

Section A. Mechanics

1. Coupled pendulums

Two simple pendulums of equal length L but unequal masses $m_1 > m_2$ hang in a uniform gravitational field g near their equilibrium positions. Let the (small) angular displacements of the pendulums from their equilibrium positions be $\theta_1 \ll 1$ and $\theta_2 \ll 1$, respectively. They are coupled by a torsional spring which applies a torque

$$\tau_1 = \kappa(\theta_2 - \theta_1)$$

to pendulum 1 (and of course the opposite torque to pendulum 2). There is no damping.

(a) Solve for the normal modes of this system and their frequencies.

(b) If this system is started at time zero with initial small displacements $\theta_1(0)$ and $\theta_2(0)$, and both pendulums initially at rest, solve for the subsequent displacements $\theta_1(t)$ and $\theta_2(t)$ as functions of time t .