

Section B. Statistical Mechanics and Thermodynamics

1. Consider a system of noninteracting spin trimers. Each trimer is a set of three spins each of which can be either “up” or “down.” Each trimer is described by the Hamiltonian

$$\hat{H} = -J(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) - \mu_0 H(\sigma_1 + \sigma_2 + \sigma_3) . \quad (1)$$

Here J is the spin coupling energy and H is an external magnetic field. The individual spin polarizations σ_i are two-state Ising variables, with $\sigma_i = \pm 1$.

- (a) Find the single trimer partition function Z_1 .
- (b) Find the magnetization per trimer $m = \mu_0 \langle \sigma_1 + \sigma_2 + \sigma_3 \rangle$.
- (c) Suppose there are N_Δ trimers in a volume V . The magnetization density is $M = N_\Delta m/V$. Find the zero field susceptibility [Hint: the definition of susceptibility is $\chi(T) = (\partial M / \partial H)_{H=0}$.]
- (d) Find the entropy $S(T, H, N_\Delta)$.
- (e) In the three limits $J \rightarrow +\infty$, $J \rightarrow 0$, and $J \rightarrow -\infty$ the expression for the entropy simplifies considerably. Give a simple physical explanation for each of these simplifications.