Section A. Mechanics

1. **Dark Matter**

   There is evidence that the visible stars in our galaxy move in a very diffuse background of dark matter. For simplicity, assume that in the neighborhood of a star, this dark matter has a mass distribution that is spherically symmetric about the star. Consider the effect of this extra matter on a planet orbiting around a star of mass M. The dark matter interacts with the planet only via gravitational attraction. Assume that near and inside the radius of planet's orbit, the mass density ρ of the dark matter is spatially uniform. Treat all motion and gravity nonrelativistically.

   (a) Compute the angular frequency of a planet in circular orbit (radius R) about the star.

   (b) Show that a nearly circular orbit will precess and compute the angular amount of precession per orbit. You may assume that the total mass of the dark matter inside a sphere of radius equal to that of the planet's orbit is small compared to the mass M of the star.