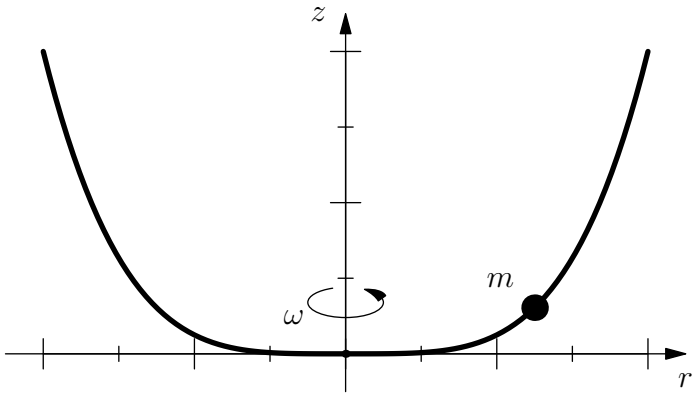
J04M.1—Bead on a Wire (J06M.3)

Problem

A bead of mass m slides without friction on a wire whose shape is

$$z(r) = a\left(\frac{r}{a}\right)^4$$

The wire rotates about the z axis with constant angular velocity ω. Earth's gravity causes acceleration g in the negative z direction.



a) Find the equation of motion for the bead in terms of coordinate r.

b) Find the equilibrium points. Say whether each is stable.

c) For the stable equilibria, find the frequency of small oscillations about equilibrium.