STABILITA' DI SISTEMI LTI

(1) LINEARITA'

(2) TEMPO INVARIANZA

Se $y(t)$ è uscita con input $u(t)$, allora $y(t-\tau)$ è uscita di IN $= u(t-\tau)$

CONCETTO DI STABILITA'

- Un sys LTI si dice Asintoticamente Stabile $\Leftrightarrow$ Tutti i poli della sua T.F. sono a parte reale < 0.
  Strettamente

- E' STABILE se la T.F. ha poli $\mathrm{Re}\{\} < 0$
  $\mathrm{Re}\{\} = 0$ con molteplicità semplice

ES

$\hookrightarrow$ 1 polo in origine $\rightarrow G(s) = \frac{1}{s}$ con $u(t) = \delta(t) \rightleftarrows U(s) = 1$

$\Rightarrow Y(s) = G(s)\,U(s) = \frac{1}{s} \Rightarrow y(t) = \mathbb{1}(t)$

gradino

- E' INSTABILE quando non è nessuno dei prec

RAMPA $\rightarrow \infty$

$G(s) = \frac{1}{s^2} \Rightarrow u(t) = \delta(t) \rightleftarrows U(s) = 1 \Rightarrow Y = G \cdot U = \frac{1}{s^2} \rightleftarrows y(t) = t \leftarrow$ INSTABILE

RISPOSTA DI UN SYS LTI

"Steady State"

$y(t)$: USCITA $\rightsquigarrow y(t) = y_{tr}(t) + y_{ss}(t)$

TRANSITORIA   REGIME

Siccome i sys che ci interessano sono Asint. Stabili, $y_{tr}(t) \rightarrow \infty$

$\Rightarrow y(t) \cong y_{ss}(t)$ ovvero $\lim_{t \to \infty} y(t) = \lim_{t \to \infty} y_{tr}(t) = 0$

RISPOSTA DI SYS DEL 1° ORDINE

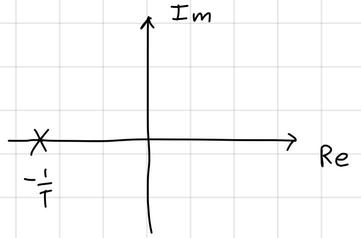
PRIMO ES:

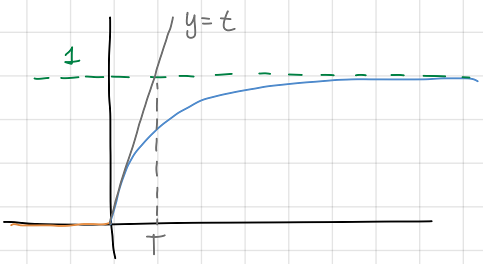
$$G(s) = \frac{1}{sT+1}$$ con T = costante di Tempo , $u(t) = \mathbb{1}(t) \rightleftharpoons U(s) = \frac{1}{s}$

$$Y(s) = G(s) \cdot U(s) = \frac{1}{sT+1} \cdot \frac{1}{s} = \frac{1/T}{\left(s+\frac{1}{T}\right)s} = \frac{\varepsilon_1}{s} + \frac{\varepsilon_2}{s+\frac{1}{T}}$$

$$\begin{cases} \varepsilon_1 = \lim\limits_{s \to 0} s \cdot Y(s) = \lim\limits_{s \to 0} \cancel{s} \cdot \dfrac{1/T}{(s+1/T)\cancel{s}} = 1 \\ \varepsilon_2 = \lim\limits_{s \to -\frac{1}{T}} \left(s+1/T\right) \cdot Y(s) = \lim\limits_{s \to -\frac{1}{T}} \cancel{\left(s+1/T\right)} \cdot \dfrac{1/T}{\cancel{(s+1/T)}\, s} \to -1 \end{cases}$$

$$\Rightarrow \quad Y(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}} \rightleftharpoons y(t) = 1 - e^{-t/T} \qquad t \geq 0$$





(1) $$y(t) = 1 - \left(1 - \frac{1}{T} e^{-\frac{1}{T}}\Big|_{t=0} \cdot t + \dots\right) = 1 - 1 + \frac{t}{T} + \dots \underset{t \ll T}{\cong} t$$

(2) Teorema V.F. L.T. :

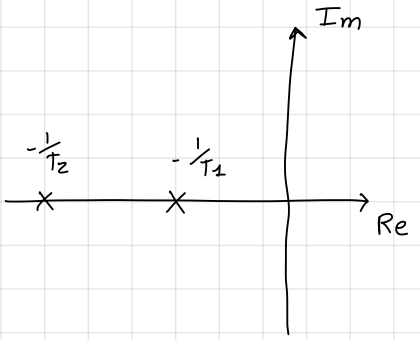
$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s) = \lim_{s \to 0} s \frac{1}{s(sT+1)} = 1$$

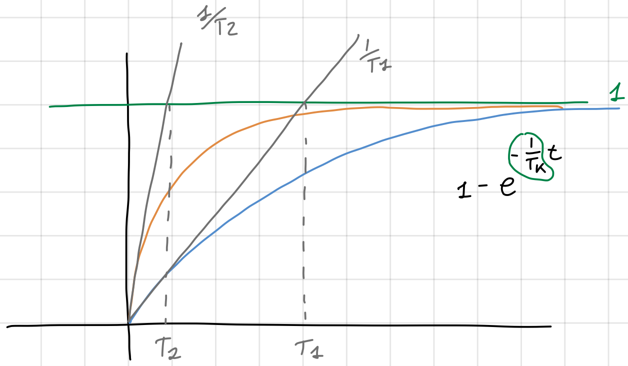
Valore a regime

(3) $$\mathcal{L}[\dot{y}] = s\,Y(s) = s \cdot \frac{1}{s(sT+1)} = \frac{1}{sT+1}$$

T.V.I : $$\lim_{t \to 0} \frac{dy}{dt} = \lim_{s \to \infty} s \cdot \frac{1}{sT+1} \to 1/T$$

Derivata in origine





IMPO: Quanto più il polo è <u>vicino</u> allo zero, tanto più impiegherà il sys ad andare a regime.

TEMPO PER ARRIVARE A REGIME

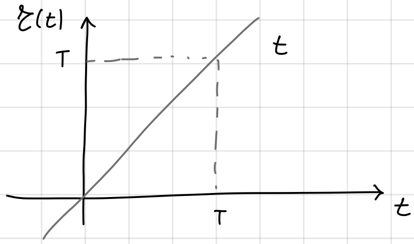
Ovviamente dipende dalla costante di Tempo $\to$ prendiamo $\boxed{4T}$

$$e(t) = 1 - y(t) = 1 - 1 - e^{-\frac{t}{T}} = e^{-\frac{t}{T}} \qquad \text{se} \quad e(4T) = e^{-\frac{4T}{T}} \cong \boxed{0.018 < 2\% \; y_{ss}}$$

$\uparrow$
errore $\rightleftharpoons$ Distanza da 1

* Come scegliere l'intervallo visualiz. MATLAB.

Risposta alla Rampa



$$R(s) = \mathcal{L}[t \cdot 1\!\!1(t)] \rightleftarrows R(s) = \frac{1}{s^2}$$

$$G(s) = 1/(sT+1)$$

$$Y(s) = ?$$

Il Sys è A Stabile

è instabile a seguito dell'input

$$Y = R \cdot G = \frac{1}{s^2} \cdot \frac{1}{sT+1} = \frac{1/T}{s^2(s+\frac{1}{T})} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{s+1/T}$$

Pow Max

Potenza MAX

$$r_2 = \lim_{s \to 0} s \cdot Y = \lim_{s \to 0} \cancel{s} \cdot \frac{1/T}{s^{\cancel{2}}(s+\frac{1}{T})} \to 1 \quad r_2$$

$$r_3 = \lim_{s \to -\frac{1}{T}} \cancel{(s+\frac{1}{T})} \cdot \frac{1/T}{s^2 \cancel{(s+\frac{1}{T})}} = \frac{1/T}{(-\frac{1}{T})^2} = T \quad r_3$$

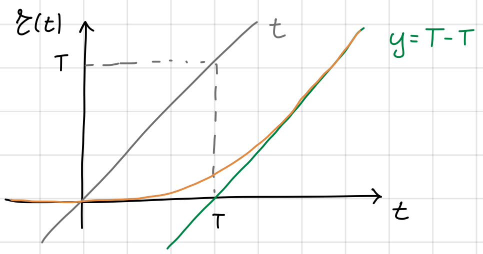
$$r_1 = \text{SOSTITUZIONE} \to \frac{r_1}{s} + \frac{1}{s} + \frac{T}{s+1/T} = \frac{1/T}{s^2(s+\frac{1}{T})} = \frac{r_1 s(s+\frac{1}{T}) + s + \frac{1}{T}}{s^2(s+\frac{1}{T})} + s^2 T$$

$$\leadsto \frac{\underbrace{\cancel{(r_1+T)}}_{\emptyset} s^2 + \underbrace{\cancel{(\frac{r_1}{T}+1)}}_{\emptyset} s + \frac{1}{T}}{s^2(s+\frac{1}{T})} = \frac{1/T}{s^2(s+\frac{1}{T})} \Rightarrow r_1 + T = 0 \Rightarrow r_1 = -T \quad r_1$$

$$\Rightarrow Y = -\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s+1/T} \rightleftarrows y(t) = -T + t + T e^{-\frac{t}{T}} \quad t \geq 0$$

Trovare l'andamento a regime e Transitorio

$$y_{ss}(t) = \lim_{t \to \infty} y(t) = \lim_{t \to \infty} -T + t + \underbrace{T e^{-\frac{t}{T}}}_{\to \emptyset} = \lim_{t \to \infty} (t - T)$$



Valore iniziale se valutato in $t=\emptyset$

$$\frac{dy}{dt} = 1 - \frac{T}{T} e^{-\frac{t}{T}} = 1 - e^{-\frac{t}{T}}$$

la derivata della risposta alla RAMPA è la risposta al gradino!

$\Rightarrow$ Per la linearità:

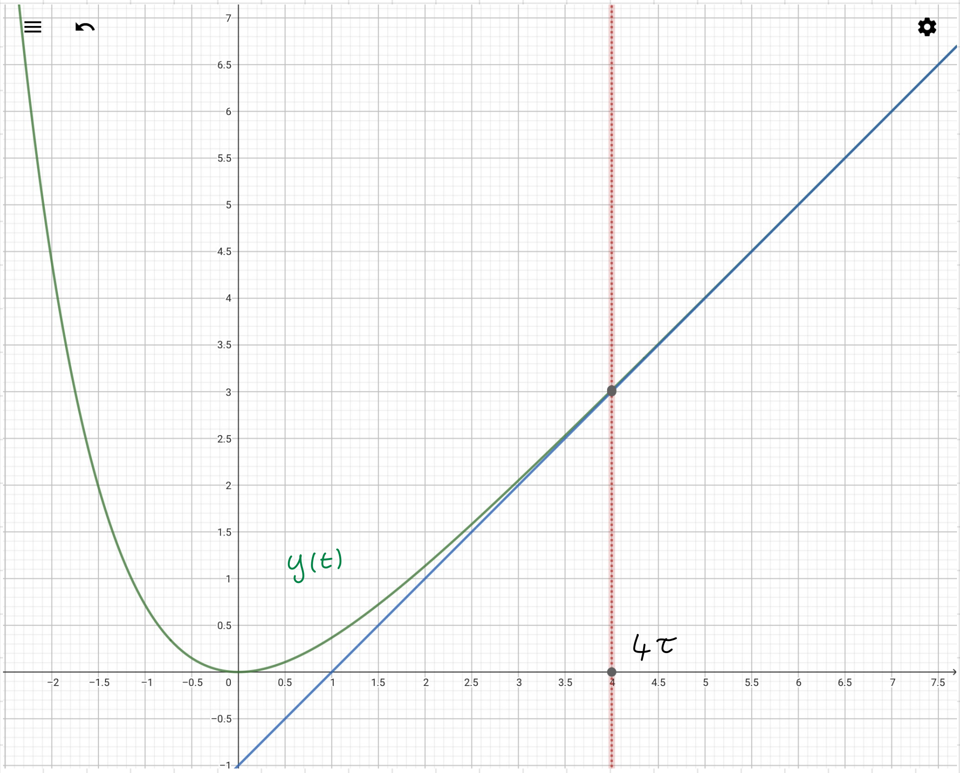
$$\boxed{\frac{d\,U(s)}{dt} \cdot G(s) = \frac{d\,Y(s)}{dt}}$$

Valore iniziale (con Teorema)

$$\lim_{s \to \infty} s \cdot \mathcal{L}\left[\frac{dy}{dt}\right] = \lim_{s \to \infty} s \cdot s \cdot Y(s) = \lim_{s \to \infty} \cancel{s^2} \cdot \frac{1/T}{\cancel{s^2}(s+\frac{1}{T})} = \emptyset$$

Errore

$$e(t) = t - y(t) = t - (-T + t + T e^{-\frac{t}{T}}) = T(1 - e^{-\frac{t}{T}})$$

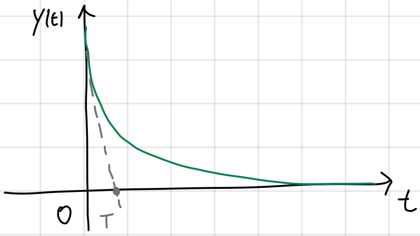


RISPOSTA IMPULSIVA

$$G(s) = \frac{1}{sT+1} = \frac{1/T}{s+\frac{1}{T}}$$

c'è 1 ma ha una dimensione!!

$$u(t) = \delta(t) \rightleftarrows R(s) = 1 \Rightarrow Y(s) = R(s)\cdot G(s) = G(s) \Rightarrow y(t) = \frac{1}{T} e^{-\frac{t}{T}} \quad t>0$$



V.I = ...
V.F. = ...

Taylor

T.V.I.

DISCONTINUA

$$y(t) = \frac{1}{T} e^{-\frac{1}{T}t}, \quad t \geq 0$$

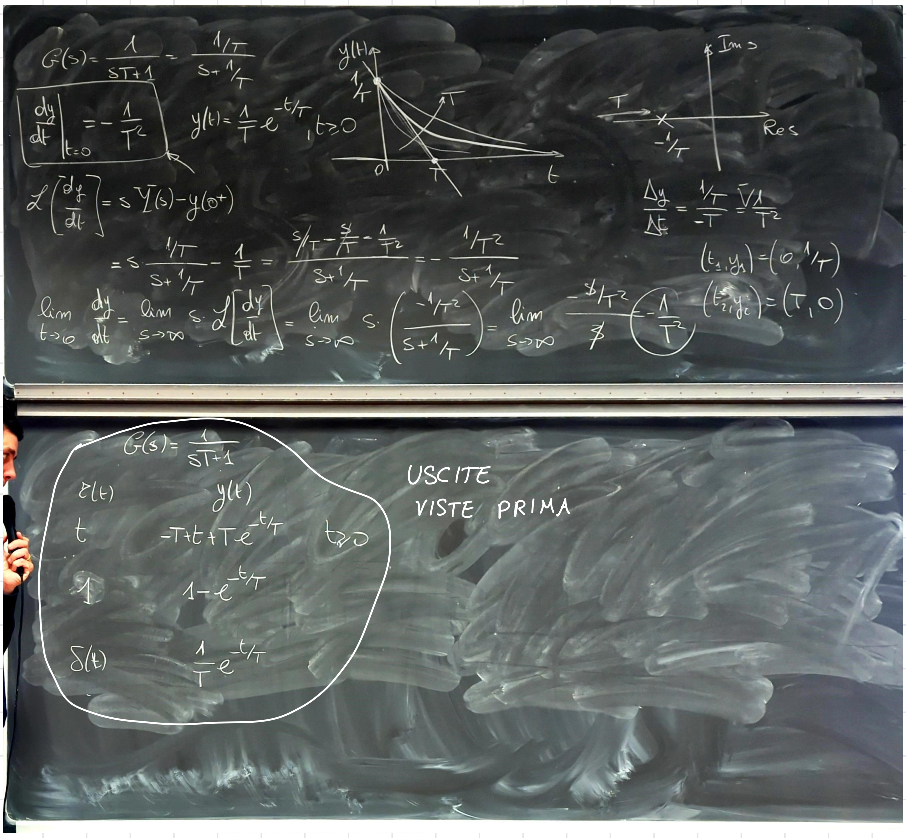
$$\left.\frac{dy}{dt}\right|_{t=0} = -\frac{1}{T^2}$$

$$\rightsquigarrow \mathcal{L}\left[\frac{dy}{dt}\right] = sY(s) - y(0^+) = s\cdot\frac{1/T}{s+1/T} - \frac{1}{T} = \frac{s/T - \frac{s}{T} - \frac{1}{T^2}}{s+1/T} = -\frac{1/T^2}{s+\frac{1}{T}}$$

$$\rightsquigarrow \text{Teorema} \quad \lim_{t\to 0}\frac{dy}{dt} = \lim_{s\to\infty} s\cdot Y(s) = \lim_{s\to\infty} -s\cdot\frac{1/T^2}{s+\frac{1}{T^2}} \to -\frac{1}{T^2}$$

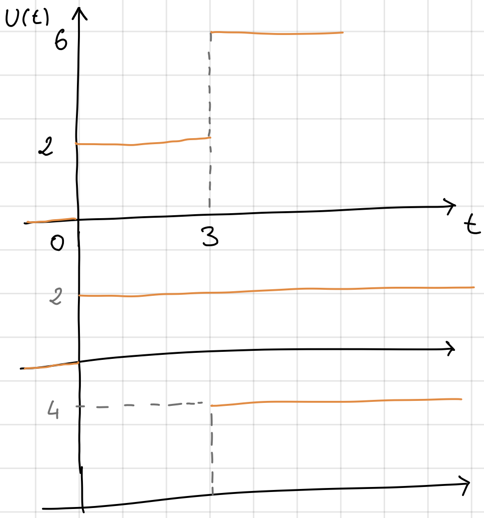
QED

FOTO



G(s) = 1/(sT+1) = (1/T)/(s+1/T)
dy/dt |t=0 = -1/T²
y(t) = 1/T e^(-t/T), t≥0
y(t)
1/T
T
0
t
Im s
Res
-1/T
ℒ[dy/dt] = s Y(s) - y(0⁺)
= s (1/T)/(s+1/T) - 1/T = (s/T - s/T - 1/T²)/(s+1/T) = - (1/T²)/(s+1/T)
Δy/Δt = (1/T)/(-T) = -1/T²
(t₁,y₁) = (0, 1/T)
(t₂,y₂) = (T, 0)
lim t→0 dy/dt = lim s→∞ s ℒ[dy/dt] = lim s→∞ s (-(1/T²)/(s+1/T)) = lim s→∞ (-s/T²)/s = -1/T²
G(s) = 1/(sT+1)
r(t)
y(t)
t
-T+t+Te^(-t/T)
t≥0
1
1-e^(-t/T)
δ(t)
1/T e^(-t/T)
USCITE
VISTE PRIMA

RISPOSTA DI UN SYS (1th) ad un segnale composto



$$G(s) = \frac{1}{s+10}$$

$$U(t) = U_1(t) + U_2(t) \quad \text{con} \quad \begin{cases} U_1(t) = 2\cdot \mathbb{1}(t) \\ U_2(t) = 4\cdot \mathbb{1}(t-3) \end{cases}$$

$$Y_1(s) = G\cdot U_1 = \frac{1}{s+10}\cdot\frac{2}{s} = \frac{r_1}{s} + \frac{r_2}{s+10}$$

$$r_1 = \lim_{s\to 0} s\cdot Y_1(s) = 1/5 \quad \rightsquigarrow \quad r_2 = \lim_{s\to -10} s\cdot Y_1(s) = -\frac{1}{5}$$

$y_1(t)$

$$\Rightarrow Y_1(s) = \frac{1}{5}\left(\frac{1}{s} - \frac{1}{s+10}\right) \rightleftharpoons y_1(t) = \frac{1}{5}\cdot\left(1 - e^{-10t}\right),\ t \geq 0$$

$$\hat{U}_2(t) = 4\cdot\mathbb{1}(t) = 2\cdot\left(2\cdot\mathbb{1}(t)\right) = 2\,U_1(t) \Rightarrow \hat{y}_2(t) = 2\,y_1(t) = \frac{2}{5}\left(1 - e^{-10t}\right),\ t \geq 0$$

senza ritardo

!!!

$$\Rightarrow y_2(t) = \hat{y}_2(t-3) = \frac{2}{5}\left(1 - e^{-10(t-3)}\right),\ t \geq 3$$

$y_2(t)$

$$\Rightarrow y(t) = \frac{1}{5}\left(1 - e^{-10t}\right)\cdot\mathbb{1}(t) + \frac{2}{5}\left(1 - e^{-10(t-3)}\right)\cdot\mathbb{1}(t-3)$$

Ans