

POLI SEMPLICI

ES 2.3

$$F(s) = \frac{s+3}{(s+1)(s+2)} = \frac{z_1}{s+1} + \frac{z_2}{s+2}$$

$$\text{den} = \frac{s^2 + 2s + s + 2}{s^2 + 3s + 2}$$

$$\Rightarrow p_1 = -1 \quad p_2 = -2$$

$$z_1 = \lim_{s \rightarrow -1} \cancel{(s+1)} \cdot \frac{s+3}{\cancel{(s+1)}(s+2)} = \frac{-1+3}{-1+2} = \textcircled{2} z_1$$

$$z_2 = \lim_{s \rightarrow -2} \cancel{(s+2)} \cdot \frac{s+3}{(s+1)\cancel{(s+2)}} = \frac{-2+3}{-2+1} = \textcircled{-1} z_2$$

$$\leadsto F(s) = \frac{2}{s+1} - \frac{1}{s+2} = 2 \cdot \frac{1}{s+1} - \frac{1}{s+2} \Rightarrow f(t) = 2 \cdot e^{-t} - e^{-2t}, t \geq 0$$

Varianti:

(a) da funzione di partenza era: $F(s) = \frac{s+3}{s^2 + 3s + 2}$

1. Trovo i poli

$$s^2 + 3s + 2 = 0 \quad p_{1,2} = \frac{-3 \pm \sqrt{9-4 \cdot 2}}{2} \rightarrow \begin{cases} p_1 = \frac{-3+1}{2} = \textcircled{-1} p_1 \\ p_2 = \frac{-3-1}{2} = \textcircled{-2} p_2 \end{cases}$$

2. Riscrivo F(s)

$$F(s) = \frac{s+3}{(s-p_1)(s-p_2)} = \frac{s+3}{(s+1)(s+2)}$$

3. Procedo come prima...

(b) Metodo esplicito

$$\frac{s+3}{(s+1)(s+2)} \stackrel{\textcircled{1}}{=} \frac{z_1}{s+1} + \frac{z_2}{s+2} = \frac{z_1 s + 2z_1 + z_2 s + z_2}{(s+1)(s+2)} = \frac{\textcircled{2} (z_1 + z_2)s + 2z_1 + z_2}{(s+1)(s+2)}$$

$$\textcircled{1} = \textcircled{2} \Rightarrow$$

$$\begin{cases} z_1 + z_2 = 1 \\ 2z_1 + z_2 = 3 \end{cases} \Rightarrow \begin{cases} z_1 = 1 - z_2 \\ 2(1 - z_2) + z_2 = 3 \end{cases} \Rightarrow \begin{cases} z_1 = 1 - z_2 \\ 2 - 2z_2 + z_2 = 3 \end{cases} \Rightarrow \begin{cases} z_1 = 1 - z_2 \\ -z_2 = 1 \end{cases} \Rightarrow \begin{cases} z_1 = 2 \\ z_2 = -1 \end{cases}$$

$$z_1 = 1 - (-1) = \textcircled{2} z_1 \Rightarrow \frac{1}{-1} = \textcircled{-1} z_2$$

ES 2.4

$$G(s) = \frac{s^3 + 5s^2 + 9s + 7}{(s+1)(s+2)}$$

$\deg(N) > \deg(D) \quad \text{!!!}$

DIVISIONE TRA POLINOMI

$$A(x) = B(x) \cdot Q(x) + R(x)$$

↑ DIVIDENDO
 ↑ DIVISORE
 ↑ QUOZIENTE
 ↑ RESTO

$ \begin{array}{r} x^3 - 2x^2 + x - 3 \\ - (x^3 + 0x^2 - x + 0) \\ \hline -2x^2 - 3 \\ + 2x^2 + 2 \\ \hline 1 \end{array} $	<div style="text-align: center;"> $x^2 + 1$ DIVISORE </div> <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: center;"> $x - 2$ QUOZIENTE </div>
$\textcircled{1}$ $R(x)$	

→ Tornando all'esercizio

$ \begin{array}{r} s^3 + 5s^2 + 9s + 7 \\ - (s^3 + 3s^2 + 2s + 0) \\ \hline 2s^2 + 7s + 7 \\ - (2s^2 + 6s + 4) \\ \hline s + 3 \end{array} $	$ \begin{array}{r} s^2 + 3s + 2 \\ s + 2 \\ \hline s + 3 \end{array} $
$\textcircled{1}$ $R(x)$	

$$\frac{s^3 + 5s^2 + 9s + 7}{(s+1)(s+2)} = \frac{s+2}{(s+1)(s+2)} + \frac{s+3}{(s+1)(s+2)}$$

$Q(x)$
 $B(x)$

$$\Rightarrow \begin{cases} p_1 = -1 \\ p_2 = -2 \end{cases}$$

$$\Rightarrow G(s) = s + 2 + \frac{z_1}{s+1} + \frac{z_2}{s+2}$$

$$z_1 = \lim_{s \rightarrow -1} (s+1) \frac{s^3 + 5s^2 + 9s + 7}{(s+1)(s+2)} = \frac{-1 + 5 - 9 + 7}{-1 + 2} = \textcircled{2} z_1$$

$$z_2 = \lim_{s \rightarrow -2} (s+2) \dots = \frac{-8 + 20 - 18 + 7}{-1} = \textcircled{-1} z_2$$

$$\Rightarrow G(s) = \textcircled{s} + 2 + \frac{2}{s+1} - \frac{1}{s+2} \Rightarrow \text{Ans } g(t) = \frac{d\delta}{dt} + 2\delta + 2e^{-t} - e^{-2t}, \quad t \geq 0$$

$$\mathcal{L}[\delta] = 1 \Rightarrow \mathcal{L}\left[\frac{dy}{dt}\right] = s \cdot Y(s) \\
 \Rightarrow \mathcal{L}\left[\frac{d\delta}{dt}\right] = s \cdot 1 = s \Rightarrow \mathcal{L}^{-1}[s] = \frac{d\delta}{dt}$$

ES 2.5

$$F(s) = \frac{2s+12}{s^2+2s+5}$$

$$P_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 5}}{2} = \frac{-2 \pm 4j}{2} \begin{matrix} \nearrow -1 + 2j \\ \searrow -1 - 2j \end{matrix}$$

$$\Rightarrow s^2+2s+5 = (s-p_1)(s-p_2) = (s+1-2j)(s+1+2j)$$

$$\Rightarrow \frac{2s+12}{s^2+2s+5} = \frac{2s+12}{(s+1-2j)(s+1+2j)} = \frac{z_1}{(s+1-2j)} + \frac{z_2}{(s+1+2j)}$$

Ci conviene scrivere come una somma di cos e sin smorzati

$$\mathcal{L}[\cos(\omega t) \cdot 1(t)] = \frac{s}{s^2 + \omega^2}$$

$$\mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2 + \omega^2}$$

ma dalla proprietà: $\mathcal{L}[e^{-\alpha t} f(t)] = F(s+\alpha)$

$$\Rightarrow \begin{cases} \mathcal{L}[\cos(\omega t) \cdot e^{-\alpha t} \cdot 1(t)] = \frac{(s+\alpha)}{(s+\alpha)^2 + \omega^2} \\ \mathcal{L}[\sin(\omega t) \cdot e^{-\alpha t} \cdot 1(t)] = \frac{\omega}{(s+\alpha)^2 + \omega^2} \end{cases} \quad \begin{matrix} \nwarrow \text{mi riconduco a questa forma} \\ \swarrow \end{matrix}$$

$$\begin{aligned} \frac{2s+12}{s^2+2s+5} &= \frac{10+2+2s}{s^2+2s+5} = \frac{10+2(s+1)}{s^2+2s+5} = \frac{10+2(s+1)}{(s+1)^2 + \underbrace{5-1}_{2^2}} = \frac{10+2(s+1)}{(s+1)^2 + \underbrace{2^2}_{\omega^2}} \\ &= \frac{\underbrace{10}_{A \cdot (s+\alpha)} + \underbrace{2}_{\omega} \cdot \underbrace{5}_{\omega^2}}{(s+1)^2 + \underbrace{2^2}_{\omega^2}} + 2 \frac{s+1}{(s+1)^2 + 2^2} \\ &= 5 \cdot \frac{\underbrace{2}_{\omega}}{\underbrace{(s+1)^2 + 2^2}_{(s+\alpha)^2 + \omega^2}} + 2 \frac{s+1}{(s+1)^2 + \underbrace{2^2}_{\omega^2}} \end{aligned}$$

cerco (s+1)^2 al denom
e^{2t} sin(2t)
e^{2t} cos(2t)

$$\Rightarrow F(s) \Rightarrow f(t) = 5 e^{-t} \sin(2t) + 2 \cos(2t) \quad \text{Ans}$$

POLI MULTIPLI

chiamato $\frac{B(s)}{A(s)}$

$$F(s) = \frac{s^2 + 2s + 3}{(s+1)^3} = \frac{\xi_1}{s+1} + \frac{\xi_2}{(s+1)^2} + \frac{\xi_3}{(s+1)^3}$$

(1) Moltiplico per il denom:

$$\left(\frac{\xi_1}{s+1} + \frac{\xi_2}{(s+1)^2} + \frac{\xi_3}{(s+1)^3} \right) \cdot (s+1)^3 = \xi_1(s+1)^2 + \xi_2(s+1) + \xi_3 \quad [a(s)]$$

$$= \xi_1 s^2 + 2\xi_1 s + \xi_1 + \xi_2 s + \xi_2 + \xi_3$$

(2) Calcolo nel polo $(s+1)^3 = 0$ per $s = -1$

$$\rightarrow (s+1)^3 \cdot F(s) \Big|_{s=-1} = \xi_3$$

(3) DERIVO $a(s)$

$$\frac{d}{ds} \left[(s+1)^3 \cdot F(s) \right] = \frac{d}{ds} (a) = 2\xi_1 s + 2\xi_1 + \xi_2 = 2\xi_1(s+1) + \xi_2 \quad [b(s)]$$

(4) Calcolo nel polo $s = -1$

$$\rightarrow \frac{d}{ds} \left[a(s) \right] \Big|_{s=-1} = \left[b(s) \right] \Big|_{s=-1} = \xi_2$$

(5) Derivo $b(s)$

$$\frac{d}{ds} \left[b(s) \right] = 2\xi_1 \quad [c(s)]$$

$$(s+1)^3 \cdot \frac{B(s)}{A(s)} = \frac{[s^2 + 2s + 3]}{(s+1)^3} \cdot (s+1)^3$$

$$\xi_2 = \frac{d}{ds} [N(s)] \Big|_{s=p_1}$$

$$\xi_1 = \frac{d}{ds} \cdot \frac{1}{2} [N(s)] \Big|_{s=p_1}$$

$$\xi_n = \frac{d^n}{ds^n} \cdot \frac{1}{n!} [N(s)] \Big|_{s=p_n}$$

In realtà calcolo il 2° elemento...

POSSO CONCLUDERE CHE...

$$\bullet \xi_1 = \left[(s+1)^3 \cdot \frac{B(s)}{A(s)} \right] \Big|_{s=-1} = \left[s^2 + 2s + 3 \right] \Big|_{s=-1} = 1 - 2 + 3 = \textcircled{2} \xi_3$$

$$\bullet \xi_2 = \left\{ \frac{d}{ds} \left[(s+1)^3 \cdot \frac{B(s)}{A(s)} \right] \right\} \Big|_{s=-1} = \frac{d}{ds} [N(s)] \Big|_{s=-1} = -2 + 2 = \textcircled{0} \xi_2$$

$$\bullet \xi_2 = \frac{1}{2} \left\{ \frac{d^2}{ds^2} \left[(s+1)^3 \cdot \frac{B(s)}{A(s)} \right] \right\} \Big|_{s=-1} = \frac{d^2}{ds^2} \left[\frac{1}{2!} N(s) \right] = \frac{1}{2} (2) = \textcircled{1} \xi_1$$

$$\Rightarrow F(s) = \overset{a}{\frac{1}{s+1}} + \overset{b}{\frac{2}{(s+1)^3}} =$$

$$(b) \mathcal{L}[F(s)] = f(t) \Rightarrow \mathcal{L}[F(s+\lambda)] = e^{-\lambda t} \cdot f(t)$$

$$\text{in (b)} \quad \lambda = 1, \quad F(s) = \frac{1}{s^3} \Leftrightarrow \frac{t^2}{2} \Rightarrow \mathcal{L}^{-1}[(b)] = t^2 e^{-t}$$

$$(a) \mathcal{L}^{-1}[(a)] = e^{-t}$$

$$\Rightarrow \mathcal{L}^{-1}[F(s)] = e^{-t} (1+t^2), \quad t \geq 0$$

ES:

$$F(s) = \frac{10}{s+2} \Leftrightarrow 10 \cdot e^{-2t}, \quad t \geq 0$$

• VALORE INIZIALE

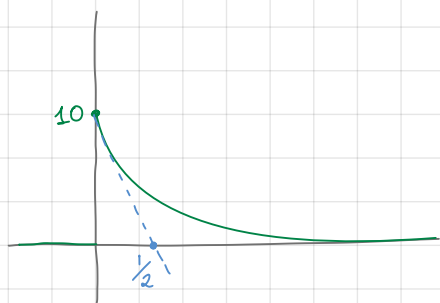
$$(a) \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} s \cdot F(s) = \lim_{s \rightarrow \infty} s \cdot \frac{10}{s+2} \rightarrow 10$$

$$(b) \lim_{t \rightarrow 0^+} f(t) = \lim_{t \rightarrow 0^+} 10 \cdot e^{-2t} \rightarrow 10$$

QED

• Valore finale

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \cdot F(s) = \lim_{s \rightarrow 0} s \cdot \frac{10}{s+2} = 0$$



ES:

$$F(s) = \frac{2s-1}{3s+2} = \frac{2}{3} \left(\frac{s - \frac{1}{2}}{s + \frac{2}{3}} \right)$$

$$= \frac{2}{3} \left(\frac{s + \frac{2}{3} - \frac{2}{3} - \frac{1}{2}}{s + \frac{2}{3}} \right)$$

$$= \frac{2}{3} \left(\frac{\cancel{s + \frac{2}{3}} - \frac{2}{3} + \frac{1}{2}}{s + \frac{2}{3}} \right)$$

$$= \frac{2}{3} \left(1 - \frac{\frac{1}{6}}{s + \frac{2}{3}} \right) = \frac{2}{3} \left(\underset{\downarrow}{1} - \frac{1}{6} \cdot \underset{\text{e}^{-\frac{2}{3}t}}{\left(\frac{1}{s + \frac{2}{3}} \right)} \right) \Rightarrow f(t) = \frac{2}{3} \delta(t) - \frac{1}{9} e^{-\frac{2}{3}t}$$

$$\mathcal{L}[\sin(\omega t) \mathbb{1}(t)] = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}[\cos(\omega t) \mathbb{1}(t)] = \frac{s}{s^2 + \omega^2}$$

ES:

$$F(s) = \frac{2s}{s^2 + 3s + 2}$$

$$= \frac{z_1}{(s+1)} + \frac{z_2}{(s+2)}$$

$$P_{1,2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2}}{2} \begin{cases} \rightarrow \frac{-3+1}{2} = \underset{\text{P}_1}{-1} \\ \rightarrow \frac{-3-1}{2} = \underset{\text{P}_2}{-2} \end{cases}$$

$$\begin{cases} z_1 = \lim_{s \rightarrow -1} \frac{2s}{(s+2)} = \frac{-2}{1} = -2 \\ z_2 = \lim_{s \rightarrow -2} \frac{2s}{(s+1)} = \frac{-4}{-1} = 4 \end{cases}$$

$$\Rightarrow F(s) = -\frac{2}{s+1} + \frac{4}{s+2}$$

$$\Rightarrow \underline{f(t) = -2 e^{-t} + 4 e^{-2t}, t \geq 0}$$

ES:

$$F(s) = \frac{s+8}{s^2 + 2s + 2}$$

$$P_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 2}}{2} = \frac{-2 \pm 2j}{2} \begin{cases} \rightarrow -1 + 2j \\ \rightarrow -1 - 2j \end{cases}$$

$$= \frac{s+8}{(s+1)^2 + 2 - 1} = \frac{s+1+7}{(s+1)^2 + 1}$$

$$= \frac{s+1}{(s+1)^2 + 1} + \frac{7}{(s+1)^2 + 1}$$

$$\mathcal{L}[\sin(\omega t) \mathbb{1}] = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}[\cos(\omega t) \mathbb{1}] = \frac{s}{s^2 + \omega^2}$$

$$\mathcal{L}^{-1}[F(s+d)] = e^{dt} \cdot f(t)$$

$$\Rightarrow F(s) \Leftrightarrow f(t) = e^{-t} \cdot \cos(t) + 7 \cdot e^{-t} \sin(t), t \geq 0$$

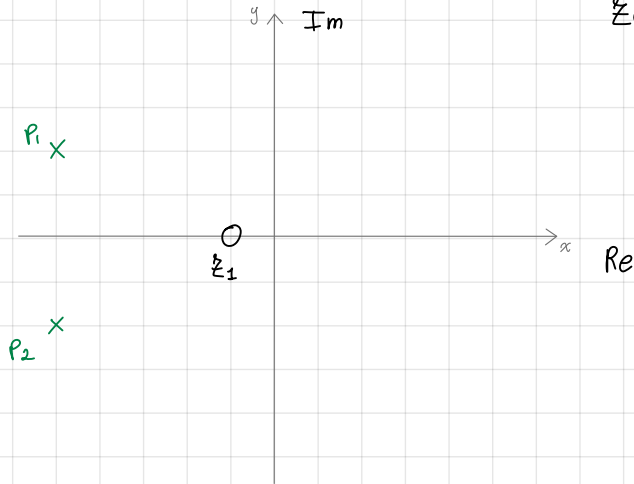
ES:

$$F(s) = \frac{10s+3}{s^3+4s^2+5s}$$

$$s(s^2+4s+5)=0 \leadsto p_1=0$$

$$-0 \quad p_{2,3} = \frac{-4 \pm \sqrt{16-4 \cdot 5}}{2} = \frac{-4 \pm 2j}{2} \rightarrow \begin{matrix} -2+j & p_2 \\ -2-j & p_3 \end{matrix}$$

$$\text{Zero: } 10s+3=0 \leadsto s = -\frac{3}{10}$$



$$F(s) = \frac{10s+3}{s^3+4s^2+5s} = \frac{10s+3}{s(s^2+4s+5)} = \frac{z_1}{s} + \frac{z_2}{(s+2-j)} + \frac{z_3}{(s+2+j)} = \frac{z_1}{s} + \frac{??}{??}$$

STESSA FORMA

$$= s^2 + 2s + 1s + 2s + 4 + 2j - js - 2j - j^2$$

$$= s^2 + 4s + 5$$

!!! I prof non da' Dim: N: $z_1 \cdot s + z_2$

$$\leadsto F(s) = \frac{z_1}{s} + \frac{z_2 s + z_3}{s^2+4s+5}$$

$$z_1 = \lim_{s \rightarrow 0} \frac{10s+3}{s^2+4s+5} = \lim_{s \rightarrow 0} \frac{10s+3}{s^2+4s+5} \leadsto \left(\frac{3}{5}\right) z_1$$

$$\frac{10s+3}{s^3+4s^2+5s} = \frac{z_1}{s} + \frac{z_2 s + z_3}{s^2+4s+5} = \frac{z_1(s^2+4s+5) + z_2 s^2 + z_3 s}{s^3+4s^2+5s} = \frac{z_1 s^2 + z_1 4s + 5z_1 + z_2 s^2 + z_3 s}{s^3+4s^2+5s}$$

$$= \frac{s^2(z_1+z_2) + s(4z_1+z_3) + 5z_1}{s^3+4s^2+5s} \Rightarrow \begin{cases} z_1+z_2=0 \leadsto z_2=-z_1 = -\frac{3}{5} \\ 4z_1+z_3=10 \leadsto -\frac{12}{5}+z_3=10 \\ 5z_1=3 \end{cases}$$

$$z_3 = \frac{62}{5}$$

$$\Rightarrow F(s) = \frac{3}{5} \cdot \frac{1}{s} + \frac{-\frac{3}{5}s + \frac{38}{5}}{s^2+4s+5}$$

$$= \frac{3}{5} \left(\frac{1}{s} + \frac{s + \frac{38}{5}}{s^2+4s+5} \right) = \frac{3}{5} \left(\frac{1}{s} - \frac{s}{s^2+4s+5} + \frac{38}{s^2+4s+5} \right)$$

$$= \frac{3}{5} \left(\frac{1}{s} - \frac{s+2}{(s+2)^2+1} + 33 \cdot \frac{1}{(s+2)^2+\underbrace{1}_{\omega^2}} \right) \Leftrightarrow \frac{3}{5} \left[1 - e^{-2t} \cos(t) + e^{-2t} + \cos(t) \right] \cdot \mathbb{1}(t)$$

ESERCIZI CAPITOLO 2

ES 2.1

$$F(s) = \frac{1}{1 - e^{-s}}$$

$$p = 1 - e^{-s} = 0 \rightarrow e^{-s} = 1 \sim e^{-(j\omega + \alpha)} = 1$$

$$e^{-\alpha} \cdot e^{-j\omega} = 1 \rightarrow e^{-\alpha} \cdot \cos(\omega) - j \sin(\omega) = 1 \rightarrow \alpha = -\ln \frac{1}{\cos(\omega) - j \sin(\omega)}$$

$$\text{per } \omega = 2k\pi \quad \cos(\omega) = 1, \sin(\omega) = 0$$

$$\Rightarrow \alpha = \pm j 2k\pi \quad \text{per } k = 1, 2, \dots, n$$

ES. 2.2

$$\begin{cases} f(t) = 0 & t < 0 \\ f(t) = t \cdot e^{-3t} & t \geq 0 \end{cases} \sim \mathcal{L}(0) = 0$$

$$\text{Siccome } \mathcal{L}[e^{-\alpha t} f(t)] = F(s + \alpha) \quad \mathcal{L}[t] = \frac{1}{s^2}$$

$$\Rightarrow F(s) = \frac{1}{(s+3)^2}, \quad t \geq 0 \quad \text{Ans}$$

ES 2.3

$$\begin{cases} f = 0 & t < 0 \\ f = \sin(\omega t + \alpha) & t \geq 0 \end{cases}$$

$$\mathcal{L}[\sin(\omega t) \cdot \mathbb{1}(t)] = \frac{\omega}{s^2 + \omega^2}$$

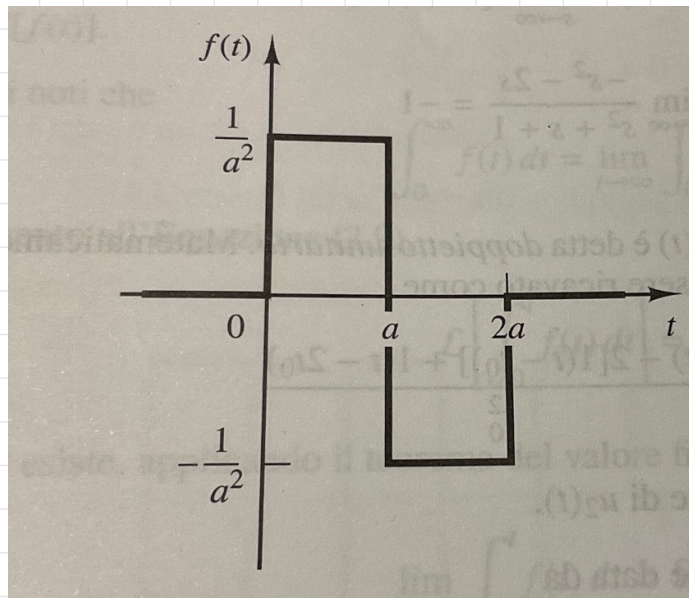
dalle formule di duplicazione: $\sin(\omega t + \alpha) = \underbrace{\sin(\omega t)}_{\mathcal{L}_1} \underbrace{\cos(\alpha)}_{K_1} + \underbrace{\sin(\alpha)}_{K_2} \underbrace{\cos(\omega t)}_{\mathcal{L}_2}$

$$\begin{cases} \sin(\omega t) = \frac{\omega}{s^2 + \omega^2} \\ \cos(\omega t) = \frac{s}{s^2 + \omega^2} \end{cases}$$

$$\Rightarrow F(s) = \cos(\alpha) \frac{\omega}{s^2 + \omega^2} + \sin(\alpha) \frac{s}{s^2 + \omega^2}$$

$$= \frac{\omega \cos(\alpha) + s \sin(\alpha)}{s^2 + \omega^2} \quad \text{Ans}$$

ES 2.4



$$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{a^2} \cdot 1(t) & 0 < t \leq a \\ -\frac{1}{a^2} \cdot 1(t) & a < t \leq 2a \\ 0 & t > 2a \end{cases}$$

Dobbiamo scriverla come SOMMA di segnali:

$$\frac{1}{a^2} 1(t) - \frac{2}{a^2} \cdot 1(t-a) + \frac{1}{a^2} 1(t-2a)$$

$$\Rightarrow F(s) = \frac{1}{sa^2} - \frac{2e^{-as}}{sa^2} + \frac{e^{-2as}}{sa^2} \quad \text{Ans 1}$$

Q2: Valore limite $F(s)$ $a \rightarrow 0$

$$\begin{aligned} \lim_{a \rightarrow 0} F(s) &= \lim_{a \rightarrow 0} \frac{1 - 2e^{-as} + e^{-2as}}{sa^2} \stackrel{\text{H\^opital}}{=} \lim_{a \rightarrow 0} \frac{2se^{-as} - 2se^{-2as}}{2sa} \\ &= \lim_{a \rightarrow 0} \frac{e^{-as} - e^{-2as}}{a} \stackrel{\text{H\^opital}}{=} \lim_{a \rightarrow 0} \frac{-se^{-as} + 2se^{-2as}}{1} \rightarrow -s + 2s = s \end{aligned}$$

Valore finale $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s)$

$$\rightarrow \lim_{s \rightarrow 0} \frac{1 - 2e^{-as} + e^{-2as}}{sa^2} \stackrel{\text{H\^opital}}{=} \lim_{s \rightarrow 0} \frac{+2ae^{-as} - 2ae^{-2as}}{a^2} \rightarrow 0 \quad \text{Valore finale}$$

V.I.

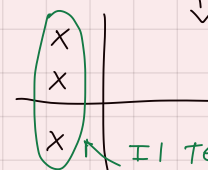
$$\lim_{s \rightarrow 0} \frac{1 - 2e^{-as} + e^{-2as}}{sa^2} \rightarrow \frac{1}{\infty} \rightarrow 0$$

Posso Applicare i Teoremi?

$$SF(s) = \frac{1 - 2e^{-as} + e^{-2as}}{a^2} = 0$$

$$\text{per } 2e^{-as} - e^{-2as} = 1$$

$$s \in \text{Imm}$$



I Teoremi
NON vale!

ES 2.5

$$F(s) = \frac{2s+1}{s^2+s+1}$$

Q: V.I. di $\frac{d}{dt} f(t)$

$$\mathcal{L}[f'(t)] = s F(s) - \underbrace{f(0)}_{\text{non conosco ma } f(t) \rightarrow \text{Non posso calcolare } f(0)}$$

\rightarrow Uso il T.V.I $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s \cdot F(s) = \lim_{s \rightarrow \infty} \frac{2s^2 + s}{s^2 + s + 1} = \lim_{s \rightarrow \infty} \frac{s^2(2 + \frac{1}{s})}{s^2(1 + \frac{1}{s} + \frac{1}{s^2})}$

$= 2 \Rightarrow f(0^+) = 2$

$$\Rightarrow \mathcal{L}[f'(t)] = \frac{2s^2 + s}{s^2 + s + 1} - 2 = \frac{2s^2 + s - 2s^2 - 2s - 2}{s^2 + s + 1} = -\frac{s+2}{s^2 + s + 1}$$

$$\Rightarrow \frac{d}{dt} f(0^+) = \lim_{s \rightarrow \infty} -s \frac{s+2}{s^2 + s + 1} = -\frac{s^2(1+2/s)}{s^2(1+0+0)} \Rightarrow -1 \text{ V.I. di } f'(t)$$