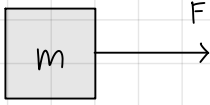


# Modellistica dei sistemi meccanici

\* Primo e secondo es  
clip audio 8/9  
e domanda orale  
2 e 3 orale → cleuco domande

Caratterizzati da MASSA molla e SMORZATORE

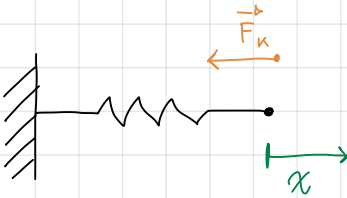
## MASSA



$$F = ma \quad [N], \quad a = \frac{dv}{dt} = \dot{v} \quad [m/s^2], \quad v = \frac{dx}{dt} = \dot{x} \quad [m]$$

$\uparrow$   
Spazio

## MOLLA



Legge di Hooke:  $F_k = -Kx$  (la molla viene Allungata)

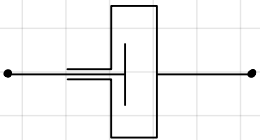
$$E_m(t) = \int_0^t \underbrace{F(t) \cdot v(t)}_{\text{Potenza}} dt = \int_0^t m \frac{dv}{dt} \cdot v(t) dt = \frac{1}{2} m \int_0^t \frac{d v^2(t)}{dt} dt = \frac{1}{2} m \left[ v^2(t) \right]_0^t$$

Energia di un corpo in movim. =  $\frac{1}{2} m v^2(t)$  (Possibile var di stato)

$$E_k(t) = \int_0^t F_k(t) \cdot v(t) dt = \frac{1}{2} K x^2(t)$$

Energia della molla =  $\frac{1}{2} K x^2(t)$  (Possibile var di stato)

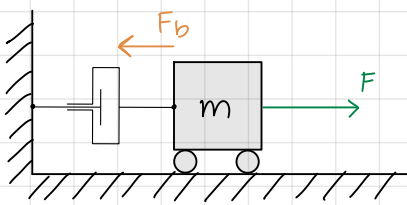
## SMORZATORE



$$v = \frac{dx}{dt} \rightarrow F_b = -bv \equiv F_b = -b \cdot \dot{x}$$

\* AUDIO DELLA LEZIONE  
Nel file nella cartella b.S. Elettrici

# SISTEMA 1

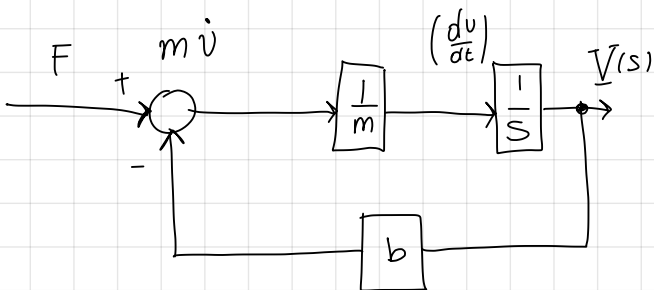
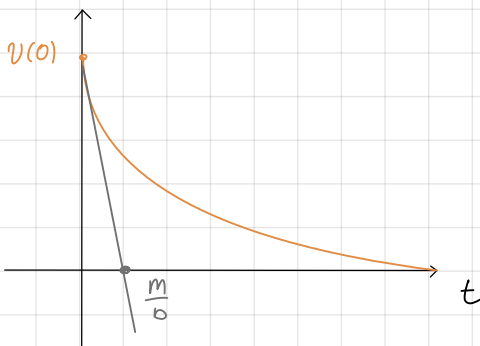
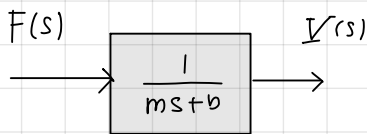
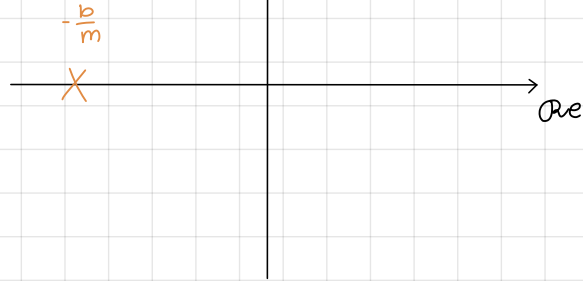


$$\begin{cases} F + F_b = m \dot{v} \\ F_b = -b v \end{cases}$$

L.T.  $\rightarrow F(s) - b \cdot V = m \cdot s V(s) \rightarrow (ms + b) V(s) = F(s)$

$\Rightarrow$

$$\frac{V(s)}{F(s)} = \frac{1}{ms + b}$$



VAR. ST.  $v$

$x = v$  ,  $u = F$  ,  $y = v$

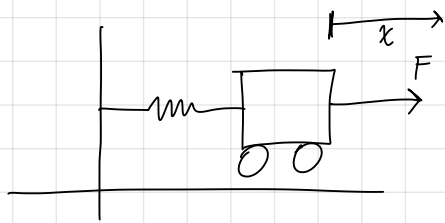
$$\begin{cases} \dot{x} = \frac{1}{m} (u - b \cdot x) \\ y = x \end{cases}$$

$$\begin{cases} \dot{x} = \frac{b}{m} x + \frac{1}{m} u \\ y = x \end{cases}$$

$$\begin{cases} \dot{x} = \underline{A} x + \underline{B} u \\ y = \underline{C} x + \underline{D} u \end{cases}$$

$\swarrow \frac{b}{m}$        $\swarrow \frac{1}{m}$   
 $\uparrow 1$        $\uparrow 0$

SENZA SMORZATORE



$$x_1 = x \quad x_2 = v \quad u = F$$

$$\begin{cases} F + F_k = m \ddot{x} \rightarrow \ddot{x} = \frac{1}{m} (F + F_k) \\ F_k = -k x \\ v = \dot{x} \end{cases}$$

$$y = x_1 \leftarrow \text{la soluzione io}$$

↑ pos in x

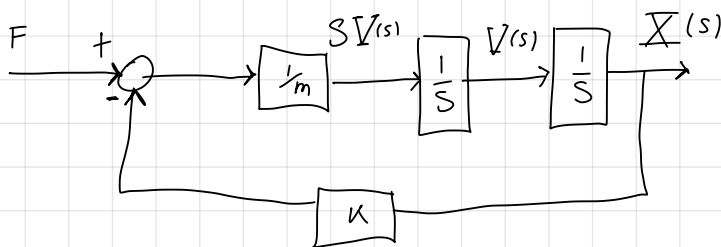
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m} (u - k x_1) \\ y = x_1 \end{cases} \rightarrow$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m} x_1 + \frac{1}{m} u \\ y = x_1 \end{cases}$$

$$\rightarrow \underline{A} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$$

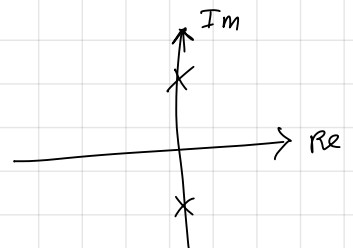
$$\underline{B} = \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \quad \underline{C} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

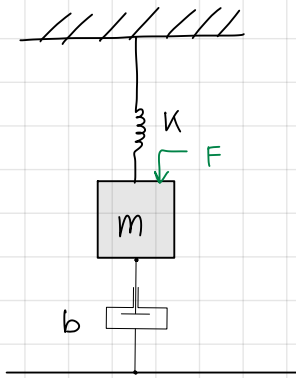


$$G(s) = \frac{X}{F} = \frac{\frac{1}{ms^2}}{1 + k \frac{1}{ms^2}} \stackrel{\text{Sin}(t)}{\parallel} = \frac{1}{ms^2 + k}$$

$$\text{con } s = \pm j \sqrt{\frac{k}{m}}$$



# CON SMORZATORE



$$\left\{ \begin{array}{l} F + F_k + F_b = m \ddot{v} \\ F_k = -kx \\ F_b = -b\dot{v} \\ v = \dot{x} \end{array} \right.$$

$$ms \dot{V}(s) = -F(s) - kX(s) - b \cdot V(s)$$

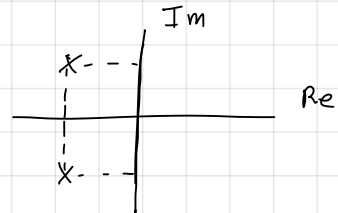
$$\text{ma } V(s) = sX(s)$$

$$\rightarrow ms \dot{V}(s) + bs X(s) + kX(s) = F(s)$$

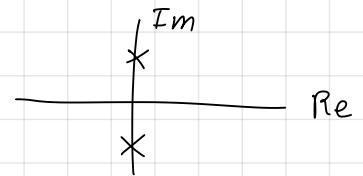
$$\rightarrow \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \rightarrow \boxed{b^2 - 4mb > 0} \text{ ovvero se } b^2 > 4mb$$

Se c'è uno smorzamento abbiamo anche la parte reale

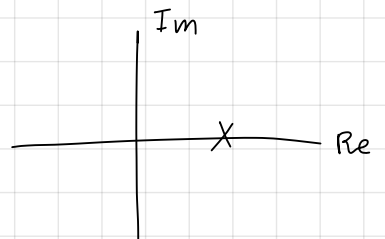
$$\lambda = a \pm jb \rightarrow \text{oscillazione smorzata}$$



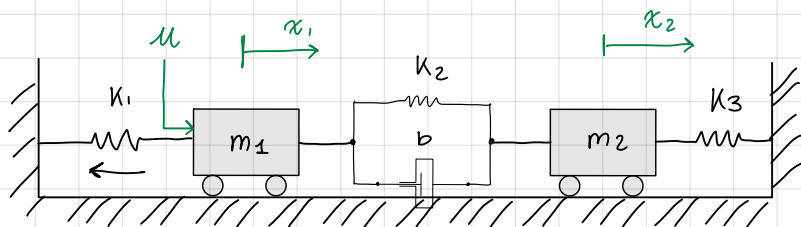
$$\lambda = 0 \pm jb \rightarrow \text{oscillazione non smorzata}$$



$$\lambda = a \rightarrow \text{decadimento esponenziale}$$



## ACCOPPIAMENTO FERROVIARIO



Scriviamo  $m a = \sum F$  Tenendo presente che  $a = \dot{v} = \ddot{x}$  e le rel car

$$\begin{cases} m_1 \ddot{x}_1 = u - k_1 x_1 - k_2 (x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2) \\ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2) - k_3 x_2 \end{cases}$$

$v_1 - v_2$        $v_1 - v_2$

manca  $\dot{x}_1$  e  $\dot{x}_2 = 0$  Pongo

$$\begin{aligned} x_3 &= \dot{x}_1 \leftarrow v_{m1} \\ x_4 &= \dot{x}_2 \leftarrow v_{m2} \end{aligned}$$

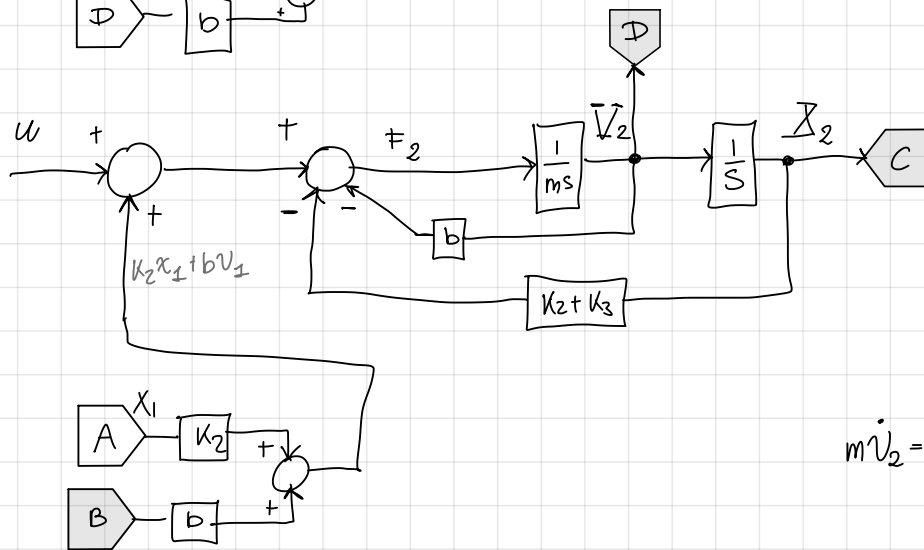
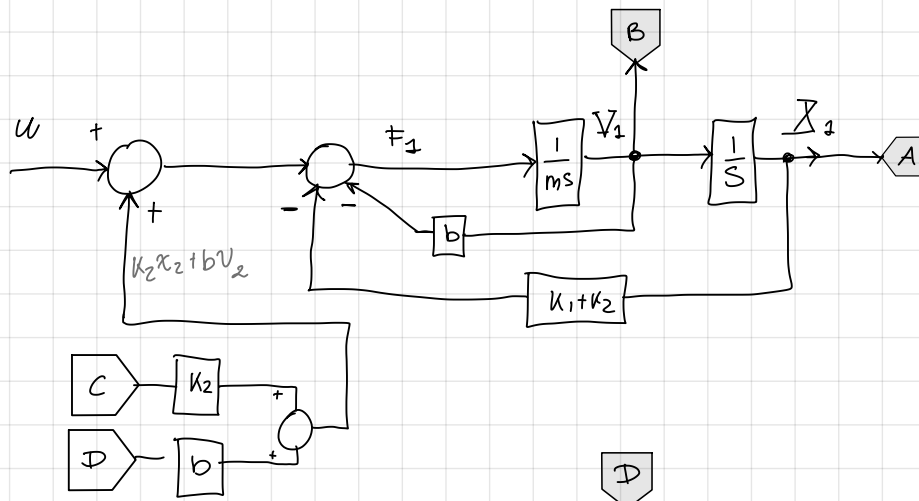
ottengo:

$$\begin{cases} \dot{x}_3 = \frac{1}{m_1} [u - k_1 x_1 - k_2 (x_1 - x_2) - b(x_3 - x_4)] \\ \dot{x}_4 = \frac{1}{m_2} [k_2 (x_1 - x_2) + b(x_3 - x_4) - k_3 x_2] \\ \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \end{cases}$$

$$\underline{A} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{b}{m_1} & \frac{b}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & \frac{b}{m_2} & -\frac{b}{m_2} \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\underline{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix} \cdot u$$

$$y = x_2 - x_1 = (0, 0, -1, 1)$$



$$m\dot{v}_2 = -(k_2 + k_3) x_2 - b v_2 + k_2 x_1 + b v_1$$