

## M99M.3—Drum Head

### Problem

A drum can be considered to be a uniform membrane of mass per unit area  $\rho$ , stretched on a rim of radius  $R$ . The tension  $\tau$ , per unit length of the membrane, is so large that it can be considered to be constant even when the membrane is slightly deformed from its equilibrium (flat) shape. (If the membrane is imagined to be cut, then  $\tau$  is the force which, applied to unit length on either side of the cut will hold the membrane in its place.)

- a) How far does the center of the membrane sag below the level of the rim when the drum is held horizontal and the acceleration of gravity is  $g$ ?
- b) Find the lowest vibrational frequency  $f$  of the membrane. Ignore gravity in this part of the problem.

A reasonable approximate solution will be accepted. However, you may want to know that the first zero of the Bessel function  $J_0(x)$ , which solves the differential equation  $J_0'' + \frac{1}{x}J_0' + J_0 = 0$ , is  $J_0(0.766\pi) = 0$ .