3. **Spins on a square**

Four spin-S spins are located at the corners of a square and interact antiferromagnetically ($J > 0$). Use the convention where the spin operators are dimensionless. The Hamiltonian is

$$H = J(\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_4 + \mathbf{S}_4 \cdot \mathbf{S}_1) \ .$$

(a) What are a complete set of good quantum numbers that can be used to fully classify all of the eigenstates of H?

(b) For spin-1/2 give the eigenenergy and the degeneracy of each energy level.

(c) For general spin S, what are the energy, degeneracy, and quantum numbers of the ground state?