2. **Graphene**

Graphene is a two-dimensional sheet of carbon atoms. Both electronic and phononic degrees of freedom contribute to the low temperature specific heat per unit area. The energies of the electron states near the Fermi energy are

$$\epsilon_{\pm}(\vec{p}) = \epsilon_0 \pm v_F p,$$

where $\vec{p} = (p_x, p_y)$ is the the momentum of the electron and $p \equiv |\vec{p}|$. (There are two energy bands, $\epsilon_+(\vec{p}) \geq \epsilon_0$ and $\epsilon_-(\vec{p}) \leq \epsilon_0$, which are degenerate at $p = 0$.) These states have a fourfold degeneracy at each value of the momentum: the usual two-fold spin degeneracy is doubled by an additional valley index.

(a) If the Fermi energy ϵ_F is $\epsilon_0 + v_F p_F$ with $p_F > 0$, what is the leading behavior of the electronic specific heat as $T \to 0$?

(b) What is the leading low-temperature electronic specific heat when $p_F = 0$?

The next calculation is independent of parts a) and b) above.

Recently freely suspended Graphene sheets have been studied. These have an unusual phonon spectrum. In addition to the two dimensional longitudinal and transverse waves with frequencies $\omega = v_L q, v_T q$ at wavenumber of magnitude $q = |\vec{q}|$, there is an extra low-frequency mode with $\omega = Kq^2$ with the atomic displacements are normal to the sheet.

(c) Obtain the leading behavior of the phonon contribution to the specific heat as $T \to 0$.

You may express your answers in terms of the numerical constants

$$C_n^{\pm} = \int_0^{\infty} dx \frac{x^n}{e^x \pm 1}$$