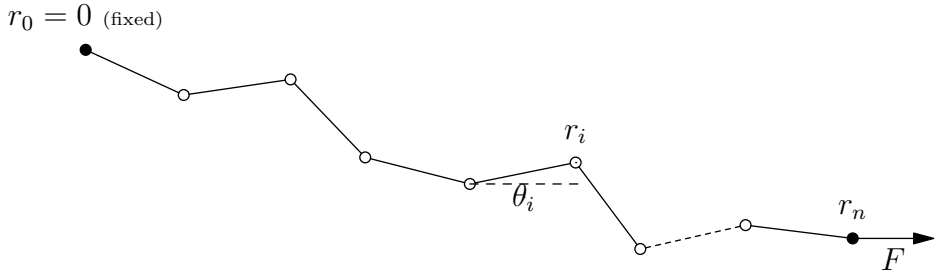
J05T.1 - Polymer Chain

Problem

A simple model of a polymer consists of $N + 1$ monomers connected by bonds of fixed length a. Le the chain be confined to a plane, so that the position of each monomer i is a vector $\boldsymbol{r}_i$ in two dimensions; alternatively the polymer can be described by angles θ_i that measure the orientation of the bond from monomer $i - 1$ to monomer i. Let the monomer $i = 0$ be fixed at $\boldsymbol{r}_0 = 0$.



a) Assume that all allowed configurations of the chain have equal energy, and that there are no externally applied forces. For $N \gg 1$, what is the mean end-to-end length of the chain? What (Gaussian) probability distribution $P(\boldsymbol{r}_N)$ of the position $\boldsymbol{r}_N$ of the free end of the polymer does this imply?

b) A force $\boldsymbol{F}$ is applied to the free end of the polymer. How does the (analog of the) Gibbs free energy $G(T, \boldsymbol{F}, N)$ of the polymer depend on $|\boldsymbol{F}|$? (Give the leading term in an expansion of $G(T, \boldsymbol{F}, N) - G(T, 0, N)$ in powers of $\boldsymbol{F}$ for $N \gg 1$.)

c) Obtain the corresponding expression for the (analog of the) Helmholtz free energy $A(T, \boldsymbol{r}_N, N)$. How do the internal energy U and entropy S depend on $\boldsymbol{F}$?

d) Show that for small forces the polymer acts like a Hooke's law spring, and compute the corresponding stiffness. When this "spring" is held stretched, where is the energy stored? (Describe the nature of this stored energy.)