2021/2022

Second list of projects
from *Numerical algorithms of algebra*

Remark: Everyone performs the task. All experiments should be prepared independently and performed in Matlab. Then write a report on the tests carried out.
Solve te system of the equation

$$Ax = b,$$

where $b = A * \texttt{ones}(1, \texttt{n})$. Note that the exact solution of this system is the vector $x = [1, 1, \ldots, 1]^T$.

- Use the Matlab command to solve the system.
- Write your own Gaussian elimination (without pivoting) to solve the system $Ax = b$. You can write the function, which parameters are n - the matrix degree, A - the matrix, b - the vector. The result of the function should be the vector x, which is the solution of the system $Ax = b$. Additionally, the fucntion can return the detrminant of the matrix A (det(A)),and the condition number of the matrix A (cond(A)) (standard Matlab functions: `cond, rand`).
- For every solution $x^{(cal)}$ calculate norms:

  $$\|x - x^{(obl)}\|, \qquad \|r\| = \|b - Ax^{(obl)}\|$$

  (Matlab function: `norm`) and the errors for the calculated solutions:

  $$\frac{\|x - x^{(cal)}\|}{\|x\|},$$

  where x is the exact solution of the equation $Ax = b$. Show the determinant of the matrix A and the condition number of the matrix A.
- Perform tests for different matrices (see in Matlab `help gallery` - look at the matrices). You can also create your matrices in the following way:

  Let B, C be the random matrices (MAtlab fucntions: `rand` or `randn`) and $B = Q_B R_B$, $C = Q_C R_C$ are qr decompositions of these matrices (Matlab function: `qr`). Then A can be calculated from the formula:

  $$A = Q_B \mathrm{diag}(d_j) Q_C,$$

  where the diagonal elements d_j are given.
- Make the right conclusions.