M98M.3—Fluid Dynamics

Problem

Answer all FIVE of the following short questions. Where a numerical answer is required use acceleration of gravity $g = 980$ cm s^{-2}, atmospheric pressure $p = 1 \times 10^6$ dynes cm^{-2} $= 1 \times 10^5$ N m^{-2}, and density of water $\rho = 1$ g cm^{-3}.

a) Water is flowing out of a hole with diameter $d = 1$ cm in the vertical side of a container. The center of the hole is $h = 1$ m below the top of the water. The diameter of the container is $D = 10$ m. Compute the speed of the water as it passes through the hole under the assumptions that the flow is laminar, and that viscous drag is negligible.

b) A balloon filled with helium (density $\rho_{He} = 2 \times 10^{-4}$ g cm^{-3}) is tied to the floor of a train car by a string of length $L = 1$ m. The car is accelerating forward on a level surface with acceleration $a = 1$ m s^{-2}. Sketch the position of the balloon in the train car and give an expression for the angle the balloon's string makes with the vertical.

c) An individual is standing on a level piston that can move freely in a cylinder. The piston is supported by water in the cylinder beneath it. The water is connected to a pipe that rises vertically beside the person. The cylinder has a circular cross section with radius $R = 1$ m. The pipe has a circular cross section with radius $r = 1$ cm. The individual and piston have total mass $M = 100$ kg. Find the difference between the heights of water in the cylinder and in the pipe.

d) A spherical soap bubble of radius $r = 1$ cm is blown from soap which has surface tension $\gamma = 50$ dynes cm^{-1} $= 0.05$ N m^{-1}. What is ΔP, the pressure difference between the inside of the bubble and the outside?

e) Four horizontal cylindrical tubes intersect as shown in the figure. The tubes have equal lengths L and radii R, with $L \gg R$. A fluid of viscosity η flows laminarly in the tubes. The ends of two opposing tubes are held at pressure $\alpha P'$, while the end of the third arm is maintained at (approximately) zero pressure. The end of the fourth tube is at pressure P'. For what values of α will the flux of fluid in the fourth tube be outward from the junction?