## M05E.2 - Coaxial Cable Motion

## **Problem**

A long, straight coaxial cable of length L has an inner conductor of radius a and an outer conductor of inner radius b. Assume the insulating region between the conductors has free-space values of the electric and magnetic permittivities. One end of the cable is attached to a load resistor R. At the other end of the cable a battery of voltage V is suddenly attached. Just before the current starts to flow, the battery, cable and resistor are all at rest with no external forces. Their total mass is M. After some time, a steady-state current flows along the inner conductor and returns along the outer one. The current may be taken as uniformly distributed throughout the inner conductor, which has conductivity  $\sigma$ . The resistance of the outer conductor can be neglected, so the return current flows as a thin surface sheet of radius b.

- a) What is the electromagnetic momentum per unit length in the coax?
- b) What is the mechanical momentum of the cable with its attached battery and resistor after the steady-state current has been established.
- c) What is the charge per unit length on the outer conductor?

Possibly useful identity for cylindrical coordinates  $\rho$ , z and  $\theta$ :

$$\nabla^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$