Section B. Electricity and Magnetism

1. A long thin non-conducting cylinder of radius r and height $h \gg r$ (figure not to scale) is concentric with a line charge of charge per unit length $-\lambda$. The cylinder has a uniform surface charge density with equal and opposite total charge per unit length $+\lambda$. The cylinder is free to rotate about its symmetry axis and has moment of inertia per unit length I/h. At times $t < 0$ the cylinder is at rest and a spatially uniform axial external magnetic field $B_0\hat{\mathbf{z}}$ is present, as shown in the figure. At time $t = 0$, the externally applied field is ramped down to zero.

   a) Compute the torque on the cylinder in terms of $\frac{dB_z(t)}{dt}$, with $B_z(t)$ the (approximately uniform) axial magnetic field *within* the cylinder.

   b) Find the angular velocity of the cylinder after the external field is reduced to zero, noting that the final field within the cylinder will be non-zero. Express your answer in terms of λ, r, B_0, I, and/or h and whatever fundamental constants are required.

   c) Recalling that the density of linear momentum stored in the electromagnetic field is proportional to the Poynting vector, express the *angular* momentum of the initial state. Demonstrate that the total angular momentum (mechanical plus electromagnetic) is conserved between the initial and the final conditions.