J98M.1—Hanging Rope

Problem

A piece of thin uniform unstretchable rope has length $2L$ and mass M. Its ends are attached to points at the same height separated by distance $2w$, and the rope hangs between them under the influence of gravity (of course, $w < L$). Let us set up coordinates (x, y) in the plane of the rope, so that the end points have equal values of y, and $x = \pm w$. You will be asked to determine the vertical coordinate of the rope, y, as a function of x.

a) Write down the functional of $y(x)$ that has to be minimized. What is the form of the constraint?

b) One may think of the functional to be minimized as an action for a 1-dimensional particle with coordinate y and time x. Find a conserved quantity.

c) For a given value of the conserved quantity, find $y(x)$. What is the equation relating the conserved quantity to w and L?