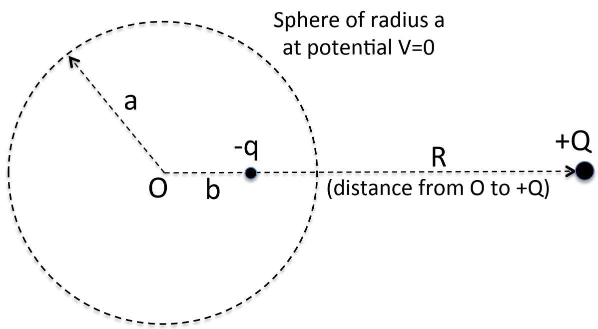
2. **Image Problems**



The 'method of images' allows us to solve many problems involving point charges and spherical conductors. In this question you are asked to obtain and then use this method in some representative applications:

(a) Consider a charge $+Q$ placed at a distance $R > a$ from the center of a sphere of radius a (for the moment, this is just a geometrical sphere, not a conductor or any other physical object). If a certain charge $-q$ of opposite sign is placed inside the sphere at the appropriate radius b (as in the figure) then the geometrical spherical surface of radius a is an equipotential surface of potential $V = 0$. What are q and b? You do not need to prove that this gives $V = 0$ over the full sphere, just get the correct image charge and position. This result can then be used in the following applications:

(b) A point charge Q is placed at a distance $R > a$ from the center of a conducting sphere of radius a. Find the force exerted on the sphere if the net total charge on the conducting sphere is Q.

(c) Consider the distribution of the total charge $Q \neq 0$ on the surface of the sphere under the conditions just described in (b). Find an equation for the largest distance R such that a zero surface charge density appears somewhere on the sphere.

(d) Two perfectly conducting spheres of radius a are placed far apart (their centers are separated by $R \gg 2a$) and kept at the same potential V_0 (this condition could be enforced by connecting the spheres with a fine wire). The full solution to this problem (which we are not asking you to do) involves an infinite sequence of image charges and image charges of image charges. For $R \gg 2a$, instead just include the leading image charge in each sphere and thus calculate the total charge on each sphere, image-corrected to leading order in the small parameter (a/R).