$$G(S) = \frac{1+S}{1+0.1\cdot S}$$

$$u(t) = \begin{cases} 0 & t<0 \\ 2-2t & t\in(0,1] \\ -2+2t & t\in(1,2] \\ 0 & t\geq 2 \end{cases}$$

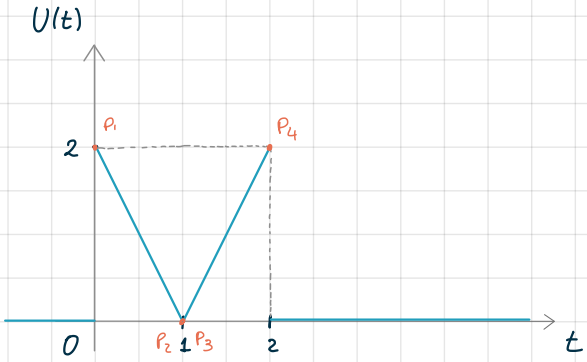
- Segnale in input

t	A
0	2
1	0

$\rightsquigarrow$ $P_1(0,2)$, $P_2(1,0)$

t	A
1	0
2	2

$\rightsquigarrow$ $P_3(1,0)$, $P_4(2,2)$

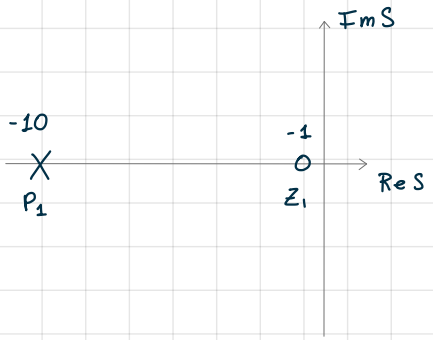


* Non in Scala

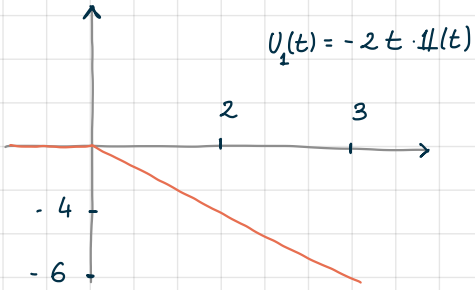
- Poli e zeri

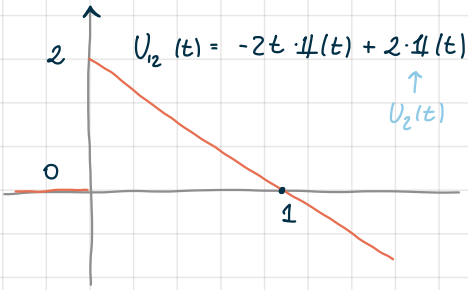
Z: $\bar{S} = -1$

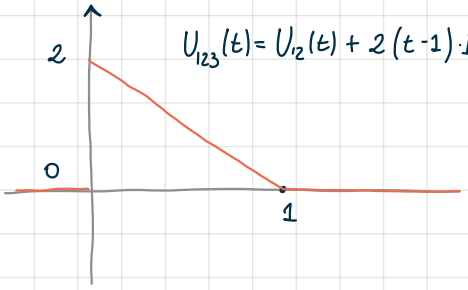
P: $\bar{S} = -\frac{1}{0.1} = -10$



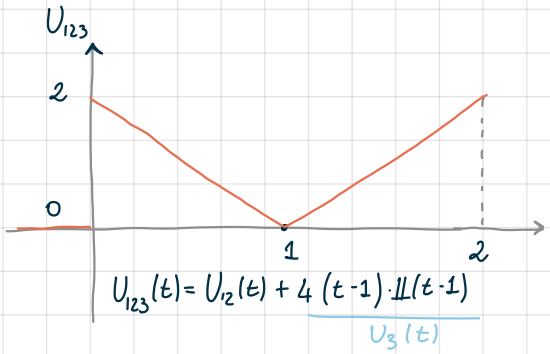
- Segnali elementari

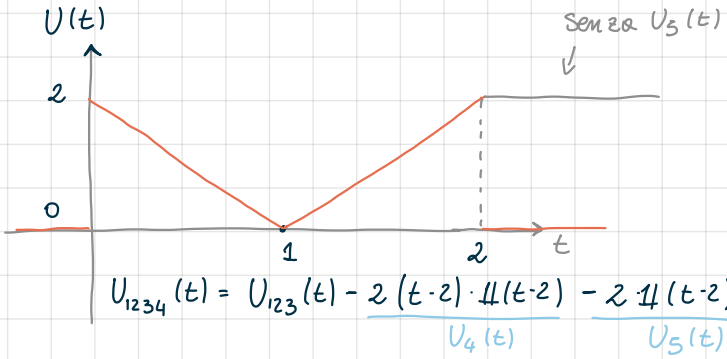






Ma siccome in 1 deve risalire, posso scrivere direttamente





$$\begin{cases} u_1(t) = -2t\cdot 1(t) \\ u_2(t) = 2\cdot 1(t) \\ u_3(t) = 4(t-1)\cdot 1(t-1) \\ u_4(t) = -2(t-2)\cdot 1(t-2) \\ u_5(t) = -2\cdot 1(t-2) \end{cases} \rightleftharpoons \begin{cases} u_1(S) = -2\ 1/S^2 \\ u_2(S) = 2\cdot 1/S \\ u_3(S) = 4\cdot 1/S^2\cdot e^{-S} \\ u_4(S) = -2\ 1/S^2\cdot e^{-2S} \\ u_5(S) = -2\cdot 1/S\cdot e^{-2S} \end{cases}$$

Time 30'

• Segnali fittizi

Siccome il segnale è composto di Rampe e gradini, mi basta trovare l'uscita del solo segnale rampa:

$$\hat{U}(t) = t \cdot 1(t) \rightleftharpoons U(S) = \frac{1}{S^2}$$

$$\Rightarrow \hat{y}(S) = G(S)\cdot \hat{U}(S) = \frac{1+S}{S^2(1+0.1S)} = \frac{r_1}{S} + \frac{r_2}{S^2} + \frac{r_3}{1+0.1S} = \frac{10+10S}{S^2(S+10)}$$

$$r_2 = \lim_{S\to 0} S^2 \cdot \frac{10+10S}{S^2(S+10)} \to \frac{10}{10} \to 1 \quad r_2$$

$$r_3 = \lim_{S\to -10} (S+10)\cdot \frac{10+10S}{S^2(S+10)} = -\frac{90}{100} = -0.9 \quad r_3$$

$$\frac{r_1 S(S+10) + r_2(S+10) + r_3 S^2}{S^2(S+10)} = \frac{10+10S}{S^2(S+10)} \rightsquigarrow \begin{cases} S^2(r_1+r_3)=0 \\ S(10r_1+r_2)=10 \\ 10 r_2 = 10 \end{cases} \to r_2 = 1 \quad \text{QED}$$

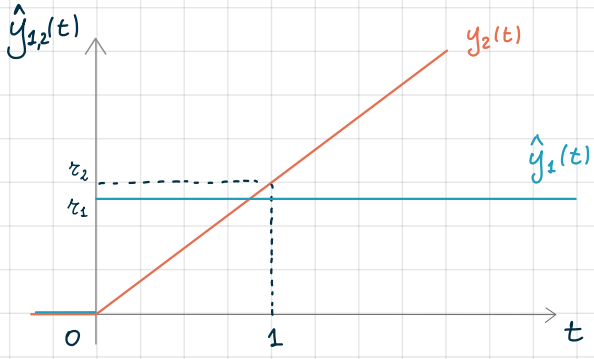
$$\to r_1 = -r_3 \Rightarrow r_1 = 0.9 \quad r_1$$

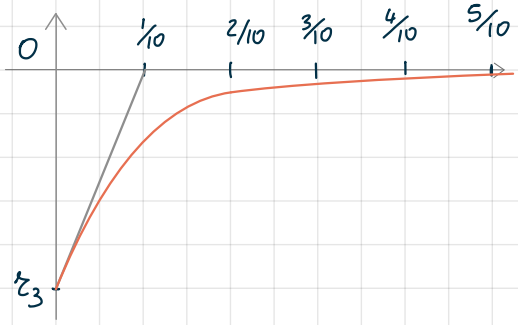
Quindi $\hat{y}(S) = \frac{1+S}{S^2 0.1(S+10)} = \frac{10S+10}{S^2(S+10)} = \underbrace{\frac{r_1}{S}}_{r_1\cdot 1(t)} + \underbrace{\frac{r_2}{S^2}}_{r_2\cdot t\cdot 1(t)} + \underbrace{\frac{r_3}{S+10}}_{r_3\cdot e^{-10t}\cdot 1(t)}$

$$\Rightarrow \hat{y}(t) = (r_1 + r_2 t + r_3 e^{-10t})\cdot 1(t)$$

* $r_3 < 0$

$$\hat{y}_3(t) = -|r_3| e^{-10t} \rightsquigarrow \tau = \frac{1}{10}$$





• Uscite Reali

→ l'uscita è data da $y(t) = \sum_{i=1}^{5} y_i(t)$

$$\begin{cases} y_1(t) = -2\cdot \hat{y}(t) \longrightarrow y_1(t) = -(2r_1 + 2r_2 t + 2r_3 e^{-10t})\cdot 1(t) \\ y_2(t) = 2\cdot \frac{d}{dt}\hat{y}(t) \longrightarrow y_2(t) = (2r_2 - 20 r_3 e^{-10t})\cdot 1(t) \\ y_3(t) = 4\cdot \hat{y}(t-1) \\ y_4(t) = -2\,\hat{y}(t-2) \\ y_5(t) = -2\cdot \frac{d}{dt}\hat{y}(t-2) \longrightarrow y_5(t) = -(2r_2 + 20 r_3 e^{-10t-2})\cdot 1(t-2) \end{cases}$$

e^{-10t-2}: costante

- Considerazioni

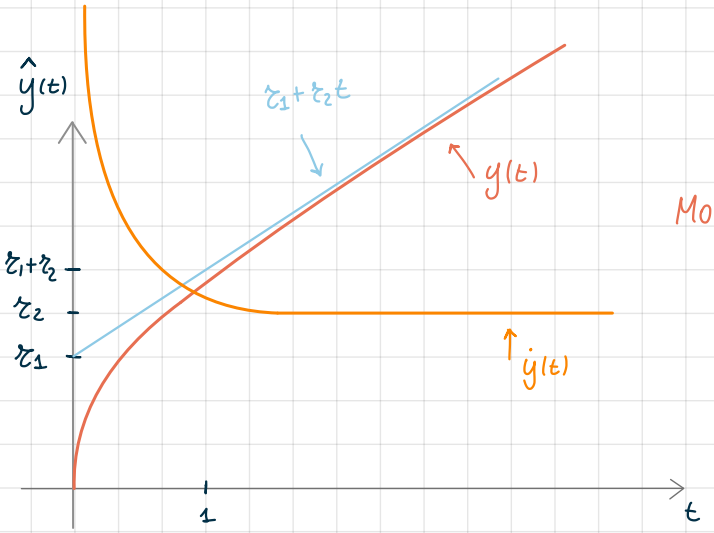
Valore iniziale

TVI: $y(t)\Big|_{t=0} = \lim_{s\to\infty} s\cdot\hat{y}(s) = \lim_{s\to\infty} s\cdot\left(\frac{r_1}{s}+\frac{r_2}{s^2}+\frac{r_3}{s+1}\right) = r_1 + \frac{r_2}{s} + \frac{s\,r_3}{s\left(1+\frac{1}{s}\right)} \to r_1 + r_3 = 0$

($\frac{r_2}{s}\to 0$, $\frac{1}{s}\to 0$)

TVF: $y(t\to\infty) = \lim_{s\to 0} s\cdot\hat{y}(s) = \lim_{s\to 0} s\cdot\left(\frac{r_1}{s}+\frac{r_2}{s^2}+\frac{r_3}{s+1}\right) = r_1 + \frac{r_2}{s} + \frac{r_3}{s+1} \to +\infty$

($\frac{r_3}{s+1}\to r_3$)



Morale della favola $y(t)$ è composta da combinazioni di $\hat{y}(t)$ graficata a Sx

$$\frac{d}{dt}\hat{y}(t) = \left(r_2 - 10 r_3 e^{-10t}\right)\cdot 1(t)$$

$$y(0) = \lim_{t\to 0} r_2 - 10 r_3 e^{-10t} = r_2 - 10 r_3 = 10$$

($e^{-10t}\to 1$)

$$y(\infty) = r_2 - 10 e^{-10t} = r_2 = 1$$

($e^{-10t}\to 0$)