

Es 2.8

$$\ddot{x} + 3\dot{x} + 2 = 0$$

1. Trasformo Tramite le regole  $\mathcal{L}[f'(t)] = s \cdot F(s) - f(0)$

$$\mathcal{L}[\ddot{x}] = \mathcal{L}\left[\frac{d}{ds} \cdot \left(\frac{d}{ds} x\right)\right] = s^2 X(s) - sx(0) - \dot{x}(0)$$

$$\rightarrow \ddot{x} + 3\dot{x} + 2 = 0 \Rightarrow s^2 X(s) - sx(0) - \dot{x}(0) + 3(sX(s) - x(0)) + 2X(s)$$

• Condizioni iniziali

$$\begin{cases} x(0) = a \\ \dot{x}(0) = b \end{cases} \rightarrow \begin{cases} s^2 X(s) - sa - b + 3sX(s) - 3a + 2X(s) = 0 \\ X(s)(s^2 + 3s + 2) = sa + 3a + b \end{cases}$$

$$\Rightarrow X(s) = \frac{sa + 3a + b}{s^2 + 3s + 2} \quad \text{Trovo poli e zeri}$$

(2) Anti trasformata

$$P_{2,2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2}}{2} \rightarrow \begin{aligned} -\frac{3+1}{2} &= -1 P_1 \\ -\frac{3-1}{2} &= -2 P_2 \end{aligned} \Rightarrow X(s) = \frac{\xi_1}{s+1} + \frac{\xi_2}{s+2}$$

$$\rightarrow \xi_1 = \lim_{s \rightarrow -1} \frac{sa + 3a + b}{s+2} = \frac{-a + 3a + b}{1} = 2a + b$$

$$\xi_2 = \lim_{s \rightarrow -2} \frac{sa + 3a + b}{s+1} = \frac{-2a + 3a + b}{-1} = -a - b$$

$$\Rightarrow X(s) = \frac{2a+b}{s+1} - \frac{a+b}{s+2} \quad \rightsquigarrow \quad x(t) = (2a+b)e^{-t} - (a+b)e^{-2t}, \quad t \geq 0$$

Costanti prese  
come valori iniziali

## ES 2.9 Equazione completa

$$\ddot{x} + 2\dot{x} + 5x = 3$$

$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = 0 \end{cases}$$

$$s^2 X(s) - \cancel{s x(0)} - \cancel{\dot{x}(0)} + 2sX(s) + 5X(s) = 3 \cdot \frac{1}{s}$$

$$\rightarrow s^2 X(s) + 2sX(s) + 5X(s) = \frac{3}{s} \quad \rightarrow X(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{z_1}{s} + ??$$

$$\begin{cases} p_1 = 0 \\ p_{2,3} = \frac{-2 \pm \sqrt{4 - 4 \cdot 5}}{2} \end{cases} \rightarrow \begin{cases} -1 + 2j & p_2 \\ -1 - 2j & p_3 \end{cases}$$

$$X(s) = \frac{z_1}{s} + \frac{z_2}{s+1-2j} + \frac{z_3}{s+1+2j} = \frac{z_1 s^2 + z_2 s + z_3}{s^2 + \cancel{2s} + \cancel{2s} + \cancel{1} + \cancel{2j} - \cancel{2j} - \cancel{4} + \cancel{4} = \frac{z_1 s^2 + z_2 s + z_3}{s^2 + 2s + 5}$$

$$z_1 = \lim_{s \rightarrow 0} s \cdot \frac{3}{s(s^2 + 2s + 5)} \rightarrow \left( \frac{3}{5} \right) z_1 \quad \leftarrow \text{Calcolo l'unico residuo calcolabile}$$

$$\frac{3}{s(s^2 + 2s + 5)} = \frac{z_1}{s} + \frac{z_2 s + z_3}{s^2 + 2s + 5} = \frac{z_1 s^2 + 2z_1 s + 5z_1 + z_2 s^2 + z_3 s}{s^2 + 2s + 5}$$

$$\rightarrow \begin{cases} s^2(z_1 + z_2) = 0 \rightarrow z_2 = -\frac{3}{5} \\ s(2z_1 + z_3) = 0 \rightarrow \frac{6}{5} + z_3 = 0 \rightarrow z_3 = -\frac{6}{5} \\ 5z_1 = 3 \rightarrow z_1 = \frac{3}{5} \text{ QED} \end{cases}$$

$$X(s) = \frac{3}{5} \cdot \frac{1}{s} + \frac{-\frac{3}{5}s - \frac{6}{5}}{s^2 + 2s + 5} = \frac{3}{5} \frac{1}{s} - \frac{3}{5} \frac{s+2}{s^2 + 2s + 5} = \frac{3}{5} \left( \frac{1}{s} - \frac{s+2}{s^2 + 2s + 5} \right)$$

$$= \frac{3}{5} \left( \frac{1}{s} - \frac{s+2}{(s+1)^2 + 2^2} \right) = \frac{3}{5} \left( \frac{1}{s} - \frac{2}{(s+1)^2 + 2^2} - \frac{s+1}{(s+1)^2 + 2^2} + \frac{4}{(s+1)^2 + 2^2} \right)$$

$$= \frac{3}{5} \frac{1}{s} - \frac{3}{5} \frac{2}{(s+1)^2 + 2^2} + \frac{s+1}{(s+1)^2 + 2^2} = \frac{3}{5} \frac{1}{s} - \frac{3}{10} \frac{2}{(s+1)^2 + 2^2} + \frac{3}{5} \frac{s+1}{(s+1)^2 + 2^2}$$

$$\Rightarrow x(t) = \frac{3}{5} - \frac{3}{10} e^{-t} \sin(2t) - \frac{3}{5} e^{-t} \cos(2t), \quad t \geq 0$$

