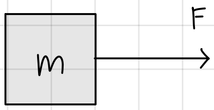
Modellistica dei sistemi meccanici

* Primo e secondo es
clip audio 8/9
e domanda orale
2° e 3° orale → elenco domande

Caratterizzati da MASSA molla e SMORZATORE

MASSA

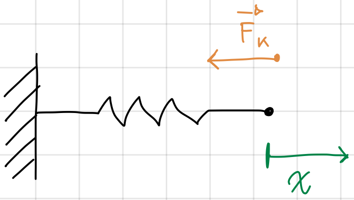


$[N]$

$F = ma \quad , \quad a = \frac{dv}{dt} = \dot{v} \; [m/s^2] \quad , \quad v = \frac{dx}{dt} = \dot{x} \; [m]$

Spazio

MOLLA



Legge di Hooke : $F_k = -K x$ (la molla viene Allungata)

$E_m(t) = \int_0^t \underbrace{F(t) \cdot v(t)}_{\text{Potenza}} \, dt = \int_0^t m \frac{dv}{dt} \cdot v(t) \, dt = \frac{1}{2} m \int_0^t \frac{d v^2(t)}{dt} dt = \frac{1}{2} m \left[v^2(t) \right]_0^t$

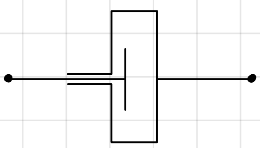
Energia di un corpo in movim.

$= \frac{1}{2} m v^2(t)$ ← Possibile Var di Stato

$E_k(t) = \int_0^t F_k(t) \cdot v(t) \, dt = \frac{1}{2} k \, x^2(t)$ ← Possibile Var di Stato

Energia della molla

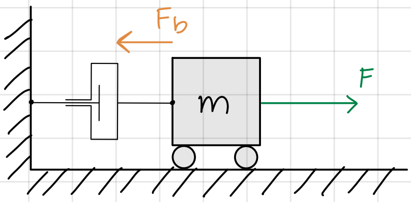
SMORZATORE



$v = \frac{dx}{dt} \rightarrow F_b = -b v \equiv F_b = -b \cdot \dot{x}$

* AUDIO DELLA LEZIONE
Nel file nella cartella b.S. elettrici

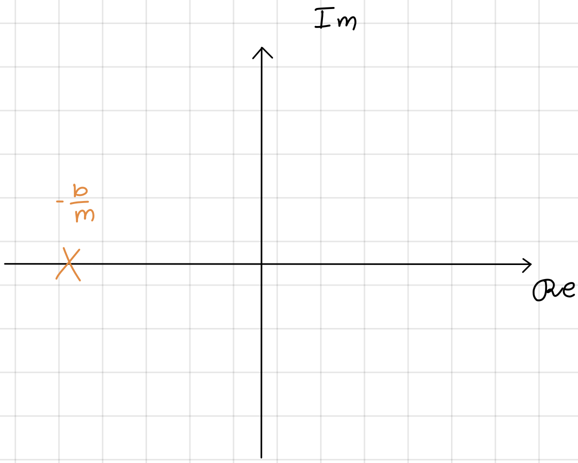
SISTEMA 1

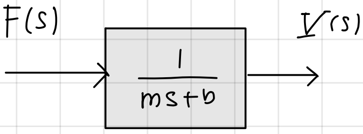


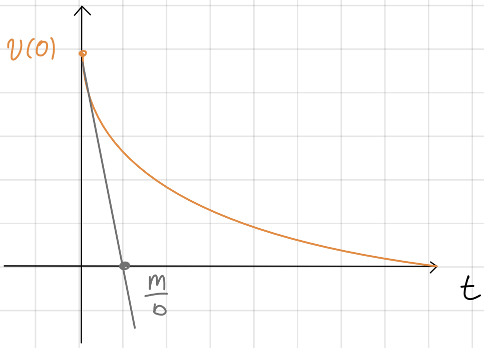
$$\begin{cases} F + F_b = m\dot{v} \\ F_b = -bv \end{cases}$$

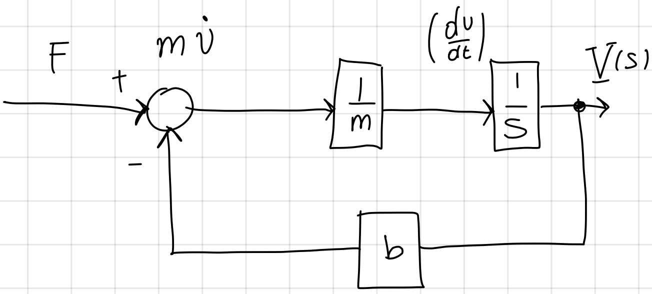
$\mathcal{L}.T. \rightarrow \quad F(s) - b \cdot V = m \cdot s\, V(s) \quad \rightarrow \quad (ms+b)\, V(s) \quad F(s)$

$\Rightarrow \quad \dfrac{V(s)}{F(s)} = \dfrac{1}{ms+b}$









VAR. ST. $\quad v$

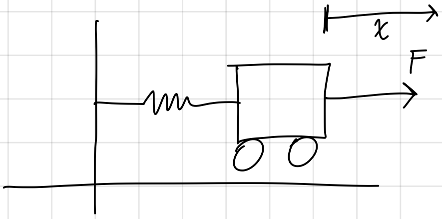
$x = v \quad , \quad u = F \quad , \quad y = v$

$$\begin{cases} \dot{x} = \frac{1}{m}(u - b \cdot x) \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \frac{b}{m} x + \frac{1}{m} u \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \underset{=}{A} x + \underset{=}{B} u \\ y = \underset{=}{C} x + \underset{=}{D} y \end{cases}$$

$\frac{b}{m} \quad \frac{1}{m}$

$1 \quad 0$

SENZA SMORZATORE



$$\begin{cases} F + F_K = m\dot{v} & \longrightarrow \quad \dot{v} = \frac{1}{m}(F + F_K) \\ F_K = -Kx \\ v = \dot{x} \end{cases}$$

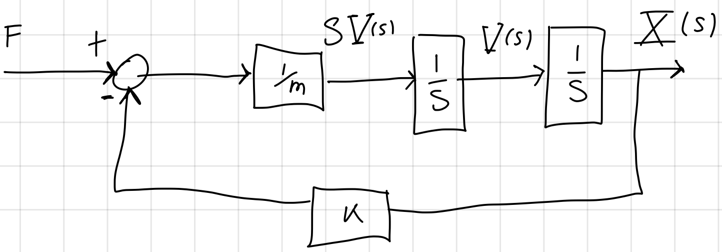
$x_1 = x \qquad x_2 = v \qquad u = F \qquad y = x_1$ ← la scelgo io

↑ pos in x

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m}(u - Kx_1) \\ y = x_1 \end{cases} \longrightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{m}x_1 + \frac{1}{m}u \\ y = x_1 \end{cases}$$

$$\longrightarrow \underset{x_1, x_2}{\underline{A}} = \begin{array}{c} \begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix} \end{array}\begin{array}{l} x_1 \\ x_2 \end{array} \qquad \underset{u}{\underline{B}} \begin{pmatrix} 0 \\ 1/m \end{pmatrix} \qquad \underset{y}{\underline{C}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

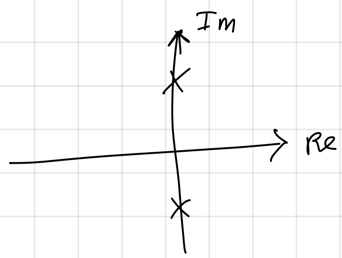
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix}}_{\underline{A}} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\underline{C}} u \qquad \underline{B}$$



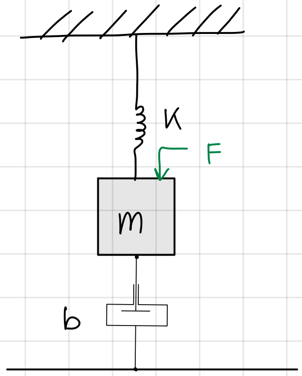
$$G(s) = \frac{X}{S} = \frac{\frac{1}{ms^2}}{1 + K\frac{1}{ms^2}} = \boxed{\frac{1}{ms^2 + K}}$$

Sin(t)

con $S = \pm j\sqrt{\frac{K}{m}}$



CON SMORZATORE



$$\begin{cases} F + F_K + F_b = m\dot{v} \\ F_K = -Kx \\ F_b = -bv \\ v = \dot{x} \end{cases}$$

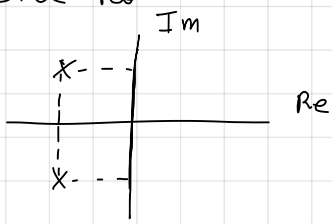
$msV(s) = -F(s) - KX(s) - b \cdot V(s)$ ma $V(s) = sX(s)$

$\rightarrow msV(s) + bsX(s) + KX(s) = F(s)$

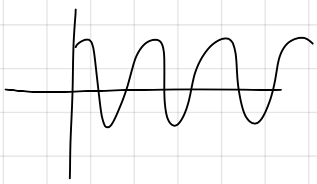
$\rightarrow \dfrac{X(s)}{F(s)} = \dfrac{1}{ms^2 + bs + K} \rightarrow \boxed{b^2 - 4m \cdot b > 0}$ ovvero se $b^2 > 4mb$

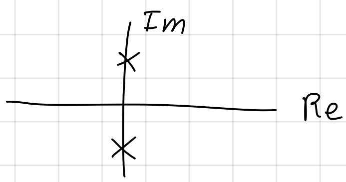
Se c'è uno smorzamento abbiamo anche la parte reale

$\lambda = a \pm jb \rightarrow$

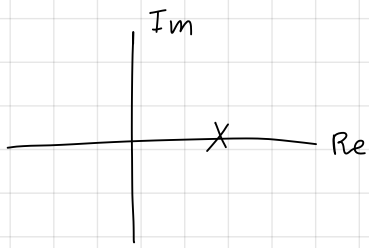


$\lambda = 0 \pm jb \rightarrow$

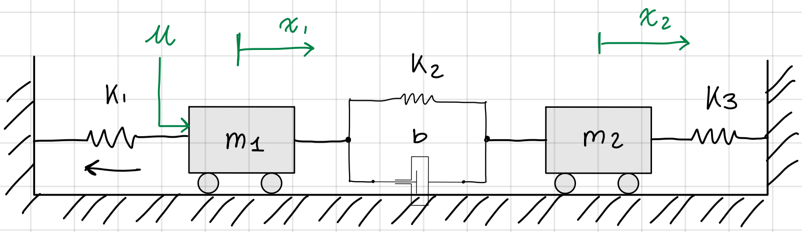




$\lambda = a$



ACCOPPIAMENTO FERROVIARIO



Scriviamo $m \cdot a = \sum F$ Tenendo presente che $a = \dot{v} = \ddot{x}$ e le rel car

$$\begin{cases} m_1 \ddot{x}_1 = u - k_1 x_1 - k_2 (x_1 - x_2) - b(\underbrace{\dot{x}_1 - \dot{x}_2}_{v_1 - v_2}) \\ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + b(\underbrace{\dot{x}_1 - \dot{x}_2}_{v_1 - v_2}) - k_3 x_2 \end{cases}$$

mancano $\dot{x}_1$ e $\dot{x}_2$ $= 0$ Pongo

$$x_3 = \dot{x}_1 \leftarrow v_{m1}$$
$$x_4 = \dot{x}_2 \leftarrow v_{m2}$$

ottengo:

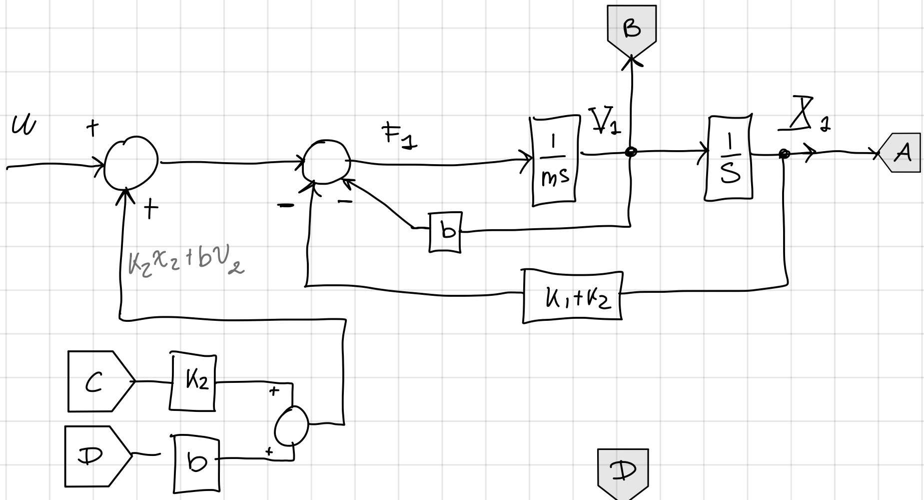
$$\begin{cases} \dot{x}_3 = \frac{1}{m_1} \left[u - k_1 x_1 - k_2 (x_1 - x_2) - b(x_3 - x_4) \right] \\ \dot{x}_4 = \frac{1}{m_2} \left[k_2 (x_1 - x_2) + b(x_3 - x_4) - k_3 x_2 \right] \end{cases}$$

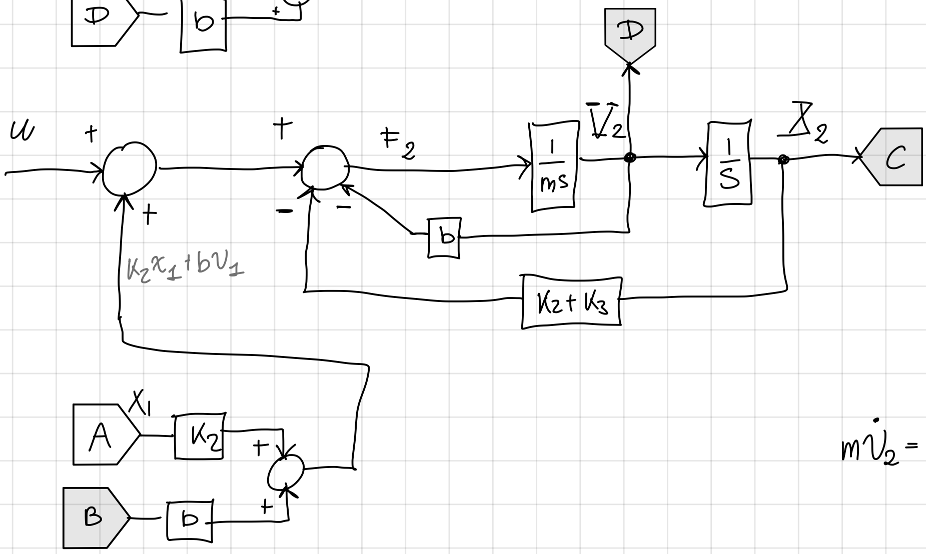
$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \end{cases}$$

$$\underline{\underline{A}} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1 + k_2}{m_1} & \frac{k_2}{m_1} & -\frac{b}{m_1} & +\frac{b}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2 + k_3}{m_2} & \frac{b}{m_2} & -\frac{b}{m_2} \end{pmatrix} \end{array} \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{array}$$

$$\underline{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix} \cdot u$$

$$y = x_2 - x_1 = (0, 0, -1, 1)$$





$$m\dot{v}_2 = -(k_2+k_3)\,x_2 - bv_2 + k_2x_1 + bv_1$$