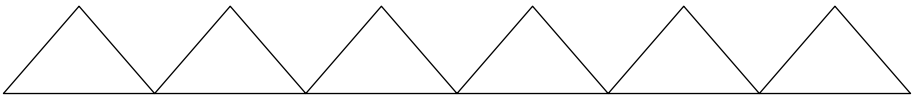
J01T.3—Triangular Chain Ising Model

Problem

Mean field treatments of ferromagnetic systems are quite useful in general, but for certain "frustrated" antiferromagnets this may not be the case. Consider the antiferromagnetic Ising model on the triangular chain shown below, with Hamiltonian

$$H = J \sum_{\langle ij \rangle} s_i s_j,$$

where $J > 0$, the spins s_i have values ± 1, and the sum runs over all nearest neighbor pairs on the lattice (vertices joined by links in the figure).



a) Consider a system of N triangles at temperature T. Assuming that all the top row spins have zero thermal expectation values, write down a mean field theory for the spins. What is the transition temperature?

b) Consider a system consisting of a single triangle at $T = 0$. How many ground states (minimum energy configurations) does it have?

c) Consider a system of N triangles at $T = 0$. How many ground states does the chain have, supposing free boundary conditions?

d) Calculate the correlation $\langle s_i s_j \rangle$, averaged over the ground states for two spins on the bottom row (N triangles). Is this consistent with the answer in part a)?

(Hint: In parts c) and d) fairly simple counting arguments can give exact results.)