RISPOSTA DEI SISTEMI DEL II ORDINE

Supponiamo di avere una generica funzione di trasferimento del secondo ordine; questa funzione G avrà due parametri importanti:

- *zetha* (ζ) ovvero il **rapporto di smorzamento**, può assumere diversi valori, a seconda dei quali cambia la risposta del sistema; andremo quindi a distinguere ogni caso.

- *Omega-n* (**ωn**) ovvero la **frequenza naturale**; questo valore misura la velocità con cui il sistema oscilla senza considerare l'effetto dello smorzamento; potremmo definirla come la velocità intrinseca del sistema.

1) La funzione di trasferimento

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Se $u(t) = \delta \rightleftarrows U(s) = 1 \Rightarrow Y(s) = G(s)$

Se $u(t) = 1\!\!1(t) \rightleftarrows U(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s} G(s)$

2) Trovo i poli

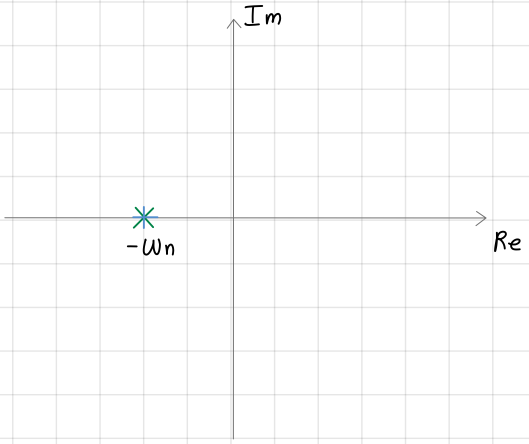
$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ per $P_{1,2} = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2} = -\zeta\omega_n \pm \sqrt{\omega_n^2(\zeta^2 - 1)}$

$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

3) A seconda del valore di ζ abbiamo diverse possibilità:

- **$\zeta = 1$ CRITICAMENTE SMORZATO**

Se $\zeta = 1 \to G(s) = \frac{\omega_n^2}{(s^2 + 2\underset{1}{\zeta}\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{(s+\omega_n)^2} \Rightarrow P_{1,2} = -\omega_n$



Infatti $P_{1,2} = -\underset{1}{\zeta}\omega_n \pm \omega_n\underbrace{\sqrt{\cancel{\zeta^2 - 1}}}_{0} = -\omega_n$

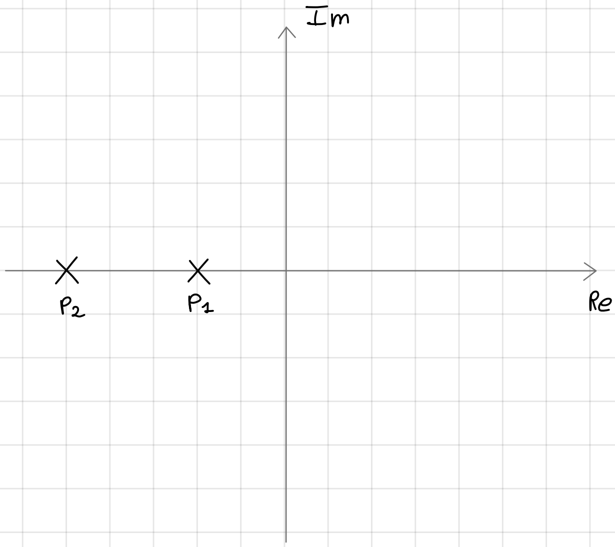
- **$\zeta > 1$ SOVRASMORZATO**

Se $\zeta > 1 \to P_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

"Un valore al quadrato meno qualcosa sotto radice sarà sempre munore del valore stesso"

ovvero $\sqrt{\zeta^2 - 1} < \zeta, \ \forall \zeta$

ES: $\zeta = 2 \Rightarrow \sqrt{\zeta^2 - 1} = 1.73 \to 1{,}73 < 2$



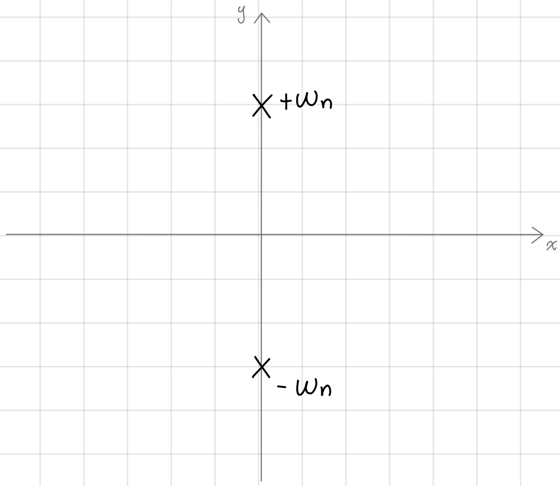
e quindi avremo due poli distinti, reali e negativi.

- $\zeta = 0$ NON SMORZATO

$$\zeta = 0 \rightarrow Y(s) = \frac{\omega_n^2}{(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{\omega_n^2}{s^2+\omega_n^2} = \omega_n \cdot \frac{\omega_n}{s^2+\omega_n^2}$$

Sin $(\omega_n t)$

$P_{1,2} = \pm j\omega_n$



- CASO GENERALE: $0 < \zeta < 1$

Se $0<\zeta<1 \rightarrow P_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$ ma se $0<\zeta<1$ $\zeta^2-1<0 \ \forall \zeta$

$\Rightarrow$ Complx : metto in evidenza $\sqrt{-1} \rightarrow -\zeta\omega_n \pm j\omega_n\sqrt{-(-\zeta^2+1)} \rightarrow P_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

Pongo $\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}} \Rightarrow \boxed{P_{1,2} = -\zeta\omega_n \pm j\omega_d}$

FREQUENZA SMORZATA

$\rightarrow$ Procedo con il calcolo dei residui...

Posso scrivere $Y(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{r_1}{s} + \frac{r_2 s + r_3}{s^2+2\zeta\omega_n s+\omega_n^2}$

Trovo i residui

$$r_1 = \lim_{s\to 0} s \cdot \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{\cancel{\omega_n^2}}{\cancel{s^2}+2\cancel{\zeta\omega_n s}+\cancel{\omega_n^2}} \rightarrow 1 \ r_1$$

$$\frac{r_1 s^2+2r_1\zeta\omega_n s+\omega_n^2 r_1+r_3 s+r_2 s^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{s^2(r_1+r_2)+s(2r_1\zeta\omega_n+r_3)+\omega_n^2 r_1}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$$\begin{cases} r_1+r_2 = 0 \rightarrow r_2 = -r_1 \Rightarrow r_2 = -1 \\ 2r_1\zeta\omega_n + r_3 = 0 \rightarrow 2\zeta\omega_n + r_3 = 0 \Rightarrow r_3 = -2\zeta\omega_n \\ \omega_n^2 r_1 = \omega_n^2 \Rightarrow r_1 = 1 \end{cases}$$

TROVATO PRIMA

$Y(s)$

$$Y(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{r_1}{s}+\frac{r_2 s+r_3}{s^2+2\zeta\omega_n s+\omega_n^2} = \boxed{\frac{1}{s} - \frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

Torno al dominio del Tempo

$$Y(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{1}{s} - \frac{s+2\zeta\omega_n}{\underbrace{(s+\zeta\omega_n)^2}_{s^2+2\zeta\omega_n s+(\zeta\omega_n)^2} + \omega_n^2-(\zeta\omega_n)^2} = \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} \;!$$

IN PIU'!

! Siccome $\omega_d = \omega_n\sqrt{1-\zeta^2} \Rightarrow \omega_d^2 = \omega_n^2(1-\zeta^2) \Longrightarrow$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} \quad (1)$$

→ ora l'obbiettivo è rimaneggiare per far uscire un sin e cosin

$$= \underbrace{\frac{1}{s}}_{1\!\!1(t)} - \underbrace{\frac{s}{(s+\zeta\omega_n)^2+\omega_d^2}}_{e^{-\zeta\omega_n}\cdot\cos(\omega_d t)} + \underbrace{\frac{2\zeta\omega_n}{\omega_d}}_{K}\underbrace{\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}}_{e^{-\zeta\omega_n t}\cdot\sin(\omega_d\cdot t)}$$

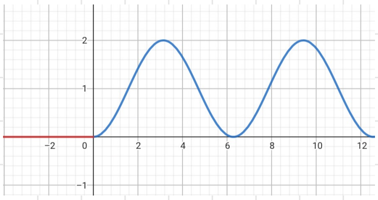
Risposta di un qualsiasi sistema di secondo ordine

$$\Rightarrow Y(s) \rightleftharpoons \boxed{y(t) = \left[1 - e^{-\zeta\omega_n}\cdot\cos(\omega_d\cdot t) + \frac{2\zeta\omega_n}{\omega_d}\cdot e^{-\zeta\omega_n t}\sin(\omega_d\cdot t)\right]\cdot 1\!\!1(t)}$$

$$y(t) = \left[1 - e^{-a\cdot b\cdot t}\cdot\cos(c\cdot t) + \frac{2a\cdot b}{c}\cdot e^{-a\cdot b\cdot t}\sin(c\cdot t)\right]\cdot 1\!\!1(t)$$

Come modificano la risposta i vari parametri?

1. $\zeta = 0$

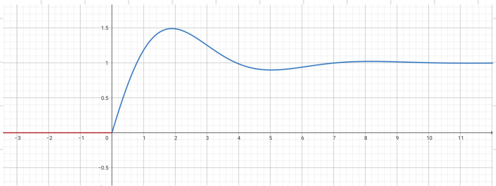


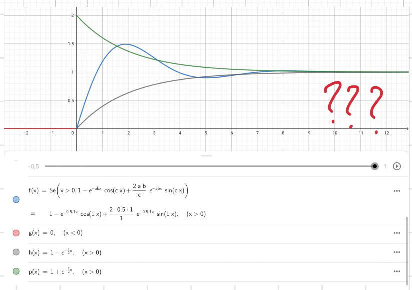
Se $\zeta = 0 \rightarrow y(t) = 1 - \cos(\omega_d\cdot t)$ NON SMORZATO

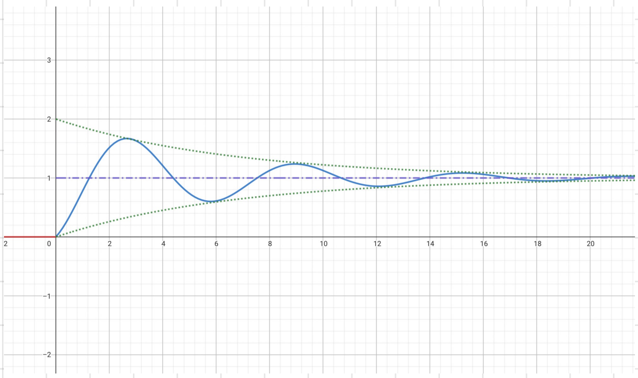
2. $0 < \zeta < 1$

Con $\omega_d = \omega_n = 1$

Se $0<\zeta<1$ $(H_p\ \zeta = \tfrac{1}{2}) \rightarrow y(t) = 1 - e^{-\frac{1}{2}t}\cos(t) + 1\cdot e^{-\frac{1}{2}t}\sin(t)$







3. $\zeta = 1$

(Come visto prima)

Se $\zeta = 1$ $\to$ Se $\zeta = 1$ $\to$ $G(s) = \dfrac{\omega_n^2}{(s^2 + 2\underbrace{\zeta}_{1}\omega_n s + \omega_n^2)} = \dfrac{\omega_n^2}{(s+\omega_n)^2} \Rightarrow P_{1,2} = -\omega_n$

$$\Rightarrow Y(s) = G(s)\cdot \underset{U(t)=\mathbb{1}(t)}{\underset{\uparrow}{U(s)}} = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{r_1}{s} + \frac{r_2}{s+\omega_n} + \frac{r_3}{(s+\omega_n)^2}$$

$\hookrightarrow P_3 = 0$

- $r_1 = \lim_{s\to 0} \cancel{s}\cdot \dfrac{\omega_n^2}{\cancel{s}(s+\omega_n)^2} \to 1$ $\quad r_1$
- $r_3 = \lim_{s\to -\omega_n} (s+\omega_n)^2 \cdot Y(s) \to -\omega_n$ $\quad r_3$

$$\Rightarrow Y(s) = \frac{r_1}{s} + \frac{r_2}{s+\omega_n} + \frac{r_3}{(s+\omega_n)^2} = \frac{\overset{1}{r_1}(s+\omega_n)^2 + r_2 s(s+\omega_n) + s\overset{-\omega_n}{r_3}}{s(s+\omega_n)^2} = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$\Rightarrow Y(s) = \underline{(s+\omega_n)^2 + r_2(s^2 + s\omega_n) - s\omega_n} \;\; // \;\; \underline{s^2(1+r_2) + s(2\omega_n + r_2\omega_n - \omega_n) + \omega_n^2}$$

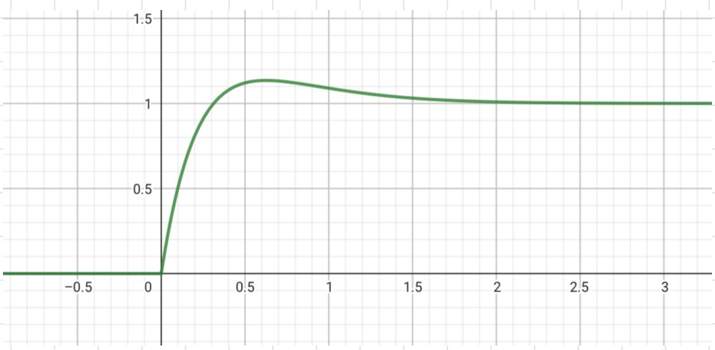
$$\Rightarrow \begin{cases} 1 + r_2 = 0 & \Rightarrow r_2 = -1 \\ 2\omega_n + r_2\omega_n - \omega_n = 0 & \to r_2\omega_n = \omega_n - 2\omega_n \to r_2 = -1 \quad \text{QED} \end{cases}$$

$$Y(s) = \frac{1}{s} - \frac{1}{s+\omega_n} + \frac{\omega_n}{(s+\omega_n)^2}$$

Siccome $\mathcal{L}[e^{-\alpha t}\cdot f(t)] = F(s+\alpha)$ e $\mathcal{L}[t] = \frac{1}{s^2}$ [Lezione 3]

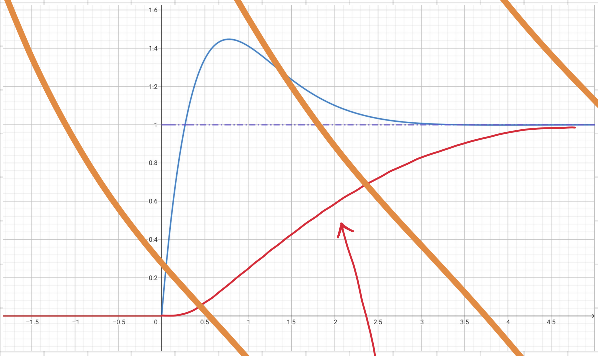
$$\Rightarrow \mathcal{L}\left[\frac{1}{(s+\alpha)^2}\right] = e^{-\alpha t}\cdot t \quad \Rightarrow \mathcal{L}^{-1}\left[\omega_n\cdot \frac{1}{(s+\omega_n)^2}\right] = \omega_n e^{-\omega_n t}\cdot t \;, t \geq 0$$

$$\Rightarrow \boxed{y(t) = 1 - e^{-\omega_n t} + \omega_n t e^{-\omega_n t}, \quad t \geq 0}$$



4. $\zeta > 1$

~~Con $\omega_d = \omega_n = 1$, $\zeta = 2$~~



~~Se $\zeta = 2 \Rightarrow y(t) = 1 - e^{-2t}\cos(t) + e^{-2t}\sin(t)$~~

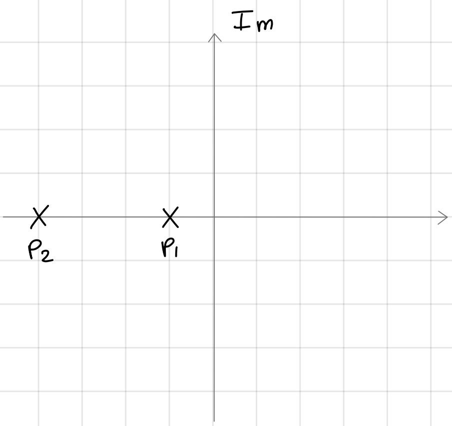
~~Dalla lezione del prof era questo l'andamento (?)~~

mai complx $\Rightarrow$ REALI E DISTINTI

$$\text{Se } \zeta > 1 \Rightarrow p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \Rightarrow \begin{cases} p_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2-1} \\ p_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2-1} \end{cases}$$

Sicuramente > 1

$$\Rightarrow \boxed{p_1 > p_2}$$



$\Rightarrow$ Due exp, con τ diversa!

e_1 è più veloce di p_2

Se i poli sono REALI E DISTINTI allora possiamo scrivere:

! I poli sono negativi !

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s + p_1)(s + p_2)$$

$$\Rightarrow G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+p_1)(s+p_2)}$$

$$\Rightarrow Y(s) = \frac{\omega_n^2}{s(s+p_1)(s+p_2)} = \frac{\varepsilon_1}{s} + \frac{\varepsilon_2}{s+p_1} + \frac{\varepsilon_3}{s+p_2}$$

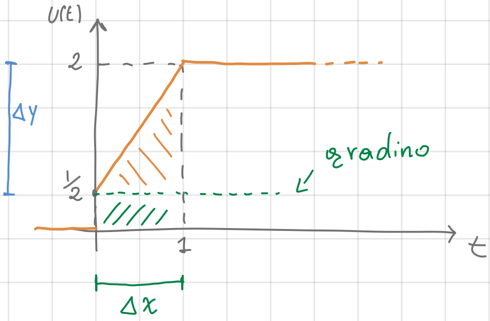
$$\Rightarrow \boxed{y(t) = 1 + \varepsilon_2 e^{p_1 t} + \varepsilon_2 e^{p_2 t}, \quad t \geq 0}$$

APPUNTI PRESI A LEZIONE

ESERCIZI

$$G(S) = \frac{2S+3}{S+5}$$

$G(S)$ ha: zeri: $\bar{z} = -\frac{3}{2}$; poli: $\bar{p} = -5$



Posso scrivere

$$u(t) = \underbrace{\frac{1}{2}\mathbb{1}(t)}_{\text{gradino}}^{U_1} + \underbrace{\overbrace{\frac{3}{2}}^{\Delta Y/\Delta x} t\cdot\mathbb{1}(t)}_{\text{rampa}}^{U_2} \underbrace{- \frac{3}{2}(t-1)\cdot\mathbb{1}(t-1)}_{\text{Rampa}}^{U_3}$$

Applico la linearità e la Tempo invarianza:

$$y(t) = y_1(t) + y_2(t) + y_3(t)$$

Considero $U_2(t)$

$$U_2(s) = \frac{3}{2s^2} \quad \rightarrow \quad Y_2(s) = \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}S+3}{s^2(S+5)} = 3 \cdot \frac{S+\frac{3}{2}}{s^2(S+5)} = \frac{r_1}{s} + \frac{r_2}{s^2} + \frac{r_3}{S+5}$$

$$r_2 = \lim_{s\to 0} s^2\, Y_2(s) = \lim_{s\to 0} \cancel{s^2} \cdot 3 \frac{s+3/2}{\cancel{s^2}(s+5)} \rightarrow \frac{9}{10} \quad r_2$$

$$r_1 = \lim_{s\to -5} \cancel{(s+5)} \cdot 3 \frac{s+3/2}{s^2\,\cancel{(s+5)}} \rightarrow -\frac{21}{50} \quad r_1$$

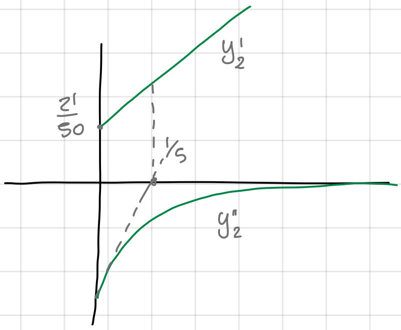
Trovo r_1:

$$3\,\frac{s+\frac{3}{2}}{s^2(s+5)} = \frac{s\,r_1(s+5)+r_2(s+5)+r_3 s^2}{s^2(s+5)} = \frac{(r_1+r_3)s^2 + \overbrace{(5r_1+r_2)}^{1}s + \overbrace{5r_2}^{\text{deve essere } 9/2}}{s^2(s+5)}$$

$$\Rightarrow r_3 = -r_1 = \frac{21}{50} \quad r_3$$

$$\Rightarrow Y_2(s) = \frac{21}{50}\,\frac{1}{s} + \frac{9}{10}\cdot\frac{1}{s^2} - \frac{21}{50}\,\frac{1}{(s+5)} \Rightarrow y_2(t) = \Big(\underbrace{\frac{21}{50} + \frac{9}{10}t}_{y_2'} \underbrace{- \frac{21}{50}e^{-5t}}_{y_2''}\Big)\cdot\mathbb{1}(t)$$

$y_2(t)$



Trovo $y_1(t)$

$$U_1(t) = \frac{1}{2}\mathbb{1}(t) = \frac{1}{3}U_2(t) = \frac{1}{3}\left(\frac{3}{2}\mathbb{1}(t)\right) = \frac{1}{3}\frac{dU_2}{dt} \Rightarrow y_1(t) = \frac{1}{3}\frac{d}{dt}y_2(t)$$

$$\Rightarrow y_1(t) = \frac{1}{3}\left(\frac{9}{10} + \cancel{5}\cdot\frac{21}{\cancel{50}_{10}}e^{-5t}\right) = \frac{3}{10} + \frac{21}{30}e^{-5t}, \quad t \geq 0$$

Trovo $y_3(t)$

$U_3(t) = -U_2(t-1) \quad = 0 \qquad y_3(t) = -y_2(t-1) = \boxed{-\frac{21}{50} - \frac{9}{10}(t-1) + \frac{21}{50} e^{-5(t-1)} \quad , t \geq 1}$ $\mathbb{1}(t-1)$ $\equiv$

Valore di regime = ?

$$Y(s) = G(s) \cdot U(s) = \frac{2s+3}{s+5} \cdot \left(\frac{1}{2s} + \frac{3}{2s^2} - \frac{3}{2s^2} e^{-s} \right)$$

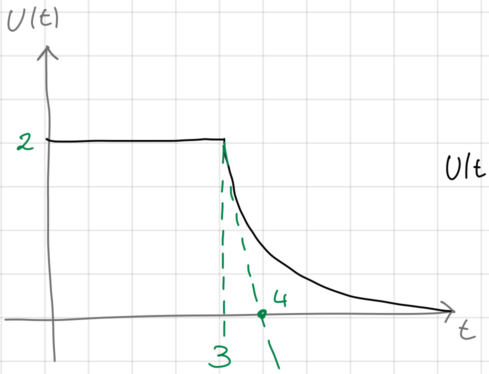
$$y_{ss}(t) = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} \frac{2s+3}{s+5} \left(\frac{1}{2} + \frac{3}{2s} - \frac{1-e^{-s}}{s} \right)$$

Taylor

$$\rightsquigarrow \lim_{s \to 0} \frac{3}{5} \left(\frac{1}{2} + \frac{3}{2} \frac{1-(1-s)}{s} \right) = \frac{6}{5}$$

Valore di regime

Esempio segnale composto



$\tau = 1$

$$U(t) = 2 \cdot \mathbb{1}(t) - 2 \cdot \mathbb{1}(t-3) + 2 \cdot e^{-(t-3)\cdot\tau} \cdot \mathbb{1}(t-3)$$

RISPOSTA TRANSITORIA PER SYS del 2^{nd} Ordine

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ω_n^2 ← Pulsazione naturale

ξ ↑ Coefficiente di smorzamento

$\rightsquigarrow$ $\bar{z}$:

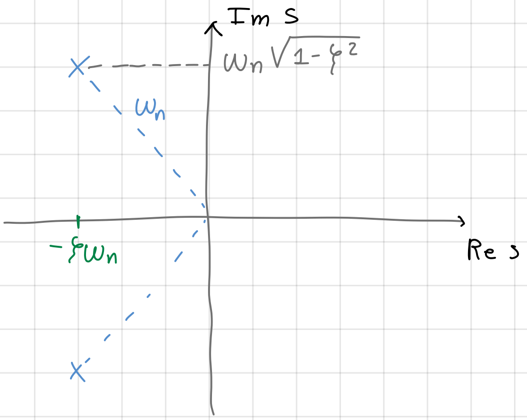
$\bar{p}$: $s^2 + 2\xi\omega_n s + \omega_n^2 \to s = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$

$\to \bar{s} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

- Se $\xi = 1$ $\rightsquigarrow$ CRITICAMENTE SMORZATO
- Se $\xi > 1$ $\rightsquigarrow$ SOVRASMORZATO PARTE REALE NEGATIVA ma 2 sol.
- Se $\xi < 1$ $\rightsquigarrow$ SOTTOSMORZATO Poli complessi e coniugati

- $\xi = 0$ NON SMORZATO

$\to -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$



ω_n è il modulo dei poli complessi e coniugati, ovvero se $\xi < 1$

* molla

Hp. $0 < \xi < 1$

$U(s) = \text{gradino} = \frac{1}{s}$

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

$$\Rightarrow Y(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{r_1}{s} + \frac{r_2 s + r_3}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$r_1 = \lim_{s \to 0} s \cdot Y(s) \to 1 \quad r_1$$

$$\frac{r_1(s^2 + 2\xi\omega_n s + \omega_n^2) + r_2 s^2 + r_3 s}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{(r_1 + r_2)s^2 + (2\xi\omega_n r_1 + r_3)s + r_1\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$r_1 + r_2 = 0 \Rightarrow r_2 = -r_1 \Rightarrow r_2 = -1 \quad r_2$$

$$2\xi\omega_n + r_3 = 0 \to r_3 = -2\xi\omega_n \quad r_3$$

$$\Rightarrow \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{1}{s} - \frac{1 + 2\xi\omega_n}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

→ Anti Trasformata

$$G(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = -\frac{}{(s+\xi\omega_n)^2+\omega_n^2-\underbrace{(\xi\omega_n)^2}} + \frac{1}{s} = -\frac{}{(s+\xi\omega_n)^2+\underbrace{\omega_n^2\cdot(1-\xi^2)}} + \frac{1}{s}$$

Tolgo il termine in più

$$= -\frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{1}{s}$$

$$\boxed{\begin{aligned}\omega_d &= \omega_n\sqrt{1-\xi^2}\\ \Rightarrow \omega_d^2 &= \omega_n^2(1-\xi^2)\end{aligned}}$$

ω_d

Pulsazione naturale smorzata

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

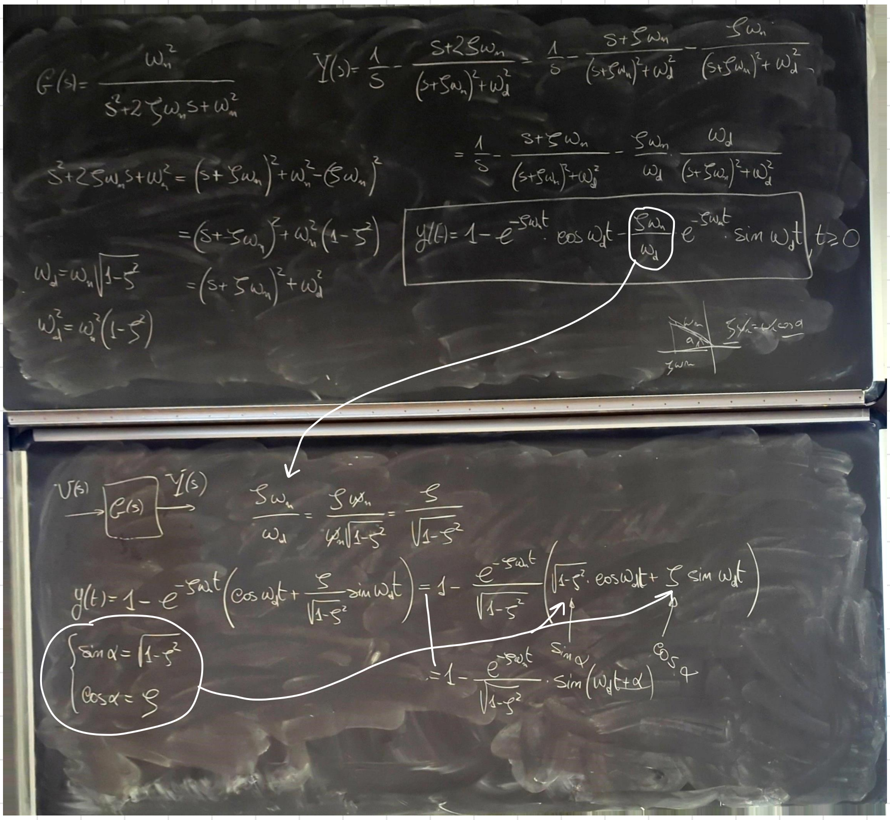
$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d} - \frac{\xi\omega_n}{\omega_d}\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\rightarrow y(t) = 1 - e^{-\xi\omega_n t}\cos(\omega_d t) - \frac{\xi\omega_n}{\omega_d}\cdot e^{-\xi\omega_n t}, \quad t \geq 0$$

Risposta di un qualsiasi sistema del 2nd ordine

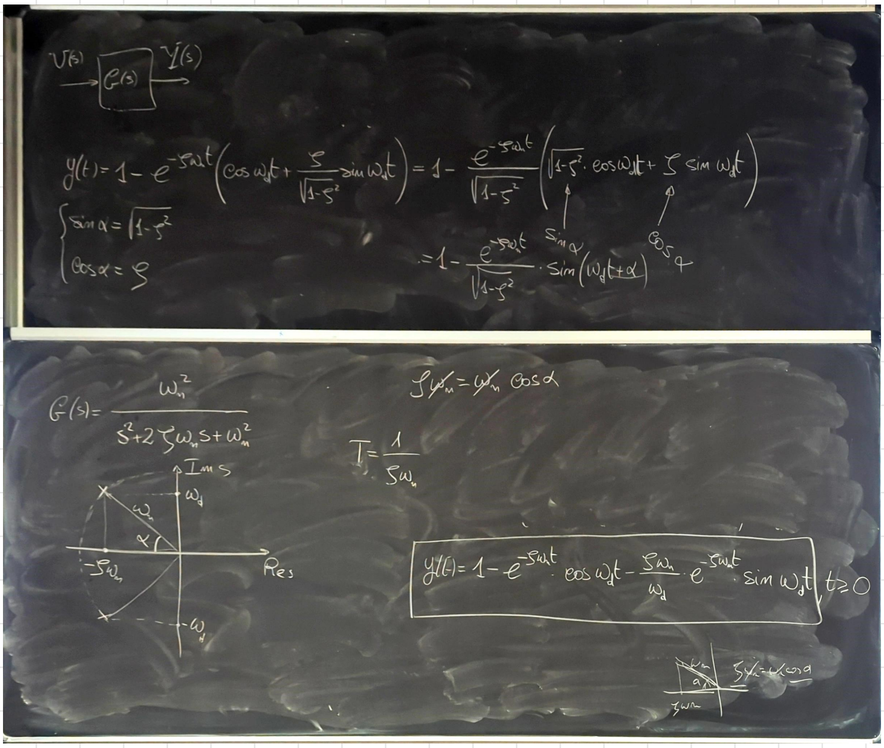
FOTO

$$\frac{\xi\omega_n}{\omega_d} = \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} = \frac{\xi}{\sqrt{1-\xi^2}} \quad \Rightarrow \quad y(t) = 1 - e$$



✱ Quanto più i poli si avvicinano a zero, tanto più la risposta tenderà ad oscillare; quando zetha è uguale a zero, il sistema non è smorzato e quindi i poli sono sull'asse immaginario, quindi il sistema può essere stabile ma non **asintoticamente**, ovvero abbiamo una sinusoide.

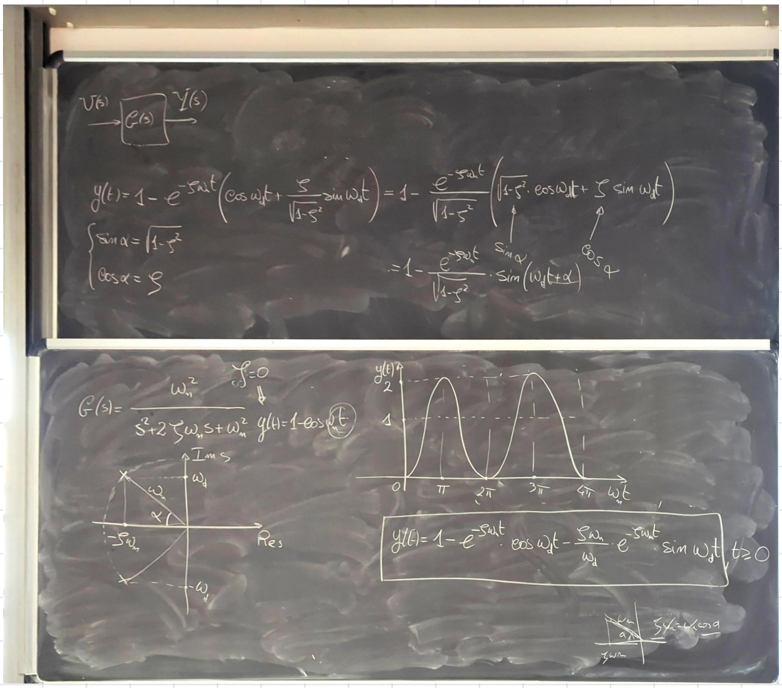
Se zetha è molto piccola, il sistema è si smorzato, ma davvero poco.
Quando zetha si avvicina ad 1, il transitorio si rimpicciolisce sempre di più fino a scomparire

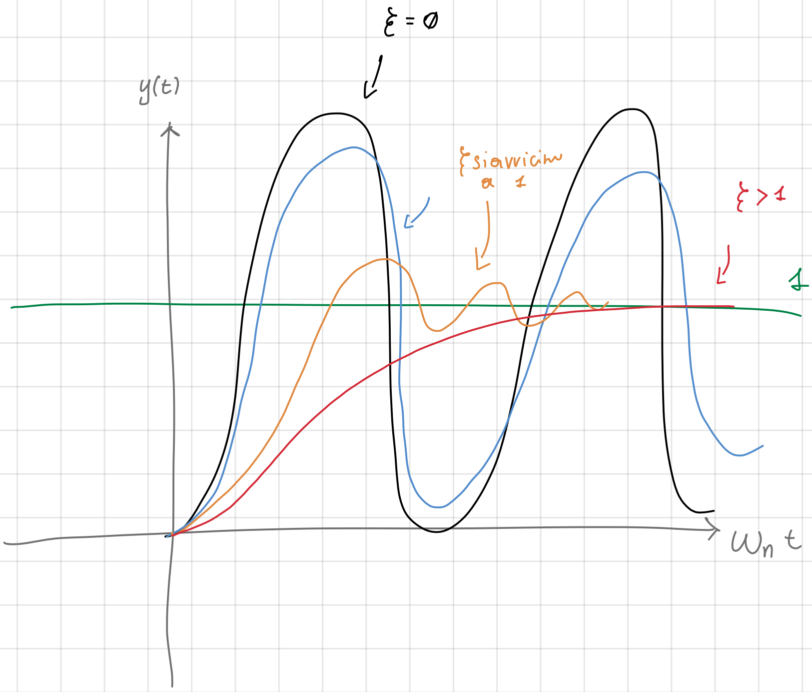


$$\omega_d = \omega_n\sqrt{1-\xi} < \omega_n$$

smorzata

Se $\xi = 0$





Risposta per $\xi = 1$ CRITICAMENTE SMORZATO

$$G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$Y(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{r_1}{s} + \frac{r_2}{s+\omega_n} + \frac{r_3}{(s+\omega_n)^2}$$

$$r_1 = \lim_{s \to 0} s\,Y(s) = 1$$

$$r_3 = \lim_{s \to -\omega_n} (s+\omega_n)^2\, Y(s) = -\omega_n$$

$$\frac{1}{s} + \frac{r_2}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2} = \frac{(s+\omega_n)^2 + r_2 s(s+\omega_n) - s\omega_n}{s(s+\omega_n)^2}$$

$$= \frac{(1+r_2)s^2 + (r_2\omega_n \overset{+2\omega_n}{} - \omega_n)s + \omega_n^2}{s(s+\omega_n)^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$Y(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{r_1}{s} + \frac{r_2}{s+\omega_n} + \frac{r_3}{(s+\omega_n)^2}$$

$$r_1 = \lim_{s \to 0} s\,Y(s) = 1$$

$$r_2 = -1$$

$$y(t) = 1 - e^{-\omega_n t} - \omega_n t\, e^{-\omega_n t}, \quad t \geq 0$$

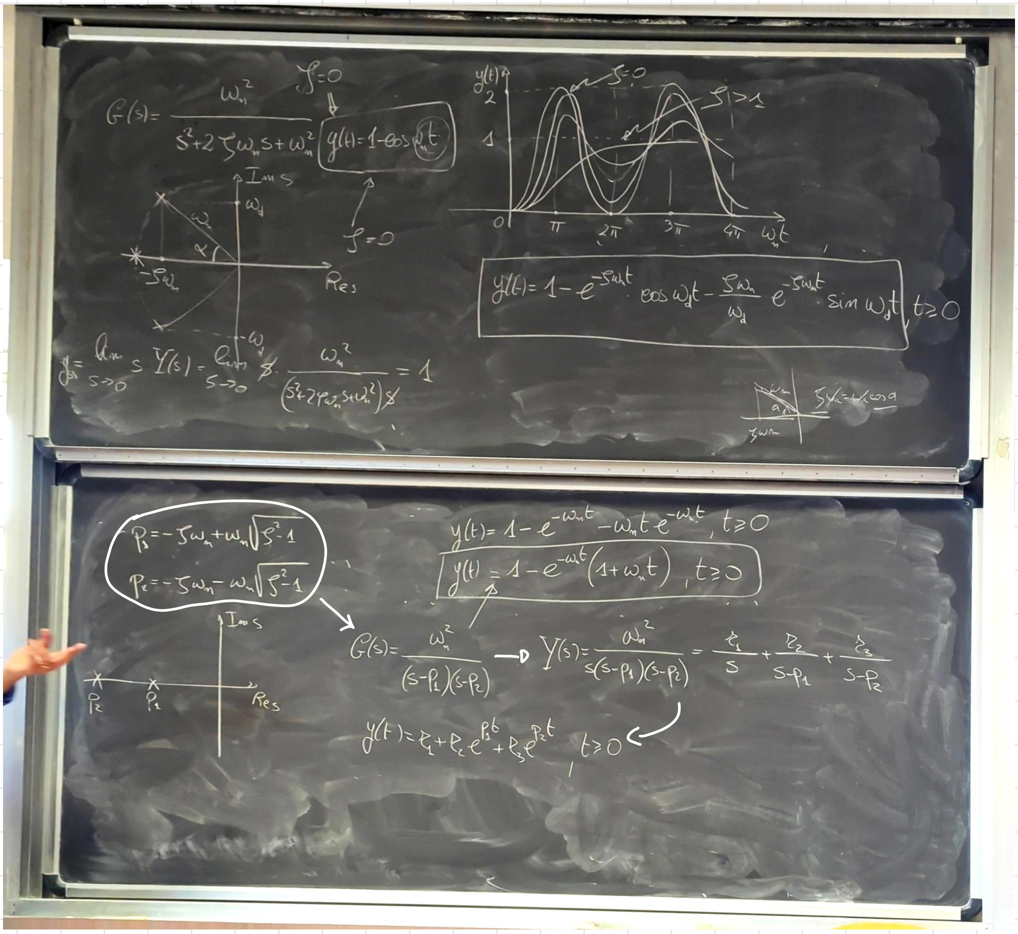
$$y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t), \quad t \geq 0$$

$$\xi = 1$$

$$\frac{1}{s} + \frac{r_2}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2} = \frac{(s+\omega_n)^2 + r_2 s(s+\omega_n) - s\omega_n}{s(s+\omega_n)^2}$$

$$= \frac{(1+r_2)s^2 + (r_2\omega_n \overset{+2\omega_n}{} - \omega_n)s + \omega_n^2}{s(s+\omega_n)^2}$$

Risposta per $\xi > 1$



$-\xi\omega_n + \omega_n\sqrt{\xi^2-1}$ ma $\sqrt{\xi^2-1} < \xi$