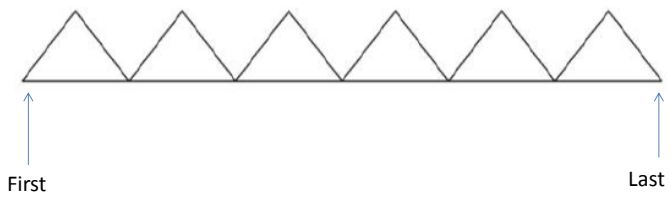
3. Frustrated and Unfrustrated Magnetism

Consider the triangular chain shown below. We will study the antiferromagnetic Ising model on this lattice defined by the Hamiltonian

$$H = J \sum_{\langle ij \rangle} S_i S_j$$

where the spins S_i live on the sites of the lattice and take values ± 1, the exchange is antiferromagnetic ($J > 0$) and the sum runs over all nearest neighbor pairs $\langle ij \rangle$ on the lattice (vertices joined by links in the figure).



(a) Consider a system consisting of a single triangle at $T = 0$. How many ground states (minimum energy configurations) does it have?

(b) Consider a system of N triangles at $T = 0$. How many ground states does the chain have, supposing free boundary conditions?

(c) Calculate the correlation $\langle S_{\text{first}} S_{\text{last}} \rangle$ averaged over the ground states for the first and last spins on the bottom row of the chain with N triangles.

(d) How would the answers to the above three questions change if the exchange were made ferromagnetic instead, i.e. $J < 0$?

(Hint: In parts (b) and (c) fairly simple counting arguments will give exact results.)