

RISPOSTA DEI SISTEMI DEL II ORDINE

Supponiamo di avere una generica funzione di trasferimento del secondo ordine; questa funzione G avrà due parametri importanti:

- *zeta* (ζ) ovvero il **rappporto di smorzamento**, può assumere diversi valori, a seconda dei quali cambia la risposta del sistema; andremo quindi a distinguere ogni caso.
- *Omega-n* (ω_n) ovvero la **frequenza naturale**; questo valore misura la velocità con cui il sistema oscilla senza considerare l'effetto dello smorzamento; potremmo definirla come la velocità intrinseca del sistema.

1) La funzione di trasferimento

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\text{Se } U(t) = f \Rightarrow U(s) = 1 \Rightarrow Y(s) = G(s)$$

$$\text{Se } U(t) = 1(t) \Rightarrow U(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s} G(s)$$

2) TROVO I POLI

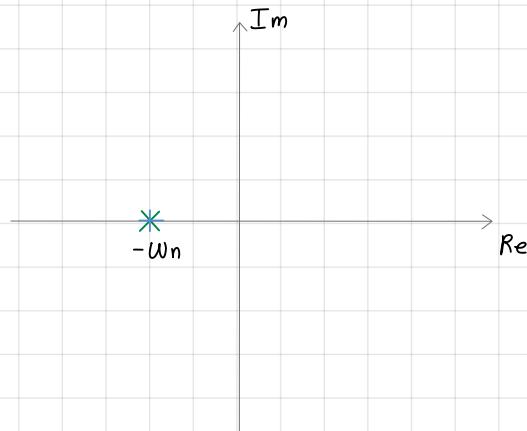
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad \text{per} \quad p_{1,2} = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2} = -\zeta\omega_n \pm \sqrt{\omega_n^2(\zeta^2 - 1)}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

3) A seconda del valore di ζ abbiamo diverse possibilità:

• $\zeta = 1$ CRITICAMENTE SMORZATO

$$\begin{aligned} \text{Se } \zeta = 1 \Rightarrow G(s) &= \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} \\ &= \frac{\omega_n^2}{(s + \omega_n)^2} = 0 \quad (p_{1,2} = -\omega_n) \end{aligned}$$



$$\text{Infatti } p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\omega_n$$

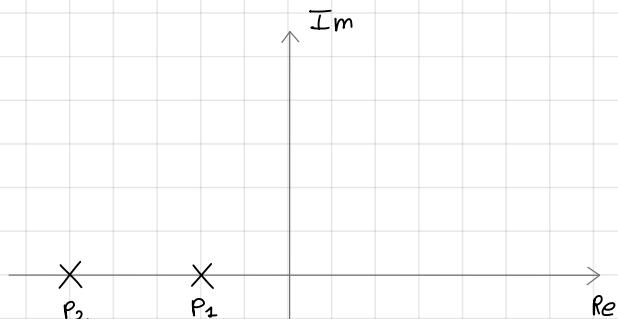
• $\zeta > 1$ SOVRASMORZATO

$$\text{Se } \zeta > 1 \Rightarrow p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

"Un valore al quadrato meno qualcosa sotto radice sarà sempre minore del valore stesso"

$$\text{ovvero } \sqrt{\zeta^2 - 1} < \zeta, \forall \zeta$$

$$\text{ES: } \zeta = 2 \Rightarrow \sqrt{\zeta^2 - 1} = 1,73 \Rightarrow 1,73 < 2$$



e quindi avremo due poli distinti, reali e negativi.

• $f = 0$ NON SMORZATO

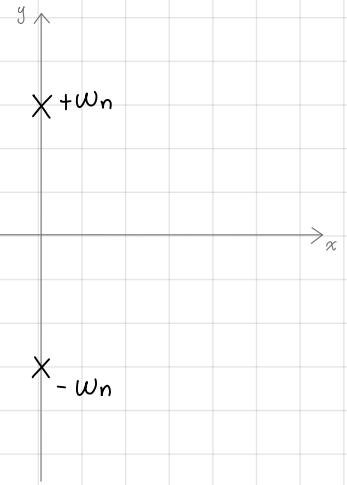
$$f=0 \rightarrow Y(s) = \frac{W_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$= \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$= \omega_n \cdot \frac{\omega_n}{s^2 + \omega_n^2}$$

$\sin(\omega_n t)$

$P_{1,2} = \pm j\omega_n$



• CASO GENERALE: $0 < f < 1$

Se $0 < f < 1 \rightarrow P_{1,2} = -f\omega_n \pm \omega_n\sqrt{f^2 - 1}$ ma se $0 < f < 1 \quad f^2 - 1 < 0 \vee f$

\Rightarrow Complex: metto in evidenza $\sqrt{-1} \rightarrow -f\omega_n \pm \omega_n\sqrt{(-f^2 + 1)}$ $\rightarrow P_{1,2} = -f\omega_n \pm j\omega_n\sqrt{1 - f^2}$

Pongo $W_d = \omega_n\sqrt{1 - f^2}$ $\Rightarrow P_{1,2} = -f\omega_n \pm jW_d$

FREQUENZA SMORZATA

\rightarrow Procedo con il calcolo dei residui...

Posso scrivere $Y(s) = \frac{W_n^2}{s(s^2 + 2f\omega_n s + \omega_n^2)} = \frac{\zeta_1}{s} + \frac{\zeta_2 s + \zeta_3}{s^2 + 2f\omega_n s + \omega_n^2}$

Trovo i residui

$$\zeta_1 = \lim_{s \rightarrow 0} s \cdot \frac{\omega_n^2}{s(s^2 + 2f\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s^2 + 2f\omega_n s + \omega_n^2} \rightarrow \textcircled{1} \zeta_1$$

$$\frac{\zeta_1 s^2 + 2\zeta_1 f\omega_n s + \omega_n^2 \zeta_1 + \zeta_3 s + \zeta_2 s^2}{s(s^2 + 2f\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s^2 + 2f\omega_n s + \omega_n^2)} = \frac{s^2(\zeta_1 + \zeta_2) + s(2\zeta_1 f\omega_n + \zeta_3) + \omega_n^2 \zeta_1}{s(s^2 + 2f\omega_n s + \omega_n^2)}$$

$$\begin{cases} \zeta_1 + \zeta_2 = 0 \rightarrow \zeta_2 = -\zeta_1 \Rightarrow \textcircled{2} \zeta_1 = -1 \\ 2\zeta_1 f\omega_n + \zeta_3 = 0 \rightarrow 2f\omega_n + \zeta_3 = 0 \Rightarrow \textcircled{3} \zeta_3 = -2f\omega_n \\ \omega_n^2 \zeta_1 = \omega_n^2 \Rightarrow \textcircled{4} \zeta_1 = 1 \end{cases}$$

TROVATO PRIMA

$Y(s)$

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2f\omega_n s + \omega_n^2)} = \frac{\zeta_1}{s} + \frac{\zeta_2 s + \zeta_3}{s^2 + 2f\omega_n s + \omega_n^2} = \frac{1}{s} - \frac{s + 2f\omega_n}{s^2 + 2f\omega_n s + \omega_n^2}$$

Torno al dominio del Tempo

$$Y(s) = \frac{W_n^2}{s(s^2 + 2j\omega_n s + \omega_n^2)} = \frac{1}{s} - \frac{s + 2j\omega_n}{(s + j\omega_n)^2 + (\omega_n^2 - (j\omega_n)^2)} = \frac{1}{s} - \frac{s + 2j\omega_n}{(s + j\omega_n)^2 + \omega_n^2(1 - j^2)}$$

IN PIU'!

⚠ Siccome $W_d = \omega_n \sqrt{1 - j^2}$ $\Rightarrow W_d^2 = \omega_n^2(1 - j^2)$ $\Rightarrow Y(s) = \frac{1}{s} - \frac{s + 2j\omega_n}{(s + j\omega_n)^2 + W_d^2}$ (1)

→ Ora l'obiettivo è rimanezzare per far uscire un sin e cosin

$$= \frac{1}{s} - \frac{s}{(s + j\omega_n)^2 + W_d^2} + \frac{2j\omega_n}{W_d} \frac{W_d}{(s + j\omega_n)^2 + W_d^2}$$

$e^{-j\omega_n t} \cdot \cos(W_d \cdot t)$ $e^{-j\omega_n t} \cdot \sin(W_d \cdot t)$

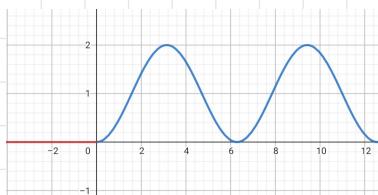
Risposta di un qualsiasi sistema di secondo ordine

$$\Rightarrow Y(s) \Rightarrow y(t) = \left[1 - e^{-j\omega_n t} \cos(W_d \cdot t) + \frac{2j\omega_n}{W_d} e^{-j\omega_n t} \sin(W_d \cdot t) \right] \cdot \mathbf{1}(t)$$

$$y(t) = \left[1 - e^{-\alpha \cdot b \cdot t} \cos(c \cdot t) + \frac{2a \cdot b}{c} e^{-\alpha \cdot b \cdot t} \sin(c \cdot t) \right] \cdot \mathbf{1}(t)$$

Come modificano la risposta i vari parametri?

1. $j = 0$

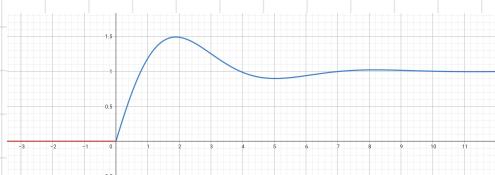


Se $j = 0 \Rightarrow y(t) = 1 - \cos(W_d \cdot t)$

NON SHORZATO

2. $0 < j < 1$

Con $W_d = \omega_n = 1$



Se $0 < j < 1$ ($H_p j = 1/2$) $\Rightarrow y(t) = 1 - e^{-\frac{1}{2}t} \cos(t) + 1 \cdot e^{-\frac{1}{2}t} \sin(t)$

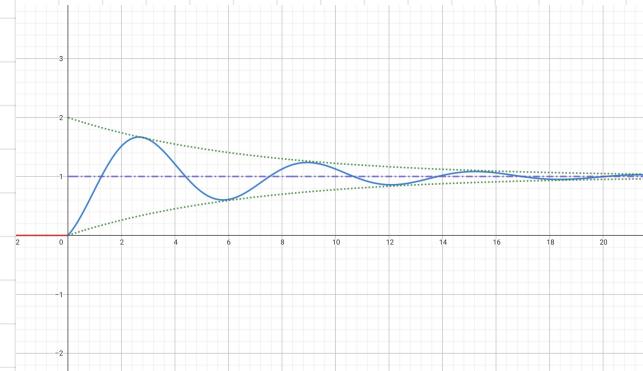
$$f(x) = 5e^{x/2} (x > 0, 1 - e^{-x}) \cos(x) + \frac{2 \cdot 5 \cdot 1}{2} e^{-x/2} \sin(x)$$

$$= 1 - e^{-x/2} \cos(x) + \frac{2 \cdot 0.5 \cdot 1}{2} e^{-x/2} \sin(x), \quad (x > 0)$$

$$g(x) = 0, \quad (x < 0)$$

$$h(x) = 1 - e^{-x}, \quad (x > 0)$$

$$p(x) = 1 + e^{-x}, \quad (x > 0)$$



3. $\mathcal{F} = 1$

(Come visto prima)

$$\text{Se } \mathcal{F} = 1 \rightarrow \text{Se } \mathcal{F} = 1 \rightarrow G(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{(s + \omega_n)^2} \Rightarrow P_{1,2} = -\omega_n$$

$$\Rightarrow Y(s) = G(s) \cdot U(s) = \frac{\omega_n^2}{s(s + \omega_n)^2} = \frac{\zeta_1}{s} + \frac{\zeta_2}{s + \omega_n} + \frac{\zeta_3}{(s + \omega_n)^2}$$

\uparrow
 $U(t) = 1(t)$

$\square P_3 = 0$

$$\cdot \zeta_1 = \lim_{s \rightarrow 0} s \cdot \frac{\omega_n^2}{s(s + \omega_n)^2} \Rightarrow \boxed{1} \zeta_1$$

$$\cdot \zeta_3 = \lim_{s \rightarrow -\omega_n} (s + \omega_n)^2 \cdot Y(s) \Rightarrow -\omega_n \zeta_3$$

$$\Rightarrow Y(s) = \frac{\zeta_1}{s} + \frac{\zeta_2}{s + \omega_n} + \frac{\zeta_3}{(s + \omega_n)^2} = \frac{\zeta_1(s + \omega_n)^2 + \zeta_2 s(s + \omega_n) + s \zeta_3}{s(s + \omega_n)^2} = \frac{\omega_n^2}{s(s + \omega_n)^2}$$

$$\Rightarrow Y(s) = \frac{(s + \omega_n)^2 + \zeta_2(s^2 + s\omega_n) - s\omega_n}{s(s + \omega_n)^2} = \frac{s^2(1 + \zeta_2) + s(2\omega_n + \zeta_2\omega_n - \omega_n) + \omega_n^2}{s(s + \omega_n)^2}$$

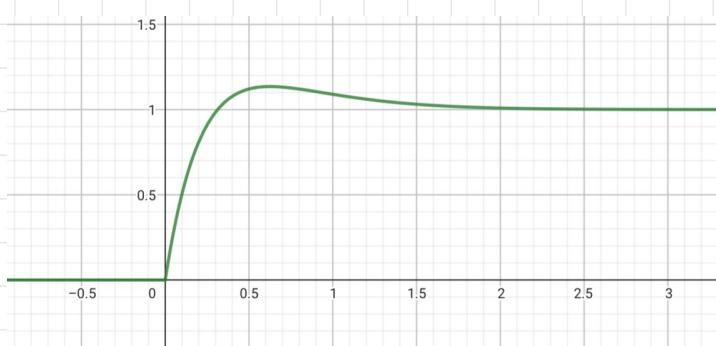
$$\Rightarrow \begin{cases} 1 + \zeta_2 = 0 \\ 2\omega_n + \zeta_2\omega_n - \omega_n = 0 \end{cases} \Rightarrow \boxed{\zeta_2 = -1}$$

$$Y(s) = \frac{1}{s} - \frac{1}{s + \omega_n} + \frac{\omega_n}{(s + \omega_n)^2}$$

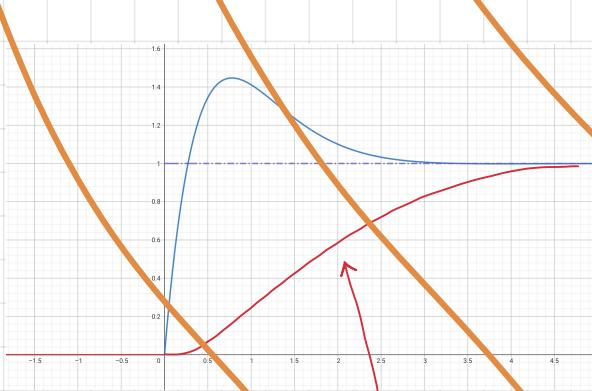
$$\text{Siccome } \mathcal{L}[e^{-\lambda t} \cdot f(t)] = F(s + \lambda) \quad \text{e } \mathcal{L}[t] = \frac{1}{s^2} \quad [\text{Lezione 3}]$$

$$\Rightarrow \mathcal{L}\left[\frac{1}{(s+2)^2}\right] = e^{-\lambda t} \cdot t \quad \Rightarrow \mathcal{L}\left[\omega_n \cdot \frac{1}{(s+\omega_n)^2}\right] = \omega_n e^{-\lambda t} \cdot t, \quad t \geq 0$$

$$\Rightarrow y(t) = 1 - e^{-\omega_n t} + \omega_n t e^{-\omega_n t}, \quad t \geq 0$$



L. $\mathcal{J} > 1$



Con $W_d = W_n = 1$, $\mathcal{J} = 2$

Se $\mathcal{J} = 2 \Rightarrow Y(t) = 1 - e^{-2t} \cos(t) + e^{-2t} \sin(t)$

Dalla lezione del prof era questo fondamento (?)

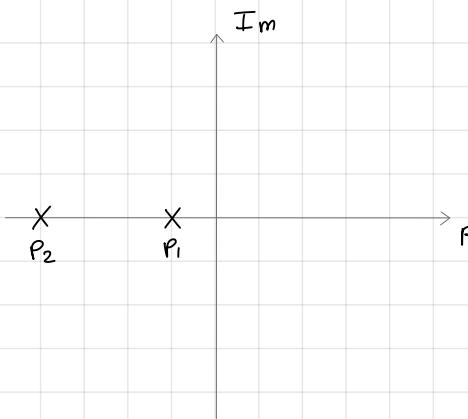
mai complx \Rightarrow REALI E DISTINTI

$$\text{Se } \mathcal{J} > 1 \Rightarrow P_{1,2} = -\mathcal{J}W_n \pm W_n \sqrt{\mathcal{J}^2 - 1}$$

Sicuramente > 1

$$\begin{cases} P_1 = -\mathcal{J}W_n + W_n \sqrt{\mathcal{J}^2 - 1} \\ P_2 = -\mathcal{J}W_n - W_n \sqrt{\mathcal{J}^2 - 1} \end{cases}$$

$$\Rightarrow P_1 > P_2$$



\Rightarrow Due exp, con τ diversa!

e_1 è più veloce di p_2

Se i poli sono REALI E DISTINTI allora possiamo scrivere:

$$s^2 + 2\mathcal{J}W_n s + W_n^2 = (s + p_1)(s + p_2)$$

! poli sono negativi!

$$\Rightarrow G(s) = \frac{W_n^2}{s^2 + 2\mathcal{J}W_n s + W_n^2} = \frac{W_n^2}{(s + p_1)(s + p_2)}$$

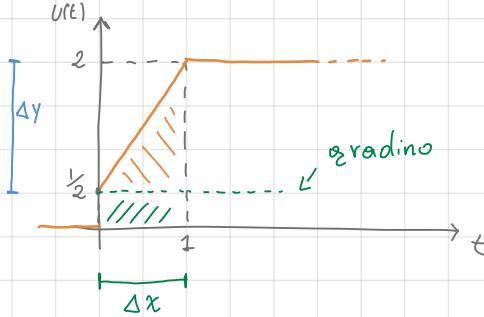
$$\Rightarrow Y(s) = -\frac{W_n^2}{s(s + p_1)(s + p_2)} = \frac{\varepsilon_1}{s} + \frac{\varepsilon_2}{s + p_1} + \frac{\varepsilon_3}{s + p_2}$$

$$\Rightarrow y(t) = 1 + \varepsilon_2 e^{p_1 t} + \varepsilon_3 e^{p_2 t}, t \geq 0$$

APPUNTI PRESI A LEZIONE

ESERCIZI

$$G(s) = \frac{2s+3}{s+5}$$



$G(s)$ fra poli: $\bar{s} = -\frac{3}{2}$

Penso scrivere

$$\bar{P} = -5$$

$$u(t) = \frac{1}{2} \mathbb{1}(t) + \frac{3}{2} t \cdot \mathbb{1}(t) - \frac{3}{2} (t-1) \cdot \mathbb{1}(t-1)$$

u_1 gradino u_2 rampa u_3 rampa

Applico la linearità e la tempo invarianza:

$$y(t) = y_1(t) + y_2(t) + y_3(t)$$

Considero $u_2(t)$

$$U_2(s) = \frac{3}{2s^2} \rightarrow Y_2(s) = \frac{3}{2} \cdot \frac{2s+3}{s^2(s+5)} = 3 \cdot \frac{s+\frac{3}{2}}{s^2(s+5)} = \frac{\xi_1}{s} + \frac{\xi_2}{s^2} + \frac{\xi_3}{s+5}$$

$$\xi_2 = \lim_{s \rightarrow 0} s^2 Y_2(s) = \lim_{s \rightarrow 0} s^2 \cdot 3 \cdot \frac{s+3/2}{s^2(s+5)} \rightarrow \frac{9}{10} \xi_2$$

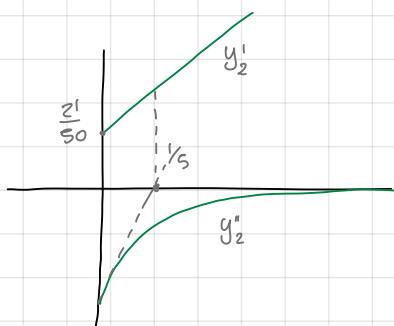
$$\xi_1 = \lim_{s \rightarrow -5} (s+5) \cdot 3 \cdot \frac{s+3/2}{s^2(s+5)} \rightarrow -\frac{21}{50} \xi_1$$

$$\text{Trono } r_1: 3 \cdot \frac{s+\frac{3}{2}}{s^2(s+5)} = \frac{s\xi_1(s+5)+\xi_2(s+5)\cdot\xi_3 s^2}{s^2(s+5)} = \frac{(\xi_1+\xi_3)s^2 + (5\xi_2+\xi_1)s + 5\xi_2}{s^2(s+5)}$$

dove essere $\frac{1}{2}$

$$\Rightarrow \xi_3 = -\xi_1 = \frac{21}{50}$$

$$\Rightarrow Y_2(s) = \frac{21}{50} \frac{1}{s} + \frac{9}{10} \cdot \frac{1}{s^2} - \frac{21}{50} \frac{1}{(s+5)} \Rightarrow y_2(t) = \left[\underbrace{\frac{21}{50} + \frac{9}{10} t}_{y_2'} \right] - \underbrace{\frac{21}{50} e^{-5t}}_{y_2''} \mathbb{1}(t)$$



Trovare $y_1(t)$

$$U_1(t) = \frac{1}{2} \mathbb{1}(t) = \frac{1}{3} U_2(t) = \frac{1}{3} \left(\frac{3}{2} \mathbb{1}(t) \right) = \frac{1}{3} \frac{dU_2}{dt} \Rightarrow y_1(t) = \frac{1}{3} \frac{d}{dt} y_2(t)$$

$$\Rightarrow y_1(t) = \frac{1}{3} \left(\frac{9}{10} + \frac{21}{50} e^{-5t} \right) = \frac{3}{10} + \frac{21}{30} e^{-5t}, \quad t \geq 0$$

Trovo $y_3(t)$

$$U_3(t) = -U_2(t-1) \Rightarrow$$

$$y_3(t) = -y_2(t-1) = -\frac{21}{50} - \frac{9}{10}(t-1) + \frac{21}{50} e^{-5(t-1)}, t \geq 1$$

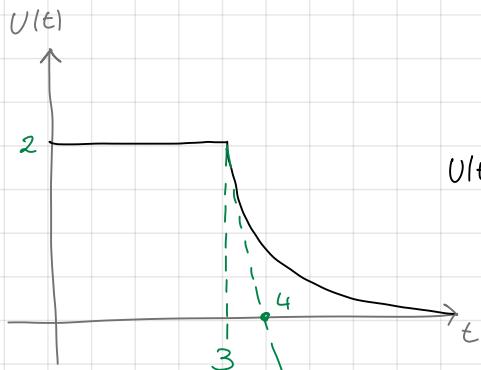
Valore di regime = ?

$$Y(s) = G(s) \cdot U(s) = \frac{2s+3}{s+5} \cdot \left(\frac{1}{2s} + \frac{3}{2s^2} - \frac{3}{2s^2} e^{-s} \right)$$

$$y_{ss}(t) = \lim_{s \rightarrow 0} s \cdot Y(s) = \lim_{s \rightarrow 0} \frac{2s+3}{s+5} \left(\frac{1}{2} + \frac{3}{2s} - \frac{1-e^{-s}}{s} \right)$$

$$\rightsquigarrow \lim_{s \rightarrow 0} \frac{3}{5} \left(\frac{1}{2} + \frac{3}{2} \frac{1-(1-s)}{s} \right) = \frac{6}{5} \quad \text{Valore di regime}$$

Esempio segnale composto



$$U(t) = 2 \cdot \mathbb{U}(t) - 2 \cdot \mathbb{U}(t-3) + 2 \cdot e^{-\frac{t-3}{\tau}}$$

$$\tau = 1$$

$$-(t-3) \cdot \tau$$

RISPOSTA TRANSITORIA PER sys del 2nd ordine

$$G(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

Pulsazione naturale

Coefficiente di smorzamento

\bar{s} :

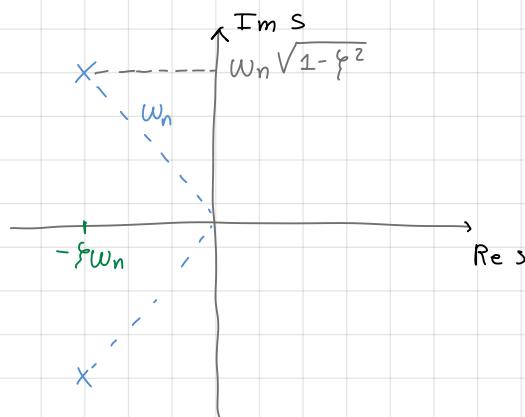
$$\bar{P}: s^2 + 2\xi w_n s + w_n^2 \rightarrow s = -\xi w_n \pm \sqrt{(\xi w_n)^2 - w_n^2}$$

$$\rightarrow \bar{s} = -\xi w_n \pm w_n \sqrt{\xi^2 - 1}$$

- Se $\xi = 1 \rightsquigarrow$ CRITICAMENTE SMORZATO
- Se $\xi > 1 \rightsquigarrow$ SOVRASMOZATO PARTE REALE NEGATIVA ma 2 sol.
- Se $\xi < 1 \rightsquigarrow$ SOTTO SMORZATO poli complessi e coniugati

• $\xi = 0$ NON SMORZATO

$$\hookrightarrow -\xi w_n \pm j w_n \sqrt{1 - \xi^2}$$

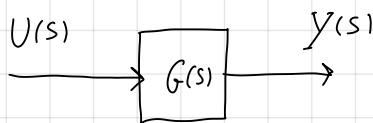


w_n è il modulo dei poli complessi e coniugati, ovvero se $\xi < 1$

* molla

Hp. $0 < \xi < 1$

$$U(s) = \text{gradino} = \frac{1}{s}$$



$$\Rightarrow Y(s) = \frac{w_n^2}{s(s^2 + 2\xi w_n s + w_n^2)} = \frac{\xi_1}{s} + \frac{\xi_2 s + \xi_3}{s^2 + 2\xi w_n s + w_n^2}$$

$$\xi_1 = \lim_{s \rightarrow 0} s \cdot Y(s) \rightarrow \textcircled{1} \xi_1$$

$$\frac{\xi_2 (s^2 + 2\xi w_n s + w_n^2) + \xi_3 s^2 + \xi_3 s}{s(s^2 + 2\xi w_n s + w_n^2)} = \frac{(\xi_1 + \xi_2)s^2 + (2\xi w_n \xi_1 + \xi_3)s + \xi_1 w_n^2}{s(s^2 + 2\xi w_n s + w_n^2)} = \frac{w_n^2}{s(s^2 + 2\xi w_n s + w_n^2)}$$

$$\xi_1 + \xi_2 = 0 \Rightarrow \xi_2 = -\xi_1 \Rightarrow \textcircled{2} \xi_2 = -1$$

$$2\xi w_n + \xi_3 = 0 \Rightarrow \xi_3 = -2\xi w_n \textcircled{3} \xi_3$$

$$\Rightarrow \frac{w_n^2}{s(s^2 + 2\xi w_n s + w_n^2)} = \frac{\frac{1}{s}}{s(s^2 + 2\xi w_n s + w_n^2)} - \frac{\frac{1 + 2\xi w_n}{s}}{s(s^2 + 2\xi w_n s + w_n^2)}$$

-o AntiTrasformata

$$\begin{aligned}
 G(s) &= \frac{w_n^2}{s(s^2 + 2\xi w_n s + w_n^2)} = -\frac{\frac{1}{s}}{(s + \xi w_n)^2 + w_n^2 - (\xi w_n)^2} = -\frac{\frac{1}{s}}{(s + \xi w_n)^2 + w_n^2 \cdot (1 - \xi^2)} + \frac{1}{s} \\
 &= -\frac{s + 2\xi w_n}{(s + \xi w_n)^2 + w_d^2} + \frac{1}{s} \\
 &= \frac{1}{s} - \frac{s + \xi w_n}{(s + \xi w_n)^2 + w_d^2} - \frac{\xi w_n}{(s + \xi w_n)^2 + w_d^2} \\
 &= \frac{1}{s} - \frac{s + \xi w_n}{(s + \xi w_n)^2 + w_d^2} - \frac{\xi w_n}{w_d} \frac{w_d}{(s + \xi w_n)^2 + w_d^2}
 \end{aligned}$$

$\Rightarrow y(t) = 1 - e^{-\xi w_n t} \cos(w_d t) - \frac{\xi w_n}{w_d} \cdot e^{-\xi w_n t}, \quad t \geq 0$

$$w_d = w_n \sqrt{1 - \xi^2}$$

$$\Rightarrow w_d^2 = w_n^2 (1 - \xi^2)$$

w_d
Pulsazione
naturale
smorzata

Risposta di
un qualsiasi
sistema del 2nd
ordine

FOTO

$$\frac{\xi \omega_n}{\omega_d} = \frac{\xi \omega_n}{\omega_n \sqrt{1-\xi^2}} = \frac{\xi}{\sqrt{1-\xi^2}} \Rightarrow y(t) = 1 - e^{-\xi t}$$

$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$Y(s) = \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} = \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = (s + \xi\omega_n)^2 + \omega_d^2 - (\xi\omega_n)^2$$

$$= (s + \xi\omega_n)^2 + \omega_d^2 (1 - \xi^2)$$

$$= (s + \xi\omega_n)^2 + \omega_d^2$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$\omega_d^2 = \omega_n^2 (1 - \xi^2)$$

$$Y(s) = 1 - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t} \cos(\omega_d t) - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t} \sin(\omega_d t), t \geq 0$$

$V(s) \xrightarrow{G(s)} Y(s)$

$$\frac{\xi\omega_n}{\omega_d} = \frac{\xi\omega_n}{\omega_n \sqrt{1 - \xi^2}} = \frac{\xi}{\sqrt{1 - \xi^2}}$$

$$y(t) = 1 - e^{-\xi\omega_n t} \left(\cos(\omega_d t) + \frac{\xi}{\sqrt{1 - \xi^2}} \sin(\omega_d t) \right) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \left(\sqrt{1 - \xi^2} \cos(\omega_d t) + \xi \sin(\omega_d t) \right)$$

$$\begin{cases} \sin \alpha = \sqrt{1 - \xi^2} \\ \cos \alpha = \xi \end{cases}$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \cdot \sin(\omega_d t + \alpha)$$



Quanto più i poli si avvicinano a zero, tanto più la risposta tenderà ad oscillare; quando zeta è uguale a zero, il sistema non è smorzato e quindi i poli sono sull'asse immaginario, quindi il sistema può essere stabile ma non **asintoticamente**, ovvero abbiamo una sinusoide.

Se zeta è molto piccola, il sistema è smorzato, ma davvero poco.

Quando zeta si avvicina ad 1, il transitorio si rimpicciolisce sempre di più fino a scomparire

$V(s) \xrightarrow{G(s)} Y(s)$

$$y(t) = 1 - e^{-\xi\omega_n t} \left(\cos(\omega_d t) + \frac{\xi}{\sqrt{1 - \xi^2}} \sin(\omega_d t) \right) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \left(\sqrt{1 - \xi^2} \cos(\omega_d t) + \xi \sin(\omega_d t) \right)$$

$$\begin{cases} \sin \alpha = \sqrt{1 - \xi^2} \\ \cos \alpha = \xi \end{cases}$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \cdot \sin(\omega_d t + \alpha)$$

$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$\Im s$

$\Re s$

$\omega_d = \omega_n \sqrt{1 - \xi^2} < \omega_n$

\uparrow

Smorzata

$$T = \frac{1}{\xi\omega_n}$$

$$y(t) = 1 - e^{-\xi\omega_n t} \cos(\omega_d t) - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t} \sin(\omega_d t), t \geq 0$$

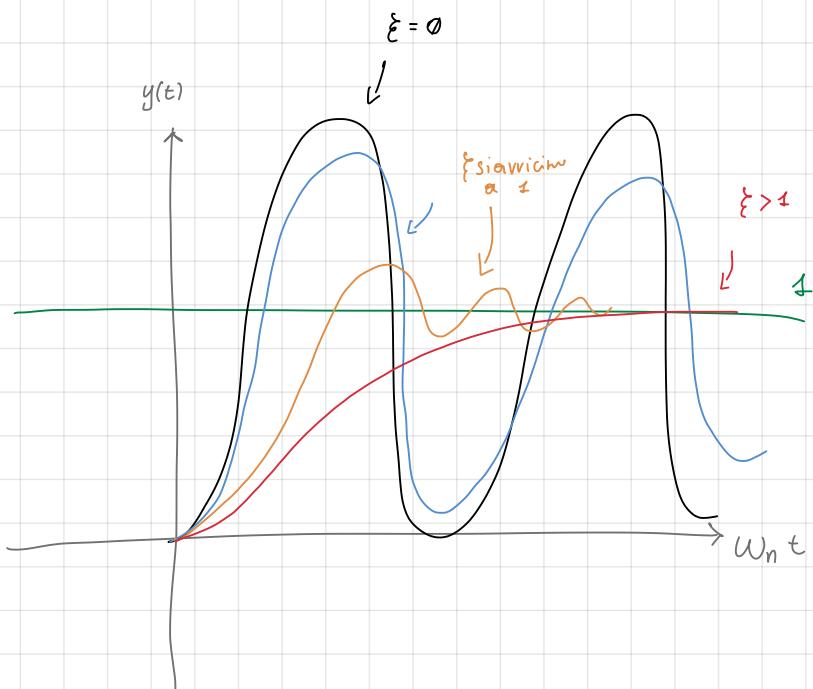
Se $\xi = 0$

Diagram of a circuit with voltage $U(s)$ and current $I(s)$.

$$y(t) = 1 - e^{-\omega_n t} \left(\cos \omega_n t + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_n t \right) = 1 - \frac{e^{-\omega_n t}}{\sqrt{1-\xi^2}} \left(\sqrt{1-\xi^2} \cos \omega_n t + \xi \sin \omega_n t \right)$$

$$\begin{cases} \sin \alpha = \sqrt{1-\xi^2} \\ \cos \alpha = \xi \end{cases}$$

$$= 1 - \frac{e^{-\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_n t + \alpha)$$



Risposta per $f=1$

CRITICAMENTE SORZATO

$$G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \omega_n)^2}$$

$$\tilde{Y}(s) = \frac{\omega_n^2}{s(s + \omega_n)^2} = \frac{\zeta_1}{s} + \frac{\zeta_2}{s + \omega_n} + \frac{\zeta_3}{(s + \omega_n)^2}$$

$$\zeta_1 = \lim_{s \rightarrow 0} s \tilde{Y}(s) = 1$$

$$\zeta_2 = \lim_{s \rightarrow -\omega_n} (s + \omega_n)^2 \tilde{Y}(s) = -\omega_n$$

$$\zeta_3 = \lim_{s \rightarrow -\omega_n} (s + \omega_n)^2 \tilde{Y}'(s) = \frac{1}{s + \omega_n} - \frac{\omega_n}{(s + \omega_n)^2} = \frac{(s + \omega_n)^2 + \zeta_3(s + \omega_n) - s\omega_n}{s(s + \omega_n)^2}$$

$$= \frac{(1 + \zeta_3)s^2 + (\zeta_3\omega_n - \omega_n)s + \omega_n^2}{s(s + \omega_n)^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \omega_n)^2}$$

$$\tilde{Y}(s) = \frac{\omega_n^2}{s(s + \omega_n)^2} = \frac{\zeta_1}{s} + \frac{\zeta_2}{s + \omega_n} + \frac{\zeta_3}{(s + \omega_n)^2}$$

$$\zeta_1 = \lim_{s \rightarrow 0} s \tilde{Y}(s) = 1$$

$$\zeta_2 = \lim_{s \rightarrow -\omega_n} (s + \omega_n)^2 \tilde{Y}(s) = -\omega_n$$

$$\zeta_3 = -1$$

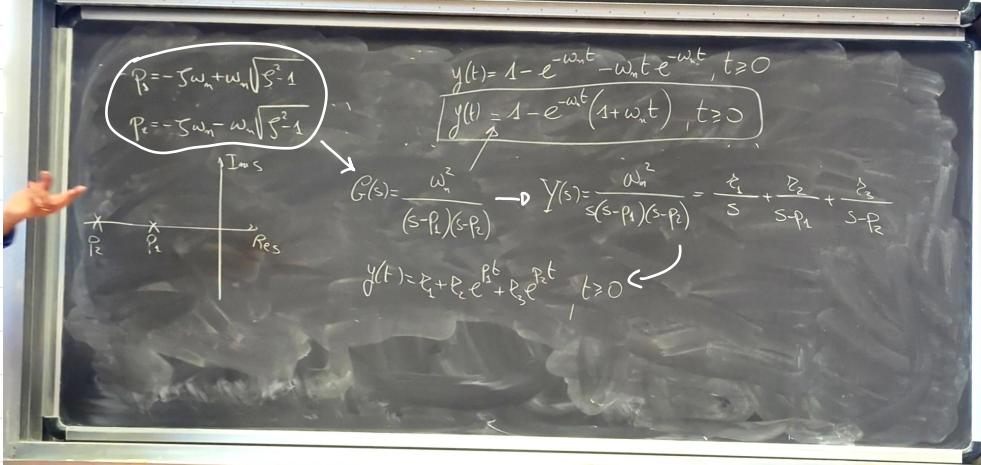
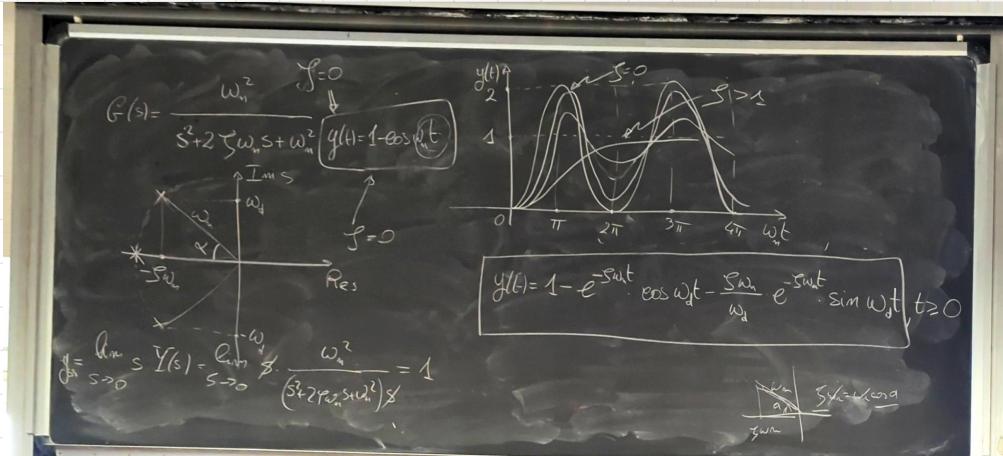
$$y(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}, t \geq 0$$

$$y(t) = 1 - e^{-\omega_n t} (1 + \omega_n t), t \geq 0$$

$$\int \frac{1}{s + \omega_n} - \frac{\omega_n}{(s + \omega_n)^2} = \frac{(s + \omega_n)^2 + \zeta_3(s + \omega_n) - s\omega_n}{s(s + \omega_n)^2}$$

$$= \frac{(1 + \zeta_3)s^2 + (\zeta_3\omega_n - \omega_n)s + \omega_n^2}{s(s + \omega_n)^2}$$

Risposta per $\xi > 1$



$$-\xi\omega_n + i\omega_n\sqrt{\xi^2 - 1} \quad \text{ma } \sqrt{\xi^2 - 1} < \xi$$