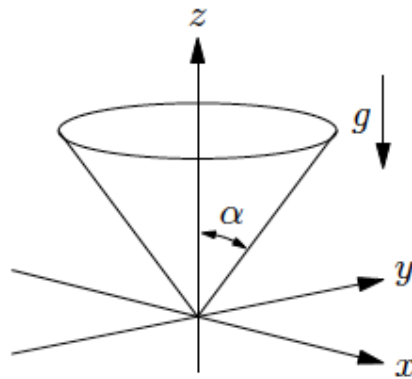


2. Particle in Cone

A point particle of mass m is constrained to slide, without friction, on the inside of a circular cone whose vertex is at the origin and whose axis is along the z -axis. The half angle at the apex of the cone is α , as shown, and there is a uniform gravitational field \vec{g} , directed downward and parallel to the axis of the cone.



- (a) Determine a set of generalized coordinates, and obtain the Lagrangian equations of motion in these coordinates. Identify any constants of the motion.
- (b) Show that motion in a circular orbit at a fixed height z_0 is a solution of the equations of motion. Obtain an expression for the frequency ω of this orbit.
- (c) Suppose that the particle moving in a circular orbit at height z_0 is given a small 'kick' in the direction away from the origin. Show that the subsequent motion consists of small oscillations about the unperturbed motion, and find the frequency Ω of those oscillations.
- (d) Under what conditions will this perturbed motion trace out a single fixed orbit (as opposed to an orbit that either precesses or closes only after multiple revolutions)?