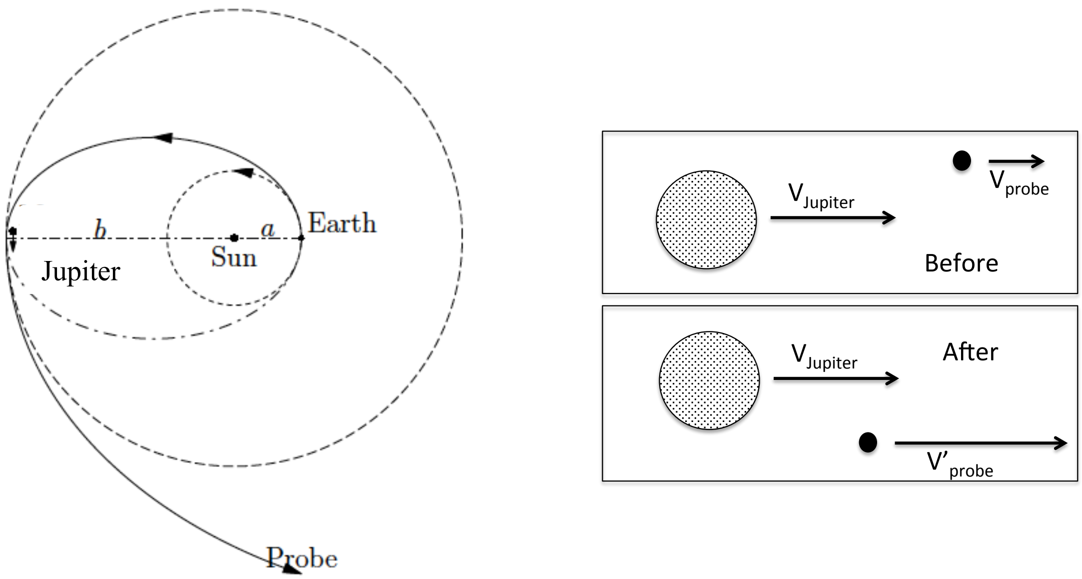
Section A. Mechanics

1. **Planetary Slingshot**



A space probe is launched from Earth into a transfer orbit that takes it very near to Jupiter. As shown in the left figure, the transfer orbit is such that the probe reaches Jupiter's orbit just as Jupiter is arriving at the same point in its orbit, with the probe moving in the same direction as Jupiter. A blowup of the Jupiter-probe encounter is shown in the right figure. Things are arranged so that instead of the probe crashing into Jupiter's surface, it just misses and swings around the planet in a very tight orbit, emerging from the encounter moving in very nearly its original direction, but with a changed speed.

Make the approximations that the orbits of Earth and Jupiter are circular with radii a and b, respectively, and that the masses of Earth and Jupiter do not affect the transfer orbit between the two planets. Assume also that the masses of the Earth and Sun can be ignored during the near collision between the probe and Jupiter, and that the masses of Earth and Jupiter can again be ignored after the near collision.

(a) Use conservation laws to evaluate the probe speed v_{probe} just before the rendezvous (v_{probe} in the frame of the Sun) in terms of Jupiter's orbital speed v_J and the radii a and b. Verify that Jupiter is moving faster than the probe.

(b) Treating the interaction between Jupiter and the probe as an elastic collision in one dimension, find the post-collision speed (relative to the Sun) of the probe, $v'_{\rm probe}$. This is of course not accurate, since the hyperbolic orbit that just grazes the planet's surface will have a scattering angle less than π, but do make this simplifying approximation.

(c) Is it possible that the probe can then escape from the solar system? Find the condition that must be met in order for escape to occur.