3. Hyperfine Splittings

An isolated hydrogen atom has a hyperfine interaction between the spins $\vec{S_1}$ of the proton and $\vec{S_2}$ of the electron of the form $J\vec{S_1} \cdot \vec{S_2}$, $J > 0$. The two spins also have magnetic moments $\alpha\vec{S_1}$ and $\beta\vec{S_2}$, and a uniform static magnetic field $\vec{B}$ is present. Assume that the electron is in its $1s$ orbital ground state, and ignore the effects of orbital motion.

(a) Find the exact energy eigenvalues of this system and sketch the hyperfine-splitting spectrum as a function of magnetic field.

(b) In the basis of states $|S_1^z, S_2^z\rangle$, find the eigenstates associated with each level.