

## Section A. Quantum Mechanics

### 1. Interacting spins

Four distinguishable spin-1/2 objects interact. The Hamiltonian is

$$H = A\vec{\sigma}_1 \cdot \vec{\sigma}_2 + B\vec{\sigma}_3 \cdot \vec{\sigma}_4 + C(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{\sigma}_3 + \vec{\sigma}_4) ,$$

where  $\vec{\sigma}_n$  are the Pauli spin operators for spin  $n$ .

(a) List a complete set of good quantum numbers for the eigenstates of this Hamiltonian.

(b) List the eigenenergies and the degeneracy of each energy level.

(c) Using the basis of the eigenstates of the  $z$ -components of each spin, show a **ground state** for the case  $A = B < 0 < C$ . For this state give its complete set of good quantum numbers.