3. **White Dwarf Star**

Model a white dwarf star as a degenerate non-interacting Fermi gas of electrons, supported against gravitational collapse by the electron degeneracy pressure. For simplicity, assume that the star is a sphere of radius R and *uniform* mass density, containing N electrons of mass m_e and N protons of mass $m_p \gg m_e$ for an approximate total mass of $M = Nm_p$.

(a) First, assume that the electrons are non-relativistic. Find their Fermi energy and thus calculate their contribution to the star's total ground-state kinetic energy.

(b) The gravitational potential energy of a uniform density sphere is

$$U_{grav} = -\frac{3GM^2}{5R} \ .$$

Find the equilibrium radius R for the ground state of this white dwarf, neglecting the protons' kinetic energy. How does this radius depend on the mass M? At roughly what mass M do the electrons in this ground state become relativistic?

(c) If instead the electrons are highly relativistic, so that their mass can be neglected, what is their Fermi energy and their total ground-state kinetic energy?

(d) Under what conditions is this highly-relativistic degenerate electron star unstable to gravitational collapse?

[This (d) is called the Chandrasehkar limit. A star that violates the limit will collapse into a neutron star or a black hole, depending on whether or not the neutron degeneracy pressure can support the star.]