M98Q.1—Two Interacting Particles

Problem

A system of two particles, one with spin s_a, the other with spin s_b, is governed by the Hamiltonian

$$H = K + V(\vec{r}_a, \vec{r}_b) + f(\vec{r}_a, \vec{r}_b)(S_3^{(a)} - S_3^{(b)}).$$

Here K is the kinetic energy operator and $\vec{S}^{(a)}$ and $\vec{S}^{(b)}$ are the spin operators. Let $\vec{S} = \vec{S}^{(a)} + \vec{S}^{(b)}$ be the total spin operator. You are not given the functions V and f, but you *are* told that $f(\vec{r}_a, \vec{r}_b) < 0$. The potential V is sufficiently attractive so that there exists at least one bound state.

a) Work out the (unique) ground state expectation value of $\vec{S}^2$, the square of the total spin angular momentum.

b) Now take the special case $s_a = 1, s_b = 1/2$. What are the possible outcomes of a measurement of $\vec{S}^2$? For the ground state, what are the probabilities of these outcomes?