3. **Low-energy scattering**

Consider the problem of s-wave ($l = 0$) scattering of a particle of mass m from an attractive square-well potential of depth V_0 and radius r_0: $V(r) = -V_0\theta(r_0 - r)$ in three dimensions.

a) First consider the problem of s-wave bound states in this potential. Show that there is a critical potential strength V_{crit} such that for $0 < V_0 < V_{crit}$ there are no bound states. To put it another way, show that a bound state first appears when $V_0 = V_{crit}$. Determine the value of V_{crit}.

b) Now consider scattering of a particle of momentum k from this potential. Set up the equation for determining the phase shift $\delta_0(k)$ and show that it implies that $\delta_0 \sim A\,k$ as $k \to 0$. Evaluate the coefficient A as a function of V_0.

c) Calculate the contribution of the s-wave phase shift to the total cross section in the limit of small k. How does the zero-energy cross section behave: (i) in the limit $V_0 \to 0$?; (ii) in the limit $V_0 \to V_{crit}$? Comment and explain.