3. **Protein folding**

The thermodynamics of protein folding plays an essential role in biology.

This problem is all at one atmosphere of pressure: Consider a large protein that folds at temperature T_1, so for $T > T_1$ it is unfolded at equilibrium, while for a range of temperature T with $T < T_1$ it is folded at equilibrium in to a compact form and thus "hides" some hydrophobic parts of the long protein molecule from the surrounding water. Assume the folded (f) state of the protein has a temperature-independent specific heat $C^{(f)}$ and that its thermal expansion is negligible so $C_P^{(f)} = C_V^{(f)}$. Assume the same is true for the unfolded (u) state, so $C_P^{(u)} = C_V^{(u)} = C^{(u)}$ is also temperature-independent. The latent heat released on unfolding at temperature T_1 is $Q > 0$. The unfolded state has the larger specific heat: $\Delta C = C^{(u)} - C^{(f)} > 0$.

(a) What is the enthalpy $(H = E + PV)$ difference, $\Delta H = H^{(u)} - H^{(f)}$, between the unfolded and folded states as a function of T? Sketch a plot of this function, assuming the above assumptions.

(b) What is the Gibbs free energy $(G = H - TS)$ difference, $\Delta G = G^{(u)} - G^{(f)}$, between the unfolded and folded states as a function of T?

(c) What conditions on the parameters mean that this protein will unfold also at some low temperatures, given all the above assumptions? Sketch ΔG vs. T for such a case with two folding transitions.