Section A. Quantum Mechanics

1. Consider a toy model of the Helium atom where the Coulombic interaction potential is replaced with a Hooke's law potential. If the nucleus of the atom is located at $\vec{r} = 0$ and the electrons of mass m have position vectors $\vec{r}_1$ and $\vec{r}_2$, the interaction potential is

$$V(\vec{r}_1, \vec{r}_2) = \frac{1}{2} m\omega^2 \left(\vec{r}_1^{\,2} + \vec{r}_2^{\,2}\right) - \frac{\lambda}{4} m\omega^2 \left(\vec{r}_1 - \vec{r}_2\right)^2 .$$

This model is exactly solvable. Assume $\lambda > 0$.

(a) What constraint must be imposed on λ for the system to be well-behaved? *[Hint: It may be useful to consider the center of mass and relative position vectors of the two electrons $\vec{u} = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{v} = \vec{r}_1 - \vec{r}_2$.]*

(b) What are the energy levels of this system when $\lambda = 1/2$?

(c) Taking into account the spin of the electrons, what are the degeneracies of the lowest four energy levels when $\lambda = 1/2$?

(d) Suppose the Helium atom is initially in the third excited state. It then undergoes a decay through an electric dipole transition to a lower-energy state. What are the possible energies of the emitted photon?