M99Q.1—Interacting Magnetic Dipoles

Problem

Two electrons are fixed at positions a distance L apart along the z-axis. Their spins interact due to the magnetic dipole interaction:

$$H = \frac{4e^2}{m^2c^2L^3}(\vec{S}_1 \cdot \vec{S}_2 - 3S_{1z}S_{2z}).$$

a) At time $t = 0$, both spins are aligned along the z-axis (so $S_{1z} = S_{2z} = \frac{\hbar}{2}$). At an arbitrary later time t, what are the possible results of a measurement of $(S_{1z} + S_{2z})$, and with what probabilities?

b) Now assume instead that at time $t = 0$, both spins are aligned along the x-axis (so $S_{1x} = S_{2x} = \frac{\hbar}{2}$). At an arbitrary later time t, what are the possible results of a measurement of $(S_{1x} + S_{2x})$, and with what probabilities?

c) Compare your answers in parts a) and b) to what would happen for two classical magnetic dipoles.