3. **Perturbed Hydrogen Atom**

A hydrogen atom interacts with a perturbing potential

$$\Delta V(x, y, z) = A(x^2 - y^2) \ ,$$

where (x, y, z) is the displacement of the electron from the nucleus. Assume $A > 0$. In all parts of this problem, ignore spins and relativistic effects.

(a) What must A be small compared to for perturbation theory to be a good approximation for the $n = 1$ and $n = 2$ eigenstates of this perturbed hydrogen atom?

(b) Give a rough estimate of the change in the ground state energy due to this perturbation, including the sign of the change. Your answer can leave a multiplicative factor that is positive and of order one undetermined.

(c) Describe qualitatively the shifts in the $n = 2$ energy levels at first order in perturbation theory. Which of the $n = 2$ levels, if any, are unshifted? What are the ratios of the different energy level shifts? What are the eigenstates of the shifted levels? Give a rough estimate of the energy shifts (again, you may leave a multiplicative factor that is positive and of order one undetermined).