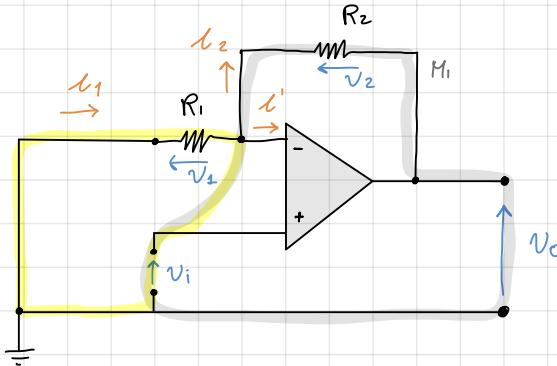
$$(1) \frac{V_i - V'}{R_1} = \frac{V' - V_o}{R_2} \Rightarrow \frac{V_i}{R_1} = - \frac{V_o}{R_2} \Rightarrow V_o = \frac{R_2}{R_1} V_i$$

INVERTENTE

Attenzione !

Il circuito visto è di tipo PID *proporzionale* ma invertente; per avere un controllore proporzionale (non invertente) basta usare $R_2=R_1$ e mettere due controllori (invertenti) in serie. Vedi nella cartella *approfondimenti*.

Amplificatore non inverteente



Trovare la relazione $v_o = K v_i$

$$\begin{cases} i_2 = \frac{v_2}{R_2} = \frac{v_i - v_o}{R_2} \\ i_1 = \frac{v_1}{R_1} = -\frac{v_i}{R_1} \end{cases}$$

\Rightarrow Siccome $i' \approx 0 \Rightarrow i_1 \approx i_2$

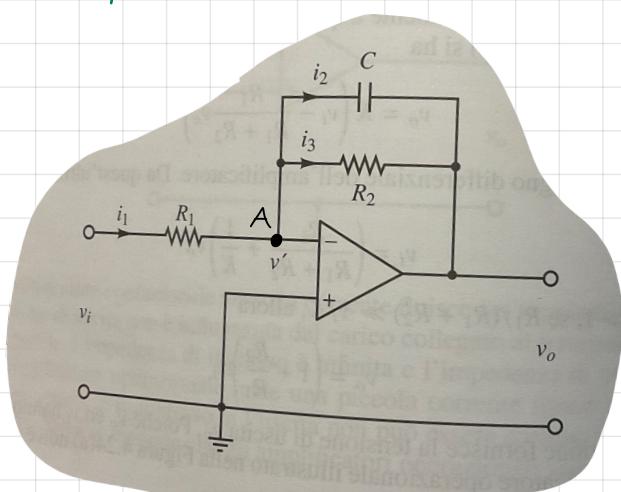
$$-\frac{v_i}{R_1} = \frac{v_i - v_o}{R_2} \Rightarrow \frac{v_o}{R_2} = \frac{v_i}{R_1} + \frac{v_i}{R_2} \Rightarrow v_o = R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_i$$

GUADAGNO

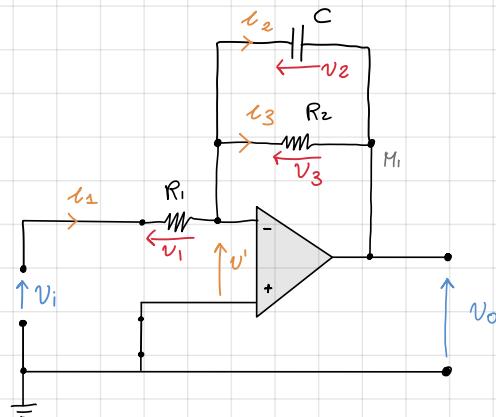
IN CONF NON INVERTENTE

ES: $K=1 \Rightarrow R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = 1 \Rightarrow \frac{1}{R_1} + \frac{1}{2} = 1 \Rightarrow \frac{1}{R_1} = \frac{1}{2} \Rightarrow R_1 = 2 \Omega$

ES 4.2 CIRCUITO RITARDATORE



Q: Determinare v_o



$$i_1 = \frac{v_1}{R_1} = \frac{v_i - v'}{R_1}$$

$$i_2 = C \frac{dv_2}{dt} \quad \text{ma} \quad v_2 = v_3 = v' - v_o \Rightarrow i_2 = C \frac{d(v' - v_o)}{dt}$$

$$i_3 = \frac{v' - v_o}{R_2}$$

Siccome $i' \approx 0 \Rightarrow LKc_A : -i_1 + i_3 + i_2 + i' = 0 \Rightarrow i_1 = i_2 + i_3$

$$\Rightarrow \frac{v_i - v'}{R_1} = C \frac{d(v' - v_o)}{dt} + \frac{v' - v_o}{R_2}$$

INVERTENTE $\Leftrightarrow v' \underset{\text{uguale approx}}{\underset{\text{}}{\overset{\circ}{\parallel}}} 0$

$$\xrightarrow{\text{---}} \frac{V_i}{R_1} = -C \dot{V}_o - \frac{1}{R_2} V_o \quad \xrightleftharpoons{\mathcal{LT}} \quad \frac{1}{R_1} V_i(s) = -CS V_o(s) - \frac{1}{R_2} V_o(s) = -\frac{SCR_2 V_o(s) + V_o(s)}{R_2}$$

$$\left| \frac{V_i(s)}{R_1} = \frac{R_2 C s + 1}{R_2} V_o(s) \right.$$

da $\mathcal{T.F.}$ e' $G(s) = \frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1(R_2 C s + 1)} = \boxed{-\frac{R_2}{R_1} \cdot \frac{1}{R_2 C s + 1}}$ $\mathcal{T.F.}$