

## Set 5

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```
library(ggplot2)
library(stats)
library(readxl)
```

### Question One

```
birth_dat <- read_excel("birth_dat.xlsx")
head(birth_dat)
```

```
## # A tibble: 6 x 5
##   Nation      Birthrate PerCapIncome PopFarms InfantMort
##   <chr>      <dbl>      <dbl>    <dbl>    <dbl>
## 1 Venezuela    46.4        392     0.4     68.5
## 2 Mexico       45.7        118     0.61    87.8
## 3 Ecuador      45.3         44     0.53   116.
## 4 Colombia     38.6        158     0.53   107.
## 5 Ceylon        37.2         81     0.53    71.6
## 6 Puerto Rico   35          374     0.37    60.2
```

```
X <- model.matrix(~ PerCapIncome + PopFarms + InfantMort, data= birth_dat)
head(X)
```

```
##   (Intercept) PerCapIncome PopFarms InfantMort
## 1           1           392     0.40     68.5
## 2           1           118     0.61     87.8
## 3           1            44     0.53    115.8
## 4           1           158     0.53    106.8
## 5           1            81     0.53     71.6
## 6           1           374     0.37     60.2
```

```
y = birth_dat$Birthrate
XtX <- t(X) %*% X
beta.hat <- solve(XtX, t(X)%*%y)
beta.hat
```

```
##           [,1]
## (Intercept) 5.553868478
## PerCapIncome 0.006566304
## PopFarms     9.104754908
## InfantMort   0.242689935
```

```
model <- lm(Birthrate ~ PerCapIncome + PopFarms + InfantMort, data = birth_dat)
coef(model)
```

```
## (Intercept) PerCapIncome      PopFarms      InfantMort
## 5.553868478 0.006566304 9.104754908 0.242689935
```

*#The results are the similar so in comparison, they are equal to each other.*

```
mexico_row <- birth_dat[birth_dat$Nation == "Mexico", ]
mexico_row
```

```
## # A tibble: 1 x 5
##   Nation Birthrate PerCapIncome PopFarms InfantMort
##   <chr>      <dbl>      <dbl>    <dbl>    <dbl>
## 1 Mexico      45.7          118      0.61     87.8
```

```
X_mexico <- model.matrix(~ PerCapIncome + PopFarms + InfantMort, data = mexico_row)
fitted_mexico <- X_mexico %*% beta.hat
fitted_mexico
```

```
##      [,1]
## 1 33.19077
```

```
new_country <- data.frame(
  PerCapIncome = 600,
  PopFarms = 0.2,
  InfantMort = 20
)
```

```
predicted_value <- predict(model, newdata = new_country)
predicted_value
```

```
##      1
## 16.1684
```

```
residuals <- residuals(model)
RSS <- (sum(residuals^2))
RSS
```

```
## [1] 1423.854
```

## Question 2

```
x <- 1:20
set.seed(463)
y <- x + rnorm(20)
```

```
model1 <- lm(y~x)
beta_model1 <- coef(model1)
beta_model1
```

```
## (Intercept)          x
## 0.0876554    1.0136850
```

```
X <- cbind(1, x)
XtX <- t(X) %*% X
beta_hat <- solve(XtX, t(X)%*%y)
beta_hat
```

```
##          [,1]
## 0.0876554
## x 1.0136850
```

```
x_2 <- x^2
model2 <- lm(y ~ x + x_2)
coef(model2)
```

```
## (Intercept)          x          x_2
## -0.298169776  1.118910079 -0.005010717
```

```
X <- model.matrix(~ x + x_2)
#head(X)
XtX1 <- t(X) %*% X
beta_hat1 <- solve(XtX1, t(X)%*%y)
beta_hat1
```

```
##          [,1]
## (Intercept) -0.298169776
## x           1.118910079
## x_2         -0.005010717
```

```
# Function to calculate estimates for polynomial terms up to degree 10
calculate_polynomial_estimates <- function(degree) {

  X_poly <- sapply(0:degree, function(d) x^d)

  # Attempting direct inversion
  tryCatch({
    XtX1 <- t(X_poly) %*% X_poly
    beta_direct <- solve(XtX1, t(X_poly)%*%y)
    return(beta_direct)
  }, error = function(e) {
    return("Direct inversion failed")
  })
}

for (degree in 0:10) {
  cat("Degree:", degree, "\n")
  print(calculate_polynomial_estimates(degree))
}
```

```

## Degree: 0
##      [,1]
## [1,] 10.73135
## Degree: 1
##      [,1]
## [1,] 0.0876554
## [2,] 1.0136850
## Degree: 2
##      [,1]
## [1,] -0.298169776
## [2,] 1.118910079
## [3,] -0.005010717
## Degree: 3
##      [,1]
## [1,] -1.134761912
## [2,] 1.545787791
## [3,] -0.054611041
## [4,] 0.001574613
## Degree: 4
##      [,1]
## [1,] -1.490596e+00
## [2,] 1.824149e+00
## [3,] -1.109391e-01
## [4,] 5.676782e-03
## [5,] -9.767067e-05
## Degree: 5
##      [,1]
## [1,] -1.280435e+00
## [2,] 1.600212e+00
## [3,] -4.334297e-02
## [4,] -2.574586e-03
## [5,] 3.384335e-04
## [6,] -8.306746e-06
## Degree: 6
## [1] "Direct inversion failed"
## Degree: 7
## [1] "Direct inversion failed"
## Degree: 8
## [1] "Direct inversion failed"
## Degree: 9
## [1] "Direct inversion failed"
## Degree: 10
## [1] "Direct inversion failed"

```

*#It fails when the degree is 6.*

```

set.seed(463)
x2 <- x + rnorm(20, sd = 0.1)
model3 <- lm(y ~ x + x2)
correlation_x_x2 <- cor(x, x2)
beta_lm3 <- coef(model3)
correlation_x_x2

```

```
## [1] 0.9999045
```

```
beta_lm3
```

```
## (Intercept)          x          x2
## 5.08423e-14 -9.00000e+00 1.00000e+01
```

```
#Both of them are highly correlated
```

```
set.seed(463)
x3 <- x + rnorm(20, sd = 0.001)
model4 <- lm(y ~ x + x3)
beta_lm4 <- coef(model4)
cor(x, x3)
```

```
## [1] 1
```

```
beta_lm4
```

```
## (Intercept)          x          x3
## 4.880861e-12 -9.990000e+02 1.000000e+03
```

c) It fails when the number of degree is 6.

d) They are highly correlated (although this might raise the concern for multicollinearity).

e) Using the correlation being “1”, there might be an issue of multicollinearity. The coefficient for x is approximately -999 and the coefficient for x3 is approximately 1000, this might show that an increase in x can be almost exactly offset by an increase in x3, but it might seem the model is assigning high sensitivity for small changes in data.