2. **Flipping a spin**

A particle of spin one-half is at rest in a static magnetic field $\vec{B} = B_0 \hat{z}$ oriented along the z-axis. The two-component wave function $\psi(t)$ of this system can be manipulated by turning on a magnetic field $\vec{B}_1(t) = B_1 \cos\omega t\, \hat{x} + B_1 \sin\omega t\, \hat{y}$ that rotates in the xy plane with a frequency ω. The time-dependent Hamiltonian that governs the evolution of $\psi(t)$ is

$$H = \mu B_0 \sigma_z + \mu B_1 (\sigma_x \cos\omega t + \sigma_y \sin\omega t)$$

where σ_{xyz} are the Pauli matrices and μ is the particle's magnetic moment. We will discuss ways of choosing parameters so that quantum evolution over a time T transforms a $\sigma_z = -1$ state into a $\sigma_z = +1$ state.

(a) As a first step toward a solution, show that the 'interaction picture' wave function $\widehat{\psi} = \exp(i\,\omega t \sigma_z/2)\psi(t)$ evolves according to a time-independent Hamiltonian H_{rot}.

(b) Let the rotating field be turned on during the time interval $[0, T]$. Find ω and T such that a $\sigma_z = -1$ state at time $t = 0$ is perfectly transformed by this time evolution into the $\sigma_z = +1$ state at time $t = T$.

We remind you of a couple of algebraic facts that may be helpful in working through this problem:

- For any 2x2 matrices A and B: $e^A B e^{-A} = B + [A, B] + \frac{1}{2!}[A, [A, B]] + \ldots$
- For unit vector $\hat{n}$ and 2x2 unit matrix I_2: $\exp(i\Omega \hat{n} \cdot \vec{\sigma}) = \cos\Omega\, I_2 + i \sin\Omega\, \hat{n} \cdot \vec{\sigma}$