3. **Dealing with Stress**

The electromagnetic energy-momentum tensor $T_{\mu\nu}$ accounts for energy and momentum densities carried by the E and B fields as well as for pressure (and other stresses) associated with the presence of electric and magnetic fields. The spatial components of the tensor are given by

$$T_{ij} = \epsilon_0 \left(E_i E_j - \delta_{ij} \frac{E^2}{2} \right) + \frac{1}{\mu_0} \left(B_i B_j - \delta_{ij} \frac{B^2}{2} \right)$$

The net electromagnetic force on the charges and currents in some volume of space can be calculated as an integral of appropriate components of T_{ij} over the boundary of that volume. In answering the following questions, be sure to clearly state how you set up the surface integral of the stress tensor. Signs are important here!

(a) Consider the classic case of a parallel plate capacitor, with two infinite parallel plates separated by distance $2d$ and carrying a uniform surface charge density σ and $-\sigma$. Find the force per unit area exerted on the plates by the electric field using the energy-momentum tensor. Explain why the answer is not just equal to the field strength inside the capacitor times σ.

(b) Consider two infinite parallel lines of charge separated by distance $2d$ and carrying uniform linear charge densities λ and $-\lambda$. Find the force between the lines of charge using the energy-momentum tensor. (You might find the integral $\int_0^\infty dx/(1+x^2)^2 = \pi/4$ useful).

(c) Now let the two line charges carry the same charge density λ. In what way does the energy-momentum tensor calculation of the force change, and what is the final result?