

2. Liquid-Gas Critical Point

The van der Waals gas is a “simple” modification to the classical ideal gas. Each molecule is assumed to occupy a volume b , so the free volume available to a given molecule is reduced to $(V - Nb)$ for a gas of N molecules in a volume V . There is also an attractive interaction between the molecules, which lowers the energy of the gas, so the Helmholtz free energy of the van der Waals (*vdW*) gas is:

$$F_{vdW}(N, T, V) = F_{ideal}(N, T, V - Nb) - aN^2/V ,$$

where F_{ideal} is the free energy of a classical ideal gas, and $a > 0$ quantifies the attractive interaction.

- (a) What is the pressure $p(N, T, V)$ of this van der Waals gas?
- (b) If you know the equation of state $p(N, T, V)$ of a more general gas with a liquid-gas critical point, what calculation would you do to locate the critical point, T_c, p_c ?
- (c) Calculate the critical point T_c, p_c of this van der Waals gas.