J04M.2—Power Law Central Force (J06M.2)

Problem

A particle of mass m moves in an attractive, power-law central force, $F \propto r^{-n}$. Define constant k so that the potential energy is

$$U(r) = -\frac{k}{r^{n-1}}.$$

The particle is set moving from a position near the force center with such initial conditions that its orbit is a circle of radius a which passes through the origin $r(\theta) = 2a\cos\theta$. The existence of this orbit imposes conditions on the force law, the energy, and the angular momentum.

a) Find the force power law n.

b) Find the angular momentum of the particle in terms of the constants k, a and m.

c) Show that the total energy, E vanishes for this orbit.

d) How long does it take for the particle to complete a full trip from the force center and back to it?

e) Remove the orbiting mass m to infinity and give it an initial velocity v_∞. Find the total cross section for the force described above to "capture" the particle (i.e. for the particle's trajectory to pass through the force center at $r = 0$).