Section B. Statistical Mechanics and Thermodynamics

1. **Particles in a Box**

First consider a single free non-relativistic particle of mass m confined to a three-dimensional volume V. The particle has no internal degrees of freedom. Let $Z_1(mT)$ denote the quantum partition function for this system at temperature T (where the partition sum is taken over the discrete energy levels of this system).

(a) Working in the classical (high temperature) regime, show that $Z_1(mT) \approx V/\lambda^3$ with a suitably defined de Broglie wavelength $\lambda(mT)$. Use this result to obtain the classical energy and heat capacity at fixed volume of this single-particle system.

(b) What (roughly) is the temperature at which this classical approximation breaks down?

(c) Now consider a system consisting of two identical, non-interacting such quantum particles in the same box. The particles either have no internal degrees of freedom in the case of bosons, or these degrees of freedom are held fixed in the case of fermions (e.g. both spins "up"). The two-particle partition function $Z_2(mT)$ contains the effects of identical-particle statistics. Show that the exact free boson and free fermion two-particle quantum partition functions $Z_2(mT)$ can in fact be expressed in a simple way (at all T) in terms of the exact one-particle quantum partition functions $Z_1(mT)$ and $Z_1(mT/2)$.

(d) Using the classical approximation $Z_1 = V/\lambda^3$ derived in the first part of this problem, calculate the leading high-temperature contribution to the energy E and the heat capacity C_V due to Bose or Fermi statistics in this two-particle system.