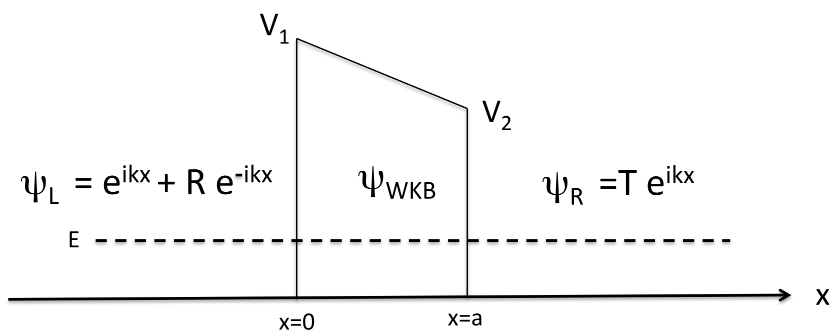
2. **Tunneling Under A Sloping Barrier**



Consider an electron of energy E moving in one dimension and tunneling from left to right under the potential barrier shown in the figure. The potential has a constant slope $dV/dx = -(V_1 - V_2)/a$ in the barrier region (as might be the case in an electric dipole layer). The electron wave function for $x < 0$ is $\psi_L = e^{ikx} + Re^{-ikx}$ and the wave function for $x > a$ is $\psi_R = Te^{ikx}$. You are to calculate T, but to do that you need to know $\psi(x)$ in the classically forbidden region $0 < x < a$. For the purposes of this problem, let us assume that the WKB approximation to $\psi(x)$ is valid everywhere under the barrier. (NB: This means that the WKB wave vector $\kappa(x) = \sqrt{2m(V(x) - E)/\hbar^2}$ satisfies $\frac{1}{\kappa}\frac{d\kappa}{dx} \ll \kappa$ everywhere under the barrier.)

(a) Write down the WKB wave function in the barrier region. Remember that there are two WKB solutions, and they will both be present with unknown amplitudes. At this stage leave any WKB integrals as implicit functions of x.

(b) To verify that you have the correct form of the WKB approximation, calculate the probability flux in the barrier region for the general case where the two amplitudes are complex. Use any simplifications that follow from the WKB validity condition $\frac{d}{dx} log\kappa(x) \ll \kappa(x)$. Is the flux independent of x? If not, revisit part (a).

(c) The WKB wave function has to join correctly at $x = 0$ and at $x = a$ to the exterior wave functions ψ_L, ψ_R. Write down the two sets of matching conditions. Once again, use any simplifications that follow from from the WKB validity condition $\frac{d}{dx} log\kappa(x) \ll \kappa(x)$.

(d) Now also assume that the barrier is wide enough so that the WKB integral $\int_0^a dx\kappa(x) \gg 1$. Use this as needed to simplify the algebra and thus solve for the leading behavior of the transmission coefficient T in this limit.