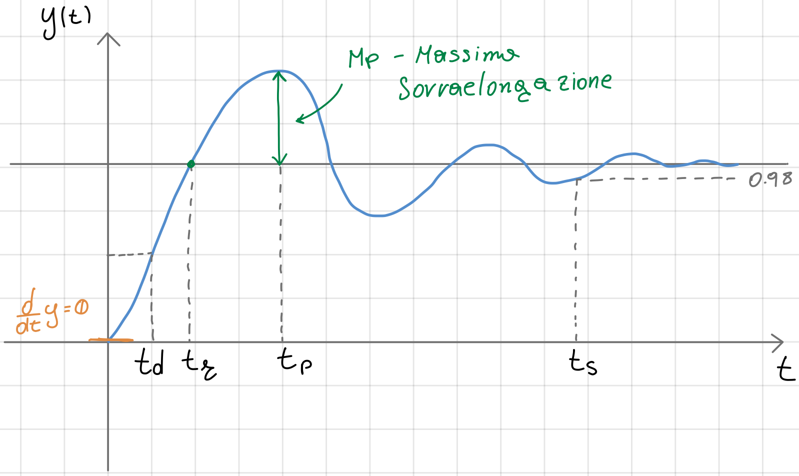
RISPOSTA AL GRADINO (DEL 2nd ORDINE)



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

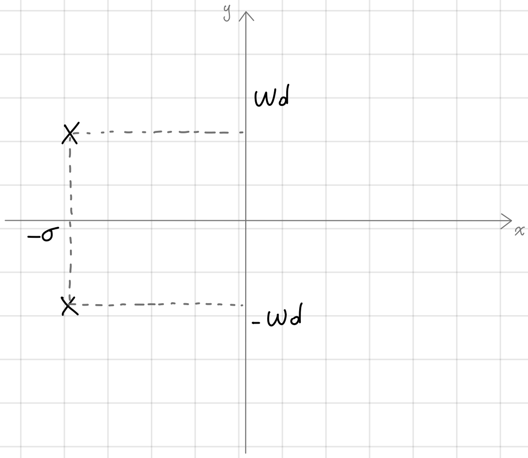
con $0 < \zeta < 1$

$$y(t) = 1 - e^{-\zeta\omega_n t}\left(\cos(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right)$$

con $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Poli complx e coniugati

$\sigma = \zeta\omega_n$



CON CHE VELOCITA' RISPONDE IL SYS ?

Tempo di picco

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \cdot 100$$

PARAMETRI

- t_r = Rise Time
- t_p = Peak Time
- t_d = delay Time → Valore per raggiungere 50% t_r
- t_s = Setteling Time

CALCOLO DEI PARAMETRI

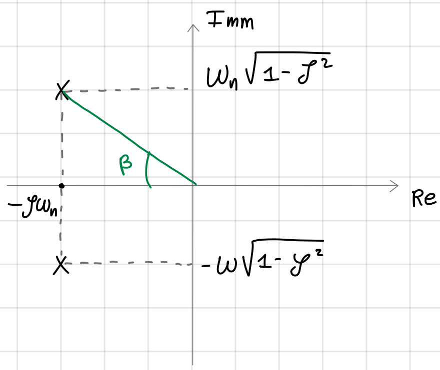
RISE TIME t_r

sempre $\neq 0$

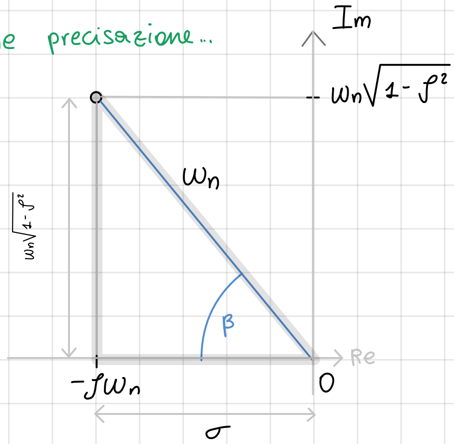
$$y(t_r) = 1 \leadsto \cancel{1} - e^{-\sigma t_r}\left(\cos(\omega_d \cdot t_r) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d \cdot t_r)\right) = \cancel{1}$$

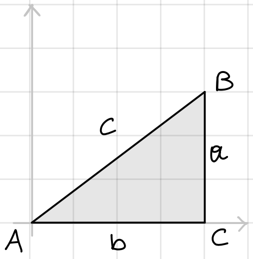
ovvero $\cos(\omega_d t_r) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d \cdot t_r) = 0 \to \sin(\omega_d \cdot t_r) = -\frac{\sqrt{1-\zeta^2}}{\zeta}\cos(\omega_d \cdot t_r)$

$$\Rightarrow \boxed{\tan(\omega_d \cdot t_r) = -\frac{\sqrt{1-\zeta^2}}{\zeta}} \quad (a)$$



Qualche precisazione...





$$\begin{cases} \cos(A) = \dfrac{b}{c} \\ \sin(A) = \dfrac{a}{c} \end{cases} \Rightarrow \tan(A) = \frac{a}{b}$$

$$\Rightarrow \boxed{\tan(\beta) = \frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\,\omega_n} = \frac{\sqrt{1-\zeta^2}}{\zeta}} \quad (b)$$

Se confrontiamo la (b) con la (a) notiamo che manca un "–", ma $\boxed{-\tan(\alpha) = \tan(\pi-\alpha)}$

Riscrivo la (a) : $\tan(\omega_d \cdot t_r) = \tan(\pi-\beta) \Rightarrow \omega_d \cdot t_r = \pi - \beta \Rightarrow \boxed{t_r = \dfrac{\pi-\beta}{\omega_d}}$

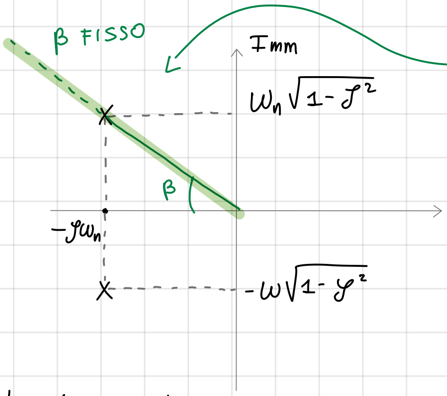
QED

Trigonometria ...

$$\cancel{\omega_n}\sqrt{1-\zeta^2} = \zeta\cancel{\omega_n}\tan(\beta) \Rightarrow \tan(\beta) = \frac{\sqrt{1-\zeta^2}}{\zeta} \Rightarrow \omega_d \cdot t_r = \pi - \beta$$

proof $\tan(\omega_d t_r) = \underset{\text{TRIG}}{\tan(\pi-\beta)} = -\tan(\beta)$

$$\Rightarrow \boxed{t_r = \frac{\pi-\beta}{\omega_d}}$$



Per Accorciare t_r

pongo $\beta = cost$

$\Rightarrow$ Mi serve ω_d più grande

Siccome $\omega_d = \omega_n\sqrt{1-\zeta^2}$

↑ cambio ω_n

PEAK TIME

$y = MAX$ per la 1st volta

> Bisogna notare che la derivata *tecnicamente* si azzera infinite volte, e la prima è in zero. Quindi quella che interessa a noi è la seconda!

$\dot{y}(t) = 0$ anche in $t=0$

$$\lim_{s\to\infty} \overset{\text{TVI}}{s} \cdot \underset{\text{Derivata}}{s} \cdot \overset{Y(s)}{G(s)\cdot \overset{\text{gradino}}{\frac{1}{s}}} = \lim_{s\to\infty} \frac{\omega_n^2 s}{s^2 + \dots} = \lim_{s\to\infty} \omega_n^2 \frac{\cancel{s}}{s^{\cancel{2}}} \to 0$$

Calcolo la derivata...

quando $\dot{y}(t) = 0$?

$$\frac{d}{dt}y(t) = \zeta\omega_n e^{-\zeta\omega_n t}\left(\cancel{\cos(\omega_d t)} + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right) - e^{-\zeta\omega_n t}\,\omega_d\left(-\sin(\omega_d \cdot t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\cancel{\cos(\omega_d t)}\right) \overset{?}{=} 0$$

proof semplific.

$$\omega_d \frac{\zeta}{\sqrt{1-\zeta^2}} = \omega_n \cancel{\sqrt{1-\zeta^2}} \cdot \frac{\zeta}{\cancel{\sqrt{1-\zeta^2}}} = \omega_n \zeta$$

* ↓ vedi giù

solo il sin può annullarsi

$$\Rightarrow \frac{d}{dt}y(t) = \underbrace{e^{-\zeta\omega_n t}}_{\neq 0\ \forall t}\left(\underbrace{\frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}} + \omega_d}_{\text{costante}}\right)\sin(\omega_d t) \overset{?}{=} 0$$

ovvero per $\sin(\omega_d \cdot t) = 0 \Rightarrow$ per $\omega_d t_p = \pi \Rightarrow \boxed{t_p = \frac{\pi}{\omega_d}}$

$$* \quad \dot{y}(t) = \zeta\omega_n e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right) - e^{-\zeta\omega_n t}\omega_d\left(-\sin(\omega_d \cdot t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\cos(\omega_d t)\right)$$

$$= \cancel{e^{-\zeta\omega_n t}\zeta\omega_n\cos(\omega_d t)} + \frac{\zeta^2\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t) + e^{-\zeta\omega_n t}\cdot\omega_d\sin(\omega_d t) + e^{-\zeta\omega_n t}\cdot\frac{\omega_d\zeta}{\sqrt{1+\zeta}}\cos(\omega_d t)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\hookrightarrow \frac{\omega_d\zeta}{\sqrt{1+\zeta}}\cos(\omega_d t) = \frac{\omega_n\cancel{\sqrt{1-\zeta^2}}}{\cancel{\sqrt{1-\zeta^2}}}\cos(\omega_d \cdot t)$$

$$= \cancel{e^{-\zeta\omega_n t}\omega_n\cos(\omega_d \cdot t)}$$

$$= \frac{\zeta^2\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t) + e^{-\zeta\omega_n t}\cdot\omega_d\sin(\omega_d t)$$

MASSIMA SOVRAELONGAZIONE

$$y(t_p) = 1 - e^{-\zeta\omega_n \cdot \frac{\pi}{\omega_d}} \left(\cancel{\cos}\left(\cancel{\omega_d} \cdot \frac{\pi}{\cancel{\omega_d}}\right)^{-1} + \frac{\zeta}{\sqrt{1-\zeta^2}} \cancel{\sin}\left(\cancel{\omega_d} \cdot \frac{\pi}{\cancel{\omega_d}}\right)^{0} \right) = \boxed{1 + e^{-\zeta\frac{\pi}{\sqrt{1-\zeta^2}}}}$$

Valore picco $y(t_p)$

per $\zeta \to 0$: $1 + e^{0} = 2$

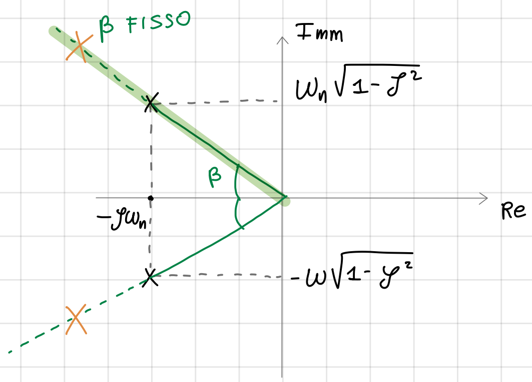
Valore a regime

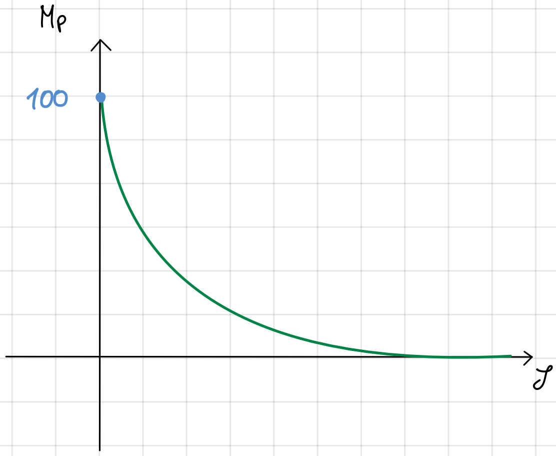
$$y(\infty) = \lim_{s\to 0} s \cdot G(s) \cdot U(s) = \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s} \Rightarrow \boxed{y(\infty) = \lim_{s\to 0} G(s)}$$

guadagno statico

$$\boxed{M_p = \frac{y(t_p) - 1}{1} \cdot 100 = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \cdot 100}$$

funzione solo di ζ $\to$ con $\omega_n = cost$ è uguale





TEMPO DI ASSESTAMENTO

* Scrivere ζ come Angolo

* $0 < \zeta < 1$

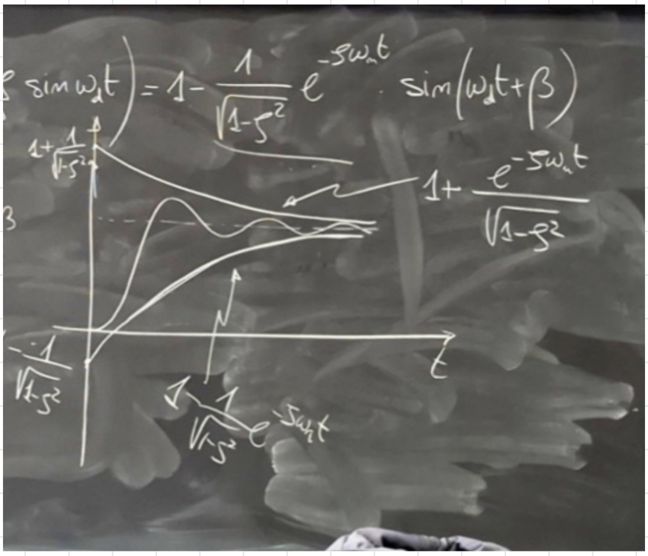
$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \left(\underbrace{\sqrt{1-\zeta^2}}_{\sin\beta} \cos(\omega_d t) + \underbrace{\zeta}_{\cos\beta} \sin(\omega_d t) \right)$$

$$\sin^2\alpha + \cos^2\alpha = 1 \quad , \forall \alpha$$

$$= 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \beta)$$

Il sin resta sempre all'interno di una coppia di curve di inviluppo

curve: $1 \pm \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}$



Si considera $4/5\ \tau$

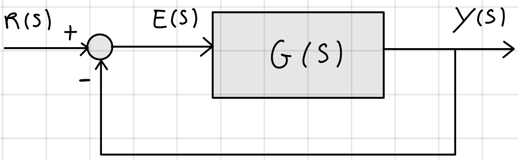
$$\Rightarrow \tau\ \frac{1}{\zeta\omega_n} \Rightarrow t_s = \frac{4}{\zeta\omega_n}$$

Recap

$$\boxed{e^{-\frac{t}{\tau}} = e^{-\alpha t} \Rightarrow \tau = \frac{1}{\alpha}}$$

ESEMPIO 5.1

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$



DATI
$\zeta = 0.6$
$\omega_n = 5$ rad/s

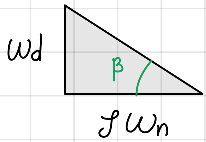
Q: Calcolare t_r, t_p, M_p e t_s con $u(t) = \mathbb{1}(t)$

$$u(t) = \mathbb{1}(t) \rightleftarrows U(s) = \frac{1}{s}$$

- $\omega_d = \omega_n\sqrt{1-\zeta^2} = 4$
- $\sigma = \zeta\omega_n = 3$

$$t_r = \frac{\pi - \beta}{\omega_d} = 0.55 \text{ secondi}$$ Ans 1

con



$\rightarrow \tan(\beta) = \frac{\omega_d}{\sigma}$

$\Rightarrow \beta = \operatorname{atan}\left(\frac{\omega_d}{\sigma}\right) = 0.93$ rad

$$t_p = \frac{\pi}{\omega_d} = 0.78 \text{ s}$$ Ans 2

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)}$$ con $y(\infty) = \lim_{s \to 0} s \cdot F(s)$ con $F(s) = Y(s) = G(s) \cdot U(s)$

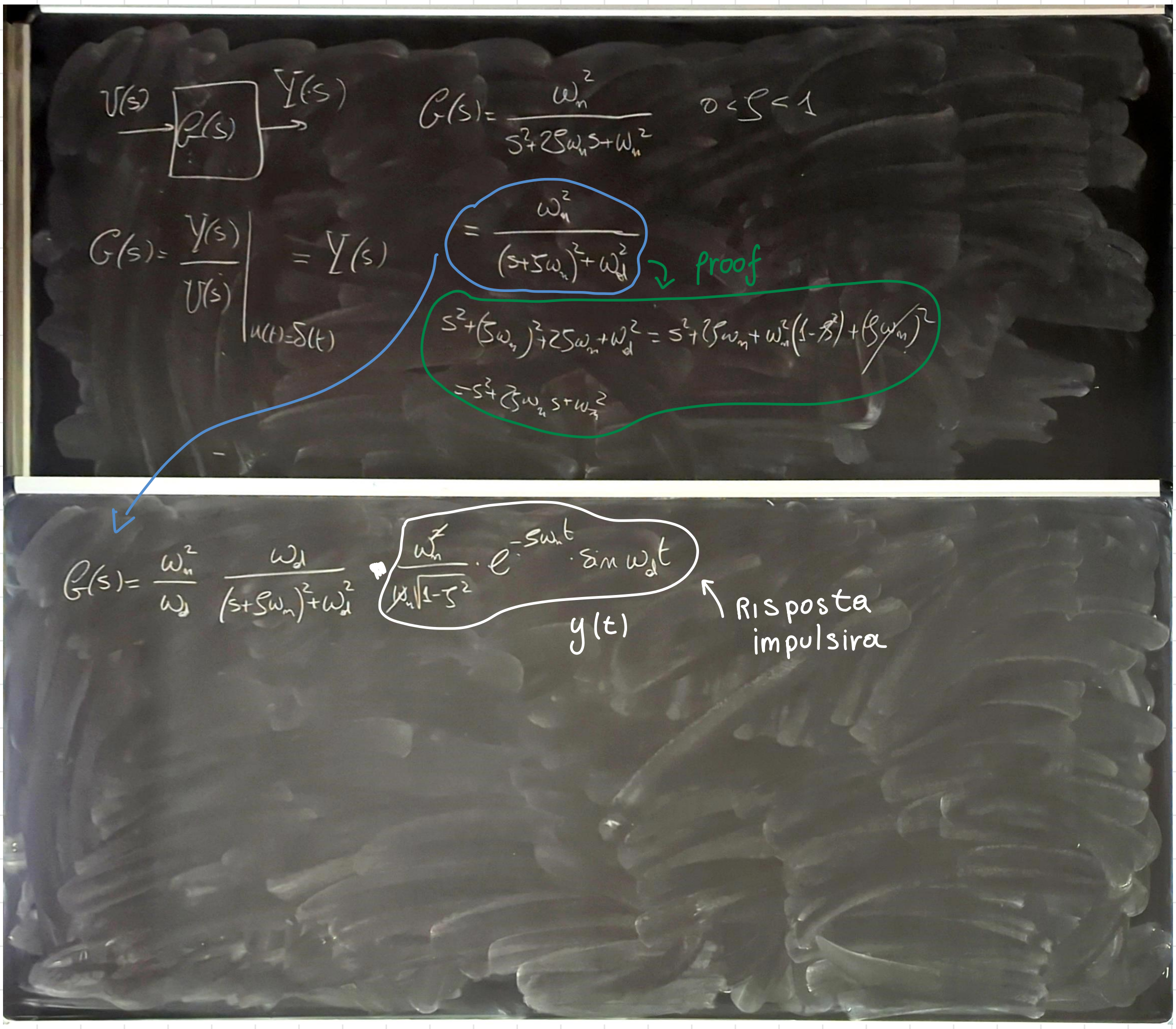
$$\rightsquigarrow Y(s) = \frac{\omega_n^2}{s^2(s+2\zeta\omega_n)} = \frac{\varepsilon_1}{s^2} + \frac{\varepsilon_2}{s+2\zeta\omega_n}$$ [LUNGO]

Calcolo invece $M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \cdot 100 = e^{-\left(\frac{\sigma}{\omega_d}\right)\pi} = 9.48 \times 10^{-2} \cong 9.5\%$

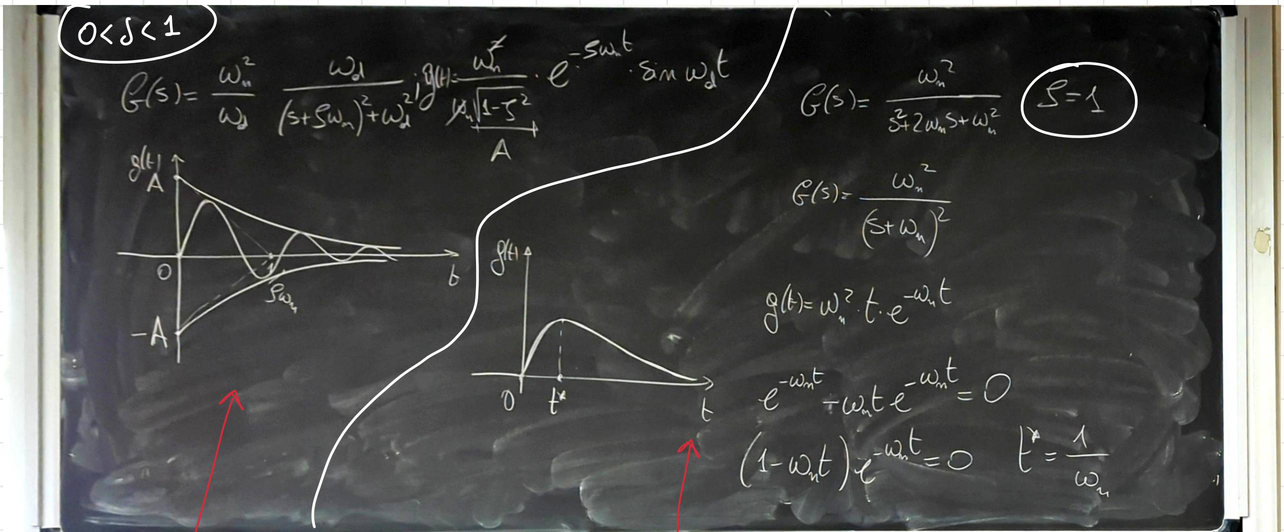
Ans 3

$$t_s = \frac{4}{\omega_d \zeta} = \frac{4}{\sigma} = 1.\bar{3} \text{ s}$$ Ans 4

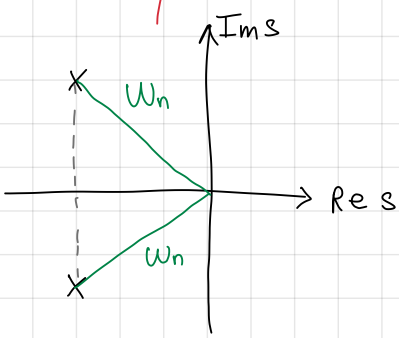
(RECAP)



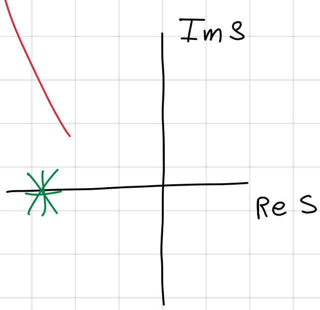
U(s)
G(s)
Y(s)
G(s) = ωn² / (s²+2ζωn s+ωn²)
0 < ζ < 1
G(s) = Y(s)/U(s) | u(t)=δ(t) = Y(s)
= ωn² / ((s+ζωn)²+ωd²)
proof
Risposta
impulsiva
y(t)



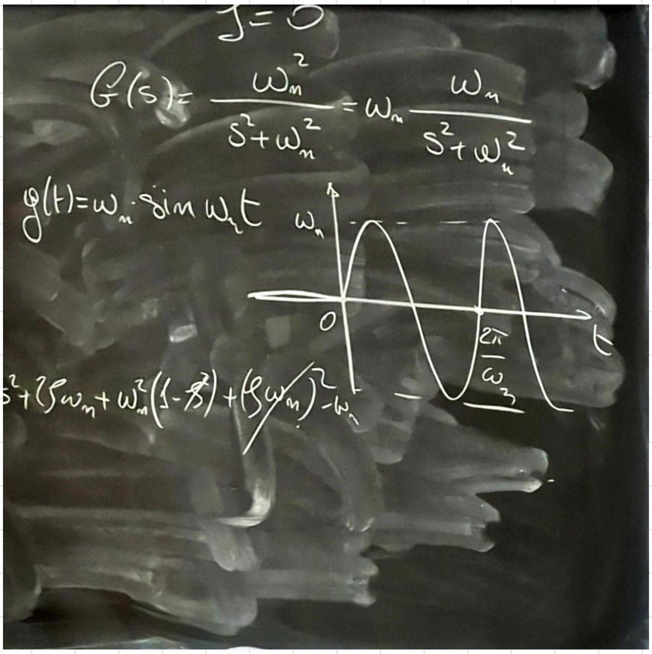
0<ζ<1
G(s) = ωn² / (s²+2ωn s+ωn²)
ζ=1
G(s) = ωn² / (s+ωn)²
g(t) = ωn² t · e^(-ωn t)
A
-A
0
t
t*
t* = 1/ωn

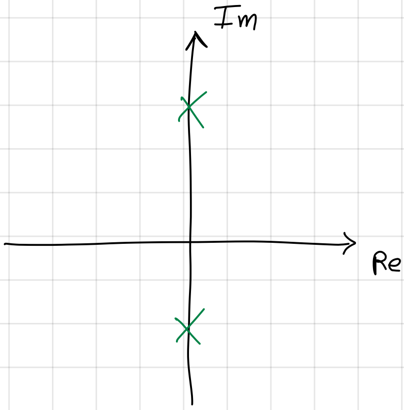


Im s
Re s
ωn
ωn



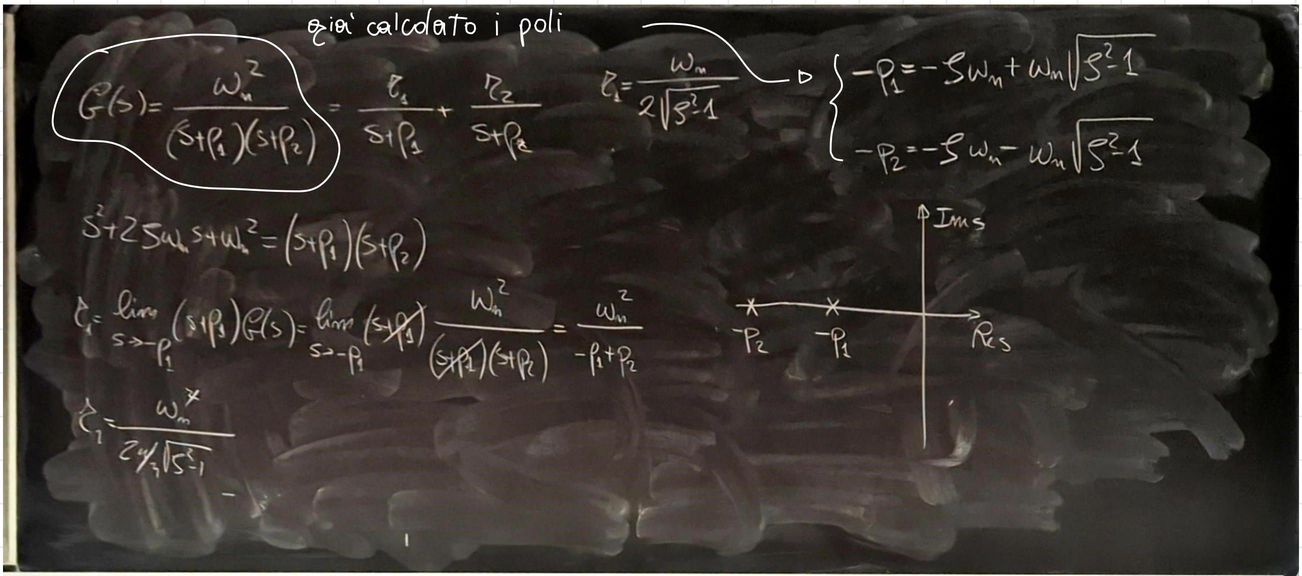
Im s
Re s

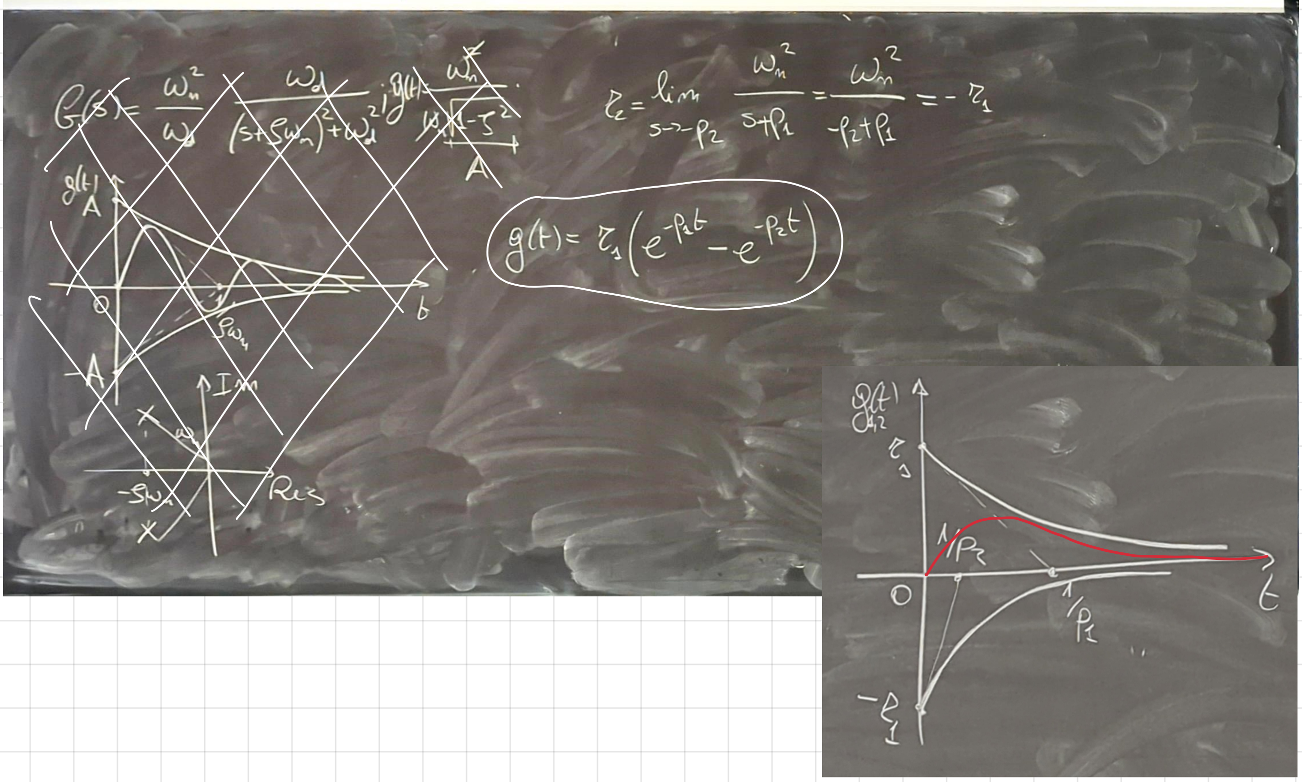




$\zeta > 0$ SOVRASMORZATO

AUDIO * Diversi CASI





COME VALUTARE UN SYS

$\frac{1}{s+p} \xrightarrow{\mathcal{L}^{-1}} e^{-pt}$

* Posizione del polo