2. **Hawking radiation**

Hawking and Bekenstein famously proposed that a black hole is in fact a thermodynamic system with a thermodynamic energy equal to its mass energy (Mc^2) and also endowed with a temperature $T(M)$ and an entropy $S(M)$. The mass can be equivalently expressed in terms of the Schwarzschild radius $R = 2GM/c^2$ (G is Newton's constant) of the black hole's spherical event horizon. Hawking further established that the entropy equals exactly one-quarter the area of the horizon (measured in Planck units), a relation that can be written as

$$S/k_B = \frac{1}{4}\frac{4\pi R^2}{\hbar G/c} = \frac{4\pi G M^2}{\hbar c} \tag{1}$$

In what follows, we will work out some consequences of the thermodynamic nature of the black hole.

a) In thermodynamics, the temperature of a system is related to the derivative of the entropy with respect to the energy. Use this basic relation to calculate the temperature of the black hole event horizon for a black hole of mass M.

b) Assuming that the black hole only emits photons with a black-body radiation spectrum from the surface of the horizon which is at this temperature, find the rate of energy loss. Express your answer in terms of fundamental constants. You might find useful the integral $\int_0^\infty x^3 dx/(e^x - 1) = \pi^4/15$.

c) Find the time it takes a black hole of mass M to evaporate due to this energy loss (thus mass loss).