Data Structures

The Practice of Haskell Programming

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Well-Typed

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Overview

- Persistent data structures
- Arrays
- Trees, sets and finite maps
- Other useful data structures

Persistent data structures

Imperative vs. functional style

Given a finite map (associative map, dictionary) `m`.

Imperative style

```
foo.put (42, "Bar"); ...
```

Functional style

```
let foo′ = insert 42 "Bar" m in ...
```

What is the difference?

Imperative vs. functional style

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Imperative: destructive update

Functional: creation of a new value

Persistent data structures

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- most operations create a new data structure
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Data structures where old version remain accessible are called **persistent**.

Persistent data structures

- In functional languages, most data structures are (automatically) persistent.
- In imperative languages, most data structures are not persistent (**ephemeral**).
- It is generally possible to also use ephemeral data structures in functional or persistent data structures in imperative languages.

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How do persistent data structures work?

Example: Haskell lists

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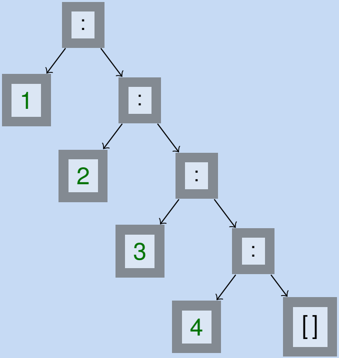
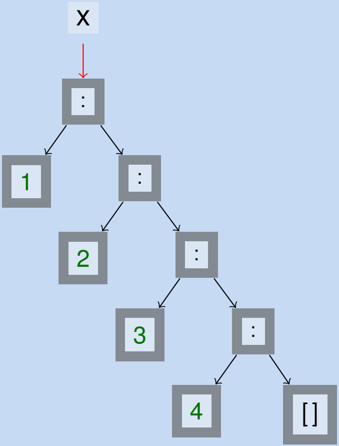
Representation in memory:



Lists are persistent

$\mathbf{let}\ x = [1, 2, 3, 4]; y = 0 : x; z = \mathrm{drop}\ 2\ y\ \mathbf{in} \ldots$



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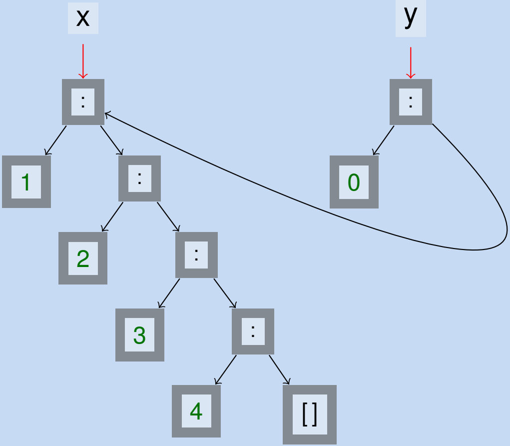

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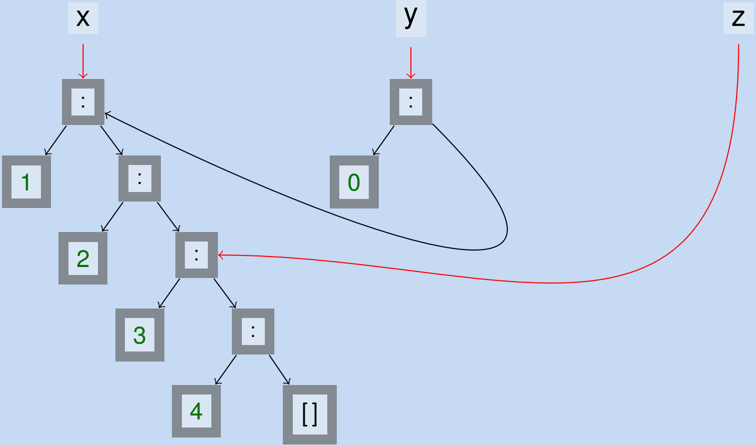
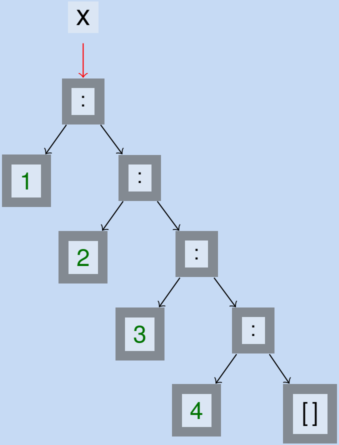


Lists are persistent – contd.

let x = [1, 2, 3, 4]; w = take 2 x **in** . . .



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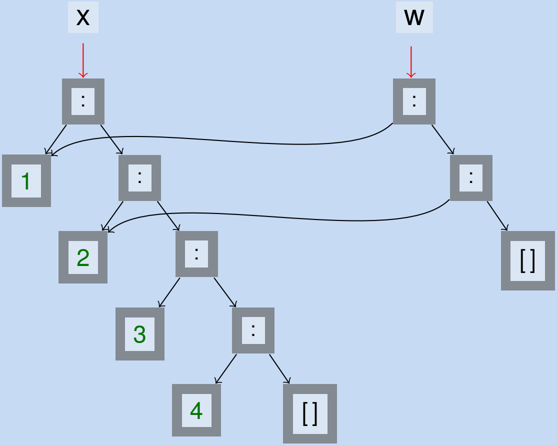

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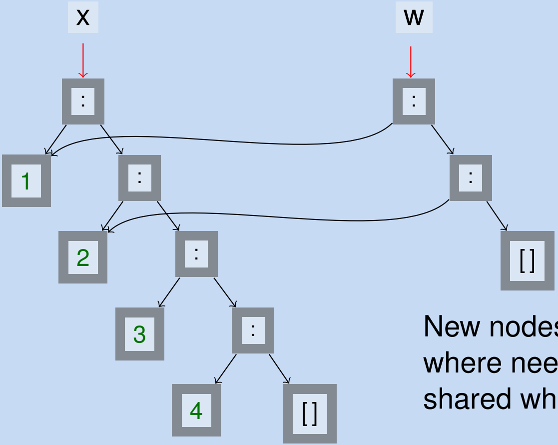

New nodes are allocated where needed; nodes are shared where possible.

Implementation of persistent data structures

- Modifications of an existing structure take place by creating new nodes and pointers.
- Sometimes, parts of a structure have to be copied, because the old version must not be modified.

Of course, we want to copy as little as possible, and reuse as much as possible.

Vacuum

Vacuum

Vacuum is a library originally developed by Matt Morrow:

- the library is a debugging tool,
- we can query and generate the internal graph representation of Haskell terms,
- useful to understand how Haskell terms are shared.

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```
〉 view (let x = [1, 2] in x ++ x)
〉 view (let x = [1, 2, 3, 4] y = 0 : x; z = drop 3 y in (x, y, z))
〉 view (let x = [1, 2, 3, 4]; w = take 2 x in (x, w))
〉 view (repeat 1)
```

Arrays

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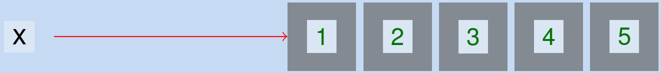
How expensive is the update operation?

- In an imperative language, we expect O(1), i.e., constant time.

Arrays

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- Arrays are stored in a contiguous block of memory.
- This allows O(1) access to each element.
- In an imperative setting, a destructive update is also possible in O(1).

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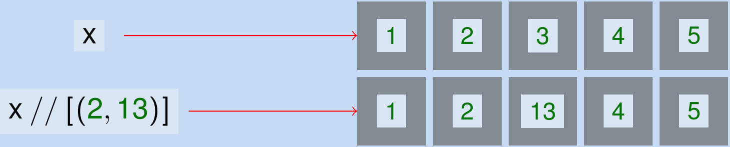

- Arrays are stored in a contiguous block of memory.
- This allows O(1) access to each element.
- In an imperative setting, a destructive update is also possible in O(1).
- But if a persistent update is desired, the whole array must be copied, which takes O(n), i.e., linear time.

Advice on arrays

Be careful when using them:

- stay away if you require a large number of incremental updates – finite maps are usually much better then;
- arrays can be useful if you have an essentially constant table that you need to access frequently;
- arrays can also be useful if you perform global updates on them anyway.

The `vector` package

There's a new, quite popular array package available from Hackage called `vector`:

- Developed by Roman Leshchinskiy.
- An interface capturing mutable and immutable arrays, boxed and unboxed arrays in a slightly more systematic way than the standard Haskell array interface allows.
- Support slicing operations.

Trees

Trees

- Arrays and hash tables are expensive in a functional (persistent) setting, because it is impossible to share substructures between different versions.
- Tree-shaped structures, however, are generally very suitable in a functional setting. Reuse of subtrees is easy to achieve. Most functional data structures therefore are some sort of trees.

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Lists are trees, too – just a very peculiar variant.

Lists

- There is a lot of syntactic sugar for lists in Haskell. Thus, lists are used for a lot of different purposes.
- Lists are the default data structure in functional languages much as arrays are in imperative languages.
- However, lists support only **very few operations efficiently**.

Operations on lists

```
[]      :: [a]                                   -- O(1)
(:)     :: a → [a] → [a]                         -- O(1)
head    :: [a] → a                               -- O(1)
tail    :: [a] → [a]                             -- O(1)
snoc    :: [a] → a → [a]                         -- O(n)
snoc    = λxs x → xs ++ [x]
(!!)    :: [a] → Int → a                         -- O(n)
(++)    :: [a] → [a] → [a]                       -- O(m), first list
reverse :: [a] → [a]                             -- O(n)
splitAt :: Int → [a] → ([a], [a])                -- O(n)
union   :: Eq a ⇒ [a] → [a] → [a]                -- O(mn)
elem    :: Eq a ⇒ a → [a] → Bool                 -- O(n)
```

Guidelines for using lists

Lists are suitable for use if:

- most operations we need are **stack operations**,
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Lists are generally not suitable:

- for random access,
- for set operations such as union and intersection,
- to deal with (really) large amounts of texts as `String`.

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Question

What is a (binary) search tree?

Finite maps

- A finite map is a function with a finite domain (type of **keys**).
- Useful for a wide variety of applications (tables, environments, “arrays”).
- Inefficient representation: **type** Map a b $=$ $[(a, b)]$.

An efficient implementation of finite maps

- Based on binary search trees.
- Available in `Data.Map` and `Data.IntMap` for `Int` as key type.
- Provided by the `containers` package that is part of the Haskell Platform.
- Keys are stored ordered in the tree, so that efficient lookup is possible.
- Requires the keys to be ordered.
- Inserting and removing elements can trigger rotations to rebalance the tree.
- Everything happens in a persistent setting.

Sets

- Sets are a special case of finite maps: essentially,

```
type Set a = Map a ()
```

- A specialized set implementation is available in `Data.Set` and `Data.IntSet`, but the idea is the same as for finite maps.

Finite map interface

This is an excerpt from the functions available in `Data.Map`:

```
data Map k a          -- abstract

insert :: (Ord k) ⇒ k → a → Map k a → Map k a        -- O(log n)
lookup :: (Ord k) ⇒ k → Map k a → Maybe a            -- O(log n)
delete :: (Ord k) ⇒ k → Map k a → Map k a            -- O(log n)
update :: (Ord k) ⇒ (a → Maybe a) →
                    k → Map k a → Map k a            -- O(log n)
union  :: (Ord k) ⇒ Map k a → Map k a → Map k a      -- O(m + n)
member :: (Ord k) ⇒ k → Map k a → Bool               -- O(log n)
size   :: Map k a → Int                              -- O(1)
map    :: (a → b) → Map k a → Map k b                -- O(n)
```

The interface for `Set` is very similar.

Implementing finite maps

Implementation

In the following, we will sketch the implementation as it is available in `Data.Map`:

```haskell
data Map k a = Tip
             | Bin !Size (Map k a) k a (Map k a)
type Size = Int
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- either a leaf called `Tip`,
- or a binary node called `Bin` containing
  - the size of the tree,
  - the key value pair,
  - and a left and right subtree.

Creating finite maps

```
empty :: Map k a
empty = Tip

singleton :: k → a → Map k a
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The function `bin` is an example of a **smart constructor** ...

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```

```
size :: Map k a → Int
size Tip                = 0
size (Bin sz _ _ _ _) = sz
```

Finding an element

```
lookup :: Ord k ⇒ k → Map k a → Maybe a
lookup key Tip              = Nothing
lookup key (Bin _ l kx x r) =
  case compare key kx of
    LT → lookup key l
    GT → lookup key r
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Comparing two elements:

```
compare :: Ord a ⇒ a → a → Ordering
data Ordering = LT | EQ | GT
```

Inserting an element

```
insert :: Ord k ⇒ k → a → Map k a → Map k a
insert kx x Tip              = singleton kx x    -- insert new
insert kx x (Bin sz l ky y r) =
  case compare kx ky of
    LT → balance (insert kx x l) ky y              r
    GT → balance                 l  ky y (insert kx x r)
    EQ → Bin sz l kx x r    -- replace old
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```

The function `balance` is an even smarter constructor with the same type as `bin`:

```
balance :: Map k a → k → a → Map k a → Map k a
```

Balancing the tree

We could just define

$$\text{balance} = \text{bin}$$

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Question

What is the problem, and when does it arise?

Balancing approach

- If the height of the two subtrees is not too different, we just use `Bin`.
- Otherwise, we perform a rotation.

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Rotation

A rearrangement of the tree that preserves the search tree property.

Rotation

```
rotateL :: Map a b → a → b → Map a b → Map a b
rotateL l kx x r@(Bin _ ly _ _ ry)
    | size ly < ratio * size ry = singleL  l kx x r
    | otherwise                 = doubleL l kx x r
rotateL _ _ _ Tip = error "rotateL Tip"
```

Depending on the shape of the tree, either a simple (single) or a more complex (double) rotation is performed.

Rotation – contd.

```
singleL :: Map a b → a → b → Map a b → Map a b
singleL t1 k1 x1 (Bin _ t2 k2 x2 t3) =
  bin (bin t1 k1 x1 t2) k2 x2 t3
```

Rotation – contd.

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```
doubleL :: Map a b → a → b → Map a b → Map a b
doubleL t1 k1 x1 (Bin _ (Bin _ t2 k2 x2 t3) k3 x3 t4) =
  bin (bin t1 k1 x1 t2) k2 x2 (bin t3 k3 x3 t4)
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Rotation – contd.

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```

```
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doubleL t1 k1 x1 (Bin _ (Bin _ t2 k2 x2 t3) k3 x3 t4) =
  bin (bin t1 k1 x1 t2) k2 x2 (bin t3 k3 x3 t4)
```

Note how easy it is to see that these rotations preserve the search tree property.

Other useful data structures

Finger trees

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- Supports search tree operations in logarithmic time.
- Supports cons and snoc in $O(1)$.
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Available in Data.Sequence from `containers` and in an extended version in the `fingertree` package on Hackage.

Byte strings and `Text`

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There are other suitable datatypes for strings:

- Byte strings are stored as compact arrays (provided by `bytestring`. Mainly suitable for low-level or binary data.
- `Text` (provided by `text`) is a convenient datatype for text that wraps `bytestring` and deals with encoding issues.
- To prevent the typical array problems, a clever form of optimization called stream fusion is being used.

More on Hackage

On Hackage, there are several additional libraries for data structures.

Some examples: heaps, priority search queues, hash maps, heterogeneous lists, zippers, tries, graphs, quadtrees, ...

Summary

- It is important to keep persistence in mind when thinking about functional data structures.
- Arrays should be used with care.
- Lists are ok for stack-like use or simple traversals.
- Good general-purpose data structures are sets, finite maps and sequences.