SISTEMI IDRAULICI

"RO"

$$\boxed{\rho = \frac{\text{PESO}}{\text{UNITA' DI VOLUME}}}$$

$[\rho] = Kg/m^3$

$[V] = m^3$

VOLUME — Metro Cubo

$q \rightleftarrows i$

$p \rightleftarrows v$

$R_{id} \rightleftarrows Re$

$$\boxed{\text{Pressione} = \text{Forza} \times \text{unità di superficie}}$$

$\to [P] = \boxed{\frac{N}{m^2}} = Pa = \frac{Kg \cdot \cancel{m}}{m^{\cancel{2}} s^2} = \boxed{\frac{Kg}{ms^2}}$

(1) (1) Pascal

$1\ atm = 10^5\ Pa = 10^5 \frac{N}{m^2}$

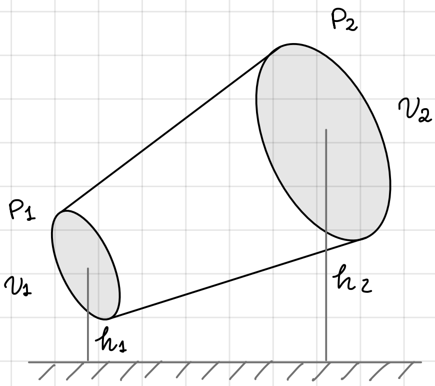
$1\ dm^2 = 10^{-2}\ m \Rightarrow \frac{10^5}{10^4} \frac{N}{m^2} \Rightarrow 1\ dm^2 = 10^{-2} \frac{N}{m^2}$

$$\boxed{q = \frac{dV}{dt}}$$

Portata Volumetrica

Variazione del Volume nel tempo con $\boxed{[q] = \frac{m^3}{s}}$

TEOREMA DI BERNOULLI



$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 \frac{1}{2} v_2^2 + \rho \cdot g\, h_2}$$

COSTANTE

* Equivalenze

GRANDEZZE IDRAULICHE

RESISTENZA IDRAULICA

\- Attrito - Resistore

$$R_{id} = \frac{\Delta P}{q}$$

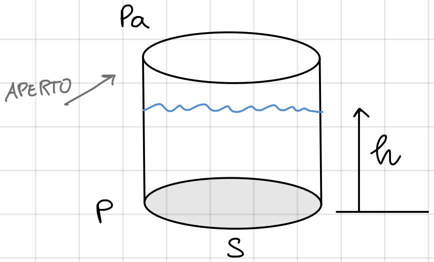
Diff di pressione

Portata

Con $\Delta P = P_1 - P_2$

CAPACITA' IDRAULICA

- Serbatoio - Condensatore



$$Pa + \rho \cdot g\, h = P \rightarrow \rho g h = \underset{\text{NON COSTANTE}}{P} - \underset{\text{COSTANTE}}{Pa}$$

$\rightarrow$ DERIVO $\rightarrow$ $\rho g \dfrac{dh}{dt} = \dfrac{dP}{dt}$

moltiplico per $\dfrac{S}{S}$ $\rightarrow$ $\dfrac{\rho g}{S} \underbrace{\dfrac{d(Sh)}{dt}}_{\text{Portata}} = \dfrac{dP}{dt} \rightarrow \dfrac{\rho g}{S} \cdot q = \dfrac{dP}{dt}$

$$\Rightarrow q = \frac{S}{\rho g} \cdot \frac{dP}{dt}$$

$$\frac{m^2}{\frac{kg}{m^3} \cdot \frac{m}{s^2}} = \frac{m^2}{\frac{kg}{m^2 s^2}}$$

Capacità idraulica $\equiv$ Capacità condensatore

INDUTTANZA IDRAULICA

$$v = L_{el}\, \dot{i}$$

Esempio: Quando la velocità all'interno di una condotta non è UNIFORME

$$m \cdot \frac{dv}{dt} = F_1 - F_2$$

$$m^2 \cdot \frac{m}{s} = \frac{m^3}{s} = q$$

$$\underbrace{\rho \cdot \underbrace{Sh}_{V}}_{m} \rightarrow \rho S h \cdot \frac{dv}{dt} = F_1 - F_2 \rightarrow \rho h \cdot \frac{d(\overbrace{Sv}^{q})}{dt} = F_1 - F_2$$

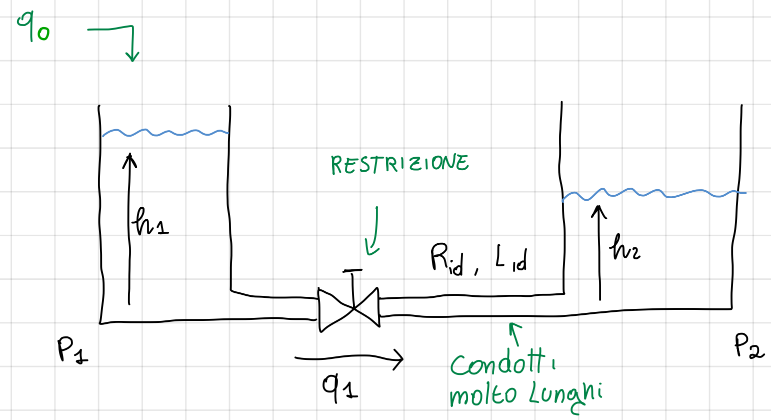
INDUTTANZA IDRAULICA

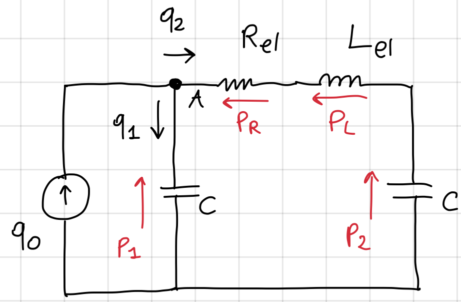
$$\rightarrow \rho h \frac{dq}{dt} = (P_1 - P_2)\, S \rightarrow \boxed{\frac{\rho h}{S}\, \dot{q} = P_1 - P_2}$$

Perchè $P = \dfrac{F}{S} \Rightarrow F = P \cdot S$

ESEMPIO: CONVERSIONE E MODELLO

$q_1 = iN = $ gen. Corr.





$-q_0 + q_1 + q_2 = 0$
$\rightarrow q_1 = q_0 - q_2$

R.C. $\begin{cases} i_C = C\,\dot{v}_C \\ v_L = L\,\dot{i}_L \end{cases}$

$$C_1 \cdot \frac{dP_1}{dt} = q_0 - q_2 \quad \leftarrow \text{equivale a } i_C$$

$$L\,\frac{dq_2}{dt} = \overbrace{-R\cdot q_2 + P_1 - P_2}^{v_L}$$

$P_L + P_2 - P_1 - P_R = 0 \Rightarrow P_L = P_R + P_1 - P_2$

$i_{C_2} = i_L$

$$C_2\,\frac{dP_2}{dt} = q_2$$

$x_1 = P_1 \quad x_2 = P_2 \quad x_3 = q_2$	
$y = h_2$	$u = q_0$

$$\rho \cdot g \cdot h \simeq P_2 \Rightarrow h = \frac{1}{\rho g} P_2 = \frac{1}{\rho g} x_2$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \rightarrow \quad A = \begin{array}{c} \begin{array}{ccc} P_1 & P_2 & q_2 \end{array} \\ \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} \end{pmatrix} \end{array} \quad B = \begin{pmatrix} 1/C_1 \\ 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 & \frac{1}{\rho g} & 0 \end{pmatrix} \quad D = \emptyset$$

Funz. di Trasferimento

$IN = q_1 \quad OUT = h_2 \rightarrow G(s) = \dfrac{H_2(s)}{Q_1(s)}$

✱ INIZIO

OBBIETTIVO: $H_1(Q_1)$

$$\mathcal{L.T.}\begin{cases} sC_1\,P_1(s) = Q_0(s) - Q_2(s) & (1) \\ sL\,Q_2(s) = -R\,Q_2(s) + P_1(s) - P_2(s) & (2) \\ sC_2\,P_2(s) = Q_2(s) & (3) \end{cases}$$

(3) in

$$H_2(s) = \frac{1}{\rho g} P_2(s) = \frac{1}{\rho g C_2} \cdot \frac{1}{s}\, Q_2(s)$$

(3) $\rightarrow$ (2) $\rightarrow$ $sL\,Q_2 + R\,Q_2 = P_1 - \underbrace{\frac{1}{sC_1}\,Q_2}_{P_2}$ (A)

dalla (1) $\quad P_1 = \dfrac{Q_0 - Q_2}{sC_1}$

→ La (B) nella (A) → $(R+SL)Q_2 + \frac{1}{SC_2}Q_2 = \frac{1}{SC_1}Q_0 - \frac{1}{SC_1}Q_2$

→ $Q_2\left(R+SL+\frac{1}{SC_2}+\frac{1}{SC_1}\right) = \frac{1}{SC_1}\cdot Q_1$ m.c.m $\frac{(R+SL)SC_1C_2+C_1+C_2}{SC_1C_2}$

mcm → $\frac{(R+SL)SC_1C_2+C_1+C_2}{\cancel{SC_1}C_2}\cdot Q_2 = \frac{1}{\cancel{SC_1}}Q_1$

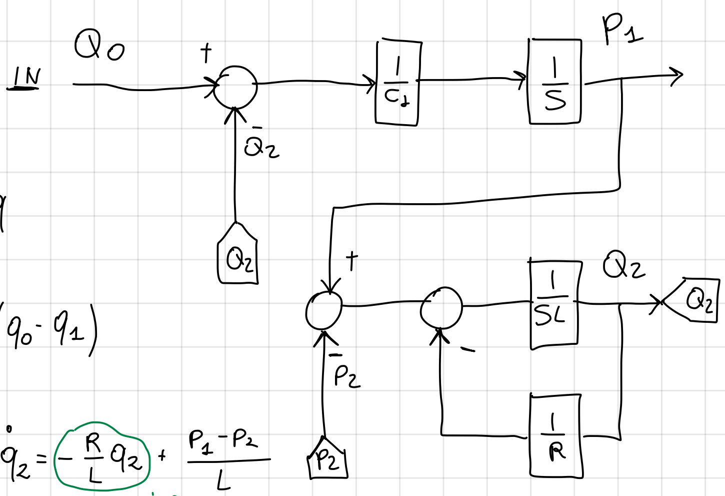
→ $Q_2(S) = \frac{C_2}{S^2LC_1C_2+SRC_1C_2+C_1C_2}\,Q_1(S)$

Effetto

⇒ $G(S) = \frac{H_2(S)}{Q_1(S)} = \frac{\frac{1}{\rho g \cancel{C_2} S}\cdot\frac{\cancel{C_2}}{S^2LC_1C_2+SRC_1C_2+C_1C_2}\cancel{Q_1(S)}}{\cancel{Q_1}} = \frac{\frac{1}{\rho g}}{S(S^2LC_1C_2+SRC_1C_2+C_1+C_2)}$

CAUSA

Schema a blocchi



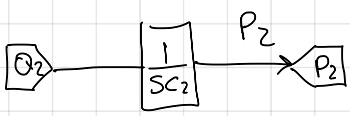
* Ricavato dalle eq

$C_1\cdot\frac{dP_1}{dt} = q_0 - q_2 \to \dot{P}_1 = \frac{1}{C}(q_0 - q_1)$

$L\frac{dq_2}{dt} = -R\cdot q_2 + P_1 - P_2 \to \dot{q}_2 = \left(-\frac{R}{L}q_2\right) + \frac{P_1-P_2}{L}$

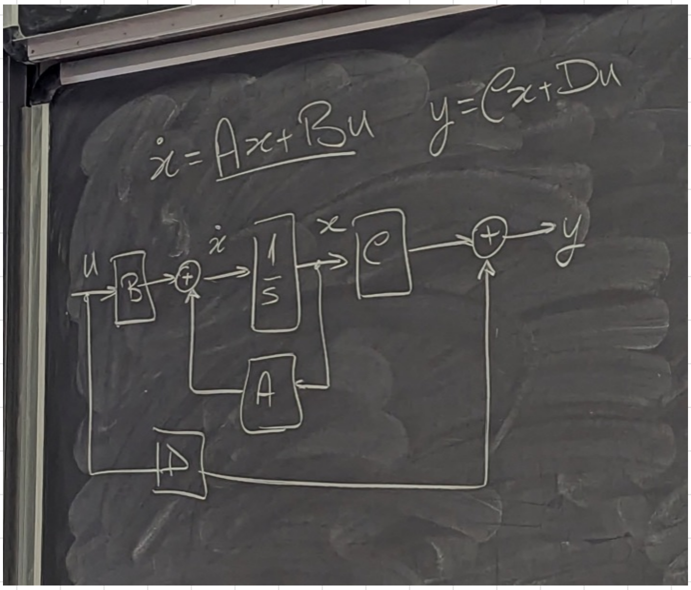
Retroazione

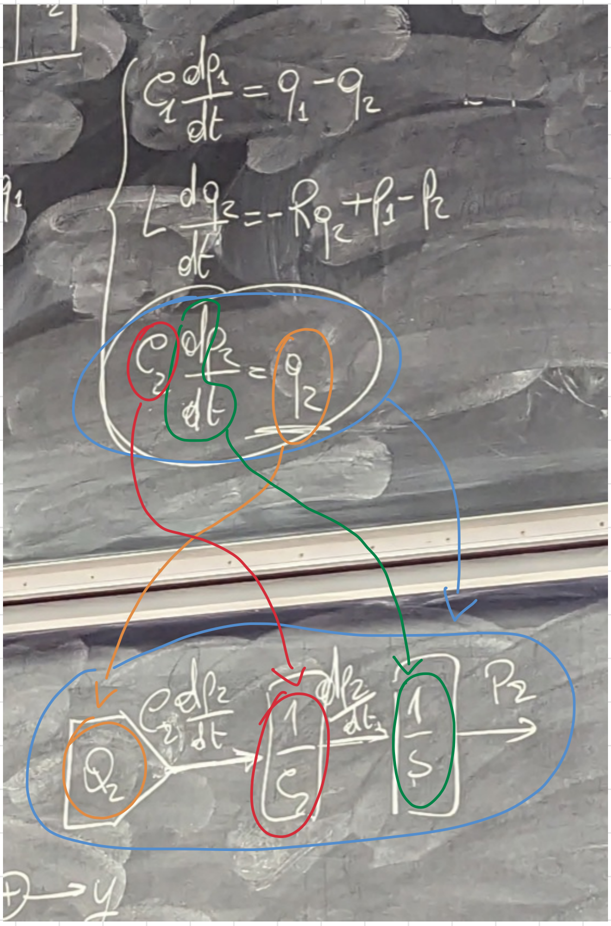
$C_2\frac{dP_2}{dt} = q_2 \to \dot{P}_2 = \frac{q_2}{C_2}$



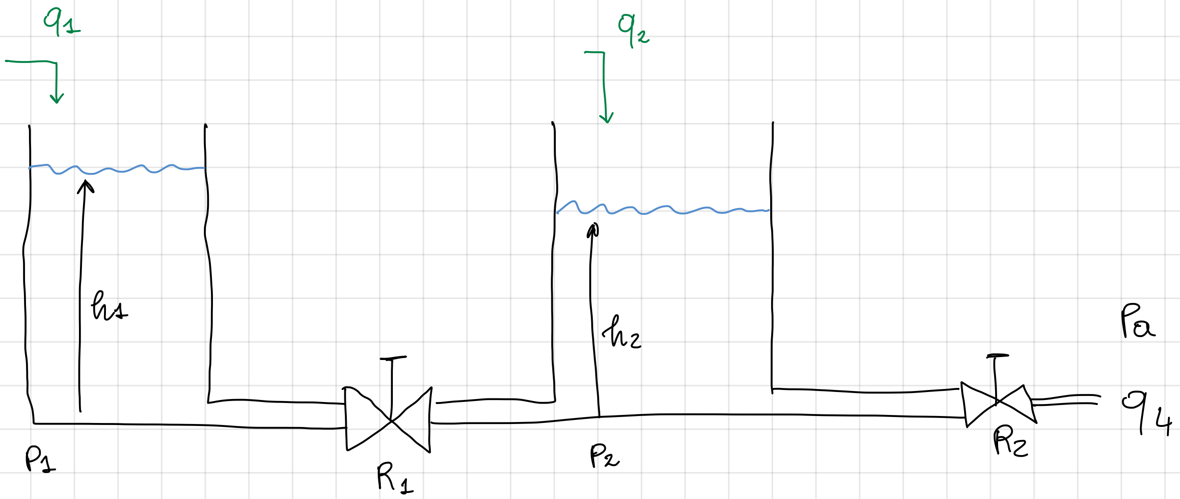
Domanda

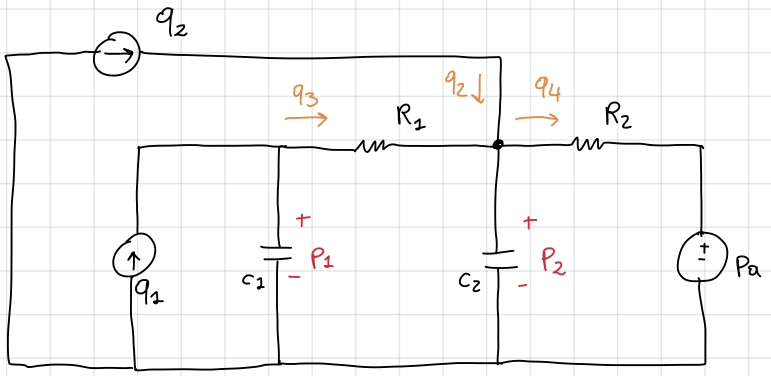
Rappr. a blocchi Con <u>MATRICI</u>





DUE SERBATOI...

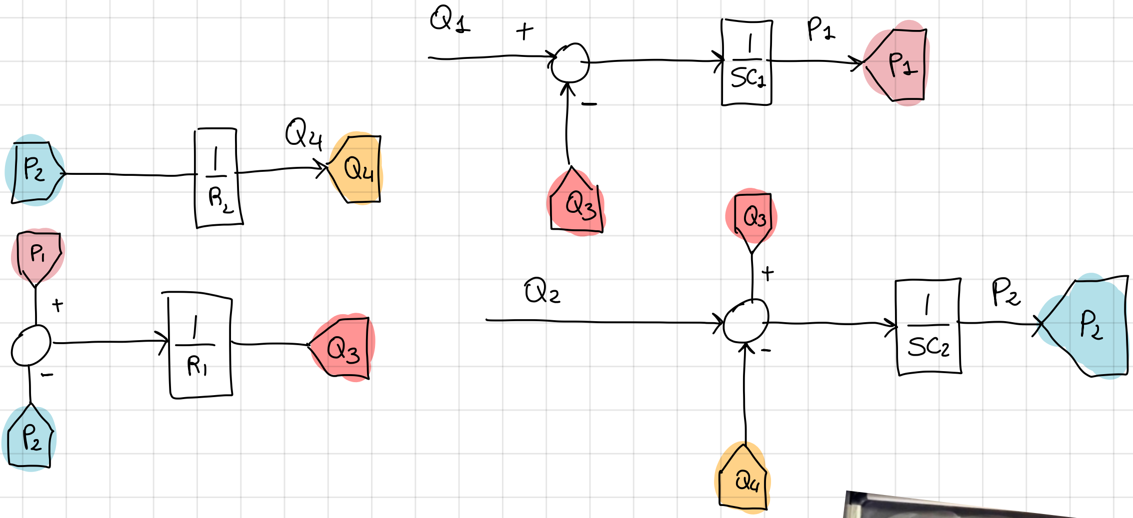


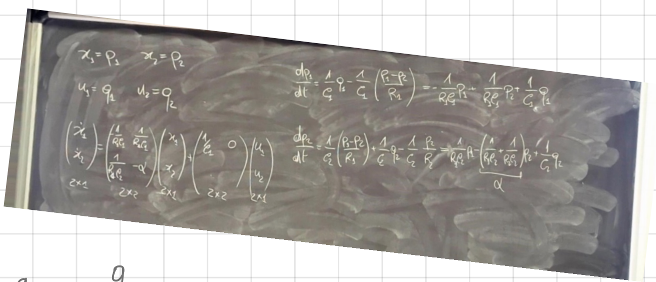


$$\begin{cases} C_1 \dfrac{dP_1}{dt} = q_1 - q_3 \\ q_3 = \dfrac{P_1 - P_2}{R_1} \\ C_2 \dfrac{dP_2}{dt} = q_3 + q_2 - q_4 \\ q_4 = \dfrac{P_2 - P_a}{R_2} \simeq \dfrac{P_2}{R_2} \end{cases}$$

$$\dot{P}_1 = \frac{q_1}{C_1} - \frac{1}{C_1}\left(\frac{P_1 - P_2}{R_1}\right)$$

$P_2 >> P_a$





$x_1 = P_1 \qquad x_2 = P_2$

$u_1 = q_1 \qquad u_2 = q_2$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{array}{cc} P_1 & P_2 \\ \left(\begin{matrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} \\ \frac{1}{R_1C_1} & -\frac{1}{R_1C_2} + \frac{1}{R_2C_2} \end{matrix}\right) \end{array}}_{2\times 2} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{array}{cc} q_1 & q_2 \\ \left(\begin{matrix} \frac{1}{C_1} & \emptyset \\ 0 & \frac{1}{C_2} \end{matrix}\right) \end{array}}_{2\times 2} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

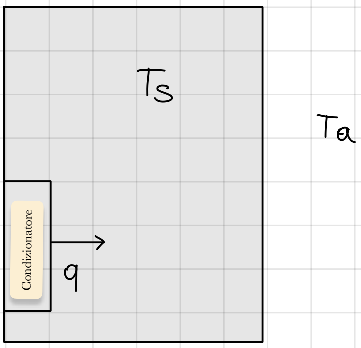
Possibile Uscita $y_1 = h_1 - h_2 = \frac{1}{\delta g} p_1 - \frac{1}{\delta g} p_2$

$\rightarrow C = \left(\frac{1}{\delta g} \quad -\frac{1}{\delta g} \right)$

Funzione di Trasferimento

Fattill Tu

MODELLI TERMICI



$$\underset{\uparrow}{C} \frac{dT}{dt} = q \nwarrow \text{Flusso Termico}$$

Capacità Termica

$\hookrightarrow q = \frac{dQ}{dt}$

$$C = \frac{Q}{\Delta T}$$

CALORE

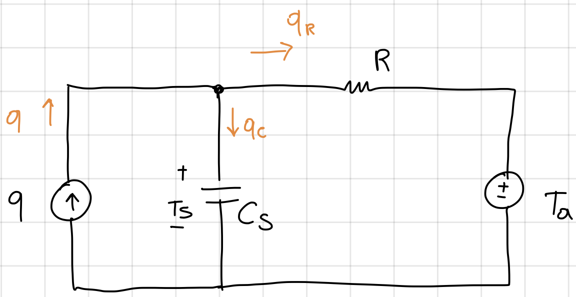
Variazione di Temp.

$$\begin{cases} i \rightarrow q \\ v \rightarrow T \end{cases}$$

Circuiti $\rightarrow$ Termic.

$$R = \frac{\Delta T}{q}$$

Resistenza Termica $\rightarrow$ funziona per $\begin{cases} \text{CONDUZIONE} \\ \text{CONVEZIONE} \end{cases}$



$$(1) \quad C_s \frac{dT_s}{dt} = q - q_R \quad \text{con} \quad q_R = \frac{T_s - T_a}{R}$$

$u_1 = q \qquad u_2 = T_a \qquad x = T_s$

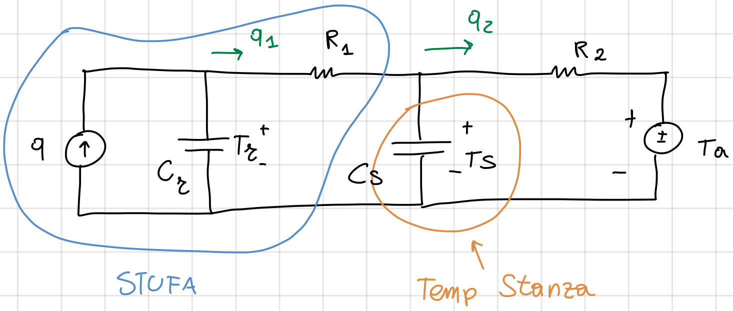
$$G(s) = \frac{T_s(s)}{Q(s)} \Rightarrow$$

$$s C_s T_s = Q - \frac{T_s - T_a}{R} \rightarrow \left(sC_s + \frac{1}{R}\right) T_s = Q + \frac{1}{R} T_a$$

$$\rightarrow \text{mcm} \rightarrow \quad T_s = \frac{R}{sRC_s + 1} Q + \frac{1}{sRC_s + 1} T_a$$

Influenza della Temp. esterna

IL RADIATORE



$$C_z \frac{dT_z}{dt} = q - q_1$$

$$q_1 = \frac{T_z - T_S}{R_1}$$

$$C_S \frac{dT_S}{dt} = q_1 - q_2$$

$$q_2 = \frac{T_S - T_a}{R_2}$$

$u_1 = T_a \qquad u_2 = q$

$x_1 = T_z \qquad x_2 = T_S$

$S C_z T_z = Q - Q_1 \quad \to \quad$ Sub $Q_1 \quad \to \quad S C_z T_z = Q - \frac{T_z}{R_1} + \frac{T_S}{R_1} \to$ Trovo T_z (1)

$S C_S T_S = Q_1 - Q_2 \quad \to \quad$ Sub $Q_2 \quad \to \quad S C_S T_S = \frac{T_z}{R_1} - \frac{T_S}{R_1} + \frac{T_S}{R_2} - \frac{T_a}{R_2}$ (2)

dalla (2) $\to \left(S C_S + \frac{1}{R_1} + \frac{1}{R_2}\right) T_S = \frac{T_z}{R_1} + \frac{T_a}{R_2}$

$\to \left(S C_S + \frac{1}{R_1} + \frac{1}{R_2}\right) T_S = \frac{1}{1 + S R_1 C_z} Q + \frac{1/R_1}{1 + S R_1 C_z} T_S + \frac{T_a}{R_2}$