Risposta di un sistema dinamico del primo ordine ad un segnale gradino unitario

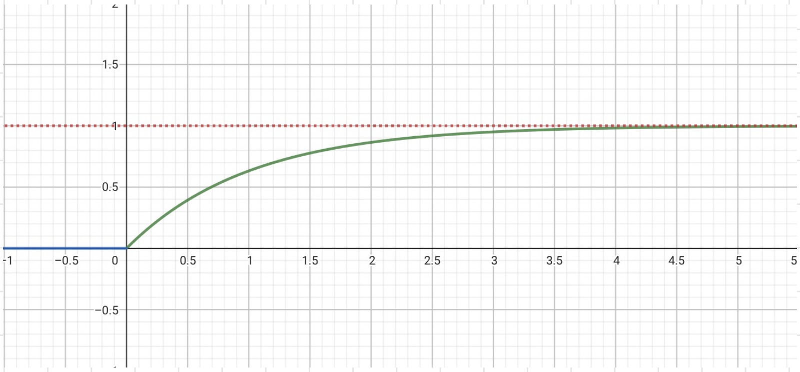
$$G(s) = \frac{1}{sT+1} \quad , \quad u(t) = \mathbb{1}(t) \rightleftarrows U(s) = \frac{1}{s}$$

$$\Rightarrow Y(s) = \frac{1}{sT+1}\cdot\frac{1}{s} = \frac{1}{s(sT+1)} = \frac{1}{T(s+\frac{1}{T})s}\cdot\frac{T}{T} = \frac{1/T}{s(s+1/T)} = \frac{r_1}{s} + \frac{r_2}{s+1/T}$$

$$\begin{cases} p_1 = 0 \\ p_2 = -\frac{1}{T} \end{cases} \Rightarrow \lim_{s\to 0} \cancel{s}\cdot\frac{1/T}{\cancel{s}(s+\frac{1}{T})} \to 1 \; r_1 \quad , \quad \lim_{s\to -\frac{1}{T}} \cancel{(s+\frac{1}{T})}\,\frac{1/T}{s\cancel{(s+1/T)}} \to -1 \; r_2$$

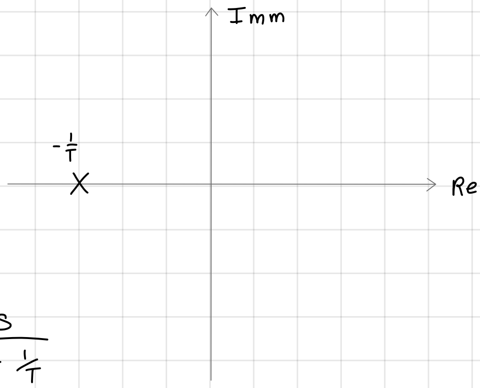
$\to$ Riscrivo $\quad \frac{1/T}{s(s+1/T)} = \boxed{\frac{1}{s} - \frac{1}{s+1/T}}$ $\;Y(s)$

trovo $y(t)$ come $y(t) = \mathcal{L}[Y(s)] = \boxed{\mathbb{1}(t) - e^{-\frac{1}{T}t}\cdot\mathbb{1}(t)} \equiv \left(1 - e^{-\frac{1}{T}t}\right)\cdot\mathbb{1}(t) \equiv 1 - e^{-\frac{1}{T}t}, \; t \geq 0$



IL SYS E' STABILE

Siccome $G(s) = \frac{1}{sT+1}$ ha $p = -\frac{1}{T}$



COME DISEGNARE LA RISPOSTA ?

Valore iniziale : $\lim_{t\to 0} y(t) = \lim_{s\to\infty} s\cdot Y(s) = \lim_{s\to\infty} s\,\frac{1}{s} - \frac{s}{s+\frac{1}{T}}$

$$= \lim_{s\to\infty} 1 - \frac{\cancel{s}^{\,1}}{\cancel{s}(1+\frac{1}{sT}_{\to 0})} = 0 \quad \text{in } t=0$$

Valore finale : REGIME $\lim_{s\to 0} \frac{\cancel{s}}{\cancel{s}}^{\,1} - \frac{s}{s+1/T}^{\,0} \to 1 \quad t\to\infty$

Pendenza della retta in 0 :

(1) Trovo la derivata con le proprietà :

$$\mathcal{L}\left[\frac{d}{dt}y(t)\right] = s\cdot Y(s) = 1 - \frac{s}{s+1/T}$$