3. **Doomsday Scenario**

A unsuspecting solid spherical planet of mass M, radius R, and uniform mass density initially rotates with angular velocity ω_0. Suddenly, an asteroid of mass αM, incident with zero angular momentum about the center of the planet, smashes into and sticks to the planet at a location which is at polar angle θ relative to the initial rotational axis. Treat the asteroid as a point mass that sticks to the surface of the planet. The new mass distribution is no longer spherically symmetric, and as a result the rotational axis will in general precess (the planet will also recoil, but this has no effect on the rotation questions we will pursue, so ignore the recoil). Recall Euler's equation for the time evolution of the angular momentum of a solid body in the absence of external torques (all quantities being expressed in a body-fixed coordinate system):

$$\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = 0$$

(a) What is the new moment of inertia tensor $I_{\alpha\beta}^{CM}$ about the new center of mass? Give it in terms of its principal axes in the body frame, and state clearly what these axes are. Recall $I = \frac{2}{5}MR^2$ for a uniform solid sphere.

(b) What is the period of precession of the rotational axis in terms of the original rotation period $2\pi/\omega_0$?