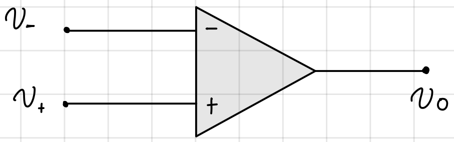
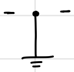
AMPLIFICATORI OPERAZIONALI

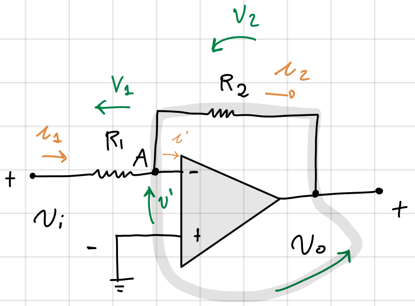


$$v_o = K\,(v_+ - v_-)$$

K ↑ guadagno



Tipica connessione INVERTENTE



$$\begin{cases} i_1 = \dfrac{v_{R_1}}{R_1} = \dfrac{v_1 - v'}{R_1} \\[2ex] i_2 = \dfrac{v' - v_o}{R_2} \end{cases}$$

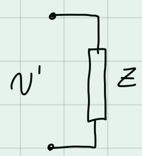
[uso la maglia v', v_o, v_2]

LKC$_A$: $-i_1 + i' + i_2 = 0 \rightarrow i_1 = i' + i_2$

nell'A.O. la corrente in entrata è ZERO

Hp. $i' \cong 0$

$\Rightarrow \boxed{i_1 \cong i_2} \Rightarrow (1) \cong (2)$



$v' = Z\,i'$ $\rightarrow$ $i' = \dfrac{v'}{Z}$ con $Z \to \infty$

$\Rightarrow i' \sim 0$

$$\rightarrow \frac{v_1 - v'}{R_1} = \frac{v' - v_o}{R_2} \Rightarrow \boxed{\frac{v_1}{R_1} = -\frac{v_o}{R_2}} \Rightarrow \boxed{v_o = -\frac{R_2}{R_1}\,v_i}$$

$v' \cong 0$

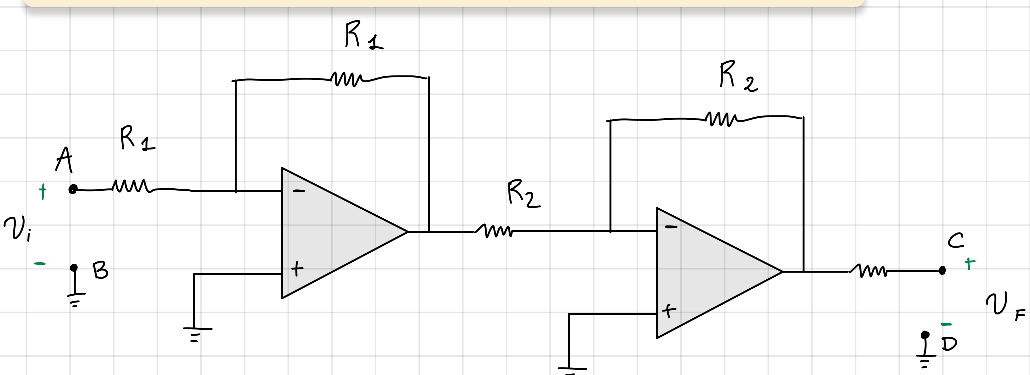
GUADAGNO a seconda delle R cambia K

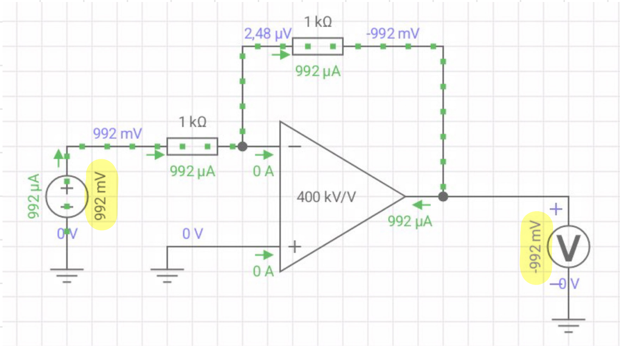
Configurazione invertente

Se metto due amplificatori operazionali (entrambi con delle resistenze uguali, anche diverse tra i due amplificatori) otteniamo esattamente la tensione iniziale (senza segno opposto)

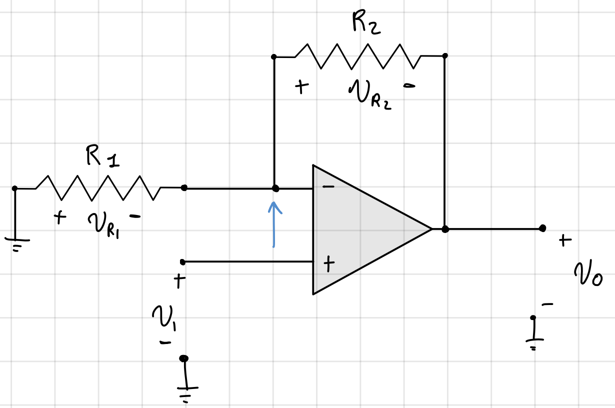
Se $R_1 = R_2$

$v_o \rightsquigarrow ▷ \rightsquigarrow -v_o$





CONFIGURAZIONE NON INVERTENTE

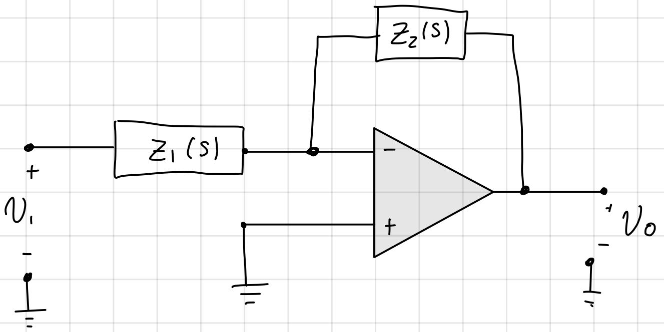


$$i_1 \simeq i_2 \rightarrow \frac{v_{R_1}}{R_1} = -\frac{v_1}{R_2}$$

$$i_2 = \frac{v_{R_2}}{R_2} = \frac{v_1 - v_o}{R_2}$$

$$\rightarrow -\frac{v_1}{R_2} = \frac{v_1 - v_o}{R_2} \rightarrow \frac{v_o}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 \Rightarrow \boxed{v_o = R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_i}$$

GUADAGNO



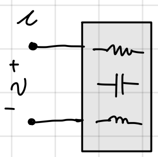
$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_1(s)}{Z_2(s)}} \equiv \text{F. DI TRANSF.}$$

TRASFORMATE

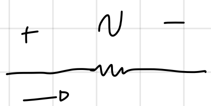
impedenze

Recap Impedenza complessa

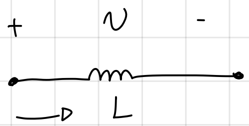
$$Z(s) = \frac{V(s)}{I(s)} \begin{matrix} \leftarrow \text{OUT} \\ \leftarrow \text{IN} \end{matrix}$$



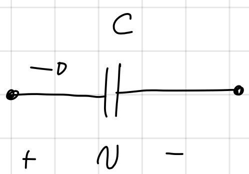
La differenza tra l'impedenza complessa ed una semplice funzione di trasferimento è che nel caso dell'impedenza blocchiamo come ingresso la corrente e come uscita la tensione.



$$\frac{v}{i} = R = Z_R(s)$$



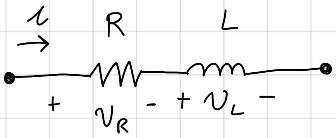
$$v = L\frac{di}{dt} \Rightarrow V(s) = sL\, I(s) \Rightarrow Z_L(s) = sL$$



$$i = C\frac{dv}{dt} \Rightarrow I(s) = sC\, V(s) \Rightarrow Z_C(s) = \frac{1}{sC}$$

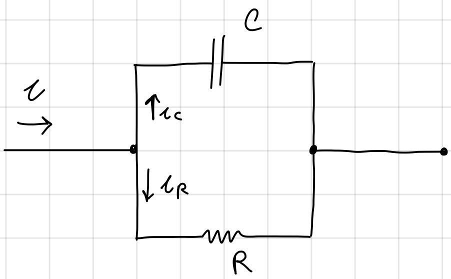
FOTO 1

COMBINAZIONI DI IMPEDENZE



$v = R i_L + L \dot{i}_L \rightarrow V(s) = R\, I(s) + sL\, I(s) = I(s)(R + sL)$

$\rightarrow$ Siccome $Z = \frac{V(s)}{I(s)} \rightarrow \boxed{Z = R + sL}$ RL Serie



$i = i_C + i_R \rightarrow i = C \dot{v}_C + \frac{v}{R}$

$\rightleftarrows I(s) = sC\, V(s) + \frac{1}{R} V = V(s)\left(sC + \frac{1}{R}\right)$

$\Rightarrow Z(s) = \frac{1}{sC + \frac{1}{R}} = \boxed{\frac{R}{RsC + 1}}$ Z(s) RC PARALL

Implementare un controllore di tipo proporzionale

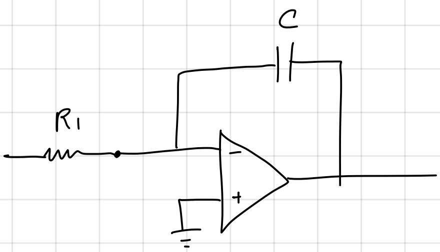
$G(s) = K_P \rightarrow$ Visto prima

$G(s) = K_P + \frac{K_I}{s} \rightarrow$ vediamo dopo

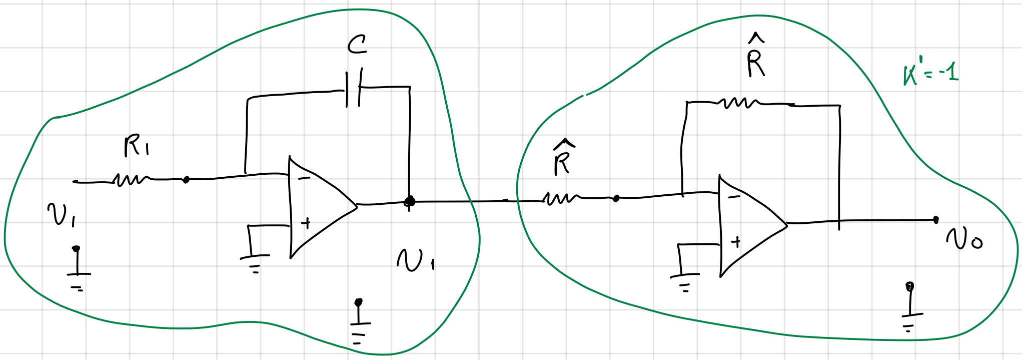
$G(s) = \frac{K_I}{s} \rightarrow$ vediamo ora

voglio impl. $G(s) = \frac{K_i}{s}$

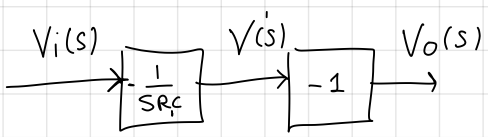
$$\begin{cases} Z_2(s) = \frac{1}{sC} \\ Z_1(s) = R_1 \end{cases}$$



$K = \frac{1}{RC}$



$K_1 = -\frac{1}{R_1 C}$

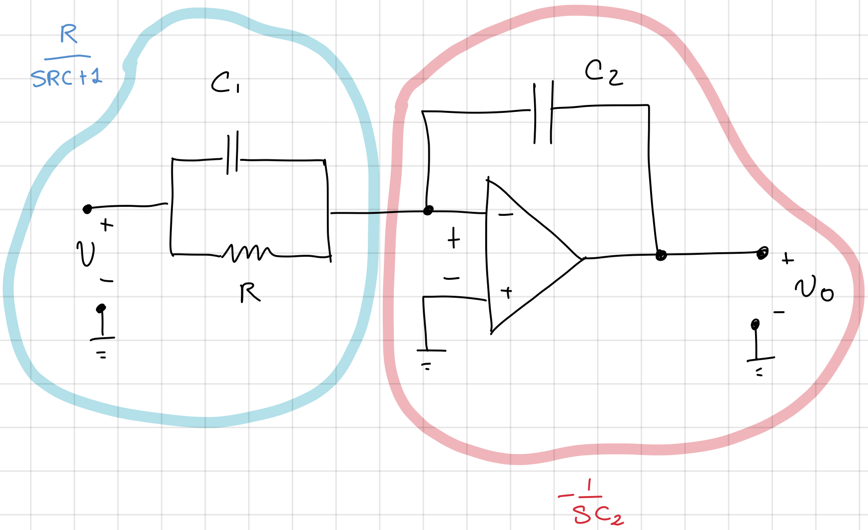


$$G(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s}$$

1 zero e 1 polo in 0

$$\frac{Z_1}{Z_2} = G(s) \quad \rightarrow \quad Z_2 = C = \frac{1}{sC_2} \leftarrow \text{POLO}$$

$$Z_1(s) = \frac{R}{sRC+1}$$



$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sRC} \cdot \frac{sRC_1+1}{R}$$

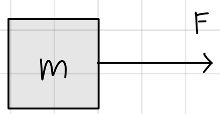
$$= \frac{s\underbrace{\frac{C_1}{C_2}}_{K_p} + \underbrace{\frac{1}{RC_2}}_{K_i}}{s}$$

Modellistica dei sistemi meccanici

* Primo e secondo es
clip audio 8/9
e domanda orale
2° e 3° orale → elenco domande

Caratterizzati da MASSA molla e SMORZATORE

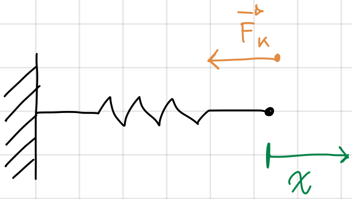
MASSA



$$\underset{[N]}{F = ma}\ ,\quad a = \frac{dv}{dt} = \underset{[m/s^2]}{\dot{v}}\ ,\quad v = \frac{dx}{dt} = \dot{x}\ \ [m]$$

↑ Spazio

MOLLA



Legge di Hooke : $F_k = -Kx$ (la molla viene Allungata)

$$E_m(t) = \int_0^t \underbrace{F(t)\cdot v(t)}_{\text{Potenza}}\,dt = \int_0^t m\frac{dv}{dt}\cdot v(t)\,dt = \frac{1}{2}m\int_0^t \frac{d\,v^2(t)}{dt}\,dt = \frac{1}{2}m\left[v^2(t)\right]_0^t$$

Energia di un corpo in morim.

$$= \frac{1}{2}m\,v^2(t)$$

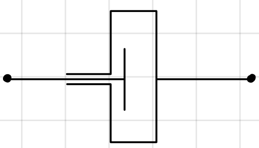
Possibile Var di Stato

$$E_k(t) = \int_0^t F_k(t)\cdot v(t)\,dt = \frac{1}{2}k\,x^2(t)$$

Energia della molla

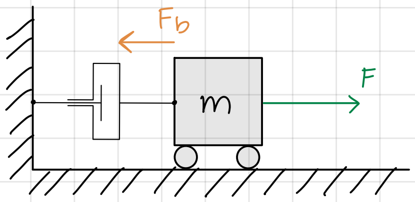
Possibile Var di Stato

SMORZATORE



$$v = \frac{dx}{dt} \rightarrow F_b = -bv \equiv F_b = -b\cdot\dot{x}$$

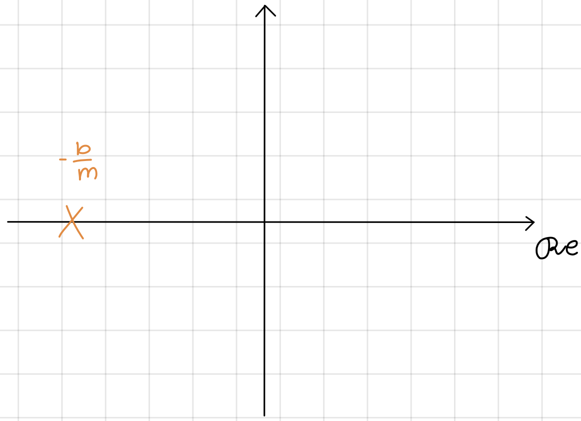
SISTEMA 1

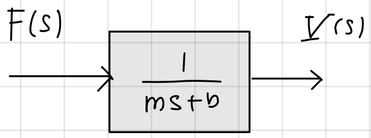


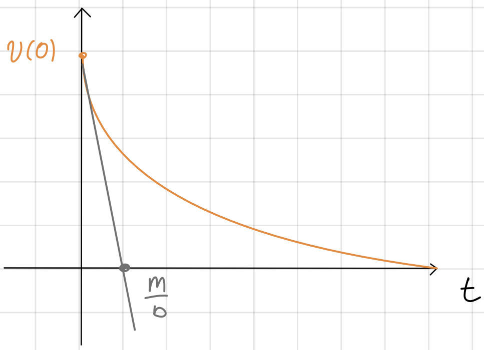
$$\begin{cases} F + F_b = m\dot{v} \\ F_b = -b\,v \end{cases}$$

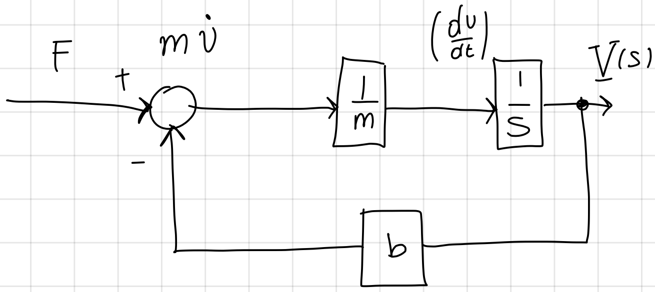
$\mathcal{L}.T. \rightarrow \quad F(s) - b \cdot V = m \cdot s\, V(s) \quad \rightarrow \quad (ms+b)\, V(s) \quad F(s)$

$\Rightarrow \quad \dfrac{V(s)}{F(s)} = \dfrac{1}{ms+b}$









VAR. ST. $\quad v$

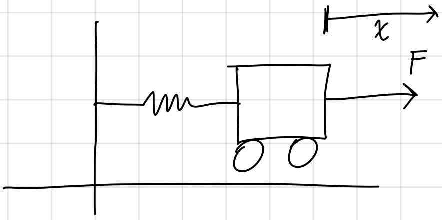
$x = v \quad , \quad u = F \quad , \quad y = v$

$$\begin{cases} \dot{x} = \frac{1}{m}(u - b\cdot x) \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \frac{b}{m}x + \frac{1}{m}u \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \underset{=}{A}x + \underset{=}{B}u \\ y = \underset{=}{C}x + \underset{=}{D}y \end{cases}$$

$\frac{b}{m}$ $\quad \frac{1}{m}$

$1 \quad \emptyset$

SENZA SMORZATORE



$$\begin{cases} F + F_K = m\dot{v} \\ F_K = -Kx \\ v = \dot{x} \end{cases} \quad \rightarrow \quad \dot{v} = \frac{1}{m}(F + F_K)$$

$x_1 = x \qquad x_2 = v \qquad u = F$

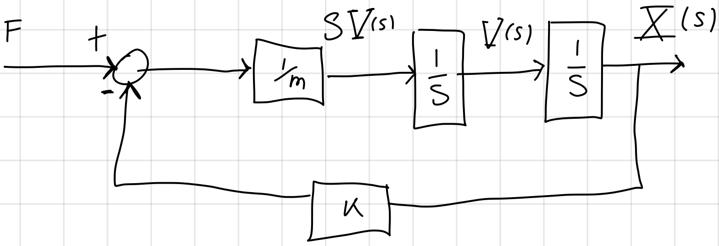
$y = x_1$ ← la scelgo io

↑ pos in x

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m}(u - Kx_1) \\ y = x_1 \end{cases} \quad \rightarrow \quad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{m}x_1 + \frac{1}{m}u \\ y = x_1 \end{cases}$$

$$\rightarrow \underset{x_1, x_2}{\underline{A}} = \begin{array}{cc} \begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix} & \begin{array}{c} x_1 \\ x_2 \end{array} \end{array} \qquad \underset{u}{\underline{B}} \begin{pmatrix} 0 \\ 1/m \end{pmatrix} \qquad \underset{y}{\underline{C}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

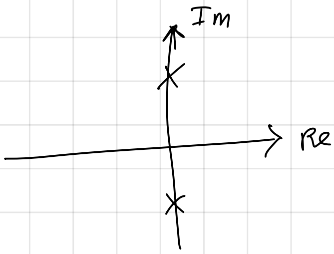
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \overset{\underline{A}}{\begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix}} \cdot \overset{\underline{B}}{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} + \overset{\underline{C}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} u$$



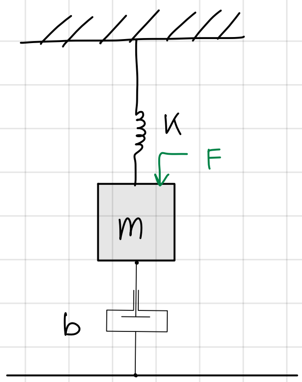
$$G(s) = \frac{X}{S} = \frac{\frac{1}{ms^2}}{1 + K\frac{1}{ms^2}} = \frac{1}{ms^2 + K}$$

Sin(t)

con $S = \pm j\sqrt{\frac{K}{m}}$



CON SMORZATORE



$$\begin{cases} F + F_K + F_b = m\dot{v} \\ F_K = -Kx \\ F_b = -bv \\ v = \dot{x} \end{cases}$$

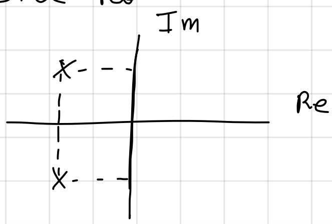
$m s V(s) = -F(s) - K X(s) - b \cdot V(s)$ ma $V(s) = s X(s)$

$\rightarrow m s V(s) + b s X(s) + K X(s) = F(s)$

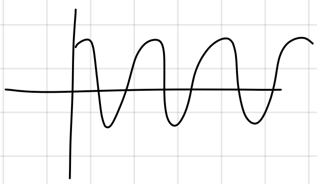
$\rightarrow \dfrac{X(s)}{F(s)} = \dfrac{1}{m s^2 + b s + K} \rightarrow \boxed{b^2 - 4 m \cdot b > 0}$ ovvero se $b^2 > 4mb$

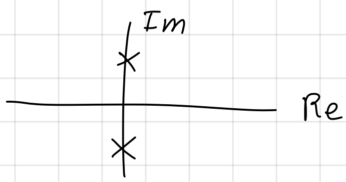
Se c'è uno smorzamento abbiamo anche la parte reale

$\lambda = a \pm j b \rightarrow$

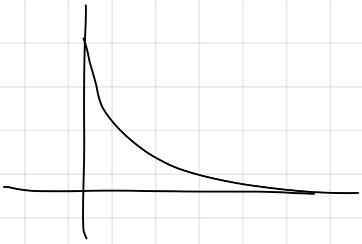


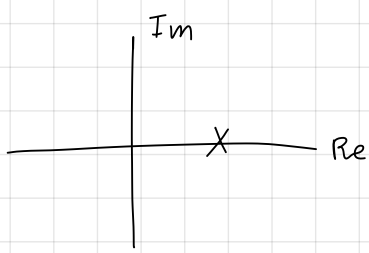
$\lambda = 0 \pm j b \rightarrow$



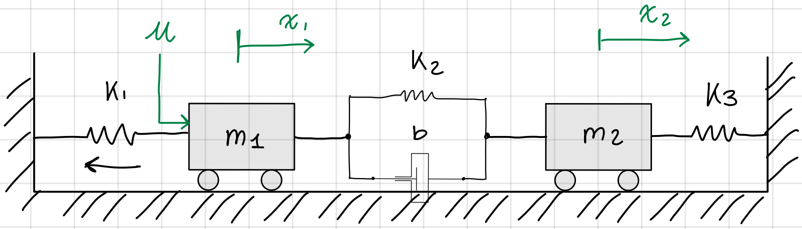


$\lambda = a$





ACCOPPIAMENTO FERROVIARIO



Scriviamo $m \cdot a = \sum F$ Tenendo presente che $a = \dot{v} = \ddot{x}$ e le rel car

$$
\begin{cases}
m_1 \ddot{x}_1 = u - K_1 x_1 - K_2 (x_1 - x_2) - b(\underbrace{\dot{x}_1 - \dot{x}_2}_{v_1 - v_2}) \\
m_2 \ddot{x}_2 = K_2 (x_1 - x_2) + b(\underbrace{\dot{x}_1 - \dot{x}_2}_{v_1 - v_2}) - K_3 x_2
\end{cases}
$$

mancano $\dot{x}_1$ e $\dot{x}_2$ $= 0$ Pongo

$$x_3 = \dot{x}_1 \leftarrow v_{m1}$$
$$x_4 = \dot{x}_2 \leftarrow v_{m2}$$

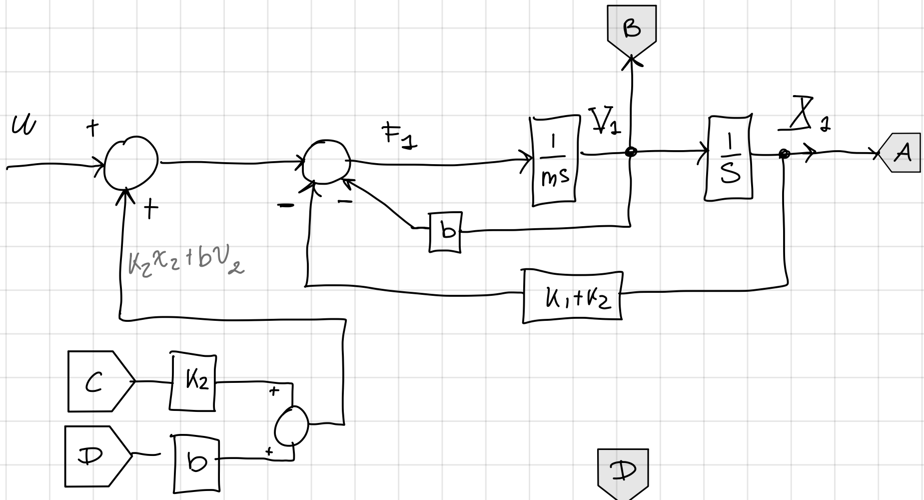
ottengo:

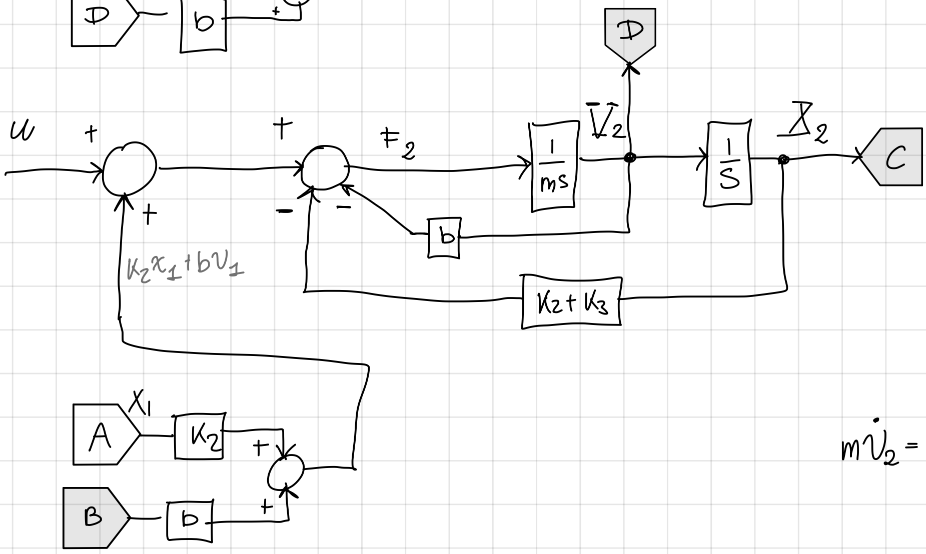
$$
\begin{cases}
\dot{x}_3 = \frac{1}{m_1} \left[u - K_1 x_1 - K_2 (x_1 - x_2) - b(x_3 - x_4) \right] \\
\dot{x}_4 = \frac{1}{m_2} \left[K_2 (x_1 - x_2) + b(x_3 - x_4) - K_3 x_2 \right]
\end{cases}
$$

$$
\begin{cases}
\dot{x}_1 = x_3 \\
\dot{x}_2 = x_4
\end{cases}
$$

$$
\underline{\underline{A}} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\frac{K_1 + K_2}{m_1} & \frac{K_2}{m_1} & -\frac{b}{m_1} & +\frac{b}{m_1} \\
\frac{K_2}{m_2} & -\frac{K_2 + K_3}{m_2} & \frac{b}{m_2} & -\frac{b}{m_2}
\end{pmatrix} \end{array}
\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{array}
\qquad
\underline{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix} \cdot u
$$

$$y = x_2 - x_1 = (0, 0, -1, 1)$$





$$m\dot{v}_2 = -(k_2+k_3)\,x_2 - bv_2 + k_2x_1 + bv_1$$