M08Q.3 - Dynamics of Spin

Problem

A spin of $s = 1/2$ has its z-component "up" at time $t = 0$. The dynamics of the spin are given by the Hamiltonian

$$H = \lambda \hbar \sigma_x \,,$$

where σ_x is the usual Pauli matrix for a spin-1/2.

a) If the z-component of this spin is measured at time $t = \tau$, what are the probabilities of each possible result of this measurement?

b) Now consider a slightly different question: the spin again starts "up" at time $t = 0$, but now its z-component is measured twice, once at time $t = \tau/2$ and then again at time $t = \tau$. The above Hamiltonian gives the spin's dynamics between the measurements, and you can assume the measurements happen instantaneously. However, the result of the first measurement at time $t = \tau/2$ is not known to you. Now what are the probabilities of each possible result of the second measurement at time $t = \tau$?