Section A. Mechanics

1. A rigid straight beam of mass M rests on and is supported by N exactly equivalent posts. The posts are located at positions $x = x_i$ along the beam ($i = 1, ..., N$), with the center of mass of the beam at $x = 0$. The posts are very stiff, but compressible, obeying Hooke's law. The positions x_i of the posts are arbitrary, but assume that all posts do bear a nonzero load and that the tops of the posts are all at precisely the same height in the absence of the load.

   a) Find the expression for the compressive force on each post in static equilibrium in terms of M, N, the $\{x_i\}$ and/or g.

   b) Consider the modes of vibration of this system that involve vertical motion. Derive the condition on the set $\{x_i\}$ under which the normal modes of vibration consist of one mode where the beam moves vertically but does not rotate and another mode where it rotates with no motion of its center of mass.