ES 7.4 Pg 394

$$G(j\omega) = \frac{e^{-j\omega L}}{1 + j\omega T} = \frac{\cos(\omega L) - j\sin(\omega L)}{1 + j\omega T} \qquad |G(j\omega)| = \frac{\sqrt{\cos^2(\omega L) + \sin^2(\omega L)}}{\sqrt{1 + (\omega T)^2}} = \frac{1}{\sqrt{1+(\omega T)^2}}$$

$$\Rightarrow |G(j\omega)|_{dB} = \cancel{20 \log(1)}^{\,0} - 20\log\left(\sqrt{1+(\omega T)^2}\right) = \underline{-20 \log\left(\sqrt{1+(\omega T)^2}\right)}$$ Modulo esatto

$$\angle G(j\omega) = \angle e^{-j\omega L} - \angle 1 + j\omega T = -\omega L - 0 - \angle \sqrt{1+(\omega T)^2} \cdot e^{j\tan^{-1}(\omega T)} = \underline{-\omega L - \tan^{-1}(\omega T)}$$ FASE esatta

Poniamo per esempio $L = \frac{1}{2}$; $T = 1$

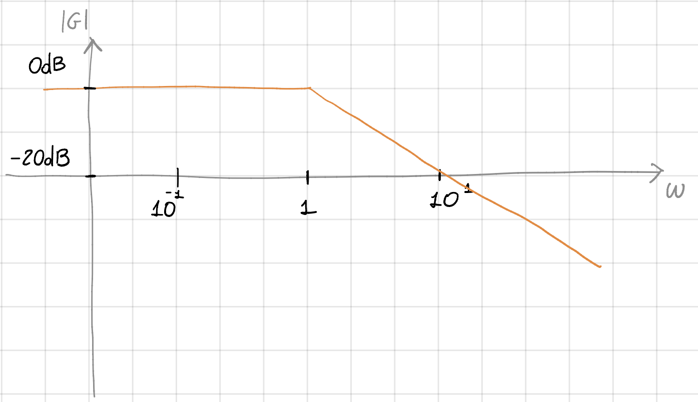
$$\Rightarrow |G(j\omega)|_{dB} = -20\log\left(\sqrt{1+\omega^2}\right) \; ; \; \angle G(j\omega) = -\tfrac{1}{2}\omega - \tan^{-1}\left(\tfrac{1}{2}\omega\right)$$

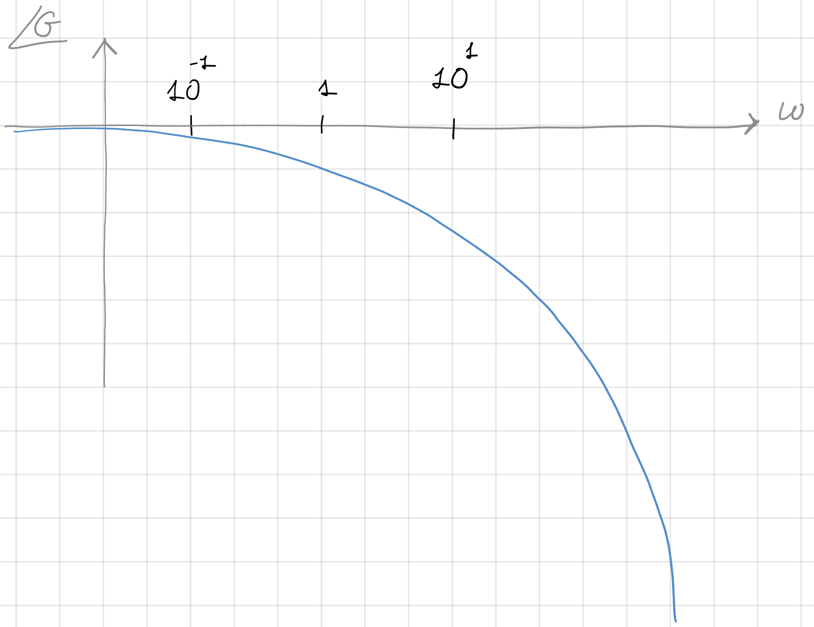
Per $\omega \to 0$ $\quad |G(j\omega)|_{dB} \cong -20\log(\sqrt{1}) = 0\,dB \qquad \angle G(j\omega) \cong -\tan^{-1}(\omega \to 0) = 0$

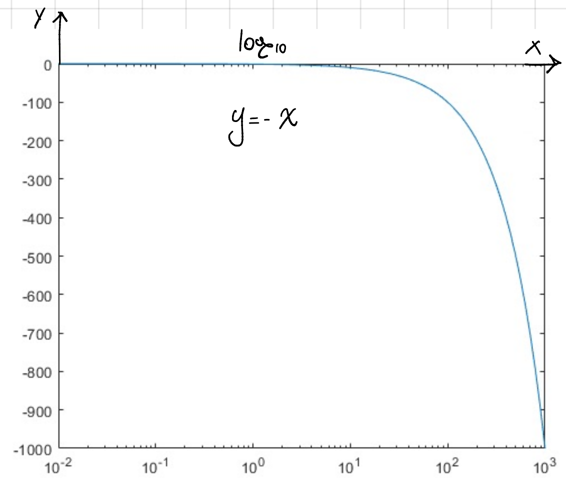
Per $\omega \to \infty$ $\quad |G(j\omega)|_{dB} \cong \underbrace{-20\log(\omega)}_{-20dB/dec} \qquad \angle G(j\omega) \cong -\omega$ ovvero tende a $-\infty$

Siccome $G(j\omega) = \dfrac{e^{-j\omega L}}{1+j\omega T} \Rightarrow G(s) = \dfrac{e^{-sL}}{1+sT} \Rightarrow \bar{s}_p = -\dfrac{1}{T} \Rightarrow \underbrace{\omega_1 = -1}_{T=1}$

Prendo come intervallo: $\bar{\omega} \in [10^{-1} ; 10^{1}]$







Problema 7.1 - Pg 503

$$G(s) = \frac{10(s+1)}{(s+2)(s+5)}$$

Guadagno (forma St.)

$$G(s) = 10 \frac{1+s}{\frac{1}{2}(1+2s)\frac{1}{5}(1+5s)} = 10^2 \cdot \frac{1+s}{(1+2s)(1+5s)}$$

Valore iniziale

guadagno Statico

Zeri: $s+1=0 \rightsquigarrow z_1=-1$ Poli $p_1=-2$; p_2-5

ZERO POLI

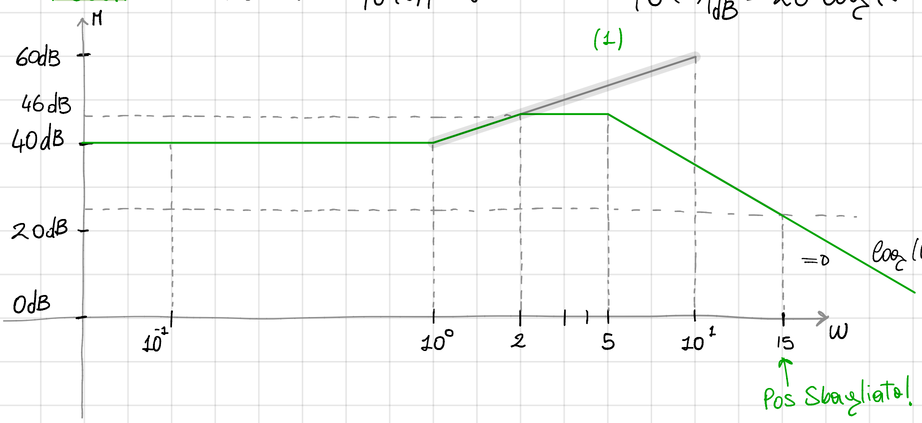
$\omega_1 = 1$, $\omega_2 = 2$, $\omega_3 = 5$

Scelta Banda

$$\bar{\omega} = [10^{-1} ; 10^{1}]$$

Moduli

$\rightsquigarrow |G(0)| = 10^2 \Rightarrow |G(0)|_{dB} = 20 \log_{10}(10^2) = 40\ dB$



Pos Sbagliata!

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

$$\frac{\log(\omega)-\log(1)}{\log(10)-\log(1)} = \frac{|G(j\omega)|_{dB}-40}{60-40}$$

$\Rightarrow \log(\omega) = \frac{|G(j\omega)|_{dB}-40}{20} \Rightarrow |G(j\omega)|_{dB} = 20\log(\omega)+40$

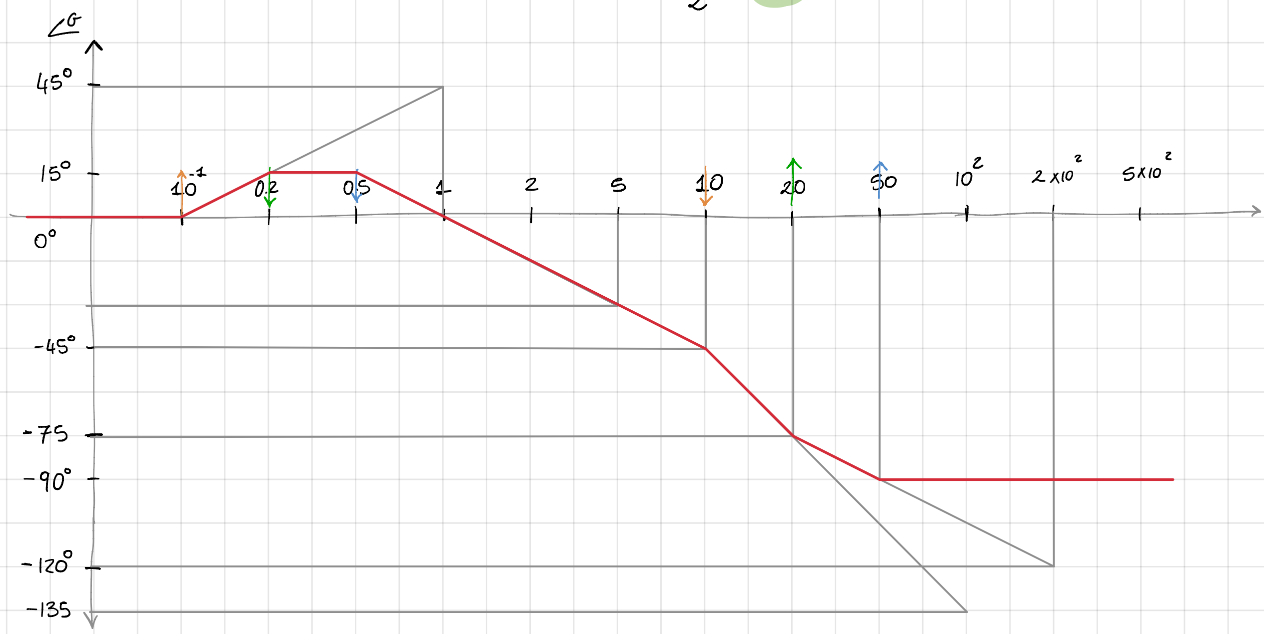
eq Retta nel Tratto (1)

$\Rightarrow |G(j\omega)|_{dB}\Big|_{\omega=2} = 46\ dB \Rightarrow 46dB - 20dB = 26dB$

Fasi

$$\angle G(j\omega) = \angle 1+j\omega - \angle 2+j\omega - \angle 5+j\omega = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

$$\begin{cases} \text{Per } \omega \to 0 \Rightarrow \angle G(j\omega) \to \tan^{-1}(0) - \tan^{-1}(0) - \tan^{-1}(0) = 0^\circ \\ \text{Per } \omega \to \infty \Rightarrow \angle G(j\omega) \to -\tan^{-1}(\omega \to \infty) = -\frac{\pi}{2} = -90^\circ \end{cases}$$



Es

$$G(s) = \frac{(s-1)(s+10)}{s(s^2+s+16)} = \frac{-(1-s)\,10\,(1+\frac{1}{10}s)}{s\,(s^2+s+16)}$$

s^2+s+16 può essere scritta nella forma $(s+a)(s+b)$?

$\leadsto \Delta = 1-4\cdot1\cdot16 = -63 < 0 \Rightarrow$ Radici complx e conj $\Rightarrow$ NO

$$s^2+s+16 = 16\left(1+\frac{1}{16}s+\frac{1}{16}s^2\right) \Rightarrow G(s) = \frac{-10}{16}\,\frac{(1-s)(1+\frac{1}{10}s)}{s(1+\frac{1}{16}s+\frac{1}{16}s^2)} = \frac{K_b}{s}\cdot\frac{(1-\tau_1 s)(1+\tau_2 s)}{(1+\frac{2\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2)}$$

2. DOVE...

$K_b = -5/8$ $\quad$ $\omega_n^2 = 16 \Rightarrow \omega_n = 4 \leadsto \frac{2\zeta}{4} = \frac{1}{16} \Rightarrow \zeta = \frac{4}{16\cdot 2} \Rightarrow \zeta = 1/8$

$\tau_1 = 1$

$\tau_2 = 1/10$

3. Punti di rottura

- $\omega_n = 4$
- $1-s=0 \to z_1 = 1$
- $1+\frac{1}{10}s = 0 \to z_2 = -10$

$$\Rightarrow \begin{cases} \omega_1 = 1 \quad z \quad \text{ReP} > 0 \\ \omega_2 = 4 \quad P \\ \omega_3 = 10 \quad z \quad \text{ReP} < 0 \end{cases}$$

4. Diagramma dei Moduli

E' presente un polo nell'origine $\Rightarrow$ il modulo inizia con una pendenza di -20dB/dec

Perchè ? Per ω molto piccolo, ovvero $\omega \ll \omega_1 = 1$ e' l'unico contributo e' quello di $\bar{G}(s) = \frac{K_b}{s}$

Termine integrale

$$\bar{G}(j\omega) = \frac{K_b}{j\omega} = 1 \to \frac{|K_b|}{\omega} = 1 \Rightarrow \omega_0 = |K_b| = \frac{5}{8} \to P_1\left(\frac{5}{8}, 0\text{dB}\right)$$

ovvero 0dB

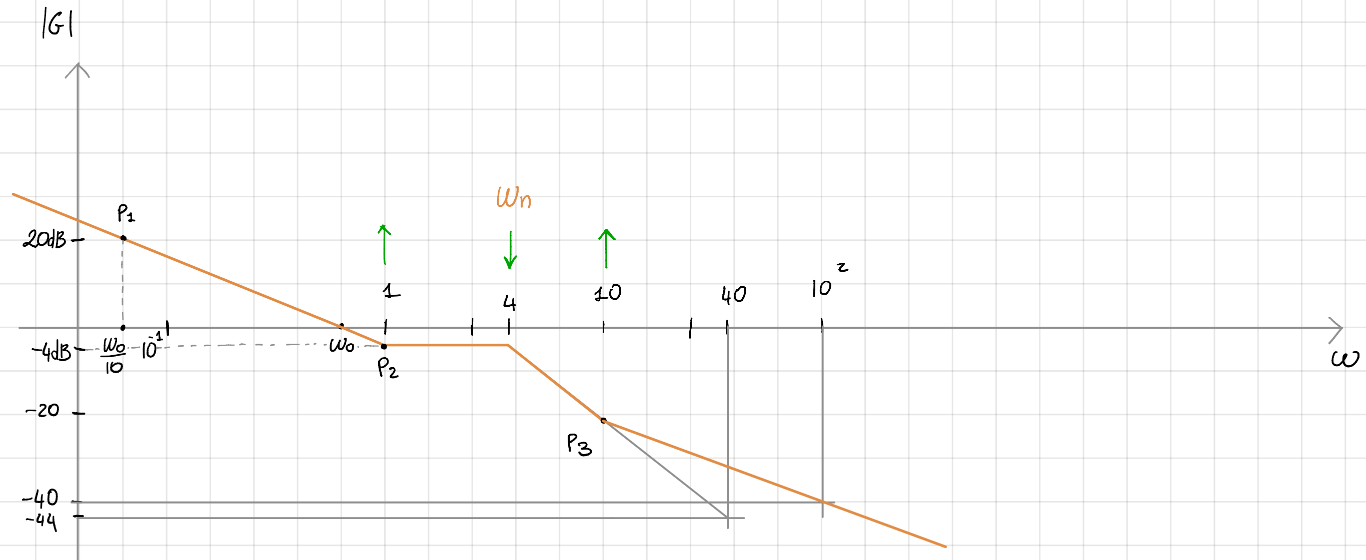
$$\frac{\log(\omega) - \log(\frac{5}{80})}{\log(5/8) - \log(\frac{5}{80})} = \frac{|G(j\omega)|_{dB} - 20}{-20} \to |G(j\omega)|_{dB} = \left[-\left(\frac{\log(\omega)+1.2}{-0.2+1.2}\right)\cdot 20\right] + 20$$

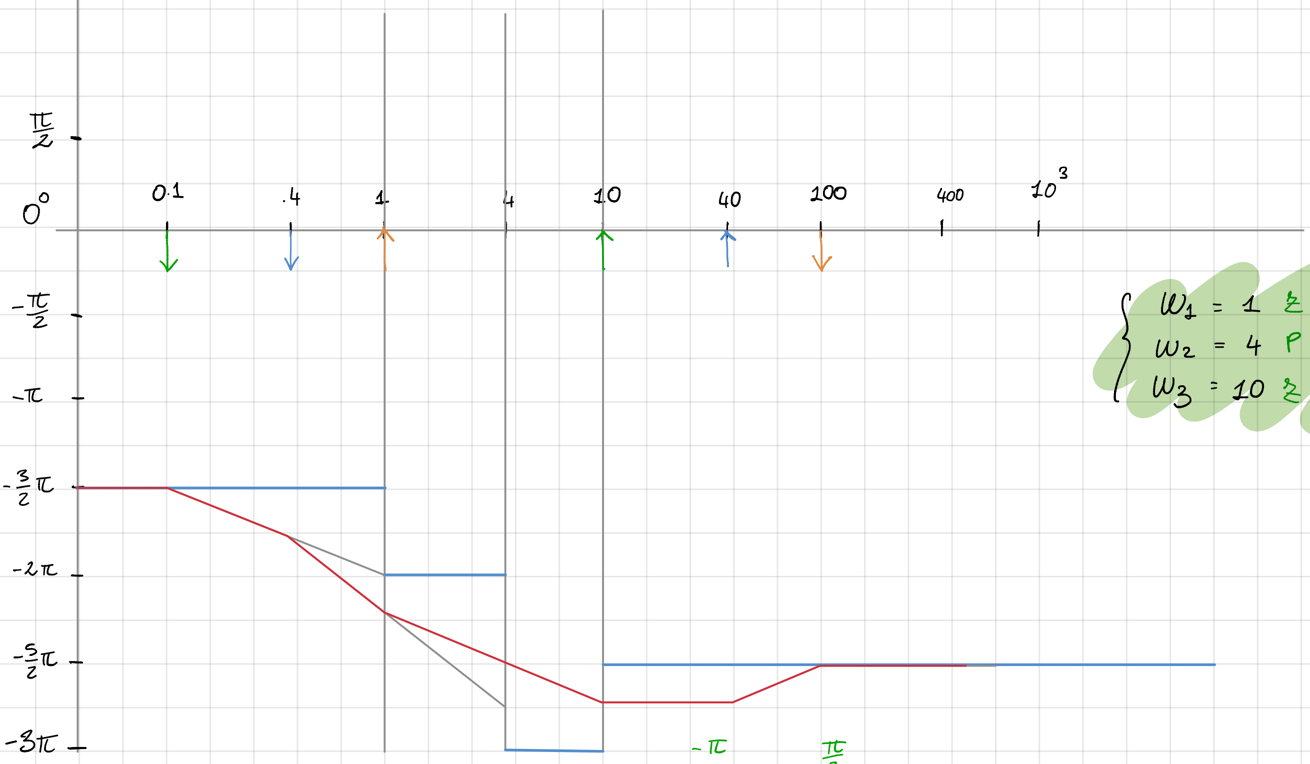
$$\Rightarrow |G(j\omega)|_{dB}\Big|_{\omega=1} = -4\,\text{dB} \leadsto P_2(1, -4\text{dB})$$

$$\frac{\log(\omega) - \cancel{\log(4)}^{\,0.6}}{\cancel{\log(40)}_{1.6} - \cancel{\log(4)}_{0.6}} = \frac{|G(j\omega)|_{dB}+4}{-44+4} \leadsto |G(j\omega)|_{dB} = -\left(\log(\omega) - 0.6\right)\cdot 40 - 4$$

1

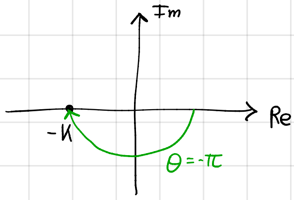
$$|G(j\omega)|_{dB}\Big|_{\omega=10} = -20\,\text{dB} \leadsto P_3 = (10, -20\text{dB})$$





$$\begin{cases} \omega_1 = 1 \quad z \quad ReP>0 \\ \omega_2 = 4 \quad P \\ \omega_3 = 10 \quad z \quad ReP<0 \end{cases}$$

Per $\omega <<$ $\Rightarrow$ $G_1(j\omega) \simeq \frac{K_0}{j\omega}$ $\Rightarrow$ $\angle G = \overset{-\pi}{\angle K_0} - \overset{\frac{\pi}{2}}{\angle j\omega}$ Siccome $K_0 < 0$



$\Rightarrow$ $\angle G = -\pi - \frac{\pi}{2} = -\frac{3}{2}\pi$ Fase iniziale

Per $1 < \omega < 4$ $\Rightarrow$ $G_2(s) \simeq \underset{-\frac{3}{2}\pi}{G_1(s)}(1-\tau_1 s)$ $\Rightarrow$ TOT $= -\frac{3}{2}\pi - \frac{\pi}{2} = 2\pi$

Zero ReP $\Rightarrow -\frac{\pi}{2}$

Per $4 < \omega < 10$ $\Rightarrow$ $G_3(s) \simeq \underset{-2\pi}{G_2(s)}\left(1+\frac{2\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2\right)$ $\Rightarrow$ TOT $= -2\pi - \pi = -3\pi$

Zero del II° $\Rightarrow -\pi$

Per $\omega \geq 10$ $\Rightarrow$ $G_4(s) = \underset{-3\pi}{G_3(s)}(1+\tau_2 s)$ $\Rightarrow$ $-3\pi + \frac{\pi}{2} = -\frac{5}{2}\pi$

Zero ReP<0 $\Rightarrow +\frac{\pi}{2}$

Esercizio

$$G(s) = \frac{100\,(s-0.2)}{(s+0.8)\,(s^2-2s+100)}$$

Forma St: $G(s) = \frac{1}{\left(1+\frac{2\delta}{\omega_n}+\frac{1}{\omega_n^2}s^2\right)}$

$G(s) = \frac{\prod^n (1+s)}{\prod^m (1+s)}$

(1) il Termine $(s^2-2s+100)$ ha radici reali?

$\rightsquigarrow \Delta = 4-4\cdot 100 = -396 < 0 \Rightarrow$ Cmplx

$$\Rightarrow G(s) = 100\cdot\frac{-0.2\left(1-\frac{s}{0.2}\right)}{0.8\left(1+\frac{s}{0.8}\right)\,100\left(1-\frac{1}{50}s+\frac{1}{100}s^2\right)}$$

$\rightsquigarrow \begin{cases} -\frac{1}{50} = \frac{2\delta}{\omega_n} \Rightarrow \delta = -\frac{10}{50}\cdot\frac{1}{2} = -\frac{1}{10} & \delta \\ \frac{1}{\omega_n^2} = \frac{1}{100} \Rightarrow \omega_n^2 = 100 \Rightarrow \omega_n = \sqrt{100} = 10 & \omega_n \end{cases}$

z_1: $1-\frac{s}{0.2} = 0 \to z_1 = +0.2$ Zero a ReP Pos

P_1: $1+\frac{s}{0.8} = 0 \to P_1 = -0.8$ Polo

$\Rightarrow \omega_1 = 0.2$ (z ReP<0), $\omega_2 = 0.8$ (P), $\omega_3 = \omega_n = 10$ (Freq)

$\Rightarrow K_b = \frac{\cancel{100}\cdot -0.2}{0.8\cdot\cancel{100}} = -\frac{1}{4}$ guadagno

$*$ Scelta della Banda

$\min(P,z) = 0.2 \Rightarrow \omega_{min} = \frac{1}{100}$

$\max(P,z) = 10 \Rightarrow \omega_{max} = 100$

(2) Possiamo dividere il diagramma in 4 Sezioni:

1° Settore: $\omega < \omega_1 = 0.2$

Modulo: $G_1(s) = K_b \Rightarrow |G(j\omega)|_{dB} = 20\log(|G_1(s)|) = 20\log(1/4) \simeq -12$

Fase: $\angle K_b = -180° = \ominus\pi$ ← Per conv.

con $K_b<0$

2° Sett $\omega_1 < \omega < \omega_2 \rightsquigarrow 0.2 < \omega < 0.8$ $\quad G_2(s) = K_b\cdot(s-0.2)$ (gain; Zero ReP<0)

Modulo: $+20$ dB/dec

Fase: $-\pi - \frac{\pi}{2} = -\frac{3}{2}\pi = -270°$

3° Sett $\omega_2 < \omega < \omega_n$ (0.8, 10) $\Rightarrow G_3(s) = K_b\frac{(s-0.2)}{(s+0.8)}$

Modulo: $+20$dB/dec $-$ 20 dB/dec = COST a [?] dB

Fase: $-\frac{3}{2}\pi - \frac{\pi}{2} = -2\pi = -360°$

4° Sett $\omega > \omega_n = 10 \Rightarrow G_4(s) = G(s)$

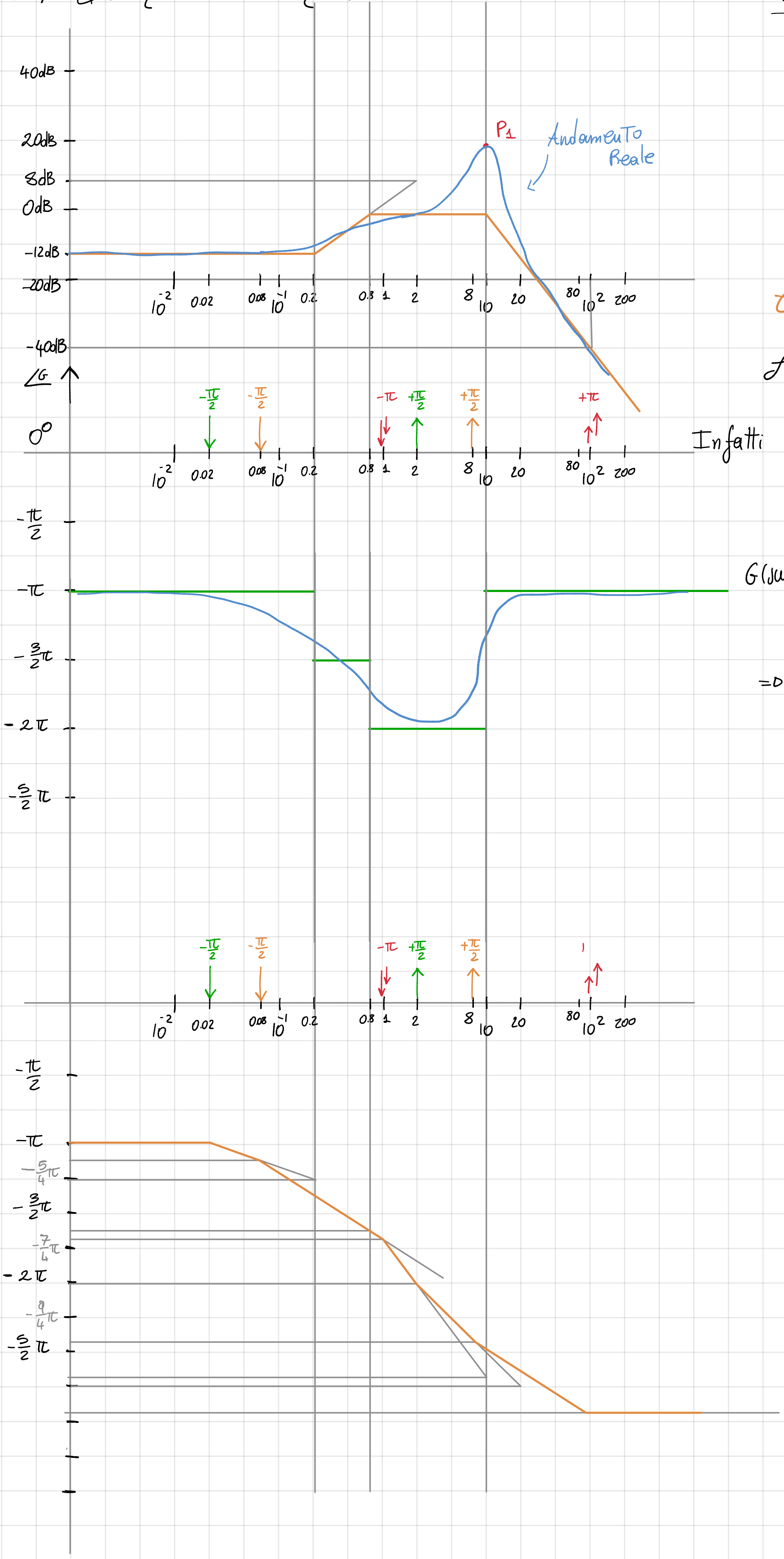
CASO NON Visto a lezione, lo do per Buono ...

Modulo 0dB/dec $-$ 40dB/dec

Siccome $\delta = -\frac{1}{10} \Rightarrow -1 < \delta < 0 \Rightarrow$ Contributo $= +\pi$

Fase $-2\pi + \pi = -\pi = -180°$

(3) Disegno dei diagrammi



$$\frac{\log(\omega) - 0.2}{2 - 0.2}\bigg|_{\omega=0.8} = \frac{|G| + 12}{8 + 12}$$

$$\Rightarrow |G| \overset{\sim}{=} \left[\left(\frac{-0.09 - 0.2}{1.8}\right)\cdot 20\right] - 12 \;\bigg|_{\omega=0.8} = \;\bigg|_{\omega=0.8}$$

Considerazioni

$\zeta = -\frac{1}{10} = -0.1 < 0.707 \Rightarrow$ c'è Risonanza!

Infatti $|G(j\omega)|_{dB}\Big|_{\omega=\omega_n} = \dots$

$$G(j\omega) = -\frac{1}{4}\cdot\frac{\left(1 - \frac{1}{0.2}j\omega\right)}{\left(1+\frac{1}{0.8}j\omega\right)\left(1 - \frac{1}{50}j\omega + \frac{1}{100}(j\omega)^2\right)}$$

$$\Rightarrow Mod\left(G(j\omega)\right)\Big|_{\omega=10} \cong 5 \underset{dB}{\cong} 14\,dB$$

$\Rightarrow P_1 = (10, 14dB) \in$ Bod

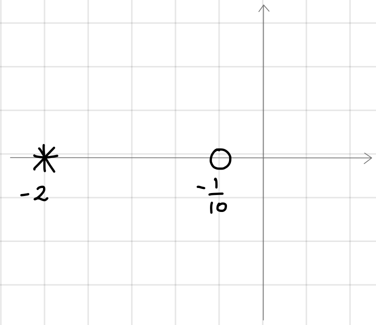
$\omega_1 = 0.2$
Z ReP<0

, $\omega_2 = 0.8$, $\omega_3 = \omega_n = 10$
P Freq

POLO DOPPIO

Es lez 25

$$G(S) = 7\,\frac{10S+1}{\left(\frac{1}{2}S+1\right)^2} = \frac{70S+7}{\frac{1}{4}S^2+S+1}$$



1) Forma St. G(S), Poli e Zeri

$$G_{St}(S) = K\cdot\frac{\prod^{m}\left(1+\frac{1}{z_i}S\right)}{\prod^{n}(1+T_iS)} \Rightarrow G(S) = 7\cdot\frac{1+10S}{\left(1+\frac{1}{2}S\right)^2}$$ già in St. F.

$z_1 = -\frac{1}{10}$ $\qquad P_{1,2} = -2$

2) Punti di rottura

ZERO : $\omega_1 = \frac{1}{10}$ ↑ $\qquad$ Poli : $\omega_{1,2} = 2 \equiv$ POLO DOPPIO : $\omega_2 = 2$ ↓↓

A ReP Pos $\Rightarrow \angle = +90°$

3) Banda di frequenze

$\omega \in [10^{-2};\ 10^{2}]$

4) Andamenti iniziali e finali

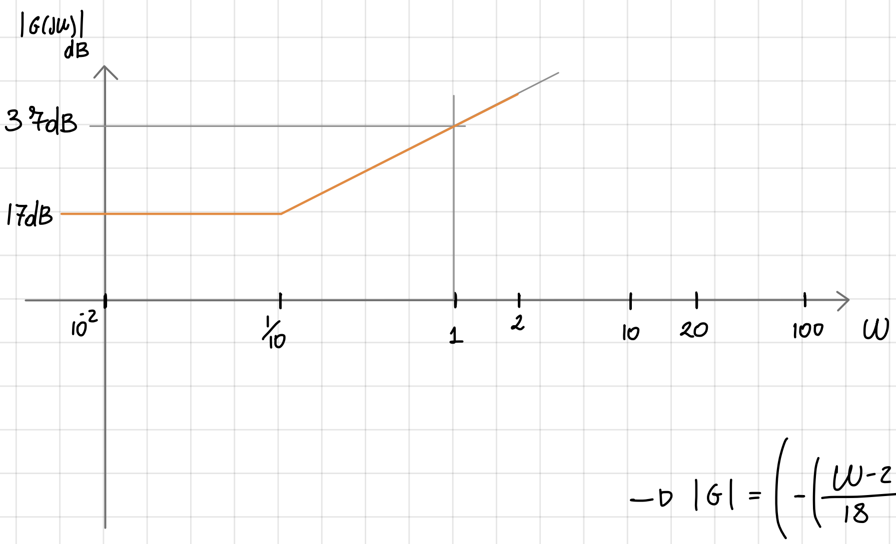
MODULI : No Poli in origine $\Rightarrow \left.\left|G(j\omega)\right|_{dB}\right|_{\omega\ll} = 20\log(7) \cong 17\,dB$

2 poli, 1 zero $\Rightarrow (-20-20+20)\,dB/dec = -20\,dB/dec$

FASI $\left.\angle G(j\omega)\right|_{\omega\ll} = 0°$

2 Poli, 1 zero $\Rightarrow -90-90+90 = -90°$

5) Traccio i diagrammi



$$\frac{\omega-0.1}{1-0.1} = \frac{|G|-17}{37-17}$$

$$|G| = \left[\left(\frac{\omega-0.1}{0.9}\right)\cdot 20\right]+17$$

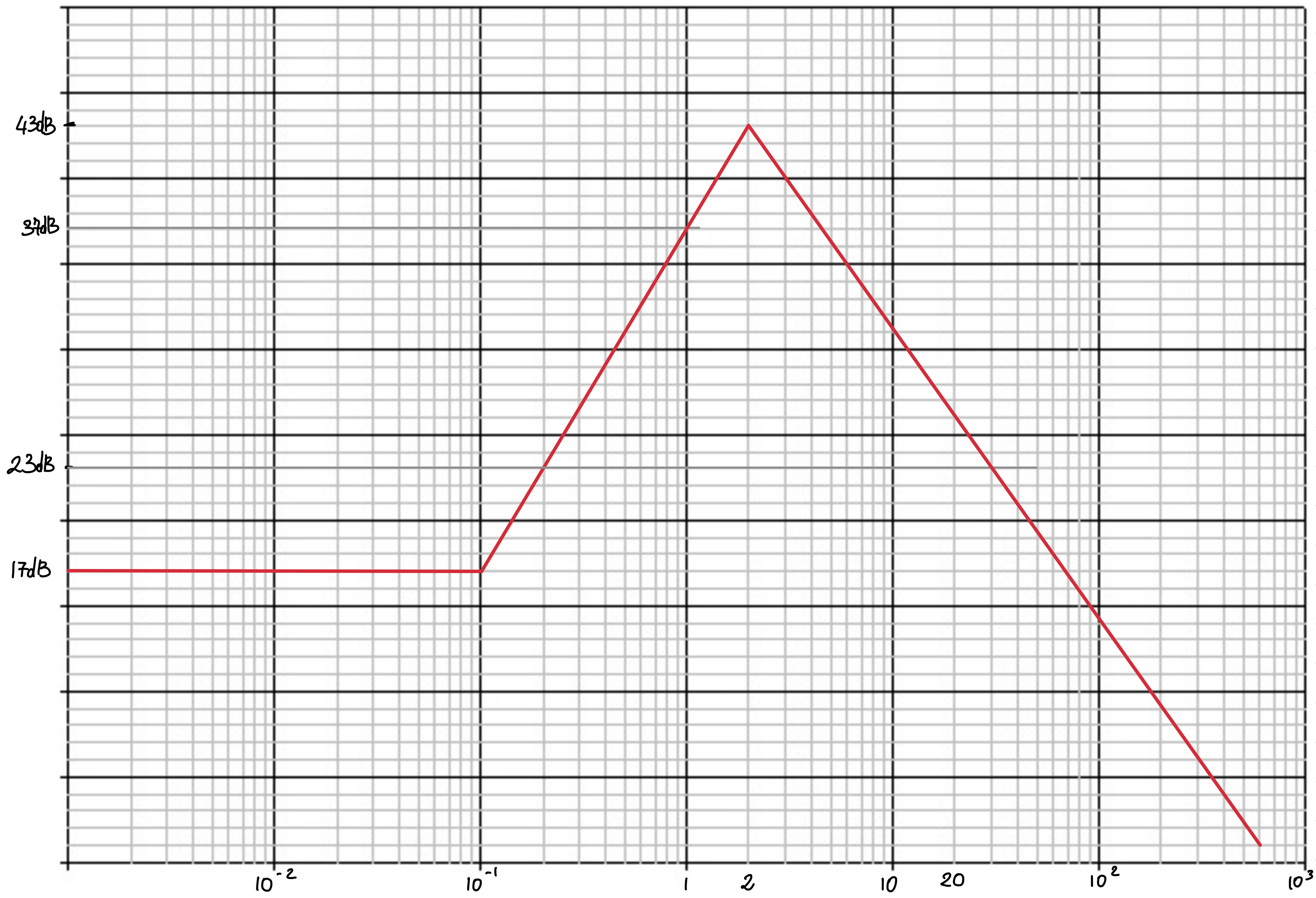
$= 59$??

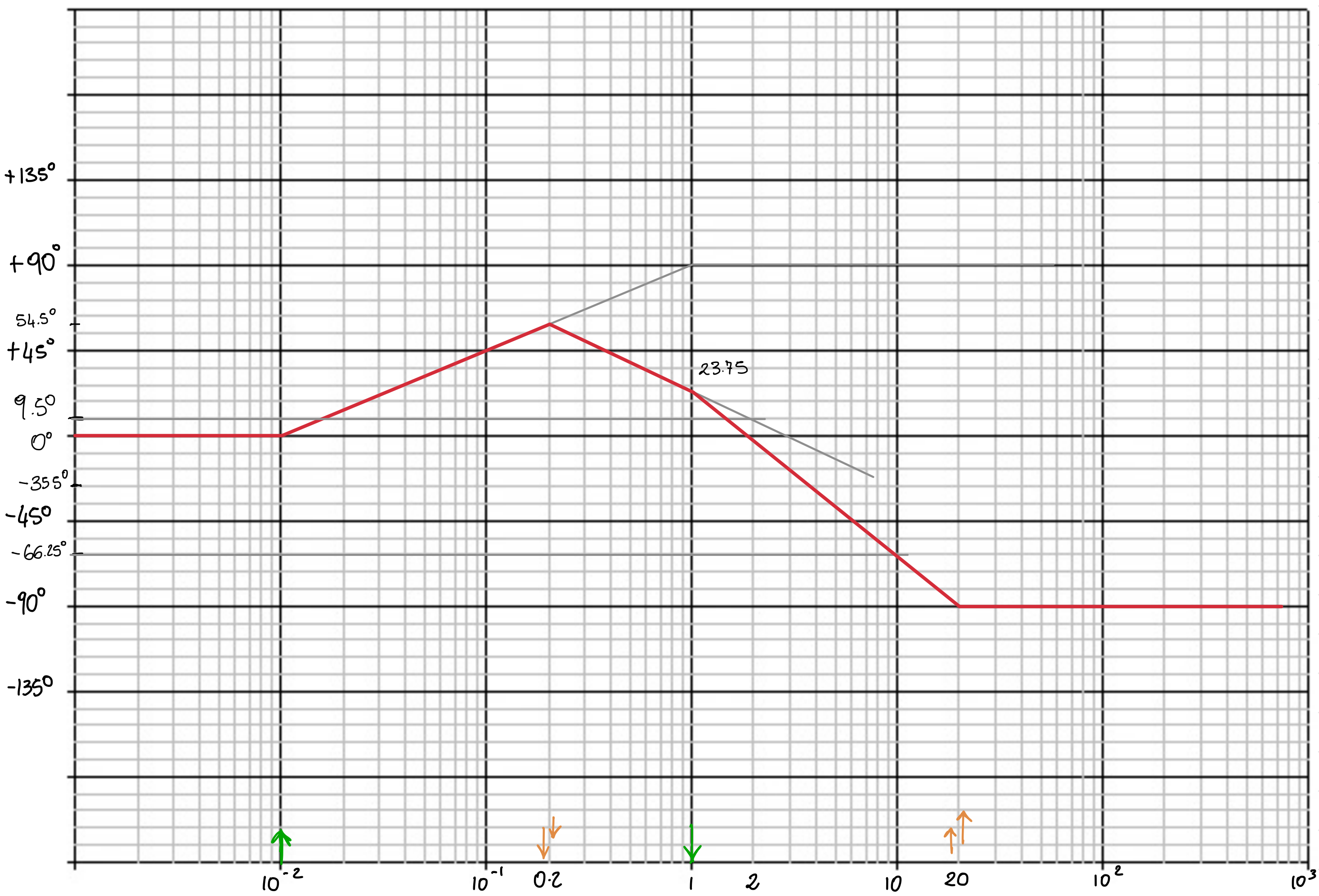
$$\frac{\omega-2}{20-2} = \frac{|G|-59}{39-59}$$

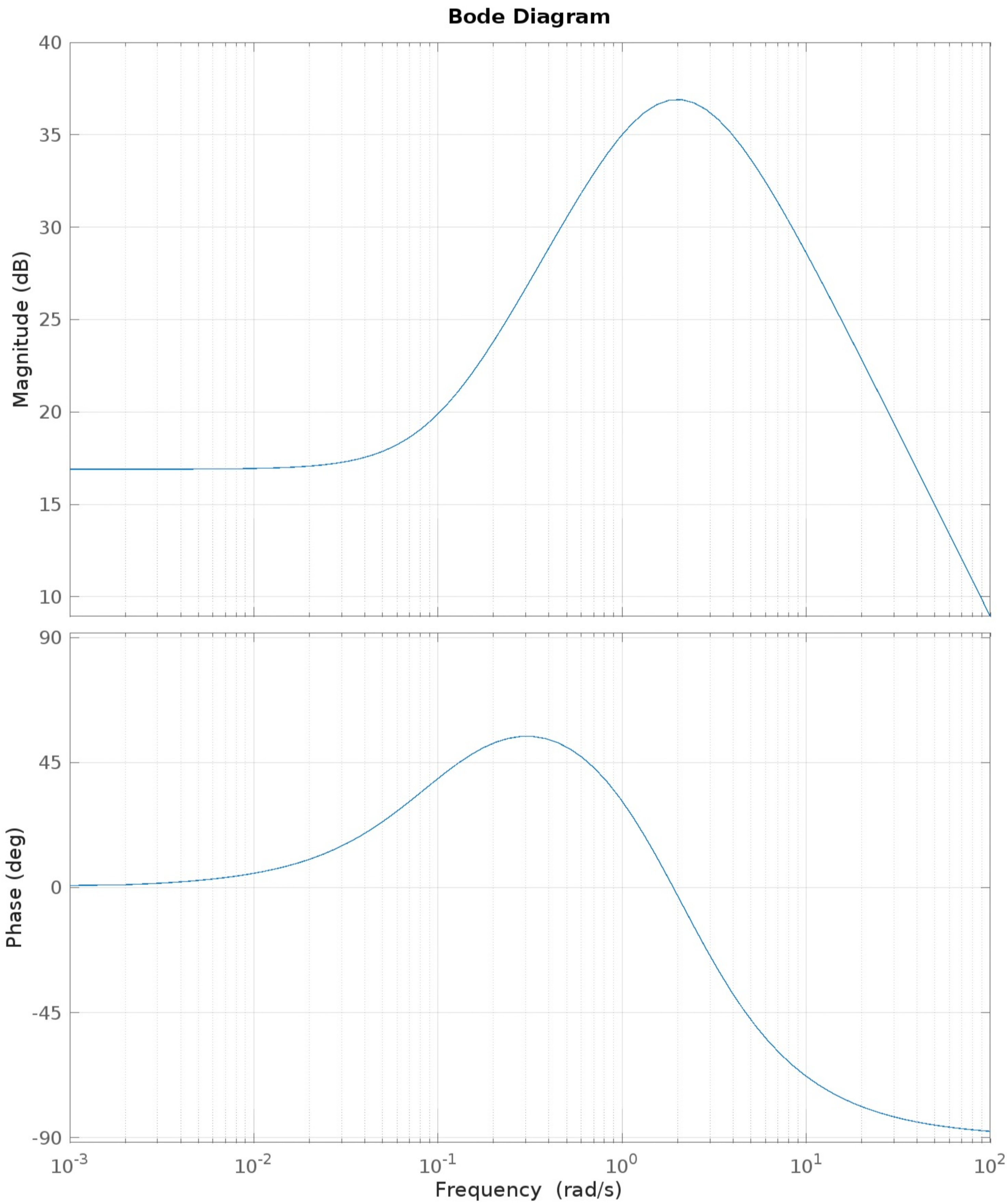
$\omega = 20$

$$\Rightarrow |G| = \left(-\left(\frac{\omega-2}{18}\right)\cdot 20\right)+59 \stackrel{!}{=}$$

DIAGRAMMA DEI MODULI






Bode Diagram
Magnitude (dB)
40
35
30
25
20
15
10
Phase (deg)
90
45
0
-45
-90
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)