ESERCIZI ANTITRASFORMATA e rappresentazione grafica

ES 1

$$F(S) = \frac{10}{S+2}$$

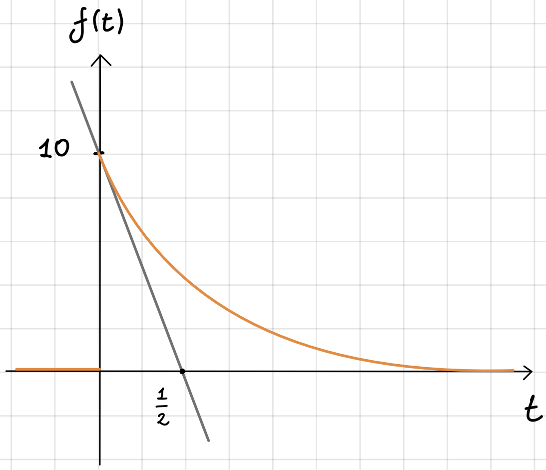
Q. Antitrasformata ed Andamento nel tempo

$$\rightarrow \quad f(t) = 10 \cdot e^{-2t} \cdot 1\!\!1(t)$$

$\rightarrow$ oppure $\begin{cases} 10 \cdot e^{-2t} & t \geq 0 \\ \emptyset & t < 0 \end{cases}$

T. Val iniz $\rightarrow$ $f(0^+) = \lim_{S \to \infty} S \cdot F(S) = \lim_{S \to \infty} \frac{10\cancel{S}}{\cancel{S}+2} = 10$

T. Val fin $\rightarrow$ $\lim_{t \to \infty} f(t) = \lim_{S \to 0} S\,F(S) = \lim \frac{10S}{S+2} \rightarrow \emptyset$



- Costante di Tempo

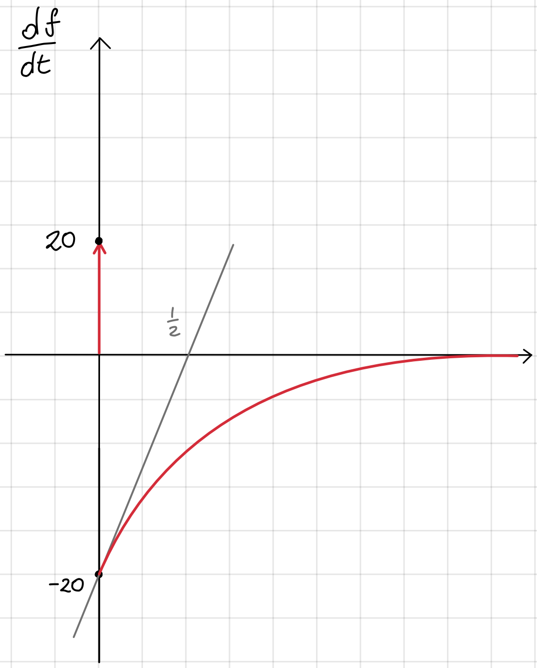
$$e^{-\alpha t} \rightarrow \frac{1}{\alpha}$$

$$f'(t) = -20\,e^{-2t} \rightarrow f'(0^+) = -20$$

$$\mathcal{L}_+\left[f'(t)\right] = s\,F(s) - f(0^+)$$ Regola della derivata

$$= \frac{10S}{S+2} - 10 = \frac{\cancel{10S} - \cancel{10S} - 20}{S+2} = -\frac{20}{S+2}$$

$\rightarrow$



$$\mathcal{L}_-\left[f'(t)\right] = S\,F(s) - f(0^-) = \frac{10S}{S+2}$$

$\rightarrow$ FRATTI SEMPLICI

$$= 10 \frac{S+2-2}{S+2} = 10 \frac{S+2-2}{S+2} = 10\left(\frac{\cancel{S+2}}{\cancel{S+2}} - \frac{2}{S+2}\right)$$

$$= 10\left(1 - \frac{2}{S+2}\right) \xrightarrow{\text{ANTI}} \mathcal{L}^{-1}\left[f'(t)\right] = 10\left(\delta(t) - 2\,e^{-2t} \cdot 1\!\!1(t)\right)$$

La differenza sta nell'impulso, perchè la derivata di una discontinuità è l'impulso

Perchè $\frac{1}{2}$ è la pendenza ?

La derivata di una funzione e consideriamo la sua derivata in t*, questa è la pendenza della retta tangente

$P_1 = (0, 10)$

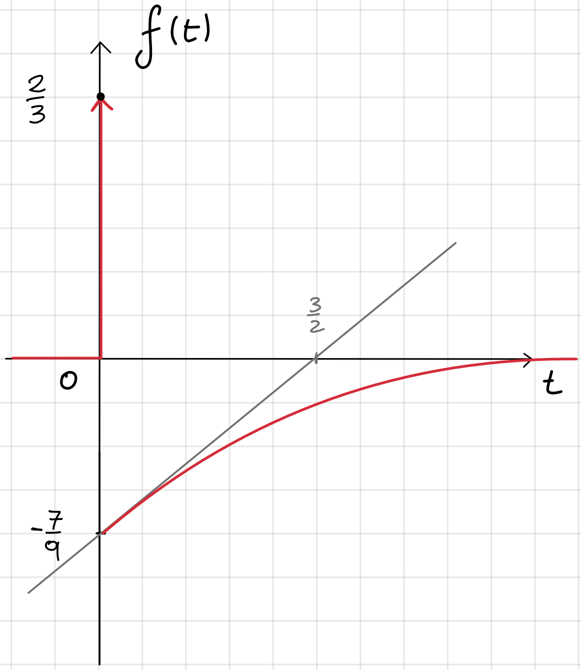
$P_2 = \left(\frac{1}{2}; 0\right)$ $\Rightarrow$ $\frac{10}{\frac{1}{2}} = 20$

ES 2:

$$F(s) = \frac{2s-1}{3s+2} = \frac{1}{3}\left(\frac{2s-1}{s+\frac{2}{3}}\right) = \frac{2}{3}\left(\frac{s-\frac{1}{2}}{s+\frac{2}{3}}\right) = \frac{2}{3}\,\frac{s+\frac{2}{3}-\frac{2}{3}-\frac{1}{2}}{s+\frac{2}{3}}$$

$$= \frac{2}{3}\left(1 - \frac{7}{6}\ \overset{?}{\ }\ \frac{1}{s+\frac{2}{3}}\right)$$

$$\Rightarrow f(t) = \frac{2}{3}\left(\delta(t) - \frac{7}{6}\, e^{-\frac{2}{3}t}\, \mathbb{1}(t)\right) \qquad \Rightarrow \tau_0 = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$



* bisognava calcolare anche il valore iniziale e finale: bisogna sempre **applicare le regole della trasformata di Laplace per ogni esercizio svolto.**

ES 3

$$F(s) = \frac{2s}{s^2+3s+2}$$

Poli e zeri

1 zero $\rightarrow$ 0 zero

2 poli $\rightarrow$ $s = \frac{-3 \pm \sqrt{9-4\cdot 1\cdot 2}}{2}$ $\rightarrow$ -2, -1 poli

$$\Rightarrow \quad F(s) = \frac{2s}{s^2+3s+2} = \frac{\tau_1}{s+1} + \frac{\tau_2}{s+2} \quad \Rightarrow \quad \frac{2s}{(s+1)(s+2)} = \frac{\tau_1}{s+1} + \frac{\tau_2}{s+2}$$

$+$ STABILE

$-$ INSTABILE!

$$\tau_1 = \lim_{s\to -1} (s+1)\, F(s) = \lim_{s\to -1} \frac{2s}{s+2} \rightarrow -2 \quad \tau_1$$

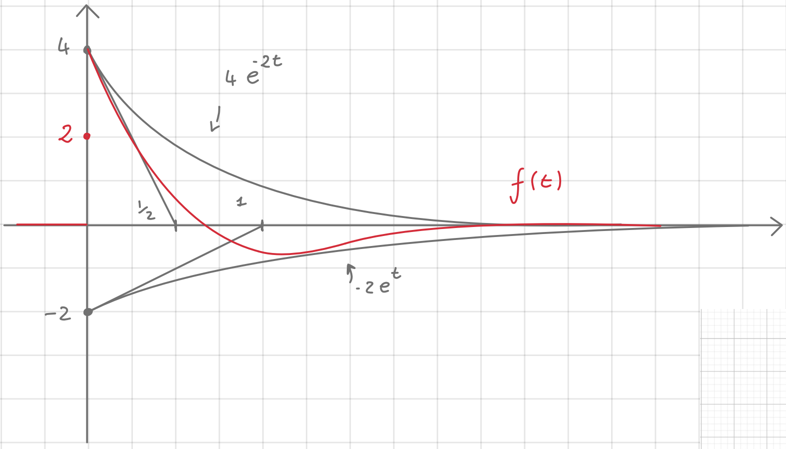
$$\tau_1 = \lim_{s\to -2} (s+2)\, F(s) = \lim_{s\to -2} \frac{2s}{s+1} \rightarrow +4 \quad \tau_2$$

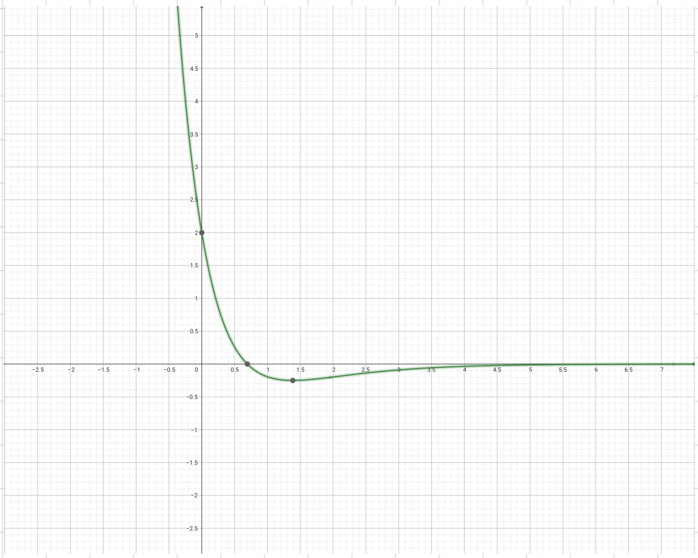
$$\Rightarrow F(s) = -\frac{2}{s+1} + \frac{4}{s+2} = -2\,\frac{1}{s+1} + 4\,\frac{1}{s+2}$$

$\mathcal{L}[e^{-\alpha t}\,\mathbb{1}(t)]$

$\alpha = 1$ $\qquad$ $\alpha = 2$

$$\Rightarrow \quad f(t) = \left(-2e^{-t} + 4e^{-2t}\right)\cdot \mathbb{1}(t) \quad \leftarrow \text{SOL}$$





ES 4 :

$$F(s) = \frac{s+8}{s^2+2s+2}$$

$$\mathcal{L}\left[\sin(\omega t)\right] = \frac{\omega}{s^2+\omega^2}$$

$$\rightarrow \mathcal{L}\left[e^{-\alpha t}\sin(\omega t)\right] = \frac{\omega}{(s+\alpha)^2+\omega^2}$$

$$= \frac{\omega}{s^2+2\alpha s+\alpha^2+\omega^2}$$

Zero di $F(s) \rightarrow s+2=0 \rightarrow s=-2$

Poli di $F(s) \rightarrow s_{1,2} = \dfrac{-2 \pm \sqrt{4-4 \cdot 1 \cdot 2}}{2} = -1 \pm j$

Riscrivo $F(s) = \dfrac{s+8}{(s+\underset{\alpha}{1})^2 + \underbrace{2-1}_{\omega=1}} \Rightarrow \alpha = 1 \quad \omega^2 = 1 \Rightarrow \omega = 1$

$s^2+2s+1+2-1 = s^2+2s+2$ ✓

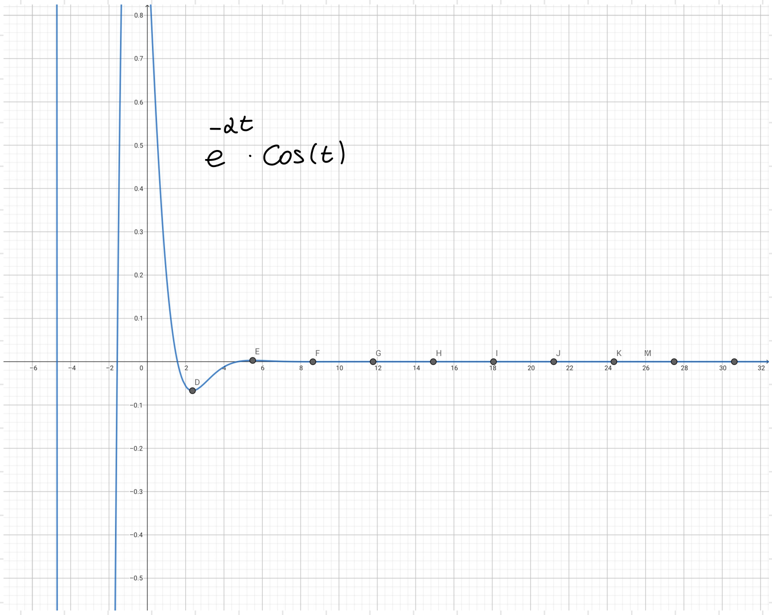
Siccome $\mathcal{L}\left[e^{-\alpha t}\cos(\omega t)\right] = \dfrac{s+\alpha}{(s+\alpha)^2+\omega^2} = \dfrac{s+\alpha}{s^2+2s\alpha+\alpha^2+\omega^2}$

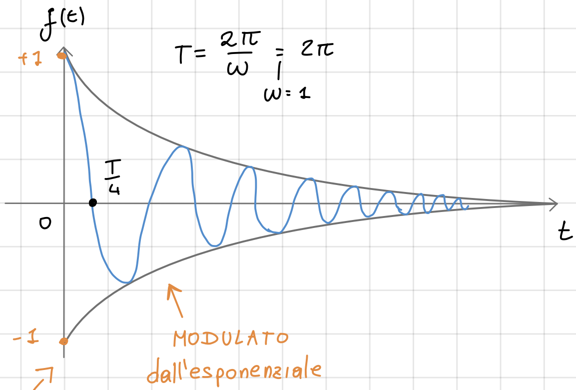
$\alpha = 1 \quad \omega = 1$

$$F(s) = \underbrace{\frac{s+1}{(s+1)^2+1}}_{e^{-\alpha t}\cos(\omega t)} + \underbrace{\frac{7}{(s+1)^2+1}}_{\sin(\omega t)}$$

$$f(t) = \left[e^{-t}\cdot\cos(t) + 7\,e^{-t}\sin(t)\right]\mathbb{1}(t) \leftarrow \text{Ans}$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s) = \lim_{s\to 0} \frac{s^2+8s}{s^2+2s+2} \rightarrow \emptyset$$





$$f(0^+) = \lim_{s\to\infty} s\,F(s) = 1$$

$$\frac{a_1 s + a_2}{s^2 + bs + c}$$

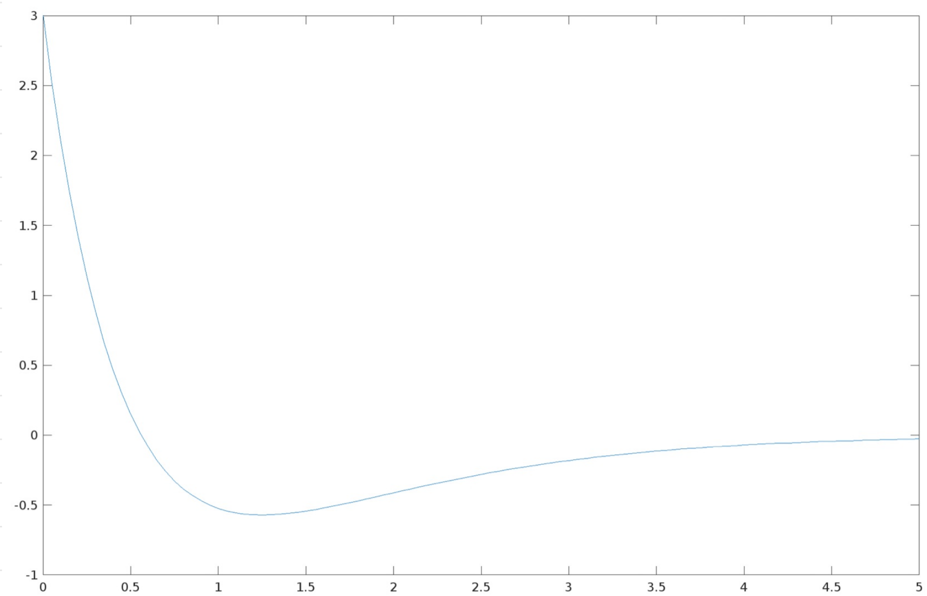
Reali $\to$ Scomposizione

imm $\to$ $$\frac{a_1 s + a_2}{\underbrace{\left(s+\frac{b}{2}\right)^2}_{\alpha} + \underbrace{c - \left(\frac{b}{2}\right)^2}_{w^2}} = a_1 \underbrace{\frac{s + \frac{b}{2}}{\left(s+\frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2}}_{\cos} + a_1 \underbrace{\frac{\frac{a_2}{a_1} - \frac{b}{2}}{//}}_{\sin}$$

MATLAB Drive > Sistemi Dinamici > SD_Lezione5.m

```matlab
% andiamo a definire dei polinomi

B = [3 -1] % polinomio al nominatore

A = [1 3 2] % polinomio al denominatore

length(A) % stampa la lunghezza del vettore

[r, p, k] = residue(B, A)     % ha come output [r, p, k]

p' %serve per fare il trasposto

% facciamo un disegno dell'anti trasformata
% (1) Definire un vettore tempo
t = 0:0.05:5 % il passo di campionamento è un decimo della costante di tempo del segnale più veloce (che ha una costante di 1/2)
% (2) definiamo la funzione
f = 7 * exp(-2 * t) - 4* exp(-t) % essendo t un vettore ci viene restituito un altro vettore fatto da tanti elementi quanti sono quelli di t

% (3) disegno la funzione
plot(t, f)
```

```
  -0.5426   -0.5559   -0.5647   -0.5698   -0.5714   -0.5702   -0.5665

Columns 29 through 35

  -0.5607   -0.5531   -0.5440   -0.5336   -0.5223   -0.5100   -0.4971

Columns 36 through 42

  -0.4837   -0.4699   -0.4559   -0.4417   -0.4274   -0.4131   -0.3989

Columns 43 through 49

  -0.3849   -0.3710   -0.3573   -0.3438   -0.3307   -0.3178   -0.3053

Columns 50 through 56

  -0.2930   -0.2812   -0.2696   -0.2585   -0.2477   -0.2372   -0.2271

Columns 57 through 63

  -0.2174   -0.2080   -0.1989   -0.1902   -0.1818   -0.1737   -0.1660

Columns 64 through 70

  -0.1586   -0.1514   -0.1446   -0.1380   -0.1317   -0.1257   -0.1199

Columns 71 through 77

  -0.1144   -0.1091   -0.1041   -0.0992   -0.0946   -0.0902   -0.0860

Columns 78 through 84

  -0.0819   -0.0781   -0.0744   -0.0709   -0.0676   -0.0644   -0.0613

Columns 85 through 91

  -0.0584   -0.0556   -0.0530   -0.0505   -0.0481   -0.0458   -0.0436

Columns 92 through 98

  -0.0415   -0.0395   -0.0376   -0.0358   -0.0341   -0.0324   -0.0309

Columns 99 through 101

  -0.0294   -0.0280   -0.0266

>> clean
```



Output